Optimal Contracts with a Risk-Taking Agent

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Risk-Taking & the Financial Crisis

- *"History is littered with examples of firms that got what they paid for."*
- A particularly insidious form of gaming is by taking on (left-tail) risk.
  - Difficult to prevent via monitoring & enormously costly
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  "compensation practices at some banking organizations have led to misaligned incentives and excessive risk-taking, contributing to bank losses and financial instability"
- Rajan (2011) argued risk-taking exarcebated the 2008 financial crisis.
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Gaming via Risk-Taking is Widespread

- **Portfolio managers** adjust the riskiness of their investments

  Chevalier & Ellison (1997), de Figueiredo et al. (2016)

- **Executives** may cut maintenance to meet earnings targets

  Repenning & Henderson (2015), Garicano & Rayo (2016) and references

- **Entrepreneurs** pursue unproven technologies or incremental progress

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What is this paper about?

Optimal contracting when

the agent can game the contract it by *gambling*

Framework

- **Model in a Nutshell.** Canonical principal-agent framework, in which:
  1. The agent chooses effort, which determines *intermediate output*.
  2. He gambles by choosing *any* mean-preserving spread of interm. output.
  3. This mean-preserving spread determines final, contractible output.
- **Mechanism.** The agent gambles to game convex incentives.
  - His payoff equals the *concave closure* of his utility under the contract.
- **Prop. 1.** Optimal contract makes agent's utility *concave* in output.
  - "Classic" principal-agent problem + *no-gaming constraint*

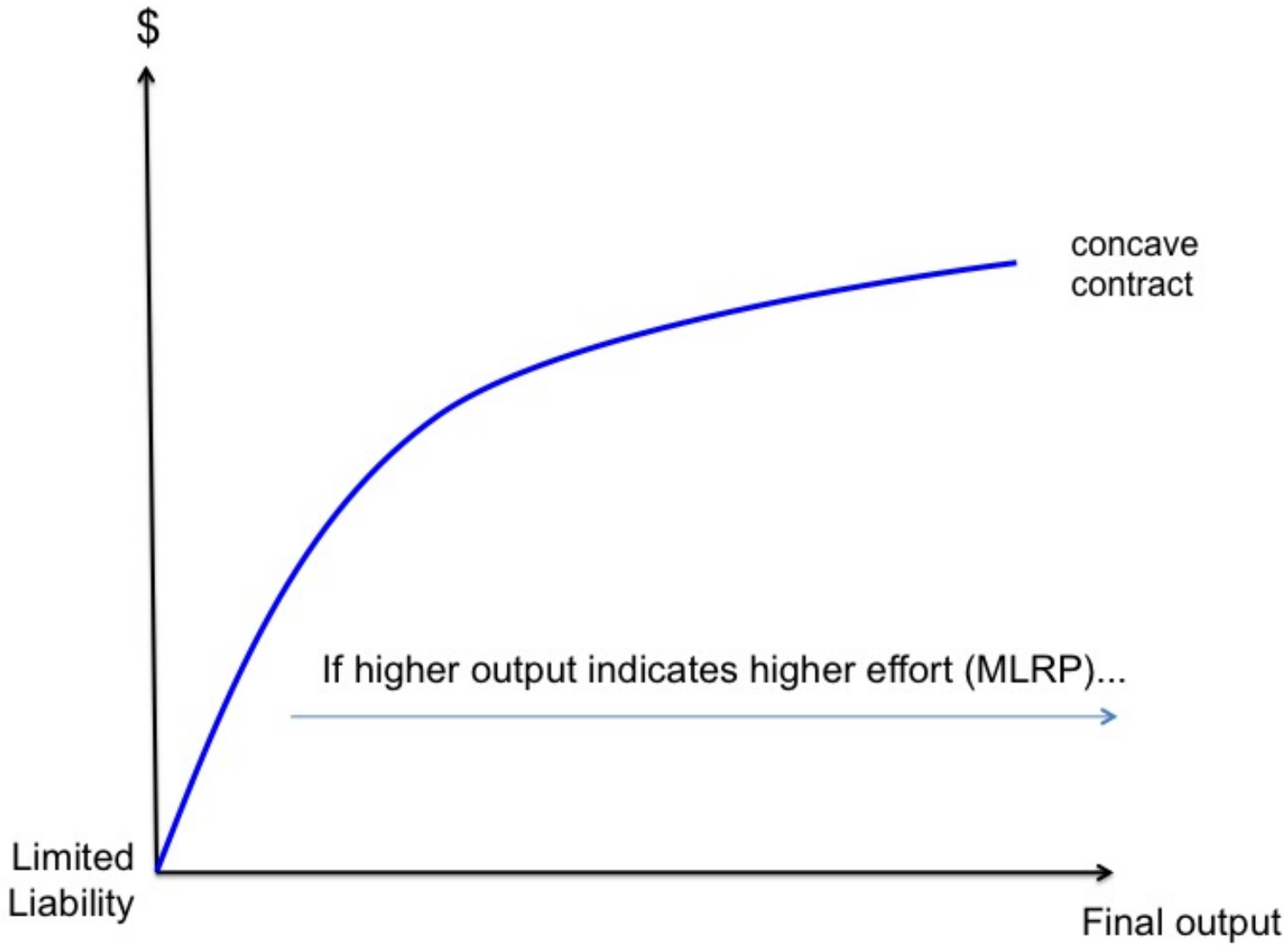
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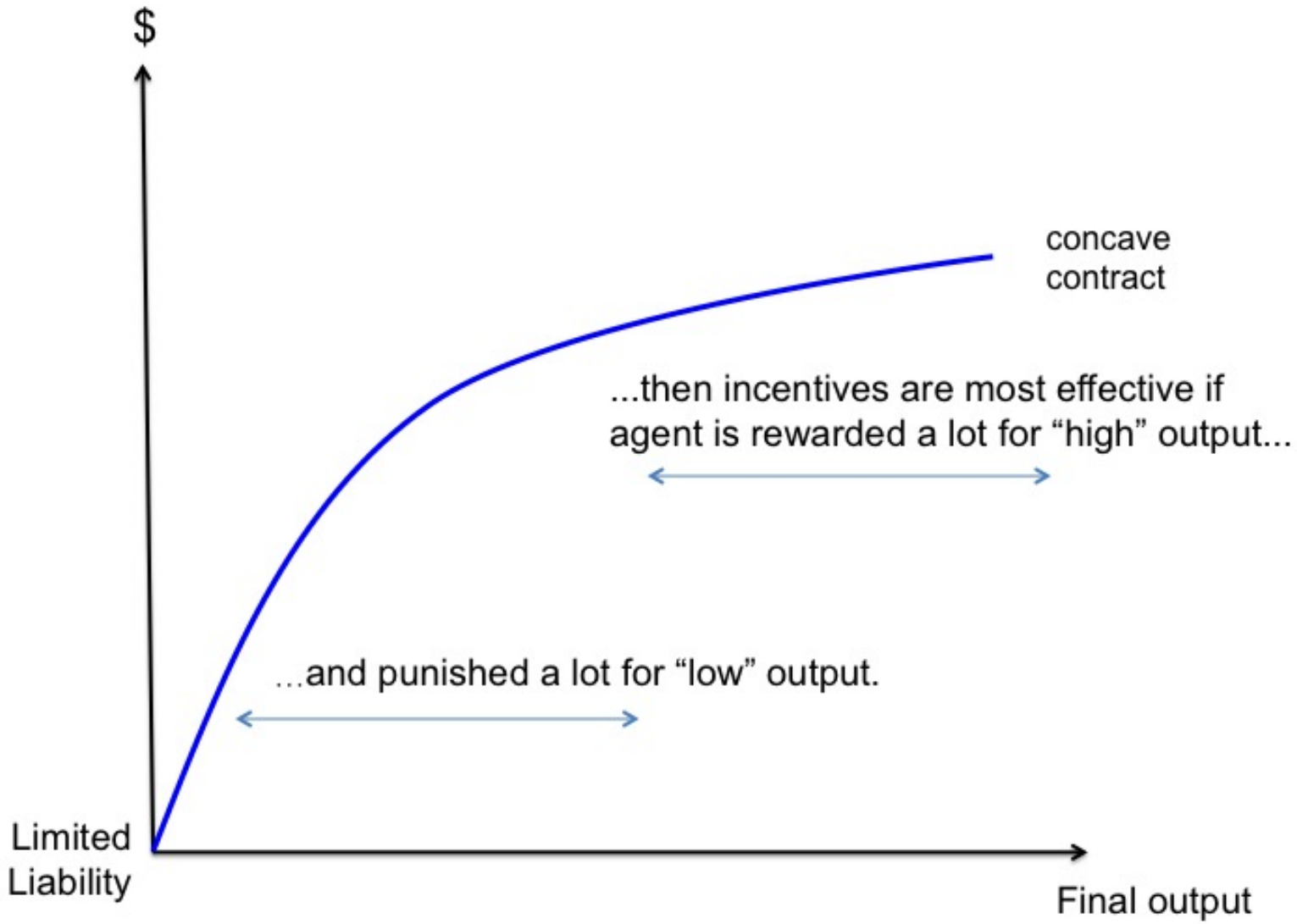
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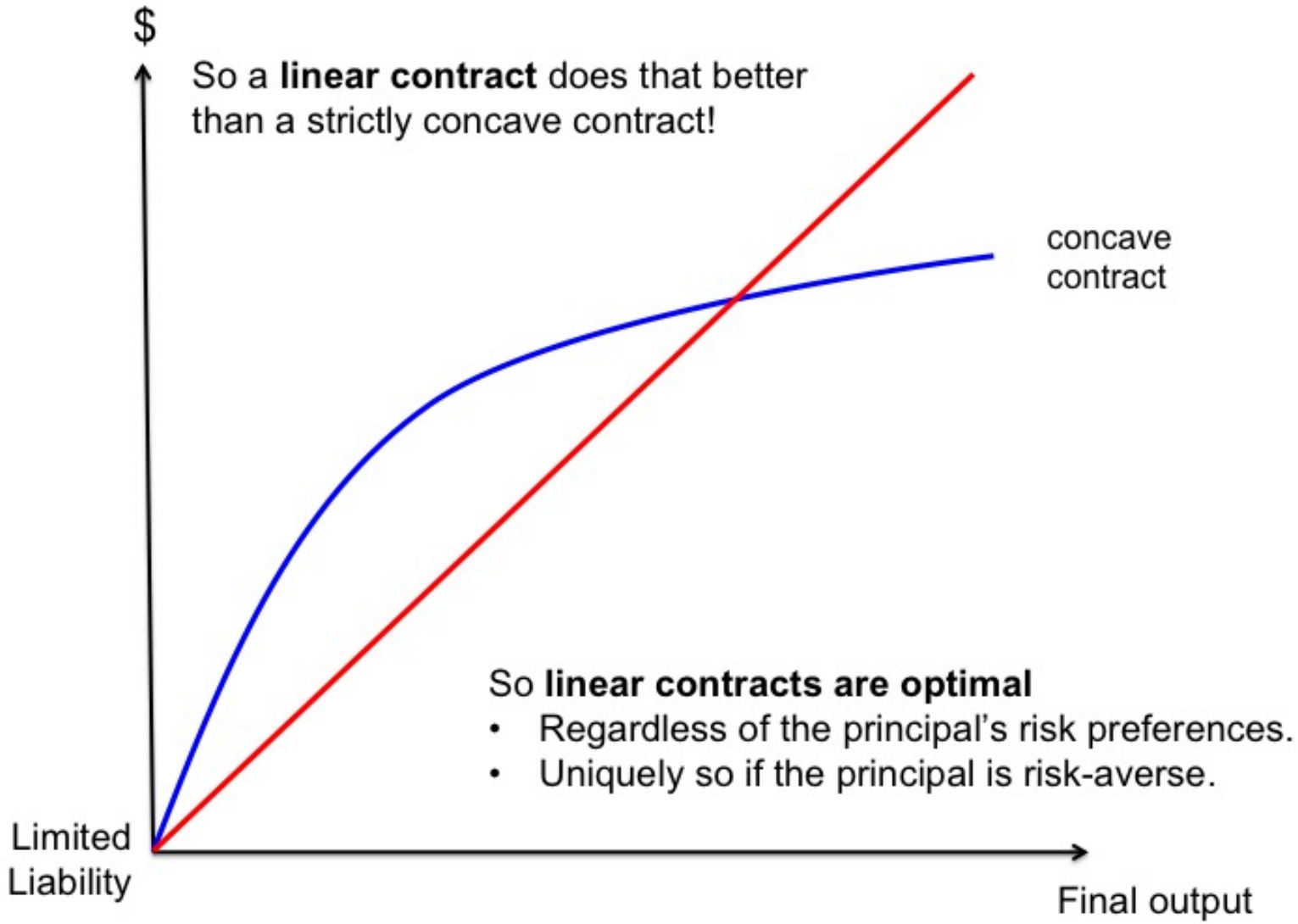
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Rough Intuition: Risk-averse Agent

- No-gaming constraint: agent's *utility* must be concave in output.
  - If agent is risk-averse, optimal contract *may* be *strictly concave*.
- We develop a set of tools to characterize the optimal contract.
  - Where no-gaming *binds*, contract makes agent's utility *linear* in output.
  - Where it is *slack*, contract resembles "classic" optimal contract.
- Two notable cases:
  1. If LL is slack, under mild conditions, optimal incentives are linear below a threshold, and coincide with the "classic" contract above threshold.
  2. If IR is slack, optimal incentives are linear below a threshold.

