International Medium of Exchange: Privilege and Duty*

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Abstract

The United States enjoys an "exorbitant privilege" that allows it to borrow at lower interest rates than the rest of the world. Meanwhile, the dollar plays a special role in the international financial system as the main international medium of exchange. We provide a new theory that links dollarization and exorbitant privilege through the need for an international medium of exchange. We consider a two-country world where international trade happens in decentralized matching markets, and must be collateralized by assets — a.k.a. currencies — issued by one of the two countries. Traders have an incentive to coordinate their currency choices and a single dominant currency arises in equilibrium. With a small heterogeneity in traders' information, the equilibrium currency choice is unique, given economic states. Nevertheless, due to feedback between the trading firms' currency choices and the households' portfolio decisions, the model has multiple steady states, where different currencies serve as the dominant international medium of exchange. The economy with the dominant currency enjoys lower interest rates and the ability to run current account deficits indefinitely. Currency regimes are stable as they are not subject to sunspots, but sufficiently large shocks or policy changes can lead to transitions from one steady state to another, with large welfare implications.

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1 Introduction

As the world's primary supplier of safe liquid assets, the United States enjoys an "exorbitant privilege" that allows it to borrow at especially low interest rates. For example, Gourinchas and Rey (2007) estimate that the United States earns an excess return on its foreign assets over foreign liabilities of about 3% a year, which in principle allows it to fund a sizable trade deficit indefinitely. At the same time, authors such as Goldberg (2011) and Gopinath (2015) have observed that a disproportionate share of international trade in both real goods and financial assets is denominated in dollars. The privilege and trade dollarization phenomena have each recent ample attention in recent research, but they have rarely been analyzed jointly.

In this paper, we develop a theory in which exorbitant privilege and trade dollarization arise together endogenously. We propose a dynamic two-country framework in which international trade happens in decentralized, bilateral markets with limited contract enforceability. Decentralized trade requires trading firms to collateralize their cross-country transactions with safe assets in order to guarantee eventual payment and delivery of goods. The firms are free to choose the currency in which they transact — and thus the currency of denomination of the needed collateral — but face a small currency mismatch cost if their counter-party uses a different currency. This gives traders incentive to coordinate their currency choices, which could lead to multiple equilibria. Instead, we introduce a small amount of strategic uncertainty to ensure that the currency choices are uniquely determined by the state of the economy. Nevertheless, traders prefer to use a means of exchange that is plentiful and therefore easier to source in their home country. The concern for funding availability leads to a feedback between the currency choice of trading firms and the asset holdings of domestic households, who like to hold assets demanded by the domestic trading firms as they pay an extra liquidity premium. As a result the model features multiple *steady-states*, each of which corresponds to a different long-run bond holdings positions and a different primary currency being used in international trade.

The economy with the dominant currency enjoys a number of benefits, including lower interest rates and the ability to run current account deficits indefinitely. Foreign demand for the dominant currency generally leads the supplier of that asset to have a negative net foreign asset position, but to earn a positive return on it, similar to the experience of the United States. Importantly, exorbitant privilege is not exogenously fixed, but is an endogenous equilibrium outcome. Maintaining the dominant currency position requires the country to

perform its "duty" to responsibly supply and support the asset that serves as international medium of exchange. Sufficiently large shocks or changes in macroeconomic policy can push the world from one currency regime to another (i.e. from one steady state attraction region to another), but regimes are uniquely determined by economic fundamentals. And, while rare, such switches have large effects on welfare.

A key component of our mechanism is the currency coordination game between foreign and domestic firms that engage in international trade. Firms in each country begin the period uncertain as to whether they will be importing or exporting and, conditional on that outcome, who their foreign trading partner will be. When trade occurs, firms encounter their trading counterparts in a bilateral, decentralized, search and matching market. Because firms are not able to fully enforce contracts internationally, they must collateralize a portion of their trade in order to complete their transactions, consistent with the evidence of Amiti and Weinstein (2011) and Ahn (2015).

Firms acquire collateral in their domestic financing market, which operates at the beginning of each period. Firms freely choose what type of financing they seek, either in the form of domestic or foreign safe assets, but know that they face a cost of currency mismatch if they eventually trade with a counterpart who holds a different type of asset. Given the uncertain identity of their eventual trading partner, and the costs of an ex-post currency mismatch, firms have an incentive to seek collateral funding in the same currency as that held by potential foreign trading partners. The coordination incentives of trading firms in both countries thus give rise to a discrete choice global game in the spirit of Morris and Shin (1998), albeit with two agent types, foreign and domestic trading firms.

Under complete information, the coordination game played by trading firms typically exhibits multiple equilibria: firms will be happy to acquire whatever type of funding they anticipate their eventual trading partners will hold. To resolve the indeterminacy of the model's predictions, we introduce a finite supply of safe assets. Firms therefore have an incentive not only to coordinate their currency choice with foreign firms, but also to choose a currency that is readily available in their home country financing markets. By introducing a small amount of uncertainty in the relative payoff of funding in each asset à la Morris and Shin (1998), we then recover a unique equilibrium currency choice that depends, crucially, on the availability of both assets in each country. In particular, trading firms tend to coordinate on the asset that is more readily available in *both* countries, as they anticipate that their trading partners will also have an easier time obtaining this type of funding.

We embed our model of currency choice and international trade within an otherwise

standard dynamic two-country model of the world economy.[1] Households produce domestic goods and use the proceeds from production to acquire both domestic consumption goods and the foreign consumption goods sold domestically by importing firms. While final goods markets are segmented across countries, a common primary good (oil) is traded freely across countries and serves as the numeraire. Moreover, households freely trade both the domestic and foreign bonds issued by the two countries' governments.

The bonds pay risk-free interest, but also may earn an extra liquidity return due to the demand for collateral stemming from international trade transactions. Each period, households offer to lend their bonds (intra-period) to domestic trading firms to use as collateral, which happens in a funding market mediated by a search and matching friction. Bonds that are successfully matched with a trading firm earn a use fee paid by that trading firm, thereby increasing the effective return of that bond.[2] Thus, households have an incentive *ceteris paribus* to hold bonds that are widely sought by their country's trading firms, as doing so increases the return the bond earns from being used as intra-period collateral. We call this additional return the *liquidity premium* of the asset, because it is a consequence of the asset's use in facilitating international exchange. In equilibrium, the price of an in-demand bond rises, depressing the interest rate paid by the country that issues it.

This environment leads to a strong feedback effect between firms' currency choice and the long-run asset position of households in the two countries. To see this, suppose for example that households in both countries expect trading firms to coordinate on using U.S. assets. If the supply of those U.S. assets is initially primarily held within the U.S., then foreign households have a strong incentive to acquire U.S. bonds, as any bond they acquire is virtually certain to find a match in their domestic funding market, and earn the associated premium. U.S. households, however, have no corresponding extra incentive to acquire foreign bonds. In the model, bonds are slow moving state variables hence these portfolio re-allocation incentives do not lead to an immediate jump in the equilibrium. However, as bond holdings adjust slowly over time and the foreign households build up their stock of U.S. assets, the wider availability of U.S. asses in all countries will push even more firms to use dollars. As a result, the model will converge towards a steady state where the dollar is a *stable* dominant currency.

We begin our analysis of model by studying the feedback between household and firm

[1] Our approach of embedding a static coordination game within a dynamic economy is inspired by Schaal and Taschereau-Dumouchel (2015), who use a similar strategy to study the potential for two absorbing states in an economy with demand externalities.

[2] The intra-period funding essentially functions as a repurchasing agreement.

choices in a simplified version of the steady state of our economy. In two propositions, we provide limiting results that show that currency choice game does indeed have a unique equilibrium if feedback from household choices are excluded. We then expand the model to include those feedbacks, demonstrating how household incentives to equalize returns among their asset holding leads to asset allocations that reinforce a conjectured shift the dominant currency.

In the simplified model, we show that when firms' desire to coordinate their currency choices is balanced by their concern regarding congestion in the funding market, the model economy exhibits five steady-states. Among these are two distinct steady-states where the U.S. asset is the dominant medium of exchange, another two mirror image steady-states where the foreign asset is dominant, and also a symmetric steady-state where both assets are used equally in trade. In one of the dollar dominant steady-states, the U.S. has a negative net foreign asset position, but earns a significant premium on its foreign assets, allowing it to run a permanent trade deficit despite its net debtor position. Moreover, that steady-state also features significant home bias in asset holdings, despite there being no frictions in asset trade. Qualitatively, this situation describes the status quo in the data remarkably well.

Having demonstrated that the model qualitatively matches the empirical patterns observed in recent decades, we then turn to quantifying our mechanism in the full economy. To do this, we select parameters to match several target moments in the portfolio composition of household, as well as a moderate exorbitant privilege and a modest cost of currency mismatch, *at the empirical relevant, dollar-dominant, steady-state*. We show the model can exactly match our target moments and is consistent with a sizable excess return on dominant (US) assets, a long-run trade deficit, and higher US consumption.

We then compare the long run welfare implications of owning the dominant asset and show that the gains, relative to a symmetric equilibrium, can be substantial. Yet, the losses from not having the dominant asset are even greater. The reason for this results is a congestion externality in funding markets: coordinated equilibria essentially waste the potential liquidity services that could otherwise have been provided by the unused, non-dominant asset. Thus, while dominant currency regimes are stable, they are inefficient.

After studying the steady-state implications of the model, we then consider the dynamics our economy. We find that all coordinated steady-states are dynamically stable, but that the (efficient) multipolar steady-state is not. In our analysis, we examine the mapping from initial conditions to long-run outcomes, showing that small initial differences can lead to long-run differences in the currency regime. Moreover, we show that transition between

currency regimes, once begun, proceed slowly at first, then accelerate, and typically conclude within about 10 years.

Finally, we consider how economic policy may affect currency regimes. We focus in particular on the effect of unanticipated changes in the supply of safe assets, highlighting that the timing of changes is crucial for their implications. For example, in one experiment, we show that a one time 5% increase in the supply of EU assets is enough to switch the economy away from a dollar-dominant steady-state, but that two increases of 2.5% — if separated by enough time — in fact *strengthen* the dollar regime.

The above analysis highlights why, through the lens of our model, the current dominant position of the dollar can be described not only as one of privilege, but also as one of duty. The U.S. enjoys the privilege of supplying the world with the dominant medium of exchange, which gives it access to significant excess returns on its foreign assets and allows it to run a permanent trade deficit. Nevertheless, it runs the danger of falling into the worst steady-state situation, where it both remains indebted to the rest of the world, and loses its currency hegemony and the associated low interest rate on its debts. Moreover, the danger of transitioning to that steady-state is increasing in the overall indebtedness of the U.S. economy – while it can run a moderate negative NFA position indefinitely, the larger the deficit becomes, the more likely it is that a shock will transition the world economy to a foreign currency coordinated steady-state in which the US is asset poor. Thus, we find that in its current position the U.S. also has the "duty" to be a relatively responsible steward to the international financial system, and to not exploit its access to excess returns, by falling deeper and deeper into debt to the rest of the world.

Relation to Existing Literature

Our work relates to several different strands of the literature. Motivated by some of the same empirical facts about the special international position of the U.S. cited above, authors have sought to provide explanations focusing on the three canonical roles of money. In this paper, we provide a model where the dominant position of U.S. assets in the international monetary system is due to their endogenous role as the main *medium of exchange*. A number of previous papers instead explore the unit of account and store of value properties of money.

Among the papers that focus on the unit of account role of money, concurrent work by Gopinath and Stein (2018) is most closely related to our own. To our knowledge, it is the only other paper that explicitly connects dollar invoicing in international trade and exorbitant privilege. The unit of account role of currency is also highlighted by Casas et al. (2016),

which considers the effects of near universal dollar-denomination on shock pass-through and expenditure switching when nominal prices are sticky.

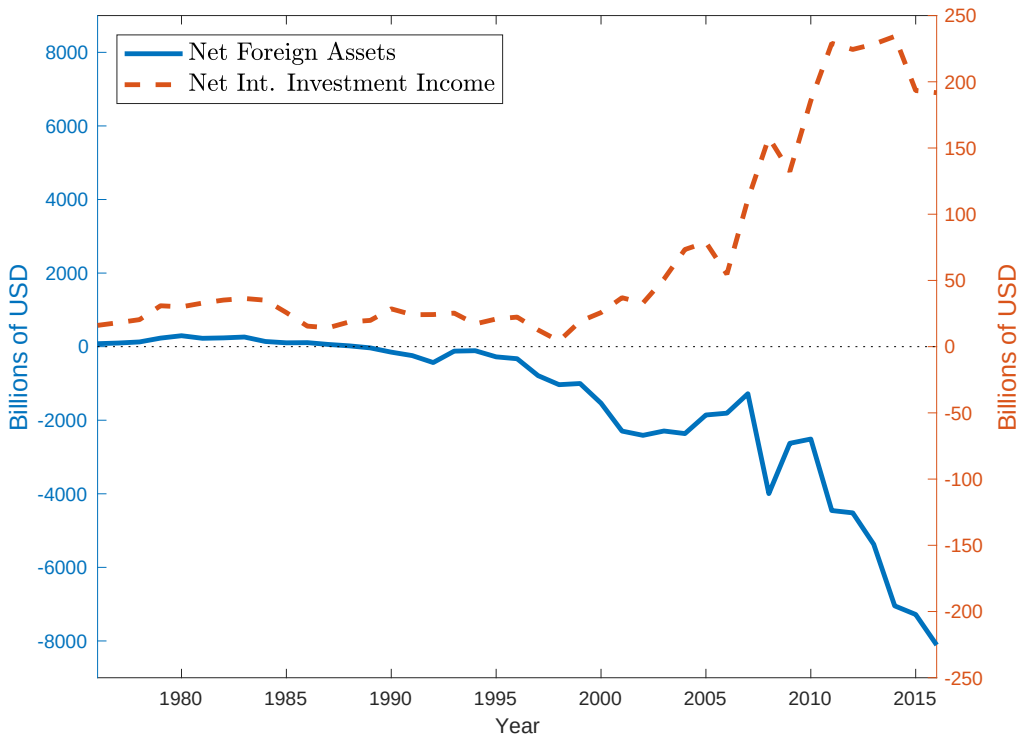
A larger portion of the related literature focuses on the capacity of countries to generate good store-of-value assets. Caballero et al. (2008) argue that the United States' superior ability — relative to developing countries — to produce store-of-value assets can explain the U.S. experience of persistent trade deficits, falling interest rates, and rising portfolio share of U.S. assets in developing countries. Maggiori (2017) models the endogenous emergence of a dominant reserve currency, in which dominance emerges because a higher level of financial development in the dominant country leads it to provide insurance vis-a-vis the rest of the world. Gourinchas et al. (2010) propose a different insurance framework, where the U.S. households are assumed to have lower risk-aversion than foreign households (perhaps due to an endogenous mechanism like the one in Maggiori, 2017) and, thus, in equilibrium end up taking on most of the world's risky assets in their portfolio. Meanwhile, Farhi and Maggiori (2016) consider the different positive and welfare implications of having a single dominant reserve asset or a multipolar system; both of which can emerge endogenously in the equilibrium in our economy. He et al. (2016) also use a global-game approach to model how, in a world with two ex ante identical assets, it may be that a single safe asset emerges in equilibrium.

