Productivity and Misallocation in General Equilibrium

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Aggregation Theorems for Efficient Economies

- Efficient economy, aggregate production function, Solow (1957):

$$\mathrm{d}\log Y = \mathrm{d}\log A + \Lambda_L \,\mathrm{d}\log L + \Lambda_K \,\mathrm{d}\log K,$$

  where Λ_L and Λ_K are labor and capital income shares.

- Efficient economy, Hulten (1978):

$$\mathrm{d}\log Y = \sum \lambda_i \,\mathrm{d}\log A_i + \Lambda_L \,\mathrm{d}\log L + \Lambda_K \,\mathrm{d}\log K,$$

  where $\lambda_i = \frac{sales_i}{GDP}$.

What We Do

- Extend these results to inefficient economies.
- General reduced-form, non-parametric formula.
- Mapping from micro to macro using a *general* structural model.
- Wide range of applications: growth accounting, costs of monopolies, costs of taxation, **effects of monetary policy**, etc.

General Framework

- Homothetic final demand.
- General input-output structure with arbitrary neoclassical production functions with A_i the TFP of producer i and wedges.
- Inelastic supply of factors, general equilibrium.
- WLOG, represent all wedges as markups μ_i via relabeling.

Revenue-Based vs. Cost-Based

Definition

Ω and $\tilde{\Omega}$ are $N \times N$ input-output matrices with ijth element:

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \quad \tilde{\Omega}_{ij} = \frac{p_j x_{ij}}{\sum_k p_k x_{ik} + \sum_f w_f F_{if}}.$$

Ψ and $\tilde{\Psi}$ are $N \times N$ Leontief inverse matrices:

$$\Psi = (I - \Omega)^{-1}, \quad \tilde{\Psi} = (I - \tilde{\Omega})^{-1}.$$

b is $N \times 1$ consumption-shares vector with ith element:

$$b_i = \frac{p_i c_i}{\sum_j p_j c_j}.$$

λ and $\tilde{\lambda}$ are $N \times 1$ Domar weights:

$$\lambda = b'\Psi, \quad \tilde{\lambda} = b'\tilde{\Psi}.$$

Revenue-Based vs. Cost-Based

Cost-based definitions capture correct notion of exposure:

- $\tilde{\Omega}_{ij}$ is direct exposure of i to j.
- $\tilde{\Psi}_{ij}$ is direct and indirect exposure of i to j.
- $\tilde{\lambda}_i$ is direct and indirect exposure of household to i.

Macro Impact of Micro Shocks

- $\mathscr{Y}(A, \mathrm{X})$: output Y given productivities A and shares $\mathrm{X}_{ij} = x_{ij}/y_j$.
- Change in equilibrium in response to shocks:

$$\mathrm{d}\log Y = \underbrace{\frac{\partial \log \mathscr{Y}}{\partial \log A}\,\mathrm{d}\log A}_{\Delta\text{Technology}} + \underbrace{\frac{\partial \log \mathscr{Y}}{\partial \mathrm{X}}\,\mathrm{d}\mathrm{X}}_{\Delta\text{Allocative Efficiency}},$$

When efficient,

$$\mathrm{d}\log Y = \underbrace{\lambda'\,\mathrm{d}\log A}_{\text{marginal cost pricing}} + \underbrace{0}_{\text{Pareto-efficiency}}.$$

Generalized Hulten – Productivity Shock

Theorem

$$\begin{aligned}\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_k} &= \frac{\partial \log \mathscr{Y}}{\partial \log A_k} + \frac{\partial \log \mathscr{Y}}{\partial X}\frac{\mathrm{d}X}{\mathrm{d}\log A_k}, \\ &= \underbrace{\tilde{\lambda}_k}_{\Delta \textit{Technology}} \quad - \underbrace{\sum_f \tilde{\Lambda}_f \frac{\mathrm{d}\log \Lambda_f}{\mathrm{d}\log A_k}}_{\Delta \textit{Allocative Efficiency}} .\end{aligned}$$

Generalized Hulten – Wedge Shock

Theorem

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log \mu_k} = \frac{\partial \log \mathscr{Y}}{\partial X}\frac{\mathrm{d}X}{\mathrm{d}\log \mu_k},$$
$$= \underbrace{-\tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f \frac{\mathrm{d}\log \Lambda_f}{\mathrm{d}\log \mu_k}}_{\Delta \textit{Allocative Efficiency}}.$$

