Banking, Trade, and the Making of a Dominant Currency*

Gita Gopinath
Harvard and NBER

Jeremy C. Stein
Harvard and NBER

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Abstract

We explore the interplay between trade invoicing patterns and the pricing of safe assets in different currencies. Our theory highlights the following points: 1) a currency's role as a unit of account for invoicing decisions is complementary to its role as a safe store of value; 2) this complementarity can lead to the emergence of a single dominant currency in trade invoicing and global banking, even when multiple large candidate countries share similar economic fundamentals; 3) firms in emerging-market countries endogenously take on currency mismatches by borrowing in the dominant currency; 4) the expected return on dominant-currency safe assets is lower than that on similarly safe assets denominated in other currencies, thereby bestowing an "exorbitant privilege" on the dominant currency. The theory thus provides a unified explanation for why a dominant currency is so heavily used in both trade invoicing and in global finance.

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1 Introduction

The U.S. dollar is often described as a dominant global currency, much as the British pound sterling was in the 19th century and beginning of the 20th century. The notion of dominance in this context refers to a constellation of related facts, which can be summarized as follows:

- **Invoicing of International Trade:** An overwhelming fraction of international trade is invoiced and settled in dollars (Goldberg and Tille (2008), Gopinath (2015)). Importantly, the dollar's share in invoicing is far out of proportion to the U.S. economy's role as an exporter or importer of traded goods. For example, Gopinath (2015) notes that 60% of Turkey's imports are invoiced in dollars, while only 6% of its total imports come from the U.S. More generally, in a sample of 43 countries, Gopinath (2015) finds that the dollar's share as an invoicing currency for imported goods is approximately 4.7 times the share of U.S. goods in imports. This stands in sharp contrast to the euro, where in the same sample the euro invoicing share and the share of imports coming from countries using the euro are much closer to one another, so that the corresponding multiple is only 1.2.

- **Bank Funding:** Non-U.S. banks raise very large amounts of dollar-denominated deposits. Indeed, the dollar liabilities of non-U.S. banks, which are on the order of $10 trillion, are roughly comparable in magnitude to those of U.S. banks (Shin (2012), Ivashina et al. (2015)). According to Bank for International Settlements (BIS) locational banking statistics, 62% of the foreign currency local liabilities of banks are denominated in dollars.

- **Corporate Borrowing:** Non-U.S. firms that borrow from banks and from the corporate bond market often do so by issuing dollar-denominated debt, more so than any other non-local "hard" currency, such as euros. According to the BIS locational banking statistics, 60% of foreign currency local claims of banks are denominated in dollars. Bräuning and Ivashina (2017) document the dominance of dollar-denominated loans in the syndicated cross-border loan market. Importantly, this dollar borrowing is in many cases done by firms that do not have corresponding dollar revenues, so that these firms end up with a currency mismatch, and can be harmed by dollar appreciation (Aguiar (2005), Du and Schreger (2014), Kalemli-Ozcan et al. (2016)).

- **Central Bank Reserve Holdings:** The dollar is also the predominant reserve currency, accounting for 64% of worldwide official foreign exchange reserves. The euro is in second place at 20% and the yen is in third at 4% (ECB Staff (2017)).

- **Low Expected Returns and UIP Violation:** Gilmore and Hayashi (2011) and Hassan (2013), among others, document that U.S. dollar risk-free assets generally pay lower expected returns (net of exchange-rate movements) than the risk-free assets of most other currencies. That is, there is a violation of uncovered interest parity (UIP) that favors the dollar as a cheap funding currency. Sometimes this phenomenon is referred to as the dollar benefiting from an "exorbitant privilege" (Gourinchas and Rey (2007)).

The goal of this paper is to develop a model that can help to make sense of this multi-faceted notion of currency dominance. Our starting point is the connection between invoicing behavior and safe asset demand. Both of these topics have been the subject of much recent (and largely separate) work, but their joint implications have not been given as much attention.[1] Yet a fundamental observation is that in a multi-currency world, one cannot think about the structure of safe asset demands without taking into account invoicing patterns. Simply put, a financial claim is only meaningfully "safe" if it can be used to buy a known quantity of some specific goods at a future date, and this necessarily forces one to ask about how the goods will be priced.

Consider, for example, a firm operating in an emerging market (EM). The firm sells all of its output domestically, but purchases some intermediate inputs from abroad, from other emerging markets. The firm also holds a buffer stock of bank deposits that it can use to make these purchases over the next several periods. In what currency would it prefer to hold its deposits? If most of imported inputs are priced in dollars—and crucially, if these dollar prices are sticky—the importer will tend to prefer deposits denominated in dollars, as these are effectively the safest claim in real terms from its perspective. In other words, while deposits in any currency may be free of default risk, in a world in which exchange rates are variable, only a dollar deposit held today can be used to purchase a certain quantity of dollar-invoiced inputs tomorrow.

It follows that when more internationally-traded goods are invoiced in dollars, there will be a greater demand for dollar deposits—or more generally, for financial claims that pay off a guaranteed amount in dollar terms. Some of these may be provided by the U.S. government, in the form of Treasury securities, but to the extent that Treasury supply is inadequate to satiate global demand, private financial intermediaries will also have an important role to play. Specifically, banks operating in other countries will naturally seek to provide safe dollar claims to their customers who want them. However, in so doing, they must satisfy a collateral constraint: a bank that promises to repay a depositor one dollar tomorrow must have assets sufficient to back that

[1] On the choice of invoicing currency, contributions include Friberg (1998), Engel (2006), Gopinath et al. (2010), Goldberg and Tille (2013). On safe asset determination in an international context, some recent papers are Hassan (2013), Gourinchas and Rey (2010), Maggiori (2017), He et al. (2016) and Farhi and Maggiori (2016). We discuss these works in more detail below.

promise. This collateral in turn, must ultimately come from the revenues on the projects that the bank lends to. And importantly, not all of these projects need be ones that produce revenues that are dollar-based. For example, a bank in an EM that is trying to accommodate a large demand for dollar deposits may seek to back these deposits by turning around and making a dollar-denominated loan to a local firm that produces non-tradeable, local-currency-denominated goods. Of course, this firm's revenues do not make particularly good collateral for dollar claims, because of exchange-rate risk: it would be more efficient to use the firm's revenues to back local-currency deposits, all else equal.

This inefficiency in collateral creation is at the heart of our results. If global demand for dollar deposits is strong enough, equilibrium inevitably involves having even those operating firms that generate revenues in other currencies serving as a marginal source of collateral for dollar deposits. Since these firms effectively have an inferior technology for producing dollar collateral relative to own-currency collateral, they can only be drawn into doing the latter if they are paid a premium for doing so, that is, if is cheaper for them to borrow in dollars than in their home currency. The intuition is of walking up a supply curve: as worldwide demand for safe dollar claims expands, we exhaust the supply that can be provided by low-cost producers (the U.S. Treasury, and firms that naturally have dollar-denominated revenues) and therefore must turn to less efficient, higher cost producers, namely firms that have to take on currency risk in order to create the collateral that backs dollar claims. As a result, the safety premium on dollar claims deposits exceeds that on local-currency deposits. Or said differently, the expected return on dollar deposits is on average lower, in violation of uncovered interest parity (UIP). This is the exorbitant privilege associated with the dollar.

Note that this line of argument turns on its head much informal reasoning about why foreign firms borrow in dollars. In particular, if one takes the UIP violation as exogenous, it seems obvious why some firms might be willing to court exchange-rate risk by borrowing in dollars—it can be worth it to do so simply because dollar borrowing is on average cheaper. But this leaves open the question of where the UIP violation comes from in the first place. Our explanation is that dollar borrowing has to be cheaper because the worldwide demand for safe dollar claims is so large that even those firms that are not particularly well-suited to it must be recruited to help provide collateral for such claims; again, the intuition here is of walking up the supply curve. This recruiting can only happen in equilibrium if it is cheaper to borrow in dollars than in local currency. Thus the primitive in our story is the share of internationally-traded goods invoiced in dollars, which in turn drives the demand for safe dollar claims; the UIP violation then emerges endogenously as the equilibrium "price" required to bring supply into line with demand.

Of course, this line of reasoning begs the question of where the dollar invoicing share comes from: what determines whether EM firms selling goods internationally price them in dollars, as opposed to their own currency or another potential dominant currency like the euro? Although a variety of factors likely come into play, we argue that there is an important feedback loop from UIP violations back to invoicing choices. Suppose for the moment that for an EM exporting firm dollar borrowing is cheaper in equilibrium than borrowing in either its own currency or in euros. All else equal, the EM firm then has an incentive to choose to invoice its exports in dollars, because doing so gives it more certainty about its next-period dollar revenues, which in turn allows it to safely borrow more in dollars, i.e., in the cheaper currency.

This then generates a link back to invoicing shares, safe asset demand and the UIP violation. To see this, consider two emerging markets i and j. An initially high dollar invoice share facing importers in i leads to an increased demand on their part for safe dollar claims, which in turn drives down dollar borrowing costs. Responding to this financing advantage, exporting firms in j are induced to invoice more of their sales to importers in country i in dollars. So the dollar invoice share facing country i importers goes up further. This same mechanism also increases the incentive for exporters in country i to price in dollars when selling to country j. In other words, a high dollar invoice share in country i tends to push up the dollar invoice share in country j, and vice-versa, through a safe asset demand-and-supply mechanism. As we show, this form of strategic complementarity can give rise to asymmetric equilibria in which a single currency becomes disproportionately dominant in both global trade and banking, even when two large candidate countries share similar economic fundamentals.

The model that we develop below formalizes this line of argument. For example, in a case where the U.S. and Europe are otherwise identical in all respects, we obtain asymmetric equilibrium outcomes where the majority of trade invoicing is done in dollars, and where most non-local-currency deposit-taking and lending by banks in other EM countries is dollar-denominated, rather than euro-denominated.

Finally, in such an asymmetric equilibrium, it seems natural to expect that the foreign-currency reserve holdings of a typical EM central bank would skew heavily towards dollars, as opposed to euros. Although we do not model this last link in the chain formally, the logic we have in mind is straightforward: given that an important role for the central bank is to act as a lender of last resort to its commercial banking system, the fact that the commercial banks' hard-currency deposits are primarily in dollars means that the central bank will need a stockpile of dollars to be able to replace any sudden loss of bank funding that occurs during a liquidity crisis. Thus the central bank's asset mix is to some extent a mirror of the commercial banks' liability structure,

and both are ultimately shaped by—and feed back on—the invoicing decisions made by exporters in other countries. This argument is consistent with the evidence in Obstfeld et al. (2010) who argue that the dramatic accumulation of reserves by central banks in emerging markets is driven in part by considerations of maintaining domestic financial stability.

Related literature: This paper aims to connect two strands of research: one on trade invoicing, and the other on safe-asset determination in an international context. The former emphasizes the role of a dominant currency as a unit of account, while the latter focuses on its role as a store of value. Our contribution is to highlight the strategic complementarity between these two roles, i.e., to show how they mutually reinforce each other. The only other work we are aware of that ties together trade invoicing and finance is contemporaneous work by Chahrour and Valchev (2017), who focus on the medium of exchange role of currencies.

We also provide a novel perspective on both trade invoicing and safe-asset determination. The literature on trade invoicing sets aside financing considerations and instead focuses on factors that influence the optimal degree of exchange rate pass-through into prices, as in the contributions of Friberg (1998), Engel (2006), Gopinath et al. (2010), Goldberg and Tille (2013). Doepke and Schneider (2017) rationalize the role of a dominant unit of account in payment contracts by the desire to avoid exchange rate risk and default risk. By contrast, we provide a complementary explanation that relates exporters' pricing decisions to their financing choices, and in particular to their desire to borrow in a cheap currency. In our model the only reason exporters choose to invoice in dollars is because by doing so they are able to more cheaply finance their projects.

On the safe asset role of the dollar and the lower expected return on the dollar relative to other currency assets, existing explanations are tied to the superior insurance properties of U.S. bonds that arise either from country size (Hassan (2013)); from the tendency of the dollar to appreciate in a crisis (Gourinchas and Rey (2010), Maggiori (2017)); from better fiscal fundamentals and liquidity of debt markets (He et al. (2016)); or from the monopoly power of the U.S. as a safe asset provider (Farhi and Maggiori (2016)). We offer a distinct explanation that is tied to the invoicing role of the dollar in international trade. In our model, it is this invoicing behavior that generates the demand for dollar safe assets and importantly, that implies that the marginal supplier of dollar claims must have a mismatch of its assets and liabilities in equilibrium.

Outline of the paper: The full model that we consider below has two large countries, the U.S. and the Euro area, a continuum of emerging market economies, and endogenous invoicing and financing decisions. To provide a clear exposition of the mechanism we build up to the full model in steps. Section 2 starts with a simple case in which there is just the U.S. and one emerging mar-

ket (EM), and in which invoice shares facing importers in the EM are exogenously specified. Here we highlight the fundamental source of the UIP violation. Section 3 endogenizes the invoicing decision of exporter firms in the EM and explains the financial incentive for invoicing in dollars. Section 4 brings in the continuum of EMs and demonstrates the strategic complementarity between their invoicing decisions and the safe asset demand that gives rise to multiple equilibria. Finally in Section 5 we add the euro as another candidate global currency and show that in spite of the symmetry in fundamentals, for some parameter values the only possible equilibrium outcomes are asymmetric, with only one global currency being used extensively by emerging-market countries to invoice their exports and to finance their projects. Section 6 concludes. All proofs not in the text can be found in the appendix.

2 Exogenous Import Invoice Shares and the UIP Violation

Suppose the world is comprised of the U.S. and one emerging market. All of the focus is on decisions made by EM agents. The U.S. only plays two simple roles. First, an exogenous fraction $\alpha_\$ < 1$ of the goods purchased by EM households are priced in dollars. And second, the U.S. supplies an exogenous net quantity $X_\$$ of safe dollar claims that are available to these same EM households. These safe claims could be, e.g. Treasury securities, or deposits in U.S.-based financial intermediaries such as banks or money-market funds.[2]

There are two kinds of agents in the EM. The first group, whom we call "importers", are households who make consumption/savings decisions, and who import some goods from abroad. The second group, whom we call "banks", can be thought of as an agglomeration of the local banking sector with those firms—and by extension, the real projects—that the banks lend to. We describe each group in detail next. The model has two dates, denoted 0 and 1.

2.1 Importers

Importer households save at time 0, and consume at both time 0 and time 1. They can save in one of three types of assets: (i) risk-free home-currency deposits, D_h, (ii) risk-free dollar deposits, $D_\$$, and (iii) risky home-currency assets, A_R, which can be thought of as either bonds with credit

[2]To put a little more flesh on this assumption: imagine that U.S. households and firms have an inelastic demand for up to $Z_\$$ units of safe assets, and no more, and that the Treasury has issued $Y_\$$ units of safe Treasury securities. Then $X_\$ = Y_\$ - Z_\$$, and the empirically-relevant case for us to consider is when $X_\$ \geqq 0$.

risk, or equities. The representative importer household maximizes:

$$C_0 + \beta \mathbb{E}_0 C_1 + \theta \log(M) \tag{1}$$

where C_0 is consumption at time 0, and C_1 is consumption at time 1. A portion of time-1 consumption is comprised of imported goods. Importantly, a fraction of these imported goods are priced in dollars, and consistent with the evidence in Gopinath (2015), the dollar price is assumed to be sticky between time 0 and time 1; this is what we mean when we say that imports are "invoiced" in dollars. And it is this dollar invoicing that gives rise to a demand for safe dollar claims.

Specifically, the $\theta \log(M)$ term in the importer utility function captures a preference for safe "money-like" assets—which we define as assets that pay a certain nominal amount at time 1 in a particular currency. This type of formulation, with a preference for safe money-like claims embedded directly in the utility function, follows a number of recent papers including Krishnamurthy and Vissing-Jorgensen (2012), Stein (2012), Sunderam (2015), Greenwood et al. (2015), and Nagel (2016).[3] However, unlike these other papers, we are dealing with a case in which there are multiple currencies, so we need to specify how to aggregate quantities of safe claims that are denominated in different currencies. We do so by assuming that M takes a Cobb-Douglas form:

$$M = \left(D_h^{\alpha_h} D_{\$}^{\alpha_{\$}}\right)^{\frac{1}{\alpha_h + \alpha_{\$}}} \tag{2}$$

where α_h and $\alpha_{\$}$ capture relative preferences for safe home-currency deposits and safe dollar-denominated deposits respectively, with $\alpha_h + \alpha_{\$} \leq 1$.[4] This formulation ensures constant returns to scale regardless of the value of $\alpha_h + \alpha_{\$}$.

[3] In taking this reduced-form approach, the literature is not always clear on what drives the primitive demand for safety. One mechanism is that safe claims are better for making payments—i.e., for transactions and settlement purposes—since they are free of adverse selection (Gorton and Pennacchi (1990)). Another is that they are attractive as a store of value for agents who are highly risk-averse and hence want to be able to ensure themselves a fixed level of future consumption (Gennaioli et al. (2012)). As will become clear, our modeling approach fits more naturally with the second mechanism, since we focus on invoicing decisions for goods with sticky prices. However, if the former were also at work, and there was a pure settlement role for a global currency like the dollar, this would likely amplify the dominant-currency effects that we model. Consider for example the case of a volatile commodity like oil. Since oil prices are not sticky, holding dollars at time 0 does not ensure the ability to buy a fixed quantity of oil at time 1. Thus, in the context of our model, there would not be a special demand for safe dollar claims on the part of an oil importer. However, in reality, to the extent that any oil purchase at time 1 must be settled in dollars, there may be a pure payments motive to hold dollars at time 0. Adding this motive to our model would presumably reinforce the sorts of effects that we are interested in.

[4] A number of other papers use a similar approach to create a single monetary aggregate from multiple underlying financial instruments, though in most cases they are aggregating over instruments that are all denominated in the same currency. See, e.g., Sunderam (2015), and Nagel (2016).

Our key premise is that these preferences across safe claims denominated in different currencies are related to the overall shares of domestic and imported goods that are invoiced in dollars versus local currency. Intuitively, the underlying notion of safety that we are trying to capture is the ability of importer households to carry out a given level of time-1 purchases. Thus if a greater share of their total time-1 expenditures has dollar prices that are fixed as of time 0, it becomes more attractive for them to hold dollar claims with a certain payoff. This gives rise to a demand for safe assets in dollars in addition to the usual demand for home-currency safe assets.