Two key features differentiate our work from all of the preceding literature cited above: (i) currency dominance emerges endogenously in equilibrium in an otherwise ex-ante symmetric world and (ii) the model features determinate equilibrium paths, with currency choice a unique function of the state variables. Previous authors have developed frameworks that feature one or the other, but not the two together. For example, some existing models can deliver dominant currencies, but do so in the context of equilibrium indeterminacy. We view (i) and (ii) as a desirable combination, as in the data currency regimes occasionally change, but have historically been quite stable.

2 Empirical Motivation

The U.S. has a unique external position. It is the world's largest debtor country, with its foreign liabilities exceeding foreign assets by more than $8 trillion dollars (roughly 40% of GDP). Yet, its net international investment income is *positive*, meaning that in effect it is making *negative* payments on its large net debt to the rest of the world. The U.S. is the only country in the world that has been able to sustain both a a negative net foreign

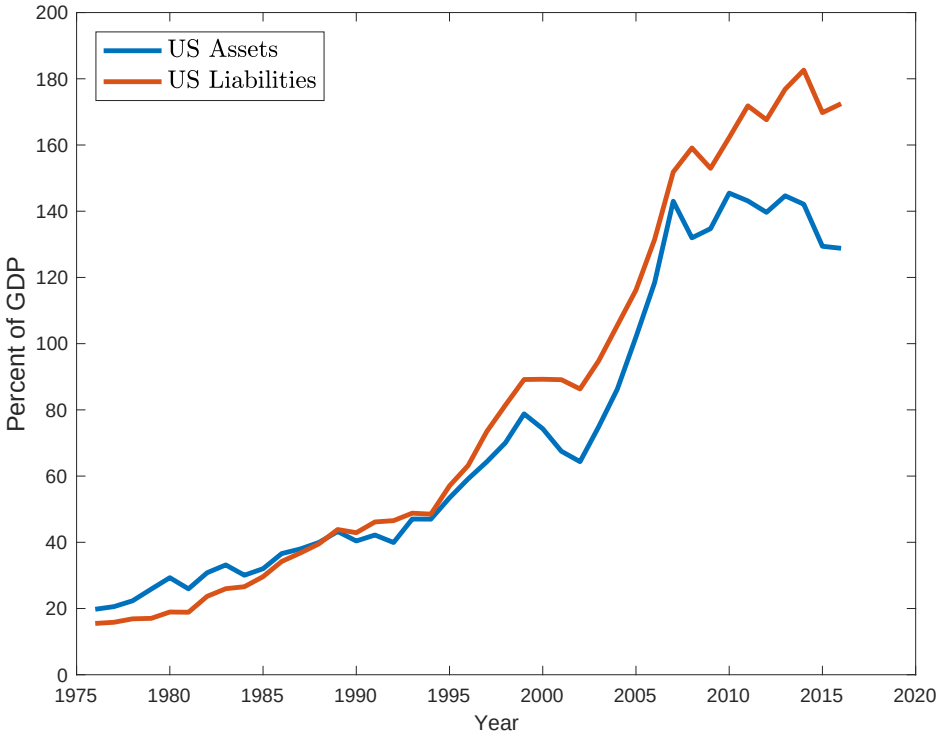
Figure 1: Evolution of U.S. External Position



assets position (NFA), and a positive net international investment income flow. Moreover, Figure 1 shows that as its NFA position has been steadily worsening over the last couple of decades, its net international income has actually been increasing. The net income is also quite significant in absolute terms – at about $250 billion, it accounts for about a third to a half of recent U.S. trade deficits. And in fact, once capital gains and other valuation changes are also taken into account (the NII measure if simply income flow, i.e. dividends and interest rate), the net return of the U.S. grows even bigger.

Gourinchas and Rey (2007) and Gourinchas et al. (2010) provide a fresh and updated look on the U.S.'s external position and its returns on it from 1952 to 2009. They show that the U.S. earns a significant excess return of about 3% on its foreign assets over its foreign liabilities, a premium that is primarily driven by differential returns within asset classes rather than different portfolio composition in assets versus liabilities. The differential returns on debt, equity and foreign direct investment are all found to be between roughly 3% to 4%. Interestingly, the "exorbitant privilege" has increased dramatically in the period after Bretton Woods – the excess return during the Bretton Woods era was only about 1.3% a year, while this differential increases all the way to 3.5% for the years 1973 to 2009. Additional

Figure 2: Evolution of U.S. External Position



corroboration is provided by Habib (2010), who studies a large panel of 49 countries over the time period 1981 to 2007, and confirms the 3% excess return earned by the U.S., and also documents that the U.S. is the only country that earns a persistently positive differential. Moreover, he also finds that the composition of the portfolio, and in particular the U.S.'s levered foreign asset position, did not contribute to its exorbitant privilege.

This clearly represents a large real transfer from the rest of the world to the U.S., and hence the usual description of it being an "exorbitant privilege". A back of the envelope calculation, based on the current gross positions of the U.S. and the post-Bretton Woods returns indicates that it could help sustain a trade deficit of up to 2.5% of GDP indefinitely. This benefit in terms of absolute dollar amounts is sensitive to the gross external asset and liability positions of the U.S.. The larger the positions on which the differential return is applied, the larger the extra income generated. Thus, given an ever increasing financial globalization (see for example Figure 2 for the increase in the gross foreign asset and liability positions of the U.S.), the benefit to the U.S. economy could potentially become even bigger.

In terms of goods and services trade, Gopinath (2015) shows that the U.S. dollar invoices almost half of global trade, and the share of transactions in dollars is five times larger than

total U.S. trade as a share of world trade. She also shows that no other currency comes anywhere close to the dominant position of the dollar in terms of intermediating third party transactions. The closest is the EUR which only intermediates 1.2 times the share of Euro-area trade. The case of Japan is particularly striking. Despite the role of the yen as a global reserve currency in its own right, 50% of all Japanese exports, and 71% of its exporters are denominated in U.S. dollars, and all the rest is in yen.

In terms of trade in financial assets, Goldberg (2011) shows that 85% of the bilateral transactions in foreign exchange markets involve the USD, testifying to its important role as a vehicle currency. Moreover, the 2015 ECB report on the international role of the euro shows that 60% of international securities are issued in U.S. dollars, and just 20% in euro. And recent work by Maggiori, Neiman and Schreger (in progress) has shown that U.S. debt is the only type of foreign debt that international investors like to buy denominated in its local currency. Strikingly, most other international debt investments are made into securities denominated in the investor's domestic currency, not the destination country's local currency. The one big exception is the U.S., which is able to sell debt securities issued in U.S. dollars to the rest of the world in large quantities, and hence most of the foreign investment into the U.S. (and hence its foreign liabilities) is indeed made up of U.S. dollar securities. Moreover, over the last 15 years there has been a significant uptick in the international investor preferences towards USD debt, and away from debt denominated in other currencies, which coincides with the apparent rise in the exorbitant privilege.

In the rest of the paper, we put these two facts, of the dominance of U.S. assets as a medium of exchange and the exorbitant privilege of the U.S.'s external position, together and develop a model where a single currency emerges as the predominant international medium of exchange in equilibrium, which leads to an endogenous exorbitant privilege being bestowed to the country that can issue it. We micro-found the need for the medium of exchange by modeling international trade markets as bilateral, anonymous search and matching markets. Our assumptions here are consistent with emerging facts in the international trade literature, which report on the intricate bilateral nature of import-export relationships (see for examples Eaton et al., 2008, 2016). The theoretical international trade literature has also recently considered incorporating such frictions, as exemplified by Eslava et al. (2015), Eaton et al. (2016) and Eslava et al. (2015).

3 Model Framework

There are two symmetric countries, which we label "US" and "EU". Each country is populated by a standard representative household, productive firms, government and a continuum of import-export firms (i.e. international trade firms) with endogenously determined mass μ_{jt}, $j \in \{us, eu\}$. Households consume both domestic and foreign goods, supply labor to the domestic productive firms and face incomplete financial markets where they can buy only home and foreign bonds. Firms combine labor and an internationally-traded commodity, call it oil, to produce a differentiated domestic good. The home and foreign bonds are issued by their governments, who fund their expenditures via taxes and issuing debt. Trade in oil and financial assets happens in frictionless Walrasian markets, and all financial transactions are settled in the frictionlessly traded commodity (oil), which we take as the numeraire.

The key novelty of the model is that international trade in real goods happens in decentralized, bilateral trade markets. Each country features an import/export sector which intermediates all flows of goods across borders; no trade in real goods happens in frictionless markets. Exporters source the domestic good from local producers and search for a foreign importer to whom they may sell the good. However, contracts are not perfectly enforceable across borders, leading to the need for collateralization of international transactions (i.e. medium of exchange). Hence, in order to participate in international trade, firms first need to obtain some safe assets from their domestic households. Trading firms are free to choose the currency denomination of the funding (i.e. safe asset) they seek, but face an additional "mismatch" cost if they end up trading with a counterpart holding a different currency.

In the following sections we describe in detail the optimization problems of all agents in the US economy, with the understanding that foreign (EU) economy is symmetric.

3.1 International Trade in Goods: The Import-Export Sector

International trade in real goods happens in decentralized search and matching markets with exporting firms from one country looking to match with importers from the other. Given a successful match, the exporting firm buys goods from the local producers, and sells them to the foreign importer, who then sells the good to the foreign household. The exporter-importer match then splits the resulting surplus via Nash bargaining. The equilibrium surplus is positive due to endogenous markups that are determined by zero profit conditions

governing entry into the import-export sector in each country.

Transactions in these decentralized international markets are collateralized on both sides, and hence before import-export firms can trade they must first be "funded" by the domestic households. This funding takes the form of the household lending some of its safe assets, either home or foreign bonds, to a firm so that it can use those assets as collateral guaranteeing its international transactions. In return, the trading firm pays a fee to the household for this service. At the end of the period, after the trading firm completes its transaction and receives its share of the resulting surplus, it pays the funding fee and returns the bond to the household. This funding arrangement effectively amounts to a within-period repurchasing agreement between trading firms and households.

The collaterallization is needed because we assume that contracts are not enforceable across borders.[3] Each transaction is collateralized on both sides, to ensure both delivery of goods (protecting importers) and payment (protecting exporters). In equilibrium, there is no default, hence the collateral assets are always returned to the household who provided them at the end of the period. The collateral essentially serves as a medium of exchange, as there can be no international transactions without it.

Each period, new import-export firms are formed, operate, return profits to the local household, and are then disbanded.[4] The problem of these firms is static, but there are three stages to the life of each firm. In stage one, prospective firms choose whether or not to pay a fixed cost ϕ and become operational this period. Moreover, at this stage the firm does not know for sure whether it is going to be an importer or an exporter, but can optimally choose the probability of being one or the other.[5] Intuitively, the firm does not know whether in the current period it will have an importing or an exporting opportunity (those arrive stochastically), but can choose how hard to look for one versus the other.

In stage two, the firm looks for funding and has the choice of whether to seek funding in either domestic or foreign currency (i.e. to seek either domestic or foreign bonds as collateral). The firm faces a search friction in obtaining funding, hence the probability of being funded is not one. We assume that the firms look for a fixed amount of funding, which we normalize

[3]For simplicity we assume there is no enforceability, which equates to an infinite enforcement cost. That is not essential to the mechanism, and the mechanism will work similarly with partial enforcement.

[4]This market structure aligns well micro data, which shows significant amount of churn in bilateral trade relationships between firms (e.g. Eaton et al. (2016)). In any case, allowing for long-term trade relationships would in fact only strengthen the main mechanism, as will become clear shortly.

[5]This assumption is convenient, but not necessary to our mechanism. If we assumed that firms know whether they will import or export at the time of currency choice, we would need to analyze a four-way, rather than a two-way, coordination game but the coordination incentive would work similarly.

to one unit of the numeraire. If successful in obtaining funding, the firm proceeds to stage three, where it discovers whether it is an importer or an exporter, and then searches for an appropriate foreign trading partner, either an exporter or an importer respectively. If that search is successful, a trading match is formed, the importer buys goods from the exporter and sells them in its domestic market, and the resulting surplus is split between the two. The international transaction must be collateralized, and this is where the respective currency denomination of the funding of the matched firms matters. In particular, if the two counterparties to the international goods trade have chosen to operate in different currencies, the firms must pay an additional transaction cost before the trade is settled. The ex post cost of currency mismatch thus generate an ex ante strategic coordination motive in the currency choice of firms.[6]

This framework is a tractable abstraction of the complex and varied trade financing arrangements that are the lifeblood of international trade in practice. Over 90% of international trade involves some form of credit, guarantee or insurance, but the particular arrangements differ substantially across industries and countries of destination and origin.[7] The burden for providing financing could fall on the importer, the exporter or both. All types of arrangements are prevalent in practice, and moreover most internationally active firms are both importing and exporting, and hence are rarely exclusively an importer or an exporter. Our modeling framework captures the key economics that obtaining credit is important to internationally active firms, though we abstract from a formal banking sector, and have the firms interact directly with households, who hold all financial assets in each economy.

Stage 3: Trading Round and Profits

We solve the problem of the trading firms starting with stage three and working backwards. In the final stage, firms discover whether they are importing or exporting this period, and then search for an appropriate foreign counterpart. Due to the search-and-matching nature of the decentralized markets and the possibility that there could be more potential exporters in country j than importers in country i, not all exporters are going to be matched with an importer. For each sub-market, we assume that the total number of successful matches is given by the den Haan et al. (2000) matching function $M^T(u,v) = \frac{uv}{(u^{\frac{1}{\varepsilon_T}}+v^{\frac{1}{\varepsilon_T}})^{\varepsilon_T}}$

[6]The particular source of the currency mismatch cost does not matter for our mechanism – it could be a literal transaction costs related to exchanging one currency for another, or due to network effects as in Doepke and Schneider (2017).

[7]For more details on the structure and importance of trade financing, see the BIS (2014) report on trade financing, and Ahn (2015), Antras and Foley (2015), and Niepmann and Schmidt-Eisenlohr (2017).

with elasticity parameter ε_T.[8]

Let $\tilde{\mu}_{us,t}$ and $\tilde{\mu}_{eu,t}$ be the mass of *funded* trading firms in the US and EU respectively, and $p^{im}_{us,t}$ and $p^{im}_{eu,t}$ be the probabilities of being an importer, as chosen optimally by the firms in Stage 1. We thus have that the probability of a country j importer matching with a country k exporter is:

$$p^{ie}_{jt} = \frac{\tilde{\mu}_{kt}(1-p^{im}_{kt})}{\left[(\tilde{\mu}_{kt}(1-p^{im}_{kt}))^{1/\xi_T} + (\tilde{\mu}_{jt}p^{im}_{jt})^{1/\xi_T}\right]^{\xi_T}}$$

and the probability that a country j exporter matches up with a country k importer is

$$p^{ei}_{jt} = \frac{\tilde{\mu}_{kt}p^{im}_{kt}}{\left[(\tilde{\mu}_{kt}p^{im}_{kt}))^{1/\xi_T} + (\tilde{\mu}_{jt}(1-p^{im}_{jt}))^{1/\xi_T}\right]^{\xi_T}}.$$

Given a successful match, the two parties split the surplus that emerges from their trade. Here, we compute this value in the case of a successful match between a US exporter and an EU importer; the remaining three possibilities can be computed in parallel. For each good that the two exchange, they can realize a maximum surplus equal to the difference in the price of the US good in the US, $P^{us}_{us,t}$, and its price in the EU, $P^{us}_{eu,t}$. If there is a currency mismatch between the two counter-parties, however, the trading surplus is reduced by an additional transaction cost $\kappa > 0$.[9] This cost gives firms an *ex ante* incentive to coordinate their currency choices.