Sources of Growth and Solow Residual

- Easy extension to changing factor supply:

$$\mathrm{d}\log Y - \tilde{\Lambda}'\,\mathrm{d}\log L = \underbrace{\tilde{\lambda}'\,\mathrm{d}\log A}_{\Delta\text{Technology}} \underbrace{-\tilde{\lambda}'\,\mathrm{d}\log\mu + \tilde{\Lambda}'\,\mathrm{d}\log\Lambda}_{\Delta\text{Allocative Efficiency}}.$$

- Solow residual:

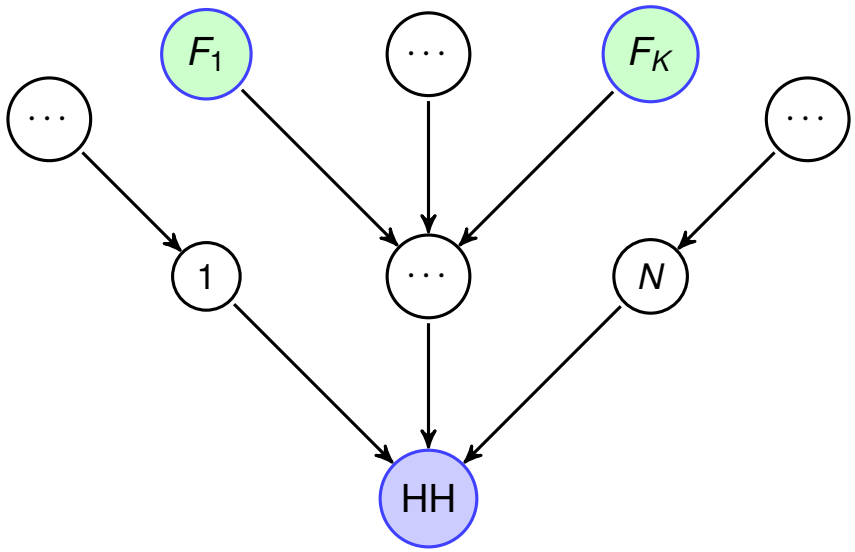
$$\mathrm{d}\log Y - \hat{\Lambda}'\,\mathrm{d}\log L = \underbrace{\tilde{\lambda}'\,\mathrm{d}\log A}_{\Delta\text{Technology}} \underbrace{-\tilde{\lambda}'\,\mathrm{d}\log\mu + \tilde{\Lambda}'\,\mathrm{d}\log\Lambda}_{\Delta\text{Allocative Efficiency}} + \underbrace{(\tilde{\Lambda} - \hat{\Lambda})'\,\mathrm{d}\log L}_{\text{undercounting factors}},$$

  where $\hat{\Lambda}$ adjusts Λ to count profit share in capital share.

- Can perform decomposition without imposing *any* parametric assumptions on production functions. Example: handles factor augmenting productivity and demand shocks with no modification.

