# Currency Wars and Trade Wars: A Welfare Analysis

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**Abstract**

I present a simple model of a global economy in which countries attempt to boost their employment and welfare by depreciating their currencies and making their goods more competitive in export markets—a "currency war"—or by imposing a tariff on imports—a "trade war." The global economy could be at full employment, or in a liquidity trap with less than full employment. The welfare impact of currency and trade wars depends on whether there is a liquidity trap and on the policy instruments that are used in the currency war (interest rate, reserve accumulation or inflation target). International coordination is generally warranted for trade policy but not for exchange rate policy. It is especially beneficial to avoid a trade war in a global liquidity trap.

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# 1 Introduction

Countries have regularly accused each other of being aggressors in a currency war since the global financial crisis. Guido Mantega, Brazil's finance minister, in 2010 accused the US of launching "currency wars" through quantitative easing and a lower dollar. "We're in the midst of an international currency war, a general weakening of currency. This threatens us because it takes away our competitiveness."[1] At the time Brazil itself was trying to hold its currency down by accumulating reserves and by imposing a tax on capital inflows. Many countries, including advanced economies such as Switzerland, have depreciated or resisted the appreciation of their currency by resorting to foreign exchange interventions. The phrase "currency war" was again used when the Japanese yen depreciated in 2013 after the Bank of Japan increased its inflation target (and more recently when it reduced the interest rate to a negative level). Bergsten and Gagnon (2012) propose that the US undertake countervailing currency intervention against countries that manipulate their currencies, or tax the earnings on the dollar assets of these countries. The election of Donald Trump added to these concerns that of a tariff war initiated by the US.

While G20 countries have regularly renewed their pledge to avoid depreciating their currencies to gain a competitive trading advantage, they have also implemented stimulatives policies that often led to depreciation. Bernanke (2015) argues that this situation should not raise concerns about currency wars as long as the depreciations are the by-product, rather than the main objective, of monetary stimulus (see also Blanchard (2016)). Mishra and Rajan (2016) find the international spillovers from monetary and exchange rate policies less benign and advocate enhanced international coordination to limit the effects of these spillovers.

The concepts of currency war and trade war are old ones but we do not have many models to analyze these wars, separately or as concurrent phenomena (more on this in the discussion of the literature below). In this paper I present a simple model in which an open economy can increase its employment and welfare by depreciating its currency and making its goods more

[1] Reported by Martin Wolf in "Currencies Clash in New Age of Beggar-thy-Neighbor," Financial Times September 28, 2010.

competitive in exports markets. I consider a symmetric world with many identical countries, each one producing its own good like in Gali and Monacelli (2005). There is downward nominal stickiness in wages like in Schmitt-Grohé and Uribe (2016). I characterize the Nash equilibria in exchange rate and trade policies and explore the case for international coordination. The main qualities that I look for in the model are tractability and analytical transparency but the model can be used to quantify the size of the effects, and in particular the welfare cost of currency and trade wars.

The main results crucially depend on whether global demand is sufficient to ensure full employment at the global level. If there is full employment at the global there is no need for international monetary cooperation but it is beneficial to agree on not using tariffs. Exchange rate policy is used to achieve full employment and tariffs are used to manipulate the terms of trade. Individual countries are not tempted to engage in a currency war since there is full employment. The temptation, rather, is for each country to manipulate the terms of trade in its favor by appreciating its currency while maintaining full employment with a tariff on imports. The outcome, in general equilibrium, is that international trade is inefficiently low and consumption is distorted towards the home good, like in the textbook model of tariff war. Under plausible calibrations the welfare cost of these trade wars is equivalent to a permanent decrease in consumption of one or two percent (significant but not overwhelming).

The results are different when global demand is insufficient. This scenario is obtained by increasing the discount factor of the representative consumer up to a point where the global economy falls in a liquidity trap. The nominal interest rate is at the zero bound and there is unemployment in all countries. Each country is tempted to boost its own employment by increasing its share in global demand but the collective implications of such beggar-thy-neighbor policies crucially depend on which policy instrument is used. There is no benefit from coordinating conventional monetary policy. There is also no benefit from coordinating unconventional monetary policy that manipulates the inflation target. In fact, the Nash equilibrium in which each country sets its inflation target competitively leads to full employment as it relaxes the zero bound constraint on the real interest rate.

The case for international coordination is the strongest when it comes to prevent a tariff war in a global liquidity trap. In a liquidity trap countries use tariffs in order to reach employment objectives. Each country finds it optimal to impose a tariff in order to switch domestic demand away from

imports and towards the home good to increase domestic employment. This makes sense in partial equilibrium but decreases each country's contribution to global demand. Global demand and employment are lower in the Nash equilibrium with tariffs. The welfare impact of a tariff war can be substantial: under my benchmark calibration the unemployment rate is increased from 10 percent to about 18 percent.

I also look at the case where countries can depreciate their currencies by restricting capital inflows and accumulating reserves (still in the case of a global liquidity trap), a situation that has been called a "capital war." I find that under my benchmark calibration a capital war leads to endogenous symmetry-breaking. A fraction of countries accumulate foreign assets to achieve a trade surplus and full employment, whereas the other countries accept a trade deficit and less than full employment. The welfare of deficit countries, however, is the same as that os surplus countries because they can borrow at a very low cost while the surplus countries receive a very low return on their external assets. Furthermore, global welfare is slightly increased by the capital war.

**Literature [sketch].** There is a long line of literature on international monetary coordination—see e.g. Engel (2016) for a review.

The case for international monetary cooperation has been studied in the New Open Macro literature by Obstfeld and Rogoff (2002), Benigno and Benigno (2006), Canzoneri, Cumby and Diba (2005) among others. These models generally focus on two countries with monetary policy rules responding to domestic productivity shocks. This line of literature has concluded that the welfare cost of domestically-oriented rules is small, in the sense that there are little welfare gains from granting monetary policy to a global social planner who maximize global welfare. However these models do not seem to capture the deviations from status-quo policies that seem to occur in times of currency wars.

Fanelli and Straub (2016) present a model in which countries can use foreign exchange interventions to affect their terms of trade. A two-period version of their model features a Nash equilibrium between advanced economies and emerging market economies where the latter accumulate reserves to depreciate their real exchange rate. This equilibrium is inefficient and there is scope for international coordination to reduce reserve accumulation.

There is an old literature about the equivalence between currency policy and exchange rate policy going back at least to Meade (1955). Equivalence

between a currency devaluation and tariff-cum-subsidy. Staiger and Sykes (2010) discuss this equivalence in the context of the recent debate about China's alleged manipulation of its exchange rate.

A more recent group of papers has explored the international spillovers associated with monetary policy when low natural rates of interest lead to insufficient global demand and liquidity traps: Eggertsson et al. (2016), Caballero, Farhi and Gourinchas (2015), Fujiwara et al. (2013), Devereux and Yetman (2014), Cook and Devereux (2013), and Acharya and Bengui (2016). This paper shares some themes with that literature, in particular the international contagion in the conditions leading to a liquidity trap. Eggertsson et al. (2016) and Caballero, Farhi and Gourinchas (2015) study the international transmission of liquidity traps using a model that shares several features with this paper, in particular the downward nominal stickiness. The main difference is that their models have overlapping generations and permanent liquidity traps. The fact that liquidity trap is permanent leads to indeterminacy in the exchange rate and the international allocation of economic activity. The overlapping generations give a more powerful role to fiscal policy but make the welfare analysis more difficult. Eggertsson et al. (2016) assume one good (and so no terms of trade).