We assume that the resulting surplus is split via Nash bargaining, with a weight α for the importer, thus the effective transaction price is:

$$P^{EX}_{us,t} = P^{us}_{us,t} + (1-\alpha)(P^{us}_{eu,t} - P^{us}_{us,t})$$

This is the wholesale price of US exports – the price at which the EU importer purchases the good from the US exporter. In turn, the EU importer then sells the good at its equilibrium EU retail price $P^{us}_{eu,t}$. In equilibrium, $P^{us}_{eu,t} > P^{EX}_{us,t}$ and hence there is a markup and an associated positive surplus to sustain trading.

The expected profits of a US exporter holding US safe assets as collateral is thus:

$$\pi^{\$,ex}_{us,t} = \frac{(1-\alpha)}{P^{EX}_{us,t}}\left[\tilde{X}_{eu,t}(P^{us}_{eu,t} - P^{us}_{us,t}) + (1-\tilde{X}_{eu,t})(P^{us}_{eu,t} - P^{us}_{us,t} - \kappa)\right],$$

[8] This functional form exhibits constant returns to scale and, unlike the Cobb-Douglas matching function, ensures matching probabilities fall between zero and one.

[9] The mismatch cost could capture the cost of transforming collateral to be consistent with the counter-party. Or, the costs of liquidity management associated with clearing a payment in an unexpected currency.

where $\tilde{X}_{eu,t}$ is the fraction of the EU trading sector that is funded with dollars, and $1-\tilde{X}_{eu,t}$ the fraction funded with euros. The term in the square brackets represents the total expected surplus per unit of goods exchanged. A fraction $(1-\alpha)$ of this surplus accrues to the exporter. The total value of surplus per unit of trade is then divided by the wholesale price of exports since the importer has limited funding (normalized to 1 unit) and can only collateralize $\frac{1}{P^{EX}_{us,t}}$ units of trade.

In the general, the profits of a country j exporter who meets a country k importer are:

$$\pi^{\$,ex}_{jt} = \frac{(1-\alpha)}{P^{EX}_{jt}}\left[\tilde{X}_{kt}(P^j_{kt}-P^j_{jt}) + (1-\tilde{X}_{kt})(P^j_{kt}-P^j_{jt}-\kappa)\right]$$

$$\pi^{€,ex}_{jt} = \frac{(1-\alpha)}{P^{EX}_{jt}}\left[(1-\tilde{X}_{kt})(P^j_{kt}-P^j_{jt}) + \tilde{X}_{kt}(P^j_{kt}-P^j_{jt}-\kappa)\right]$$

depending on whether the firm is funded by dollars or euro respectively. We can similarly derive the profits of the importers by multiplying by $\frac{\alpha}{1-\alpha}$.

$$\pi^{\$,im}_{jt} = \frac{\alpha}{P^{EX}_{kt}}\left[\tilde{X}_{kt}(P^k_{jt}-P^k_{kt}) + (1-\tilde{X}_{kt})(P^k_{jt}-P^k_{kt}-\kappa)\right]$$

$$\pi^{€,im}_{jt} = \frac{\alpha}{P^{EX}_{kt}}\left[(1-\tilde{X}_{kt})(P^k_{jt}-P^k_{kt}) + \tilde{X}_{kt}(P^k_{jt}-P^k_{kt}-\kappa)\right]$$

Stage 2: Funding Stage

At this stage, the trading firms choose what type of funding to seek. We refer to funding with US safe assets as "dollar" funding, and funding with EU safe assets as "euro" funding. Firms seek their funding in matching markets, and not all firms succeed in finding funding each period. Supply in each of these markets is furnished by the domestic household, which lends its bond holdings of a particular currency. On the demand side are the domestic firms who choose to search for that currency.

Bonds promise payment of one unit of the issuing country's consumption good. Thus, the total value of the US bonds available for lending in country j at time t is given by $P_{us,t}B^{\$}_{jt}Q^{\$}_t$, where $B^{\$}_{jt}$ are the holdings of US bonds in the country j household's portfolio, $P_{us,t}$ is the price of the US consumption basket, and $Q^{\$}_t$ is the real price of the bond that pays off one unit of US consumption tomorrow.

Let X_{jt} be the fraction of country-j trading firms choosing to seek US bonds, so that the total mass of country-j trading firms searching the domestic US bond market is $\mu_{jt}X_{jt}$.

The total number of matches is given according to the constant returns to scale matching function, so that the probability that a country j trading firm seeking US bonds finds a suitable supplier is

$$p_{jt}^{\$} = \frac{M^F\left(\mu_{jt}X_{jt}, P_{us,t}B_{jt}^{\$}Q_t^{\$}\right)}{\mu_j X_{jt}},$$

where the matching function $M^F(u,v)$ describes the matching technology in the funding market. Similarly, the probability that a country j trading firm seeking EU bonds finds a match is

$$p_{jt}^{€} = \frac{M^F\left(\mu_{jt}(1-X_{jt}), P_{eu,t}B_{jt}^{€}Q_t^{€}\right)}{\mu_j(1-X_{jt})}.$$

In order to make their currency choice, firms compare their expected profits conditional on either being funded with dollars or euros. At this stage, they do not yet know whether they will be importers or exporters, or whether they will be able to find a successful trading matches in the next stage. Hence, they form expectations over the trading profits that they would receive, conditional on choosing one type of funding over the other.

The expected profit of a country-j trading firm funded with US assets is

$$\tilde{\Pi}_{jt}^{\$} = p_{jt}^{im}p_{jt}^{ie}\pi_{jt}^{\$,im} + (1-p_{jt}^{im})p_{jt}^{ei}\pi_{jt}^{\$,ex}.$$

The first of the two terms in the above sum equals the expected profit of being a dollar-funded importer. It equals the probability of being an importer, times the probability of then finding a successful match with a foreign exporter, times the resulting profits from that match. The second component is the expected profit of being a dollar-funded exporter. The corresponding expected profits of a country j trading firm funded with EU assets instead is:

$$\tilde{\Pi}_{jt}^{€} = p_{jt}^{im}p_{jt}^{ie}\pi_{jt}^{€,im} + (1-p_{jt}^{im})p_{jt}^{ei}\pi_{jt}^{€,ex}.$$

In the event that the trading firm finds the funding it seeks, it pays a fee $r_j^{\$}$ or $r_j^{€}$ for the funding services of dollars or euros respectively. Thus, the expected profit of a country-j firm seeking dollar funding is given by

$$\Pi_{jt}^{\$} = p_{jt}^{\$}(\tilde{\Pi}_{jt}^{\$} - r_j^{\$}), \tag{1}$$

which is simply the probability of obtaining dollar funding, $p_{jt}^{\$}$, times the expected profit net of the dollar funding costs, $(\tilde{\Pi}_{jt}^{\$} - r_j^{\$})$. Similarly, we can compute the expected profit of

a country-j firm seeking Euro funding:

$$\Pi^{€}_{jt} = p^{€}_{jt}(\tilde{\Pi}^{€}_{jt} - r^{€}_{j}). \tag{2}$$

The only equilibrium requirement for the funding fees $r^{\$}_{j}$ and $r^{€}_{j}$ are that they leave firms with a positive surplus relative to the alternative of declining funding and doing no trade. In parallel with the labor match and searching literature, these prices could be fixed exogenous parameters — so long as they fall within the surplus range of the trading firms — or they could be endogenously determined by assuming some bargaining paradigm, like Nash bargaining. For simplicity, we follow the first of these paths and fix the funding prices to a common value, $r = r^{\$}_{j} = r^{€}_{j}$.

Stage 1: Firm Formation

At stage one, the firm optimizes over the probability of being an importer, which leads to the following optimality condition:

$$\frac{\partial \max\{\Pi^{\$}_{jt}, \Pi^{€}_{jt}\}}{\partial p^{im}_{jt}} = 0. \tag{3}$$

The solution to equation (3) above determines the share of importers and exporters in equilibrium.

Given this and all of the above choices, prospective firms then decide whether or not to pay the fixed cost $\phi > 0$ in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$\max\{\Pi^{\$}_{jt}, \Pi^{€}_{jt}\} - \phi = 0,$$

is satisfied, where $\Pi^{\$}_{jt}$ and $\Pi^{€}_{jt}$ are evaluated at the optimal choice of p^{im}_{jt}. The entry condition thus determine the equilibrium size, μ_{jt}, of the import-export sector in each country.

Currency Choice Equilibrium

The payoff structure described above incorporates a strategic complementarity in currency choice across countries. In particular, trading firms in the US have an incentive—whether they expect to be importers or exporters—to hold the same currency that (they believe) trading firms in the EU are holding. These payoffs also include a strategic substi-

tutability of currency choice among firms in the same country; firms choosing to be funded with high-demand currency face a decreasing probability of finding the funding they desire. Finally, all trading firms have a desire *ceteris paribus* to hold a currency with a relatively large asset base, as a large supply of the asset to the market increases the likelihood of finding suitable funding.

So long as the payoffs of the two currency choices are sufficiently similar, the currency choice game described described above exhibits a multiplicity of equilibria. In this paper, however, we are interested in studying dynamic multiplicity, i.e. a situation in which the *static* currency game exhibits a unique equilibrium, while the *dynamic* general equilibrium economy has several steady-states. To achieve uniqueness in the currency choice game, we follow Morris and Shin (1998) in using a vanishing information friction to select a unique equilibrium that depends on economic states and fundamentals, and then embed the model of trading firms within a standard dynamic economy that determines the evolutions of state variables.

We assume that, in addition to the economic payoffs described above, trading firms in both countries perceive a small additive disturbance, θ_{it}, to the payoff of seeking dollars over euros.[10] This disturbance contains both an aggregate and an idiosyncratic component,

$$\theta_{it} = \theta_t + \varepsilon_{it}.$$

These θ_t and ε_{it} are mutually orthogonal, iid shocks, with $\theta_t \sim N(0, \sigma_\theta^2)$ and $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$. Agents are assumed to observe their own shock, θ_{it}, but not the contributions of the common and idiosyncratic components. In this context, the firm's posterior belief about θ_t is Normal, is centered around

$$E[\theta_t|\theta_{it}] = \omega\theta_{it},$$

where $\omega \equiv \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$, and has posterior-variance

$$Var[\theta_t|\theta_{it}] \equiv \tilde{\sigma}_\theta^2 = \frac{\sigma_\theta^2\sigma_\varepsilon^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}.$$

Given that an increase in θ_{it} increases the expected payoff of seeking dollar funding to all agents,[11] we restrict our analysis to the space of monotone strategies in which trading

[10] The additive formulation is particularly tractable. To achieve a similar effect in earlier drafts of this paper, we assumed that the effective supply of the asset was subject to a small shock.

[11] Unlike in Morris and Shin (1998), this is true both because θ_{it} directly increases the incentive to hold

firms from both countries adopt the US asset so long as their private shock exceeds country-specific thresholds $\bar{\theta}_{us,t}$ and $\bar{\theta}_{eu,t}$, respectively. Given a value for these cutoffs, the resulting fraction of each country's trading firms adopting the US asset are

$$X_{jt} \equiv \int_0^1 \mathbb{1}(\theta_{it} >= \bar{\theta}_{jt}) di = 1 - \Phi\left(\frac{\bar{\theta}_{jt} - \theta_t}{\sigma_\varepsilon}\right),$$

where $\Phi(\cdot)$ denotes the standard normal CDF.

Given the conjectured monotonicity, the equilibrium cutoff $\bar{\theta}_{jt}$ is the value of the signal that leaves the country-j importer-exporter indifferent between choosing one asset or the other, given everyone else's strategy. Denoting with $\phi(\theta_t|\theta_{it})$ firm i's posterior distribution over θ_t, the equilibrium cutoff values $(\bar{\theta}_{us,t}, \bar{\theta}_{eu,t})$ solve the equations

$$E\left[\Pi^{\$}_{jt}(\bar{\theta}_{jt}, \bar{\theta}_{j't}, \theta_t, \omega_t) - \Pi^{€}_{jt}(\bar{\theta}_{jt}, \bar{\theta}_{j't}, \theta_t, \omega_t) \middle| \bar{\theta}_{jt}\right] + \bar{\theta}_{jt} = 0, \tag{4}$$

for $j \in \{us, eu\}$ and j' denoting the country that is not j.

3.2 Domestic Sector

We embed our model of the import-export sector within a standard two-country open economy model. The domestic sector of the economy is summarized below.

Households

The household sector in country j consists of a representative consumer-worker, who seeks to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{jt}^{1-\sigma}}{1-\sigma} - \frac{L_{jt}^{1+\nu}}{1+\nu}\right)$$

where the consumption basket C_{jt} is a CES aggregator of US (C_{jt}^{us}) and EU (C_{jt}^{eu}) goods:

$$C_j(C_{jt}^{us}, C_{jt}^{eu}) = \left(a_j^{us\,\frac{1}{\eta}} C_{jt}^{us\,\frac{\eta-1}{\eta}} + a_j^{eu\,\frac{1}{\eta}} C_{jt}^{eu\,\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{5}$$

with the price index

$$P_{jt} = \left(a_j^{us}(P_{jt}^{us})^{1-\eta} + a_j^{us}(P_{jt}^{eu})^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{6}$$

dollars and because higher θ_{it} indicates that other firms have also received a shock favoring dollars.

In addition, the household chooses how to allocate its savings among US and EU bonds, which are risk-free in the units of their denomination. Since trade in the differentiated US and EU products happen in decentralized markets, as described above, the bond payments are settled in terms of the international numeraire, which is the only real good that is frictionlessly traded across countries. As a result, the bonds do not promise delivery of real goods, but rather enough resources to purchase the promised consumption good. Thus, payments on the US bond are indexed to the price of US consumption – one unit of US bonds purchased at time $t-1$ yields a payment of $P_{us,t}$ – and the EU bond similarly returns $P_{eu,t}$. Lastly, each period the households of both countries are also endowed with one unit of oil.

