Use It Or Lose It: Efficiency Gains from Wealth Taxation*

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Abstract

This paper studies the quantitative implications of wealth taxation (tax on the *stock* of wealth) as opposed to capital income taxation (tax on the income *flow* from capital) in an overlapping-generations incomplete-markets model with rate of return heterogeneity across individuals. With such heterogeneity, capital income and wealth taxes have opposite implications for efficiency and some key distributional outcomes. Under capital income taxation, entrepreneurs who are more productive, and therefore generate more income, pay higher taxes. Under wealth taxation, on the other hand, entrepreneurs who have similar wealth levels pay similar taxes regardless of their productivity, which expands the base and shifts the tax burden toward unproductive entrepreneurs. This reallocation increases aggregate productivity and output. In the simulated model calibrated to the US data, a revenue-neutral tax reform that replaces capital income tax with a wealth tax raises welfare by about 8% in consumption-equivalent terms. Moving on to optimal taxation, the optimal wealth tax is positive, yields even larger welfare gains than the tax reform, and is preferable to optimal capital income taxes. Interestingly, optimal wealth taxes result in more even consumption and leisure distributions (despite the wealth distribution becoming more dispersed), which is the opposite of what optimal capital income taxes imply. Consequently, wealth taxes can yield both efficiency *and* distributional gains.

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1 Introduction

In this paper, we revisit the question of optimal capital taxation in an environment with two empirically motivated features. First, because wealth holdings are extremely concentrated in the United States, a small fraction of the population pays most of the capital taxes. For example, in 2010, the top 1% of households ranked by wealth paid 44%, and the top 10% of households paid almost 80%, of capital income taxes. Thus, it is important to account for the concentration of wealth—including at the very top—for a sound quantitative analysis of capital taxation. Therefore, the first feature of the economic model we study is that it aims to reproduce some salient features of the U.S. wealth distribution so as to be suitable for a quantitative analysis of capital taxes.

Second, it seems plausible to conjecture that the mechanism by which this concentration is generated also matters for the analysis of capital taxation. As we show in this paper, this conjecture is correct: different mechanisms generating the same basic facts about inequality nevertheless have very different—and sometimes opposite—implications for the effects of capital taxation. This observation brings us to the second feature of the model: we build on recent theoretical advances and empirical evidence—reviewed in greater detail in the next section—that provide support for the importance of heterogeneity in investment returns for explaining the observed wealth concentration, including the Pareto right tail, which is a salient feature of the wealth distribution in many countries (Vermeulen (2016)). Fagereng et al. (2016) provide evidence for large permanent differences in the rates of return across households in Norway. Using administrative data linking 10 million firms to their owners, Smith et al. (2017) provide evidence for persistent differences in firm profitability among privately owned firms that is tied to the business owner. Both papers show that these large differences remain even after controlling for risk and size. A recent literature has shown that return heterogeneity can generate not only a concentrated wealth distribution, but also a Pareto tail as observed in the data (Benhabib et al. (2011, 2013, 2014)). Moreover, if return heterogeneity is persistent over time, then this class of models also generates behavior consistent with the evolution of inequality over time (Gabaix et al. (2016)).

To be clear, we do not claim that other mechanisms for generating wealth inequality (such as heterogeneity in discount factors) are not important. Rather, we note that despite the fast growing literature on models with return heterogeneity, to our knowledge, the implications of these models for capital taxation have not been studied quantitatively

and thus are not well understood. Against this backdrop, the main contribution of this paper is to fill this gap and provide a quantitative analysis of optimal taxes in this class of models. Specifically, we analyze how a capital income tax differs from a wealth tax.

Before describing our findings in more detail, here is a brief overview of the model. We study an overlapping generations economy inhabited by individuals who derive utility from consumption and leisure. The key ingredient of the model is persistent heterogeneity in investment/entrepreneurial skills, which, together with incomplete financial markets that prevent free flow of funds across agents, allows some individuals to earn persistently higher returns on their wealth than others.[1] Individuals can borrow from others in a bond market to invest in their firm over and above their own saved resources. The same bond market can also be used as a savings device, which will be optimal for individuals whose entrepreneurial skill (and hence private return) is low or have too much wealth or both.

Each individual/entrepreneur produces a differentiated intermediate good using a linear technology and individual-specific productivity levels and these intermediates are combined in a Dixit-Stiglitz aggregator by the final goods producing firm, which pins down each entrepreneur's production scale and profits. In our calibrated economy, most individuals earn the bulk of their income from wages, and only a small fraction (10–20%, depending on the exact definition) of individuals produce large enough output to be considered an entrepreneur/investor. Individuals also face idiosyncratic labor income risk, mortality risk, borrowing constraints in the bond market, and other features, although we show that plausible variations in these additional details do not change the main conclusions of the paper. Finally, we also consider intergenerational links between parents and children through accidental bequests and the transmission of entrepreneurial and labor market ability. These also turn out not to be too important. The calibrated model matches salient features of the wealth distribution in the U.S.—in particular, the degree of wealth concentration in the data as well as the patterns of wealth accumulation over the life cycle for the very rich. Further, the extent of capital misallocation generated in the model is in line with the US data (e.g., as reported in Bils et al. (2017)).

Our analysis produces three sets of results. First, we begin with a revenue-neutral tax reform that replaces the current US tax system of capital income taxation with a

[1] While we model this persistence in a rich fashion, allowing both intergenerational correlation and stochastic evolution over the life cycle, our main substantive results on the desirability of wealth taxes is robust as long as return heterogeneity is fairly persistent.

flat wealth tax, keeping taxes on labor and consumption unchanged. This reform raises average welfare significantly—equivalent to about 7% of consumption (per person per year) for newborn individuals in our baseline calibration. Furthermore, welfare gains are quite evenly distributed across the population—they are not concentrated only among the wealthy.

Second, we move to an optimal tax analysis, in which a utilitarian government chooses linear taxes on labor income and on wealth to maximize the ex ante expected lifetime utility of a newborn. We repeat the same analysis, this time having the government choose linear taxes on labor and capital income, and compare the implications of each optimal tax system to each other as well as to the current US benchmark. The main result from the first experiment is that a positive tax on the stock of wealth is optimal. The tax rate on wealth is relatively high, about 3%, which allows the government to reduce labor income taxes (from about 22.5% down to 14.5%), which are more distorting than wealth taxes in this environment. The combination of reduction in labor taxes and rise in before-tax wages boosts work incentives and further raises output and welfare. Most of this welfare gain comes from increasing efficiency in the allocation of capital toward more productive entrepreneurs, and a relatively modest component comes from further capital accumulation in response to changing incentives provided by wealth taxes.

Turning to the second experiment, we find that a negative tax (or a subsidy) on capital income is optimal and the rate is high: about –35%. This contrasts with some well-known results in similar life cycle models with incomplete markets where a large and positive tax rate on capital income was found to be optimal (c.f. Conesa et al. (2009)). The main difference is the return heterogeneity present in this model, and we verify that eliminating it from our framework restores the positive and large tax rate found in previous work. This result shows that persistent heterogeneity in returns across individuals that generates high wealth inequality has distinct implications for not only wealth taxation but also for the optimal taxation of capital income.

Third, we find that among the two optimal tax systems, the one with wealth taxes yields higher welfare (9.5% of consumption per year for newborns) than the one with capital income taxes (6.5%). A decomposition analysis shows that the gains under wealth taxes come from both a rise in the *level* of consumption (driven by higher after-tax wages) and a decline in the *inequality* of consumption and leisure. Thus, optimal wealth taxes yield both first- and second-order gains. This is not the case with optimal capital income taxes: although they deliver an even larger rise in output, providing capital subsidies

requires higher taxes on labor income, which yields only a smaller rise in *after-tax* wages. Furthermore, subsidies on capital, together with the small rise in consumption levels, leads to higher inequality (both in wealth but also more importantly in consumption and leisure) yielding distributional losses, which offsets some of the gains from levels—unlike under optimal wealth taxes. Overall, we find a series of interesting differences and contrasts between optimal wealth and capital income taxes in this environment.

Finally, we have conducted a large number of sensitivity analyses to gauge the robustness of these conclusions. In particular, we have considered a progressive labor income tax, optimal wealth taxes with an exemption level, looser borrowing constraints, different assumptions about the stochastic process for entrepreneurial ability, various changes in key parameters, among others. While these changes affect the various magnitudes of welfare gains (as could be expected), they do not overturn any of the main substantive conclusions of our analysis.

The rest of the paper is organized as follows. Section 2 starts with a simple one period example to illustrate some key differences between capital income and wealth taxes. Section 3 lays out the full-blown OLG model, and Section 4 describes the parameterization and model fit. Sections 5 and 6 present the quantitative results about the tax reform and optimal taxation, respectively. Section 7 discusses various sensitivity analyses, Section 8 concludes.

Related Literature [Incomplete]

This paper is most closely related to two strands of literature. The first one is the literature on capital taxation when financial markets are incomplete, tax instruments are restricted (in plausible ways), and/or individuals are finitely lived. A number of studies found that it may be desirable to tax capital income and that the rate can be positive and large (Hubbard et al. (1986), Aiyagari (1995), Imrohoroglu (1998), Erosa and Gervais (2002), Garriga (2003), Conesa et al. (2009), Kitao (2010)). The main difference of our analysis is the presence of heterogeneous returns, which was not modeled in this earlier literature, and we show that incorporating it into the analysis alters some key conclusions (e.g., it becomes optimal to subsidize capital income instead of taxing it, wealth taxes work differently—along many dimensions—from capital income taxes, among others).

Turning to wealth taxes, the "use-it-or-lose-it" mechanism has been discussed by some previous authors, although we are not aware of a quantitative analysis of all its effects as done in this paper. Among these, Maurice Allais was probably one of the

best-known proponents of wealth taxes. He observed, for example, that "[a] tax on capital stock represents a bonus to production and penalizes the inefficient owner, passive, for whom income taxes encourage inaction (Allais (1977, p. 501, translated))." More recently, Piketty (2014) has revived the debate on wealth taxation and proposed using a combination of capital income and wealth taxes to balance these efficiency and inequality tradeoffs. Piketty mostly focused on equity considerations, but also described the efficiency gain benefits of the use-it-or-lose-it mechanism without providing a formal analysis.

The second literature that this paper is related to concerns models of wealth inequality, especially those in which inequality is generated through return heterogeneity. Some of the earlier work in this area built micro-founded models in which return heterogeneity resulted from differences in entrepreneurial skills (Quadrini (2000) and Cagetti and De Nardi (2006)) or limited stock market participation (Guvenen (2006)). Regardless of the precise source, both types of models are shown to generate substantial wealth inequality as observed in the US data. A more recent literature has shown that return heterogeneity can generate not only a concentrated wealth distribution but also a Pareto tail as observed in the data (Benhabib et al. (2011, 2013, 2014), Gabaix et al. (2016)).

Several other papers have also used frameworks with entrepreneurial or firm heterogeneity to address different questions. Buera et al. (2011) uses a framework with entrepreneurial heterogeneity, very similar to ours, to explain the aggregate productivity and financial development across countries. In terms of policy analyses, Cagetti and De Nardi (2009) evaluate the effect of eliminating estate taxation and Benhabib et al. (2011) study the effect of capital income and estate taxes on wealth inequality. Neither of these papers however, analyzes the differences between capital income and wealth taxes, nor studies optimal capital taxation as we do in this paper.

2 A Simple One-Period Example

To fix ideas and illustrate some of the key differences between wealth taxes and capital income taxes, we start with a stylized 1-period example. The example is summarized in Table I. Consider two brothers, named Frodo and Michael, who each has \$1000 of wealth at time zero. Frodo has low entrepreneurial skills, and so he earns a rate of return of $r_F = 0\%$ on his investments, whereas Michael is a highly skilled business man, and so he earns a rate of return of $r_M = 20\%$. Both brothers invest all their wealth

in their business and make no other decisions (such as consumption or saving choice). To introduce taxation, suppose that there is a government that needs to finance an expenditure of $G = \$50$ through tax revenues collected at the end of the period.

Now, suppose that the government taxes capital income at a flat rate. To raise \$50, the required tax rate is 25% on income and is paid entirely by Michael, who is the skilled entrepreneur and the only one earning any capital income. Consequently, the after-tax return is 0% for Frodo and 15% for Michael. By the end of the period, Frodo's wealth remained unchanged, whereas Michael experienced an increase from \$1,000 to \$1,150 after paying his taxes.

Next, suppose that the government decided to raise the same revenue with a wealth tax. Now the base of taxation is broader, because Frodo does have wealth and cannot avoid taxation as he did under the capital income tax. Specifically, the tax base covers the entire wealth stock, or \$2200, at the end of the period. The tax rate on wealth is $\$50/\$2,200 \approx 2.27\%$. More importantly, Frodo's tax bill is now \$23, up from zero, whereas Michael's tax bill is cut by almost half, from \$50 before down to \$27. The after-tax rate of return is, respectively, $(\$0 - \$23)/\$1000 \approx -2.3\%$ for Frodo and $(\$200 - \$27)/\$1000 \approx 17.3\%$ for Michael. Notice that the dispersion in after-tax returns is higher under wealth taxes and the end-of-period wealth inequality is also higher: $\$1,173/\$977 \approx 1.20$ versus $\$1,150/\$1,000 = 1.15$ before. Most crucially, the more productive entrepreneur (Michael), ends up with a larger fraction of aggregate wealth: 54.6% vs. 53.5% under capital income taxes.

