The Production of Cognitive and Non-cognitive Human Capital in the Global Economy

Chong Xiang, Purdue University
Stephen Ross Yeaple, Penn State University and NBER

November 2017

Abstract

A country's welfare is tied to its ability to foster the accumulation of cognitive and non-cognitive human capital. However, we do not fully understand how countries produce them and even struggle with their measurement. For instance, international test scores neglect non-cognitive human capital. In this paper, we develop a multi-country, open-economy general-equilibrium framework where heterogeneous individuals make optimal occupational choices. Because people in non-cognitive (cognitive) occupations primarily use their non-cognitive (cognitive) human capital, by observing people's occupational choices we can quantify how well countries produce their cognitive and non-cognitive human capital. We show that countries differ substantially in the underlying productivities of cognitive and non-cognitive human capital. We then aggregate over these two dimensions to construct a single measure of human-capital productivity, and to demonstrate its implications for cross-country differences in output per worker. Our results have important policy implications: many high-test-score countries fair poorly according to our productivity metrics, and education policies that *increase* test score may *decrease* aggregate output. Our results depend on how much countries trade the services of cognitive and non-cognitive human capital: if barriers to such trade were eliminated, the countries with strong comparative advantages in producing cognitive or non-cognitive human capital would reap large gains in aggregate output.

1 Introduction

Human capital is central to both economics and other social sciences. Therefore, understanding how well countries produce their human capital is critical for both academic research and for policy. One way these questions are currently answered is to look at the scores of international assessment tests, like PISA. The U.S. generates low test scores despite having one of the highest levels of per-capita educational spending in the world. These low test scores have alarmed policy makers in the U.S.,[1] and motivated major policy changes (e.g. No Child Left Behind of 2001 and Race to the Top of 2009). Like the U.S., many other countries worry that their test scores are *too low* (e.g. U.K., Canada, Slovakia and Qatar).[2]

Oddly, many countries whose students excel in international exams worried that their educational systems overemphasize formal examination proficiency at the expense of less tangible forms of education with the result that their students spend too much time studying for exams![3] This concern has influenced policy: the Education Ministry in China declared a ban on homework assignments for young children in August 2013, and South Korea declared a 10 pm curfew on private tutoring. The fear is that the educational systems emphasize testing to such a degree that students do not effectively develop other useful skills, such as leadership, co-operation, and communication. While the importance of these non-cognitive skills has been clearly established in academic research (e.g. Heckman and Rubinstein 2001), their quantification and measurement remain challenging, because many of them do not show up in test scores (e.g. Heckman and Kautz 2012). Hanushek and Woessmann (2011) recognize that "the systematic measurement of such skills has yet to be possible in international comparisons".

In this paper, we present a multi-country, open-economy general equilibrium (GE) framework that we have developed to quantify how countries produce human capital

[1] e.g. President Obama said that the nation that "out-educates us today will out-compete us tomorrow."

[2] For example, in February 2014, Elizabeth Truss, the U.K. education minister, visited Shanghai, China, whose test score is much higher than the U.K.'s, to "learn a lesson a math".

[3] For example, the Wall Street Journal reports that "A typical East Asian high school student often must follow a 5 a.m. to midnight compressed schedule, filled with class instruction followed by private institute courses, for up to six days a week, with little or no room for socializing" (February 29, 2012), and that "many students prepare for [the national college] entrance exams from an early age, often studying up to 16 hours a day for years to take these tests" (November 10, 2011).

along multiple dimensions. The starting point of our framework is the observation that peoples' occupational choices reveal information about their skills at different types of tasks, and part of these skills have been developed through their education. For example, a manager issues directions and guidance to subordinates, a secretary follows these orders, while an engineer uses the knowledge in math and science to solve problems. We follow previous research (e.g. Autor, Levy and Murname 2003) and classify occupations as non-cognitive and cognitive. Because the people in non-cognitive (cognitive) occupations are primarily drawing on their non-cognitive (cognitive) human capital, by observing people's occupational choices we can quantify how well countries produce their human capital along these dimensions.

To be specific, we write down production functions of cognitive and non-cognitive human capital, and use the TFP's (Total Factor Productivity) of these production functions to quantify countries' productivities in accumulating cognitive and non-cognitive human capital. Our inspiration is the strong and intuitive intellectual appeal of TFP and its ubiquitous uses to measure the qualities of production technologies for countries, industries and firms. Intuitively speaking, a country with a high cognitive (non-cognitive) productivity produces a large quantity of cognitive (non-cognitive) human capital, holding fixed resources inputs.

In addition, researchers have long recognized that incentives matter for educational outcomes,[4] which is closely related to human capital production. We accommodate incentives in our model by having heterogeneous workers make optimal occupational choices given their own comparative advantages in non-cognitive and cognitive skills, as in Willis and Rosen (1979). These individuals' comparative advantages, in turn, are determined by their innate abilities at birth and human capital accumulated. When workers decide how much human capital to accumulate, they factor in the returns of human capital on the labor market, recognizing that non-cognitive and cognitive occupations require different types of human capital. This implies that in our model, individual workers' human capital accumulation is affected by their occupational choices, which, in turn,

[4]In empirical studies using micro data, researchers have long recognized that incentives, in the form of money or even candy, improve the scores of IQ tests (Heckman and Kautz 2012). In a recent large-scale field experiment in Mexico, Behrman, Parker, Todd, and Wolpin (2015) show that providing monetary incentives to students has substantial and immediate effects on their test scores.Researchers have also shown that instructor incentives matter for the scores of high-stake tests (e.g. Neal and Schanzenbach 2010). PISA, whose scores we use, is not a high-stake test.

depend on the non-cognitive and cognitive productivities of the economy.

For cognitive productivities we use test scores as the starting point, leveraging on the widely available test-score data and building on the insight of the empirical literature on international test scores (e.g. Hanushek and Woessmann 2011). We then peel back the confounding factors of resources inputs and incentives under the guidance of our GE model, to reveal the countries' underlying productivities in fostering cognitive human capital. Importantly, this procedure remains the same whether our model is closed-economy, open-economy with free trade, or open-economy with positive trade costs. Our results show that countries' cognitive-productivity rankings are substantially different than their PISA-score rankings. In particular, those with the highest test scores do not necessarily have the highest cognitive productivities (e.g. S. Korea, Hong Kong).

Our GE model draws on the insight of the empirical literature that examines non-cognitive skills using micro data (e.g. Kuhn and Weinberger 2005, Heckman and Kautz 2012). In our model, the ratio of non-cognitive productivity to cognitive productivity of the country's educational system, i.e. its comparative advantage for non-cognitive human capital, drives workers' occupational choices, and so comparative advantage in human capital accumulation is revealed by the ratio of occupation employment shares. Intuitively speaking, the fact that many individuals in country k choose the non-cognitive occupation suggests that country k has a strong comparative advantage for non-cognitive human capital. If, in addition, country k has a high absolute advantage in cognitive human capital accumulation, then this country must also have an absolute advantage in non-cognitive human capital accumulation.

Here international trade plays an important role. In the closed-economy setting, the relative return of non-cognitive human capital ultimately depends on country k's comparative advantage for non-cognitive human capital. Therefore, data on occupation employment shares are sufficient to back out this comparative advantage. With international trade, however, the relative return of non-cognitive human capital is determined globally. Intuitively speaking, the fact that country k is a large net importer of the service of non-cognitive human capital suggests that it has a strong comparative advantage for non-cognitive human capital, since the non-cognitive workers in k have chosen their occupation despite import competition. Our model delivers an analytical expression for the comparative advantage of human capital production, where the effects of trade are summarized by its factor content in terms of cognitive and non-cognitive human capital. Of course, high barriers to factor content trade shrink the difference between open- vs.

closed-economy settings.

Looking at our data, we show that non-cognitive productivities are very similar across open- and closed-economy settings. In particular, countries' non-cognitive-productivity rankings have zero correlation with their PISA score rankings, and many countries with low test scores have high non-cognitive productivities (e.g. the U.S. and U.K.). Therefore, non-cognitive productivities are a novel dimension of the quality of human-capital production that is not revealed by test scores.

Our model then allows us to condense the multi-dimensional differences in cognitive and non-cognitive productivities into a single metric, which we call overall educational quality. This metric is the weighted power mean of cognitive and non-cognitive productivities, the weights being the employment shares of cognitive and non-cognitive occupations. The power coefficients of this metric depend on the following three parameters: the dispersion of workers' innate abilities, which governs the supply-side elasticity of the economy; the substitution-elasticity across different types of human capital in aggregate production, which governs the demand-side elasticity of the economy; and the output elasticity in the production of human capital. To identify these key parameters, we draw on the parsimonious relationships predicted by our model among publicly available data, such as test score, output per worker, employment shares of non-cognitive and cognitive occupations, and factor content of trade. Our unique focus on cross-country differences in the production of human capital, distinguish our work from the quantitative literature on worker heterogeneity and income dispersion (e.g. Ohsornge and Trefler 2007, Hsieh, Hurst, Jones and Klenow 2016, Burnstein, Morales and Vogel 2016).

We can now graph the combinations of cognitive and non-cognitive productivities that produce the same overall educational quality, borrowing the idea of the isoquant. This iso-education-quality figure illustrates the large differences in how countries produce their cognitive and non-cognitive human capital. To draw out the economic significance of such differences, we show that the ratio of output per worker between any pair of countries can be decomposed into a power function of the ratio of overall educational quality, multiplied by a power function of the ratio of output TFP. Implementing this exact decomposition using raw data and our model parameters, we show that the differences in human-capital productivities across countries have large implications for output per worker. For example, Germany's output per worker is 62.96% of the U.S. level (data), of which 88.34% can be attributed to human-capital productivities (model parameters) and 71.26% to output TFP (model parameters).

Our paper has some outward similarity to the literature that accounts for variation across countries in income per capita. This literature has focused on the appropriate way to aggregate labor that varies in the number of years of education (e.g. Mankiw, Romer and Weil 1992, Klenow and Rodriguez-Clare 1997, Caselli 2005). Some of the more recent literature, such as Jones (2014) and Malmberg (2017), have improved on this literature by allowing different educational levels to be imperfect substitutes. None of these papers addresses the issues that years of education do not distinguish between cognitive versus non-cognitive skills, or that individuals optimally choose both the quantities and types of human capital to accumulate. On the other hand, an applied micro literature examines the formation of individual skills using worker-level data (e.g. Cunha, Heckman and Schennach 2010, Jackson, Johnson and Persico 2015). We examine the different ways in which high-income (and some middle-income) countries produce different *types* of human capital, and the implications of such differences for aggregate output and for education policies.

More broadly, the ways countries produce their human capital are related to their educational systems, which often have deep historic roots and so are an important part of these countries' institutions. We thus also contribute to the institutions literature (e.g. Hall and Jones 1999, Acemoglu, Johnson and Robinson 2001) by quantifying key characters of the educational institution and drawing out their implications for aggregate output.

Our model and our results have policy implications. Ever since the 1983 report by the National Commission on Excellence in Education, there have been heated debates in the U.S. about the pros and cons of focusing on test scores. We bring the rigor of economic modeling and quantitative analyses into these discussions. In our model, education policies that focus on test scores tend to increase cognitive human capital, but create dis-incentives against non-cognitive human capital and may decrease its quantity in the aggregate economy. Our model quantifies these pros and cons and calculates the net effect on aggregate output. e.g. we show that if the U.S. were to have Hong Kong's cognitive and non-cognitive productivities, the U.S. test score would rise but U.S. aggregate output would fall. Such calculations also provide a benchmark for the cost effectiveness and payoffs of education policies, and clarify that aggregate output is a better goal for education policies than test score. In doing so, we contribute to a large empirical literature using micro data to evaluate the effects of education policies on individual outcome (e.g. Figlio and Loeb 2011).

Finally, we use our model to calculate how much countries' aggregate output would be if there were no frictions to trade. Our calculations show large gains from trade liberalization, especially for the countries with strong comparative advantages in producing cognitive (e.g. S. Korea would gain 30.1% to 44.1% of its output) or non-cognitive human capital (e.g. the Netherlands would gain 18.8% to 55.6%). The magnitudes of these gains-from-trade estimates are comparable to the literature (e.g. Costinot and Rodriguez-Clare 2014), even though our model has perfect competition and leaves out such mechanisms as variety gains and gains from resource reallocation across firms. Intuitively, we obtain large gains from trade among high-income countries because they differ substantially in how they produce cognitive and non-cognitive human capital. In addition, factor endowments are endogenous in our model: individuals can change occupations and change the types of human capital they accumulate.

The remainder of this paper is organized as follows. Section 2 discusses the key facts that motivate our theoretical framework. Section 3 sketches this theoretical framework. Section 4 outlines the identification of our structural parameters. Section 5 draws out the implications of our non-cognitive and cognitive productivities. Section 6 explores the quantitative implications of our model. Section 7 concludes.

2 Non-cognitive and Cognitive Occupations, and Other Motivating Data Patterns

A simple way to assess a country's proficiency in human capital production is to use internationally comparable PISA test scores with educational spending per student, as is shown in Figure 1. This figure shows that more input (spending) leads to more output (test score), with substantial deviations from the best linear predictor (crude measure of productivity).

Missing from this naïve assessment is that the non-cognitive skills that are important in a modern work place are not well assessed by examinations, and that a country's ability to foster these skills will be hard to compare internationally. Moreover, to the extent that a country has a comparative advantage in producing easily measured skills, this country will look productive along this dimension, in part because workers will optimally choose to acquire these skills more at the expense of less quantifiable skills.

We now demonstrate that occupations differ in the extent to which performance

on test scores matters for workplace productivity. We use leadership to measure non-cognitive occupations. If the O*NET characteristic "providing guidance and direction to subordinates ..." is important for an occupation, we classify it as non-cognitive, and we classify all the other occupations as cognitive. We focus on leadership because it gives us intuitive and plausible correlation patterns in the micro data used by previous studies and also in our own micro data. To be specific, Kuhn and Weinberger (2005) use U.S. data to show that those who have leadership experiences during high school have higher wages later in their lives. In addition, we use the framework of Neal and Johnson (1996) to show below (see Table 1) that the wages of leadership occupations are less correlated with test scores than those of the other occupations.

The data used in Table 1 is the 1979 NLSY (National Longitudinal Survey of Youth). The dependent variable is the log of individuals' wages in 1991, and the main explanatory variable is their AFQT score (Armed Force Qualification Test) in 1980, before they enter the labor force. Column 1 shows that the coefficient estimate of AFQT score is positive and significant, and this result replicates Neal and Johnson (1996).[5] Columns 2 and 3 show that AFQT score has a smaller coefficient estimate for the subsample of non-cognitive occupations than for the subsample of cognitive occupations.[6] To show this pattern more rigorously, we pool the data in column 4 and introduce the interaction between AFQT score and the non-cognitive-occupation dummy. The coefficient estimate of this interaction term is negative and significant.[7] In column 5 we use the O*NET characteristic of enterprising skills as an alternative measure for leadership. The interaction between enterprising skills and AFQT score is negative but not significant. [8]

Having classified occupations as non-cognitive and cognitive using the U.S. O*NET, we next bring in employment data by 3- or 4-digit occupations from the International

[5]We include both men and women in Table 1, while Neal and Johnson (1996) do the estimation separately for men and women. We have experimented with this and obtained very similar results. We also use the same sample cuts as Neal and Johnson (1996) (see the Appendix for the details).

[6]Note that the coefficient estimates for AFQT square are not significant.

[7]Note that (1) we include the non-cognitive dummy itself, plus the college dummy and its interaction with AFQT score; (2) the non-cognitive dummy itself has a positive and significant coefficient estimate, consistent with Kuhn and Weinberger (2005).

[8]We have also experimented with using the following O*NET characteristics to measure non-cognitive occupations: investigative skills, originality, social skills, and artistic talents. The results for originality, investigative skills and social skills are counter-intuitive, and artistic talents account for a very small fraction of the labor force. See the Appendix for more details.

Labor Organization (ILO). We keep only the countries whose raw data are in ISCO-88 (International Standard Classification of Occupations), because O*NET occupations can be easily mapped into ISCO-88 occupations but the mappings among other occupation codes are very scarce (e.g. we cannot find the mapping between Canadian and U.S. occupation codes). This leaves us with a single cross-section of 34 countries, and most of them are in 2000. Examples of non-cognitive occupations include business professionals (ISCO-88 code 2410), managers of small enterprises (1310), building frame and related trades workers (7120), nursing and midwifery professionals (3230), etc. Examples for cognitive occupations include architects, engineers and related professionals (2140), finance and sales professionals (3410), secretaries (4110), motor vehicle drivers (8320), etc.

We then merge in mean PISA scores in reading, math and science from the official PISA website,[9] and the ratios of private plus public expenditures on education to GDP in 2004 from the UNESCO Global Education Digest of 2007.[10] Finally, we add other variables, such as labor-force size and aggregate output, from standard sources, such as NIPA (National Income and Product Account) and PWT (Penn World Tables). Because we do not have physical capital in our model, as we show in section 3, we use labor income, or compensation of employees from NIPA, as our measure for aggregate output.[11] In addition, in middle-income countries (e.g. Thailand), subsistence farming and/or the

[9] We use PISA scores because they are widely reported in the media, and have influenced education policies in many countries. In addition, PISA samples students in a nationally representative way, covers many countries, and controls qualities of the final data (e.g. the 2000 UK scores and 2006 US reading scores are dropped because of quality issues). Finally, while PISA scores are for high-school students, they are highly correlated with the scores of adult tests (e.g. Hanushek and Zhang 2009, our Data Appendix and Table A3). Compared with PISA, adult tests cover substantially fewer countries (they would cut our sample size by at least 25%) and have substantially lower response rates (e.g. Brown et al. 2007, our Data Appendix).

[10] When PISA first started in 2000, only the reading test was administered, and only a small set of countries participated (e.g. the Netherlands did not participate). In order to obtain PISA scores in all three subjects for every country in our sample, we calculate simple averages over time by country by subject, using all years of available data; e.g. Germany's PISA math score is the simple average of 03, 06, 09 and 2012, U.K.'s reading score the average of 06, 09 and 2012, etc. In the Appendix we show that PISA scores have small over-time variation but large cross-section variation.

[11] We experimented with stripping capital from GDP by assuming a Cobb-Douglas production function and using the parameter values from the macro literature (e.g. Klenow and Rodriguez-Clare 1997). The aggregate output of this second approach has a correlation of 0.9994 with our main output variable.

informal sector might be an issue, since it is unclear how to think about the contributions of human capital there. We use the 28 high-income countries as our main sample, and examine the extended sample that also includes middle-income countries in section 8. These high-income countries account for 42.94% of world GDP in 2000. Table 2 provides summary statistics of our main variables of interest, and Table 3 lists the countries and years in our sample. Note that all our data come from public sources.

To further motivate our model, we now demonstrate that the countries with relative abundance in non-cognitive human capital are in fact net exporters of its service. We follow the literature (e.g. Nunn 2007) and examine the correlation between the patterns of trade and the interactions between relative factor abundance and factor-use intensities. For each country in our sample, we collect aggregate import and export for the 31 NAICS manufacturing industries in the 2000 U.S. census, and the 9 1-digit service industries in the UN service-trade database.

To measure trade patterns, we calculate net export divided by the sum of import and export by industry by country. For each country, we measure its relative abundance in non-cognitive human capital, physical capital and skilled labor as, respectively, the non-cognitive employment share, the ratio of physical capital stock to population, and the fraction of college-educated labor force. For each industry, we measure the intensities of non-cognitive human capital, physical capital and skilled labor using U.S. data.[12] Finally, we control for industry fixed effects and country fixed effects.

Table 4 reports the results. Column (1) includes only the interaction for non-cognitive human capital. We add the interaction for physical capital in column (2), and then the interaction for skilled labor in column (3). The interaction for non-cognitive human capital has positive and significant coefficient estimates in all specifications.

3 A Model of Human Capital Production with Heterogeneous Workers

In this section we develop our model for the production of human capital and illustrate the intuition of our key parameters. A key feature of our model is that heterogeneous workers optimally choose both the quantities and types of human capital to accumulate.

[12] See our Appendix for more details.

We also show how the model can make contact with observable country outcomes with an eye toward quantification, in preparation for section 4.

3.1 Model Structure

There are K countries, indexed by k, each endowed with L^k units of heterogeneous labor. Workers are endowed with non-cognitive and cognitive attributes ε_n and ε_c, drawn from the following Frechet distribution

$$F(\varepsilon_n, \varepsilon_c) = \exp\left(-\left(T_c\varepsilon_c^{-\theta} + T_n\varepsilon_n^{-\theta}\right)^{1-\rho}\right),\ \theta \equiv \frac{\widetilde{\theta}}{1-\rho} \tag{1}$$

In the context of the correlation patterns that we discussed in section 2, we think about the attributes n and c as two distinct packages of skills, rather than two individual skills. These two packages may have common elements. In equation (1), the parameter ρ captures the degree to which non-cognitive and cognitive packages are correlated. When $\rho = 0$, they are independent; when $\rho > 0$, they have positive correlation; and when $\rho \to 1$, they become perfectly collinear. The parameter θ captures the dispersion of attributes across workers. As θ rises, the distribution becomes more compressed, and so there is less worker heterogeneity. Note that for the distribution to have finite variance, we require $\theta > 1$. Finally, T_c and T_n, both positive, capture the locations of the attributes distribution; e.g. as T_c rises, the distribution of cognitive abilities shifts to the right, so that the average worker has better innate cognitive abilities. We assume that ρ, θ, T_c, and T_n do not vary across countries.[13]

To minimize the number of moving parts, we follow Hsieh et al. (2016) and specify the following human-capital production function. Workers accumulate human capital of type i, $i = n$ (non-cognitive) or c (cognitive), according to the technology

$$h_i(e) = h_i^k e^\eta,\ i = c, n. \tag{2}$$

In equation (2), e is an individual worker's spending on human capital accumulation, in units of the final good (we specify its production below). The parameter η captures decreasing returns in the production of human capital, and guarantees an interior solution for workers' optimal human-capital spendin, e. We assume that η is common across

[13] The assumption over ρ and θ is standard in the literature using Roy model models. The assumption that the Ts are same is that there are no inherent genetic differences across countries.

countries. The parameters h_n^k and h_c^k are country k's productivities in non-cognitive and cognitive human capital, and they capture the strength of country k's educational institution along these two dimensions, net of resources inputs.

We treat h_n^k and h_c^k as exogenous, because the educational institution has deep historic roots in many countries. For example, in the U.S., private universities and colleges are a main feature of the educational institution, and their legal rights and status were enshrined by the Supreme Court in 1819 in Dartmouth-College-vs-Woodward.[14] In S. Korea, and many other East Asian countries, the national exam has been a cornerstone of the educational institution for over 1,000 years.[15] We capture, and quantify, such cross-country differences in educational institutions as h_n^k and h_c^k, and so we place no restriction on their values.

Both non-cognitive and cognitive tasks are needed to produce the final good. When a worker chooses task i, or occupation i, her output is

$$h_i(e)\varepsilon_i, \ i = n, c \tag{3}$$

where $h_i(e)$ is the quantity of the worker's human capital, accumulated according to the technology (2), and ε_i her attribute, drawn from the distribution in (1).[16]

The representative firm hires workers in both cognitive and non-cognitive occupations to maximize output

$$y^k = \Theta^k \left(A_c \left(L_c^k \right)^{\frac{\alpha-1}{\alpha}} + A_n \left(L_n^k \right)^{\frac{\alpha-1}{\alpha}} \right)^{\frac{\alpha}{\alpha-1}} \tag{4}$$

[14] In 1816, New Hampshire enacted state law to convert Dartmouth College from a private institution to a state institution. The case went to the U.S. Supreme Court, the legal issue being whether Dartmouth's original charter with the King of England should be upheld after the American Revolution. In 1819, the Supreme Court sided with Dartmouth, and this decision also guaranteed the private status of other early colonial colleges, such as Harvard, William and Mary, Yale, and Princeton (e.g. Webb, Metha, and Jordan 2013).

[15] China used archery competitions to help make promotion decisions for certain bureaucrateic positions before 256 B.C.E. and established the imperial examination system as early as 605 A.D. and this remained in use for over 1,000 years. In this system, one's score in the national exam determines whether or not he is appointed to a government official, and if so, his rank. Through trade, migration, and cultural exchanges, China's imperial examination system spread to neighboring countries; e.g. Korea established a similar system in 958 A.D. (Seth, 2002).

