Structural Change in Investment and Consumption: A Unified Approach*

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Abstract

Existing models of structural change typically assume that all of investment is produced within the goods producing sector. We show that this assumption is strongly counterfactual. In the US economy, services value added account for a growing share of final investment expenditure and are now a larger share of investment than goods value added. We develop and analyze a new model that offers a unified treatment of structural change in both the investment and consumption sectors. Relative to the existing literature our model has distinct answers for the conditions required for balanced growth as well as the asymptotic behavior of growth. In particular, constant TFP growth in all sectors is inconsistent with balanced growth and the sector with slowest TFP growth may absorb all resources.

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JEL classification: O11; O14.

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1 Introduction

A growing literature has proposed extensions of the one-sector growth model in order to jointly study structural change and growth. In this paper we develop and analyze a new model that offers a unified treatment of structural change in both the investment and consumption sectors. We use our framework to revisit three core questions studied in the structural change literature. What properties hold along a balanced growth path? Under what conditions does a balanced growth path exist? And what determines the asymptotic rate of growth in the economy?

The standard framework in the literature on growth and structural change extends the standard one sector growth model to allow for multiple consumption goods. Although this framework allows one to explicitly address structural change within the final consumption sector, it does not allow one to address structural change in the overall economy without additional assumptions regarding activity in the investment sector. The literature has typically proceeded by assuming that all final investment reflects value added in the manufacturing sector.[1] The starting point for our analysis is a broader assessment of the structural transformation facts in the context of the post WWII US during 1947–2016. We first show that the standard assumption is squarely at odds with the data–services constitute a large and growing share of final investment. Second, structural change within the investment sector features the same qualitative patterns as structural change within the consumption sector: we observe an increase in the expenditure share of services at the same time that the relative price of services has increased. And third, the expenditure share of goods is much higher in investment than in consumption.

Motivated by these facts, we develop a new model of structural change and growth in which both final consumption and final investment are produced by combining output from the goods and services sectors. Notably, our model treats the production of final (aggregate) consumption and final (aggregate) investment in a symmetric manner. The framework we develop could be extended to allow for further disaggregation both in terms of final expenditure and value added sectors. For example, it could be of interest to further disaggregate investment into equipment and structures, and to disaggregate services into low and high skilled (see, e.g., Buera and Ka-

[1] See, for example the analysis in Kongsamut et al. (2001), Ngai and Pissarides (2007) and Buera and Kaboski (2009).

boski (2012) and Buera et al. (2015)) or high productivity growth and low productivity growth (see Duernecker et al. (2017)). Here we focus on the simplest version of our framework so as to facilitate exposition. Our model features three sources of technological change: improvements in the TFP of the goods producing sector, improvements in the TFP of the services producing sector, and improvements in the TFP of the technology that combines goods and services to produce investment. This last source of technological change captures the possibility of investment specific technical change.[2]

Having developed the model we proceed to examine its theoretical properties. We characterize the properties of a Generalized Balanced Growth Path for this economy and ask under what conditions a Generalized Balanced Growth Path will exist. The behavior of our unified model differs from the existing literature along two key dimensions. First, we show that constant growth in all sectoral TFPs is inconsistent with generalized balanced growth and structural change. The fact that we model investment as a nonlinear aggregate of goods and services is key to this result. As noted in Herrendorf et al. (2014), nonlinear aggregation in the consumption sector does not create any issues regarding balanced growth in two sector models because balanced growth is effectively anchored by the behavior of investment and capital. In fact, in those models all that matters for balanced growth is constant TFP growth in the investment sector.

Second, our model has distinct implications for the asymptotic growth rate along a balanced growth path. Ngai and Pissarides (2007) found that the sector with slow productivity growth would not take over the entire economy in a model with capital accumulation if investment occurs in a sector with higher productivity growth. This no longer holds in our framework. Despite having capital, our framework is thus consistent with the pessimistic view expressed by Baumol (1967) in which the sector with the slowest productivity growth would come to dominate the entire economy.

Our simple framework features CES aggregators in both the consumption and investment sectors. While somewhat specialized, we show that specification can quantitatively capture the salient features of the secular trends in the US data in the post WWII period, though it requires

[2]Greenwood et al. (1997) argued for the importance of this form of technical change. Our formulation more closely follows Oulton (2007).

that there is little scope for substitution between goods and services in both the consumption and the investment sector. This result extends the earlier analysis of Herrendorf et al. (2013) to the case of structural change within the investment sector. We also carry out an empirical assessment of our theoretical expression stating a necessary for balanced growth. In particular, we estimated all three TFP growth terms using standard methods and evaluate the extent to which our theoretical condition holds and the role of changes in each of the sectoral TFPs. We find that our theoretical condition does hold approximately in the data. But interestingly, it holds despite the fact that the three TFP terms very quite dramatically over time. This supports the view that constant growth of sectoral TFPs should not be assumed in the context of looking for balanced growth paths in multi–sector models.

Our paper is related to several papers in the literature. By considering the possibility of structural change along a balanced growth path it follows the earlier work by Kongsamut et al. (2001), Ngai and Pissarides (2007), and Acemoglu and Guerrieri (2008).[3] Because our analysis abstracts from nonhomotheticities in preferences it more closely resembles Ngai and Pissarides (2007). In fact, as we will discuss later, their framework can be seen as a special case of our framework. Our focus on decomposing investment into the components of goods and services also appears in Garcia-Santana et al. (2016), but they focus on the impact of transition dynamics associated with growth miracles and investment booms on structural change dynamics.

An outline of the paper follows. In the next section we present the key facts from the US time series data. Section 3 presents our model and Section 4 characterizes some properties of the competitive equilibrium. Section 5 studies the features of a GBGP and presents conditions for the existence of a GBGP with structural change. Section 6 examines some aspects of the model and its theoretical properties from an empirical perspective. Section 7 concludes.

[3] More recent contributions include Boppart (2016) and Comin et al. (2015). See also the survey of this literature in Herrendorf et al. (2014).

