Asset Pricing with Return Extrapolation*

Lawrence J. Jin and Pengfei Sui†

January 12, 2018

ABSTRACT

We present a new model of return extrapolation. It is a Lucas-type general equilibrium model, in which the agent has Epstein-Zin preferences and extrapolative beliefs. Unlike earlier return extrapolation models, this model can be brought to the data quantitatively, allowing for a direct comparison with rational expectations models. Moreover, it matches important facts about both aggregate asset prices and extrapolative expectations in surveys. Consistent with recent empirical findings, the model highlights an important source of stock market movements: changes in subjective expectations about future dividend growth explain most of the variation of the price-dividend ratio.

JEL classification: G02, G12
Keywords: Expectations, Return Extrapolation, Stock Market Movements

*We thank Nicholas Barberis, John Campbell, Stefano Cassella, Michael Ewens, Robin Greenwood, David Hirshleifer, Philip Hoffman, Jonathan Ingersoll, Theresa Kuchler, Jiacui Li, Cameron Peng, Paulo Rodrigues, Paul Sangrey, Andrei Shleifer, and seminar participants at Caltech, Maastricht University, Tilburg, University of California, Irvine, the Young Economists Symposium at Yale, the Caltech Junior Faculty Behavioral Finance Conference, and the NBER Behavioral Finance meeting for helpful comments. Please send correspondence to Lawrence J. Jin, California Institute of Technology, 1200 E. California Blvd. MC 228-77, Pasadena, CA, 91125; telephone: 626-395-4558. E-mail: lawrence.jin@caltech.edu.

†Both authors' affiliation is the California Institute of Technology.

In behavioral finance, there is growing interest in "return extrapolation," the idea that investors' beliefs about an asset's future returns are a positive function of the asset's recent past returns. Models with return extrapolation have two appealing features. First, they are consistent with survey evidence on the beliefs of real-world investors.[1] Second, they show promise in matching important asset pricing facts, such as volatility and predictability in the aggregate market, momentum and reversals in the cross-section, and bubbles (Barberis, Greenwood, Jin, and Shleifer (2015, 2017); Hong and Stein (1999)).

One issue with existing models of return extrapolation, however, is that they can only be compared to the data qualitatively. Earlier models, such as Cutler, Poterba, and Summers (1990) and DeLong, Shleifer, Summers, and Waldmann (1990), highlight the conceptual importance of return extrapolation, but they are not dynamic consumption-based models and not designed to match asset pricing facts quantitatively. Barberis et al. (2015) is a dynamic model that tries to make sense of both survey expectations and aggregate stock market prices. However, the simplifying assumptions in the model make it difficult to evaluate the model's fit with the empirical facts. For instance, their model adopts a framework with constant absolute risk aversion (CARA) preferences and a constant interest rate. Given these assumptions, many ratio-based quantities that we study in asset pricing (e.g., the price-dividend ratio) do not have well-defined distributions in the model and therefore do not have properties that match what we observe in the data.

In this paper, we overcome this weakness by proposing a new model of aggregate stock market prices based on return extrapolation. It is a representative-agent Lucas-type general equilibrium model with Epstein-Zin preferences. The goal of the paper is to match important facts about both aggregate stock market prices and survey expectations, and to allow for a direct quantitative comparison with rational expectations models of the stock market.

The model's main mechanism stems from the interaction between return extrapolation and Epstein-Zin preferences. Suppose the stock market has had high past returns. In such a case, return extrapolation leads investors to forecast high future returns. Under Epstein-Zin preferences, the separation between the elasticity of intertemporal substitution and risk aversion gives rise to a strong intertemporal substitution effect. As a result, the agent's forecast of high future returns leads her to push up current prices significantly, generating excess volatility. A feedback loop emerges from this mechanism. If current returns are high, that makes the agent think that future returns will also be high, which leads her to push up prices, increasing current returns further, and so on. In general, there is a danger that this feedback loop could explode. Nonetheless, the agent's risk aversion serves as a caution against heightened demand for the stock market. Furthermore, in the model, the agent believes in mean reversion of her expectations about stock market returns; she knows her optimism will decline in the future. Taken together, these two forces prevent price

[1] Among others, Vissing-Jorgensen (2004), Bacchetta, Mertens, and van Wincoop (2009), Amromin and Sharpe (2013), Greenwood and Shleifer (2014), Koijen, Schmeling, and Vrugt (2015), and Kuchler and Zafar (2016) document that many individual and institutional investors have extrapolative expectations: they believe that the stock market will continue rising in value after a sequence of high past returns, and that it will continue falling in value after a sequence of low past returns.

explosion; the model remains stable.[2]

One important feature of the model is that for less than half of the time, the agent's beliefs about future stock market returns are overly optimistic; for more than half of the time, her beliefs are overly pessimistic. When the agent is optimistic about future returns, her risk aversion counteracts her beliefs and therefore cushions against the subsequent decline of the stock market price, making the agent less likely to be overly optimistic. On the other hand, when the agent is pessimistic about future returns, her risk aversion strengthens her beliefs and amplifies the subsequent increase of the stock market price, making the agent more likely to be overly pessimistic. This asymmetry in the interaction between the agent's beliefs and risk aversion implies that the true equity premium is more likely to be *higher* than the perceived equity premium. That is, the true long-run equity premium is above the perceived long-run equity premium. This feature helps the model generate a high equity premium even when the elasticity of intertemporal substitution is significantly lower than one, a result that the long-run risks models cannot deliver.

Quantitatively, the model fits the long-run properties of stock market prices and returns: it generates a high equity premium, significant excess volatility of stock market returns, a Sharpe ratio similar to the empirical value, an interest rate that has a low level and low volatility, and a price-dividend ratio whose average level is close to the empirical one. At the same time, the model matches the dynamic behavior of stock market prices: it produces the long-horizon predictability of stock market returns that we observe in the data, and generates a low correlation between stock market returns and consumption growth. By using realized returns and dividend growth in a Campbell-Shiller decomposition, the model replicates the empirical finding of the volatility test literature that the variation of the price-dividend ratio comes primarily from discount rate variation.

Furthermore, the model matches important facts about the aggregate stock market by fitting the agent's beliefs to the extrapolative expectations documented in surveys. When the agent's expectations about future returns are regressed on past twelve-month returns, the model produces a regression coefficient and a t-statistic that match the empirical estimates from surveys. In addition, the agent's beliefs are calibrated to match the survey evidence on the relative weights investors put on recent versus distant return realizations when forming beliefs about future returns. Nonetheless, this calibration exercise also points to some challenges: when calibrated with the survey expectations data, the model predicts a persistence of the price-dividend ratio that is much lower than its empirical value. In other words, to match the empirical persistence of the price-dividend ratio, investors need to form beliefs about future returns based on many years of past returns. However, the available survey evidence suggests that they focus on just the past year or two.

Finally, the model yields some novel implications. The model highlights an "expectations transmission mechanism" that is generally overlooked in the literature. When past price growth of the stock market has been high, the agent's beliefs about future dividend growth are positively affected

[2]On the contrary, models like Cutler et al. (1990) and Barberis et al. (2015) require introducing fully rational investors in order to counteract the behavioral investors and preserve equilibrium.

by her optimistic beliefs about future price growth; they rise at a pace that exceeds the agent's expectations about future price growth.[3] This mechanism posits an important source of stock market movements. By using the agent's *subjective* expectations about stock market returns and dividend growth in a Campbell-Shiller decomposition, the model shows that changes in subjective expectations about future dividend growth—rather than changes in subjective expectations about future returns—explain most of the volatility of the price-dividend ratio. These model implications are consistent with the recent empirical findings of de la O and Myers (2017): they find that during periods when the price-dividend ratio of the U.S. stock market is high, expectations of aggregate dividend growth are much higher than the expectations of future stock market returns. As a result, changes in investors' subjective expectations of future dividend growth explain the majority of stock market movements.

Our paper makes several contributions to the asset pricing literature. First, we present a new return extrapolation model that allows us to make quantitative comparisons with the data. As with the habit formation model of Campbell and Cochrane (1999), the long-run risks models of Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012), and the rare disasters models of Barro (2006), Gabaix (2012), Wachter (2013), our model is developed in a Lucas economy with a representative agent. This model structure allows for a direct comparison between our model and models with rational expectations. Consistent with survey evidence, the agent in our model expects high stock market returns after a sequence of high past returns. By contrast, models with rational expectations imply that the agent would expect *lower* or flat returns in the future. Therefore, our model, unlike the rational expectations models, not only matches the behavior of aggregate market asset prices, it can also fit the survey evidence on investor beliefs.

Second, our paper stresses the importance of expectations data in understanding aggregate stock market behavior. Both the agent's expectations about future dividend growth and her expectations about future returns vary positively with the price-dividend ratio, with the former explaining most of the variation in the price-dividend ratio. This result contrasts with the traditional view that variation in cash flow news does not explain stock market movements and therefore suggests that investors' subjective cash flow expectations contain useful information about the stock market.

Third, to highlight the importance of the feedback loop described above in matching asset pricing facts, we further compare our model to a model with *fundamental* extrapolation, the notion that some investors hold extrapolative expectations about future dividend growth of the stock market. With fundamental extrapolation, high past dividend growth generates optimistic beliefs about future dividend growth and hence pushes up current prices, but higher prices do not further affect the agent's beliefs about future dividends: the feedback loop is absent. As a result, for the same parameter values, the fundamental extrapolation model generates a much lower average equity premium. This feedback loop also points to a methodological contribution of the paper. While most belief-based models specify belief formation exogenously, the feedback loop in return extrapolation models involves endogenous belief formation. Our differential equations approach

[3] We provide a detailed explanation of this finding in Sections I and II.

provides a solution to this greater modelling challenge.

Fourth, to the best of our knowledge, our model is the first belief-based behavioral asset pricing theory that analyzes how investors separately form beliefs about dividend growth of the stock market and beliefs about aggregate consumption growth. We assume that the bias in the agent's beliefs about consumption growth only derives from the bias in her beliefs about dividend growth. Given the low correlation observed in the data between consumption growth and dividend growth, the bias in the agent's beliefs about consumption growth is small. This assumption is consistent with the lack of empirical evidence on consumption extrapolation. The agent's approximately correct beliefs about consumption growth allow the model to generate low interest rate volatility and a low correlation between consumption growth and stock market returns.

Finally, our belief-based model allows for different time-series properties of quantities objectively measured by outside econometricians and quantities subjectively perceived by investors. For instance, while the true equity premium and Sharpe ratio are countercyclical, the *perceived* equity premium and Sharpe ratio are both procyclical; this is consistent with the findings of Amromin and Sharpe (2013). The model also generates a *negative* equity premium when investor sentiment is high, a prediction that is consistent with the findings of Greenwood and Hanson (2013), Baron and Xiong (2015), Cassella and Gulen (2017), and Yang and Zhang (2016).

Our paper adds to a new wave of theories that attempt to understand the role of belief formation in driving the behavior of asset prices and the macroeconomy (Fuster, Hebert, and Laibson (2011); Choi and Mertens (2013); Alti and Tetlock (2014); Hirshleifer, Li, and Yu (2015); Barberis et al. (2015); Jin (2015); Ehling, Graniero, and Heyerdahl-Larsen (2015); Vanasco, Malmendier, and Pouzo (2015); Collin-Dufresne, Johannes, and Lochstoer (2016a,b); Greenwood, Hanson, and Jin (2016)). Our paper also adds to a growing literature on understanding the source of stock price movements (Cochrane (2008); Cochrane (2011); Chen, Da, and Zhao (2013); de la O and Myers (2017)). Furthermore, it is fundamentally related to theories of model uncertainty and ambiguity aversion such as Bidder and Dew-Becker (2016). These models typically assume that agents have uncertainty in beliefs about the dynamic properties of the consumption process or the dividend process. As a result, they are closely linked to the notion of fundamental extrapolation in the behavioral finance literature, but do not match survey evidence on return expectations. Finally, our paper speaks to the debate between Bansal et al. (2012) and Beeler and Campbell (2012) which focuses on excess predictability: the notion that, in the long-run risks literature, future consumption and dividend growths are excessively predicted by current variables such as the price-dividend ratio (see also, Collin-Dufresne et al. (2016b)). Our model does not give rise to excess predictability, because return extrapolation in the model only generates *perceived* but not true long-run risks, and therefore the true consumption and dividend growths remain unpredictable.

The paper proceeds as follows. In Section I, we lay out the basic elements of the model and characterize its solution. In Section II, we examine the model's implications in detail. Section III provides a comparative statics analysis. Section IV discusses differences between our model and rational expectations models. Section V further compares the model to a model with fundamental

extrapolation. Section VI concludes and suggests directions for future research. All technical details are in the Appendix.

I. The Model

In this section, we first describe the model setup and characterize its solution, and then derive equilibrium quantities that are important for understanding the implications of the model.

Model Setup.—We consider an infinite-horizon Lucas economy in continuous time with a representative agent. The Lucas tree is a claim to an aggregate consumption process. We assume it is a geometric Brownian motion

$$dC_t/C_t = g_C dt + \sigma_C d\omega_t^C. \tag{1}$$

And we denote the price of the Lucas tree at time t as P_t^C.

Besides the Lucas tree, there are two other tradeable assets in the economy; they are the main focus of our analysis. The first asset is the stock market which is a claim to an aggregate dividend process given by

$$dD_t/D_t = g_D dt + \sigma_D d\omega_t^D. \tag{2}$$

And we denote the price of the stock market at time t as P_t^D.[4] Both ω_t^D and ω_t^C are standard Brownian motions. We assume the instantaneous correlation between dD_t and dC_t is ρ: $\mathbb{E}_t[d\omega_t^D \cdot d\omega_t^C] = \rho dt$. The second asset is an instantaneous riskless asset. This asset is in zero net supply, and its interest rate r_t is determined in equilibrium.

In terms of the agent's preferences, we follow Epstein and Zin (1989, 1991) and assume that the agent has a recursive intertemporal utility

$$U_t = \left[(1-e^{-\delta dt})C_t^{1-\psi}dt + e^{-\delta dt}\left(\mathbb{E}_t^e[\tilde{U}_{t+dt}^{1-\gamma}]\right)^{(1-\psi)/(1-\gamma)}\right]^{1/(1-\psi)}, \tag{3}$$

where δ is the subjective discount rate, $\gamma > 0$ is the relative risk aversion, and $\psi > 0$ is the reciprocal of the elasticity of intertemporal substitution. When ψ equals γ, (3) reduces to a power utility case. Note that the certainty equivalence in (3) is computed under the representative agent's subjective beliefs, which, as we specify later, incorporate the notion of return extrapolation. Also note that Epstein-Zin preferences allow for a strong interaction between preferences and beliefs that is crucial for the model to quantitatively match the empirical facts; Sections II and V discuss this point in detail.

[4] Since the aggregate consumption process in the model is exogenous, the dividend payment from the stock market does not further affect consumption. As a result, we can think of the stock market as an asset in zero net supply with a shadow price determined in equilibrium. This is a common assumption adopted by many other consumption-based models such as Campbell and Cochrane (1999) and Barberis, Huang, and Santos (2001).

The subjective Euler equation, or the first-order condition, is

$$\mathbb{E}_t^e \left[e^{-\delta(1-\gamma)dt/(1-\psi)} \left(\frac{\tilde{C}_{t+dt}}{C_t} \right)^{-\psi(1-\gamma)/(1-\psi)} \tilde{M}_{t+dt}^{(\psi-\gamma)/(1-\psi)} \tilde{R}_{j,t+dt} \right] = 1. \tag{4}$$

Here $\tilde{M}_{t+dt}$ is the gross return on the optimal portfolio held by the agent from time t to time $t+dt$. In a Lucas economy with a representative agent, the optimal portfolio in equilibrium is the Lucas tree itself, and therefore

$$\tilde{M}_{t+dt} = \frac{\tilde{P}_{t+dt}^C + \tilde{C}_t dt}{P_t^C} = \frac{\tilde{P}_{t+dt}^C + \tilde{C}_{t+dt} dt}{P_t^C} + o(dt). \tag{5}$$

On the other hand, $\tilde{R}_{j,t+dt}$ is the gross return on *any* tradeable asset j in the market from time t to time $t+dt$; as mentioned above, the two tradeable assets we focus on are the stock market and the riskless asset.

We now turn to the key part of the model: the representative agent's beliefs about the stock market returns. According to surveys, real-world investors form beliefs about future stock market returns by extrapolating past returns (Vissing-Jorgensen (2004); Bacchetta et al. (2009); Amromin and Sharpe (2013); Greenwood and Shleifer (2014); Koijen et al. (2015); Kuchler and Zafar (2016)). One natural way to incorporate this notion of return extrapolation is through a regime-switching learning model. Specifically, we suppose that the agent believes that the expected growth rate of stock market prices is governed by $(1-\theta)g_D + \theta\tilde{\mu}_{S,t}$, where $\tilde{\mu}_{S,t}$ is a latent variable which switches between a high value μ_H in a high-mean price growth regime H and a low value μ_L ($\mu_L < \mu_H$) in a low-mean price growth regime L with the following transition matrix[5]

$$\begin{array}{c} \\ \tilde{\mu}_{S,t} = \mu_H \\ \tilde{\mu}_{S,t} = \mu_L \end{array} \begin{array}{c} \begin{array}{cc} \tilde{\mu}_{S,t+dt} = \mu_H & \tilde{\mu}_{S,t+dt} = \mu_L \end{array} \\ \begin{pmatrix} 1-\chi dt & \chi dt \\ \lambda dt & 1-\lambda dt \end{pmatrix} \end{array}. \tag{6}$$

Here χ and λ are the intensities for the transitions of regime from H to L and from L to H, respectively. And the parameter θ ($0 \leq \theta \leq 1$) controls the extent to which the agent's beliefs are extrapolative: setting θ to zero reduces the model to a rational benchmark.