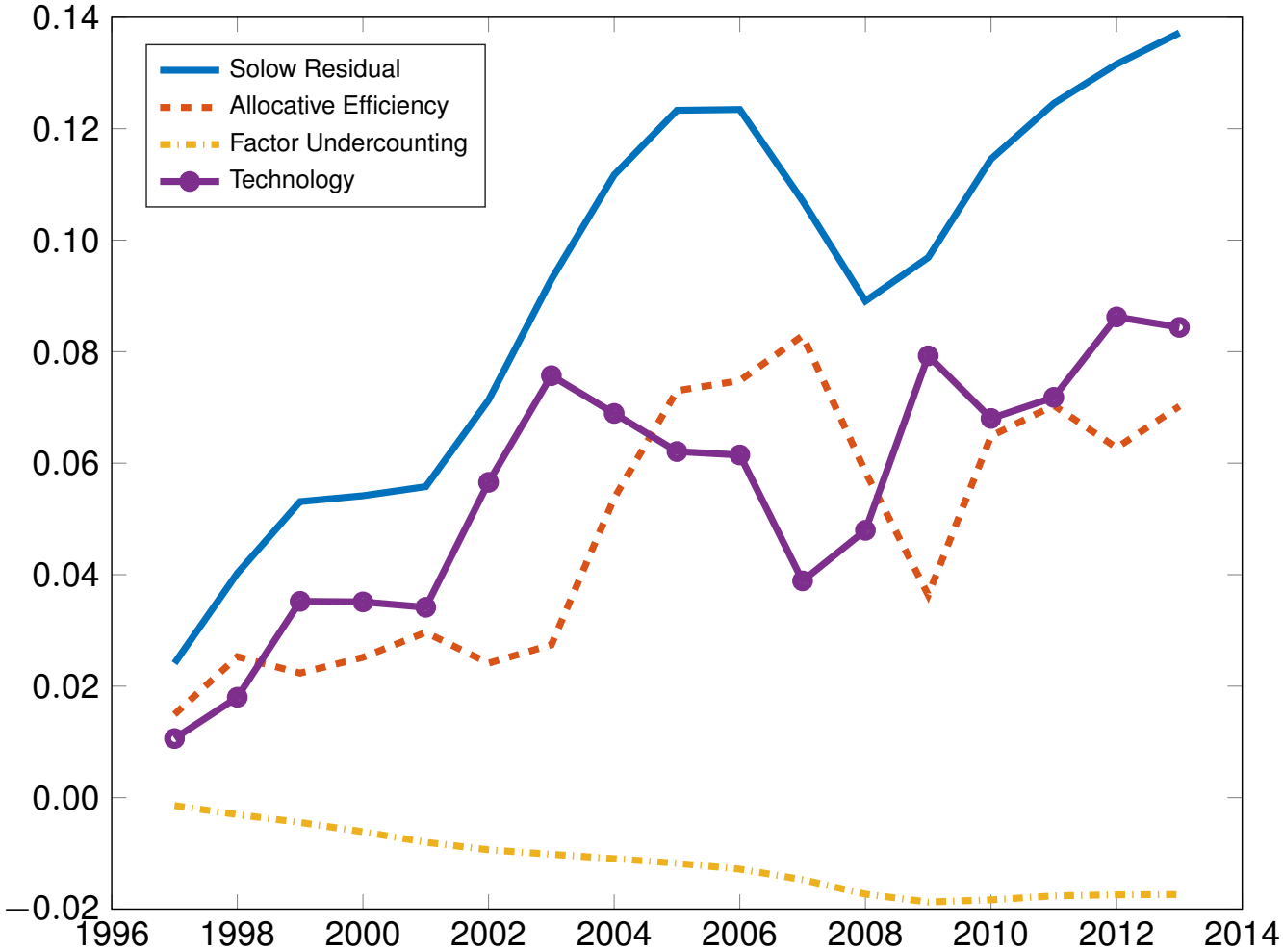
Alternative Decompositions

- Alternative decompositions of Basu-Fernald (2002) and Petrin-Levinsohn (2012).
- These decompositions do not use input-output information.
- Revealing example of acyclic economies:



- These decompositions detect changes in allocative efficiency, even though allocation is efficient.

Sources of Growth and Solow Residual



- Using the Gutiérrez and Philippon (2016) markup data.
- Similar with De Loecker and Eeckhout (2017) and Lerner Indices.

Parametric Model

- General nested CES economy.
- "Relabel" each CES nest to be a new sector with elasticity ε_i.
- Structure can actually represent *any* nested CES economy with arbitrary pattern of nests and wedges.

Definition

$$Cov_{\tilde{\Omega}^{(j)}}\left(\tilde{\Psi}_{(k)}, \Psi_{(L)}\right) = \sum_i \tilde{\Omega}_{ji}\tilde{\Psi}_{ik}\Psi_{iL} - \left(\sum_i \tilde{\Omega}_{ji}\tilde{\Psi}_{ik}\right)\left(\sum_i \tilde{\Omega}_{ji}\Psi_{iL}\right).$$

Macro Impact of Micro Productivity Shocks: One Factor

Proposition

Suppose there is only one factor (with index L). Then

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_k} = \tilde{\lambda}_k - \frac{\mathrm{d}\log \Lambda_L}{\mathrm{d}\log A_k},$$