The paper uses the same kind of nominal stickiness as Schmitt-Grohé and Uribe (2016). The nominal wage is rigid downwards and the failure of the nominal wage to adjust may lead to unemployment. Schmitt-Grohé and Uribe (2016) study an open-economy model with tradable and nontradable goods. I look instead at a model with a continuum of country-specific goods a la Gali-Monacelli. This allows me to talk about the terms of trade and competitiveness in a way that seems relevant for currency wars.

The paper is related to and Korinek (2016). That paper presents a unified framework for analyzing whether international spillovers call for international policy coordination. Korinek gives a set of conditions under which the spillovers are efficient and coordination is uncalled for. My model does not satisfy one condition—that countries do not have monopoly power—since each country tries to manipulate its terms of trade.

# 2 Assumptions

The model is in infinite discrete time $t = 1, 2, ...$ It represents a world composed of a continuum of atomistic countries indexed by $j \in (0, 1)$. Each

country produces its own good, has its own currency and implements an independent monetary policy. The goods structure is similar to Gali and Monacelli (2005). The nominal wage is rigid downwards as in Schmitt-Grohé and Uribe (2016, 2017). There is no uncertainty.

Each country is populated by a mass of identical consumers. I first describe the preferences of the representative consumer and drop the country index to alleviate notations.

**Preferences.** The utility of the representative consumer is defined recursively by,

$$U_t = u\left(C_t\right) + \beta_t U_{t+1}, \tag{1}$$

where $C_t$ is consumption. The discount factor is time-varying and will be used to generate changes in demand.

The consumer consumes the good that is produced domestically (the home good) as well as a basket of foreign goods. The consumer cares about the CES index,

$$C = \left[\alpha_H^{1/\eta} C_H^{(\eta-1)/\eta} + \alpha_F^{1/\eta} C_F^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)},$$

(with $\alpha_H + \alpha_F = 1$) where $C_H$ is the consumption of home good, and $C_F$ is the consumption of foreign good. The consumption of foreign good is a CES index of the goods produced in all the countries,

$$C_F = \left[\int_0^1 C_k^{(\gamma-1)/\gamma} dk\right]^{\gamma/(\gamma-1)}.$$

Imports are defined by the same index for all countries. This index defines what I call the "global good" in the following. The elasticity of substitution between foreign goods will be assumed to be larger than one, $\gamma > 1$.

**Production and labor market.** The home good is produced with a linear production function that transforms one unit of labor into one unit of good, $Y_t = L_t$. Like in Schmitt-Grohé and Uribe (2016) or Eggertsson et al. (2016) the representative consumer is endowed with a fixed supply of labor $\overline{L}$

$$L_t \leq \overline{L}. \tag{2}$$

There is full employment if this constraint is satisfied as an equality.

The nominal wage in period $t$ is denoted by $W_t$ and the gross inflation rate in the nominal wage (or home good price) is

$$\Pi_t = \frac{W_t}{W_{t-1}}.$$

Downward nominal stickiness in the wage is captured by the constraint,

$$\Pi_t \geq \underline{\Pi}. \tag{3}$$

The economy can be in two regimes: full employment ($L_t = \overline{L}$), or less than full employment, in which case the nominal wage is at its lowest possible level ($L_t < \overline{L}$ and $\Pi_t = \underline{\Pi}$). The constraints on the labor market can be summarized by (2), (3) and

$$\left(\overline{L} - L_t\right)\left(\Pi_t - \underline{\Pi}\right) = 0. \tag{4}$$

This leads to a kinked Phillips curve where inflation becomes neutral once there is full employment.

**Budget constraints.** Consumers can trade one-period bonds denominated in global consumption good. The budget constraint of the consumer is

$$E_t \frac{B_{t+1}}{1+r_t} + W_t C_{Ht} + \left(1+\tau_t^m\right) E_t C_{Ft} = W_t L_t + E_t B_t + Z_t, \tag{5}$$

where $E_t$ denotes the offshore domestic-currency price of the global good, $\tau_t^m$ is a tax on imports, $B_t$ is the payoff on the bonds accumulated in $t-1$, $r_t$ is the real interest rate in terms of the global good, and $Z_t$ is the lump-sum rebate of the proceeds of the trade taxes. I have used the fact that the price of the home good is equal to the wage because of the linearity in the production function.

The demand for home labor is,

$$L_t = C_{Ht} + \left[\left(1+\tau_t^x\right)\frac{W_t}{E_t}\right]^{-\gamma} C_{Ft}^W, \tag{6}$$

where $C_F^W = \int C_{Fk} dk$ is global gross imports and $\tau_t^x$ is the tax on exports. The first term is the labor to produce home consumption of home good and the second term is the labor used to produce exports. There is a tax on exports $\tau_t^x$, so that the home-currency price of the home good is $\left(1+\tau_t^x\right) W_t$ in export markets.

It will be convenient to define three terms of trade,

$$S_t \equiv \frac{W_t}{E_t},\ S_t^m \equiv \frac{S_t}{1+\tau_t^m} \text{ and } S_t^x \equiv (1+\tau_t^x)\, S_t, \tag{7}$$

where $S_t$ denotes the undistorted terms of trade, and $S_t^m$ and $S_t^x$ are the tax-distorted terms of trade that apply to the home imports and exports respectively. The home demand for the home good and for imports are respectively given by,

$$C_{Ht} = \alpha_H \left[\frac{S_t^m}{p\left(S_t^m\right)}\right]^{-\eta} C_t, \tag{8}$$

$$C_{Ft} = \alpha_F \left[\frac{1}{p\left(S_t^m\right)}\right]^{-\eta} C_t, \tag{9}$$

where $p(S^m) = \left[\alpha_H \left(S^m\right)^{1-\eta} + \alpha_F\right]^{1/(1-\eta)}$ is the price of home consumption in terms of the global good. Using $Z_t = \tau_t^m E_t C_{Ft} + \tau_t^x W_t \left(L_t - C_{Ht}\right)$ and equations (6), (7), (8), and (9) to substitute out $Z_t$, $L_t$, $C_{Ht}$, $C_{Ft}$, $W_t$ and $E_t$ from the representative consumer's budget constraint (5) gives

$$B_{t+1} = (1+r_t)\left(B_t - M_t\right), \tag{10}$$

$$M_t = \alpha_F p(S_t^m)^{\eta} C_t - \left(S_t^x\right)^{1-\gamma} C_{Ft}^W. \tag{11}$$

$M_t$ denotes the country's net imports in terms of global good. Equation (10) is the balance-of-payments identity. Equation (11) gives net imports as a function of its determinants. Net imports increase with domestic consumption, $C_t$, and decrease with global consumption, $C_{Ft}^W$. Net imports increase if the country loses competitiveness on the side of exports (an increase in $S_t^x$) or on the side of imports (an increase in $S_t^m$).

Finally, the demand for home labor (6) can be re-written in terms of the same determinants as for net imports,

$$L_t = \left(S_t^x\right)^{-\gamma} C_{Ft}^W + \alpha_H \left[\frac{p\left(S_t^m\right)}{S_t^m}\right]^{\eta} C_t. \tag{12}$$

The demand for home labor increases with home and global consumptions but is reduced by a loss in competitiveness.

**Calibration.** I adopt the following specifications

$$u\left(C\right) = C^{1-1/\sigma} / \left(1 - 1/\sigma\right)$$

The parameter values that will be used in quantitative illustration are reported in Table 1. The discount rate is set to a level that implies a steady-state real interest rate of 4 percent (a period is a year). The elasticity of intertemporal substitution of consumption, $\sigma$, is set to 0.5, a standard value in the literature. The labor endowment is normalized to 1. The values for the elasticity of substitution between home and foreign goods, $\eta$, and between foreign goods, $\gamma$, is consistent with Feenstra et al. (2014). There is moderate home bias.

