Asset Pricing with Return Extrapolation*

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ABSTRACT

We develop a representative agent general equilibrium model with return extrapolation and recursive preferences. Our model is the first return extrapolation model that can be taken to the data in a serious way; it allows for detailed model calibration and direct comparisons with the leading rational expectations models of the stock market. The model matches investors' extrapolative expectations observed in surveys. It also matches facts about asset prices such as a large equity premium, low interest rate volatility, strong excess volatility and predictability for equity returns, and a low correlation between consumption growth and stock returns. Extrapolative beliefs generate perceived persistence in dividend and consumption growth that, under recursive preferences, serves as an important source of discount rate variation.

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Rational expectations models—models such as the habit formation model of Campbell and Cochrane (1999), the long-run risks models of Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012), and the rare disasters models of Barro (2006) and Gabaix (2012)—have been the leading candidates to make sense of many stylized facts about the aggregate stock market. These facts include the equity premium puzzle of Mehra and Prescott (1985), the excess volatility puzzle of LeRoy and Porter (1981) and Shiller (1981), the evidence on predictability of equity returns documented by Campbell and Shiller (1988) and Fama and French (1988), low correlations between consumption growth and stock returns noted by Hansen and Singleton (1982, 1983), as well as negative autocorrelations of equity returns presented in Poterba and Summers (1988).

However, recent survey evidence on actual investor expectations of stock market returns have raised challenges to these rational expectations models. Among others, Vissing-Jorgensen (2004), Bacchetta, Mertens, and van Wincoop (2009), Amromin and Sharpe (2013), Greenwood and Shleifer (2014), Koijen, Schmeling, and Vrugt (2015), and Kuchler and Zafar (2016) document that many individual and institutional investors have extrapolative expectations: they believe that the stock market will continue rising in value after a sequence of high past returns, and that it will continue falling in value after a sequence of low past returns. On the contrary, models with rational expectations imply that investors would expect *lower* or flat returns, instead of higher returns, after a sequence of high past returns. Moreover, rational expectations about future returns are positively correlated with realized returns over the same time period, whereas survey expectations are *negatively* correlated with realized returns. Given the discrepancy between survey expectations and models with rational expectations, there is a clear need for a behavioral benchmark model of asset pricing—one that is consistent with survey evidence on investor beliefs—that can be directly compared on quantitative grounds to more traditional asset pricing models.

In this paper, we take up this challenge. We develop a Lucas-type general equilibrium model with return extrapolation and Epstein-Zin preferences. In the model, the price of the aggregate equity market and the interest rate are jointly determined in equilibrium by investor preferences and subjective beliefs. The model generates a large and countercyclical equity premium, a low and procyclical interest rate, a sizable and countercyclical Sharpe ratio, low interest rate volatility, strong excess volatility for equity, predictability of equity returns using price-dividend ratios, negative autocorrelations of equity returns, persistence of price-dividend ratios, as well as low correlations between consumption growth and stock returns. We directly compare these model implications with empirical data, and find that the majority of them quantitatively match the data. More important, the model further matches investors' extrapolative beliefs and their memory structure derived directly from survey evidence.[1] Most rational expectations models have difficulty in matching empirical predictions related to survey expectations.

The basic mechanism behind our model is an endogenous two-way feedback loop: investor beliefs about returns drive asset prices, and asset prices in turn affect investor beliefs. When past returns

[1]Specifically, memory structure refers to the speed at which investors' memory about past returns decays when these investors form beliefs about future returns.

of the stock market have been high, investors who extrapolate past returns become more optimistic about future returns and push up the stock market prices. With higher prices, current returns become higher, and therefore investors become even more optimistic. This two-way feedback loop naturally generates excess volatility. On top of this basic mechanism, a significant multiplier effect arises from the interaction between extrapolative beliefs and Epstein-Zin preferences. With the elasticity of intertemporal substitution that is greater than one—a value that is much higher than the reciprocal of risk aversion in our model—the intertemporal substitution effect dominates the wealth effect. Given this, when past returns of the stock market have been high, investors *perceive* high returns and high dividend growth moving forward, and therefore increase their holding of the stock market. This further pushes up the stock market price and gives rise to an even stronger feedback loop. At the same time, the separation of the elasticity of intertemporal substitution and risk aversion allows for low interest rates and helps generate a large equity premium.

