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Financing Ventures*

by

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Abstract

The relationship between venture capital and growth is examined using an endogenous growth model incorporating dynamic contracts between entrepreneurs and venture capitalists. At each stage of financing, venture capitalists evaluate the viability of startups. If viable, VCs provide funding for the next stage. The success of a project depends on the amount of funding. The model is confronted with stylized facts about venture capital; viz., the average cash-on-cash multiple and statistics by funding round concerning the success rate, failure rate, investment rate, equity shares, and the value of an IPO. Raising capital gains taxation reduces growth and welfare.

Keywords: capital gains taxation, dynamic contract, endogenous growth, evaluating, funding rounds, growth regressions, IPO, monitoring, startups, research and development, venture capital



*Address correspondence to Juan M. Sanchez at vediense©gmail.com. This write-up is on research in progress and hence is preliminary and incomplete.

1 Introduction

The importance of venture capital in the U.S. economy has skyrocketed over the last 50 years. Investment by venture capitalists was roughly $303 million in 1970. This soared to $59 billion by 2015 (both numbers are in 2009 dollars). The rise in venture capital (VC) financing is shown in the right-hand side panel of Figure 1. While the share of VC funding in total investment is still relatively small, around 2 percent in 2015, its punch far exceeds its weight. The fraction of public firms that have been backed at some time by VCs is now around 20 percent, compared with just 4 percent in 1970–see the left-hand side panel of Figure 1. Such firms presently account for about 20 percent of market capitalization. Today VCs are a significant player in job creation and technological innovation. Public firms that were once backed by VCs currently make up a significant portion of employment and an even larger share of R&D spending, as opposed to virtually nothing in 1970, as the left-hand side panel of Figure 2 makes clear. The right-hand side of the figure displays their enormous contribution to the generation of patents, both in raw and quality-adjusted terms.

The VC industry has been an incubator of numerous breathtaking technological giants in the information and communication technology sector as well as the biotechnology sector, plus a dazzling array of innovating stars in the service industry. Former VC-backed firms are household names. Table 1 lists the top 30 VC-backed public companies by market capitalization. Figure 3 plots the relative significance of "banks" and "venture capital," as reflected by the usage of these terms in English language books. As can be seen, the term venture capital was virtually unused in 1930. The relative significance of venture capital vis-à-vis banks has increased considerably since then.

To address the importance of venture capital in the U.S. economy, an endogenous growth model is developed. At the heart of the growth model is a dynamic contract between an entrepreneur and a venture capitalist. The venture capitalist invests in the entrepreneur's startup as an active participant. He evaluates the worthiness of the project stage by stage and invests according. The contract is designed so that it is not in the entrepreneur's interest to divert funds away from their intended purpose. The venture capitalist can imperfectly

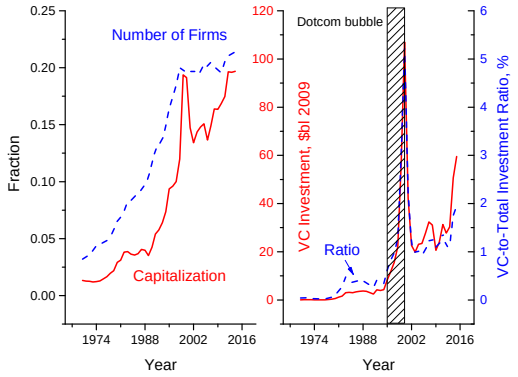


Figure 1: The rise in venture capital, 1970 to 2015. The right-hand side panel shows investment by venture capitalists. The left-hand side panel plots both the fraction of public firms financed by venture capitalists and the share of VC-backed public firms in market capitalization. See the Data Appendix for the sources of all data used in the paper.

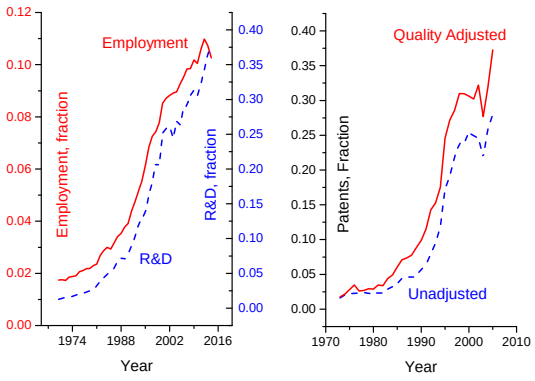


Figure 2: The share of VC-backed firms in employment, R&D spending, and patents. The data in the left-hand side panel is from 1970 to 2014, while that in the right-hand panel spans 1973 to 2005.

Top 30 VC-Backed Companies

1	Apple Inc	11	Amgen Inc	21	Fedex Corp
2	Cisco Systems Inc	12	Yahoo Inc	22	Juniper Networks Inc
3	Microsoft Corp	13	Genentech Inc	23	Nextel Communications Inc
4	Alphabet Inc	14	Celgene Corp	24	Gap Inc
5	Facebook Inc	15	Ebay Inc	25	Viacom Inc
6	Oracle Corp	16	Compaq Computer Corp	26	Veritas Software Corp
7	Amazon.Com Inc	17	Starbucks Corp	27	Salesforce.Com Inc
8	Sun Microsystems Inc	18	Micron Technology Inc	28	Alexion Pharmaceuticals Inc
9	Gilead Sciences Inc	19	Applied Materials Inc	29	Adobe Systems Inc
10	Dell Inc	20	Regeneron Pharmaceuticals	30	Twitter Inc

Table 1: The table shows the top 30 VC-backed companies by market capitalizaton. These companies are identified by matching firm names in VentureXpert with CompuStat.

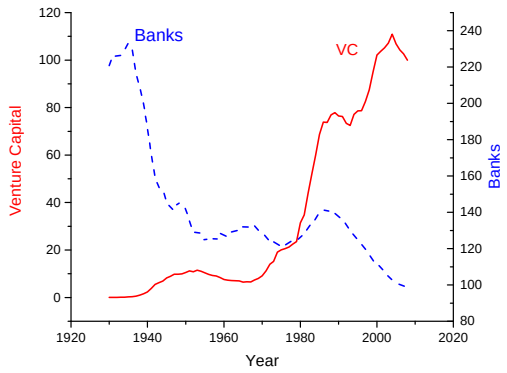


Figure 3: Banks and Venture Capital, 1930-2008. The figure plots the use of the words "banks" and "venture capital" in English language books using the Google Ngram Viewer. For each series, the value in 2008 is normalized to 100.

monitor at a cost the entrepreneur's use of funds and this helps to ensure incentive compatibility. The contract specifies by funding round the amount of investment that the venture capitalist will do, the evaluation strategy to gauge the project's worthiness, the level of monitoring to avoid malfeasance, and the shares of each party's equity in a potential IPO. The predicted features of the contract are compared with some stylized facts about venture capital: (i) the average cash-on-cash multiple, (ii) the success and failure rates by funding round, (iii) investment by funding round, (iv) the value of an IPO by duration of the project, and (v) the venture capitalist's share of equity by funding round. Despite the importance of venture capital, the majority of firms in the U.S. economy are not financed through this channel. So, the analysis includes a traditional sector that produces the majority of output using capital that can be thought of as being financed through regular banks. The key participants in a venture capital partnership receive the majority of their compensation in the form of stock options and convertible equity. As such, they are subject primarily to capital gains taxation. The analysis examines how innovative activity is affected by the capital gains tax rate.

Dynamic contract models have now been used for some time to study consumption/savings cum effort decisions with moral hazard. An early example is Phelan and Townsend (1991), with more recent work being represented by Karaivanov and Townsend (2014). Dynamic contract frameworks that focus on firms, and venture capital in particular, are rarer. On this, Bergemann and Hege (1998), Clementi and Hopenhayn (2006), and Cole, Greenwood, and Sanchez (2016) develop contracting structures that share some similarities with the one presented here. In Bergemann and Hege (1998) a venture capitalist also learns about a project's type, good or bad, over time. The odds of a good project's success are a linear function of investment. The entrepreneur can secrete some of funds intended for investment, so there is a moral hazard problem. Given the linear structure of their model, which generates corner solutions, analytical results obtain. In an extension, the venture capitalist can monitor investment or not. If he monitors, then any irregularities are uncovered with certainty. The analysis is done in partial equilibrium. While illuminating some economics about

venture capital, it would be hard to take their streamlined structure to the data. While not focusing on venture capital, the Clementi and Hopenhayn (2006) model also reformulates as one where an entrepreneur can secrete investment. The lender cannot monitor the borrower. Again, the analysis is done in partial equilibrium.

The current paper borrows Cole, Greenwood, and Sanchez's (2016) flexible monitoring technology. The more the VC invests in auditing the higher are the odds that he will detect any irregularities. The VC can also invest in evaluating a project each period to learn about its type, good or bad, something not allowed in Bergemann and Hege (1998). This feature is important because it allows the odds that a project is good to rise over time. This works to generate an upward sloping funding profile over time. The odds of a good project's success are an increasing, concave function of investment in development. Additionally, venture capital is taken to be a competitive industry; this is similar to Cole, Greenwood, and Sanchez's (2016) assumption that financial intermediation, more generally, is competitive.

Additionally, the current analysis is done within the context of an endogenous growth model. Cole, Greenwood, Sanchez (2016) focus on the impact that financial intermediation, more broadly defined, has on cross-country technological adoption and income levels. As in Akcigit, Celik, and Greenwood (2016), there is a distribution of competitive firms operating in general equilibrium. This distribution is continually shifting rightward with technological progress in the economy. A new entrepreneur decides how far to push his productivity relative to the frontier; this is somewhat reminiscent of Parente (1994). The position of the frontier is determined by a classic Romer (1986) type externality. The last three papers have no startups. None of the above papers compare the predictions of their models with the venture capital process in the United States. And none of them examine how innovative activity is affected by the rate of capital gains taxation.

There is, of course, work on venture capital that does not take a dynamic contract perspective. Silveira and Wright (2016) build a canonical search model of the process where entrepreneurs are matched with VCs, something abstracted from here. Upon meeting, the parties bargain in Nash fashion over the each one's investment and how to split the pro-

ceeds. Jovanovic and Szentes (2013) focus on a setting where the incubation period for a project is unknown. Unlike entrepreneurs, VCs have deep pockets and can weather supporting a project over a prolonged period of time, if they so choose. A contract specifies the initial investment by the VC and some fixed split of the profits. The analysis focuses on characterizing and measuring the excess return earned by VCs, due to their scarcity.

2 The Rise of Venture Capital as Limited Partnerships

Financing cutting-edge technologies has always been problematic.[1] It is difficult to know whether new ideas are viable, if they will be saleable, and how best they should be brought to market. Also, it is important to ensure that entrepreneurs' and investors' incentives are aligned. Traditional financial institutions, such as banks and equity/securities markets, are not well suited to engage in this sort of finance. Historically speaking, the introduction of new technologies was privately financed by wealthy individuals. The investors were plugged into networks of inventive activity, which they used to learn about new ideas, vet them, and draw on the expertise needed to operationalize them.

The Brush Electric Company provided such a network for inventors and investors in Cleveland around the turn of the 20th century. Electricity was one of the new inventions that was born during the Second Industrial Revolution. Individuals linked with the Brush Electric Company network spawned ideas for arc lighting, liquefying air, smelting ores electrically, electric cars and trolleys, among other things. The shops at Brush Electric were a meeting place for inventors. They could develop and debug new ideas with help from others. Investors connected with the Brush network learned about promising new ideas from the scuttlebutt at the shops. They became partners/owners in the firms that they financed. Interestingly, in the mid-West at the time, prolific inventors (those with more than 15 patents) who were principals in companies were much more likely to keep their patents or assign them to the company where they were principals as opposed to other types of inventors, who typically

[1] This section draws *heavily* on Lamoreaux, Levenstein, and Sokoloff (2007) for the period prior to World War II and on Kenney (2011) for the one afterward.

sold them to businesses where they had no concern. This aligned the incentives of innovators and investors.

World War II and the start of the Cold War ushered in new technologies, such as jets, nuclear weapons, radars, rockets, etc. There was a splurge of spending by the Defense Department. A handful of venture capital firms were formed to exploit the commercialization of scientific advances. American Research and Development (ARD), founded by General Georges Doriot and others, was one of these. ARD pulled in money from mutual funds, insurance companies, and through an initial public stock offering. The founders knew that it was important for venture capitalists to provide advice to the fledging enterprises in which they were investing. In 1956 ARD invested $70,000 in Digital Equipment Corporations (DEC) in exchange for a 70 percent equity stake. ARD's share was worth $38.5 million when DEC went public in 1966, which represented an annual return of 100 percent. While this investment was incredibly successful, the organizational form of ARD did not come to dominate the industry. The compensation structure of ARD made it difficult for the company to retain the venture capital professionals needed to evaluate startups and provide the guidance necessary for success.

An alternative organizational form came to emblemize the industry; viz., the limited partnership. This is exemplified by the formation of Davis and Rock in 1961. These partnerships allowed venture capital professionals to share in the gains from startups along with the entrepreneurs and investors. Limited partnerships served to align venture capitalists' interests along with those of entrepreneurs, investors, and key employees. Money was put in only at the beginning of the partnership. The general partners received management fees as a salary, plus a share of the capital gains from the investments, say 40 percent, with the limited partners earning 60 percent. The limited partners had no say in the decisions of the general partners. The partnerships were structured for a limited length of time, say 7 to 10 years. The returns from the partnership were paid out to the investors only when the partnership was dissolved–there were no dividends, interest payments, etc. Therefore, the returns upon dissolution were subject only to capital gains taxation at the investor level.