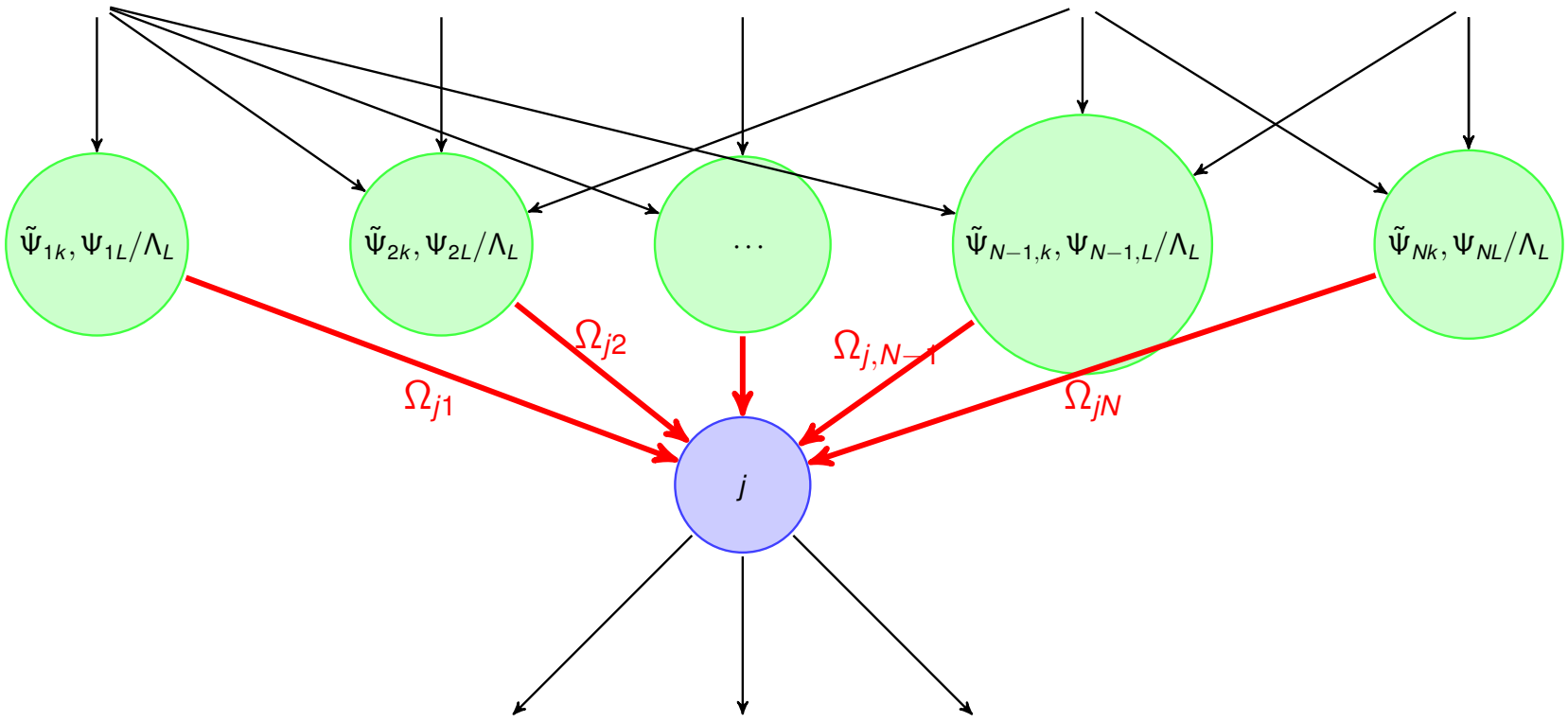
which is equal to

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_k} = \underbrace{\tilde{\lambda}_k}_{\Delta \textit{Technology}} \underbrace{- \sum_j (\theta_j - 1)\mu_j^{-1}\lambda_j Cov_{\tilde{\Omega}(j)}\left(\tilde{\Psi}_{(k)}, \frac{\Psi_{(L)}}{\Lambda_L}\right)}_{\Delta \textit{Allocative Efficiency}}.$$

- Change in allocative efficiency opposite of change in labor share.
- Centrality measure mixing network and elasticities.
- Upstream and downstream distortions matter.

Explaining Covariance Operator

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_k} = \tilde{\lambda}_k - \sum_j \underbrace{(\theta_j - 1)\mu_j^{-1}\lambda_j Cov_{\tilde{\Omega}(j)}\left(\tilde{\Psi}_{(k)}, \frac{\Psi_{(L)}}{\Lambda_L}\right)}.$$



- High $\tilde{\Psi}_{ik}$: i's highly exposed to k.
- High Ψ_{iL}/Λ_L: most of i's revenues are ultimately paid to workers.