[16] Equation (3) assumes that occupation i uses skill i. We have experimented with having occupations use both skills, with occupation i being more intensive in skill i. This alternative specification adds little insight but much complexity so we have gone with the simpler set up.

In equation (4), Θ^k is country k's total-factor productivity (TFP), and A_c and A_n common technological parameters. The parameter $\alpha > 0$ is the substitution elasticity between non-cognitive and cognitive skills. L_n^k and L_c^k are the sums of individual workers' outputs of non-cognitive and cognitive tasks, which are specified in equation (3). L_n^k and L_c^k can also be interpreted as country k's aggregate supplies of non-cognitive and cognitive human capital. While we assume that final goods cannot be traded, we allow for the possibility that labor inputs L_c^k and L_n^k can be traded as factor content embodied in intermediate inputs.

The key prices in country k are the price of an effective unit of cognitive labor w_c^k, the price of an effective unit of non-cognitive labor w_n^k, and the price of the final output, P^k. Given cost minimization of the perfectly competitive final goods producers, the price of the final good (4) is given by

$$P^k = \frac{1}{\Theta^k}\left((A_c)^{\alpha}\left(w_c^k\right)^{1-\alpha} + (A_n)^{\alpha}\left(w_n^k\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{5}$$

All markets are perfectly competitive. The timing happens as follows. First, workers choose how much and what type (cognitive or non-cognitive) of education to obtain. Second, final goods producers choose how many workers of each type to employ and how much of each type of intermediate (cognitive intensitive or non-cognitive intensive) to import. Finally, all markets clear.

3.2 Equilibrium Conditions

We first analyze how workers allocate their time to study given their optimal occupational choice. We then consider the aggregate supply of effective labor of cognitive and non-cognitive labor. Having characterized the supply of labor of each type, we then consider the supply side taking into account the supplies of each type of labor supplied via international trade.

Recall that the human capital investment is in terms of final output so that the proper maximization problem facing an individual that will choose occupation i is

$$\max_e \left\{w_i h_i^k e^{\eta}\epsilon_i - P^k e\right\}$$

and so the optimal choice of human capital investment, after substituting for the price

index and accounting for the normalization, is then

$$e(\epsilon_i) = \left(\eta \frac{w_i^k}{P^k} h_i^k \epsilon_i\right)^{\frac{1}{1-\eta}}. \tag{6}$$

Equation (6) says that, intuitively, individuals in country k accumulate large quantities of human capital if real wages are high. It also shows that the final-good price index, P^k, has the same effects on $e(\epsilon_i)$ for both the cognitive and non-cognitive occupations. This means that P^k does not affect the comparative advantage for non-cognitive human capital.

We now plug the worker' optimal choice in (6) into her maximization problem, and obtain the following expression for her highest net income in occupation i

$$I_i(\varepsilon_i) = (1-\eta)\eta^{\frac{\eta}{1-\eta}} \left(\frac{w_i^k}{P^k} h_i^k \varepsilon_i^k\right)^{\frac{1}{1-\eta}} \tag{7}$$

Equation (6) and (7) show that net income, $I_i(\varepsilon_i)$, is proportional to educational spending, $e_i(\varepsilon_i)$. This result will be handy for our analyses below. In addition, equation (7) implies that the worker chooses occupation n if and only if $w_c^k h_c^k \varepsilon_c^k \leq w_n^k h_n^k \varepsilon_n^k$. This is a classic discrete-choice problem (e.g. McFadden 1974). Using the Frechet distribution (1) we show, in the Appendix, that

Proposition 1 *The employment share of occupation i equals*

$$p_i^k = \frac{T_i (w_i^k h_i^k)^\theta}{T_c (w_c^k h_c^k)^\theta + T_n (w_n^k h_n^k)^\theta},\ i = c, n. \tag{8}$$

Equation (8) says that the non-cognitive employment share, p_n^k, is high, if workers have a strong comparative advantage in non-cognitive innate abilities (high T_n/T_c), non-cognitive skills have a high relative return in the labor market (high w_n^k/w_c^k), or country k has a strong comparative advantage in fostering non-cognitive human capital (high h_n^k/h_c^k). In (8), θ plays an important role. As θ rises and workers become more homogeneous, given changes in w_i^k or h_i^k lead to bigger shifts in the proportion of workers that opt to work in different occupations. Equation (8) characterizes individuals' optimal choices for the types of human capital accumulated, and plays a key role in our model.

To solve the model, we start by calculating the average net income of non-cognitive and cognitive workers, which analytically involves taking the expected value of equation (7), with respect to ε_i, conditional on type i, $i = n, c$. We show, in the Appendix, that

Proposition 2 *The average net income is the same for non-cognitive and cognitive workers; i.e.*

$$\begin{aligned} I_n^k &= I_c^k = \gamma(1-\eta)\eta^{\frac{\eta}{1-\eta}} \left[T_c \left(\frac{w_c^k}{P^k} h_c^k \right)^{\theta} + T_n \left(\frac{w_n^k}{P^k} h_n^k \right)^{\theta} \right]^{\frac{1}{\theta(1-\eta)}}, \qquad (9) \\ \textit{where } \gamma &= \Gamma\left(1 - \frac{1}{\theta(1-\rho)(1-\eta)} \right). \end{aligned}$$

Proposition 2 is a common feature of the solution to discrete choice problems where the underlying distribution is Frechet (e.g. Eaton and Kortum 2002). In equation (8), the term in the square brackets is the denominator of the employment-share expression, (8). $\Gamma(.)$ is the Gamma function and so γ is a constant. Similar aggregation over educational obtainment at the individual level, combined with optimal choice of occupation, implies the following corollary:

Corollary 1 *The average educational expenditure is the same for non-cognitive and cognitive workers and is equal to*

$$E_n^k = E_c^k = \gamma \left[\eta \left(T_c \left(\frac{w_c^k}{P^k} h_c^k \right)^{\theta} + T_n \left(\frac{w_n^k}{P^k} h_n^k \right)^{\theta} \right)^{\frac{1}{\theta}} \right]^{\frac{1}{1-\eta}}. \qquad (10)$$

By the Corollary we now use E^k, without an occupation subscript, to denote the average educational spending in country k. Proposition 2 and its corollary will prove useful in pinning down η, the elasticity of the output of human capital with respect to input, as we show in section 4.

Finally, we define the local supply of effective labor of type i as L_i^{kS}. We show in the Appendix that

Proposition 3 *Given occupational and educational choices of workers, the aggregate supply of locally provided labor of type i in country k is*

$$L_i^{kS} = L^k p_i^k E(h_i^k e^{\eta} | Occp.i) = \frac{L^k p_i^k}{w_i^k} \left(\eta^{\eta} (P^k)^{1-\eta} \left(T_c \left(\frac{w_c^k}{P^k} h_c^k \right)^{\theta} + T_n \left(\frac{w_n^k}{P^k} h_n^k \right)^{\theta} \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\eta}} \gamma. \qquad (11)$$

To complete our characterization of labor supply, we use equations (8) and (11) to derive the relative supply of non-cognitive labor, which is given by

$$\frac{L_n^{kS}}{L_c^{kS}} = \frac{T_n}{T_c}\left(\frac{h_n^k}{h_c^k}\right)^{\theta}\left(\frac{w_n^k}{w_c^k}\right)^{\theta-1} \tag{12}$$

Intuitively, the relative supply of non-cognitive labor, L_n^{kS}/L_c^{kS}, is increasing in the availability of raw talent in the country, the comparative advantage of that country in non-cognitive human capital, h_n^k/h_c^k, and the relative return of non-cognitive human capital, w_n^k/w_c^k. As foreshadowed by our discussion of Proposition 1, it is clear from equation (12) that θ is the supply elasticity: as workers' skills become more homogeneous, a given change in h_n^k/h_c^k or w_n^k/w_c^k affects the occupational choices of more workers, and so solicits a larger response in L_n^{kS}/L_c^{kS}. We will obtain the value of θ in section 4 below.

Finally, equation (11), together with $w_c^k L_c^k + w_n^k L_n^k = P^k y^k$, implies that the income share of cognitive workers in the population is given by

$$\frac{w_i^k L_i^{kS}}{P^k y^k} = p_i^k. \tag{13}$$

This completes the labor supply side of the model.

We now turn our attention to the demand side. Cost minimization by final goods producers facing technology (4) determines the demand for cognitive and non-cognitive labor. The first order conditions imply that the relative demand for non-cognitive labor is given by

$$\frac{L_n^{kD}}{L_c^{kD}} = \left(\frac{A_c}{A_n}\frac{w_n^k}{w_c^k}\right)^{-\alpha} \tag{14}$$

Equation (14) is a standard relative demand equation where the key demand elasticity is given by α. Cost minimization using equation (4) also implies that the cost share of labor input of type i in country k is given by)

$$s_i^k = \frac{A_i^{\alpha}\left(w_i^k\right)^{1-\alpha}}{A_c^{\alpha}\left(w_c^k\right)^{1-\alpha} + A_n^{\alpha}\left(w_n^k\right)^{1-\alpha}}. \tag{15}$$

We now use (13) and (15) to show that the aggregate supplies, L_c^{kS} and L_n^{kS}, differ from aggregate demand, L_c^{kD} and L_n^{kD}, by international trade. We define GDP-normalized net exports of type i human capital as

$$nx_i^k = \frac{w_i^k\left(L_i^{kS} - L_i^{kD}\right)}{P^k y^k} = p_i^k - s_i^k \tag{16}$$

Equation (16) makes it clear that if $s_i^k > p_i^k$ country k must be a net importer of type i human capital. With balance-of-trade, $nx_c^k + nx_n^k = 0$. In order to relate the aggregate quantities of cognitive and non-cognitive human capital to trade and occupation employment shares, it is convenient to define net exports as ratios

$$x_i^k = \frac{L_i^{kS} - L_i^{kD}}{L_i^{kS}} = \frac{nx_i^k}{p_i^k}, i = c, n \tag{17}$$

For example, if $x_c^k = -0.5\%$, country k imports cognitive human capital, the quantity of which is 0.5% of its aggregate supply. Using equations (12)-(17), we have

$$\frac{L_n^{kD}}{L_c^{kD}} = \left(\frac{A_c}{A_n}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{p_n^k}{p_c^k}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{1-x_n^k}{1-x_c^k}\right)^{\frac{\alpha}{\alpha-1}} \tag{18}$$

We can think about the intuition of equation (18) in two steps. First, assume that there is no trade; i.e. $x_n^k = x_c^k = 0$. Equation (18) says that if we observe, in the data, that many in country k choose the non-cognitive occupation (i.e. p_n^k is high and p_c^k low), we can make the inference that country k has a large relative quantity of non-cognitive human capital. With international trade, some of country k's residents sell their services abroad in exchange for imports of the services of other countries' residents, and the last term of equation (18) shows us how to make adjustments to p_n^k and p_c^k, which show the employment shares of country k's residents.

We can also relate the relative returns for human capital to occupation employment shares and trade. Working with equations (12)-(16) and imposing that trade balances, we have

$$\frac{w_c^k}{w_n^k} = \left(\frac{A_c}{A_n}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k}\right)^{\frac{1}{\alpha-1}}. \tag{19}$$

To see the intuition of equation (19), we again start with autarky; i.e. $nx_c^k = nx_n^k = 0$. In this case, (19) says that the relative factor return, w_c^k/w_n^k, is inversely related to relative factor abundance, which can be measured by p_c^k/p_n^k according to equation (18). With international trade, then, (19) says that this autarky relationship should be adjusted accordingly. The more trade there is, the larger are the magnitudes of nx_c^k and nx_n^k, and the more we deviate from the autarky relationship between w_c^k/w_n^k and p_c^k/p_n^k.[17]

Equations (18)-(19) will prove useful in allowing us to back out cognitive and non-cognitive productivities from the data. Below, we will further close the model for two

[17] Under free trade, w_c^k/w_n^k is independent of country k's occupation employment shares because it is equalized for all countries.

special cases: autarky and free trade. For now, we do not concern ourselves with the international market clearing, but turn instead to an assessment of the economic significance of the differences in h_c^k and h_n^k across countries.

3.3 Output per Worker, Aggregate Educational Quality and Output TFP

To show the connections between h_c^k and h_n^k and income differences across countries, we derive an analytical expression that decomposes the differences in output per worker into a component that reflects differences in human-capital productivities, and another that reflects differences in output TFP. To do so, we first define a base country 0 against which any particular country can be compared. We show, in the Appendix, that output per worker in country k relative to the base country 0 is

$$\frac{y^k/L^k}{y^0/L^0} = \left[\frac{\Theta^k}{\Theta^0}\left(p_c^0\left(\left(\frac{p_c^0 - nx_c^0}{p_c^k - nx_c^k}\right)^{\frac{1}{\alpha-1}}\frac{h_c^k}{h_c^0}\right)^{\theta} + p_n^0\left(\left(\frac{p_n^0 - nx_n^0}{p_n^k - nx_n^k}\right)^{\frac{1}{\alpha-1}}\frac{h_n^k}{h_n^0}\right)^{\theta}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}} \tag{20}$$

To show the intuition of equation (20), we consider the special cases of autarky and free trade.

Special Case: Closed Economy In this case, we can express the relative return in terms of country k's parameters, and equation (19) simplifies to

$$\frac{w_n^k}{w_c^k} = \left[\frac{T_c}{T_n}\left(\frac{h_c^k}{h_n^k}\right)^{\theta}\left(\frac{A_n}{A_c}\right)^{\alpha}\right]^{\frac{1}{\phi}} \tag{21}$$

where $\phi \equiv \theta + \alpha - 1 > 0$. As a result, equation (20) simplifies to (see the Appendix for the details)

$$\frac{y^k/L^k}{y^0/L^0} = \left[\frac{\Theta^k}{\Theta^0}\right]^{\frac{1}{1-\eta}} \times \left[\Omega^k\right]^{\frac{1}{1-\eta}} \tag{22}$$

where

$$\Omega^k \equiv \left(p_c^0\left(\frac{h_c^k}{h_c^0}\right)^{\frac{\theta(\alpha-1)}{\phi}} + p_n^0\left(\frac{h_n^k}{h_n^0}\right)^{\frac{\theta(\alpha-1)}{\phi}}\right)^{\frac{\phi}{\theta(\alpha-1)}} \tag{23}$$

The first term in equation (22), $\left[\Theta^k/\Theta^0\right]^{1/(1-\eta)}$, is the variation across countries in their output per worker that is due to Hick's neutral productivity differences. The sources of these differences could be associated with many factors, such as efficient court systems and business regulations. Note that because higher output TFP lowers the relative price of the final good and makes it easier to produce human capital, its effect on output per worker is amplified by the power $1/(1-\eta)$.

The second term, $[\Omega^k]^{1/(1-\eta)}$, is the portion of per capita income differences across countries that is due to Ω^k, the overall education quality. Ω^k is a weighted power mean of the cognitive and non-cognitive productivities, with the weights being the occupational employment shares of the base country. This measure is akin to an index, summarizing the multi-dimensional differences in cognitive and non-cognitive productivities into a single numerical value, and it captures the contribution of the overall quality of the educational institution to output per capita.

Because the powers in Ω^k are determined by the demand and supply elasticities, θ and α, they play important roles in determining how overall education quality, Ω^k, relates to cognitive and non-cognitive productivities, h_c^k and h_n^k. As both θ, $\alpha \to \infty$, Ω^k goes to the maximum of h_c^k and h_n^k. This is intuitive, as workers become equally capable at both perfectly substitutable tasks. In this case, being strong in producing one type of human capital but weak in producing the other type has few consequences for a country's well-being. As $\alpha \to -\infty$, however, the aggregate production function becomes Leontief, Ω^k goes to the minimum of h_c^k and h_n^k, and excelling along a single dimension in human-capital production does little good for national well-being. For the more empirically relevant case found in our data (see below) Ω^k is reasonably well approximated as a geometric mean, where the relative importance of cognitive and non-cognitive productivities is determined by the occupational shares. In this case, both cognitive and non-cognitive productivities are important, and so a country with high productivity along one dimension but low productivity along the other tends to have low overall education quality.[18] We will further illustrate this intuition in section 5, by drawing a curve along which Ω^k stays constant. We call this the iso-education-quality curve.

[18] Note that the less substitutable are the two types of skills in the population (smaller α) the bigger the penalty toward poor performance in just one of the dimensions of human-capital investments.

Special Case: Free Trade In the case of free trade, all countries produce final output using the same mixture of resources but are free to specialize in the occupation types in which they excel., i.e. there must be a single global price per unit of cognitive (w_c) and non-cognitive (w_n) human capital. Given effective factor price equalization, it immediately follows that there is a common numeraire for all countries can be defined. It is useful to define this numeraire as a bundle of inputs into final production, i.e. $\left(\left(A_c\right)^{\alpha}\left(w_c\right)^{1-\alpha}+\left(A_n\right)^{\alpha}\left(w_n\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}=1$, and that the price of final output in country k is given by $P^k=\left(\Theta^k\right)^{-1}$.

In this case, the relative return, the left-hand side of equation (19), is the same across countries, and so gets cancelled out of cross-country comparisons. As a result, equation (20) simplifies to (see the Appendix for the details)

$$\frac{y^k/L^k}{y^0/L^0}=\left(\frac{\Theta^k}{\Theta^0}\Omega^k\right)^{\frac{1}{1-\eta}}, \tag{24}$$

where

$$\Omega^k=\left(p_c^0\left(\frac{h_c^k}{h_c^0}\right)^{\theta}+p_n^0\left(\frac{h_n^k}{h_n^0}\right)^{\theta}\right)^{\frac{1}{\theta}}. \tag{25}$$

Comparing the free-trade decomposition with that of the closed economy given by equation (22), we see that the key difference is in the power coefficients in the construction of the power mean of cognitive and non-cognitive productivities. Critically, the power coefficients under free trade do not include α as local labor market demand does not have to equal local labor market supply. This has the effect of increasing the size of these power coefficients relative to the closed economy case; i.e. as if $\alpha\to\infty$ in equation (22). As a result, being relatively inefficient at producing one type of human capital is less of a drag on output per worker. Intuitively, in a world of free trade, imbalance in human capital productivities helps countries specialize and is a source of welfare gains. We will further illustrate this intuition in section 6, by showing how the iso-education-quality curve changes its shape in the movement from closed economy to free trade.

3.4 Changes in Output per Worker

For comparative statics, we derive an analytical expression that relates changes in output per worker to changes in model parameters and observables in the data. Let the super-

script "1" denote the initial equilibrium and "2" the subsequent one. We use equations (4), (11) and (18) to show that

$$\left(\frac{y^{k2}/L^{k2}}{y^{k1}/L^{k1}}\right)^{1-\eta} = \frac{\Theta^{k2}}{\Theta^{k1}}\frac{h_c^{k2}}{h_c^{k1}}\left(\frac{p_c^{k2}}{p_c^{k1}}\right)^{1-\frac{1}{\theta}}\frac{1-x_c^{k2}}{1-x_c^{k1}}\frac{\left(1+\frac{1-x_n^{k2}}{1-x_c^{k2}}\frac{p_n^{k2}}{p_c^{k2}}\right)^{\frac{\alpha}{\alpha-1}}}{\left(1+\frac{1-x_n^{k1}}{1-x_c^{k1}}\frac{p_n^{k1}}{p_c^{k1}}\right)^{\frac{\alpha}{\alpha-1}}} \tag{26}$$

On the left-hand side of equation (26), the intuition of the power $1-\eta$ is round-about production, the same as that for the power of $1/(1-\eta)$ on the right-hand side of equations (22) and (24). On the right-hand side, the first term is the change in output TFP. The next three terms represent the change in the aggregate quantity of cognitive human capital, L_c^{kD}, and the last term represents the change in the relative quantity of non-cognitive human capital, L_n^{kD}/L_c^{kD}.

To compare our analytical results with the literature, suppose that equilibrium 1 is data, where countries trade, subject to trade costs, and that equilibrium 2 is autarky, where $x_c^{k2} = x_n^{k2} = 0$. In addition, there is no change in the values of L^k, h_c^k, h_n^k, or Θ^k. This is the same scenario analyzed in many studies of gains from trade (e.g. Arkolakis, Costinot and Rodriguez-Clare 2012, or ACR 2012). In this special case, equation (26) simplifies to

$$\frac{y^{k2}}{y^{k1}} = \left[\left(p_c^{k1}\right)^{\frac{\theta-1}{\theta}}\left(1-x_c^{k1}\right)\frac{\left(1+\frac{1-x_n^{k1}}{1-x_c^{k1}}\frac{p_n^{k1}}{p_c^{k1}}\right)^{\frac{\alpha}{\alpha-1}}}{\left(1+\frac{p_n^{k1}}{p_c^{k1}}\left(\frac{1-x_n^{k1}}{1-x_c^{k1}}\right)^{\frac{\theta}{\theta+\alpha-1}}\right)^{\frac{\alpha}{\alpha-1}-\frac{\theta-1}{\theta}}}\right]^{-\frac{1}{1-\eta}} \tag{27}$$

Equation (27) computes the output loss in the hypothetical movement from the observed equilibrium in the data back to autarky, and it uses the employment shares and trade values at the initial equilibrium, p_c^{k1}, p_n^{k1}, x_n^{k1} and x_c^{k1}, plus parameter values, all of which are observables. In this sense, it is the counterpart of the ACR-2012 formular for our model. Below, in section 6, we will draw on equation (26) again for our counterfactual exercises.

3.5 Cognitive and Non-cognitive Productivities: Meaurement

In this sub-section, we lay down preparatory work for quantification in section 4 below, by showing how our model can make contact with observables in the data. We begin

by backing out a country's comparative advantage in human capital production from data. Rearranging equation (8) we can solve for a country's relative human capital accumulation productivities:

$$\frac{h_c^k}{h_n^k} = \left(\frac{p_c^k/T_c}{p_n^k/T_n}\right)^{\frac{1}{\theta}} \frac{w_n^k}{w_c^k}.$$

Intuitively, if a large fraction of the population is employed in cognitive occupations despite a low relative wage in that occupation, it must be that accumulating cognitive human capital is relatively easy. Now, substituting for relative wages using (19), and dividing country k's expression relative to a base country 0, we obtain

$$\frac{h_c^k/h_n^k}{h_c^0/h_n^0} = \left(\frac{p_c^k/p_n^k}{p_c^0/p_n^0}\right)^{\frac{1}{\theta}+\frac{1}{\alpha-1}} \left(\frac{f_k}{f_0}\right)^{\frac{1}{\alpha-1}}, f_k = \frac{1-x_c^k}{1-x_n^k} \tag{28}$$

To see the intuition of equation (28), we start with autarky, when it simplifies to

$$\frac{p_c^k/p_n^k}{p_c^0/p_n^0} = \left(\frac{h_c^k/h_n^k}{h_c^0/h_n^0}\right)^{\frac{\theta(\alpha-1)}{\phi}}, \tag{29}$$

where $\phi \equiv \theta + \alpha - 1 > 0$. Equation (29) shows the importance of the key demand-side elasticity, α. Suppose h_c^k/h_n^k increases; i.e. country k has a stronger comparative advantage in producing cognitive human capital. By equation (8), this has a direct effect on the relative employment share of cognitive occupations, p_c^k/p_n^k, as well as an indirect effect on it, through the movement of the relative return to cognitive human capital, w_c^k/w_n^k. Equation (29) says that the net effect depends on α. If $\alpha > 1$, demand is elastic, and so the movement in the relative return is small, and the direct effect dominates. Therefore, a larger relative employment share of cognitive occupations reflects a greater comparative advantage for cognitive human capital. When $\alpha \leq 1$, however, this result does not hold. We show, in section 4 below, that data indicates $\alpha > 1$.

We now go back to equation (28). It has the flavor of revealed comparative advantage: we can back out a country's comparative advantage for cognitive human capital, the left-hand side of (28), using the data and parameter values on the right-hand side of (28). The first term there captures the effects of the endogenous choices of workers and the optimal hiring decisions of the final goods producers. If we observe, in the data, that many have chosen the cognitive occupation in country k, we can infer that country k has a strong comparative advantage for cognitive human capital. The second term on the right-hand side of (28) captures the effects of international trade. If we observe, in

the data, that country k imports, in the net, the service of cognitive human capital (i.e. $x_c^k < 0$), we can infer that country k has a stronger comparative advantage for cognitive human capital than its employment shares suggest, because the cognitive workers in k have chosen their occupation despite import competition.

