Firm-to-Firm Trade:

Imports, Exports, and the Labor Market[1]

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Abstract

Customs data and firm-level production data reveal both the heterogeneity and the granularity of individual buyers, and sellers. We seek to capture these firm-level features in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

1 Introduction

International economists have begun to exploit data generated by customs records, which describe the finest unit of trade transactions. These records expose the activity of individual buyers and sellers that underlie the aggregate trade flows, which had been the object of earlier quantitative analysis in international trade.

Some striking regularities emerge. One that has received attention previously (e.g., Eaton, Kortum, and Kramarz (2011), Eaton, Kortum, and Sotelo (2013)) is the tight connection between market size, market share, and the number of individual exporters. Figures 1 illustrates this relationship for French manufacturing exports to other members of the European Union (EU) in 2005.[1] Figure 1 reports a destination's market size, as measured by its manufacturing absorption, on the x-axis, and the number of French manufacturing firms selling there, on the y-axis. On a log-log scale, the slope is 0.52 (standard error 0.06).[2]

Combining data on French exporters and their buyers reveals new facts. Figure 2 plots the average number of buyers per French exporter across the other EU members, again with market size on the x-axis. The relationship is positive, with a slope of 0.34 (0.04), indicating another important margin of export expansion with market size. Nonetheless, as the y-axis in Figure 2 reveals, the average exporter has few buyers, motivating the granular theory of import demand we pursue in this paper.

These data also allow us to count the number of *relationships* between French exporters and their buyers in any destination. Regressing this measure on both market size and French

[1] Data sources are described in Appendix A.

[2] Figures 1-3 are all plotted on a log-log scale, and the corresponding regressions are run in logs (with standard errors of the coefficient estimates in parentheses).

market share (the share of a destination's absorption spent on manufacturing imports from France) we obtain a slope of 0.83 (0.06) on market size and 1.04 (0.16) on market share. Most striking, this last coefficient is indistinguishable from 1, inviting us to normalize the number of relationships by market share before plotting against market size. The result, in Figure 3, illustrates the very tight connection between relationships and the size of the destination market.

The small number of buyers in each destination shown in Figure 2 hides the extreme heterogeneity in the number reached by any given exporter. Table 1 reveals this underlying variation across French exporters to four EU destinations of diverse size. Note that the median number of buyers never exceeds 2 even in Germany, the largest EU market. But, numbers at the top end exceed one hundred.[3] Our model must account for this highly skewed distribution of buyers per firm.

Most existing theory has taken a monolithic approach to modeling technology, with all firms in a sector employing factors and intermediate inputs in the same way. But the data reveal substantial heterogeneity with respect to inputs as well. Figure 4 portrays the distribution of the share of production labor in total costs (including the cost of intermediates) across French manufacturing firms (the y-axis show the percent of firms with a share of production labor less than or equal to the value shown on the x-axis).

We seek to capture both the heterogeneity and the granularity in individual buyer-seller relationships in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who

[3] Our findings in Table 1 and Figure 2 on buyers per firm match evidence from Norwegian exporters reported in Bernard, Moxnes, and Ultveit-Moe (2017), their Figures 1 and 2 in particular.

may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

In contrast to standard production theory, we model a firm's technology as combining a set of tasks. Each task can be performed by labor, which can be of different types appropriate for different tasks. But labor competes with intermediate goods produced by other firms which can also perform these tasks. Firms may thus look very different from one another in terms of their production structure, depending on the sellers of intermediate goods that they happen to encounter. A firm's cost in a market thus depends not only on its underlying efficiency, but also on the costs of its suppliers. An implication is that an aggregate change, such as a reduction in trade barriers, can reduce the share of labor in production by exposing producers to more and cheaper sources of supply.

Our model is complementary to recent work of Oberfield (2017) in which a producer's cost depends not only on its own efficiency but the efficiencies of its upstream suppliers. It is also complementary to recent work of Chaney (2014) and Eaton, Eslava, Jinkins, Krizan, and Tybout (2014), with trade the consequence of individual links formed between buyers and sellers over time. In order to embed the framework into general equilibrium, however, our analysis here remains static, more in line with the two-stage model of production in Bernard, Moxnes, and Ultveit-Moe (2017).[4] Our model also relates to Garetto (2013), in that firms

[4]Bernard, Moxnes, and Saito (2015) apply this model to micro data from Japan to evaluate the effects of a new high-speed train line on firms' supplier networks.

and workers compete directly to provide inputs for firms.[5]

We proceed as follows. Section 2 develops our model. Section 3 analyzes its implications for aggregate outcomes such as the distribution of wages. Section 4 turnss to firm-level outcomes such as the number of relationships between sellers in one country and buyers in another. Section 5 is an initial peak at the model's quantitative implications. Section 6 concludes.

2 A Model of Production through Random Encounters

Consider a world with a set of $i = 1, 2, ..., \mathcal{N}$ countries. Each country has an endowment of L_i^l workers of type $l = 1, 2, ..., L$.

2.1 Technology

A producer j in country i can make a quantity of output $Q_i(j)$ by combining a discrete set of tasks, $k = 1, ..., K$. Any task k consists of n_k subtasks, indexed by ω (which we refer to as a k-subtask). The production function, for a firm with numbers of k-subtasks given by $\{n_k\}_{k=1}^K$,

[5] In addition to the work already cited, our paper relates closely to a number of active areas. One is recent work on exports and the labor market, including Caliendo and Rossi-Hansberg (2012), Egger and Kreickemeier (2009), Felbermayr, Prat, and Schmerer (2008), Helpman, Itskhoki, and Redding (2010), and Hummels, Jørgenson, Munch, and Xiang (2014). Another is quantitative work focussing on firm-level imports, including Biscourp and Kramarz (2007), Blaum, Lelarge, and Peters (2014), Bricongne, Lionel, Gaulier, Taglioni, and Vicard (2012), Caliendo, Monte, and Rossi-Hansberg (2015), Frías, Kaplan, and Verhoogen (2009), Helpman, Itskhoki, Muendler, and Redding (2017), Irarrazabal, Moxnes, and Ulltveit-Moe (2013), Klein, Moser, and Urban (2010), Kramarz (2009), and Kramarz, Martin, and Mejean (2015). A third is other theories of networks or input-output interactions, including Acemoglu and Autor (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Lucas (2009), and Luttmer (2015).

is:

$$Q_i(j;\{n_k\}) = z_i(j)\prod_{k=1}^{K} b_k \left(\frac{1}{\beta_k} \left(\sum_{\omega=1}^{n_k} x_{k,i}(j,\omega)^{(\sigma_k-1)/\sigma_k} \right)^{\sigma_k/(\sigma_k-1)} \right)^{\beta_k}, \tag{1}$$

where $z_i(j)$ is the overall efficiency of producer j, $x_{k,i}(j,\omega)$ is the input of k-subtask ω, σ_k is the elasticity of substitution between k-subtasks, b_k is a constant (which we use later to neutralize the effect of n_k on task k productivity), and β_k is the Cobb-Douglas share of task k, satisfying $\beta_k > 0$ and

$$\sum_{k=1}^{K} \beta_k = 1.$$

An important special case of our framework assumes that $n_k = 1$ for all tasks k and for all producers j. In that case (1) reduces to a Cobb-Douglas production function. The point of introducing more than a single subtask ω is to more flexibly match our data on firm-to-firm trade. Allowing heterogeneity across firms along this dimension captures the observation that some firms have a very large number of suppliers. An elasticity of substitution greater than one among subtasks allows for the possibility that buyers purchase more when transacting with more efficient suppliers.

Consider k-subtask ω. It can be performed either by the unique type of labor appropriate for that task, denoted $l(k)$, or with an input produced by a firm. We allow $K \geq L$, so that one type of labor might be able to perform tasks of several different types. We denote the set of tasks that labor of type l can perform as Ω_l. Worker productivity performing k-subtask ω for a given firm is $q_{k,i}(j,\omega)$. If the firm hires labor it pays the wage for workers of type $l(k)$. The producer also is in contact with a set of suppliers of an intermediate good that can also perform the subtask. From producer j's perspective, labor and the available inputs are perfect substitutes for performing any subtask. Hence it chooses whatever performs the

subtask at lowest cost.

We assume that producers can hire labor in a standard Walrasian labor market at the market wage $w_{k,i} = w_i^{l(k)}$. In finding intermediates, however, buyers match with only an integer number of potential suppliers, either because of search frictions or because only a handful of producers make an input appropriate for this particular firm. We could make various assumptions about the price at which the intermediate is available. Because it yields the simplest set of results, we assume Nash bargaining in which the buyer has all the bargaining power, so that the price is pushed down to unit cost.[6]

Let $c_{k,i}^{\min}(j,\omega)$ denote the lowest price available to firm j for an intermediate to perform k-subtask ω. The price it pays to perform this subtask is thus:

$$c_{k,i}(j,\omega) = \min\left\{\frac{w_{k,i}}{q_{k,i}(j,\omega)}, c_{k,i}^{\min}(j,\omega)\right\}$$

and the firm's unit cost of delivering a unit of its own output to destination n is:

$$c_{ni}(j;\{n_k\}) = \frac{d_{ni}}{z_i(j)}\prod_{k=1}^{K}\frac{1}{b_k}\left(\left(\sum_{\omega=1}^{n_k} c_{k,i}(j,\omega)^{-(\sigma_k-1)}\right)^{-1/(\sigma_k-1)}\right)^{\beta_k}, \tag{2}$$

where $d_{ni} \geq 1$ is the iceberg transport cost of delivering a unit of output from source i to destination n, with $d_{ii} = 1$ for all i.

In order to derive a closed form solution for the distribution of costs in this setting, we impose specific distributions for producer efficiency, the efficiency of labor in performing a task, and the distribution of the prices of intermediate inputs.

[6] An implication is that there are no variable profits. Our model thus cannot accommodate fixed costs, either of market entry as in Melitz (2003) or in accessing markets for inputs, as in Bernard, Moxnes, and Ultveit-Moe (2017) or Antras et al. (2017). An alternative, which would allow for variable profits and hence fixed costs, is Bertrand pricing. While we found this alternative analytically tractable, we deemed the added complexity not worth the benefit.