To sum up, wealth taxation has two main effects that are opposite to capital income taxes. First, by shifting some of the tax burden to the less productive entrepreneur, it allows the more productive one to keep more of his wealth, thereby reallocating the aggregate capital stock towards the more productive agent. Second, wealth taxes do not compress the after-tax return distribution nearly as much as capital income taxes do, which effectively punish the successful entrepreneur and reward the inefficient one. In a (more realistic) dynamic setting, such as the one we study in the next section, this feature will yield an endogenous response in savings rates, further increasing the reallocation of capital to the more productive agent, leading to a rise in productivity and output. At the same time, this reallocation process also increases wealth concentration, which may conflict with distributional goals of the society. So, overall, relative to the capital income tax, wealth tax generates efficiency gains but can lead to distributional losses. As we shall see in the quantitative analysis, however, distributional losses are not

TABLE I – Capital Income Tax vs. Wealth Tax

	Capital Income Tax		**Wealth Tax**	
	$r_1 = 0\%$	$r_2 = 20\%$	$r_1 = 0\%$	$r_2 = 20\%$
Wealth	\$1,000	\$1,000	\$1,000	\$,1000
Pre-tax income	\$0	\$200	\$0	\$200
Tax rate	$\tau_k = \frac{\$50}{\$200} = 0.25$		$\tau_a = \frac{\$50}{\$2{,}200} = 2.27\%$	
Tax liability	\$0	\$50	$\$1,000 \times \tau_a \approx \$23$	$\$1,200 \times \tau_a \approx \$27$
After-tax rate of return	0%	$\frac{\$200-\$50}{\$1,000} = 15\%$	$-\frac{\$23}{\$1,000} = -2.3\%$	$\frac{\$200-\$27}{\$1,000} = 17.3\%$
After-tax wealth ratio	$\frac{W_2}{W_1} = \frac{\$1,150}{\$1,000} = 1.15$		$\frac{W_2}{W_1} = \frac{\$1,173}{\$977} = 1.20$	

a robust feature of wealth taxes and are mitigated or reversed (into gains) when a proper production function is introduced and wage income is added to the model. In that case, wealth taxes yield both efficiency and distributional gains.

Before we conclude this example, an important remark is in order. If this one-period example were to be repeated for many periods, all aggregate wealth—both in the capital income tax and the wealth tax cases—will eventually be owned by the more productive investor, Michael. As it turns out, as long as there are variations in the rates of return, the main arguments in favor of a wealth tax, highlighted in the simple model, remain valid. Variations in the rates of return are realistic features of the data: both over the life cycle (the fortunes of entrepreneurs do fluctuate over time) and from one generation to the next (the entrepreneurial ability of children often differs from that of their parents). Thus, we incorporate these features in the rich dynamic model we consider next.

3 Full OLG Model

We study an economy populated by overlapping generations of finitely-lived individuals, two sectors (producing intermediate-goods and the final good, respectively), and a government that raises revenues through various taxes.

3.1 Individuals

Individuals face mortality risk and can live up to a maximum of H years. Let ϕ_h be the unconditional probability of survival up to age h and let $s_h \equiv \phi_h/\phi_{h-1}$ be the

conditional probability of surviving from age $h-1$ to h. When an individual dies, she is replaced by an offspring that inherits her wealth.

Individuals derive utility from consumption, c, and leisure, ℓ, and maximize expected lifetime utility without any bequests motives (for now):[2]

$$\mathbb{E}_0 \left(\sum_{h=1}^{H} \beta^{h-1} \phi_h u(c_h, \ell_h) \right).$$

Individuals make four decisions every period: (i) leisure time vs. labor supply to the market (until retirement age, $R < H$), (ii) consumption today vs saving for tomorrow, (iii) portfolio choice: how much of his own assets/wealth to invest in his own business versus how much to lend to others in the bond market, and (iv) how much to produce (of an intermediate good) as an entrepreneur. We now describe the endowments of various skills, production, technologies, and the market arrangements, and then spell out each of the four decisions in more detail.

3.2 Skill Endowments and their Evolution

Each individual is endowed with two types of skill: one that determines his productivity in the labor market as a worker and another that determines his productivity in entrepreneurial activities. We now describe these two skills and how they evolve across generations and over the life cycle and how they enter the two activities undertaken by the individuals.

I. Entrepreneurial productivity

Let z_{ih} denote the entrepreneurial productivity of individual i at age h, which has two components: $\overline{z}_i$, which is fixed over the life cycle but changes across generations (inherited from the parent), and a second component that varies stochastically over the life cycle. Specifically, a newborn inherits $\overline{z}_i$ imperfectly from her parent:

$$\log(\overline{z}_i^{\text{child}}) = \rho_z \log(\overline{z}_i^{\text{parent}}) + \varepsilon_{\overline{z}_i},$$

where $\varepsilon_{\overline{z}_i}$ is an i.i.d. normal innovation with mean zero and variance $\sigma^2_{\varepsilon_{\overline{z}}}$. Because $\overline{z}_i$ is imperfectly inherited, some children with low entrepreneurial skills will inherit large

[2] We consider a bequest motive in the robustness analysis later.

amounts of wealth from their successful parent, and vice versa, causing misallocation of productive resources.

Whereas $\overline{z}_i$ captures an individual's more permanent traits, we also want to allow for the fact that these entrepreneurial skills can be augmented with external factors (such as a lucky head-start on a new idea, good health and energy that can allow skills to be fully utilized) or hampered again by factors (such as competitors entering the field, opportunity cost of time rising due to family and other factors, negative health shocks, among others). To allow for these variations, we allow the individual to be in different "phases" of productivity, modeled as a three-state Markov chain that can take on the values high, low, and zero: $\mathbb{I}_{ih} \in \{\mathcal{H}, \mathcal{L}, 0\}$ at h. Together with $\overline{z}_i$ this determines the entrepreneurial productivity of an individual at a given age:

$$z_{ih} = f(\overline{z}_i, \mathbb{I}_{ih}) = \begin{cases} (\overline{z}_i)^{\lambda} & \text{if } \mathbb{I}_{ih} = \mathcal{H} \qquad \text{where } \lambda > 1 \\ \overline{z}_i & \text{if } \mathbb{I}_{ih} = \mathcal{L} \\ 0 & \text{if } \mathbb{I}_{ih} = 0 \end{cases}$$

and transition between these states is governed by the transition matrix:

$$\Pi_z = \begin{bmatrix} 1 - p_1 - p_2 & p_1 & p_2 \\ 0 & 1 - p_2 & p_2 \\ 0 & 0 & 1 \end{bmatrix}.$$

Finally, individuals whose permanent ability is above the median permanent ability—i.e., $\overline{z} > \overline{z}_{med} = 1$—start life in state $\mathbb{I}_{ih} = H$ while the rest start in state $\mathbb{I}_{ih} = L$. Overall, this structure is intended to capture the fact that many individuals who are extremely wealthy go through a very high growth phase especially in the early stages of their business, followed by a slowdown as their business matures or their competitors catch up.

II. Labor market productivity

At a given age individuals differ in their labor market productivity, y_{ih}, which consists of three components

$$\log y_{ih} = \underbrace{\theta_i}_{\text{permanent}} + \underbrace{\kappa_h}_{\text{lifecycle}} + \underbrace{e_{ih}}_{\text{AR(1)}}$$

where θ_i is an individual fixed effect, κ_h is a life-cycle component that is common to all individuals and e_{ih} follows an AR(1) process during working years ($h < R$):

$$e_{ih} = \rho_e e_{i,h-1} + \epsilon_e,$$

where ϵ_e is an i.i.d. shock with mean zero and variance $\sigma^2_{\epsilon_e}$. Individual-specific labor market ability θ is imperfectly inherited from parents:

$$\theta^{child} = \rho_\theta \theta^{parent} + \epsilon_\theta,$$

where ϵ_θ is an i.i.d. Gaussian shock with mean zero and variance $\sigma^2_{\epsilon_\theta}$.

Let $n_{ih} = 1 - \ell_{ih}$ denote the labor hours supplied in the market. Individuals supply their labor services to the final goods producer, so they make up the aggregate labor supply,

$$L = \int_i (y_{ih} n_{ih})\, di, \tag{1}$$

used in the aggregate production function (2) described in a moment. Therefore, for a given market wage rate per efficiency units of labor, w, an individual's labor income is given by $w y_{ih} n_{ih}$.

3.3 Production Technology

I. Final Goods Producer

The final good, Y, is produced according to a Cobb-Douglas technology,

$$\mathcal{Y} = Q^\alpha L^{1-\alpha}, \tag{2}$$

where L is the aggregate labor input defined in (1), and Q is the CES composite of intermediate inputs, x_i:[3]

$$Q = \left(\int_i x_i^\mu di \right)^{1/\mu}. \tag{3}$$

Each x_i is produced by a different individual/entrepreneur in a way that is specified in a moment. The final goods producing sector is competitive, so the profit maximization

[3]To distinguish Q from the unadjusted capital stock $K := \int_i k_i di$, we will often refer to the former as the "quality-adjusted capital stock" since its level depends on the allocation of the capital stock across entrepreneurs (and reflects the extent of misallocation).

problem is:

$$\max_{\{x_i\},L} \left(\int_i x_i^{\mu} di \right)^{\alpha/\mu} L^{1-\alpha} - \int_i p_i x_i di - wL,$$

where p_i is the price of the intermediate good i. The first order optimality conditions yield the inverse demand (price) function for each intermediate input:

$$p_i(x_i) = \alpha x_i^{\mu-1} Q^{\alpha-\mu} L^{1-\alpha},$$

and the wage rate

$$w = (1-\alpha) Q^{\alpha} L^{-\alpha}.$$

II. Intermediate Goods Producers

There is a continuum of intermediate goods, each produced by a different individual/entrepreneur according to linear technology:

$$x_{ih} = z_{ih} k_{ih}$$

where k_{ih} is the final good (consumption/capital) used in production by entrepreneur i and z_{ih} is her stochastic and idiosyncratic entrepreneurial productivity at age h.

3.4 Markets and the Government

Financial markets. There is a bond market where one-period borrowing and lending takes place at a risk-free rate of r. Individuals with sufficiently high entrepreneurial productivity relative to their private assets may choose to borrow in this market to finance their business. Similarly, those with low productivity relative their assets may find it optimal to lend for a risk-free return. Following a large literature, we impose borrowing constraints to capture information frictions or commitment problems, which we do not model explicitly (among others, Cagetti and De Nardi (2006); Buera et al. (2011)). In particular, an individual with asset level a faces a financial constraint

$$k \leq \vartheta(z_{ih}) \times a,$$

where $\vartheta(z_{ih}) \in [1, \infty]$. The (potential) dependence of ϑ on z_{ih} is to allow for the fact that more productive agents could potentially borrow more against their personal assets.[4]

[4] We allow for this possibility to capture the idea that the market could (perhaps partially) observe individuals' productivity level and know they are able to produce a lot and pay back their debt. We

When $\vartheta = 1$, the financial constraint is extreme, since individuals can only use their own assets in production. When $\vartheta = \infty$ there is no longer a financial constraint since there is no longer a restriction on the amount that an individual can borrow.

Taxation. In the benchmark economy that aims to represent the current US tax system, the government is assumed to impose flat taxes at rate τ_c on consumption (expenditures), τ_l on labor income, and τ_k on capital income. In the tax experiment we consider, we will study a revenue-neutral switch to an alternative system where the government will replace taxes on capital income (i.e., set $\tau_k \equiv 0$) with flat taxes on individuals' wealth stock, τ_a, leaving labor and consumption tax rates intact. In the robustness analysis, we will consider various forms of progressivity in taxes (especially on labor income and on wealth).

Social security. When an individual retires at age R, she starts receiving social security income $y^R(\theta, e)$ that depends on her type θ in the following way:

$$y^R(\theta, e) = \Phi(\theta, e)\,\overline{E},$$

where Φ is the agent's replacement ratio, a function that depends on the agent's permanent type θ and the last transitory shock to labor productivity, and $\overline{E}$ which corresponds to the average earnings of the working population in the economy.

3.5 Individual's problem

For clarity of notation, in this subsection we suppress the individual subscript i. The production problem of each individual is static in nature and can be solved in isolation of her other decisions.