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Extensions and Reinterpretations

- Three extensions with risk-neutral players.

1. What if agent gambles *before* the exogenous uncertainty is resolved?
   - Leads to different but related constraints on contracts.
   - A linear contract is optimal under mild conditions.
2. What if gambling is costly?
   - Tools extend (for our formulation of risk-taking costs)
   - Optimal contract is convex, and converges to linear as costs vanish.
3. What if the agent can game by shifting output over time?
   - Reinterpret model as intertemporal gaming of stationary contracts.
   - A linear contract is optimal.

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Related Literature

- Large empirical literature on risk-taking and intertemporal gaming.
  - Brown, Harlow & Starks (1996), Chevalier & Ellison (1997), Brav et al. (2005), Matta & Beamish (2008), Rajan (2010), de Figueiredo et al. (2014), Repenning & Henderson (2015), Shue & Townsend (2017)
  - Oyer (1998), Larkin (2014), Repenning & Henderson (2015)
- Some theory on agents using risk-taking to game contracts:
  - **Diamond (1998)** and Garicano & Rayo (2016)
  - Palomino & Pratt (2003)
  - DeMarzo et al. (2014), Hébert (2015), Makarov & Plantin (2016)
- Theory on the optimality of simple (linear) contracts:
  - **Holmström & Milgrom (1987)** and Edmans & Gabaix (2011)
  - Chassang (2013), **Carroll (2015)**, Antic (2016)

Model

- Set of possible outcomes $\mathcal{Y} = [\underline{y}, \bar{y}]$.
- Players:
  - Weakly risk-averse principal
  - Weakly risk-averse agent with liability M and outside option u_0
- Timing:
  1. Principal offers a contract $s(y)$.
  2. Agent accepts / rejects contract, and chooses effort a.
  3. Intermediate output $x \sim F(\cdot|a)$ is privately observed by the agent.
     - F satisfies strict MLRP, some regularity conditions, and $\mathbb{E}_{F(\cdot|a)}[x] = a$.
  4. Agent chooses distribution $G_x \in \Delta(\mathcal{Y})$ subject to $\mathbb{E}_{G_x}[y] = x$.
  5. Final output $y \sim G_x$ is realized and the agent is paid $s(y)$.
- Payoffs:
  - Principal: $\pi(y - s(y))$, where $\pi'' \leq 0 < \pi'$.
  - Agent: $u(s(y)) - c(a)$, where $u'' \leq 0 < u'$ and $c', c'' > 0$.

Problem Formulation and a Simplifying Result

- Principal solves the following constrained maximization problem:

$$\max_{a,\, G\in\mathcal{G},v(\cdot)} \quad \mathbb{E}_{F(\cdot|a)}\left[\mathbb{E}_{G_x}\left[\pi\left(y-u^{-1}\left(v\left(y\right)\right)\right)\right]\right] \tag{Obj$_F$}$$

$$\text{s.t.} \quad a, G \in \arg\max_{\tilde{a},\, \tilde{G}\in\mathcal{G}} \left\{\mathbb{E}_{F(\cdot|\tilde{a})}\left[\mathbb{E}_{\tilde{G}_x}\left[v\left(y\right)\right]\right] - c(\tilde{a})\right\} \tag{IC$_F$}$$

$$\mathbb{E}_{F(\cdot|a)}\left[\mathbb{E}_{G_x}\left[v\left(y\right)\right]\right] - c(a) \geq u_0 \tag{IR$_F$}$$

$$v\left(\cdot\right) \geq u(-M)\,, \tag{LL$_F$}$$

where $v(y) = u(s(y))$, $G = \{G_x\}_{x\in\mathcal{Y}}$.

Proposition 1:

- Suppose $\{a, G, v(\cdot)\}$ solves (Obj$_F$) - (LL$_F$).
- Then $\left\{a, G^{Degenerate}, v^c(\cdot)\right\}$, where v^c denotes concave closure of v, satisfies (Obj$_F$) - (LL$_F$), and gives the principal weakly higher profit.

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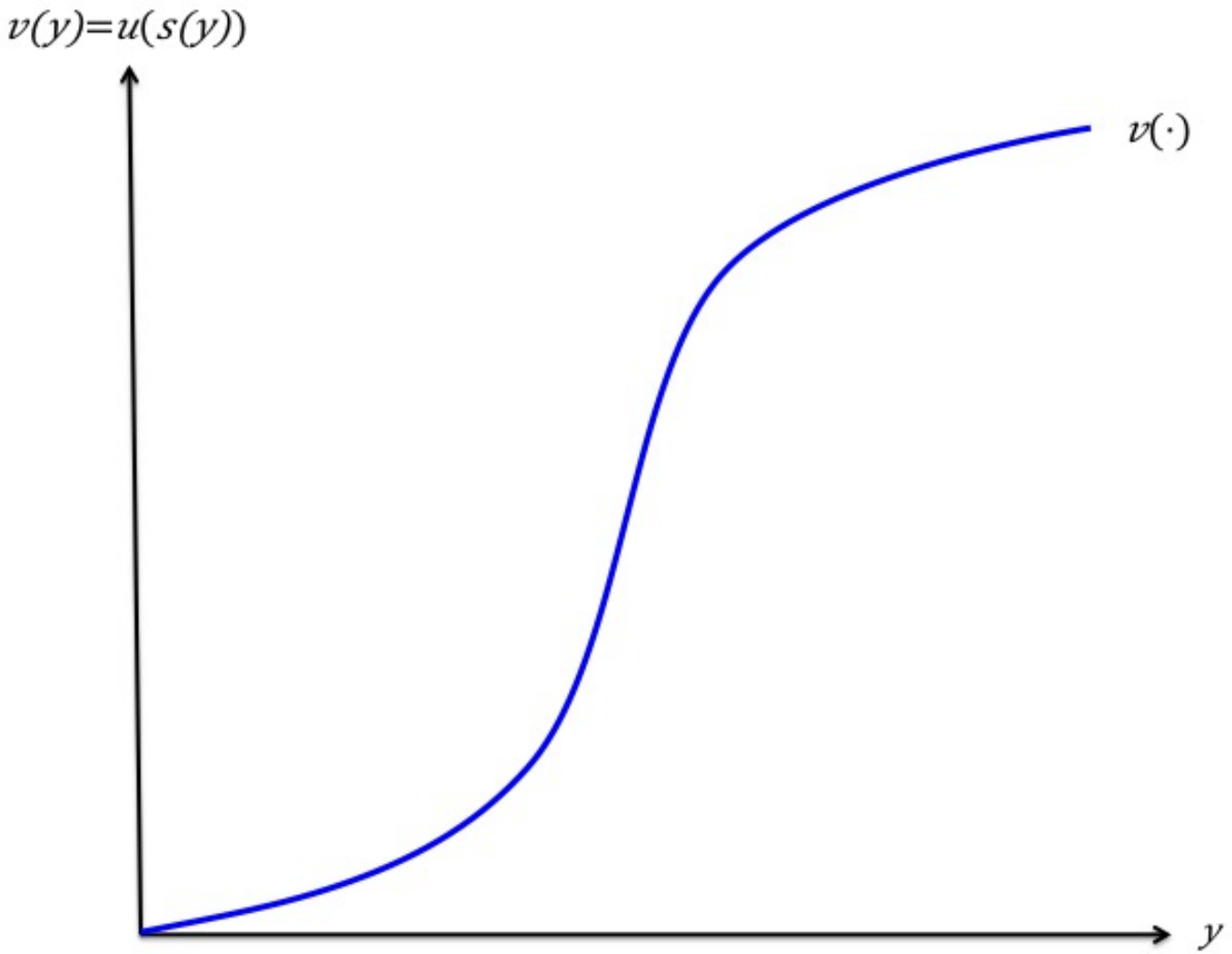
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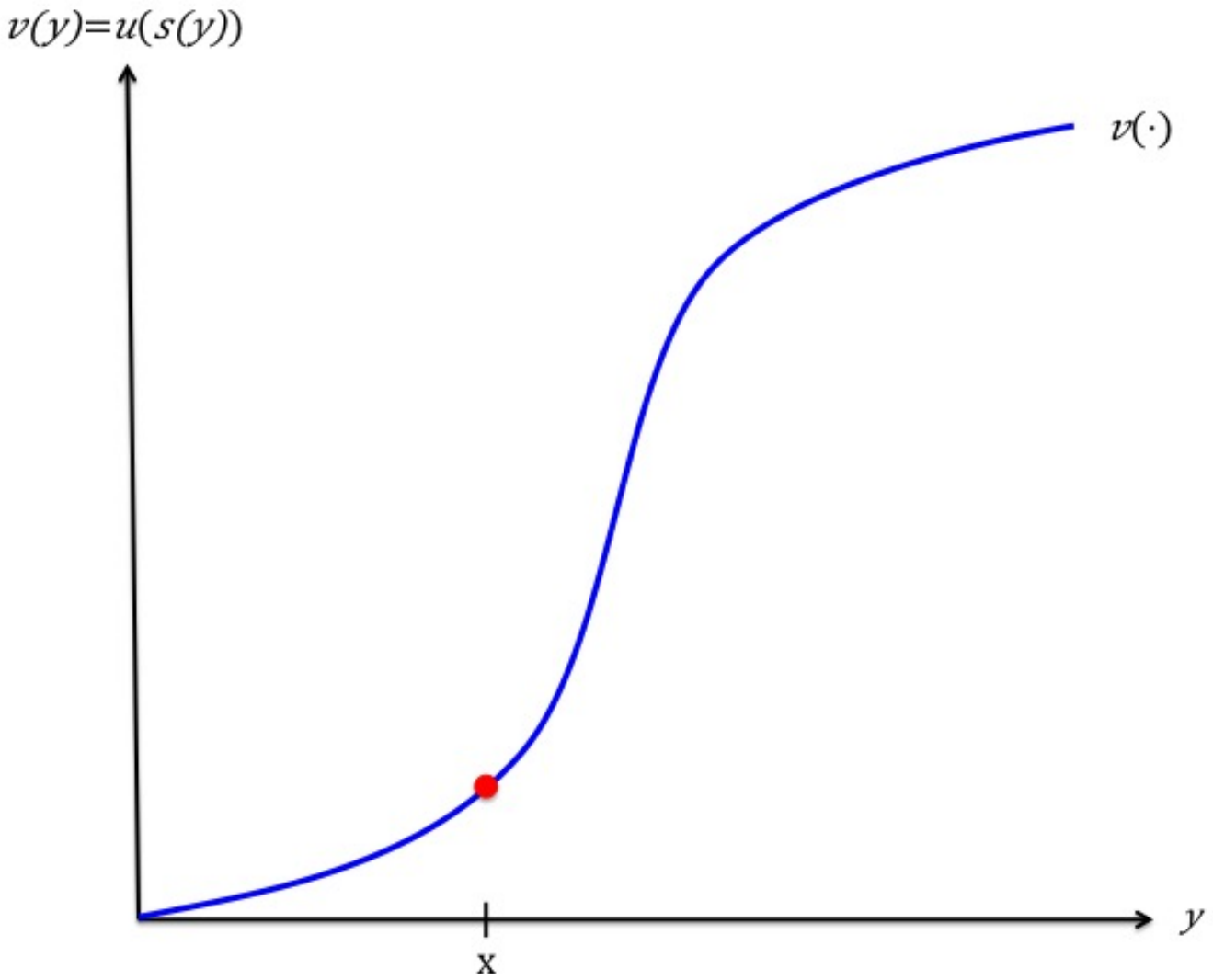
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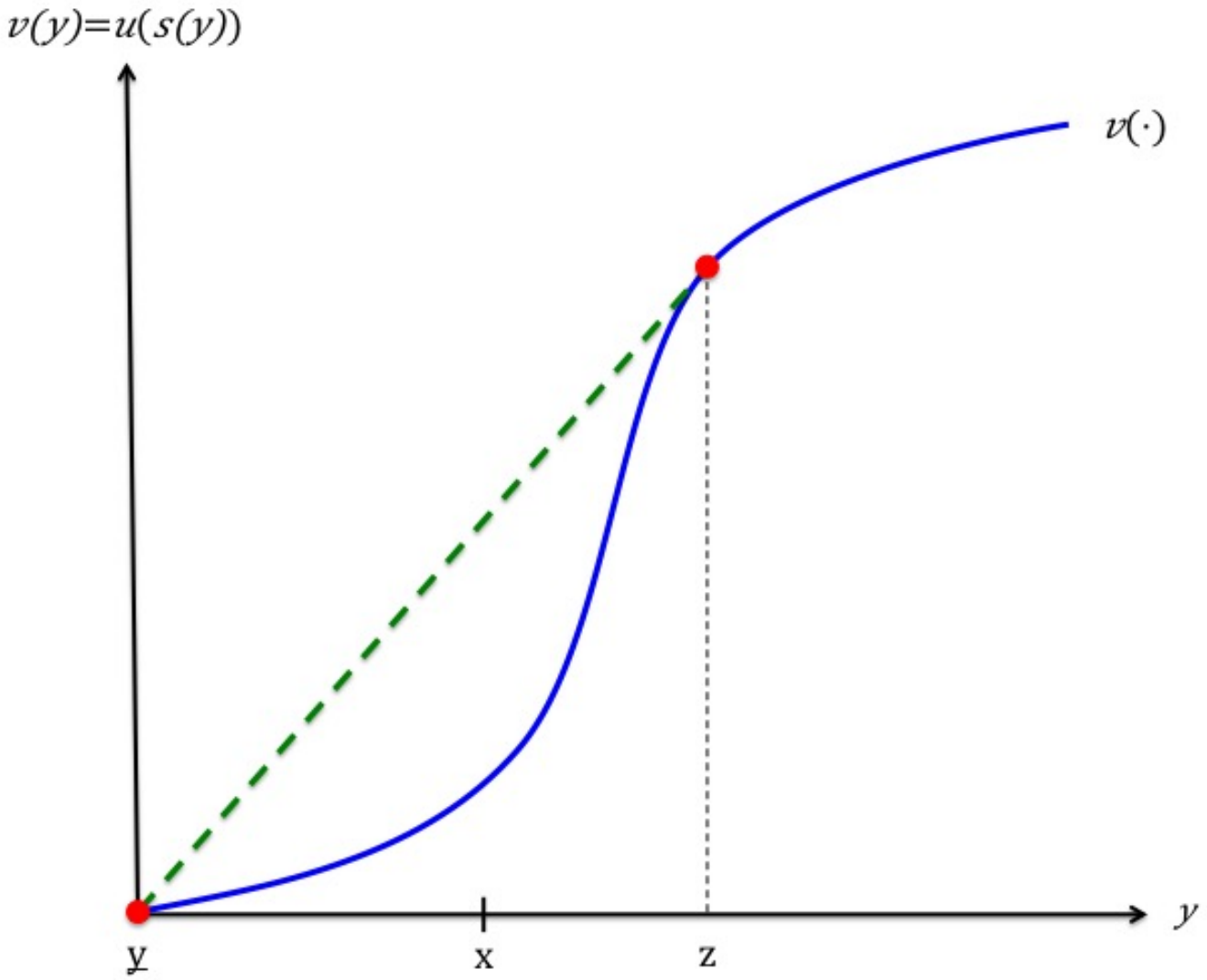
Intuition for Concave Incentives