2 Structural Change in Investment and Consumption: The Evidence

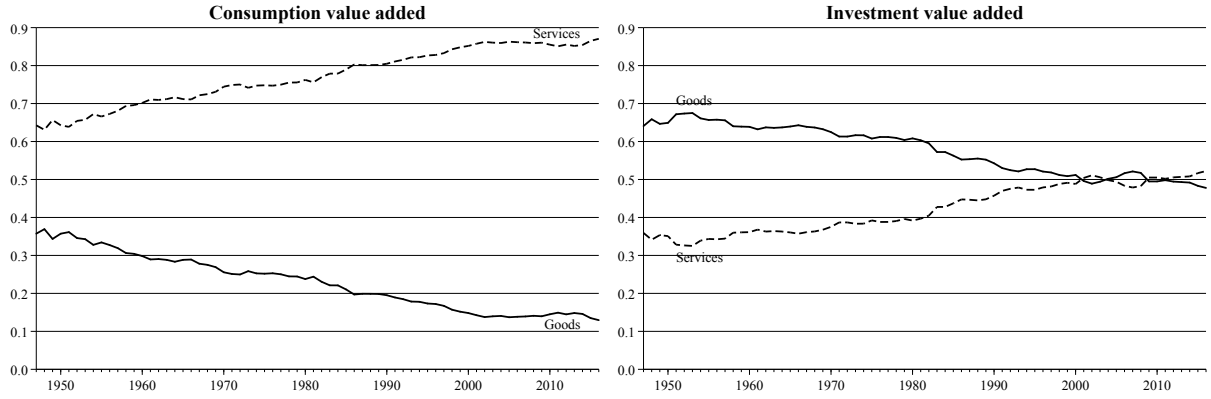
Our goal in this section is to complement existing presentations of the stylized facts of structural change and motivate the framework that we develop. Our analysis offers a unified analysis of structural change facts for both the final consumption and final investment sectors. We combine US industry data from WORLD KLEMS with the annual input-output tables from the BEA to decompose final expenditures into its value added components. In principle one could do this decomposition along many dimensions. But as noted in the introduction, to facilitate exposition and focus attention on the novel implications, in what follows we consider two value added components: goods and services, and two final expenditure categories: consumption and investment.[4]

In implementing these decompositions we define the goods sector to consist of agriculture, mining, manufacturing, public utilities and construction. Services consists of the remaining industries–wholesale and retail trade, transportation, business services, personal services and government. The analysis that we carry out in the following sections will focus on the case of a closed economy. To connect the closed economy model to the data requires that we allocate net exports between consumption and investment. Because net exports are not that large, the rule for allocating them is not of first order significance. In what follows we allocate all of net export to our measure of consumption. The benefit of doing this is that the notion of investment and capital in the model will correspond to the notion of investment and capital as measured in the data. Figure 1 shows the key facts for our two by two decomposition.

Consumption and investment show a similar pattern, with an increase in the services value added share and a decrease in the goods value added share. But both the initial level and the extent of the changes vary significantly across expenditure categories: the services value added share in consumption begins at roughly .60 and increases to about .80, whereas for investment the increase is from around .40 to around .50.

[4]In much of the structural change literature it is common to consider three components of consumption: agriculture, non-agricultural goods (typically referred to as manufacturing) and services. Because we focus on the post WWII US and agriculture is a relatively unimportant component of structural change during this period, we combine agriculture and manufacturing into a single goods producing sector to facilitate exposition.

Figure 1: Sector shares in consumption and investment

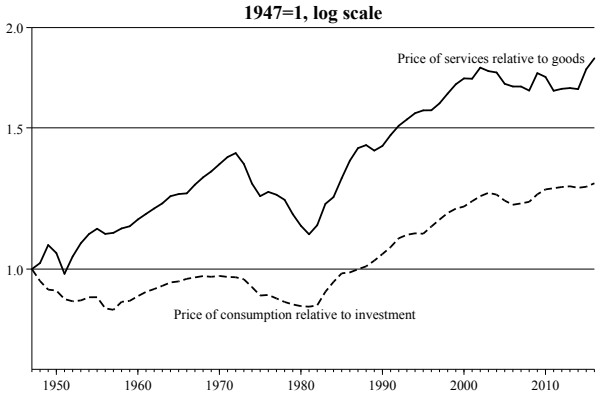


We stress three key properties relative to the existing literature on structural change. First, assuming that investment is produced entirely by the goods sector is strongly at odds with the data; in fact, by the end of the sample the goods value added share is less than the services value added share in investment. Second, the value added shares exhibit important changes over time in both consumption and investment. This suggests that any analysis of structural change at the aggregate level needs to confront the reality that structural change occurs both within the consumption producing sector and the investment producing sector. Third, the value added shares differ significantly between investment and consumption, suggesting that it is important that consumption and investment be modelled separately.

Relative prices will play an important role in the analysis in later sections, and so we also present time series evidence on the relative price of goods and services, as well as the relative price of consumption to investment.

The figures reveal two key secular trends that are familiar from the existing literature, but for completeness are worth repeating here. First, there has been a marked increase in the price of services relative to goods. This increase accelerates in the period after 1980. The somewhat unusual behavior of this relative price in the 1970s is driven by a dramatic spike in the prices of agriculture and oil during the early 1970s. While this suggests that a more detailed analysis might warrant further disaggregation, the somewhat anomalous behavior of the 1970s should not distract us from the clear secular trends over this period. Second, there is also a marked increase in the price of consumption relative to services. Once again, the behavior is quite

Figure 2: Relative prices



distinct in the pre-1970 and post 1980-period, with little trend in the first subperiod and a marked positive trend in the second subperiod. Once again the 1970s show some anomalous behavior, though perhaps less so than in the previous case.