The two-way feedback loop described above is a main reason for earlier models like Cutler, Poterba, and Summers (1991) and Barberis, Greenwood, Jin, and Shleifer (2015) to introduce fully rational investors in order to counteract the biased beliefs from behavioral investors. In the absence of rational investors, the positive feedback loop in these models may lead to divergence in price, and as a result, equilibrium may not exist. In this paper, we show that carefully modelling a representative agent with return extrapolation and Epstein-Zin preferences can preserve equilibrium. A stronger two-way feedback loop leads to higher contemporaneous risks and higher perceived persistence of risks. These risks interacting with Epstein-Zin preferences counteract the extrapolative beliefs, and hence preserve the existence of equilibrium.

The paper makes several contributions to the literature on asset pricing theories. First, our model is the first return extrapolation model that can be taken to the data in a serious way. As for the habit formation models, the long-run risks models, and the rare disasters models, our model is developed in a Lucas economy with a representative agent. This brings belief-based behavioral asset pricing models to the same level of sophistication as leading rational expectations models about the stock market in the literature. Compared to these models, not only can our model quantitatively match the behavior of aggregate market asset prices, it can also quantitatively match the survey evidence on investor beliefs; we provide a detailed comparison between our model and rational expectations models in Section IV.

Second, to the best of our knowledge, our model is the first among belief-based behavioral asset pricing theories that seriously analyzes investor beliefs about dividend growth of the stock market and consumption growth separately. As an empirical fact, the correlation between consumption growth and dividend growth is low. When investors extrapolate past returns of the stock market to form beliefs about future returns, to what extent do they also extrapolate past consumption growth? Our model formally addresses this question and studies the asset pricing implications of "asset-specific" belief formation. One such implication is that our model performs well in generating low correlations between consumption growth and stock market returns. The intuition behind this finding is as follows. Return extrapolation, the primary force in the model that drives systematic

time variation in stock returns, is directly applied to the risky claim of dividend streams, but much less directly to the consumption stream due to the low correlation between consumption growth and dividend growth. As a result, a large component of consumption movements does not affect stock returns.

Third, the model highlights an important source of discount rate variation. In the context of the model, the decomposition method proposed in Campbell and Shiller (1988) suggests that under the objective probability measure, stock price movements primarily come from discount rate variation, consistent with the empirical literature on asset pricing. However, our belief-based behavioral model shows quantitatively that, discount rate variation could be driven by investors' misperception that stock price movements primarily correspond to cash flow news; regressing future dividend growths on the current dividend-price ratio from the representative agent's perspective, the coefficient is significantly negative. In other words, return extrapolation generates perceived persistence in dividend and consumption growths that serves as an important source for discount rate variation.

Fourth, to better understand the quantitative implications of the two-way feedback loop, we compare our model to a model with *fundamental extrapolation*, the notion that some investors hold extrapolative expectations about dividend growth of the stock market. For the same parameter values, we find that the model with fundamental extrapolation generates on average a much lower equity premium. With fundamental extrapolation, the two-way feedback loop does *not* arise: realized dividend growth generates optimistic beliefs about future dividend growth and hence pushes up prices, but higher prices do not further affect investors' beliefs about future dividends.

The paper also makes some methodological contribution to the literature. While most belief-based models specify belief formation exogenously, the two-way feedback loop described above in return extrapolation models require *endogenous* belief formation. This imposes a greater modelling challenge, and our differential equations approach provides a solution to it.