We now turn to backing out a country's absolute advantage in producing human capital from data. To do this, we assume that country k's average score on international exams, such as PISA, is informative about its aggregate supply of cognitive human capital. Country k's cognitive productivity can then be obtained by appropriately adjusting this test score for educational expenditures and occupational choices. Specifically, for some positive constant b, we assume that

$$S^k = b\frac{L_c^{kS}}{L^k}, \tag{30}$$

where L_c^{kS} is given by (11). Proposition 2 implies that $w_c^k L_c^k = p_c^k P_y^k y^k$, which together with (8) and (10), can be combined to yield the following expression for country k's cognitive human capital accumulation productivity relative to a base country:

$$\frac{S^k}{S^0} = \left(\frac{E^k}{E^0}\right)^{\eta} \left(\frac{p_c^k}{p_c^0}\right)^{1-\frac{1}{\theta}} \left(\frac{h_c^k}{h_c^0}\right) \tag{31}$$

As expression (31) makes clear, a good showing on international tests can happen for multiple reasons. First, a high test score could be obtained by a high level of spending on education per capita, E^k. The effect of E^k on cognitive human capital, and so test score, is raised to the power of η, because the production technology of human capital, (2), is subject to diminishing returns.

The second term in (31) captures the effects of incentives and selection, and they arise in general equilibrium because heterogeneous individuals make optimal choices for human capital investment. To see these effects, suppose that many choose the cognitive occupation in country k; i.e. p_c^k is high. This means that the cognitive occupation is an attractive career choice, and so individuals have strong incentives to accumulate cognitive human capital. This incentive effect implies high average test score for country k, and its magnitude is raised to the power of 1.

On the other hand, workers are heterogeneous, and so a high p_c^k implies that many individuals with low innate cognitive abilities have self-selected into the cognitive occupation. Their presence tends to lower the average cognitive human capital, and so the

test score. The magnitude of this selection effect is p_c^k raised to the power of $-1/\theta$. If θ is large, the distribution of innate abilities becomes more compressed. This means less individual heterogeneity and so the selection effect is weaker. Note that because $\theta > 1$ the incentive effect always dominates. We allow the data to steer us to the most appropriate value for θ, and it will turn out that the value does indeed exceed one.

Finally, cognitive productivity, h_c^k, soaks up all the other reasons why the test score is high for country k, net of the effects of resources, and incentives minus selection. In this sense, h_c^k is country k's TFP in producing cognitive human capital. An important implication of equation (31) is that, in order to isolate h_c^k, one has to adjust test scores for the convoluting factors of educational expenditures and occupational employment shares.

4 Values of Structural Parameters

As we have shown in the previous section, given the elasticities, θ, η, and α, and data L^k, labor-force size, y_k, aggregate output, p_n^k, non-cognitive employment share, and p_c^k, cognitive employment share by country, we can extract the values of final-good TFP, Θ^k, and the TFPs of human capital production, h_n^k and h_c^k. As these data are readily available, the challenge is to obtain the values of these elasticities. Specifically, our data includes labor force by cognitive and non-cognitive occupation, international test scores, real GDP per capita, and the cognitive and non-cognitive factor content of trade. Descriptive statistics are shown in Table 2.

4.1 Elasticity of Human Capital Production, η

We begin with η, the elasticity of human capital attainment with respect to educational expenditure. Corollary 1 and Proposition 3 imply that

Proposition 4 *Country k spends fraction η of its aggregate output on education; i.e.*

$$E^k L^k = \eta y^k. \tag{32}$$

Proof. By equations (9) and (10), $w_i^k L_i^k = L^k p_i^k E^k/\eta$, and so $\eta\left(\sum_i w_i^k L_i^k\right) = L^k E^k\left(\sum_i p_i^k\right) = E^k L^k$. In our model aggregate output equals aggregate income, and so $\eta\left(\sum_i w_i^k L_i^k\right) = \eta y^k$. ■

By equation (32), η is the ratio of aggregate educational spending, $E^k L^k$, to aggregate output, y_k. Therefore, we set its value to match the mean share of public plus private educational expenditure in output, 0.1255 (see Table 2); i.e. $\eta = 0.1255$.

4.2 Supply Elasticity, θ

We now turn to θ, which measures the dispersion of innate abilities across workers and also governs the elasticity of the aggregate supplies of human capital. Using the results of the previous proposition and equation (31), we obtain

$$\ln\left(\frac{S^k}{(y^k/l^k)^\eta}\right) = D + \left(1 - \frac{1}{\theta}\right)\ln p_c^k + \ln h_c^k \tag{33}$$

where D is a constant. Equation (33) decomposes the cross-country variation in the average test score, S^k, into resource inputs, $(y^k/L^k)^\eta$, incentives (minus selection), p_c^k, and cognitive productivity, h_c^k.[19]

Equation (33) also instructs us to construct variables and to look for novel correlation patterns that previous research has not examined. We follow these instructions in Figure 2. The vertical axis is log PISA math score, normalized by the logarithm of output per worker raised to the power of η. The horizontal axis is log cognitive employment share. We weigh the data in the scatterplot using aggregate output.[20]

Figure 2 clearly illustrates that, consistent with equation (33), the countries in which workers are clustered in cognitive occupations are the countries that score well on tests (normalized by resources inputs), which can measure primarily cognitive achievement. The best-fit line has $R^2 = 0.288$ and a slope coefficient of 0.717. This novel correlation pattern provides an important validation that incentives indeed matter for the accumulation of human capital, a key mechanism of our general-equilibrium model.

Figure 2 also allows us to interpret the correlation pattern as structural parameters of our model, because it follows the exact specification of equation (33). The slope coefficient of the best-fit line corresponds to the coefficient of log cognitive employment share, $\left(1 - \frac{1}{\theta}\right)$, implying that $\theta = 3.4965$. This estimate for θ provides yet another validation of our model, which, as we discussed in section 3, requires $\theta > 1$. The

[19] Relative to (30), (33) has output per worker rather than educational expenditure per worker, because we have more data points on output per worker than for average educational expenditure per capita.

[20] The countries in our sample vary a lot in their size (e.g. Switzerland, Germany, and the United States.)

countries' deviations from the best-fit line then correspond to the log of their cognitive productivities, h_c^k.

Furthermore, Figure 2 illustrates the intuition for the identification of θ. As we discussed earlier, with individual heterogeneity, selection moderates the effect of incentives on average cognitive human capital. A small θ implies high heterogeneity and strong selection effect. This means we should observe limited variation in the normalized test scores despite substantial variation in cognitive employment shares; i.e. log cognitive employment share should have a small slope coefficient in Figure 2. Therefore, we identify θ through the strength of the selection effect, the magnitude of which is $-1/\theta$ according to our model.

Table 5 shows the results of fitting our data using (33), implemented as a regression with aggregate output as weight. Column (1) corresponds to the best-fit line in Figure 2. In column (2) we add Australia and New Zealand but dummy them out,[21] and in column (3) we use labor-force size as weight. The results are very similar to column (1). In column (4) we use PISA reading score. The coefficient becomes smaller, 0.521, and remains significant, implying that $\theta = 2.0877$. Column (5) has PISA science score and the results are similar to column (4). Column (6) uses the O*NET characteristic of enterprising skills as an alternative measure of leadership, and so non-cognitive occupations. The coefficient is positive but not significant, and this pattern echoes column (3) of Table 2.[22]

Table 5 produces a range of values for θ, 2.0877 ~3.4965. We use $\theta = 3.4965$ in the rest of the paper and show below that our estimates are very similar to the literature, and that we get very similar results if we use other values for θ (e.g. 2.0877) instead. As we will see in the robustness section, estimates of θ that are obtained in a manner consistent with our model but appearing in other papers (e.g. Burstein et al, 2016) are very similar in magnitude.

Finally, we then calculate the residuals and construct cognitive productivities, h_c^k, according to (33). Like the output-TFP estimates in the macro literature (e.g. Hall and Jones, 1999), our estimates for cognitive productivities are relative, and so we normalize the U.S. value to 1.

[21] As discussed in subsection 2.2, these countries have different occupation classification codes in their raw data.

[22] We present the results of alternative measures of non-cognitive occupations in Appendix Table 4A.

4.3 Demand Elasticity, α

For the value of α, the substitution elasticity on the demand side, we turn to the aggregate production function (4). Specifically, we substitute out the quantities of human capital, L_c^k and L_n^k, using the first order conditions for final good producer profit maximization, and equations (19), (30) and (31). After some algebra, we obtain

$$\log\left(\frac{y^k}{S^k L^k}\frac{1}{1-x_c^k}\right) = F + \frac{\alpha}{\alpha-1}\log\left(1+\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k}\right) + \log\Theta^k \tag{34}$$

where the constant F has no cross-country variation.

Equation (34) is an input-output relationship, and its intuition is similar to that of equation (26). The output is y^k, and there are two inputs. The first is the quantity of cognitive human capital, represented by $L^k S^k$, since test score, S^k, represents average cognitive human capital of country k's residents by equation (30). We adjust it by $\left(1-x_c^k\right)$ to take into account net export of the services of cognitive human capital. The second input is the relative quantity of non-cognitive human capital used in production, and it is a monotonic function of $\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k}$ by equation (18). Therefore, equation (34) shows how aggregate output, normalized by the quantity of cognitive human capital, varies with the relative quantity of non-cognitive human capital, and this variation identifies α.

The estimation of (34), then, is similar to the estimation of the aggregate production function.[23] The coefficient of $\log\left(1+\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k}\right)$ gives us α, and the residuals give us Θ^k, the output TFP. In the estimation, our data disciplines our model for two reasons. First, (34) instructs us to use the average test score as one input and the ratio of employment shares as the relative quantity of another input. These are novel ways to measure the quantities of human capital that previous research has not considered.

Table 6 shows the results of fitting our data using (34), implemented as a regression with aggregate output as weight. The structure of Table 5 is similar to Table 4 and so are the flavors of the results. Columns (1), (4) and (5) use PISA math, reading and science scores, respectively. Column (2) drops Austalia and New Zealand, and column (3) uses labor-force size as weight. The coefficients are all significant, ranging from 2.605 to 2.802. Using 2.802 we infer that $\alpha = 1.5549$. Column (6) uses enterprising skills as

[23] As in the macro literature, we implicitly assume that output TFP is uncorrelated with relative quantity, which in our case is determined by the comparative advantage of human capital production. While progress has been made in the micro literature with respect to identification, it has been slower in the cross-country macro literature.

the alternative measure for non-cognitive occupations, and the coefficient is positive but not significant, echoing Tables 2 and 4. In column (7), we consider the case of autarky, setting $x_c^k = nx_n^k = nx_n^k = 0$ in (34). The results are very similar to column (1), implying that $\alpha = 1.4706$. As we show in the robustness section below, this elasticity is similar to those used in recent papers (e.g. Hurst et al, 2016).

We then calculate the residuals and construct the output TFP, Θ^k, according to (34), normalizing the U.S. value to 1. We check the correlation coefficients between our output TFP estimates and those reported in the literature. They are all positive and significant, ranging from 0.4466 (Klenow and Rodriguez-Clare 1997) to 0.6687 (PWT 8.0), and provide an external validation for our approach.[24]

5 Cognitive and Non-cognitive Productivities, and Overall Education Quality

The elasticities estimated in the previous section, combined with the data and equations (28) and (31), imply the full set of values of h_c^k and h_n^k. These values are relative, like output TFP, and we choose the U.S. as the benchmark country and set $h_c^k = h_n^k = 1$. Table 7 summarizes our parameter values and how we obtain them. In this section we present the values of h_c^k and h_n^k, discuss their policy implications and draw out their economic significance.

5.1 Cognitive Productivity

Figure 3 plots the countries' rankings in h_c^k against their rankings in PISA math score, and Table 3 lists these rankings by country. These two rankings are positively correlated (0.5101), since both test score and cognitive productivity measure the quality of human capital production along the cognitive dimension. However, Figure 3 shows that they are quite different for many countries. We highlight these differences using the 45 degree line.

These differences arise because test score is an outcome, and so a noisy measure for the underlying quality of cognitive-human-capital production. Equation (33) highlights two sources of noisiness. The first is resources, $(y^k/L^k)^\eta$. Other things equal, a country with

[24] See Appendix Table A4 for all the pairwise correlation coefficients.

more resources inputs is expected to produce better outcome. The second is incentives (minus selection), $\left(1-\frac{1}{\theta}\right)\ln p_c^k$. The country where individuals are strongly incentivized to learn cognitive skills will perform well in international tests. Equation (33) then allows us to use test score, S^k, as the starting point, and remove the effects of resources and incentives, to arrive at our cognitive productivity, h_c^k. Therefore, cognitive productivity is a cleaner measure for the underlying quality of cognitive education than test score.

Consider, first, Poland, Czech Republic, Hungary and Slovakia. They have decent PISA scores, ranked outside of top 10. However, our model says that this outcome should be viewed in the context of low output per worker in these countries, and so limited resources for human capital production. Therefore, the qualities of their educational institutions are better than their test scores suggest, and they all rank within top 10 based on cognitive productivities.

Now consider Hong Kong, South Korea and Switzerland. They are superstars in PISA scores, all ranked within top 5. However, our model says that this outcome should be viewed in the context of high cognitive employment shares and so strong incentives to accumulate cognitive human capital. Therefore, the qualities of their educational institutions are not as good as their test scores suggest, and their rankings drop to 10, 12 and 14, respectively, by cognitive productivities.

Finally, we look at the U.S. First, the U.S. has very high output per worker. The abundance of resources makes the low U.S. PISA scores even harder to justify. Second, the employment share of cognitive occupations is relatively low in the U.S., implying weak incentives to accumulate cognitive human capital. The effects of resources and incentives thus offset each other, leaving the U.S. ranking in cognitive productivities very close to its ranking in PISA scores, near the bottom in our set of 28 countries. In our Introduction, we discussed the worries and concerns about the quality of the U.S. educational system. Figure 3 quantifies these concerns and shows that they are well justified, when we look at the cognitive dimension. We now move on to the non-cognitive dimension.

5.2 Non-cognitive Productivity

Figure 4 plots the countries' rankings in h_n^k against their rankings in PISA math score, and Table 3 lists the rankings by country. Figure 4 clearly shows that the PISA-math rankings are simply not informative about non-cognitive productivity rankings (correla-

tion = −0.0602 with p-value = 0.7609). Thus non-cognitive productivities allow us to compare countries' educational systems in a novel dimension, hidden from PISA scores.

In our Introduction, we discussed the concerns in S. Korea and many East Asian countries that the educational systems emphasize exams so much that students are unable to develop non-cognitive skills. Our results in Figure 4 quantify this issue and suggest that these concerns are well grounded. S. Korea and Hong Kong, super starts in terms of PISA scores, round up the very bottom among our 28 countries. They have low non-cognitive productivities for two reasons. First, they have decent, but not stellar, cognitive productivities, as shown in Figure 3. In addition, in these countries, many choose the cognitive occupations, implying that these countries have weak comparative advantages for non-cognitive human capital, by equation (28).

Figure 4 also shows that PISA-math rankings substantially understate the proficiency of the U.S. and U.K. in fostering non-cognitive skills. The U.S. ranks in the middle of our 28 countries and the U.K. ranks No. 4. Many in the U.S. have long argued against focusing exclusively on test scores in education.[25] Figure 4 provides quantifications for this argument, showing that the U.S. indeed has a comparative advantage for non-cognitive skills. As for the U.K., it ranks ahead of Hong Kong in both non-cognitive (Figure 4) and cognitive productivities (Figure 3), and it seems reasonable to assume that Hong Kong and Shanghai, China, have similar educational systems. If the former U.K. education minister, Elizabeth Truss, had known about these rankings in 2014, would she have traveled to Shanghai to "learn a lesson in math"?

In summary, our estimates for cognitive and non-cognitive productivities provide better numerical metrics than test scores for the qualities of education. As another example, Figures 3 and 4 suggest that the educational systems of Finland, Netherlands and Belgium are far more worthy of emulation than those of South Korea and Hong Kong. Below we condense the multi-dimensional differences in cognitive and non-cognitive productivities into a single index for the overall educational quality, and quantify its contribution to output per worker.

[25] For example, the National Education Association states that, in response to NCLB and RTT, "We see schools across America dropping physical education ... dropping music ... dropping their arts programs ... all in pursuit of higher test scores. This is not good education."

5.3 Aggregate Educational Quality

In this section, we present decompositions of output per worker across countries into a component that is due to pure TFP differences (Θ^k in the model) and to the quality of a country's educational system (Ω^k). We present two sets of decompositions. The first are the estimates that follow directly from the model that accounts for observed levels of factor content trade.[26] The second is the case of the closed economy where the net factor content of trade is constrained to be zero. In each set of calculations, we re-estimate h_c^k and h_n^k to be consistent with the assumed degree of tradability.

The results are shown in Table 8. The first column of table 8 shows the actual level of GDP per capita by country relative to the United States (i.e.$\frac{y^k/L^k}{y^0/L^0}$). Columns 2 and 3 correspond to estimates of TFP, $\left[\frac{\Theta^k}{\Theta^0}\right]^{\frac{1}{1-\eta}}$, and of educational quality, $\left[\Omega^k\right]^{\frac{1}{1-\eta}}$, implied by observed factor content of trade. Columns 4 and 5 correspond to estimates obtained under the assumption of no factor content trade.

There are several messages to be gleaned from the estimates in table 8. First, the productivities estimated in columns 4 and 5 suggest large variation across countries in terms of both educational quality and TFP. To interpret the estimates, consider Germany. The overall quality of Germany's educational institution is lower than the U.S., the effect of which puts Germany's output per worker at 88.34% of the U.S. level (column (3)). On top of this, Germany also has lower output TFP than the U.S., the effect of which places its output per worker at 71.26% of the U.S. level (column (2)). Aggregating these two effects, Germany's output per worker is 62.96% (= 88.34% x 71.26%) of the U.S. level (column (1)).[27]

Now consider the large differences in overall educational qualities across countries. For example, columns 4 and 5 show that although S. Korea's educational system delivers high test scores, it puts S. Korea's output per worker at 71.42% of the U.S. level, other things equal. Finland, on the other hand, has the strongest educational institution in our sample, which puts Finland's output per worker at 154.58% of the U.S. level, ceteris

[26] In this case, there is no clean decomposition of the quality of educational system in a closed form that presents the relative contributions of h_c^k and h_n^k. A mixture of data and h_c^k and h_n^k estimates allow the calculation to be made, however.

[27] Columns (2) and (3) in Table 8 are based on $\theta = 3.4965$ and $\alpha = 1.4706$. Table 9 shows that we obtain very similar values and country rankings for overall education quality under alternative values of α and θ.

paribus. These results suggest that educational policies and reforms have very large potential payoffs, as well as danger, in terms of aggregate output.[28]

Second, the numbers in columns 2 and 3 are very close to columns 4 and 5. This tells us that the closed economy model is a very good approximation for the actual estimates. This is because the observed level of factor content of trade relative to occupational choice differences across countries is very small. Hence, the educational productivities for cognitive and non-cognitive human capital accumulation that have been estimated taking into account factor content trade are very close to those that would obtain in a closed economy.

A nice feature of the decomposition for the case of autarky is that it allows us to plot h_c^k and h_n^k for the countries in our sample to understand why the rankings appear in table 8 as they do. We do this in figure 5. The line that passes through the United States in this diagram is what we call an iso-education-quality curve. These are the combinations of h_c^k and h_n^k that yield a constant level of educational quality Ω^k, the expression of which is given in equation (23). As the curve passes through the United States, our benchmark country for which $h_c^k = h_n^k = 1$, we can see which countries fare better and which fare worse while the curvature of the curve tells us about the trade offs of increasing cognitive educational productivity for non-cognitive educational productivity. It also illustrates the countries whose overall education qualities are similar to the U.S. (e.g. Sweden and Denmark), those with higher overall education qualities than the U.S. (e.g. the U.K. and Finland), and those with lower overall education qualities (e.g. Italy and S. Korea).

The curvature and shape of the iso-education-quality curve are determined by equation (23). Intuitively speaking, given the values of α and θ, equation (23) says that both cognitive and non-cognitive productivities are important in overall education quality. We see, in Figure 5, that the countries with high productivity along one dimension but low productivity along the other (e.g. Germany and Hong Kong) lie below the iso-education-quality curve, meaning that they have lower overall education quality than the U.S. This is because the imbalance of their productivities holds down their overall performance. There is, however, a silver lining: this imbalance would be a very useful asset under free trade, as we show in section 6.

[28] The countries' rankings in overall education quality are very similar to their rankings in non-cognitive productivity (correlation is 0.9425). The correlation between overall-education-quality rankings and PISA-math rankings is 0.0909 (p value = 0.6456).

5.4 Extensions and Robustness

5.4.1 Parameter Values

We first compare our parameter values with the literature. Hsieh et al (2016)'s model features the same Frechet distribution of innate abilities as ours, but for identification they use worker-level data and explore wage dispersion within occupations and labor-force participation; i.e. their data and identification strategy are completely different from ours. Despite such differences, Hsieh et al (2016)'s θ estimate ranges from 2.1 to 4, matching our range of 2.0877 to 3.4965. On the other hand, Burnstein et al. (2016) features a CES aggregate production function, like us, but for identification they use cross-section and over-time variations in occupational wages and employment in micro data. Although Burnstein et al. (2016)'s data and identification strategy are completely different from ours, their substitution-elasticity estimate ranges from 1.78 to 2, similar to our value of 1.4706 to 1.5549.[29]

We now perform sensitivity analyses and report the results in Table 9. The 2nd and 3rd columns show the values of α and θ, and the rest of the table report the correlation coefficients of h_c^k, h_n^k, Θ^k and Ω^k with our benchmark values. The first row of Table 9 re-caps these benchmark values. The second row shows the values we obtain for the case of closed economy, where we set factor content of trade to zero. The results suggest that the closed-economy model is a good approximation of the data we observe. The third and fourth rows show how our parameter values change, relative to the benchmark, if we vary the value of θ to 2.0877 or vary both θ and α to 2.0877 and 2. We obtain similar results in both cases.

5.4.2 Middle-Income Countries

In this sub-section we extend our sample to also include the middle-income countries that have 3- or 4-digit ISCO-88 occupation data. This increases the number of countries we have from 28 to 34. We focus on the results under the closed-economy setting, to keep our discussions succinct.

Column (7) of Table 4 shows the implementation of equation (33) using the extended sample. The results are similar to column (2), implying that we obtain similar values

[29] The substitution-elasticity parameter is not identified in Hsieh et al. (2016). Burnstein et al. (2016), on the other hand, do not model the production of human capital.

for θ and h_c^k for the extended sample. Column (8) of Table 5 show the implementation of equation (34) using the extended sample. The results are similar to column (1), indicating that we obtain similar values for α and Θ^k, and so h_n^k, for the extended sample. The last row of Table 9 reports our parameter values for the extended sample, and shows their high correlation coefficients with our benchmark values. To help visualize these similarities, we use these parameter values of the extended sample to graph the iso-education-quality curve in Figure 6. We label the names of the middle-income countries in all capital letters. Figure 6 is similar to Figure 5. It also enriches Figure 5, showing that overall education quality varies substantially among the middle-income countries.

6 Comparative Statics and Policy Implications

Having demonstrated the economic significance of the differences in human capital productivities, h_c^k and h_n^k, in section 5, we now draw out the policy implications of having multiple types of human capital in our GE model.

6.1 Closed Economy

We start by deriving analytical expressions for how changes in the qualities of the educational institution affect test scores and aggregate output for any country k under the assumption that economies are closed, since the data suggest that autarky is a reasonable approximation for global factor trade. These exercises are easy to implement using our model. First, equations (22) and (23) provide closed form solutions for output per worker, and map changes in human capital productivities into changes in output per worker relative to any arbitrary base country 0. This "country 0" in our model can be specified as the initial equilibrium of country k itself. Second, equations (29), (32) and (33), together with the identity $p_c^k + p_n^k = 1$, imply that the change in test score is a log-linear function of the changes in cognitive and non-cognitive productivities (see the Appendix for the proof)

$$(1-\eta)d\ln S^k = (1+Bp_c^k)d\ln h_c^k - (Bp_n^k)d\ln h_n^k, B = \frac{(\theta-1)(\alpha-1)-\alpha\eta}{\theta+\alpha-1} > 0, \quad (35)$$

where $B = 0.2496$ according to our parameter values.[30] Equation (35) says that an increase in test score, S^k, can be achieved by either an increase in cognitive productivity,

[30] This is based on $\theta = 3.4965$. If $\theta = 2.0877$, $B = 0.1279$.

h_c^k, and/or a reduction in non-cognitive productivity, h_n^k. The latter works because an educational institution with a very low level of non-cognitive productivity simply denies most people the option of accumulating non-cognitive human capital. This creates very strong incentives to accumulate cognitive human capital, showing up as an increase in test score. As a result, a rise in test score may result from a better educational institution along the cognitive dimension, or a worse one along the non-cognitive dimension. While the former is a blessing, the latter is a curse in disguise, as we illustrate below.