The budget constraints of the importers are:

$$\begin{aligned} C_0 &\leq Z_0 - Q_h D_h - \mathcal{E}_0 Q_\$ D_\$ - Q_R A_R \\ C_1 &\leq Z_1 + D_h + \mathcal{E}_1 D_\$ + \xi A_R \end{aligned}$$

where: Z_0 and Z_1 are endowment income in periods 0 and 1 respectively; Q_h is the time-0 price of a deposit that pays off a certain one unit in the local currency at time 1; $Q_\$$ is the time-0 price of a deposit that pays off a certain one unit in dollars at time 1; and Q_R is the time-0 price of a risky local-currency claim with a stochastic payoff of ξ at time 1 with $\mathbb{E}_0(\xi) = 1$.[5] $\mathcal{E}_0$ and $\mathcal{E}_1$ are time-0 and time-1 exchange rates. We take the exchange rates as exogenous and for simplicity normalize them so that $\mathbb{E}_0(\mathcal{E}_1) = \mathcal{E}_0 = 1$.[6]

The importers' first-order conditions for D_h, $D_\$$, and A_R yield:

$$Q_h = \beta + \theta \frac{\alpha_h}{(\alpha_h + \alpha_\$) D_h} \tag{3}$$

$$Q_\$ = \beta \frac{\mathbb{E}_0(\mathcal{E}_1)}{\mathcal{E}_0} + \theta \frac{\alpha_\$}{(\alpha_h + \alpha_\$) D_\$} = \beta + \theta \frac{\alpha_\$}{(\alpha_h + \alpha_\$) D_\$} \tag{4}$$

$$Q_R = \beta \mathbb{E}_0(\xi) = \beta \tag{5}$$

[5] In writing the time-1 budget constraint, we are assuming for simplicity that final time-1 consumption is entirely denominated in local-currency units. To square this with the fact that households are consuming a basket that includes imported goods, we can think of the household sector as owning a set of retail firms that stand between them and exporters from other countries. These retailers purchase the imports from abroad at time 1—at prices that are fixed in dollars and other producer-country currencies—and then turn around and sell these imported goods to the households at sticky local-currency prices. In this interpretation, the utility for safe claims M that we model in part reflects the demands of the retailers for dollars in order to purchase the imports. The profits of the retailers, inclusive of any exchange-rate variation, then enter into the time-1 endowment Z_1 of the households. An alternative approach dispenses with the retailers, so that households purchase imports directly from abroad, and rewrites the time-1 budget constraint to recognize that the price of the time-1 consumption basket depends on exchange-rate movements between 0 and 1. This alternative leads to slightly more complicated expressions for Q_h, $Q_\$$ and Q_R than those in equations (3)-(5), but all the results that follow are unchanged.

[6] We assume that the endowment Z_0 is large enough so that in equilibrium the constraint $C_0 \geq 0$ never binds.

Two observations follow immediately from these first-order conditions. First, the price of the risky asset Q_R is lower than the price of either of the safe claims, meaning that the expected return on the risky asset is higher; this is because the risky asset does not provide any monetary services, i.e., it does not enter into the M aggregator. Second, the prices of the two safe claims, Q_h and $Q_\$$, need not be equalized. Since there is no expected currency appreciation or depreciation, a failure of these two prices to be equalized amounts to a violation of uncovered interest parity (UIP)—that is, a potentially higher (or lower) return on dollar-denominated deposits than on local-currency deposits. We cannot yet sign any UIP violation however, since this will depend on the equilibrium quantities of deposits in each currency, which we endogenize below.[7]

Remark 1 *Exogenous Exchange Rates?*

We are taking exchange rates as exogenous, and also assuming that there is no expected appreciation or depreciation between time 0 and time 1. This is not important for our key conclusions. The first-order conditions in equations (3)-(4) fundamentally pin down the net-of-exchange-rate expected returns on the different assets in the economy. With expected exchange-rate changes set to zero, this means that equations (3)-(4) simply determine own-currency rates of return; the analysis is therefore best thought of as suited to making 'on average' statements about the rate in different currencies. An alternative approach would be to add active monetary policy to the model, thereby allowing rates in each country to be displaced from their average values in response to aggregate demand shocks. In this case, variants of equations (3)-(4) would still hold, meaning that there would still be the same violations of UIP described by these equations. But now, if interest rates rose in the U.S. due to contractionary monetary policy, the dollar would have to be expected to weaken going forward so as to maintain the same relative expected return on dollar claims. This is how exchange rates might be endogenized in the richer version of the model. Note however, that we would still be making the same statements about on-average interest-rate differentials—i.e. rate differentials when monetary policy in both countries was at its neutral level.[8]

[7]It should be noted that the violations of UIP that we model need not be associated with violations of covered interest parity (CIP). This is because the underlying factor that drives a UIP violation is the preference that savers have for a financial claim that pays out a certain amount in a given currency. For example, savers may be willing to pay a premium for a sure dollar return at time 1. But they are indifferent between two ways of getting to that sure dollar return. In particular, they are indifferent between a dollar deposit that pays out one dollar for sure and a synthetic dollar deposit that involves a domestic currency deposit coupled with a foreign-exchange forward contract that, taken together, promise the same one dollar with certainty. This indifference on the part of depositors will tend to enforce covered interest parity.

[8]Either version of our model is silent with respect to any higher frequency aspect of UIP violations such as the forward premium puzzle, according to which relative expected return to holding a given country's currency

2.2 Banks

We model the representative EM bank as an entity that is endowed with N projects that collectively pay a risky return of γN in domestic currency in period 1, where γ is a random variable. Each project requires a unit of home-currency investment at time 0 that the bank finances through borrowing with one of three types of liabilities: safe local-currency claims B_h; safe dollar-denominated claims $B_\$$; and risky local-currency bonds B_R. The bank is a price-taker in each of the three markets. Importantly, because the bank's projects are risky, there is an upper bound on how much it can promise in terms of safe claims. In other words, it faces a collateral constraint on its production of B_h and $B_\$$. Specifically, define γ_L to be the worst realization of the productivity shock γ, and $\bar{\mathcal{E}} > 1$ to be the most depreciated value of the local currency.[9] Then the maximum quantities of safe claims B_h and $B_\$$ that the bank can issue are constrained by the condition: $\bar{\mathcal{E}} B_\$ + B_h \leq \gamma_L N$

A central piece of intuition that emerges from this collateral constraint is that the bank has a comparative advantage in manufacturing local-currency safe claims relative to dollar-denominated safe claims. This is because the bank's underlying collateral is a collection of projects that pay off in local currency. Given the risk of currency depreciation, an amount of local collateral that is sufficient to back one unit of safe local-currency claims is only enough to back $1/\bar{\mathcal{E}}$ units of safe dollar claims.

The bank's problem is therefore:

$$\max_{B_h, B_\$, B_R} \mathbb{E}_0 \left[\gamma N - B_h - B_\$ - \xi B_R\right]$$

subject to,

$$Q_h B_h + Q_\$ B_\$ + Q_R B_R \geq N \tag{6}$$

$$\bar{\mathcal{E}} B_\$ + B_h \leq \gamma_L N \tag{7}$$

Define λ and μ to be the Lagrange multipliers on the financing constraint eq. (6) and the collateral constraint eq. (7), respectively. The first-order conditions for the problem imply:

increases when the interest rate in that currency rises (Engel (2014)). Instead our focus is on cross-country rate of return differentials, of which we take the "exorbitant privilege" to be a leading example.

[9] To be clear, $\bar{\mathcal{E}}$ is in units of domestic currency/dollar, so a higher value indicates a weaker domestic currency.

$$B_\$: \qquad Q_\$ = \frac{\mu\bar{\mathcal{E}} + 1}{\lambda} \tag{8}$$

$$B_h : \qquad Q_h = \frac{\mu + 1}{\lambda} \tag{9}$$

$$A_R : \qquad Q_R = \frac{1}{\lambda} \tag{10}$$

These conditions yield the following proposition.

Proposition 1 ***[Exorbitant Privilege]*** *In an interior equilibrium in which the bank issues all three forms of debt, we have that $Q_\$ > Q_h > Q_R$.*

$$\frac{Q_\$ - \beta}{Q_h - \beta} = \bar{\mathcal{E}}$$

In other words, UIP is violated, and dollar deposits benefit from an "exorbitant privilege" relative to local-currency deposits: they have a higher price and a lower expected return.

The proposition is a direct consequence of the bank's comparative disadvantage in creating dollar safe claims out of local-currency-denominated collateral. Because of this disadvantage, the bank will only be willing to fund these local projects with dollar borrowing if doing so is cheaper than funding with domestic deposits. However, it still remains to check, as we do just below, whether the bank does in fact fund its local-currency projects with dollar claims in equilibrium. Intuitively, it will do so only if the local demand for dollars is large relative to the exogenous supply of safe dollar claims $X_\$$ that are available from abroad.

2.3 Market Clearing

In order to solve for the equilibrium of the model, we note that total safe dollar claims available to EM importers are the sum of those produced by the bank borrowing against local-currency collateral, and those exogenously supplied from abroad: $D_\$ = B_\$ + X_\$$. At the same time, safe local-currency claims can only be collateralized by local projects, meaning that $D_h = B_h$. Assuming that the safe asset constraint binds, this implies that $\bar{\mathcal{E}}(D_\$ - X_\$) + D_h = \gamma_L N$.

Using equations (3)-(10), we can now solve for the prices and quantities of safe claims that obtain in an interior equilibrium in which $B_\$ > 0$:

$$
\begin{aligned}
D_h &= \frac{\alpha_h}{\alpha_\$ + \alpha_h}\left(\gamma_L N + \bar{\mathcal{E}} X_\$\right) \\
D_\$ &= \frac{\alpha_\$}{\alpha_\$ + \alpha_h}\frac{\left(\gamma_L N + \bar{\mathcal{E}} X_\$\right)}{\bar{\mathcal{E}}} \\
Q_h &= \beta + \frac{\theta}{\left(\gamma_L N + \bar{\mathcal{E}} X_\$\right)} \\
Q_\$ &= \beta + \frac{\theta \bar{\mathcal{E}}}{\left(\gamma_L N + \bar{\mathcal{E}} X_\$\right)}
\end{aligned}
$$

And again, in this case where the bank issues a positive amount of dollar claims backed by local collateral, we have a failure of UIP with $Q_h < Q_\$$. In order for the bank to in fact be in the interior region where $B_\$ > 0$, it must be that $D_\$ > X_\$$, which can be rewritten as:

$$
\frac{\alpha_\$}{\alpha_h + \alpha_\$} > \frac{\bar{\mathcal{E}} X_\$}{\gamma_L N + \bar{\mathcal{E}} X_\$} \tag{11}
$$

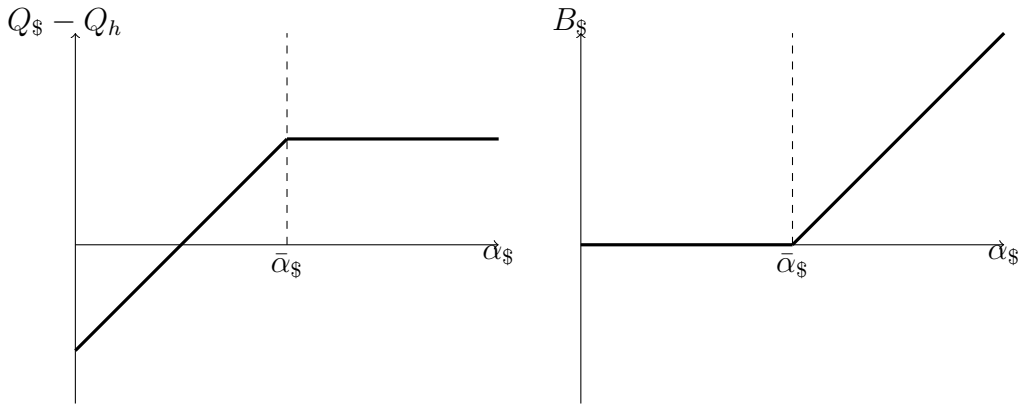
Simply put, if the dollar invoice share is large enough relative to the supply $X_\$$ of safe dollar claims available from abroad, the bank will necessarily get drawn into the business of manufacturing dollar deposits backed by local-currency projects, which in turn requires the rate of return on these dollar deposits to be lower than that on own-currency deposits.

We can now fully characterize equilibrium outcomes in this simple version of the model:

Proposition 2 *[**Import Invoice Shares and Exorbitant Privilege**] Define $\bar{\alpha_\$}$ as the value of $\alpha_\$$ where eq.* (11) *holds with equality: $\bar{\alpha_\$} = \frac{\alpha_h \bar{\mathcal{E}} X_\$}{\gamma_L N}$. The full solution to the model in the case where the invoice shares $\alpha_\$$ and α_h are exogenously specified is given by:*

$$
D_h = B_h = \begin{cases} \gamma_L N & \text{if } \alpha_\$ < \bar{\alpha}_\$ \\ \frac{\alpha_h}{\alpha_h + \alpha_\$}\left(\gamma_L N + \bar{\mathcal{E}} X_\$\right) & \text{if } \alpha_\$ \geq \bar{\alpha}_\$ \end{cases}
$$

$$
D_\$ = \begin{cases} X_\$ & \text{if } \alpha_\$ < \bar{\alpha}_\$ \\ \frac{\alpha_\$}{(\alpha_h + \alpha_\$)\bar{\mathcal{E}}}\left(\gamma_L N + \bar{\mathcal{E}} X_\$\right) & \text{if } \alpha_\$ \geq \bar{\alpha}_\$ \end{cases}
$$



(a) $Q_\$ - Q_h$ (b) $B_\$$

Figure 1: Invoicing Shares, UIP Deviations, and Dollar Borrowing

$$B_\$ = \begin{cases} 0 & \text{if } \alpha_\$ < \bar{\alpha}_\$ \\ \frac{\alpha_\$}{(\alpha_h+\alpha_\$)\bar{\mathcal{E}}}\left(\gamma_L N + \bar{\mathcal{E}} X_\$\right) - X_\$ & \text{if } \alpha_\$ \geq \bar{\alpha}_\$ \end{cases}$$

$$Q_\$ - Q_h = \begin{cases} \frac{\theta}{\alpha_\$+\alpha_h}\left(\frac{\alpha_\$}{X_\$} - \frac{\alpha_h}{\gamma_L N}\right) & \text{if } \alpha_\$ < \bar{\alpha}_\$ \\ \frac{\theta(\bar{\mathcal{E}}-1)}{\left(\gamma_L N+\bar{\mathcal{E}} X_\$\right)} & \text{if } \alpha_\$ \geq \bar{\alpha}_\$ \end{cases}$$

Figure 1 illustrates, plotting the magnitude of the UIP deviation $(Q_\$ - Q_h)$ (in panel (a)) and the quantity of dollar funding by the banking system $B_\$$ (in panel (b)) versus the dollar invoice share in imports $\alpha_\$$. Note that $(Q_\$ - Q_h)$ has to become significantly positive—in particular, it has to reach a value of $\frac{\theta(\bar{\mathcal{E}}-1)}{\left(\gamma_L N+\bar{\mathcal{E}} X_\$\right)}$ before the banks start using local-currency collateral to back dollar claims. This is because the cost of doing even the first unit of this kind of currency conversion is discretely positive, and is proportional to $(\bar{\mathcal{E}} - 1)$, which is effectively a proxy for the variability of the exchange rate.

Proposition 2 and Figure 1 highlight our first key point: that in equilibrium, there is a fundamental link between the dollar's role as a global invoicing currency, and the low return on safe dollar claims, i.e., the exorbitant privilege. To the extent that the dollar enjoys a large invoicing share, this increases the demand on the part of importers for safe dollar deposits. Equilibrium then requires these claims to have a higher price, or equivalently, to offer a lower rate of return. This is true because when the demand is high enough, the marginal supply of safe dollar

claims must be produced with a relatively inefficient technology—that is, it must be backed by the collateral coming from non-dollar-denominated projects.

Remark 2 *Banks and Non-financial Firms*

The agents that we have been calling "EM banks" invest directly in real projects that yield returns in local-currency units. Thus they are more accurately thought of as an agglomeration of banks and the local non-financial firms that the banks lend to. To create a separation between these two types of entities, and a more well-defined account of the role of financial intermediation, assume that any individual non-financial firm can invest in a single project that pays a random amount γ/p if the project succeeds, which happens with probability p, and zero otherwise. This individual project-level success or failure draw is idiosyncratic, and uncorrelated across firms. Thus no single non-financial firm can issue any amount of safe claims, because there is always some chance that its project will yield zero. However, a bank that pools a large number N of these uncorrelated projects will be assured of a worst case payout of $\gamma_L N$, as we have been assuming.[10] Hence, as originally pointed out by Gorton and Pennacchi (1990), there is a specific pooling-and-tranching role for banks in creating safe claims.

However, this observation raises a further question of who bears the exchange rate risk. In the model, a bank that issues dollar deposits against its local-currency collateral bears some exchange-rate risk: if the dollar appreciates against the local currency, it will see its profits decline. But if the word "bank" is really a metaphor for the combined local banking and non-financial sectors, which of the two do we expect will actually wind up bearing the bulk of the currency risk? In other words, one possibility is that non-financial firms borrow from banks using local-currency debt, in which case the banks assume the currency mismatch. Alternatively, the non-financial firms could borrow using dollar-denominated debt, in which case they would be the ones bearing the currency risk, while the banks would be insulated. For the internal logic of the model, either interpretation works, since in either case the exchange-rate risk acts to limit the ultimate amount of safe dollar claims that can be produced from a given amount of local-currency collateral. As a matter of empirical reality, the existing evidence suggests that a significant amount of the exchange-rate risk is borne by the non-financial corporate sector in emerging markets (Galindo et al. (2003), Du and Schreger (2014)). So when we develop propositions about the degree of exchange-rate mismatch in the "banking" sector in what follows, these propositions are best taken as statements that refer at least in part to mismatch among non-financial firms.

[10] This particular formulation follows Stein (2012).

3 Exporter Firms and Endogenous Invoicing

The next step is to allow exporter firms in the EM to choose how to invoice their sales to other countries, while temporarily maintaining the assumption that the invoice shares facing its importers are exogenously fixed. Bearing in mind the interpretation that the banks in the model are really agglomerations of banks and operating firms, we now assume that the EM banks have two types of projects. First, there are N_0 projects which, as before, necessarily produce home-currency revenues; these can be thought of as representing investments undertaken by firms that sell all of their output domestically. Second, there are N projects that can produce either dollar revenues or home-currency revenues. These latter projects are meant to capture the pricing decisions facing exporter firms in the EM: they have the choice of whether to invoice their sales in either dollars or their home currency. Moreover, if they do more of the former—and if prices are sticky—their dollar revenues will be more predictable, and hence will make better collateral for backing safe dollar claims.