**Table 1. Benchmark calibration**

| $\beta$ | $\sigma$ | $\overline{L}$ | $\eta$ | $\gamma$ | $\alpha_H$ |
|---|---|---|---|---|---|
| $1.04^{-1}$ | 0.5 | 1 | 1 | 3 | 0.6 |

# 3 Policies

Domestic social planner. There are three policy areas: exchange rate (or monetary) policy, capital account policy (or capital controls), and trade policy. I assume that the social planner might want to round the corners of the trilemma.

This section first specifies the policy instruments available to a small open economy and show that they are well-defined in the sense that they determine one unique allocation. The second part of this section establishes equivalence results between exchange rate policy and trade policy.

## 3.1 Policy instruments

The policies of country $j$ are characterized by a set of paths $(E_{jt}, B_{jt}, \tau^m_{jt}, \tau^x_{jt})_{t\geq 1}$. The first instrument, $E_{jt}$, implements exchange rate policy. The second instrument, $B_{jt}$, corresponds to capital account policy, and trade policy consists in the trade taxes. I now give more details about each policy in turn.

- **Exchange rate policy** $(E_{jt})_{t\geq 1}$**.** Exchange rate policy consists in setting the path for the domestic-currency price of the global good. By the law of one price for the global good the bilateral exchange rate

between currency $j$ and currency $k$ (the price of currency $k$ in terms of currency $j$) is given by[2]

$$E_{jk} = \frac{E_j}{E_k}.$$

Other things equal increasing $E_j$ depreciates the domestic currency against all other currencies by the same amount. It is the sense in which setting $E_{jt}$ can be interpreted as exchange rate policy. By a slight abuse of language I will sometimes call $E_{jt}$ the exchange rate for country $j$. It is natural to characterize monetary policy in this way (rather than, say, with a Taylor rule) because the paper focuses on currency wars. Note that there is a difference between an exchange rate *policy*, which is a particular path for the exchange rate, and an exchange rate *regime*, which specifies how the exchange rate path responds to changes in the economic conditions. We will say that the country has a floating exchange rate regime if it freely adjusts the exchange rate path $(E_{jt})_{t\geq 1}$to changes in the economic conditions.

- **Capital account policy** $(B_{jt})_{t\geq 1}$**.** The domestic social planner sets the country's level of foreign assets $B_{jt}$. This could be interpreted for example as a Chinese-style capital account regime (before the recent liberalization) in which the government monopolizes financial transactions with the rest of the world and foreign assets are mostly composed of reserves (Jeanne, 2013).

  Alternatively the capital controls imposed by the domestic social planner could take the form of a tax $\tau^b$ on capital inflows. With such a tax the intertemporal allocation of domestic consumption leads to the Euler equation,

$$\frac{u'(C_{jt})}{p(S_{jt})} = \beta_t (1 + r_t) \left(1 + \tau^b_{jt}\right) \frac{u'(C_{jt+1})}{p(S_{jt+1})}. \tag{13}$$

  Free capital mobility is the special case where the tax is always set to zero. These two ways of specifying capital account policy are equivalent. Any path $(B_{jt})_{t\geq 1}$ can be implemented with a tax $(\tau^b_{jt})_{t\geq 1}$and conversely any tax path corresponds to a foreign asset path. I specify policy in terms of net foreign assets because this is more tractable

[2]The law of one price does not involve the trade taxes because the same taxes are applied independently of the currency in which trade is invoiced.

but most of the results are easy to transpose to the case where capital controls are tax-based.

- **Trade policy** $(\tau_{jt}^m, \tau_{jt}^x)_{t\geq 1}$**.** Trade policy consists of tariffs on imports and on exports.

The next step is to check that policy is well specified in the sense that a given policy path leads to one unique allocation. The path for foreign assets implies a path for net imports $(M_{jt})_{t\geq 1}$ since $M_{jt} = B_{jt} - B_{jt+1}/(1+r_t)$. Then using (11) to substitute out $C_{jt}$ from (12) one obtains an expression for labor demand in terms of global imports, the country's imports and the terms of trade,

$$L_{jt} = \frac{\left(S_{jt}^x\right)^{-\gamma}\left[\alpha_F + \alpha_H S_{jt}^x \left(S_{jt}^m\right)^{-\eta}\right] C_{Ft}^W + \alpha_H \left(S_{jt}^m\right)^{-\eta} M_{jt}}{\alpha_F}. \tag{14}$$

Another important relationship links the nominal exchange rate, inflation and the terms of trade,

$$\Pi_{jt} = \frac{E_{jt}}{E_{jt-1}} \frac{S_{jt}}{S_{jt-1}}. \tag{15}$$

The difference between the rate of inflation in the home good and the rate of inflation in the global good (both in terms of home currency) is the rate of increase in the terms of trade, or the rate of decrease in home competitiveness.

The equilibrium allocation can be derived by iterating on equations (14) and (15) starting from initial conditions $E_{j0}$ and $S_{j0}$. The question in each period $t \geq 1$ is whether the rate of nominal depreciation is sufficient to ensure full employment. The iteration in the first period runs as follows (the following iterations are similar). Given $\tau_{j1}^m$ and $\tau_{j1}^x$, equation (14) determines the terms of trade $\widetilde{S}_{j1}$ that are consistent with full employment ($L_{j1} = \overline{L}$). Given $E_{j0}$ and $S_{j0}$ from the previous period, and the exchange rate $E_{j1}$, plugging $\widetilde{S}_{j1}$ into (15) gives the inflation rate conditional on full employment, $\widetilde{\Pi}_{j1} = \frac{E_{j1}}{E_{jt0}} \frac{\widetilde{S}_{j1}}{S_{j0}}$. If this inflation rate is larger than $\underline{\Pi}$ there is full employment, $S_{j1} = \widetilde{S}_{j1}$ and $\Pi_{j1} = \widetilde{\Pi}_{j1}$. If the implied inflation rate is strictly smaller than $\underline{\Pi}$ then $\Pi_{j1} = \underline{\Pi}$, and the terms of trade are given by $S_{j1} = \underline{\Pi} \frac{E_{j0}}{E_{j1}} S_{j0} > \widetilde{S}_{j1}$ and there is less than full employment. The level of consumption $C_{j1}$ can then be derived from (11). Iterating in this way it is possible to derive the whole allocation $(C_{jt})_{t\geq 1}$, $(S_{jt})_{t\geq 1}$, $(L_{jt})_{t\geq 1}$ and $(\Pi_{jt})_{t\geq 1}$.

**Proposition 1** *For any policy path* $(E_{jt}, B_{jt}, \tau^m_{jt}, \tau^x_{jt})_{t\geq 1}$ *there is at most one set of paths* $(C_{jt})_{t\geq 1}$, $(S_{jt})_{t\geq 1}$, $(L_{jt})_{t\geq 1}$, $(\Pi_{jt})_{t\geq 1}$ *and* $(M_{jt})_{t\geq 1}$ *satisfying the equilibrium conditions (2), (3), (4), (10), (11), (12) and (15).*

**Proof.** See discussion above. ■

This proposition ensures that our policy instruments are well-defined. Any policy path is associated with one unique allocation. One can show that the same proposition applies if capital account policy is specified in terms of a tax on capital flows $\tau^b_{jt}$ instead of the net foreign assets (see appendix).

## 3.2 Equivalence between exchange rate policy and trade policy

As mentioned in the introduction a long-standing question in the macroeconomic and trade literature is that of the conditions under which exchange rate manipulation can replicate the impact of tariffs. The relationship between trade policy and exchange rate policy is clarified in the following proposition.