The VC industry also rewarded founders, CEOs and key employees using stock options. Thus, they, too, were subject to capital gains taxation and not taxation on labor income. The short time horizon created pressure to ensure a venture's success rapidly.

Banks and other financial institutions are not well suited to invest in cutting-edge new ventures. While banks are good at evaluating lending risk, they have limited ability to judge the skill of entrepreneurs, the worth of new technologies, and the expertise to help commercialize them. The Glass-Steagll Banking Act of 1933 prohibited them from taking equity positions in industrial firms–the act was repealed in 1999. Allstate Insurance Company created a private placements program in the 1960s to undertake venture capital type investments. It abandoned the program because it could not compensate the venture capital professionals enough in order to retain them. The Employee Retirement Income Security Act of 1974 prevented pension funds (and dissuaded other traditional fiduciaries) from investing in high-risk ventures. The act was reinterpreted in the 1980s to allow pension funds to invest in venture capital operating companies, which provided a fillip for the VC industry.

3 Empirical Evidence on Venture Capital and Firm Performance

How does VC affect firm growth and technological innovation? The VC industry is a successful incubator of high-tech and high-growth companies. VC-backed public companies have higher R&D-to-sales ratios than their non-VC-backed counterparts. Following an IPO, they also grow faster in terms of employment and sales. VC-backed companies are embraced as the "golden geese" by the investors. They are valued higher than their non-VC-backed counterparts around the time of an IPO. In addition, VC is a potent apparatus for financing technological innovation. VC funding is positively associated with patenting activity by firms. Moreover, patenting depends more on VC funding in those industries where the dependence on external financing is high.

3.1 Venture Capital and Firm Growth

Some regression analysis is now undertaken to evaluate the performance of VC-backed and non-VC-backed firms along four dimensions for the year after an IPO: the R&D-to-sales ratio, the growth rate of employment, the growth of sales revenue, and the market value of firms. The results are presented in Table 2. The regressions are based on an unbalanced panel of U.S. public companies between 1970 and 2014. To compare VC-backed companies with their non-VC-backed counterparts, a VC dummy is entered as an independent variable that takes the value of one, if the company is funded by VC before its IPO. In all regressions, industry dummies, year dummies, and a year dummy for the IPO are included. In addition, a cross term is added between the VC dummy and the number of years since the firm's IPO.

As shown by the first row of regression coefficients, VC-backed companies are more R&D intensive and grow faster than their non-VC-backed counterparts. On average the R&D-to-sales ratio of a public VC-backed company is higher than its non-VC-backed counterpart by 5.2 percentage points, and it grows faster by 4.9 percentage points in terms of employment and 7.0 percentage points in terms of sales revenue. These superior performances translate into higher market values: VC-backed companies are valued 37.3 percent higher than their non-VC-backed counterparts. The difference in performance, however, gradually dwindles over the years, as can be seen from the negative signs of the regression coefficients in the second row. As a consequence, the performance of VC- and non-VC-backed public companies tend to converge in the long run, though the speed of convergence is fairly low, as revealed by the magnitude of the regression coefficients on the second row.

3.2 Venture Capital and Innovation

Some regression analysis is now undertaken to assess the role of VC in encouraging technological innovation. Specifically, the impact of VC funding on patent performance at an annual periodicity is evaluated, both at the firm and industry level. The regression analysis is based on all companies funded by venture capitalists between 1970 and 2015. These VC-funded

VC- versus Non-VC-Backed Public Companies

Dependent Variable	*R&D / sales*	*employment growth*	*sales growth*	ln*(firm value)*
VC (= 1, if backed by VC)	0.0521***	0.0490***	0.0696***	0.373***
	(0.00169)	(0.00206)	(0.00270)	(0.0141)
VC ×years since IPO	-0.000780***	-0.00304***	-0.00406***	-0.0110***
	(0.000132)	(0.000165)	(0.000215)	(0.00110)
ln(employment)	-0.0133***	-0.00567***	-0.00641***	0.851***
	(0.000248)	(0.000254)	(0.000335)	(0.00170)
Observations	84,116	148,834	149,672	168,549
R-squared	0.383	0.084	0.108	0.737

Table 2: All specifications include year dummies, industry dummies (at the 4-digit SIC), and a year dummy for the IPO. Standard errors are in parentheses and significance at the 1 percent level is denoted by ***.

patentees are identified by matching firm names in VentureXpert with PatentsView.

Firm-Level Regressions. In the firm-level regression analysis, the primary independent variable is (the natural logarithm of) annual VC funding while the dependent variable is a measure of patenting performance, both in the year, and the year after, the firm receives the funding. The primary independent variable may suffer from both measurement error and selection issues. So, in some of the regressions, two instrumental variables are used. The first IV is the (maximum) rate of capital gains taxation in the state where the VC-funded company is located. The second IV is a Rajan and Zingales (1998) type measure of the dependence on external finance of the industry in which the firm operates. The measure reflects the extent to which outside funds are used in the industry for expenditures on property, plant and equipment, R&D, advertising and employee training. Both of these datums are exogenous at the level of a startup. In all of the regressions, controls are added for the number of the patents held by the firm at the beginning of the year, the age of the firm, the total amount of private and federally funded R&D of the industry in which the firm operates. Additionally, both a year and industry dummy are entered. Last, since both innovation and VC activities are remarkably clustered in California and Massachusetts, a "cluster dummy" for a firm headquartered in California and Massachusetts is included.

The results of the regression analysis are reported in Table 3. Panel A of Table 3 conducts the analysis along the extensive margin analysis; i.e., it examines whether the firm obtains any patents after receiving funding from a VC. In regressions (1) and (2), the dependent variable is a dummy variable that takes the value of one, if the firm files any successful patent applications at the U.S. Patents and Trademark Office (USPTO) within one year after it receives funding. Regressions (3) and (4) focus on the "breakthrough" patents, a measure pioneered by Kerr (2010). "Breakthrough" patents refers to those in the right tail of the citation distribution. Here the dependent variable in regressions (3) and (4) is a dummy variable that takes the value of one, if the firm files any patents in the top 10% of the citation distribution in its cohort (i.e., those patents with the same technological class and same application year). Panel B of Table 3 turns to the intensive margin. In regressions (5) and (6) the dependent variable is the natural logarithm of the number of patents. The natural logarithm of the number of patents is weighted by citations in regressions (7) and (8).

As can be seen from the positive regression coefficients of VC funding in panel A, a firm is more likely to file a patent and come up with a "breakthrough" patent the larger is the funding from a VC, although the impact of VC funding is somewhat smaller in spurring "breakthrough" patents than ordinary patents. According to the IV estimates in regressions (6) and (8), a 10 percent increase in VC funding will induce a 3.6 percent boost in patenting one year after funding, and this number goes up to 6.7 percent when the number of patents is adjusted by quality. In addition, across all the regressions in Table 3, the estimates are consistently higher in the IV regressions.

Industry-Level Regressions. The above firm-level regressions are now recast at the 4-digit industry level. The main explanatory variable is now the (natural logarithm of the) aggregate amount of VC investment across all industries between 1970 and 2015. The dependent variable is the (natural logarithm of the) number of patents filed by all VC-backed companies in the industry one year after they receive VC funding. To capture the heterogeneous dependence on external finance across industries, a cross term is added between

VC Funding and Patenting: Firm-Level Regressions

Panel A: Extensive Margin Analysis				
Dependent Variable	*1{patent > 0}*		*1{"breakthrough patent" > 0}*	
	Probit	IV	Probit	IV
	(1)	(2)	(3)	(4)
ln(VC funding)	0.141***	0.682***	0.133***	0.635***
	(0.0108)	(0.0590)	(0.0112)	(0.0979)
Observations	9,166	8,132	9,149	8,122
Panel B: Intensive Margin Analysis				
Dependent Variable	ln*(patent)*		ln*(patent, quality adj)*	
	OLS	IV	OLS	IV
	(5)	(6)	(7)	(8)
ln(VC funding)	0.115***	0.363*	0.155***	0.674*
	(0.00907)	(0.187)	(0.0164)	(0.356)
Observations	5,828	5,207	5,032	4,519
R-squared	0.244		0.123	

Table 3: See the main text for a description of the dependent and independent variables. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

industry VC funding and the industry's dependence on external finance. This specification emulates Rajan and Zingales (1998) in the sense that they exploit the variation of financial development across countries, whereas the current analysis taps into fluctuations of aggregate VC investment across time. As in the firm-level regressions, the main independent variable may suffer from both measurement error and selection issues. An instrumental variable is used to address this. The IV follows Kortum and Lerner (2000) and is based on the deregulation of pension funds in 1979, as highlighted in Section 2. To be specific, a "deregulation dummy," which takes the value of one after 1979, is used as an instrumental variable. In all of the industry-level regressions, controls are added for the total amounts of private R&D and federally funded R&D in the industry. A 2-digit industry dummy variable is also included. Since the deregulation dummy is used as an IV, year dummies cannot be used anymore, so common shocks to all industries are controlled for by adding NBER recession dummies as a proxy for the business cycle, and the federal funds rate as a proxy for the tightness of the monetary policy.

VC Funding and Patenting: Industry-Level Regressions

Dependent Variable	ln*(patent)*		ln*(patent, quality adj)*	
	OLS	IV	OLS	IV
ln(agg VC funding)	0.200***	0.151***	0.129***	0.115*
	(0.0381)	(0.0569)	(0.0454)	(0.0681)
ln(agg VC funding) × ind financial dependence	0.1854***	0.1852***	0.192***	0.191***
	(0.00965)	(0.00976)	(0.0117)	(0.0118)
Observations	1,971	1,971	1,890	1,890
R-squared	0.378		0.362	

Table 4: See the main text for a description of the dependent and independent variables. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

The industry-level regressions are presented in Table 4. As can be seen from the first row of the regression coefficients, the positive signs on aggregate VC funding complement the findings at the firm level. VC investment contributes positively to patenting performance at the industry level. According to the IV estimate in column 2, at the median level of financial dependence across industries, a 10 percent increase in aggregate VC funding will induce a 1.57 percent boost in industry-level patenting within a year. This elasticity is 0.194 in the prepackaged software industry, which accounted for 23 percent of VC investment. In addition, the impact of VC is heterogeneous across industries, as revealed by the cross term between VC funding and the dependence on external finance–see the second row. Since the regression coefficients on the cross terms turn out to be positive, the impact of the fluctuations in aggregate VC investment is more pronounced the higher is the industry's dependence on external finance. For industries in the top quartile of financial dependence the elasticity is 0.339 versus 0.111 in the bottom quartile.[2]

[2] To be conservative, the number for the upper quartile excludes an unrealistic high elasticity for the insurance carrier industry, where there are only two VC-funded firms.

4 The Model

At center of the analysis is the interplay between an entrepreneur and a venture capitalist. Each period entrepreneurs bring ideas, of a type of their choosing, to a venture capitalist to obtain funding. The entrepreneur uses the funds to research and develop the idea into a successful project, potentially speaking. If successful, the project will be floated on the stock market or sold to another firm. This yields a reward that will be a function of the idea's type. Some ideas brought by entrepreneurs to the venture capitalist are good, others are bad. Only a good idea has a payoff, and even then, this might not happen. Neither party knows whether an idea is good or bad. The venture capitalist can evaluate projects over time at a cost and potentially detect the bad ones. Funding for a bad project is terminated. Projects that aren't known to be bad are given money. Some of these will be successful, while others will not. The probability of success is an increasing function of the level of investment in development undertaken by the entrepreneur. How much of the money the entrepreneur uses for development is private information. The venture capitalist can imperfectly monitor development investment at a cost in an attempt to detect any malfeasance. The relationship between an entrepreneur and a venture capital is governed by an incentive-compatible financial contract. Any profits from floating a VC-funded enterprise are subject to capital gains taxation. All revenue from capital gains taxation is rebated back to the populace in lump-sum transfer payments.

The analysis focuses on balanced-growth paths. The aggregate level of productivity in a period is denoted by $\mathbf{x}$. This represents the aggregate state of the economy. Along a balanced-growth path, $\mathbf{x}$ will grow at the gross rate $\mathbf{g}_x > 1$ so that

$$\mathbf{x}' = \mathbf{g}_x \mathbf{x}.$$

The gross growth rate of aggregate productivity, $\mathbf{g}_x$, is an endogenous variable in equilibrium. It will be a function of the efficiency of the venture capital system.

4.1 Floated Firms

A successful VC-backed firm produces output, o, according to the production process

$$o = x^{\zeta} k^{\kappa} l^{\lambda}, \text{ with } \zeta + \kappa + \lambda = 1, \tag{1}$$

where k and l are the amounts of capital and labor used in production. The variable x represents the firm's productivity and this denotes its type. This structure is borrowed from Akcigit, Celik, and Greenwood (2016). It results in the firm earning pure profits that are linear in its productivity, x. The lure of capturing these profits is what motivates entrepreneurs and venture capitalists. Labor is hired at the wage rate w and capital at the rental rate r. The firm's per period takings are

$$\begin{aligned} T(x;\mathbf{x}) &= \max_{k,l}\{x^{\zeta} k^{\kappa} l^{\lambda} - rk - wl\} \\ &= x(1-\kappa-\lambda)[(\frac{\kappa}{r})^{\kappa}(\frac{\lambda}{w})^{\lambda}]^{1/\zeta}. \end{aligned} \tag{P1}$$

Clearly, as wages rise, which will be a function of the aggregate level of productivity, $\mathbf{x}$, takings will shrink for a given level of the firm's productivity, x. Operating firms last stochastically with the time-invariant survival rate $\mathfrak{s}$. A successful VC-backed project is sold for $I(x;\mathbf{x})$, either through an IPO or an M&A, just before production starts. The (gross) reward for a successful IPO is

$$I(x;\mathbf{x}) = \sum_{t=1}^{\infty} (\mathfrak{s}\delta)^{t-1} T(x;\mathbf{g}_x^{t-1}\mathbf{x}), \tag{2}$$

where δ is the market discount factor.