We denote by η the fraction of the N projects that are invoiced in dollars, with the remaining fraction $(1-\eta)$ being invoiced in home currency. We also assume that there is a cost to the bank-exporter coalition associated with doing more dollar invoicing, and that this cost is given by $\frac{\phi}{2}N\eta^2$. One concrete way to interpret the cost is that it proxies for the risk aversion of the ultimate owners of the EM's exporter firms. If these owners are themselves EM residents, whose consumption basket is mostly home currency denominated, risk aversion will lead them to prefer a profit stream that is also home currency denominated. Hence the preference for home currency invoicing, all else equal.[11]

With these assumptions in place, the modified problem for the bank can be written as:

$$\max_{B_h, B_\$, B_R, \eta} \mathbb{E}_0 \left[\gamma N_0 + \gamma(1-\eta)N + \mathcal{E}\gamma\eta N - B_h - \mathcal{E}B_\$ - \xi B_R - \frac{\phi}{2}N\eta^2 \right]$$

subject to,

$$Q_h B_h + Q_\$ B_\$ + Q_R B_R \geq N + N_0 \tag{12}$$

$$\bar{\mathcal{E}} B_\$ + B_h \leq \gamma_L N_0 + (1-\eta)\gamma_L N + \bar{\mathcal{E}}\eta\gamma_L N \tag{13}$$

$$B_h \leq \gamma_L N_0 + (1-\eta)\gamma_L N \tag{14}$$

[11] Note that even if all dollar-invoiced projects are used to back safe dollar deposits, there is still a residual dollar profit stream that accrues to some other set of claimants, whom we might think of as domestic EM shareholders. This is because, given the inherent riskiness of all projects, none can be financed entirely with risk-free deposits. Thus, there is always a risky residual claim, and the currency exposure of this residual claim depends on the invoicing decision, i.e. on the currency denomination of the revenues.

There are a couple of points to note about this revised formulation. First, the collateral constraint eq. (13) now reflects the fact that by invoicing in dollars, the bank-exporter coalition is able to increase the total quantity of safe dollar claims it can create. Again, this is because when it sets prices in dollars, and these prices are sticky, the lower bound on future dollar revenues is higher. Second, we have added an additional constraint in eq. (14) which says that all local-currency safe claims must be backed by projects with local-currency revenues. This rules out a perverse outcome where exporters first bear a cost to invoice their projects in dollars, and then turn around and use these dollar revenues to back local-currency safe claims.[12]

Define λ, μ and κ to be the Lagrange multipliers on the three constraints in (12), (13) and (14) respectively. The first-order conditions for the bank's problem are given by:

$$B_\$: \qquad Q_\$ = \frac{\mu\bar{\mathcal{E}} + 1}{\lambda} \tag{15}$$

$$B_h : \qquad Q_h = \frac{\mu + 1 + \kappa}{\lambda} \tag{16}$$

$$B_R : \qquad Q_R = \frac{1}{\lambda} \tag{17}$$

$$\eta : \qquad \eta = \frac{\left[\mu\left(\bar{\mathcal{E}} - 1\right) - \kappa\right]\gamma_L}{\phi} = \frac{\gamma_L}{\beta\phi}\left(Q_\$ - Q_h\right) \tag{18}$$

Equation (18) captures the key wrinkle in this variant of the model: now, as soon as the UIP deviation $(Q_\$ - Q_h) > 0$, it must be that $\eta > 0$, i.e., there is some amount of dollar invoicing by EM exporters in equilibrium. Intuitively, the marginal cost to an exporter of doing the first unit of dollar invoicing is zero. Therefore, at least some will occur so long as there is any benefit to doing so in terms of providing exporters with the dollar revenues that make it easier for them to tap cheaper dollar financing.

With this apparatus in hand, we can generalize Proposition 2. Now as $\alpha_\$$ increases from zero to one, we pass through three distinct regions of the parameter space, rather than just two. In the first, lower-$\alpha_\$$ region, we have $(Q_\$ - Q_h) < 0$ and $B_\$ = 0$. That is, banks do not finance any of their projects with safe dollar claims, because the interest rate on dollar deposits is higher than that on local-currency deposits. In the second, intermediate-$\alpha_\$$ region, we have $(Q_\$ - Q_h) > 0$, $\eta > 0$, and $B_\$ = \eta\gamma_L N$. Here there is some amount of dollar invoicing by exporters, and dollar-invoiced projects are the only source of collateral that is used to back safe dollar claims—no

[12]Such an outcome is endogenously ruled out as soon as one notes that the local currency can appreciate, as well as depreciate, against the dollar. For example, denoting the most appreciated value of the local currency by $\underline{\mathcal{E}} < 1$, one can never use a unit of dollar revenues to back more than $\underline{\mathcal{E}}$ units of local-currency safe claims. Incorporating this constraint explicitly into the optimization is formally identical to incorporating eq. (14).

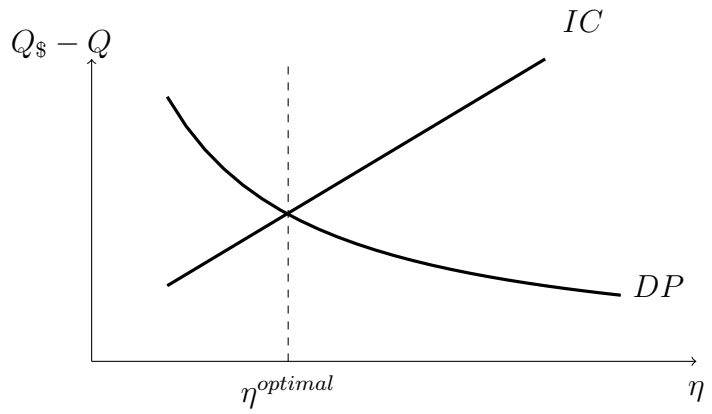


Figure 2: Determination of Dollar Export Share η

dollar claims are backed by home currency projects. Finally, in the third, upper-$\alpha_\$$ region, we have $(Q_\$ - Q_h) > 0$, $\eta > 0$, and $B_\$ > \eta\gamma_L N$. That is, dollar deposits are backed both by dollar-invoiced projects, as well as by the remaining local-currency projects, as they were in the earlier setting. Or said differently, here the banks (or the locally-oriented firms they lend to) take on some degree of currency mismatch, as they did in Proposition 2

In the second and third regions there is a unique positive solution for η given exogenous parameters. The determination of η is depicted in Figure 2. The upward-sloping IC line (for "Invoicing Choice") corresponds to eq. (18), which says that exporters' incentive to price in dollars is increasing in the magnitude of the UIP violation $(Q_\$ - Q_h)$. The downward-sloping DP curve (for "Dollar Premium") says that the magnitude of the UIP violation in turn depends on the production of dollar safe claims, and hence is declining in the amount of dollar-invoiced exports. This latter curve is derived by combining the demand for safe assets, equations (3), (4) and (5) with equations (15), (16, (17) and the collateral constraint equation (13). The resulting expressions for $(Q_\$ - Q_h)$ are:

$$Q_\$ - Q_h = \frac{1}{\alpha_h + \alpha_\$}\left(\frac{\theta\alpha_\$}{(\eta\gamma_L N + X_\$)} - \frac{\theta\alpha_h}{\gamma_L N_0 + (1-\eta)\gamma_L N}\right)$$

in the second region of the parameter space, and

$$Q_\$ - Q_h = \frac{\theta(\bar{\mathcal{E}} - 1)}{\gamma_L(N_0 + (1-\eta)N) + \bar{\mathcal{E}}\eta\gamma_L N + \bar{\mathcal{E}}X_\$}$$

in the third region. The unique equilibrium value of η is then given by the intersection of the IC line and the DP curve.

The full solution to this version of the model is characterized in Proposition 3, as follows:

Proposition 3 ***[Endogenous Invoicing]*** *Define the two cut-offs $\underline{\alpha}_\$$ and $\bar{\alpha}_\$$ as:*

$$\bar{\alpha}_\$ = \frac{\alpha_h \bar{\mathcal{E}} \left(\eta^* \gamma_L N + X_\$\right)}{\gamma_L N_0 + (1-\eta^*)\gamma_L N} \tag{19}$$

$$\underline{\alpha}_\$ = \frac{\alpha_h X_\$}{\gamma_L (N_0 + N)} \tag{20}$$

The solution to the model can then be characterized as:

$$\eta = \begin{cases} 0 & \textit{if } \alpha_\$ < \underline{\alpha}_\$ \\ \in [0, \eta^*] & \textit{if } \underline{\alpha}_\$ \leq \alpha < \bar{\alpha}_\$ \\ \eta^* & \textit{if } \alpha_\$ \geq \bar{\alpha}_\$ \end{cases} \tag{21}$$

$$D_h = \begin{cases} \gamma_L (N_0 + N) & \textit{if } \alpha < \underline{\alpha}_\$ \\ \gamma_L N_0 + (1-\eta)\gamma_L N & \textit{if } \underline{\alpha}_\$ \leq \alpha < \bar{\alpha}_\$ \\ \frac{\alpha_h}{\alpha_h + \alpha_\$} K^* & \textit{if } \alpha \geq \bar{\alpha}_\$ \end{cases} \tag{22}$$

$$D_\$ = \begin{cases} X_\$ & \textit{if } \alpha < \underline{\alpha}_\$ \\ \eta \gamma_L N + X_\$ & \textit{if } \underline{\alpha}_\$ \leq \alpha < \bar{\alpha}_\$ \\ \frac{\alpha_\$}{(\alpha_h + \alpha_\$)\mathcal{E}} K^* & \textit{if } \alpha \geq \bar{\alpha}_\$ \end{cases} \tag{23}$$

$$Q_\$ - Q_h = \begin{cases} \frac{\theta}{\alpha_h + \alpha_\$} \left(\frac{\alpha_\$}{X_\$} - \frac{\alpha_h}{\gamma_L (N + N_0)} \right) < 0 & \textit{if } \alpha < \underline{\alpha}_\$ \\ \frac{\theta}{\alpha_h + \alpha_\$} \left(\frac{\alpha_\$}{(\eta \gamma_L N + X_\$)} - \frac{\alpha_h}{\gamma_L N_0 + (1-\eta)\gamma_L N} \right) > 0 & \textit{if } \underline{\alpha}_\$ \leq \alpha < \bar{\alpha}_\$ \\ \frac{\theta(\bar{\mathcal{E}} - 1)}{K^*} > 0 & \textit{if } \alpha \geq \bar{\alpha}_\$ \end{cases} \tag{24}$$

where

$$\eta^* = \frac{-\beta\phi \left(\gamma_L (N_0 + N) + \bar{\mathcal{E}} X_\$\right) + \sqrt{\beta^2 \phi^2 \left(\gamma_L (N_0 + N) + \bar{\mathcal{E}} X_\$\right)^2 + 4\beta\phi\gamma_L^2 N (\bar{\mathcal{E}} - 1)^2 \theta}}{2\beta\phi\gamma_L N (\bar{\mathcal{E}} - 1)}$$

$$K^* \equiv \gamma_L(N_0 + (1-\eta^*)N) + \bar{\mathcal{E}}\eta^*\gamma_L N + \bar{\mathcal{E}}X_{\$} \tag{25}$$

■

Note that through market clearing $B_h = D_h$ and $B_{\$} = D_{\$} - X_{\$}$. Figure 3 illustrates Proposition 3, showing how the equilibrium values of the dollar export share η (in panel (a)), the dollar premium $(Q_{\$} - Q_h)$ (in panel (b)) and dollar borrowing $B_{\$}$ (in panel (c)) all vary as the exogenous dollar import-invoice share increases.

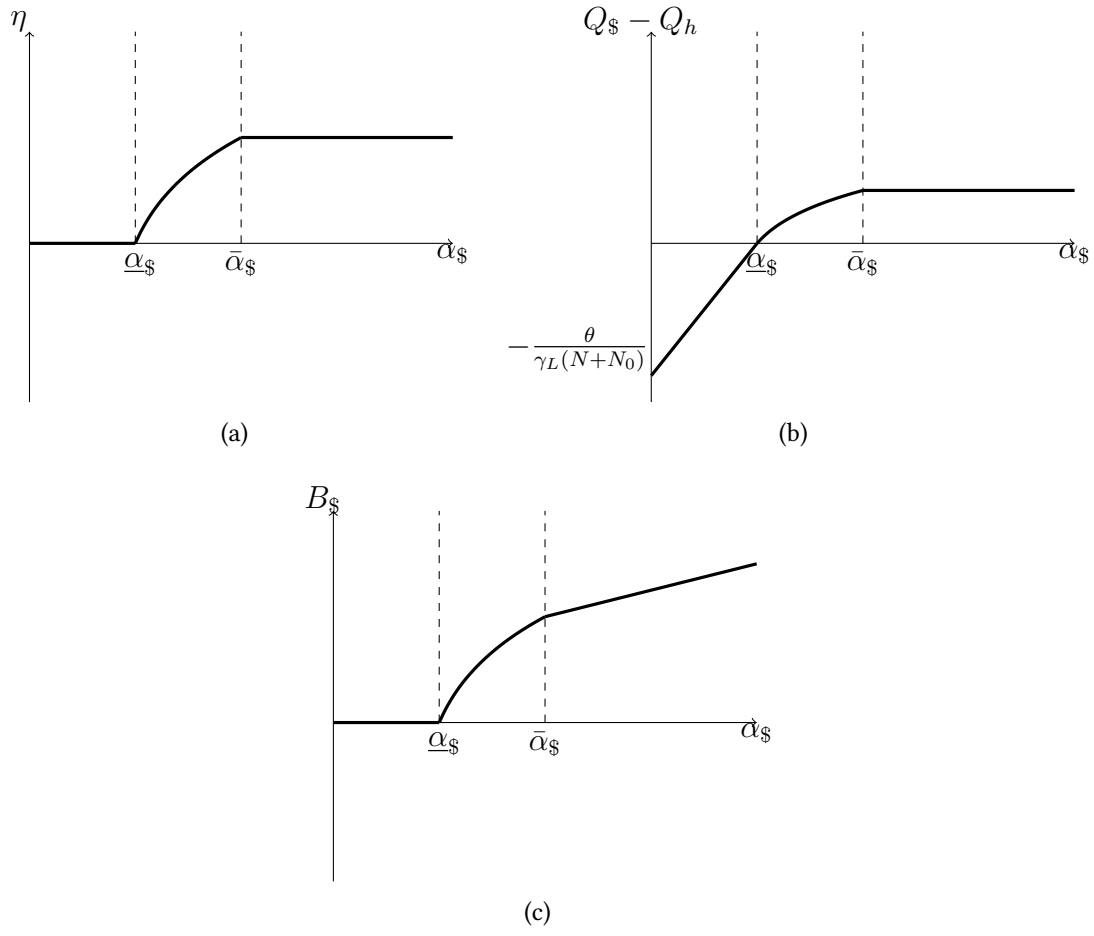


Figure 3: Equilibrium Values As Dollar Invoice Share Varies

This figure and the associated proposition summarize the second key message of the paper: we offer a novel argument for why EM firms choose to invoice their exports in dollars. The existing literature has no role for financing considerations and instead focuses on factors that influence the optimal degree of cost pass-through into prices, such as the contributions of Friberg (1998), Engel

(2006), Gopinath et al. (2010), Goldberg and Tille (2013). An alternate explanation, as developed in Rey (2001) and Devereux and Shi (2013), is that the dollar is used as a vehicle currency to minimize transaction costs of exchange.

By contrast, here we set aside all these factors and provide a complementary explanation that relates exporters' pricing decisions to their desire to borrow in a cheap currency. Indeed, in our model the *only* reason exporters choose to invoice in dollars is because by doing so they are able to more cheaply finance their projects.[13]

Remark 3 *Why is the Export-Pricing Decision Relevant if Exporters Can Hedge?*

At first glance, one might think that there is no need for an exporter firm that wants to insulate its dollar revenues to invoice its sales in dollars; it could instead invoice in home currency and then overlay a foreign exchange swap to convert the proceeds from the sale into dollars. Or said a bit differently, invoicing in dollars bundles together a goods-pricing decision with a risk-management decision, and in principle these two decisions could be unbundled, in which case the model's predictions for invoicing behavior would be less clear cut.

A recent theoretical and empirical literature (Rampini and Viswanathan (2010), Rampini et al. (2017), among others) has argued that, due to financial contracting frictions, hedging of this sort by both operating firms and financial intermediaries tends to be quite constrained. The broad idea of this work is that when a firm wishes to enter (say) a forward contract to hedge its FX risk, it needs to post adequate collateral to ensure that it will be able to perform should the hedge move against it. In a world of financial frictions, posting such collateral is necessarily expensive, as it draws resources away from real investment activities.

To see why such frictions can make invoicing in dollars preferred to FX hedging in our setting, consider the following example. An exporter in Mexico plans to offer machines for sale in Brazil. It can either price these machines in Mexican pesos, and then enter into a forward contract with a derivatives dealer to convert the pesos into dollars; or it can price the machines in dollars. In the former case, it needs to be able to assure the derivatives dealer that the sale of the machines will actually happen and will generate the stipulated revenues, and that these revenues will not be diverted by the exporter before the dealer can get its hands on them. If this is difficult or expensive to do, the exporter will be required to post a significant amount of collateral in order to enter the hedging transaction. Moreover, if it is already liquidity-constrained, this posting of collateral will in turn compromise its ability to do real investment. In contrast, if the exporter

[13] Baskaya et al. (2017) use micro data for Turkish firms and banks to to show that there is indeed a failure of UIP and bank loans denominated in dollars are cheaper than those in Turkish lira.

invoices in dollars, these problems of assuring performance disappear. Effectively, by bundling the two decisions, it sources its hedge from somebody (the Brazilian importer) who is already fully protected from default on the part of the exporter, because the importer does not have to turn over any cash until it receives its machines, and is not promised anything other than the machines in any state of the world. Compare this with the derivatives dealer who makes a payment in one state (when the dollar depreciates against the peso) in the hopes of receiving a potentially default-prone payment in another state (when the dollar appreciates against the peso).

4 Endogenous Invoice Shares and Multiple Equilibria

In the previous section we endogenized the invoicing choices of exporter firms but did not link these decisions to the shares $\alpha_{\$}$ and α_h that determine the preferences of importers for safe assets. In this section we close the loop. To do so we extend the model to include many emerging markets that trade with each other. Specifically, we now consider a world comprised of one large economy—namely the U.S.—and a continuum of small open economies (EMs) of measure one. The EM we described in the previous section is one of this continuum and therefore of measure zero. This extension of course introduces multiple exchange rates. To keep the analysis tractable we assume that households in each EM demand safe assets only in their own local currency and in dollars. The idea is that local-currency consumption and dollar-invoiced imports are always a non-negligible fraction of expenditures in each EM country; the latter because the U.S. is discretely large. By contrast, imports from any single other EM are only an infinitesimal share of the expenditure bundle. Therefore, if we think of there being a small fixed cost of setting up a deposit account in each currency, citizens of country i will only want to do so in dollars and in country-i currency, rather than having to set up an infinite number of such accounts to cover all the currencies of the world.