Thus, a household faces the following budget constraint (with prices expressed in terms of the numeraire),

$$\begin{aligned} P_{jt}C_{jt} + (1-\Delta^{\$}_{jt})P_{us,t}Q^{\$}_t B^{\$}_{jt} + (1-\Delta^{€}_{jt})P_{eu,t}Q^{€}_t B^{€}_{jt} + P_{us,t}Q^{\$}_t \frac{\tau^{\$}_j}{2}(B^{\$}_{jt}-B^{\$}_{jt-1})^2 + P_{eu,t}Q^{€}_t \frac{\tau^{€}_j}{2}(B^{€}_{jt}-B^{€}_{jt-1})^2 \\ = P_{us,t}B^{\$}_{jt-1} + P_{eu,t}B^{€}_{jt-1} + W_{jt}L_{jt} + R_t + \Pi_{jt} + T_{jt}, \end{aligned} \tag{7}$$

where Π_{jt} is the total profit of country j's trading sector, which is returned lump-sum to the representative household, $R_t \equiv 1$ represents the common price of the unit endowment of the numeraire good which we normalize to one, $Q^{\$}_t$ and $Q^{€}_t$ are the prices of the US and the EU bonds respectively, and $\Delta^{\$}_{jt}$ and $\Delta^{€}_{jt}$ represent the endogenous liquidity premia (i.e. lending fees) earned by the bonds lent within the period to trading firms. Lastly, there are also quadratic portfolio adjustment costs. Adjustment costs are parameterized to be zero in steady-state, and serve to prevent excess volatility of capital flows, without affecting the average level of bond holdings. We will consider model specifications with and without portfolio adjustment costs, and the main qualitative implications do not differ.

The first order conditions of the household yield the Euler equations:

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{us,t+1}}{P_{us,t}} \frac{1}{Q^{\$}_t(1-\Delta^{\$}_{jt}+\tau^{\$}_j(B^{\$}_{j,t+1}-B^{\$}_{jt}))} \right] \tag{8}$$

$$1 = \beta E_t \left[\left(\frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{eu,t+1}}{P_{eu,t}} \frac{1}{Q^{€}_t(1-\Delta^{€}_{jt}+\tau^{€}_j(B^{€}_{j,t+1}-B^{€}_{jt}))} \right] \tag{9}$$

Domestic Production

All output in the economy is tradable, and is produced by competitive domestic firms according to the production function

$$Y_{jt} = A_{jt} L_{jt}^{\alpha_l} \tilde{K}_{jt}^{1-\alpha_l}. \tag{10}$$

where $\tilde{K}_{jt}$ represents the firms' use of international numeraire good in production. Standard first-order condition imply that

$$W_{jt} = P_{jt}^{j} \alpha_l \left(\frac{L_{jt}}{\tilde{K}_{jt}} \right)^{\alpha_l - 1} \tag{11}$$

$$1 = R_t = P_{jt}^{j} (1 - \alpha_l) \left(\frac{L_{jt}}{\tilde{K}_{jt}} \right)^{\alpha_l}. \tag{12}$$

Notice that in general $\tilde{K}_{jt} \neq 1$, i.e. that domestic use of the commodity input good need not equal the country's unit endowment.

Government

The government faces expenditures fixed in proportion to domestic production:

$$G_{jt} = \phi_{j,g} Y_{jt}. \tag{13}$$

For simplicity, we begin by assuming that the government levies lump-sum taxes so at keep the stock of debt $\bar{B}^c > 0$ constant.

Market Clearing

Market clearing in the goods market requires that domestic production is either consumed at home or exported abroad via the export sector. Letting the measures of household in the US and EU be μ and $1 - \mu$ respectively, we have

$$(1 - \phi_{us,g}) Y_{us,t} = C_{us,t}^{us} + \frac{(1 - \mu)}{\mu} C_{eu,t}^{us} \tag{14}$$

$$(1 - \phi_{eu,g}) Y_{eu,t} = \frac{\mu}{(1 - \mu)} C_{us,t}^{eu} + C_{eu,t}^{eu} \tag{15}$$

Market clearing for the frictionlessly traded input good (oil) is

$$\mu\left(\tilde{K}_{us,t}+\Delta^{k}_{us,t}\right)+(1-\mu)\left(\tilde{K}_{eu,t}+\Delta^{k}_{eu,t}\right)+\bar{\kappa}=1. \tag{16}$$

where $\Delta^{k}_{us,t}$ and $\Delta^{k}_{eu,t}$ represent the capital resources devoted to establishing trading firms (i.e. covers the fixed cost of entry),

$$\Delta^{k}_{jt}=\phi\mu_{jt},$$

where μ_{ct} is the equilibrium measure of trading firms that enter the market. The parameter $\bar{\kappa}$ denotes the total amount of resources used up in transaction costs due to currency mismatch.

Bond market clearing requires that the foreign and domestic holding of bonds combine to equal the fixed aggregate supply of government debt.

$$B^{\$}_{us,t+1}+\frac{(1-\mu)}{\mu}B^{\$}_{eu,t+1}=\bar{B}^{\$} \tag{17}$$

$$\frac{\mu}{(1-\mu)}B^{€}_{us,t+1}+B^{€}_{eu,t+1}=\bar{B}^{€} \tag{18}$$

Finally, note that because of the frictions in cross-border trade, the law of one price does not hold across countries. Specifically,

$$P^{eu}_{us,t}=\Delta_{us,t}P^{eu}_{eu,t} \tag{19}$$

$$P^{us}_{eu,t}=\Delta_{eu,t}P^{us}_{us,t} \tag{20}$$

where the wedge Δ_{jt} is an equilibrium object pinned down by free entry and the equilibrium of the coordination game played by traders.

Given processes for $\left\{A_{jt},\Delta^{\$}_{jt},\Delta^{€}_{jt},\Delta_{us,t},\Delta_{eu,t}\right\}$, an equilibrium in the domestic sector is described by the aggregate prices, $\left\{Q^{\$}_{t},Q^{€}_{t}\right\}$, the set of country specific prices $\left\{P_{jt},P^{us}_{jt},P^{eu}_{jt},W_{jt}\right\}$, and the set of country specific allocations $\left\{C_{jt},C^{us}_{jt},C^{eu}_{jt},B^{\$}_{jt},B^{€}_{jt},L_{jt},Y_{jt},G_{jt},\tilde{K}_{jt}\right\}$ that satisfy equations (5) through (20). Accounting for equations that appear for country j and j', this yields a total of 28 equations in 28 unknowns.

4 Steady-state Multiplicity

In this section we explore a crucial feature of our model, the existence of multiple steady-states. This does not mean that economy has multiple equilibria; indeed, at any point in the state space, the equilibrium path is unique. Rather, paths from different starting points converge to different long-run absorbing states. The source of steady-state multiplicity is the feedback between the choice of funding currency by the trading firms, and the bond holdings of the households. Firms desire to fund themselves with a currency that is broadly available, and households wish to hold assets that are demanded by domestic trading firms. Since assets holdings are (slow moving) state variables, equilibrium jumps within a period are precluded – i.e. for given bond states, there is only one equilibrium currency choice in the trading game and thus no indeterminacy. Nevertheless, the equilibrium of currency choice game will endogenously reinforce several different possible initial bond holding states.

To better understand this multiplicity, in this section we present and analyze an approximation to the model's steady-state that highlights the key feedback effects behind our results. For clarity of presentation, we make two key simplifications: (i) we treat prices and the measure of operating firms as exogenous parameters that equal one, and (ii) we specify the matching functions in the form suggested by den Haan et al. (2000), with elasticity equal to one. This allows us to reduce the steady-state relationships to a set of six, self-contained equations – two bond return indifference conditions that determining the portfolio choice of both households, two market clearing conditions for bonds, and the two indifference conditions describing the equilibrium currency choice in each country. These six equations capture the key feedback mechanisms that generate the steady-state multiplicity, and the simplification allows us to tractably illustrate how it works. In our quantitative analysis of the full model, we find that ignoring the price effects has little effect.

Steady-state Portfolios

In our environment, long-run household bond allocations can be derived from the steady-state versions of the Euler equations (8) and (9). To see this, rearrange to obtain:

$$\frac{1}{\beta} = \frac{1}{Q^{\$}(1-\Delta_j^{\$})} \tag{21}$$

$$\frac{1}{\beta} = \frac{1}{Q^{€}(1-\Delta_j^{€})}. \tag{22}$$

for $j \in \{us, eu\}$. These conditions equate the inverse of the time discount parameter, β, to the effective returns on the bonds, which include both the interest rate $\frac{1}{Q_t^\$}$ and the extra return from repo-ing the bonds to the trading firms $\frac{1}{1-\Delta_j^\$}$, i.e. the liquidity premium. Since both households face the same bond prices, equations (21) and (22) imply liquidity that the premia terms themselves must be equalized:

$$\Delta_{us}^\$ = \Delta_{eu}^\$,$$

$$\Delta_{us}^€ = \Delta_{eu}^€.$$

Equations (21) and (22) also highlight a well-known issue in international models with incomplete markets. If the liquidity premia terms are exogenous, then steady-state portfolios are either indeterminate — if all four premia terms are equal, the assets are perfect substitutes for all agents; or completely specialized — if premia terms are not equal, agents will have a strict preference for one asset over the other.

In contrast, our *endogenous* premia terms help to deliver equilibria with determinate, interior allocations of world assets. To see this, observe that

$$\Delta_j^\$ = p_j^{\$,h} r_j^\$ = \frac{M^F\left(X_j, B_j^\$\right)}{B_j^\$} r = \frac{X_j}{B_j^\$ + X_j} r \tag{23}$$

$$\Delta_j^€ = p_j^{€,h} r_j^€ = \frac{M^F\left((1-X_j), B_j^€\right)}{B_j^€} r = \frac{1-X_j}{B_j^€ + 1 - X_j} r. \tag{24}$$

where the last equality substitutes in the HRW matching function with elasticity of one. In this case, the probability of finding a match is simply the ratio of the searches on one side of the market to the total number of searchers on both sides.

The resulting expressions for the equilibrium liquidity premia have several intuitive features. Premia are increasing in the number of firms searching for a particular type of currency in each country. So, for example, if more US firms are looking for dollars (i.e. higher X_{us}), the probability that a dollar bond held in the US finds a match goes up, increasing the dollar bond liquidity premium in the US. On the other hand, if fewer US firms look for dollars, then the probability of a dollar bond matching falls, and the dollar liquidity premium goes down. Conversely, if $B_{us}^\$$ is high, and the market is already well-supplied with many dollar assets, the liquidity premia earned by those dollar assets falls.

Imposing market clearing in bond markets, equations (23) and (24) can be rearranged to

deliver expressions for asset holdings in terms of firms funding choices:

$$B_j^{\$} = B\frac{X_j}{X_j + X_{j'}} \tag{25}$$

$$B_j^{€} = B\frac{1 - X_j}{1 - X_j + 1 - X_{j'}} \tag{26}$$

These portfolio allocations are also quite intuitive. When there is relatively more demand for dollar funding in country j as compared to j', then a larger proportion of the available dollar bonds are held in country j. Equations (25) and (26) thus summarize how, in equilibrium, the currency choice of the trading firms influences the steady-state portfolios of households.

Currency Choice Incentives

We now characterize the indifference conditions that determine the equilibrium choices of funding currency. In making their choice, firms compare the expected profits from funding themselves in dollars to the expected profit from funding in euros. To keep things tractable, we assume that firms' face equal probability of being an importer or an exporter and thus stand to earn a fixed amount of gross profit π from trading.

Gross profits must then be adjusted for expected currency mismatch costs, which are a function of $\tilde{X}_j$, the fraction of firms funded with dollars. For simplicity, we approximate $\tilde{X}_j$ with X_j, according to

$$\tilde{X}_j = \frac{p_j^{\$} X_j}{p_j^{\$} X_j + p_j^{€}(1 - X_j)} = \frac{X_j}{X_j + \frac{p_j^{€}}{p_j^{\$}}(1 - X_j)} \approx X_j.$$

The approximation is exact whenever the funding probabilities in different currencies are identical. In our benchmark calibration, these probabilities are both quite close to one 1. Intuitively, this is because the size of the funding available (i.e. the outstanding supply of bonds) is much larger than the funding demanded by the trading sector, given reasonable calibration of the size of trade.

Our (approximate) net profit of choosing dollar funding for firm i in country j is thus:

$$V_{ij}^{\$} = \underbrace{\frac{B_j^{\$}}{B_j^{\$} + X_j}}_{\text{Prob. finding \$}} [\pi - \kappa(1 - X_{j'})] - \underbrace{\frac{B_j^{€}}{B_j^{€} + X_j}}_{\text{Prob. finding €}} [\pi - \kappa X_{j'}] + \theta_i.$$

The first term is the payoff of going with dollars – it is equal to the probability of finding

dollar funding times the trading payoff when using dollars. The dollar payoff, in turn, is gross profit from trading, π, minus the expected currency mismatch cost given by κ times the number of firms using euros in country j'. The second term is the payoff of choosing euros, and θ_i is the stochastic shock that perturbs the payoffs and generates the strategic uncertainty needed for unique equilibrium in this game, given finding probabilities.

Currency choice with exogenous funding probabilities

We now prove that strategic uncertainty in the baseline game is indeed enough to achieve uniqueness when funding probabilities are exogenous. This results emphasizes that the steady-state multiplicity that we find is a consequence of the ~~endogeneity of funding probabilities via household bond choices~~ endogenous funding probabilities, which create a feedback channel between the choices of the firms and the households.

When $p_j^{\$}$ and $p_j^{€}$ are given, a firm experiencing shock θ_i has expected net profits of

$$\begin{aligned} E(\bar{V}_j^{\$}|\theta_i) &= p_j^{\$}\left[\pi - \kappa(1 - E(X_{j'}|\theta_i))\right] - p_j^{€}\left[\pi - \kappa E(X_{j'}|\theta_i)\right] + \theta_i \\ &= \underbrace{(p_j^{\$} - p_j^{€})\pi - p_j^{\$}\kappa}_{\equiv M_0^j} + \underbrace{\kappa(p_j^{\$} + p_j^{€})}_{\equiv M_1^j} E(X_{j'}|\theta_i) + \theta_i. \end{aligned}$$

Intuitively, the equation above says that, in order to evaluate its optimal currency decision, the trading firm must form a conjecture about the fraction of firms in the foreign country that seeks dollars, $X_{j'}$. The optimal cutoff values, $(\bar{\theta}_{us}, \bar{\theta}_{eu})$, are the solutions to the analogue to equation (4),

$$E(\bar{V}_j^{\$}|\bar{\theta}_j) = p_j^{\$}\left[\pi - \kappa(1 - E(X_{j'}|\bar{\theta}_j))\right] - p_j^{€}\left[\pi - \kappa E(X_{j'}|\bar{\theta}_j)\right] + \bar{\theta}_j = 0, \tag{27}$$

for $j \in \{us, eu\}$.

The only unknown in equation (27) is $X_j = 1 - \Phi\left(\frac{\bar{\theta}_j - \theta}{\sigma_\varepsilon}\right)$, which depends on the realization of θ. The expectation of $X_{j'}$ conditional on $\bar{\theta}_j$ involves integrating a normal CDF against the normal PDF, a case for which standard formulas exist. This expectation is

$$E[X_{j'}|\bar{\theta}_j] = 1 - \Phi\left(\frac{1}{\sigma_\varepsilon\sqrt{1 + \sigma_{\tilde{\theta}}^2}}\left(\bar{\theta}_{j'} - \sigma_{\tilde{\theta}}^2\bar{\theta}_j\right)\right).$$

Substituting this result, the two conditions describing the equilibrium are

$$\bar{\theta}_{us} = M_0^{us} - M_1^{us} + M_1^{us}\Phi\left(\gamma\left(\bar{\theta}_{eu} - \sigma_{\tilde{\theta}}^2\bar{\theta}_{us}\right)\right) \tag{28}$$

$$\bar{\theta}_{eu} = M_0^{eu} - M_1^{eu} + M_1^{eu}\Phi\left(\gamma\left(\bar{\theta}_{us} - \sigma_{\tilde{\theta}}^2\bar{\theta}_{eu}\right)\right). \tag{29}$$

where $\gamma \equiv \frac{1}{\sigma_\varepsilon\sqrt{1+\sigma_{\tilde{\theta}}^2}}$.