4.2 Startups

Each period a flood of entrepreneurs in the amount $\mathfrak{e}$ approaches venture capitalists in order to obtain funding for their ideas. An entrepreneur incurs an opportunity cost in the

amount $w\mathfrak{o}$ to run a project. The component $\mathfrak{o}$ of this cost is distributed across potential entrepreneurs according to the non-normalized distribution function, $O(\mathfrak{o})$. This distribution function $O(\mathfrak{o})$ is assumed to be Pareto so that

$$O(\mathfrak{o}) = 1 - (\upsilon/\mathfrak{o})^{\nu}, \text{ with } \nu, \upsilon > 0. \tag{3}$$

Only those potential entrepreneurs who expect the payoff from a startup to exceed their opportunity cost, $w\mathfrak{o}$, will approach a venture capitalist for funding. This criteria will determine the number of funded entrepreneurs $\mathfrak{e}$.

Out of pool of new entrepreneurs, the fraction ρ will have good ideas, implying that the fraction $1-\rho$ have bad ones. A startup of type x turns into a going concern with productivity x, if successful. The odds of success in a period depend on the investment in devlopment that the entrepreneur undertakes. In particular, a probability of success, σ, can be secured by undertaking development investment of the amount $D(\sigma; \mathbf{x})$, where D is an increasing, convex function in σ. The development cost function $D(\sigma; \mathbf{x})$ is given the form

$$D(\sigma; \mathbf{x}) = w(\frac{1}{1-\sigma} - 1)\sigma/\chi_D.$$

Note that the marginal cost of doing development starts at zero, when $\sigma = 0$, and goes to infinity, as σ approaches one. The cost of doing development rises with the level of wages, w, which will be a function of the aggregate level of productivity, $\mathbf{x}$. Think about χ_D as capturing the efficiency of investment in development.

Suppose that the venture capitalist fronts the entrepreneur funds to do development in the amount $D(\sigma; \mathbf{x})$. The actual level of investment that the entrepreneur will do is private information. That is, the entrepreneur may decide to invest $D(\widetilde{\sigma}; \mathbf{x}) \leq D(\sigma; \mathbf{x})$ in development, so that the odds of success are $\widetilde{\sigma}$, and use the difference $D(\sigma; \mathbf{x}) - D(\widetilde{\sigma}; \mathbf{x})$ for his own consumption. By monitoring the entrepreneur, the venture capitalist can try to prevent this from happening. If the startup is successful, the entrepreneur must pay the venture capitalist the amount p.

There is also a fixed cost, ϕ_t, connected with running an age-t startup project. This fixed cost rises with the level of wages in the economy. In particular,

$$\phi_t = w\mathbf{g}_w^{t-1}\phi(t),$$

where $\mathbf{g}_w > 1$ is the gross growth rate in wages (which will be a function of $\mathbf{g}_x$). Additionally, the fixed cost changes by the stage of the project, as reflected by the function $\phi(t)$. The shape of the function $\phi(t)$ will be parameterized using a polynomial that is pinned down from the U.S. data.

A new entrepreneur is free to choose the type of startup, x, that he wants to develop. In particular, when deciding on the project, the entrepreneur picks x subject to research cost function of the form

$$i = wR(\frac{x}{\mathbf{x}}) = w(\frac{x}{\mathbf{x}})^{\iota}/\chi_R.$$

where $i \geq 0$ is the initial investment in developing the project. The entrepreneur can choose how far ahead is the productivity of his firm, x, from the average level of productivity in the economy, $\mathbf{x}$. The cost is $R(x/\mathbf{x})$ in terms of labor, which translates into $wR(x/\mathbf{x})$ in terms of output. This structure provides a mechanism for endogenous growth in the model.

4.3 Venture Capitalists

Venture capitalists provide funding to entrepreneurs. They raise the money to do this from savers, to whom they promise a gross rate of return of $1/\delta$. When a startup is successful, the venture capitalist collects a payoff from the IPO in the amount p. At the beginning of each period, $\mathfrak{e}$ entrepreneurs approach a venture capitalist to secure funding for their ideas. The determination of $\mathfrak{e}$ in equilibrium is discussed later.

Out of the pool of qualified entrepreneurs, or out of $\mathfrak{e}$, some will have good ideas and others bad ones. The venture capitalist can potentially discover a bad project by evaluating it. Assume that the VC can detect a bad project with probability β, according to the cost function, $E(\beta; \mathbf{x})$, where E is an increasing, convex function in β. The evaluation function

$E(\beta; \mathbf{x})$ has a similar form to the one for $D(\sigma; \mathbf{x})$. Specifically,

$$E(\beta; \mathbf{x}) = w(\frac{1}{1-\beta} - 1)\beta/\chi_E.$$

The productivity of the evaluation process is governed by χ_E.

The VC provides the entrepreneur the amount $D(\sigma; \mathbf{x})$ to do development. The entrepreneur may decide do to some smaller amount $D(\widetilde{\sigma}; \mathbf{x}) \leq D(\sigma; \mathbf{x})$ and siphon off the difference in funds, $D(\sigma; \mathbf{x}) - D(\widetilde{\sigma}; \mathbf{x})$. The venture capitalist can attempt to dissuade this fraud by engaging in monitoring. Assume that the VC can pick the odds μ of detecting fraud in an age-t venture according to the strictly increasing, convex cost function, $M_t(\mu; \mathbf{x})$, where

$$M_t(\mu; \mathbf{x}) = w\mathbf{g}_w^{t-1}(\frac{1}{1-\mu} - 1)\mu/\chi_{M,t}.$$

The cost of monitoring rises with wages in the economy. Additionally, monitoring costs change by the stage of the project, as reflected by the term $\chi_{M,t}$; again, $\chi_{M,t}$ represents the productivity of this auditing process at stage t. Presumably, as the VC becomes more familiar with the project, $\chi_{M,t}$ will rise with t. This features implies that the incentive problem will become less severe over time and helps to generate an upward sloping funding profile. A polynomial will be used to fit $\chi_{M,t}$ to the U.S. data. While motivated by the prototypical costly state verification paradigms of Townsend (1979) and Williamson (1986), the monitoring technology employed here is different. In those frameworks, getting monitored is a random variable–in Williamson (1986) everybody declaring a bad outcome is monitored while in Townsend (1979) some fraction are. The audit will detect any fraud with certainty. By contrast, here everybody gets monitored, but the detection of any fraud is a probabilistic event.

Last, the type of oversight of entrepreneurs by venture capitalists modeled here (viz., evaluating and monitoring) appears to be important. Bernstein, Giroud and Townsend (2016) show how the introduction of new airline routes, which reduces the cost for a VC of overseeing a startup, leads to an increase in the quality and quantity of patents and a higher

likelihood of a successful acquisition or IPO.

4.4 The Financial Contract

The financial contract between the entrepreneur and the venture capitalist is cast now. Venture capital is a competitive industry so the entrepreneur shops around to secure the financial contract with the best terms. The VC covers the cost of development, evaluation, monitoring, and research. There are no profits on venture capital activity in equilibrium. The profits that accrue to the entrepreneur are subject to the rate of capital gain taxation, τ. The analysis presumes that there is a maximum of T rounds of potential funding. The timing of events within a generic funding round is shown in Figure 4. The research for the idea is done at the start of the funding cycle or in period zero. At the beginning a funding round the VC evaluates projects and purges the ones that are found to be bad. Goods projects are then given an injection of cash for development. The VC monitors the use of these funds, If malfeasance is detected, the project is terminated. Some projects will be successful. These are floated in the next period on the stock market. The unsuccessful projects then start another funding rounds (assuming the age of project is no greater than T).

Let β_t represent the odds of detecting a bad age-t project and σ_t denote the probability of success for a good one. Now, suppose that a unit measure of new entrepreneurs approach the VC for funding. As this cohort ages, the numbers of good and bad projects will evolve as shown in the table below. For example, of the people initially applying for funding the number ρ will have good projects and $1-\rho$ will have bad ones. The VC will evaluate the applicants and eliminate $(1-\rho)\beta_1$ bad projects, so that $(1-\rho)(1-\beta_1)$ bad ones will still remain. Of the good projects, the number $\rho\sigma_1$ will be successful. So, at the beginning of the second round there will be $\rho(1-\sigma_1)$ good projects in the pool. After the second period evaluation, $(1-\rho)(1-\beta_1)(1-\beta_2)$ bad projects will still be around. Table 5 elaborates how the number of good and bad projects evolve as the funding rounds progress.

Evolution of Project Types across Funding Rounds

Age	*Number Good*	*Number Bad*
1	ρ	$(1-\rho)(1-\beta_1)$
2	$\rho(1-\sigma_1)$	$(1-\rho)(1-\beta_1)(1-\beta_2)$
3	$\rho(1-\sigma_1)(1-\sigma_2)$	$(1-\rho)(1-\beta_1)(1-\beta_2)(1-\beta_3)$
$\vdots$	$\vdots$	$\vdots$
t	$\rho\Pi_{j=1}^{t-1}(1-\sigma_j)$	$(1-\rho)\Pi_{j=1}^{t}(1-\beta_j)$

Table 5: The table shows how the number of good and bad projects change across funding rounds assuming that the VC starts with a unit mass of ventures.

The odds of an age-t project being good are

$$\Pr(\text{Good}|\text{Age}=t)=\frac{\rho\Pi_{j=1}^{t-1}(1-\sigma_j)}{\rho\Pi_{j=1}^{t-1}(1-\sigma_j)+(1-\rho)\Pi_{j=1}^{t}(1-\beta_j)}. \tag{4}$$

As time goes by, more and more bad projects are purged from the pool. The number of goods projects will also fall due to the successes. Thus, the odds of being good can rise or fall with age, depending on which type of projects are exiting the pool the fastest, at least theoretically speaking. If the odds of being good in the current period are $\rho\Pi_{j=1}^{t-1}(1-\sigma_j)$, then the odds of being good and still being around next period are $\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\times(1-\sigma_t)$. The odds of being good and still being around $t+i$ periods ahead are $\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\times\Pi_{j=t}^{t+i-1}(1-\sigma_j)$.

The contract between the entrepreneur and the venture capitalist will specify for the length of the relationship: (i) the investments in development as reflected by the σ_t's; (ii) the payments that an entrepreneur who finds success at age t must make to the intermediary, or the p_t's; (iii) the precision of evaluation, as given by the β_t's; and (iv) the exactness of monitoring as measured by the μ_t's. The contract is summarized by the outcome of the following maximization problem in sequence space:

$$C(x;\mathbf{x})=\max_{\{p_t,\sigma_t,\mu_t,\beta_t\}}(1-\tau)\sum_{t=1}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t], \tag{P2}$$

subject to:

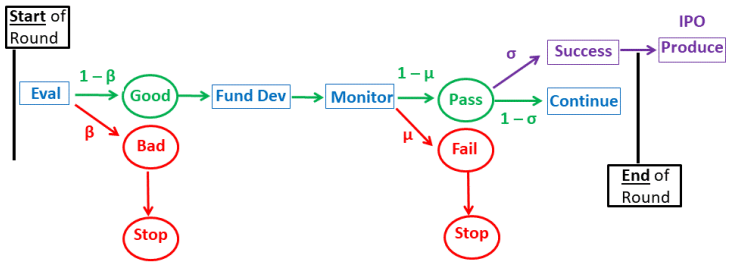


Figure 4: The timing of events within a typical funding round. The research underlying the idea is done at beginning of the funding cycle (or period zero) and is not shown.

1. The age-t incentive constraints

$$\begin{aligned}
&\Pr(\text{Good}|\text{Age}=t)\times(1-\tau)\times\{\delta\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t] \\
&\qquad +(1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x})-p_i]\} \\
&\qquad\qquad \geq (1-\mu_t)\max_{\widetilde{\sigma}_t}\Bigg(D(\sigma_t)-D(\widetilde{\sigma}_t) \\
&\quad +\Pr(\text{Good}|\text{Age}=t)\times(1-\tau)\times\{\delta\widetilde{\sigma}_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t] \\
&\qquad\qquad +(1-\widetilde{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x})-p_i]\}\Bigg),
\end{aligned} \tag{5}$$

for $t=1,\cdots,T$, where $\Pr(\text{Good}|\text{Age}=t)$ is given by (4);

2. The age-0 zero-profit condition

$$\rho\sum_{t=1}^{T}\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t p_t-\sum_{t=1}^{T}[\rho\Pi_{j=1}^{t-1}(1-\sigma_j)+(1-\rho)\Pi_{j=1}^{t}(1-\beta_j)]\delta^{t-1}[D(\sigma_t)+\phi_t+M_t(\mu_t)]$$
$$-\sum_{t=1}^{T}[\rho\Pi_{j=1}^{t-1}(1-\sigma_j)+(1-\rho)\Pi_{j=1}^{t-1}(1-\beta_j)]\delta^{t-1}E(\beta_t)-wR(\frac{x}{\mathbf{x}})=0. \tag{6}$$

The objective function in (P2) reflects the fact that venture capital is a competitive industry. A contract must maximize the expected return for the entrepreneur, subject to two constraints. The maximized value of objective function, $C(x;\mathbf{x})$, specifies the worth of the financial contract for the entrepreneur. The term $I(x;\mathbf{g}_x^t\mathbf{x})-p_t$ gives the payoff to the entrepreneur should the enterprise be floated at stage t. The payoff could come from executing stock options or convertible shares. It is taxed at the capital gains rate, τ.