Exporters in each of the EMs can choose to invoice their exports in either their own currency or in dollars.[14] We assume that the dollar invoice share facing importers in EM country i is given by

$$\alpha_{\$i} \equiv a + b \int_{j \neq i} \eta_j dj$$

where $a > 0$ and $b > 0$ are two constants with $a + b < 1$, and where η_j is the fraction of the N projects in country j that are priced to generate dollar revenues, as chosen by exporters in country j. Simply put, if exporters in the rest of the world price more of their exports in dollars,

[14] They will never want to invoice in a third currency, as will become clear.

importers in i who import from these countries have a higher share of dollar-invoiced goods in their own expenditures.

The key exogenous parameters in the model are now a and b, as opposed to $\alpha_{\$i}$. What is the economic interpretation of these parameters? Suppose we think of country i as importing goods from other EMs and from the U.S. Moreover, assume that U.S. exporters always price in dollars, no matter what. In this case, the parameter a corresponds to the share of U.S. goods in country-i expenditures, and the parameter b corresponds to the share of goods from all other EM countries $j \neq i$ in country-i expenditures. In terms of the mechanics of the model, a acts as an exogenous anchor on import-invoice shares, while b serves as a feedback coefficient—meaning that the higher is b, the stronger is the feedback from the rest of the EM world's export-pricing decisions to import-invoice shares, and vice-versa, and hence the stronger are the strategic-complementarity effects that can give rise to multiple equilibria.

By keeping a constant across all EM countries, we are effectively assuming that all EMs are equally exposed to the U.S. as a trading partner. This makes for a convenient simplification, though it is straightforward to generalize. Finally, we also assume that the market for dollar deposits is integrated, meaning that country-i citizens can obtain safe dollar claims from anywhere in the world. This ensures that the interest rates on dollar deposits offered by banks is the same across countries. By contrast, home-currency markets are segmented across the countries. These assumptions imply that the market-clearing conditions are given by:

$$B_{hi} = D_{hi} \tag{26}$$

$$B_{Ri} = A_{Ri} \tag{27}$$

$$\int_i B_{\$i} di + X_{\$} = \int_i D_{\$i} di \tag{28}$$

As just noted, for sufficiently large values of the invoicing-feedback coefficient b, we can obtain multiple equilibria, with differing degrees of dollar invoicing. Intuitively, if exporters in all countries $j \neq i$ price a lot of their sales in dollars, this raises the dollar invoice share $\alpha_{\$i}$ facing country-i importers—and more so if b is larger. Given this higher value of $\alpha_{\$i}$, country-$i$ importers demand more dollar-denominated deposits, which tends to push down dollar interest rates. These low dollar rates in turn validate the original decision on the part of country-j exporters to price in dollars; they do so precisely because it helps them to tap more of the cheap dollar funding. This line of reasoning explains how we can sustain an equilibrium where the dollar is used relatively intensively in both trade and banking. Conversely, a less dollar-intensive equilibrium can also be self-sustaining. In this case, there is less invoicing in dollars, which lowers the demand on the

part of importers for safe dollar claims, and therefore leads to higher interest rates on safe dollar claims. These higher rates in turn validate the choice on the part of exporters to do less in the way of pricing their exports in dollars.

Proposition 4, which is illustrated in Figures 4 and 5, formalizes this intuition. The proposition again divides the parameter space into three regions, but now the exogenous parameter that defines the regions is a, not $\alpha_\$$. Recall again that a can be interpreted as the U.S. share in expenditures of all EM countries.

Proposition 4 ***[Multiple equilibria with varying degrees of dollar invoicing]*** *Define two cut-offs $\underline{a}$ and $\bar{a}$ as:*

$$\underline{a} \equiv \frac{\alpha_h \bar{\mathcal{E}} \left(\eta^* \gamma_L N + X_\$\right)}{\gamma_L N_0 + (1-\eta^*)\gamma_L N} - b\eta^* \tag{29}$$

$$\bar{a} \equiv \frac{\alpha_h X_\$}{\gamma_L (N_0 + N)} \tag{30}$$

If the invoicing-feedback coefficient b is large enough—specifically, if

$$b > \frac{1}{\eta^*}\left(\frac{\alpha_h \bar{\mathcal{E}} \left(\eta^* \gamma_L N + X_\$\right)}{\gamma_L N_0 + (1-\eta^*)\gamma_L N} - \frac{\alpha_h X_\$}{\gamma_L (N_0 + N)}\right)$$

we can describe the solution of the model according to three regions. In the low-a region where $a < \underline{a}$, the only stable equilibrium is one in which $\eta = 0$. In the high-a region where $a > \bar{a}$, the only stable equilibrium is one in which $\eta = \eta^*$. And in the intermediate-a region where $\underline{a} < a \leq \bar{a}$, there are multiple stable equilibria: one with $\eta = 0$, and one with $\eta = \eta^*$. The values of all of the other endogenous variables in these two equilibria are the same as given by the corresponding expressions for the lower and upper ranges in Proposition 3. ■

There are two broad messages to take away from Proposition 4 and the accompanying figures. First, as the share of EM imports from the U.S.—proxied for by the parameter a—gradually increases from zero, we eventually must get a discrete jump in the global role of the dollar, by $\underline{a}$ at the earliest, or by $\bar{a}$ at the latest . This jump occurs when other countries besides the U.S. start pricing some of their exports in dollars as well. When they do so, the dollar premium jumps also, and the lower interest rate on dollar safe claims is precisely what helps to support the decision of non-U.S. exporters to price their sales in dollars. Second, because of these strategic complementarities, there can be some indeterminacy in the outcome when imports from the U.S. are in

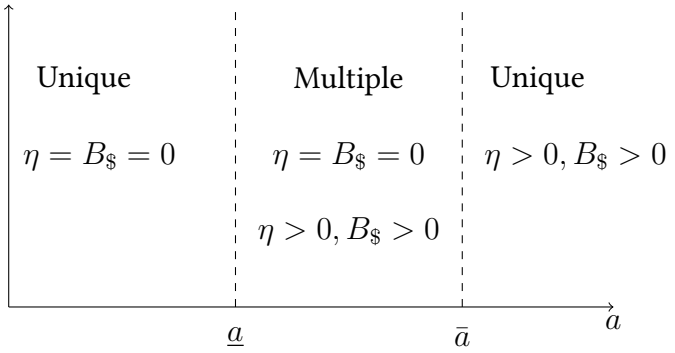
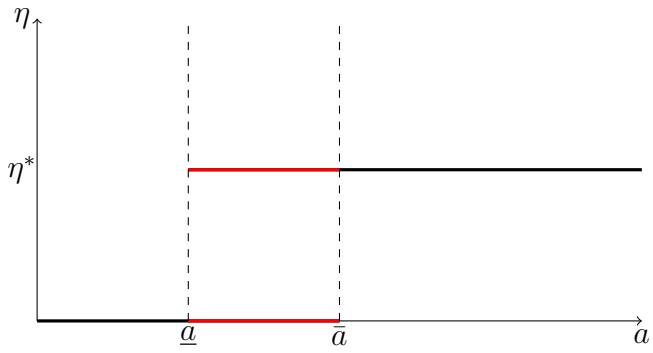
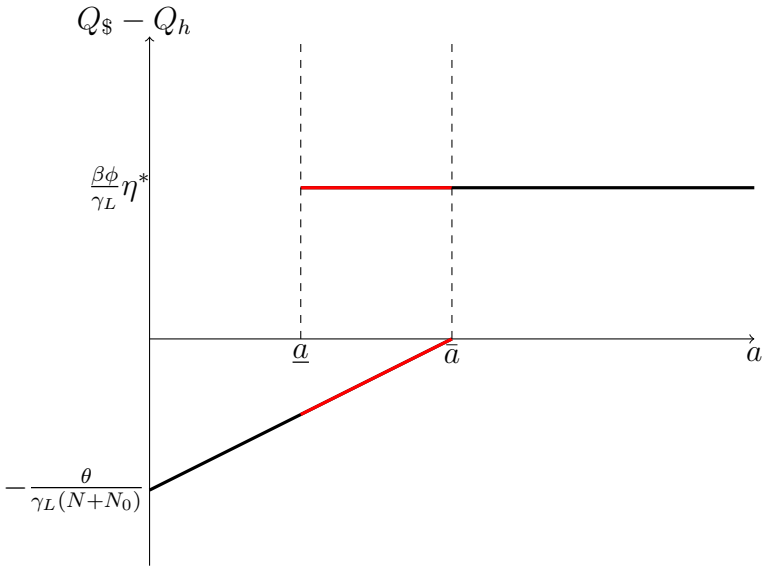


Figure 4: Regions of Uniqueness and Multiplicity



(a)



(b)

Figure 5: Equilibrium Values as Share of Imports From U.S. Varies

a middle range. This indeterminacy may leave the door open for historical factors to pin down what actually happens; we return to this point in more detail below.

5 The Dollar vs. the Euro: Will One Currency Dominate?

In this section we explore the possibility of the emergence of a *single* globally dominant currency out of several possible alternatives. To do so we need to create a level playing field where we pit two candidate currencies against one another, and then ask what the potential outcomes are. This is what we do next. In particular, we now consider a symmetric setting where there are two possible global currencies, the dollar and the euro, with identical economic fundamentals. And the question we are going to be most interested in is this: are there circumstances where, in spite of the symmetry in fundamentals, the only possible equilibrium outcomes are asymmetric, with one global currency being used extensively by emerging-market countries to invoice their exports and to finance projects, and the other global currency not being used at all in this way?

It turns out that such asymmetric outcomes arise naturally in our framework, and they are driven by the same invoicing-feedback mechanism that led to multiple equilibria in Proposition 4 above. Intuitively, once one currency—say the dollar—gets a bit of an edge in invoice share, this tends to feed on itself: as more global trade is invoiced in that currency, there is more demand for it as a safe store of value. This in turn makes it a cheaper currency to borrow in, which leads exporters in search of lower borrowing costs to invoice their sales in that currency. Such a virtuous circle can entrench the dollar as the dominant currency, and at the same time freeze out the euro, even if there is initially no fundamental difference between the two.

5.1 Augmenting the Model

To capture this all in the model, we make several adaptations that allow us to incorporate the euro alongside the dollar. There is now an equal-sized exogenous external supply of dollar and euro safe assets available to emerging markets, that is $X_{\$} = X_{€} = X$. The goods purchased by importers in EM i can now be invoiced in either dollars or euros. The share of imports invoiced in dollars is given by $\alpha_{\$i} = a + b \int_{j \neq i} \eta_{\$j} dj$, where as before $\eta_{\$j}$ is the fraction of the N export projects in country j that are invoiced in dollars. Similarly, the share of EM i imports invoiced in euros is $\alpha_{€i} = a + b \int_{j \neq i} \eta_{€j} dj$. The domestic share α_{hi} remains exogenously fixed, as before.

We assume complete symmetry everywhere, so these expressions hold for any EM. Note that this implies that the parameter a now not only proxies for the share of U.S. goods in total EM

expenditures, it also proxies for the Euro area share, which is therefore assumed to be the same. This symmetry is designed to create a level-playing-field benchmark.

Importers in EM country i maximize the same utility function as before (given by equation (1)) but now the money aggregator M depends on the quantities of dollar, euro and local-currency deposits. That is:

$$M_i = \left(D_{hi}^{\alpha_{hi}} D_{\$i}^{\alpha_{\$i}} D_{€i}^{\alpha_{€i}}\right)^{\frac{1}{\sum \alpha_i}} \tag{31}$$

where $\sum \alpha_i = \alpha_{hi} + \alpha_{\$i} + \alpha_{€i}$. The budget constraints are now given by:

$$C_{i,0} \leq Z_{i,0} - Q_{hi} D_{hi} - \mathcal{E}_{\$i,0} Q_{\$} D_{\$i} - \mathcal{E}_{€i,0} Q_{€} D_{€i} - Q_{Ri} A_{Ri}$$

$$C_{i,1} \leq Z_{i,1} + D_{hi} + \mathcal{E}_{\$i,1} D_{\$i} + \mathcal{E}_{€i,1} D_{€i} + \xi A_{Ri}$$

The first-order conditions for D_{hi}, $D_{\$i}$, $D_{€i}$, and $A_{R,i}$ yield:

$$Q_{hi} = \beta + \theta \frac{\alpha_{hi}}{(\sum \alpha_i) D_{hi}} \tag{32}$$

$$Q_{\$} = \beta + \theta \frac{\alpha_{\$i}}{(\sum \alpha_i) D_{\$i}} \tag{33}$$

$$Q_{€} = \beta + \theta \frac{\alpha_{€i}}{(\sum \alpha_i) D_{€i}} \tag{34}$$

$$Q_{R,i} = \beta \tag{35}$$

Note that, as before, $\mathbb{E}_0(\mathcal{E}_{\$i,1}) = \mathbb{E}_0(\mathcal{E}_{€i,1}) = \mathcal{E}_{\$i,0} = \mathcal{E}_{€i,0} = 1$, and we continue to assume that the dollar and euro deposit markets are integrated, implying a common price for dollar and euro deposits.

To characterize the problem of the representative bank we need to spell out two further assumptions. First, we assume that the dollar and the euro are equally volatile with respect to the currencies of all EMs, and therefore that the maximally appreciated value of each is the same. That is, $\bar{\mathcal{E}}_{€i} = \bar{\mathcal{E}}_{\$i} = \bar{\mathcal{E}}$. This assumption has the effect of making it equally costly to use local-currency projects as collateral for either dollar or euro safe claims. Again, the goal here is to do everything we can to create a level playing field between the dollar and the euro based on fundamental considerations.

Second, when a fraction $\eta_{\$i}$ of the N export projects are priced in dollars, and a fraction $\eta_{€i}$ are priced in euros, we assume that this imposes a cost on the bank-exporter coalition of $\frac{\phi}{2} N(\eta_{\$i}^2 + \eta_{€i}^2 + 2c\eta_{\$i}\eta_{€i})$ where $0 < c < 1$. The motivation for this functional form is the same as that in the previous section: the ultimate shareholders of the export firms are risk-averse domestic agents who prefer local-currency income given their consumption basket. The one new

wrinkle is that with two non-local currencies, we now allow exporters to enjoy a diversification gain when they invoice in a mix of dollars and euros, as opposed to invoicing in only one of the two. This gain is decreasing in the parameter c, which can be thought of as a proxy for the covariance of the dollar and euro exchange rates versus the local EM currency.

With these assumptions in place, the augmented version of the bank's problem can be stated as:

$$\begin{aligned}\max_{B_{hi},B_{\$i},B_{€i},B_{Ri},\eta_{\$i},\eta_{€i}} \mathbb{E}_0[&\gamma(N_0+N)+\gamma N\eta_{\$i}(\mathcal{E}_{\$i,1}-1)+\gamma N\eta_{€i}(\mathcal{E}_{€i,1}-1)\\ &-B_{hi}-\mathcal{E}_{\$i,1}B_{\$i}-\mathcal{E}_{€i,1}B_{€i}-\xi B_{Ri}\\ &-\frac{\phi}{2}N(\eta_{\$i}^2+\eta_{€i}^2+2c\eta_{\$i}\eta_{€i})]\end{aligned}$$

subject to,

$$Q_hB_{hi}+Q_\$B_{\$i}+Q_€B_{€i}+Q_{Ri}B_{Ri}\geq N+N_0 \tag{36}$$

$$\bar{\mathcal{E}}(B_{\$i}+B_{€i})+B_i\leq\gamma_L(N_0+(1-\eta_{\$i}-\eta_{€i})N)+(\eta_{\$i}+\eta_{€i})\bar{\mathcal{E}}\gamma_LN \tag{37}$$

$$B_i\leq\gamma_L(N_0+(1-\eta_{\$i}-\eta_{€i})N) \tag{38}$$

The first order conditions with respect to $\eta_{\$i}$ and $\eta_{€i}$ are

$$\begin{aligned}\eta_{\$i}&=\frac{\gamma_L}{\beta\phi}(Q_\$-Q_{hi})-c\eta_{€i}\\ \eta_{€i}&=\frac{\gamma_L}{\beta\phi}(Q_€-Q_{hi})-c\eta_{\$i}\end{aligned}$$

Finally, the market-clearing conditions are now given by:

$$D_{hi} = B_{hi} \qquad \forall i \tag{39}$$

$$A_{Ri} = B_{Ri} \qquad \forall i \tag{40}$$

$$\int_i D_{\$i} = \int_i B_{\$i}+X \tag{41}$$

$$\int_i D_{€i} = \int_i B_{€i}+X \tag{42}$$

Before formally stating the full solution to this version of the model, it is useful to preview the types of outcomes that one can expect. Broadly speaking, depending on the value of the exogenous parameter a, three kinds of equilibria can arise. The first is a symmetric zero-η equilibrium,

where exporters do no pricing in either dollars or euros: $\eta_{\$} = \eta_{€} = 0$. The second is a symmetric positive-η equilibrium, where exporters do some pricing in both dollars and euros: $\eta_{\$} = \eta_{€} > 0$. And the third is an asymmetric dominant-currency equilibrium, where exporters exclusively use only one of the two currencies (in addition to the relevant local currency) to price their exports: $\eta_{\$} > 0, \eta_{€} = 0$ or $\eta_{€} > 0, \eta_{\$} = 0$

For any given value of a, it is possible that more than one of these types of equilibria can be sustained. For example, for some values of a, it might be the case that we can have both a symmetric zero-η equilibrium, as well as an asymmetric dominant-currency equilibrium. Nevertheless, the symmetric zero-η equilibrium is more likely to arise when a is relatively low, while the symmetric positive-η equilibrium is more likely to arise when a is high. And the asymmetric dominant-currency equilibria are most prevalent for intermediate values of a. Intuitively, this is because the parameter a proxies for the exogenous component of non-local-currency invoicing, and hence the generalized demand for safe claims denominated in some non-local currency, be it the euro or the dollar. When this demand is very low, this tends to produce outcomes where neither the dollar nor the euro plays an important role in global trade. And when it is very high, we can get situations where both are prominently used. But in the intermediate region—and this is of particular interest to us—it can effectively be the case that while there is enough safe-asset demand to sustain one global currency, there is not enough to sustain two. This is what can lead to there being a single dominant currency.