Individual/Entrepreneur's Problem

First, as an entrepreneur, the individual chooses the optimal capital level to maximize profit:

$$\begin{aligned} \pi(a, z) = &\max_{k \leq \vartheta(z)a} \{p(zk) \times zk - (r + \delta)k\} \\ &\text{s.t. } p(zk) = \mathcal{R} \times (zk)^{\mu - 1}, \end{aligned} \tag{4}$$

model this feature as a possibly realistic aspect of financial markets that mitigates the constraints on investment and the extent of misallocation, thereby reducing the role of wealth taxes that we study later. With homogenous constraints, $\vartheta(z_{ih}) = \overline{\vartheta}$, the impact of wealth taxes are larger as we show in the sensitivity analysis in Section 7.

where δ is the depreciation rate of capital, $z = f(\overline{z}_i, \mathbb{I}_{ih})$, and $\mathcal{R} = \alpha Q^{\alpha-\mu} L^{1-\alpha}$, which yields the solution:

$$k\left(a,z\right) = \min\left\{\left(\frac{\mu\mathcal{R}z^{\mu}}{r+\delta}\right)^{\frac{1}{1-\mu}}, \vartheta(z)a\right\}. \tag{5}$$

Then, the maximized profit function is:

$$\pi\left(a,z\right) = \begin{cases} \mathcal{R}\times\left(z\vartheta(z)a\right)^{\mu} - \left(r+\delta\right)\vartheta(z)a & \text{if } k\left(a,z\right) = \vartheta(z)a \\ \left(1-\mu\right)\mathcal{R}z^{\mu}\left(\frac{\mu\mathcal{R}z^{\mu}}{r+\delta}\right)^{\frac{\mu}{1-\mu}} & \text{if } k\left(a,z\right) < \vartheta(z)a \end{cases}. \tag{6}$$

The after-tax non-labor income, $Y(a,z,\tau_k,\tau_a)$, is given by after-tax profits from their firm and interest payments obtained from the financial market:

$$Y(a,z,\tau_k,\tau_a) = \left[a + \left(\pi\left(a,z\right) + ra\right)\left(1-\tau_k\right)\right]\left(1-\tau_a\right). \tag{7}$$

Individual's Dynamic Programming Problem

The individual's problem then is given by:

$$\begin{aligned} V_h(a,\mathbf{S}) = \max_{c,n,a'} & \, u\left(c,1-n\right) + \beta s_{h+1} E\left[V_{h+1}(a',\mathbf{S}') \mid \mathbf{S}\right] \\ \text{s.t.} \;\; & \left(1+\tau_c\right)c + a' = Y\left(a,z,\tau_k,\tau_a\right) + y^W(\theta,e) \\ & a' \geq 0, \end{aligned}$$

where $\mathbf{S} = (\overline{z}, \mathbb{I}, \theta, e)$ is the vector of exogenous states of an individual and

$$y^W(\theta,e) = \begin{cases} (1-\tau_l) w y_h n & \text{if } h < R \qquad \text{where } \log y_h = \theta + \kappa_h + e \\ y^R(\theta,e) & \text{if } h \geq R. \end{cases}$$

We assume that $e_h = e_{h-1}$ for $h \geq R$, thus the retirement income is essentially conditioned on the earnings shock in period $R-1$.

3.6 Social Security Pension System

The government taxes capital income, labor income, and consumption in order to finance government expenditures G and pension payments to retirees. Let SSP denote

the aggregate value of "social security pension" payments:

$$SSP \coloneqq \int_{h \geq R, a, \mathbf{S}} y^R(\theta, e)\, \Gamma(h, a, \mathbf{S})$$

where Γ is the stationary distribution of agents over all possible states. To characterize $y^R(\theta, e)$ we define:

$$\overline{E} = \frac{wL}{I_1^R}$$

where w is the wage rate per efficiency unit, $I_1^R \leq 1$ is the measure of agents in working age, and L is the total number of effective hours worked in the economy:

$$L = \int_{h < R, a, \mathbf{S}} y_h(\theta, e)\, n_h(a, \mathbf{S})\, \Gamma(h, a, \mathbf{S}).$$

Note that the summation is taken over efficiency units y_h. The replacement ratio is progressive and satisfies:

$$\Phi(\theta, e) = \begin{cases} 0.9 \frac{y_1^R(\theta,e)}{\overline{y}_1^R} & \text{if } \frac{y_1^R(\theta,e)}{\overline{y}_1^R} \leq 0.3 \\ 0.27 + 0.32\left(\frac{y_1^R(\theta,e)}{\overline{y}_1^R} - 0.3\right) & \text{if } 0.3 < \frac{y_1^R(\theta,e)}{\overline{y}_1^R} \leq 2 \\ 0.91 + 0.15\left(\frac{y_1^R(\theta,e)}{\overline{y}_1^R} - 2\right) & \text{if } 2 < \frac{y_1^R(\theta,e)}{\overline{y}_1^R} \leq 4.1 \\ 1.1 & \text{if } 4.1 < \frac{y_1^R(\theta,e)}{\overline{y}_1^R} \end{cases}$$

where $y_1^R(\theta, e)$ is the average efficiency units that an agent of type θ gets conditional on having a given $e_R = e$.

$$y_1^R(\theta, e_R) = \frac{1}{R} \int_{h < R, a, \mathbf{S}} y_h(\theta, e)\, \Gamma(h, a, \mathbf{S}).$$

The integral is taken with respect to the stationary distribution of agents by age and is taken over all possible asset holdings, types z, and histories of e such that e_R is the one given in the left hand side. Finally $\overline{y}_1^R$ is the average of $y_1^R(\theta, e)$ across θ and e.

3.7 Equilibrium

Let $c_h(a, \mathbf{S})$, $n_h(a, \mathbf{S})$ and $a_{h+1}(a, \mathbf{S})$ denote the optimal decision rules and $\Gamma(h, a, \mathbf{S})$ be the stationary distribution of agents. A competitive equilibrium is given by the following conditions:

1. Consumers maximize given $p(x)$, w, r and taxes.

2. The solution to the final goods producer gives pricing function $p(x)$ and wage rate w.

3. $Q = \left(\int_{h,a,\mathbf{S}} (z \times k(a,z))^{\mu} \Gamma(h,a,\mathbf{S}) \right)^{1/\mu}$ and $L = \int_{h,a,\mathbf{S}} (y_h n_h(a,\mathbf{S})) \Gamma(h,a,\mathbf{S})$, where $\log y_h = \theta + \kappa_h + e$.

4. The government budget balances. We will compare the following two alternatives:

   (a) Taxing capital and labor income, in which case the government's budget becomes

$$\begin{aligned} G + SSP &= \tau_k \int_{h,a,\mathbf{S}} (\pi(a,z) + ra) \Gamma(h,a,\mathbf{S}) \\ &+ \tau_L \int_{h,a,\mathbf{S}} (w y_h n_h(a,\mathbf{S})) \Gamma(h,a,\mathbf{S}) \\ &+ \tau_c \int_{h,a,\mathbf{S}} c_h(a,\mathbf{S}) \Gamma(h,a,\mathbf{S}) \end{aligned}$$

   where

$$SSP = \int_{h \geq R,a,\mathbf{S}} y^R(\theta,e) \Gamma(h,a,\mathbf{S}).$$

   (b) Taxing wealth stock and labor income, in which case the government's budget becomes

$$\begin{aligned} G + SSP &= \tau_a \int_{h,a,\mathbf{S}} ((1+r)a + \pi(a,z)) \Gamma(h,a,\mathbf{S}) \\ &+ \tau_L \int_{h,a,\mathbf{S}} (w y_h n_h(a,\mathbf{S})) \Gamma(h,a,\mathbf{S}) \\ &+ \tau_c \int_{h,a,\mathbf{S}} c_h(a,\mathbf{S}) \Gamma(h,a,\mathbf{S}) \end{aligned}$$

5. The bond market clears:

$$0 = \int_{h,a,\mathbf{S}} (a - k(a,z)) \Gamma(h,a,\mathbf{S})$$

4 Quantitative Analysis

4.1 Model Parameterization

The benchmark model is calibrated to the US data. The model period is one year.

Government policy. The current US tax system is modeled as a triplet of tax rates: on capital income (τ_k), labor income (τ_l), and consumption expenditures (τ_c). Following McDaniel (2007) who measures these tax rates for the US economy, we set the capital income tax rate to $\tau_k = 25\%$, the labor income tax rate to $\tau_\ell = 22.4\%$, and the consumption tax rate to $\tau_c = 7.5\%$.

Demographics. Individuals enter the economy at age 20 and can live up to age 100 (i.e., a maximum of 80 periods). They retire at age 66 (model age $R = 46$). The conditional survival probabilities from age h to $h+1$ are taken from Bell and Miller (2002) for the US data.

Preferences. In the baseline analysis, we consider a Cobb-Douglas utility function:

$$u(c, \ell) = \frac{(c^\gamma \ell^{1-\gamma})^{1-\sigma}}{1-\sigma}.$$

We set $\sigma = 4$ following Conesa et al. (2009). We then choose γ and β (the subjective time discount factor) to generate an average of 40 hours of market work per week for the working-age population (i.e., $\ell = 0.6$, assuming 100 hours of discretionary time per week) and a wealth-to-output ratio of 3, which requires $\gamma = 0.46$ and $\beta = 0.9475$. In the sensitivity analysis, we also consider a separable utility function and vary these parameters to gauge their effects on the results.

Labor market efficiency. The deterministic life-cycle profile, κ_h, is modeled as a quadratic polynomial that generates a 50% rise in average labor income from age 21 to age 51.[5] The annual persistence of the autoregressive process for labor income, ρ_e, is set to 0.9.[6] The standard deviation of the innovation, σ_e, is set to 0.2. The intergenerational correlation of the fixed effect of labor market efficiency, ρ_θ, is set to 0.5, which is broadly consistent with the estimates in the literature (see Solon (1999) for a survey). Finally, with these parameters fixed, we set $\sigma_{\epsilon_\theta} = 0.305$ so as to match our empirical target of a cross-sectional standard deviation of log labor earnings of 0.80 (Guvenen et al. (2015)).

[5] $\kappa_{ih} = \frac{60(h-1)-(h-1)^2}{1800}$

[6] See Guvenen (2007) and others.

Entrepreneurial productivity. The evolution of entrepreneurial ability across generations is governed by the parameters ρ_z and $\sigma_{\varepsilon_{\overline{z}}}$. Unfortunately, there is not much empirical evidence on either parameter from the US data that we are aware of. In light of this, we turn to evidence from other countries. In particular, Fagereng et al. (2016) estimate individual fixed effects in rates of return over a 20-year period for parents and their children from administrative panel data on Norwegian households. They report a small correlation of about 0.1, which we take as our empirical value of ρ_z. We also conducted robustness analysis using a value of $\rho_z = 0.5$ but did not find any substantive differences. As for, $\sigma_{\varepsilon_{\overline{z}}}$ we choose it so as to match the share of aggregate wealth held by the top 1% of the wealth distribution.

In calibrating the stochastic component of entrepreneurial ability, one concern we have in mind is the inability of many models of wealth inequality to generate the speed at which the super wealthy—or the self-made billionaires—emerge in the data. In contrast, in these models the extreme wealth concentration emerges at a very slow pace and often requires hundred of years. Thus, one target we match is the fraction of self-made billionaires in the Forbes 400 list. The classification adopted by Forbes is shown in Table A.1 in the appendix. We define a self-made billionaire to be one who came from an upper-middle-class or lower-income family (Categories 8–10 in Table A.1). By this definition 54% of individuals on the list are self made. The model counterpart is defined as an individual who inherits less than one million dollars and goes on to become a billionaire. We set $\lambda = 5$, $p_1 = 0.05$, and $p_2 = 0.03$, which generates a self-made ratio of billionaires of 50%.

Production. We target a labor share of output of 0.60 by setting $\alpha = 0.4$. The curvature parameter of the CES aggregator of intermediate inputs, μ, is set to 0.9. With this value, our model generates the Pareto tail of the wealth distribution as it is observed in the U.S. data (see Figure 1). Later, we will provide robustness checks on its value. The depreciation rate of capital is set to 5%.

Financial constraint. We allow firms with higher productivity to borrow more. In particular, we choose

$$\vartheta(\overline{z}_i) = 1 + 1.5(i-1)/8 \text{ for } i = 1, ..., 9.$$

Note that we have 9 grid points for the permanent component of z.

TABLE II – Benchmark Parameters

Parameters Calibrated Outside of the Model		
Parameter		Value
Capital income tax rate	τ_k	0.25
Labor income tax rate	τ_L	0.224
Consumption tax rate	τ_c	0.075
Exponent of labor tax function (baseline)	ψ	0.00
Wealth tax rate	τ_a	0.00
Autocorrelation for idiosyncratic labor efficiency	ρ_e	0.9
Std. for idiosyncratic labor efficiency	σ_{ϵ_e}	0.2
Interg. correlation of labor fixed effect	ρ_θ	0.5
Intermediate goods aggregate share in production	α	0.4
Curvature parameter of CES production func.	μ	0.9
Depreciation rate	δ	0.05
Curvature of utility function	σ	4.0
Maximum age	H	81
Retirement age	R	45
Survival probabilities	ϕ_h	Bell and Miller (2002)
Parameters Calibrated Jointly in Equilibrium		
Discount factor	β	0.9475
Consumption share in utility	γ	0.460
Std. dev. of entrepreneurial ability	$\sigma_{\varepsilon_{\bar{z}}}$	0.072
Std. dev. of individual fixed effect	σ_θ	0.305
Productivity boost	λ	5.0

TABLE III – Targeted Moments

	US Data	Benchmark
Top 1%	0.36	0.36
Wealth-to-output ratio	3.00	3.00
Std. dev. of log earnings	0.80	0.80
Avg. Hours	0.40	0.40
Fraction self made	54%	50%

Table II summarizes the parameters that we calibrate independently (top panel) and those that are calibrated jointly (bottom panel) in equilibrium to match the moments shown in Table III.

TABLE IV – Statistics of the Benchmark Model

	US Data	Benchmark
Bequest/Wealth	1–2%	0.99%
GDP share of total tax revenue	0.295	0.25
Revenue share of capital tax	0.280	0.25
GDP share of capital tax	0.083	0.063
Mean return on wealth	6.9	8.33
Aggregate Debt/GDP	0.68	1.27

4.2 Performance of the benchmark model

Table IV shows the model's performance in matching moments that are not targeted in the calibration. A few observations are in order. First, the model generates bequest-to-wealth ratio that is broadly consistent with the data despite all bequests being accidental in the model. Second, tax revenues as a fraction of GDP and the capital tax share of total tax revenues the model generates are close to their counterparts in the data. Finally, the Federal Reserve Statistical Release (2015) report that the total non-financial business liability is $12.2 trillion, which implies an aggregate debt-to-GDP ratio of 0.68. Asker et al. (2011) report an average debt-to-asset ratio of 0.20 for publicly-listed firms and a ratio of 0.31 for private firms in the United States. Given that the capital-to-output ratio is 3 in our model, their figures correspond to an aggregate debt-to-output ratio of between 0.6 to 0.93. It is worth noting that aggregate debt-to-GDP ratio in the model is higher than in the data. We have deliberately chosen a looser borrowing limit, especially for more productive firms, so that our model does not overstate the extent of capital misallocation. (As can be expected, tightening the constraints yields even higher welfare gains from wealth taxation, so we opt for this more conservative choice in the baseline analysis.)