Given this perceived regime-switching model—this is not the true model—if recent stock market price growth has been high, the agent thinks it is likely that the high-mean price growth regime is generating prices so therefore she forecasts high price growth in the future. Conversely, if recent price growth has been low, the agent thinks it is likely that the low-mean price growth regime is generating prices and therefore forecasts low price growth in the future. Formally, at each point in time, the agent computes the expected value of the latent variable $\tilde{\mu}_{S,t}$ given the history of past price growth: $S_t \equiv \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^P]$. That is, she applies the optimal filtering theory (see, for

[5] The models of Barberis, Shleifer, and Vishny (1998), Veronesi (1999), and Jin (2015) also adopt a regime-switching learning structure.

instance, Lipster and Shiryaev (2001)) and obtains

$$\begin{aligned} dS_t &= (\lambda\mu_H + \chi\mu_L - (\lambda+\chi)S_t)dt + (\sigma^D_{P,t})^{-1}\theta(\mu_H - S_t)(S_t - \mu_L)d\omega^e_t \\ &\equiv \mu^e_S(S_t)dt + \sigma_S(S_t)d\omega^e_t, \end{aligned} \tag{7}$$

where $d\omega^e_t \equiv [dP^D_t/P^D_t - (1-\theta)g_D dt - \theta S_t dt]/\sigma^D_{P,t}$ is a standard Brownian innovation term from the agent's perspective. As a result, she perceives the evolution of the stock market price P^D_t as

$$dP^D_t/P^D_t = \mu^{D,e}_P(S_t)dt + \sigma^D_P(S_t)d\omega^e_t, \tag{8}$$

where

$$\mu^{D,e}_P(S_t) = (1-\theta)g_D + \theta S_t. \tag{9}$$

In summary, the agent's expectation about price growth $\mu^{D,e}_P(S_t)$ is a linear combination of a rational component g_D and a behavioral component S_t; hereafter we call S_t the sentiment variable.

The evolution of sentiment in (7) captures return extrapolation: high past price growth dP^D_t/P^D_t pushes up the perceived shock $d\omega^e_t$, which leads the agent to raise her expectation of the sentiment variable S_t, causing her expectation about future price growth $\mu^{D,e}_P(S_t)$ to rise.[6]

Although the subjective evolution of sentiment (7) is derived through optimal learning, the representative agent, it should be emphasized, does *not* hold rational expectations. With rational expectations, the agent will realize in the long run that the regime-switching model (6) is incorrect: she can look at the historical distribution of $d\omega^e_t$ and realize that it does not fit a normal distribution with a mean of 0 and a variance of dt. Instead, the agent in our behavioral model always believes that the regime-switching model is correct. In reality, it is possible that investors in the market learn over time that their mental model is incorrect. At the same time, new investors who hold extrapolative beliefs may continuously enter the market. The stable belief system in (6) is an analytically convenient way to capture these dynamics. Alternatively, if equations (6) and (7) represent the *true* data generating process, then the agent does hold rational expectations. In that case, the model becomes a fully rational model with incomplete information.[7] We discuss the implications of such a model in Section IV.

So far we have been focusing on the agent's beliefs about stock market prices. These beliefs also have direct implications for the agent's beliefs about dividend growth. If we write the perceived

[6] To capture return extrapolation, there are many ways to specify the evolution of S_t. We derive S_t from a regime-switching learning model based on two reasons. First, such a learning model fundamentally captures base rate neglect, an important form of representative heuristic (Tversky and Kahneman (1974)). To see this, note that the perceived regimes or states, H and L, are not part of the true states of the economy. As a result, assigning positive probability weights to these regimes reflect the bias that the investor neglects the zero base rate associated with such regimes. On the other hand, the earlier work of Barberis et al. (1998) has associated the regime-switching model to sample size neglect, another form of representativeness heuristic. Second, bounding S_t by a finite range (μ_L, μ_H) reduces the analytical difficulty for solving the model.

[7] Information is incomplete in the sense that the agent does not directly observe the latent variable $\tilde{\mu}_{S,t}$.

dividend process as

$$dD_t/D_t = g_D^e(S_t)dt + \sigma_D d\omega_t^e, \tag{10}$$

we can connect the agent's expectation about dividend growth $g_D^e(S_t)$ to her expectation about the stock market price growth $\mu_P^{D,e}(S_t)$. To formally make this connection, first we observe that, all the ratio-based quantities in our model (e.g., the price-dividend ratio for the stock market) are each a function of the sentiment variable S_t; we can define $f(S_t) \equiv P_t^D/D_t$. We then apply Ito's lemma on both sides of the equation $f(S_t) = P_t^D/D_t$ and match terms to obtain

$$g_D^e(S_t) = \underbrace{(1-\theta)g_D + \theta S_t}_{\text{expectation of price growth}} \quad \underbrace{-(f'/f)\mu_S^e(S_t)}_{\text{expectation of sentiment evolution}} \quad \underbrace{+\sigma_D^2 - \sigma_P^D(S_t)\sigma_D - \tfrac{1}{2}(f''/f)(\sigma_S(S_t))^2}_{\text{Ito correction terms}}, \tag{11}$$

where

$$\sigma_P^D(S_t) = \frac{\sigma_D + \sqrt{\sigma_D^2 + 4\theta(\mu_H - S_t)(S_t - \mu_L)(f'/f)}}{2} > \sigma_D. \tag{12}$$

Equation (11) highlights an "expectations transmission mechanism" that has been generally overlooked in the literature. It says that the agent's expectation about dividend growth equals the sum of her expectation about the stock market price growth, her expectation about how the price-dividend ratio evolves with respect to changes in sentiment, and the Ito correction terms that are related to the agent's risk aversion and the volatility of dividend growth, price growth, and changes in sentiment. In this way, the agent's expectation about price growth affects her expectation about dividend growth.

It takes two steps to explain the economic intuition behind (11). The first step is to explain why the agent's expectation about dividend growth—the left hand side of (11)—is tied to her expectation about price growth—the first term on the right hand side of (11). To do this, iterating forward the Euler equation (4) with the stock market as the tradeable asset gives

$$\frac{P_t^D}{D_t} = \mathbb{E}_t^e \left[\int_t^\infty e^{-\delta(1-\gamma)(s-t)/(1-\psi)} \left(\frac{\tilde{C}_s}{C_t} \right)^{-\psi(1-\gamma)/(1-\psi)} \tilde{M}_{t\to s}^{(\psi-\gamma)/(1-\psi)} \left(\frac{\tilde{D}_s}{D_t} \right) ds \right], \tag{13}$$

where $\tilde{M}_{t\to s}$ denotes the continuously compounded gross return for holding the Lucas tree from time t to time s $(> t)$.[8] That is, the stock market price equals the sum of discounted future dividends under the agent's subjective expectations. Therefore, the current level of price-dividend ratio—the left hand side of (13)—is linked to the agent's expectations about future dividend growth—which is on the right hand side of (13). Because of this link, for the agent to perceive high future price

[8] The transversality condition holds in this economy as the stock market prices are bounded by a finite range.

growth following high past price growth, she must anticipate higher-than-usual dividend growth in the near future, as this is the main way to push up near-future prices. In other words, the agent's expectation about dividend growth $g_D^e(S_t)$ is connected to her expectation about price growth $(1-\theta)g_D + \theta S_t$. In addition, knowing future dividend growth will be high, the agent drives up the current price, generating a higher price-dividend ratio. Therefore, the price-dividend ratio is a positive function of sentiment.

The second step explains why the agent's expectation about dividend growth can be *more* responsive to changes in sentiment than her expectation about price growth. To see this, note from (7) that, under the regime-switching learning model, the agent perceives sentiment to be mean-reverting: $\mu_S^e(S_t)$ is a negative function of S_t. Given that sentiment drives the price-dividend ratio, the agent also perceives the price-dividend ratio to be mean-reverting. Furthermore, since the price-dividend ratio is a positive function of sentiment, the price-dividend ratio is high when sentiment is high. Putting these together, the agent anticipates that the price-dividend ratio will decline from a high value when she expects high future price growth. That is, the agent expects dividend to grow at a faster rate than the stock market price; the difference between the agent's expectation about dividend growth and her expectation about price growth is captured by the second term on the right hand side of (11).

In sum, these two steps together imply that when the agent expects high future price growth, she also expects high dividend growth, at a pace that exceeds her expectations about future price growth. This belief dynamic has important implications regarding the source of stock market movements. We provide further discussion in Section II.

To complete the description of the model, we need to further specify the agent's beliefs about consumption growth. To do this, first note that with a local correlation of ρ between consumption growth and dividend growth, we can rewrite the aggregate consumption process of (1) as

$$dC_t/C_t = g_C dt + \sigma_C \left(\rho d\omega_t^D + \sqrt{1-\rho^2} d\omega_t^{\perp} \right), \tag{14}$$

where $\omega_t^{\perp}$ is a Brownian motion that is locally uncorrelated with ω_t^D, the Brownian shock on dividends. We then assume that the agent perceives the consumption process as

$$dC_t/C_t = g_C^e(S_t) dt + \sigma_C \left(\rho d\omega_t^e + \sqrt{1-\rho^2} d\omega_t^{\perp} \right). \tag{15}$$

That is, we replace the true Brownian shock on dividends $d\omega_t^D$ by the agent's perceived Brownian shock $d\omega_t^e$ and factor the difference between these two Brownian shocks into $g_C^e(S_t)$, the agent's subjective expectation about consumption growth. Conceptually, this amounts to assuming that the bias in the agent's beliefs about consumption growth only comes from the bias in her beliefs about dividend growth.[9] In doing so, we derive the agent's expectation about dividend growth as

[9] For any alternative assumption, one needs to explain why the investor has incorrect beliefs about consumption above and beyond her incorrect beliefs about dividends.

$$g_C^e(S_t) - g_C = \rho\sigma_C\sigma_D^{-1}(g_D^e(S_t) - g_D). \tag{16}$$

Empirically, the correlation between consumption growth and dividend growth is low—ρ is positive but low—and the consumption growth is much less volatile than the dividend growth—σ_C is much smaller than σ_D. As a result, (16) implies that the bias in the agent's expectation about consumption growth—the difference between $g_C^e(S_t)$ and g_C—is small. This is in keeping with the lack of empirical evidence on consumption extrapolation.[10] Moreover, the agent's approximately correct beliefs about consumption growth allow the model to generate low interest rate volatility and a low correlation between consumption growth and stock market returns, both of which are consistent with the data (Campbell (2003); Hansen and Singleton (1982, 1983)).

Model Solution.—The subjective Euler equation in (4) shows that when pricing the stock market, the gross return for holding the Lucas tree is also part of the pricing kernel. This observation has two implications. First, both the price-dividend ratio $f(S_t) = P_t^D/D_t$ and the wealth-consumption ratio P_t^C/C_t are functions of the sentiment variable S_t; we can define $l(S_t) \equiv P_t^C/C_t$. Second, the two functions f and l are interrelated through Euler equations, so they need to be solved simultaneously. Specifically, using the Euler equation to price the stock market—setting $\tilde{R}_{j,t+dt}$ in (4) to the gross return on the stock market—we obtain

$$0 = \left[\begin{array}{l} -\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(f'/f) + \frac{\psi-\gamma}{1-\psi}(l'/l)]\mu_S^e + \frac{1}{2}[(f''/f) + \frac{\psi-\gamma}{1-\psi}(l''/l)]\sigma_S^2 \\ +\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(l'/l)^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(l'/l) - \gamma\rho\sigma_C\sigma_D - \gamma\rho\sigma_C\sigma_S(f'/f) \\ +\frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(l'/l) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2(l'/l)(f'/f) + \sigma_D\sigma_S(f'/f) + \frac{\psi-\gamma}{1-\psi}l^{-1} + f^{-1} \end{array}\right]. \tag{17}$$

Similarly, using the Euler equation to price the Lucas tree—setting $\tilde{R}_{j,t+dt}$ in (4) to the gross return on the Lucas tree—we obtain

$$0 = \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(l'/l)\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(l''/l)\sigma_S^2 \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(l'/l)^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(l'/l) + \frac{1-\gamma}{1-\psi}l^{-1} \end{array}\right]. \tag{18}$$

Substituting μ_S and σ_S from (7), g_D^e and σ_P^D from (11) and (12), and g_C^e from (16) into equations (17) and (18), we then obtain a system of two ordinary differential equations that jointly determines

[10] Consistent with the way we model the agent's expectations about consumption growth, Kuchler and Zafar (2016) find that survey expectations are "asset-specific:" respondents who become pessimistic about their employment situation after experiencing unemployment are not pessimistic about other economic outcomes, such as stock prices or interest rates. Similarly, Huang (2016) finds that investors who become optimistic about an industry's future returns after having positive prior investment experience in the industry do not invest heavily in a dissimilar industry.

the evolutions of f and l.[11] The detailed derivation of (17) and (18) is in the Appendix.

Regarding the boundary conditions for solving the differential equations, note that in (17) and (18), the second derivative terms are all multiplied with σ_S, and that σ_S goes to zero as S approaches either μ_H or μ_L. As a result, μ_H and μ_L are both singular points, and therefore no boundary condition is required.

Equations (17) and (18) cannot be solved analytically. We apply a projection method with Chebyshev polynomials to solve them numerically. We leave the details of the numerical procedure to the Appendix.

Important Equilibrium Quantities.—With the model solution at hand, we derive equilibrium quantities that are important for understanding the model implications. Specifically, we derive the dynamics of the interest rate, the objective and subjective expectations of stock market returns, and the steady-state distribution of the sentiment variable.

For the interest rate, we use the Euler equation in (4) to price the riskless asset—we set $\tilde{R}_{j,t+dt}$ to the gross return on the riskless asset $1 + r_t dt$—and obtain

$$r_t = \tfrac{1-\gamma}{1-\psi}\delta + \gamma g_C^e - \tfrac{\gamma(\gamma+1)}{2}\sigma_C^2 - \tfrac{\psi-\gamma}{1-\psi} \times \left[\begin{array}{l} (\mu_S^e - \gamma\rho\sigma_C\sigma_S)(l'/l) + \frac{1}{2}\sigma_S^2(l''/l) \\ + \frac{2\psi-\gamma-1}{2(1-\psi)}\sigma_S^2(l'/l)^2 + l^{-1} \end{array} \right]. \tag{19}$$

The interest rate is linked to the agent's time preferences, her subjective expectation about consumption growth, precautionary saving, as well as how the wealth-consumption ratio P_t^C/C_t responds to changes in sentiment.[12]

To understand the risk-return tradeoff in the model, we compute, at each point in time, both the agent's expectation about future stock market returns and the (objectively measured) rational expectation about future stock market returns. From equations (8) and (9), the log excess return on the stock market from time t to time $t + dt$ is

$$\begin{aligned} r_{t+dt}^{D,e} dt &\equiv \ell\mathrm{n}(P_{t+dt}^D + D_{t+dt}dt) - \ell\mathrm{n}(P_t^D) - r_t dt \\ &= [(1-\theta)g_D + \theta S_t + f^{-1} - \tfrac{1}{2}(\sigma_P^D)^2 - r_t]dt + \sigma_P^D d\omega_t^e. \end{aligned} \tag{20}$$

Therefore, the agent's expectation about the log excess return is

$$\mathbb{E}_t^e[r_{t+dt}^{D,e}] = (1-\theta)g_D + \theta S_t + f^{-1} - \tfrac{1}{2}(\sigma_P^D)^2 - r_t, \tag{21}$$

and the subjective Sharpe ratio is $[(1-\theta)g_D + \theta S_t + f^{-1} - \frac{1}{2}(\sigma_P^D)^2 - r_t]/\sigma_P^D$.

[11] When $\theta = 0$, our model reduces to a fully rational benchmark. In this case, equations (17) and (18) give rise to

$$f = \left[\delta + \psi g_C - g_D - \tfrac{\gamma(\psi+1)}{2}\sigma_C^2 + \gamma\rho\sigma_C\sigma_D\right]^{-1}, \quad l = \left[\delta + (\psi-1)g_C - \tfrac{\gamma(\psi-1)}{2}\sigma_C^2\right]^{-1}.$$

[12] When $\theta = 0$, $r = \delta + \psi g_C - \frac{\gamma(\psi+1)}{2}\sigma_C^2$.

Next, to compute the rational expectation about the stock market return, we compare (2) with (10) and obtain a relationship between the true and perceived Brownian shocks

$$d\omega_t^e = d\omega_t^D - (g_D^e(S_t) - g_D)dt/\sigma_D. \tag{22}$$

We substitute (22) into (20) and derive

$$r_{t+dt}^{D,e}dt = [(1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma_P^D(g_D^e - g_D) - \tfrac{1}{2}(\sigma_P^D)^2 - r_t]dt + \sigma_P^D d\omega_t^D. \tag{23}$$

So the rational expectation about the log excess return on the stock market is

$$\mathbb{E}_t[r_{t+dt}^{D,e}] = (1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma_P^D(g_D^e - g_D) - \tfrac{1}{2}(\sigma_P^D)^2 - r_t. \tag{24}$$

And the objectively measured Sharpe ratio of the stock market return is $[(1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma_P^D(g_D^e - g_D) - \frac{1}{2}(\sigma_P^D)^2 - r_t]/\sigma_P^D$.

All the ratio-based quantities in this model such as the agent's expectation about stock market returns and the interest rate are functions of the sentiment variable S_t. Given this, to provide a statistical assessment of the model's fit with the empirical facts, we also compute the steady-state distribution for the sentiment variable S_t *objectively* measured by outside econometricians. To that end, we first obtain the objective evolution of sentiment by substituting the change-of-measure equation (22) into the subjective evolution of sentiment in (7)

$$dS_t = [\mu_S^e(S_t) + \sigma_D^{-1}\sigma_S(S_t)(g_D - g_D^e(S_t))]dt + \sigma_S(S_t)d\omega_t^D. \tag{25}$$

Compared to the subjective evolution of sentiment, the objective evolution exhibits a larger degree of mean reversion: the term $\sigma_D^{-1}\sigma_S(S_t)(g_D - g_D^e(S_t))$ in (25) is a negative function of sentiment.