First, following Melitz (2003) and Chaney (2008), each country has a measure of potential producers. The measure of potential producers in country i with efficiency $z_i(j) \geq z$ is:

$$\mu_i^Z(z) = T_i z^{-\theta}, \tag{3}$$

where $T_i \geq 0$ is a parameter reflecting the magnitude of country i's endowments of technology and $\theta \geq 0$ their similarities.

Second, worker productivity performing a task for a given producer $q_{\omega,i}(j)$ is a random variable Q drawn from the distribution:

$$F(q) = \Pr[Q \leq q] = e^{-q^{-\phi}}, \tag{4}$$

where $\phi \geq 0$ reflects the similarity of labor productivities across tasks and firms. For purposes that will become apparent below we restrict $\phi \leq \theta$.

Third, the measure of potential producers from i who can produce a good at a unit cost below c is given by:

$$\mu_{ii}(c) = T_i \Xi_i c^{\theta}, \tag{5}$$

where $\Xi_i \geq 0$. It follows that the measure of potential producers from i who can deliver to n at a cost below c is:

$$\mu_{ni}(c) = \mu_{ii}(c/d_{ni}) = d_{ni}^{-\theta}\mu_{ii}(c) = T_i \Xi_i d_{ni}^{-\theta} c^{\theta}.$$

Our specifications of the heterogeneity in producer efficiency given in (3) and the distribution of labor productivity given in (4) are primitives of the model, with T_i, θ, and ϕ exogenous parameters. We show below, however, that the resulting heterogeneity in unit costs c given by (5) arises endogenously from our other assumptions, with Ξ_i determined by underlying

technology, labor market conditions, and access to intermediates in different countries of the world, as well as to trade barriers between countries.

A potential producer only survives if it meets a customer who buys from it. A customer could be a final consumer (a household) or a firm which uses the producer's output as an input. The measure of final consumers in market n is the exogenous measure of households:

$$L_n = \sum_l L_n^l.$$

The measure of *active* producers in market n is determined endogenously by the *potential* producers there that are able to make a sale (either in market n or in some other destination).

2.2 Preferences

Final demand is by households supported by workers of different skill levels spending their wage income (since there are no profits in our model). Analogous with the tasks that firms need to perform, households have an integer number of needs indexed by k, that are combined with a Cobb-Douglas share α_k. We thus treat consumers in parallel to firms as buyers. In particular, like a producer's tasks, a household's needs can be fulfilled either by goods purchased from a producer or directly by labor.

2.3 Matching Buyers and Sellers

In contrast with standard Walrasian models, we assume that matching between buyers and sellers is random. Even though there are a continuum of possible sellers and buyers, an individual seller matches with only an integer number of potential buyers and an individual buyer matches with only an integer number of potential sellers. The matching literature (e.g.,

Mortensen and Pissarides, 1994) typically posits that in a market with more potential buyers and sellers, the likelihood of a match between any given potential buyer and potential seller is smaller.[7]

While in our case the measure of potential sellers implied by (5) is unbounded, the measure of sellers with unit cost below c is always bounded. Therefore, we treat the likelihood of a match involving a seller with unit cost c as limited by the measure of sellers with unit cost below c.

Matching frictions in the model are captured by the parameters $\lambda_{k,ni}$, which governs the rate of contact between sellers in i and buyers in n seeking to perform a k-subtask. For tractability we impose the following separability assumption:

$$\lambda_{k,ni} = \lambda_k \lambda_{ni}.$$

We thus posit that the intensity with which a seller in i with unit cost c in delivering to country n encounters a buyer seeking to fulfill a k-subtask is:

$$e_{k,ni}(c) = \lambda_k \lambda_{ni} L_n^{-\varphi} \mu_n(c)^{-\gamma}. \tag{6}$$

Here:

$$\mu_n(c) = \sum_i \lambda_{ni} \mu_{ni}(c) = \Upsilon_n c^{\theta},$$

captures the availability of suppliers with cost below c in country n, with associated parameter:

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i.$$

[7] Matching in our framework can be interpreted literally as coming into contact with each other, but it also could relate to the appropriateness of a seller's product for the buyer's purpose. In this sense we can think of products as differentiated not only by seller, but by buyer as well.

The key new parameters are (i) $\varphi \geq 0$, which governs how other buyers impede the ability of a given buyer to match with a seller and (ii) $0 \leq \gamma < 1$, which governs how lower cost sellers impede the ability of a seller to match with a buyer.

Aggregating across potential suppliers from each source i, with different costs of delivering to n, the number of "quotes" that a buyer in n receives for a k-subtask with a price below c is distributed Poisson with parameter:

$$\begin{aligned}\rho_{k,n}(c) &= \sum_i \int_0^c e_{k,ni}(x) d\mu_{ni}(x) = \lambda_k L_n^{-\varphi} \int_0^c \mu_n(x)^{-\gamma} \sum_i \lambda_{ni} d\mu_{ni}(x) \\ &= \lambda_k L_n^{-\varphi} \int_0^c \mu_n(x)^{-\gamma} d\mu_n(x) = \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \mu_n(c)^{1-\gamma}.\end{aligned}$$

This Poisson parameter grows arbitrarily large with c, so that many potential suppliers are available to serve any given buyer, although perhaps at a very high cost.

We now consider the measure of matches between customers in country n seeking to fulfill a k-subtask and sellers with unit cost below c when selling in n. This measure is simply the product of the measure of firms buying in country n and the expected number of suppliers per buyer for k-subtasks:

$$m_{k,n}(c) = (L_n + M_n) E_k \rho_{k,n}(c) = \frac{\lambda_k E_k}{1-\gamma} (L_n + M_n) L_n^{-\varphi} \mu_n(c)^{1-\gamma},$$

where

$$E_k = \sum_{n_k=1}^{\infty} P(n_k) n_k$$

is the expected number of k-subtasks per buyer ($P(n_k)$ is the probability of a buyer having exactly n_k subtasks of task k).

The firm can perform a k-subtask at a cost below c_k unless the cost of hiring workers directly and the lowest quote both exceed c_k. From the Poisson density, we know that with

probability $\exp\left(-\rho_{k,n}(c_k)\right)$ the buyer will encounter no quotes below c_k. It will cost more than c_k to hire workers to perform the task if $w_{k,n}/Q > c_k$, which occurs with probability $F(w_{k,n}/c_k)$. Since the two events are independent, the distribution of the lowest cost to fulfill a k-subtask is:

$$G_{k,n}(c_k) = 1 - F(w_{k,n}/c_k)e^{-\rho_{k,n}(c_k)}.$$

To work out the implications of this distribution for the resulting distribution of production costs, we restrict:

$$\gamma = \frac{\theta - \phi}{\theta} \text{ and therefore } \phi = \theta\left(1 - \gamma\right).$$

With this restriction, the parameter governing heterogeneity in the distribution of costs of intermediates is the same as the parameter governing heterogeneity in the distribution of worker efficiency (4) at a given k-subtask for a given buyer. In particular, the distribution of the cost to the buyer of fulfilling a k-subtask becomes:

$$G_{k,n}(c_k) = 1 - e^{-\Xi_{k,n}c_k^{\phi}}, \tag{7}$$

where

$$\Xi_{k,n} = \nu_{k,n} + w_{k,n}^{-\phi} \tag{8}$$

and

$$\nu_{k,n} = \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \Upsilon_n^{1-\gamma}. \tag{9}$$

2.4 Preliminary Results

At this point we can already derive results about labor shares and trade shares.

With probability $w_{k,n}^{-\phi}/\Xi_{k,n}$ the buyer hires workers to perform a k-subtask while with probability $\nu_{k,n}/\Xi_{k,n}$ it purchases an intermediate from the lowest-cost supplier. Notice that these probabilities are independent of the unit cost c.

While $w_{k,n}^{-\phi}/\Xi_{k,n}$ is the probability that a k-subtask is performed by labor in country n, since there are a continuum of producers, it is also the aggregate share of labor in performing task k in country n.[8] The aggregate share of labor of type l in total production costs is consequently:

$$\beta_n^l = \sum_{k \in \Omega_l} \beta_k w_{k,n}^{-\phi}/\Xi_{k,n}$$

and the overall labor share in production costs is:

$$\beta_n^L = \sum_l \beta_n^l.$$

Note that, even though our basic technology is Cobb-Douglas across tasks k, the labor share depends on wages and other factors.

To derive bilateral trade shares, given factor prices, we need to solve for $\rho_{k,ni}(c)$, the Poisson parameter for the number of "quotes" from a potential supplier in i that a buyer in n receives for a k-subtask with a price below c. This parameter is:

$$\rho_{k,ni}(c) = \int_0^c e_{k,ni}(x) d\mu_{ni}(x) = \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \Upsilon_n^{-\gamma} \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i c^{\phi}.$$

Summing over all the countries from which suppliers might come:

$$\begin{aligned} \sum_i \rho_{k,ni}(c) &= \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \Upsilon_n^{-\gamma} \left(\sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i \right) c^{\phi} \\ &= \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \Upsilon_n^{1-\gamma} c^{\phi} = \rho_{k,n}(c). \end{aligned}$$

[8] Similarly, in Eaton and Kortum (2002) the probability π_{ni} that destination n buys a good from a source i is also source i's share in destination n's spending.

Thus, the probability that a supplier comes from country i is:

$$\pi_{ni} = \frac{\rho_{k,ni}(c)}{\rho_{k,n}(c)} = \frac{\lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i}{\Upsilon_n} = \frac{\lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i}{\sum_{i'} \lambda_{ni'} d_{ni'}^{-\theta} T_{i'} \Xi_{i'}}. \tag{10}$$

Just as in Eaton and Kortum (2002), with our continuum of producers, in the aggregate π_{ni} is the bilateral trade share of source i in the purchases of destination n. At this point we can't fully interpret this equation, as we have not yet derived Ξ_i.

We proceed by showing first how the cost measure (5) arises from our model of firm-to-firm trade. We then turn to consumer demand and then to intermediate demand before closing the model in general equilibrium.