Next, we analyze the implications of our model for the wealth distribution. The model generates a clear Pareto tail of the wealth distribution as in the U.S. data. Figure 1 illustrates the Pareto tail from the benchmark calibration, which generates a slightly thicker tail than in the data. In other words, the wealth concentration in the percentiles

FIGURE 1 – Pareto Tail - Wealth above 1 Million

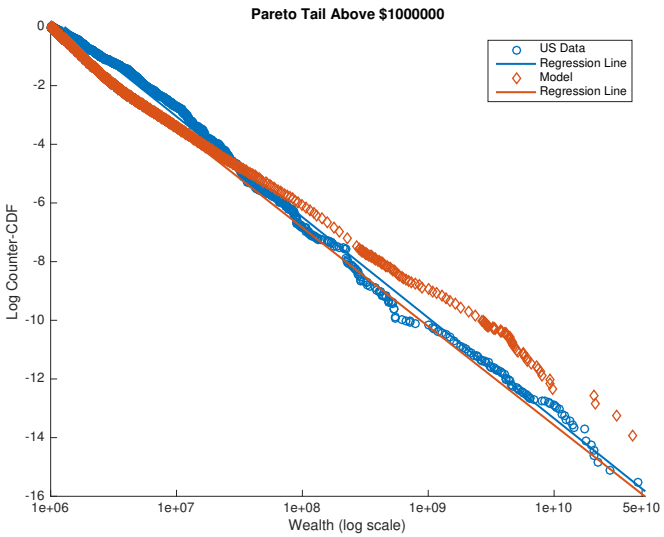


TABLE V – Wealth Concentration in the Benchmark Model

	US Data	Benchmark
Top 0.1%	0.14	0.23
Top 0.5%	0.27	0.31
Top 1%	0.36	0.36
Top 10%	0.75	0.66
Top 50%	0.99	0.97
Wealth Gini	0.82	0.78

of the wealth above the top 1% is higher in our model than in the data. For example, as seen in Table V, the top 0.1% richest's wealth share is 0.14 in the U.S. data, but the model generates 0.23. On the other hand, wealth shares of the top 10%, 20%, 40%, and 50%'s richest are somewhat understated in the model relative to the U.S. data. The shape of the Pareto tail is closely linked to the curvature parameter μ, which determines the degree to which returns fall as an individual becomes richer (or, to be more precise, the capital employed in his business grows). In the robustness analysis, we have experimented with different values of μ and found that the Pareto shape is preserved for values of μ higher than 0.8 while for lower values it turns concave.

TABLE VI – Deviation of percentiles of the distribution of lifetime returns relative to the median

	p99.9-p50	p99-p50	p90-p50	p75-p50	p25-p50	p10-p50
Norwegian Data	19.9%	9.7%	4.1%	2.1%	-1.3%	-2.4%
Working life	19.4%	13.3%	7.8%	4.5%	-2.9%	-3.7%
Ages 20-24	55.4%	32.7%	13.3%	4.9%	-5.6%	-10.0%
Ages 25-65	19.9%	13.6%	8.0%	4.7%	-2.9%	-3.4%

Note: *Lifetime returns are weighted by the individual's wealth at each age. All numbers are before tax. All numbers are presented as differences from the median. The Norwegian data is taken from Fagereng et al. (2016), Table 4.*

Lifetime returns in the benchmark model

The heterogeneity in the rates of returns is an important mechanism in the model for generating a wealth distribution that is consistent with the data in numerous dimensions. Therefore, it is of interest to compare the dispersion in the rates of return in the model and in the data. Even though, the empirical evidence is scarce, Fagereng et al. (2016) report the rates if returns in the Norwegian data. Rather encouragingly, the dispersion observed in the model matches well with the facts reported in Fagereng et al. (2016).

The return at age h for individual i is given by:

$$\text{Return}_{ih} = 100\frac{ra_{ih} + \pi\left(a_{ih}, z_{ih}\right)}{a_{ih}}$$

where π is given as in equation (4). The lifetime return for individual i is computed as the weighted average over the individual's working life, weighted by the individual's wealth at each age:

$$\text{Return}_i = \sum_{h=1}^{R} \frac{a_{ih}}{\sum_{h=1}^{R} a_{ih}} \text{Return}_{ih.}$$

Table VI reports various percentiles in the lifetime rates of return distribution in the data and in the model, relative to the median return in the data and in the model, respectively. The lifetime rate of return at the 99.9th percentile, relative to the median return, is around 20% both in the model and in the data. The lifetime returns at other percentiles above the median, however, are slightly higher than the returns observed in the data–e.g., the lifetime return at the 99th percentile is around 10% in the data and around 13-14% in the model. As expected, the rates of return are substantially higher at high percentiles when individuals are young. As productive individuals experience

TABLE VII – Percentiles of the distribution of lifetime returns

	p99.9	p99	p90	p75	p50	p25	p10
Norwegian Data	19.3%	9.1%	3.5%	1.5%	-0.55%	-1.8%	-2.9%
Working life	18.8%	12.8%	7.2%	3.9%	-0.55%	-3.4%	-4.3%
Ages 20-24	64.3%	41.6%	22.1%	13.8%	8.9%	3.3%	-1.1%
Ages 25-65	18.8%	12.5%	7.0%	3.6%	-1.0%	-4.0%	-4.4%

Note: *Lifetime returns are weighted by the individual's wealth at each age. All numbers are before tax. All numbers are adjusted to match the median in the Norwegian data by subtracting the median of the average return in the model and adding the median return in the Norwegian data. The Norwegian is taken from Fagereng et al. (2016), Table 4.*

significant growth early in the life cycle, between the ages of 20 and 24, they experience rates of return as high as 55% at the 99th percentile.

Overall, the distribution of lifetime rates of returns in the model is consistent with the distribution observed in the Norwegian data. For completeness, we also provide Table VII, which reports lifetime rates of return percentiles in the model relative to the median return in the Norwegian data, and the overall message remains unchanged.

Misallocation in the benchmark model

Our benchmark economy is distorted due to the existence of financial frictions in the form of borrowing constraints, and we can measure the effects of these distortions on aggregate TFP and output and compare them to those obtained in other studies. A large and growing literature frames the discussion on misallocation in terms of various wedges, such as capital, labor, and output wedges. The analysis in Hsieh and Klenow (2009) is particularly useful since, in a similar model environment, they study the degree of misallocation and its effect on TFP in manufacturing in China, India, and the United States. Hsieh and Klenow (2009) use detailed firm-level data from the US Census of Manufacturers (1977, 1082, 1987, 1992, and 1997) and find that the TFP gains from removing all distortions (wedges), which equalizes the "Revenue Productivity" (TFPR) within each industry, is 36% in 1977, 31% in 1987, and 43% in 1997.

We can follow the approach of Hsieh and Klenow and compute the same measures of misallocation for the US as in their analysis. Instead of modeling and capturing the effect of a particular distortion, or distortions, the approach in Hsieh and Klenow (2009), and the related misallocation literature, is to infer the underlying distortions and wedges in the economy by studying the extent to which the marginal revenue products of capital and labor differ across firms in the economy (or in a particular industry). This is based

on the insight that absent any distortions, the marginal revenue products of capital and labor have to be equalized across all firms.[7]

Appendix B provides the details as to how we map our model into the wedge analysis environment in Hsieh and Klenow (2009). Their analysis measures the improvement in total output as a result of an improvement in TFP in all industries. In our model, this corresponds to the improvement in TFP in the Q sector. We find that removing the capital wedges would increase total output, through its effect on TFP in the Q sector, by 20%—this is approximately half of the gains reported by Hsieh and Klenow. However, in ongoing research Bils et al. (2017) propose a method for correcting measurement error in micro data and find that TFP gains from removing distortions in the US are rather in the range of 20%, very much in line with the results from our benchmark economy. Therefore, we conclude that the level of distortion in our model environment is not far from the actual amount of distortion present in the US economy.

5 Results: Tax Reform

In this section, we study the effects of a simple tax reform in which the government eliminates capital income taxes (setting $\tau_k = 0$) from the baseline economy, keeps τ_l and τ_c unchanged, and levies a flat-rate wealth tax so as to keep the tax revenue fixed at its the level in the baseline economy. An important detail, however, is that the pension benefits, as described in Section 3.6, are a function of the average labor income in the economy, so any change in the level of income implies a change in the level of aggregate social security payments and hence would lead to an unbalanced budget if revenue is kept constant.

To deal with this issue, we consider two cases. In the first case, which is our main "revenue neutral" tax reform experiment, we keep the pension income of every individual fixed at its baseline value after the wealth tax reform. In the second case (balanced budget tax reform), we allow pension benefits to scale up or down with the level of average labor income in the economy, while choosing the level of wealth taxes to keep the government budget balanced. Except where we note explicitly, all results we discuss pertain to the first case—the revenue neutral tax reform.

[7] This is the case in the monopolistic competition models, such as in Hsieh and Klenow (2009). Alternatively, in environments such as in Lucas (1978) and Restuccia and Rogerson (2008), in which firms feature decreasing returns to scale, but produce the same homogeneous good, in the non-distorted economy the marginal products of capital and labor have to be equalized.

TABLE VIII – Changes in the Return Distribution

	P10	P50	P90	P95	P99
	Before-tax				
Benchmark	2.00	2.00	17.28	22.35	42.36
Wealth Tax	1.74	1.74	14.62	19.04	36.91
	After-tax				
Benchmark	1.50	1.50	12.96	16.76	31.77
Wealth Tax	0.59	0.59	13.32	17.69	35.35

Note: *Each cell reports the rate of return in percentages.*

TABLE IX – Key Variables

	Data	Benchmark	Tax Reform
Top 1%	0.36	0.36	0.46
Top 10%	0.75	0.66	0.72
Wealth/Output	3.00	3.00	3.25
Average hours	0.40	0.40	0.41
Std of log earnings	0.80	0.80	0.80
Bequest/Wealth	1–2%	0.99	1.07

5.1 Changes in After-tax Returns and Reallocation of Wealth

To illustrate the key mechanisms at play after the tax reform is implemented, we first present various percentiles of the after-tax return distribution shown in Table VIII. After-tax returns increase at upper percentiles and decrease at lower percentiles of the return distribution. This increase in the dispersion in after-tax returns increases the concentration of wealth at the top, since on average more productive agents hold a larger fraction of aggregate wealth.

Table IX shows some key statistics on wealth in the benchmark and the tax-reform economies. Consistent with the changes in after-tax returns, the wealth distribution becomes more concentrated at the top under the wealth tax: the share of wealth held by the top 1% increases from 36% to 46%, while the fraction held by the top 10% increases from 66% to 72%. The wealth-to-output ratio also increases from 3.0 to 3.25.

It is also instructive to analyze the reallocation of wealth when the capital income tax is replaced with a wealth tax. Table X reports, for a particular top x% of the wealth distribution, the percentage change in the fraction of agents with a particular entrepreneurial productivity. For example, among the top 1% in the wealth distribution,

TABLE X – Tax Reform from τ_k to τ_a: Change in Wealth Composition

	Productivity group (Percentile)						
Top x%	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
1	-12.0	-13.0	-10.8	10.5	11.2	9.8	6.9
5	-8.2	-3.3	1.6	8.3	8.9	8.1	6.2
10	-6.4	-1.3	2.9	6.4	6.9	6.3	5.0
50	-2.5	0.9	1.8	1.6	1.2	1.1	1.1

Note: *The table shows the percentage change induced by the tax reform from τ_k to τ_a of the share of agents in each entrepreneurial productivity group among the top x% wealth holders (i.e. agents above the x^{th} percentile of the wealth distribution). Each entry is computed as $100 \times \frac{s_{ij}(\tau_a) - s_{ij}(\tau_k)}{s_{ij}(\tau_k)}$, where i indexes groups of top x% wealth holders, j indexes entrepreneurial productivity groups and τ the tax regime.*

the fraction of individuals in the top 10% of the productivity distribution increased at the expense of less productive agents, resulting in a reallocation of wealth towards more productive entrepreneurs. This increased aggregate efficiency results in higher capital, Q, output, and wages, as we analyze next.

Aggregate variables. Table XI lists the values of the aggregate variables in the baseline economy and their percentage change after the wealth tax reform. First, aggregate capital increases by 19.4% with the tax reform. Moreover, Q (effective or quality-adjusted capital) increases even more, by 24.8%. The larger increase in Q relative to $\bar{k}$ reflects the fact that wealth is more concentrated in the hands of more productive agents under the wealth tax, reflecting the efficiency gains associated with the wealth tax. The increase in Q drives up other aggregate variables as well. The aggregate output increases by 10.1%, labor supply increases by 1.3%, and the wage rate increases by 8.7%. The general equilibrium increase in the wage rate is critical in distributing more evenly the welfare gains from the tax reform to the whole population since labor efficiency is more evenly distributed than wealth.