**Proposition 2** *Any allocation* $(C_{jt})_{t\geq 1}$, $(L_{jt})_{t\geq 1}$, $(\Pi_{jt})_{t\geq 1}$ *achieved by policy* $\left(E_{jt}, B_{jt}, \tau^m_{jt}, \tau^x_{jt}\right)_{t\geq 1}$ *can also be achieved without export tax by policy* $\left(E'_{jt}, B_{jt}, \tau^{m\prime}_{jt}, 0\right)_{t\geq 1}$ *with* $E'_{jt} = E_{jt}/\left(1+\tau^x_{jt}\right)$ *and* $1+\tau^{m\prime}_{jt} = \left(1+\tau^m_{jt}\right)\left(1+\tau^x_{jt}\right)$.

**Proof.** See appendix ■

The fact that a tax on imports has the same impact as a tax on exports is known as Lerner's symmetry theorem in the trade literature (Lerner, 1936). Real allocations are determined by the tax-distorted prices of the home good in terms of global good both in export markets and in the domestic market. Real allocations are left unchanged if a decrease in the export tax is perfectly offset by an increase in the domestic wage in terms of global good coupled with an increase in the import tax of the same amount. In order to keep domestic inflation at the same level the real appreciation must come from a nominal appreciation.

The proposition allows us to identify the conditions under which trade policy and exchange rate policy are equivalent (Meade, 1955). Consider the trade policies that subsidize exports at the same rate as they tax imports, that is

$$\left(1+\tau^m_{jt}\right)\left(1+\tau^x_{jt}\right) = 1. \tag{16}$$

This implies that $S_t^m = S_t^x$, i.e., the trade taxes induce the same distortion for imports and exports. Then Proposition 2 implies that the impact of the trade policy on real allocations can be replicated, without any trade tax, by the exchange rate policy $E'_{jt} = \left(1 + \tau_{jt}^m\right) E_{jt}$. That is, there is an equivalence between exchange rate policy and trade policy provided that the latter belongs to the class of policies that introduce the same price distortion in domestic and exports markets. There is no equivalence between trade policy and exchange rate policy in general.

Another implication of Proposition 2 is that one of the policy instruments is redundant. If the country has a floating exchange rate regime it does not need to have taxes on both imports and exports. Hereafter I will assume that the social planner does not tax exports[3]

$$\tau_{jt}^x = 0.$$

From now on policy will be defined as a set of paths $(E_{jt}, B_{jt}, \tau_{jt})_{t\geq 1}$, where I drop the subscript $m$ on the tax on imports to alleviate the notations.

# 4 Optimal policy in a small open economy

Assume that policy is set by a benevolent social planner who takes the global economic conditions $\left(C_{Ft}^W, r_t\right)_{t\geq 1}$ as given. I assume that the social planner maximizes the welfare function,

$$\widetilde{U}_t = u\left(C_t\right) - \varepsilon\left(\Pi_t - \Pi^*\right)^2 + \beta_t \widetilde{U}_{t+1}. \tag{17}$$

This is the same as (1) except for the term $\varepsilon f\left(\Pi_t - \Pi^*\right)^2$, which captures the cost of deviating from an inflation target $\Pi^*$. This term is added to make inflation determinate under full employment. As we will see the policy chosen by the social planner does not depend on the value of $\varepsilon$ so that we can assume that $\varepsilon$ is vanishingly small.[4]

The domestic social planner's preferred policy is a path $(E_{jt}, B_{jt}, \tau_{jt})_{t\geq 1}$ that maximizes the objective (17) given the current and expected global conditions.

[3]Real-world policymakers seem more inclined to impose taxes on imports than on exports.

[4]As long as there is home bias in consumption, one could assume, without essential change to the results but at the cost of complications, that the social planner's objective depends on CPI inflation rather than inflation in the price of the home good.

The problem of the social planner can be simplified by defining her indirect utility function in terms of net imports. The social planner's indirect utility is the solution to the following problem,

$$(P)\begin{cases} \widetilde{u}(M) = \max_{S,S^m,\Pi} u(C) - \varepsilon(\Pi - \Pi^*)^2, \\ C = \left(S^{1-\gamma}C_F^W + M\right)/\left(\alpha_F p(S^m)^\eta\right), \\ L = S^{-\gamma}C_F^W + \alpha_H\left[\frac{p(S^m)}{S^m}\right]^\eta C \leq \overline{L}, \\ \Pi \geq \underline{\Pi}, \\ \left(\overline{L} - L\right)(\Pi - \underline{\Pi}) = 0. \end{cases}$$

The constraints correspond to equations (11), (12), (3) and (4) respectively. We have taken a shortcut by assuming that the social planner sets the endogenous variables $S, S^m$, and $\Pi$ rather than the exogenous policy instruments $E$ and $\tau$.[5] It is easy to see that the social planner's problem $(P)$ is solved by achieving full employment and the inflation target. If $L < \overline{L}$ it is possible to reduce the terms of trade $S$, which increases employment, consumption and welfare. The assumption $\gamma > 1$ ensures that the value of the country's exports in terms of global good increases with the volume of exports. Otherwise the country would manipulate its terms of trade by reducing employment. Given full employment, it is then trivial that inflation $\Pi$ should be set to the target $\Pi^*$.

The social planner then sets the terms of $S$ and $S^m$ so as maximize consumption $C$ subject to full employment, $L = \overline{L}$. A simple computation shows that this implies

$$\frac{S^m}{S} = \frac{\gamma - 1}{\gamma},$$

or equivalently, that imports are taxed at rate $\tau = 1/(\gamma - 1)$. The social planner taxes imports so as to manipulate the terms of trade like in the classical optimal tariff literature.[6] Conditional on full employment, taxing imports

[5]There is a one-to-one mapping between the endogenous variables and the policy instruments. Given the previous period nominal wage, $W_{-1}$, and the current level of net imports, $M$, the social planner can determine whether the economy is at full employment or not by setting the exchange rate $E$ and the tax on imports $\tau$. An exchange rate level $E$ leads to unemployment if the implied terms of trade, $S = \underline{\Pi}W_{-1}/E$ and $S^m = S/(1+\tau)$, are consistent with $L < \overline{L}$. Conditional on unemployment, changing $E$ affects the terms of trade and the level of employment. Conditional on full employment, setting $E$ determines the rate of inflation $\Pi$.

[6]The formula for the optimal tax rate was originally derived by (Johnson, 1953) in a static model with balanced trade.

shifts domestic demand towards the home good, which reduces exports and increases the terms of trade. There is a simple allocation of instruments to objectives in this model: exchange rate policy is used to achieve full employment and the inflation target while trade policy is used to manipulate the terms of trade.

The social planner's problem can then be written in Bellman form

$$\begin{aligned} \widetilde{U}_t(B_t) &= \max_{M_t}\left[\widetilde{u}_t(M_t) + \beta_t \widetilde{U}_{t+1}(B_{t+1})\right], \\ \text{s.t. } B_{t+1} &= (1+r_t)(B_t - M_t). \end{aligned}$$

The time subscript on the utility function reflects that the social social planner's problem depends on global demand $C_F^W$, which is time-dependent in general. The dynamic problem is formally equivalent to that of an optimizing consumer in an open economy, with consumption being replaced by net imports. The solution is time consistent. My new results so far are summarized in the following proposition.