Denote the objective steady-state distribution for sentiment as $\xi(S)$. Based on (25), we then derive $\xi(S)$ as the solution to the Kolmogorov forward equation (the Fokker-Planck equation)

$$\begin{aligned} 0 &= \tfrac{1}{2}\tfrac{d^2}{dS^2}\left(\sigma_S^2(S)\xi(S)\right) - \tfrac{d}{dS}\left([\mu_S^e(S_t) + \sigma_D^{-1}\sigma_S(S_t)(g_D - g_D^e(S_t))]\xi(S)\right) \\ &= (\sigma_S')^2\xi + \sigma_S\sigma_S''\xi + 2\sigma_S\sigma_S'\xi' + \tfrac{1}{2}\sigma_S^2\xi'' \\ &\quad - [(\mu_S^e)' + \sigma_D^{-1}\sigma_S'(g_D - g_D^e) - \sigma_D^{-1}\sigma_S(g_D^e)']\xi - [\mu_S^e + \sigma_D^{-1}\sigma_S(g_D - g_D^e)]\xi', \end{aligned} \tag{26}$$

where σ_S and g_D^e are from (7) and (11), respectively, and the expressions for σ_S', σ_S'', $(\mu_S^e)'$ and $(g_D^e)'$ are provided in the Appendix. In addition, the steady-state distribution must integrate to one.

II. Model Implications

In this section, we examine the model implications. We begin by setting the benchmark values for the model parameters. There are three types of parameters: asset parameters, utility parameters, and belief parameters. For the asset parameters, we set $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$. These values are consistent with those used in Campbell and Cochrane (1999), Barberis et al. (2001), Bansal and Yaron (2004), and Beeler and Campbell (2012).[13] For the utility parameters, we set γ, the relative risk aversion parameter, to 10. As pointed out in Bansal et al. (2012) and Bansal and Yaron (2004), the long-run risks literature—a literature that depends significantly on the parameter values of Epstein-Zin preferences for its model implications—typically assigns a value of 10 or below for γ. Bansal and Yaron (2004), for instance, set γ to either 10 or 7.5.[14] For ψ, the reciprocal of the elasticity of intertemporal substitution, there exists a wide range of estimates in the asset pricing literature. The majority of previous works suggests that ψ should be lower than one, but several other papers argue the opposite.[15] Given these previous works, we set ψ to 0.9, a value that implies an elasticity of intertemporal substitution slightly above one. We explain in Section IV that our model implications are quantitatively robust even with an elasticity of intertemporal substitution significantly lower than one. Finally, for δ, the subjective discount rate, we assign a value of 2%.

We now turn to the belief parameters. We set θ, the parameter that controls the extent to which the representative agent is behavioral, to 0.5. Recall that $\theta = 0$ means the agent is fully rational, whereas $\theta = 1$ means the agent is fully behavioral. Therefore, 0.5 seems to be a natural default value for θ. We set χ and λ, the perceived transition intensities between the high- and low-mean price growth regimes, both to 0.18. This parameter value implies that the agent looks back a couple of years when forming beliefs about future returns. And this is in keeping with the survey evidence in Greenwood and Shleifer (2014) and other papers; we make this point more precise later in this section. Finally, we set μ_H and μ_L, the mean price growth in the high and low regimes, to 15% and -15%, respectively.

We summarize our default parameter values in Table 1. For parameters that either have dispersed estimates in the literature or can not be directly observed from the data, we provide a comparative statics analysis in Section III to examine the sensitivity of the model's implications with respect to changes in these parameter values.

[Place Table 1 about here]

In the remaining part of this section, we first look at two building blocks for the model implications: a set of important empirical quantities, each as a function of sentiment; and the steady-state distribution of sentiment. We then analyze the model's implications for investor expectations. Fi-

[13] The parameter values for g_C and g_D are set such that both log consumption and log dividend grow at an annual rate of 1.84%; this rate is used in Barberis et al. (2001).

[14] An estimate of 10 for γ is also the maximum magnitude that Mehra and Prescott (1985) find reasonable.

[15] See Bansal et al. (2012) for a discussion of this point.

nally, we discuss the model's implications for asset prices.

Building Blocks.—We plot, in Figure 1, the price-dividend ratio of the stock market f, the volatility of stock market returns σ_P^D, the rational expectation about the log excess return $\mathbb{E}[r^{D,e}]$ (the conditional equity premium), and the interest rate r, each as a function of the sentiment variable S.

[Place Figure 1 about here]

Figure 1 shows that the model generates substantial excess volatility: in the model, the volatility of dividend growth is 11%, while the volatility of stock market returns is typically above 20%. This result highlights a strong interaction between return extrapolation and Epstein-Zin preferences. With return extrapolation, high past returns lead the agent to forecast high future returns. Under Epstein-Zin preferences, the separation between the elasticity of intertemporal substitution and risk aversion gives rise to a strong intertemporal substitution effect. As a result, the agent's forecast of high future returns leads her to push up the current price significantly, generating excess volatility. The interaction between beliefs and preferences is quantitatively important. Without return extrapolation, Epstein-Zin preferences alone with *i.i.d.* dividend growth and consumption growth do not lead to any excess volatility. Without Epstein-Zin preferences—that is, setting both ψ and γ to 10 while keeping all the other parameter values unchanged—return extrapolation alone only generates 17% of the excess volatility.

Figure 1 also shows that the model generates a strong procyclical pattern for the price-dividend ratio of the stock market. As we described in Section I, an expectations transmission mechanism leads the agent to forecast high future dividend growth after high past returns. Given that the stock market price is the sum of the agent's expectations about discounted future dividends, high expectations about future dividend growth lead the agent to push up the current price, making the price-dividend ratio procyclical with respect to changes in sentiment.

In addition, the model generates a strong countercyclical pattern for the equity premium. Suppose the stock market has had high past returns. The agent's expectation about future returns then increases, pushing up the stock market price relative to dividends. Given that the sentiment variable on average tends to revert back to its mean, future stock market prices tend to decrease, leading to low future returns. At the same time, the agent's high expectation about future returns also makes her optimistic about future consumption growth, although to a much lesser extent. This in turn causes the agent to push up the equilibrium interest rate. Together, both a low (rational) expectation of stock market returns and a high interest rate contribute to a low equity premium during high-sentiment periods. Of these two forces, the first one dominates: moving S_t from its mean to the top 25% percentile level leads to a total decrease of 9.7% for the equity premium, out of which 9.5% comes from the decrease in the expected log return of the stock market.

The second building block for the model implications is the objectively measured steady-state distribution of sentiment. Figure 2 plots this steady-state distribution. Under the true probability

measure, sentiment exhibits a strong degree of mean reversion for two reasons. First, the agent believes that sentiment will naturally revert: the agent thinks that the expected growth rate of stock market prices tends to switch over time from one regime to the other. Second, the agent's expectations are incorrect: precisely when the agent thinks that the future price growth is high, future price growth tends to be low endogenously, causing sentiment to mean-revert faster than what the agent perceives. Altogether, the steady-state distribution has a mean of 2.0% and a standard deviation of 2.7%: the chance of sentiment approaching the extreme values of μ_H and μ_L is close to zero.

[Place Figure 2 about here]

The mean reversion of sentiment helps to keep the model stable. Recall that the interaction between return extrapolation and Epstein-Zin preferences gives rise to a feedback loop: if the current price growth is high, the agent thinks that future price growth will also be high, which leads her to push up the current price, causing the current price growth to further rise, and so on. In general, there is a danger that this feedback loop could explode. However, in our model, the agent believes that sentiment tends to revert back to its mean, so she will not become infinitely optimistic about future price growth of the stock market. In addition, the agent's risk aversion prevents her from infinitely increasing the demand for the stock market. Therefore, the cumulative impact of this feedback loop on investor expectations and asset prices remains finite.

Model's Implications for Investor Expectations.—One main goal of the model is to match survey expectations about stock market returns. Empirically, Greenwood and Shleifer (2014) regress survey expectations about future stock market returns on past twelve-month cumulative raw returns across various survey expectations measures. They find that the regression coefficient is positive and statistically significant. For comparison, we want to run the same regression in the context of the model. One caveat, however, is that we are uncertain about what survey respondents think the definition of return is. Does it include the dividend yield or not? And is it a raw return or an excess return? Given this caveat, we examine *four* measures of return expectations: $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, the agent's expectation about the percentage return on the stock market, $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, the agent's expectation about the percentage return in excess of the interest rate, $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$, the agent's expectation about the price growth of the stock market, and $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$, the agent's expectation about the price growth in excess of the interest rate. The latter two measures are also plausible candidates because investors may not actively think about dividend yields when answering survey questions.[16]

[Place Table 2 about here]

[16] Hartzmark and Solomon (2017) provide empirical evidence for the idea that investors do not take dividend yield into account when calculating returns. Barberis et al. (2015) also take this interpretation when calibrating their model parameters to survey expectations.

Table 2 reports the regression coefficient and the t-statistic, the intercept, as well as the adjusted R-squared, for regressing each of the four measures of return expectations described above on either the past twelve-month cumulative raw return or the current log price-dividend ratio, over a sample of 15 years or 50 years. Each reported value—for instance, the regression coefficient—is averaged over 100 trials, with each trial being a regression using monthly data simulated from the model. Here we make two observations. First, the magnitude of the agent's extrapolative beliefs about future stock market returns matches the empirical values suggested by Greenwood and Shleifer (2014). Regressing the agent's expectation about future price growth ($\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$) on past twelve-month cumulative raw returns for a 15-year simulated sample, the regression coefficient is 4.0% with a Newey-West adjusted t-statistic of 8.4. Running the same regression for a 50-year simulated sample, the regression coefficient is 4.0% with a t-statistic of 12.1. As a comparison, for a 5-year sample of sparse data from the Michigan survey, the regression coefficient is 3.9% with a t-statistic of 1.68; for a 15-year sample of data from the Gallup survey, the regression coefficient, after some conversion, is 8% with a t-statistic of 8.81.

Second, by comparing the regression coefficients and the t-statistics across the four measures of return expectations, we find that including the dividend yield in the calculation of return reduces the regression coefficient by about a half, but does not significantly affect the t-statistic. Therefore, even though we model return extrapolation as extrapolating past price growth, the agent also holds extrapolative expectations about the total return. Furthermore, subtracting the interest rate from the expectation of returns has little impact on the regression results because of low interest rate volatility. In summary, across all four measures of return expectations, the agent extrapolates past stock market returns when forming expectations about future returns. In Section IV, we compare these regression results with those from the rational expectations models of Campbell and Cochrane (1999), Bansal and Yaron (2004), and Bansal et al. (2012).

Next, we study the formation of the agent's expectations by examining whether expectations depend primarily on recent past returns or also on more distant ones. Specifically, we estimate the following non-linear least squares regression

$$\text{Expectation}_t = a + b\sum\nolimits_{j=1}^{n} w_j R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)} + \varepsilon_t, \tag{27}$$

where Expectation_t is the agent's time-t expectation about stock market returns, $R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)}$ is the raw return from time $t-j\Delta t$ to $t-(j-1)\Delta t$, and $w_j \equiv e^{-\phi(j-1)\Delta t}/\sum_{l=1}^{n} e^{-\phi(l-1)\Delta t}$. In Equation (27), each past realized return is assigned a weight. The weight decreases exponentially the further back we go into the past, and the coefficient ϕ measures the speed of this exponential decline. When ϕ is high, the agent's expectation is determined primarily by recent past returns; when it is low, even distant past returns have a significant impact on the agent's current expectation.

[Place Table 3 about here]

Table 3 reports the regression coefficient a, the intercept b, the adjusted R-squared, and most importantly, the parameter ϕ. As before, we examine four measures of return expectations, $\mathbb{E}_t^e[(dP_t^D +$

$D_t dt)/(P_t^D dt)]$, $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$, $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, and $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$. Each reported value is averaged over 100 trials, with each trial being a regression using simulated data with a monthly frequency over either 15 years or 50 years. We set Δt, the time interval for each past return in (27), to 1/12 (one month), and we set n, the total number of past returns on the right hand side of (27), to 600.[17]

Across the four measures of return expectations, the estimation of ϕ is stable: it is around 0.42. This value means that a monthly return three years ago is weighted about 25% as much as the most recent return; that is, the agent looks back a couple of years when forming beliefs about future returns. For comparison, Barberis et al. (2015) estimate ϕ using survey data and obtain a value of 0.44. Basically, we calibrate the model to the survey expectations by setting a value of 0.18 for both χ and λ.

The behavioral finance literature currently has no consensus on the value of ϕ. On the one hand, Greenwood and Shleifer (2014) and Kuchler and Zafar (2016) find that investor expectations only depend on recent returns from the past few years. On the other hand, Malmendier and Nagel (2011, 2013) and Vanasco et al. (2015) suggest that distant past events may also play an important role when investors form beliefs. Reconciling this discrepancy is beyond the scope of the paper. Here, we provide two possible explanations. First, investors may simultaneously adopt two mechanisms when forming beliefs: one that focuses on recent past events such as daily stock market fluctuations, the other that focuses on infrequent but salient events such as a stock market crash. Second, the horizon over which investors form expectations may affect how far they look back into the past. Survey expectations are mainly about stock market returns over the next six to twelve months, and therefore investors look back a couple of years. Equity holdings, on the other hand, may span the next couple of decades, and therefore investors look back a few decades when making decisions about equity holdings.

Model Implications on Asset Prices.—We now evaluate the model's fit with empirical facts by analyzing its implications on asset prices. We begin with examining the model's fit with the long-run properties of stock market prices and returns. Table 4 reports the model's predictions for six important moments, and compares them side by side with the empirical values. In general, the model matches the facts: it generates a high equity premium, significant excess volatility, a Sharpe ratio similar to the empirical value, an interest rate that has a low level and low volatility, and a price-dividend ratio whose average level is close to the empirical one.

[Place Table 4 about here]

To understand these results, recall that the interaction between return extrapolation and Epstein-Zin preferences gives rise to a feedback loop, which in turn generates significant excess volatility. Three factors affect the perceived long-run equity premium. First and most intuitively,

[17] We choose $n = 600$ so that further increasing n would have a minimal impact on the estimated values of the parameter ϕ and the adjusted R-squared.

excess volatility interacts with the agent's risk aversion and causes the perceived equity premium to increase. Second, return extrapolation gives rise to *perceived* persistence of both the aggregate dividend process and, to a lesser extent, the aggregate consumption process.[18] Under Epstein-Zin preferences, this perceived persistence is significantly priced, pushing up the perceived equity premium. Finally, the separation between the elasticity of intertemporal substitution and the relative risk aversion keeps the equilibrium interest rate low and hence helps to keep the equity premium high.

The true long-run equity premium is higher than the perceived long-run equity premium. When the agent is optimistic about future returns, her risk aversion counteracts her beliefs, cushioning against the subsequent decline of the stock market price and therefore making the agent less likely to be overly optimistic. On the other hand, when the agent is pessimistic about future returns, her risk aversion strengthens her beliefs, amplifying the subsequent increase of the stock market price and making the agent more likely to be overly pessimistic. This asymmetry implies that for more than half of the time, the agent's beliefs about future stock market returns are overly pessimistic—the true equity premium is higher than the perceived equity premium; for less than half of the time, her beliefs are overly optimistic—the true equity premium is lower than the perceived equity premium. As a result, the true long-run equity premium is above the perceived long-run equity premium. Quantitatively, the true average equity premium is 4.9% when measured as the rational expectation of log excess returns; it is 8.6% when measured as the rational expectation of excess returns $\mathbb{E}[(dP_t^D + D_t dt)/(P_t^D dt) - r_t]$. The perceived long-run equity premium, on the other hand, is 1.6% when measured as the subjective expectation of log excess returns; it is 5.1% when measured as the subjective expectation of excess returns $\mathbb{E}^e[(dP_t^D + D_t dt)/(P_t^D dt) - r_t]$.

The model generates a *negative* equity premium during high-sentiment periods: the equity premium averaged over the top quartile of the sentiment distribution is −13.05%. This implication is consistent with the recent empirical findings of Greenwood and Hanson (2013), Baron and Xiong (2015), Cassella and Gulen (2017), and Yang and Zhang (2016): these papers document that risk premium turns negative during high-sentiment periods. Note that in general, rational expectations models—for instance, the long-run risks models and the habit formation models—do not generate a negative equity premium at any time.[19] In our model, however, subjective expectations and objective expectations of stock market returns differ significantly during high-sentiment or low-sentiment periods: when the sentiment level is high, the agent expects high stock market returns moving forward, but precisely because of her incorrect beliefs, future stock market returns are low on average, generating a negative equity premium.

We further examine the model's fit with the dynamic behavior of stock market prices. We start by investigating the source of stock market movements. Figure 3 plots both the agent's expectation

[18] The agent is averse to persistent shocks when $\gamma > \psi$; with our choice of parameter values, this condition is satisfied.

[19] Strictly speaking, a rational expectations model can still generate a negative equity premium if the stock market negatively correlates with some other risk factors in the agent's portfolio and therefore serves as a diversification device.

about stock market returns, $\mathbb{E}^e[(dP_t^D + D_t dt)/(P_t^D dt)]$ and the agent's expectation about dividend growth, $\mathbb{E}^e[dD_t/(D_t dt)]$, each as a function of the sentiment variable S.

[Place Figure 3 about here]

The expectations transmission mechanism described before suggests an important pattern: the agent's expectation about price growth is less responsive to changes in sentiment than her expectation about dividend growth. Note that the total return equals the sum of the price growth and the dividend yield, and that the dividend yield decreases as sentiment increases. Therefore, the agent's expectation about stock market returns is even less responsive to changes in sentiment compared to her expectation about price growth. Quantitatively, Figure 3 suggests that a one-standard deviation (2.7%) increase of sentiment from its mean (2.0%) pushes up the agent's expectation about stock market returns from 7.44% to 8.22%—a small increase of 0.78%—while it pushes up the agent's expectation about dividend growth from −0.04% to 6.48%—a much larger increase of 6.52%. Also recall from Figure 1 that a one-standard deviation increase of sentiment from its mean pushes up the price-dividend ratio of the stock market from 19.16 to 21.75. Putting these results together, we observe that the movements in the price-dividend ratio primarily comes from changes in subjective expectations about future dividend growth.

To analyze the source of stock market movements statistically, we follow the procedure in Campbell and Shiller (1988) to decompose the log price-dividend ratio of the stock market:

$$\ell\mathrm{n}(P_t^D/D_t) \approx \sum_{j=0}^{\infty} \xi^j \left(\Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)} - r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}\right) - (\ell\mathrm{n}(\xi) + (1-\xi)\zeta)/(1-\xi), \tag{28}$$

where ζ is the in-sample average of the annual log dividend-price ratio, $\xi = e^{-\zeta}/(\Delta t + e^{-\zeta})$, $\Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ is the log dividend growth from time $t+j\Delta t$ to $t+(j+1)\Delta t$, and $r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ is the log gross return over the same period.

