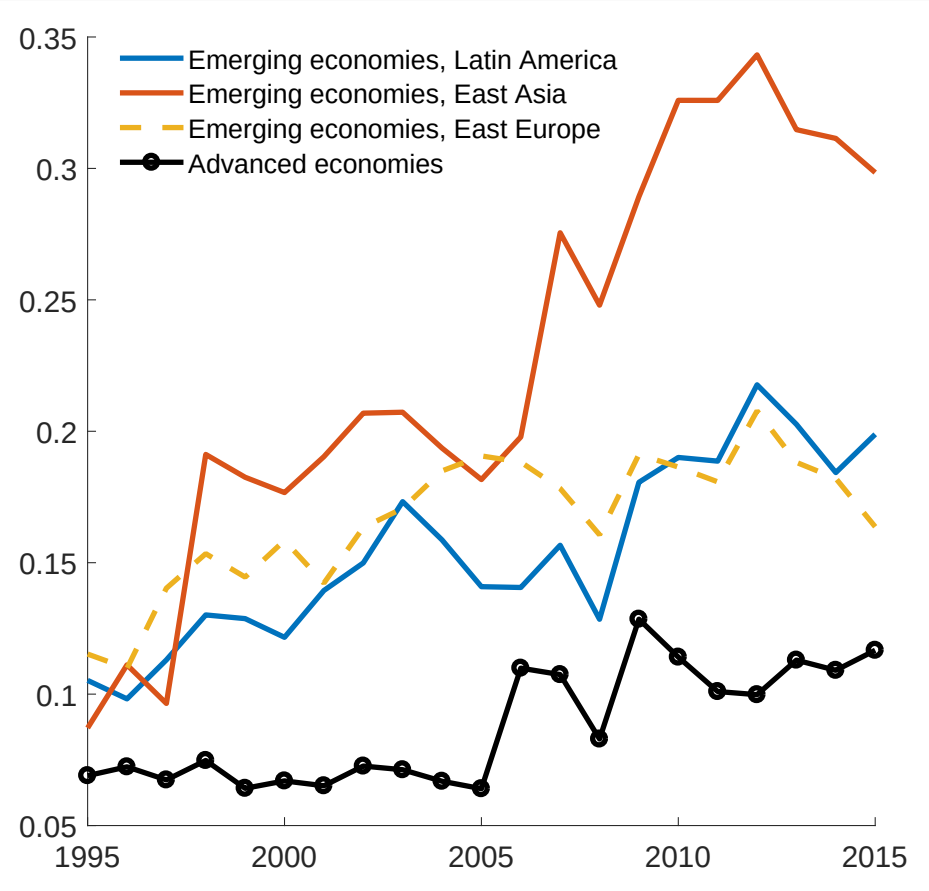
Foreign Reserve Management

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Motivation

Over the past 20 years massive increase in foreign reserves holdings by Central Banks around the world



Reserves-GDP

Motivation (ctd)

Why do countries hold foreign reserves? Two motives in the literature

1. *Precautionary role*: reserves used as a buffer for bad shocks
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This paper: Given exchange rates and monetary policy objectives, How should a Central Bank manage its reserve portfolio?

Preview: Interest Parity & Central Bank Balance Sheet

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  - If exchange rate policy is in conflict with ZLB, central bank has to absorb capital inflows and buy reserves
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- Today: A theory of which assets should central banks buy
  - Generalize ABBP 2017 with a **CB portfolio choice**, a broad monetary policy objective and financial structure

Preview: Main Results

Two goals:

- Improve allocation of consumption across time and states
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- If economy is relatively closed:
  - Policy: buy assets that pay when exchange rate appreciates
  - Key idea: increase *intratemporal* distortions to reduce *intertemporal* distortions
- If economy is relatively open:
  - Policy: make sure arbitrageurs hold domestic risk free bonds
  - Key idea: dispersion of arbitrage returns magnify losses

Framework

- Two-period model, $t \in \{1, 2\}$
  - Small open economy (central bank + households)
  - International Financial Market
  - Foreign Intermediaries
- Uncertainty realized at $t = 2$
  - $s \in S \equiv \{s_2, ..., s_N\}, \pi(s)$
- One (tradable) good, law of one price, foreign price normalized to 1