We provide two propositions for this version of the funding game. Our first proposition shows that for any value of κ, there exists a level of idiosyncratic variation below which the equilibrium of the funding game is unique. This result is an extension of the analysis of Morris and Shin (1998) to a game with two types of agents playing against each other and shows that the global game refinement works in much the same way as in the one-dimensional game.

Proposition 1. *For any $\kappa > 0$, there exits a threshold value, $\bar{\sigma}_\varepsilon^2$, such that if $\sigma_\varepsilon^2 < \bar{\sigma}_\varepsilon^2$ the currency game has a unique equilibrium.*

Proof. See the Appendix. □

Our second proposition shows that if errors in the signal and the fluctuations in the fundamental approach zero at appropriate rates, the exogenous funding model continues to have a unique equilibrium. Hence, in the limit where agents are exposed to only negligible uncertainty about the fundamentals driving currency choice, the associate strategic uncertainty always delivers a unique equilibrium.

Proposition 2. *Let $\sigma_{\tilde{\theta}}^2 = c\sigma_\varepsilon^{2x}$ for any strictly positive scalar c and $x < 1/2$. Then, the currency choice game has a unique equilibrium in the limit $\sigma_\varepsilon^2 \to 0$.*

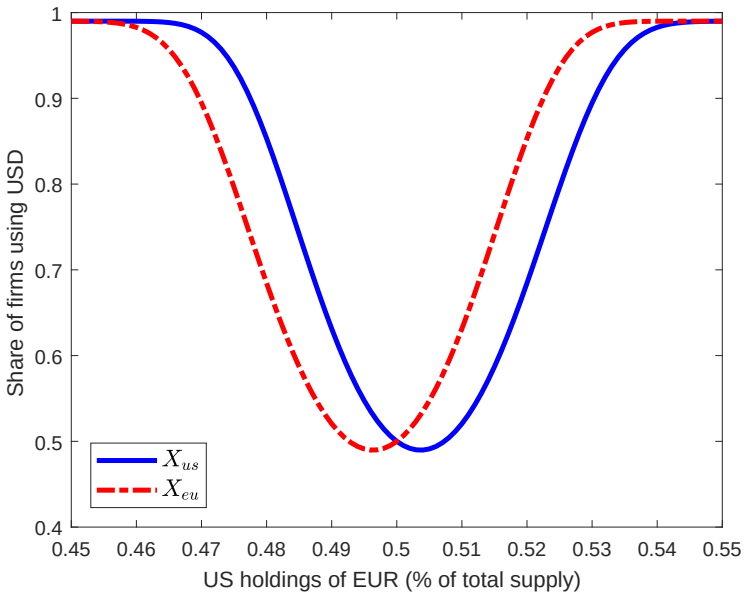
Proof. See the Appendix. □

Our propositions establish that, no matter the mismatch cost κ, the game approaching the limit of no uncertainty has a unique equilibrium. We will perform all of our numerical analysis close to this limit, ensuring that any steady-state multiplicity we find is driven by interplay between asset allocations and the funding probabilities, and not by indeterminacy in the within-period funding choice of firms.

Currency choice with endogenous funding probabilities

We now turn to the key force driving our result, the endogeneity of funding probabilities. In this version of the model, firms know the equilibrium bond holdings of the households,

Figure 3: Steady State Currency Use



$B_j^{\$}$ and $B_j^{€}$, and take into account how shifting bond allocations affect the likelihood they will secure funding of each type. Thus, the indifference condition (27) becomes

$$E(V_j^{\$}|\bar{\theta}_j) = E\left(\frac{B_j^{\$}}{B_j^{\$}+X_j}\left[\pi-\kappa(1-X_{j'})\right]-\frac{B_j^{€}}{B_j^{€}+X_j}\left[\pi-\kappa X_{j'}\right]\middle|\bar{\theta}_j\right)+\bar{\theta}_j=0, \tag{30}$$

This endogenous funding probability version of the game is less tractable, because the random terms X_j now appear in non-linear fashion in equation (30). We thus explore this game through numerical analysis, showing how the key forces described above influence currency choice and steady-state multiplicity.

To begin our analysis, Figure 3 describes the equilibrium of the currency choice game when bond holdings are shifted exogenously from one country to another. This allows us to isolate the impact of bond holdings on currency choice, without first imposing optimality on the part of households.

In the figure, we suppose that dollar assets are equally distributed across countries and then vary the distribution of euro assets across countries, keeping the world supply of both assets identical and constant. The center point on the x-axis thus describes a perfectly symmetric asset allocation: both US and EU households hold equal shares of the world portfolio. In this case, firms in both countries find dollars and euros equally appealing from an availability perspective, and therefore their currency coordination game begins from

a point where neither asset has an initial advantage. Given the nature of the strategic uncertainty, the resulting equilibrium is a perfectly symmetric, *non-coordinated* currency choice: firms in both countries use dollars with a probability of one half.

Crucially, shifting euro bond holdings in *either* direction — towards the US or towards the EU — actually leads a coordinated movement in favor using dollars for trade finance. To see why, first observe that given market clearing it is not possible for either bond to be relatively abundant in both countries: if one country has fewer dollar bonds, then the other must have more. Thus, at any other point, except for in the middle of the graph, one type of asset is relatively more abundant than the other in both countries. Necessarily, though, the abundant asset in each country is different – if dollars are relatively more available in the US, then via market clearing, it must be the case that euros are relatively more available in the EU. Intuitively, the choice of global currency is driven by the firms in the asset poor country, where funding less available overall.

To see this, take as an example the case where only 45% of European bonds are held in the US. In this case, trade financing opportunities are *overall more scarce* in the United States, and thus, at the margin funding availability considerations act as bigger motivation for US firms, as compared to EU firms. With euro funding relatively scarce relative to dollars, the US firms shift towards dollar use. In turn, the coordination incentive leads EU firms to follow and also move towards dollars, which is followed by another (now smaller) shift in US funding, and etc., to the point where firms in both countries lean strongly towards dollars. Note that the EU firms themselves face more favorable conditions for finding euro funding, but they also have more abundant funding overall, and hence do not care as much about the scarcity of funding. Thus, the EU firms become the followers in the coordination game. The same logic would apply, though now initiated with the EU, had the EU held a minority of euro bonds (i.e. a shift to the right in the figure). In this way, a relatively modest initial shift in bond holdings by households can deliver large changes in equilibrium funding choices.

In turn, the change in the currency choices would feedback into the households' bond portfolios through equations (25) and (26) in a way that reinforces the firms' choices. In particular, as dollar use rises in the US relatively more than in the EU, relatively more dollar bonds will be held by US households, thus supporting that currency choice as a potential steady state.

Figure 4 illustrated the possible steady-state values of bond holding for different values for the currency mismatch cost, κ.[12] The axes in each panel represent the two relevant state

[12]For this example we use the following parameters: $\pi = B = 1$, $\kappa = 0.05$, $\sigma_\varepsilon^2 = 1e^{-8}, \sigma_\theta^2 = 1e^{-5}$.

variables of the model – the home and foreign bond positions with which a household enters a given period. We plot the picture from the view point of the US household, with its holdings of EU bonds on the x-axis, and its holdings of US bonds on the y-axis.

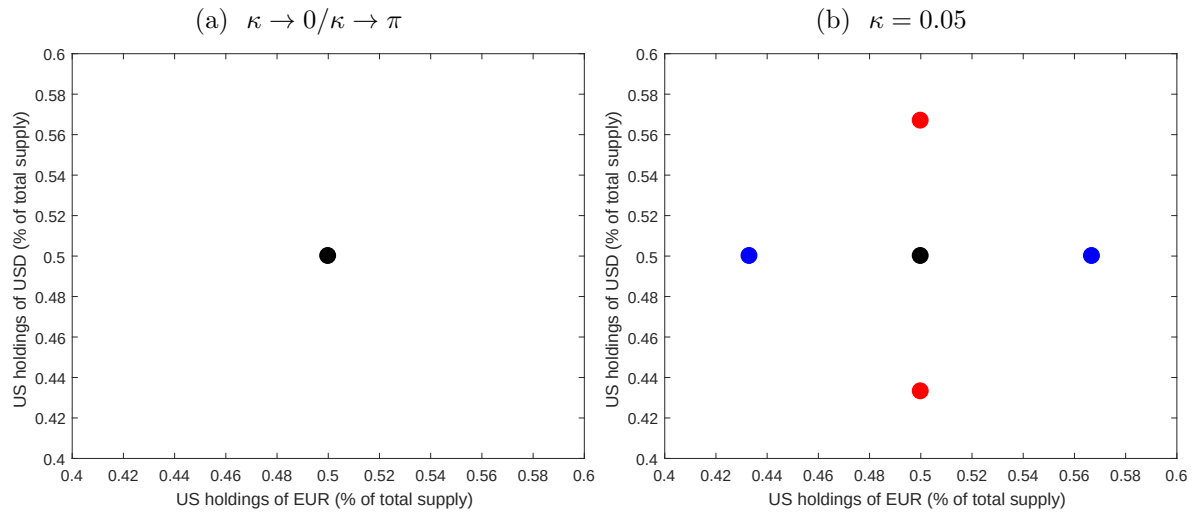
Panel (a) of the figure describes a case in which firms have virtually no incentive to coordinate their currency choice (κ is very low). In this case, firm wish to pick an asset that is not only in large supply in their country, but also one that is not already being heavily used by other domestic trading firms. Thus, firms perceive a (within country) strategic substitutability in currency choice, and the only equilibrium is the perfectly symmetric outcome: a non-coordinated equilibrium with asset holdings and currency choice equally distributed between euros and dollars.

Panel (b) of the figure describes a case with a small but non-trivial cost of currency mismatch, and it corresponds to the type of calibration we consider in our quantitative analysis in Section 5. In this case, additional coordinated steady-states emerge in which asset allocations and currency choice are no longer symmetric. The location of the points describes the corresponding asset allocation, while the color of the dots the dominant currency in that steady state – blue for dollars and red for euros. The left-most blue dot, for example, describes a dollar-coordinated equilibrium in which both US and EU firms primarily use dollars, and US holdings of Euro bonds are lower than EU holding of US bonds. In other words, this is a situation where the dollar is dominant and the US has a negative NFA position.

Our description of Figure 3 has explained why such an asset allocation leads to dominance of the dollar in trading firms' currency choices, but why is this an equilibrium from the household perspective? The answer is that the asymmetry in bond holdings is supported by an asymmetry in the degree of dollarization in the two countries. In particular, because in this case the US holds relatively few euro assets, US firms will seek dollars at an even higher rate than in the EU (see Figure 3). But to satisfy this relatively higher demand, the US households will must have portfolios that are relatively concentrated in dollars, while the EU households will be happy to pick up the excess euro assets, as there are still somewhat more EU firms using euros. Thus, the asymmetric coordinated steady-state is self-sustaining.

Finally, Panel (a) of the same figure also describes a case with a very large currency coordination incentive, as in that case we are back to a single, unique steady state – the perfectly symmetric one. How can this be? When coordination incentives are strong enough, the strategic incentives related to funding probabilities become less important, so that the game looks very much like the simple exogenous coordination game. And this, we have

Figure 4: Steady-States – varying κ



already proven, is game which has a unique — and symmetric — equilibrium.

Taken together, Panels (a) and (b) of the figure show that the existence of coordinated steady states rely on qualitative balance between currency coordination incentives and concerns about asset availability, a balance that is achieved at intermediate values of κ. In the quantitative section of this paper, we show that realistic calibrations of the economy exhibit precisely this sort of balance. One experiment that we have not shown here is that of changing overall bond supply. It turns out that changing bond supply is almost isomorphic to changing κ, because reducing or increasing bond supply has the effect of mitigating or amplifying the importance of asset availability, and it is the only the relative importance of these forces that is crucial to our results.

Coordinated Steady-State Characteristics

We have demonstrated that there are two types of potential steady-states – one type in which trade is coordinated on using a single currency and a symmetric type where both currencies are used equally. In this section, we will focus attention on the dollar-centric steady-state where the US is running a negative NFA position and preview some of its key characteristics. This steady state best describes the data, and we will show more detailed analysis of the rest in the quantitative section. The dollar coordinated steady-state has a number of key features, but it all starts with the fact that most firms in both countries are using dollars as their funding currency, and hence most of the trade is intermediated in

dollars. This is also a good representation of the actual invoice currency distribution we see in the data (e.g. Gopinath, 2015).

To assess the implications of this steady-state, recall the return equalization implies,

$$\frac{Q^{\$}}{Q^{€}} = \frac{1-\Delta^{€}}{1-\Delta^{\$}}.$$

Thus, a higher dollar liquidity premium ($\Delta^{\$}$) leads to a high price of the US bonds and a lower effective interest rate ($\frac{1}{Q^{\$}}$) paid by the US. Since the EU bonds lack the liquidity premium of the US bonds, they sell at a lower price and pay higher interest. Rearranging, this return differential is given by,

$$\frac{1}{Q^{€}} - \frac{1}{Q^{\$}} = \frac{\Delta^{\$} - \Delta^{€}}{\beta}$$

This difference in the effective interest rates of the two assets gives rise to a transfer of real resources from the EU economy to the US economy each period, and this transfer is the reason why such excess returns have been referred to as an "exorbitant privilege".[13] We can see that most clearly by expressing the trade balance of the US as the negative of the capital account, to obtain:

$$\underbrace{B^{\$}_{eu}(1+R^{\$}) - B^{€}_{us}(1+R^{€})}_{\text{US Financial Account Deficit}} = \text{ US Trade Balance}$$

Since the US bonds pay a lower interest rate, the US incurs lower expenditures on its foreign debts, and hence could potentially sustain a negative trade balance indefinitely. The actual numbers depend on the parameterization, but the bigger is the interest rate differential at steady-state, the larger the potential deficit that is sustainable indefinitely. Essentially, in this coordinated steady-state, the EU households have strong incentives to acquire US bonds, and hence they are willing to transfer resources every period to the US economy in the form of a US trade deficit. In this economy, persistent trade deficits are not a sign of economic stress or potential balance of payments issues, but in fact a sign of the dominant position that the domestic currency enjoys in the international financial system.

As a result of the ability of the US to sustain permanent trade deficits, the US households enjoy higher permanent incomes and hence have a higher consumption, lower work effort and

[13]Note that this implies Uncovered Interest Parity (UIP) is violated on average. In fact, in the model UIP is violated both unconditionally and conditionally as in the data, but this is not the focus of this paper.

overall higher welfare than the EU households. In conclusion, the coordinated steady-states are ones where the country with the dominant currency enjoys (i) high asset prices, (ii) excess returns on its foreign assets, (iii) a permanent trade deficit and (iv) higher consumption and welfare.