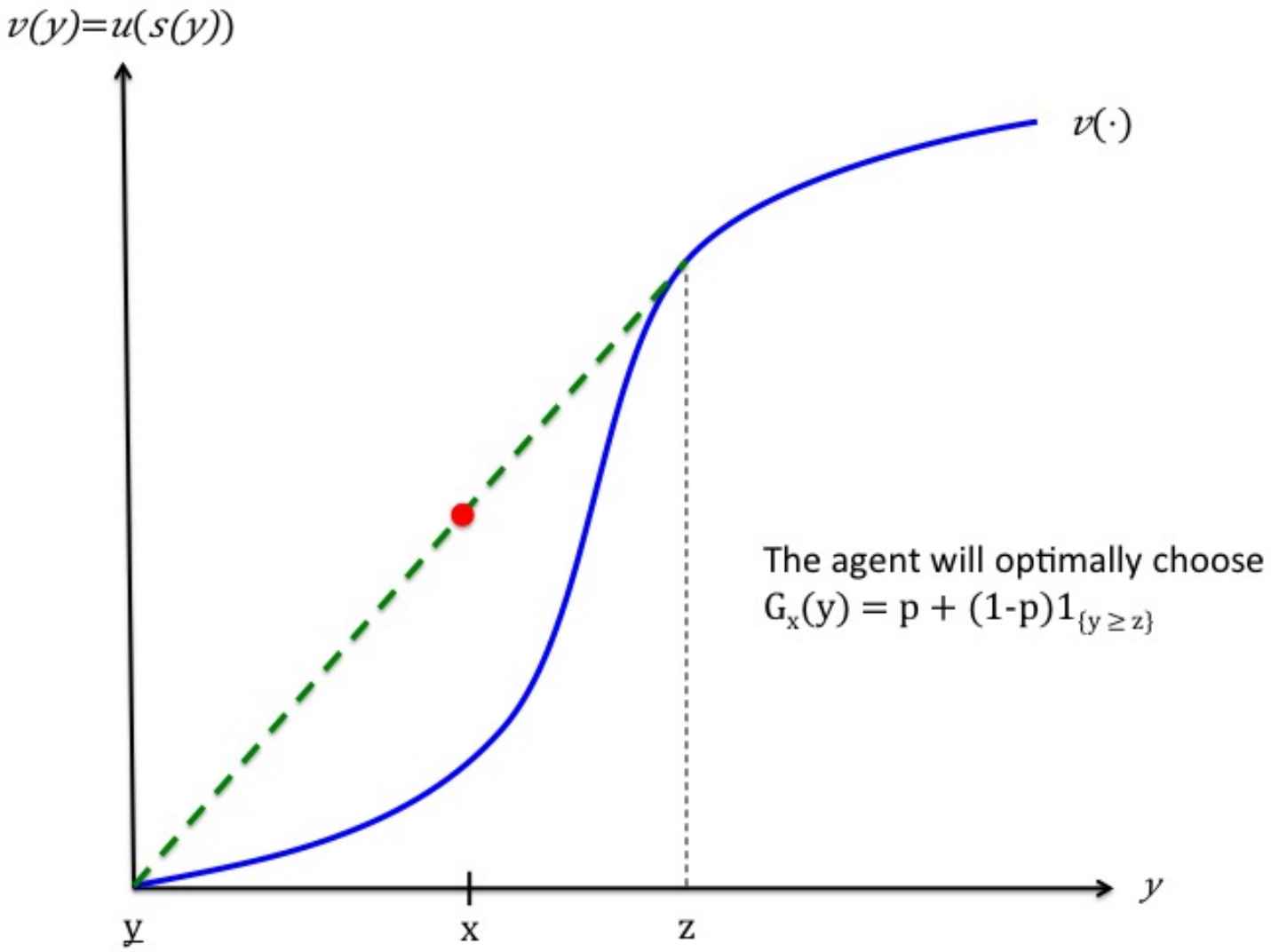
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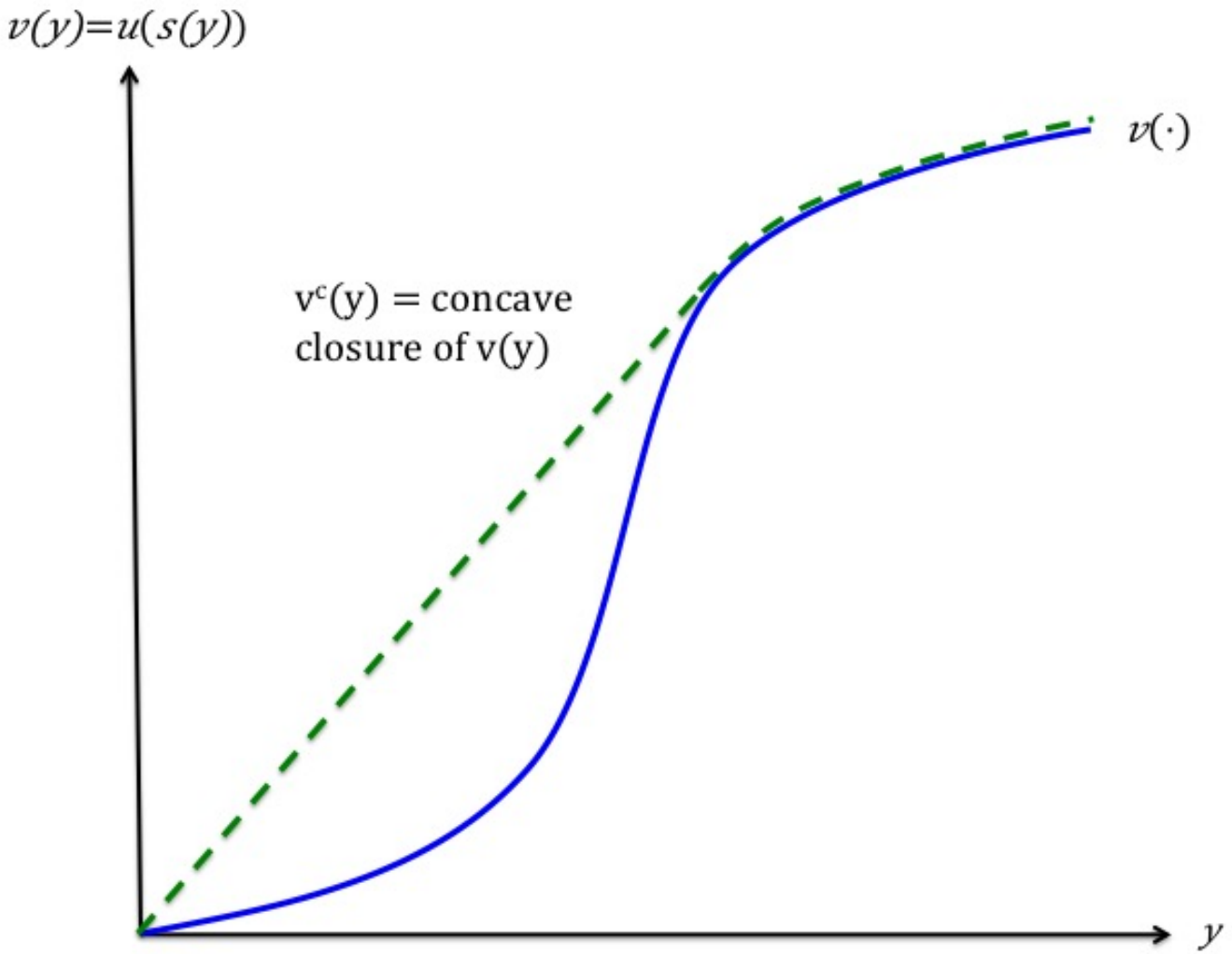
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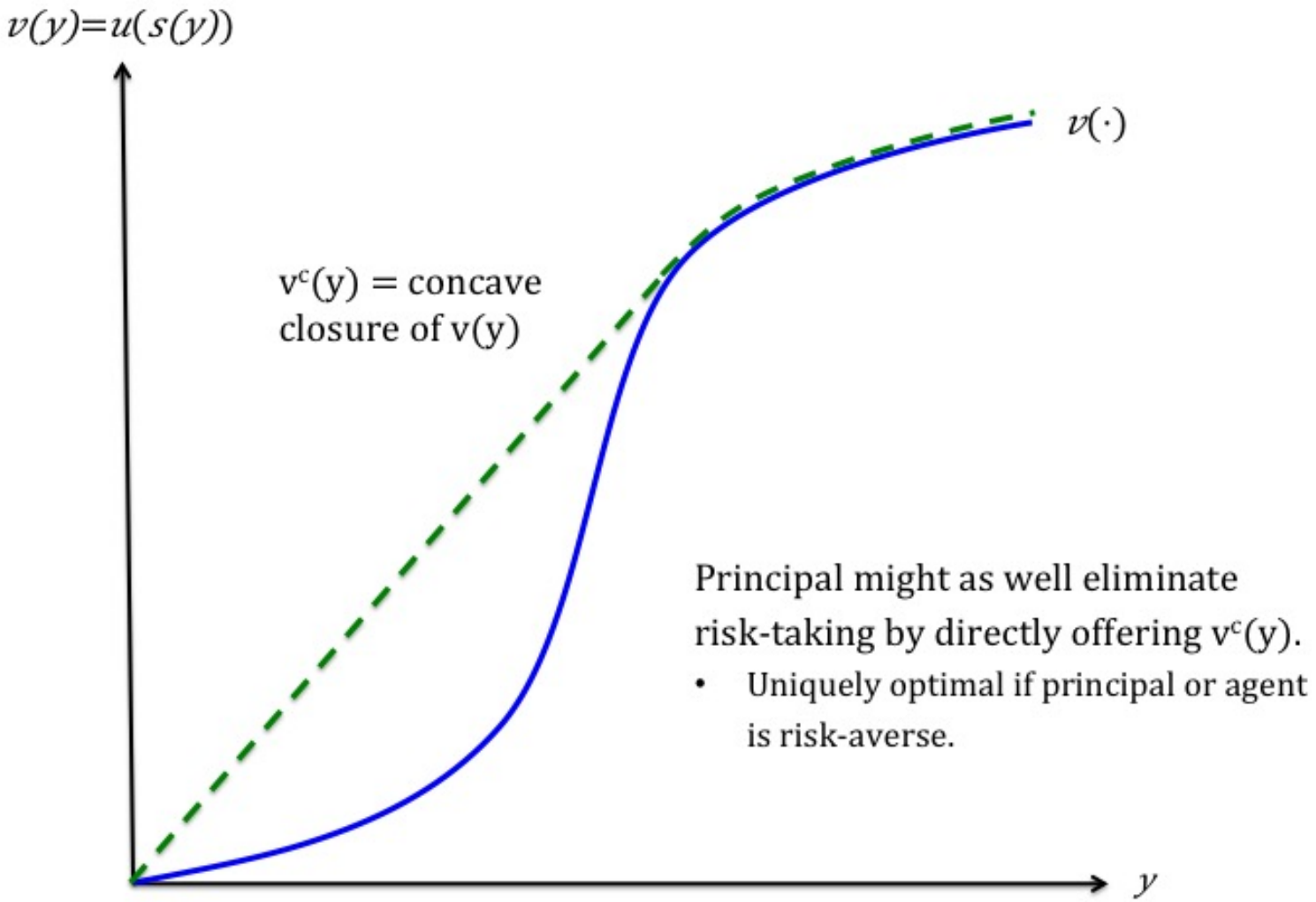
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Principal might as well eliminate risk-taking by directly offering $v^c(y)$.