Proposition 5 *[**Dominant Currency**] The solution is characterized by the following three equilibria, the existence of which depends on parameter values:*

1. *No dominant currency equilibrium:*

$$
\begin{aligned}
D^n_{\$} &= D^n_{€} = X \\
D^n_h &= \gamma_L(N_0 + N) \\
B^n_{\$} &= B^n_{€} = 0 \\
Q^n_{\$} &= Q^n_{€} = \beta + \theta \frac{a}{(\alpha_h + 2a)X} \\
Q^n_h &= \beta + \theta \frac{\alpha_h}{(\alpha_h + 2a)\gamma_L(N_0 + N)} \\
\eta^n_{\$} &= \eta^n_{€} = 0
\end{aligned}
$$

where the superscript 'n' stands for 'no dominant currency'. For this to be an equilibrium it must be $(Q^n_{\$} - Q^n_h) = (Q^n_{€} - Q^n_h) < 0$

2. *Asymmetric (Single) dominant currency equilibrium:*

$$
\begin{aligned}
D^s_{\$} &= \frac{a+b\eta^s}{\alpha_h+a+b\eta^s}\frac{K^s}{\bar{\mathcal{E}}}, \qquad D^s_{€} = X \\
D^s_h &= \frac{\alpha_h}{\alpha_h+a+b\eta^s}K^s \\
B^s_{\$} &= D_{\$} - X, \qquad B^s_{€} = 0 \\
Q^s_{\$} &= \beta + \frac{\theta\bar{\mathcal{E}}}{K^s}\left(\frac{\alpha_h+a+b\eta^s}{\alpha_h+2a+b\eta^s}\right) \\
Q^s_{€} &= \beta + \frac{\theta}{X}\frac{a}{(\alpha_h+2a+b\eta^s)} \\
Q^s_h &= \beta + \frac{\theta}{K^s}\left(\frac{\alpha_h+a+b\eta^s}{\alpha_h+2a+b\eta^s}\right) \\
\eta_{\$} &= \eta^s = \frac{\gamma_L}{\phi\beta}\left(Q^s_{\$} - Q^h_h\right), \qquad \eta_{€} = 0
\end{aligned}
$$

$$K^s = \gamma_L N_0 + (1-\eta^s)\gamma_L N + \eta^s\bar{\mathcal{E}}\gamma_L N + \bar{\mathcal{E}}X$$

where the superscript 's' stands for 'single dominant currency'. For this to be an equilibrium it must be that $\left[\frac{\gamma_L}{\phi\beta}(Q^s_{€} - Q^s_h) - c\eta^s\right] < 0$ *and* $B^s_{\$} > \eta^s\gamma_L N$.

3. *Symmetric (Both) dominant currency equilibrium:*

$$
\begin{aligned}
D^b_{\$} &= D^b_{€} = \frac{(a+b\eta^b)}{(\alpha_h+2a+2b\eta^b)}\frac{K^b}{\bar{\mathcal{E}}}, \\
D_h &= \frac{\alpha_h}{\alpha_h+2a+2b\eta^b}K^b \\
Q_{\$} &= Q_{€} = \beta + \frac{\theta\bar{\mathcal{E}}}{K^b} \\
Q_h &= \beta + \frac{\theta}{K^b} \\
\eta_{\$} &= \eta_{€} = \eta^b = \frac{\gamma_L}{\phi\beta(1+c)}\left(Q_{\$} - Q_h\right)
\end{aligned}
$$

$$K^b = \gamma_L N_0 + (1-2\eta^b)\gamma_L N + 2\eta^{*b}\bar{\mathcal{E}}\gamma_L N + 2\bar{\mathcal{E}}X$$

where the superscript 'b' stands for the case where 'both' the dollar and euro are dominant currencies. For this to be an equilibrium it must be that $B^b_{\$} = B^b_{€} > \eta^b\gamma_L N$.

■

As in the previous section, we are interested in the values of the parameter a for which each of

these equilibria can exist. Using the conditions listed above, we can derive the following four cut-offs. First, $\underline{a}^s$ defines the lower end of the asymmetric-equilibrium region: it is the cut-off such that for $a < \underline{a}^s$ an equilibrium with one positive η cannot be sustained, and we can only sustain the no-dominant-currency equilibrium. Second, $\bar{a}^s$ defines the upper end of the asymmetric-equilibrium region: it is the cut-off above which an equilibrium with only one positive η again cannot be sustained, in this case leaving as the only possible outcome the dual-dominant-currency equilibrium. Third, $\bar{a}^n$ is the cut-off above which a no-dominant-currency equilibrium with both $\eta = 0$ cannot be sustained. And finally, $\bar{a}^b$ is the cut-off below which a dual-dominant-currency equilibrium with both $\eta > 0$ cannot be sustained. The formulas for these four cut-offs are as follows:

$$\begin{aligned}
\bar{a}^n &= \frac{\alpha_h X}{\gamma_L(N_0+N)} \\
\frac{(\underline{a}^s + \eta^s(\underline{a}^s)b)}{\alpha_h + \underline{a}^s + b\eta^s(\underline{a}^s)} \frac{K^s(\underline{a}^s)}{\bar{\mathcal{E}}} &= \eta^s(\underline{\mathrm{a}}^s)\gamma_L N + X \\
\bar{a}^s &= \frac{(\alpha_h + b\eta^s(\bar{a}^s)b))(\theta\gamma_L X + c\eta^s(\bar{a}^s)\phi\beta K^s(\bar{a}^s)X)}{\theta\gamma_L(K^s(\bar{a}^s) - X) - 2c\eta^s(\bar{a}^s)\phi\beta K^s(\bar{a}^s)X} \\
\frac{(\bar{a}^b + b\eta^b)}{(\alpha_h + 2\bar{a}^b + 2b\eta^b)} \frac{K^b}{\bar{\mathcal{E}}} &= \eta^b\gamma_L N + X
\end{aligned}$$

where,

$$K^s(a) = \gamma_L N_0 + (1 - \eta^s(a))\gamma_L N + \eta^s(a)\bar{\mathcal{E}}\gamma_L N + \bar{\mathcal{E}}X$$

$$K^b = \gamma_L N_0 + (1 - 2\eta^b)\gamma_L N + 2\eta^b\bar{\mathcal{E}}\gamma_L N + 2\bar{\mathcal{E}}X$$

Since some of the cut-off formulas do not have closed-form solutions, we cannot provide a sharp analytical characterization of how the cut-offs line up. However, in Figure 6 below we depict one intuitively natural ordering which arises for a range of plausible parameter values (although our experimentation suggests that other orderings are also possible). What is particularly noteworthy about this ordering is that there is an intermediate range of values of a—namely, where $\bar{a}^n < a < \bar{a}^b$—where the only possible equilibrium is one with a single dominant currency.

5.2 Numerical Example

In this section we provide a detailed numerical example that generates the same ordering of cut-offs as in Figure 6. The parameters used are listed in Table 1.

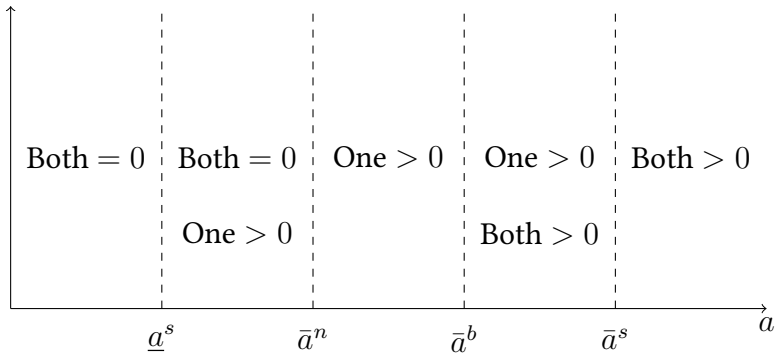


Figure 6: Equilibria supported as a function of 'a'

Parameter	N	N_0	X	α_h	ϕ	θ	β	γ_L	$\bar{\mathcal{E}}$	b	c
Value	7	7	3	0.2	0.1	1.4	0.8	0.7	2	0.5	0.8

Table 1: Parameter Values

As the figures show, in the no-dominant-currency case, which is the short-dashed (blue) line labeled "Both=0", we have that $\eta = 0$ and $B_{\$} = B_{€} = 0$. There is no incentive to invoice in a global currency, as $(Q_{\$} - Q) = (Q_{€} - Q) < 0$. In this range as a increases the negative gap between the dollar (euro) bond price and the EM bond declines as a consequence of the exogenous increase in demand for dollar and euro safe assets. The dollar invoicing share in importer preferences, defined as $\hat{\alpha}_{\$} = \left(\alpha_{\$} / \sum_{k\in\{\$,€,h\}} \alpha_k\right)$, and the euro invoicing share, $\hat{\alpha}_{€} = \left(\alpha_{€} / \sum_{k\in\{\$,€,h\}} \alpha_k\right)$, both increase by the same amount with the exogenous increase in a.

In the case of a single dominant currency, depicted by the long-and-short dashed (yellow) line marked "One>0", there is positive invoicing in one of the two global currencies, whose η is plotted. For the purposes of discussion, we assign this dominant role to the dollar. The euro on the other hand is not used in trade invoicing, and EM banks do not create any safe euro claims. This difference in dollar and euro invoicing leads to a divergence between $\hat{\alpha}_{\$}$ and $\hat{\alpha}_{€}$, with the former jumping sharply relative to the no-dominant-currency case because of the endogenous increase in dollar invoicing, while the latter falls. Indeed,$\hat{\alpha}_{\$}$ exceeds $\hat{\alpha}_{€}$ for all values of a for which this asymmetric equilibrium is sustainable. Consistent with this, in this equilibrium, $(Q_{\$} - Q)$ is always positive and exceeds $(Q_{€} - Q)$. More subtly, $(Q_{€} - Q)$ is negative for lower values of a and then turns positive, but even at this point there is still no incentive to invoice in euros as long as $(Q_{€} - Q) < c\eta_{\$}$. The figures also illustrate that in this equilibrium the bank-exporter coalition bears a dollar currency mismatch—in the sense that dollar deposits exceed dollar-denominated collateral— while there is no euro mismatch. In addition, the dollar's use in trade invoicing $\alpha_{\$}$

greatly exceeds the U.S. share in world trade a, while that same ratio equals one for the euro. This is very much in line with the empirical evidence on trade invoicing.

The case of dual dominant currencies is graphed as the solid (orange) line labeled "Both>0". Now η represents invoicing in both dollars and euros, and it is symmetric and constant over this range. The size of the exorbitant privilege and the extent of currency mismatch are now also identical across dollars and euros. The vertical lines demarcate the regions that support the different equilibria.

5.3 Which Currency Dominates? The Role of History

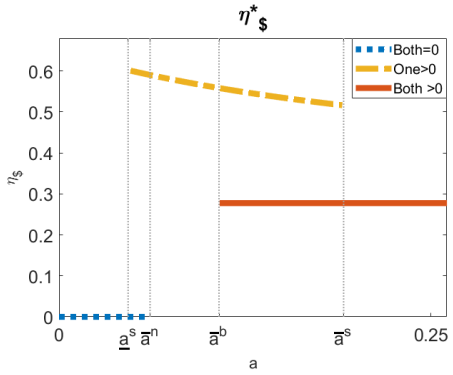
As discussed above, we are particularly interested in asymmetric dominant-currency equilibria, where exporters exclusively use only one of the two currencies (in addition to the relevant local currency) to price their exports: either $\eta_{€} = 0, \eta_{\$} > 0$, or $\eta_{€} > 0, \eta_{\$} = 0$. Our interest is motivated by the fact that the former configuration aligns very closely with what we observe in reality. In particular, although the U.S. and Eurozone economies are the two largest in the world, trade invoicing by all countries other than these two skews almost entirely to the dollar: as noted in the introduction, the volume of international trade that is invoiced in dollars is several times that of imports coming from the U.S., while the volume of trade that is invoiced in euros is very similar to that of imports coming from the Eurozone.[15] Since the η's correspond precisely to export-pricing decisions made in countries other than the U.S. and Europe, it appears that we are in a situation that is strikingly similar to what the model envisions in an equilibrium with $\eta_{€} = 0$.

However, while the model suggests that we may well wind up in an asymmetric equilibrium where one currency dominates in this lopsided fashion, it is unable to speak to *which* currency that will be, given that it treats the U.S. and Europe as being identical on all fundamental dimensions. Taken literally, the model says that the outcome is indeterminate.

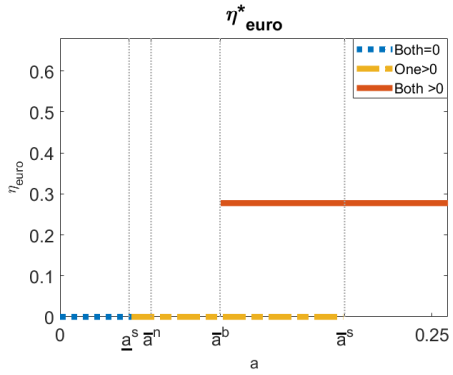
To break this indeterminacy, it may be useful to assign a role to history. Here is what we have in mind. If one steps away from the symmetric case where the U.S. and European shares in imports of other countries are the same—i.e., where $a_{\$} = a_{€}$—there can for a wide range of parameter values be just a single deterministic equilibrium outcome. Specifically, if $a_{\$}$ is much larger than $a_{€}$, it may well be that the only equilibrium is one in which $\eta_{€} = 0, \eta_{\$} > 0$.[16] In the case of the U.S. and Europe, something like this might have been a good description of the situation that existed before the formation of Eurozone in 1999, when all the member countries

[15] The Euro Area refers to the 19 countries that use the euro as their common currency. Defined this way, the largest "countries" by GDP as of 2016 were, in descending order: the U.S., the Euro Area, China, Japan, and the U.K.

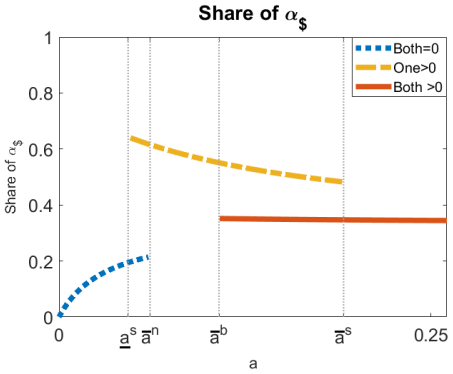
[16] To see this point explicitly, note that the unique outcome in the high-a region in Proposition 4 is just the limiting case of such an equilibrium, where we keep $a_{\$}$ large while allowing $a_{€}$ to go to zero.



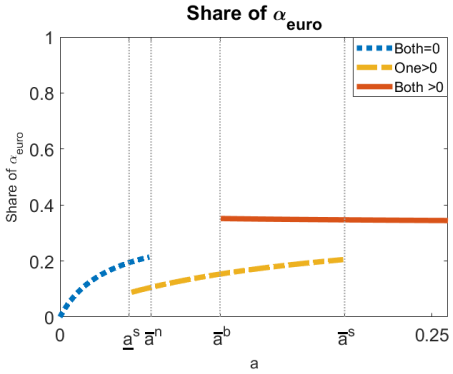
(a) \$ invoicing



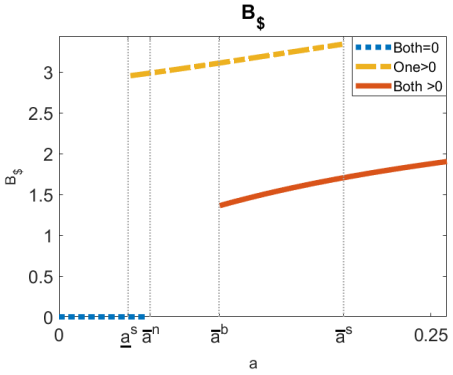
(b) € invoicing



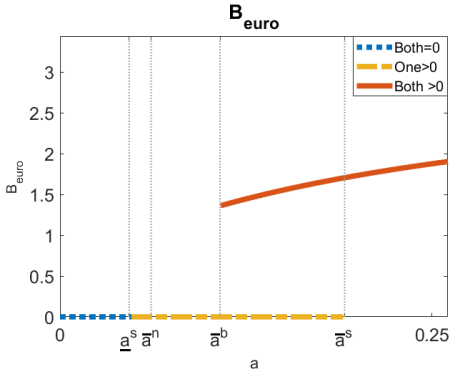
(c) \$ invoicing share



(d) € invoicing share

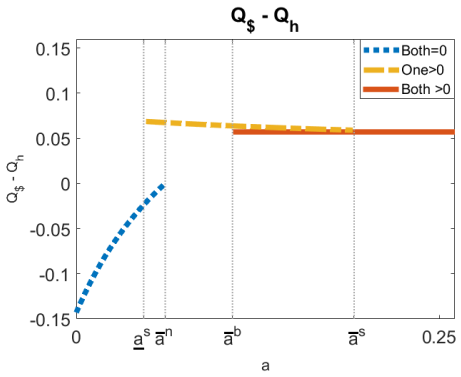


(e) Bank dollar deposits

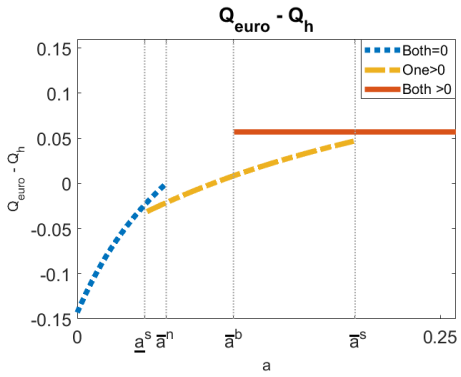


(f) Bank euro deposits

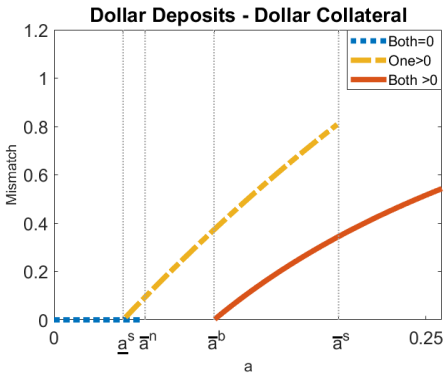
Figure 7: Numerical Example



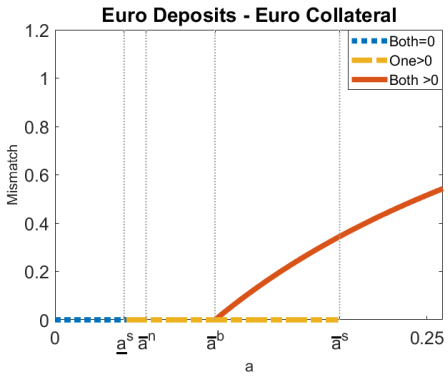
(a) Dollar Exorbitant Privilege



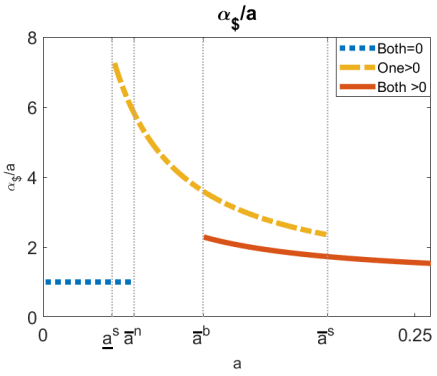
(b) Euro Exorbitant Privilege



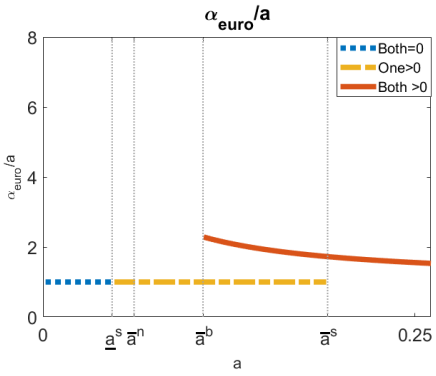
(c) Dollar Mismatch



(d) Euro Mismatch



(e) Dollar Trade Invoicing



(f) Euro Trade Invoicing

Figure 7: Numerical Example (continued)

had their own currencies, and the largest individual member, Germany, had a GDP only about a fifth that of the U.S. So applied to the pre-Eurozone period, our model might well have predicted that the only possible equilibrium outcome was one in which the dollar was the lone dominant currency.