Asset markets: complete but segmented

International financial markets (IFM)

- Full set of Arrow-Debreu securities in foreign currency:
  - Security s: 1 unit of foreign currency in state s, 0 otherwise
  - Price $q(s)$ in terms of foreign currency at $t = 1$

Domestic financial market

- Full set of Arrow-Debreu securities in domestic currency
  - Security s: 1 unit of domestic currency in state s, 0 otherwise
  - Price $p(s)$ in terms of domestic currency at $t = 1$

Intermediaries

- Trade securities with SOE & IFM

Households

- Endowment: $(y_1, \{y_2(s)\})$, transfers: $(\{T_2(s)\})$
- Borrowing limits $\{\bar{f}^\star(s), \bar{a}^\star(s)\}$

$$\max_{c_1, \{c_2(s), a(s), f(s)\}} \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\}$$

subject to:

$$y_1 = c_1 + \sum_{s \in S} \left[q(s) f(s) + p(s) \frac{a(s)}{e_1} \right]$$

$$y_2(s) + T_2(s) + f(s) + \frac{a(s)}{e_2(s)} = c_2(s) \quad \forall s \in S$$

$$f(s) \geq -\bar{f}(s), \quad a(s) \geq -\bar{a}(s) e_2(s) \quad \forall s \in S$$

e_1, $e_2(s)$: exchange rates at $t = 1$ and $t = 2$

$f(s), a(s)$: holdings of foreign and domestic security s

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Foreign Intermediaries

- Endowed with capital $\bar{w}$
- Borrowing limits $\bar{f}^\star(s), \bar{a}^\star(s)$

$$\max_{\{c_1^\star, c_2^\star(s), a^\star(s), f^\star(s)\}} c_1^\star + \sum_{s\in S} \pi(s)\Lambda(s)c_2^\star(s)$$

subject to:

$$\bar{w} = c_1^\star + \sum_{s\in S} p(s)\frac{a^\star(s)}{e_1} + \sum_{s\in S} q(s)f^\star(s)$$

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Consider $\Lambda(s) = \frac{q(s)}{\pi(s)}$

Central Bank

- CB policy objective is taken as given: $(i, e_1, \{e_2(s)\})$

$$1 + i = \left(\sum_{s \in S} p(s) \right)^{-1} \qquad \text{(NIRC)}$$

- CB achieves its objective by managing its balance sheet: invest $\{A(s), F(s)\}$; and transfers $\{T_2(s)\}$ to households, subject to budget constraints.

$$\sum_{s \in S} p(s) \frac{A(s)}{e_1} + \sum_{s \in S} q(s) F(s) = 0$$

$$T_2(s) = \frac{A(s)}{e_2(s)} + F(s) \quad \forall s \in S$$

First-best allocation

First best allocation $(c_1^{fb}, \{c_2^{fb}(s)\})$ solves:

$$\max_{(c_1, \{c_2(s)\})} \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\}$$

subject to

$$y_1 - c_1 - \sum_{s \in S} q(s) c_2(s) = 0$$

Euler equation:

$$u'(c_1) q(s) = \beta \pi(s) u'(c_2(s))$$

Absent monetary policy constraints $\{i, e_1, e_2(s)\}$, CB can always implement this allocation (intermediary capital is irrelevant)

Characterizing equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_1}{e_2(s)p(s)}}{\frac{1}{q(s)}} - 1$$

$\kappa(s) > 0 \Rightarrow$ domestic security paying in state s yields higher return

- Households: borrow up to limit in foreign currency security and invest in domestic one.

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Intermediaries solution: invest all available funds in highest return security

Characterizing equilibria: Resource constraint

$$y_1 - c_1 + \sum_{s \in S} q(s)[c_2(s) - y_2(s)] - \sum_{s \in S} a^*(s) \frac{p(s)}{e_1} \kappa(s) = 0$$

Gains of Intermediaries are losses for the SOE

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Using investors' optimal portfolio, losses

$$\bar{\kappa} \left[\bar{w} + \sum_{s \in S} q(s) \bar{f}(s) \right] + \sum_{s \in S} \bar{a}^\star(s) e_2(s) \frac{p(s)}{e_1} (\bar{\kappa} - \kappa(s))$$