Equation (5) is the incentive compatibility constraint for an age-t project. The left-hand side gives the expected return to entrepreneur when he undertakes the level of investment linked with σ_t. The first term in brackets are the Bayesian odds of being the good type at the beginning of period t, conditional on the entrepreneur still dealing with the venture capitalist. The right-hand side gives the return when the entrepreneur deviates and picks the level of development connected with $\widetilde{\sigma}_t$. The level of development represented by $\widetilde{\sigma}_t$ maximizes the value of the deviation. The return from deviating will only materialize if the entrepreneur is not caught cheating, which has the odds $1-\mu$; if the deviating entrepreneur is caught cheating, which occurs with probability μ, then the contract is terminated and he receives nothing. The incentive constraint has a dynamic element to it. If the entrepreneur invests less in research today, he lowers the odds that a good project will be successful in the current period. He increases the probability that a good project will be successful in the future; thus, an intertemporal tradeoff is involved.

The last equation, or (6), is the zero-profit constraint. Observe that there is a fixed cost, ϕ_t, connected with operating an age-t startup project. Last, the venture capitalist

must cover the initial development cost, $wR(x/\mathbf{x})$. Since venture capital is competitive, the expected returns from lending will exactly offset the expected costs.

Now, it easy to see that the ability of the venture capitalist to monitor the entrepreneur is important. Focus on the incentive constraint (5). If $\mu_t = 1$, say because the cost of monitoring is zero, then the left-hand side of the constraint will always exceed the right-hand. This transpires no matter what the solution for $\widetilde{\sigma}_t$ is, as dictated by the right-hand side of (5). In this situation, the first-best solution to problem (P2) can be obtained. Alternatively, suppose $\mu_t = 0$, because the cost of monitoring is infinite. Then, the incentive compatible contract specifies that $\sigma_t = \widetilde{\sigma}_t$. To see this, pull the $D(\sigma_t)$ term over onto the left-hand side of (5). Note that the terms on left- and right-hand sides are then the same, except that they involve σ_t on the left and $\widetilde{\sigma}_t$ on the right. But, $\widetilde{\sigma}_t$ maximizes the right-hand side, implying that right-hand side must then equal the left-hand side. This can only be the case if $\sigma_t = \widetilde{\sigma}_t$, which limits the contract a lot, and may result in an allocation far away from the first-best one. So, if no monitoring is done, then the incentive constraint holds tightly. Can the incentive constraint be slack? Suppose it is slack, implying that the associated Lagrange multiplier is zero. Then, no monitoring will be done, because it would have no benefit while it is costly. But, as just discussed, when $\mu_t = 0$ the constraint must hold tightly, a contradiction. Therefore, the incentive constraint (5) always binds.

Lemma 1 *(The VC constantly monitors the entrepreneur) The incentive constraint (5) holds tightly for all funding rounds with* $0 < \mu_t < 1$.

Remark 1 *(One-shot versus multi-shot deviations) The incentive constraints in (5) prevent one-shot deviations from occuring in any funding round. Lemma 4 in the theory appendix establishes that this is equivalent to using a single consolidated time-*0 *incentive constraint with multi-shot deviations.*

Remark 2 *(Self financing) If an entrepreneur has any funds, he should invest them all. This does not change the generic form of the contract problem. The entrepreneur's funds can merely be subtracted off of the expected present value of the fixed costs in (6)–see Cole,*

Greenwood, and Sanchez (2016, Lemmas 1 and 6). What matters is how much the entrepreneur borrows, net of his own investment. The entrepreneur's funds can be incorporated in problem (P2) by normalizing the fixed costs.

4.5 The Choice of Idea

The entrepreneur is free to pick the type of venture, x, that he pitches to the venture capitalist. He selects the one that maximizes his expected profits. Therefore, x will solve

$$V(\mathbf{x}) = \max_{x} C(x; \mathbf{x}), \tag{P3}$$

where the value of the entrepreneur's contract, for a type-x project when aggregate productivity is $\mathbf{x}$, or $C(x; \mathbf{x})$, is specified by problem (P2). The faster profits rise with x, the higher will be the value of x picked by the entrepreneur. So, if better intermediation implies that profits rise more steeply with x, then venture capital will increase growth. Note that cost of researching x, or $wR(x/\mathbf{x})$, is embedded in the zero-profit condition (6) connected with problem (P2). This problem will give a decision rule of the form

$$x = X(\mathbf{x})\mathbf{x}.$$

The function $V(\mathbf{x})$ gives an entrepreneur's expected payoff from a startup.

4.6 The Flow of New Startups

Recall that an entrepreneur incurs an opportunity cost in the amount $w\mathfrak{o}$ to run a project. Therefore, only those new entrepreneurs with $w\mathfrak{o} \leq V(\mathbf{x})$ will choose to engage in a startup. Now, $\mathfrak{o}$ is distributed according the non-normalized distribution function $O(\mathfrak{o})$. Therefore, $O(V(\mathbf{x})/w)$ entrepreneurs will approach the venture capitalist for funding. Consequently,

the number of new entrants, $\mathfrak{e}$, is given by

$$\mathfrak{e} = O(V(\mathbf{x})/w). \tag{7}$$

4.7 Non-VC Sector

Most firms are not funded by venture capitalists. To capture this, suppose there are always $\mathfrak{m}$ firms operating that were not funded by VCs. All firms in the non-VC sector are same. These non-VC firms produce using a production function that is identical to a VC firm with one exception; their productivity differs. Specifically, they produce in line with

$$o = z^{\zeta} k^{\kappa} l^{\lambda}, \text{with } \zeta + \kappa + \lambda = 1,$$

where z represents their productivity. Suppose that

$$z = \omega \mathbf{x}, \text{ with } \omega < 1.$$

Thus, firms in the non-VC profit of the economy are on average less productive that the ones in the VC part, but will be dragged along by latter. The non-VC firm profit maximization problem is

$$\max_{k,l} \{z^{\zeta} k^{\kappa} l^{\lambda} - rk - wl\}. \tag{8}$$

One can think about these firms as raising the funds for capital through traditional intermediation at the gross interest rate $1/\delta$–VC-funded firms also raise capital this way after they are floated. On this, Midrigan and Xu (2014) argue that producing establishments can quickly accumulate funds internally and thus rapidly grow out of any borrowing constraints. Therefore, modeling producing firms as having frictionless access to capital markets may not be grossly at variance with reality.

4.8 Balanced-Growth Equilibrium

The analysis focuses on characterizing a balanced-growth path for the model. Along a balanced the growth path the rental rate on capital, r, is some fixed number. In particular, the rental rate on capital will be $r = 1/\delta - \mathfrak{d}$, where δ is the market discount factor and $\mathfrak{d}$ is the depreciation factor on capital. Along a balanced-growth path the market discount factor, δ, in turn is given by

$$\delta = \widehat{\delta}\mathbf{g}_w^{-\varepsilon}, \tag{9}$$

where $\widehat{\delta}$ is the representative agent's discount factor and ε denotes his coefficient of relative risk aversion.[3] A VC-funded firm with a productivity level of x will hire labor in the amount

$$l(x;w) = \left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta} x, \tag{10}$$

where again w and r are the wage and rental rates. For a non-VC-funded firm just replace the x with a z in the above formula.

In general equilibrium, the labor market must clear each period. Suppose that there is one unit of labor available in aggregate. To calculate the aggregate demand for labor sum over all operating firm's demands for labor, both in the VC- and non-VC-backed sectors. Now, no firms will operate in the VC-backed sector with productivity level x, since this type is not operational yet. Let $\mathfrak{n}_t$ represent the number of VC-backed firms that are operating with an idea, x_{-t}, that was generated t periods ago. Attention will now be turned to specifying the number $\mathfrak{n}_t$.

Each period $\mathfrak{e}$ new entrepreneurs will be funded by the venture capitalist. Hence, $\mathfrak{n}_1 = \mathfrak{e}\rho\sigma_1$ firms will operate with the idea generated one period ago, x_{-1}. Likewise, there will

[3] That is, in the background there is a representative consumer/worker who inelastically supplies one unit of labor and has a utility function (in period 1) of the form

$$\sum_{t=1}^{\infty}\widehat{\delta}^{t-1}c_t^{1-\epsilon}/(1-\epsilon),$$

where c_t is his consumption in period t.

$\mathfrak{n}_2 = \mathfrak{e}\rho\sigma_1\mathfrak{s} + \mathfrak{e}\rho(1-\sigma_1)\sigma_2$ firms operating with the two-period-old idea, x_{-2}. So, the number of firms operating with the idea x_{-t}, from $t \leq T$ periods ago, is

$$\mathfrak{n}_t = \mathfrak{e}\sum_{i=1}^{t} \rho\Pi_{j=1}^{i-1}(1-\sigma_j)\sigma_i\mathfrak{s}^{t-i}, \text{ for } t = 1, \cdots, T. \tag{11}$$

The venture capital capitalist only funds entrepreneurs for T periods. Consequently, the number of operational firms with an idea from more than T periods ago is

$$\mathfrak{n}_{T+j} = \mathfrak{s}^j\mathfrak{n}_T, \text{for } j \geq 1. \tag{12}$$

The total number of VC-backed firms in the economy, $\mathfrak{n}$, is given by

$$\mathfrak{n} = \sum_{t=1}^{T}\mathfrak{n}_t + \sum_{t=T+1}^{\infty}\mathfrak{n}_t = \sum_{t=1}^{T}\mathfrak{n}_t + \frac{\mathfrak{n}_T\mathfrak{s}}{1-\mathfrak{s}}.$$

Equilibrium in the labor market requires that

$$\sum_{t=1}^{T}\mathfrak{n}_t l(x_{-t}; w) + \sum_{t=T+1}^{\infty}\mathfrak{n}_t l(x_{-t}; w) + \mathfrak{m}l(z; w) = 1,$$

where again $\mathfrak{m}$ is the measure of firms in the non-VC sector. Along a balanced-growth path, the productivity of the latest idea will grow at rate $\mathbf{g}_x$. Therefore, the above condition can be recast as

$$\sum_{t=1}^{T}\mathfrak{n}_t l(x_{-1}\mathbf{g}_x^{1-t}; w) + \sum_{t=T+1}^{\infty}\mathfrak{n}_t l(x_{-1}\mathbf{g}_x^{1-t}; w) + \mathfrak{m}l(\omega\mathbf{x}; w) = 1.$$

Using equations (10) and (12), this can be expressed as

$$\left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta}[x_{-1}(\sum_{t=1}^{T}\mathfrak{n}_t\mathbf{g}_x^{1-t} + \frac{\mathfrak{n}_T\mathfrak{s}\mathbf{g}_x^{-T}}{1-(\mathfrak{s}/\mathbf{g}_x)}) + \mathfrak{m}\omega\mathbf{x}] = 1.$$

Therefore wages, w, are given by

$$w = \lambda \left(\frac{\kappa}{r}\right)^{\kappa/(\zeta+\lambda)} \underbrace{[x_{-1}(\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g}_x^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g}_x^{-T}}{1-(\mathfrak{s}/\mathbf{g}_x)})}_{=\mathfrak{n}\mathbf{x}} + \mathfrak{m}\omega\mathbf{x}]^{\zeta/(\zeta+\lambda)}, \tag{13}$$

where aggregate productivity, $\mathbf{x}$, is defined below:

$$\mathbf{x} \equiv \frac{x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g}_x^{1-t} + \mathfrak{n}_T \mathfrak{s} \mathbf{g}_x^{-T}/(1-(\mathfrak{s}/\mathbf{g}_x)]}{\sum_{t=1}^{T} \mathfrak{n}_t + \mathfrak{n}_T \mathfrak{s}/(1-\mathfrak{s})} = \frac{x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g}_x^{1-t} + \mathfrak{n}_T \mathfrak{s} \mathbf{g}_x^{-T}/(1-(\mathfrak{s}/\mathbf{g}_x)]}{\mathfrak{n}}.$$

As can be seen, wages rise with the aggregate level of productivity, $\mathbf{x}$, which grows at rate $\mathbf{g}_x$. Therefore, wages will grow at the gross growth rate $\mathbf{g}_x^{\zeta/(\zeta+\lambda)}$, so that

$$\frac{w'}{w} \equiv \mathbf{g}_w = \mathbf{g}_x^{\zeta/(\zeta+\lambda)}.$$

All new entrepreneurs will pick the same type of project, x. Now,

$$\mathbf{g}_x = \mathbf{x}'/\mathbf{x} = x'/x.$$

In a stationary equilibrium, the distribution function over VC-funded firms using an age-t idea will remain constant; that is, $\mathfrak{n}_t' = \mathfrak{n}_t$. The demand for capital by a type-x VC-backed firm is

$$k(x;w) = (\frac{\kappa}{r})^{(1-\lambda)/\zeta} (\frac{\lambda}{w})^{\lambda/\zeta} x.$$

From this it is easy to deduce that $k(\boldsymbol{g}_x x; \mathbf{g}_w w) = \mathbf{g}_w k(x;w)$. The same is true for a non-VC backed firms; just replace x with z to get $k(\boldsymbol{g}_x z; \mathbf{g}_w w) = \mathbf{g}_w k(z;w)$. Let the aggregate capital stock in the current period be represented by $\mathbf{k}$ and that for next period by $\mathbf{k}'$. Then, $\mathbf{k}' = \sum_{t=1}^{\infty} \mathfrak{n}_t k(\mathbf{g}_x x_{-t}; \mathbf{g}_w w) + \mathfrak{m} k(\boldsymbol{g}_x z; \mathbf{g}_w w) = \mathbf{g}_w [\sum_{t=1}^{\infty} \mathfrak{n}_t k(x_{-t};w) + \mathfrak{m} k(z;w)] = \mathbf{g}_w \mathbf{k}$, so that the aggregate capital stock grows at gross rate $\mathbf{g}_w$. A similar argument can be used to show that aggregate output grows at the same rate.