Now suppose that after the Eurozone forms, it is large enough so that given current parameter values, the model admits two equilibrium outcomes: one where the dollar is dominant and one where the euro is dominant. Which of the two is likely to actually obtain? To the extent that there is any history-dependence, it would naturally seem to be the dollar-dominant equilibrium. In other words, any time we are faced with multiple possible equilibrium outcomes at some date t, a plausible selection mechanism would be to go back in time to the first date prior to t when one of those equilibria is uniquely pinned down by the model, and posit that it then remains as the focal equilibrium until the parameters change to the point where it is no longer viable.

If one accepts this line of reasoning, it suggests that even if the European economy grows to the point where it catches up with—or even somewhat surpasses—the U.S., this may not be enough to dislodge the now-entrenched dollar from its dominant-currency perch. According to the dynamic equilibrium-selection process outlined above, this might require the European economy to get substantially bigger than the U.S., to the point where $\eta_{€} > 0, \eta_{\$} = 0$ becomes the unique equilibrium outcome. Alternatively, even with no catch-up of Europe relative to the U.S., with enough growth on the part of both we could conceivably get to a point where both $a_{\$}$ and $a_{€}$ are so big—i.e. where both countries are so important as a share of world imports—that the only possible equilibrium is one where both the dollar and the euro are used by other countries to invoice their exports. That is, we could wind up in a situation where the only possible outcome is a symmetric one with $\eta_{\$} = \eta_{€} > 0$. And of course, exactly the same observations apply if, instead of Europe, one asks about the prospects for the Chinese renminbi to become a globally-dominant currency: even as its fundamentals approach those of the U.S., it is likely to be handicapped by history, which we would argue can play an important role in selecting the equilibrium in a setting like that of our model.

5.4 Cross-Country Empirical Evidence

A fundamental starting premise of the paper is that a currency's role as a unit of account is complementary to its role as a store of value. In this section, we test a simple cross-sectional version of this hypothesis.

To get started, note that in any equilibrium of our model, the first-order conditions of the

importers imply that:

$$\frac{D_{\$,i}}{D_{€,i}} = \frac{\alpha_{\$,i}}{\alpha_{€,i}} \cdot \frac{Q_{€} - \beta}{Q_{\$} - \beta} \tag{43}$$

In other words, if importers in country i have a greater share of their imports invoiced in dollars—relative to euros—than importers in country j, they will hold a greater share of their deposits in dollars as well. To operationalize a test of this prediction, we need to make two assumptions. First, country-i importers keep their deposits in country-i banks. Second, recalling that $D_{\$,i} = B_{\$,i} + X_{\$,i}$, where $X_{\$,i}$ denotes holdings by country-i importers of U.S. Treasury securities and the like, we need to assume that investments by importers in these securities are intermediated through the banking system, rather than being directly held. This implies that the asset side of bank balance sheets in country i includes not only their real projects, but also the $X_{\$,i}$.

With these assumptions in place, the mapping from the theory to the data is straightforward. From Gopinath (2015) we have data at the country level on import invoice shares in different currencies. From the BIS Locational Banking Statistics, we can compute for any country in the dataset the fraction of foreign-currency local banking liabilities that are denominated in dollars. To be clear, for a given country i, these numbers are based on all banks domiciled in i, be they locally-headquartered banks or subsidiaries of foreign bank holding companies.

In the top panel of Figure 8 we plot this measure against the share of a country's foreign-currency-invoiced imports that are denominated in dollars. For the twelve countries for which both of these data are available, there is indeed a strong positive relation between the two variables, with a slope coefficient equal to 0.79 and an $R^2 = 0.75$. In the lower panel of Figure 8 we attempt to create an alternative measure of banking activity that is somewhat cleaner relative to the theory we have in mind; we do so by restricting attention to banking liabilities in the sub-category "loans and deposits", and where the counterparty (i.e., the party making the loan to the bank in question) is itself a non-bank institution. The aim here is to eliminate inter-bank lending transactions, and other wholesale sources of funding that are less likely to come from local depositors. This refinement cuts our sample down from twelve to ten countries, but leads to a very similar picture: the slope coefficient is now 0.78 and the $R^2 = 0.83$. Thus either way, there seems to be a strong empirical association at the country level between dollar-invoiced imports and dollar-denominated bank deposits.

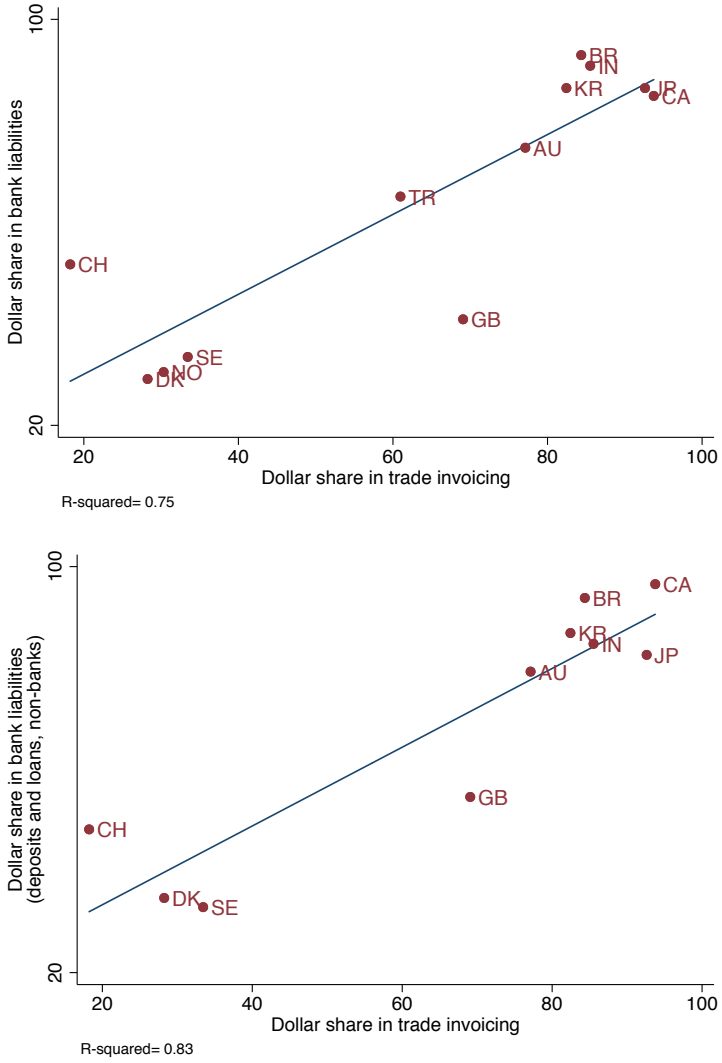


Figure 8: Dollar Share in Trade Invoicing and Banks Local Foreign Currency Liabilities

6 Conclusion

The central theme of this paper is that there is fundamental connection between the dollar's role as the currency in which non-U.S. exporters predominantly invoice their sales, and its prominence in global banking and finance. Moreover, these two roles feed back on and reinforce each other. Going in one direction, a large volume of dollar invoicing in international trade creates an increased demand for safe dollar deposits, thereby conferring an exorbitant privilege on the dollar in terms of reduced borrowing costs. Going in the other direction, these low dollar-denominated borrowing costs make it attractive for non-U.S. exporters to invoice their sales in dollars, so that they can more easily tap the cheap dollar funding. The end result of this two-way feedback can be an asymmetric entrenchment of the dollar as the global currency of choice, even when other countries are roughly similar to the U.S. in terms of economic fundamentals such as their share of overall world-wide imports.

Looking to the future, the self-reinforcing asymmetric equilibrium outcomes that we have highlighted carry a double-edged message about the dollar's potential prospects in a changing world. Consider, for example, what might happen as the Chinese economy eventually surpasses that of the U.S. in aggregate GDP and in the volume of its own exports to other countries. In the medium term, our model might predict that the dollar's dominance would prove to be relatively resilient, and that the renminbi would have a hard time gaining much traction as an invoicing currency for exporters operating in other countries. However, in the longer run, if the gap between China and the U.S. widens far enough, we could eventually get to a point where a renminbi-dominant equilibrium becomes inevitable. At this point, the dollar may potentially fall off the world stage to a very substantial extent, much as the British pound sterling did in the early part of the 20th century. In other words, change may be slow to come, but when it finally does, the forces in our model suggest that the change may well be quite dramatic in magnitude.

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7 Appendix (In Progress)

7.1 Proof of Proposition 3

It is convenient to divide the solution into three cases.

[Case 1] $\eta = \eta^* > 0$ **and** $B_h < \gamma_L N_0 + (1-\eta)\gamma_L N$

It should be first noted that $B_h < \gamma_L N_0 + (1-\eta)\gamma_L N$ implies $\kappa = 0$. This leads to

$$Q_\$ = \frac{\mu\bar{\mathcal{E}} + 1}{\lambda}, \qquad Q_h = \frac{\mu + 1}{\lambda}, \qquad Q_R = \frac{1}{\lambda} \tag{44}$$

from the bank's first order conditions. Thus we need $\mu > 0$ to sustain $\eta^* > 0$ in equilibrium since, otherwise, $Q_\$ = Q_h = \frac{1}{\lambda}$ and $\eta^* = \frac{\gamma_L}{\beta\phi}(Q_\$ - Q_h) = 0$. It follows from $\mu > 0$ that the constraint associated with μ should then be binding:

$$\bar{\mathcal{E}} B_\$ + B_h = \gamma_L(N_0 + (1-\eta^*)N) + \bar{\mathcal{E}}\eta^*\gamma_L N \tag{45}$$

This expression can be converted into

$$\bar{\mathcal{E}} D_\$ + D_h = \gamma_L(N_0 + (1-\eta^*)N) + \bar{\mathcal{E}}\eta^*\gamma_L N + \bar{\mathcal{E}} X_\$ \tag{46}$$

by adding $\bar{\mathcal{E}}\bar{X}_\$$ to the both sides and using $D_\$ = B_\$ + X_\$$. Define the parameter K^* as the right-hand side expression of (46) i.e. $K^* \equiv \gamma_L(N_0 + (1-\eta)N) + \bar{\mathcal{E}}\eta\gamma_L N + \bar{\mathcal{E}} X_\$$. Next, from the representative importer's first order conditions, we have

$$\begin{aligned} Q_h &= \beta + \theta\frac{\alpha_h}{(\alpha_h + \alpha_\$)D_h} \\ Q_\$ &= \beta + \theta\frac{\alpha_\$}{(\alpha_\$ + \alpha_h)D_\$} \\ Q_R &= \beta \end{aligned}$$

Combining these conditions with (44), we can obtain

$$\frac{Q_\$ - \beta}{Q_h - \beta} = \bar{\mathcal{E}} = \frac{\alpha_\$}{\alpha_h}\frac{D_h}{D_\$} \tag{47}$$

$$\frac{Q_\$ - \beta}{Q_€ - \beta} = 1 = \frac{\alpha_\$}{\alpha_€}\frac{D_€}{D_\$} \tag{48}$$

Substituting $\bar{\mathcal{E}} D_{\$} \frac{\alpha_h}{\alpha_{\$}}$ for D_h in (46), we can derive the following equilibrium relations,

$$
\begin{aligned}
D_{\$} &= \frac{\alpha_{\$}}{\alpha_{\$} + \alpha_h} \frac{K^*}{\bar{\mathcal{E}}} && (49)\\
D_h &= \frac{\alpha_h}{\alpha_{\$} + \alpha_h} K^* && (50)\\
Q_{\$} - Q_h &= \frac{\theta(\bar{\mathcal{E}} - 1)}{\gamma_L(N_0 + (1-\eta^*)N) + \bar{\mathcal{E}}\eta^*\gamma_L N + \bar{\mathcal{E}} X_{\$}} && (51)\\
\eta^* &= \frac{\gamma_L}{\beta\phi}\left(Q_{\$} - Q_h\right) && (52)
\end{aligned}
$$

The preceding four equations in $D_{\$}, D_h, Q_{\$} - Q_h$, and η have a unique solution, because from the last two equations we have one positively sloped and the other negatively sloped. Merging the last two equations and arranging the terms, we have

$$\kappa_1 \eta^2 + \kappa_2 \eta + \kappa_3 = 0$$

where

$$
\begin{aligned}
\kappa_1 &= \gamma_L N \beta\phi(\bar{\mathcal{E}} - 1)\\
\kappa_2 &= \beta\phi\left(\gamma_L(N_0 + N) + \bar{\mathcal{E}} X_{\$}\right)\\
\kappa_3 &= -\gamma_L \theta(\bar{\mathcal{E}} - 1)
\end{aligned}
$$

$$\eta^* = \frac{-\beta\phi\left(\gamma_L(N_0+N) + \bar{\mathcal{E}}X_{\$}\right) + \sqrt{\beta^2\phi^2\left(\gamma_L(N_0+N) + \bar{\mathcal{E}}X_{\$}\right)^2 + 4\gamma_L^2 N\beta\phi(\bar{\mathcal{E}}-1)^2\theta}}{2\gamma_L N\beta\phi(\bar{\mathcal{E}}-1)}$$

Here, we have ignored the negative root. Note that η does not depend on $\alpha_{\$}$ conditional on being an interior solution. We can then back out $B_{\$}$ and B_h from $B_{\$} = D_{\$} - X_{\$}$ and $B_h = D_h$. Finally, to ensure that $B_h < (1-\eta^*)\gamma_L N + \gamma_L N_0$, we need

$$\frac{\alpha_h}{\alpha_{\$} + \alpha_h} K^* < (1-\eta^*)\gamma_L N + \gamma_L N_0$$

Hence, this equilibrium is sustainable if and only if $\alpha_{\$} > \bar{\alpha}_{\$}$ where

$$\bar{\alpha}_{\$} = \frac{\alpha_h \bar{\mathcal{E}}(\eta^* \gamma_L N + X_{\$})}{\gamma_L N_0 + (1-\eta^*)\gamma_L N}$$

[Case 2] $\eta > 0$ **and** $B_h = \gamma_L N_0 + (1-\eta)\gamma_L N$

We next consider the case where $\eta > 0$ and $\alpha_\$ \leq \bar{\alpha}_\$$. It follows from Case 1 that $B_h = (1-\eta)\gamma_L N + \gamma_L N_0$, and thus $\kappa \geq 0$. The bank's first order conditions can then be expressed as

$$Q_\$ = \frac{\mu\bar{\mathcal{E}} + 1}{\lambda}, \qquad Q_h = \frac{\mu + 1 + \kappa}{\lambda}, \qquad Q_R = \frac{1}{\lambda} \tag{53}$$

Again, we need $\mu > 0$ for $Q_\$ > Q_h$, which leads to

$$\begin{aligned} \bar{\mathcal{E}} B_\$ + B_h &= \gamma_L N_0 + (1-\eta)\gamma_L N + \bar{\mathcal{E}}\eta\gamma_L N \\ B_\$ &= \eta\gamma_L N \end{aligned}$$

Plugging these expressions into the first order conditions, we have

$$\begin{aligned} D_\$ &= \eta\gamma_L N + X_\$, \quad D_h = B_h \\ Q_\$ - Q_h &= \frac{\alpha_\$}{\alpha_h + \alpha_\$}\frac{\theta}{(\eta\gamma_L N + X_\$)} - \frac{\alpha_h}{\alpha_h + \alpha_\$}\frac{\theta}{(\gamma_L N_0 + (1-\eta)\gamma_L N)} \qquad (54) \\ \eta &= \frac{\gamma_L}{\beta\phi}(Q_\$ - Q_h) \qquad (55) \end{aligned}$$

Again, the last two equations pin down the equilibrium value of η. To confirm whether η is continuously increasing in $\alpha_\$$, let us define

$$f(\eta|\alpha_\$) \equiv \frac{\beta\phi}{\gamma_L}\eta - \frac{\alpha_\$}{\alpha_h + \alpha_\$}\frac{\theta}{(\eta\gamma_L N + X_\$)} + \frac{\alpha_h}{\alpha_h + \alpha_\$}\frac{\theta}{(\gamma_L N_0 + (1-\eta)\gamma_L N)} \tag{56}$$

The solution of (56) corresponds to the equilibrium value of η. Note here that $f(\eta|\alpha_\$)$ decreases in $\alpha_\$$. Define the lower cutoff

$$\underline{\alpha}_\$ \equiv \frac{\alpha_h X_\$}{\gamma_L(N + N_0) + X_\$}$$

In the middle range $\alpha_\$ \in [\underline{\alpha}_\$, \bar{\alpha}_\$]$, we have

$$\begin{aligned} f(0|\alpha_\$) &= -\frac{\theta\alpha_\$}{X_\$(\alpha_h + \alpha_\$)} + \frac{\theta\alpha_h}{\gamma_L(N_0 + N)(\alpha_h + \alpha_\$)} < f(0|\underline{\alpha}_\$) = 0 \qquad (57) \\ f(\eta^*|\alpha_\$) &> f(\eta^*|\bar{\alpha}_\$) = 0 \qquad (58) \end{aligned}$$