Our belief-based model allows for different time-series behavior for quantities objectively measured by outside econometricians and quantities subjectively perceived by investors. For instance, while the true equity premium and Sharpe ratio are countercyclical, the *perceived* equity premium and Sharpe ratio are both procyclical; this is consistent with the findings of Amromin and Sharpe (2013). The difference between subjective expectations and objective expectations also allows the model to generate a *negative* equity premium when investor sentiment is very high, a prediction that is consistent with the recent findings of Greenwood and Hanson (2013), Baron and Xiong (2015), Cassella and Gulen (2015), and Yang and Zhang (2016).

The paper adds to a new wave of theories that attempt to understand the role of belief formation in driving the behavior of asset prices and the macroeconomy (Fuster, Hebert, and Laibson (2011), Choi and Mertens (2013), Alti and Tetlock (2014), Hirshleifer, Li, and Yu (2015), Barberis et al. (2015), Jin (2015), Ehling, Graniero, and Heyerdahl-Larsen (2015), Vanasco, Malmendier, and Pouzo (2015), Collin-Dufresne, Johannes, and Lochstoer (2016a,b), and Greenwood, Hanson, and Jin (2016)). The most related is the model developed by Barberis et al. (2015). Although their

model also features return extrapolation in a consumption-based asset pricing model, it adopts a framework with constant absolute risk aversion preferences and a constant interest rate. These simplifying assumptions make it difficult to take their model to the data in a serious way. For instance, ratio-based quantities such as the dividend-price ratio do not have a steady-state distribution in Barberis et al. (2015), which, as a result, prohibits standard regression tests. In addition, their model does not separate consumption and dividend as we do in our model, and this further hinders the ability of their model in matching asset pricing facts. On the contrary, our model can be directly taken to the data and compared with more traditional asset pricing models.

The paper proceeds as follows. In Section I, we lay out the basic elements of the model and characterize its solution. In Section II, we examine the model implications in detail. Section III provides some comparative statics results. Section IV discusses differences between the model and rational expectations models. Section V further compares the model with a fundamental extrapolation model. Section VI concludes and suggests directions for future research. All technical details are in the Appendix.

I. The Model

In this section, we lay out the model, characterize its solution, and derive equilibrium quantities that are important for understanding the model implications.

Consider an infinite-horizon economy with a representative agent. Following Epstein and Zin (1989, 1991), we assume that the representative agent has a recursive intertemporal utility

$$U_t = \left[(1-e^{-\delta\Delta t})C_t^{1-\psi}\Delta t + e^{-\delta\Delta t}\left(\mathbb{E}_t^e[\tilde{U}_{t+\Delta t}^{1-\gamma}]\right)^{(1-\psi)/(1-\gamma)}\right]^{1/(1-\psi)}, \tag{1}$$

where δ is the subjective discount factor, $\gamma > 0$ is the relative risk aversion, $\psi > 0$ is the reciprocal of the elasticity of intertemporal substitution. When ψ equals γ, (1) reduces to a standard power utility case. Expectations are taken under the representative agent's subjective beliefs, which, as we specify later, incorporate return extrapolation.

In discrete time, the subjective Euler equation is

$$\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)\Delta t/(1-\psi)}\left(\frac{\tilde{C}_{t+\Delta t}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)}\tilde{M}_{t+\Delta t}^{(\psi-\gamma)/(1-\psi)}\tilde{R}_{j,t+\Delta t}\right] = 1, \tag{2}$$

where $\tilde{M}_{t+\Delta t}$ is the gross return on the optimal portfolio from time t to time $t+\Delta t$, and $\tilde{R}_{j,t+\Delta t}$ is the gross return on any tradeable asset j in the market over the same horizon.