2.5 Deriving the Cost Distribution

Each c_k is distributed independently according to (7). From (3) and (2), the measure of potential producers from source i that can produce at a unit cost below c (given the number of sub-tasks n_k for each task) is:

$$\begin{aligned} \mu_{ii}(c; \{n_k\}) &= T_i c^\theta \prod_k b_k^\theta \left[\int_0^\infty ... \int_0^\infty \left(\sum_{\omega=1}^{n_k} c_\omega^{-(\sigma_k - 1)} \right)^{\theta \beta_k / (\sigma_k - 1)} dG_{k,i}(c_1) ... dG_{k,i}(c_{n_k}) \right] \\ &= T_i \Xi_i c^\theta \end{aligned} \tag{11}$$

where:

$$\Xi_i = \prod_k b_k^\theta \Psi_{k,i}$$

and

$$\begin{aligned}
\Psi_{k,i} &= \int_0^\infty ... \int_0^\infty \left(\sum_{\omega=1}^{n_k} c_\omega^{-(\sigma_k-1)} \right)^{\theta\beta_k/(\sigma_k-1)} \phi \prod_{\omega=1}^{n_k} \Xi_{k,i} c_\omega^{\phi-1} e^{-\Xi_{k,i} c_\omega^\phi} dc_\omega \\
&= \int_0^\infty ... \int_0^\infty \left(\sum_{\omega=1}^{n_k} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-(\sigma_k-1)/\phi} \right)^{\theta\beta_k/(\sigma_k-1)} \prod_{\omega=1}^{n_k} e^{-x_\omega} dx_\omega \\
&= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{n_k-1}} \left[\int_0^\infty e^{-x_{n_k}} \left(\sum_{\omega=1}^{n_k} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-(\sigma_k-1)/\phi} \right)^{\theta\beta_k/(\sigma_k-1)} dx_{n_k} \right] dx_{n_k-1} ... dx_1,
\end{aligned}$$

where in the second and third line we have changed the variables of integration to $x_\omega = \Xi_{k,i} c_\omega^\phi$.

To simplify further, we impose the restriction that $\theta\beta_k = \sigma_k - 1$, giving us:

$$\begin{aligned}
\Psi_{k,i} &= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{n_k-1}} \left[\int_0^\infty e^{-x_{n_k}} \left(\sum_{\omega=1}^{n_k} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} \right) dx_{n_k} \right] dx_{n_k-1} ... dx_1 \\
&= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{n_k-1}} \left[\sum_{\omega=1}^{n_k-1} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} + \int_0^\infty e^{-x_{n_k}} \left(\frac{x_{n_k}}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} dx_{n_k} \right] dx_{n_k-1} ... dx_1 \\
&= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{n_k-2}} \left[\int_0^\infty e^{-x_{n_k-1}} \left(\sum_{\omega=1}^{n_k-1} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} \right) dx_{n_k-1} \right] dx_{n_k-2} ... dx_1 \\
&\quad + \Gamma\left(1 - \theta\beta_k/\phi\right) \Xi_{k,i}^{\theta\beta_k/\phi}, \qquad (12)
\end{aligned}$$

where we require that parameter values satisfy $\theta\beta_k/\phi < 1$.

Since the remaining integral has the same form as what we started with, we can repeat the steps above again and again to obtain:

$$\Psi_{k,i} = n_k \Gamma\left(1 - \theta\beta_k/\phi\right) \Xi_{k,i}^{\theta\beta_k/\phi}.$$

Setting

$$b_k = \left[n_k \Gamma\left(1 - \theta\beta_k/\phi\right) \right]^{-1/\theta},$$

we have:

$$\Xi_i = \prod_k b_k^\theta \Psi_{k,i} = \prod_k \Xi_{k,i}^{\theta\beta_k/\phi}.$$

We can either solve for the vector of Ξ_i from the system of equations:

$$\Xi_i = \prod_{k=1}^{K} \left(\frac{\lambda_k}{1-\gamma} L_i^{-\varphi} \left(\sum_{i'} \lambda_{ii'} d_{ii'}^{-\theta} T_{i'} \Xi_{i'} \right)^{1-\gamma} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k}$$

or, substituting in (9), we can solve for the vector of Υ_n from the system of equations:

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \prod_k \left(\frac{\lambda_k}{1-\gamma} L_i^{-\varphi} \Upsilon_i^{1-\gamma} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k} \tag{13}$$

for $n = 1, 2, ..., \mathcal{N}$. Given wages and exogenous parameters of the model, the Υ_n are thus the solution to the set of equations (13). Appendix B provides sufficient conditions for a unique solution to the Υ_n's and an iterative procedure to compute them.

Since $\mu_{ii}(c; \{n_k\})$ is invariant to $\{n_k\}$, we can safely ignore the heterogeneity in the number of subtasks per task, writing this measure as $\mu_{ii}(c)$ instead, which applies to all firms from i.

2.6 Households

Final demand is by different types of workers spending their wage income (since there are no profits in our model). As mentioned above, we model their preferences in parallel to our assumptions about production. Consumers have an integer number K of needs, with each need having a Cobb-Douglas share α_k in preferences, with $\alpha_k > 0$ and

$$\sum_{k=1}^{K} \alpha_k = 1.$$

Proceeding as above, a consumer faces a distribution of costs for fulfilling each subtask of need k given by (7). The probability that a k-subtask is fulfilled by labor is again $\upsilon_{k,i}$, which, with our continuum of consumers, is the share of labor in fulfilling need k. The share of labor of type l used by consumers in their total spending is thus:

$$\alpha_i^l = \sum_{k \in \Omega_l} \alpha_k \upsilon_{k,i}$$

and the share of labor in consumer spending in country i is:

$$\alpha_i^L = \sum_l \alpha_i^l.$$

As with the share of labor in production costs, the share of labor in final spending depends on wages and other factors.

When a consumer in country n fulfills a k-subtask by purchasing a good, the probability that the good come from country i is given by π_{ni} in expression (10). With our continuum of consumers π_{ni} thus represents the share of country i in country n's final spending.

2.7 Consumer Welfare

Two workers with the same income won't typically have the same level of utility, as they encounter different goods and worker productivities in satisfying their needs. The income (hence expenditure) $Y(V)$ needed to obtain expected utility V in market n, for an individual with needs fulfilled by numbers of k-subtasks given by $\{n_k\}_{k=1}^K$, is thus:

$$Y(V;\{n_k\}) = V \prod_k \frac{1}{a_k} \left[\int_0^\infty ... \int_0^\infty \left(\sum_{\omega=1}^{n_k} c_\omega^{-(\sigma_k-1)} \right)^{-\alpha_k/(\sigma_k-1)} dG_{k,n}(c_1)...dG_{k,n}(c_{n_k}) \right].$$

In parallel to the derivation of the cost distribution (12), we can simplify this expression to obtain:

$$Y(V;\{n_k\}) = V \prod_k \frac{1}{a_k} n_k \Gamma\left(1+\alpha_k/\phi\right) \Xi_{k,n}^{-\alpha_k/\phi}.$$

To neutralize the effect of n_k, we set $a_k = n_k \Gamma\left(1+\frac{1}{\phi}\alpha_k\right)$ so that the expected expenditure function becomes:

$$Y(V) = V \prod_{k=1}^K \left(\Xi_{k,n}\right)^{-\frac{1}{\phi}\alpha_k}.$$

We can write the result more compactly as:

$$Y(V) = P_n^C V,$$

where

$$P_n^C = \prod_{k=1}^{K} \left(\Xi_{k,n}\right)^{-\frac{1}{\phi}\alpha_k}$$

is the consumer price index.

3 Aggregate Equilibrium

We now have in place the assumptions we need to solve for the aggregate equilibrium. We first solve for equilibrium in the production of intermediates, given wages, and then turn to labor-market equilibrium, which determines those wages.

3.1 Production Equilibrium

With balanced trade, total final spending X_n^C is labor income:

$$X_n^C = \sum_{l=1}^{L} w_n^l L_n^l = \sum_{k=1}^{K} w_{k,n} L_{k,n}. \tag{14}$$

Total production in country i equals total revenue in supplying consumption goods and intermediates around the world:

$$Y_i = \sum_{n=1}^{\mathcal{N}} \pi_{ni} \left[\Phi_n^C X_n^C + \Phi_n^I Y_n\right]$$

where $\Phi_n^C = 1 - \alpha_n^L$ and $\Phi_n^I = 1 - \beta_n^L$, the shares of goods in final spending and in production spending, respectively.

We can write this result in matrix form as:

$$\boldsymbol{Y} = \boldsymbol{\Pi}\left(\boldsymbol{\Phi}^C \boldsymbol{X}^C + \boldsymbol{\Phi}^I \boldsymbol{Y}\right)$$

where:

$$\boldsymbol{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ . \\ Y_{\mathcal{N}} \end{bmatrix}, \quad \boldsymbol{X}^C = \begin{bmatrix} X_1^C \\ X_2^C \\ . \\ . \\ . \\ X_{\mathcal{N}}^C \end{bmatrix}$$

$$\boldsymbol{\Phi}^j = \begin{bmatrix} \Phi_1^j & 0 & ... & 0 & 0 \\ 0 & \Phi_2^j & ... & 0 & 0 \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ 0 & 0 & ... & \Phi_{\mathcal{N}-1}^j & 0 \\ 0 & 0 & ... & 0 & \Phi_{\mathcal{N}}^j \end{bmatrix} \quad j = C, I$$

and:

$$\boldsymbol{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{21} & ... & \pi_{\mathcal{N}-1,1} & \pi_{\mathcal{N}1} \\ \pi_{12} & \pi_{22} & ... & \pi_{\mathcal{N}-1,2} & \pi_{\mathcal{N}2} \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ . & . & . \ . \ . & . & . \\ \pi_{1,\mathcal{N}-1} & \pi_{2,\mathcal{N}-1} & ... & \pi_{\mathcal{N}-1,\mathcal{N}-1} & \pi_{\mathcal{N},\mathcal{N}-1} \\ \pi_{1\mathcal{N}} & \pi_{2\mathcal{N}} & ... & \pi_{\mathcal{N}-1,\mathcal{N}} & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}$$

We can then solve for $\boldsymbol{Y}$:

$$\boldsymbol{Y} = (\boldsymbol{I}_{\mathcal{N}} - \boldsymbol{\Pi}\boldsymbol{\Phi}^I)^{-1}\boldsymbol{\Pi}\boldsymbol{\Phi}^C\boldsymbol{X}^C$$

where $\boldsymbol{I}_{\mathcal{N}}$ is the $\mathcal{N} \times \mathcal{N}$ identity matrix.