- Uniquely optimal if principal or agent is risk-averse.

Reformulating the Problem

- Therefore, the principal's problem can be simplified to:

$$\begin{aligned}
\max_{a,v(\cdot)} \quad & \mathbb{E}_{F(\cdot|a)}\left[\pi\left(y - u^{-1}(v(y))\right)\right] && \text{(Obj)}\\
\text{s.t.} \quad & a \in \arg\max_{\tilde{a}} \left\{\mathbb{E}_{F(\cdot|\tilde{a})}\left[v(y)\right] - c(\tilde{a})\right\} && \text{(IC)}\\
& \mathbb{E}_{F(\cdot|a)}\left[v(y)\right] - c(a) \geq u_0 && \text{(IR)}\\
& v(y) \geq \underline{u} \text{ for all } y \in \mathcal{Y} && \text{(LL)}\\
& v(\cdot) \text{ weakly concave.} && \text{(Conc)}
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Lemma 1:

- Fix $a \geq 0$ and assume $\underline{u} > -\infty$. A profit-maximizing contract exists.
- It is unique if at least one party is strictly risk averse.

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Risk-Neutral Agent: A Linear Contract is Optimal

- Define

$$s_a^L(y) = c'(a)(y - \underline{y}) - \underbrace{\min\left\{M, c'(a)(a - \underline{y}) - c(a) - u_0\right\}}_{\text{constant to satisfy (IR) and (LL)}}$$

  - Cheapest *linear contract* that satisfies (IC), (IR) and (LL) for effort a.

- Define a^{FB} by $c'(a^{FB}) = 1$. This is the first-best effort level.

Proposition 2: Risk-neutral agent

- Assume $u(s) \equiv s$.
- If a^* is optimal, then $a^* \leq a^{FB}$ and $s_{a^*}^L(\cdot)$ is optimal.