In a Lucas economy, the optimal portfolio in equilibrium is the Lucas tree itself, and therefore

$$\tilde{M}_{t+\Delta t} = \frac{\tilde{P}_{t+\Delta t}^C + \tilde{C}_t\Delta t}{P_t^C} = \frac{\tilde{P}_{t+\Delta t}^C + \tilde{C}_{t+\Delta t}\Delta t}{P_t^C} + o(\Delta t), \tag{3}$$

where P_t^C is the price of the Lucas tree at time t.

Consider the continuous-time limit of the economy described above. We assume that the aggregate consumption process evolves as

$$dC_t/C_t = g_C dt + \sigma_C d\omega_t^C, \tag{4}$$

and that the aggregate dividend process for the equity market evolves as

$$dD_t/D_t = g_D dt + \sigma_D d\omega_t^D, \tag{5}$$

where both ω_t^C and ω_t^D are a standard Brownian motion. The instantaneous correlation between dC_t and dD_t is ρ: $\mathbb{E}_t[d\omega_t^C \cdot d\omega_t^D] = \rho dt$. And the price of the future dividend stream at time t is denoted as P_t^D.

We now begin to make specific assumptions about the beliefs of the representative agent. Recent survey evidence suggests that real-world investors form beliefs about future returns of the equity market by extrapolating the market's past returns (see Vissing-Jorgensen (2004), Bacchetta et al. (2009), Amromin and Sharpe (2013), Greenwood and Shleifer (2014), Koijen et al. (2015), and Kuchler and Zafar (2016)). To formalize this, we assume that, from the representative agent's perspective, the evolution of P_t^D is

$$dP_t^D/P_t^D = \mu_P^{D,e}(S_t)dt + \sigma_P^D(S_t)d\omega_t^e, \tag{6}$$

where

$$\mu_P^{D,e}(S_t) = (1-\theta)g_D + \theta S_t. \tag{7}$$

That is, the representative agent's belief $\mu_P^{D,e}(S_t)$ is a linear combination of a rational component g_D and a behavioral component S_t, which we call the sentiment variable.[2] The parameter θ controls the degree of extrapolation: setting θ to zero reduces the model to a rational benchmark.

Furthermore, to capture the notion of return extrapolation, our belief structure needs to be such that when past returns are high, the representative agent perceives a high sentiment level S_t, and therefore a higher return moving forward. One natural way to incorporate this notion is through a regime-switching learning model. Specifically, suppose the representative agent believes that the expected growth rate on the risky asset is governed by a latent variable $\tilde{\mu}_{S,t}$ and that $\tilde{\mu}_{S,t}$ follows a regime-switching process between a high-state return μ_H and a low-state return μ_L ($\mu_H > \mu_L$) with the following transition matrix[3]

$$\begin{array}{c} \\ \tilde{\mu}_{S,t} = \mu_H \\ \tilde{\mu}_{S,t} = \mu_L \end{array} \begin{array}{c} \begin{array}{cc} \tilde{\mu}_{S,t+dt} = \mu_H & \tilde{\mu}_{S,t+dt} = \mu_L \end{array} \\ \begin{pmatrix} 1-\chi dt & \chi dt \\ \lambda dt & 1-\lambda dt \end{pmatrix} \end{array}. \tag{8}$$

[2] The growth rate g_D is the rational expectation of dP_t^D/P_t^D only in an economy without any biased beliefs.

[3] A similar regime-switching type of learning structure are analyzed in Barberis, Shleifer, and Vishny (1998), Veronesi (1999), and Jin (2015).

Here χ and λ are the intensities for the transitions from the high-return state to the low-return state and from the low-return state to the high-return state, respectively. Setting $S_t \equiv \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^P]$, we then apply optimal filtering theory (see, for instance, Lipster and Shiryaev (2001)) and obtain

$$\begin{aligned} dS_t &= (\lambda\mu_H + \chi\mu_L - (\lambda+\chi)S_t)dt + (\sigma_{P,t}^D)^{-1}\theta(\mu_H - S_t)(S_t - \mu_L)d\omega_t^e \\ &\equiv \mu_S^e(S_t)dt + \sigma_S(S_t)d\omega_t^e, \end{aligned} \tag{9}$$