**Proposition 3** *The welfare-maximizing policy* $(E_{jt}, B_{jt}, \tau_{jt})_{t\geq 1}$ *ensures full employment in all periods and imposes a constant tax on imports,*

$$\forall t,\ L_{jt} = \overline{L},\ \Pi_{jt} = \Pi^*\ \textit{and}\ \tau_{jt} = 1/(\gamma - 1).$$

*The optimal policy is time consistent.*

**Proof.** See discussion above. ■

In general full employment requires a flexible exchange rate. For example country $j$ responds to a fall in global demand by improving the competitiveness of its exports, which requires an exchange rate depreciation. An important property of the model is that the social planner does not opt for free capital mobility even if there is a flexible exchange rate ("rounding the corners of the trilemma"). Contrary to Mundell's trilemma, the use of capital controls is not limited to the case where the country pegs the exchange rate.

This property comes from the fact that the social planner's marginal valuation of foreign assets is not in general the same as private agents' valuation. As shown in the appendix in the case $\tau = 0$, the social planner's valuation of foreign assets is given by

$$\widetilde{U}'(B) = \frac{u'(C)}{p(S)}\left(1 + S^{-\gamma} C_F^W \frac{\partial S}{\partial M}\right). \tag{18}$$

It is larger than the private valuation of foreign wealth, which is equal to $u'(C)/p(S)$. The social planner takes into account that conditional on full employment, higher net foreign wealth leads to higher terms of trade, $\partial S/\partial M > 0$. The case for capital controls comes from the fact that the wedge between the private valuation and the social valuation of net foreign wealth can fluctuate over time. The social planner taxes capital inflows in the periods when the private sector undervalues net foreign wealth more.

The optimal capital controls are small and will not play any role in the Nash equilibria considered in the rest of this paper (except in section 6.3). To illustrate, Figure 1 shows the behavior of the main variables of interest in response to a transitory decrease in global imports demand. The exogenous shock is a 10 percent transitory decrease in global demand in period 2, which is announced in period 1. It is assumed that the country has the same inflation target as the rest of the world so that absent the global demand shock the country's exchange rate would remain constant. The figure shows that the social planner responds to the negative demand shock by a transitory depreciation of the currency which lowers the terms of trade and limits the fall in net exports. The country also uses capital controls so as to facilitate depreciation in period 2 and limit depreciation in period 1, but the magnitude of the tax on ca;ital flows is very small.

# 5 Nash equilibria and the benefits of coordination

In this section we look at Nash equilibria in which each domestic social planner chooses his country's policy taking the other countries' policies as given. I focus on symmetric Nash equilibria (SNE) in which all countries have the same discount factors $(\beta_t)_{t\geq 1}$ and the same inititial conditions, although their inflation targets $\left(\Pi_j^*\right)_{j\in[0,1]}$ could be different. I characterize the policy areas in which there is scope for international policy coordination. The SNE is compared with the allocation implemented by a global social planner who maximizes the welfare of the representative country.

A Nash equilibrium (NE) is composed of (i) initial conditions $(B_{j1})$, $(E_{j0})$, $(S_{j0})$; (ii) paths for the global interest rate $(r_t)_{t\geq 1}$ and global imports $\left(C_{Ft}^W\right)_{t\geq 1}$; (iii) country policies $(E_{jt}, B_{jt}, \tau_{jt})_{t\geq 1}$; and (iii) allocations and prices $(C_{jt})_{t\geq 1}$, $(L_{jt})_{t\geq 1}$, $(\Pi_{jt})_{t\geq 1}$, $(S_{jt})_{t\geq 1}$ for all countries $j$ such that:

- markets clear: $\int B_{jt}dj = 0$ for all $t$ ,
- global imports are the sum of imports across countries, $C_{Ft}^{W} = \alpha_F \int p\left(S_{jt}\right)^{\eta} C_{jt}dj$, for all $t$
- country allocations satisfy the equilibrium conditions and maximize domestic welfare given $(r_t)_{t\geq 1}$ and $\left(C_{Ft}^{W}\right)_{t\geq 1}$.

A SNE then corresponds to the special case where all countries start with zero foreign assets, $B_{j1} = 0$. Symmetry implies that the terms of trade are equal to 1 for all countries, $\forall j, t \;\; S_{jt} = 1$. The following proposition states some properties of a SNE and compares it with the global social planner allocation.

**Proposition 4** *In a symmetric Nash equilibrium, all countries set the rate of exchange rate depreciation to their inflation targets, $E_{jt}/E_{jt-1} = \Pi_j^*$. Countries do not use capital controls but impose the same tariff $\tau_{jt} = 1/\left(\gamma - 1\right)$ on imports. A global social planner implements the same exchange rate and capital account policies as in the SNE but reduces the tax on imports to zero.*

**Proof.** Equation (15), with $S_{jt} = S_{jt-1} = 1$ and $\Pi_{jt} = \Pi_j^*$ by Proposition 3 imply $E_{jt}/E_{jt-1} = \Pi_j^*$ for all $j$ and $t$. Proposition 3 also implies that there is full employment in each country. The Euler equation for social planner $j$'s problem,

$$\widetilde{u}'\left(M_{jt}\right) = \beta_t\left(1 + r_t\right)\widetilde{u}'\left(M_{jt+1}\right),$$

where the indirect utility function $\widetilde{u}\left(\cdot\right)$ is defined by problem $(P)$. There is no time index on $\widetilde{u}\left(\cdot\right)$ because the global demand for imports $C_F^W$ is constant. In a symmetric equilibrium one has $M_{jt} = 0$ for all $j$ and $t$ so that the Euler equation implies

$$1 + r_t = \frac{1}{\beta_t}.$$

Then given that $C_{jt}$ and $S_{jt}$ are constant and do not depend on $t$, equation (13) implies $\tau_{jt}^{b} = 0$, i.e., there is free capital mobility. A global social planner that sets the policies of all countries maximizes the consumption of the representative country. This is achieved by full employment and setting the tax on imports to zero. ■

In a SNE, each country implements the policies that are optimal from the point of view of a small open economy as characterized in Proposition 3. As a result there is full employment, inflation is equal to the target and the optimal tariff is imposed on imports. In addition the national social planners do not impose capital controls, i.e., they do not interfere in the private agents' decisions to accumulate or deccumulate foreign assets. As we saw in the previous section the need to use capital controls comes from time variation in the wedge between the private valuation and the social valuation of foreign assets. In a SNE the global economy is in a steady state so that this wedge is constant.

The only area in which international policy coordination is beneficial is trade policy. The Nash equilibrium leads to a textbook tariff war in which all countries try to manipulate the terms of trade in their favor. This is impossible in general equilibrium and the only impact of the tariffs is to distort consumption.

The quantitative impact of a trade war can be assessed as follows. Under our benchmark calibration the equilibrium tariff is $\tau$ is equal to 50 percent. In a tariff war the consumption of home good increases from $0.6\overline{L}$ to $0.69\overline{L}$, the consumption of global good shrinks from $0.4\overline{L}$ to $0.31\overline{L}$, and total consumption falls from $\overline{L}$ to $0.98\overline{L}$. Thus the welfare loss from a tariff war (and the welfare gains from international coordination of trade policies) is equivalent to a two percent permanent gain in consumption.

# 6 Global liquidity trap

We have so far ignored the zero bound constraint (ZBC) on the nominal interest rate. This section reconsiders the case for coordinating exchange rate policy, trade policy and capital controls when this constraint is binding, i.e., when the economy is in a liquidity trap

First, let us determine the conditions under which the ZBC is binding in the SNE. Denoting the nominal interest rate by $i_t$, arbitrage between nominal bonds and real bonds (due to free capital mobility) mplies,

$$\begin{aligned} 1+i_t &= (1+r_t)\,\frac{E_{t+1}}{E_t}, \\ &= (1+r_t)\,\frac{W_{t+1}/S_{t+1}}{W_t/S_t}, \end{aligned} \tag{19}$$

or

$$S_{jt} = \frac{1 + i_{jt}}{1 + r_t} \frac{S_{jt+1}}{\Pi_{jt+1}}. \tag{20}$$

Given the terms of trade and inflation expected for the next period, the social planner can decrease the terms of trade (depreciate the home currency) by lowering the nominal interest rate. The extent of depreciation is limited however by the zero-bound constraint $i_t \geq 0$.

