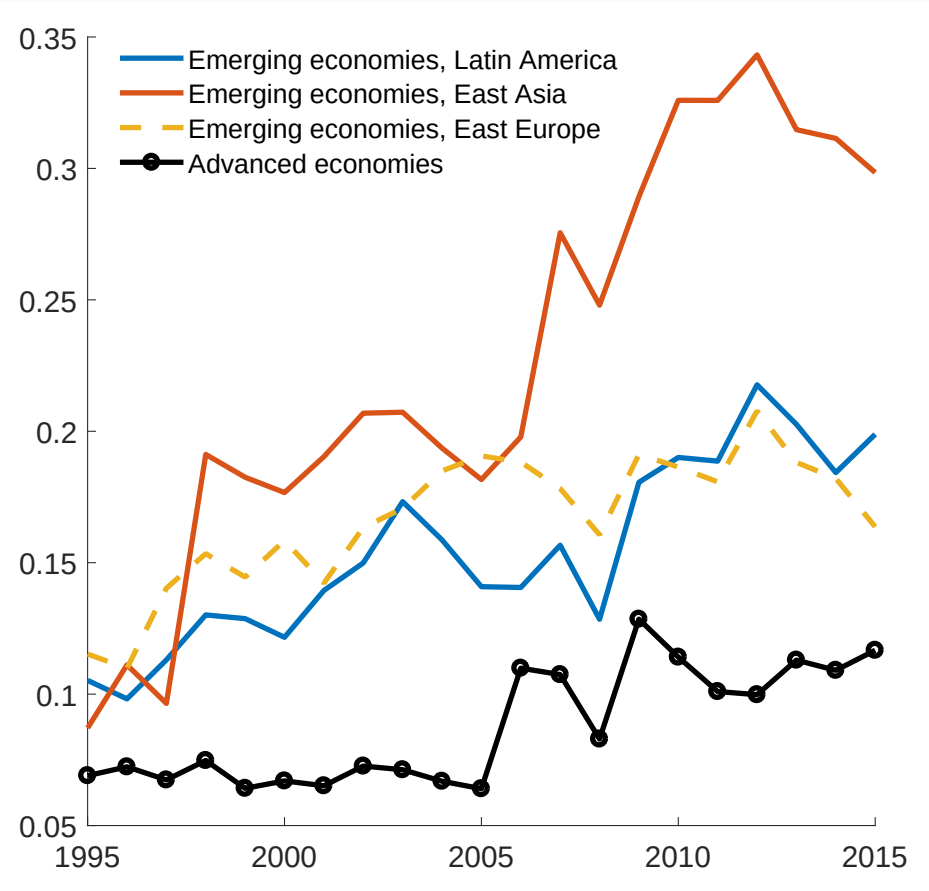
Foreign Reserve Management

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Motivation

Over the past 20 years massive increase in foreign reserves holdings by Central Banks around the world



Reserves-GDP

Motivation (ctd)

Why do countries hold foreign reserves? Two motives in the literature

1. *Precautionary role*: reserves used as a buffer for bad shocks
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This paper: Given exchange rates and monetary policy objectives, How should a Central Bank manage its reserve portfolio?

Preview: Interest Parity & Central Bank Balance Sheet

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- Today: A theory of which assets should central banks buy
  - Generalize ABBP 2017 with a **CB portfolio choice**, a broad monetary policy objective and financial structure

Preview: Main Results

Two goals:

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- If economy is relatively closed:
  - Policy: buy assets that pay when exchange rate appreciates
  - Key idea: increase *intratemporal* distortions to reduce *intertemporal* distortions
- If economy is relatively open:
  - Policy: make sure arbitrageurs hold domestic risk free bonds
  - Key idea: dispersion of arbitrage returns magnify losses

Framework

- Two-period model, $t \in \{1, 2\}$
  - Small open economy (central bank + households)
  - International Financial Market
  - Foreign Intermediaries
- Uncertainty realized at $t = 2$
  - $s \in S \equiv \{s_2, ..., s_N\}, \pi(s)$
- One (tradable) good, law of one price, foreign price normalized to 1

Asset markets: complete but segmented

International financial markets (IFM)

- Full set of Arrow-Debreu securities in foreign currency:
  - Security s: 1 unit of foreign currency in state s, 0 otherwise
  - Price $q(s)$ in terms of foreign currency at $t = 1$

Domestic financial market

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  - Price $p(s)$ in terms of domestic currency at $t = 1$