where $d\omega_t^e \equiv [dP_t^D/P_t^D - (1-\theta)g_D dt - \theta S_t dt]/\sigma_{P,t}^D$ is *perceived* by the representative agent to be a standard Brownian innovation term. Indeed, the evolution of sentiment in (9) captures the notion of return extrapolation that after a sequence of good past returns, the investor becomes more optimistic about future returns: high past return realizations dP_t^D/P_t^D push up perceived shocks $d\omega_t^e$ and therefore lead the investor to raise her expectations S_t.[4]

It is important to emphasize that although the subjective evolution of sentiment (9) is constructed through optimal learning, the representative agent does *not* hold rational expectations. This is because in the long run, the representative agent could have realized that the regime-switching model (8) is an incorrect belief model: she could have looked at the historical distribution of $d\omega_t^e$ and have noticed that it does not fit into a normal distribution with mean of 0 and variance of dt. In reality, while existing investors may learn over time that (8) is an incorrect belief model, new investors who hold extrapolative beliefs are likely to enter the financial markets over time. Setting a dogmatic and stable belief system (8) for the representative agent is an analytically convenient way to capture this notion. If, on the other hand, equations (8) and (9) represent the *true* data generating process, then the representative agent does hold rational expectations. In this case, our model reduces to a fully rational model with incomplete information.[5] We discuss the implications of such a model in Section IV.

Without loss of generality, the price-dividend ratio P_t^D/D_t can be written as a function of the sentiment variable $P_t^D/D_t \equiv f(S_t)$. Writing the perceived dividend process as

$$dD_t/D_t = g_D^e(S_t)dt + \sigma_D d\omega_t^e \tag{10}$$

[4]There are apparently many ways to specify the evolution of S_t to capture the notion of return extrapolation. We choose the regime-switching learning structure based on several reasons. First, this learning structure fundamentally captures base rate neglect, an important form of representative heuristic (Tversky and Kahneman (1974)). To see this, notice that the perceived regimes or states, μ_H and μ_L, are not part of the true states of the economy. As a result, assigning positive probability weights to these regimes reflect the bias that the investor neglects the base rate of zero for such regimes. Also notice that earlier work of Barberis et al. (1998) has associated the regime-switching model to sample size neglect, another form of representativeness heuristic. Second, given the regime-switching process perceived by the investor, applying Bayesian learning is a natural and optimal way to discipline our belief specification. Lastly, bounding S_t at μ_H and μ_L reduces the analytical difficulty when solving the model.

[5]Information is incomplete in the sense that the agent does not directly observe the latent variable $\tilde{\mu}_{S,t}$.

and applying Ito's lemma on both sides of $P_t^D/D_t = f(S_t)$, we obtain

$$\begin{aligned}&\left\{\mu_{S,t}^e(f'/f)dt + (f'/f)\sigma_{S,t}d\omega_t^e + \tfrac{1}{2}(f''/f)\sigma_{S,t}^2 dt\right\}\\ =&[(1-\theta)g_D + \theta S_t - g_{D,t}^e + \sigma_D^2 - \sigma_{P,t}^D\sigma_D]dt + (\sigma_{P,t}^D - \sigma_D)d\omega_t^e.\end{aligned} \tag{11}$$

Matching terms allows for expressing g_D^e and σ_P^D explicitly as functions of S, f and its derivatives

$$\begin{aligned}\sigma_P^D(S) =&\frac{\sigma_D + \sqrt{\sigma_D^2 + 4\theta(\mu_H - S)(S-\mu_L)(f'/f)}}{2},\\ g_D^e(S) =&(1-\theta)g_D + \theta S + \sigma_D^2 - \sigma_P^D(S)\sigma_D - \mu_S^e(S)(f'/f) - \tfrac{1}{2}(f''/f)(\sigma_S(S))^2.\end{aligned} \tag{12}$$