Equation (28) says that the movement of price-dividend ratio comes from either the movement of future dividend growth or the movement of future returns. The first component is called "cash flow news" by empiricists, and the second component is called "discount rate news." As pointed out by de la O and Myers (2017), there are two approaches to empirically address the relative importance of these two components. The first approach looks at future *realized* dividend growth and returns, and it computes

$$\begin{aligned} 1 \approx & \underbrace{\frac{\mathbb{C}\mathrm{ov}\left(\sum_{j=0}^{\infty}\xi^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}, \ell\mathrm{n}(P_t^D/D_t)\right)}{\mathbb{V}\mathrm{ar}\left(\ell\mathrm{n}(P_t^D/D_t)\right)}}_{CF_{objective}} \\ & + \underbrace{\frac{\mathbb{C}\mathrm{ov}\left(-\sum_{j=0}^{\infty}\xi^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}, \ell\mathrm{n}(P_t^D/D_t)\right)}{\mathbb{V}\mathrm{ar}\left(\ell\mathrm{n}(P_t^D/D_t)\right)}}_{DR_{objective}}. \end{aligned} \tag{29}$$

The first term on the right hand side of (29), $CF_{objective}$, is the contribution of changes in cash flow news to stock market movements, and the second term, $DR_{objective}$, is the contribution of changes in discount rate news to the stock market movements. These two terms approximately sum to one. By using future realized dividend growth and returns, this approach effectively imposes rational expectations. Most empirical studies that have conducted a Campbell-Shiller decomposition take this approach.

The second approach, on the other hand, takes subjective expectations on both sides of (28) and computes

$$1 \approx \underbrace{\frac{\mathbb{C}\text{ov}\left(\mathbb{E}_t^e[\sum_{j=0}^{\infty}\xi^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}], \ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{CF_{subjective}} + \underbrace{\frac{\mathbb{C}\text{ov}\left(-\mathbb{E}_t^e[\sum_{j=0}^{\infty}\xi^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}], \ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{DR_{subjective}}. \tag{30}$$

This approach examines the relationship between stock market movements and subjective expectations about future stock market returns and future dividend growth. The first term on the right hand side of (30), $CF_{subjective}$, is the contribution of changes in subjective expectations about cash flow news to stock market movements, and the second term, $DR_{subjective}$, is the contribution of changes in subjective expectations about discount rate news to stock market movements.

[Place Table 5 about here]

Table 5 reports the four coefficients, $CF_{objective}$, $DR_{objective}$, $CF_{subjective}$, $DR_{subjective}$, as well as their corresponding adjusted R-squared. These coefficients and R-squared are calculated using 10,000 years of monthly data simulated from the model. By using future realized dividend growth and stock market returns and therefore imposing rational expectations, we obtain $DR_{objective} = 0.98$ with a R-squared of 0.209 and $CF_{objective} = 0.02$ with a R-squared of 1.2×10^{-4}. Therefore, the model replicates the empirical finding of the volatility test literature that the variation of the price-dividend ratio comes primarily from discount rate variation (see Cochrane (2008) and Cochrane (2011)).

On the other hand, by relaxing the rational expectations assumption and using the agent's subjective expectations about dividend growth and returns, we obtain $DR_{subjective} = -0.08$ with a R-squared of 0.982 and $CF_{subjective} = 1.08$ with a R-squared of 0.984. This result draws a very different picture from earlier findings that use realized dividend growth and returns. First, stock market movements come primarily from changes in the agent's subjective expectations about future dividend growth: changes in subjective expectations about future cash flow news explain most of the variation of the price-dividend ratio, whereas changes in the rational expectations about future cash flow news do not. Second, changes in the agent's subjective expectations about future

returns make a small but *negative* contribution to explaining the variation of the price-dividend ratio—the subjective expectation about future returns is high rather than low when the price-dividend ratio is high—whereas changes in the rational expectations about future returns explain most of the variation of the price-dividend ratio. Empirically, de la O and Myers (2017) find that $DR_{subjective} = -0.09$ and $CF_{subjective} = 1.09$. These values match the theoretical values our model delivers.

Next, we examine the model's implication on the predictability of stock market returns. Empirically, Campbell and Shiller (1988) and Fama and French (1988) document that a regression of future log excess returns on the current log price-dividend ratio gives a negative and significant regression coefficient. Moreover, the predictive power of the price-dividend ratio is greater when future returns are calculated over longer horizons.

[Place Table 6 about here]

Table 6 reports the regression coefficient β_j and the adjusted R-squared for a regression of the log excess return of the stock market from time t to time $t+j$ on the current log price-dividend ratio

$$r^{D,e}_{t\to t+j} = \alpha_j + \beta_j \ell\mathrm{n}(P^D_t / D_t) + \varepsilon_{j,t} \tag{31}$$

over various time horizons j. We calculate the regression coefficients and the R-squared using 10,000 years of monthly data simulated from the model, and compare them side by side with the empirical values. Consistent with the data, β_j is negative and its magnitude increases as the time horizon j increases. The intuition for this model implication is as follows. A sequence of high past returns raises the agent's expectation about future returns, which in turn pushes up the current price-dividend ratio. Since the agent's expectation—the sentiment variable—tends to revert back to its mean, subsequent returns are low on average, giving rise to a negative regression coefficient in (31).

The magnitudes of the regression coefficient and the R-squared generated from the model are broadly consistent with the empirical values. One difference, however, is that in the model, the R-squared begins to decrease as the time horizon j increases beyond three years, whereas in the data, the R-squared keeps rising over longer horizons. To understand this difference, recall that we calibrate the model to the survey expectations by setting λ and χ to 0.18: the agent looks back a couple of years when forming beliefs about future returns. Given this parameter choice, the mean reversion of sentiment tends to occur over the first few years. Therefore, over longer horizons, no additional mean reversion in the agent's beliefs contributes to the predictability of stock market returns.

We now study the autocorrelations of asset prices. Empirically, log excess returns exhibit negative autocorrelations at lags longer than one year (Poterba and Summers (1988)), and price-dividend ratios are highly persistent at short lags. Table 7 presents the theoretical values and empirical values for these autocorrelations. It shows that the model captures both facts, but only qualitatively: the negative autocorrelations of log excess returns are generated by the mean rever-

sion of sentiment, whereas the persistence of the price-dividend ratio is driven by the persistence of sentiment. Quantitatively, however, the model generates a persistence for the price-dividend ratio that is much *lower* than the empirical value. The available survey evidence suggests that investors focus on just the past year or two when forming beliefs about future returns. Nonetheless, to match the empirical persistence of the price-dividend ratio, the agent needs to form beliefs about future returns based on many years of past returns.

[Place Table 7 about here]

We leave a careful reconciliation of the survey expectations about stock market returns and the observed persistence of the price-dividend ratio for future research. One possibility, however, is to introduce an interaction between financial frictions and investor sentiment. As documented by Lopez-Salido, Stein, and Zakrajsek (2016), various types of financial frictions are empirically more persistent than investor sentiment. As a result, financial frictions can serve as a propagation mechanism to prolong the impact of survey expectations on asset prices.

Finally, we investigate the model's implication for the correlation between stock market returns and consumption growth. Empirically, Hansen and Singleton (1982, 1983) document that this correlation is low. Nonetheless, most consumption-based asset pricing models generate a high, if not perfect, correlation between stock market returns and consumption growth. By imposing rational expectations, these models leave the consumption-based pricing kernels as the only source for stock market movements. In other words, stock market movements are driven by changes in consumption. As a result, the correlation between stock market returns and consumption growth is high.[20]

[Place Table 8 about here]

Table 8 reports both the model-implied values and the empirical values for the correlations between consumption growth and stock market returns. Consistent with the data, the model produces a low correlation: the correlation between annual log consumption growth and annual log excess returns is 0.19 in the model, and similarly it is 0.09 in the data. In comparison, the model of Campbell and Cochrane (1999) generates a correlation of 0.79, a much higher value. In our model, the agent extrapolates past price growth on the stock market, but she extrapolates past consumption growth to a much lesser extent: the bias in the agent's beliefs about consumption growth is small. In other words, the comovement between the agent's beliefs about returns and her beliefs about consumption growth is low. As a consequence, the model produces a low correlation between consumption growth and stock market returns.

The model also generates a small but negative correlation between the current consumption growth and the stock market return in the subsequent period, an implication that is consistent

[20] One exception is Barberis et al. (2001). They use "narrow framing," one component of prospect theory developed by Kahneman and Tversky (1979), to generate a low correlation between stock market returns and consumption growth. Specifically, they use power utility as the agent's preferences over consumption, but use prospect theory as the agent's preferences over financial wealth.

with the data. Recall that consumption growth and dividend growth are positively correlated. If the current consumption growth is high, dividend growth is also high on average, which leads the agent to drive up the current price, increasing the current return and the current sentiment level. In the subsequent period, sentiment reverts towards its mean value, giving rise to a low stock market return.

We conclude this section by discussing the role of rational arbitrageurs. The model has a representative agent whose beliefs are biased. One natural question to ask is: if we introduce rational arbitrageurs, to what extent can they counteract the mispricing caused by the behavioral agent and therefore attenuate the significance of the model implications? Developing such a two-agent model is beyond the scope of the paper. However, three observations suggest that our model implications will remain intact after taking rational arbitrageurs into account.

First, in an economy with both rational and behavioral agents who have recursive preferences, the behavioral agents may eventually dominate the market: there is a positive probability that they take up most of the wealth in the economy in the long run. This is a key finding in Borovicka (2016). It suggests that our model's implications can be the limiting implications of a model with both rational and behavioral agents in the initial period. Second, heterogeneity in investor beliefs is an additional source for excess volatility, so it can further amplify—rather than attenuate—our model implications.[21] Lastly, as pointed out by Barberis et al. (2015), extrapolative expectations are persistent in a dynamic model, which means that the behavioral agents have persistently heightened demand for the stock market following high stock market returns. The persistence of this demand prevents near-future stock market returns from becoming too low, which reduces the incentive of the rational agents to counteract mispricing. In other words, the persistence of extrapolative expectations limits the overall impact of rational arbitrageurs on asset prices.

III. Comparative Statics

In this section, we examine the sensitivity of the model implications with respect to changes in parameter values. We focus on parameters that either have dispersed estimates in the literature or can not be directly observed from the data. Specifically, for the utility parameters, we study how changes in γ, the relative risk aversion parameter, and ψ, the reciprocal of the elasticity of intertemporal substitution, affect the equity premium and the volatility of stock market returns. For the belief parameters, we focus on θ, the parameter that controls the extent to which the agent is behavioral, and χ and λ, the perceived transition intensities between the high- and low-mean price growth regimes. We look at how changes in θ affect the equity premium, the volatility of returns, the price-dividend ratio, and the average interest rate. We also examine how changes in θ, χ, and λ affect return predictability and the persistence of the price-dividend ratio.

[21] See Xiong (2013) for more discussion about this amplification mechanism.

Utility Parameters.—Figure 4 plots the long-run average of the equity premium and the volatility of stock market returns, each as a function of γ or ψ. The relative risk aversion is positively related to the equity premium but negatively related to the volatility of returns. Lower risk aversion naturally leads the agent to require a lower equity premium for risk compensation; reducing γ from 10 to 5, the model still explains 75% of the observed equity premium. At the same time, lower risk aversion strengthens the feedback loop since it increases the agent's demand for risky assets. Therefore, it leads to higher return volatility.

[Place Figure 4 about here]

Within the examined range, changes in the elasticity of intertemporal substitution does not significantly affect the average equity premium and the average volatility of returns. As a result, our model implications are quantitatively robust to changes in ψ. We provide a detailed explanation of this observation in the next section when we compare our model with the model of Bansal and Yaron (2004).

Belief Parameters.—Figure 5a plots the long-run average of the equity premium, the volatility of stock market returns, the price-dividend ratio, and the interest rate, each as a function of θ, the parameter that measures the extent to which the agent is behavioral. Setting θ to zero reduces the model to its rational benchmark. In this case, the equity premium is 0.23%; the return volatility equals the fundamental volatility of 11%; the price-dividend ratio stays constant at 135; and the interest rate stays at 2.35%. With $\theta = 0$, the model fails to match the long-run properties of the stock market. Increasing θ from zero allows the feedback loop described above to emerge, which generates excess volatility, pushes up the perceived equity premium, and significantly reduces the price-dividend ratio. At the same time, a higher θ—and the agent's more extrapolative beliefs about the stock market returns—leads the agent to perceive a higher persistence in dividend growth, which, under Epstein-Zin preferences, is significantly priced, causing the perceived equity premium to further rise. Finally, a higher θ pushes the true equity premium further above the perceived one.

[Place Figure 5a about here]

Overall, an 1% increase in θ leads to a 0.09% increase in the equity premium and a 0.29% increase in the volatility of returns. On the other hand, the effect of θ on the interest rate is small. A higher θ increases the extrapolation bias in the agent's beliefs about stock market returns, but does not significantly affect her beliefs about consumption growth which determines the equilibrium interest rate.

Figure 5b examines how changes in θ, χ, and λ affect the predictability of stock market returns and the persistence of the price-dividend ratio. Here the predictability of returns is measured by the slope coefficient for a regression of the next year's log excess return on the current log price-dividend ratio, and the persistence of the price-dividend ratio is measured by the one-year autocorrelation of log price-dividend ratios.

[Place Figure 5b about here]

Figure 5b shows that higher values of θ, χ, and λ lead to stronger predictability of returns and a lower persistence of the price-dividend ratio. Higher values of χ and λ—that is, higher perceived transition intensities between the high- and low-mean price growth regimes—suggest that the agent focuses on a shorter history of past return realizations when forming beliefs about future returns. A higher value of θ has a similar implication: it suggests that the agent exhibits a stronger extrapolation bias, which means that the agent deviates more from a rational agent whose beliefs depend on both recent and distant past returns. In other words, with a higher θ, the agent relies more heavily on recent past returns when forming beliefs about future returns. Therefore, higher values of θ, χ, and λ all lead to a stronger degree of mean reversion in sentiment, which in turn gives rise to stronger predictability of returns and a lower persistence of the price-dividend ratio.

The comparative statics results in Figure 5b are consistent with recent empirical findings. At the aggregate level, Cassella and Gulen (2017) find that during periods when investors' expectations about future returns depend on both recent and distant past returns, the price-dividend ratio does not strongly predict the next year's return. Conversely, during periods when investors' expectations depend primarily on recent past returns, the price-dividend ratio strongly predicts the next year's return. In our model, higher values of θ, χ, and λ lead to a higher ϕ, and therefore the agent's expectations about future returns depend more heavily on recent past returns. In the meantime, they also lead to stronger return predictability. Together, the model implies that when the agent form beliefs based on a short history of past returns, the predictability of returns is strong. To give an example, increasing θ from 0.05 to 0.5 changes ϕ from 0.37 to 0.43. At the same time, it changes β_1, the slope coefficient for a regression of the next year's log excess return on the current log price-dividend ratio, from -0.32 to -0.72; the corresponding R-squared increases from 0.001 to 0.13. In the cross-section, Da, Huang, and Jin (2017) show that stocks associated with a larger extrapolation bias—beliefs of forecasters on these stocks depend more strongly on recent past returns—exhibit stronger return reversals. Applying our model to individual stocks gives the same prediction.

IV. Comparison with Rational Expectations Models

In this section, we provide a quantitative comparison between our model and several models with rational expectations. First, we look at a rational expectations model that is most analogous to our behavioral model, one in which the regime-switching process characterized in Section I represents the *true* data generating process. We then examine differences between our model and the rational expectations models of Campbell and Cochrane (1999), Bansal and Yaron (2004), and Bansal et al. (2012).

The True Regime-Switching Model.—The rational expectations model most analogous to our

model is the one that assumes the regime-switching process characterized in equations (6) and (7) as the true data generating process. In this case, the true evolution of the stock market price is

$$dP_t^D/P_t^D = [(1-\theta)g_D + \theta\tilde{\mu}_{S,t}]dt + \sigma_P^D(S_t)d\omega_t^P, \tag{32}$$

where ω_t^P is a standard Brownian motion. As with the behavioral model, the agent in this model does not directly observe the latent variable $\tilde{\mu}_{S,t}$. Instead, she uses past stock market prices to form a Bayesian estimate of $\tilde{\mu}_{S,t}$: $S_t = \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^P]$. That is, the perceived evolution of stock market price in (8) is fully rational. We further assume that the perceived dividend process (10) and the perceived consumption process (15) are also rational.

By construction, this rational expectations model shares the same equilibrium prices with our behavioral model: the solutions to the differential equations of (17) and (18) also apply to this model. Nonetheless, these two models have statistical properties that are significantly different. One difference, for instance, lies in the models' implications for the predictability of stock market returns.

[Place Table 9 about here]

Table 9 reports the regression coefficient β_j and the adjusted R-squared for a regression of the log excess return of the stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$ over various time horizons j (one to seven years), now using the true regime-switching model. Table 9 shows that the model fails to produce the predictability of stock market returns documented in Campbell and Shiller (1988) and Fama and French (1988): both the regression coefficients and the R-squared are close to zero. In contrast, Table 6 suggests that the behavioral model produces the observed predictability of stock market returns.

[Place Figure 6 about here]

The lack of return predictability in the true regime-switching model is due to two reasons. First, with rational expectations, the agent's beliefs about stock market returns are on average correct. Therefore, following high past price growth, the agent properly anticipates high future price growth, which pushes down the dividend yield in equilibrium, leading to flat returns in subsequent periods. Second, with rational expectations, the evolution of sentiment is more volatile and less mean-reverting compared to the one in the behavioral model. This further reduces the statistical significance of the return predictability regression. Figure 6 plots the evolution of sentiment in the true regime-switching model: the standard deviation of the sentiment distribution is 4.7%, almost twice as high as the standard deviation of sentiment in the behavioral model.