3 Model

The model is a multi–sector version of the standard one sector neoclassical growth model, formulated in continuous time. Motivated by the presentation in the previous section, we will view the production of goods and services as the two core production activities, with consumption and investment in turn produced by combining goods and services. The general approach that we follow is to start with sectoral valued added production functions and view the production for final demand as combining value added from the underlying sectors. Specifically, output of goods and services (denoted by Y_{gt} and Y_{st}, respectively) are each produced according to Cobb-Douglas production functions with the same capital shares but potentially subject to different rates of technological progress:

$$Y_{jt} = A_{jt} K_{jt}^{\theta} L_{jt}^{1-\theta}, \;\; j = g, s,$$

where $0 < \theta < 1$ and the A_{jt} represent exogenous technological progress. At this point we do not place any restrictions on the evolution of technological progress over time. The assumption that

the underlying production functions are Cobb-Douglas with identical capital shares is common to the literature on structural change, as it allows for the existence of a balanced growth path. The case in which the two technologies do not have the same capital share has previously been studied in a different context by Acemoglu and Guerrieri (2008), who show that a balanced growth path in that setting exists only asymptotically. We impose this restriction in order to better relate our results to the literature.

The outputs of the goods and services sectors are in turn used as intermediate inputs to produce final consumption and final investment. Final consumption C_t is a CES aggregate of consumption of goods (C_{gt}) and services (C_{st}):

$$C_t = \left(\omega_{cg}^{\frac{1}{\varepsilon_c}} C_{gt}^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + \omega_{cs}^{\frac{1}{\varepsilon_c}} C_{st}^{\frac{\varepsilon_c - 1}{\varepsilon_c}}\right)^{\frac{\varepsilon_c}{\varepsilon_c - 1}}$$

where $\varepsilon_c \geq 0$ is the elasticity of substitution between goods and services in producing final consumption, and $\omega_{cg}, \omega_{cs} \geq 0$ with $\omega_{cg} + \omega_{cs} = 1$.[5] For our purposes it does not matter whether we think of the household as buying goods and services in the market and combining them inside the home to produce C_t or alternatively assume that a firm purchases goods and services as intermediate inputs, combines them into the aggregate C_t and then sells the aggregate consumption good directly to the household. In what follows we adopt the latter formulation and so will view the above relationship as a production function for a firm that operates in the market.

Symmetrically with the production of aggregate consumption, and motivated by the data reviewed in the previous section, we assume that the final investment good in this economy (X_t) is also produced using inputs of goods and services, but with its own CES aggregator:

$$X_t = A_{xt}\left(\omega_{xg}^{\frac{1}{\varepsilon_x}} X_{gt}^{\frac{\varepsilon_x - 1}{\varepsilon_x}} + \omega_{xs}^{\frac{1}{\varepsilon_x}} X_{st}^{\frac{\varepsilon_x - 1}{\varepsilon_x}}\right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}}$$

where A_{xt} represents investment–specific technological change, $\varepsilon_x \geq 0$ is the elasticity of sub-

[5] Our consumption aggregator is homothetic and so does not exhibit any income effects. As we will see in the next section, and consistent with the existing literature, allowing for income effects does slightly improve the fit of the model to the data. We adopt a homothetic aggregator for consumption to allow us to better focus on the novel implications of our formulation of investment.

stitution between goods and services in producing final investment, and $\omega_{xg}, \omega_{xs} \geq 0$ with $\omega_{xg} + \omega_{xs} = 1$. Importantly, we do not restrict that ε_c and ε_x are the same. While the assumption of CES production functions is analytically convenient, we will show later that these specifications are able to do a good job of capturing the salient empirical patterns presented in the previous section.

There is an infinitely lived representative household with preferences defined by:

$$\int_0^\infty e^{-\rho t} \log(C_t) dt$$

$\rho > 0$ is the discount rate. The household is endowed with one unit of time at each instant, which will be supplied inelastically.

The law of motion for capital is given by:

$$\dot{K}_t = X_t - \delta K_t$$

where $\delta > 0$ is the depreciation rate. The economy begins at time 0 with $K_0 > 0$ units of capital owned by the household. We assume that capital and labor are freely mobile across sectors, so that feasibility for this economy requires:

$$C_{gt} + X_{gt} \leq Y_{gt}$$

$$C_{st} + X_{st} \leq Y_{st}$$

$$K_{gt} + K_{st} \leq K_t$$

$$L_{gt} + L_{st} \leq 1$$

4 Equilibrium

We study the competitive equilibrium for the above economy, assuming that the household accumulates capital and rents it to the firms. For simplicity we assume a representative firm in each of the goods and services sectors, as well as a representative firm that produces each of final consumption and final investment. At each point in time there will be six markets: four

markets for outputs and two factor markets. Factor prices for capital and labor are denoted as R_t and W_t respectively. Prices for goods and services will be denoted by P_{gt} and P_{st}, respectively. The price of the final consumption aggregate will be denoted by P_{ct} and the price of the final investment good will be normalized to unity in each period. Arbitrage implies that the interest rate will be given by $R_t - \delta$. Given that the equilibrium concept is standard, we do not provide a formal definition of equilibrium.

In the remainder of this section we provide a partial characterization of the equilibrium. In the first step we derive analytic expressions for all of the output prices (i.e., prices of all outputs relative to investment) in terms of model primitives. As a by product of the first step we also generate expressions for relative expenditure shares on inputs in each of the final consumption and final investment sectors in terms of model primitives. These derivations rely only on the production side of the economy. In the second step we use these expressions to derive an expression for a (pseudo) aggregate production function that expresses final output evaluated using current prices as a function of aggregate inputs and current TFP. In the third step we derive an expression to characterize the nature of structural change along the equilibrium path.