5 Quantifying the Mechanism

In this section, we calibrate our model in order to examine its quantitative implications for both steady-state and dynamic transition. We fix a set of parameters to standard values, then use the remaining parameters for which we have relatively weak priors to target five steady-state moments, computed within the US-dominant steady-state. The model is able to exactly replicate our target moments. We then explore implications both for other potential (non-targeted) steady-states, as well as the potential for dynamic transitions to these alternative steady-states given unexpected shocks to the economy.

Steady-State

The set of exogenously-fixed parameters is listed in Table 1. These values are either standard in the literature, or fixed at values that favor symmetric outcomes in the economy. To highlight a few, we set the household's consumption preference parameters such that there is no bias in consumption originating from preferences – i.e. we set $\alpha_j^{us} = \alpha_j^{eu} = 0.5$.[14] We pick an elasticity of substitution between domestic and foreign goods equal to 4, in line with micro level estimates in the trade literature. Our monthly discount factor of $\beta = .997$ is consistent with an annual risk-free rate of 3.6%. We also assume log preferences ($\gamma = 1$), and a labor share of $\alpha_l = 0.66$. The relatively low value for the trade matching function parameter, $\varepsilon_T = 0.20$, ensures that roughly 90% of funded firms find a trading partner. Finally, our choices of $\sigma_\theta^2 = 1e^{-5}$ and $\sigma_\varepsilon^2 = 1e^{-8}$ put us near to our limiting result, and approach the boundary of what is numerically feasible.

Table 2 lays out the set of moments that we target using the remaining five parameters. These moments are selected to capture the following features of US data. (1) gross safe assets that are somewhat less than half of US gross domestic product; (2) a substantial net foreign liability within the safe asset class, i.e. more US governments bonds held by foreigners than US entities holdings of foreign bonds; (3) a trade share of 60%; (4) an excess return on safe

[14] In this case, any real exchange rate movements must come from the frictions in trade markets, either from law of one price deviations or because equilibrium consumption baskets favor cheaper domestic goods.

Table 1: Exogenously Fixed Parameters

Parameter	Concept	Value
$A_{us} = A_{eu}$	Productivity	1.00
$a_c^{us} = a_c^{eu}$	Pref. for domestic vs foreign goods	0.50
η	Elasticity of substitution between consumption goods	4.00
α_l	Labor share in production	0.66
α	Exporters bargaining parameter	0.50
β	Time preference	0.997
γ	Risk Aversion	1.00
ν	Labor elasticity parameter	2.00
ε_T	Elasticity of trade matching function	0.20
μ	Country size	0.50
$\phi_{us,g} = \phi_{eu,g}$	Government spending	0
σ_θ^2	Firm's prior variance of θ_t	1e-5
σ_ε^2	Variance of noise in signals	1e-8

assets of 1.75%; and (5) a cost of currency mismatch that is a small fraction of the overall transaction made by trading firms.

Mappings from our model to data are subtle, and our choice of target moments reflect several important judgments Moments (1) and (2) roughly correspond to the aggregate US balance sheet positions in safe government debt in 2006, a benchmark year prior to the financial crisis (data from the IMF). In the model, all assets are safe and have the potential to serve a liquidity function in trade. In reality, however, many assets are risky, and not all assets have the potential to serve a liquidity role. In order to ensure a conservative calibration of our model, we have decided to focus only on data on the US's holdings of safe government debt, which presumes that only a relatively narrow class of assets could serve the liquidity purpose envisioned by our model.[15]

Our target for a trade share of 60% corresponds to the overall *world trade share* according to the IMF tables (average over the 2000s before the financial crisis). In reality, the trade share of the US and the EU, which we have labeled as the two countries in our model, are smaller. However, while for tractability purposes we model a two-country economy, in reality the use of dollars in international trade is not only driven by US and EU trade, but by all countries worldwide. Had we targeted a trade share strictly equal to the average of the observed US and EU trade shares, we would have found the model is better suited to match

[15]Expanding the class of assets that fit the model's definition of safe and liquid would increase the welfare gains we estimate below, as it will increase the absolute size of the exorbitant privilege.

Table 2: Steady-state targets (US-centric steady-state)

Concept	Value
Gross foreign (safe) assets	40% of GDP
Gross foreign (safe) liabilities	50% of GDP
Trade share	60% of GDP
Exorbitant Privilege	1.75% annually
Mismatch cost	1.5% of transaction value

a somewhat smaller privilege, but other results would remain qualitatively similar.

Our target for the exorbitant privilege of 1.75% annually is roughly one-half of the total excess return estimated by Gourinchas and Rey (2007). Those authors find that nearly all of the over three percent privilege they find occurs within asset classes. Since these asset classes are quite broad, however, it is natural to suspect that a substantial portion of observed privilege is compensation for risk rather than the liquidity premia we model here. In the appendix, we show that nearly all of our qualitative results follow if we instead target a premia of 2.5% or 0.75% annually.

Finally, our target for mismatch cost is selected based on our prior that firms should perceive only a modest loss in the case that they match with a partner using a different currency. While this cost is certainly higher than the explicit margins faced in the markets for liquid currencies, we believe actual firm behavior suggests they perceive non-trivial implicit costs of currency mismatch, potentially including the cost of managing liquidity to avoid delays in transactions, exchange rate risk, and the cost of incorporating denomination choice when negotiating transactions.

We target these five moments with the five remaining free parameters. Listed in Table 3, these parameters are κ, the cost of currency mismatch, ϕ, the fixed cost of entering the trading sector, ε_f, the elasticity of the funding matching function, $\bar{B}$, the total supply of bonds in each country, and r, the fee paid by funded trading firms for intra-period loans of bonds. While these parameters are not easy to interpret in isolation, the estimated values imply economically plausible results. Our estimate of ε_f is extremely close to the value of unity we assumed in our simplified analysis. The entry cost, ϕ, implies an equilibrium markup of imports of roughly 30% over production cost in the originating country. And, the estimated cost of funding, r, implies that firms spend less than 5% of the surplus their in trade to reimburse their funding costs.

Using these parameters, we find a total of five steady-states. These steady-states corre-

Table 3: Implied Parameters

Parameter	Concept	Value
κ	Currency mismatch cost	0.0499
ϕ	Fixed cost of entry into trading sector	0.0854
ε_f	Elasticity of funding matching function	1.0785
$\bar{B}^{\$} = \bar{B}^{€}$	Supply of liquid assets	9.1571
r	Funding fees	0.0129

spond qualitatively to those described in Section 4: one symmetric, two distinct steady-states where the dollar emerges as the dominant currency, and two mirror image steady-states where the euro emerges as the dominant currency. Of these, all but the symmetric steady-state are dynamically stable.[16] Below we discuss the characteristics of the symmetric steady-state and the two dollar-centric steady-states, with the understanding that the euro-centric steady-states are mirrors of those.

Like its simplified analogue, our quantitative model delivers two dollar-coordinated steady-states. The key difference between the two dollar-coordinated steady-states is in the overall net foreign asset position of the US, and otherwise the two dollar-coordinated steady-states are very similar in terms of the extent of dollar use in trade and the excess return on foreign assets. One of these steady-states is such that the US has a steady-state negative Net Foreign Asset (NFA) position, and in the other one it has a positive NFA position. Hence we will refer to the first one as the dollar coordinated steady-state where the US is "poor", in the sense of having low stock of assets and a net debtor positions, and the latter as the one where the US is relatively "rich", and has a net creditor positions. The key implied moments of all three types of steady-states are listed in Table 4.

Comparing the two dollar coordinated steady-states, we see that the main difference comes in the stock of foreign bonds held by the US household. In the low NFA steady-state, the US portfolio displays a non-trivial home bias of 0.12 which reflects the fact that it has relatively few foreign assets, while the EU households have significant US asset holdings.[17] The EU households have relatively large holdings of US bonds because those are needed to

[16] We discuss the stability of the steady-states in our analysis of the dynamic properties of the model in the next section.

[17] To measure the portfolio home bias we use the standard index

$$\text{Home Bias} = 1 - \frac{\text{Share of Foreign Bonds in HH Portfolio}}{\text{Share of Foreign Bonds in World Supply}}.$$

Table 4: Steady-state Summary

Moments	USD Coord., low NFA	USD Coord., high NFA	Symmetric
Consumption	0.821	0.826	0.820
Trade Balance	-0.50%	-1.22%	0.00%
$400(r^{\$} - r^{€})$	1.75%	1.75%	0.00%
Implied revenue/GDP	0.85%	0.86%	0.00%
NFA	-10.0%	10.0%	0.00%
Gross Foreign Assets ($B^{€}_{us}$)	40.0%	60.0%	50.0%
Gross Foreign Liab. ($B^{\$}_{eu}$)	50.0%	50.0%	50.0%
Home Bias	0.12	-0.09	0.00
X_{us}	0.987	0.980	0.500
X_{eu}	0.981	0.987	0.500

fund their domestic trading firms which have coordinated on using dollars as their primary source of funding. As a result, in this steady-state, the holdings of US assets is relatively equally distributed across the world, with both US and EU households holding about half of the available supply of US bonds each. This relatively equal distribution of US assets plays a crucial role in supporting the coordination on the dollar as the main medium of international exchange, as it ensures that it is a viable funding source for firms in both countries. If firms in one of the countries were not able to acquire US collateral sufficiently easily, then the coordination on the dollar would not be possible.

Though dollar funding dominates, not every firm seeks dollar funding – as we can see from the last two rows of the table, 1% of US firms and 2% of EU firms operate in euros. While both of these are small numbers, the difference is large in percentage terms, with twice as many EU firms using euros relative to US firms. As a result, there is a relatively bigger demand for euros in the EU, and hence in steady-state most of the EU assets are being held by the EU household. In this way, we arrive at a steady-state where the US is significantly indebted to the rest of the world, because the US has relatively little incentive to own foreign assets while US assets are highly sought after in international markets.

Crucially, the US is able to sustain a negative NFA position in steady-state thanks to the excess return it earns on its foreign assets relative to its foreign liabilities. This excess return comes about as a result of the liquidity premium that the US bonds earn relative

to EU bonds, because firms across the world have coordinated on using dollars as their main funding source. Our targeted 1.75% excess return is able to generate a steady-state trade deficit of almost 0.5% of GDP. This is a substantial figure, and represents a significant transfer of resources from the rest of the world to the US.

This steady-state arguably captures many features of international financial positions as found in the data. The model was designed to generate an excess return on US foreign assets, but beyond that, it also implies that the US could sustain both a persistent trade deficit and a significantly negative NFA position, as we have seen happen over the last few decades. Moreover, this steady-state implies not only large gross capital flows across countries, but also features a non-trivial amount of portfolio home bias. While the table only lists the US positions, the EU households are also home biased, despite their relatively large holdings of dollar assets. The reason that EU households also display home bias is the low US holdings of foreign assets, which leaves EU households holding most of the supply of EU safe assets.

It is interesting to also briefly consider how the high NFA dollar coordinated equilibrium differs from the one we just discussed. As we can see, that equilibrium features the same excess return on euro assets over dollar assets, which is underpinned by the same liquidity premium differential. The main difference is in the gross foreign holdings of the US household, and in the relative use of euros across both countries. In this steady-state, the US households are "rich" and have large EU bond holdings, which drive a significantly positive NFA position.

This second coordinated steady-state is also sustainable because, just as in the low NFA steady-state, dollar funding is relatively available in the EU, which is now the country with relatively small share of total assets and therefore plays the role of "leader" in the currency coordination game. Because of this, euro use and distribution are the mirror image of the other steady-state: euros are now relatively more heavily used in the US than in the EU, which leads to a relatively large holding of EU assets by US households. Thus, in this steady-state, the US households are relatively rich and have a substantial positive NFA position.

The final column of the table describes the symmetric (multipolar) steady-state. This is a steady-state where both assets are perfectly symmetrically distributed across the world, hence both households' portfolios are perfectly diversified and feature no home bias. Moreover, firms use both currencies equally in their trading activities, hence there is no exorbitant privilege, and no excess returns on either asset.

Table 5: Welfare Comparison

steady-State	Consumption equivalent
Dollar Coordination, high US NFA	0.98%
Dollar Coordination, low US NFA	0.25%
Symmetric	-
Euro Coordination, high US NFA	-0.76%
Euro Coordination, low US NFA	-1.51%

Implications for Welfare

Clearly the different steady-states deliver different levels of welfare for the two countries. To quantify these, we compute the corresponding compensating consumption differential, as a fraction of the symmetric steady-state consumption. We do this analysis from the perspective of the US household, but the table can also be read in reverse for the EU household's perspective.

Welfare comparisons are presented in Table 5, all stated using the symmetric steady-state as a baseline Unsurprisingly, the best steady-state for the US household is the one where both the USD is the dominant currency *and* the US households are rich, hence the US has a positive NFA position. In that case, the US households are quite wealthy and their welfare improvement over the symmetric steady-state is equal to 0.98% of steady-state consumption. While this represents a very large effect, arguably we have not seen this steady-state in the data. On the other hand, the dollar coordinated steady-state where the US runs a negative NFA position appears to be quite empirically relevant, and here the US also enjoys welfare gain of 0.25% of steady-state consumption. This remains a significant number, and shows the large potential importance of the exorbitant privilege.

Perhaps the most striking results of Table 5, however, is the asymmetry between relatively small gains earned by the country issuing a dominant currency compared to the loss faced by the country that does not issue the dominant currency. The gap between the best case and worst case steady-states is 2.5%, but the loss from being the non-dominant currency is roughly 0.50% of permanent consumption larger than the gains from dominance.

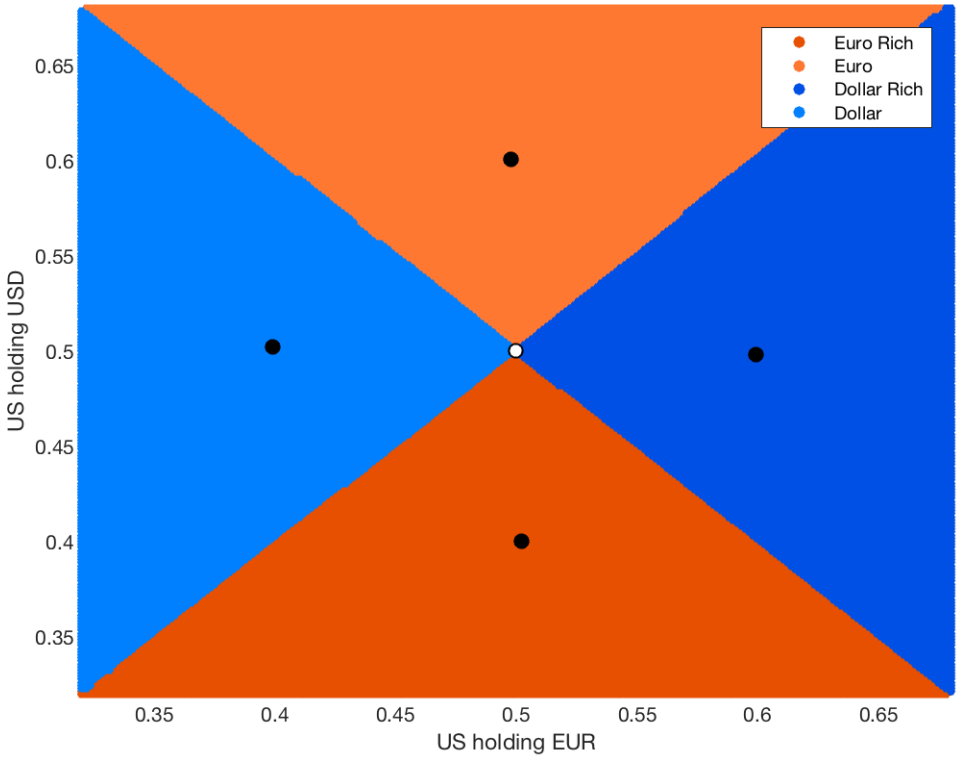
The asymmetry indicates a surprising result from our quantitative model: coordinated steady-states are inefficient. To see how this can be, recall that while mismatched currency pairings entail a deadweight loss, additional deadweight losses occur whenever a trading firm fails to find funding. But such occurrences are much more common in a coordinated steady-

state because trading firms are seeking nearly all their funding from a single asset (e.g. the dollar) effectively wasting the liquidity services that the alternative asset (e.g. the euro) could have provided in a symmetric steady-state. This asymmetry is thus a consequence of particular type of congestion externality that, in a manner akin to the literature of labor search, leads to inefficient outcomes in matching markets.