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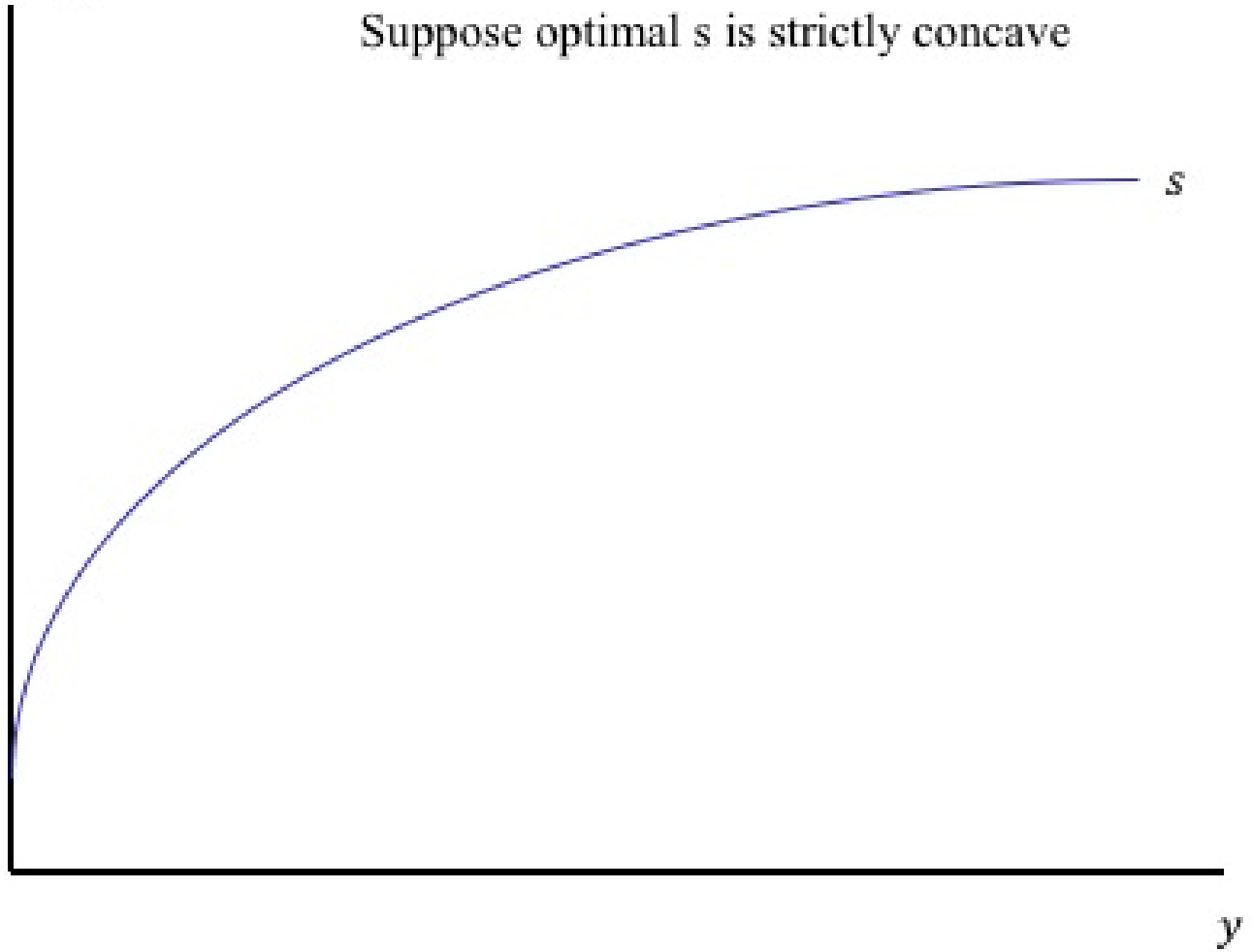
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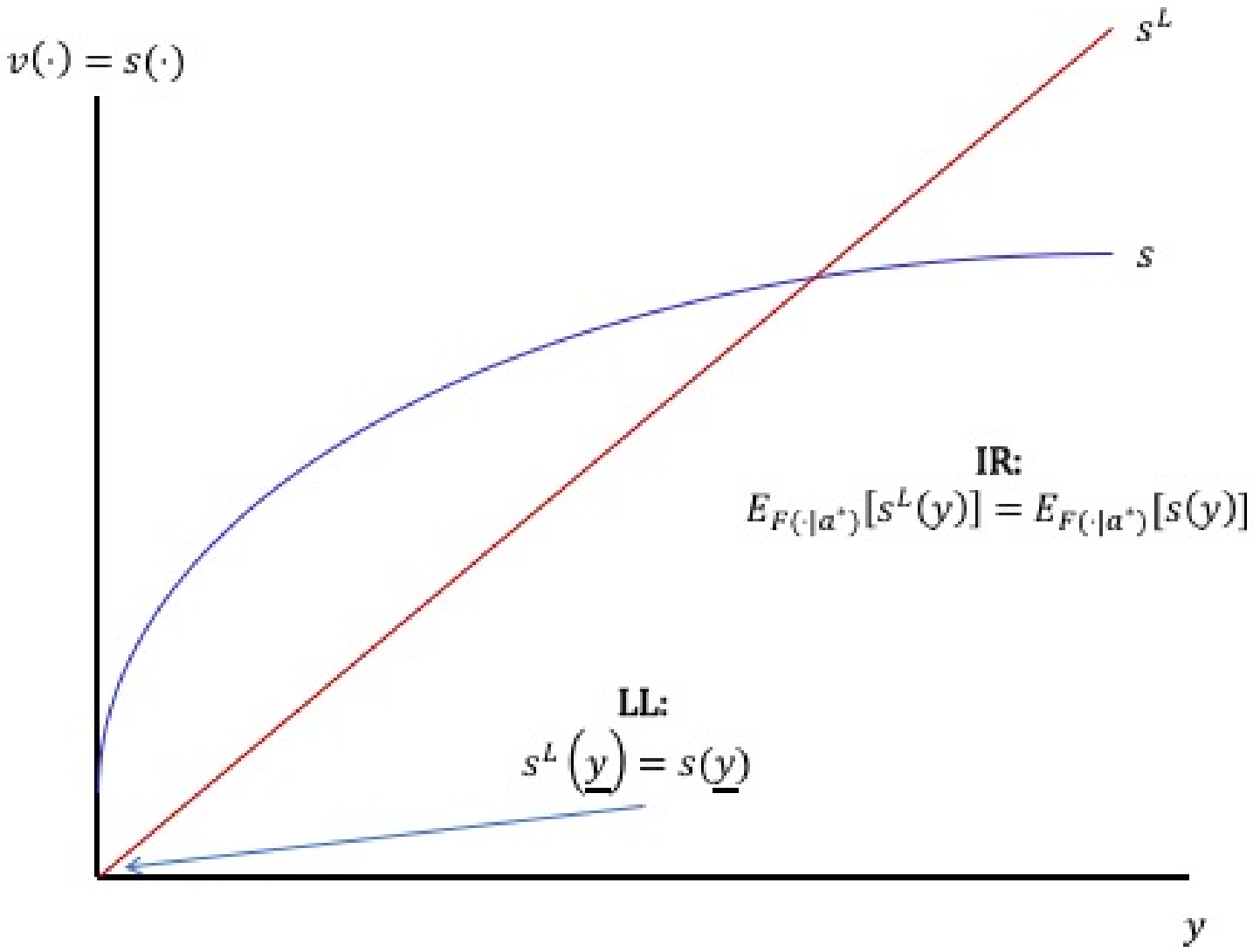
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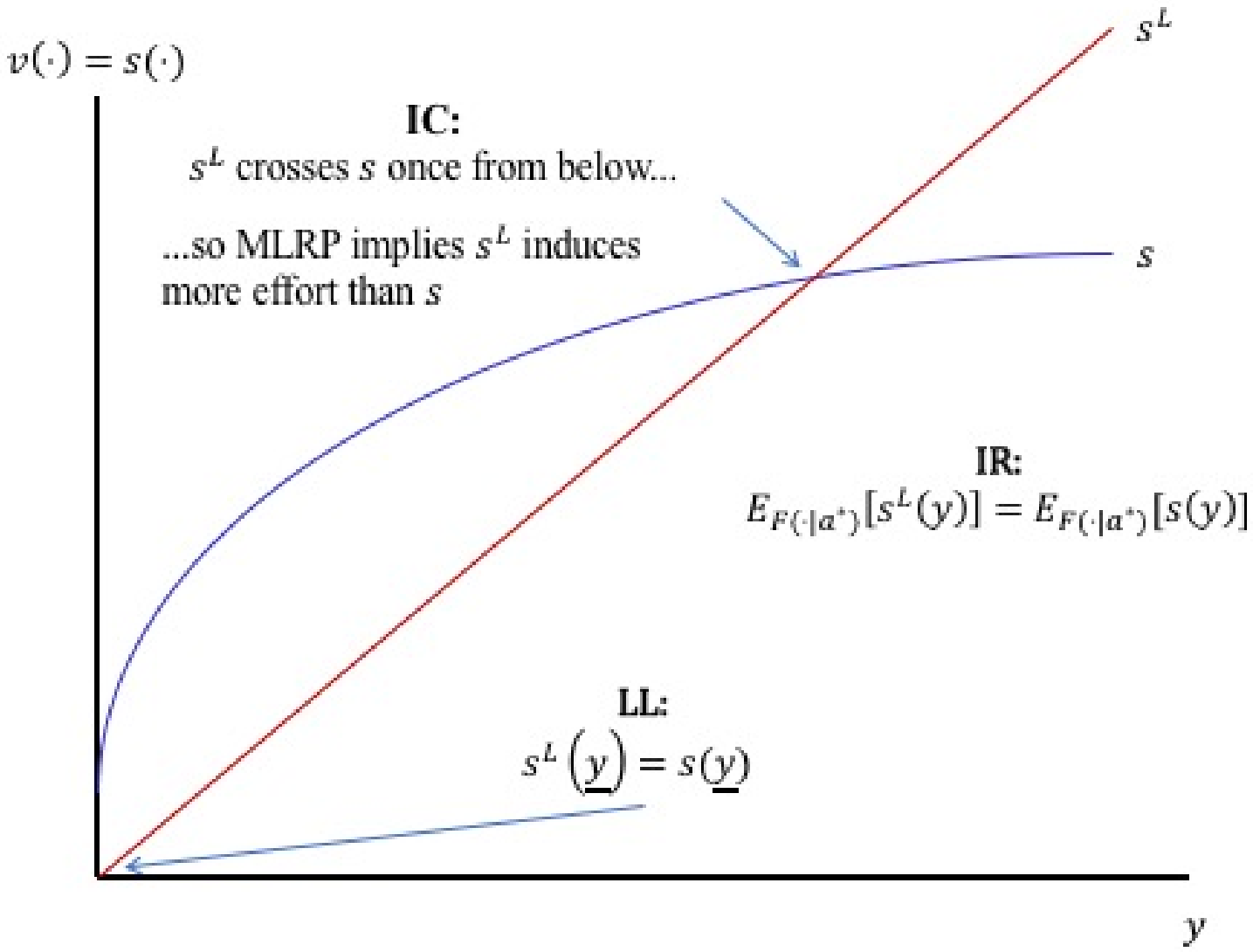
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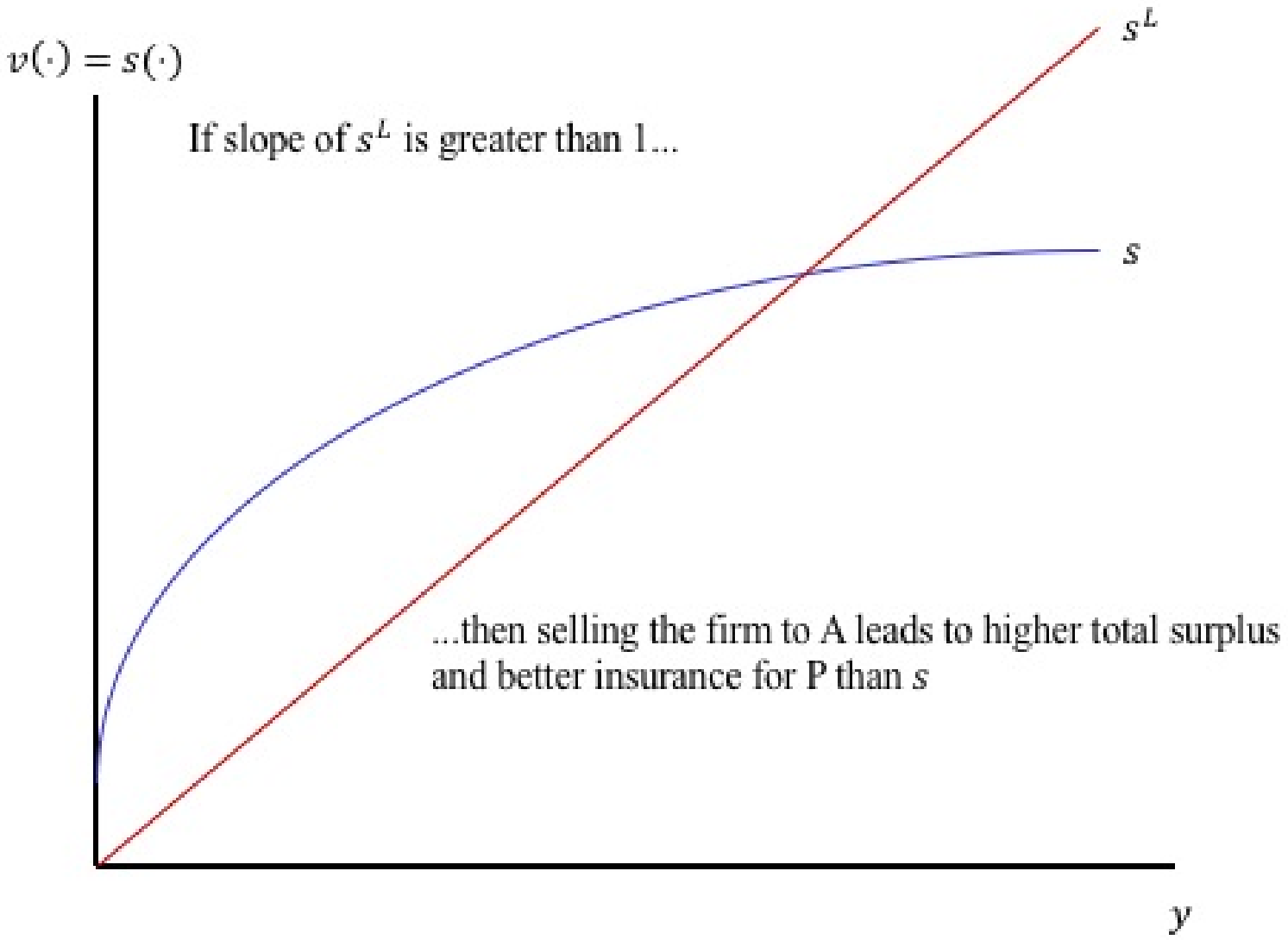
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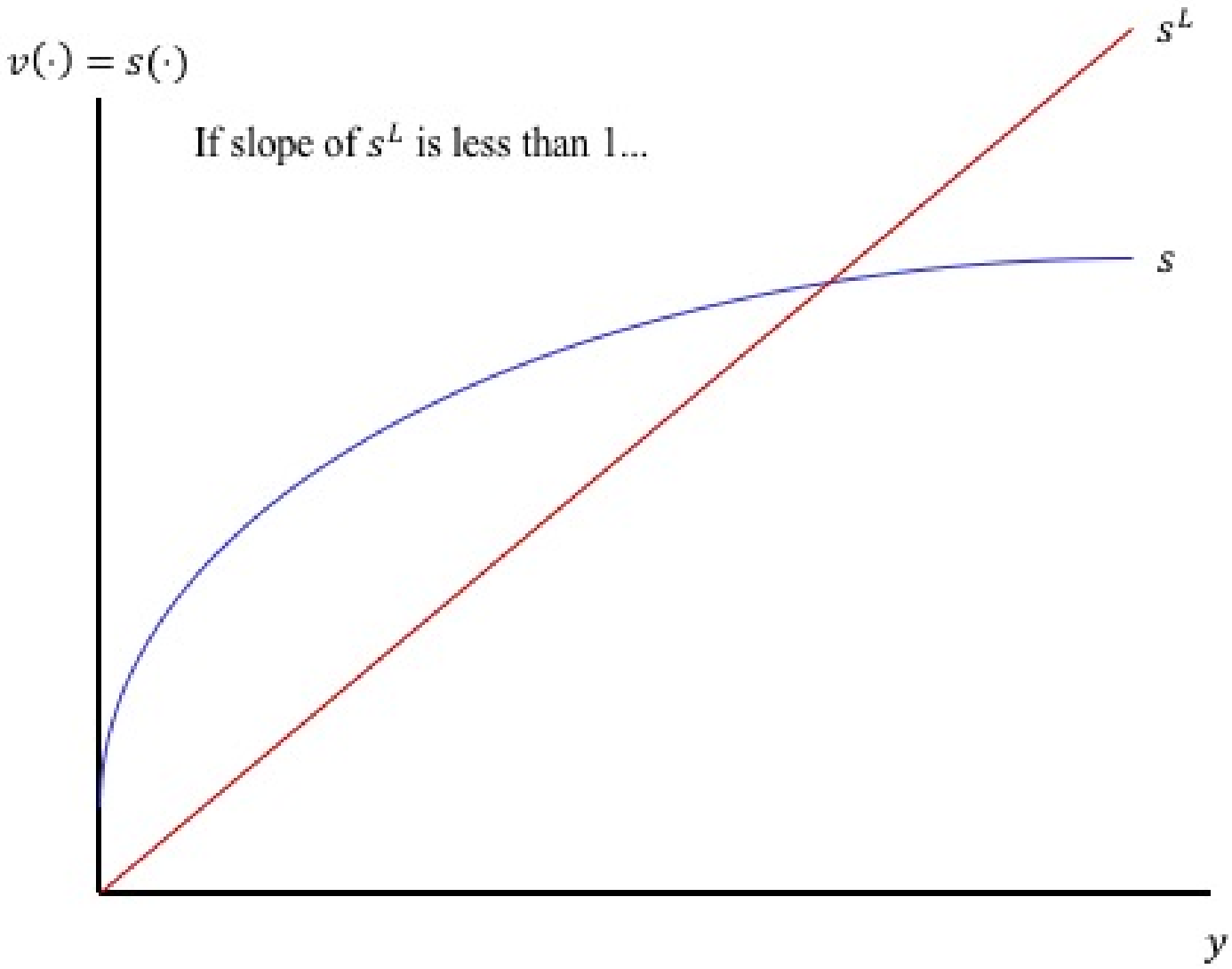
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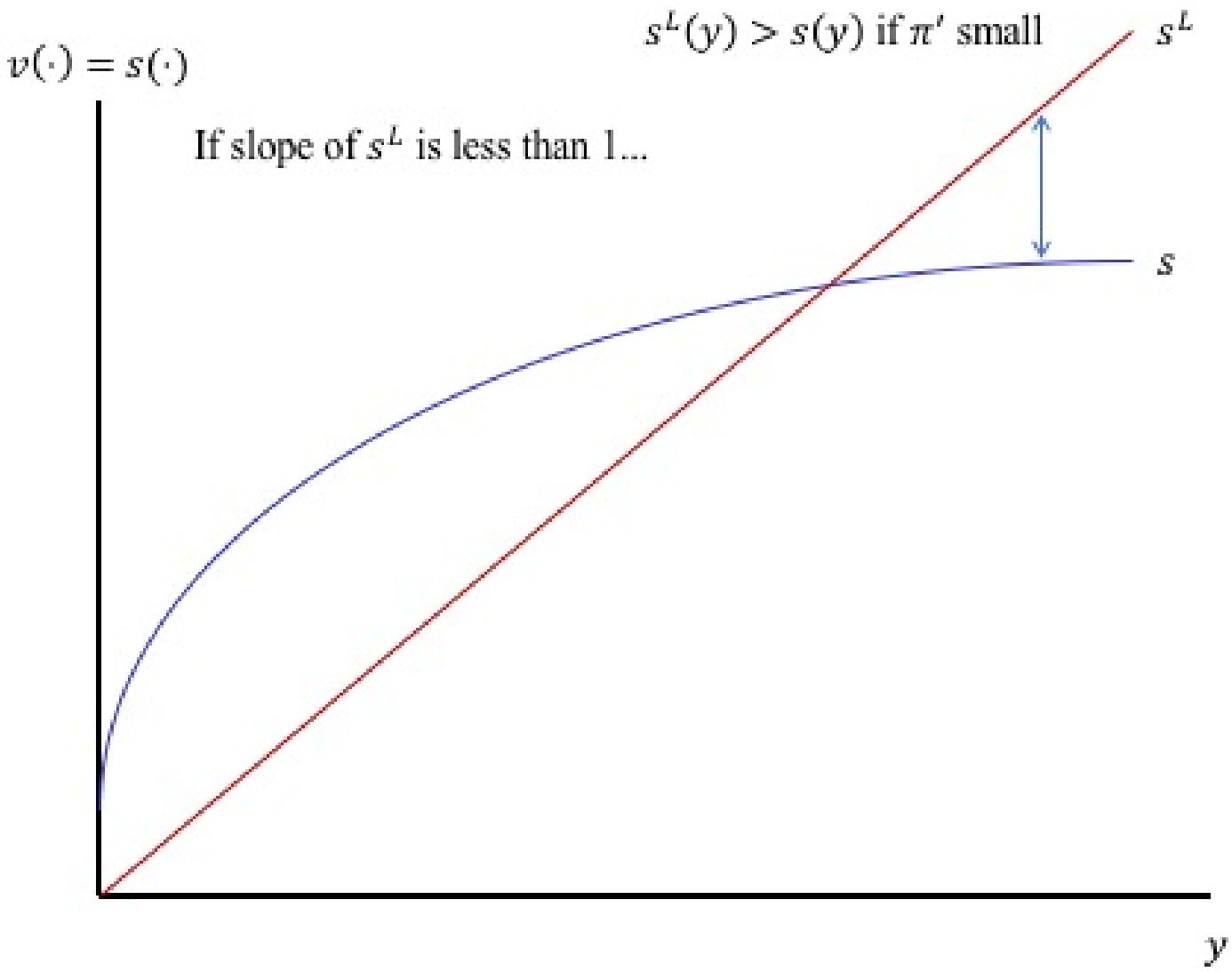