4.1 Output Prices

We start with the first order conditions for labor and capital for the two firms producing goods and services. For $j = g, s$ these are given by:

$$\theta P_{jt} A_{jt} K_{jt}^{\theta-1} L_{jt}^{1-\theta} = R_t \tag{1}$$

$$(1-\theta) P_{jt} A_{jt} K_{jt}^{\theta} L_{jt}^{-\theta} = W_t \tag{2}$$

Taking the ratio of the two first order conditions for a given sector j and rearranging gives:

$$\left(\frac{K_{jt}}{L_{jt}}\right)^{-1} = \frac{1-\theta}{\theta}\frac{R_t}{W_t} \tag{3}$$

It follows that the capital-labor ratio will be equalized across the two sectors. Given that aggregate labor is unity, it follows that $(K_{jt}/L_{jt}) = K_t$ for all times t, for $j = g, s$. Using this fact, the

two first order conditions for capital imply:

$$P_{gt}A_{gt} = P_{st}A_{st} \tag{4}$$

and hence:

$$\frac{P_{gt}}{P_{st}} = \frac{A_{st}}{A_{gt}}. \tag{5}$$

so that changes in relative prices are the inverse of changes in relative TFP for these two sectors. This is a standard result in the structural change literature when sectors differ only in the extent of TFP growth.

Next we derive relations between the prices of goods and services and the prices of the two final outputs. Because both final output production functions are constant returns to scale and profits must therefore equal zero in a competitive equilibrium, it follows that P_{ct} must equal the cost minimizing unit production cost of a unit of consumption, and similarly that P_{xt}, which is normalized to unity, must be the cost minimizing unit production cost for a unit of investment. It is straightforward to show that these conditions imply:

$$P_{ct} = \left(\omega_{cg}P_{gt}^{1-\varepsilon_c} + \omega_{cs}P_{st}^{1-\varepsilon_c}\right)^{\frac{1}{1-\varepsilon_c}}, \tag{6}$$

$$1 = \frac{\left(\omega_{xg}P_{gt}^{1-\varepsilon_x} + \omega_{xs}P_{st}^{1-\varepsilon_x}\right)^{\frac{1}{1-\varepsilon_x}}}{A_{xt}} \tag{7}$$

These results allow us to fully characterize output prices. Specifically, the zero profit condition for the final investment good imposes a relation between the prices of goods and services. But we previously showed that relative TFPs in the goods and services sectors determine relative prices of goods and services, thus giving us two equations in the two unknown prices.[6]

[6]Alternatively, the zero profit condition for the investment sector can be rewritten as an expression linking the price of goods and the relative price of goods to services. Since the relative prices is determined by relative TFPs, this equation gives us the price of goods in terms of primitives.

Straightforward algebra yields:

$$P_{gt} = \frac{A_{xt}\left(\omega_{xg}A_{gt}^{\varepsilon_x-1} + \omega_{xs}A_{st}^{\varepsilon_x-1}\right)^{\frac{1}{\varepsilon_x-1}}}{A_{gt}} \tag{8}$$

$$P_{st} = \frac{A_{xt}\left(\omega_{xg}A_{gt}^{\varepsilon_x-1} + \omega_{xs}A_{st}^{\varepsilon_x-1}\right)^{\frac{1}{\varepsilon_x-1}}}{A_{st}} \tag{9}$$

Using these in the expression for the price of the final consumption good yields:

$$P_{ct} = \frac{A_{xt}\left(\omega_{xg}A_{gt}^{\varepsilon_x-1} + \omega_{xs}A_{st}^{\varepsilon_x-1}\right)^{\frac{1}{\varepsilon_x-1}}}{\left(\omega_{cg}A_{gt}^{\varepsilon_c-1} + \omega_{cs}A_{st}^{\varepsilon_c-1}\right)^{\frac{1}{\varepsilon_c-1}}} \tag{10}$$

The calculations that yield unit production costs as a function of input prices also generate standard expressions for relative input expenditure in each of the two final output sectors. These are given by:

$$\frac{P_{gt}C_{gt}}{P_{st}C_{st}} = \frac{\omega_{cg}}{\omega_{cs}}\left(\frac{P_{gt}}{P_{st}}\right)^{1-\varepsilon_c} \tag{11}$$

and

$$\frac{P_{gt}X_{gt}}{P_{st}X_{st}} = \frac{\omega_{xg}}{\omega_{xs}}\left(\frac{P_{gt}}{P_{st}}\right)^{1-\varepsilon_x} \tag{12}$$

Given that the price of goods relative to services is the inverse of the TFP in goods relative to services, these last two equations tell us the ratio of expenditure shares in each of the final output sectors in equilibrium as a function of primitive.

4.2 Aggregate Output

Define aggregate final output as measured in units of final investment as $Y_t = X_t + P_{ct}C_t$. It is convenient to derive a pseudo aggregate production function for Y_t. Because payments to inputs will exhaust income in each of the final output producing sectors, we can also write:

$$Y_t = P_{gt}X_{gt} + P_{st}X_{st} + P_{gt}C_{gt} + P_{st}C_{st}$$

Feasibility in turn implies that $X_{gt} + C_{gt} = Y_{gt}$ and $X_{st} + C_{st} = Y_{st}$. Substituting these and using the fact that $P_{gt}A_{gt} = P_{st}A_{st}$ gives:

$$\begin{aligned} Y_t &= P_{gt}A_{gt}K_{gt}^{\theta}L_{gt}^{1-\theta} + P_{st}A_{st}K_{st}^{\theta}L_{st}^{1-\theta} \\ &= P_{gt}A_{gt}\left(K_{gt}^{\theta}L_{gt}^{1-\theta} + K_{st}^{\theta}L_{st}^{1-\theta}\right) \end{aligned}$$

Using the fact that the capital to labor ratio in each sector is equal to the aggregate capital to labor ratio we have:

$$\begin{aligned} Y_t &= P_{gt}A_{gt}\left[L_{gt}\left(\frac{K_{gt}}{L_{gt}}\right)^{\theta} + L_{st}\left(\frac{K_{st}}{L_{st}}\right)^{\theta}\right] \\ &= P_{gt}A_{gt}\left(L_{gt}K_t^{\theta} + L_{st}K_t^{\theta}\right) = P_{gt}A_{gt}K_t^{\theta} \end{aligned}$$

Note that since $P_{gt}A_{gt} = P_{st}A_{st}$ this last line could also be written using the price of services and TFP in the service sector. Either way, using our previously derived expression for P_{gt} or P_{st} in terms of primitives this last expression can in turn be written as:

$$Y_t = A_{xt}\left(\omega_{xg}A_{gt}^{\varepsilon_x - 1} + \omega_{xs}A_{st}^{\varepsilon_x - 1}\right)^{\frac{1}{\varepsilon_x - 1}} K_t^{\theta}. \tag{13}$$

As remarked earlier, this relation expresses aggregate output as a function of aggregate inputs and TFPs.