3.2 Labor-Market Equilibrium

With balanced trade, final spending in country i, X_i^C is given by (14). Equilibrium in the market for labor of type l in country i solves the expression:

$$w_i^l L_i^l = \alpha_i^l X_i^C + \beta_i^l Y_i.$$

where the first term on the right-hand side corresponds to labor demanded directly by households and the second term to labor demanded by firms. These sets of equations, for each type of labor l in each country i, determine the wage w_i^l.

4 Implications for Individual Producers

While our analysis so far has allowed us to investigate the implications of various changes in exogenous variables on equilibrium aggregate outcomes, we have more work to do to find out what happens to individual producers. We start by calculating the measure of producers active in each market. We then turn to the number of firms selling in each market and the number of relationships, as discussed in the introduction. These implications of the model have close analogs in our data on customers, in other EU countries, of French manufacturers.

In connecting the theory to data, we reinterpret final customers as retailers and wholesalers rather than individual households. Furthermore, since our data is for French manufacturers, whereas their customers are firms in any sector, we will distinguish between the measure of manufacturers in country n, which we denote by M_n, and all other buyers, whose measure we denote by O_n.[9] Thus, we focus the theory on the behavior of manufacturing firms, and hence on the tasks that can be carried out by manufactured goods.

Our data on buyers are for each of 24 EU destinations. We will also treat France itself as a separate destination, but our data for France are more limited on the buyer side. We treat all buyers outside the EU as being in a single destination $\mathcal{N}$.

While we replace L_n with O_n for non-manufacturing customers, we continue to treat the congestion effect as being a function of the measure of households, L_n. Both L_n and O_n are treated as being exogenous to our theory.

[9] We still assume that the non-manufacturing buyers B_n have the same production technology as manufacturers.

4.1 Active Manufacturers

A producer is active as long as it has a customer that buys from it. How many buyers a firm has depends not only on its efficiency z, but on its luck in finding low-cost suppliers and its luck in running into buyers who don't have better alternatives.

Consider a supplier from i with a unit cost c in the destination market n. The number of buyers that it connects with, seeking to carry out a k-subtask, is distributed Poisson with parameter:

$$\begin{aligned} E_k \left(O_n + M_n\right) e_{k,ni}(c) &= E_k \left(O_n + M_n\right) \lambda_k \lambda_{ni} L_n^{-\varphi} \mu_n(c)^{-\gamma} \\ &= \left(O_n + M_n\right) E_k \lambda_k \lambda_{ni} L_n^{-\varphi} \left(\Upsilon_n c^{\theta}\right)^{-\gamma}, \end{aligned}$$

where recall that E_k is the expected number of tasks of type k across all buyers.

Having met a buyer, this supplier will make the sale with probability $e^{-\Xi_{k,n} c^{\phi}}$, the probability that that there is no lower quote. Combining these two results the number of customers in n buying from a supplier from i with unit cost (of delivering to destination n) of c for a k-subtask is distributed Poisson with parameter $\eta_{k,ni}(c)$, given by:

$$\eta_{k,ni}(c) = \left(O_n + M_n\right) E_k \lambda_k \lambda_{ni} L_n^{-\varphi} \left(\Upsilon_n c^{\theta}\right)^{-\gamma} e^{-\Xi_{k,n} c^{\phi}},$$

where, recall,

$$\Xi_{k,n} = \nu_{k,n} + w_{k,n}^{-\phi}.$$

Note that $\eta_{k,ni}(c)$ is decreasing in the producer's unit cost (of delivery) c for two reasons. First, as long as $\gamma > 0$, a low-cost producer typically finds more potential customers. Second, each potential customer is more likely to have no better option. Note also that, given Υ_n and $w_{k,n}$, the Poisson parameter is at first increasing and then decreasing in λ_k. If it's impossible

to meet customers ($\lambda_k = 0$) then it's impossible to make a sale. Thus, starting from 0, an increase in λ_k increases the likelihood of a sale. But an increase in λ_k also means that a potential buyer is more likely to have found another seller with a lower cost. At some point (which is earlier for a firm with a high c), as λ_k rises, this second effect dominates, so that further increases reduce expected sales.

Since purchases are independent across k, the total number of customers in n for a producer in i with unit cost (of delivery to n) of c is distributed Poisson with parameter:

$$\eta_{ni}(c) = \sum_{k=1}^{K} \eta_{k,ni}(c) = \lambda_{ni} \left(O_n + M_n\right) L_n^{-\varphi} \left(\Upsilon_n c^{\theta}\right)^{-\gamma} \sum_{k=1}^{K} E_k \lambda_k e^{-\Xi_{k,n} c^{\phi}}. \tag{15}$$

By the properties of the Poisson distribution, $\eta_{ni}(c)$ is also the expected number of customers for a potential producer from i selling a product at unit cost c in market n.

Consider customers worldwide for a producer in country i with cost of delivery to its home market of c. Its unit cost in country n is therefore cd_{ni}. The total number of customers around the world for this producer is distributed Poisson with parameter:

$$\begin{aligned} \eta_i^W(c) &= \sum_{n=1}^{\mathcal{N}} \eta_{ni}(cd_{ni}) \\ &= \sum_{n=1}^{\mathcal{N}} \lambda_{ni} \left(O_n + M_n\right) L_n^{-\varphi} \left(\Upsilon_n d_{ni}^{\theta} c^{\theta}\right)^{-\gamma} \sum_{k=1}^{K} E_k \lambda_k e^{-\Xi_{k,n} d_{ni}^{\phi} c^{\phi}}. \end{aligned}$$

Thus, the probability that a potential producer from source i with unit cost c fails to make a sale anywhere is $\exp(-\eta_i^W(c))$.

A manufacturer in country n will exist if it has at least one customer, either another manufacturer or a firm outside of manufacturing, somewhere in the world. Integrating over the cost distribution of potential manufacturers in source i (those from i that can deliver to i

at cost c):

$$\begin{aligned} M_i &= \int_0^\infty (1 - e^{-\eta_i^W(c)}) d\mu_{ii}(c) \\ &= T_i \Xi_i \int_0^\infty (1 - e^{-\eta_i^W(c)}) \theta c^{\theta-1} dc. \end{aligned} \tag{16}$$

Since $\eta_i^W(c)$ itself depends on the measure of customers in each market n, which itself depends on M_n, we need to iterate to find a solution for all the M_i's.

4.2 The Measure of Sellers

A manufacturer in i will sell to country n if it has at least one customer there, either another manufacturer or a firm from some other sector. The probability that it will have at least one customer in n, if it can sell there at cost c, is simply $1 - \exp(-\eta_{ni}(c))$. We can thus calculate N_{ni}, the measure of manufacturers from i selling in n, by integrating over the cost distribution (of costs in n for firms from i):

$$N_{ni} = \int_0^\infty (1 - e^{-\eta_{ni}(c d_{ni})}) d\mu_{ii}(c) = d_{ni}^{-\theta} \int_0^\infty (1 - e^{-\eta_{ni}(c)}) d\mu_{ii}(c). \tag{17}$$

Note how the iceberg cost d_{ni} enters directly while the information friction only enters indirectly (and non-linearly) through $\eta_{ni}(c)$. We need to evaluate this integral numerically. The total measure of producers selling to firms in n is:

$$N_n = \sum_{i=1}^{\mathcal{N}} N_{ni}.$$

4.3 The Measure of Relationships

So far we've considered whether or not a firm has any customers in a market, but not how many. We refer to the total number of firm seller-buyer transactions as the number of re-

lationships. The measure of relationships between manufacturers from i and buyers in n is:

$$\begin{aligned}
R_{ni} &= \int_0^\infty \eta_{ni}(c)d\mu_{ni}(c) = T_i\Xi_i d_{ni}^{-\theta}\int_0^\infty \eta_{ni}(c)\theta c^{\theta-1}dc \\
&= \pi_{ni}\Upsilon_n\left(O_n+M_n\right)L_n^{-\varphi}\Upsilon_n^{-\gamma}\int_0^\infty \sum_{k=1}^K E_k\lambda_k c^{-\gamma\theta}e^{-\Xi_{k,n}c^\phi}\theta c^{\theta-1}dc \\
&= \pi_{ni}\left(O_n+M_n\right)\sum_{k=1}^K \nu_{k,n}E_k\left(1-\gamma\right)\int_0^\infty e^{-\Xi_{k,n}c^\phi}\theta c^{\phi-1}dc \\
&= \pi_{ni}\left(O_n+M_n\right)\sum_{k=1}^K \nu_{k,n}E_k\,\frac{-1}{\Xi_{k,n}}e^{-\Xi_{k,n}c^\phi}\Big|_0^\infty \\
&= \pi_{ni}\left(O_n+M_n\right)\sum_{k=1}^K E_k\frac{\nu_{k,n}}{\Xi_{k,n}}.
\end{aligned}$$

Recall that π_{ni} is proportional to $\lambda_{ni}d_{ni}^{-\theta}$, so that relationships R_{ni} are affected by these frictions distinctly from how they affect exporters N_{ni}.

The measure of relationships is simply the product of the measure of buyers in n, the sum across tasks of the expected number of outsourced k-subtasks ($E_k\nu_{k,n}/\Xi_{k,n}$), and the share of spending in n going to sellers from i. The total measure of relationships involving buyers from destination n is:

$$R_n = \sum_i R_{ni} = \left(O_n+M_n\right)\sum_{k=1}^K E_k\frac{\nu_{k,n}}{\Xi_{k,n}}.$$

4.4 Buyers per Seller

Dividing the measure of relationships between sellers in i and buyers in n by the measure of firms from i selling in n, we get the mean number of buyers per seller (customers per firm):

$$\bar{B}_{ni} = \frac{R_{ni}}{N_{ni}}.$$

It represents the mean of the integer-valued random number B_{ni} of customers in n buying from a firm from i. First consider a firm from i selling in n at cost c.