Dynamics

A natural question for our environment is what could lead to changes in the world currency regime. To better understand which potential switches are most likely and why, we now turn to analyzing the dynamic properties of our economy. For now, we focus on the perfect foresight dynamics and transition paths.[18]

Figure 5: Steady-State Attraction Regions



In Figure 5 we plot the attraction regions of all five steady-states. We construct the figure by starting the dynamic economy from different initial points along a fine grid in this

[18]The dynamic model is highly non-linear, so characterizing dynamics requires a global solution. We solve the model using a parameterized expectations approach. We parameterize expectations in our procedure using multilinear projection over a very fine grid of bond holdings.

two-dimensional space, and simulating for many periods. We then color code each initial bond holding position according to the steady-state to which it converges. This gives us a picture of the overall attraction regions for the steady-states. The solid black dots within each region correspond to the steady-state of their respective regions, while the open dot (center) corresponds to the unstable symmetric steady-state.

The figure shows that all four asymmetric steady-states are stable and have non-trivial attraction regions.[19] Thus, if we know that the economy is close to one steady-state at time t, we would expect that it stays in that neighborhood for a significant amount of time. Small shocks are not enough to cause a switch from one steady-state to another. The stability of the steady-states is due to the endogenous response of the liquidity premia $\Delta_j^{\$}$ and $\Delta_j^{€}$. In response to small shocks, those liquidity premia adjust in such a way as to incentivize the households to choose bond positions that will bring the economy back to the local steady-state. We explain this process through an example below.

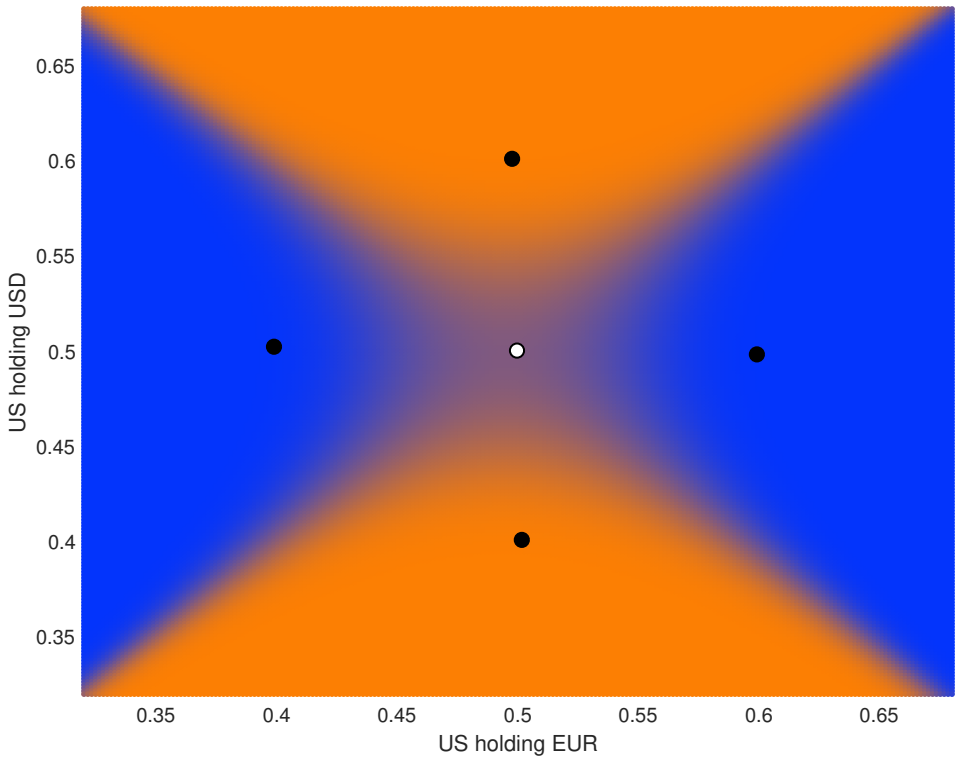
Figure 6 summarizes world average currency choices at each point in the state space. Darker blue indicates more use of dollars, while darker orange indicates more use of euros. The blue and orange regions generally follow the same patterns seen in Figure 5: points that converge to a dollar steady-state already exhibit relatively high dollar usage today. The relatively narrow band of purple, which corresponds to a mixed currency choice, shows that currency choice in the economy can change quickly whenever states are near the boundary of the attraction regions we computed in Figure 5.

To illustrate potential switches, and the likely transition paths, Figure 7 plots the transition paths from two points that are close to boundary of the attraction regions for the Dollar-USpoor and the Euro-EUpoor steady-states. In particular, we choose one point that is just inside the attraction region of the dollar-dominant steady-state where the US is a net debtor, and another point that is just inside the attraction region of the euro-dominant steady-state where the US is a net creditor (hence the EU is a net debtor). In the top left panel, we show the transition paths after just a few periods, and then proceeds clockwise, by adding more and more periods to the transition paths.

The top left panel of the figure show that, though the starting points are almost indistinguishable, equilibrium paths diverge almost immediately. If the economy is within the dollar-dominant attraction region, the US household decumulates EU assets, initially trading them for more US assets. At this initial point, however, the US is (just barely) the

[19] This is also confirmed by solving the linear approximations to the model at each steady-state, and then computing the implied eigenvalues of the linear system.

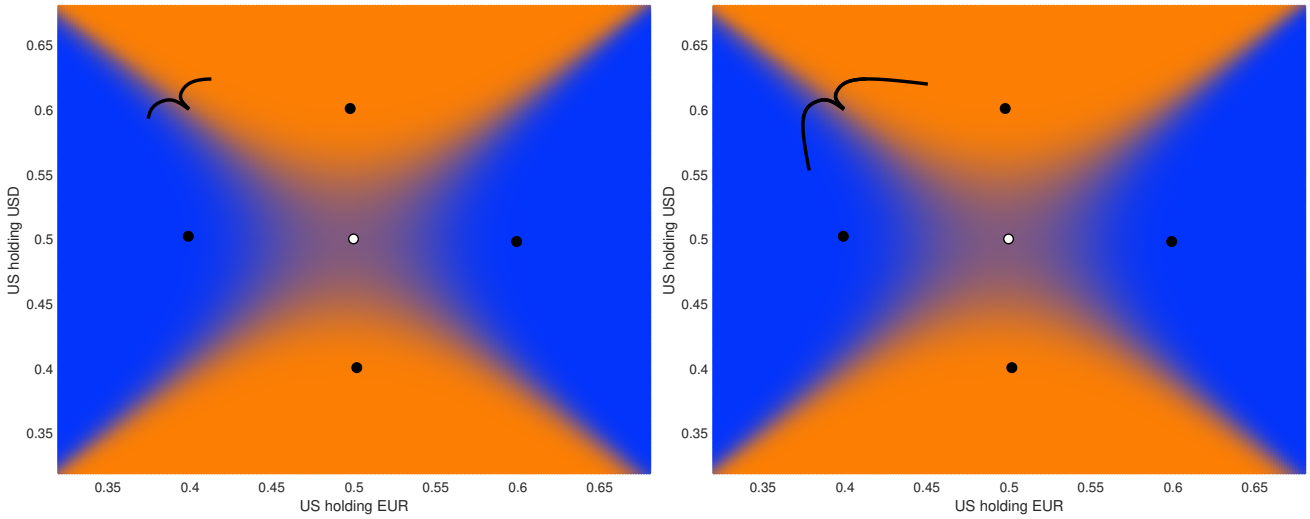
Figure 6: Dollar Usage



relatively asset-poor country, so that the relative abundance of dollars pushes its domestic trading sector towards dollars. This initial move towards dollars also pushes EU firms towards dollars, increasing EU household's desire to hold these assets. Eventually, due to this increased European demand, the US starts to decumulates both EU and US assets, and the higher availability of dollar assets abroad reinforces the European firm's desire to seek dollar funding. As the dollar becomes ever more dominant as the world medium of exchange, the process accelerates, pushing the economy towards the left-most (dollar dominant) steady-state. In the case of the path converging to the euro-dominant steady-state, the logic precedes identically, only with the EU and US roles reversed.

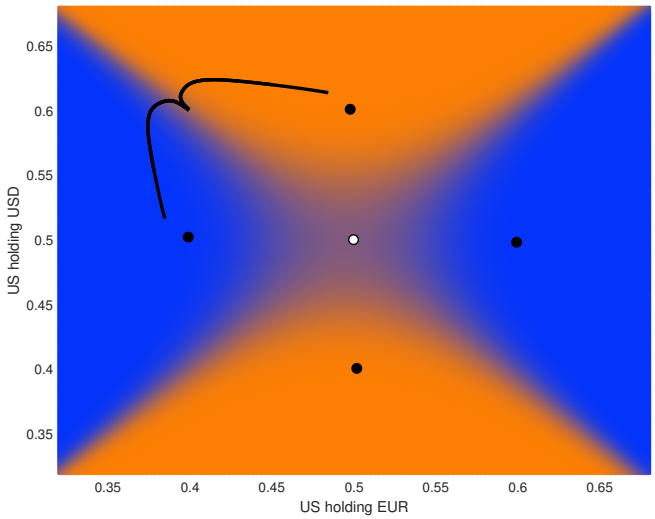
Figure 7 illustrates a crucial insight of our model: small initial differences can lead to permanent (or long-lived) changes in both the dominant currency and corresponding asset positions of countries. Figure 8 plots the transition of several key identical variables from these nearly identical starting points over time. In the figure, the x-axis gives years passed since initializing the states, which underscores that the full transition could be quite slow.

The top left panel of the figure shows that the transition form a non-coordinated (multipolar) currency paradigm to coordinated paradigm is complete within about 10 years. In

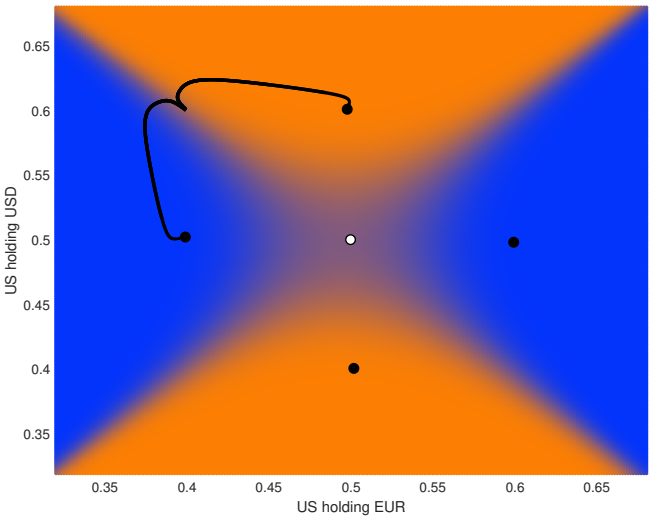


(a)

(b)



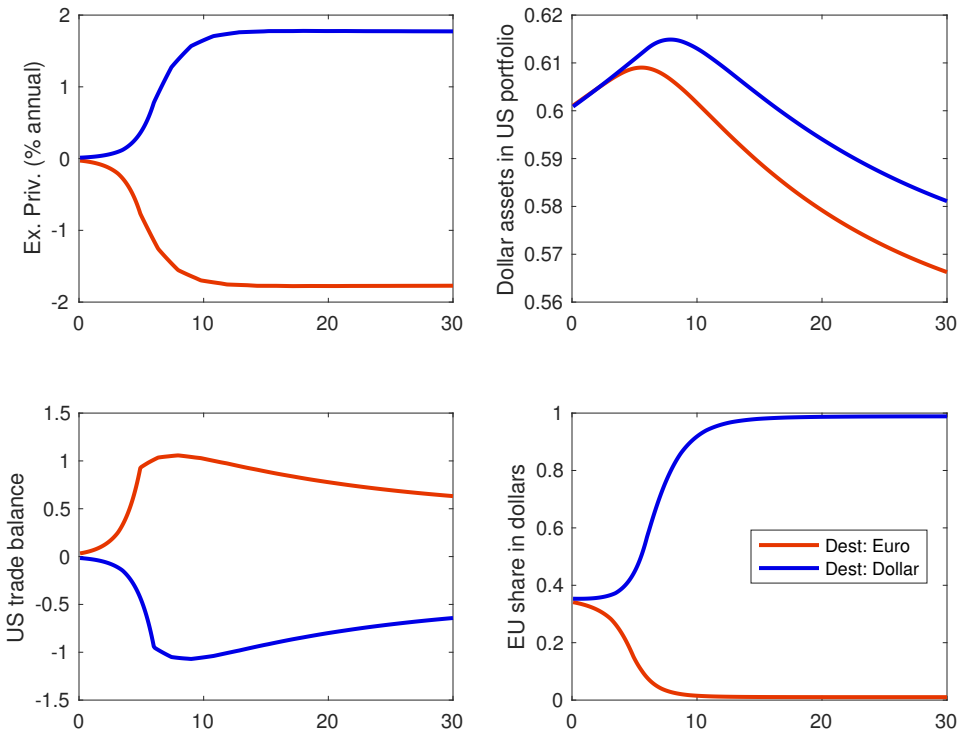
(c)



(d)

Figure 7: Transition Paths

Figure 8: Transition Paths

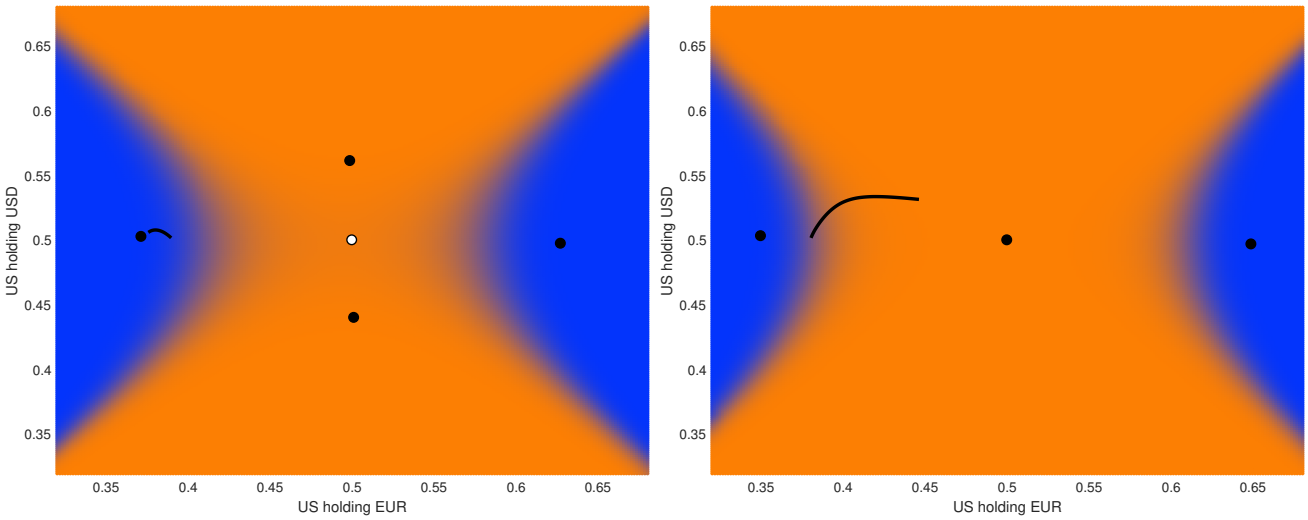


the first five of those years, changes in currency usage are modest (bottom right panel). Change then accelerates quickly between years 5 and 10. Dollar bonds holdings of the US peak in the middle of this acceleration period, then fall gradually to a new lower level, with a permanent negative trade balance that is substantially larger in the short run relative to the long run.