Campbell and Cochrane (1999).—The habit formation model of Campbell and Cochrane (1999) is a rational expectations model with complete information. In their model, the agent's subjective expectations are the same as rational expectations. To examine the model's implication for investor

expectations, Table 10 reports the regression coefficients and t-statistics for a regression of rational expectations about future returns on either the past twelve-month return or the current log price-dividend ratio. The regressions are based on simulated data from Campbell and Cochrane (1999). The regression coefficient on the past twelve-month return is negative and insignificant for simulated samples of either 15 years or 50 years.

[Place Table 10 about here]

In Campbell and Cochrane (1999), high past returns primarily result from high past dividend growth, which also implies high past consumption growth since dividend growth and consumption growth are positively correlated. In this case, the difference between the consumption level and the habit level increases, reducing the effective risk aversion of the agent and therefore lowering the expected returns moving forward. This mechanism leads to a negative slope coefficient for a regression of the agent's expectations—this is also the rational expectations—about future stock market returns on past returns, in contrast to the positive coefficient obtained using either survey data or simulated data from our behavioral model. At the same time, since past dividend growth—the main determinant of past returns—has a low correlation with past consumption growth—the main determinant of future returns, the correlation between past returns and future returns is also low, giving rise to the low t-statistic in Table 10 for the regression coefficient on past returns.

Bansal and Yaron (2004).—Table 11 reports the regression coefficients and t-statistics for regressing four measures of rational expectations of return—raw return or excess return, with or without dividend yield—either on the past twelve-month return or on the current log price-dividend ratio. The regressions are based on simulated data from Bansal and Yaron (2004). Interestingly, their model generates, to some extent, extrapolative expectations about future raw returns: a regression of the agent's expectations—this is also the rational expectations—about the next twelve-month total returns on past twelve-month total returns yields a coefficient of 2.5% with a t-statistic of 2.4 for a 15-year simulated sample; the regression coefficient is 3.0% with a t-statistic of 3.8 for a 50-year simulated sample.

[Place Table 11 about here]

In Bansal and Yaron (2004), consumption growth and dividend growth share a stochastic yet persistent component. High past stock market returns are typically caused by positive shocks on this common component, which, given its persistence, implies high dividend growth moving forward, hence pushing up future raw returns. This mechanism gives rise to the agent's extrapolative beliefs about future raw returns. Nevertheless, precisely because consumption growth and dividend growth share a common persistent component, high dividend growth tends to coincide with high consumption growth, which implies a high interest rate in equilibrium. That is, when raw returns are high, the interest rate is also high. As a result, the agent's expectation about future *excess*

returns is not extrapolative: regressing the agent's expectation about future excess returns on past returns, the regression coefficient and the t-statistic are both close to zero.

These regression results highlight one fundamental difference between our model and the model of Bansal and Yaron (2004). In our model, the agent extrapolates past returns on the stock market, but extrapolates past consumption growth much less. The comovement between the agent's beliefs about returns and her beliefs about consumption growth is low. On the contrary, in Bansal and Yaron (2004), the comovement between the agent's beliefs about stock market returns—they rationally drive returns—and her beliefs about consumption growth—they determine the interest rate in equilibrium—is high. Therefore, the agent in our model is extrapolative of both raw returns and excess returns, whereas the agent in Bansal and Yaron (2004) is only extrapolative of raw returns.

Furthermore, as shown in Figure 4, our model produces an equity premium and return volatility that do not vary significantly with respect to changes in the elasticity of intertemporal substitution. On the contrary, the long-run risks models cannot generate a high equity premium with a low elasticity of intertemporal substitution. For instance, setting the elasticity of intertemporal substitution to 0.5, our model generates an equity premium of 7.1% (measured as the rational expectation of excess returns), while the model of Bansal and Yaron (2004) produces an equity premium between 1% and 2%. Given this contrast, our model does not face the challenge of defending a greater than one elasticity of intertemporal substitution.

To understand this model difference, first note that sentiment variable, the state variable in our model, is much less persistent than the stochastic component of dividend and consumption growth in Bansal and Yaron (2004), allowing the equilibrium interest rate in our model to be less responsive to changes in the elasticity of intertemporal substitution while reducing the equity premium at the same time. On the other hand, the perceived dividend growth in our model depends *more* strongly on the state variable of sentiment, leading to a higher perceived equity premium; in the model of Bansal and Yaron (2004), dividend growth depends less strongly on the stochastic growth component.[22] In addition, the true average equity premium in our model is above the perceived one, allowing the equity premium to be high even when the elasticity of intertemporal substitution is low.

Bansal et al. (2012).—Table 12 repeats the regression analyses of Table 11 using the model of Bansal et al. (2012). Compared to the original model of Bansal and Yaron (2004), Bansal et al. (2012) introduces additional time variation in the long-run risks that further reduces the model's ability to generate extrapolative expectations: when regressing the expectation of raw returns on past returns, the coefficient now becomes insignificant.

[Place Table 12 about here]

We complete our discussion for this section by making two remarks. First, the persistence

[22] Bansal and Yaron (2004) set their "leverage parameter" ϕ at 3.5. In comparison, with return extrapolation, our model effectively sets ϕ at 14.5, a much higher value.

introduced by the long-run risks models on consumption growth and dividend growth leads to excess predictability, the notion that future consumption and dividend growths are excessively predicted by current variables such as the price-dividend ratio (see Beeler and Campbell (2012) and Collin-Dufresne et al. (2016b) for a detailed discussion). Our model does not give rise to excess predictability: return extrapolation in the model only generates perceived but not true long-run risks, and therefore the true consumption and dividend growths remain unpredictable.

Second, the rational expectations models and our model generate different implications regarding the source of stock market movements. In models with rational expectations, the volatility of the price-dividend ratio comes from either the variation in discount rate news or the variation in cash flow news: by using either the agent's subjective expectations or the rational expectations, a Campbell-Shiller decomposition yields the same result. In our model, they yield very different results. By using rational expectations, the volatility of the price-dividend ratio comes from the variation in discount rate news. However, by using the agent's subjective expectations, the volatility of the price-dividend ratio comes from changes in the expectations of cash flow news. The recent empirical findings of de la O and Myers (2017) support these model implications.

V. Fundamental Extrapolation

A literature in behavioral finance focuses on fundamental extrapolation, the notion that some investors hold extrapolative expectations about *fundamentals* such as dividend growth or GDP growth (Barberis et al. (1998); Fuster et al. (2011); Choi and Mertens (2013); Alti and Tetlock (2014); Hirshleifer et al. (2015)). In this section, we provide a quantitative comparison between our model and a model with fundamental extrapolation. To facilitate the comparison, we keep the two models almost identical. The only difference is that, in the model with fundamental extrapolation, sentiment is constructed from past dividend growth, whereas in the model with return extrapolation, sentiment is constructed from past price growth. Below we first briefly describe the key assumptions in this fundamental extrapolation model. We then discuss the model implications.

Model Setup.—With fundamental extrapolation, the agent believes that the expected growth rate of *dividends*—instead of the expected growth rate of stock market prices in the case of return extrapolation—is governed by $(1-\theta)g_D + \theta\tilde{\mu}_{S,t}$, where $\tilde{\mu}_{S,t}$ is a latent variable that follows a regime-switching process described in Section I. The agent does not directly observe the latent variable $\tilde{\mu}_{S,t}$. Instead, she computes its expected value given the history of past dividend growth: $S_t \equiv \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^D]$. She then applies the optimal filtering theory and derives

$$\begin{aligned} dS_t =& (\lambda\mu_H + \chi\mu_L - (\lambda+\chi)S_t)dt + \sigma_D^{-1}\theta(\mu_H - S_t)(S_t - \mu_L)d\omega_t^e \\ \equiv& \mu_S^e(S_t)dt + \sigma_S(S_t)d\omega_t^e, \end{aligned} \tag{33}$$

where $d\omega_t^e \equiv [dD_t/D_t - (1-\theta)g_D dt - \theta S_t dt]/\sigma_D$ is a standard Brownian innovation term from the

agent's perspective. That is, she perceives the evolution of dividend as

$$dD_t/D_t = g_D^e(S_t)dt + \sigma_D d\omega_t^e, \tag{34}$$

where

$$g_D^e(S_t) = (1-\theta)g_D + \theta S_t. \tag{35}$$

In other words, the agent's expectation about dividend growth $g_D^e(S_t)$ is a linear combination of a rational component g_D and a sentiment component S_t constructed from past dividend growth. On the other hand, the perceived evolution of stock market price can be derived as

$$dP_t^D/P_t^D = \mu_P^{D,e}(S_t)dt + \sigma_P^D(S_t)d\omega_t^e, \tag{36}$$

where

$$\begin{aligned} \sigma_P^D(S) =& \sigma_D + (f'/f)\sigma_D^{-1}\theta(\mu_H - S)(S - \mu_L), \\ \mu_P^{D,e}(S) =& (f'/f)\mu_S^e + \tfrac{1}{2}(f''/f)\sigma_S^2 + (1-\theta)g_D + \theta S - \sigma_D^2 + \sigma_D\sigma_P^D(S). \end{aligned} \tag{37}$$

As before, f is defined as the price-dividend ratio of the stock market.

As with the return extrapolation model, equations (17) and (18) determine f and l, the price-dividend ratio and the wealth-consumption ratio, except that μ_S, σ_S, g_D^e, $\mu_P^{D,e}$, and σ_P^D are now from (33), (35), and (37).

Model Implications.—We first examine the model's implications for investor expectations. Table 13 reports the regression coefficient and the t-statistic, the intercept, as well as the adjusted R-squared, for regressing the four measures of return expectations on either the past twelve-month cumulative raw returns or the current log price-dividend ratio. With fundamental extrapolation, the regression coefficient on past returns and the t-statistic are both close to zero.

[Place Table 13 about here]

Suppose past dividend growth has been high. On the one hand, it results in high past returns. On the other hand, fundamental extrapolation leads the agent to expect high dividend growth moving forward, but *not* high returns: the agent's expectation of returns is determined by her perception of risk, which does not change significantly following high dividend growth. Therefore, a fundamental extrapolation model with a representative agent faces challenges in explaining survey expectations about returns. In contrast, our return extrapolation model is constructed to explain these survey expectations.[23]

[Place Table 14 about here]

[23] A fundamental extrapolation model with *heterogeneous* agents—for instance, one with both an agent who extrapolates past dividend growth and an agent who is fully rational—can potentially generate extrapolative expectations of returns for the behavioral agent in the model. See the model of Ehling et al. (2015) as an example.

Table 14 analyzes the model's fit with the long-run properties of the stock market. Using the same parameters that allow the return extrapolation model to well explain the important moments of the stock market, the fundamental extrapolation model generates a much lower equity premium and lower excess volatility. Quantitatively, fundamental extrapolation produces 37.9% of the equity premium and 74.4% of excess volatility that return extrapolation produces.

This quantitative comparison highlights the importance of the feedback loop described above in matching asset pricing facts. With return extrapolation, the feedback loop emerges because extrapolative beliefs are applied to the stock market return, a variable that is *endogenously* determined in equilibrium. With fundamental extrapolation, however, the feedback loop is absent because extrapolative beliefs are applied to dividend growth, a variable that is exogenous in the model: high past dividend growth makes the agent optimistic about future dividend growth and therefore pushes up the current price, but the higher price does not further affect the agent's beliefs about future dividend growth.

The feedback loop also points to a methodological contribution of the paper. Equation (33) shows that in a fundamental extrapolation model, sentiment S, the state variable that drives asset prices dynamics, can be exogenously specified without solving the equilibrium; this greatly simplifies the model. On the other hand, with return extrapolation, sentiment S determines—and is endogenously determined by—asset prices. As a result, such a model requires solving beliefs and asset prices simultaneously, and therefore imposes a greater modelling challenge. Our numerical approach to solving a system of differential equations provides a solution to this challenge.

VI. Conclusion

We build a new return extrapolation model that can be brought to the data quantitatively. With the agent's beliefs calibrated to fit the extrapolative expectations documented in surveys, the model matches the long-run properties of stock market prices: it generates a high average equity premium, significant excess volatility, a low average interest rate, low interest rate volatility, and a price-dividend ratio whose average level is similar to the empirical one. The model also matches the dynamic behavior of stock market prices: it produces the long-horizon predictability of stock market returns, and generates the observed low correlation between stock market returns and consumption growth. Overall, the model implications are quantitatively robust to changes in the elasticity of intertemporal substitution. In contrast, long-run risks models fail to generate a high equity premium with a low elasticity of intertemporal substitution.

The model makes novel implications. The model shows a fundamental connection between the agent's beliefs about returns and her beliefs about dividend growth: when past returns have been high, not only does the agent expect high returns, she also expects high dividend growth, at a pace that exceeds her expectations about future returns. This belief dynamic makes the variation in the agent's expectations about dividend growth an important source of stock market movements: changes in the agent's subjective expectations about future dividend growth account for most of

the volatility of the price-dividend ratio. This result differs from the traditional view that variation in cash flow news does not explain stock market movements and hence stresses the importance of expectations data in understanding aggregate stock market behavior. These model implications are consistent with recent empirical findings.

Our analysis has left several important issues for future work. First, when calibrated with the survey expectations data, the model predicts a persistence of the price-dividend ratio that is much lower than its empirical value. To reconcile the survey expectations about stock market returns with the observed persistence of the price-dividend ratio, we need a deeper understanding about how investors form beliefs. Second, the extrapolation frameworks are closely linked to theories of model uncertainty. A careful investigation of this connection may produce useful insights to both literatures. Finally, our representative-agent model neglects an important channel that affects asset prices: the time-varying fraction of wealth held by behavioral agents. Explicitly incorporating rational agents into our framework can lead to additional implications.

Appendices

A. *Derivation of the Differential Equations (17) and (18)*

For the subjective Euler equation (4), setting $\tilde{R}_{j,t+dt}$, the return on the tradeable asset, to the gross return on the stock market, the equation becomes

$$\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)dt/(1-\psi)}\left(\frac{\tilde{C}_{t+dt}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)}\left(\frac{\tilde{P}^C_{t+dt}+\tilde{C}_t dt}{P^C_t}\right)^{(\psi-\gamma)/(1-\psi)}\frac{\tilde{P}^D_{t+dt}+\tilde{D}_{t+dt}dt}{P^D_t}\right]=1. \tag{A.1}$$

Using Taylor expansion, (A.1) becomes

$$\begin{aligned}&\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)dt/(1-\psi)}\left(\tilde{C}_{t+dt}\right)^{-\psi(1-\gamma)/(1-\psi)}(\tilde{P}^C_{t+dt})^{(\psi-\gamma)/(1-\psi)}\tilde{P}^D_{t+dt}\left(1+\tfrac{\psi-\gamma}{1-\psi}\tfrac{\tilde{C}_{t+dt}}{\tilde{P}^C_{t+dt}}dt+\tfrac{\tilde{D}_{t+dt}}{\tilde{P}^D_{t+dt}}dt\right)\right]\\&=C_t^{-\psi(1-\gamma)/(1-\psi)}(P^C_t)^{(\psi-\gamma)/(1-\psi)}P^D_t.\end{aligned} \tag{A.2}$$

Rearranging terms gives

$$0=\mathbb{E}_t^e\left[\begin{array}{l}d(\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}Df)+\frac{\psi-\gamma}{1-\psi}\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(2\psi-\gamma-1)/(1-\psi)}Dfdt\\+\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}Ddt\end{array}\right], \tag{A.3}$$

where $\Theta(C,t)\equiv e^{-\delta(1-\gamma)t/(1-\psi)}C^{-\psi(1-\gamma)/(1-\psi)}$. By Ito's lemma, (A.3) leads to

$$0=\mathbb{E}_t^e\left[\begin{array}{l}-\frac{\delta(1-\gamma)}{1-\psi}dt-\gamma(dC/C)+(dD/D)+(df/f)+\frac{\psi-\gamma}{1-\psi}(dl/l)+\frac{\gamma(\gamma+1)}{2}(dC/C)^2\\+\frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(dl/l)^2-\frac{\gamma(\psi-\gamma)}{1-\psi}(dC/C)(dl/l)-\gamma(dC/C)(dD/D)-\gamma(dC/C)(df/f)\\+\frac{\psi-\gamma}{1-\psi}(dl/l)(dD/D)+\frac{\psi-\gamma}{1-\psi}(dl/l)(df/f)+(df/f)(dD/D)+\frac{\psi-\gamma}{1-\psi}l^{-1}dt+f^{-1}dt\end{array}\right]. \tag{A.4}$$

Using (7), (10), and (15) to further simplify (A.4) gives (17).