TABLE XI – Tax Reform: Macro Variables in the Baseline Economy and After Reform

		Benchmark	Tax Reform	
			τ_a	τ_a + SS
Capital income tax rate	τ_k	25%	0.0	0.0
Wealth tax rate rate	τ_a	0	1.13%	1.54%
Consumption tax rate	τ_c	7.5%	7.5%	7.5%
		Level	(Δ% from benchmark)	
Aggregate capital	$\bar{k}$	3.50	19.4	12.3
Intermediate goods	Q	3.51	24.8	18.4
Wage	w	1.25	8.7	6.4
Output	$\mathcal{Y}$	1.17	10.1	7.9
Labor	L	0.56	1.3	1.4
Consumption	C	0.83	10.0	8.4

Note: *The last column labeled "$\tau_a + SS$" reports the results from the "balanced budget" experiment in which pensions payments are allowed to change as average labor income changes with the tax reform.*

5.2 Welfare Analysis

In order to quantify the welfare consequences of the tax reform, we use the following two measures. The first measure is constructed at the individual level and then aggregated up. In particular, we first compute the consumption equivalent welfare for each individual and then integrate it over the population, using the stationary distribution in the benchmark economy:[8]

$$V_0((1 + CE_1(\mathbf{s}_0))c^*_{\text{US}}(\mathbf{s}_0), \ell^*_{\text{US}}(\mathbf{s}_0)) = \mathbb{V}_0(c(\mathbf{s}_0), \ell(\mathbf{s}_0))$$

$$\overline{CE}_1 \equiv \sum_{\mathbf{s}_0} \Gamma_{\text{US}}(\mathbf{s}_0) \times CE(\mathbf{s}_0),$$

[8]Given our utility function specification, the welfare consequences of switching from the benchmark economy to a counterfactual economy with a wealth tax for a individual in state $\mathbf{S}$ with age h and wealth a is given by

$$CE_h(a, \mathbf{S}) = 100 \times \left[\left(\frac{V_h(a, \mathbf{S}; \tau^{policy})}{V_h(a, \mathbf{S}; \tau^{bench})} \right)^{1/\gamma(1-\sigma)} - 1 \right].$$

This measure specifically gives what fraction of consumption an individual is willing to pay in order to move from the steady state of the economy with a capital income tax to the steady state of the economy with a wealth tax.

TABLE XII – Average Welfare Gains from Tax Reform

	Baseline		Baseline + SS reform	
	$\overline{CE}_1$	$\overline{CE}_2$	$\overline{CE}_1$	$\overline{CE}_2$
Average CE for newborns	7.40%	7.86%	5.58%	4.71%
Average CE	3.14%	5.14%	4.95%	4.10%
% in favor of reform	67.8%		94.8%	

where V_0 and $\mathbb{V}_0$ are the lifetime value functions in the benchmark (US) capital income tax economy and the counterfactual wealth tax economy, respectively.

The first measure allows us to discuss individual-specific outcomes and to understand "who gains, and who loses, and by how much" from the tax reform. The second measure is simpler, and more similar to the famous Lucas (1987) calculation: it measures the fixed proportional consumption transfer to all individuals in the benchmark economy so that the average utility is equal to that in the tax-reform economy:

$$\sum_{\mathbf{s}_0} \Gamma_{\text{US}}(\mathbf{s}_0) \times V_0((1+\overline{CE}_2)c^*_{\text{US}}(\mathbf{s}_0), \ell^*_{\text{US}}(\mathbf{s}_0)) = \sum_{\mathbf{s}_0} \Gamma(\mathbf{s}_0) \times \mathbb{V}_0(c(\mathbf{s}_0), \ell(\mathbf{s}_0)).$$

Table XII summarizes the results from the welfare analysis. The average welfare gain is 3.14% for the whole population using the CE_1 measure and 5.14% using the CE_2 measure. The average welfare gain for newborn individuals is higher: 7.40% and 7.86%, respectively, for the two different welfare measures. Overall, 68% of all individuals across the whole population in the benchmark economy prefer to be in an economy with a wealth tax.

Panel A in Table XIII shows the average welfare gains (CE_1) for the baseline tax reform by age group and entrepreneurial ability computed using the stationary distribution under the capital income tax. Young individuals, at the age of 20, that are at the top of the productivity distribution experience the largest welfare gains – they are able to grow faster and get higher after-tax returns under the wealth tax than under the capital income tax. Young individuals at the bottom of the productivity distribution also experience substantial welfare gains even though they hold very little wealth – those gains are due to higher wages in the wealth-tax economy. The welfare gains decline with age and even become negative for individuals over the age of 65. Low productive agents

TABLE XIII – Welfare Gain by Age Group and Entrepreneurial Ability

(A) Baseline Tax Reform

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	7.0	7.3	7.9	8.9	10.6	11.6	12.4
21–34	6.5	6.3	6.3	6.6	7.0	6.9	5.7
35–49	5.1	4.4	3.9	3.3	1.7	0.4	-2.2
50–64	2.3	1.8	1.4	0.8	-0.6	-1.7	-3.5
65+	-0.2	-0.3	-0.4	-0.6	-1.2	-1.7	-2.7

(B) Tax Reform with Social Security Reform

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	4.9	5.3	6.0	7.2	9.3	10.4	11.4
21–34	4.7	4.6	4.8	5.4	6.1	6.3	5.2
35–49	4.2	3.7	3.4	2.8	1.4	0.0	-2.8
50–64	4.9	4.3	4.0	3.2	1.4	0.0	-2.3
65+	7.2	6.7	6.4	5.8	4.3	3.2	1.2

Note: *Each entry reports the average welfare gain (CE_1) from the tax reform from τ_k to τ_a of agents in a given age and entrepreneurial productivity group. The average is computed with respect to the benchmark distribution.*

do save for precautionary reasons and for retirement and imposing a wealth tax instead of a capital income tax late in life results in lower after-tax returns and is costly for them. Older high productivity entrepreneurs, on the other hand, experience low welfare gains, and even welfare losses, since some of them have lost their productivity and the wealth tax is costlier for them than the capital income tax. Retirees mostly lose from the reform since their benefits are fixed at the benchmark level, and they mostly face lower after-tax return on their savings under the wealth-tax economy. These considerations are reflected in the observed support for the reform from various part of the age-productivity distribution, as reported in Panel A in Table XIV.

Panel B in Table XIII reports the welfare gains in the case when the pension benefits are adjusted for changes in the average labor income in the economy and the wealth tax

TABLE XIV – Fraction with Positive Welfare Gain by Age Group and Entrepreneurial Ability

(A) Baseline Tax Reform

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	96.1	95.8	97.2	98.0	98.7	98.9	99.0
21–34	97.3	96.3	95.8	95.0	92.6	89.9	82.5
35–49	95.8	92.7	89.5	83.9	70.7	60.7	43.7
50–64	79.4	74.5	70.2	62.9	51.1	44.1	34.4
65+	8.0	9.5	9.5	8.8	7.3	6.2	4.8

(B) Tax Reform with Social Security Reform

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	94.3	94.6	95.9	97.3	98.6	98.9	99.0
21–34	95.9	94.7	94.4	94.0	91.7	89.1	82.0
35–49	95.4	92.3	89.5	84.2	71.4	61.4	44.4
50–64	96.6	93.7	90.7	83.7	70.1	61.1	48.5
65+	99.5	98.6	97.5	92.8	82.0	73.9	60.3

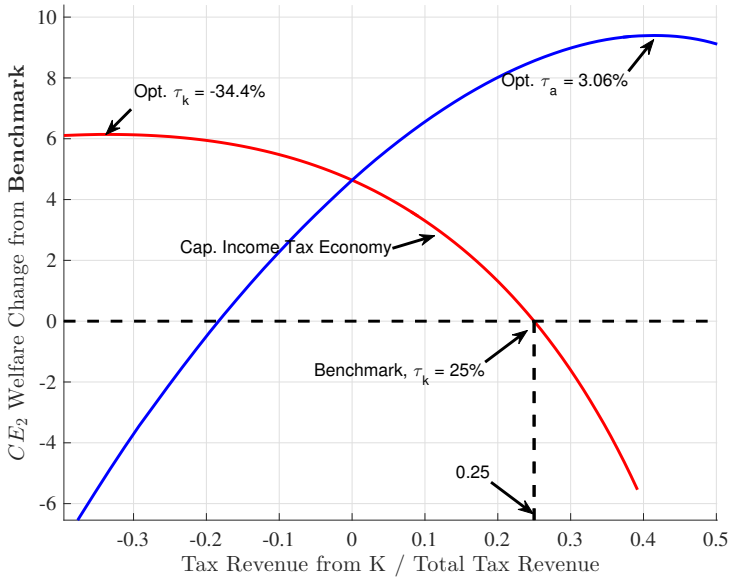
Note: *Each entry reports the share of agents in a given age and entrepreneurial productivity group that would experience a positive welfare gain (CE_1) from the tax reform from τ_k to τ_a. The shares are computed with respect to the benchmark distribution.*

is chosen in order to keep the government budget balanced. The main difference of note is the fact that individuals over the age of 65 now experience welfare gains rather than welfare losses. They are benefiting from the higher efficiency in the economy under the wealth tax since their pensions reflect the higher average labor income in the economy. This results in larger support for the reform from those groups of the population, as reported in Panel B in Table XIV.

6 Results: Optimal Taxation

The discussion so far illustrates that a wealth tax is a better way of taxing capital than a capital income tax. A natural question, however, is whether taxing capital in

FIGURE 2 – Welfare Gain from Optimal Taxes



this framework would be a part of the optimal tax schedule to begin with. We study quantitatively this question by performing two experiments: (i) we find the optimal taxes in an environment where the government uses proportional labor income taxes and proportional capital income taxes, and (ii) we find the optimal taxes in an environment where the government uses proportional labor income taxes and proportional wealth taxes. Figure 2 illustrates the average welfare gain (CE_2) of the newborn, relative to the benchmark, as we vary the taxes on capital/wealth. The red line corresponds to the welfare gain in the capital income tax economy and the blue line corresponds to the one in the wealth tax economy. The x-axis corresponds to the tax revenue from capital as a fraction of total tax revenue. Note that total tax revenue ($G + SSP$) is fixed in this experiment. Thus, as we vary the taxes on capital, the labor income tax adjusts to balance the government budget. The benchmark capital income tax economy with capital income tax economy corresponds to 0.25 on the x-axis since the capital tax revenue as a fraction of total tax revenue is 0.25 in that economy.

The first observation from Figure 2 is that the average welfare gain of the newborn increases as the capital *income* tax is reduced below its benchmark level in the capital income tax economy so that the optimal capital income tax turns out to be –34.4%, which is in sharp contrast to the findings of the recent literature on capital income taxation, most notably Conesa et al. (2009) who find that the optimal capital income tax is 36%. In the wealth-tax economy, the average welfare of the newborn increases as we increase

TABLE XV – Optimal taxation: statistics

	τ_k	τ_ℓ	τ_a	$\frac{Thresh.}{Y_a}$	% Taxed	Top 1%	$\overline{CE}_2$ (%)
Benchmark	25%	22.4%	–	–	100%	0.36	–
Tax reform	–	22.4%	1.13%	0	100%	0.46	7.86
Opt. τ_k	–34.4%	36.0%	–	–	100%	0.56	6.28
Opt. τ_a	–	14.1%	3.06%	0	100%	0.47	9.61
Opt. τ_a – Threshold	–	14.2%	3.30%	6.3%	63%	0.48	9.83

the wealth tax, and the optimal wealth tax is positive and substantial at (3.06%). At the optimal wealth tax, the tax revenue from capital/wealth is more than 40% of the total tax revenue, which is higher than the benchmark level of 25%. Table XV summarizes some key statistics from this experiment. First, note that the optimal capital income tax of –34.4% is associated with a high labor income tax of 36%. The optimal wealth tax of 3.06% on the other hand is associated with a labor income tax of 14.1%. The optimal wealth tax delivers the highest welfare gain, 9.61%, while under the optimal capital income tax the welfare gain, 6.28%, is lower than in the tax reform experiment, 7.86%.

We have also studied the optimal wealth tax allowing for a threshold level below which the wealth is not taxed. In this experiment, the government maximizes welfare by choosing jointly the wealth threshold level, the wealth tax rate that applies above that threshold, and the labor income tax rate. We find that the optimal threshold level is 6.3% of average income in the economy and the optimal wealth tax rate is 3.3%. In this case, only 63% of the population pays wealth taxes. The aggregate welfare gain from its implementation is 9.83%, which is higher than the 9.61% welfare gain from the optimal linear wealth tax. (Remember that the welfare gain from the optimal linear wealth tax is 9.61%). The additional aggregate welfare gain is small relative to the overall welfare gains from the implementation of the wealth tax instead of capital income tax. However, there are some important differences in distribution of welfare gains and political support for wealth taxes between a linear wealth tax system and a wealth tax system with a threshold, which we report in Section 6.2.

6.1 Efficiency gains (losses) from wealth tax (capital income tax) and optimal taxes

These results can be intuitively explained using the information provided in Panels A-D of Figure 3. As Panel A illustrates, raising taxes on capital–either through a capital income tax or a wealth tax–reduces aggregate capital $\overline{k}$ and Q. However, there are two notable differences between these two ways of raising taxes. First, aggregate capital $\overline{k}$ decreases less under the wealth tax system than under the capital income tax system. Second, Q declines more than $\overline{k}$ under the capital income tax system while it declines less than $\overline{k}$ under the wealth tax system.