Now, recall that

$$x = X(\mathbf{x})\mathbf{x},$$

and

$$\mathbf{x} = x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g}_x^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g}_x^{-T}}{1-(\mathfrak{s}/\mathbf{g}_x)}]/\mathfrak{n}.$$

Therefore,

$$\mathbf{g}_x = \frac{x}{x_{-1}} = X(\mathbf{x})[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g}_x^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g}_x^{-T}}{1-(\mathfrak{s}/\mathbf{g}_x)}]/\mathfrak{n}. \tag{14}$$

This is a nonlinear equation in $\mathbf{g}_x$.

Definition 1 *(Balanced-Growth Path) For a given subjective discount factor and coefficient of relative risk aversion, $\widehat{\delta}$ and ε, a balanced-growth path consists of (i) a financial contract, $\{p_t, \sigma_t, \mu_t, \beta_t\}$, between entrepreneurs and venture capitalists; (ii) a set of labor inputs, $l(x;w)$ and $l(z;w)$, for VC- and non-VC-funded firms; (iii) values for the contract, an IPO, and a startup, $C(x;\mathbf{x})$, $I(x;\mathbf{x})$, and $V(\mathbf{x})$; (iv) a project type, x, for new entrepreneurs; (v) a wage rate, w; (vi) a gross growth rate of aggregate productivity, $\mathbf{g}_x$; (vii) a flow in of new entrepreneurs, $\mathfrak{e}$; (viii) a distribution for VC-funded firms, $\{\mathfrak{n}_t\}_{t=1}^{\infty}$; and (ix) a market discount factor, δ, such that:*

(1) The financial contract, $\{p_t, \sigma_t, \mu_t, \beta_t\}$, solves problem (P2), given the function I and $x, \mathbf{g}_x$, and $\mathbf{x}$. The solution to this problem gives the expected return to a new entrepreneur from the contract, $C(x;\mathbf{x})$.

(2) The VC-funded firm maximizes its profits, given x, r and w, as specified by problem (P1). This determines the value of an IPO, I, as presented in (2). The solution to the firm's maximization problem gives the rule for hiring labor (10). Analogously, a non-VC-funded maximizes its profits, given x, r and w, as specified by problem (8).

(3) A new entrepreneur picks the type of his project, x, to solve problem (P3), given the value of contract, $C(x;\mathbf{x})$, as a function of x and $\mathbf{x}$. This determines the expected value of a startup, $V(\mathbf{x})$.

(4) Aggregate productivity, $\mathbf{x}$, grows at the rate $\mathbf{g}_x$ specified by (14).

(5) The market-clearing wage rate, w, is given by (13) and grows at the rate $\mathbf{g}_w = \mathbf{g}_x^{\zeta/(\zeta+\lambda)}$.
(6) The flow in of new entrepreneurs, $\mathfrak{e}$, is regulated by (3) and (7), taking as given the value of a startup, $V(\mathbf{x})$.
(7) The distribution for VC-funded firms, $\{\mathfrak{n}_t\}_{t=1}^{\infty}$, is specified by (11) and (12).
(8) The market discount factor is governed by (9), given $\mathbf{g}_w$.

The lemma below establishes that the setup will have a balanced-growth path.

Lemma 2 *(Balanced Growth) Let $x' = \mathbf{g}_x x$ and $\mathbf{x}' = \mathbf{g}_x \mathbf{x}$, for all time. In the contract specified by (P2) the new solution will be given by $\sigma_t' = \sigma_t, \mu_t' = \mu_t, \beta_{t+1}' = \beta_{t+1}$, $\widetilde{\sigma}_t' = \widetilde{\sigma}_t$, $p_t' = \mathbf{g}_w p_t$, and $C(x'; \mathbf{x}') = \mathbf{g}_w C(x; \mathbf{x})$. The gap between the frontier, x, and and average productivity, $\mathbf{x}$, as measured by $x/\mathbf{x}$, will be time invariant. The flow in of new entrepreneurs, $\mathfrak{e}$, is a constant.*

Proof. See Theory Appendix. ■

5 Calibration

As discussed in Section 2, venture capital partnerships are of a limited duration, usually between 7 to 10 years. So, the analysis assumes that an entrepreneur's contract with a venture capitalist has 7 potential funding rounds each lasting 1.5 years. Thus, partnerships are structured to last at most 10.5 years. The decreasing returns to scale parameter in the production function (P1) is taken from Midrigan and Xu (2014), which requires setting $\zeta = 0.15$. The exponents for the inputs are set so that capital earns 1/3 of nonprofit income and labor receives 2/3. The survival rate of a firm is selected so that on average a publicly listed firm lives 25 years, as in the U.S. economy. The depreciation rate on capital, $1 - \mathfrak{d}$, is taken to 7 percent. Last, Henrekson and Sanandaji (2016) report that the key personnel connected with venture capital startups are taxed at a 15 percent capital gains rate. So, set $\tau = 0.15$.

The model is calibrated to match several facts. Over the period 1948 to 2015, GDP per hours worked in the U.S. economy grew at 1.8 percent per year. This fact is targeted in the calibration procedure. The long-run interest rate is taken to 2 percent. A standard value of 2 is assigned for the coefficient of relative risk aversion. The market discount factor is the reciprocal of the equilibrium interest rate and it will change as the growth rate of the economy, $\mathbf{g}_w$, changes. At the calibrated equilibrium, the representative agent's annual discount factor is determined by the formula to $\widehat{\delta} = (1 - .02)/(1.018)^{-2}$; cf. (9). This yields a yearly discount rate of 2.5%.

To calibrate the elasticity of the research cost function, ι, the following regression is run using VentureXpert data

$$\ln(\text{IPO VALUE}) = \underset{(0.106)}{0.829^{***}} \times \ln(\text{VC FUNDING}) + \text{CONTROLS}, \text{ obs} = 1{,}153, \qquad (15)$$

where the controls are the ln(# of employees), age at IPO, and a 2-digit industry dummy variable. Three instrumental variables are also used: capital gain taxes (which vary across states and time), dependence on external finance (which varies across industries), and the deregulation dummy. The impact of a change in VC funding on IPO value is also calculated for the model. This calculation is broken down into two steps. First, the elasticity of $I(x; \mathbf{x})$ with respect to x is computed. Second, the elasticity of VC funding with respect to x is toted up numerically. The ratio of these two elasticities gives the elasticity of market value with respect to VC funding. Thus, the following object is computed for the model:

$$\text{IPO VALUE ELASTICITY} = \frac{d \ln \text{IPO}/d \ln x}{d \ln(\text{VC FUNDING})/d \ln x}.$$

The process for the efficiency of monitoring, $\chi_{M,t}$, by the project's age, t, is taken to be a cubic:

$$\chi_{M,t} = \log(a_1 \times t + a_2 \times t^2 + a_3 \times t^3).$$

This requires specifying three parameters, namely a_1, a_2 and a_3. These parameters will be identified by fitting this process to match the VC's share of equity by the duration of project–this pattern is taken up below. The more efficient monitoring is, the higher will be the VC's share of equity. The time profile for the fixed cost, $\phi(t)$, will be governed by the quartic shown below

$$\phi(t) = \exp(b_0 + b_1 \times t + b_2 \times t^2 + b_3 \times t^3 + b_4 \times t^4).$$

Five parameters, b_0, b_1, b_2, b_3, and b_4, govern this process. The pattern of VC investment by funding round–discussed below–determines these parameters.

Next, projects that are funded by venture capitalists have an average success rate per funding round of 1.1 percent and a failure rate of 4.7 percent. The calibration procedure attempts to match these two statistics. To construct these statistics for the model, note that the success rate in period t is just the number of IPOs divided by the mass of surviving firms:

$$\textsc{Success Rate}_t = \frac{\textsc{IPOs}_t}{\textsc{Surviving Firms}_t} = \frac{\sigma_t \rho \Pi_{j=1}^{t-1}(1-\sigma_j)}{\rho \Pi_{j=1}^{t-1}(1-\sigma_j) + (1-\rho)\Pi_{j=1}^{t}(1-\beta_j).}.$$

The analogous definition for the failure rate in funding round t is

$$\textsc{Failure Rate}_t = \frac{\textsc{Failures}_t}{\textsc{Surviving Firms}_t} = \frac{\beta_t (1-\rho) \Pi_{j=1}^{t-1}(1-\beta_j).}{\rho \Pi_{j=1}^{t-1}(1-\sigma_j) + (1-\rho)\Pi_{j=1}^{t}(1-\beta_j).}.$$

Conjectured cash-on-cash multiples are used by venture capitalists to access the expected returns on a project. The cash multiple is calculated by dividing the profit of a project by the funds invested. The calibration procedure aims to hit an average cash multiple of 5.5. For the model, the expected cash multiple is

$$\textsc{Cash Multiple} = \frac{\sum_{t=1}^{T} \rho \Pi_{j=1}^{t-1}(1-\sigma_j)\sigma_t p_t}{wR(x) + \sum_{t=1}^{T}[\rho \Pi_{j=1}^{t-1}(1-\sigma_j) + (1-\rho)\Pi_{j=1}^{t}(1-\beta_j)][D(\sigma_t) + \phi_t]}.$$

The calibration also targets the observed pattern of investment and the venture capitalist's share of equity by funding rounds.

On average a VC-backed company is 57.2 log points larger in terms of employment than a non-VC-backed firm. This is a calibration target. For the model, the employment ratio is

$$\text{Employment Ratio} = \frac{\left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta}\mathfrak{n}\mathbf{x}/\mathfrak{n}}{\left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta}\mathfrak{m}\omega\mathbf{x}/\mathfrak{m}} = \frac{1}{\omega}.$$

The last calibration target is the share of VC-funded firms in total employment, which was 7.3 percent in the U.S. data for the period 2001 to 2005. The counterpart for the model is

$$\text{VC Share Employment} = \frac{\mathfrak{n}}{\mathfrak{n}+\mathfrak{m}\omega}.$$

The upshot of the calibration procedure is now discussed. The parameter values used in the calibration are presented in Table 6. First, the model does a respectable job mimicking the cash-on-cash multiple, although it is a bit short of the target, as can be seen from Table 7. The model matches the average success and failure rates very well; again, see Table 7. The share of VC-backed firms in total employment generated by the model is also very close to the data. And, the model matches perfectly the VC-backed to non-VC backed employment size ratio.

Next, note how investment in a project by a venture capitalist increases with the funding round–see the top panel of Figure 5. This time profile is a calibration target. Given the limited life span of venture capital partnership, there is considerable pressure to bring a project to fruition as quickly as possible. This is true in the model too, which displays the same increasing profile of funding. Two features help to generate this. The first is that bad projects get purged over time through the evaluation process. The second is that the cost of monitoring drops as the VC becomes more familiar with project, which reduces the incentive problem. Without these features funding would fall over time. Last, since investment is rising over time one would expect that the venture's capitalist's share of the

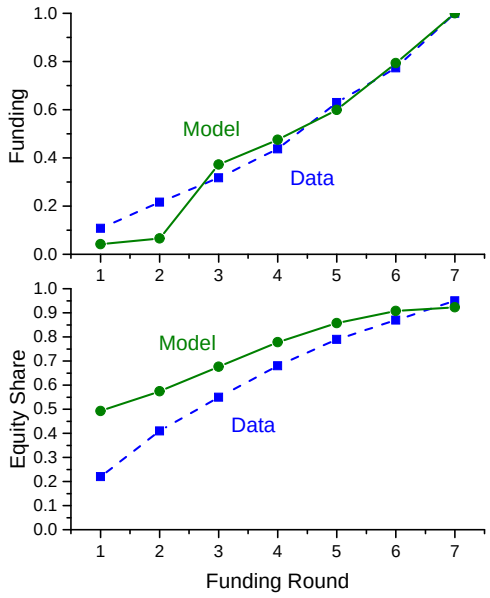


Figure 5: Investment and equity share by funding round–data and model. The upper panel shows the venture capitalist's investment by funding round. Funding in the last round is normalized to one. The lower panel charts the venture capitalist's share of equity by funding round.

enterprise will be too. The bottom panel of Figure 5 illustrates this. The model does well on this account. Again, the calibration procedure focuses on this feature of the data.