Suppose the U.S. can implement some policy reform to boost its PISA score by 2.58%, in order to advance 5 places in PISA math rankings. This puts U.S. PISA math score at U.K's level. To illustrate the intended consequence of this policy, assume that U.S. non-cognitive productivity, h_n^{US}, remains unchanged. Equation (35) tells us that U.S. cognitive productivity rises by 2.12%, and equation (22) tells us that U.S. aggregate output rises by 1.81%. The increase in output provides an upper bound estimate for the amount of resources to be spent on the reform, or an estimate for the potential returns of the reform if we know the amount of resources spent. This exercise illustrates that our model is a useful tool for the cost-benefit analysis of education policies.

Our model is also useful for clarifying the objective of education policies. In the U.S., both No Child Left Behind of 2001 and Race To the Top of 2009 are motivated by the concern for low test scores, and both measure student performance using test scores. Our model shows that test score and output may move in the opposite direction, because there are multiple types of human capital and heterogeneous individuals respond to policy changes by changing the types of human capital they accumulate. Suppose that the U.S. implements less ambitious education reforms than in scenario 1 above, and succeeds in raising U.S. PISA score by 0.258%. To illustrate the unintended consequence of this policy, assume that U.S. cognitive productivity, h_c^{US}, remains unchanged. Then by equation (35), U.S. non-cognitive productivity, h_n^{US}, decreases by 4.08%, and by equation (22), U.S. aggregate output decreases by 1.02%. This exercise illustrates that an increase in test score could mask a reduction in the overall quality of the educational institution. As a result, aggregate output is a better objective for education policies than test score.

Indeed, many educational reforms that are promoted to raise test scores have been criticised because of the fear that improvement along one dimension may come at the expense of decline along another. Our model quantifies the pros and cons of education policy reforms. For instance, many in the U.S. advocate emulating the heavily test-based

educational systems and practices of east Asian countries, such as those in S. Korea and Hong Kong. While this may increase cognitive learning, it can also induce poor performance in non-cognitive human capital. Our calculations in section 4 show that Hong Kong's cognitive productivity is 1.13 times the U.S. level, but her non-cognitive productivity is 0.40 times the U.S. level. Equation (35) then says that should the U.S. get Hong Kong's educational system, test score would increase by 22.50%, putting the U.S. as the world champion in PISA scores. However, equations (22) and (23) and Table 8 tell us that despite this accomplishment in test scores, U.S. aggregate output would decrease by 11.13%!

6.2 Closed Economy to Free Trade

As we know from equations (22) and (24), the relative quality of a country's educational institutions depends on the extent of openness to factor service trade. While the data suggest that the world is closer to autarky than to free trade, it is instructive to see what overall education quality and output per worker would be across countries were international trade frictionless.[31]

We start our counterfactual exercises by computing the relative return of cognitive human capital, w_c/w_n, under the free-trade equilibrium. To do so, we obtain the global relative supply of cognitive human capital by summing equation (11) across countries, and obtain the global relative demand by setting $w_c^k/w_n^k = w_c/w_n$ for all k in equation (14). Equating global relative supply to relative demand, we have

$$\left(\frac{A_c}{A_n}\right)^{\alpha}\left(\frac{w_c}{w_n}\right)^{1-\alpha} = \frac{T_c}{T_n}\left(\frac{w_c}{w_n}\right)^{\theta} \frac{\sum_k L^k \left(\Theta^k\right)^{\frac{\eta}{1-\eta}} \left(h_c^k\right)^{\theta} \left(\frac{T_c}{T_n}\left(\frac{w_c}{w_n}h_c^k\right)^{\theta} + \left(h_n^k\right)^{\theta}\right)^{\frac{1}{\theta(1-\eta)}-1}}{\sum_k L^k \left(\Theta^k\right)^{\frac{\eta}{1-\eta}} \left(h_n^k\right)^{\theta} \left(\frac{T_c}{T_n}\left(\frac{w_c}{w_n}h_c^k\right)^{\theta} + \left(h_n^k\right)^{\theta}\right)^{\frac{1}{\theta(1-\eta)}-1}} \quad (36)$$

Once we obtain the value of w_c/w_n from equation (36), we can plug it into equation (8)

[31] Our motivations for this world-is-flat counterfactual are as follows. First, while service trade has been growing faster than goods trade (e.g. wto.org), it has seen less liberalizations than goods trade, and so has more scope for further liberalization. Second, new technology is rapidly decreasing the cost of service trade; e.g. in the U.S., the employment share of contract-firm workers reaches 14% in 2005 (the Wall Street Journal, A9, Sep. 15, 2007), surpassing the employment share of the manufacturing sector.

to compute the occupation employment shares, and into equation (19) to compute the net exports of human-capital services.

We can then draw on equation (26) to compute the change in output. Equilibrium 1, the initial equilibrium, is closed economy; the subsequent equilibium, 2, is free trade; and there is no change in the values of L^k, h_c^k, h_n^k, or Θ^k. Equation (26) thus simplifies to

$$\frac{y^{k2}}{y^{k1}} = \left[\left(\frac{p_c^{k2}}{p_c^{k1}} \right)^{1-\frac{1}{\theta}} \left(1 - x_c^{k2}\right) \left(\frac{1/s_c^2}{1/p_c^{k1}} \right)^{\frac{\alpha}{\alpha-1}} \right]^{\frac{1}{1-\eta}}, \tag{37}$$

where s_c is the cost share of cognitive human capital in global production, as defined by equation (15) with $w_c^k = w_c$ and $w_n^k = w_n$ for all k. The output change given by equation (37) shows countries' output gains if they were to move from autarky to free trade.

In implementing our computation, we use the parameter values identified under the assumption of closed economy, since we are comparing free trade with closed economy. One might be concerned that our results depend on the values of A_c/A_n and T_c/T_n, which we have not identified. We show, in the Appendix, that although the nominal variable w_c/w_n does depend on A_c/A_n and T_c/T_n, the values of all the real variables, such as employment shares, net exports and output, are invariant to A_c/A_n and T_c/T_n.

We report the gains-from-trade results in column (4) of Table 10. To help visualize the patterns of these results, we plot them against non-cognitive and cognitive productivities in Figure 7. In this 3D plot, the countries in the middle, who have balanced cognitive and non-cognitive productivities, have limited gains from trade. However, gains from trade are large for the countries on the edges of the figure, who have strong comparative advantages in either producing cognitive human capital or who have strong comparative advantages in producing non-cognitive human capital. For example, Hong Kong would see a 17.1% gain in output, S. Korea 44.1%, the Netherlands 18.8%, and Belgium 18.2%.

To explore the intuition for these countries' large gains from trade, we go back to overall education quality. While imbalance in human capital productivities contributes to low overall education quality under closed economy, by equation (23), it helps countries specialize and so is a source of welfare gains under free trade, by (25). To illustrate the change in overall education quality, we plot, in Figure 8, the iso-educational-quality curve under free trade. Figure 8 has the same values of cognitive and non-cognitive productivities as Figure 5. However, the iso-educational-quality curve is generated by

equation (25) in Figure 8, vs. (23) in Figure 5. We see that this curve bends sharply towards the origin in Figure 8, in contrast to Figure 5. We also see, in Figure 8, that several countries that are exceptional along the cognitive dimension in human capital production (e.g. S. Korea and Hong Kong) lie above the iso-education-quality curve, meaning that they would have higher overall education quality than the U.S. under free trade. The results are highly intuitive: countries that have highly unbalanced educational productivities benefit dramatically from being able to specialize in the occupations in which they excel.

We now clarify the connection between changes in overall education quality and gains from trade

$$\frac{y^{k2}}{y^{k1}} = \frac{y^{US2}}{y^{US1}} \frac{y^{k2}/y^{US2}}{y^{k1}/y^{US1}} = \frac{y^{US2}}{y^{US1}} \frac{(\Omega^{k2})^{\frac{1}{1-\eta}}}{(\Omega^{k1})^{\frac{1}{1-\eta}}}. \tag{38}$$

Equation (38) says that country k's gains from trade is equal to the change in its overall education quality multiplied by U.S. gains from trade. This is intuitive, since the U.S. is our benchmark country for overall education quality.[32] Column (1) of Table 10 reports the overall education quality in closed economy, $(\Omega^{k1})^{\frac{1}{1-\eta}}$, and is the same as column (5) of Table 8. Column (2) shows $(\Omega^{k2})^{\frac{1}{1-\eta}}$, the overall education quality under free trade, and column (3) shows the ratios of free-trade values to closed-economy values. We see that column (3) of Table 10 is very similar to column (4), because U.S. gains from trade are small; i.e. the large gains from trade we saw in Figure 7 are mostly driven by changes in overall education quality.

To be clear, U.S. gains from trade are small because the U.S. has the largest labor-force size and second highest output TFP in our sample, where several large countries, such as Japan and China, are missing. The inclusion of these countries may imply larger gains for the U.S. Since we lack occupation employment data for Japan, we assume that Japan has the same occupation shares and the same cognitive and non-cognitive productivities as S. Korea. We then use Japan's output-per-worker data and equation (22) to calculate Japan's output TFP. Likewise, we assume that China is the same as Hong Kong except for labor-force size and output TFP. We then re-compute the free-trade equilibrium and gains from trade using equations (36) and (37).

We plot these gains from trade against non-cognitive and cognitive productivities in Figure 9. As compared with Figure 7, Figure 9 shows a similar overall pattern: the

[32] In our computation for gains from trade, the results obtained using (38) are identical to those obtained using (37).

countries with strong comparative advantages see large gains, while those in the middle see small gains. Figure 9 is also different from Figure 7 in several aspects, due to the addition of Japan and China. First, the U.S. gains are larger, at 8.1%. Second, the countries with strong comparative advantages for producing cognitive human capital have smaller gains;[33] e.g. 6.6% for Hong Kong and 30.1% for S. Korea. Finally, the countries with strong comparative advantages for producing non-cognitive human capital have substantially larger gains; e.g. 40.5% for the U.K., 44.0% for Finland, and 55.6% for the Netherlands.

6.3 US Reform with Free Trade

Our results in the previous sub-section indicate that some high-PISA-score countries, such as S. Korea and Hong Kong, would have higher overall education quality than the U.S. under free trade. What would happen to U.S. output if the U.S. were to emulate these countries?

To answer this question, we again draw on equation (26). Both the initial and subsequent equilibria are free-trade, and there is no change in the values of L^k or Θ^k. Equation (26) simplifies to

$$\frac{y^{k2}}{y^{k1}} = \left[\frac{h_c^{k2}}{h_c^{k1}} \left(\frac{1/p_c^{k2}}{1/p_c^{k1}} \right)^{\frac{1}{\theta}} \left(\frac{s_c^2}{s_c^1} \right)^{-\frac{1}{\alpha-1}} \right]^{\frac{1}{1-\eta}}. \tag{39}$$

The intuition of equation (39) is as follows. The cost share of cognitive human capital, s_c, depends on the relative return, w_c/w_n, by equation (15). This means that the change in s_c in equation (39) represents the change in relative prices, or country k's terms of trade. On the other hand, the change in h_c^k represents the change in country k's absolute advantage in human capital production, while the change in p_c^k represents the change in k's comparative advantage. Thus these two terms in (39) represent the changes in country k's endowments and PPF (Production Possibility Frontier).

Using equations (36) and (39),[34] we show that, if the U.S. were to have Hong Kong's

[33] Japan's gains are the same as S. Korea's, and China's the same as Hong Kong's.

[34] In the literature, it is popular to use Deckle, Eaton and Kortum (2008), or DEK, to do comparative statics. We derive the DEK equations in the Appendix. In practice, we obtain identical results whether we: (1) use (36) for the initial equilibrium and then apply DEK for the change; or (2) use (36) for both the initial and subsequent equilibria.

cognitive and non-cognitive productivities under free trade, U.S. output would fall by 1.54%. The output loss increases to over 15% from the roughly 11% loss in the closed-economy setting, if we include China and Japan in the free-trade equilibrium. These results are intuitive, because the emulation of Hong Kong works against the U.S. comparative advantage. Before this policy change, the U.S. specialized, taking advantage of the opportunity to trade with S. Korea, Japan and China. After the change, however, the U.S. would be competing against these countries. These results are also the mirror image of our results in the previous sub-section. Given that countries would reap large output gains in the movement to free trade, it follows that the erosion of their comparative advantages could lead to large output losses.

7 Conclusion

The measurement of human capital accumulation across countries is fraught with difficulties. Merely counting the number of students, years of education, or money spent does not correct for quality differences across countries. International test scores offer a degree of comparability of student outcomes but suffer from the fact that they only provide a measure of outcomes along easily codified, cognitive knowledge. Many are concerned that excessive attention paid to test scores not only results in resources that are wasted "teaching to the test" but that students enter the labor market with poorly developed non-cognitive human capital.

We adapt the Roy (1951) framework to measure cognitive and non-cognitive productivities of national educational systems and to analyze their importance to in explaining differences in real income per capita across countries. The theoretical framework integrates data on international test scores, educational spending per capita, occupational choices, and international trade. Our stylized model is analytically tractable, and provides the following novel insights.

We show that hard to measure non-cognitive human capital is quantitatively important for measuring educational quality. Many countries that perform well on international tests appear to have substandard performance on non-cognitive human capital and this is large enough to drag down their aggregate educational productivity. Our study not only suggests institutional problems in many countries but it also shows that careless attempts to raise performance on international test scores could reduce welfare. This

points to the importance of spelling out the impacts of education policies on aggregate output when we formulate their objectives and conduct cost-benefit analyses. Here, our model provides a useful tool.

While we show that globalization and associated trade in factor services are critical in assessing the quality of a country's educational institutions, the data suggest that the world is much closer to autarky than it is to free trade. For the moment at least, educational institutions that focus on one type of human capital to the great detriment of another are the source of substantially lower income per capita; i.e. imbalance is a source of weakness. However, the countries with imbalanced human-capital productivities would reap very large output gains if countries were to engage in free trade of the services of human capital; i.e. under free trade, imbalance would be a source of strength.

References

[1] Acemoglu, Daron, Simon Johnson, and James A. Robinson. "The Colonial Origins of Comparative Development: An Empirical Investigation." *American Economic Review* 91.5 (2001): 1369-1401.

[2] Autor, David H., Frank Levy and Richard J. Murname, 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration", *Quarterly Journal of Economics* 118(4).

[3] Behrman, J. R., Parker, S. W., Todd, P. E., & Wolpin, K. I. (2015). Aligning learning incentives of students and teachers: results from a social experiment in Mexican high schools. *Journal of Political Economy*, 123(2), 325-364.

[4] Burnstein, Ariel, Eduardo Morales and Jonathan Vogel, 2016. "Changes in Between-Group Inequality: Computers, Occupations and International Trade", mimeo.

[5] Casselli, F. 2005. "Accounting for Cross-Country Income Differences." in Handbook of Economic Growth.

[6] Choi, Yochul, David Hummels, and Chong Xiang. 2009. "Explaining Import Quality: the Role of the Income Distribution". J*ournal of International Economics*, 78, 293-303.

[7] Cunha, Flavio, James Heckman and Susanne Schennach. "Estimating the Technology of Cognitive and Non-cognitive Skill Formation", *Econometrica* 78(3), May 2010, 883-931.

[8] Deckle, Jonathan Eaton, and Samuel Kortum. 2008. "Global Rebalancing with Gravity." *International Monetary Fund Staff Papers* 55: 511-540

[9] Eaton, Jonathan and Samuel Kortum, 2002. "Technology, Geography and Trade", *Econometrica*, 70(5), 1741-1779.

[10] Figlio, David and Susanna Loeb, 2011. "School Accountability", in *Handbook of the Economics of Education*, Volume 3, Edited by Eric Hanushek, Stephen Machin and Ludger Woessmann, Elsevier North-Holland: Amsterdam, 383-421.

[11] Hall, Robert E. and Charles I. Jones, "Why Do Some Countries Produce So Much More Output per Worker than Others?", *Quarterly Journal of Economics*, February 1999, Vol. 114, pp. 83-116.

[12] Hanushek, E., "Education Production Functions", The New Palgrave Dictionary of Economics, 2008.

[13] Hanushek, Eric, and Ludger Woessman. 2011. "How Much do Educational Outcomes Matter in OECD Countries?" *Economic Policy* 26(67): 427-491.

[14] Heckman, James J., and Tim Kautz. "Hard evidence on soft skills." *Labour economics* 19.4 (2012): 451-464.

[15] Heckman, James J. and Yona Rubinstein. 2001. "The Importance of Noncognitive Skills: lessons from the GED Testing Program", *American Economic Review* Papers & Proceedings 91(2): 145-149.

[16] Hsieh, Chang-Tai, Erik Hurst, Charles Jones and Peter Klenow. 2013. "The Allocation of Talent and U.S. Economic Growth", NBER working paper 18693.

[17] Hummels, David, Rasmus Jorgensen, Jakob Munch, and Chong Xiang, 2011, "The wage and employment effects of outsourcing: evidence from Danish matched worker-firm data", NBER working paper 17496.

[18] Hummels, David, Rasmus Jørgensen, Jakob Munch, and Chong Xiang. "The Wage Effects of Offshoring: Evidence from Danish Matched Worker-Firm Data", *American Economic Review*, 104 (6), June 2014, 1597-1629.

[19] Jackson, Kirabo , Rucker C. Johnson, and Claudia Persico. 2016. "The Effects of School Spending on Educational and Economic Outcomes: Evidence from School Finance Reforms," *The Quarterly Journal of Economics* 131(1): 157-218.

[20] Jones, Benjamin. 2014. "The Human Capital Stock: A Generalized Approach." *American Economic Review* 104(11): 3752-3777.

[21] Klenow, Peter, and Andres Rodriquez-Clare. 1997. "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" NBER Working paper.

[22] Kuhn, Peter, and Catherine Weinberger. 2005. "Leadership Skills and Wages." *Journal of Labor Economics* 23(3): 395-436.

[23] Liu, Runjuan and Daniel Trefler, 2011. A Sorted Tale of Globalization: White Collar Jobs and the Rise of Service Offshoring, NBER working paper 17559.

[24] Malmberg, Hannes. 2017. "Human Capital and Development Accounting Revisited." mimeo Institute for International Economic Studies.

[25] Neal, Derek A., and William R. Johnson, "The Role of Premarket Factors in Black –White Wage Differences", *Journal of Political Economy* 104 (5), October 1966, 869-895.

[26] Nunn, Nathan. "Relationship-specificity, incomplete contracts, and the pattern of trade." *The Quarterly Journal of Economics* (2007): 569-600.

[27] Ohsornge, and Daniel Trefler 2007. Journal of Political Economy.

[28] Pierce, Justin and Peter Schott, 2009, "A Concordance Between Ten-Digit U.S. Harmonized System Codes and SIC/NAICS Product Classes and Industries". NBER working paper 15548.

[29] Roy, Arthur. 1951. "Some Thoughts on the Distribution of Earnings." *Oxford Economic Papers* 3(2): 135-146.

[30] Willis, Robert and Sherwin Rosen. "Education and Self-Selection", J*ournal of Political Economy* 87(5), October 1979, S7-S36.

8 Theory Appendix 1

8.1 Proposition 1

To simplify notation, we drop the superscript k. In addition, let $\omega_c = w_c h_c$, $\omega_n = w_n h_n$, $F_c = \frac{\partial F(.)}{\partial \varepsilon_c}$, and $F_{nc} = \frac{\partial^2 F(.)}{\partial \varepsilon_n \partial \varepsilon_c}$. Using the definition of p_n, we have

$$\begin{aligned} p_n &= \Pr(\omega_n \varepsilon_n \geq \omega_c \varepsilon_c) = \int_0^\infty \int_{\frac{\omega_c}{\omega_n}\varepsilon_c}^\infty F_{nc} d\varepsilon_n d\varepsilon_c \\ &= \int_0^\infty [F_c(\varepsilon_c, \varepsilon_n \to \infty) - F_c(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c)] d\varepsilon_c \\ &= \int_0^\infty F_c(\varepsilon_c, \varepsilon_n \to \infty) d\varepsilon_c - \int_0^\infty F_c(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c) d\varepsilon_c \end{aligned}$$

Using the Frechet distribution (1), we have

$$F_c(\varepsilon_c, \varepsilon_n) = AFT_c\varepsilon_c^{-\theta-1}, A = (1-\rho)\theta(T_n\varepsilon_n^{-\theta} + T_c\varepsilon_c^{-\theta})^{-\rho}$$

(1) When $\varepsilon_n \to \infty$, $A = (1-\rho)\theta(T_c\varepsilon_c^{-\theta})^{-\rho}$ and $F = \exp[-(T_c\varepsilon_c^{-\theta})^{1-\rho}]$. Therefore,

$$\begin{aligned} F_c(\varepsilon_c, \varepsilon_n &\to \infty) = (1-\rho)\theta(T_c\varepsilon_c^{-\theta})^{-\rho} \exp[-(T_c\varepsilon_c^{-\theta})^{1-\rho}][T_c\varepsilon_c^{-\theta-1}] \\ &= \theta(1-\rho)(T_c)^{1-\rho}\varepsilon_c^{-\theta(1-\rho)-1} \exp[-(T_c)^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}] \end{aligned}$$

and

$$\begin{aligned} \int_0^\infty F_c(\varepsilon_c, \varepsilon_n &\to \infty) d\varepsilon_c = \int_0^\infty \theta(1-\rho)(T_c)^{1-\rho}\varepsilon_c^{-\theta(1-\rho)-1} \exp[-(T_c)^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}] d\varepsilon_c \\ &= \int_0^\infty \frac{d \exp[-(T_c)^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]}{d\varepsilon_c} = \left(\exp[-(T_c)^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]\right)\Big|_0^\infty = 1 \end{aligned}$$

(2) When $\varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c$,

$$A = (1-\rho)\theta[T_n\varepsilon_c^{-\theta}(\frac{\omega_c}{\omega_n})^{-\theta} + T_c\varepsilon_c^{-\theta})^{-\rho} = (1-\rho)\theta(\varepsilon_c^{-\theta})^{-\rho}B^{-\rho}, B = T_n(\frac{\omega_c}{\omega_n})^{-\theta} + T_c$$

and,

$$F(\varepsilon_c, \varepsilon_n = \frac{\omega_c}{\omega_n}\varepsilon_c) = \exp\{-[T_n\varepsilon_c^{-\theta}(\frac{\omega_c}{\omega_n})^{-\theta} + T_c\varepsilon_c^{-\theta}]^{1-\rho}\} = \exp[-B^{1-\rho}(\varepsilon_c^{-\theta})^{1-\rho}]$$

Therefore,

$$\begin{aligned} F_c(\varepsilon_c, \varepsilon_n &= \frac{\omega_c}{\omega_n}\varepsilon_c) = (1-\rho)\theta(\varepsilon_c^{-\theta})^{-\rho}B^{-\rho}\exp[-B^{1-\rho}(\varepsilon_c^{-\theta})^{1-\rho}][T_c\varepsilon_c^{-\theta-1}] \\ &= (1-\rho)\theta T_c\varepsilon_c^{-\theta(1-\rho)-1}B^{-\rho}\exp[-B^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}] \end{aligned}$$

and

$$\begin{aligned} \int_0^\infty F_c(\varepsilon_c, \varepsilon_n &= \frac{\omega_c}{\omega_n}\varepsilon_c)d\varepsilon_c = \int_0^\infty (1-\rho)\theta T_c\varepsilon_c^{-\theta(1-\rho)-1}B^{-\rho}\exp[-B^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]d\varepsilon_c \\ &= T_cB^{-1}\int_0^\infty \frac{d\exp[-B^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]}{d\varepsilon_c} \\ &= \left(T_cB^{-1}\exp[-B^{1-\rho}\varepsilon_c^{-\theta(1-\rho)}]\right)\Big|_0^\infty = T_cB^{-1} \end{aligned}$$

(3) Using (1) and (2) above we have

$$p_n = 1 - T_cB^{-1} = \frac{T_n(\omega_c)^{-\theta}(\omega_n)^\theta}{T_c + T_n(\omega_c)^{-\theta}(\omega_n)^\theta} = \frac{T_n(\omega_n)^\theta}{T_c(\omega_c)^\theta + T_n(\omega_n)^\theta}$$

This is equation (8).