Macro Impact of Micro Markup Shocks: One Factor

Proposition

Suppose there is only one factor indexed by L. Then

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log \mu_k} = -\tilde{\lambda}_k - \frac{\mathrm{d}\log \Lambda_L}{\mathrm{d}\log \mu_k},$$

which is equal to

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log \mu_k} = \underbrace{-\tilde{\lambda}_k - \left[\sum_j (1-\theta_j)\mu_j^{-1}\lambda_j Cov_{\tilde{\Omega}_j}\left(\tilde{\Psi}_{(k)}, \frac{\Psi_{(L)}}{\Lambda_L}\right) - \lambda_k \frac{\Psi_{kL}}{\Lambda_L}\right]}_{\Delta Allocative\ Efficiency}.$$

- First two terms act like a negative productivity shock.
- Third term captures that increase in markups releases labor.

Macro Impact of Micro Productivity Shocks: Multiple Factors

The following linear system describes the elasticities of factor shares:

$$\frac{\mathrm{d}\log\Lambda}{\mathrm{d}\log A_k} = \Gamma\frac{\mathrm{d}\log\Lambda}{\mathrm{d}\log A_k} + \delta^{(k)},$$

with

$$\Gamma_{F,L} = \sum_j(\theta_j - 1)\lambda_j\mu_j^{-1}Cov_{\tilde{\Omega}^{(j)}}\left(\tilde{\Psi}_{(F)}, \frac{\Psi_{(L)}}{\Lambda_L}\right),$$

and

$$\delta_F^{(k)} = \sum_j(\theta_j - 1)\lambda_j\mu_j^{-1}Cov_{\tilde{\Omega}^{(j)}}\left(\tilde{\Psi}_{(k)}, \frac{\Psi_{(F)}}{\Lambda_F}\right).$$

Given the elasticities of factor shares, we have

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_k} = \tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f\frac{\mathrm{d}\log\Lambda_f}{\mathrm{d}\log A_k}.$$

- Similar for markup/wedge shocks.

Gains from Eliminating Markups

- Calibrated firm-level model with input-output connections and mark-up data from Gutierrez & Philippon (GP), De Loecker and Eeckhout (DE), and Lerner Index (LI).

	Benchmark	CD+CD	VA Benchmark	VA CD + CD
GP	20%	4%	8%	1%
LI	17%	4 %	7%	1%
DE	35%	7%	18 %	3%