We can dig deeper by considering the probability that a firm from i with cost c in n has x buyers there:

$$p_{ni}(x;c) = \Pr[B_{ni}(c) = x] = \frac{e^{-\eta_{ni}(c)}\left[\eta_{ni}(c)\right]^x}{x!},$$

for $x = 0, 1, ...$ The mean of $B_{ni}(c)$ is simply:

$$\bar{B}_{ni}(c) = \sum_{x=1}^{\infty} x p_{ni}(x;c) = \sum_{x=1}^{\infty} x \frac{e^{-\eta_{ni}(c)}\left[\eta_{ni}(c)\right]^x}{x!} = \eta_{ni}(c). \tag{18}$$

The mean number of customers is clearly decreasing in c. Integrating this term over the distribution of costs in n for firms from i that find at least one customer there, we come full circle to:

$$\bar{B}_{ni} = \int_0^{\infty} \bar{B}_{ni}(c) \frac{d\mu_{ni}(c)}{\int_0^{\infty}(1 - e^{-\eta_{ni}(x)})d\mu_{ni}(x)} = \frac{1}{N_{ni}} \int_0^{\infty} \eta_{ni}(c) d\mu_{ni}(c) = \frac{R_{ni}}{N_{ni}}.$$

For any $x \geq 1$, we can integrate over the cost measure to obtain the measure of firms from i with x buyers in n:

$$N_{ni}(x) = \int_0^{\infty} p_{ni}(x;c) d\mu_{ni}(c) = \int_0^{\infty} \frac{e^{-\eta_{ni}(c)}\left[\eta_{ni}(c)\right]^x}{x!} d\mu_{ni}(c)$$

Thus, the fraction of firms from i selling in n who have x buyers is:

$$p_{ni}(x) = \frac{N_{ni}(x)}{N_{ni}},$$

for $x = 1, 2, ...$[10] We can observe this distribution, and how it varies across destinations for French firms.

[10] An alternative way to express the measure of firms from i with at least one firm buyer in n is:

$$N_{ni} = \sum_{x=1}^{\infty} N_{ni}(x).$$

4.5 Buyers per Seller, Conditional

By analogy to moments in Eaton, Kortum, and Kramarz (2011), we can calculate $\bar{B}_{ni}$ conditional on firms that also sell in some third country m. Such firms, selected for their low cost, should have more buyers in n, on average, than those not selling in m. Pursuing this strategy, we first compute the measure of relationships in n for firms from i that also sell in m:

$$R_{ni,m} = \int_0^\infty \eta_{ni}(cd_{ni}) \left(1 - e^{-\eta_{mi}(cd_{mi})}\right) d\mu_{ii}(c).$$

In terms of buyers per firm, conditional on firms from i that sell in both n and m, we have:

$$B_{ni|m} = \frac{R_{ni,m}}{N_{(nm)i}},$$

where the measure of firms from i selling in both n and m is:

$$N_{(nm)i} = \int_0^\infty (1 - e^{-\eta_{ni}(cd_{ni})})(1 - e^{-\eta_{mi}(cd_{mi})}) d\mu_{ii}(c).$$

Here, we have exploited the fact that, given the firm's cost, having at least one buyer in n is independent of having at least one buyer in m.

4.6 Sellers Per Buyer

A firm has an average of E_k subtasks to fulfill for each task k. For a firm in country n, the probability of outsourcing a k-subtask is $\nu_{k,n}/\Xi_{k,n}$. The probability that the supplier comes

An alternative for the number of relationships is:

$$R_{ni} = \sum_{x=1}^{\infty} x N_{ni}(x).$$

Thus a third way to think of mean buyers per seller is

$$\bar{B}_{ni} = \sum_{x=1}^{\infty} x \frac{N_{ni}(x)}{N_{ni}} = \sum_{x=1}^{\infty} x p_{ni}(x).$$

from country i is π_{ni}. Let S_{ni} be the random number of suppliers from country i for a firm in country n. The expected number of such suppliers is:

$$E\left[S_{ni}\right] = \pi_{ni} \sum_{k=1}^{K} E_k \frac{\nu_{k,n}}{\Xi_{k,n}}.$$

What we observe, however, is the expected number of suppliers from i for a firm in n, conditional on the firm having at least one supplier from i.

The probability of a firm in n having at least one supplier from i is:

$$\Pr\left[S_{ni} \geq 1\right] = 1 - \prod_{k=1}^{K} \left(\sum_{n_k=1}^{\infty} P(n_k) \left(1 - \pi_{ni} \frac{\nu_{k,n}}{\Xi_{k,n}}\right)^{n_k} \right).$$

Under the assumption that the number of k-subtask per task is a constant $n_k = E_k$, for all k, this equation simplifies to:

$$\Pr\left[S_{ni} \geq 1\right] = 1 - \prod_{k=1}^{K} \left(1 - \pi_{ni} \frac{\nu_{k,n}}{\Xi_{k,n}}\right)^{n_k}.$$

In that case, the theoretical analog of the observable moment is:

$$E\left[S_{ni} | S_{ni} \geq 1\right] = \frac{\pi_{nF} \sum_{k=1}^{K} n_k \frac{\nu_{k,n}}{\Xi_{k,n}}}{1 - \prod_{k=1}^{K} \left(1 - \pi_{ni} \frac{\nu_{k,n}}{\Xi_{k,n}}\right)^{n_k}}. \tag{19}$$

5 Estimation

To be completed. See Appendix C for our current procedure.

6 Conclusion

Taking into account the granularity of individual buyer-seller relationships expands the scope for firm heterogeneity in a number of dimensions. Aside from differences in raw efficiency,

firms experience different luck in finding cheap inputs. These two sources of heterogeneity combine to create differences in the firm's cost to deliver to different markets around the world. But within each market firms have different degrees of luck in connecting with buyers. We can thus explain why a firm may happen to sell in a small, remote market while skipping over a large one close by. It also explains why one firm may appear very successful in one market and sell very little in another, while another firm does just the opposite.

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7 Appendix A: Data Source

The empirical analysis is conducted using detailed export data covering the universe of French exporting firms. The data have been provided by the French Customs, and have been used by Kramarz, Martin, and Mejean (2014). The full data set covers all export transactions that involve a French exporter and an importing firm located in the European Union. In this paper, we use only the data for the year 2005.

Many researchers before us have used individual trade data from the French Customs. Typically, the data used in such empirical analyses are annual measures disaggregated at the level of the exporting firm, as in Eaton, Kortum, and Kramarz (2011) among others. Some papers, such as Biscourp and Kramarz (2007) and Blaum, Lelarge, and Peters (2014), also use data at the level of the importer. An exception is Bricongne, Fontagné, Gaulier, Taglioni, and Vicard (2012) who use data that record, for each exporting firm, each transaction in each month, although not identifying the exact buyer. In this respect, the data we use are more precise since they not only record the transaction but also the exact identity of the buyer. For each transaction, the dataset gives us the identity of the exporting firm (its name and its SIREN identifier), the identification number of the importer (an anonymized version of its VAT number), the date of the transaction (month and year), the product category of the transaction (at the 8-digit level of the combined nomenclature), the value and the quantity of the shipment. For the analysis here, records will be aggregated across transactions within a year, for each exporter-importer-product triplet. Such measurement is possible because, whereas goods are perfectly free to move across countries within the European Union, firms selling goods outside France are still compelled to fill a Customs form. Such forms are used to

repay VAT for transactions on intermediate consumptions. Hence, our data are exhaustive. However, small exporters are allowed to fill a "simplified" form that does not require the product category of exported goods. The "simplified" regime can be used by firms with total exports in the EU below 100,000 euros in 2005 (and 150,000 euros thereafter). In 2005, we have data for 46,928 French firms exporting 7,807 8-digit products to 571,149 buyers located in the EU. Total exports by these firms amounts to 207 billions of euros. Such exports account for 58 percent of French total exports. The total number of observations is 3,983,909.

8 Appendix B: Computing Υ

[***THIS APPENDIX IS NOT UPDATED***] We derive conditions under which there is a unique solution for Υ, given wages, that can be computed by simple iteration. To ensure a solution it helps to have a sufficient share of tasks in which outsourcing is not possible ($\lambda_k = 0$). Denote the set of such tasks as Ω^0 and its complement (among the set of all tasks $\{1, 2, ..., K\}$) as Ω^P. We require:

$$\beta^P = \sum_{k \in \Omega^P} \beta_k < 1.$$

As a warm-up exercise, we start with the case of a single country ($\mathcal{N} = 1$), so that Υ is a scalar. We then turn to the general case with multiple countries, in which Υ is an $\mathcal{N} \times 1$ vector.

8.1 The Case of a Single Country

With a single country, the solution for Υ is a fixed point

$$\Upsilon = f(\Upsilon)$$

of the function f defined as:

$$f(x) = T\prod_{k=1}^{K}\left(\frac{\theta}{\phi}\lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi}\right)^{\frac{\theta}{\phi}\beta_k}.$$

Employing our assumption that $\lambda_k = 0$ for all tasks $k \in \Omega^0$, we can write:

$$f(x) = T\left(\prod_{k\in\Omega^0}(w_k)^{-\theta\beta_k}\right)\prod_{k\in\Omega^P}\left(\frac{\theta}{\phi}\lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi}\right)^{\frac{\theta}{\phi}\beta_k}.$$

It is convenient to work in logs. Thus $\ln\Upsilon$ is the fixed point

$$\ln\Upsilon = F(\ln\Upsilon)$$

of the function:

$$F(y) = A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right),$$

where

$$A = \ln T - \sum_{k\in\Omega^0}\theta\beta_k \ln w_k,$$

and

$$u_k = \frac{\theta}{\phi}\lambda_k$$

There exists a unique fixed point of F if it is a contraction. To show that it is, we can check Blackwell's sufficient conditions, monotonicity and discounting. For monotonicity, note that $x \leq y$ implies:

$$F(x) = A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}x} + w_k^{-\phi}\right) \leq A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right) = F(y).$$

For discounting, $a > 0$ implies:

$$\begin{aligned}
F(y+a) &= A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}(y+a)} + w_k^{-\phi}\right) = A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(e^{\frac{\phi}{\theta}a}u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right) \\
&= A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k\left[\frac{\phi}{\theta}a + \ln\left(u_k e^{\frac{\phi}{\theta}y} + e^{-\frac{\phi}{\theta}a}w_k^{-\phi}\right)\right] = A + \beta^P a + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + e^{-\frac{\phi}{\theta}a}w_k^{-\phi}\right) \\
&\leq A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right) + \beta^P a = F(y) + \beta^P a.
\end{aligned}$$

We can thus compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with $y^{(0)} = 0$. This method is justified, since the contraction mapping theorem guarantees that:

$$\lim_{t\to\infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that $\ln \Upsilon$ is increasing in technology T, decreasing in any task-specific wage w_k, and increasing in any task-specific arrival of price quotes λ_k.