8.2 Proposition 2

To simplify notation, we drop the superscript k. We note that the Frechet distribution is max stable; i.e. the max of Frechet variables is still Frechet. To be specific, consider the random variable $\varepsilon^* = \max\{w_ch_c\varepsilon_c, w_nh_n\varepsilon_n\}$. By our discussions in section 3, $\varepsilon^* = w_nh_n\varepsilon_n$ if and only if the individual chooses occupation n.

We now obtain the cdf of the distribution of ε^*

$$\begin{aligned} \Pr(\varepsilon^* &\leq y) = \Pr(w_ch_c\varepsilon_c \leq y \text{ and } w_nh_n\varepsilon_n \leq y) \\ &= F(\frac{y}{w_ch_c}, \frac{y}{w_nh_n}) \\ &= \exp[-B_1y^{-\theta(1-\rho)}], B_1 = \left(T_c\left(\frac{w_c}{P}h_c\right)^\theta + T_n\left(\frac{w_n}{P}h_n\right)^\theta\right)^{1-\rho} \end{aligned}$$

where we have used the Frechet distribution (1) in the second equality.

Consider the mean of non-cognitive workers' net income, I_n, conditional on choosing the non-cognitive occupation, n. By the expression of I_n, (7), we know that it is proportional to the mean of $(w_n h_n \varepsilon_n)^{\frac{1}{1-\eta}}$, conditional on choosing occupation n. This conditional mean is, by Bayesian rule, the mean of $(w_n h_n \varepsilon_n)^{\frac{1}{1-\eta}}$ for those choosing occupation n, divided by the employment share p_n. The mean of $(w_n h_n \varepsilon_n)^{\frac{1}{1-\eta}}$ for those choosing occupation n, in turn, is the mean of $(\varepsilon^*)^{\frac{1}{1-\eta}}$ for all workers times the employment share p_n. As a result, the conditional mean of I_n is proportional to the mean of $(\varepsilon^*)^{\frac{1}{1-\eta}}$, which equals

$$\int_0^\infty y^{\frac{1}{1-\eta}} \frac{d \exp[-B_1 y^{-\theta(1-\rho)}]}{dy} = \int_0^\infty y^{\frac{1}{1-\eta}} \exp[-B_1 y^{-\theta(1-\rho)}] B_1 \theta(1-\rho) y^{-\theta(1-\rho)-1} dy$$

We then use change-of-variables to calculate the value of this expression, because the Gamma function is defined as

$$\Gamma(a+1) = \int_0^\infty t^a e^{-t} dt,$$

where a is a constant. Let $x = B_1 y^{-\theta(1-\rho)}$. Then $y = (\frac{x}{B_1})^{-\frac{1}{\theta(1-\rho)}}$, and $dy = -\frac{1}{\theta(1-\rho)} B_1^{\frac{1}{\theta(1-\rho)}} x^{-\frac{1}{\theta(1-\rho)}-1} dx$. In addition, as $y \to 0$, $x \to \infty$; as $y \to \infty$, $x \to 0$. Therefore,

$$\begin{aligned}
& \int_0^\infty y^{\frac{1}{1-\eta}} \frac{d \exp[-B_1 y^{-\theta(1-\rho)}]}{dy} \\
= & \int_0^\infty y^{\frac{1}{1-\eta}} \exp[-B_1 y^{-\theta(1-\rho)}] B_1 \theta(1-\rho) y^{-\theta(1-\rho)-1} dy \\
= & \int_\infty^0 (\frac{x}{B_1})^{-\frac{1}{\theta(1-\rho)(1-\eta)}} e^{-x} B_1 \theta(1-\rho) (\frac{x}{B_1})^{\frac{1+\theta(1-\rho)}{\theta(1-\rho)}} [-\frac{1}{\theta(1-\rho)}] B_1^{\frac{1}{\theta(1-\rho)}} x^{-\frac{1}{\theta(1-\rho)}-1} dx \\
= & \int_0^\infty (\frac{x}{B_1})^{-\frac{1}{\theta(1-\rho)(1-\eta)}+\frac{1}{\theta(1-\rho)}+1-\frac{1}{\theta(1-\rho)}-1} e^{-x} dx \\
= & B_1^{\frac{1}{\theta(1-\rho))(1-\eta)}} \int_0^\infty x^{-\frac{1}{\theta(1-\rho)(1-\eta)}} e^{-x} dx = B_1^{\frac{1}{\theta(1-\rho))(1-\eta)}} \Gamma(1 - \frac{1}{\theta(1-\rho)(1-\eta)}) \\
= & \gamma [T_c (w_c h_c)^\theta + T_n (w_n h_n)^\theta]^{\frac{1}{\theta(1-\eta)}}, \gamma = \Gamma(1 - \frac{1}{\theta(1-\rho)(1-\eta)})
\end{aligned}$$

Therefore, the average net income of non-cognitive workers, I_n, equals $(1-\eta)\eta^{\frac{\eta}{1-\eta}} \gamma [T_c (\frac{w_c}{P} h_c)^\theta + T_n (\frac{w_n}{P} h_n)^\theta]^{\frac{1}{\theta(1-\eta)}}$.This is equation (9).

8.3 Proposition 3

We again drop the superscript k. We start with the expression $h_i e^{\eta}\varepsilon_i = (\eta w_i)^{\frac{\eta}{1-\eta}} (h_i\varepsilon_i)^{\frac{1}{1-\eta}}$, which we show in the text, right above Proposition 3. Using this expression and the equation of net income, (7), we get

$$h_i e^{\eta}\varepsilon_i = (\eta w_i)^{\frac{\eta}{1-\eta}} \frac{I_i}{(1-\eta)\eta^{\frac{\eta}{1-\eta}}(w_i)^{\frac{1}{1-\eta}}} = \frac{I_i}{(1-\eta)w_i}$$

This means that

$$\begin{aligned} L_i &= Lp_i E\left(h_i e^{\eta}\varepsilon_i | \text{Occupation } i\right) = Lp_i \frac{1}{(1-\eta)w_i} E(I_i | \text{Occupation } i) \qquad (40) \\ &= \frac{Lp_i}{(1-\eta)w_i}(1-\eta)\eta^{\frac{\eta}{1-\eta}}\gamma[T_c(w_c h_c)^{\theta} + T_n(w_n h_n)^{\theta}]^{\frac{1}{\theta(1-\eta)}} \end{aligned}$$

where the last equality is by Proposition 2. To simplify this expression, we use Proposition 1 to get

$$T_c(\frac{w_c}{P}h_c)^{\theta} + T_n(\frac{w_n}{P}h_n)^{\theta} = \frac{T_i(\frac{w_i}{P}h_i)^{\theta}}{p_i}$$

This allows us to obtain

$$\begin{aligned} L_i &= \frac{Lp_i}{(1-\eta)w_i}(1-\eta)\eta^{\frac{\eta}{1-\eta}}\gamma[T_c\left(\frac{w_c}{P}h_c\right)^{\theta} + T_n\left(\frac{w_n}{P}h_n\right)^{\theta}]^{\frac{1}{\theta(1-\eta)}} \\ &= \frac{Lp_i}{w_i}\eta^{\frac{\eta}{1-\eta}}\gamma[\frac{T_i(\frac{w_i}{P}h_i)^{\theta}}{p_i}]^{\frac{1}{\theta(1-\eta)}} = \eta^{\frac{\eta}{1-\eta}}\gamma Lp_i(h_i)^{\frac{1}{1-\eta}}(\frac{w_i}{P})^{\frac{\eta}{1-\eta}}(\frac{T_i}{p_i})^{\frac{1}{\theta(1-\eta)}} \\ &= \gamma Lp_i\left[h_i\left(\eta\frac{w_i}{P}\right)^{\eta}\left(\frac{T_i}{p_i}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}} \end{aligned}$$

8.4 Derivations of Various Equations

8.4.1 Equations (26) and (27)

By equation (34), the production function can be re-written, using x_i^k, as (since test score, $S^k = bL_c^{kS}/L^k$).

$$y^k = \Theta^k(A_c)^{\frac{\alpha}{\alpha-1}}L^k S^k b^{-1}\left(1-x_c^k\right)\left(1+\frac{1-x_n^k}{1-x_c^k}\frac{p_n^k}{p_c^k}\right)^{\frac{\alpha}{\alpha-1}}$$

We can now substitute out S^k using equation (31)

$$\begin{aligned} y^k &= \Theta^k (A_c)^{\frac{\alpha}{\alpha-1}} L^k (E^k)^{\eta} (p_c^k)^{1-\frac{1}{\theta}} \gamma^{1-\eta} (T_c)^{\frac{1}{1-\eta}} h_c^k \left(1-x_c^k\right) \left(1+\frac{1-x_n^k}{1-x_c^k}\frac{p_n^k}{p_c^k}\right)^{\frac{\alpha}{\alpha-1}} \\ \Leftrightarrow \quad \frac{y^k}{L^k} &= \Theta^k (A_c)^{\frac{\alpha}{\alpha-1}} \gamma^{1-\eta} \eta^{\eta} (T_c)^{\frac{1}{1-\eta}} h_c^k (\frac{y^k}{L^k})^{\eta} (p_c^k)^{1-\frac{1}{\theta}} \left(1-x_c^k\right) \left(1+\frac{1-x_n^k}{1-x_c^k}\frac{p_n^k}{p_c^k}\right)^{\frac{\alpha}{\alpha-1}} \\ \Leftrightarrow \quad (\frac{y^k}{L^k})^{1-\eta} &= \Theta^k (A_c)^{\frac{\alpha}{\alpha-1}} \gamma^{1-\eta} \eta^{\eta} (T_c)^{\frac{1}{1-\eta}} h_c^k (p_c^k)^{1-\frac{1}{\theta}} \left(1-x_c^k\right) \left(1+\frac{1-x_n^k}{1-x_c^k}\frac{p_n^k}{p_c^k}\right)^{\frac{\alpha}{\alpha-1}} \end{aligned}$$

This expression implies equation (26).

Now let "T" denote the initial equilibrium and replace the superscript "1", and "A" the counterfactual equilibrium and replace "2". With $x_c^{k2} = x_n^{k2} = 0$ and no change in the values of L^k, h_c^k, h_n^k, or Θ^k, (26) can be written as

$$\frac{y^{kT}}{y^{kA}} = \left[(p_c^{kT})^{1-\frac{1}{\theta}} \left(1-x_c^k\right) \frac{\left(1+\frac{1-x_n^k}{1-x_c^k}\frac{p_n^{kT}}{p_c^{kT}}\right)^{\frac{\alpha}{\alpha-1}}}{(p_c^{kA})^{1-\frac{1}{\theta}} \left(1+\frac{p_n^{kA}}{p_c^{kA}}\right)^{\frac{\alpha}{\alpha-1}}} \right]^{\frac{1}{1-\eta}} \tag{41}$$

For the counterfactual variables we have

$$\begin{aligned} \frac{p_n^{kA}}{p_c^{kA}} &= \left(\frac{h_n^k}{h_c^k}\frac{w_n^{kA}}{w_c^{kA}}\right)^{\theta} \frac{T_n}{T_c} \\ p_c^{kA} &= \left(1+\frac{p_n^{kA}}{p_c^{kA}}\right)^{-1} \end{aligned}$$

We also know, from equations (8) and (19), that

$$\begin{aligned} \frac{h_n^k}{h_c^k} &= \left(\frac{p_n^{kT} T_c}{p_c^{kT} T_n}\right)^{\frac{1}{\theta}} \frac{w_c^{kT}}{w_n^{kT}} \\ \frac{w_c^{kT}}{w_n^{kT}} &= \left(\frac{A_c}{A_n}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{p_n^{kT}\left(1-x_n^k\right)}{p_c^{kT}(1-x_c^k)}\right)^{\frac{1}{\alpha-1}} \end{aligned}$$

Finally, by equation (21), we have

$$\begin{aligned} \frac{w_n^{kA}}{w_c^{kA}} &= \left[\frac{T_c}{T_n}\left(\frac{A_n}{A_c}\right)^{\alpha}\left(\frac{h_c^k}{h_n^k}\right)^{\theta}\right]^{\frac{1}{\theta+\alpha-1}} \\ &= \left[\frac{p_c^{kT}}{p_n^{kT}}\left(\frac{A_n}{A_c}\right)^{\alpha}\left(\frac{w_n^{kT}}{w_c^{kT}}\right)^{\theta}\right]^{\frac{1}{\theta+\alpha-1}}. \end{aligned}$$

where the second inequality uses the expression for $\frac{h_n^k}{h_c^k}$.

Assembling this stuff, we have

$$\begin{aligned}
\frac{p_n^{kA}}{p_c^{kA}} &= \frac{T_n}{T_c}\left(\frac{h_n^k}{h_c^k}\frac{w_n^{kA}}{w_c^{kA}}\right)^\theta = \frac{T_n}{T_c}\frac{p_n^{kT}}{p_c^{kT}}\frac{T_c}{T_n}\left(\frac{w_c^{kT}}{w_n^{kT}}\frac{w_n^{kA}}{w_c^{kA}}\right)^\theta \\
&= \left(\frac{p_n^{kT}}{p_c^{kT}}\right)^{\frac{\alpha-1}{\theta+\alpha-1}}\left(\left(\frac{w_c^{kT}}{w_n^{kT}}\right)^{\frac{\alpha-1}{\theta+\alpha-1}}\left(\frac{A_n}{A_c}\right)^{\frac{\alpha}{\theta+\alpha-1}}\right)^\theta \\
&= \frac{p_n^{kT}}{p_c^{kT}}\left(\frac{1-x_n^k}{1-x_c^k}\right)^{\frac{\theta}{\theta+\alpha-1}}
\end{aligned}$$

Plugging these expressions into (41), we obtain equation (27).

8.4.2 Equation (32)

Using equation (11), we can show that

$$\begin{aligned}
\eta w_c^k L_c^k &= \eta L^k p_c^k\left((\eta(P^k)^{-1})^\eta\left(T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right)^{1/\theta}\right)^{1/(1-\eta)}\gamma \\
&= \eta L^k \frac{T_c\left(w_c^k h_c^k\right)^\theta}{T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta}\left((\eta(P^k)^{-1})^\eta\left(T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right)^{1/\theta}\right)^{1/(1-\eta)}\gamma \\
&= \gamma L^k \eta^{\frac{1}{1-\eta}}((P^k)^{-1})^{\frac{\eta}{1-\eta}} T_c\left(w_c^k h_c^k\right)^\theta\left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta}\frac{1}{1-\eta}-1}
\end{aligned}$$

By analogy we have

$$\eta w_n L_n^k = \gamma L^k \eta^{\frac{1}{1-\eta}}((P^k)^{-1})^{\frac{\eta}{1-\eta}} T_n\left(w_n^k h_n^k\right)^\theta\left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta}\frac{1}{1-\eta}-1}$$

Adding up these equations we get

$$\begin{aligned}
\eta w_c L_c^k + \eta w_n L_n^k &= \gamma L^k \eta^{\frac{1}{1-\eta}}((P^k)^{-1})^{\frac{\eta}{1-\eta}}\left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]\left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta}\frac{1}{1-\eta}-1} \\
&= \gamma L^k \eta^{\frac{1}{1-\eta}}((P^k)^{-1})^{\frac{\eta}{1-\eta}}\left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta}\frac{1}{1-\eta}}
\end{aligned}$$

Using the output identity $P^k y^k = w_c^k L_c^k + w_n^k L_n^k$, we have

$$\begin{aligned}
\eta y^k &= \eta\frac{w_c^k L_c^k + w_n^k L_n^k}{P^k} \\
&= (P^k)^{-1}\gamma L^k \eta^{\frac{1}{1-\eta}}((P^k)^{-1})^{\frac{\eta}{1-\eta}}\left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta}\frac{1}{1-\eta}} \\
&= \gamma L^k \eta^{\frac{1}{1-\eta}}((P^k)^{-1})^{\frac{1}{1-\eta}}\left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta}\frac{1}{1-\eta}}
\end{aligned}$$

This expression and equation (10) imply that $E^k L^k = \eta y^k$.

8.4.3 Equation (31)

Using equations (30), (10) and (11), we can show that

$$\begin{aligned} S^k &= b\frac{L_c^{kS}}{L^k} \\ &= b\frac{p_c^k}{w_c^k}\left((\eta(P^k)^{-1})^\eta\left(T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)\right)^{1/\theta}\right)^{1/(1-\eta)}\gamma \\ &= b\frac{p_c^k}{w_c^k}\frac{E^k}{\eta(P^k)^{-1}} \\ \iff\ & w_c^k = b\frac{p_c^k}{S^k}\frac{E^k}{\eta(P^k)^{-1}} = b\frac{p_c^k}{S^k}\frac{E^k}{\eta}\frac{1}{(P^k)^{-1}} \end{aligned}$$

We now use equation (8) to obtain that $T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right) = \frac{T_i(w_i^k h_i^k)^\theta}{p_i}$. This expression allows us to substitute out the term $T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)$ in equation (11), giving us, together with equation (30), that

$$\begin{aligned} S^k &= b\frac{L_c^k}{L^k} \\ &= bp_c^k\left(h_c^k(\eta(P^k)^{-1}w_c^k)^\eta\left(\frac{T_c}{p_c^k}\right)^{1/\theta}\right)^{1/(1-\eta)}\gamma \\ &= b(p_c^k)^{1-\frac{1}{\theta(1-\eta)}}\gamma\eta^{\frac{\eta}{1-\eta}}(T_c)^{\frac{1}{\theta(1-\eta)}}((P^k)^{-1}w_c^k)^{\frac{\eta}{1-\eta}}(h_c^k)^{\frac{1}{1-\eta}} \end{aligned}$$

We then substitute out w_c^k using $b\frac{p_c^k}{S^k}\frac{E^k}{\eta}\frac{1}{(P^k)^{-1}}$ to obtain

$$\begin{aligned} S^k &= b(p_c^k)^{1-\frac{1}{\theta(1-\eta)}}\gamma\eta^{\frac{\eta}{1-\eta}}(T_c)^{\frac{1}{\theta(1-\eta)}}((P^k)^{-1}b\frac{p_c^k}{S^k}\frac{E^k}{\eta}\frac{1}{(P^k)^{-1}})^{\frac{\eta}{1-\eta}}(h_c^k)^{\frac{1}{1-\eta}} \\ &= (\frac{1}{S^k})^{\frac{\eta}{1-\eta}}b^{\frac{1}{1-\eta}}\gamma\eta^{\frac{\eta}{1-\eta}}(T_c)^{\frac{1}{\theta(1-\eta)}}(p_c^k)^{1-\frac{1}{\theta(1-\eta)}+\frac{\eta}{1-\eta}}(\frac{E^k}{\eta})^{\frac{\eta}{1-\eta}}(h_c^k)^{\frac{1}{1-\eta}} \\ \Leftrightarrow\ & S^k = b\gamma^{1-\eta}\eta^\eta(T_c)^{\frac{1}{(1-\eta)}}(p_c^k)^{1-\frac{1}{\theta}}(E^k)^\eta h_c^k \end{aligned}$$

Taking the ratio of this expression with respect to country 0, we get equation (31).

8.4.4 Equation (35)

The comparative static exercise involves changing h_c^k and h_n^k, holding the other parameters fixed, and tracing out the responses of the endogenous variables. First, the identity

$p_n^k + p_c^k = 1$ implies that

$$d \ln p_n^k = -(d \ln p_c^k) \frac{p_c^k}{p_n^k}$$

Next, equations (29), (33) and (34) imply, respectively, that

$$(d \ln p_c^k) - d \ln p_n^k = \frac{\theta(\alpha - 1)}{\theta + \alpha - 1} (d \ln h_c^k - d \ln h_n^k)$$

$$d \ln S^k - \eta d \ln y^k = (1 - \frac{1}{\theta}) d \ln p_c^k + d \ln h_c^k$$

and

$$d \ln y^k - d \ln S^k = -\frac{\alpha}{\alpha - 1} d \ln p_c^k$$

These four equations are all log linear, and we can solve for $d \ln y^k$, $d \ln S^k$, $d \ln p_c^k$, and $d \ln p_n^k$ in terms of $d \ln h_c^k$ and $d \ln h_n^k$. The solution for $d \ln S^k$ is equation (35).

8.4.5 Equation (37)

We derive the connection between $1 + \frac{1-x_n^k}{1-x_c^k} \frac{p_n^k}{p_c^k}$ and the cost share, s_c.

$$\begin{aligned} \frac{1}{s_c} &= \frac{(A_c)^\alpha (w_c)^{1-\alpha} + (A_n)^\alpha (w_n)^{1-\alpha}}{(A_c)^\alpha (w_c)^{1-\alpha}} \\ &= 1 + \frac{(A_n)^\alpha (w_n)^{1-\alpha}}{(A_c)^\alpha (w_c)^{1-\alpha}} \\ &= 1 + \frac{A_n}{A_c} (\frac{L_n^{kD}}{L_c^{kD}})^{\frac{\alpha-1}{\alpha}} \\ &= 1 + \frac{1 - x_n^k}{1 - x_c^k} \frac{p_n^k}{p_c^k} \end{aligned} \tag{42}$$

In the derivation, the first equality follows from equation (15), the third follows from (14), and the last from (18). Plug equation 42) into (26) and we obtain (37).

8.4.6 The Equilibrium Using (36) Is Invariant to T_c/T_n and A_c/A_n

1. We first show that the ways we obtain closed-economy parameter values place restrictions on how T_c/T_n is related to A_c/A_n; i.e. there is only one degree of freedom in the values of T_c/T_n and A_c/A_n, not two.

We plug equation (21) into (8) to obtain

$$\frac{p_c^k}{p_n^k} = (\frac{T_c}{T_n})^{\frac{\alpha-1}{\alpha+\theta-1}}(\frac{A_c}{A_n})^{\frac{\alpha\theta}{\alpha+\theta-1}}(\frac{h_c^k}{h_n^k})^{\frac{\theta(\alpha-1)}{\alpha+\theta-1}}$$

or

$$(\frac{T_c}{T_n})^{\alpha-1}(\frac{A_c}{A_n})^{\alpha\theta} = (\frac{p_c^k}{p_n^k})^{\alpha+\theta-1}(\frac{h_c^k}{h_n^k})^{-\theta(\alpha-1)}$$

This expression holds for every country k, including the U.S., for which $h_c^{US} = h_n^{US} = 1$. Therefore,

$$(\frac{T_c}{T_n})^{\alpha-1}(\frac{A_c}{A_n})^{\alpha\theta} = (\frac{p_c^{US}}{p_n^{US}})^{\alpha+\theta-1} \tag{43}$$

Equation (43) says that $(\frac{T_c}{T_n})^{\alpha-1}(\frac{A_c}{A_n})^{\alpha\theta}$ is a constant. Let

$$C_2 = (\frac{A_c}{A_n})^{\alpha}(\frac{w_c}{w_n})^{1-\alpha}, C_1 = \frac{T_c}{T_n}\left(\frac{w_c}{w_n}\right)^{\theta}$$

Using these definitions and (43), we obtain

$$C_1^{\alpha-1}C_2^{\theta} = (\frac{T_c}{T_n}\left(\frac{w_c}{w_n}\right)^{\theta})^{\alpha-1}((\frac{A_c}{A_n})^{\alpha}(\frac{w_c}{w_n})^{1-\alpha})^{\theta} = (\frac{T_c}{T_n})^{\alpha-1}(\frac{A_c}{A_n})^{\alpha\theta} = (\frac{p_c^{US}}{p_n^{US}})^{\alpha+\theta-1}; \tag{44}$$

i.e. $C_1^{\alpha-1}C_2^{\theta}$ is also a constant.

2. We now show that under the free trade equlibrium, C_1 and C_2 are invariant to T_c/T_n and A_c/A_n. To be specific, as we change T_c/T_n, w_c/w_n adjusts, but the change in w_c/w_n offsets the initial change in T_c/T_n, leaving C_1 and C_2 unchanged.