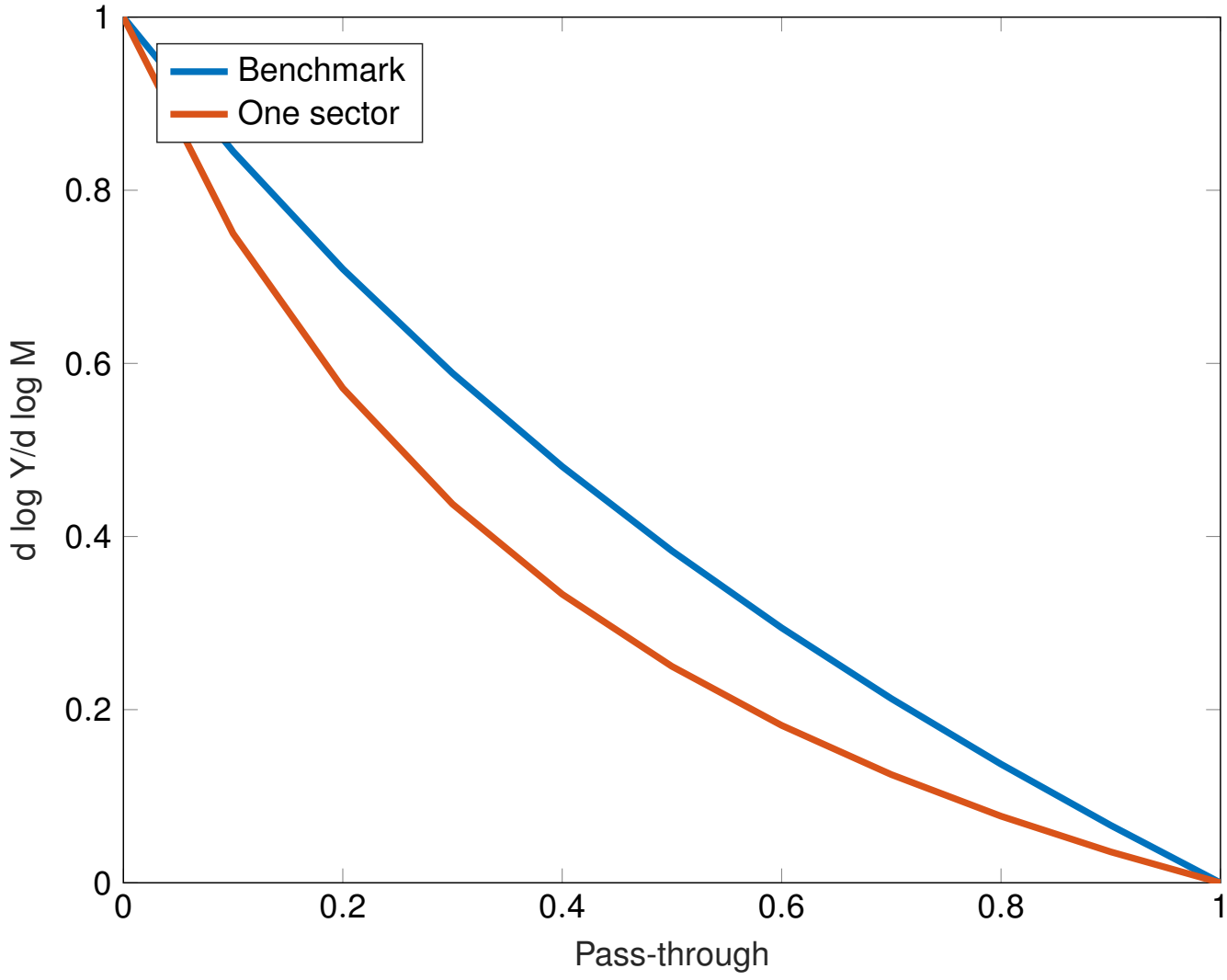
- Elasticities matter.
- Input-output structure matters.
- Value-added production functions misleading!

Nominal Rigidities

- Use framework to analyze sticky price models with heterogeneity, production networks, *and* inefficient steady state.
- Markups adjust so that prices don't.
- Pass-through function: $\mathrm{d}\log p_i = \phi_i(\mu_i)\,\mathrm{d}\log \text{marginal cost}_i$.
- Money and cash-in-advance constraint.
- Labor only factor with Frisch elasticity of supply $\nu = 0.5$.
- Characterize effects of changes in money supply.
- Details and formulas in paper.

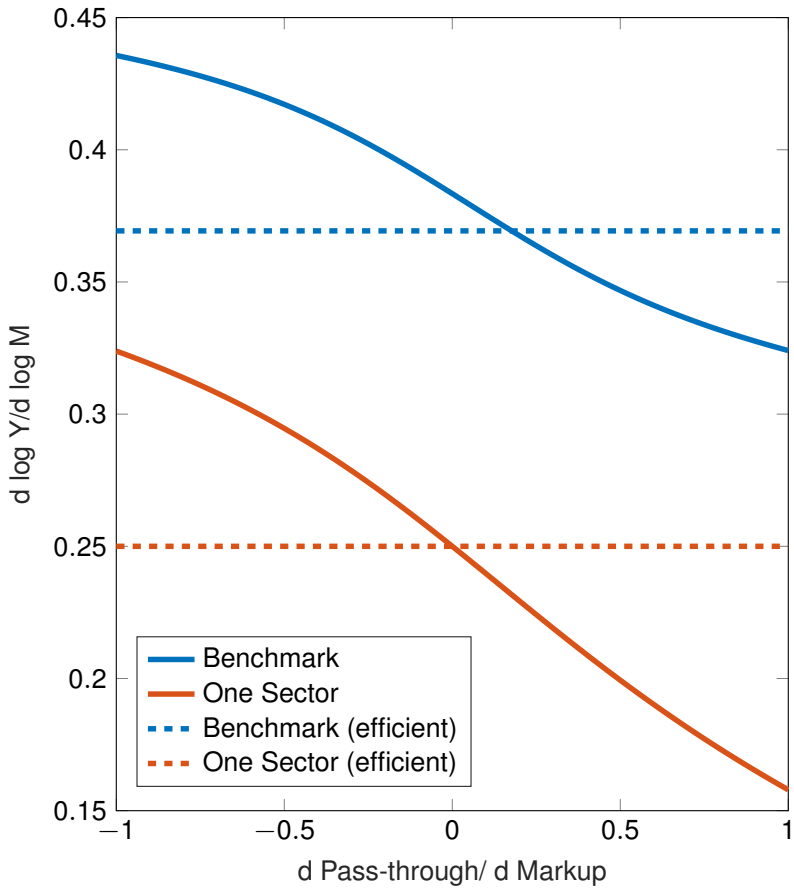
Macro-stickiness from Intermediate Inputs