Intermediaries

- Trade securities with SOE & IFM

Households

- Endowment: $(y_1, \{y_2(s)\})$, transfers: $(\{T_2(s)\})$

$$\max_{c_1, \{c_2(s), a(s), f(s)\}} \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\}$$

subject to:

$$y_1 = c_1 + \sum_{s \in S} \left[q(s) f(s) + p(s) \frac{a(s)}{e_1} \right]$$

$$y_2(s) + T_2(s) + f(s) + \frac{a(s)}{e_2(s)} = c_2(s) \quad \forall s \in S$$

$$f(s) \geq -\bar{f}(s), \quad a(s) \geq -\bar{a}(s) e_2(s) \quad \forall s \in S$$

e_1, $e_2(s)$: exchange rates at $t = 1$ and $t = 2$

$f(s), a(s)$: holdings of foreign and domestic security s

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Foreign Intermediaries

- Endowed with capital $\bar{w}$

$$\max_{\{c_1^\star, c_2^\star(s), a^\star(s), f^\star(s)\}} c_1^\star + \sum_{s \in S} \pi(s) \Lambda(s) c_2^\star(s)$$

subject to:

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Consider $\Lambda(s) = \frac{q(s)}{\pi(s)}$ (same SDF as IFM)

Central Bank

- CB has an objective for the nominal interest rate and exchange rates that we take as given: $(i, e_1, \{e_2(s)\})$

$$1 + i = \left(\sum_{s \in S} p(s) \right)^{-1} \qquad \text{(NIRC)}$$

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- Given objectives, CB chooses policies to maximize welfare

Characterizing equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_1}{e_2(s)p(s)}}{\frac{1}{q(s)}} - 1$$

$\kappa(s) > 0 \Rightarrow$ domestic security paying in state s yields higher return

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$$y_1 - c_1 + \sum_{s \in S} q(s)[c_2(s) - y_2(s)] - \sum_{s \in S} a^*(s) \frac{p(s)}{e_1} \kappa(s) = 0$$

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- Two potential sources of arbitrage losses for SOE
  - Borrowing at high rates of return
  - Buying assets with low rates of return

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Given CB objective $(i, e_1, \{e_2(s)\})$, consider the return differential between risk-free home and foreign bonds

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We study three regimes $(i, e_1, \{e_2(s)\})$

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Capital Inflow Regime

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ABBP, 2017 + CB portfolio choice

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  Details
- Each $\{\kappa(s)\}$ corresponds to a reserve policy

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NIRC binds from below:

$$(1+i)^{-1} \geq \mathbb{E}\left(\frac{e_1}{e_2(s)}\right)\mathbb{E}\left(\frac{1}{1+\kappa(s)}\right) + \mathrm{Cov}\left(\frac{e_1}{e_2(s)}, \frac{1}{1+\kappa(s)}\right)$$

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Assume that output volatility is low, we have that

- If currency appreciates in good times, this means buying assets that pay-off in good times!

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Assume that output volatility is low, we have that

- If currency appreciates in good times, this means buying assets that pay-off in good times!

Prescriptions for reserve policy

Recall that a higher $\kappa(s) > 0$, means that CB must accumulate the foreign security that pay in state s:

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Financially open economy (large $\bar{w}$)

Recall losses (assume $\bar{a}^\star = 0$) : $\max_s\{\kappa(s)\}\left[\bar{w} + \sum_{s\in S} q(s)\bar{f}(s)\right]$

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- Some leeway about actual CB portfolio
  - e.g. purchase foreign risk-free bonds

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If $\bar{a}^\star(s) > 0$, equalizing gaps remain optimal

- Extra loss term $\sum_{s\in S} \bar{a}^\star(s)e_2(s)\frac{p(s)}{e_1}(\max_s\{\kappa(s)\} - \kappa(s))$

Conclusion

- Reserve management under exchange rate objectives
- In the capital inflow regime, optimal portfolio hinges on *degree of openness*
  - Relatively closed economies:
    - Invest in foreign assets that pay when the currency appreciates
  - Relatively open economies:
    - Make sure intermediaries demand domestic risk free bonds
- Agenda
  - Implementation with specific assets (e.g. bonds and equity)
  - Closed economy implications

CB must open positive "gaps"

For some $s, \kappa(s) > 0$

Under $\kappa(s) \leq 0$

$$\sum_{s \in S} p(s) = \sum_{s \in S} q(s) \frac{e_1}{e_2(s)(1+\kappa(s))} \geq \sum_{s \in S} q(s) \frac{e_1}{e_2(s)} = \frac{1+\Delta(i)}{1+i}$$

Since $\Delta(i) > 0$,

$$\left[\sum_{s \in S} p(s)\right]^{-1} < (1+i)$$

Interest rate is too low relative to NIRC.

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A key condition: arbitrage return on risk-free bond

Investor with one unit of the consumption good

- Invest it in domestic risk free bond:

Cost today: 1 Benefit tomorrow: $\left\{ \left(\frac{e_1}{\sum_s p(s)} \right) \frac{1}{e_2(s)} \right\}$

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Characterizing equilibria: Balance of Payment

- Trade deficits and net foreign assets:

$$\underbrace{c_1 - y_1}_{\text{trade deficit}} = \underbrace{\frac{\sum_s p(s)a^\star(s)}{e_1}}_{\text{foreign liabilities}} - \underbrace{\sum_s q(s)\left[f(s) + F(s)\right]}_{\text{foreign assets}}$$