Using the definitions of C_1 and C_2, we can rewrite equation (36) as

$$C_2 = C_1\frac{\sum_k L^k \left(\Theta^k\right)^{\frac{\eta}{1-\eta}} \left(h_c^k\right)^{\theta}\left(C_1\left(h_c^k\right)^{\theta} + \left(h_n^k\right)^{\theta}\right)^{\frac{1}{\theta(1-\eta)}-1}}{\sum_k L^k \left(\Theta^k\right)^{\frac{\eta}{1-\eta}} \left(h_n^k\right)^{\theta}\left(C_1\left(h_c^k\right)^{\theta} + \left(h_n^k\right)^{\theta}\right)^{\frac{1}{\theta(1-\eta)}-1}}$$

From this expression, we see that (36) depends on the two variables of C_1 and C_2, not the three variables of T_c/T_n, A_c/A_n and w_c/w_n.

Suppose we start with, say, $T_c/T_n = 1$. We obtain $\frac{A_c}{A_n}$ using equation (43), and then obtain w_c/w_n by solving equation (36). These initial set of values of T_c/T_n, $\frac{A_c}{A_n}$ and w_c/w_n give us initial values of C_1 and C_2.

Now suppose we specify a different value, say, $T_c/T_n = 10$. We pick the value of w_c/w_n such that C_1 remains unchanged, using the definition of C_1. Next, we obtain a

different value of $\frac{A_c}{A_n}$ using equation (43). We then use the new values of $\frac{A_c}{A_n}$ and w_c/w_n to compute C_2, according to the definition of C_2.

We can say two things about the value of C_2, obtained this way. (1) It satisfies equation (44), together with C_1. This is because the definitions of C_1 and C_2 plus equation (43) imply (44). (2) It is the same value as when $T_c/T_n = 1$, because C_1 remains unchanged, and so does the right-hand side of equation (44).

So we see that, although the values of T_c/T_n, $\frac{A_c}{A_n}$ and w_c/w_n have all changed, from the initial values associated with $T_c/T_n = 1$ to the new values associated with $T_c/T_n = 10$, the values of C_1 and C_2 remain unchanged. In other words, C_1 and C_2 and equation (36) are invariant to T_c/T_n.

3. We now show that the other real variables depend on C_1 and C_2 and so are invariant to T_c/T_n and A_c/A_n.

First, by equation (8), the ratio of employment shares is pinned down by C_1, h_c^k, h_n^k and θ, all of which are invariant to T_c/T_n.

Second, by (15), s_c is pinned down by C_2 and so is invariant to T_c/T_n.

Third, we can solve for factor content of trade using equation (42) and balance of trade $nx_c^k + nx_n^k = 0 \Leftrightarrow p_c^k x_c^k + p_n^k x_n^k = 0$

$$x_c^k = 1 - s_c(p_c^k)^{-1} \tag{45}$$

As a result, x_c^k and x_n^k are invariant to T_c/T_n.

Finally, by equation (37), the output ratio depends on employment shares, factor content of trade, and s_c. Therefore, the output ratio is also invariant to T_c/T_n.

8.4.7 Equation (39)

Equations (42), (45) and (26) imply (39).

8.4.8 DEK Equations

Define changes to variable x as $\widehat{x} = x'/x$. Let $w_{cn} = w_c/w_n$ and $\widehat{w}_{cn} = w'_{cn}/w_{cn}$.

By equation (11)

$$L_c^k = L^k p_c^k \left(h_c^k (\eta w_c \Theta^k)^\eta \left(\frac{T_c}{p_c^k} \right)^{1/\theta} \right)^{1/(1-\eta)} \qquad \gamma = \gamma L^k (p_c^k)^{1-\frac{1}{\theta(1-\eta)}} (h_c^k)^{\frac{1}{1-\eta}} (\eta w_c \Theta^k)^{\frac{\eta}{1-\eta}} (T_c)^{\frac{1}{\theta(1-\eta)}}$$

This implies that

$$\frac{\sum_k L_c^{k\prime}}{\sum_k L_c^k} = \sum_j \frac{L_c^k}{\sum_k L_c^k} \widehat{L}_c^k = (\widehat{w}_c)^{\frac{\eta}{1-\eta}} \sum_k SH_c^k \widehat{L}^k (\widehat{\Theta}^k)^{\frac{\eta}{1-\eta}} \left(\widehat{h}_c^k\right)^{\frac{1}{1-\eta}} \left(\widehat{p}_c^k\right)^{1-\frac{1}{\theta(1-\eta)}}$$

where SH_c^k is country k's share in global cognitive income

$$\frac{p_c^k y^k (\Theta^k)^{-1}}{\sum_j p_c^j y^j (\Theta^j)^{-1}} = SH_c^k$$

By symmetry,

$$\frac{\sum_k L_n^{k\prime}}{\sum_k L_n^k} = (\widehat{w}_n)^{\frac{\eta}{1-\eta}} \sum_k SH_n^k \widehat{L}^k (\widehat{\Theta}^k)^{\frac{\eta}{1-\eta}} \left(\widehat{h}_n^k\right)^{\frac{1}{1-\eta}} \left(\widehat{p}_n^k\right)^{1-\frac{1}{\theta(1-\eta)}}$$

As a result, the change in relative supply is

$$\frac{\sum_k L_c^{k\prime}}{\sum_k L_c^k} / \frac{\sum_k L_n^{k\prime}}{\sum_k L_n^k} = (\widehat{w}_{cn})^{\frac{\eta}{1-\eta}} \frac{\sum_k SH_c^k \widehat{L}^k (\widehat{\Theta}^k)^{\frac{\eta}{1-\eta}} \left(\widehat{h}_c^k\right)^{\frac{1}{1-\eta}} \left(\widehat{p}_c^k\right)^{1-\frac{1}{\theta(1-\eta)}}}{\sum_k SH_n^k \widehat{L}^k (\widehat{\Theta}^k)^{\frac{\eta}{1-\eta}} \left(\widehat{h}_n^k\right)^{\frac{1}{1-\eta}} \left(\widehat{p}_n^k\right)^{1-\frac{1}{\theta(1-\eta)}}}$$

On the other hand, relative demand is $\frac{L_c^{kD}}{L_n^{kD}} = (\frac{A_c}{A_n})^{\alpha} (\frac{w_c}{w_n})^{-\alpha}$, and so its change is equal to $(\widehat{w}_{cn})^{-\alpha}$. Putting these two pieces together we have

$$(\widehat{w}_{cn})^{-\alpha} = (\widehat{w}_{cn})^{\frac{\eta}{1-\eta}} \frac{\sum_k SH_c^k \widehat{L}^k (\widehat{\Theta}^k)^{\frac{\eta}{1-\eta}} \left(\widehat{h}_c^k\right)^{\frac{1}{1-\eta}} \left(\widehat{p}_c^k\right)^{1-\frac{1}{\theta(1-\eta)}}}{\sum_k SH_n^k \widehat{L}^k (\widehat{\Theta}^k)^{\frac{\eta}{1-\eta}} \left(\widehat{h}_n^k\right)^{\frac{1}{1-\eta}} \left(\widehat{p}_n^k\right)^{1-\frac{1}{\theta(1-\eta)}}} \tag{46}$$

We now express $\widehat{p}_c^k$ and $\widehat{p}_n^k$ in terms of $\widehat{w}_{cn}$, plus other parameters. We start by defining that $p_{cn}^k = p_c^k / p_n^k$. It is easy to see that

$$p_c^k = \frac{p_{cn}^k}{1 + p_{cn}^k}, p_n^k = 1 - p_c^k = \frac{1}{1 + p_{cn}^k}$$

Therefore,

$$\widehat{p}_c^k = \frac{p_c^{k\prime}}{p_c^k} = \widehat{p}_{cn}^k \frac{1 + p_{cn}^k}{1 + p_{cn}^{k\prime}}$$

And we know that

$$\frac{1 + p_{cn}^{k\prime}}{1 + p_{cn}^k} = \frac{1}{1 + p_{cn}^k} + \frac{p_{cn}^{k\prime}}{p_{cn}^k} \frac{p_{cn}^k}{1 + p_{cn}^k} = p_n^k + p_c^k \widehat{p}_{cn}^k$$

This means that

$$\widehat{p}_c^k = \frac{\widehat{p}_{cn}^k}{p_n^k + p_c^k \widehat{p}_{cn}^k}$$

Using the employment-share equations, it is easy to show that

$$\widehat{p}_{cn}^k = \left(\widehat{w}_{cn}\right)^\theta \left(\widehat{h}_c^k\right)^\theta \left(\widehat{h}_n^k\right)^{-\theta}$$

and that

$$\widehat{p}_n^k = \widehat{p}_c^k / \widehat{p}_{cn}^k$$

Plugging these expressions into (46), and we obtain a single equation for a single unknown, $\widehat{w}_{cn}$.

9 Theory Appendix 2: General Output-per-Worker Decomposition

Start with

$$p_c^k P^k y^k = w_c^k L_c^k$$

so

$$\frac{y^k}{L^k} = \frac{1}{P^k}\left(\frac{\eta}{P^k}\right)^{\frac{\eta}{1-\eta}} \left[T_c\left(w_c^k h_c^k\right)^\theta + T_n\left(w_n^k h_n^k\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}} \gamma$$

and so relative to the base country we have

$$\frac{y^k/L^k}{y^0/L^0} = \left(\frac{P^0}{P^k}\right)^{\frac{\eta}{1-\eta}} \left[p_c^0\left(\frac{w_c^k h_c^k}{w_c^0 h_c^0}\right)^\theta + p_n^0\left(\frac{w_n^k h_n^k}{w_n^0 h_n^0}\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}}$$

rearranging

$$\frac{y^k/L^k}{y^0/L^0} = \left(\frac{w_c^k/P^k}{w_c^k/P^k}\right)^{\frac{1}{1-\eta}} \left[p_c^0\left(\frac{h_c^k}{h_c^0}\right)^\theta + p_n^0\left(\frac{w_c^0}{w_c^k}\frac{w_n^k h_n^k}{w_n^0 h_n^0}\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}}$$

Substituting for the relative wages we have

$$\frac{y^k/L^k}{y^0/L^0} = \left(\frac{w_c^k/P^k}{w_c^k/P^k}\right)^{\frac{1}{1-\eta}} \left[p_c^0\left(\frac{h_c^k}{h_c^0}\right)^\theta + p_n^0\left(\frac{\left(\frac{p_n^0+nx_c^0}{p_c^0-nx_c^0}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{p_n^k+nx_c^k}{p_c^k-nx_c^k}\right)^{\frac{1}{\alpha-1}}}\frac{h_n^k}{h_n^0}\right)^\theta\right]^{\frac{1}{\theta(1-\eta)}}$$

and for the relative real wage of cognitive labor

$$\frac{y^k/L^k}{y^0/L^0} = \left(\frac{w_c^k/P^k}{w_c^k/P^k}\right)^{\frac{1}{1-\eta}} \left[p_c^0 \left(\frac{h_c^k}{h_c^0}\right)^{\theta} + p_n^0 \left(\frac{\left(\frac{p_n^0+nx_c^0}{p_c^0-nx_c^0}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{p_n^k+nx_c^k}{p_c^k-nx_c^k}\right)^{\frac{1}{\alpha-1}}} \frac{h_n^k}{h_n^0} \right)^{\theta} \right]^{\frac{1}{\theta(1-\eta)}}$$

From the price index we have

$$\begin{aligned} P^k &= \frac{1}{\Theta^k} \left((A_c)^{\alpha} (w_c^k)^{1-\alpha} + (A_n)^{\alpha} (w_n^k)^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \\ &= \frac{w_c^k}{\Theta^k} \left((A_c)^{\alpha} + (A_n)^{\alpha} \left(\frac{w_c^k}{w_n^k}\right)^{\alpha-1} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

and so

$$\frac{w_c^k}{P^k} = \Theta^k \left((A_c)^{\alpha} + (A_n)^{\alpha} \left(\frac{w_c^k}{w_n^k}\right)^{\alpha-1} \right)^{\frac{1}{\alpha-1}}$$

and subbing for relative prices, we have

$$\begin{aligned} \frac{w_c^k}{P^k} &= \Theta^k \left((A_c)^{\alpha} + (A_n)^{\alpha} \left(\left(\frac{A_c}{A_n}\right)^{\frac{\alpha}{\alpha-1}} \left(\frac{p_n^k + nx_c^k}{p_c^k - nx_c^k}\right)^{\frac{1}{\alpha-1}} \right)^{\alpha-1} \right)^{\frac{1}{\alpha-1}} \\ &= \Theta^k \left((A_c)^{\alpha} + (A_c)^{\alpha} \frac{p_n^k + nx_c^k}{p_c^k - nx_c^k} \right)^{\frac{1}{\alpha-1}} \\ &= \Theta^k (A_c)^{\frac{\alpha}{\alpha-1}} \left(p_c^k - nx_c^k\right)^{-\frac{1}{\alpha-1}} \end{aligned}$$

so relative real wage is

$$\frac{w_c^k/P^k}{w_c^0/P^0} = \frac{\Theta^k}{\Theta^0} \left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0}\right)^{-\frac{1}{\alpha-1}}$$

So substitute this into the decomposition to obtain

$$\begin{aligned}
\frac{y^k/L^k}{y^0/L^0} &= \left(\frac{\Theta^k}{\Theta^0}\left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0}\right)^{-\frac{1}{\alpha-1}}\right)^{\frac{1}{1-\eta}} \left[p_c^0\left(\frac{h_c^k}{h_c^0}\right)^{\theta} + p_n^0\left(\frac{\left(\frac{p_n^0+nx_c^0}{p_c^0-nx_c^0}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{p_n^k+nx_c^k}{p_c^k-nx_c^k}\right)^{\frac{1}{\alpha-1}}}\frac{h_n^k}{h_n^0}\right)^{\theta}\right]^{\frac{1}{\theta(1-\eta)}} \\
&= \left[\frac{\Theta^k}{\Theta^0}\left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0}\right)^{-\frac{1}{\alpha-1}}\left(p_c^0\left(\frac{h_c^k}{h_c^0}\right)^{\theta} + p_n^0\left(\frac{\left(\frac{p_n^0+nx_c^0}{p_c^0-nx_c^0}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{p_n^k+nx_c^k}{p_c^k-nx_c^k}\right)^{\frac{1}{\alpha-1}}}\frac{h_n^k}{h_n^0}\right)^{\theta}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}} \\
&= \left[\frac{\Theta^k}{\Theta^0}\left(p_c^0\left(\left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0}\right)^{-\frac{1}{\alpha-1}}\frac{h_c^k}{h_c^0}\right)^{\theta} + p_n^0\left(\left(\frac{p_c^k - nx_c^k}{p_c^0 - nx_c^0}\right)^{-\frac{1}{\alpha-1}}\frac{\left(\frac{p_n^0+nx_c^0}{p_c^0-nx_c^0}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{p_n^k+nx_c^k}{p_c^k-nx_c^k}\right)^{\frac{1}{\alpha-1}}}\frac{h_n^k}{h_n^0}\right)^{\theta}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}} \\
&= \left[\frac{\Theta^k}{\Theta^0}\left(p_c^0\left(\left(\frac{p_c^0 - nx_c^0}{p_c^k - nx_c^k}\right)^{\frac{1}{\alpha-1}}\frac{h_c^k}{h_c^0}\right)^{\theta} + p_n^0\left(\left(\frac{p_n^0 + nx_c^0}{p_n^k + nx_c^k}\right)^{\frac{1}{\alpha-1}}\frac{h_n^k}{h_n^0}\right)^{\theta}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1-\eta}}
\end{aligned}$$

9.1 Closed Economy Decomposition

Starting with the final good production function, we have

$$\begin{aligned}
y^k &= \Theta^k\left(A_c\left(L_c^k\right)^{\frac{\alpha-1}{\alpha}} + A_n\left(L_n^k\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}} \\
&= \Theta^k L_c^k\left(A_c + A_n\left(\frac{L_n^k}{L_c^k}\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}}
\end{aligned}$$

The first-order condition for optimal input choice requires

$$\frac{L_c^k}{L_n^k} = \left(\frac{p_c^k}{p_n^k}\frac{A_n}{A_c}\right)^{\frac{\alpha}{\alpha-1}}.$$

Substituting this expression into the output equation yields

$$y^k = \Theta^k L_c^k\left(\frac{A_c}{p_c^k}\right)^{\frac{\alpha}{\alpha-1}}$$

Educational and occupational choice requires that $w_c^k L_c^k = p_c^k y^k$. Substituting this expression into the output equation, we obtain

$$w_c^k = \Theta^k \left(p_c^k\right)^{-\frac{1}{\alpha-1}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \tag{47}$$

Rearranging the educational expenditure equation,

$$E_c^k = \left(\eta w_c^k h_c^k\right)^{\frac{1}{1-\eta}} \left(\frac{T_c}{p_c^k}\right)^{\frac{1}{\theta(1-\eta)}} \gamma,$$

and substituting $E^k = \eta y^k / L^k$, we can substitute w_c^k in equation (47) to obtain after rearranging

$$\frac{y^k}{L^k} = \left(\Theta^k h_c^k \left(p_c^k\right)^{-\frac{\phi}{\theta(\alpha-1)}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \left(T_c\right)^{\frac{1}{\theta}} \eta\right)^{\frac{1}{1-\eta}} \frac{\gamma}{\eta},$$

where we have defined $\phi \equiv \alpha + \theta - 1$. Substituting out p_c^k using its definition, we obtain

$$\frac{y^k}{L^k} = \left(\Theta^k h_c^k \left(1 + \frac{T_n \left(h_n^k\right)^\theta}{T_c \left(h_c^k\right)^\theta} \left(\frac{w_n^k}{w_c^k}\right)^\theta\right)^{\frac{\phi}{\theta(\alpha-1)}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \left(T_c\right)^{\frac{1}{\theta}} \eta\right)^{\frac{1}{1-\eta}} \frac{\gamma}{\eta}.$$

Finally, factor market clearing implies

$$\frac{w_c^k}{w_n^k} = \left[\frac{T_n}{T_c} \left(\frac{A_c}{A_n}\right)^\alpha \left(\frac{h_n^k}{h_c^k}\right)^\theta\right]^{\frac{1}{\phi}}.$$

Substituting this expression into the GDP per capita equation and simplifying, we obtain an expression with no endogenous variables

$$\frac{y^k}{L^k} = \left(\Theta^k h_c^k \left(1 + \left(\frac{T_n \left(h_n^k\right)^\theta}{T_c \left(h_c^k\right)^\theta}\right)^{\frac{\alpha-1}{\phi}} \left(\frac{A_n}{A_c}\right)^{\alpha \frac{\theta}{\phi}}\right)^{\frac{\phi}{\theta(\alpha-1)}} \left(A_c\right)^{\frac{\alpha}{\alpha-1}} \left(T_c\right)^{\frac{1}{\theta}} \eta\right)^{\frac{1}{1-\eta}} \frac{\gamma}{\eta}.$$

Comparing GDP per capita in country k to a base country (or to the initial values for that country in a comparative static, we have

$$\frac{y^k/L^k}{y^0/L^0} = \left(\frac{\Theta^k}{\Theta^0} \frac{h_c^k \left(1 + \left(\frac{T_n \left(h_n^k\right)^\theta}{T_c \left(h_c^k\right)^\theta}\right)^{\frac{\alpha-1}{\phi}} \left(\frac{A_n}{A_c}\right)^{\alpha \frac{\theta}{\phi}}\right)^{\frac{\phi}{\theta(\alpha-1)}}}{h_c^0 \left(1 + \left(\frac{T_n (h_n^0)^\theta}{T_c (h_c^0)^\theta}\right)^{\frac{\alpha-1}{\phi}} \left(\frac{A_n}{A_c}\right)^{\alpha \frac{\theta}{\phi}}\right)^{\frac{\phi}{\theta(\alpha-1)}}}\right)^{\frac{1}{1-\eta}}.$$

Combining the occupational share equations and labor market clearing conditions for the base country, we have

$$\left(\frac{A_n}{A_c}\right)^{\frac{\theta\alpha}{\phi}}\left(\frac{T_n}{T_c}\right)^{\frac{\alpha-1}{\phi}}=\left(\frac{\left(h_c^0\right)^\theta}{\left(h_n^0\right)^\theta}\right)^{\frac{\alpha-1}{\phi}}\frac{p_n^0}{p_c^0}.$$

Substituting this expression into the relative GDP per capita expressions and simplifying, we arrive at our decomposition:

$$\frac{y^k/L^k}{y^0/L^0}=\left(\frac{\Theta^k}{\Theta^0}\left(p_c^0\left(\frac{h_c^k}{h_c^0}\right)^{\frac{\theta(\alpha-1)}{\phi}}+p_n^0\left(\frac{h_n^k}{h_n^0}\right)^{\frac{\theta(\alpha-1)}{\phi}}\right)^{\frac{\phi}{\theta(\alpha-1)}}\right)^{\frac{1}{1-\eta}}.$$

9.2 Free Trade Decomposition

Under free trade, a single w_c and w_n prevails everywhere. The value of output must be equal to the value of income and so

$$P^k y^k = w_c L_c^k + w_n L_n^k, P^k = (\Theta^k)^{-1}$$

The supply of type i labor in country k is given by

$$L_i^k=\frac{L^k p_i^k}{w_i}\left((\eta\Theta^k)^\eta\left(T_c\left(w_c h_c^k\right)^\theta+T_n\left(w_n h_n^k\right)^\theta\right)^{1/\theta}\right)^{1/(1-\eta)}\gamma$$

So we can write real output per capita in country k relative to a base country 0 as

$$\frac{y^k/L^k}{y^0/L^0}=\frac{P^0}{P^k}\left(\left(\frac{\Theta^k}{\Theta^0}\right)^\eta\left(\frac{T_c\left(w_c h_c^k\right)^\theta+T_n\left(w_n h_n^k\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta+T_n\left(w_n h_n^0\right)^\theta}\right)^{\frac{1}{\theta}}\right)^{\frac{1}{1-\eta}}$$

Since $P^k=\left(\Theta^k\right)^{-1}$, we have

$$\frac{y^k/L^k}{y^0/L^0}=\left(\frac{\Theta^k}{\Theta^0}\left(\frac{T_c\left(w_c h_c^k\right)^\theta+T_n\left(w_n h_n^k\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta+T_n\left(w_n h_n^0\right)^\theta}\right)^{\frac{1}{\theta}}\right)^{\frac{1}{1-\eta}}$$

Rearranging, we obtain

$$\frac{y^k/L^k}{y^0/L^0}=\left(\frac{\Theta^k}{\Theta^0}\left(\frac{T_c\left(w_c h_c^0\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta+T_n\left(w_n h_n^0\right)^\theta}\left(\frac{T_c\left(w_c h_c^k\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta}\right)+\frac{T_n\left(w_n h_n^0\right)^\theta}{T_c\left(w_c h_c^0\right)^\theta+T_n\left(w_n h_n^0\right)^\theta}\left(\frac{T_n\left(w_n h_n^k\right)^\theta}{T_n\left(w_n h_n^0\right)^\theta}\right)\right)^{\frac{1}{\theta}}\right)$$

now replacing the expressions with occupation shares from the base country, we obtain

$$\frac{y^k/L^k}{y^0/L^0} = \left(\frac{\Theta^k}{\Theta^0} \left(p_c^0 \left(\frac{h_c^k}{h_c^0} \right)^\theta + p_n^0 \left(\frac{h_n^k}{h_n^0} \right)^\theta \right)^{\frac{1}{\theta}} \right)^{\frac{1}{1-\eta}}$$

So, the difference with the closed economy is in the exponents. Note, however, that since this is holding fixed relative prices it cannot be thought used to talk about comparative statics as was the case in the closed economy.

10 Data Appendix

1. Sample Cuts for NLSY-79 Data

Following Neal and Johnson (1996) we: (1) use the 1989 version of AFQT and drop the observations with missing AFQT scores; (2) drop those whose wage exceeds \$75 or below \$1 in 1991; and (3) drop those who are older than 17 when they take the AFQT.

2. O*NET Data

The following is the list of O*NET task ID's of the measures we discuss in the text. Leadership is 4.A.4.b.4, and enterprising 1.B.1.e. Enterprising skills involve "starting up and carrying out projects" and "leading people and making many decisions".