I now take into account the zero bound constraint (ZBC) on the nominal interest rate, $i_t \geq 0$. The Nash equilibrium of Proposition 4 does not necessarily satisfy the zero bound constraint since

$$1 + i_{jt} = (1 + r_t) \Pi_j^* = \frac{\Pi_j^*}{\beta_t}.$$

The ZBC cannot be satisfied in the SNE described in Proposition 4 if $\beta_t > \Pi_j^*$. In this case there is unemployment even if the social planner sets the interest rate at zero.

To simplify the analysis I will assume that all countries have the same inflation target $\Pi^*$ and that the ZBC is binding in the first period only, i.e.,

$$\beta_1 > \Pi^* > \beta_2 = \beta_3 = ...$$

From period $t = 2$ onwards the economy is in the steady state equilibrium with full employment and no tariff. I then consider the Nash equilibrium and the case for international policy coordination in period 1, when the global economy is in a liquidity trap. In section 6.1 it is assumed that countries can use exchange rate policy in an environment of free capital mobility and without trade taxes. I allow countries to use tariffs on imports in section 6.2. Finally, Section 6.3 looks at the case where countries may close the financial account and so are no longer constrained by interest parity in setting exchange rates.

## 6.1 Currency wars in a global liquidity trap

Consider a symmetric Nash equilibrium in which each country $j$ sets a time-consistent exchange rate path $(E_{jt})_{t \geq 1}$ taking the exchange rate policies of other countries as given. By equation (20) the period-1 terms of trade in country $j$ are given by

$$S_{j1} = \frac{1 + i_j}{1 + r_1} \frac{S_{j2}}{\Pi^*}, \tag{21}$$

where $i_j$ is the nominal interest rate in country $j$ and I have used the fact that the period-2 inflation rate is $\Pi^*$ since there is full employment in that period. The country can lower its terms of trade by reducing its nominal interest rate and depreciating its currency.

It is then possible to see that in a symmetric Nash equilibrium all countries have some unemployment and set their nominal interest rate to zero. In a symmetric equilibrium countries have the same level of employment and they cannot be all at full employment because this would imply $1+r_1 = 1/\beta_1$ and the nominal interest rate $i_1 = \Pi^*/\beta_1 - 1$ would not be positive. Given that all countries have unemployment they set their nominal interest rates to zero. Otherwise they could increase their employment and welfare by lowering their nominal interest rates. It then follows from setting $i_j = 0$ and $S_{j1} = S_{j2} = 1$ in equation (21) that the equilibrium real interest rate is given by

$$1 + r_1 = \frac{1}{\Pi^*},$$

and the Euler equation then implies that the global level of employment and consumption satisfy

$$u'(C_1) = \frac{\beta_1}{\Pi^*} u'\left(\overline{L}\right), \tag{22}$$

This equation indeed implies some unemployment: $C_1 = L_1 < \overline{L}$ since $\beta_1 > \Pi^*$.

The equilibrium might be described like a "currency war" in which each country depreciates its currency to make home-produced goods more competitive both at home and abroad. However there is no benefit from international policy coordination. A global social planner subject to the ZBC would also set the nominal interest rate to zero in all countries since this the level that maximizes global employment, consumption and welfare.

**Proposition 5** *(Currency war in a global liquidity trap) If $\beta_1 > \Pi^*$ in a symmetric Nash equilibrium the global economy falls in a liquidity trap in period 1, in which all countries have some unemployment and set their nominal interest rate to zero. There is no benefit from international policy coordination.*

Lowering the interest rate to depreciate the currency has an expenditure-switching effect and an expenditure-augmenting effect. When all countries reduce the interest rate by the same amount there is no expenditure switching

(since exchange rates do not change) but an increase in global expenditures and employment. This is efficient.

I have assumed that the exchange rate policy was time consistent but countries can increase their welfare by committing to a higher rate of inflation (or nominal depreciation) between period 1 and period 2. Given that the nominal interest rate is at zero, increasing inflation lowers the real interest rate. This is the benefit of "committing to being irresponsible" (Krugman).

**Proposition 6** *(Endogenous inflation targets) Assume that each country chooses its inflation target $\Pi_j^*$ in a global liquidity trap. In a symmetric Nash equilibrium all countries choose an inflation target larger than $\beta_1$ and there is full employment in all periods. There is no benefit from international policy coordination.*

International coordination has no benefits for the same reason as in the previous proposition. When all countries increase their inflation targets by the same amount, there is no expenditure-switching effect, only an expenditure augmenting effect which is efficient at the global level.

## 6.2 Trade wars

I now assume that in addition to setting the exchange rate (subject to the ZBC) all countries can tax imports at rate $\tau_j$ in a global liquidity trap. The tariffs are imposed only in period 1 and not afterwards. All the inflation targets are equal to the same level $\Pi^*$. The nominal interest rate is set to zero in all countries and there is still free capital mobility, implying that the real interest rate is given by $1 + r_1 = 1/\Pi^*$. The problem of country $j$ is

$$\begin{cases} \max_{S_{j1}, S_{j1}^m} u\left(C_{j1}\right) + \beta_1 U\left(B_{j2}\right) \\ B_{j2} = \frac{1}{\Pi^*}\left[S_{j1}^{1-\gamma} C_{F1}^W - \alpha_F p\left(S_{j1}^m\right)^\eta C_{j1}\right], \\ \frac{u'(C_{j1})}{p\left(S_{j1}^m\right)} S_{j1}^m = \frac{\beta_1}{\Pi^*} \frac{u'(C_{j2})}{p(S_{j2})}, \\ L_{j1} = S_{j1}^{-\gamma} C_{F1}^W + \alpha_H \left[\frac{p\left(S_{j1}^m\right)}{S_{j1}^m}\right]^\eta C_{j1} \leq \overline{L}, \\ S_{j1} \geq S_{j2}, \end{cases}$$

where $S_{j2}$ and $C_{j2}$ are policy functions of $B_{j2}$. The benefit of taxing imports (lowering $S_{j1}^m$) is that this raises the demand for labor and national income,

but the tax also depresses domestic consumption. The optimal tariff optimally trades off the benefit of higher employment against the cost of lower consumption.

In a symmetric equilibrium one must have $S_1 = S_2 = 1$ and $C_{j2} = \overline{L}$ so that the level of global consumption is given by,

$$u'(C_1) = \frac{\beta_1}{\Pi^*} p \left( \frac{1}{1+\tau} \right) (1+\tau) u'\left(\overline{L}\right).$$

Comparing with (22) one can see that global consumption is reduced by the tariff. The reason is that the tariff acts as a tax on period-1 consumption. Tariffs have another cost in addition to distorting consumption: they also reduce global demand and employment. In a symmetric equilibrium, therefore, all countries are made worse off by the use of tariffs.

Countries nevertheless use tariffs in a symmetric Nash equilibrium. The equilibrium tariff can be determined by computing the optimal tariff $\tau_j$ of country $j$ when all other countries apply tariff $\tau$. Increasing $\tau$ reduces global imports $C_{F1}^W$ and the demand for labor in country $j$. Country $j$ then need to increase its own tariff to maintain domestic employment but the marginal cost of repressing consumption becomes larger and larger as the global level of tariffs increases. The symmetric Nash equilibrium is achieved when $\tau_j = \tau$.