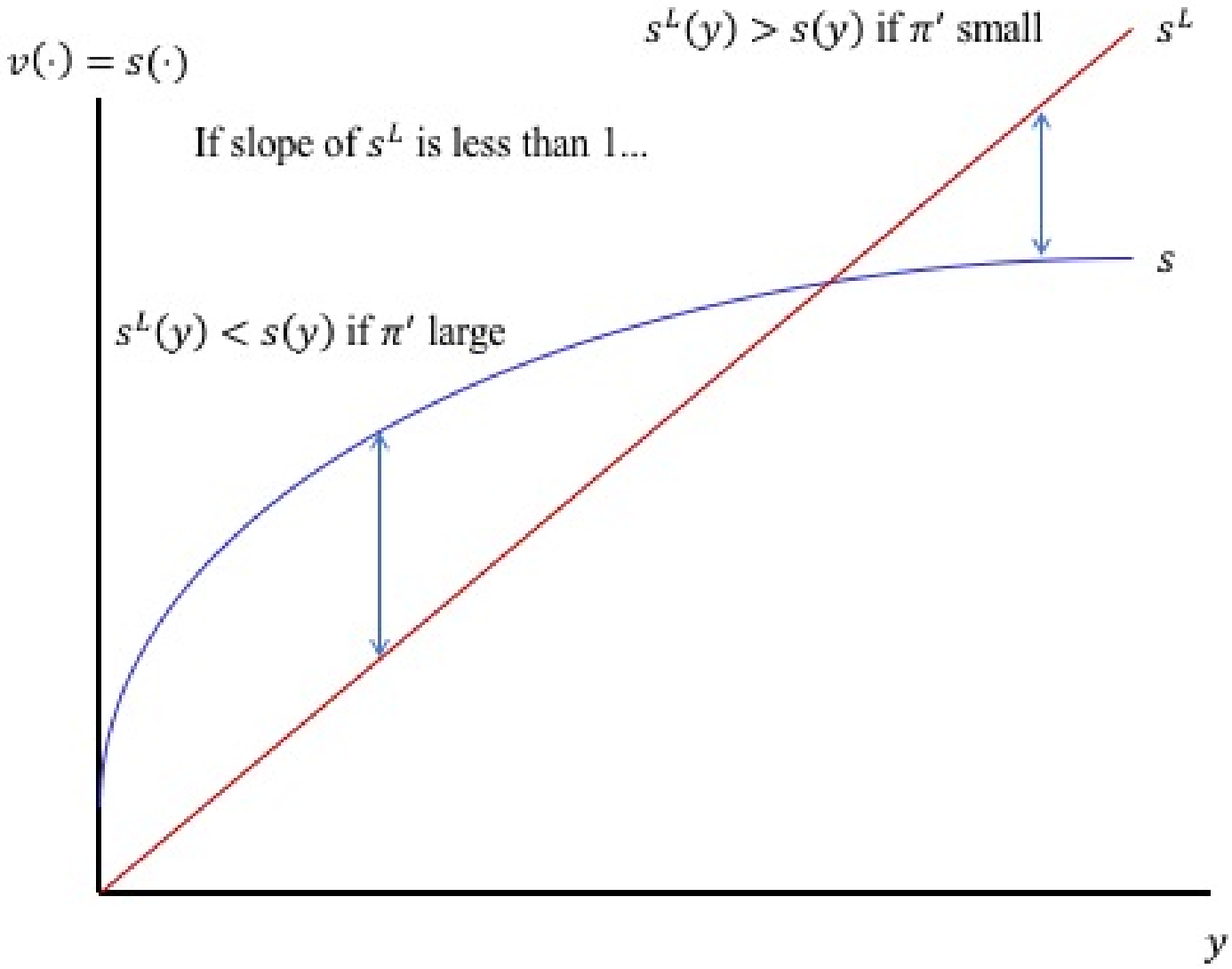

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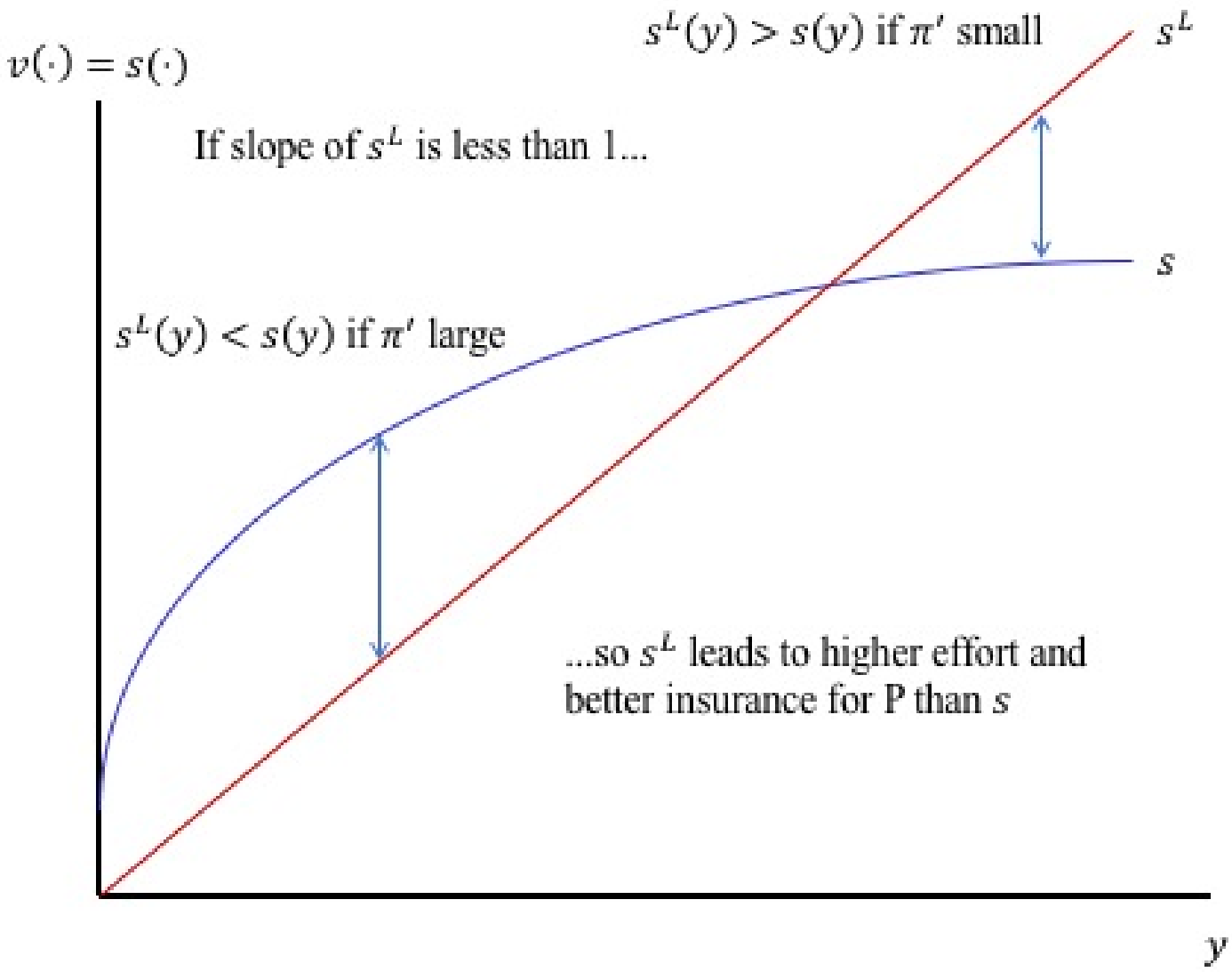