In addition, we have experimented with the following candidate measures. (1) Originality is about coming up with "unusual or clever ideas about a given topic or situation", or developing "creative ways to solve a problem". 1.A.1.b.2. (2) Social skills involve "working with, communicating with, and teaching people". 1.B.1.d. (3) Artistic talents show up when "working with forms, designs and patterns", where "the work can be done without following a clear set of rules". 1.B.1.c 2. (4) Investigative skills involve "working with ideas" and "searching for facts and figuring out problems mentally", and require "an extensive amount of thinking"; 1.B.1.b. The results are in Table A1.

When we use originality, social skills or investigative skills to measure non-cognitive skills, the AFQT coefficient of the non-cognitive sub-sample is larger than the cognitive sub-sample. This is counter-intuitive. On the other hand, for the artistic-talent sub-sample, the AFQT coefficient is negative, meaning that the artists with higher test scores have lower wages. However, out of the NLSY-79 sample of over 3000, there are only 30 artists, less than 1% of the sample size.

3. ILO Employment-by-Occupation Data

We map the O*NET occupation codes into the ISCO-88 codes using the crosswalk at the National Crosswalk center ftp://ftp.xwalkcenter.org/DOWNLOAD/xwalks/. We drop the following observations from the ILO raw data because of data quality issues. 1. All data from Cyprus, because the data source is official estimate (source code "E"). 2. Year 2000 for Switzerland, because over 1 million individuals, a large fraction of the Switzerland labor force, are "not classified". 3. Uganda, Gabon, Egypt, Mongolia, Thailand, Poland in 1994 and Romania in 1992, because the aggregate employment of the sub-occupation categories does not equal the number under "Total". 4. Estonia in 1998, S. Korea in 1995, and Romania in 2000, because the data is in 1-digit or 2-digit occupation codes.

Most countries have a single year of data around 2000. In Figure A1 we plot the non-cognitive employment share for all the countries that have multiple years of data. Within countries the non-cognitive employment share shows very limited variation over time. As a result, for this set of countries we keep the single year of data closest to 2000; e.g. 1990 for Switzerland, 2000 for U.S. and Australia, etc. By construction, the non-cognitive and cognitive employment shares sum to 1 by country.

4. Test Score Data

We have tabulated over-time changes of PISA scores within countries and found very little variation. For example, for the U.S. reading score the mean is 499.26 and the standard deviation is 3.93. We list these summary statistics by country by subject in Table A2.

There have been several international tests on adults: IALS (International Adult Literacy Survey), administered in 1994-1998, ALLS (Adult Literacy and Life Skills Survey), conducted in 2002-2006, and PIAAC (Program for the International Assessment of Adult Competencies), conducted in 2013. The response rate of IALS, 63%, is substantially lower than the initial wave of PISA in 2000, 89% (Brown et al. 2007). ALLS was designed as a follow-up to IALS, but only 5 countries participated. Of the 28 countries in our sample, only 18 participated in IALS, and only 21 in PIAAC. This would represent a 36% and 25% reduction in the number of observations, respectively.

We regress the 2012 PISA scores on 2013 PIAAC scores, for reading and math, for all the countries that participated in both tests, including those that are not in our sample. We obtain, respectively, the coefficient estimate of 0.938 and 1.067, and R-square of 0.508 and 0.527.

5. Correlation Coefficients of Output TFP Estimates

In Table A4 we report the full correlation table among our output TFP estimates, Θ^k, and those reported in the literature. Ours = our estimates for Θ^k; HJ98 = Hall and Jones (1998) TFP (A); KRC97 = Klenow and Rodriguez-Clare (1997); EK96 = Eaton and Kortum (1996); HR97 = Harrigan (1997); PWT_90 = Penn World Tables 8.0, current PPP, year 1990; PWT_00 = PWT 8.0, current PPP, 2000; EK 02 = Eaton and Kortum (2002). The correlation coefficients between our Θ^k and the literature's estimates, reported in the first column of Table A4 and in boldface, are comparable to those among the literature's estimates, reported in the rest of Table A4.