The impact of a tariff war can be quantitatively large. Under the benchmark calibration, when $\beta_1$ is at a level that leads to a global liquidity trap with an unemployment rate of 10 percent, the symmetric Nash equilibrium level of tariff is $\tau = 95\%$. As a result consumption is reduced from $C = 0.9\overline{L}$ to $0.79\overline{L}$ and the unemployment rate increases from 10 percent to 17.2 percent.

## 6.3 Capital wars

I now assume that there is no tariff but relax the assumption of free capital mobility. Countries may now close their financial accounts and use the additional freedom to devalue their currencies in period 1. From period 2 onwards the global economy is in a steady state where countries have no reason to restrict capital mobility. I thus focus on the impact of capital controls in period 1.

With a closed financial account a country can set its exchange rate $E_1$ independently of its nominal interest $i_1$. It can set its exchange rate $E_1$ as

high as it wants and set its terms of trade

$$S_1 = \frac{W_1}{E_1},$$

as low as it wants (as long as there is unemployment so that $W_1 = \underline{\Pi} W_0$ does not respond to the exchange rate). The nominal interest rate is independently set to zero.

The problem of country $j$ is

$$\begin{cases} \max_{S_{j1}} u\left(C_{j1}\right) + \beta_1 U\left(B_{j2}\right) \\ B_{j2} = \left(1 + r_1\right)\left[S_{j1}^{1-\gamma} C_{F1}^W - \alpha_F p\left(S_{j1}\right)^\eta C_{j1}\right], \\ \frac{u'(C_{j1})}{p(S_{j1})} S_{j1} = \frac{\beta_1}{\Pi^*} \frac{u'(C_{j2})}{p(S_{j2})}, \\ L_{j1} = S_{j1}^{-\gamma} C_{F1}^W + \alpha_H \left[\frac{p(S_{j1})}{S_{j1}}\right]^\eta C_{j1} \leq \overline{L}, \end{cases}$$

where, again, $S_{j2}$ and $C_{j2}$ are policy functions of $B_{j2}$.

It is interesting to compare this problem with that of the trade war. There is no wedge now between the terms of trade applying to imports and to exports: the same exchange rate applies to both markets. A depreciation has a larger impact on the level of activity than a tariff on imports because it stimulates exports. But it remains true that depreciating the currency and gaining competitiveness comes at the cost of repressing domestic consumption. This is an important difference with conventional monetary stimulus in which depreciating the currency through a lower interest rate also expands domestic demand. The currency depreciation that can be achieved by capital controls are more "mercantilist" than conventional monetary stimulus because they tend to repress domestic demand.

Another difference is that the gross real interest rate $1 + r_1$ is no longer equal to $1/\Pi^*$. The global real interest rate is determined by the equilibrium of the global financial market, i.e.

$$\int B_{j1} dj = 0.$$

[Preliminary summary of results follows.] A symmetric Nash equilibrium does not generally exist. This is because in a symmetric allocation individual countries have incentives to deviate and boost their levels of activity by a currency depreciation. There is an asymmetric Nash equilibrium. The global economy endogenously divides itself between two groups of countries. The

first group of countries (the surplus countries) choose to have full employment and a large trade surplus. The second group of countries (the deficit countries) has less than full employment and a trade deficit. The sizes of the two groups are endogenous. Countries have the same welfare in the two groups (since they are free to choose the group that they belong to). The deficit countries suffer from unemployment but benefit from borrowing at a low interest rate. The surplus countries have a high level of activity but a very poor return on their external savings.

Simulations show that welfare is slight higher for all countries in the capital war equilibrium than in the equilibrium without capital controls. Unlike the case of trade wars, there is no reason to prevent capital wars by international cooperation in this model.

The capital war equilibrium may be reminiscent of the complaints of surplus and deficit countries in the real world (think the US vs. China). It is important to remember though that in the model both surplus and deficit countries use capital controls. If only a fraction of countries use capital controls, these countries increase their welfare at the cost of the welfare of countries that do not use controls.

# 7 Conclusions

Directions for further research. It would be interesting to study fiscal policy in the context of this model. Because we have a model in which each country is populated by an infinitely-lived representative agent fiscal policy will have only a small impact.

## APPENDIX

**Proof of Proposition 1 when the social planner taxes capital flows.** Proposition states that a policy path determines one unique allocation when the instrument for the capital account policy is the level of net foreign assets. I show here that this is also true if the policy instrument is instead a tax on capital inflows.

I drop the country index to alleviate notations. The marginal utility from one unit of global consumption good is denoted by

$$\lambda_t \equiv \frac{C_t^{-1/\sigma}}{p\left(S_t^m\right)}.$$

Using the Euler equation (13) $\lambda_t$ satisfies,

$$\lambda_t = \beta_t \left(1 + r_t\right) \left(1 + \tau_t^b\right) \lambda_{t+1},$$

where $\tau_t^b$ is the tax on capital inflows. This implies

$$\lambda_t = \kappa_t \lambda_1,$$

where $\kappa_t = \left[\prod_{s=1}^{t-1} \beta_s \left(1 + r_s\right) \left(1 + \tau_s^b\right)\right]^{-1}$ for $t \geq 2$ is taken as given by the country. This equation defines the whole sequence $\left(\lambda_t\right)_{t>1}$ given the initial value $\lambda_1$.

The demand for labor (12) then implies

$$L_t = \left(S_t^x\right)^{-\gamma} C_{Ft}^W + \alpha_H \left[\frac{p\left(S_t^m\right)}{S_t^m}\right]^{\eta} \lambda_t^{-\sigma} p\left(S_t^m\right)^{-\sigma}. \tag{23}$$

Conditional on $\lambda_1$ and initial conditions $S_0$ and $E_0$ we can iterate on equation (15) to construct the paths $\left(S_t\right)_{t\geq 1}$, $\left(C_t\right)_{t\geq 1}$, $\left(\Pi_t\right)_{t\geq 1}$and $\left(L_t\right)_{t\geq 1}$. The iteration in period 1 goes as follows (the following iterations are similar). First, determine the terms of trade $\widetilde{S}_1$ such that there is full employment in period 1, i.e., such that $L_1 = \overline{L}$ in equation (23). Then use (15) to infer the inflation rate under full employment. If this inflation rate is higher than $\underline{\Pi}$, that is if $E_1 \widetilde{S}_1 \geq \underline{\Pi} W_0$, then $S_1 = \widetilde{S}_1$ and inflation is given by $\Pi_1 = E_1 \widetilde{S}_1 / W_0$. If this inflation rate is lower than $\underline{\Pi}$, that is if $E_1 \widetilde{S}_1 < \underline{\Pi} W_0$, then there is

some unemployment, the inflation rate is $\Pi_1 = \underline{\Pi}$ and by (15) the terms of trade are $S_1 = \underline{\Pi} W_0/E_1$. Since $S_1 > \widetilde{S}_1$ there is less than full employment and $L_1$ is given by (23). The consumption results from $C_t = \lambda_t^{-\sigma} p \left(S_t^m\right)^{-\sigma}$. By construction these paths satisfy conditions (2), (3), and (4).