The first inequality $f(0|\alpha_\$) < f(0|\underline{\alpha}_\$)$ is due to the relation that $f(\eta|\alpha_\$)$ decreases in $\alpha_\$$. The last equality $f(\eta^*|\bar{\alpha}_\$) = 0$ follows from the fact that, once we plug $\bar{\alpha}_\$$ and η^* into (56), the equation $f(\eta^*|\bar{\alpha}_\$) = 0$ coincides with the previous characterization $\kappa_1\eta^2 + \kappa_2\eta + \kappa_3 = 0$ in Case 1. Thus, invoking the Intermediate Value Theorem, (57) and (58) imply that there must exist a real-valued

solution $\eta[\alpha_\$] \in [0, \eta^*]$ that satisfies $f(\eta[\alpha_\$]|\alpha_\$) = 0$. It follows from

$$f'(\eta|\alpha_\$) = \frac{\beta\phi}{\gamma_L} + \gamma_L N \frac{\theta\alpha_\$}{(\eta\gamma_L N + X_\$)^2(\alpha_h + \alpha_\$)} + \gamma_L N \frac{\theta\alpha_h}{(\gamma_L N_0 + (1-\eta)\gamma_L N)^2(\alpha_h + \alpha_\$)} > 0, \ \forall \eta$$

that the solution must be unique. Finally, using the Implicit Function Theorem and differentiating the both sides of

$$0 = \frac{\beta\phi}{\gamma_L}\eta[\hat{\alpha}_\$] - \frac{\theta\alpha_\$}{(\eta[\hat{\alpha}_\$]\gamma_L N + X_\$)(\alpha_h + \alpha_\$)} + \frac{\theta\alpha_h}{(\gamma_L N_0 + (1-\eta[\hat{\alpha}_\$])\gamma_L N)(\alpha_h + \alpha_\$)}$$

with respect to $\hat{\alpha}_\$ \equiv \frac{\alpha_\$}{\alpha_h + \alpha_\$}$, we obtain

$$\frac{\partial \eta[\hat{\alpha}_\$]}{\partial \hat{\alpha}_\$} = \frac{\frac{\theta}{\eta[\hat{\alpha}_\$]\gamma_L N + X_\$} + \frac{\theta}{\gamma_L N_0 + (1-\eta[\hat{\alpha}_\$])\gamma_L N}}{\frac{\beta\phi}{\gamma_L} + \gamma_L N \frac{\theta\hat{\alpha}_\$}{(\eta[\hat{\alpha}_\$]\gamma_L N + X_\$)^2} + \gamma_L N \frac{\theta(1-\hat{\alpha}_\$)}{(\gamma_L N_0 + (1-\eta[\hat{\alpha}_\$])\gamma_L N)^2}} > 0$$

This expression implies that $\eta[\hat{\alpha}_\$]$ is monotonically increasing in $\hat{\alpha}_\$$ and thus increasing in $\alpha_\$$ over the interval $[\underline{\alpha}_\$, \bar{\alpha}_\$]$. Combined with the equations $f(\eta^*|\bar{\alpha}_\$) = 0$ and $f(0|\underline{\alpha}_\$) = 0$, we have confirmed that η connects from 0 at the lower cutoff to η^* at the upper cutoff, continuously and monotonically.

[Case 3] $\eta = 0$ **and** $B_h = \gamma_L(N_0 + N)$

In this equilibrium, $B_h = \gamma_L(N_0 + N)$, $B_\$ = 0$ and $D_\$ = X_\$$. From the first order conditions of the importers we have,

$$\begin{aligned} Q_h &= \beta + \frac{\theta\alpha_h}{\gamma_L(N_0 + N)(\alpha_\$ + \alpha_h)} \\ Q_\$ &= \beta + \frac{\theta\alpha_\$}{X_\$(\alpha_\$ + \alpha_h)} \end{aligned}$$

To ensure that this is indeed an equilibrium, we need $Q_\$ - Q_h \leq 0$ since, otherwise, $\eta = \frac{\gamma_L}{\beta\phi}(Q_\$ - Q_h) > 0$. This requires that

$$\alpha_\$ \leq \frac{\alpha_h X_\$}{\gamma_L(N_0 + N)}$$

In other words, we can define a threshold $\underline{\alpha}_\$ = \frac{\alpha_h X_\$}{\gamma_L(N_0+N)}$ such that this equilibrium is sustainable if and only if $a \leq \underline{\alpha}_\$$.

7.2 Proof of Proposition 4

Let η_{-i} denote $\int_{j\neq i}\eta^j dj$. We divide the solution into three cases as before and consider its stability separately.

[Case 1] $\eta_i = \eta^* > 0$ **and** $B_{hi} < \gamma_L N_0 + (1-\eta)\gamma_L N$ **for all** i

Again, as in Proposition 3, we turn to $D_{hi} = \bar{\mathcal{E}} D_{\$i}\frac{\alpha_h}{\alpha_\$} = \bar{\mathcal{E}} D_\$ \frac{\alpha_h}{a+b\eta_{-i}}$ and

$$\bar{\mathcal{E}} D_\$ + D_h = \gamma_L(N_0 + (1-\eta_i)N) + \bar{\mathcal{E}}\eta_i\gamma_L N + \bar{\mathcal{E}} X_\$$$

to derive the following equilibrium relations,

$$\begin{aligned}
D_\$ &= \frac{\alpha_\$}{a+b\eta_{-i}+\alpha_h}\frac{K^*}{\bar{\mathcal{E}}} \\
D &= \frac{\alpha_h}{a+b\eta_{-i}+\alpha_h}K^* \\
Q_\$ - Q_{hi} &= \frac{\theta(\bar{\mathcal{E}}-1)}{\gamma_L(N_0+(1-\eta_i)N) + \bar{\mathcal{E}}\eta_i\gamma_L N + \bar{\mathcal{E}} X_\$} \qquad (59)\\
\eta_i &= \frac{\gamma_L}{\beta\phi}\left(Q_{\$i} - Q_{hi}\right) \qquad (60)
\end{aligned}$$

where

$$K^* \equiv \gamma_L(N_0 + (1-\eta^*)N) + \bar{\mathcal{E}}\eta^*\gamma_L N + \bar{\mathcal{E}} X_\$$$

It should be noted that, even though we incorporate endogenous invoicing shares through $\alpha_\$ = a + b\eta_{-i}$, the core equations (59) and (60) that pin down the optimal η_i remain unchanged from Proposition 3. Also, because of the equality of $Q_\$$ across i this equalizes Q_i across i, which then equalizes η across i. Solving the system of equations, we obtain

$$\eta_i = \eta^* = \frac{-\beta\phi\left(\gamma_L(N_0+N) + \bar{\mathcal{E}} X_\$\right) + \sqrt{\beta^2\phi^2\left(\gamma_L(N_0+N)+\bar{\mathcal{E}} X_\$\right)^2 + 4\gamma_L^2 N\beta\phi(\bar{\mathcal{E}}-1)^2\theta}}{2\gamma_L N\beta\phi(\bar{\mathcal{E}}-1)}$$

as before. Essentially, η_i does not depend on other countries' invoicing decisions. It is then straightforward that Case 1 always yields a stable and symmetric equilibrium.
Finally, to ensure that $\kappa = 0$, we need $B_h < (1-\eta^*)\gamma_L N + \gamma_L N_0$. It follows from the equilibrium value of B_h that this condition holds if and only if $\alpha_\$ > \underline{\alpha}_\$$ such that

$$\underline{\alpha}_\$ = \frac{\alpha_h\bar{\mathcal{E}}(X_\$ + \eta^*\gamma_L N)}{\gamma_L N_0 + (1-\eta)\gamma_L N}$$

This also ensures that $B_\$ > \eta^* \gamma_L N > 0$ as $\bar{\mathcal{E}} B_\$ + B_h = \gamma_L (N_0 + (1-\eta^*)N) + \bar{\mathcal{E}} \eta^* \gamma_L N$ in Case 1. The cutoff can now be expressed with respect to a in the new setting.

$$\underline{a} = \frac{\alpha_h \bar{\mathcal{E}} \left(\eta \gamma_L N + X_\$ \right)}{\gamma_L N_0 + (1-\eta)\gamma_L N} - b\eta$$

[Case 2] $\eta_i > 0$ **and** $B_{hi} = \gamma_L N_0 + (1-\eta_i)\gamma_L N$ **for all** i

We next show that an equilibrium with no mismatch is unstable under a certain parametric restriction. From the bank's optimization problem, we have

$$\begin{aligned} B_{\$i} &= \eta_i \gamma_L N \\ B_{hi} &= \gamma_L N_0 + (1-\eta_i)\gamma_L N \end{aligned}$$

which leads to

$$\begin{aligned} D_{\$i} &= \eta \gamma_L N + X_\$, \quad D_{hi} = B_{hi} \\ Q_\$ - Q_{hi} &= \frac{\alpha_\$}{\alpha_h + \alpha_\$} \frac{\theta}{\left(\eta_i \gamma_L N + X_\$ \right)} - \frac{\alpha_h}{\alpha_h + \alpha_\$} \frac{\theta}{\left(\gamma_L N_0 + (1-\eta_i)\gamma_L N \right)} \qquad (61) \\ \eta_i &= \frac{\gamma_L}{\beta \phi} (Q_\$ - Q_{hi}) \qquad (62) \end{aligned}$$

The only difference from Case 2 of Proposition 3 is that we now have $\alpha_\$ = a + b\eta_{-i}$ due to the endogenous invoicing shares. The system of the last two equations can be restated as

$$\eta_i = \frac{\gamma_L}{\beta \phi} \left(\frac{a + b\eta_{-i}}{\alpha_h + a + b\eta_{-i}} \frac{\theta}{\left(\eta_i \gamma_L N + X_\$ \right)} - \frac{\alpha_h}{\alpha_h + a + b\eta_{-i}} \frac{\theta}{\left(\gamma_L N_0 + (1-\eta_i)\gamma_L N \right)} \right) \qquad (63)$$

It is clear from this expression that the left-hand side is monotonically increasing in η_i, whereas the right-hand side is monotonically decreasing in η_i. Essentially, this equation pins down a unique η_i, which allows us to use Implicit Function Theorem conveniently. Let $A[\eta_{-i}] \equiv \frac{a + b\eta_{-i}}{\alpha_h + a + b\eta_{-i}}$ and $1 - A[\eta_{-i}] \equiv \frac{\alpha_h}{\alpha_h + a + b\eta_{-i}}$ to simplify the expression. Differentiating the both sides of (63) by

η_{-i}, we have

$$\frac{\partial \eta_i}{\partial \eta_{-i}} = \frac{\gamma_L \theta}{\beta \phi} \Bigg(\frac{\partial A[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\eta_i \gamma_L N + X_\$} - A[\eta_{-i}] \frac{\gamma_L N}{(\eta_i \gamma_L N + X_\$)^2} \frac{\partial \eta_i}{\partial \eta_{-i}} \\ - \frac{\partial (1 - A[\eta_{-i}])}{\partial \eta_{-i}} \frac{1}{\gamma_L N_0 + (1-\eta_i)\gamma_L N} - (1 - A[\eta_{-i}]) \frac{\gamma_L N}{(\gamma_L N_0 + (1-\eta_i)\gamma_L N)^2} \frac{\partial \eta_i}{\partial \eta_{-i}} \Bigg)$$

Arranging the terms, we have

$$\frac{\partial \eta_i}{\partial \eta_{-i}} = \frac{\gamma_L \theta}{\beta \phi} \frac{\frac{\partial A[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\eta_i \gamma_L N + X_\$} - \frac{\partial (1 - A[\eta_{-i}])}{\partial \eta_{-i}} \frac{1}{\gamma_L N_0 + (1-\eta_i)\gamma_L N}}{1 + A[\eta_{-i}] \frac{\gamma_L N}{(\eta_i \gamma_L N + X_\$)^2} + (1 - A[\eta_{-i}]) \frac{\gamma_L N}{(\gamma_L N_0 + (1-\eta_i)\gamma_L N)^2}}$$

where

$$\frac{\partial A[\eta_{-i}]}{\partial \eta_{-i}} = \frac{\alpha_h b}{(\alpha_h + a + b\eta_{-i})^2} > 0$$
$$\frac{\partial (1 - A[\eta_{-i}])}{\partial \eta_{-i}} = -\frac{\alpha_h b}{(\alpha_h + a + b\eta_{-i})^2} < 0$$

Thus, we can clearly see that there is complementary in strategies as

$$\frac{\partial \eta_i}{\partial \eta_{-i}} > 0$$

under any parametric values. The derivative can be further simplified to

$$\frac{\partial \eta_i}{\partial \eta_{-i}} = \frac{\gamma_L \theta}{\beta \phi} \frac{\frac{\alpha_h b}{\alpha_h + a + b\eta_{-i}} \left(\frac{1}{\eta_i \gamma_L N + X_\$} + \frac{1}{\gamma_L N_0 + (1-\eta_i)\gamma_L N} \right)}{\alpha_h + a + b\eta_{-i} + (a + b\eta_{-i}) \frac{\gamma_L N}{(\eta_i \gamma_L N + X_\$)^2} + \alpha_h \frac{\gamma_L N}{(\gamma_L N_0 + (1-\eta_i)\gamma_L N)^2}}$$

In particular, if $\frac{\theta}{\phi}$ is large enough such that

$$\frac{\gamma_L \theta}{\beta \phi} \frac{\frac{\alpha_h b}{\alpha_h + a + b\eta_{-i}} \left(\frac{1}{\eta_i \gamma_L N + X_\$} + \frac{1}{\gamma_L N_0 + (1-\eta_i)\gamma_L N} \right)}{\alpha_h + a + b\eta_{-i} + (a + b\eta_{-i}) \frac{\gamma_L N}{(\eta_i \gamma_L N + X_\$)^2} + \alpha_h \frac{\gamma_L N}{(\gamma_L N_0 + (1-\eta_i)\gamma_L N)^2}} > 1 \tag{64}$$

for all $\eta_i \in [0, 1]$, we have

$$\frac{\partial \eta_i}{\partial \eta_{-i}} > 1$$

In other words, any equilibrium of Case 2 is unstable under this sufficient condition.

[Case 3] $\eta_i = 0$ **and** $B_{hi} = \gamma_L(N_0 + N)$**, for all** i

In this equilibrium, $B_{hi} = \gamma_L(N_0 + N)$, $B_{\$i} = 0$ and $D_{\$i} = X_\$$. From the first order conditions of the importers we have,

$$\begin{aligned} Q_h &= \beta + \frac{\theta\alpha_h}{\gamma_L(N_0+N)(\alpha_\$ + \alpha_h)} \\ Q_\$ &= \beta + \frac{\theta\alpha_\$}{X_\$(\alpha_\$ + \alpha_h)} \end{aligned}$$

To ensure that this is indeed an equilibrium, we need $Q_\$ - Q_h \leq 0$ since, otherwise, $\eta = \frac{\gamma_L}{\beta\phi}(Q_\$ - Q_h) > 0$. This requires that

$$\alpha_\$ \leq \frac{\alpha_h X_\$}{\gamma_L(N_0+N)}$$

In other words, we can define a threshold $\underline{\alpha}_\$ = \frac{\alpha_h X_\$}{\gamma_L(N_0+N)}$ such that this equilibrium is sustainable if and only if $a \leq \underline{\alpha}_\$$. The stability holds as η_i does not depend on η_{-i} over this range.

7.3 Proof of Proposition 5

We divide the solution into four cases and consider their stability properties separately at the end of each part. Superscript n, s and b are suppressed in equilibrium values unless necessary.

[Case 1] Symmetric Equilibrium with Mismatch: $\eta_{\$i} = \eta_{€i} = \eta^b > 0$ **and** $B_{hi} < \gamma_L N_0 + (1 - 2\eta^b)\gamma_L N$ **for all** i

Note here that $B_h < \gamma_L N_0 + (1 - 2\eta^b)\gamma_L N$ implies $\kappa = 0$. The bank's first order conditions can then be expressed by

$$Q_{\$i} = Q_{€i} = \frac{\mu_i\bar{\mathcal{E}} + 1}{\lambda_i}, \qquad Q_{hi} = \frac{\mu_i + 1}{\lambda}, \qquad Q_{Ri} = \frac{1}{\lambda_i} \tag{65}$$

Thus, we need $\mu_i > 0$ to sustain $\eta^b > 0$ as in Proposition 3. The constraint associated with μ_i should then be binding:

$$\bar{\mathcal{E}}(B_{\$i} + B_{€i}) + B_{hi} = \gamma_L(N_0 + (1 - \eta_{\$i} - \eta_{€i})N) + \bar{\mathcal{E}}(\eta_{\$i} + \eta_{€i})\gamma_L N \tag{66}$$

which can be converted into

$$\bar{\mathcal{E}}(D_{\$i} + D_{€i}) + D_{hi} = \gamma_L(N_0 + (1 - \eta_{\$i} - \eta_{€i})N) + \bar{\mathcal{E}}(\eta_{\$i} + \eta_{€i})\gamma_L N + 2\bar{\mathcal{E}}X \tag{67}$$

Define K^b as the right-hand side expression of (67). Next, from the representative importer's first order conditions, we have

$$\begin{aligned} Q_{hi} &= \beta + \theta \frac{\alpha_h}{(\alpha_h + \alpha_\$ + \alpha_€) D_{hi}} \\ Q_\$ &= \beta + \theta \frac{\alpha_\$}{(\alpha_\$ + \alpha_h + \alpha_€) D_{\$i}} \\ Q_€ &= \beta + \theta \frac{\alpha_€}{(\alpha_\$ + \alpha_h + \alpha_€) D_{€i}} \\ Q_R &= \beta \end{aligned}$$

Combining these conditions with (65), we have

$$\frac{Q_\$ - \beta}{Q_{hi} - \beta} = \bar{\mathcal{E}} = \frac{\alpha_\$}{\alpha_h} \frac{D_{hi}}{D_{\$i}} \tag{68}$$

$$\frac{Q_\$ - \beta}{Q_€ - \beta} = 1 = \frac{\alpha_\$}{\alpha_€} \frac{D_{€i}}{D_\$} \tag{69}$$

Substituting $\bar{\mathcal{E}} D_\$ \frac{\alpha_h}{\alpha_\$}$ and $D_€ \frac{\alpha_€}{\alpha_\$}$ for D_h and $D_€$ in (67) respectively, we can derive the following equilibrium relations

$$\begin{aligned} D_{\$i} = D_{€i} &= \frac{(a + b\eta_{-i}) K^b}{\bar{\mathcal{E}}(2a + 2b\eta_i + \alpha_h)} \qquad \forall i \\ D_{hi} &= \frac{\alpha_h K^b}{2a + 2b\eta_{-i} + \alpha_h} \qquad \forall i \\ Q_\$ - Q_{hi} &= \frac{\theta(\bar{\mathcal{E}} - 1)}{\gamma_L(N_0 + (1 - 2\eta_i)N) + 2\bar{\mathcal{E}}\eta_i\gamma_L N + 2\bar{\mathcal{E}}X} \qquad (70) \\ \eta_i &= \frac{\gamma_L}{\phi\beta}(Q_\$ - Q_{hi}) - c\eta_i \qquad (71) \end{aligned}$$

where η_{-i} denotes $\eta_{-i} \equiv \int_{j \neq i} \eta_{\$j} dj = \int_{j \neq i} \eta_{€j} dj$. Notice here that $\eta_{\$i} = \eta_{€i}$ should hold in equilibrium as $Q_\$ = Q_€$ in the set-up. The system of the last two equations, (70) and (71), can then be converted into

$$\kappa_1 \eta_i^2 + \kappa_2 \eta_i + \kappa_3 = 0$$

where

$$\begin{aligned} \kappa_1 &= 2(1 + c)\gamma_L N \beta \phi (\bar{\mathcal{E}} - 1) \\ \kappa_2 &= \beta\phi(1 + c)\left(\gamma_L(N + N_0) + 2\bar{\mathcal{E}}X\right) \\ \kappa_3 &= -\gamma_L \theta(\bar{\mathcal{E}} - 1) \end{aligned}$$

Solving this equation

$$\eta_i = \eta^b = \frac{-\kappa_2 + \sqrt{\kappa_2^2 - 4\kappa_3\kappa_1}}{2\kappa_1}$$

where η^b is notation for optimal η when 'both' dollar and euro assets are produced by banks. To ensure this is an equilibrium, we need

$$B_h < \gamma_L N_0 + (1 - 2\eta^b)\gamma_L N \Leftrightarrow B_\$ = B_\euro > \eta^b \gamma_L N \tag{72}$$

in light of the collateral constraint (66). Define $\bar{a}^b$ to be the cut-off value at which eq. (72) holds with equality. That is,

$$\frac{(\bar{a}^b + b\eta^b)K^b}{\bar{\mathcal{E}}(2\bar{a}^b + 2b\eta^b + \alpha_h)} - X = \eta^b \gamma_L N \tag{73}$$

Because η^b is independent of a and the left-hand side is increasing in a we have that for $a > \bar{a}^b$ there is an equilibrium with $\eta_\$ = \eta_\euro > 0$. Finally, stability always holds in Case 1 as the best response η_i is pinned down by eq. (70) and (71), which are not dependent on η_{-i}.