Setting $\tilde{R}_{j,t+dt}$ in (4) to the gross return on the Lucas tree, the subjective Euler equation (4) becomes

$$\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)dt/(1-\psi)}\left(\frac{\tilde{C}_{t+dt}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)}\left(\frac{\tilde{P}^C_{t+dt}+\tilde{C}_t dt}{P^C_t}\right)^{(1-\gamma)/(1-\psi)}\right]=1. \tag{A.5}$$

Rearranging terms yields

$$0=\mathbb{E}_t^e\left[d(\Theta C^{(1-\gamma)/(1-\psi)}l^{(1-\gamma)/(1-\psi)})+\tfrac{1-\gamma}{1-\psi}\Theta C^{(1-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}dt\right]. \tag{A.6}$$

By Ito's lemma, (A.6) leads to

$$0 = \mathbb{E}_t^e \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta dt - (\gamma-1)(dC/C) + \frac{\gamma(\gamma-1)}{2}(dC/C)^2 + \frac{1-\gamma}{1-\psi}(dl/l) \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(dl/l)^2 + \frac{(1-\gamma)^2}{1-\psi}(dC/C)(dl/l) + \frac{1-\gamma}{1-\psi}l^{-1}dt \end{array} \right]. \tag{A.7}$$

Using (7) and (15) to further simplify (A.7) gives (18). ■

B. *Steady-State Distribution for Sentiment*

Below we provide all the terms necessary for solving the Kolmogorov forward equation (26). From the expression of σ_S in (7)

$$\begin{aligned} \sigma_S' =& \frac{\theta\sigma_P^D(\mu_H+\mu_L-2S) - \theta(\mu_H-S)(S-\mu_L)(\sigma_P^D)'}{(\sigma_P^D)^2}, \\ \sigma_S'' =& \frac{\theta(\mu_H-S)(S-\mu_L)\{2[(\sigma_P^D)']^2 - \sigma_P^D(\sigma_P^D)''\}}{(\sigma_P^D)^3} - 2\theta\frac{\sigma_P^D(\sigma_P^D)'(\mu_H+\mu_L-2S)+(\sigma_P^D)^2}{(\sigma_P^D)^3}. \end{aligned} \tag{B.1}$$

For the expression of μ_S^e in (7) and the expression of g_D^e in (11)

$$\begin{aligned} (\mu_S^e)' =& -(\lambda+\chi), \\ (g_D^e)' =& \theta - \sigma_D(\sigma_P^D)' - \mu'_S(f'/f) - \mu_S[f''/f - (f')^2/f^2] \\ & - \sigma_S\sigma_S'(f''/f) - \tfrac{1}{2}\sigma_S^2[f'''/f - f'f''/f^2], \end{aligned} \tag{B.2}$$

where σ_P^D is from (11), and $(\sigma_P^D)'$ and $(\sigma_P^D)''$ are

$$\begin{aligned} (\sigma_P^D)' =& \frac{\theta(\mu_H+\mu_L-2S)(f'/f) + \theta(\mu_H-S)(S-\mu_L)[f''/f-(f')^2/f^2]}{\sqrt{\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)}}, \\ (\sigma_P^D)'' =& -\frac{2\{\theta(\mu_H+\mu_L-2S)(f'/f) + \theta(\mu_H-S)(S-\mu_L)[f''/f-(f')^2/f^2]\}^2}{[\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)]^{3/2}} \\ & + \frac{-2\theta f'/f + 2\theta(\mu_H+\mu_L-2S)[f''/f-(f')^2/f^2]}{\sqrt{\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)}} \\ & + \frac{\theta(\mu_H-S)(S-\mu_L)[f'''/f - 3(f'f'')/f^2 + 2(f')^3/f^3]}{\sqrt{\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)}}. \end{aligned} \tag{B.3}$$

■

C. *Numerical Procedure for Solving the Equilibrium*

We use a projection method with Chebyshev polynomials to jointly solve the two differential equations (17) and (18). The value of the sentiment variable S ranges from μ_L to μ_H, whereas the domain for Chebyshev polynomials is [–1, 1]. Therefore, we transform S to a new state variable z

$$z \equiv aS + b, \qquad \text{where} \quad a = \frac{2}{\mu_H - \mu_L}, \; b = -\frac{\mu_H + \mu_L}{\mu_H - \mu_L}, \tag{C.1}$$

and we define $h(z) \equiv f(S(z))$ and $j(z) \equiv l(S(z))$. Equations (17) and (18) can be rewritten as

$$0 = \left[\begin{array}{l} -\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(h'/h) + \frac{\psi-\gamma}{1-\psi}(j'/j)]a\mu_S^e + \frac{1}{2}[(h''/h) + \frac{\psi-\gamma}{1-\psi}(j''/j)]a^2\sigma_S^2 \\ +\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(aj'/j)^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(aj'/j) - \gamma\rho\sigma_C\sigma_D - \gamma\rho\sigma_C\sigma_S(ah'/h) \\ +\frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(aj'/j) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2a^2(j'/j)(h'/h) + \sigma_D\sigma_S(ah'/h) + \frac{\psi-\gamma}{1-\psi}j^{-1} + h^{-1} \end{array} \right] \tag{C.2}$$

and

$$0 = \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(aj'/j)\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(a^2j''/j)\sigma_S^2 \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(aj'/j)^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(aj'/j) + \frac{1-\gamma}{1-\psi}j^{-1} \end{array} \right]. \tag{C.3}$$

We approximate h and j by

$$\hat{h}(z) = \sum_{r=0}^{n} a_r T_r(z), \qquad \hat{l}(z) = \sum_{r=0}^{m} b_r T_r(z), \tag{C.4}$$

where $T_r(z)$ is the r^{th} degree Chebyshev polynomial of the first kind.[24] The projection method chooses the coefficients $\{a_r\}_{r=0}^n$ and $\{b_r\}_{r=0}^m$ so that the differential equations are *approximately* satisfied. One criterion for a sufficient approximation is to minimize the weighted sum of squared errors

$$\begin{aligned} &\sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l} -\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(\hat{h}'/\hat{h}) + \frac{\psi-\gamma}{1-\psi}(\hat{j}'/\hat{j})]a\mu_S^e + \frac{1}{2}[(\hat{h}''/\hat{h}) + \frac{\psi-\gamma}{1-\psi}(\hat{j}''/\hat{j})]a^2\sigma_S^2 \\ +\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(a\hat{j}'/\hat{j})^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(a\hat{j}'/\hat{j}) - \gamma\rho\sigma_C\sigma_D \\ -\gamma\rho\sigma_C\sigma_S(a\hat{h}'/\hat{h}) + \frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(a\hat{j}'/\hat{j}) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2a^2(\hat{j}'/\hat{j})(\hat{h}'/\hat{h}) + \sigma_D\sigma_S(a\hat{h}'/\hat{h}) \\ +\frac{\psi-\gamma}{1-\psi}\hat{j}^{-1} + \hat{h}^{-1} \end{array} \right]_{z=z_i}^2 \\ &+\sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(a\hat{j}'/\hat{j})\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(a^2\hat{j}''/\hat{j})\sigma_S^2 \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(a\hat{j}'/\hat{j})^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(a\hat{j}'/\hat{j}) + \frac{1-\gamma}{1-\psi}\hat{j}^{-1} \end{array} \right]_{z=z_i}^2, \end{aligned} \tag{C.5}$$

where $\{z_i\}_{i=0}^N$ are the N zeros of $T_N(z)$. By the Chebyshev interpolation theorem, if N is suffi-

[24] See Mason and Handscomb (2003) for a detailed discussion of the properties of Chebyshev polynomials.

ciently larger than n and m, and if the sum of weighted square in (C.5) is sufficiently small, the approximated functions $\hat{h}(z)$ and $\hat{l}(z)$ are sufficiently close to the true solutions.

For the numerical results in the main text, we set $m = 40$, $n = 40$, $N = 400$. We then apply the Levenberg-Marquardt algorithm and obtain a minimized sum of squared errors less than 10^{-11}. The small size of the total error indicates convergence of the numerical solution. The solution is also insensitive to the choice of n, m, or N. Together, these findings suggest that the numerical solutions are a sufficient approximation for the true h and j functions.

The same numerical procedure is applied to solving the Kolmogorov forward equation (26). ■

REFERENCES

Alti, Aydogan, and Paul Tetlock, 2014, Biased beliefs, asset prices, and investment: A structural approach, *Journal of Finance* 69, 325–361.

Amromin, Gene, and Steven A. Sharpe, 2013, Expectation of risk and return among household investors: Are their sharpe ratios countercyclical?, Working paper, Federal Reserve Board, Washington DC.

Bacchetta, Philippe, Elmar Mertens, and Eric van Wincoop, 2009, Predictability in financial markets: What does survey expectations tell us?, *Journal of International Money and Finance* 28, 406–426.

Bansal, Ravi, Dana Kiku, and Amir Yaron, 2012, An empirical evaluation of the long-run risks model for asset prices, *Critical Finance Review* 183–221.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer, 2015, X-capm: An extrapolative capital asset pricing model, *Journal of Financial Economics* 115, 1–24.

Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer, 2017, Extrapolation and bubbles, *Journal of Financial Economics* forthcoming.

Barberis, Nicholas, Ming Huang, and Tano Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* 116, 1–53.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307–343.

Baron, Matthew, and Wei Xiong, 2015, Credit expansion and neglected crash risk, Working paper, Princeton University.

Barro, Robert, 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Beeler, Jason, and John Y. Campbell, 2012, The long-run risks model and aggregate asset prices: An empirical assessment, *Critical Finance Review* 141–182.

Bidder, Rhys, and Ian Dew-Becker, 2016, Long-run risk is the worst-case scenario, *American Economic Review* 106, 2494–2527.

Borovicka, Jaroslav, 2016, Survival and long-run dynamics with heterogeneous beliefs under recursive preferences, Working paper, New York University.

Campbell, John Y., 2003, Consumption-based asset pricing, in George M. Constantinides, Milton Harris, and Rene M. Stulz, eds., *Handbook of the Economics of Finance*, volume 1B, chapter 13, 801–885 (Elsevier: Amsterdam).

Campbell, John Y., and John Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, John Y., and Robert Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

Cassella, Stefano, and Huseyin Gulen, 2017, Extrapolation bias and the predictability of stock returns by price-scaled variables, *Review of Financial Studies* forthcoming.

Chen, Long, Zhi Da, and Xinlei Zhao, 2013, What drives stock market movements?, *Review of Financial Studies* 16, 841–876.

Choi, James, and Thomas Mertens, 2013, Extrapolative expectations and the equity premium, Working paper, New York University and Yale University.

Cochrane, John H., 2008, The dog that did not bark: A defense of return predictability, *Review of Financial Studies* 21, 1533–1575.

Cochrane, John H., 2011, Presidential address: Discount rates, *Journal of Finance* 66, 1047–1108.

Collin-Dufresne, Pierre, Michael Johannes, and Lars A. Lochstoer, 2016a, Asset pricing when 'this time is different', Working paper, Ecole Polytechnique Federale de Lausanne and Columbia Business School.

Collin-Dufresne, Pierre, Michael Johannes, and Lars A. Lochstoer, 2016b, Parameter learning in general equilibrium: The asset pricing implications, *American Economic Review* 106, 664–698.

Cutler, David M., James M. Poterba, and Lawrence H. Summers, 1990, Speculative dynamics and the role of feedback traders, *American Economic Review Papers and Proceedings* 80, 63–68.

Da, Zhi, Xing Huang, and Lawrence Jin, 2017, Extrapolative beliefs in the cross-section: What can we learn from the crowds?, Working paper, University of Notre Dame, Washington University at St. Louis, and California Institute of Technology.

de la O, Ricardo, and Sean Myers, 2017, Subjective cash flows and discount rates, Working paper, Stanford University.

DeLong, Bradford, Andrei Shleifer, Lawrence Summers, and Robert Waldmann, 1990, Positive feedback investment strategies and destablizing rational speculation, *The Journal of Finance* 45, 379–395.

Ehling, Paul, Alessandro Graniero, and Christian Heyerdahl-Larsen, 2015, Asset prices and portfolio choice with learning from experience, Working paper, BI Norwegian Business School and London Business School.

Epstein, Larry, and Stanley Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Epstein, Larry, and Stanley Zin, 1991, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis, *Journal of Political Economy* 99, 263–286.

Fama, Eugene, and Kenneth French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3–25.

Fuster, Andreas, Benjamin Hebert, and David Laibson, 2011, Natural expectations, macroeconomic dynamics, and asset pricing, *NBER Macroeconomics Annual* 26, 1–48.

Gabaix, Xavier, 2012, Variable rare disasters: An exactly solved framework for ten puzzles in macro–finance, *Quarterly Journal of Economics* 127, 645–700.

Greenwood, Robin, Sam Hanson, and Lawrence Jin, 2016, A model of credit market sentiment, Working paper, California Institute of Technology and Harvard University.

Greenwood, Robin, and Samuel G. Hanson, 2013, Issuer quality and corporate bond returns, *Review of Financial Studies* 26, 1483–1525.

Greenwood, Robin, and Andrei Shleifer, 2014, Expectations of returns and expected returns, *Review of Financial Studies* 27, 714–746.

Hansen, Lars, and Kenneth Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica* 50, 1269–1286.

Hansen, Lars, and Kenneth Singleton, 1983, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, *Journal of Political Economy* 91, 249–265.

Hartzmark, Samuel M., and David H. Solomon, 2017, Reconsidering returns, Working paper, University of Chicago and Boston College.

Hirshleifer, David, Jun Li, and Jianfeng Yu, 2015, Asset pricing in production economies with extrapolative expectations, *Journal of Monetary Economics* 76, 87–106.

Hong, Harrison, and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143–2184.

Huang, Xing, 2016, Mark twain's cat: Investment experience, categorical thinking, and stock selection, Working paper, Michigan State University.

Jin, Lawrence J., 2015, A speculative asset pricing model of financial instability, Working paper, California Institute of Technology.

Kahneman, Daniel, and Amos Tversky, 1979, Prospect theory: An analysis of decision under risk, *Econometrica* 47, 263–292.

Koijen, Ralph S. J., Maik Schmeling, and Evert B. Vrugt, 2015, Survey expectations of returns and asset pricing puzzles, Working paper, London Business School.

Kuchler, Theresa, and Basit Zafar, 2016, Personal experiences and expectations about aggregate outcomes, Working paper, New York University and Federal Reserve Bank of New York.

Lipster, Robert, and Albert Shiryaev, 2001, *Statistics of Random Processes I, General Theory*, second edition (Springer-Verlag, New York).

Lopez-Salido, David, Jeremy C. Stein, and Egon Zakrajsek, 2016, Credit-market sentiment and the business cycle .

Malmendier, Ulrike, and Stefan Nagel, 2011, Depression-babies: Do macroeconomic experiences affect risk-taking?, *Quarterly Journal of Economics* 126, 373–416.

Malmendier, Ulrike, and Stefan Nagel, 2013, Learning from inflation experiences, *Quarterly Journal of Economics* forthcoming.

Mason, John C., and David C. Handscomb, 2003, *Chebyshev Polynomials* (Chapman & Hall/CRC).

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Poterba, James, and Lawrence Summers, 1988, Mean reversion in stock prices: Evidence and implications, *Journal of Financial Economics* 22, 27–59.

Tversky, Amos, and Daniel Kahneman, 1974, Judgment under uncertainty: Heuristics and biases, *Science* 185, 1124–1131.

Vanasco, Victoria, Ulrike Malmendier, and Demian Pouzo, 2015, A theory of experience effects, Working paper, Stanford University and University of California, Berkeley.

Veronesi, Pietro, 1999, Stock market overreaction to bad news in good times: A rational expectations equilibrium model, *Review of Financial Studies* 12, 975–1007.

Vissing-Jorgensen, Annette, 2004, Perspectives on behavioral finance: Does "irrationality" disappear with wealth? Evidence from expectations and actions, *NBER Macroeconomics Annual 2003* 18, 139–194.

Wachter, Jessica A., 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility?, *Journal of Finance* 68, 987–1035.

Xiong, Wei, 2013, Bubbles, crises, and heterogeneous beliefs, in Jean-Pierre Fouque, and Joe Langsam, eds., *Handbook for Systemic Risk*, 663–713 (Cambridge University Press).

Yang, David C., and Fan Zhang, 2016, The household equity share and expected market returns, Working paper, University of California, Irvine.

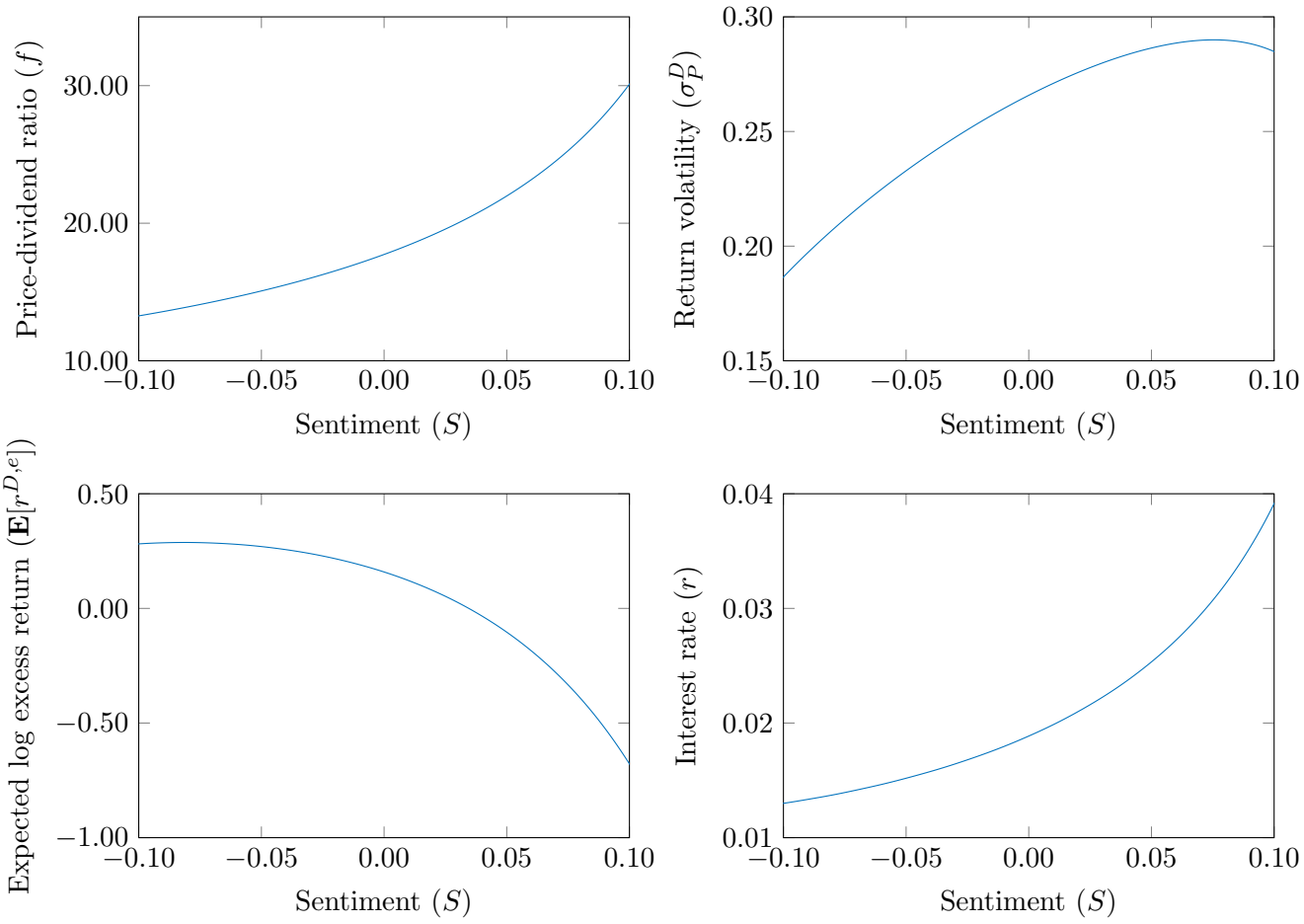


Figure 1. Important Equilibrium Quantities Each as a Function of Sentiment. The figure plots the price-dividend ratio of the stock market f, the volatility of stock market returns σ_P^D, the rational expectation about the log excess return $\mathbb{E}[r^{D,e}]$ (the conditional equity premium), and the interest rate r, each as a function of the sentiment variable S. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