4.3 Structural Change

The key aspect of structural change that concerns us is the composition of economic activity across goods and services. Structural change can be measured either as changes in sectoral value added shares or changes in sectoral employment shares. We first show that in our model these two measures are identical in equilibrium, so that it does not matter which one we focus on. To see this, we note that the nominal value added shares for goods and services, which we

denote as V_{jt}, $j = g, s$, is given by:

$$V_{jt} = \frac{P_{jt}Y_{jt}}{Y_t} = \frac{P_{jt}Y_{jt}}{P_{gt}Y_{gt} + P_{st}Y_{st}}$$

where the second equality again derives from the fact that with constant returns to scale, payments to factors of production exhaust all income. Substituting sectoral production functions, making use of the fact that the capital to labor ratio in each sector is the same as the aggregate capital to labor ratio as well as that $P_{gt}A_{gt} = P_{st}A_{st}$ we have:

$$V_{jt} = \frac{P_{jt}A_{jt}K_t^{\theta}L_{jt}}{P_{gt}A_{gt}K_t^{\theta}L_{gt} + P_{st}A_{st}K_t^{\theta}L_{st}} = L_{jt}$$

It follows that sectoral nominal value added shares and sectoral employment shares are identical.

In what follows we focus on the behavior of V_{gt}. From above we have:

$$V_{gt} = \frac{P_{gt}Y_{gt}}{Y_t}$$

Using $Y_{gt} = C_{gt} + X_{gt}$ and carrying out some simple manipulations gives:

$$\begin{aligned} V_{gt} &= \frac{P_{gt}C_{gt} + P_{gt}X_{gt}}{Y_t} = \frac{\frac{P_{gt}C_{gt}}{P_{ct}C_t}P_{ct}C_t + \frac{P_{gt}X_{gt}}{X_t}X_t}{Y_t} = \frac{P_{gt}C_{gt}}{P_{ct}C_t}\frac{P_{ct}C_t}{Y_t} + \frac{P_{gt}X_{gt}}{X_t}\frac{X_t}{Y_t} \\ &= \frac{P_{gt}C_{gt}}{P_{gt}C_{gt} + P_{st}C_{st}}\frac{P_{ct}C_t}{Y_t} + \frac{P_{gt}X_{gt}}{P_{gt}X_{gt} + P_{st}X_{st}}\frac{X_t}{Y_t} = \frac{1}{1 + \frac{P_{st}C_{st}}{P_{gt}C_{gt}}}\frac{P_{ct}C_t}{Y_t} + \frac{1}{1 + \frac{P_{st}X_{st}}{P_{gt}X_{gt}}}\frac{X_t}{Y_t} \end{aligned}$$

We previously solved for relative expenditure shares in terms of primitives. For concreteness, assume that each of ε_c and ε_x is less than unity and that technical change is more rapid in the goods sector than in the services sector, so that A_{gt}/A_{st} is increasing over time and P_{gt}/P_{st} is decreasing over time.[7] It follows that within each final output sector we have the standard patterns emphasized in the literature on structural change in the presence of uneven technical change: in both the final investment sector and the final consumption sector, the relative expenditure

[7]We will show later on that the data support each of these assumptions when interpreted through the lens of our model.

share on services will be increasing. Equivalently, an increasing fraction of the total labor that is used to produce the goods and services that are used as inputs into the production of final consumption will be allocated to producing services, and similarly for the investment sector.

However, from the perspective of the overall economy, structural change will also potentially be influenced by the changing composition of final output between investment and consumption. If the final output shares are constant over time, then the above effects would imply that labor is moving systematically from the goods producing sector to the services producing sector. But if there is a change in final output shares between consumption and investment, then one cannot infer the nature of structural change without additional assumptions.

In the next section we will show that along a generalized balanced growth path in which the rental rate of capital is constant, the final output shares are indeed constant, so that the nature of structural change along such a path can be inferred without additional assumptions.

5 Generalized Balanced Growth

Having displayed some of the properties that will hold along any equilibrium path, in this section we examine generalized balanced growth paths. In the first subsection we characterize the properties of such a path, and in the second subsection we study the existence of such a path.

5.1 Characterizing A Generalized Balanced Growth Path

We define a generalized balanced growth path to be an equilibrium in which the rental rate of capital is constant.[8] In this subsection we consider the properties of such a path. To pursue this we will also have to make use of the conditions that come from the household side of the equilibrium. The household seeks to maximize the present discounted integral of utility subject to its budget equation:

$$P_{ct}C_t + \dot{K}_t + \delta K_t = R_t K_t + W_t \tag{14}$$

[8]This is the same property considered by both Kongsamut et al. (2001) and Ngai and Pissarides (2007).

Defining total consumption expenditure in period t, $P_{ct}C_t$, as E_t, the Euler equation for the household can be written as:

$$\frac{\dot{E}_t}{E_t} = R_t - \delta - \rho \tag{15}$$

and the transversality condition is given by:

$$\lim_{t\to\infty} e^{-\rho t}\frac{K_t}{E_t} = 0 \tag{16}$$

The first proposition establishes that if a generalized balanced growth path exists, it must be the case that E_t, Y_t, K_t, and X_t all grow at the same constant rate. For future reference we note that whereas both K_t and X_t are physical quantities, the other two terms – E_t and Y_t – include both prices and quantities.