8.2 Multiple Countries

Consider generalizing the argument above to a world of many countries, trading intermediates and final goods with each other. Now Υ is an $\mathcal{N} \times 1$ vector satisfying

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left(\frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i^{\frac{\phi}{\theta}} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k},$$

for $n = 1, ..., \mathcal{N}$.

Let $\ln \Upsilon$ be the corresponding vector with $\ln \Upsilon_n$ in place of Υ_n for $n = 1, ..., \mathcal{N}$. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the mapping F, whose n'th element is:

$$\begin{aligned} F_n(y) &= \ln \left[\sum_i \exp \left(\ln \left(T_i d_{ni}^{-\theta} \right) + \sum_k \frac{\theta}{\phi} \beta_k \ln \left(u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right] \\ & \ln \left[\sum_i \exp \left(A_{ni} + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right], \end{aligned}$$

where

$$A_{ni} = \ln\left(T_i d_{ni}^{-\theta}\right) - \sum_{k\in\Omega^0} \theta\beta_k \ln w_{k,i}$$

and

$$u_{k,i} = \frac{\theta}{\phi}\lambda_{k,i}.$$

We can check Blackwell's conditions again. For monotonicity, it is readily apparent that for a vector $x \leq y$ we have $F_n(x) \leq F_n(y)$ for all $n = 1, ..., \mathcal{N}$. For discounting, consider $a > 0$ so that

$$\begin{aligned}
F_n(y+a) &= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}(y_i+a)} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \left[\frac{\phi}{\theta}a + \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right]\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right)\right] \\
&\leq \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] + \beta^P a \\
&= F_n(y) + \beta^P a.
\end{aligned}$$

Thus, even with multiple countries, we can still compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with an $\mathcal{N} \times 1$ vector $y^{(0)}$ (which could simply be a vector of zeros). This method is justified, since the contraction mapping theorem guarantees (just as in the scalar case) that:

$$\lim_{t\to\infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that each element of $\ln \Upsilon$ is increasing in technology anywhere T_i, decreasing in any task-specific wage $w_{k,i}$ in any country, and increasing in any task-specific arrival of price quotes $\lambda_{k,i}$ in any country. An important caveat, however, is that these comparative statics take task-specific wages as given, so do not predict general-equilibrium outcomes.

9 Appendix C: Estimation Algorithm

Here we describe an algorithm for estimation. Our strategy will be to condition on key endogenous aggregate variables such as the matrix of bilateral trade share and the vector of manufacturing labor shares. We have these measures for an unlimited number of countries. We have the relationships data only for 24 EU countries (with France as the supplier but not the buyer).

To carry out our algorithm, we project the relationships data R_{nF} onto French market share π_{nF} and market size X_n for the 24 EU countries for which we have these data. For France and the Rest of World (country 26) we can measure π_{nF} and X_n, which we use to predict R_{nF} for these two. Doing so, we proceed as if we have R_{nF} for all countries.

9.1 Restrictions on the Model

We make the following restrictions to the model as layed out above:

- Task-specific contact rates satisfy: $\lambda_0 = 0$ and $\lambda_k = \lambda$ for $k = 1, ..., K$.
- Set $E_k = 1$ for $k = 1, ..., K$.

- Task shares are: β_0 and $\beta_k = (1-\beta_0)/K$. We interpret β_0 as the share of purchased services used in production by a firm.

- There are two types of labor. Service workers perform task 0 and manufacturing workers perform all other tasks (if they are not outsourced).

These simplification leave us with the following vector of parameters to estimate (recall that $\gamma = 1 - \phi/\theta$):

$$\Theta = \left\{\theta, \phi, \varphi, \lambda_1, \lambda_{\{n,F;\ n=1,\ldots,24\}}, \beta_0, K\right\}.$$

We now turn to an algorithm for computing this simplified version of the model, conditioning on trade shares and labor shares.

Before proceeding, we define a few new terms. Labor's share in manufacturing is $\beta_n^{1,L}$ while the share of manufacturing inputs is:

$$\beta_n^{1,M} = 1 - \beta_0 - \beta_n^{1,L} \leq 1.$$

Considering all tasks:

$$\beta_0 + \beta_n^{1,L} + \beta_n^{1,M} = 1.$$

9.2 Basic Algorithm

We present the algorithm as a series of steps:

1. Fix Θ.

2. Exploit data on labor shares in manufacturing. In this restricted version of the model, labor's share (in the aggregate) is:

$$\beta_n^{1,L} = (1-\beta_0)\frac{w_{1,n}^{-\phi}}{\nu_{1,n} + w_{1,n}^{-\phi}} = (1-\beta_0)\frac{\Xi_{1,n} - \nu_{1,n}}{\Xi_{1,n}},$$

which we can solve for:

$$\nu_{1,n} = \frac{1-\beta_0-\beta_n^{1,L}}{1-\beta_0} = \frac{\beta_n^{1,M}}{1-\beta_0}\Xi_{1,n}.$$

Linking back to more fundamental parameters of the model:

$$\Xi_{1,n} = \frac{(1-\beta_0)\,\nu_{1,n}}{\beta_n^{1,M}} = (1-\beta_0)\,\frac{\theta}{\phi}\frac{\lambda_1 L_n^{-\varphi}}{\beta_n^{1,M}}\Upsilon_n^{\phi/\theta}.$$

While we don't know Υ_n, its value will cancel out in the expressions below.

3. Invert the expression for relationships:

$$R_{nF} = \pi_{nF}\,(O_n+M_n)\,K\frac{\nu_{1,n}}{\Xi_{1,n}} = \pi_{nF}\,(O_n+M_n)\,K\frac{\beta_n^{1,M}}{1-\beta_0},$$

to get the measure of buyers:

$$O_n + M_n = \frac{(1-\beta_0)\,R_{nF}}{K\pi_{nF}\beta_n^{1,M}}. \tag{20}$$

4. Calculate a transformed version of $\eta_{ni}(c)$. In several expression below we require:

$$\begin{aligned}\eta_{ni}(cd_{ni}) &= \lambda_{ni}\,(O_n+M_n)\,L_n^{-\varphi}\left(\Upsilon_n c^\theta d_{ni}^\theta\right)^{-(1-\phi/\theta)}\lambda_1 K e^{-\Xi_{1,n}c^\phi d_{ni}^\phi} \\ &= \lambda_{ni}\frac{(1-\beta_0)\,R_{nF}L_n^{-\varphi}}{\pi_{nF}\beta_n^{1,M}}\left(\Upsilon_n c^\theta d_{ni}^\theta\right)^{-(1-\phi/\theta)}\lambda_1 e^{-\frac{\theta}{\phi}\frac{(1-\beta_0)\lambda_1 L_n^{-\varphi}}{\beta_n^{1,M}}\left(\Upsilon_n c^\theta d_{ni}^\theta\right)^{\phi/\theta}}.\end{aligned}$$

Note that K cancels. To cancel Υ_n as well, make the change of variable $y = T_i\Xi_i c^\theta$ and construct:

$$\begin{aligned}\tilde{\eta}_{ni}(y) &= \eta_{ni}\left(\left(\frac{y}{T_i\Xi_i}\right)^{1/\theta}d_{ni}\right) \\ &= \lambda_{ni}\frac{(1-\beta_0)\,R_{nF}L_n^{-\varphi}}{\pi_{nF}\beta_n^{1,M}}\left(\frac{\lambda_{ni}}{\pi_{ni}}y\right)^{-(1-\phi/\theta)}\lambda_1 e^{-\frac{\theta}{\phi}\frac{(1-\beta_0)\lambda_1 L_n^{-\varphi}}{\beta_n^{1,M}}\left(\frac{\lambda_{ni}}{\pi_{ni}}y\right)^{\phi/\theta}}.\end{aligned} \tag{21}$$

The integrals that follow can be expressed in terms of $\tilde{\eta}_{ni}(y)$.

5. Solve for the measure of active manufacturers in each country. Start with (16):

$$M_i = T_i \Xi_i \int_0^{\infty} (1 - e^{-\eta_i^W(c)}) \theta c^{\theta-1} dc.$$

Now change the variable of integration to $y = T_i \Xi_i c^{\theta}$, so that $dy = T_i \Xi_i \theta c^{\theta-1} dc$:

$$M_i = \int_0^{\infty} \left(1 - e^{-\eta_i^W \left(\left(\frac{y}{T_i \Xi_i} \right)^{1/\theta} \right)} \right) dy.$$

Employing (21) we get:

$$\eta_i^W \left(\left(\frac{y}{T_i \Xi_i} \right)^{1/\theta} \right) = \sum_{n=1}^{\mathcal{N}} \eta_{ni} \left(\left(\frac{y}{T_i \Xi_i} \right)^{1/\theta} d_{ni} \right) = \sum_{n=1}^{\mathcal{N}} \tilde{\eta}_{ni}(y),$$

which delivers a simple expression for the measure of manufacturers:

$$M_i = \int_0^{\infty} \left(1 - e^{-\sum_{n=1}^{\mathcal{N}} \tilde{\eta}_{ni}(y)} \right) dy. \tag{22}$$

Notice how data on π_{ni} eliminates the need for d_{ni}, Υ_n, and $T_i \Xi_i$. Also, note that K cancels out of the resulting equation.

6. Check if the measure of nonmanufacturing buyers is positive by comparing (20) and (22). We may need to include the constraint that $O_n \geq 0$ in the objective function.