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$\Pi^\star$ encodes two potential sources of arbitrage losses for SOE

- Borrowing at high rates of return
- Buying assets with low rates of return

CB objective and $\Delta(i)$

Given CB objective $(i, e_1, \{e_2(s)\})$, consider the return differential between risk-free home and foreign bonds

$$\Delta(i) \equiv \mathbb{E}\left[\Lambda(s)\left(\frac{e_1}{e_2(s)}(1+i) - (1+i^\star)\right)\right]$$

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We study three regimes $(i, e_1, \{e_2(s)\})$

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Capital inflow regime more likely if currency is safe heaven

Capital Inflow Regime

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Which assets $\{F(s)\}$ should CB buy?

- What are the trade-offs involved?
- What are the key goals that should be pursued?

ABBP, 2017 + CB portfolio choice

CB must open positive "gaps"

Using definition of $i^\star$ and $\Lambda(s)$

$$0 < \Delta(i) = \left[\sum_{s \in S} q(s) \frac{e_1}{e_2(s)}\right](1+i) - 1$$

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Combining definition of $\kappa(s)$ and NIRC:

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If $\kappa(s) \leq 0$ for all s, NIRC is violated (domestic rate too low)

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$$F(s) = c(s) - y(s) + a^\star(s)$$

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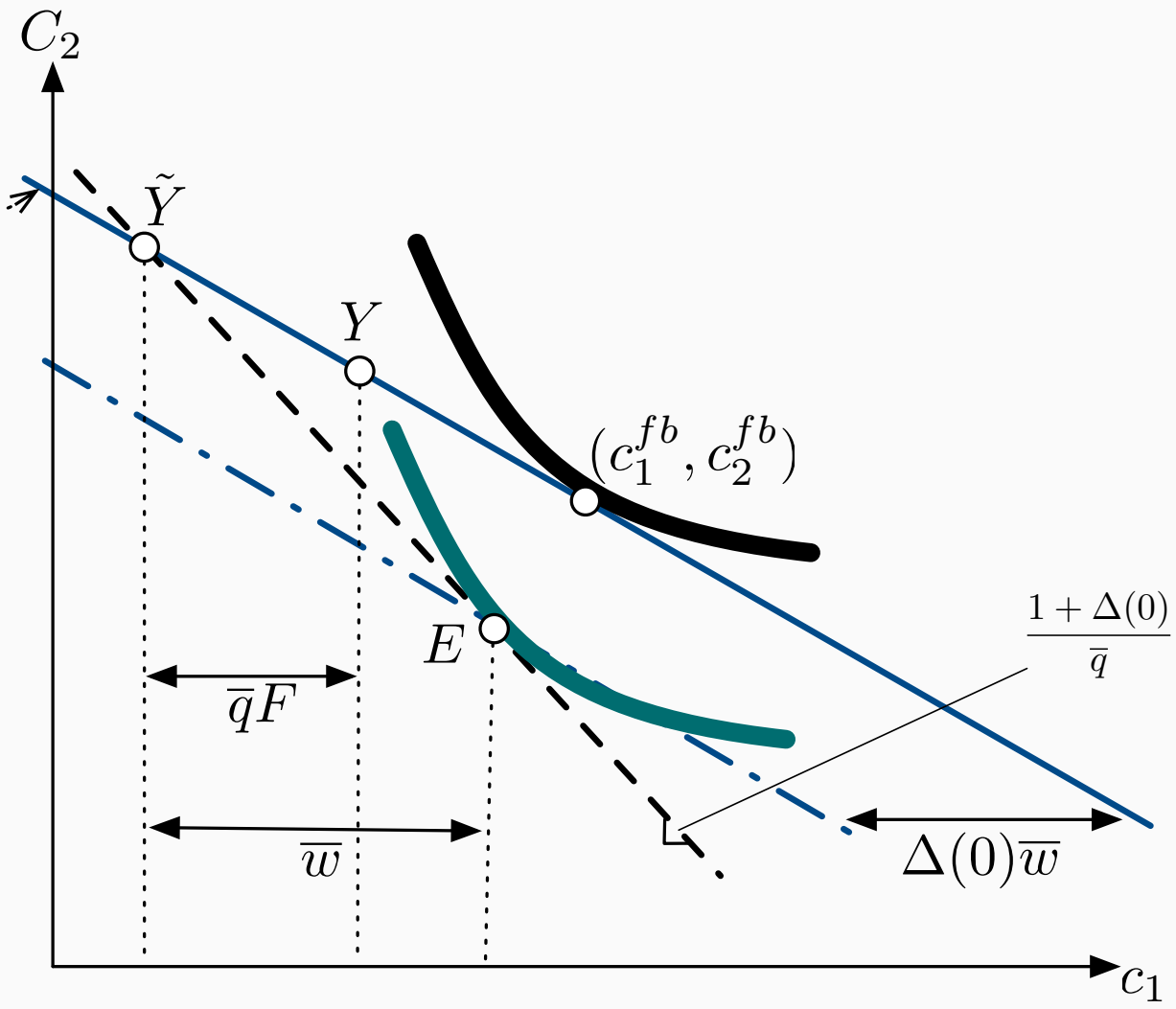
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Implementation of equal gaps with risk-free assets