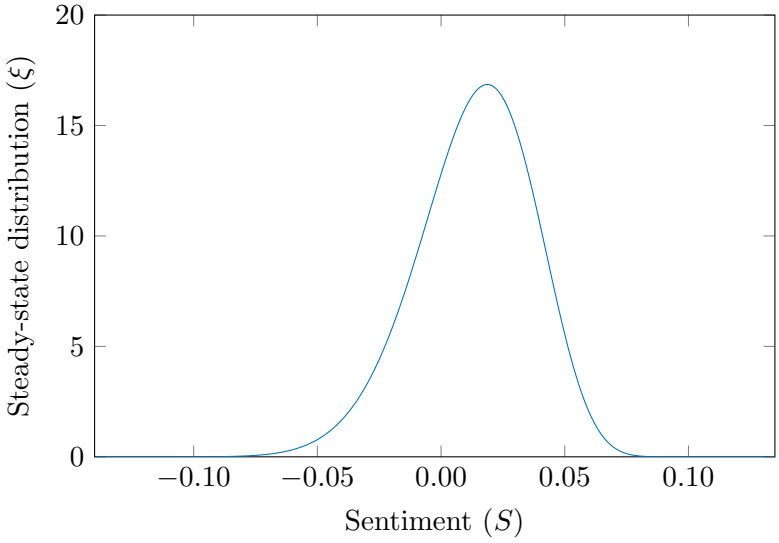


Figure 2. Objectively Measured Steady-State Distribution of Sentiment. The figure plots the objective steady-state distribution of sentiment ξ as a function of the sentiment variable S. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

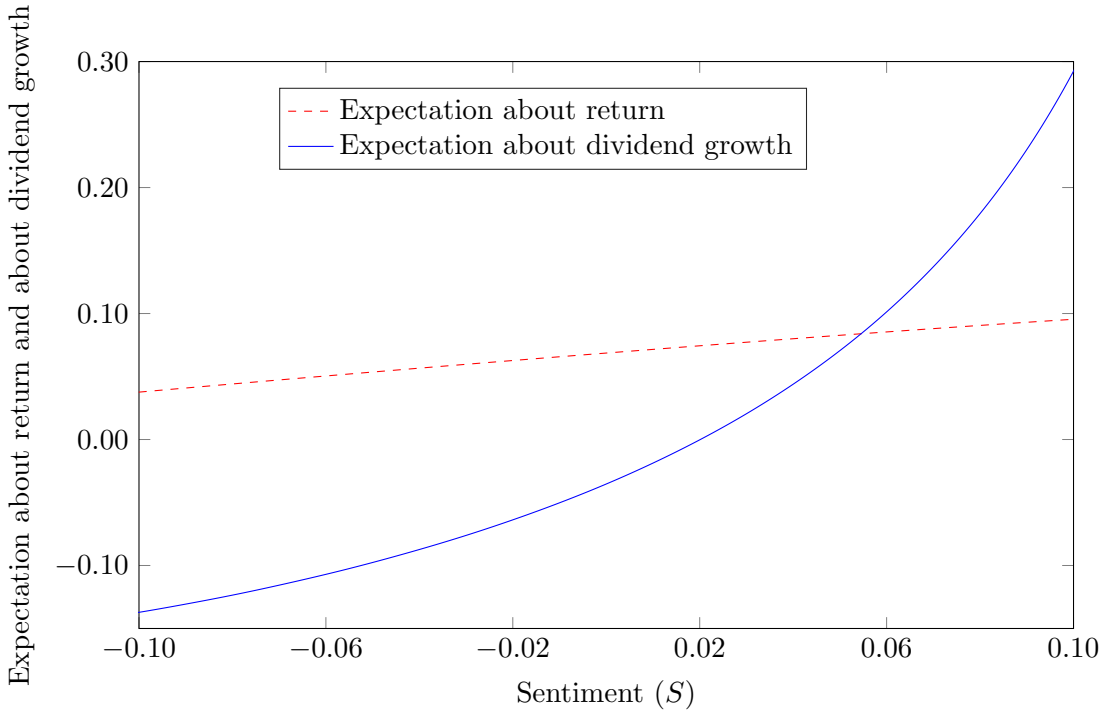


Figure 3. Agent's Expectation about Stock Market Returns and about Dividend Growth. The dashed line plots the agent's expectation about stock market returns, $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] = (1-\theta)g_D + \theta S_t + 1/f$, as a function of the sentiment variable S_t. The solid line plots the agent's expectation about dividend growth, $\mathbb{E}_t^e[dD_t/(D_t dt)]$, as a function of the sentiment variable S_t. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

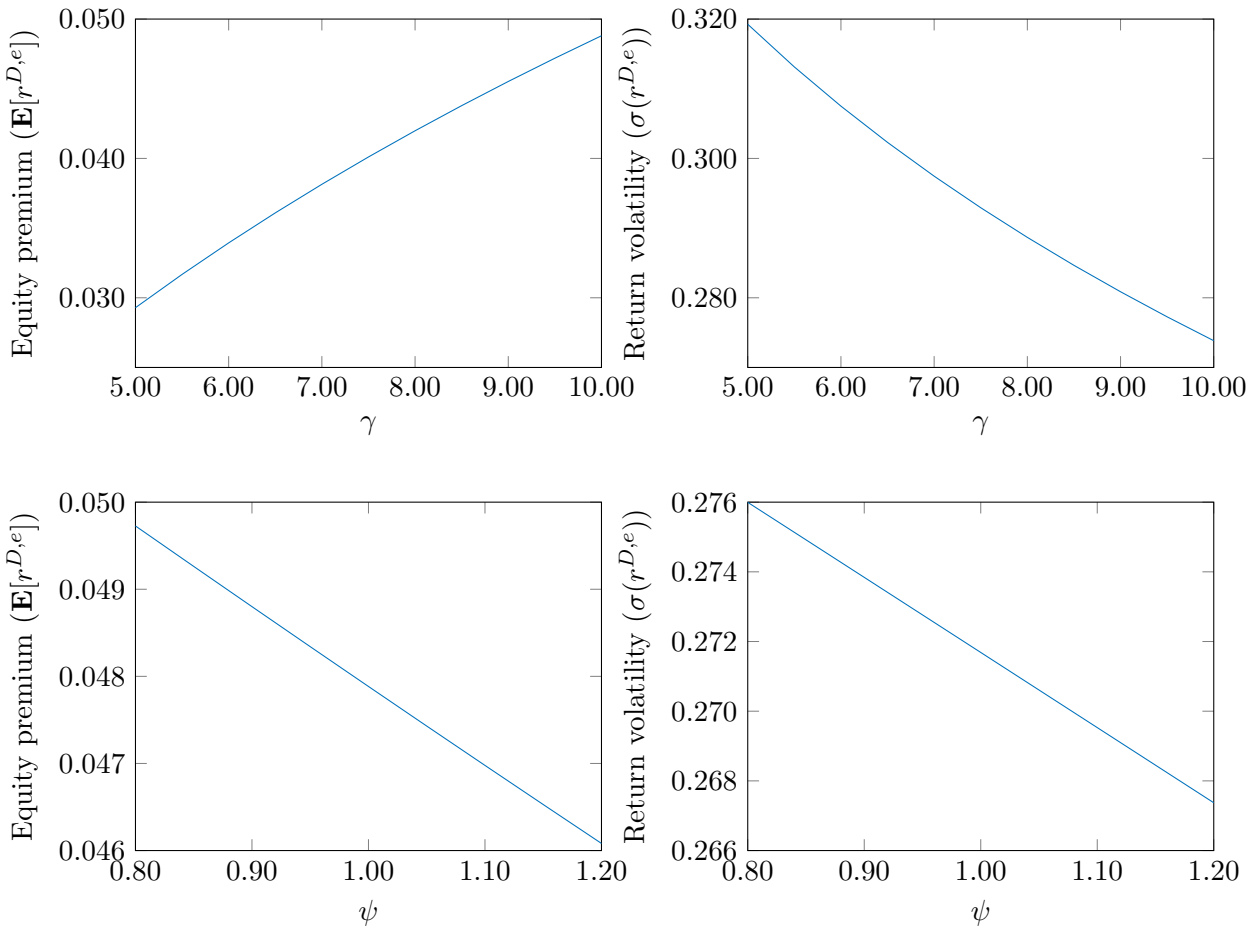


Figure 4. Comparative Statics: Utility Parameters. The upper panel plots the average equity premium $\mathbb{E}[r^{D,e}]$ and the average volatility of stock market returns $\sigma(r^{D,e})$, each as a function of γ, the relative risk aversion parameter. The lower panel plots the average equity premium $\mathbb{E}[r^{D,e}]$ and the average volatility of stock market returns $\sigma(r^{D,e})$, each as a function of ψ, the reciprocal of the elasticity of intertemporal substitution. The default values for γ and ψ are 10 and 0.9, respectively. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

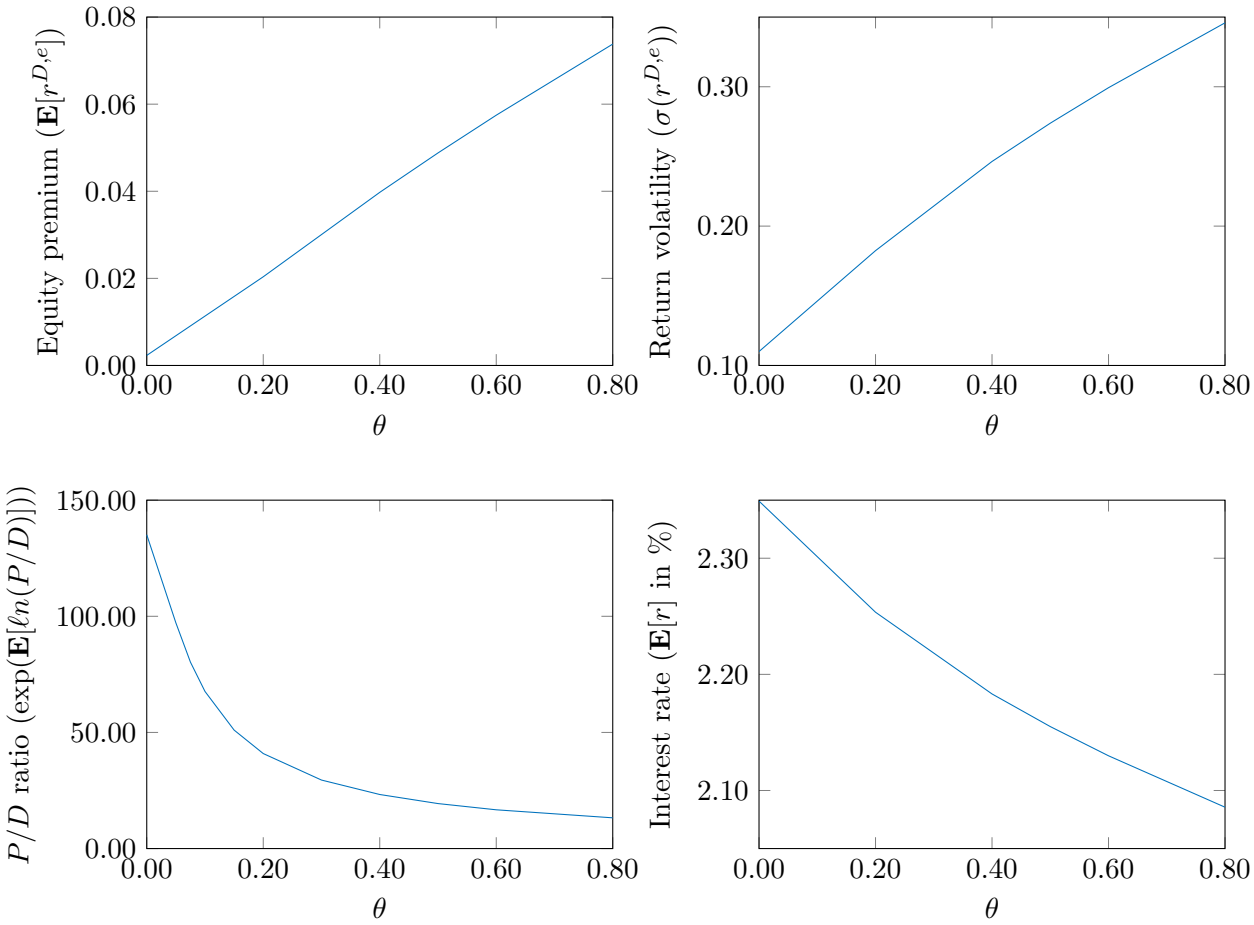


Figure 5a. Comparative Statics: Belief Parameters (I). The figure plots the average equity premium $\mathbb{E}[r^{D,e}]$, the average volatility of stock market returns $\sigma(r^{D,e})$, the average price-dividend ratio $\exp(\mathbb{E}[\ell \mathrm{n}(P/D)])$, and the average interest rate $\mathbb{E}[r]$ (in percentage), each as a function of θ, the parameter that controls the extent to which the agent is behavioral. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

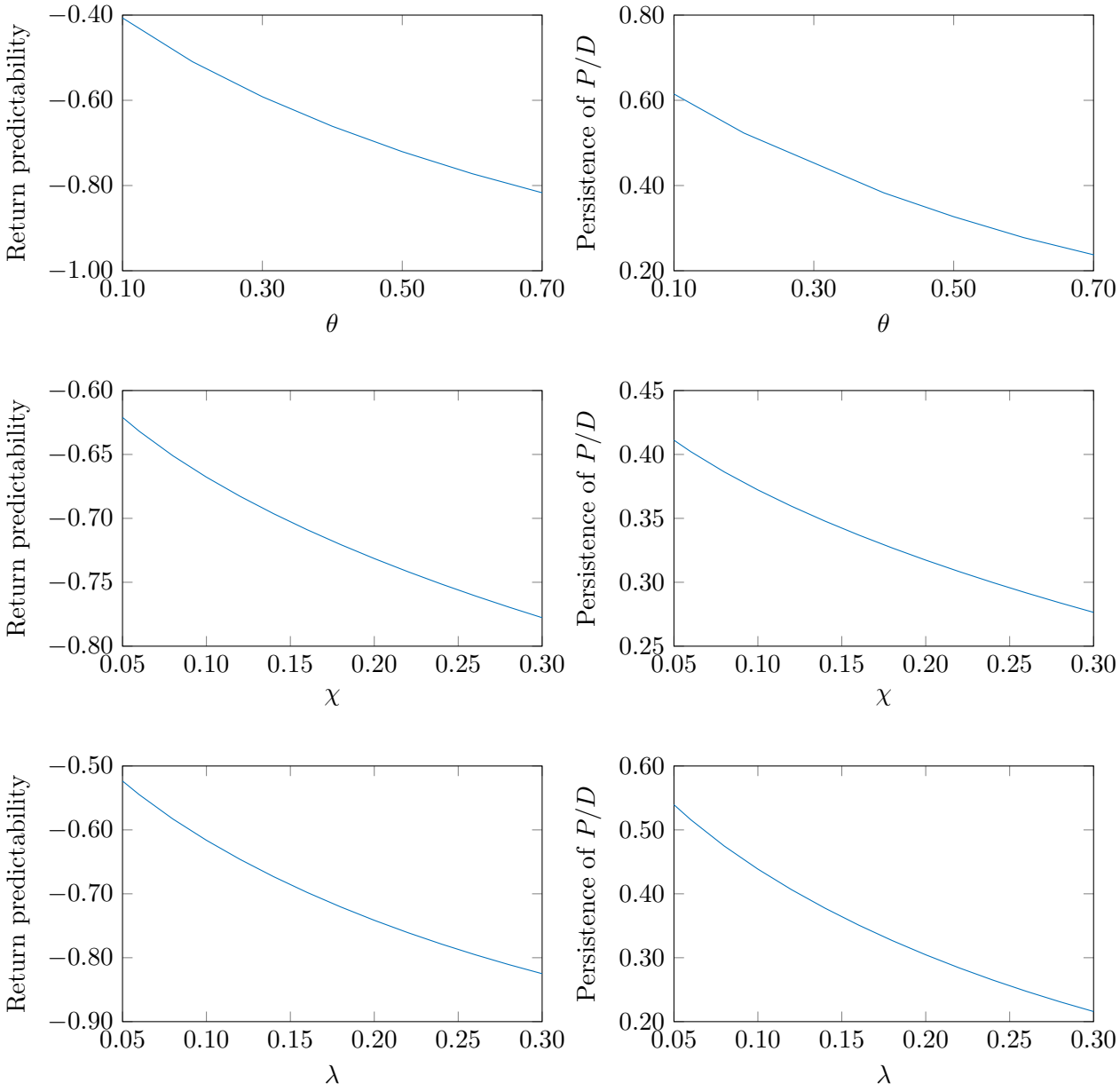


Figure 5b. Comparative Statics: Belief Parameters (II). The figure plots the predictability of stock market returns and the persistence of the price-dividend ratio, each as a function of θ, χ, or λ. The predictability of returns is measured by the slope coefficient for a regression of the next year's log excess return on the current log price-dividend ratio. The persistence of the price-dividend ratio is measured by the one-year autocorrelation of log price-dividend ratios. The default values for θ, χ, and λ are 0.5, 0.18, and -0.18, respectively. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

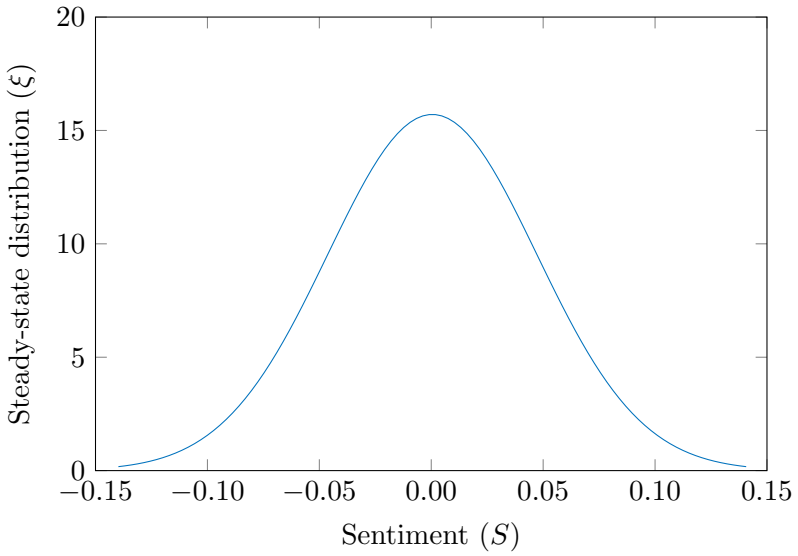


Figure 6. Steady-State Distribution of Sentiment in the True Regime-Switching Model. The figure plots the steady-state distribution of sentiment ξ as a function of the sentiment variable S in the true regime-switching model. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

Table 1. Parameter Values.