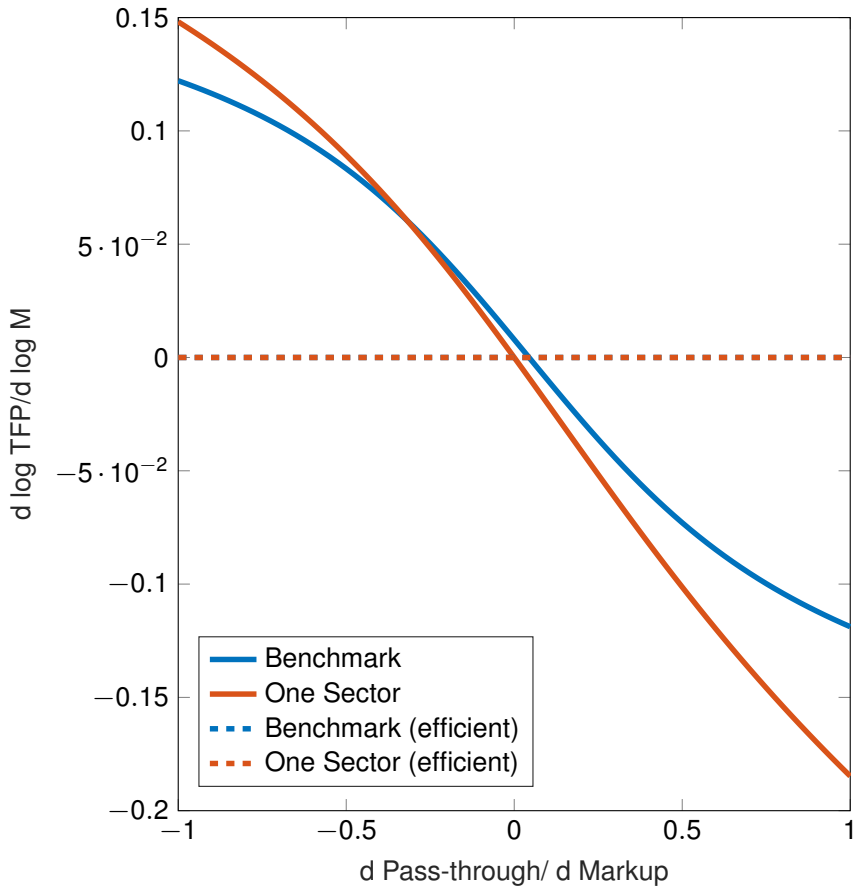
- Intermediate inputs increases macro-stickiness (Basu 1995).
- Amplifies effects of monetary policy.



TFP Effects when Inefficient Steady-state

- Inefficient steady-state, so TFP response to monetary policy (consistent with Evans 1990, Hall 1988, etc.)
- Amplifies or mitigates effects of monetary policy depending on correlation price adjustment vs. level of markups.
- Amplification when high markup firms adjust less as in Melitz-Ottaviano (2008), Atkeson-Burstein (2008), Bergin-Feenstra (2001).





Conclusion

- Reduced-form aggregation theorem for economies with frictions.
- Structural aggregation theorems.
- Wide range of applications in different contexts.
- Some implications for monetary policy:
  - Intermediate inputs amplify macro-stickiness.
  - TFP responds to monetary policy.

De Loecker, J. and J. Eeckhout (2017). The rise of market power and the macroeconomic implications. Technical report, National Bureau of Economic Research.

Gutiérrez, G. and T. Philippon (2016). Investment-less growth: An empirical investigation. Technical report, National Bureau of Economic Research.

Solow, R. M. (1957). Technical change and the aggregate production function. *The review of Economics and Statistics*, 312–320.