The time profiles for the success and failure rates are not targeted in the calibration procedure. As can be seen from the top panel of Figure 6, in the data the odds of success decline by funding round or with time. The model shares this feature of the data. Failure rates also decline with time. The model does fairly well on this dimension too. Now, turn to the bottom panel of Figure 6. Observe that the value of an IPO drops with the incubation time for the project. In the model, as time passes the value of a project declines because aggregate productivity catches up with the productivity of an entrepreneur's venture; "the thrill is gone," so to speak. It is a bit surprising that the framework can match almost perfectly this feature of the data, which is not targeted.

PARAMETER VALUES

Parameter Value	*Description*	*Identification*
Firms		
$\kappa = 1/3 \times 0.85 = 0.283$	Capital's share	Standard
$\lambda = 2/3 \times 0.85 = 0.567$	Labor's share	Standard
$1 - \mathfrak{d} = 0.07$	Depreciation rate	Standard
$\mathfrak{s} = 0.96$	Firm survival rate	Expected life of Compustat firms
$\chi_R = 7.7$	Research efficiency, x	Growth rate
$\iota = 2$	Research cost elasticity, x	Regression
$\nu = 0.01958$	Pareto shape parameter	Share of entrepreneurs
$\upsilon = 0.005$	Pareto scale parameter	Normalization
Consumers		
$\varepsilon = 2$	CRRA	Standard
$\widehat{\delta} = 1.011$	Discount factor	2% risk-free rate
VC		
$T = 7$	Number of funding rounds	Partnership length (10.5 years)
$\rho = 0.2$	Fraction of goods ideas	Cash Multiple
$\chi_D = 0.017$	Development efficiency, σ	Average success rate
$\chi_E = 0.140$	Evaluation efficiency, β	Average exit rate
$a = \{-0.111, 0.321, -0.019\}$	Monitoring efficiency, μ	Equity share by round
$b = \{-1.192, 1.909, -0.613, 0.087, -0.004\}$	Fixed costs, ϕ	VC funding by round
$\tau = 0.15$	Capital gains tax rate	Henrekson and Sanandaji (2016)
Non-VC		
$\mathfrak{m} = 1.7$	Number non-VC firms	Relative empl. non-VC firms
$\omega = 0.56$	Relative prod of non-VC firms	Relative size of non-VC firms

Table 6: The parameter values used in the baseline simulation.

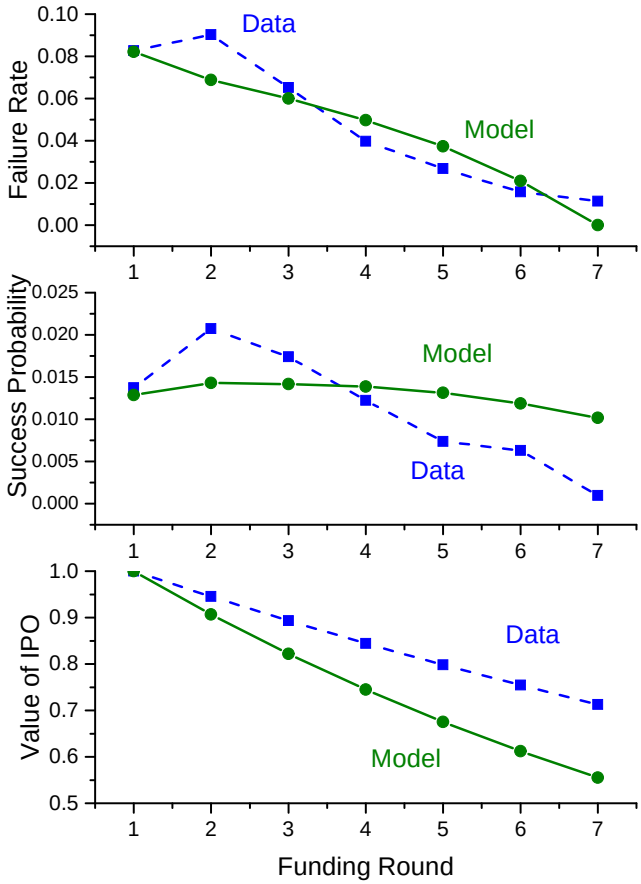


Figure 6: The odds of success and failure by funding round and the value of an IPO by the duration of funding–data and model. The value of an IPO that occurs during first funding round is normalized to one. All of these profiles are not targeted in the calibration.

Calibration Targets

Target	*Source*	*Data*	*Model*
Economic growth	BEA	1.80	1.75
Cash Multiple	Gompers et al (2016, Table 12)	5.50	4.73
Success Rate	Puri and Zarutskie (2012, Table VI.B)	1.1	1.3
Failure Rate	Puri and Zarutskie (2012, Table VI.B)	4.7	4.6
Share entrepreneurs	U.S. Census Bureau Business Dynamics Statistics	10	9.5
VC funding	Crunchbase	Figure 5	
Equity Share	Crunchbase	Figure 5	
IPO Value Elasticity	Regression (15)	0.86	0.90
VC Share Employment	Puri and Zarutskie (2012, Table I)	7.3	7.5
Employment ratio	Puri and Zarutskie (2012, Table IV)	57.2	57.2

Table 7: All numbers, except for the cash multiple, are in percentages. See the data appendix for a description of the data in Figure 5

6 Thought Experiments

6.1 Changes in Monitoring Efficiency, $\chi_{M.t}$

How important is the venture capitalist's ability to monitor the use of funds by entrepreneurs? To undertake this thought experiment, the efficiency of the monitoring profile, $\{\chi_{M,1}, \cdots, \chi_{M,7}\}$, is changed by scalar. The upshot of the experiment is shown in Figure 7. As efficiency in monitoring is improved there is an increase in the average odds of detecting fraud across funding rounds–see the top panel. The VC's share of equity rises, on average, because it is now easier for the VC to ensure that funds are not diverted. Compliance with the contract can be still be guaranteed when the entrepreneur is given a lower share of an IPO. As a result of improved monitoring, the VC can increase investment, which is reflected by a higher share of VC investment in GDP–middle panel. The VC must still earn zero profits. Part of the increased return to the VC is soaked up by letting the new entrepreneur be more ambitious about his choice of technique, which raises the initial cost of development, $R(x, \mathbf{x})$; the rest by the increased investment. So, the economy's growth rate moves up, which results in a welfare gain (measured in terms of consumption)–see the bottom panel.[4]

[4] See Akcigit, Celik, and Greenwood (2016, Section 5.1) for detail on how the welfare gain is computed.

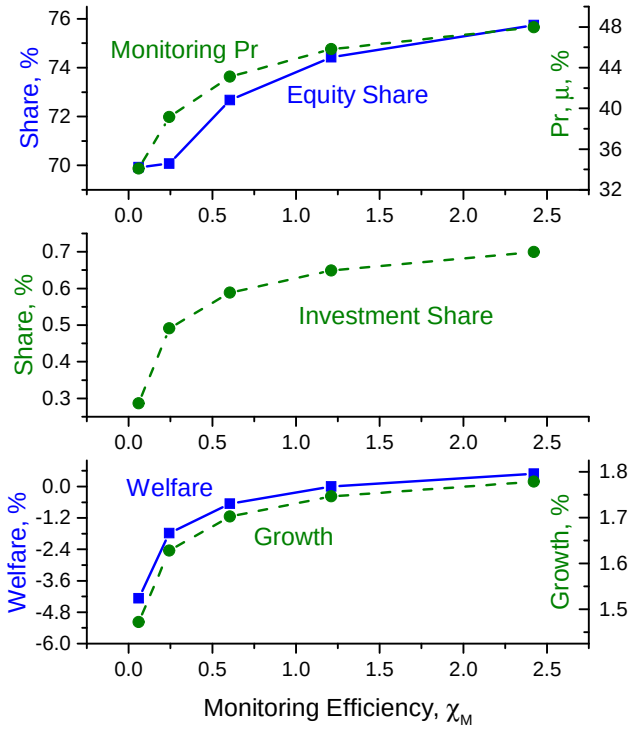


Figure 7: Efficiency in monitoring, χ_M. The top panel shows how the average probability of detecting fraud and the VC's share of equity vary with efficiency in monitoring. The middle panel illustrates how the ratio of VC investment (in startups) to GDP responds. Growth and welfare are displayed in the bottom panel.

6.2 Changes in Evaluation Efficiency, χ_E

The importance of efficiency in evaluation is examined now. The results are displayed in Figure 8. As evaluation becomes more efficient, the odds of detecting a bad project increase. Hence, the average failure rate across funding rounds moves up–see the top panel. The success rate rises, both due to the purging of bad projects and the resulting increased investment by the VC. The betterment in the pool of projects with improved evaluation is shown in the middle panel. At the last round the fraction of good projects, in the pool of funded ventures, rises with χ_E. The fact that it is more profitable to invest is reflected by an upward movement in the VC-investment-to-GDP ratio. Economic growth and welfare move up in tandem as evaluation efficiency improves–the bottom panel.

7 Capital Gains Taxation

Most VC-funded firms in the United States are setup as partnerships. CEOs, central employees, founders, and investors are paid in terms of convertible equity and stock options. These financial assets payoff only under certain well-specified contingencies and serve to align the incentives of key participants. Interestingly, the returns on convertible equity and stock options are taxed in the United States at the capital gains rate, which is 15 percent. The IRS lets companies assign an artificially low value to these instruments when they are issued. So, effectively, participants are only subject to taxation at the time of an acquisition/IPO. In other countries the rate of taxation on VC-funded startups is much higher. For example, it is 30 percent in France, 47.5 percent in Germany, and 72 percent in Italy. Figure 9 shows how VC investment as a percentage of GDP tends to fall with the tax rate on VC activity. In a cross-country regression analysis, Henrekson and Sanandaji (2016, Table 4) report a strong negative correlation between capital gains tax rates and VC investment as a percentage of GDP.

The impact of capital gains taxation in the model is illustrated in Figure 10. As the capital gains tax rate rises, not surprisingly, the share that VC-backed firms contribute to

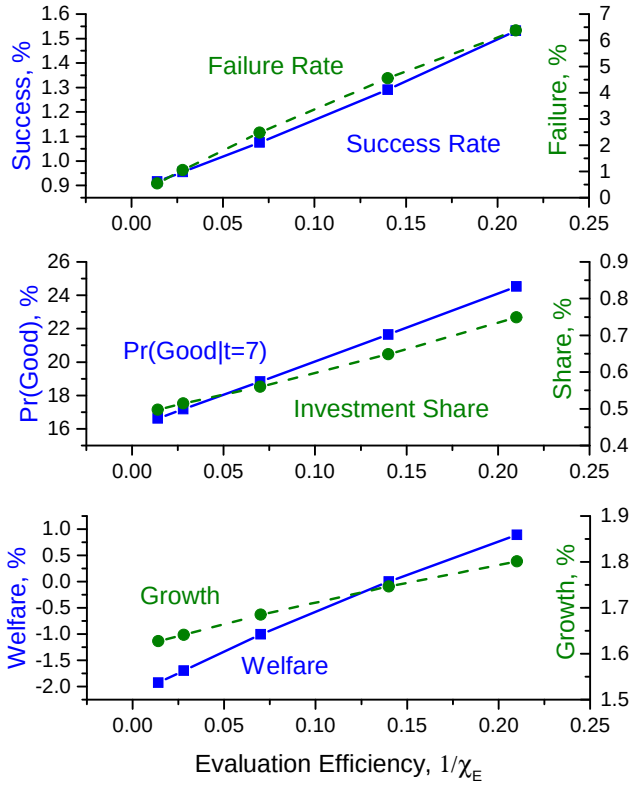


Figure 8: Efficiency in evaluation, χ_E. The top panel shows how the average failure and success rates across funding rounds vary with efficiency in evaluation. The middle panel illustrates how odds of being good in the seventh round and the ratio of VC investment (in startups) to GDP respond. Growth and welfare are illustrated in the bottom panel.

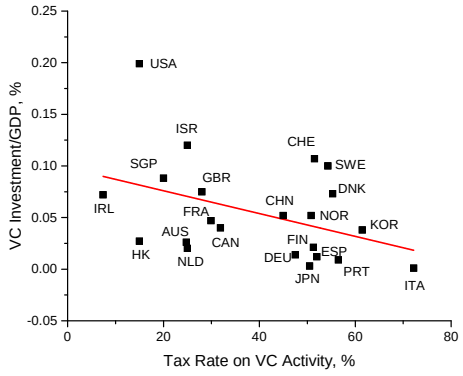


Figure 9: The cross-country relationship between the tax rate on VC activity and the ratio of VC investment to GDP, both expressed as percentages.