We first explain the second result since it is critical for understanding the first one. Consider a simplified version of our model where before-tax gross return is given as $1 + Pz$ where z is entrepreneurial productivity and P is the price of Q. The after-tax gross returns are given as $1 + Pz\,(1 - \tau_k)$ and $(1 + Pz)\,(1 - \tau_a)$ under the capital income and wealth taxes, respectively. Consider two individuals as in our simple example, i.e. Mike and Fredo such that $z_M > 0$ and $z_F = 0$. The first observation we want to point out is that an increase in the capital income tax has no effect on Fredo's after tax gross return since $z_F = 0$, but reduces Mike's after-tax return. Thus, the capital income tax mainly distorts the wealth accumulation of more productive agents, which reduces their wealth share, increases misallocation of capital, and leads to larger decline in Q than $\overline{k}$. With the wealth tax on the other hand, $(1 + Pz)\,(1 - \tau_a)$ is affected at the same rate for both agents for a given P. However, with a higher wealth tax, Q goes down and P increases. Now consider these two individual's after-tax returns: $(1 + Pz_F)\,(1 - \tau_a)$ versus $(1 + Pz_M)\,(1 - \tau_a)$. The general equilibrium increase in P partially offsets the decline in the after-tax return $(1 + Pz_M)\,(1 - \tau_a)$ for Mike when the wealth tax is increased. However, Fredo's after-tax return does not benefit from the increase in P since $z_F = 0$. Thus, a higher τ_a has a smaller negative impact on the more productive Mike's after-tax return. This mechanism reallocates wealth to productive agents, and reduces the misallocation of capital, and leads to a smaller decline in Q than $\overline{k}$ as the wealth tax is increased.

Since the distortionary effects of capital taxes is much smaller under the wealth tax than the capital income tax, the government can increase the wealth tax without significantly distorting output (and wages) as seen in Panel B, and can reduce the labor income tax so that the after-tax wage increases with the wealth tax. Panel C shows that

TABLE XVI – Optimal Taxation: Percentage Change in Aggregate Variables

	$\%\Delta K$	$\%\Delta Q$	$\%\Delta L$	$\%\Delta Y$	$\%\Delta w$	$\%\Delta w$ (net)	Δr	Δr (net)	$\%\Delta TFP$
Tax reform	19.37	24.79	1.28	10.10	8.70	8.70	–0.25	–0.90	4.60
Opt. τ_k	68.97	79.57	–1.16	25.51	26.97	4.72	–1.51	–0.87	6.29
Opt. τ_a	2.76	10.26	3.90	6.40	2.41	13.42	0.68	–1.92	7.29
Opt. τ_a -Threshold	0.41	8.12	3.67	5.42	1.70	12.48	0.78	-2.07	7.70

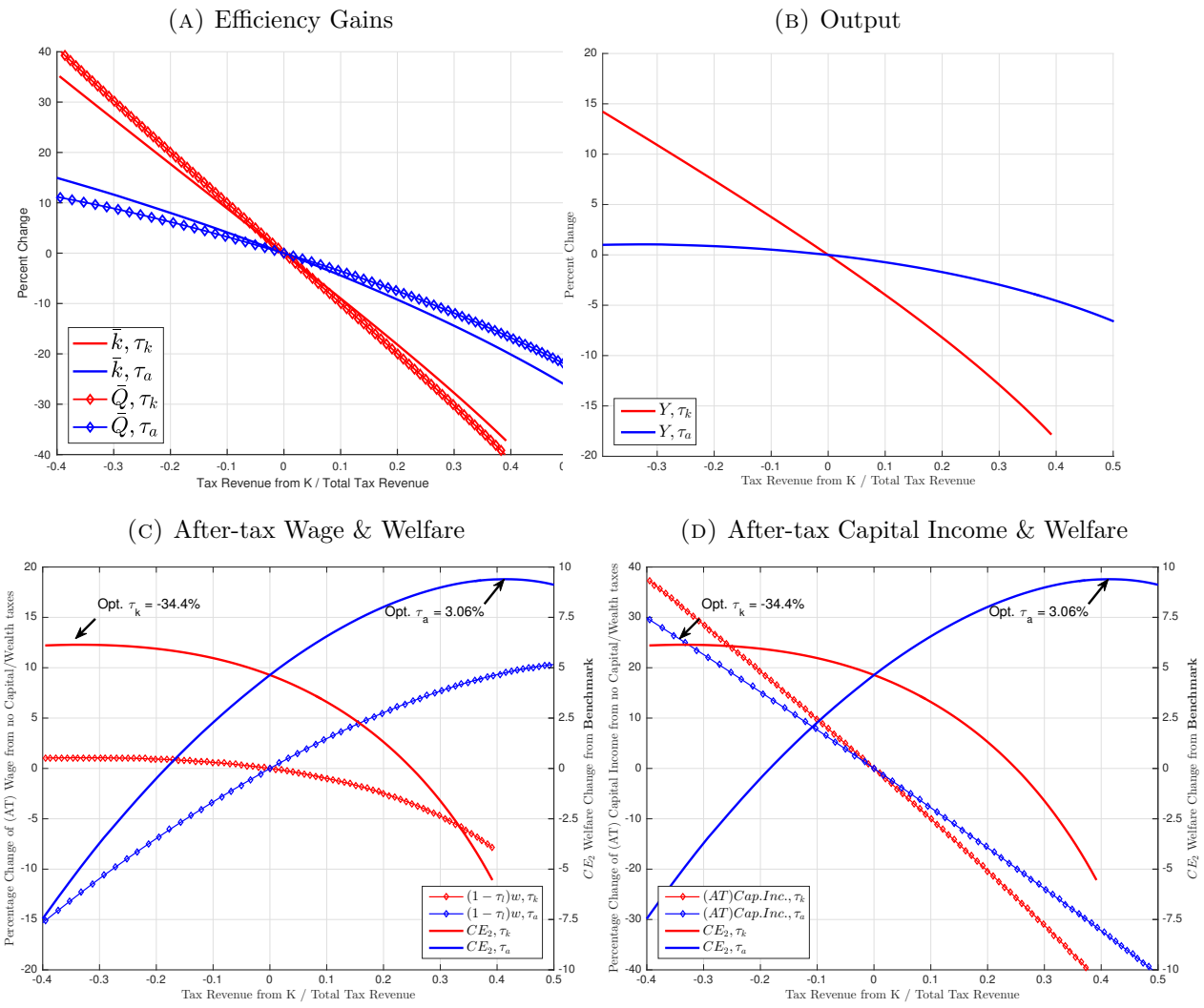
Note: *Percentage changes are computed with respect to the benchmark economy without wealth taxes and capital income taxes of 25%. Changes in the interest rate are computed in percentage points. The net wage is defined as* $(1-\tau_l)\,w$*, and the net interest rate is defined as* $(1+(1-\tau_k)\,r)\,(1-\tau_a)-1$*. The TFP variable is measured in the intermediate goods market.*

the after-tax wage rate indeed increases with the wealth tax but declines with the capital income tax. Panel D illustrates that capital income is declining with capital taxes under both tax systems but it declines by less under the wealth tax system. Thus, for a given tax revenue from capital, since the after-tax wage and capital income are higher under wealth taxes, people will accumulate more assets and aggregate capital will be higher under the wealth tax.

The mechanisms described above are also closely linked to the optimal tax level found under these two tax systems. Individuals whose resources mostly consist of labor income will gain from the wealth tax. Those whose resources are mainly from wealth, will lose from it. Since wealth is much more concentrated in the hands of very few agents and labor income is more evenly distributed across the population, our welfare measure, which weighs rich and poor at the same rate and maximizes the welfare of a newborn whose income is more influenced by wages, picks up a rate that is close to the rate that actually maximizes the after-tax wage rate. This point is illustrated in Panel C. Similarly, under the capital income tax economy, the after-tax wage is maximized when the capital income tax is negative. Thus, we obtain a negative optimal capital income tax.

Table XVI shows the percentage change in aggregate variables relative to their benchmark levels once the optimal taxes are implemented. As can be seen from the table, the optimal capital income tax leads to much larger increases in output and wages. However, after-tax wages increase significantly more under the optimal wealth tax.

FIGURE 3 – Optimal Taxes on Capital



6.2 Distribution of Welfare Gains and Political Support

The two panels in Table XVII illustrate the welfare gains, by age and entrepreneurial ability, when the benchmark capital income tax is replaced with the following two tax systems: optimal capital income tax and optimal (linear) wealth tax.[9] Welfare gains are typically higher for younger agents in all of these tax systems. However, there are some important differences. First, focusing on the working age population, we observe that the welfare gains are typically higher under wealth taxes than under capital income taxes for

[9]The results based on an optimal (linear) wealth tax with a threshold limit are similar to those in the optimal wealth tax, and we refer to them when appropriate.

agents with lower entrepreneurial ability. This is directly related to the fact that after-tax wages are much higher under optimal wealth taxes than under optimal capital income taxes. Second, retirees typically experience welfare losses with the implementation of the optimal tax system under both tax systems. However, welfare losses are higher in the optimal wealth tax case. This is mainly because the after-tax interest rate is lower in this case: for example, Table XVI shows that the after-tax interest rate r(net) is 1.92% lower under wealth taxes than in the benchmark economy, while it is lower by only 0.87% under capital income taxes relative to the benchmark. Thus, retirees whose retirement benefits are fixed at the benchmark level and whose capital income declines due to the decline in the after-tax interest rate experience larger welfare losses when wealth taxes are implemented rather than capital income taxes. We also analyzed separately the optimal wealth tax with a threshold and found that in that case many of the low ability retirees experience lower welfare losses since they no longer pay taxes on wealth as their wealth is not that high.

Table XVIII reports the fraction of households with positive welfare gains for each age-ability group. Red numbers correspond to less than 50% support within a group. We notice that the fraction of retirees that prefer wealth taxes is smaller than the fraction of retirees that prefer capital income taxes – that reduces the support for wealth taxes. Thus while, overall, 69.7% of the population prefers to be in the capital income tax economy, 60.7% of the population prefers to be in the wealth tax economy, and the retirees are key for understanding the larger support for capital income taxes. Once we introduce a threshold in the wealth tax, the support for the wealth tax increases among the retirees and 78.9% of the population are now in favor of the optimal wealth tax with a threshold, as shown in Table (XIX).

TABLE XVII – Welfare Gain by Age Group and Entrepreneurial Ability

(A) Optimal **Capital Income Taxes**

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	4.0	5.6	7.2	9.5	13.0	14.8	16.1
21–34	3.7	5.0	6.2	7.9	10.4	11.4	11.2
35–49	2.7	3.3	3.8	4.0	3.5	2.7	0.7
50–64	1.1	1.4	1.6	1.5	0.6	-0.2	-1.9
65+	-0.1	0.1	0.2	0.2	-0.2	-0.7	-1.6

(B) Optimal **Wealth Taxes**

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	10.0	9.7	10.1	11.1	13.1	14.3	15.3
21–34	9.2	7.9	7.3	7.1	6.6	5.9	3.1
35–49	6.8	4.9	3.7	2.1	-1.3	-3.9	-8.8
50–64	2.7	1.4	0.6	-0.8	-3.7	-5.8	-9.3
65+	-0.6	-0.9	-1.2	-1.8	-3.2	-4.3	-6.3

Note: *Each entry reports the average welfare gain (CE_1) from the corresponding optimal tax experiment of agents in a given age and entrepreneurial productivity group. The average is computed with respect to the benchmark distribution.*

TABLE XVIII – Fraction with Positive Welfare Gain by Age Group and Entrepreneurial Ability

(A) Optimal **Capital Income Taxes**

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	95.4	98.6	99.3	99.6	99.8	99.8	100.0
21–34	96.3	97.7	97.7	97.3	96.0	94.9	92.3
35–49	91.7	92.8	91.1	87.8	80.3	74.5	63.7
50–64	74.2	76.2	73.8	69.4	60.3	53.8	43.8
65+	13.8	18.6	18.7	18.2	16.6	15.2	13.0

(B) Optimal **Wealth Taxes**

	Productivity group (Percentile)						
Age	0-40	40-80	80-90	90-99	99-99.9	99.9-99.99	99.99+
20	94.5	93.1	93.3	94.6	95.8	96.1	95.8
21–34	95.7	92.6	90.5	88.8	84.2	79.4	67.0
35–49	91.3	82.8	76.5	68.2	53.6	44.6	34.0
50–64	72.6	62.9	56.1	49.4	39.8	34.5	27.2
65+	2.1	2.3	1.8	1.4	0.9	0.7	0.4

Note: *Each entry reports the share of agents in a given age and entrepreneurial productivity group that would experience a positive welfare gain (CE_1) from from the corresponding optimal tax experiment. The shares are computed with respect to the benchmark distribution.*

TABLE XIX – Welfare Gains and Political Support

	$\overline{CE}_2$ (%)	Vote (%)
Benchmark	–	–
Tax reform	7.86	67.8
Opt. τ_k	6.28	69.7
Opt. τ_a	9.61	60.7
Opt. τ_a – Threshold	9.83	78.9

6.2.1 Decomposition of welfare gains

Following Conesa et al. (2009), we decompose the aggregate welfare gain into a component arising from changes in consumption and a component arising from changes in leisure. The effect of consumption changes on welfare can be further decomposed into components arising from the change in average consumption and changes in the distribution of consumption.[10] Table XX reports these decomposition results. First, notice that the 9.61 percent welfare gain under the optimal wealth tax (τ_a) is due to an 11.02 percent welfare gain in consumption and a 1.27 percent welfare loss in leisure. Second, focusing on consumption, we observe that both an increase in the level and an improvement in the distribution positively contribute to the total welfare gain: by 8.28 percent and 2.53 percent, respectively. This is an important point worth emphasizing – despite the fact that wealth inequality becomes much higher under the optimal wealth tax, the distribution of consumption becomes more equal relative to our benchmark, which contributes to the overall welfare gain from wealth taxes. This pattern is different from the determinants of the 5.90 percent welfare gains due to consumption under the capital income tax (τ_k)—a large 21.04 percent is due to an increase in the average level of consumption which is offset by a 12.51 percent welfare loss due to a substantial increase in consumption inequality.