- C_2 is the value of $\{c_2(s)\}$ in terms of period 2 goods
- $\bar{q}$ is the price of risk-free asset

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- We study this next

The Central Bank's problem: choose $(c_1, \{c_2(s), \kappa(s)\})$ to solve

$$V = \max \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\}$$

$$s.t. \quad y_1 - c_1 - \sum q(s) c_2(s) = L^{\star}(\{\kappa(s)\}, \bar{\kappa}) \qquad \text{(IRC)}$$

$$1 - \sum_s \frac{q(s) e_1}{(1 + \kappa(s)) e_2(s)} = i \qquad \text{(NIRC)}$$

$$1 + \kappa(s) = \frac{q(s) u_1'(c_1)}{\beta \pi(s) u'(c_2(s))} \ \forall s \qquad (\kappa(s))$$

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$$V(\tilde{\kappa}) = \max \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\}$$

$$s.t. \quad y_1 - c_1 - \sum q(s) c_2(s) = L^{\star}(\{\kappa(s)\}, \tilde{\kappa}) \qquad \text{(IRC)}$$

$$1 - \sum_s \frac{q(s) e_1}{(1 + \kappa(s)) e_2(s)} = i \qquad \text{(NIRC)}$$

$$1 + \kappa(s) = \frac{q(s) u_1'(c_1)}{\beta \pi(s) u'(c_2(s))} \ \forall s \qquad (\kappa(s))$$

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- Solve problem for given $\tilde{\kappa}$. Check ignored constraints
- Solve $V = \max_{\tilde{\kappa}} V(\tilde{\kappa}), \quad \bar{\kappa} = \text{argmax}\, V(\tilde{\kappa})$

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The CB has two goals:

1. Minimize intertemporal distortions
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Today: Assume $\bar{f} = \bar{a} = 0$ and $q(s) = \beta\pi(s)$

Financially closed economy

Optimal policy:

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- Nominal bond is too attractive $\Rightarrow$ "excessive" savings
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- CB has to buy $F(s)$ to deliver consumption goods in that state

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Assume that output volatility is low, we have that

- If currency appreciates in good times, this means buying assets that pay-off in good times!

Financially open economy

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  - Goal of minimizing losses prevail. How to do it?

$$\min_{\{\kappa(s)\}} \left\{ \max_s \{\kappa(s)\} \right\} \text{ subject to:}$$

$$0 \leq 1 + i - \sum_s \frac{q(s)e_1}{(1+\kappa(s))e_2(s)} \qquad \text{(NIRC)}$$

- Optimal policy calls for equal gaps $\kappa(s) = \kappa \; \forall s$
  - only allocation in which investors demand risk free bonds
- Some leeway about actual CB portfolio
  - e.g. purchase foreign risk-free bonds

Conclusion

- Reserve management under exchange rate objectives
- In the capital inflow regime, optimal portfolio hinges on *degree of openness*
  - Relatively closed economies:
    - Invest in foreign assets that pay when the currency appreciates
  - Relatively open economies:
    - Make sure foreign investors demand domestic risk free bonds
- Agenda
  - Implementation with specific assets (e.g. bonds and equity)
  - Closed economy implications