Proposition: Assume that a generalized balanced growth path exists. Then E_t, Y_t, K_t, and X_t must all grow at the same rate.

Proof: Note first that if R_t is constant, the household's Euler equation (15) implies that E_t must grow at a constant rate. Denoting this rate by γ, we have that γ satisfies:

$$\gamma = R - \delta - \rho.$$

Second, because both the goods and services sectors have Cobb-Douglas production functions with the same capital share, total payments to capital will be a fraction θ of total output: $RK_t = \theta Y_t$. A constant value for R thus also implies that Y_t/K_t is constant. Using this expression to substitute for R in the Euler equation gives:

$$\frac{Y_t}{K_t} = \frac{\gamma + \delta + \rho}{\theta} \tag{17}$$

The fact that Y_t and K_t grow at the same rate does not necessarily imply that they grow at constant rates. But we next show that along a GBGP it is indeed the case that K_t grows at a constant rate, and furthermore that this rate is γ. Using $\dot{K}_t = X_t - \delta K_t$ with the resource constraint $X_t = Y_t - E_t$ gives:

$$\frac{\dot{K}_t}{K_t} = \frac{Y_t}{K_t} - \delta - \frac{E_t}{K_t}$$

Substituting for Y_t/K_t using equation (17) gives:

$$\frac{\dot{K}_t}{K_t} = \frac{\gamma + \delta + \rho}{\theta} - \delta - \frac{E_t}{K_t} \tag{18}$$

To show that $\dot{K}_t/K_t$ is constant and equal to γ, we argue by way of contradiction. First, assume that at some time T that $\dot{K}_t/K_t > \gamma$. Since E_t grows at rate γ this implies that E_t/K_t is decreasing. Equation (18) then implies that $\dot{K}_t/K_t$ is increasing, so that $\dot{K}_t/K_t$ will exceed γ for all t greater than T. It follows that the limit of E_t/K_t must be zero and hence that:

$$\lim_{t\to\infty} \frac{\dot{K}_t}{K_t} = \frac{\gamma + \delta + \rho}{\theta} - \delta > \gamma + \rho$$

But this implies that the transversality condition is violated, since it requires that K_t grow at a rate less than $\gamma + \rho$.

Suppose alternatively that at some time T we have $\dot{K}_t/K_t < \gamma$. Arguing as above, E_t/K_t is now increasing, implying that $\dot{K}_t/K_t$ must be decreasing, so that $\dot{K}_t/K_t$ will be less than γ for all $t > T$. This implies that E_t/K_t will tend to infinity and hence that:

$$\lim_{t\to\infty} \frac{\dot{K}_t}{K_t} = -\infty$$

It follows that the growth rate of K_t is negative and bounded away from zero beyond some finite date, implying that K_t must become negative in finite time. This violates feasibility.

We conclude that K_t grows at the constant rate γ. Since Y_t grows at the same rate as K_t it follows that Y_t also grows at the same constant rate γ. It remains to show that X_t grows at rate γ. Combining $\dot{K}_t = X_t - \delta K_t$ with the fact that K_t grows at the constant rate γ gives:

$$\frac{X_t}{K_t} = \gamma + \delta,$$

so that X/K is constant. It follows that X_t must also grow at the constant rate γ. **QED**

Having established the properties of a GBGP we can now revisit the issue of structural change along the GBGP. As noted in the previous section, although knowing the direction of relative price changes for goods and services combined with the elasticity of substitution

between goods and services in production is sufficient to determine the nature of structural change for the production of final consumption and final investment individually, changes in the composition of final output between consumption and investment could create opposing effects. But what we have just shown is that along a GBGP, the composition of final output is constant, and thus there is no effect of changing composition. It follows that if both of the ε_j are between 0 and 1 and A_{gt} grows faster than A_{st} that both the value added share and the employment share of the services sector will increase over time.

5.2 Existence of a GBGP

The previous subsection characterized the properties that would hold along a GBGP. But under what conditions will such a path exist? This is the question that we take up in this subsection. In the previous section we established that along a GBGP, Y_t and K_t must grow at the same constant rate. To understand the significance of this it is useful to recall our derivation of an aggregate production function:

$$Y_t = A_{xt}\left(\omega_{xg}A_{gt}^{\varepsilon_x-1} + \omega_{xs}A_{st}^{\varepsilon_x-1}\right)^{\frac{1}{\varepsilon_x-1}} K_t^{\theta}$$

From this expression we see that if Y_t and K_t are both to grow at a constant rate then it must be the case that $A_{xt}(\omega_{xg}A_{gt}^{\varepsilon_x-1}+\omega_{xs}A_{st}^{\varepsilon_x-1})$ also grows at a constant rate. If ε_x is equal to unity then this simply requires that the rate of investment–specific technological change is constant. This is effectively the same result that obtains in a two-sector model that has separate consumption and investment sectors and generalizes the analysis in Herrendorf et al. (2014). But in this case there is no structural change in the investment sector. If we assume that ε_x is not equal to one, then in general at most two of the three growth rates of A_{xt}, A_{gt}, and A_{st} can be constant. Specifically, if we were to restrict attention to constant rates of investment–specific technological change and of technological change in both the goods and services sector, this condition could only hold if both the goods and services sectors experienced the same rate of technological progress. But if this is the case then there are no changes in the relative price of goods and services and hence there will be no structural change.

While the literature on structural change and balanced growth has tended to focus on the case in which sectoral TFPs grow at constant rates (see, for example, the papers by Kongsamut et al. (2001) and Ngai and Pissarides (2007)) there is no natural reason to focus on this case. In a one sector model, constant growth at the aggregate level necessarily requires constant growth in aggregate TFP. But in a multi–sector model with non-linear aggregators, the fact that certain aggregates grow at (approximately) constant rates does not translate to (approximately) constant rates of growth in sectoral TFPs. In fact, from the previous subsection, along a GBGP both X_t and E_t will grow at the same constant rate. If these conditions hold and structural change is occurring, then it follows that sectoral TFPs *cannot* all be growing at constant rates. These conditions can be checked in the data, and we will do this in the next section.