Parameter	Variable	Value
Asset parameters:		
Expected consumption growth	g_C	1.91%
Standard deviation of consumption growth	σ_C	3.80%
Expected dividend growth	g_D	2.45%
Standard deviation of dividend growth	σ_D	11%
Correlation between dD and dC	ρ	0.2
Utility parameters:		
Relative risk aversion	γ	10
Reciprocal of EIS	ψ	0.9
Subjective discount rate	δ	0.02
Belief parameters:		
Degree of extrapolation	θ	0.5
Perceived transition intensity from H to L	χ	0.18
Perceived transition intensity from L to H	λ	0.18
Upper bound of sentiment	μ_H	0.15
Lower bound of sentiment	μ_L	–0.15

Table 2. Investor Expectations.

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$			
$R^D_{t-12\to t}$	0.022	0.023			0.040	0.040		
	(8.2)	(11.7)			(8.4)	(12.1)		
$\ell\mathrm{n}(P/D)$			0.068	0.069			0.120	0.121
			(29.5)	(39.2)			(36.8)	(48.9)
Constant	0.07	0.07	–0.13	–0.13	0.02	0.02	–0.33	–0.34
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.57	0.54	0.98	0.98	0.58	0.55	0.99	0.99

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$			
$R^D_{t-12\to t}$	0.013	0.013			0.030	0.030		
	(6.6)	(9.0)			(7.8)	(11.1)		
$\ell\mathrm{n}(P/D)$			0.039	0.039			0.091	0.091
			(12.4)	(16.6)			(22.3)	(28.7)
Constant	0.05	0.05	–0.06	–0.06	–0.002	–0.002	–0.27	–0.27
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.51	0.47	0.92	0.91	0.56	0.53	0.97	0.97

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing the agent's expectation about future stock market returns either on the past twelve-month cumulative raw return $R^D_{t-12\to t}$ or on the current log price-dividend ratio $\ell\mathrm{n}(P_t/D_t)$, over a sample of 15 years or 50 years. In the top panel, the expectation measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, and the expectation measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the expectation measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, and the expectation measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$. Each reported value is averaged over 100 trials, and each trial represents a regression using monthly data simulated from the model. The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 3. Determinants of Investor Expectations.

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$		$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$	
ϕ	0.432	0.417	0.428	0.420
a	0.064	0.064	0.004	0.004
b	1.15	1.18	2.04	2.07
Sample size	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.98	0.98	0.99	0.98
	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$		$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$	
ϕ	0.414	0.408	0.418	0.420
a	0.047	0.047	–0.013	-0.010
b	0.70	0.67	1.58	1.55
Sample size	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.92	0.90	0.97	0.96

The table reports the parameter ϕ, the intercept a, the regression coefficient b, and the adjusted R-squared, for running the non-linear least squares regression

$$\text{Expectation}_t = a + b \sum_{j=1}^{n} w_j R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)} + \varepsilon_t,$$

over a sample of 15 years or 50 years, where $w_j \equiv e^{-\phi(j-1)\Delta t}/\Sigma_{l=1}^{n} e^{-\phi(l-1)\Delta t}$. Here $\Delta t = 1/12$ and $n = 600$. In the top panel, the expectation measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, and the expectation measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the expectation measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, and the expectation measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$. Each reported value is averaged over 100 trials, and each trial represents a regression using monthly data simulated from the model. The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

Table 4. Basic Moments.

Statistic	Theoretical value	Empirical value
Equity premium ($\mathbb{E}[r^{D,e}]$)	4.88%	3.90%
Return volatility ($\sigma(r^{D,e})$)	27.4%	18.0%
Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$)	0.20	0.22
Interest rate ($\mathbb{E}[r]$)	2.16%	2.92%
Interest rate volatility ($\sigma(r)$)	0.33%	2.89%
Price-dividend ratio ($\exp(\mathbb{E}[\ell\text{n}(P/D)])$)	19.4	21.1

The table reports six important moments about stock market prices and returns: the long-run average of the equity premium (the rational expectation of log excess return, $\mathbb{E}[r^{D,e}]$), the average volatility of stock market returns (the volatility of log excess return, $\sigma(r^{D,e})$), the Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$), the average interest rate ($\mathbb{E}[r]$), interest rate volatility ($\sigma(r)$), and the average price-dividend ratio of the stock market ($\exp(\mathbb{E}[\ell\text{n}(P/D)])$). The theoretical values for these moments are computed over the objectively measured steady-state distribution of sentiment S. The model parameters are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. For the empirical values, five out of six are from Campbell and Cochrane (1999); the empirical value for interest rate volatility is not reported in Campbell and Cochrane (1999), so we report the value from Beeler and Campbell (2012).

Table 5. Campbell-Shiller Decomposition.

Coefficient	Realized dividend growth and returns	
	Value	Adjusted R-squared
$DR_{objective}$	0.98	0.209
$CF_{objective}$	0.02	1.2×10^{-4}

Coefficient	Subjective expectations about dividend growth and returns	
	Value	Adjusted R-squared
$DR_{subjective}$	-0.08	0.982
$CF_{subjective}$	1.08	0.984

The table reports the four coefficients, $CF_{objective}$, $DR_{objective}$, $CF_{subjective}$, and $DR_{objective}$, defined in equations (29) and (30), as well as their corresponding adjusted R-squared. These coefficients and R-squared are calculated using 10,000 years of monthly data simulated from the model. At each point in time, for a given level of sentiment, subjective expectations about dividend growth and returns are calculated as the average values of 100 trials. Each trial is 50 years of monthly simulated data under the agent's expectations with the given initial level of sentiment. For realized dividend growth and returns, both $\sum_{j=0}^{\infty} \xi^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ and $\sum_{j=0}^{\infty} \xi^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ are approximated using 50 years of monthly simulated data. From the simulated data, $\xi = 0.9957$. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 6. Return Predictability Regressions.

Horizon (years)	Theoretical value		Empirical value	
	10× coefficient	Adjusted R-squared	10× coefficient	Adjusted R-squared
1	–7.2	0.13	–1.3	0.04
2	–9.5	0.16	–2.8	0.08
3	–10.1	0.15	–3.5	0.09
5	–10.6	0.13	–6.0	0.18
7	–11.0	0.12	–7.5	0.23

The table reports the regression coefficient β_j and the adjusted R-squared for a regression of the log excess return of stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$

$$r_{t\to t+j}^{D,e} = \alpha_j + \beta_j \ell\text{n}(P_t^D/D_t) + \varepsilon_{j,t},$$

where j = 1, 2, 3, 5, and 7 (years). The theoretical values are calculated using 10,000 years of monthly data simulated from the model. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 7. Autocorrelations of Log Excess Returns and Log Price-dividend Ratios.

Lag (years)	Theoretical value			Empirical value		
	$\ell n(P^D/D)$	$r^{D,e}$	$\Sigma_{i=1}^{j}\rho(r_t^{D,e}, r_{t-i}^{D,e})$	$\ell n(P^D/D)$	$r^{D,e}$	$\Sigma_{i=1}^{j}\rho(r_t^{D,e}, r_{t-i}^{D,e})$
1	0.33	–0.28	–0.28	0.78	0.05	0.05
2	0.11	–0.09	–0.37	0.57	–0.21	–0.16
3	0.05	–0.02	–0.39	0.50	–0.08	–0.09
5	0.00	–0.01	–0.40	0.32	–0.14	–0.28
7	–0.02	–0.01	–0.41	0.29	0.11	–0.15

The table reports, over various lags j, the autocorrelations of log price-dividend ratios and log excess returns, as well as the partial sum of the autocorrelations of log excess returns. The operator $\rho(x, y)$ computes the sample correlation between variable x and variable y. The theoretical values are calculated using 10,000 years of monthly data simulated from the model; for each month, we compound the next 12 months of log excess returns to obtain an annual log excess return. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 8. Correlation between Consumption Growth and Stock Returns.

Correlation	Theoretical value			Empirical value
	monthly	quarterly	annual	annual
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t-1}/C_{t-2}))$	–0.01	–0.02	–0.02	–0.05
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t}/C_{t-1}))$	–0.01	–0.03	–0.06	–0.08
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+1}/C_{t}))$	0.20	0.20	0.19	0.09
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+2}/C_{t+1}))$	0.00	–0.00	0.01	0.49
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+3}/C_{t+2}))$	–0.00	–0.00	0.01	0.05

The table reports correlations between log consumption growth and log excess returns of the stock market. The log consumption growth and log excess returns are computed at either a monthly, a quarterly, or an annual horizon. Correlations are either contemporaneous or with a lead-lag structure; for instance, at a monthly frequency, $\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+2}/C_{t+1}))$ is the correlation between the current monthly log excess return and the log consumption growth in the subsequent month. The theoretical values are calculated using 10,000 years of monthly data simulated from the model. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 9. Return Predictability Regressions in the True Regime-Switching Model.

Horizon (years)	Theoretical value		Empirical value	
	10× coefficient	10^3×Adjusted R-squared	10× coefficient	Adjusted R-squared
1	0.2	0.16	–1.3	0.04
2	0.4	0.31	–2.8	0.08
3	0.5	0.32	–3.5	0.09
5	0.8	0.54	–6.0	0.18
7	0.7	0.34	–7.5	0.23

The table reports the regression coefficient β_j and the adjusted R-squared for a regression of the log excess return of stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$

$$r_{t\to t+j}^{D,e} = \alpha_j + \beta_j \ell\text{n}(P_t^D/D_t) + \varepsilon_{j,t},$$

where j = 1, 2, 3, 5, and 7 (years). The theoretical values are calculated using 10,000 years of monthly data simulated from the true regime-switching model described in Section IV. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 10. Investor Expectations in Campbell and Cochrane (1999).

	$\mathbb{E}_t[(dP_t^D + D_t dt)/(P_t^D dt)]$				$\mathbb{E}_t[dP_t^D/(P_t^D dt)]$			
$R^D_{t-12\to t}$	–0.002	–0.001			–0.013	–0.011		
	(–1.7)	(–1.2)			(–1.7)	(–1.4)		
$\ell\text{n}(P/D)$			–0.009	–0.008			–0.059	–0.062
			(–64.4)	(–68.4)			(–51.6)	(–38.7)
Constant	0.03	0.03	0.05	0.05	0.09	0.09	0.25	0.26
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.085	0.017	0.992	0.991	0.090	0.020	0.995	0.987

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing the rational expectation about future stock market returns either on the past twelve-month cumulative raw return $R^D_{t-12\to t}$ or on the current log price-dividend ratio $\ell\text{n}(P_t/D_t)$, over a sample of 15 years or 50 years. The expectation measure for the first four columns is $\mathbb{E}_t[(dP_t^D + D_t dt)/(P_t^D dt)]$, and the expectation measure for the last four columns is $\mathbb{E}_t[dP_t^D/(P_t^D dt)]$. Each reported value is the estimator median over 1,000 trials, and each trial represents a regression using monthly data simulated from Campbell and Cochrane (1999). The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameters take their default values from Table 1 of Campbell and Cochrane (1999). Given that the interest rate in their model is constant, the regression coefficients and the t-statistics remain the same if we subtract the interest rate from stock market returns.

Table 11. Investor Expectations in Bansal and Yaron (2004).

	Expectation of return				Expectation of return w/o divd.			
$R^D_{t-12\to t}$	0.025	0.030			0.025	0.031		
	(2.4)	(3.8)			(2.5)	(4.0)		
$\ell n(P/D)$			0.068	0.067			0.069	0.067
			(5.6)	(7.6)			(5.8)	(7.6)
Constant	1.06	1.05	0.88	0.88	1.06	1.05	0.88	0.89
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.140	0.142	0.500	0.451	0.136	0.145	0.529	0.451

	Expectation of excess return				Expectation of excess return w/o divd.			
$R^D_{t-12\to t}$	–0.001	0.000			0.000	0.001		
	(–0.1)	(0.1)			(–0.1)	(0.2)		
$\ell n(P/D)$			–0.008	–0.012			–0.010	–0.011
			(–0.8)	(–1.3)			(–0.8)	(–1.2)
Constant	0.06	0.06	0.08	0.10	0.06	0.06	0.09	0.09
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.026	0.008	0.087	0.054	0.026	0.008	0.093	0.053

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing four measures of rational expectation of return—raw return or excess return, with or without dividend yield—either on the past twelve-month return or on the current log price-dividend ratio, over a sample of 15 years or 50 years. The conditional expectation of subsequent returns, the dependent variable in each regression, is computed by averaging realized returns across simulations over a twelve-month horizon for a given state of the economy. Each reported value is the estimator median over 1,000 trials, and each trial represents a regression using monthly data simulated from Bansal and Yaron (2004). The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameters take their default values from Table II and IV of Bansal and Yaron (2004).

Table 12. Investor Expectations in Bansal et al. (2012).

	Expectation of return				Expectation of return w/o divd.			
$R^D_{t-12\to t}$	0.006	0.010			0.006	0.011		
	(0.7)	(1.3)			(0.7)	(1.3)		
$\ell n(P/D)$			–0.006	–0.041			–0.006	–0.038
			(–0.3)	(–3.1)			(–0.2)	(–2.9)
Constant	1.08	1.08	1.11	1.22	1.08	1.08	1.11	1.21
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.031	0.021	0.129	0.147	0.033	0.020	0.139	0.136
	Expectation of excess return				Expectation of excess return w/o divd.			
$R^D_{t-12\to t}$	–0.003	0.000			–0.004	–0.001		
	(–0.3)	(0.1)			(–0.5)	(–0.1)		
$\ell n(P/D)$			–0.071	–0.095			–0.036	–0.054
			(–5.2)	(–10.7)			(–3.4)	(–5.2)
Constant	0.09	0.08	0.30	0.37	0.08	0.08	0.19	0.24
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.021	0.010	0.432	0.599	0.026	0.008	0.226	0.254

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing four measures of rational expectation of return—raw return or excess return, with or without dividend yield—either on the past twelve-month return or on the current log price-dividend ratio, over a sample of 15 years or 50 years. The conditional expectation of subsequent returns, the dependent variable in each regression, is computed by averaging realized returns across simulations over a twelve-month horizon for a given state of the economy. Each reported value is the estimator median over 1,000 trials, and each trial represents a regression using monthly data simulated from Bansal et al. (2012). The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameters take their default values from Table 1 of Bansal et al. (2012).

Table 13. Investor Expectations in the Fundamental Extrapolation Model.

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$			
$R^D_{t-12\to t}$	0.004	0.003			0.010	0.010		
	(0.21)	(0.63)			(1.15)	(2.04)		
$\ell n(P/D)$			0.017	0.019			0.040	0.042
			(0.88)	(1.62)			(2.08)	(3.52)
Constant	0.05	0.05	–0.01	–0.02	0.03	0.03	–0.12	–0.13
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.14	0.04	0.24	0.10	0.17	0.08	0.31	0.22

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$			
$R^D_{t-12\to t}$	–0.004	–0.004			0.003	0.002		
	(–0.88)	(–1.13)			(0.07)	(0.42)		
$\ell n(P/D)$			–0.009	–0.007			0.014	0.016
			(–0.49)	(–0.54)			(0.71)	(1.35)
Constant	0.03	0.03	0.06	0.05	0.006	0.006	–0.05	–0.05
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.15	0.05	0.22	0.06	0.14	0.04	0.23	0.08

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing the agent's expectation about future stock market returns either on the past twelve-month cumulative raw return $R^D_{t-12\to t}$ or on the current log price-dividend ratio $\ell n(P_t/D_t)$, over a sample of 15 years or 50 years. In the top panel, the expectation measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, and the expectation measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the expectation measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, and the expectation measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$. Each reported value is averaged over 100 trials, and each trial represents a regression using monthly data simulated from the fundamental extrapolation model described in Section V. The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 14. Basic Moments in the Fundamental Extrapolation Model.

Statistic	Theoretical value	Empirical value
Equity premium ($\mathbb{E}[r^{D,e}]$)	1.85%	3.90%
Return volatility ($\sigma(r^{D,e})$)	20.4%	18.0%
Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$)	0.08	0.22
Interest rate ($\mathbb{E}[r]$)	2.27%	2.92%
Interest rate volatility ($\sigma(r)$)	0.23%	2.89%
Price-dividend ratio ($\exp(\mathbb{E}[\ell n(P/D)])$)	44.4	21.1

The table reports six important moments about stock market prices and returns: the long-run average of the equity premium (the rational expectation of log excess return, $\mathbb{E}[r^{D,e}]$), the average volatility of stock market returns (the volatility of log excess return, $\sigma(r^{D,e})$), the Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$), the average interest rate ($\mathbb{E}[r]$), interest rate volatility ($\sigma(r)$), and the average price-dividend ratio of the stock market ($\exp(\mathbb{E}[\ell n(P/D)])$). The theoretical values for these moments are computed over the objectively measured steady-state distribution of sentiment in the fundamental extrapolation model described in Section V. The model parameters are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. For the empirical values, five out of six are from Campbell and Cochrane (1999); the empirical value for interest rate volatility is not reported in Campbell and Cochrane (1999), so we report the value from Beeler and Campbell (2012).