The only thing left in order to prove the uniqueness of the allocation is to show that $\lambda_1$ is uniquely determined. This can be done by writing the intertemporal budget constraint,

$$B_1 = \sum_{t=0}^{+\infty} \frac{\lambda_1^{-\sigma} \alpha_F p \left(S_t^m\right)^{\eta-\sigma} \kappa_t^{-\sigma} - \left(S_t^x\right)^{1-\gamma} C_{Ft}^W}{R_t}, \tag{24}$$

where $R_1 = 1$ and $R_t = \left[\prod_{s=1}^{t-1} (1+r_s)\right]$ for $t \geq 2$ is the compounded interest factor. The right-hand side of this equation is strictly decreasing with $\lambda_1$ if $\sigma \leq \eta$. Assume $\lambda_1$ is replaced by $\lambda_1' > \lambda_1$, which raises $\lambda_t$ to $\lambda_t' > \lambda_t$ for all $t$. The iteration expression for the terms of trade is

$$S_t = \max\left(\widetilde{S}_t, \underline{\Pi} \frac{E_{t-1}}{E_t} S_{t-1}\right).$$

Increasing $\lambda_t$ lowers $\widetilde{S}_t$, and so $S_t$, for all $t$: $S_t' \leq S_t$. The r.h.s of (24) is lowered by raising $\lambda_1$. Hence there is at most one level of $\lambda_1$ satisfying the intertemporal budget constraint.

Conversely, one can derive a unique path $(E_t)_{t\geq 1}$ from $(S_t)_{t\geq 1}$, $(\Pi_t)_{t\geq 1}$ and the initial condition $E_0$ using (15).

**Proof of Proposition 2.** Let us denote with primes the endogenous paths associated with policy $\left(E_{jt}', B_{jt}, \tau_{jt}^{m\prime}, 0\right)_{t\geq 1}$. The implied paths $(C_{jt}, L_{jt}, \Pi_{jt})_{t\geq 1}$ and $\left(C_{jt}', L_{jt}', \Pi_{jt}'\right)_{t\geq 1}$ are derived through the iterative process described in the previous section. The time-$t$ iteration uses equation (14) to determine the levels of $S_{jt}$, $S_{jt}^x$ and $S_{jt}^m$ that are consistent with full employment, and equation (15) to check whether full employment is consistent with (3). We prove the proposition by showing that the two policy path imply the same allocation at each iteration $t$, except for the new undistorted terms of trade, given by

$$S_{jt}' = \left(1 + \tau_{jt}^x\right) S_{jt}.$$

The new policy path leaves $B_{jt}$ and so $M_{jt}$ unchanged. Using (7), $\tau_{jt}^{x\prime} = 0$ and $1 + \tau_{jt}^{m\prime} = \left(1 + \tau_{jt}^m\right)\left(1 + \tau_{jt}^x\right)$ one can see that the ratio of the terms of

trade relevant to exports and to imports are the same in the original policy path as in the new one,

$$\forall t,\ \frac{S_{jt}^{x\prime}}{S_{jt}^{m\prime}} = \frac{S_{jt}^{x\prime}}{S_{jt}^{m\prime}}.$$

It follows that the tax-distorted terms of trade implying full employment in equation (14) are the same with the two policies,

$$\widetilde{S}_{jt}^{m\prime} = \widetilde{S}_{jt}^{m} \text{ and } \widetilde{S}_{jt}^{x\prime} = \widetilde{S}_{jt}^{x}.$$

so that $\widetilde{S}'_{jt} = \widetilde{S}_{jt}^{x\prime} = \widetilde{S}_{jt}^{x} = \left(1+\tau_{jt}^{x}\right)\widetilde{S}_{jt}$. Using this equality the inflation rate consistent with full employment in the new policy is given by

$$\begin{aligned}
\widetilde{\Pi}'_{jt} &= \frac{E'_{jt}}{E'_{jt-1}}\frac{\widetilde{S}'_{jt}}{\widetilde{S}'_{jt-1}}, \\
&= \frac{E_{jt}}{E_{jt-1}}\frac{\widetilde{S}_{jt}}{\widetilde{S}_{jt-1}}, \\
&= \widetilde{\Pi}_{jt}.
\end{aligned}$$

The inflation rate is thus the same under the two policies conditional on full employment. If $\widetilde{\Pi}'_{jt} = \widetilde{\Pi}_{jt} < \underline{\Pi}$ then the inflation rates are again the same, $\Pi'_{jt} = \Pi_{jt} = \underline{\Pi}$, and there is less than full employment. Using $S'_{jt-1} = \left(1+\tau_{jt-1}^{x}\right)S_{jt-1}$ the new terms of trade are given by,

$$\begin{aligned}
S'_{jt} &= \underline{\Pi}\frac{E'_{jt-1}}{E'_{jt}}S'_{jt-1}, \\
&= \underline{\Pi}\frac{E_{jt-1}}{E'_{jt}}S_{jt-1}, \\
&= \left(1+\tau_{jt}^{x}\right)S_{jt}.
\end{aligned}$$

Hence $S_{jt}^{x\prime} = S'_{jt} = S_{jt}^{x}$ and $S_{jt}^{m\prime} = S'_{jt}/\left(1+\tau_{jt}^{m\prime}\right) = S_{jt}^{m}$. Since the trade-distorted terms of trade are the same under the two policies, it follows from (14) that employment is also the same, $L'_{jt} = L_{jt}$. Finally, it follows from (11) that consumption is the same under the two policies, $C'_{jt} = C_{jt}$.

**Proof of equation (18).** The social valuation of foreign assets is given by

$$\widetilde{U}'_{t}\left(B_{t}\right) = \widetilde{u}'_{t}\left(M_{t}\right) = u'\left(C_{t}\right)\frac{\partial C_{t}}{\partial M_{t}},$$

where, if $\tau_t = 0$ and $S_t^m = S_t$, the partial derivative can be computed from

$$C_t = \frac{\overline{L} S_t + M_t}{p\left(S_t\right)}.$$

This gives equation (18). Then we have

$$\frac{\partial S}{\partial M} = \frac{S^{\gamma}}{\eta M S^{\gamma-1} + \frac{\gamma}{\alpha_H}\left[\frac{p(S)}{S}\right]^{1-\eta} C_F^W - (1-\eta) C_F^W} > 0.$$

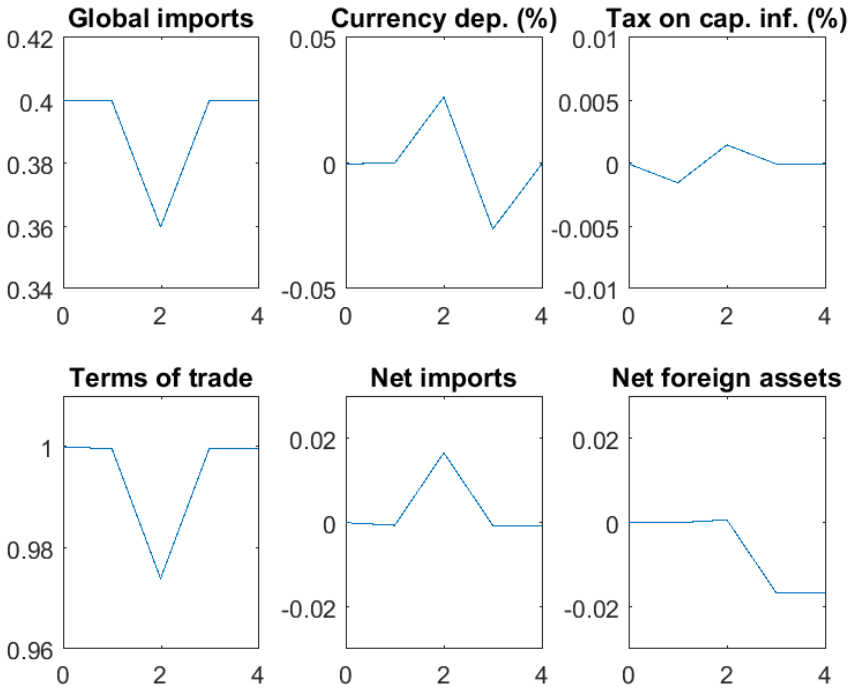



Figure 1: Optimal policy response to a temporary fall in global demand

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