5.3 Asymptotic Behavior

Baumol (1967) was the first to note that if sectors experience differential productivity growth and resources systematically move from high productivity growth sectors to low productivity growth sectors, the economy will exhibit a secular decline in aggregate productivity growth. Taken to the extreme, this argument suggested that if there is one sector with no productivity growth, it will eventually dominate the entire economy. Importantly, Baumol's formal analysis abstracted from capital. Ngai and Pissarides (2007) showed that this implication is overturned if one introduces capital and assumes that capital is produced solely by one of the sectors with positive or high productivity growth. Loosely speaking, because the capital to output ratio grows at a constant rate along a balanced growth path, the capital producing sector cannot disappear, even asymptotically.

But key to this result in Ngai and Pissarides is that capital is produced solely by one sector. In our framework, motivated by the evidence in Section 2, this condition is no longer met. To see why, recall that we previously established two properties along a balanced growth path. First, nominal value added shares are the same as employment shares, and second, that the capital to output ratio is constant, where output is measured in units of investment. Now consider the special case of our model in which only goods are used to produce investment. This corresponds to the assumptions in Ngai and Pissarides. The second property above implies that

investment is a constant share of output, and given the production structure, the first observation implies that the employment share of the goods sector is therefore also constant. That is, the goods sector will not disappear asymptotically. It is true that all labor used to produce the consumption good will asymptotically be allocated to the service sector.

In contrast, consider our more general specification in which both goods and services are used to produce investment. Additionally, in the spirit of the original analysis by Baumol, assume that $\varepsilon_x < 1$, $\varepsilon_c < 1$, $\dot{A}_{gt} > \dot{A}_{st}$ and that A_{gt} is bounded away from A_{st}. For concreteness assume that A_{xt} is constant. In this case, while it remains true that investment will be a constant share of output, the value added share of services will approach one for both final consumption and final investment, implying that all labor will asymptotically be in the service sector.

6 Some Empirical Analysis

In this section we complement the preceding theoretical analysis of the model of Section 3 with an empirical assessment of two issues. First, to what extent are the CES aggregators in our formulation able to capture the secular trends in the post WWII US data? And second, to the extent that we observe something that looks like approximate balanced growth in the data, what is the nature of the technological change that accounts for this. Put somewhat differently, what is the behavior of the various terms in the theoretical condition necessary for generalized balanced growth?

We begin with the first question. Our analysis focuses on the expressions for goods and services expenditure shares by final expenditure sector that we previously derived, which we repeat here for convenience:

$$\frac{P_{gt}C_{gt}}{P_{st}C_{st}} = \frac{\omega_{cg}}{\omega_{cs}}\left(\frac{P_{gt}}{P_{st}}\right)^{1-\varepsilon_c} \tag{19}$$

and

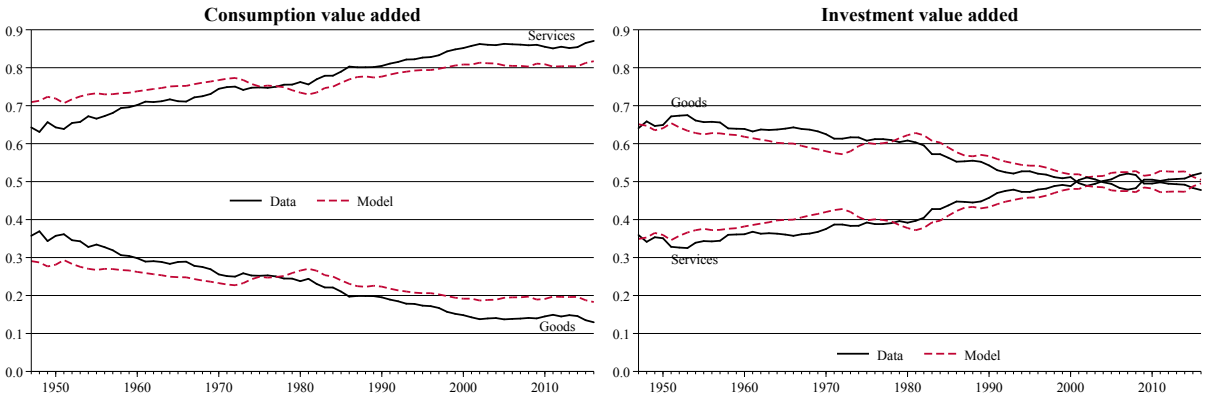
$$\frac{P_{gt}X_{gt}}{P_{st}X_{st}} = \frac{\omega_{xg}}{\omega_{xs}}\left(\frac{P_{gt}}{P_{st}}\right)^{1-\varepsilon_x} \tag{20}$$

To assess whether the assumed CES structure is empirically reasonable for our purposes we

Table 1: Preference and technology parameters

ε_c	ω_{cg}	ω_{cs}	ε_x	ω_{xg}	ω_{xs}
0.00	0.19	0.81	0.00	0.52	0.48

Figure 3: Expenditure shares – data versus model



ask whether there are values for the share and elasticity parameters (the ω_{ij} and ε_j) such that when taking relative prices as given by the data are able to capture the key secular changes in expenditure shares. To implement this we use the data as presented in Section 2 and estimate the share and elasticity parameters via iterated generalized least squares. The estimated values are given in Table 1. This is the same procedure that we used in Herrendorf et al. (2013).

The striking result is that both elasticity parameters are estimated to be zero, implying that both aggregators are Leontief. The result that $\varepsilon_c = 0$ is not surprising given the results obtained earlier in Herrendorf et al (2013). But the estimation results imply that the aggregator in the investment sector features a similarly low degree of substitutability.[9] Figures 3 shows the fitted values for the expenditure shares along with their corresponding values in the data.