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s^L crosses s once from below...
...so MLRP implies s^L induces more effort than s
s^L
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y


$v(\cdot) = s(\cdot)$
s^L
If slope of s^L is greater than 1...
s
...then selling the firm to A leads to higher total surplus
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Optimal Effort Level: A Comparative Static

- To determine the optimal effort level, the principal solves

$$a^* \in \arg\max_a \left\{ \mathbb{E}_{F(\cdot|a)} \left[\pi \left(y - c'(a) \left(y - \underline{y} \right) - constant \right) \right] \right\}$$

- *Exercise*: Decrease the lower support of $\mathcal{Y}$, $\underline{y}$, while ensuring that our assumptions on $F(\cdot|a)$ are satisfied; *i.e.*, full support and mean a.

Corollary 3.

1. As $\underline{y} \to -\infty$, the optimal effort level $a^* \to 0$.
2. If $\pi(y) \equiv y$, then the optimal effort level a^* increases in $\underline{y}$.

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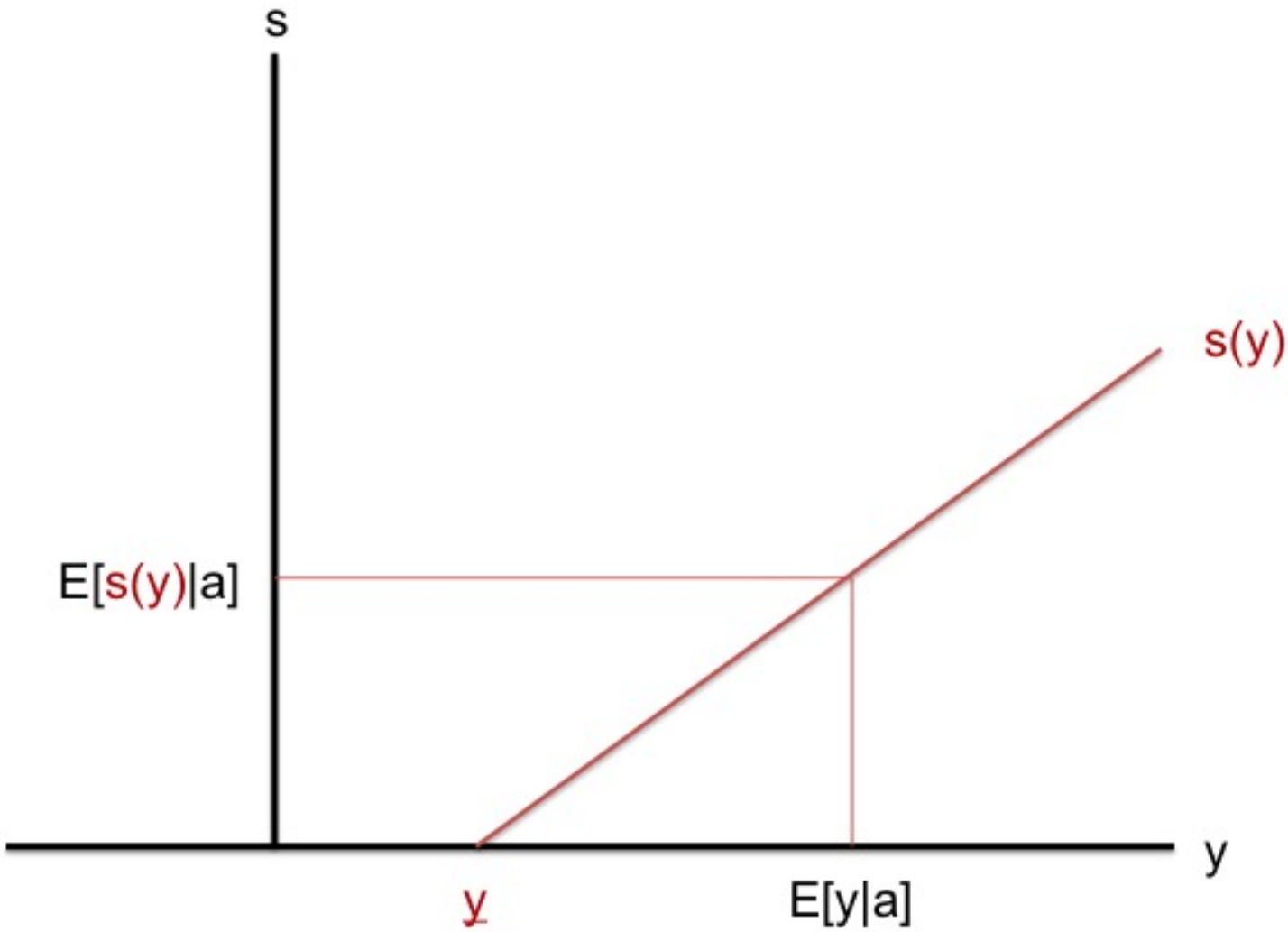
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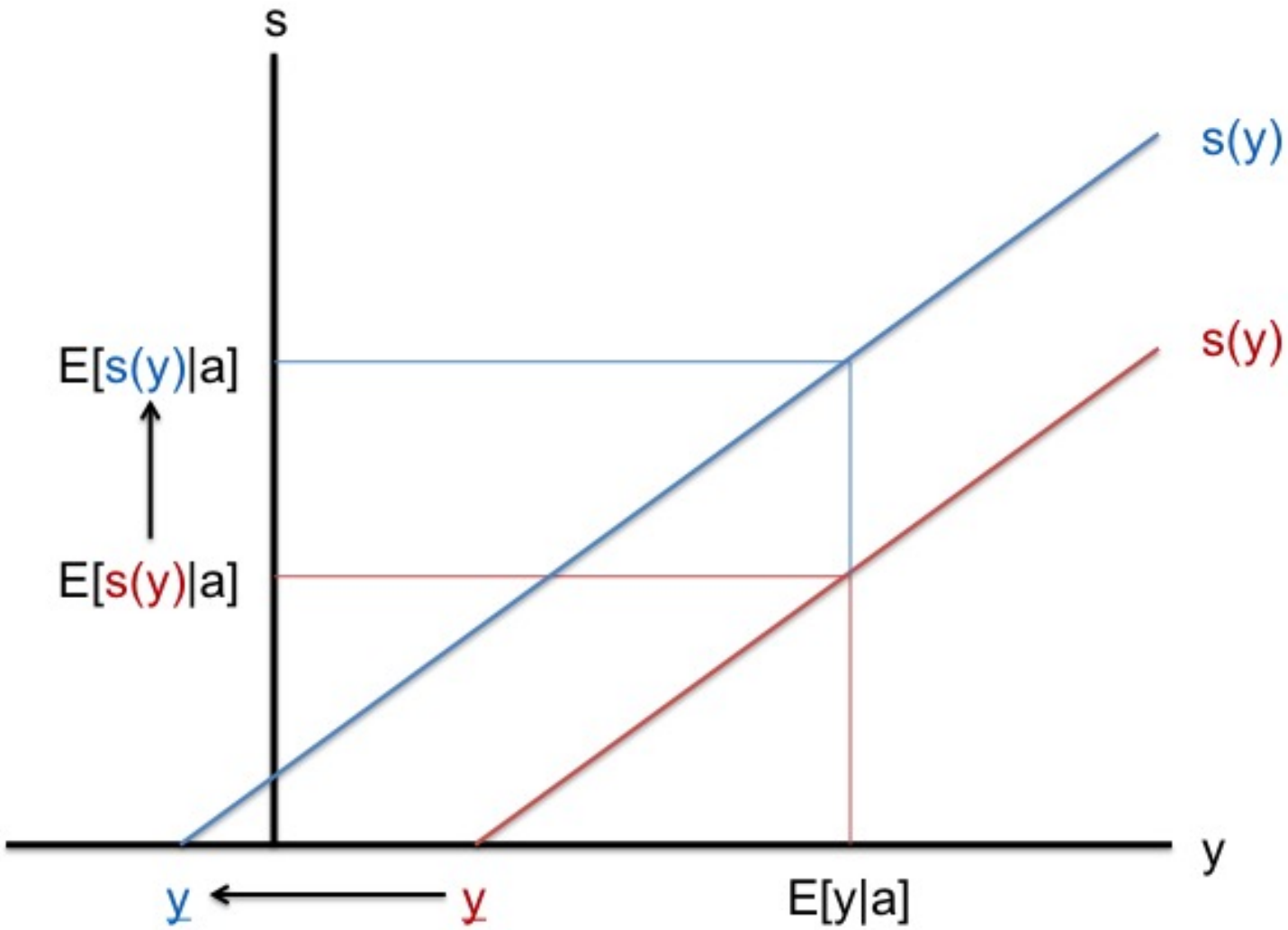
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Comparative Static: Illustrated



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If the Principal is loves risk...

- Rajan (2011) argues that anticipating bailouts, financial institutions may have had an incentive to encourage excessive risk-taking.

Corollary 1.

- Assume $\pi(\cdot)$ is strictly increasing with concave closure $\pi^c(\cdot)$.
- If a^* is optimal, then $a^* \leq a^{FB}$, and $s^L_{a^*}(\cdot)$ is optimal.

- The principal wants the agent to choose

$$G^P_x \in \Delta(\mathcal{Y}) \text{ such that } \mathbb{E}_{G^P}[\pi(y)] = \pi^c(x).$$

- From Prop. 2, $s^L_{a^*}(\cdot)$ maximizes $\pi^c(y - s(y))$ subject to (IC)-(Conc).
- Given $s^L_{a^*}(\cdot)$, the agent is indifferent across all G.