[10] Let CE be the aggregate welfare gain, and CE_C and CE_L be the components of the aggregate welfare gain arising from changes in consumption and leisure respectively. CE_C is given by

$$V_0((1+CE_C(\mathbf{s}))c^*_{\text{US}}(\mathbf{s}),\ell^*_{\text{US}}(\mathbf{s})) = \widetilde{\mathbb{V}}_0(c(\mathbf{s}),\ell^*_{\text{US}}(\mathbf{s}))$$

and CE_L is given by

$$V_0((1+CE_L(\mathbf{s}))c^*_{\text{US}}(\mathbf{s}),\ell^*_{\text{US}}(\mathbf{s})) = \widetilde{\mathbb{V}}_0(c^*_{\text{US}}(\mathbf{s}),\ell(\mathbf{s})).$$

Note that $1+CE = (1+CE_C)(1+CE_L)$. Furthermore, CE_C can be decomposed into level $CE_{\overline{C}}$ and distribution component CE_{σ_C} as

$$V_0((1+CE_{\overline{C}}(\mathbf{s}))c^*_{\text{US}}(\mathbf{s}),\ell^*_{\text{US}}(\mathbf{s})) = \widehat{\mathbb{V}}_0(\widehat{c}(\mathbf{s}),\ell^*_{\text{US}}(\mathbf{s}))$$

where $\widehat{c}(\mathbf{s}) = c^*_{\text{US}}(\mathbf{s})\frac{\overline{C}}{\overline{C}^*_{US}}$ and

$$\widehat{\mathbb{V}}_0\left((1+CE_{\sigma_C})\,\widehat{c}(\mathbf{s}),\ell^*_{\text{US}}(\mathbf{s})\right) = \widetilde{\mathbb{V}}_0(c(\mathbf{s}),\ell^*_{\text{US}}(\mathbf{s}))$$

where one can show that $1+CE_C = (1+CE_{\overline{C}})(1+CE_{\sigma_C})$. Similar decomposition applies to leisure.

TABLE XX – Decomposition of Welfare Gain – CE_2 for Newborn

	Tax Reform	Opt. τ_k	Opt. τ_a
$CE_2(NB)$ (%)	7.86	6.28	9.61
	Consumption		
Total	8.27	5.90	11.02
Level	10.01	21.04	8.28
Dist.	–1.58	–12.51	2.53
	Leisure		
Total	–0.38	0.36	–1.27
Level	–0.66	0.73	–2.21
Dist.	0.27	–0.38	0.76

7 Robustness

In this section, we explore the robustness of our results by conducting a sensitivity analysis with respect to the following changes in the economic environment: 1) the labor income tax is allowed to be progressive, 2) the stochastic component of entrepreneurial ability is eliminated (no shock case), 3) the constraint on borrowing for the entrepreneur is eliminated, i.e. $\vartheta = \infty$, 4) $\mu = 0.8$, 5) estate tax is allowed, 6) wealth is measured as present value rather than book value, and 7) the rate of return heterogeneity is eliminated by setting $z_i = 1$ for all i and $\mu = 1$ (we refer to the case as "CKK" since this framework then becomes very similar to the framework used in Conesa et al. (2009)). In all of these cases, we follow the same calibration procedure as in our benchmark economy – i.e., we target the same set of moments with the same set of parameters, except in (i) the "no shock case" when we do not target the fraction of self-made billionaires, and (ii) the CKK case where the model is unable to match the wealth concentration in the data. The results from the tax reform experiments are presented in Table XXI, and the results from the optimal tax experiments are presented in Table XXII. The message from all of these experiments is that our substantive conclusions are robust to any of these changes in the economic environment.

Progressive labor income tax. We introduce progressive labor income taxation, letting the after-tax labor income to be $(1-\tau_l)(wy_h n)^{\psi}$, and following Heathcote et al. (2014)we set $\psi = 0.815$. τ_l is chosen so that the average labor income tax rate is 0.224 –

the same as in our benchmark. In the tax reform experiment, we keep the labor income tax unchanged. As seen in Table (XXI) the results are quantitatively quite similar to those in our benchmark. In the optimal tax experiment, we search for the optimal level and progressivity of the labor income tax τ_l and ψ jointly with capital taxes. We find that the optimal progressivity of the labor income tax should be higher, which is reflected in a smaller ψ. The optimal levels of the capital income and wealth taxes are quite similar to those in the benchmark calibration.

No shock. When we eliminate the stochastic component of entrepreneurial ability, we increase the dispersion of the permanent component in order to generate the same amount of wealth concentration. However, this version of the model can only generate 18.5% self-made richest individuals. We find that the welfare gains from the tax reform are smaller than in the benchmark, but still very large. The optimal capital income tax is slightly negative at -2.33% while the optimal wealth tax is still positive and large at 2.21%.

No borrowing constraint: $\vartheta = \infty$ In this case, the marginal returns are equalized across individuals, and the misallocation of capital is completely eliminated. Yet, surprisingly, replacing the capital income tax with a wealth tax does increase welfare. Table XXI shows that aggregate capital $\overline{k}$ and effective capital Q increase by 6.28%.[11] The increase in aggregate capital is generating the welfare gain from switching to a wealth tax. In order to illustrate why aggregate capital increases, consider an individual's after-tax non-labor income when the financial constraint is eliminated. The entrepreneurial profit is given as

$$\pi^*(z) = \max_k \left\{ \mathcal{R} \times (zk)^\mu - (r+\delta)k \right\}.$$

The after-tax non-labor income, $Y(a, z, \tau_k, \tau_a)$, is given by

$$Y(a,z,\tau_k,\tau_a) = \begin{cases} (1 + r(1-\tau_k))\, a + \pi^*(z)(1-\tau_k) & \text{under capital income tax} \\ \\ (1+r)(1-\tau_a)\, a + \pi^*(z)(1-\tau_a) & \text{under wealth tax.} \end{cases}$$

When the capital income tax is replaced with a wealth tax, there are two opposing mechanisms at play. We will illustrate these mechanisms for a given interest rate and distribu-

[11] Note that they increase at the same rate since there is no misallocation.

tion of agents across states. First, we can show that $(1+r)(1-\tau_a)a < (1+r(1-\tau_k))a$, which will reduce capital accumulation under wealth taxes. Second, $\pi^*(z)(1-\tau_a) > \pi^*(z)(1-\tau_k)$ – in fact, $\pi^*(z)(1-\tau_a)$ will be much larger than $\pi^*(z)(1-\tau_k)$ for high z types since $\tau_k = 25\%$ and τ_a is around 2.5%. The second mechanism will increase capital accumulation, especially for the most productive agents with high $\pi^*(z)$ since their after-tax profits will increase substantially.[12] Ultimately, the second mechanism dominates resulting in an increase in the aggregate capital stock when the economy switches from a capital income to a wealth tax.

Turning to the optimal tax experiment, we find that the optimal capital income tax is positive at 13.6%, but still smaller than the benchmark level of 25%. The optimal wealth tax is 1.57%, which is close to the benchmark tax reform level of 1.65%. And finally, the optimal wealth tax delivers a higher welfare gain than the optimal capital income tax.

Estate taxes. We incorporate in the model an estate tax of 40%, with an exemption level of bequests below $5 Million, in order to capture the level of estate taxation in the US. We recalibrate the benchmark economy and conduct the tax reform and optimal tax experiments holding the estate taxes fixed. The results are not much different from those in the benchmark model. The welfare gains are larger in all three experiments: tax reform, optimal capital income tax, and optimal wealth tax and all our conclusions remain unchanged.

Curvature parameter in the CES production function: $\mu = 0.80$. Holding other parameters fixed, a higher curvature (lower μ) in the CES production function implies lower efficiency gains from switching to a wealth tax since high productivity agents will face diminishing marginal productivity more quickly as they accumulate more wealth under the wealth tax. However, with a higher curvature, the model generates lower wealth concentration. Thus, recalibration requires a higher dispersion in $\bar{z}_i$ ($\sigma_{\bar{\varepsilon}_{\bar{z}}}$) in order to match the wealth concentration in the top 1%. Table XXI shows that the welfare gain from switching to a wealth tax is very similar to the one in our benchmark. Table XXII shows that the welfare gain is larger for the optimal capital income tax (7.38%) and lower

[12] Note that $G = \tau_k \sum (ra + \pi^*(z))\Gamma(a,z,:) = \tau_k (rK + \pi^*(z))$ under capital income tax and $G = \tau_a \sum ((1+r)a + \pi^*(z))\Gamma(a,z,:) = \tau_a ((1+r)K + \pi^*(z))$ under wealth tax. Using these equations we can show that 1) $\tau_a << \tau_k$, thus $\pi^*(z)(1-\tau_a) > \pi^*(z)(1-\tau_k)$ and 2) $1+r(1-\tau_k) = 1+r-\frac{G}{K+\frac{\sum \pi^*(z)}{r}}$ is greater than $(1+r)(1-\tau_a) = 1+r-\frac{G}{K+\frac{\sum \pi^*(z)}{1+r}}$.

for the optimal wealth tax (8.32%) as compared to those in the benchmark case, and that the welfare gain under the optimal wealth tax remains larger than the one under the optimal capital income tax.

Present value. We measure wealth in the model based on the book value, a , of individual's assets. This is the approach followed in the related literature as well. However, some of the wealth moments and statistics from the data could potentially be based on the market (or discounted present) value rather than book value of assets. As a result, we have experimented with a case when wealth measures in the model are based on the expected present value of future earnings from the firm, discounted by the average rate of return in the economy. In this version of the model the dispersion of wealth turns out to be higher for given set of parameter values than in the benchmark. Thus, we recalibrate the model and reduce the dispersion in $\bar{z}_i$ ($\sigma_{\varepsilon_{\bar{z}}}$) in order to match the wealth concentration in the top 1%. This recalibration reduces somewhat the welfare gains in all our experiments, although they still remain large.

Comparison to Conesa et al. (2009). One of the major differences between our model and the one studied in Conesa et al. (2009) is the rate of return heterogeneity. For comparison, we eliminate the return heterogeneity by setting $z = 1$ for all individuals and $\mu = 1$. In this case, as we have mentioned earlier, capital income taxes and wealth taxes are equivalent. Column CKK in Table XXI confirms this result – there are no changes in allocations nor any welfare gains from switching to a wealth tax. When we study optimal capital income taxes in this case, we find that the optimal capital income tax rate is 25.4%, which is consistent with the 36% value found in Conesa et al. (2009). This confirms their result that in an OLG model with idiosyncratic labor income risk and incomplete markets, the optimal capital income tax is positive and substantial. However, note that we find a smaller optimal capital income tax in our CKK experiment than they do. The reason for that is that accidental bequests are inherited by newborn individuals in our version of the CKK model while they are distributed equally to the whole population in their framework. Thus, in their framework, newborns start their life with less wealth and as a result prefer a higher tax on capital income, which implies a lower labor income tax. Since the optimal policy maximizes the average utility of newborn, their framework generates a higher optimal capital income tax. We confirm this by distributing accidental bequests equally to all population, in which case the optimal capital income tax increases to 42.4%.

Then why do we find, in our benchmark model with *rate of return heterogeneity*, the optimal capital income tax to be negative and the optimal wealth tax to be positive? In both Conesa et al. (2009) and in our model, a higher capital income tax reduces capital accumulation and leads to lower output. However, in our model, a higher capital income tax hurts productive agents disproportionately, leading to more misallocation, and further reductions in output. Therefore, the capital income tax is much more distortionary in our environment with rate of return heterogeneity than in the environment in Conesa et al. (2009). With a wealth tax, the tax burden is shared between productive and unproductive agents, leading to a smaller misallocation and a lower decline in output as we increase wealth taxes. Thus, the government can increase the wealth tax without reducing output much, allowing it at the same time to reduce the labor income tax resulting in higher after-tax wages and thus higher welfare gains.

Transitions. Our analysis focuses on steady states and makes our results readily comparable to those in important recent papers on capital taxation such as Conesa et al. (2009). Steady-state welfare gains often are due to higher capital stocks, achieved through a transition period during which consumption is lower in order to allow the economy to invest towards building a larger capital stock. Thus, taking the transition period into account would usually lower the computed welfare gains of moving from one steady state to another. In our framework, with rate of return heterogeneity, however, it turns out that in the case of optimal wealth taxes, not capturing the transition period would not change quantitatively our results. We observe that the optimal wealth tax implies only a 2.76% increase in capital stock relative to the benchmark, and as a result the optimal wealth tax does not require much lower consumption during the transition. Most of the gain is coming from a better allocation of capital. The case of an optimal wealth tax with a threshold – which delivers even larger welfare gains – requires an even smaller steady-state increase in capital stock of 0.41%. Therefore, although potentially interesting and worth exploring in future work, incorporating transitions would not dramatically alter our main results.

We should point out that any analysis on the optimal capital income tax would be much more affected by a transition period. The optimal capital income tax requires a 69% increase in the capital stock relative to the benchmark and reaching that steady state requires a large sacrifice in consumption. Therefore, we expect the welfare gain to be much lower in this case.