The consumption process is not perfectly correlated with the dividend process. Therefore, we need to address, both conceptually and mathematically, how the representative agent perceives the consumption process. First, notice that, without loss of generality, (4) can be alternatively expressed as

$$dC_t/C_t = g_C dt + \sigma_C\left(\rho d\omega_t^D + \sqrt{1-\rho^2}d\omega_t^\perp\right), \tag{13}$$

where $\omega_t^\perp$ is a Brownian motion that is locally uncorrelated with ω_t^D. To obtain the perceived consumption process, we impose the assumption that the representative agent has correct beliefs about $\omega_t^\perp$. In other words, we assume that the biases in beliefs about consumption only come from the biases in beliefs about dividends.[6] Consistent with this assumption, Kuchler and Zafar (2016) find that survey expectations are asset-specific: respondents who become pessimistic about their employment situation after experiencing unemployment are not pessimistic about other economic outcomes, such as stock prices or interest rates. Similarly, Huang (2016) finds that investors who become optimistic about an industry's future returns after having positive prior investment experience in the industry do not invest heavily in a dissimilar industry.

Given this assumption, the consumption process perceived by the representative agent is

$$dC_t/C_t = g_C^e(S_t)dt + \sigma_C\left(\rho d\omega_t^e + \sqrt{1-\rho^2}d\omega_t^\perp\right). \tag{14}$$

By comparing (5), (10), (13), and (14), we know

$$g_C^e(S_t) - g_C = \rho\sigma_C\sigma_D^{-1}(g_D^e(S_t) - g_D). \tag{15}$$

Empirically, ρ is a small and positive number, and σ_C is much smaller than σ_D. As a result, the difference between $g_C^e(S_t)$ and g_C is much smaller than the difference between $g_D^e(S_t)$ and g_D. In other words, in the model, the extrapolative expectations the investor has about stock returns do not transfer strongly to any biased beliefs about the consumption growth. As we will further discuss in Section II, this result has two implications. First, the perceived consumption process

[6] For any alternative assumption, one needs to explain *why* the investor has incorrect beliefs about consumption that are uncorrelated with her incorrect beliefs about dividends.

does not differ significantly from the true consumption process, and this keeps the volatility of the equilibrium interest rate low, which is consistent with the data. Second, the low correlation between movements in sentiment and movements in consumption leads to a low correlation between changes in log consumption and log excess returns, which is also consistent with the data (Hansen and Singleton (1982, 1983)).

From (2) it is apparent that when pricing the dividend stream, the gross return of the optimal portfolio is required in the Euler equation under Epstein-Zin preferences. Given this, the price-consumption ratio P_t^C/C_t also becomes a function of the sentiment variable $P_t^C/C_t \equiv l(S_t)$, and f and l are interrelated through Euler equations. Writing the perceived price process for consumption stream P_t^C as

$$dP_t^C/P_t^C = \mu_P^{C,e}(S_t)dt + \sigma_{P,1}^C(S_t)d\omega_t^e + \sigma_{P,2}^C(S_t)d\omega_t^\perp \tag{16}$$

and applying Ito's lemma on both sides of $P_t^C/C_t \equiv l(S_t)$ gives

$$\begin{aligned}&\left\{\mu_{S,t}^e(l'/l)dt + (l'/l)\sigma_{S,t}d\omega_t^e + \tfrac{1}{2}(l''/l)\sigma_{S,t}^2 dt\right\}\\ =&[\mu_{P,t}^{C,e} - g_{C,t}^e + \sigma_C^2 - \rho\sigma_{P,1}^C\sigma_C - \sqrt{1-\rho^2}\sigma_{P,2}^C\sigma_C]dt + (\sigma_{P,1}^C - \rho\sigma_C)d\omega_t^e + (\sigma_{P,2}^C - \sqrt{1-\rho^2}\sigma_C)d\omega_t^\perp.\end{aligned} \tag{17}$$