7. Having solved for the $\{M_i\}$, given a guess of Θ, we now propose a set of moments that will allow us to identify the other parameters. These moments will be constructed for the sample of 24 EU countries. They all involve (21), rewritten with France as the exporter:

$$\begin{aligned} \tilde{\eta}_{nF}(y) &= \eta_{nF} \left(\left(\frac{y}{T_F \Xi_F} \right)^{1/\theta} d_{nF} \right) \\ &= \lambda_{nF} \frac{(1-\beta_0) R_{nF} L_n^{-\varphi}}{\pi_{nF} \beta_n^{1,M}} \left(\frac{\lambda_{nF}}{\pi_{nF}} y \right)^{-(1-\phi/\theta)} \lambda_1 e^{-\frac{\theta}{\phi} \frac{(1-\beta_0)\lambda_1 L_n^{-\varphi}}{\beta_n^{1,M}} \left(\frac{\lambda_{nF}}{\pi_{nF}} y \right)^{\phi/\theta}}. \end{aligned} \tag{23}$$

$$\tilde{\eta}_{nF}(y) = \lambda_{nF} \frac{(1-\beta_0) R_{nF} L_n^{-\varphi}}{\pi_{nF} \beta_n^{1,M}} \left(\frac{\lambda_{nF}}{\pi_{nF}} y \right)^{-(1-\phi/\theta)} \lambda_1 e^{-\frac{\theta}{\phi} \frac{(1-\beta_0)\lambda_1 L_n^{-\varphi}}{\beta_n^{1,M}} \left(\frac{\lambda_{nF}}{\pi_{nF}} y \right)^{\phi/\theta}}$$

8. The first of these moments is entry, by destination, of French firms:

$$N_{nF} = T_F \Xi_F \int_0^\infty (1 - e^{-\eta_{nF}(cd_{nF})}) \theta c^{\theta-1} dc.$$

Applying the change of variables $y = T_F \Xi_F c^\theta$, so that $dy = \theta T_F \Xi_F c^{\theta-1} dc$, and substituting in (23):

$$N_{nF} = \int_0^\infty \left(1 - e^{-\tilde{\eta}_{nF}(y)}\right) dy,$$

which depends only on parameters and data.

9. The second of these moments is the distribution, by destination, of the number of buyers per French firm. In particular, the fraction of French entrants into n who have x buyers is:

$$\begin{aligned} \frac{N_{nF}(x)}{N_{nF}} &= \frac{T_F \Xi_F}{N_{nF}} \int_0^\infty \frac{e^{-\eta_{nF}(cd_{nF})}}{x!} \left[\eta_{nF}(cd_{nF})\right]^x \theta c^{\theta-1} dc \\ &= \frac{1}{N_{nF}} \int_0^\infty \frac{e^{-\tilde{\eta}_{nF}(y)}}{x!} \left[\tilde{\eta}_{nF}(y)\right]^x dy \end{aligned}$$

for $x = 1, 2, ...$

10. The third of these moments is the expected number of buyers in n per French firm exporting to both n and m:

$$\begin{aligned} B_{nF|m} &= \frac{R_{nF,m}}{N_{(nm)F}} = \frac{T_F \Xi_F}{N_{(nm)F}} \int_0^\infty \eta_{nF}(cd_{nF}) \left(1 - e^{-\eta_{mF}(cd_{mF})}\right) \theta c^{\theta-1} dc \\ &= \frac{1}{N_{(nm)F}} \int_0^\infty \tilde{\eta}_{nF}(y) \left(1 - e^{-\tilde{\eta}_{mF}(y))}\right) dy, \end{aligned}$$

where:

$$\begin{aligned} N_{(nm)F} &= T_F \Xi_F \int_0^\infty (1 - e^{-\eta_{nF}(cd_{nF})})(1 - e^{-\eta_{mF}(cd_{mF})}) \theta c^{\theta-1} dc \\ &= \int_0^\infty \left(1 - e^{-\tilde{\eta}_{nF}(y)}\right) \left(1 - e^{-\tilde{\eta}_{mF}(y)}\right) dy. \end{aligned}$$

We can calculate this moment across both $n \neq F$ and for all m (including $m = F$).

11. The fourth of these moments is the mean number of French suppliers per buyer in country n with at least one such supplier. Let D_{nF} be the random number of French suppliers for a firm in n. Its expectation is:

$$E\left[D_{nF}\right] = \pi_{nF} K \frac{\nu_{1,n}}{\Xi_{1,n}} = \pi_{nF} K \frac{\beta_n^{1,M}}{1-\beta_0}.$$

We observe buyers in n only when they have at least one French supplier. The probability of a firm in n having at least one French supplier is:

$$\Pr\left[D_{nF} \geq 1\right] = 1 - \left(1 - \pi_{nF} \frac{\beta_n^{1,M}}{1-\beta_0}\right)^K.$$

Thus the theoretical analog of the observable moment is:

$$E\left[D_{nF} | D_{nF} \geq 1\right] = \frac{\pi_{nF} K \frac{\beta_n^{1,M}}{1-\beta_0}}{1 - \left(1 - \pi_{nF} \beta_n^{1,M}\right)^K}$$

12. Our final moment is based on the size of manufacturing employment in the economy. Let $L_{0,n}$ be service sector employment and $L_{1,n}$ be manufacturing employment, with $L_{0,n} + L_{1,n} = L_n$. The share of aggregate labor income due to employment in services is:

$$\frac{w_{0,n} L_{0,n}}{w_{0,n} L_{0,n} + w_{1,n} L_{1,n}} = \frac{\beta_0}{\beta_0 + \beta_n^{1,L}},$$

Ignoring payments to capital, we can also measure this ratio as:

$$\frac{w_{0,n} L_{0,n}}{w_{0,n} L_{0,n} + w_{1,n} L_{1,n}} = 1 - \frac{Y_n^M}{Y_n},$$

where Y_n^M is manufacturing value added and Y_n is GDP. This moment will help us identify β_0.

13. We iterate on a Θ to minimize the distance between these theoretical moments and their empirical counterparts.

Table 1: Customers per French Exporter

	Destination Market			
	Lithuania	Denmark	UK	Germany
Market Size ($billions)	18	94	882	1480
Customers per Exporter:				
Mean	1.5	2.6	5.8	9.6
Percentiles:				
25th	1	1	1	1
50th	1	1	2	2
75th	2	2	5	7
90th	3	5	12	18
95th	4	8	21	33
99th	7	19	65	117

Data are for 2005.

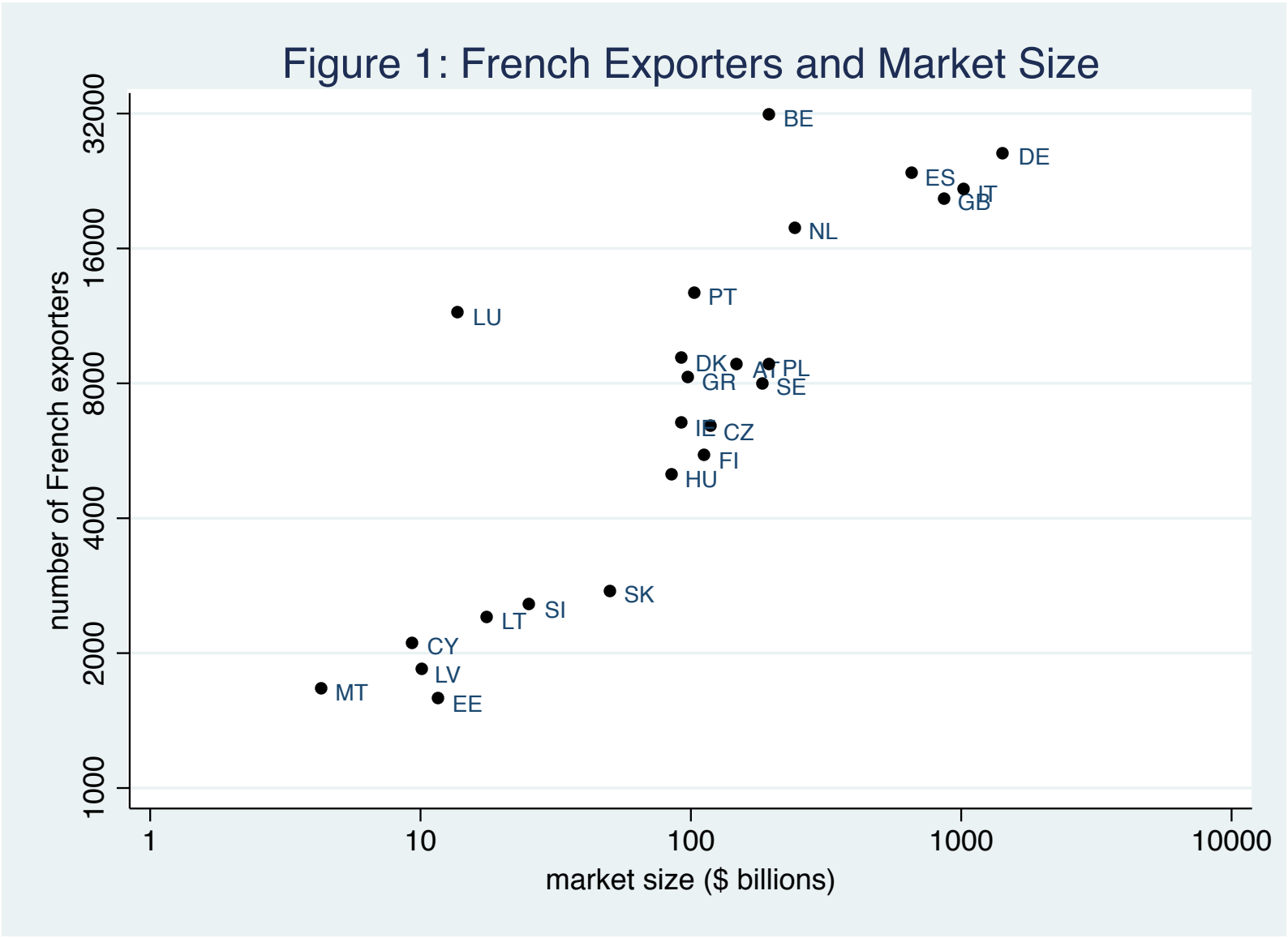

Figure 1: French Exporters and Market Size
number of French exporters
1000
2000
4000
8000
16000
32000
1
10
100
1000
10000
market size ($ billions)
BE
DE
ES
IT
GB
NL
PT
LU
DK
AT
PL
GR
SE
IE
CZ
FI
HU
SK
SI
LT
CY
LV
MT
EE