TABLE XXI – Robustness: **Tax Reform** Experiments

	Baseline	Prog. Labor Tax	No Shock	No Const.	$\mu = 0.8$	Estate Taxes	Present Value	CKK
τ_a	1.13%	0.90%	1.23%	1.65%	1.24%	0.95%	1.26%	1.92%
				Welfare Gain from Tax Reform				
CE_1 (All)	3.14	2.79	2.29	0.44	3.07	3.56	2.47	0
CE_1 (New born)	7.40	6.48	5.46	1.86	7.54	8.22	6.07	0
CE_2 (All)	5.14	4.68	2.92	0.36	5.06	5.85	4.21	0
CE_2 (New born)	7.86	7.06	5.36	1.43	7.85	8.80	6.48	0
Vote (%)	67.7	69.0	68.3	55.9	70.2	68.4	66.9	-
				Percentage Change in Aggregates				
$\overline{k}$	19.37	21.27	9.56	6.28	16.43	21.05	15.60	0
Q	24.79	25.61	22.37	6.28	21.25	27.90	19.87	0
w	8.70	9.25	7.66	2.10	7.77	9.75	7.08	0
Y	10.10	10.01	9.54	3.02	8.38	11.25	8.18	0
L	1.28	0.69	1.75	0.91	0.57	1.37	1.04	0
C	10.01	10.01	11.25	2.93	8.33	11.31	8.17	0

TABLE XXII – Robustness: **Optimal Tax** Experiments

	τ_k	τ_ℓ	ψ	τ_a	Top 1%	$\overline{CE}_2$ (%)	Vote (%)
Baseline Model							
Baseline US	25%	22.4%	1	–	0.36		
Opt. τ_k	–34.4%	36.0%	1	–	0.56	6.28	69.7
Opt. τ_a	–	14.1%	1	3.06%	0.47	9.61	60.7
Prog. Lab. Tax							
Benchmark	25%	15.0%	0.815		0.36		
Opt. τ_k	–38.8%	29.3%	0.720	–	0.61	9.31	52.1
Opt. τ_a	–	12.7%	0.720	2.40%	0.53	10.71	**48.9**
Constant z over life cycle							
Opt. τ_k	–2.33%	29.0%	1	–	0.47	3.27	83.1
Opt. τ_a	–	18.5%	1	2.21%	0.46	5.80	61.6
No Constraint							
Opt. τ_k	13.6%	26.0%	1	–	0.39	0.41	59.9
Opt. τ_a	–	22.7%	1	1.57%	0.42	1.43	56.6
$\mu = 0.8$							
Opt. τ_k	-38.6%	37.7%	1	-	0.52	7.38	67.1
Opt. τ_a	-	18.6%	1	2.12%	0.44	8.32	66.0
Estate Taxes							
Opt. τ_k	-32.2%	33.7%	1	—	0.56	9.26	72.5
Opt. τ_a	-	13.0%	1	3.12%	0.49	11.02	60.7
Present Value							
Opt. τ_k	–18.3%	33.56%	1		0.46	4.16	70.3
Opt. τ_a	–	16.45%	1	2.64%	0.43	7.38	60.4
Conesa et al							
Opt. τ_k	25.4%	22.26%	1	—	0.09	0.46	42.8%
Opt. τ_a	-	22.33%	1	1.93%	0.09	0.25	42.8%

8 Discussions and conclusions

Many countries currently have or have had wealth taxes: France, Spain, Norway, Switzerland, Italy, Denmark, Germany, Finland, Sweden, among others. However, the rationale for such taxes are often vague – fairness, reducing inequality – and not studied formally. Here, we propose a case for wealth taxes based on efficiency and distributional benefits and quantitatively evaluate its impact. In particular, we analyze the quantitative implications of wealth taxation (tax on the stock of wealth) as opposed to capital income taxation (tax on the income flow from capital) in an overlapping-generations incomplete-markets model with rate of return heterogeneity across individuals. With such heterogeneity, capital income and wealth taxes have opposite implications for efficiency and some key distributional outcomes. Under capital income taxation, entrepreneurs who are more productive, and therefore generate more income, pay higher taxes. Under wealth taxation, on the other hand, entrepreneurs who have similar wealth levels pay similar taxes regardless of their productivity, which expands the base and shifts the tax burden toward unproductive entrepreneurs. This reallocation increases aggregate productivity and output. In the simulated model calibrated to the US data, a revenue-neutral tax reform that replaces capital income tax with a wealth tax raises welfare by about 8% in consumption-equivalent terms. Moving on to optimal taxation, the optimal wealth tax is positive, yields even larger welfare gains than the tax reform, and is preferable to optimal capital income taxes. Interestingly, optimal wealth taxes result in more even consumption and leisure distributions (despite the wealth distribution becoming more dispersed), which is the opposite of what optimal capital income taxes imply. Consequently, wealth taxes can yield both efficiency and distributional gains.

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TABLE A.1 – Forbes Self-made Index

	Description	Fraction 2015
1	Inherited fortune but not working to increase it	7.00
2	Inherited fortune and has a role managing it	4.75
3	Inherited fortune and helping to increase it marginally	5.50
4	Inherited fortune and increasing it in a meaningful way	5.25
5	Inherited small or medium-size business and made it into a ten-digit fortune	8.50
6	Hired or hands-off investor who didn't create the business	2.25
7	Self-made who got a head start from wealthy parents and moneyed background	10.00
8	**Self-made who came from a middle- or upper-middle-class background**	**32.00**
9	**Self-made who came from a largely working-class background; rose from little to nothing**	**14.50**
10	**Self-made who not only grew up poor but also overcame significant obstacles**	**7.75**
	Our definition of "Self-made:" Groups 8 to 10	**54.25**

A Additional Tables

B Misallocation in the Benchmark Economy

Our benchmark economy is distorted due to the existence of financial frictions in the form of borrowing constraints, and we can measure the effects of these distortions on aggregate TFP and output and compare them to those obtained in other studies. A large and growing literature frames the discussion on misallocation in terms of various wedges, such as capital, labor, and output wedges. The analysis in Hsieh and Klenow (2009) is particularly useful since, in a similar model environment, they study the degree of misallocation and its effect on TFP in manufacturing in China, India, and the United States. Hsieh and Klenow use detailed firm-level data from the US Census of Manufacturers (1977, 1082, 1987, 1992, and 1997) and find that the TFP gains from removing all distortions (wedges), which equalizes the "Revenue Productivity" (TFPR) within each industry, is 36% in 1977, 31% in 1987, and 43% in 1997.

We will follow the approach in Hsieh and Klenow (2009) and will compute the same measures of misallocation for the US as in their analysis. It is useful to briefly describe their approach as it applies to our framework.[13] The final goods producer behaves competitively and uses an aggregated good, Q, and labor, L, in the production of the final good

$$Y = Q^{\alpha} L^{1-\alpha},$$

[13] Appendix **??** summarizes in great detail our benchmark model environment and the existing financial friction.

where Q aggregates the intermediate goods x_i in the following way

$$Q = \left(\int_i x_i^{\mu} di \right)^{1/\mu}.$$

Each intermediate-goods producer i produces a differentiated intermediate good using the production function $x_i = z_i k_i$, where z_i is the individual i's entrepreneurial ability and k_i is the amount of capital.

Instead of modeling and capturing the effect of a particular distortion, or distortions, the approach of Hsieh and Klenow, and the related misallocation literature, is to infer the underlying distortions and wedges in the economy by studying the extent to which the marginal revenue products of capital and labor differ across firms in the economy (or in a particular industry). This is based on the insight that absent any distortions, the marginal revenue products of capital and labor have to be equalized across all firms.[14]

TFP in the Q sector. We will first focus on the Q-sector, the sector that produces the composite intermediate input Q by aggregating all the intermediate goods x_i. Under this alternative capital-wedge approach, the problem of each intermediate-goods producer is

$$\pi_i = \max_{k_i} p\left(z_i k_i\right) z_i k_i - \left(1 + \tau_i^k\right)\left(R + \delta\right) k_i \ ,$$

where τ_i^k is a firm-specific capital wedge. The only input in the production function of the intermediate-goods producer is capital, and as a result only one wedge can be identified in the analysis. We choose to specify that wedge to be the capital wedge, but in principle it should be understood as capturing the effect of an output wedge.

The revenue TFP in sector Q for each firm i is

$$TFPR_{Q,i} \equiv \frac{p\left(x_i\right) x_i}{k_i} = \frac{1}{\mu}\left(1 + \tau_i^k\right)\left(R + \delta\right).$$

The aggregate TFP in sector Q can be expressed as

$$TFP_Q = \left(\int_i \left(z_i \frac{\overline{TFPR_Q}}{TFPR_{Q,i}} \right)^{\frac{\mu}{1-\mu}} di \right)^{\frac{1-\mu}{\mu}},$$

[14] This is the case in the monopolistic competition models, such as in Hsieh and Klenow (2009). Alternatively, in environments such as in Lucas (1978) and Restuccia and Rogerson (2008), in which firms feature decreasing returns to scale, but produce the same homogeneous good, in the non-distorted economy the marginal products of capital and labor have to be equalized.

where the average $TFPR_Q$ is given by

$$\overline{TFPR_Q} = \left(\int \frac{1}{TFPR_{Q,i}} \frac{p\left(x_i\right) x_i}{p_q Q} di \right)^{-1}.$$

In the non-distorted economy, without capital wedges, the level of TFP in the Q sector is

$$TFP_Q^* = \left(\int_i \left(z_i\right)^{\frac{\mu}{1-\mu}} di \right)^{\frac{1-\mu}{\mu}} \equiv \overline{z}.$$

Therefore, we can measure the improvement in TFP in the Q sector, Ω_Q, as a result of eliminating the capital wedges, or equivalently, as a result of eliminating the borrowing constraints:

$$\Omega_Q = \frac{TFP_Q^*}{TFP_Q} = \left(\int_i \left(\frac{\overline{z}}{z_i} \frac{TFPR_{Q,i}}{\overline{TFPR_Q}} \right)^{\frac{\mu}{1-\mu}} di \right)^{\frac{1-\mu}{\mu}}.$$

Table B.2 reports Ω_Q for various economies—the TFP in the Q sector in the non-distorted economy is 58% higher than in the benchmark economy, 51% higher than in the economy with a wealth tax, 54% higher than in the economy with consumption tax, 49% higher than in the economy with an optimal capital income tax, and 47% higher than in the economy with an optimal wealth tax.

Wealth taxes give the higher TFP gains, allowing for better allocation of capital across firms, even without eliminating the borrowing constraints. The tax reform experiment to wealth taxes implies a TFP gain of 4.6% and optimal wealth taxes give a TFP gain of 7.3% with respect to our benchmark economy.

This can also be seen in the dispersion of TFPR of the different models. Recall that absent any constraints on the firms the TFPR would be equated across all of them, so there is higher misallocation in the economy the higher the dispersion of TFPR across firms. Table B.2 reports the standard deviation of TFPR and some of its percentiles.

TABLE B.2 – Hsieh and Klenow (2009) Efficiency Measure - Benchmark Model

	Benchmark	Tax Reform (τ_a)	Opt. Taxes (τ_k)	Opt. Taxes (τ_a)
TFP_Q	1.001	1.047	1.064	1.074
$\frac{TFP_Q^*}{TFP_Q}$	1.582	1.514	1.489	1.475
Mean TFPR	0.145	0.131	0.106	0.145
StD TFPR	0.054	0.048	0.039	0.053
p99.9	0.68	0.61	0.5	0.66
p99	0.35	0.32	0.27	0.35
p90	0.19	0.17	0.14	0.19
p50	0.14	0.12	0.1	0.14
p10	0.1	0.09	0.07	0.1

Comparison with the Hsieh and Klenow (2009) results for the US

In order to compare these results with the results reported in Hsieh and Klenow (2009) for the US, we need to note that the improvement in aggregate output, Ω_Y, as a result of eliminating the capital wedges in the economy can be expressed as

$$\Omega_Y = \frac{Y^*}{Y} = \left(\frac{TFP_Q^*}{TFP_Q}\right)^{\alpha} \left(\frac{K^*}{K}\right)^{\alpha} \left(\frac{L^*}{L}\right)^{1-\alpha}.$$

Since the model with capital wedges is static, the effect of the removal of the capital wedges on aggregate capital, K, and labor supply, L cannot be taken into account. The analysis in Hsieh and Klenow (2009), measures the improvement in total output as a result of an improvement in TFP in all industries. In our model, this corresponds to the improvement in TFP in the Q sector. Therefore, removing the capital wedges would increase total output, through its effect on TFP in the Q sector, by 20%.[15]

Two things are important to point out. First, the magnitude of the misallocation in our benchmark economy is substantial, although a bit lower than the one measured in Hsieh and Klenow (2009) using micro data from manufacturing firms: 36% in 1977, 31% in 1987, and 43% in 1997. Our benchmark economy is parametrized based on moments from the entire economy, not just the manufacturing sector. Second, our benchmark model is a dynamic model and any changes in the financial frictions will affect aggregate capital accumulation and aggregate labor supply. The misallocation calculations above do not take those changes into account. It is clear, however, that eliminating the financial

[15]Note that $\tilde{\Omega}_Y = \Omega_Q^{\alpha} = \Omega_Q^{0.40} = 1.20$.

friction would increase the aggregate capital stock K and lead a larger increases in total output than measured above. The effect on aggregate labor supply is less obvious.