Origins of Transitions

A natural question for our environment is what sort of changes in the economy — and of what size — might lead to switch from one steady-state to another? Figure 9 provides one example of a change in the economy that could potentially lead to such a shift. Panel (a) of the figure depicts an exogenous (unanticipated) 2.5% increase in the stock of EU bonds being supplied to the world economy, while Panel (b) depicts a change of twice that magnitude.

Panel (a) plots the dynamic paths of the portfolio compositions over the period. Because the shock is relatively small, it is not large enough to overcome the initial bias of international portfolios that favors the dollar-coordinated equilibrium. Thus, the economy moves gradually towards a new dollar-dominated steady-state, which has moved even *further* towards the left



(a) Increase EU asset supply by 2.5%

(b) Increase EU asset supply by 5.0%

Figure 9: Change in bond supply

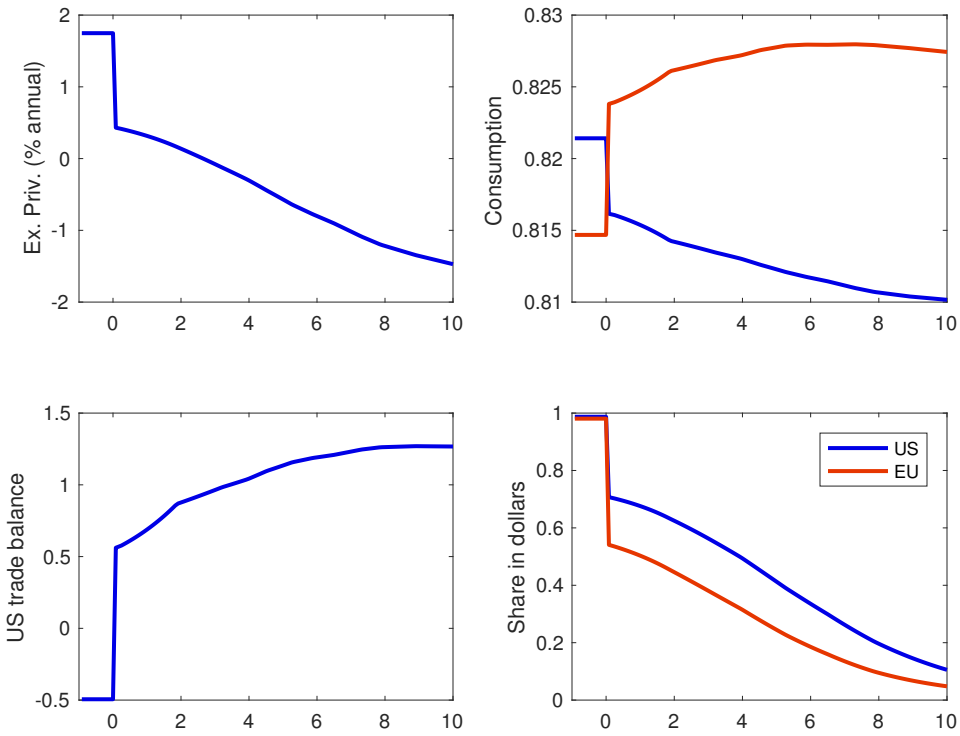
of the figure, indicating an even larger home bias in the US portfolio. In this case, the dollar survives as the medium of exchange in international trade and a subsequent increase in the EU bond supply will lead a similar response, with persistent dollar dominance an even larger bias in the portfolio allocations.

Panel (b) plots the dynamics in the case of a larger initial change in EU bond supply. In this case, given initial asset holdings, the increase in EU bonds *is* enough to convince many firms to switch their funding choice. This creates an incentive for American households to accumulate EU assets (rather than decumulate) and the economy transitions to the right towards a euro-dominate equilibrium.

Figure 10 plot response of several endogenous variables to this change, and shows that interest rate differential between EU and US bonds falls on impact, but then continues to fall over time as the economy converges to the new equilibrium. Transition to the new dominant currency is again complete in roughly 10 years.

One insight highlighted by the results in Figures 9 - 10 is that both the size and the *timing* of changes may be crucial for whether economy experiences a regime shift in the dominant currency. For, in the case of the incremental change depicted in panel (a), state variables are given time to adjust towards the dollar-dominated steady-state before further shocks perturb that equilibrium. In the case of a large change, depicted in Panel (b), the shift is large for the economy to cross the attraction threshold, so that it immediately begins

Figure 10: Transition Paths



to converge to the new euro-dominated steady-state.

6 Conclusions

This paper proposes a feedback mechanism between household portfolio choices and the funding choices made by firms engaged in trade. An asset that is widely used for liquidity purposes earns an excess return. The liquidity premium is higher in countries where the asset is relatively scarce, leading it to be relatively more broadly-held internationally. Conversely, a broadly-held asset is more likely to be chosen by trading firms to perform the liquidity function. Hence, a multiplicity of steady-states emerges corresponding to different dominant currencies and different long-run asset positions. A country issuing the dominant asset benefits from lower interest rates on its foreign asset position, corresponding to the liquidity premium earned by the assets it issues. Such a privilege can persist indefinitely, but it is not unconditional: changes in the environment or policy may lead the dominant country to lose it privilege, potentially falling into a long-lasting period of low wealth and low returns on its foreign asset position.

A Proof of Propositions

Proof of Proposition 1. Define $\alpha \equiv \sigma_\theta^{-2}$ and $\beta \equiv \sigma_\varepsilon^{-2}$. Solving (29) for $\bar{\theta}_{us}$ we get

$$\bar{\theta}_{us} = \frac{\alpha}{\alpha+\beta}\bar{\theta}_{eu} + \frac{1}{\gamma}\Phi^{-1}(\frac{\bar{\theta}_{eu} - (M_0^{eu} - M_1^{eu})}{M_1^{eu}})$$

So we can combine the two equilibrium conditions as

$$\frac{\alpha}{\alpha+\beta}\bar{\theta}_{eu} + \frac{1}{\gamma}\Phi^{-1}(\frac{\bar{\theta}_{eu} - (M_0^{eu} - M_1^{eu})}{M_1^{eu}}) = M_0^{us} - M_1^{us} + M_1^{us}\Phi\left(\gamma\left(\bar{\theta}_{eu} - \frac{\beta}{\alpha+\beta}\bar{\theta}_{us}\right)\right)$$

Further substituting $\bar{\theta}_{us}$ out of the RHS, we have

$$\frac{\alpha}{\alpha+\beta}\bar{\theta}_{eu} + \frac{1}{\gamma}\Phi^{-1}(\frac{\bar{\theta}_{eu} - (M_0^{eu} - M_1^{eu})}{M_1^{eu}}) = M_0^{us} - M_1^{us} + M_1^{us}\Phi\left(\gamma\frac{\alpha^2+2\alpha\beta}{(\alpha+\beta)^2}\bar{\theta}_{eu} - \frac{\beta}{\alpha+\beta}\Phi^{-1}(\frac{\bar{\theta}_{eu} - (M_0^{eu} - M_1^{eu})}{M_1^{eu}})\right)$$

The LHS is an increasing function of $\bar{\theta}_{eu}$ that asymptotes to $-\infty$ as $\bar{\theta}_{eu} \to M_0^{eu} - M_1^{eu}$, and to ∞ as $\bar{\theta}_{eu} \to M_0^{eu}$. The RHS is a function bounded above by M_0^{us} and below by $M_0^{us} - M_1^{us}$. Thus an equilibrium always exists.

A sufficient condition for the uniqueness of the equilibrium is that the RHS is a decreasing function of $\bar{\theta}_{eu}$. This is true if the argument of the normal cdf Φ is decreasing. Taking a derivative of that expression in respect to $\bar{\theta}_{eu}$ we obtain

$$\begin{aligned}
\gamma\frac{\alpha^2+2\alpha\beta}{(\alpha+\beta)^2} - \frac{\beta}{\alpha+\beta}\frac{1}{\phi(\Phi^{-1}(\frac{\bar{\theta}_{eu}-(M_0^{eu}-M_1^{eu})}{M_1^{eu}}))}\frac{1}{M_1^{eu}} &< \gamma\frac{\alpha^2+2\alpha\beta}{(\alpha+\beta)^2} - \frac{\beta}{\alpha+\beta}\frac{1}{\phi(0)}\frac{1}{M_1^{eu}} \\
&= \gamma\frac{\alpha^2+2\alpha\beta}{(\alpha+\beta)^2} - \frac{\beta}{\alpha+\beta}\frac{\sqrt{2\pi}}{M_1^{eu}} \\
&= \sqrt{\frac{\beta}{(\alpha+\beta)(\alpha+2\beta)}}(\alpha^2+2\alpha\beta) - \frac{\beta}{\alpha+\beta}\frac{\sqrt{2\pi}}{M_1^{eu}}
\end{aligned}$$

This expression is negative if and only if

$$M_1^{eu} < \frac{\sqrt{2\pi(\alpha+2\beta)(\alpha+\beta)\beta}}{\alpha^2+2\alpha\beta}$$

or using the definition of M_1^{eu}

$$\kappa < \frac{1}{M_{\$}^{eu} + M_{€}^{eu}} \frac{\sqrt{2\pi(\alpha+2\beta)(\alpha+\beta)\beta}}{\alpha^2 + 2\alpha\beta} \tag{31}$$

If we had used other substitutions, we could have arrived at a similar condition in terms of $M_{\$}^{us} + M_{€}^{us}$ instead:

$$\kappa < \frac{1}{M_{\$}^{us} + M_{€}^{us}} \frac{\sqrt{2\pi(\alpha+2\beta)(\alpha+\beta)\beta}}{\alpha^2 + 2\alpha\beta}$$

either one is a sufficient condition for a unique equilibrium. Both imply that for any $\kappa > 0$, there exists a β high enough such that one of the above inequalities is satisfied, and hence the equilibrium is necessarily unique. □

Proof of Proposition 2. Take the limit of the RHS of (31) as $\beta \to \infty$ (keeping $\alpha = c\beta^x$):

$$\begin{aligned}
\lim_{\beta\to\infty} \frac{\sqrt{2\pi(\alpha+2\beta)(\alpha+\beta)\beta}}{\alpha^2 + 2\alpha\beta} &= \lim_{\beta\to\infty} \frac{\sqrt{2\pi(c\beta^x+2\beta)(c\beta^x+\beta)\beta}}{c^2\beta^{2x} + 2c\beta^{1+x}} \\
&= \lim_{\beta\to\infty} \frac{\sqrt{2\pi(c\beta^{x-1}+2)(c\beta^{x-1}+1)}}{c^2\beta^{2x-\frac{3}{2}} + 2c\beta^{x-\frac{1}{2}}} \\
&= \infty
\end{aligned}$$

Thus, in the limit equilibrium is unique for any finite value of κ. □

B Calibration Robustness

We also calibrated the model to match the same target moments, save for different values of the exorbitant privilege earned by US assets. The tables below show that (1) estimated parameters are almost unchanged across targets, except for the cost of funding which is higher the higher is level privilege that we target; and (2) the qualitative features of the economy are virtually unchanged, except for the size of the revenue stream (and associated trade balance) that accompanies ownership of the dominant currency.

Table 6: Implied Parameters - Low EP Target

Parameter	Concept	Value
κ	Currency mismatch cost	0.0495
ϕ	Fixed cost of entry into trading sector	0.0924
ε_f	Elasticity of funding matching function	1.0631
$\bar{B}^{\$} = \bar{B}^{€}$	Supply of liquid assets	9.126
r	Funding fees	0.00548

Table 7: Implied Parameters - High EP Target

Parameter	Concept	Value
κ	Currency mismatch cost	0.0502
ϕ	Fixed cost of entry into trading sector	0.0800
ε_f	Elasticity of funding matching function	1.0938
$\bar{B}^{\$} = \bar{B}^{€}$	Supply of liquid assets	9.181
r	Funding fees	0.01848

Table 8: Steady-state Summary - Low EP target

Moments	USD Coord., low NFA	USD Coord., high NFA	Symmetric
Consumption	0.814	0.820	0.816
Trade Balance	-0.00%	-0.73%	0.00%
$400(r^{\$} - r^{€})$	0.75%	0.75%	0.00%
Implied revenue/GDP	0.36%	0.37%	0.00%
NFA	-10.0%	10.0%	0.00%
Gross Foreign Assets ($B^{€}_{us}$)	40.0%	60.0%	50.0%
Gross Foreign Liab. ($B^{\$}_{eu}$)	50.0%	50.0%	50.0%
Home Bias	0.11	-0.09	0.00
X_{us}	0.987	0.981	0.500
X_{eu}	0.981	0.987	0.500

Table 9: Steady-state Summary - High EP target

Moments	USD Coord., low NFA	USD Coord., high NFA	Symmetric
Consumption	0.827	0.831	0.823
Trade Balance	-0.86%	-1.59%	0.00%
$400(r^{\$} - r^{€})$	2.50%	2.49%	0.00%
Implied revenue/GDP	1.22%	1.24%	0.00%
NFA	-10.0%	10.0%	0.00%
Gross Foreign Assets ($B^{€}_{us}$)	40.0%	60.0%	50.0%
Gross Foreign Liab. ($B^{\$}_{eu}$)	50.0%	50.0%	50.0%
Home Bias	0.11	-0.09	0.00
X_{us}	0.987	0.980	0.500
X_{eu}	0.980	0.987	0.500

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