Figure 2: Buyers per French Exporter, by Destination

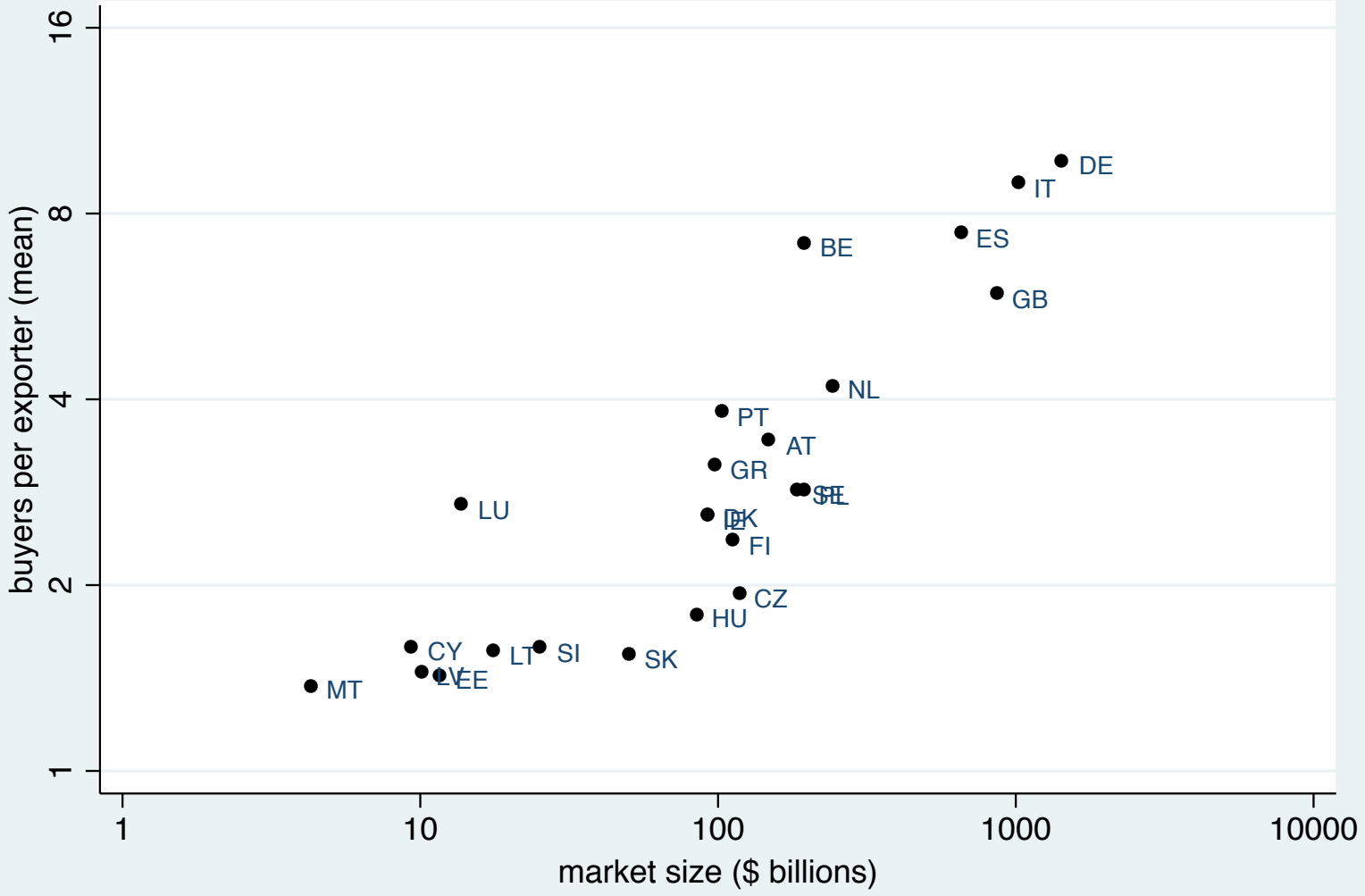


Figure 3: French Relationships and Market Size

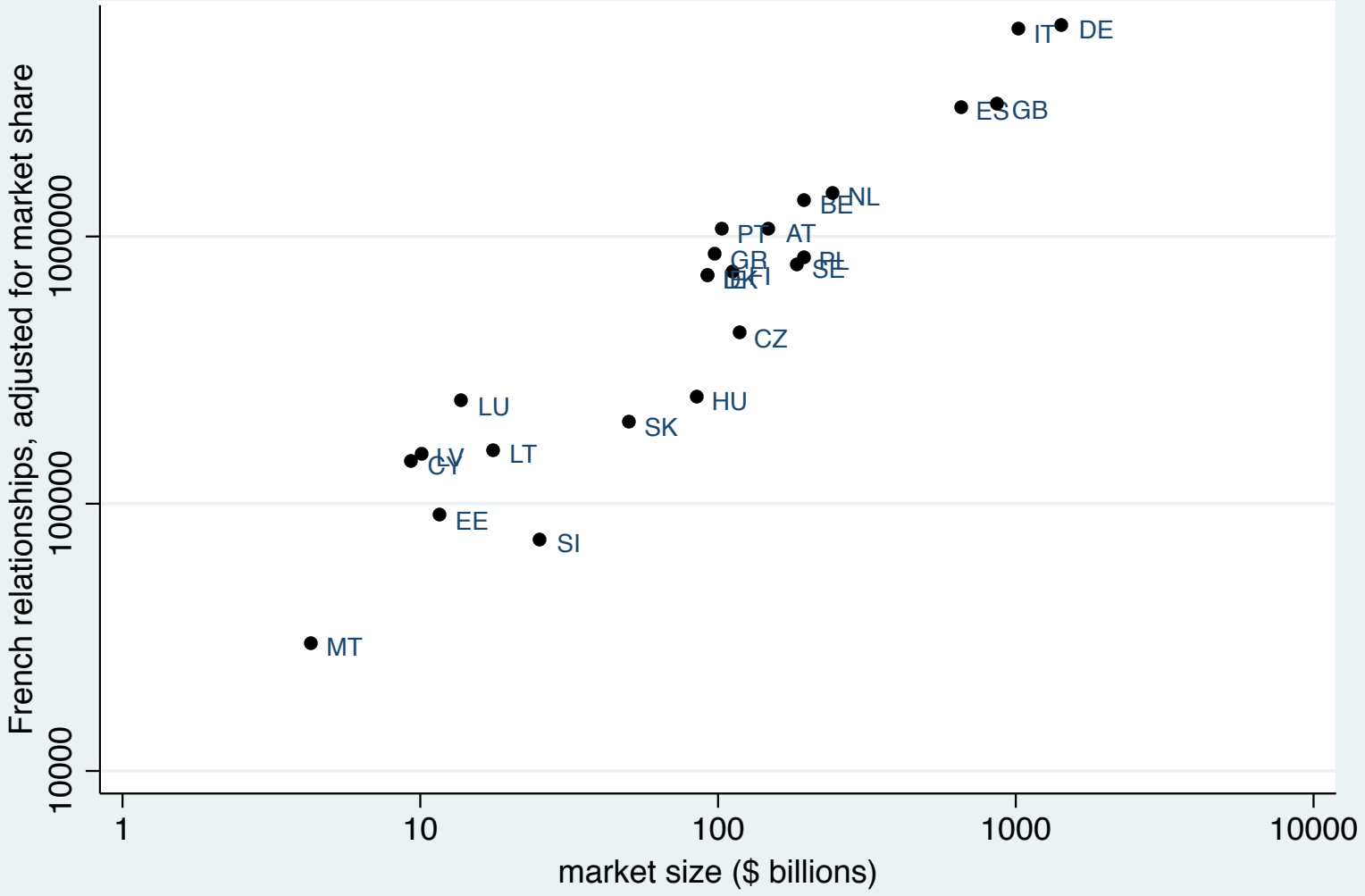
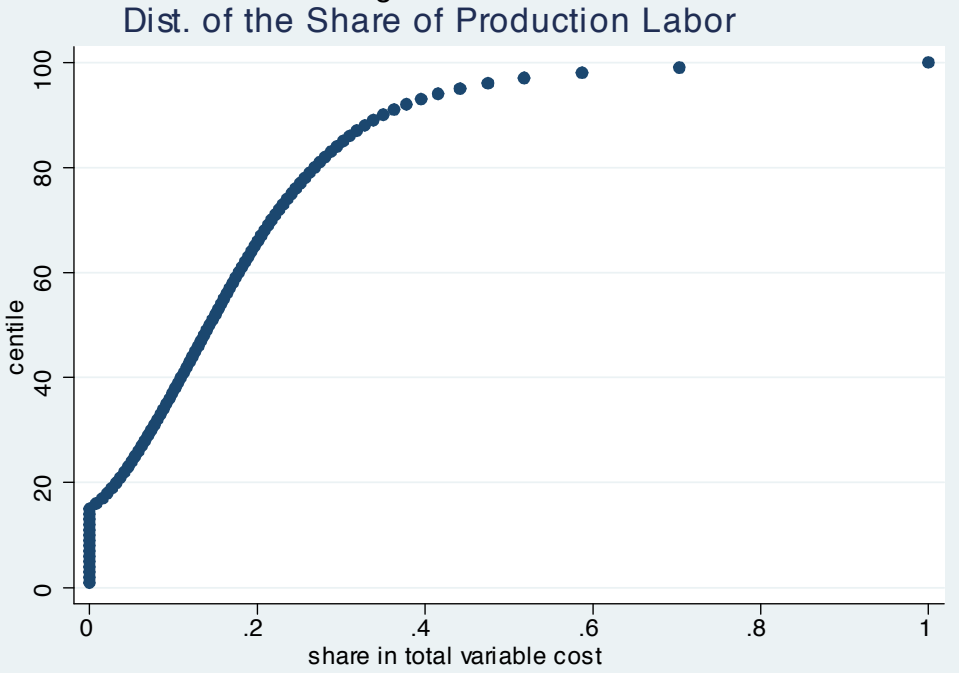


Figure 4


Dist. of the Share of Production Labor
centile
0
20
40
60
80
100
0
.2
.4
.6
.8
1
share in total variable cost