[Case 2] Symmetric Equilibrium with no Mismatch: $\eta_{\$i} = \eta_{\euro i} = \eta_i > 0$ **and** $B_{hi} = \gamma_L N_0 + (1 - 2\eta_i)\gamma_L N$ **for all** i

Let $\eta_i \equiv \eta_{\$i} = \eta_{\euro i}$ denote the symmetric invoicing. We now consider the case where $B_\$ = B_\euro = \eta_i \gamma_L N$ and $B_{hi} = \gamma_L N_0 + (1 - 2\eta_i)\gamma_L N$. As shown in Case 1, this corresponds to the region $a \leq \bar{a}^b$. Plugging these into the importers' first order conditions, we have

$$Q_\$ - Q_{hi} = \frac{\alpha_\$}{\alpha_h + \alpha_\$ + \alpha_\euro} \frac{\theta}{(\eta_i \gamma_L N + X_\$)} - \frac{\alpha_h}{\alpha_h + \alpha_\$ + \alpha_\euro} \frac{\theta}{(\gamma_L N_0 + (1 - 2\eta_i)\gamma_L N)} \tag{74}$$

$$\eta_i = \frac{\gamma_L}{\beta\phi}(Q_\$ - Q_{hi}) - c\eta_i \tag{75}$$

This system of equations can be restated as

$$\begin{aligned}(1 + c)\eta_i =& \frac{\gamma_L}{\beta\phi}\left(\frac{a + b\eta_{-i}}{\alpha_h + 2a + 2b\eta_{-i}} \frac{\theta}{(\eta_i \gamma_L N + X_\$)} - \frac{\alpha_h}{\alpha_h + 2a + 2b\eta_{-i}} \frac{\theta}{(\gamma_L N_0 + (1 - 2\eta_i)\gamma_L N)}\right) \\ \equiv& \frac{\gamma_L}{\beta\phi}\left(A_1[\eta_{-i}] \frac{\theta}{(\eta_i \gamma_L N + X_\$)} - A_2[\eta_{-i}] \frac{\theta}{(\gamma_L N_0 + (1 - 2\eta_i)\gamma_L N}\right)\end{aligned} \tag{76}$$

which pins down a unique η_i as the right-hand side is increasing in η_i while the left-hand side is decreasing in η_i. We can then invoke Implicit Function Theorem. Differentiating the both sides

with respect to η_{-i}, we have

$$(1+c)\frac{\partial \eta_i}{\partial \eta_{-i}} = \frac{\gamma_L \theta}{\beta \phi}\left(\frac{\partial A_1[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\eta_i \gamma_L N + X_{\$}} - A_1[\eta_{-i}] \frac{\gamma_L N}{(\eta_i \gamma_L N + X_{\$})^2} \frac{\partial \eta_i}{\partial \eta_{-i}} \right.$$
$$\left. - \frac{\partial A_2[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\gamma_L N_0 + (1-2\eta_i)} - A_2[\eta_{-i}] \frac{2\gamma_L N}{(\gamma_L N_0 + (1-2\eta_i)\gamma_L N)^2} \frac{\partial \eta_i}{\partial \eta_{-i}} \right)$$

This yields

$$\frac{\partial \eta_i}{\partial \eta_{-i}} = \frac{\gamma_L \theta}{\beta \phi} \frac{\frac{\partial A_1[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\eta_i \gamma_L N + X_{\$}} - \frac{\partial A_2[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\gamma_L N_0 + (1-2\eta_i)}}{1 + c + A_1[\eta_{-i}] \frac{\gamma_L N}{(\eta_i \gamma_L N + X_{\$})^2} + A_2[\eta_{-i}] \frac{2\gamma_L N}{(\gamma_L N_0 + (1-2\eta_i)\gamma_L N)^2}} \tag{77}$$

where

$$\frac{\partial A_1[\eta_{-i}]}{\partial \eta_{-i}} = \frac{b\alpha_h}{(\alpha_h + 2a + 2b\eta_{-i})^2} > 0$$
$$\frac{\partial A_2[\eta_{-i}]}{\partial \eta_{-i}} = -\frac{2b\alpha_h}{(\alpha_h + 2a + 2b\eta_{-i})^2} < 0$$

Thus, $\frac{\partial \eta_i}{\partial \eta_{-i}} > 0$ holds under any parameter values. In particular, if $\frac{\theta}{\phi}$ is large enough such that

$$\frac{\gamma_L \theta}{\beta \phi} \frac{\frac{\partial A_1[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\eta_i \gamma_L N + X_{\$}} - \frac{\partial A_2[\eta_{-i}]}{\partial \eta_{-i}} \frac{1}{\gamma_L N_0 + (1-2\eta_i)}}{1 + c + A_1[\eta_{-i}] \frac{\gamma_L N}{(\eta_i \gamma_L N + X_{\$})^2} + A_2[\eta_{-i}] \frac{2\gamma_L N}{(\gamma_L N_0 + (1-2\eta_i)\gamma_L N)^2}} > 1$$

for any $\eta_i \in [0,1]$, we have

$$\frac{\partial \eta_i}{\partial \eta_{-i}} > 1$$

which makes this equilibrium unstable.

[Case 3] Asymmetric Equilibrium: $\eta_{\$} = \eta^s > 0, \eta_{€} = 0$ **or** $\eta_{€} = \eta^s > 0, \eta_{\$} = 0$

Let's consider the case $B_{€} = 0$. We now have,

$$\int_i D_{€,i} = X$$

Combining the market clearing condition for € along with the demand condition for each i

$$Q_{€} = \beta + \theta \frac{\alpha_{€,i}}{(\alpha_{\$,i} + \alpha_{€,i} + \alpha_{h,i}) D_{€,i}} = \beta + \theta \frac{a}{(\alpha_h + 2a + b\eta) D_{€,i}}$$

we have

$$\begin{aligned} D_{€,i} &= X \\ Q_{€} &= \beta + \theta \frac{a}{(\alpha_h + 2a + b\eta)X} \end{aligned}$$

To ensure this is an equilibrium we need, $Q_{€} - Q_{hi} < 0$. Because of the equality of $Q_{\$}$ across i this equalizes Q_i across i, which then equalizes η across i. We need to ensure that $(Q_{\$} - Q_{hi}) > 0$. Also, again, it follows from $\bar{\mathcal{E}} = \frac{Q_{\$i} - \beta}{Q_{hi} - \beta} = \frac{D_{hi}}{D_{\$i}} \frac{\alpha_{\$i}}{\alpha_{hi}}$ that $D_{hi} = \bar{\mathcal{E}} D_{\$i} \frac{\alpha_{hi}}{\alpha_{\$i}}$. Plugging this into the constraint

$$\bar{\mathcal{E}} D_{\$i} + D_{hi} = \gamma_L (N_0 + (1 - \eta)N) + \bar{\mathcal{E}} \eta \gamma_L N + \bar{\mathcal{E}} X$$

where $\eta = \eta_{\$i}$ denotes dollar invoicing across countries. We can then obtain the following equilibrium relations:

$$\begin{aligned} D_{\$} &= \frac{a + b\eta}{\bar{\mathcal{E}}} \frac{K^s}{\alpha_h + a + b\eta} \qquad \forall i \\ D &= \frac{\alpha_h K^s}{\alpha_h + a + b\eta} \qquad \forall i \\ D_{€} &= X \qquad \forall i \\ Q_{\$} &= \beta + \frac{\theta \bar{\mathcal{E}}}{K^s} \left(\frac{\alpha_h + a + b\eta}{\alpha_h + 2a + b\eta} \right) \\ Q_h &= \beta + \frac{\theta}{K^s} \left(\frac{\alpha_h + a + b\eta}{\alpha_h + 2a + b\eta} \right) \end{aligned}$$

where

$$K^s = \gamma_L N_0 + (1 - \eta)\gamma_L N + \eta \bar{\mathcal{E}} \gamma_L N + \bar{\mathcal{E}} X$$

Again, we can derive the optimal η from the system of equations below

$$\begin{aligned} \eta &= \frac{\gamma_L}{\phi \beta} (Q_{\$} - Q_{hi}) \\ Q_{\$} - Q_{hi} &= \frac{\theta(\alpha_h + a + b\eta)}{K^s(\alpha_h + 2a + b\eta)} \left(\bar{\mathcal{E}} - 1 \right) > 0 \end{aligned}$$

The problem is converted into solving a cubic equation

$$\kappa_1 \eta^3 + \kappa_2 \eta^2 + \kappa_3 \eta + \kappa_4 = 0$$

where

$$\begin{aligned}
\kappa_1 &= \beta\phi b(\bar{\mathcal{E}}-1)\gamma_L N \\
\kappa_2(a) &= (\alpha_h+2a)\gamma_L N\beta\phi(\bar{\mathcal{E}}-1)+\phi\beta b(\gamma_L(N+N_0)+\bar{\mathcal{E}}X) \\
\kappa_3(a) &= (\alpha_h+2a)\beta\phi\left(\gamma_L(N+N_0)+\bar{\mathcal{E}}X\right)-b\gamma_L\theta(\bar{\mathcal{E}}-1) \\
\kappa_4(a) &= -\gamma_L\theta(a+\alpha_h)(\bar{\mathcal{E}}-1)
\end{aligned}$$

Let η^s denote the interior optimum share of invoicing in dollars ('single' dominance currency), with the remainder invoiced in local currency. Plugging this back into equilibrium demand for deposits, we have

$$\begin{aligned}
D_{\$} &= \frac{a+b\eta^s}{\bar{\mathcal{E}}}\frac{K^s}{\alpha_h+a+b\eta^s} \qquad \forall i \\
D &= \frac{\alpha_h K^s}{\alpha_h+a+b\eta^s} \qquad \forall i \\
D_{€} &= X \qquad \forall i \\
Q_{\$} &= \beta+\frac{\theta\bar{\mathcal{E}}}{K^s}\left(\frac{\alpha_h+a+b\eta^s}{\alpha_h+2a+b\eta^s}\right) \\
Q_h &= \beta+\frac{\theta}{K^s}\left(\frac{\alpha_h+a+b\eta^s}{\alpha_h+2a+b\eta^s}\right)
\end{aligned}$$

Next, we define cut-off $\underline{a}^s$ and $\bar{a}^s$. $\underline{a}^s$ is the cut-off such that for $a<\underline{a}^s$ an equilibrium with one positive η cannot be sustained; it can only sustain (0,0). $\bar{a}^s$ is the cut off such that to the right of it an equilibrium with only one positive η cannot be sustained.

$$\bar{a}^s=\frac{(\alpha_h+b\eta^s(\bar{a}^s))(\theta\gamma_L X+c\eta^s(\bar{a}^s)\phi\beta K^s X)}{\theta\gamma_L(K^s-X)-2c\eta^s(\bar{a}^s)\phi\beta K^s(\bar{a}^s)X} \tag{78}$$

$$\frac{(\underline{a}^s+\eta^s(\underline{a}^s)b)}{\bar{\mathcal{E}}}\frac{K^s(\underline{a}^s)}{2(\alpha_h+a+b\eta)}-X=\eta^s(\underline{\mathrm{a}}^s)\gamma_L N \tag{79}$$

where the first equality comes from $\frac{\gamma_L}{\phi\beta}(Q_{€}-Q_{hi})-c\eta^s=0$ and the last equality follows from $B_{\$}=\eta^s\gamma_L N$ at the two cutoffs respectively.

Finally, we can show the stability of asymmetric equilibrium, provided that $\bar{a}^s>\underline{a}^s$. Return-

ing to the best response function of country i, we have

$$\eta_i = \frac{\gamma_L}{\phi\beta}\left(Q_{\$} - Q_{hi}\right)$$
$$Q_{\$} - Q_{hi} = \frac{\theta(\alpha_h + a + b\eta_{-i})}{K^s(\alpha_h + 2a + b\eta_{-i})}\left(\bar{\mathcal{E}} - 1\right) > 0$$

where K^s is a function of η_i

$$K^s = \gamma_L N_0 + (1-\eta_i)\gamma_L N + \eta_i \bar{\mathcal{E}}\gamma_L N + \bar{\mathcal{E}}X$$

Let C denote $C[\eta_{-i}] \equiv \frac{\phi\beta}{\gamma_L}\frac{\theta(\alpha_h+a+b\eta_{-i})}{(\alpha_h+2a+b\eta_{-i})}(\bar{\mathcal{E}}-1)$ a constant in the quadratic equation. Then we have $\frac{\partial C[\eta_{-i}]}{\partial \eta_{-i}} > 0$ and the optimal η_i comes from solving

$$(\bar{\mathcal{E}} - 1)\gamma_L N\eta_i^2 + \gamma_L(N + N_0)\eta_i + C[\eta_{-i}] = 0$$

The positive root of the above equation is

$$\eta_i = \frac{-\gamma_L(N+N_0) + \sqrt{\gamma_L^2(N+N_0)^2 - 4(\bar{\mathcal{E}}-1)\gamma_L N C[\eta_{-i}]}}{2(\bar{\mathcal{E}}-1)\gamma_L N}$$

which is a decreasing function of $C[\eta_{-i}]$. It is now straightforward to show that

$$\frac{\partial \eta_i}{\partial \eta_{-i}} < 0$$

Essentially, an asymmetric equilibrium is stable regardless of parameter values.

[Case 4] No dominant currency: $\eta_{\$i} = \eta_{€i} = 0$ **and** $B_{\$i} = B_{€i} = 0$ **for all** i

$$\int_i D_{€i} = X \qquad \int_i D_{\$i} = X$$
$$D_{\$i} = D_{€i} = X \qquad \forall i$$
$$Q_{\$} = Q_{€} = \beta + \theta\frac{a}{X(\alpha_h + 2a)}$$

From the safe asset constraint we have,

$$D_{hi} = B_{hi} = \gamma_L\left(N_0 + N\right)$$

$$Q_{hi} = \beta + \theta \frac{\alpha_h}{(\alpha_h + 2a) D_h} = \beta + \theta \frac{\alpha_h}{\gamma_L \left(N_0 + N\right) \left(\alpha_h + 2a\right)}$$

For this to be an equilibrium we need,

$$Q_{\$} - Q_{hi} = Q_{€} - Q_{hi} \leq 0$$

which requires that,

$$a < \frac{\alpha_h X_{\$}}{\gamma_L (N_0 + N)}$$

Define

$$\bar{a}^n = \frac{\alpha_h X_{\$}}{\gamma_L (N_0 + N)}$$

[Summary] Characterization of Cutoffs

We have four cut-offs defined as follows:

$$\begin{aligned}
\bar{a}^n &= \frac{\alpha_h X}{\gamma_L (N_0 + N)} \\
\frac{(\underline{a}^s + \eta^s(\underline{a}^s) b)}{\bar{\mathcal{E}}} \frac{K^s(\underline{a}^s)}{\alpha_h + a + b\eta^s(\underline{a}^s)} &= \eta^s(\underline{\mathbf{a}}^s) \gamma_L N + X \\
\bar{a}^s &= \frac{(\alpha_h + b\eta^s(\bar{a}^s))(\theta \gamma_L X + c\eta^s(\bar{a}^s)\phi\beta K^s X)}{\theta\gamma_L(K^s - X) - 2c\eta^s(\bar{a}^s)\phi\beta K^s(\bar{a}^s) X} \\
\frac{(\bar{a}^b + b\eta^b) K^b}{\bar{\mathcal{E}}(2\bar{a}^b + 2b\eta^b + \alpha_h)} &= \eta^b \gamma_L N + X
\end{aligned}$$

where

$$K^s(a) = \gamma_L N_0 + (1 - \eta^s(a))\gamma_L N + \eta^s(a)\bar{\mathcal{E}}\gamma_L N + \bar{\mathcal{E}} X$$

$$K^b(a) = \gamma_L N_0 + (1 - 2\eta^b(a))\gamma_L N + 2\eta^b(a)\bar{\mathcal{E}}\gamma_L N + 2\bar{\mathcal{E}} X$$