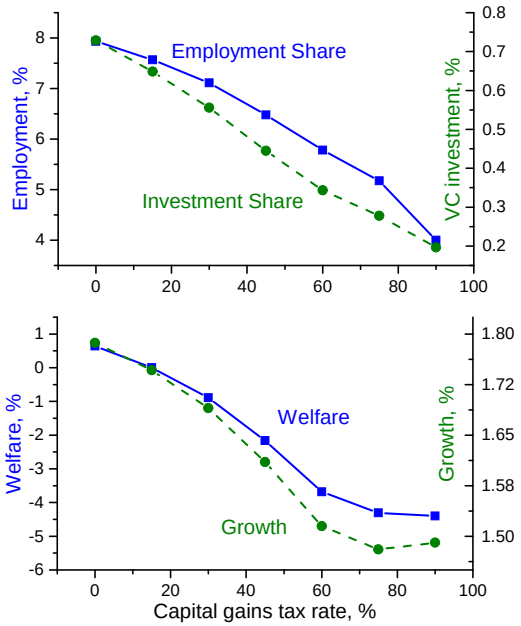


Figure 10: Impact of capital gains taxation. The upper panel shows the impact of capital gains taxation on the share of VC-funded firms in total employment and on the ratio of VC investment (in startups) to GDP. The lower panel illustrates the impact of capital gains taxation on economic growth and welfare.

total employment declines–focus on the top panel. It drops from about 8 percent, when capital gains are not taxed, to 4 percent, at a 90 percent rate. Likewise, VC investment in startups, as a share of GDP, declines from 0.7 to 0.2 percent. Note that the share of VC investment in GDP is very small, both in the data and model. The implied elasticity of the share of VC investment to the capital gains tax rate is -0.52. This compares with Henrekson and Sanandaji (2016, Table 5) estimates that hover around -1.00. So, the predicted effects from the model about the impact of capital gains taxation are on the conservative side. As the capital gains tax rate moves up from 0 to 90 percent, economic growth falls from 1.78 to 1.49 percent. This might not seem like much, but reducing the capital gains tax rate from 15 percent to zero produces a welfare gain of 0.65 percent, and increasing it from 15 to 75 percent generates a welfare loss of 4.4 percent, all measured in terms of consumption.

8 What about Growth?

Is the recent rise in VC investment reflected in growth statistics? The answer to this question is nuanced. On the one hand, at the country level VC investment appears to be positively linked with economic growth. A scatter plot between economic growth and VC investment for G7 countries is shown in the upper panel of Figure 11. These are developed nations. As can be seen, there is a clear positive association between these two variables. The analysis is extended to G20 countries in the bottom panel of the figure. Now, the scatter plot includes some poorer countries, where VC investment isn't so prevalent. There is still a positive association, but not surprisingly it is weaker.

To conduct a more formal analysis, some regression analysis is conducted A sample of 37 economies over the period 1995 to 2014 is used. This sample covers 99 percent of world VC investment and 88 percent of world GDP. In addition, this two-decade sampling period is divided into 4 sub-periods, each lasting 5 years. A country is included in the sample if its share of world VC investment between 1995 and 2014 is not less than 0.05 percent.[5] The dependent variable in the regression analysis is the median of the growth rate of real GDP per capita in each period, while the main explanatory variable is the natural logarithm of the median VC investment-to-GDP ratio in each period. The regressions include the initial levels of real GDP per capita and the Barro and Lee (2013) human capital index. These control variables are the two main factors demonstrated to be important in the empirical literature of the determinants of economic growth. Moreover, period dummies are included to control for aggregate shocks to all countries. An IV approach is also taken to address the endogeneity issues. Two IVs are used. The first is the median VC investment-to-GDP ratio for each country during the decade preceding sampling period (i.e., 1985 to 1994), following the strategy pioneered in Barro and Lee (1994). The second is a dummy variable for the legal origin of the country, which is equal to one for common-law countries. The idea is that common-law systems foster better financial development than the civil law ones, because of

[5] An exception is Bermuda, which accounted for 0.18% of world VC investment. Bermuda is excluded because it is a tax haven. Companies set up offices there, while undertaking virtually no business activity, to avoid corporate income taxation.

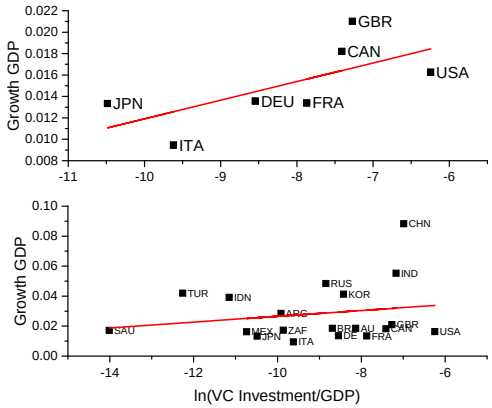


Figure 11: Economic Growth and VC Investment, 1995-2014. The upper panel shows the relationship between VC investment and growth in G7 countries, while the bottom panel does the same thing for the G20.

higher judicial independence from the government and the flexibility of the courts to adapt to changing conditions in the common-law countries–see Beck, Demirguc-Kunt, and Levine (2005).

The main regression results are reported in Table 8. As the table shows, VC and growth are positively correlated. Take the IV estimate for the G7 countries in the last regression in Panel A. This signifies that a ten percent increase in the VC investment-to-GDP ratio will be connected with a 0.024 percentage point increase in growth. This may seem small, but it implies that increasing the VC investment-to-GDP ratio from the Japanese level (0.003 percent) to the U.S. level (0.19 percent) would increase growth by 1.01 percentage points.[6]

On the other hand, the impact of venture capital may not be readily apparent in growth statistics for several reasons. First, technological revolutions, such as the information age, may cause disruptions in an economy. Old forms of businesses are displaced by new forms. Online retailing is displacing brick and mortar stores for example. Greenwood and Yorukoglu (1997) discuss how the dawning of the first and second industrial revolutions were associated with productivity slowdowns and suggest that the same phenomena characterize the infor-

[6] Relatedly, Sampsa and Sorenson (2011) estimate, using a panel of U.S. metropolitan statistical areas, that venture capital positively affects startups, employment, and regional income.

VC Investment and Growth: Cross-Country Regressions

Dependent Variable	*Growth of GDP*			
	OLS	IV		
		Pre ln(VC Inv/GDP)	Legal Origin	Both
Panel A: G7				
ln(VC Inv/GDP)	0.186**	0.253***	0.227**	0.240***
	(0.0782)	(0.0910)	(0.0899)	(0.0816)
Observations	28	28	28	28
R-squared	0.695			
Panel B: 37-Country Sample				
	0.228**	1.156**	0.421*	0.463*
	(0.112)	(0.501)	(0.254)	(0.260)
Observations	148	120	148	120
R-squared	0.295			

Table 8: See the main text for a description of the dependent and independent variables. Pre ln(VC Inv/GDP) refers to the pre-sample VC-investment-to-GDP ratio. Standard errors are in parentheses. ***, **, and * denote significance at the 1, 5 and 10 percent levels.

mation age. Second, measuring investments and outputs in the information age is difficult. Think about the introduction of cell phones, as discussed in Hulten and Nakumura (2017). Cell phones substitute for traditional land lines, audio players, cameras, computers, navigation systems, and watches, inter alia. Cell phones have free apps. Between 1988 and 2015, land lines fell from 1.7 to 0.3 percent of personal consumption expenditure. Since cell phones constitute 0.15 percent of personal consumption expenditure, this would be measured as a drop or slowdown in GDP. An iPhone 5 would have cost more than 3.56 million dollars to build in 1991.[7] Likewise, global camera production dropped from 120 million units to 40 million from over the 2007 to 2014 period. Additionally, investment may be in intangibles, such as software, R&D, retraining workers, reconfiguring products and organizational forms, branding new products, etc. Corrado, Hulten and Sichel (2009) estimate that investment in such intangibles is now as large as that in tangibles. Including intangible investment in

[7] This guesstimate was done by Bret Swanson, who calculates that the flash memory, processor, and broadband communications of an iPhone 5 would have cost 1.44, 0.62, and 1.5 million dollars in 1991. The cost of these three components adds up to 3.56 million dollars. On top of that, considering the other components (camera, iOS operating system, motion detectors, display, apps, etc), it would have cost more than 3.56 million dollars to build an iPhone 5 in 1991.

GDP accounting could increase estimates of growth by 10 to 20 percent. McGrattan and Prescott (2005) argue that, after taking intangibles into account, the 1990s was a boom period. Third, technologies flow across national boundaries. So, even countries that don't innovate will experience growth from the adoption of new technologies. Out of France, Germany, Japan, the United Kingdom, and the United States, Eaton and Kortum (1999) find that only the United States derived most of its growth from domestic innovation. Comin and Hobijn (2010) document that diffusion lags for new technologies have shrunk over time. Fourth, firms may park the profits from new innovation offshore to avoid taxation. Accounting for this could increase productivity growth by 0.25 percentage points over the 2004 to 2008 period, according to Guvenen et al (2017).

9 Conclusion

Venture capital appears to be important for economic growth. Funding by VCs is positively associated with patenting activity. VC-backed firms have higher IPO values when they are floated. Following flotation they also have higher R&D-to-sales ratios. VC-backed firms also grow faster in terms of employment and sales.

An endogenous growth model of the venture capital process is constructed and taken to the data. In the framework, entrepreneurs take ideas to venture capitalists for funding. Venture capitalists evaluate projects to access their ongoing viability and monitor them to avoid malfeasance. The terms of investment in development, evaluation, monitoring, and the equity share of the venture capitalist are governed by a dynamic contract between the entrepreneur and a venture capitalist. The model is capable of matching several stylized facts of the venture capital process by funding rounds. In particular, it mimics the funding-round profiles for the success and failure rates of projects, investment by the venture capitalist, the venture capitalist's share of equity, and the value of an IPO by the time it takes to go market. This is done while matching the share of VC-backed firms in total employment and the average size of a VC-backed firm relative to a non-VC-backed one.

The key personnel involved with starting up the enterprises funded by venture capitalists are rewarded in the form of convertible equity and stock options. In the United States, they are subject only to capital gains taxation. The rate at which VC-funded startups are taxed in the United States is low relative to other developed countries. Does this promote innovative activity? The analysis suggests that raising the tax on VC-funded startups from the U.S. rate of 15 percent to an Italian rate of 75 percent would shave 0.25 percentage points off of growth and lead to a consumption equivalent welfare loss of 4.3 percent.

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10 Data Appendix

10.1 Figures

- *Figure 1: The Rise in Venture Capital.* Investment by venture capitalists is obtained from the VentureXpert database of Thomson ONE. The fraction of public firms backed by VC companies is drummed up by matching firm names in VentureXpert and CompuStat, the latter available from Wharton Research Data Services.[8]

- *Figure 2: The Share of VC-Backed Companies.* The employment and R&D shares of VC-backed public companies are calculated by matching firm names in VentureXpert and CompuStat, as in Figure 1. The share of patents for VC-backed public companies is computed by matching firm names in VentureXpert and the NBER Patent Data Project.[9]

- *Figure 3: Banks and Venture Capital, 1930-2008.* The data on the use of the words "banks" and "venture capital" relative to all words in English language books derives from the Google Ngram Viewer. The year 2008 has been normalized to 100 for both series.

- *Figure 5: Investment and Equity Share.* Investment at each funding round is based on the VC-funded deals in Crunchbase between 1981 and 2015. The vertical axis is the mean of level funding in a round across all deals, from round 1 (i.e., series A) to round 7 (i.e., series G). It is converted into constant 2009 million dollars using the GDP deflator. The mean duration of a funding round in Crunchbase is 1.4 years, which is taken to 1.5 years here. The share of equity transferred to the VC at each funding round is calculated as the ratio of VC funding at each round to the post-money valuation of the company after the VC investment. For each funding round, the mean value of equity share across all deals is used, and the vertical axis is the cumulated

[8] Source link: https://wrds-web.wharton.upenn.edu/wrds/index.cfm?
[9] Source link: https://sites.google.com/site/patentdataproject/Home

shares of equity transferred to VC.

- *Figure 6: The odds of success and failure by funding round and the value of an IPO by the duration of funding.* The underlying data source is Puri and Zarutskie (2012, Table VI.B, p. 2271). The success rate refers to firms that have an IPO or that are acquired by another firm. The acquisitions in Puri and Zarutskie (2012) are converted into successes by multiplying by 0.629. This is based on the fact that the cash multiple for acquisitions is 37.1% lower than for IPOs, as reported in Achleitner et al. (2012). In addition, the success and failure rates by funding round are obtained by interpolating the original annual data using a cubic spline to get a periodicity of 1.5 years. The value of an IPO as a function of the duration of VC funding derives from the regression discussed below.

- *Figure 9*: The source is Henrekson and Sanandaji (2016, Table 1) .

- *Figure 11: Economic Growth and VC Investment.* VC investment and the growth rate of real GDP per capita are based on VentureXpert of Thomson ONE and the World Development Indicators of the World Bank, respectively.

10.2 Tables

- *Table 1: Top 30 VC-Backed Companies.* As in Figure 1, the list of VC-backed public companies is gathered by matching firm names in VentureXpert and CompuStat.

- *Table 2: VC versus Non-VC-Backed Public Companies.* The VC-backed public companies are singled out by matching firm names in VentureXpert and CompuStat. Since the R&D-to-sales ratio and growth rates can be very volatile across firms, the top and bottom 5 percent of the outliers are trimmed in this regression. The results are robust to changing the trimming threshold (at the level of 1 percent versus 5 percent).

- *Table 3: VC and Patenting, Firm-Level Regressions.* The VC-funded patentees are

identified by matching firm names in VentureXpert and PatentsView.[10] The capital gain taxes are accessed from TAXSIM, an NBER tax simulation program.[11] In calculating the dependence on external finance, 30 percent of selling, general and administrative expense is taken as intangible investment. The industry-level of private and federally funded R&D is collected from the Business R&D and Innovation Survey by the National Science Foundation.[12] A truncation adjustment for citations is made following Bernstein (2015). The industry dummies in this regression are at the 2-digit SIC level.