The fitted values track the secular change within the investment sector very well, whereas the fitted values for consumption explain only about half of the change in expenditure shares within consumption. As we noted earlier, the latter result is not entirely surprising since we have

[9]We note that the estimation procedure moves to the lower boundary of the allowable parameter space for both elasticity parameters and so standard errors are not available.

abstracted from income effects by virtue of considering a homothetic aggregator and our earlier work that focused solely on consumption did find a role for income effects.[10] Additionally, recall that we have aggregated agriculture and manufacturing into a single goods sector and that nonhomotheticities are typically found to be important for the decline of agriculture. The fit for investment is of greater interest in the current context given that this is the more novel aspect of our framework, and the figures show that our CES aggregator does a very good job of capturing the secular change in the US over this time period.

Next we consider the necessary condition for balanced growth–that $A_{xt}(\omega_{xg}A_{gt}^{\varepsilon_x-1}+\omega_{xs}A_{st}^{\varepsilon_x-1})$ grow at a constant rate– from an empirical perspective. To do this we use the estimated values for ω_{xg} and ω_{xs} from the previous exercise and compute series for the $A'_{jt}s$ using standard growth accounting methods as in Solow (1957). To solve for A_{gt} and A_{st} we use labor input in efficiency units from the WORLD KLEMS data, capital stock data from the fixed asset series produced by the BEA, and factor shares from WORLD KLEMS. To estimate TFP growth in the investment sector we use:[11]

$$\frac{\dot{X}_t}{X_t}=\frac{\dot{A}_{xt}}{A_{xt}}+\frac{P_{gt}X_{gt}}{X_t}\frac{\dot{X}_{gt}}{X_{gt}}+\frac{P_{st}X_{st}}{X_t}\frac{\dot{X}_{st}}{X_{st}}$$

from which we compute the implied growth rate series for A_{xt}. Note that we do not impose our functional forms when computing the TFP series.

Figure 4 shows the implied series for the level of each TFP term.

Given our estimates of ω_{xj}, $j=g,s$ and ε_x, plus our estimated series for the A_{jt}, $j=x,g,s$, we evaluate the value of $A_{xt}\left(\omega_{xg}A_{gt}^{\varepsilon_x-1}+\omega_{xs}A_{st}^{\varepsilon_x-1}\right)^{\frac{1}{\varepsilon_x-1}}$. Recall that constant growth in this quantity is necessary in order to have a GBGP. Figure 5 plots the log of this value over time.

The figure shows that this quantity indeed grows on average at a constant rate over the postwar period. We emphasize two points. First, the construction of this figure did not directly use the information that the average growth of chained GDP in the US data is approximately constant. Second, although this terms does grow at an approximately constant rate over the time

[10] We note that adding net exports to consumption tends to worsen the fit of the model.

[11] The continuous–time divisia index was approximated with the Törnqvist index in the data.

Figure 4: Sectoral TFPs

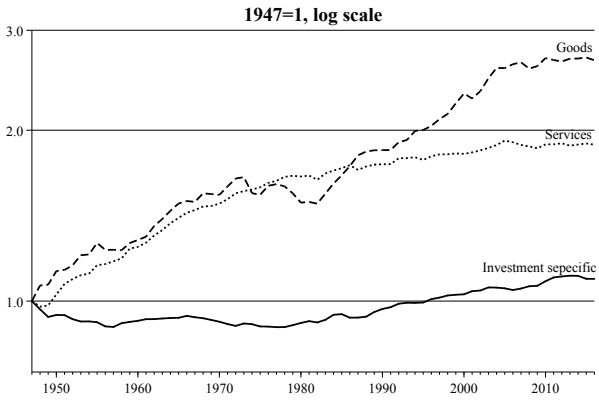
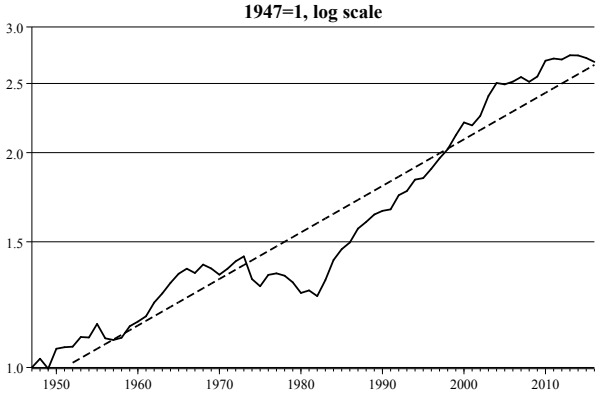


Figure 5: Aggregate TFP



period being studied, none of the individual TFP terms exhibits approximate constant growth. In particular, while the growth in A_{st} declines over time, the growth in A_{xt} actually increases over time.

7 Conclusion

In this paper we have proposed a new framework for studying structural change and growth that treats consumption and investment in a symmetric manner. In particular, we model production at the level of sector value added and treat both final consumption and final investment as aggregates of the underlying sectoral value added. We study a simple form of this framework in which we have two final demand sectors–consumption and investment, and two underlying sectors producing value added–goods and services. We focus on this simple framework to best illustrate the distinctive features of this framework. We derive conditions to guarantee the existence of a generalized balanced growth path, and in particular show that constant growth in all sectoral TFPs is generically inconsistent with balanced growth. We also show that the sector with the lowest productivity growth may asymptotically dominate the entire economy. We also show that a version of the model with a CES aggregator in the investment sector can account for the salient trends in the value added composition of final investment.

We believe that richer versions of this model will prove useful in refining our view of the nature of structural change and the forces that drive it. In particular, we think it will be of interest to separately model structures and equipment within the investment sector. To the extent that goods represent a higher share of investment in equipment than they do in final consumption, it is interesting to ask whether the higher TFP growth in equipment is truly specific to equipment or if it instead reflects higher TFP growth in goods combined with a higher share of goods. Similarly, we think it is of interest to consider further decompositions of the service sector. We believe this framework will help us better isolate the underlying sources of TFP growth.

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