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Problem Formulation with a Risk Averse Agent

- Assume the principal is risk-neutral.
- Contract $v(\cdot)$ implements $a \geq 0$ at max. profit / min. cost if it solves:

$$\min_{v(\cdot)} \quad \mathbb{E}_{F(\cdot|a)}\left[u^{-1}(v(y))\right]$$

$$\text{s.t.} \quad \mathbb{E}_{F(\cdot|a)}\left[v(y)\right] - c(a) \geq u_0 \qquad \text{(IR)}$$

$$a \in \arg\max_{\tilde{a}} \mathbb{E}_{F(\cdot|\tilde{a})}\left[v\left(y\right)\right] - c(\tilde{a}) \qquad \text{(IC)}$$

$$v(y) \geq \underline{u} \text{ for all } y \in \mathcal{Y} \qquad \text{(LL)}$$

$$v(\cdot) \text{ weakly concave.} \qquad \text{(NG)}$$

- Replace (IC) with weaker condition that local incentives are slack:

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Building Blocks

- Ignoring (LL) and (NG) for now, we can write the Lagrangian as

$$L(\lambda,\mu)=\min_{v(\cdot)}\int\left[u^{-1}(v(y))-\lambda v(y)-\mu v(y)\frac{f_a(y|a)}{f(y|a)}\right]f(y|a)dy+\ldots,$$

  where λ and μ are shadow values on (IR) and (IC-FOC).

- Differentiating with respect to $v(y)$ yields

$$\underbrace{\frac{1}{u'\left(u^{-1}(v(y))\right)}}_{\triangleq\rho^{-1}(v(y))}-\lambda-\mu\underbrace{\frac{f_a(y|a)}{f(y|a)}}_{\triangleq l(y|a)}$$

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Rough Intuition

- Define the net cost of marginally increasing $v(y)$ by

$$n(y) = \rho^{-1}(v(y)) - \lambda - \mu l(y|a)$$

  - If $n(y) > 0$, then principal wants to *decrease* $v(y)$.
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- Holmström (1979): $v(\cdot)$ optimal iff $n(y) \equiv 0$ (for some λ, μ).
- Rough intuition for our characterization (LL slack):
  - If for some y, setting $n(y) = 0$ violates (NG), then $v(y)$ is locally linear.
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$$\int_{y_L}^{y_H} n(y) f(y|a) dy = 0 .$$

  - Outside linear segments, (NG) is slack, and $n(y) = 0$ at such y.

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Implication #1 of No-Gaming Constraint – (LL) Slack

Proposition 3.

- Let $v^*(\cdot)$ implement a at max. profit, and assume (LL) is slack.
- Suppose $\rho(\lambda + \mu l(\cdot|a))$ is convex for $y < y_l$, and concave otherwise.
- Then there exist $\hat{y} > y_l$, $\underline{v}$, and $\alpha > 0$ such that

$$v^*(y) = \begin{cases} \underline{v} + \alpha(y - \underline{y}) & \text{if } y < \hat{y}, \text{ and} \\ \rho(\lambda + \mu l(y|a)) & \text{otherwise,} \end{cases}$$

where $\int_{\underline{y}}^{\hat{y}} n(y) f(y|a) dy = 0$.

- $\rho(\lambda + \mu l(\cdot|a))$ is convex-concave $\forall \lambda$ and μ if, for example,
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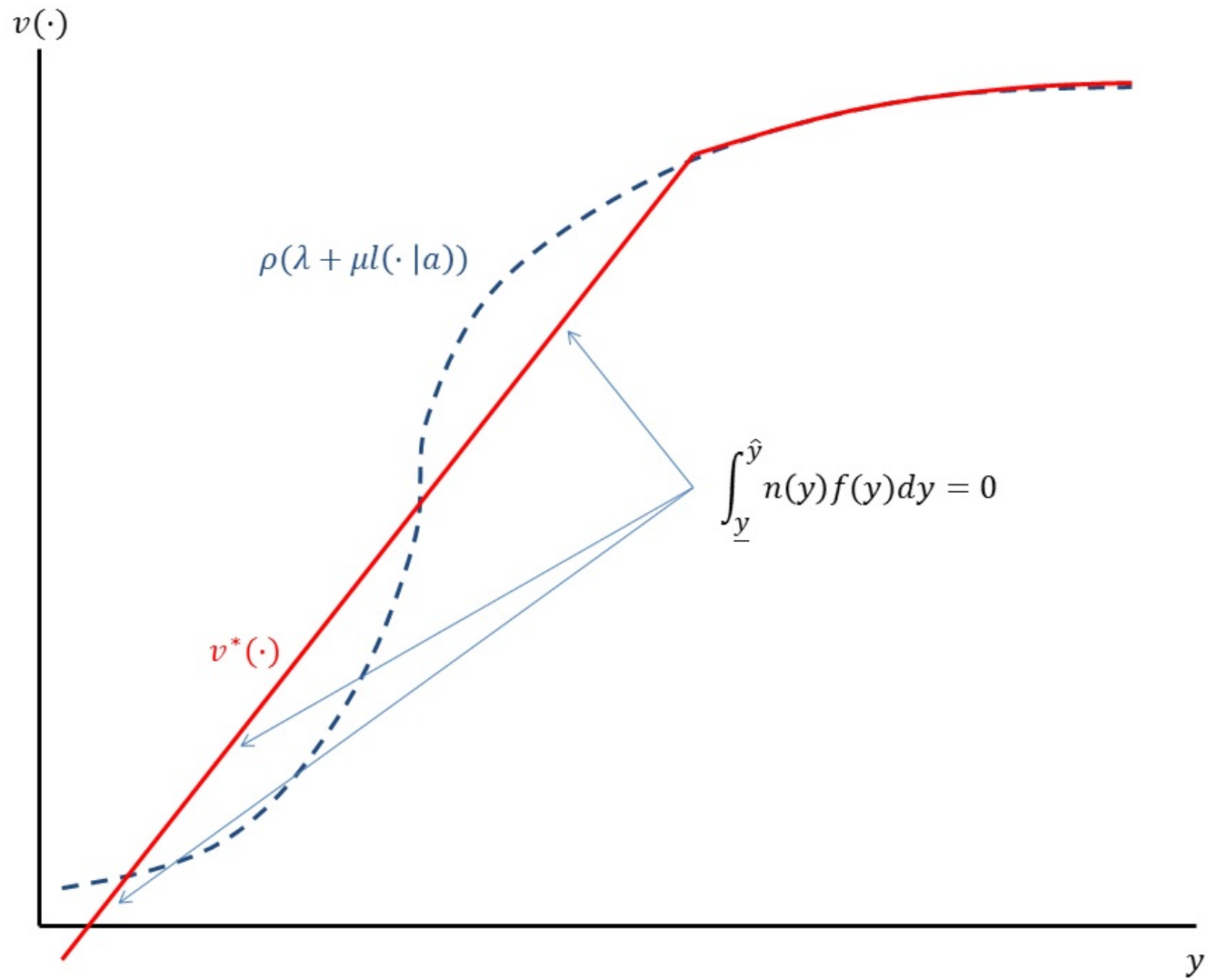
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Proposition 3 – Intuition



Implication #2 of No-Gaming Constraint – (IR) Slack

Proposition 4.

- Let $v^*(\cdot)$ implement a at max. profit, and assume (IR) is slack.
- Then $v^*(\cdot)$ is linear on $[\underline{y}, y_0]$, where $l(y_0|a) = 0$.

- For any y such that $l(y|a) < 0$, principal wants to reduce pay.
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Risk-Taking before Intermediate Output is Realized

▸ Summary

- What if the agent gambles *before* exogenous uncertainty is resolved?

Timing:

1. Principal offers a contract $s(y)$.
2. Agent chooses effort $a \geq 0$ and distribution $G \in \Delta(\mathcal{Y})$ s.t $\mathbb{E}_G[x] = a$.
3. Outcome of gamble $x \sim G$, and final output $y \sim F(\cdot|x)$ are realized.
   - $F(\cdot|x)$ satisfies strict MLRP in x and $\mathbb{E}_{F(\cdot|x)}[y] = x$.

- Can interpret x as profitability conditional on economic conditions, and $F(\cdot|x)$ as capturing residual uncertainty.
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Analysis

- For fixed $s(\cdot)$, denote agent's expected pay conditional on x by

$$V_s(x) = \mathbb{E}_{F(\cdot|x)}[s(y)]$$

- The agent will choose his risk profile G such that

$$\max_G \{\mathbb{E}_G[V_s(x)] \text{ s.t. } \mathbb{E}_G[x] = a\} = V_s^c(a),$$

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- For optimal effort a^*, $s(y) = c'(a^*)\left(y - \underline{y}\right) + \textit{constant}$ is optimal

- If principal could choose $V_s^c(\cdot)$ directly, this boils down to baseline problem with a degenerate $F(\cdot|a)$, and a linear $V_s^c(\cdot)$ is optimal.
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Costly Gaming

- Finding difficult-to-detect ways to game contract requires effort.
- Assume that to choose distribution G_x, the agent incurs cost

$$\mathbb{E}_{G_x}\left[d\left(y\right)\right] - d\left(x\right)$$

where $d\left(\cdot\right)$ is some smooth, increasing, convex function.

  - *Example:* If $d(y) = y^2$, then agent's cost equals the variance of G_x.
  - *Idea:* A more dispersed distribution is costlier to implement.

Costly Gaming

- Finding difficult-to-detect ways to game contract requires effort.
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Optimal Contract

- Conditional on the realization of the final output y, payoffs are:

$$\text{Agent:} \quad \underbrace{s(y) - d(y)}_{\equiv \tilde{v}(y)} - \underbrace{\left(c(a) - \mathbb{E}_{F(\cdot|a)}[d(x)]\right)}_{\equiv \tilde{c}(a) \text{ (assume incr. \& convex)}}$$

$$\text{Principal:} \quad y - s(y) = \underbrace{y - d(y)}_{\equiv \tilde{\pi}(y) \text{ (concave)}} - \tilde{v}(y)$$

- For every x, agent optimally chooses G_x s.t $\mathbb{E}_{G_x}[\tilde{v}(y)] = \tilde{v}^c(x)$.

Proposition 7.

- For optimal a^*, $s(y) = \tilde{c}'(a^*)(y - \underline{y}) + d(y) + \textit{constant}$ is optimal.

- Following Prop. 2, optimal $\tilde{v}(\cdot)$ is linear & $s(y) = \tilde{v}(y) + d(y)$.

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A Model for Intertemporal Gaming

- Principal and agent contract during $[0,1]$ and do not discount time.
- Timing:
  1. Principal offers a stationary contract $s(y)$.
     - Agent receives $s(y(t))\,dt$ if output during $(t, t+dt)$ is $y(t)$.
  2. Agent chooses effort $a \geq 0$.
  3. Total output $x \sim F(\cdot|a)$ is privately observed by the agent.
  4. Agent chooses flow output $y_x(t)$ subject to $\int_0^1 y_x(t)\,dt = x$.
  5. Final output $\{y_x(t)\}_{t\in[0,1]}$ and payoffs are realized.
- Payoffs:
  - Principal: $\pi = \int_0^1 [y_x(t) - s(y_x(t))]\,dt$
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Analysis

- Given any x and contract $s(\cdot)$, the agent solves:

$$\max_{y_x} \left\{ \mathbb{E} \int_0^1 s(y_x(t))dt \text{ s.t. } \int_0^1 y_x(t)dt = x \right\} = s^c(x)$$

- Agent will exploit convex incentives by bunching output, and concave incentives by smoothing output over time.
- Equivalently, one can think of the agent choosing

$$G_x(y) = \text{fraction of time for which } y_x(t) \leq y \text{ s.t. } \mathbb{E}_{G_x}[y] = x$$

  - By Lemma 1, agent will optimally choose G_x s.t $\mathbb{E}_{G_x}[s(y)] = s^c(x)$.

Proposition 8.

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Takeaways

- Tractable model of gaming by risk taking.
- Linear contracts are optimal if the agent is risk neutral.
- Characterization if the agent is risk averse.
- Why might risk-taking occur?
  - Principal may be unable to commit, or might benefit from risk-taking
  - Competition?
  - Dynamics?