Figure 1 Test Score and Educational Spending Per Capita

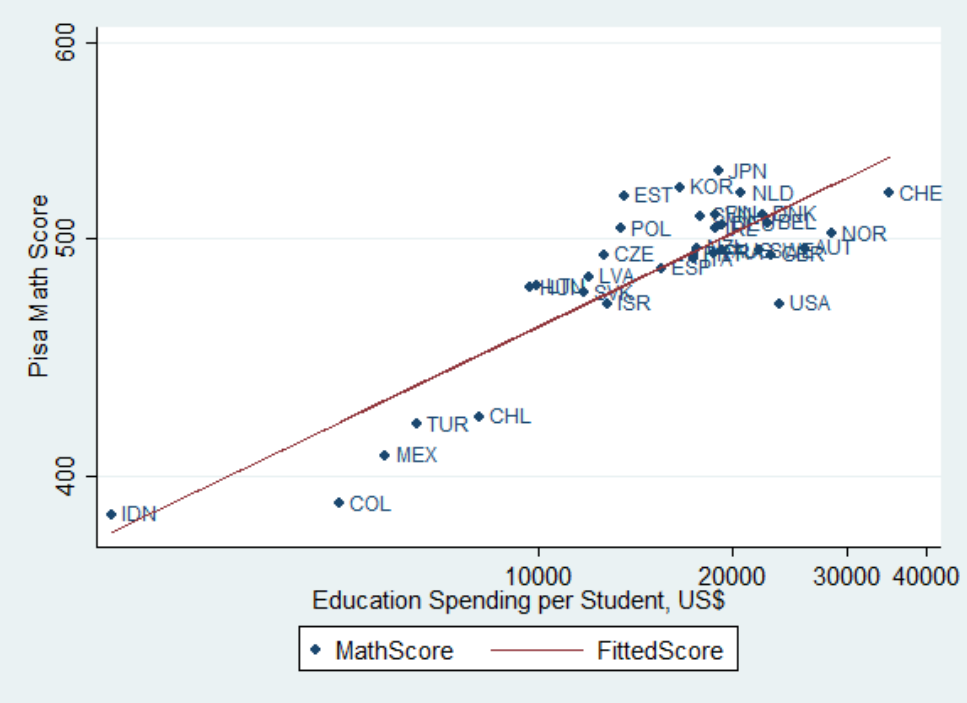


Figure 2 Normalized Test Scores and Cognitive Employment Shares

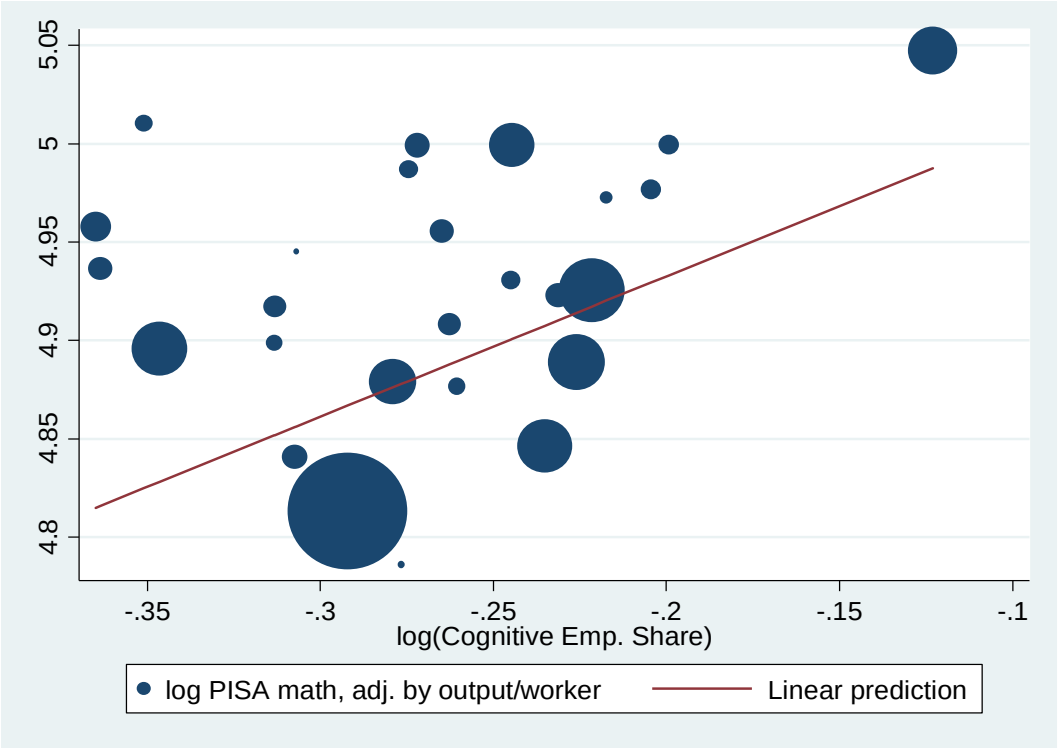


Figure 3 Cognitive-Productivity Ranking vs. PISA-Math Ranking

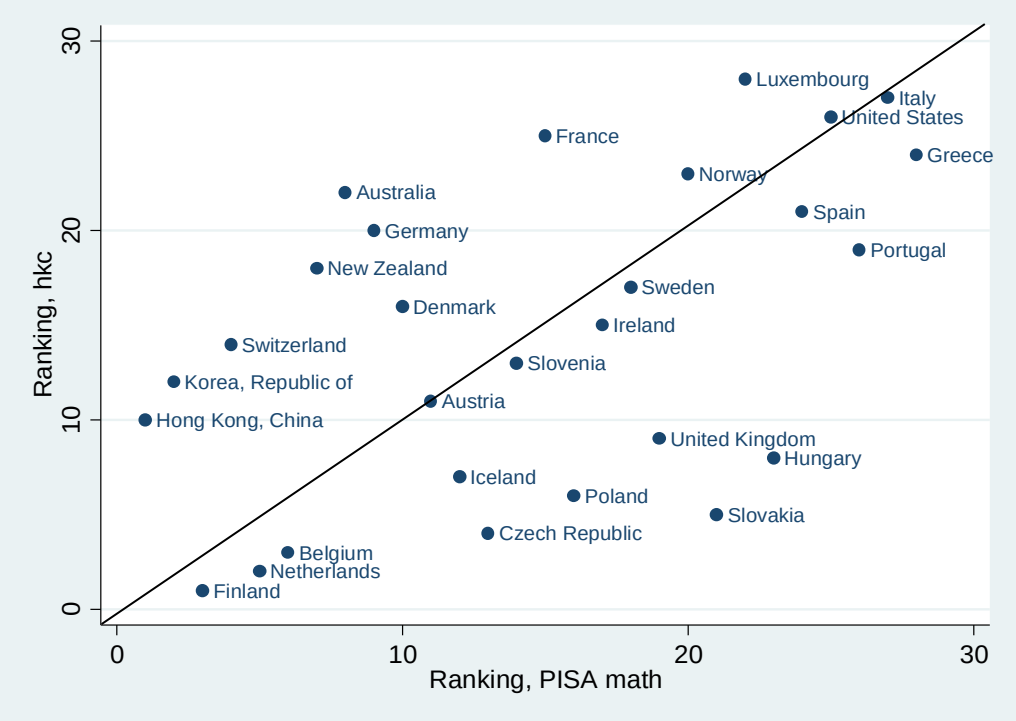


Figure 4 Non-Cognitive-Productivity Ranking vs. PISA-Math Ranking

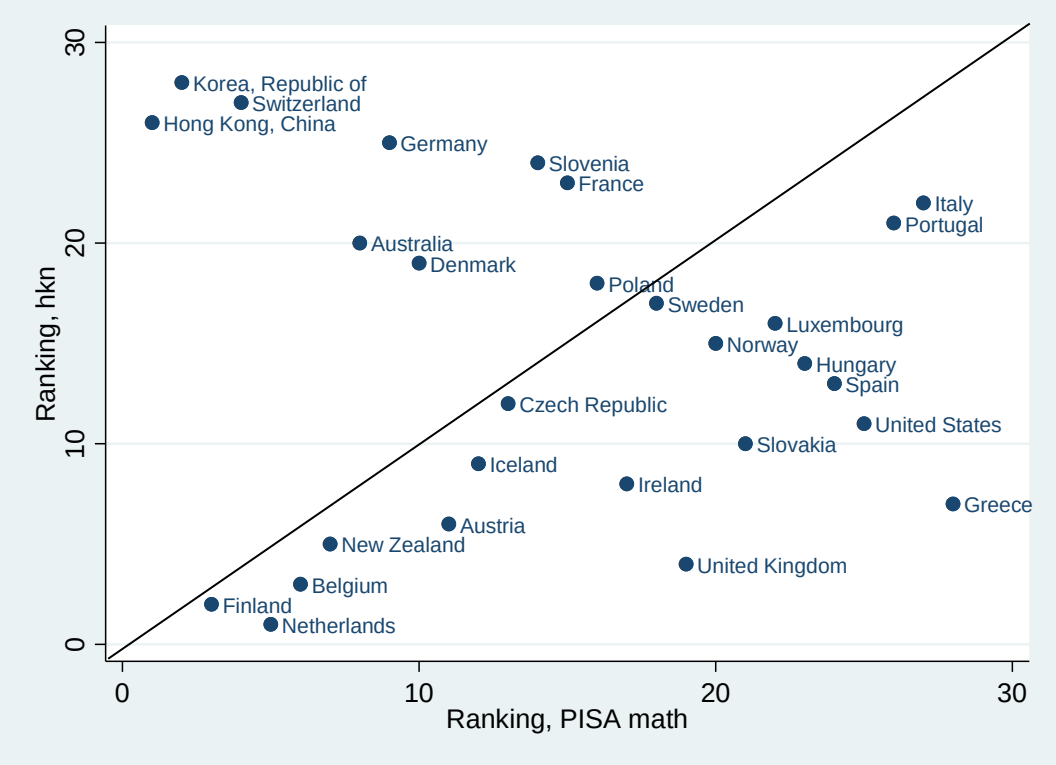


Figure 5 Overall Education Quality

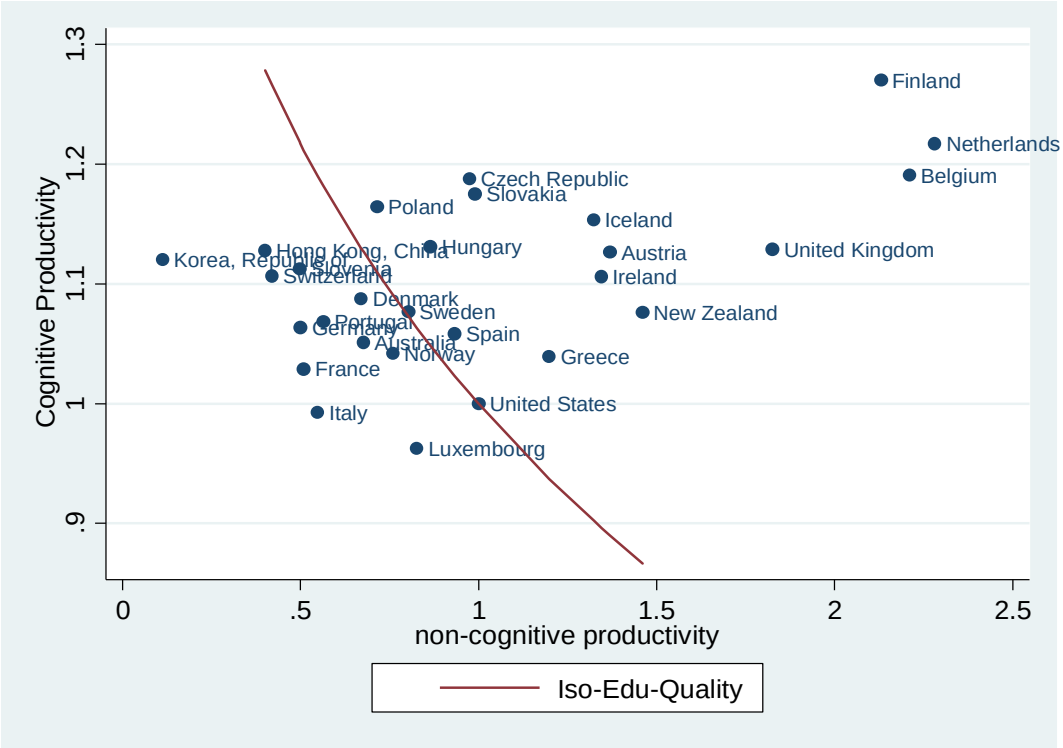


Figure 6 Overall Education Quality: Extended Sample

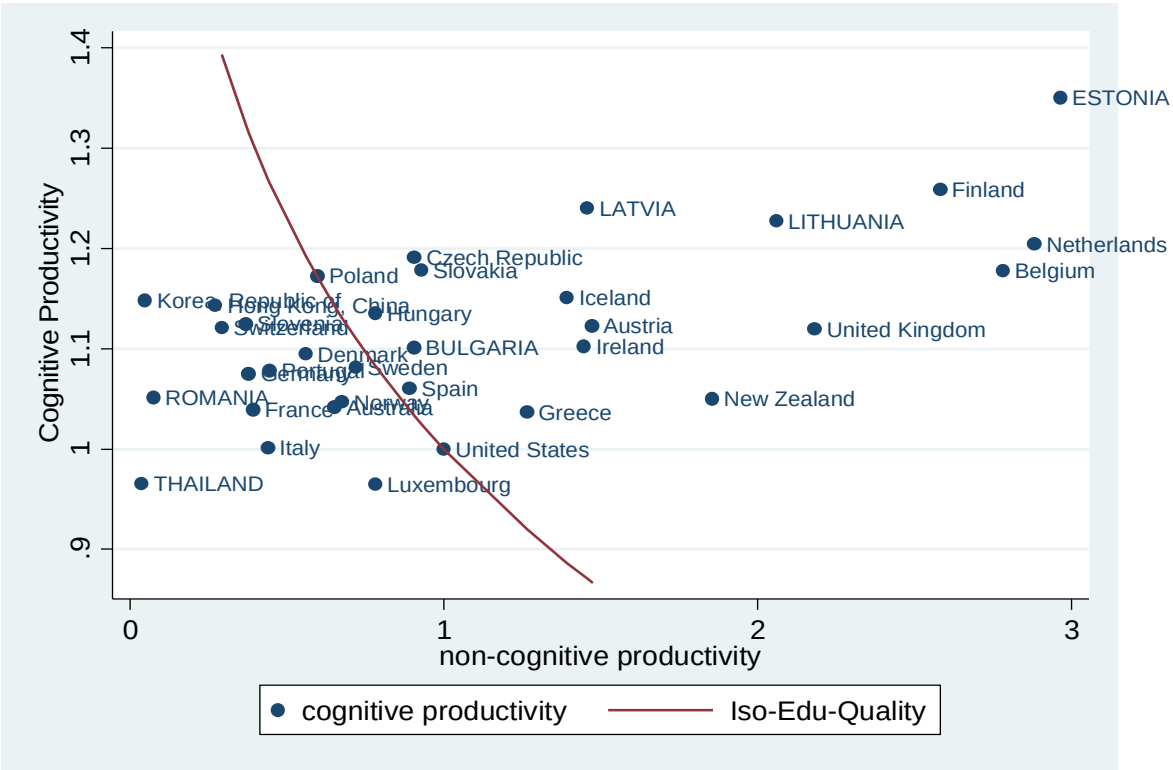


Figure 7 Output Gains, Autarky to Free Trade, Sample Countries

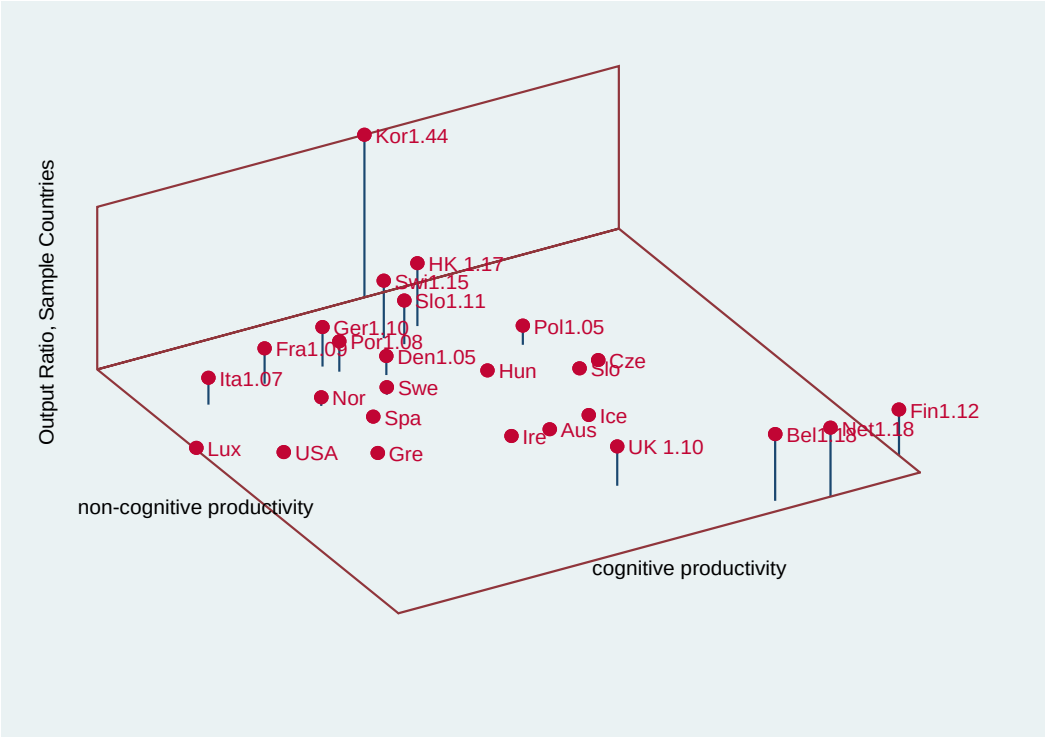


Figure 8 Overall Education Quality: Free-trade Counterfactual

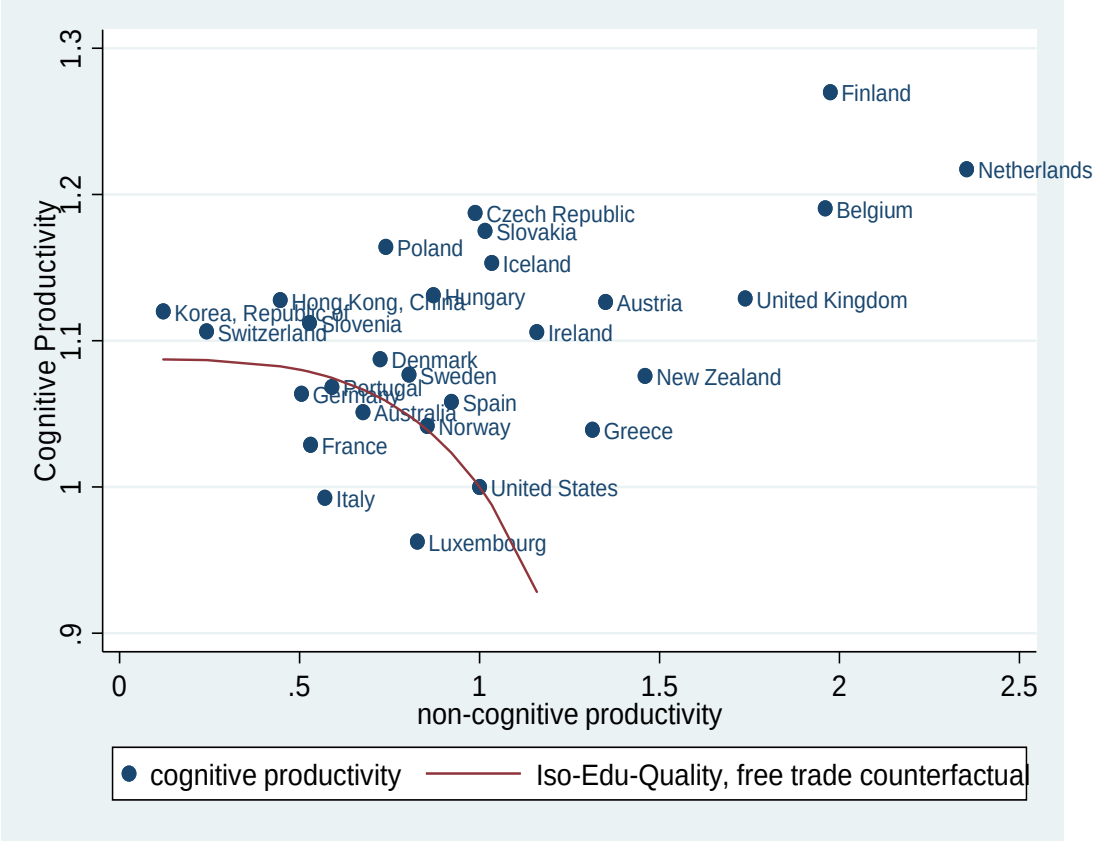


Figure 9 Output Gains, Autarky to Free Trade, Adding China & Japan

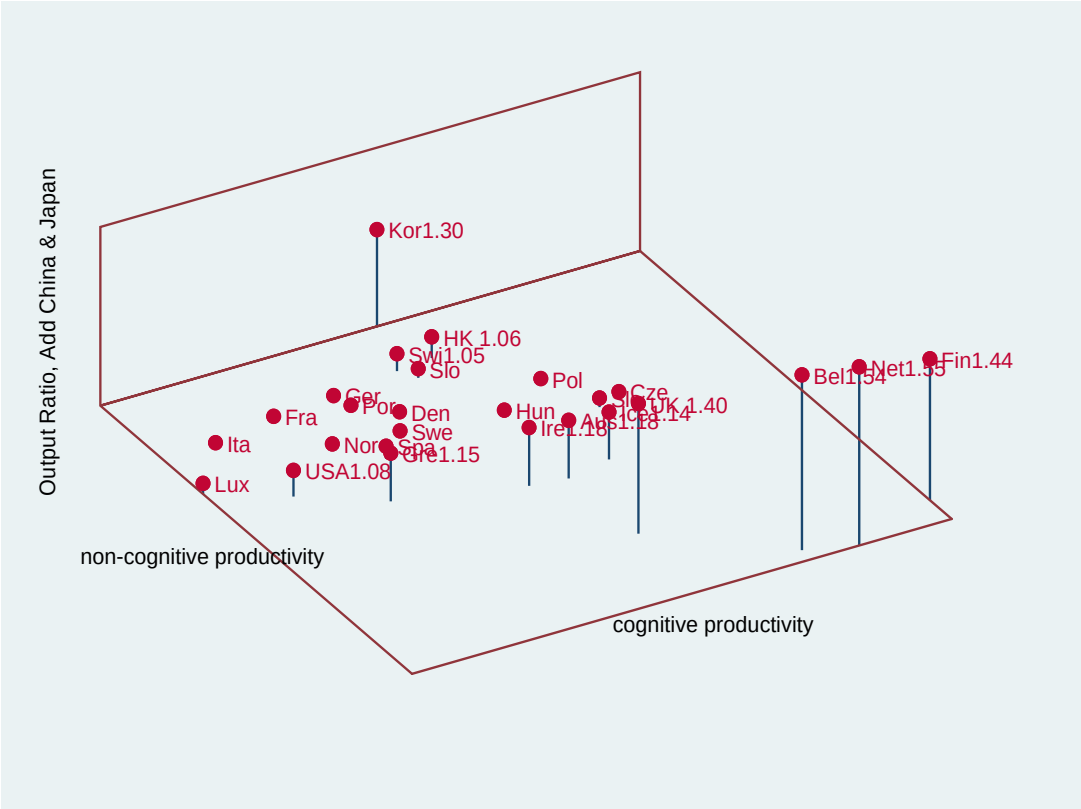


Table 1 Test Score and Wages of Non-cognitive and Cognitive Occupations

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Replicate	Non-Cog. SubSample	Cog. SubSample	Interaction	Alt. Leadership
Black	-0.0537***	-0.0937**	-0.0381*	-0.0661***	-0.0641***
	(0.0196)	(0.0365)	(0.0228)	(0.0191)	(0.0192)
Hispanics	0.0425**	0.0164	0.0482*	0.0413**	0.0414**
	(0.0211)	(0.0378)	(0.0251)	(0.0206)	(0.0206)
Age	0.0349***	0.0483***	0.0285***	0.0323***	0.0316***
	(0.00708)	(0.0129)	(0.00833)	(0.00689)	(0.00690)
Non-cog. Occp.				0.121***	0.127***
				(0.0163)	(0.0186)
College				0.187***	0.195***
				(0.0264)	(0.0263)
AFQT	0.183***	0.157***	0.183***	0.137***	0.125***
	(0.00964)	(0.0182)	(0.0113)	(0.0115)	(0.0113)
AFQT2	-0.0130	-0.0199	-0.00717	-0.0369***	-0.0358***
	(0.00802)	(0.0143)	(0.00961)	(0.00950)	(0.00956)
AFQT x Non-Cog.				-0.0345**	-0.00749
				(0.0159)	(0.0182)
AFQT x College				0.0525**	0.0495**
				(0.0245)	(0.0244)
Constant	6.233***	6.148***	6.281***	6.218***	6.232***
	(0.112)	(0.205)	(0.132)	(0.109)	(0.109)
Obs. No.	3,210	951	2,259	3,210	3,210
R2	0.168	0.151	0.163	0.214	0.211

Notes: The dependent variable is log wage, and the sample is NLSY 79. Standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1.

Table 2 Summary Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Labor Force Size	28	12541.24	23132.62	156.43	120464.70
Non-cog. Emp. Share	28	0.2425	0.0514	0.1157	0.3775
Cognitive Emp. Share	28	0.7575	0.0514	0.6225	0.8843
Agg. Output ($000)	28	4.59E+08	1.18E+09	4130208	6.25E+09
Edu. Exp./Output	20	0.1255	0.0194	0.0985	0.1695
PISA Reading Score	28	498.96	18.30	468.93	539.34
PISA Math Score	28	503.73	22.17	455.80	553.40
PISA Science Score	28	506.81	19.70	470.07	554.28
$\lvert x_c^k \rvert$	28	0.0194	0.0156	0.0001	0.0626
$\lvert x_n^k \rvert$	28	0.0357	0.0348	0.0011	0.1396

Table 3 Sample Countries, Years and Rankings

Country	Year	Cog-Prod Rank	PISA Math Rank	Non-Cog Prod Rank
Australia	2000	22	8	20
Austria	2000	11	11	6
Belgium	2000	3	6	3
Czech Republic	2000	4	13	12
Denmark	2000	16	10	19
Finland	2000	1	3	2
France	2000	25	15	23
Germany	2000	20	9	25
Greece	2000	24	28	7
Hong Kong	2001	10	1	26
Hungary	2000	8	23	14
Iceland	2000	7	12	9
Ireland	2000	15	17	8
Italy	2000	27	27	22
S. Korea	2000	12	2	28
Luxembourg	2000	28	22	16
Netherlands	2000	2	5	1
New Zealand	1996	18	7	5
Norway	2000	23	20	15
Poland	2000	6	16	18
Portugal	2000	19	26	21
Slovakia	2000	5	21	10
Slovenia	2000	13	14	24
Spain	2000	21	24	13
Sweden	2000	17	18	17
Switzerland	1990	14	4	27
United Kingdom	2000	9	19	4
United States	2000	26	25	11

Table 4 Patterns of Trade

	Dep. Var. = net exp./(imp.+exp.)		
	(1)	(2)	(3)
Non-cog abundance x non-cog intensity	15.989	15.979	10.615
	(2.92)	(2.92)	(2.02)
Cap abundance x cap intensity		0.000	0.000
		(0.10)	(0.22)
Skill abundance x skill intensity			9.173
			(4.71)
constant	-1.108	-1.113	1.976
	(-3.30)	(-3.28)	(2.77)
industry FE	yes	yes	yes
country FE	yes	yes	yes
R^2	0.369	0.369	0.401
# obs.	1103	1103	1103

Table 5 Value of θ

Dependent Variable = normalized test score, equation (30)

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)
lnp_c^k	0.717***	0.714***	0.696***	0.521***	0.512**	0.677*	0.565**
	(0.230)	(0.224)	(0.223)	(0.165)	(0.201)	(0.357)	(0.198)
ASNZ		0.213**	0.208***	0.189***	0.189**	0.175**	0.210**
		(0.0773)	(0.0574)	(0.0570)	(0.0695)	(0.0842)	(0.0682)
Constant	5.076***	5.075***	5.072***	5.032***	5.040***	5.032***	5.020**
	(0.0624)	(0.0607)	(0.0608)	(0.0448)	(0.0546)	(0.0784)	(0.0547)
Observations	26	28	28	28	28	28	34
R^2	0.288	0.347	0.393	0.384	0.292	0.196	0.302

Notes: ASNZ is the dummy for Australia and New Zealand, whose raw occupation-employment data are in different classification codes as compared with the other countries in our sample.

Table 6 Value of α

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\ln(1+\frac{p_n^k - nx_n^k}{p_c^k - nx_c^k})$	2.802**	2.784**	2.746**	2.616**	2.605**	3.418*	3.125**	3.872**
	(1.191)	(1.224)	(1.171)	(1.140)	(1.177)	(1.840)	(1.224)	(1.100)
ASNZ	-1.074**		-1.080***	-1.051**	-1.051**	-0.976**	-1.094**	-1.246***
	(0.425)		(0.311)	(0.407)	(0.420)	(0.434)	(0.423)	(0.379)
Constant	3.548***	3.553***	3.564***	3.590***	3.583***	3.554***	3.465***	3.004***
	(0.324)	(0.332)	(0.320)	(0.310)	(0.320)	(0.404)	(0.332)	(0.274)
Obs. No.	28	26	28	28	28	28	28	34
R2	0.263	0.177	0.342	0.264	0.251	0.209	0.282	0.363

Notes: ASNZ is the dummy for Australia and New Zealand, whose raw occupation-employment data are in different classification codes as compared with the other countries in our sample.

Table 7 Summary of Parameter Values and Identification

Parameters	Intuition	Values	Identification
η	Elasticity in Human Cap Prod	0.1255	Edu. spending as share of output, (32)
θ	Dispersion of Innate Ability	2.0877~3.4965	Strength of selection effect, (33)
α	Sub Elasticity in Agg Production	1.4706~1.5549	Agg. production function, (34)
Θ^k	Output TFP	Table 8	Same as α, (34)
h_c^k	TFP, Cog. Human Cap.	Figures 3, 5, 6 & 8	Normalized test score and cog. emp. share (33)
h_n^k	TFP, Non-cog. Human Cap.	Figures 4, 5, 6 & 8	Revealed comp advantage by relative emp. share and trade (28)

Table 8 Contributions of Overall Education Quality to Output per Worker

		Trade Cost		Closed Econonomy	
	(1)	(2)	(3)	(4)	(5)
Countries	Output Per Worker	Contribution of Output TFP	Contribution of Overall Edu Quality	Contribution of Output TFP	Contribution of Overall Edu Quality
Austria	0.6434	0.5645	1.1399	0.5297	1.2147
Belgium	0.6892	0.5301	1.3001	0.4636	1.4867
Czech Republic	0.3293	0.2982	1.1045	0.2860	1.1513
Denmark	0.5979	0.7037	0.8496	0.6187	0.9664
Finland	0.5037	0.3326	1.5145	0.3259	1.5458
France	0.7329	0.8720	0.8405	0.8517	0.8606
Germany	0.6296	0.7088	0.8883	0.7126	0.8834
Greece	0.5190	0.5144	1.0089	0.4761	1.0901
Hong Kong	0.6864	0.7840	0.8755	0.7724	0.8887
Hungary	0.3517	0.3375	1.0419	0.3292	1.0684
Iceland	0.5110	0.5139	0.9943	0.4168	1.2261
Ireland	0.6642	0.5136	1.2930	0.5583	1.1896
Italy	0.6761	0.7982	0.8470	0.7977	0.8476
S. Korea	0.4304	0.6267	0.6868	0.6027	0.7142
Luxembourg	1.4376	1.5674	0.9172	1.5674	0.9172
Netherlands	0.6712	0.4624	1.4515	0.4387	1.5300
Norway	0.7289	0.8704	0.8374	0.7589	0.9605
Poland	0.3045	0.2979	1.0219	0.2917	1.0438
Portugal	0.3845	0.4195	0.9165	0.4216	0.9121
Slovakia	0.2979	0.2732	1.0901	0.2600	1.1459
Slovenia	0.3929	0.4406	0.8918	0.4275	0.9191
Spain	0.6087	0.5979	1.0180	0.5913	1.0293
Sweden	0.5937	0.6389	0.9292	0.5917	1.0034

Table 8 Continued

		Trade Cost		Closed Econonomy	
	(1)	(2)	(3)	(4)	(5)
Countries	Output Per Worker	Contribution of Output TFP	Contribution of Overall Edu Quality	Contribution of Output TFP	Contribution of Overall Edu Quality
Switzerland	0.5855	0.9880	0.5926	0.6641	0.8816
United Kingdom	0.6349	0.5040	1.2597	0.4758	1.3345
United States	1.0000	1.0000	1.0000	1.0000	1.0000

Notes: Columns (4) and (5) are obtained using equations (15) and (16), and columns (6) and (7) obtained using equation (28).

Table 9 Robustness Exercises

	Values		Correlation Coefficients			
	θ	α	Cog Productivity	Non-cog Productivity	Overall Edu Quality	Output TFP
Benchmark	3.4907	1.5549	1.0000	1.0000	1.0000	1.0000
Closed-economy	3.4907	1.4706	1.0000	0.9948	0.9675	0.9940
Alternative θ value	2.0887	1.5549	0.9844	0.9983	1.0000	1.0000
Alternative θ & α values	2.0887	2.0000	0.9844	0.9884	0.9778	0.9923
Mid. Income Countries	2.2975	1.3481	0.9926	0.9942	0.9985	0.9957

Notes: This table reports the correlation coefficients between the values and country rankings of cognitive productivity, non-cognitive productivity, and overall educational quality under our main specification and under alternative parameter values and settings.

Table 10 Change in Overall Education Quality and Gains from Trade

	Overall Edu. Quality			Gains From Trade	
Countries	Closed-Economy (1)	Free-trade Counterfactual (2)	Change (3) = (2)/(1)	Sample Countries (4)	Adding China & Japan (5)
Austria	1.2147	1.2358	1.0174	1.0175	1.1810
Belgium	1.4867	1.7565	1.1814	1.1816	1.5453
Czech Republic	1.1513	1.1618	1.0091	1.0092	1.0232
Denmark	0.9664	1.0161	1.0514	1.0516	1.0003
Finland	1.5458	1.7387	1.1248	1.1250	1.4401
France	0.8606	0.9432	1.0960	1.0962	1.0158
Germany	0.8834	0.9783	1.1074	1.1075	1.0221
Greece	1.0901	1.1006	1.0096	1.0098	1.1502
Hong Kong	0.8887	1.0405	1.1709	1.1710	1.0662
Hungary	1.0684	1.0867	1.0172	1.0173	1.0114
Iceland	1.2261	1.2371	1.0090	1.0092	1.1474
Ireland	1.1896	1.2104	1.0175	1.0176	1.1816
Italy	0.8476	0.9094	1.0730	1.0732	1.0055
S. Korea	0.7142	1.0292	1.4411	1.4413	1.3012
Luxembourg	0.9172	0.9219	1.0051	1.0052	1.0332
Netherlands	1.5300	1.8167	1.1873	1.1875	1.5558
Norway	0.9605	0.9830	1.0235	1.0236	1.0061
Poland	1.0438	1.0982	1.0522	1.0523	1.0004
Portugal	0.9121	0.9872	1.0824	1.0825	1.0092
Slovakia	1.1459	1.1535	1.0067	1.0068	1.0286
Slovenia	0.9191	1.0280	1.1185	1.1186	1.0289
Spain	1.0293	1.0328	1.0033	1.0034	1.0397
Sweden	1.0034	1.0238	1.0204	1.0205	1.0084
Switzerland	0.8816	1.0188	1.1557	1.1558	1.0547
United Kingdom	1.3345	1.4772	1.1070	1.1071	1.4046
United States	1.0000	1.0000	1.0000	1.0001	1.0815

Notes: XXXX

Figure A1 Non-Cognitive Employment Share Over Time for Select Countries

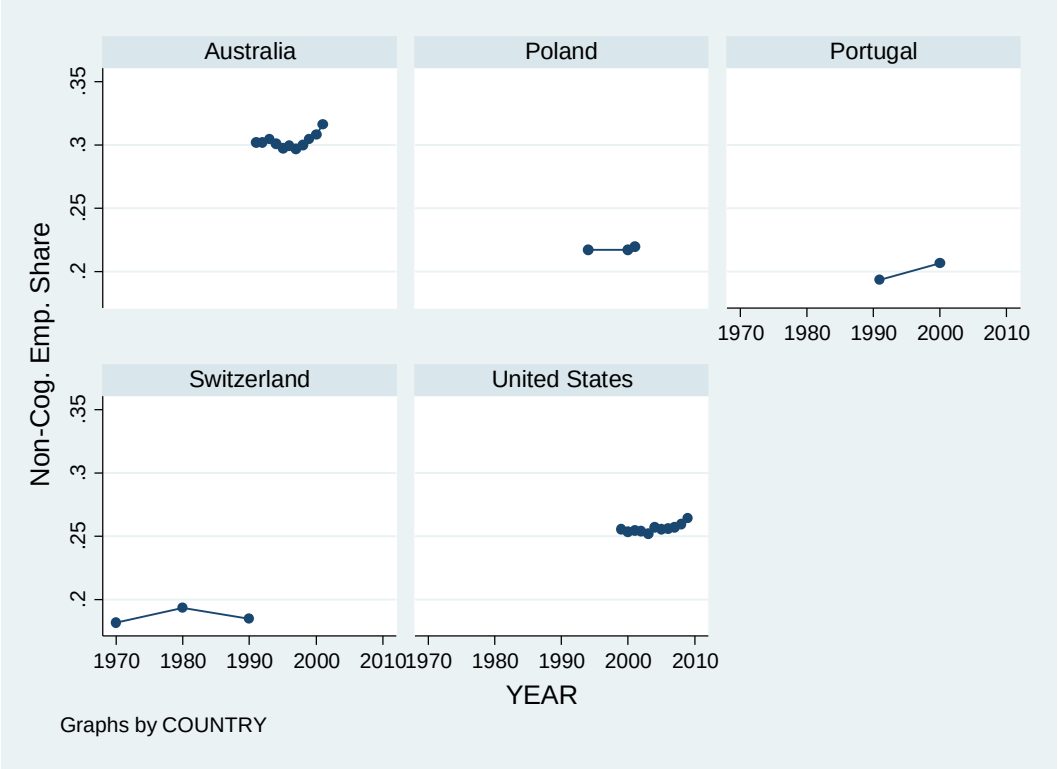


Table A1 Neal-Johnson Regressions for Alternative Measures of Non-Cognitive Skills

VARIABLES	Originality	Not Originality	Social-skill	Not Social-skill	Artistic	Not Artistic	Investigative	Not Investigative
Black	-0.0735*	-0.0463**	0.0238	-0.0515**	-1.490*	-0.0533***	0.010	-0.060***
	(0.0395)	(0.0216)	(0.0683)	(0.0202)	(0.799)	(0.0195)	(0.091)	(0.02)
Hispanics	0.0380	0.0398*	0.119	0.0364*	-0.586*	0.0422**	0.036	0.039*
	(0.0402)	(0.0240)	(0.0788)	(0.0215)	(0.331)	(0.0212)	(0.092)	(0.022)
Age	0.0569***	0.0220***	0.0557**	0.0325***	0.0752	0.0345***	0.027	0.036***
	(0.0136)	(0.00798)	(0.0254)	(0.00722)	(0.0844)	(0.00710)	(0.030)	(0.007)
AFQT	0.182***	0.154***	0.204***	0.185***	-0.713**	0.184***	0.188***	0.171***
	(0.0210)	(0.0109)	(0.0370)	(0.00979)	(0.333)	(0.00965)	(0.060)	(0.010)
$AFQT^2$	0.00428	-0.0382***	-0.00483	-0.0172**	0.299*	-0.0120	-0.043	-0.019**
	(0.0149)	(0.00996)	(0.0341)	(0.00807)	(0.150)	(0.00809)	(0.032)	(0.008)
Constant	5.942***	6.414***	5.732***	6.292***	6.061***	6.239***	6.642***	6.212***
	(0.216)	(0.126)	(0.403)	(0.114)	(1.357)	(0.112)	(0.481)	(0.114)
Obs. No.	1,096	2,114	382	2,828	30	3,180	158	3052
R^2	0.164	0.126	0.127	0.181	0.188	0.170	0.106	0.148

Table A2 Within-Country, Over-Time Variations of PISA Scores

	Mean			Std. Dev.		
Country	Math	Reading	Science	Math	Reading	Science
Australia	515.67	518.66	525.21	8.69	7.63	3.23
Austria	505.54	490.64	508.31	0.07	1.05	3.58
Belgium	519.86	505.92	507.27	6.80	2.96	2.81
Canada	526.10	527.32	529.54	5.96	4.37	4.57
Czech Republic	504.52	486.78	507.22	10.63	6.20	6.25
Denmark	507.66	494.94	497.90	7.08	1.75	1.79
Estonia	515.74	506.00	533.54	4.34	8.91	7.04
Finland	537.98	539.34	554.28	13.21	9.63	8.94
France	499.53	497.95	497.47	7.55	7.35	1.99
Germany	508.27	495.06	520.06	5.66	8.67	4.25
Greece	455.80	473.15	470.07	9.02	8.53	3.33
Hungary	487.04	485.37	500.29	6.68	5.86	5.23
Iceland	505.03	493.19	488.18	9.20	10.38	9.01
Ireland	498.24	515.65	512.77	7.43	12.05	8.00
Italy	473.90	481.49	485.92	11.95	9.04	9.41
Japan	530.66	515.24	539.19	5.92	17.19	7.68
S. Korea	547.42	537.98	532.64	4.78	11.42	9.09
Luxembourg	490.53	479.69	487.16	1.83	6.39	3.72
Netherlands	529.32	509.88	523.05	6.49	2.85	1.57
New Zealand	516.13	520.89	526.01	11.06	5.89	9.02
Norway	493.09	499.30	493.64	4.18	8.63	6.72
Poland	499.49	500.41	510.57	12.23	14.46	14.17
Portugal	476.53	479.42	485.51	12.06	8.77	9.87
Slovak Republic	492.15	468.93	483.30	7.47	6.25	10.52
Slovenia	502.35	486.27	514.91	1.83	7.10	3.59
Spain	483.22	480.58	491.04	2.27	12.12	4.69
Sweden	495.98	503.74	494.41	13.27	13.58	9.28
Switzerland	530.28	500.46	514.46	3.07	5.34	2.63
United Kingdom	493.93	496.19	514.20	1.52	2.74	0.53
United States	481.50	499.26	496.11	5.41	3.93	6.64

Table A3 Correlation between 2012 PISA and 2013 PIAAC scores

	PISA Reading	PISA Math
PIAAC Literacy	0.938	
	(5.18)	
PIAAC Numeracy		1.067
		(5.38)
Constant	249.047	215.948
	(5.13)	(4.13)
Obs. No.	28	28
R^2	0.508	0.527

Table A4 Correlation Coefficients for Output TFP Estimates

	Ours	HJ98	KRC97	EK 96	HRG95	PWT_90	PWT_00
Ours	1						
HJ98	**0.5549**	1					
	0.0033						
KRC97	**0.4466**	0.8412	1				
	0.0424	0					
EK 96	**0.5858**	0.5348	0.7109	1			
	0.0171	0.0328	0.002				
HRG95	**0.6298**	0.5841	0.5394	0.068	1		
	0.0942	0.1284	0.1677	0.8729			
PWT_90	**0.6192**	0.8792	0.7401	0.6976	0.6126	1	
	0.0004	0	0.0001	0.0027	0.1064		
PWT_00	**0.6687**	0.6878	0.2565	0.3856	-0.5382	0.7089	1
	0.0001	0.0001	0.2617	0.1402	0.1688	0	
EK 02	**0.6222**	0.4159	0.4828	0.7655	0.3538	0.6114	0.4646
	0.0077	0.0968	0.0496	0.0009	0.3899	0.0091	0.0602

Notes: Ours = our estimates for Θ^k; HJ98 = Hall and Jones (1998) TFP (A); KRC97 = Klenow and Rodriguez-Clare (1997); EK96 = Eaton and Kortum (1996); HR97 = Harrigan (1997); PWT_90 = Penn World Tables 8.0, current PPP, year 1990; PWT_00 = PWT 8.0, current PPP, 2000; EK 02 = Eaton and Kortum (2002).