- *Table 4: VC and Patenting, Industry-Level Regressions.* The product of the deregulation dummy and dependence on external finance is used as the IV for the cross term between VC funding and dependence on external finance. The industry panel is based on the 4-digit SIC. The industry dummies in this regression are at 2-digit SIC level.

- *Table 8: VC Investment and Growth, Cross-Country Regressions.* The full sample covers 37 economies between 1995 and 2014. As in Figure *11,* VC investment is from VentureXpert and the GDP growth rate is from the World Development Indicators. The Barro and Lee (2013) human capital index is a measure of the educational attainment at the country level. The IVs are the median VC investment-to-GDP ratio (in natural logarithm) for each country between 1985 and 1994, and a dummy variable for legal origin (equal to one for common-law countries) à la Beck, Demirguc-Kunt, and Levine (2005).

10.3 Duration of VC Funding and the Value of an IPO

The relationship between the firm's value at an IPO and the number of years it received funding from the VC is examined using regression analysis. The regressions are based on public companies funded by VCs between 1970 and 2015. These VC-backed companies

[10] Source link of PatentsView: http://www.patentsview.org/download/.
[11] Source link of TAXSIM: http://users.nber.org/~taxsim/state-rates/.
[12] Source link of BRDIS: https://www.nsf.gov/statistics/srvyindustry/#tabs-2.

are identified by matching firm names in CompuStat with VentureXpert. The dependent variable in the regressions is the natural logarithm of the market value of the firms at IPO (in 2009 dollars). A three-year average is used for market value because of the notorious volatility of share prices following an IPO. IPOs are excluded when they take more than 11 years for the firms to go public after receiving the first funding from VCs. This is for two reasons: (i) the sampling period is formulated to be consistent with the model where the maximum duration for each VC investment is 10.5 years, and (ii) only 4.5 percent of the observations occur after 11 years with the data being very noisy. The main explanatory variable is the number of years between the firm's first VC funding and the date of its IPO.

VC Funding and Years to Go Public

Dependent Variable	ln(*Firm Value at IPO, real)*	
	1	2
years btw first VC funding and IPO	-0.0470***	-0.0385***
	(0.0161)	(0.0146)
firm age at IPO		-0.0246***
		(0.00495)
# of employees at IPO (log)		0.709***
		(0.0375)
year dummy for IPO	N	Y
industry effect	N	Y
Observations	1,042	1,006
R-squared	0.008	0.627
Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1		

11 Theory Appendix

Proofs for Lemmas 2 and 4 are supplied in turn here. Lemmas 2 establishes the existence of a balanced growth path. Lemma 4 shows that solving the contract problem (P2) subject

to a sequence of one-shot incentive constraints is equivalent to solving it subject to a single consolidated time-0 incentive constraint that allows for multi-shot deviations. Lemma 4 proves this, using Lemma 3 as an intermediate step.

11.1 Balanced Growth

Lemma 2 *(Balanced Growth) There exists a balanced growth of the form outlined in Definition 1.*

Proof. Suppose that $\{p_t, \sigma_t, \mu_t, \beta_t\}$ solves the old problem. It will be shown that $\{\mathbf{g}_w p_t, \sigma_t, \mu_t, \beta_t\}$ solves the new one. First, observe that if $x' = \mathbf{g}_x x$ and $\mathbf{x}' = \mathbf{g}_x \mathbf{x}$, then $I(x'; \mathbf{g}_x^t \mathbf{x}') = \mathbf{g}_w I(x; \mathbf{g}_x^t \mathbf{x})$. This occurs because $T(x'; \mathbf{x}_t') = \mathbf{g}_w T(x; \mathbf{x}_t)$. This can be seen from (P1) because x will rise by $\mathbf{g}_x$ and wages by $\mathbf{g}_w$. If $p_t^{'} = \mathbf{g}_w p_t$, then it is immediate from the objective function in (P2) that $C(x'; \mathbf{x}') = \mathbf{g}_w C(x; \mathbf{x})$. Now, consider the incentive constraint (5). At the conjectured solution the left-hand side will blow up by the factor $\mathbf{g}_w$. So, will the right-hand side because $D(\sigma_t') - D(\widetilde{\sigma}_t') = \mathbf{g}_w[D(\sigma_t) - D(\widetilde{\sigma}_t)]$, since all costs are specified as a function of w. Therefore, the new solution still satisfies the incentive constraint. Move now to the zero-profit constraint (6). Again, the left-hand side will inflate by the factor $\mathbf{g}_w$, since $E(\beta_t^{'}) = \mathbf{g}_w E(\beta_t)$, $p_t^{'} = \mathbf{g}_w p_t$, $\phi_t^{'} = \mathbf{g}_w \phi_t$, $M_t(\mu_t') = \mathbf{g}_w M_t(\mu_t)$, and $D(\sigma_t') = \mathbf{g}_w D(\sigma_t)$. This is trivially true for the right-hand side. Hence, the zero-profit constraint holds at the new allocations. Last, it is easy to deduce from the right-hand side of (5) that the old solution for $\widetilde{\sigma}_t$ will still hold. This can be seen by using the above line of argument while noting that $D_1(\widetilde{\sigma}_t') = \mathbf{g}_w D_1(\widetilde{\sigma}_t)$. To sum up, at the conjectured new solution the objective function and the constraints all scale up by the same factor of proportionality $\mathbf{g}_w$. By cancelling out this factor of proportionality, the new problem reverts back to the old one. Last, it is now easy to see that problem (P3) is homogeneous of degree one in x and $\mathbf{x}$. Therefore, if $x/\mathbf{x}$ remains constant along a balanced-growth path, then the initial development cost of the project will rise at the same rate as wages, $\mathbf{g}_w$. Additionally, $V(\mathbf{x})$ will grow the same rate as wages, w, so from (7) it is apparent that $\mathfrak{e}$ will remain constant. ■

11.2 One-Shot Deviations versus Multi-Shot Deviations

This is an intermediate step toward solving Lemma 4. To this end, it will be shown that if the incentive constraint (5) holds for period t, when the entrepreneur has not deviated up to period $t-1$, then it will also hold when he follows some arbitrary path of deviations up to stage $t-1$. Let α_t represent that the probability that a project is good at stage t as defined by (4). These odds evolve recursively according to

$$\alpha_{t+1} = \frac{(1-\sigma_t)\alpha_t}{(1-\sigma_t)\alpha_t + (1-\beta_{t+1})(1-\alpha_t)},$$

where $\alpha_1 = \rho/[\rho + (1-\rho)(1-\beta_1)]$. For use in proving Lemma 3, note that α_{t+1} is increasing in α_t and decreasing in σ_t. This implies that if the entrepreneur deviates in period t, so that $\widetilde{\sigma}_t < \sigma_t$, he will be more optimistic about the future, as α_{t+1} will be higher. This increases the value of α's for future periods as well. With this notation, the period-t incentive constraint (5) then reads

$$\begin{aligned}\alpha_t\{\delta\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\} \\ \geq (1-\mu_t)\max_{\widetilde{\sigma}_t}\Bigg(D(\sigma_t) - D(\widetilde{\sigma}_t) \\ + \alpha_t\{\delta\widetilde{\sigma}_t[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\widetilde{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\}\Bigg).\end{aligned}$$

Lemma 3 *If the incentive constraint (5) holds for period t, when the entrepreneur has not deviated up to and including period $t-1$, then it will also hold when he follows some arbitrary path of deviations up to and including stage $t-1$.*

Proof. Suppose that the entrepreneur deviates in some manner up to stage $t-1$. Let $\widehat{\alpha}_t$ be the prior associated with this path of deviations. Since the $\widetilde{\sigma}$'s will be less that than the σ's,

it follows that $\widehat{\alpha}_t > \alpha_t$. Let $\widehat{\sigma}_t$ be the *optimal* period-t deviation associated with $\widehat{\alpha}_t$. Now,

$$\begin{aligned}
&\alpha_t\{\delta\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\} \\
&\qquad\qquad \geq (1-\mu_t)\Bigg(D(\sigma_t) - D(\widehat{\sigma}) \\
&\quad + \alpha_t\{\delta\widehat{\sigma}_t[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\widehat{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\}\Bigg),
\end{aligned}$$

because $\widetilde{\sigma}_t$ is maximal when the prior is α_t while $\widehat{\sigma}_t$ is not. Next, replace α_t with $\widehat{\alpha}_t$ to get

$$\begin{aligned}
&\widehat{\alpha}_t\{\delta\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\} \\
&\qquad\qquad \geq (1-\mu_t)\Bigg(D(\sigma_t) - D(\widehat{\sigma}_t) \\
&\quad + \widehat{\alpha}_t\{\delta\widehat{\sigma}[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\widehat{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\}\Bigg),
\end{aligned}$$

since $\widehat{\alpha}_t > \alpha_t$. Last, if the prior is $\widehat{\alpha}_t$, then $\widehat{\sigma}_t$ is maximal, so that the above equation can be rewritten as

$$\begin{aligned}
&\widehat{\alpha}_t\{\delta\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\} \\
&\qquad\qquad \geq (1-\mu_t)\max_{\widehat{\sigma}_t}\Bigg(D(\sigma_t) - D(\widehat{\sigma}_t) \\
&\quad + \widehat{\alpha}_t\{\delta\widehat{\sigma}_t[I(x;\mathbf{g}_x^t\mathbf{x}) - p_t] + (1-\widehat{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x}) - p_i]\}\Bigg).
\end{aligned}$$

■

11.3 The Consolidated Time-0 Incentive Constraint

The consolidated period-0 incentive constraint is

$$\sum_{t=1}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t] \;\geq\; \max_{\{\widetilde{\sigma}_t\}_{t=1}^{T}}\{\sum_{t=1}^{T}\delta^{t-1}[\rho\Pi_{j=1}^{t-1}(1-\widetilde{\sigma}_j)+(1-\rho)\Pi_{j=1}^{t}(1-\beta_j)] \times(1-\mu_t)[D(\sigma_t)-D(\widetilde{\sigma}_t)] + \sum_{t=1}^{T}\rho\Pi_{j=1}^{t-1}(1-\widetilde{\sigma}_j)\delta^t\widetilde{\sigma}_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t]\}. \tag{16}$$

Lemma 4 *(Equivalence of contracts) A contract* $\{p_t,\sigma_t,\mu_t,\beta_t\}$ *solves problem (P2) subject to the sequence of one-shot incentive constraints (5) if and only if it solves (P2) subject to the consolidated time-*0 *incentive constraint (16).*

Proof. (*Necessity*) Suppose that an allocation satisfies the one-shot incentive compatibility constraints (5) but that it violates the consolidated one (16). This implies that at some stage in the consolidated constraint it pays to deviate and pick a $\widetilde{\sigma}_t\neq\sigma_t$. Pick the last period of deviation (which may be T). It must be true that $\widetilde{\sigma}_t$ solves the maximization problem

$$(1-\mu_t)\max_{\widetilde{\sigma}_t}\Bigg(D(\sigma_t)-D(\widetilde{\sigma}_t) + \widehat{\alpha}_t\{\delta\widehat{\sigma}[I(x;\mathbf{g}_x^t\mathbf{x})-p_t]+(1-\widetilde{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x})-p_i]\}\Bigg),$$

where $\widehat{\alpha}_t$ is the prior associated with the path of σ's up to period $t-1$, which may include previous deviations. But, as was shown in Lemma 3, this is less than value of sticking with the contract or

$$\widehat{\alpha}_t\{\delta\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t]+(1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_x^i\mathbf{x})-p_i]\},$$

when the period-t one-shot incentive constraint (5) holds, as assumed.

(*Sufficiency*) Suppose $\{\sigma_t\}_{t=1}^T$ satisfies the consolidated incentive constraint, but one violates the one-shot incentive constraint at stage k. Then, using (4) and (5), it follows that

$$\rho\Pi_{j=1}^{k-1}(1-\sigma_j)\delta^{k-1}\{\delta\sigma_k[I(x;\mathbf{g}_x^k\mathbf{x})-p_k]+(1-\sigma_k)\sum_{t=k+1}^{T}\Pi_{j=k+1}^{t-1}(1-\sigma_j)\delta^{t+1-k}\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t]\}$$

$$=\sum_{t=k}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t]$$

$$<\delta^{k-1}(1-\mu_k)\Bigg([\rho\Pi_{j=1}^{k-1}(1-\sigma_j)+(1-\rho)\Pi_{j=1}^{k}(1-\beta_j)][D(\sigma_k)-D(\widetilde{\sigma}_k)]$$

$$+\rho\Pi_{j=1}^{k-1}(1-\sigma_j)\{\delta\widetilde{\sigma}_k[I(x;\mathbf{g}_x^k\mathbf{x})-p_k]+(1-\widetilde{\sigma}_k)\sum_{t=k+1}^{T}\Pi_{j=k+1}^{t-1}(1-\sigma_j)\delta^{t+1-k}\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t]\}\Bigg). \tag{17}$$

The left-hand side gives the payoff in the contract at the optimal solution from stage k on, when using the consolidated incentive constraint, while the right-hand side represents the payoff from a one-shot deviation at stage k. Now, the objective function for the contract can be written as

$$\sum_{t=1}^{k-1}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t]+\sum_{t=k}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g}_x^t\mathbf{x})-p_t].$$

Evaluate this at the optimal solution for contract when using (16) instead of (5). Next, in this objective function replace the payoff from stage k on, as represented by the left-hand side of (17), with payoff from the one-shot deviation, as given by the right-hand side. This deviation would increase the value of the objective function for the entrepreneur, which is a contradiction. ■