Matching terms allows for expressing $\mu_P^{C,e}$, $\sigma_{P,1}^C$, and $\sigma_{P,2}^C$ explicitly as functions of S_t, l and its derivatives

$$\begin{aligned}\sigma_{P,2}^C &= \sqrt{1-\rho^2}\sigma_C,\\ \sigma_{P,1}^C(S) &= \rho\sigma_C + (l'/l)\sigma_S(S),\\ \mu_P^{C,e}(S) &= g_C^e(S) - \sigma_C^2 + \rho\sigma_{P,1}^C(S)\sigma_C + \sqrt{1-\rho^2}\sigma_{P,2}^C\sigma_C + \mu_S^e(S)(l'/l) + \tfrac{1}{2}(l''/l)(\sigma_S(S))^2,\end{aligned} \tag{18}$$

where $\sigma_P^D(S)$ and $g_C^e(S)$ are derived in (12) and (15) respectively, and $\mu_S^e(S)$ and $\sigma_S(S)$ are from (9).

Based on the subjective Euler equation of (2), the evolutions of f and l are governed by

$$0 = \left[\begin{array}{l} -\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(f'/f) + \frac{\psi-\gamma}{1-\psi}(l'/l)]\mu_S^e + \frac{1}{2}[(f''/f) + \frac{\psi-\gamma}{1-\psi}(l''/l)]\sigma_S^2\\ +\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(l'/l)^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(l'/l) - \gamma\rho\sigma_C\sigma_D - \gamma\rho\sigma_C\sigma_S(f'/f)\\ +\frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(l'/l) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2(l'/l)(f'/f) + \sigma_D\sigma_S(f'/f) + \frac{\psi-\gamma}{1-\psi}l^{-1} + f^{-1}\end{array}\right] \tag{19}$$

and

$$0 = \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(l'/l)\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(l''/l)\sigma_S^2\\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(l'/l)^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(l'/l) + \frac{1-\gamma}{1-\psi}l^{-1}\end{array}\right]. \tag{20}$$

Substituting μ_S and σ_S from (9), σ_P^D and g_D^e from (12), and g_C^e from (15) into equations (19) and (20), we obtain a system of two ordinary differential equations that jointly determines f and

l.[7] The detailed derivation of (19) and (20) is in the Appendix.

In both (19) and (20), the second derivative terms are all multiplied with σ_S, and σ_S goes to zero as S approaches either μ_H or μ_L. As a result, μ_H and μ_L are both singular points, and no boundary condition is required when jointly solving these differential equations. Equations (19) and (20) cannot be solved analytically. We apply a projection method with Chebyshev polynomials to solve them numerically. We discuss the details of the numerical procedure in the Appendix.

Below we derive some key variables that are important for understanding the model implications.

A. Interest Rate Dynamics

Given the price-consumption ratio l for the consumption stream and the price-dividend ratio f for the dividend stream, we can determine the equilibrium shadow interest rate. From (2) we know that for a riskless bond $\tilde{R}_{j,t+\Delta t} \equiv B_{t+\Delta t}/B_t = 1 + r_t \Delta t$. Substituting this and the optimal return from (3) into (2) and letting Δt go to zero, we obtain

$$0 = \mathbb{E}_t^e \left[d(\Theta C^{(\psi-\gamma)/(1-\psi)} l^{(\psi-\gamma)/(1-\psi)} B) + \tfrac{\psi-\gamma}{1-\psi} \Theta C^{(\psi-\gamma)/(1-\psi)} l^{(\psi-\gamma)/(1-\psi)} B l^{-1} dt \right], \tag{21}$$

where $\Theta(C,t) \equiv e^{-\delta(1-\gamma)t/(1-\psi)} C^{-\psi(1-\gamma)/(1-\psi)}$. Applying Ito's lemma,

$$0 = \mathbb{E}_t^e \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta dt - \gamma(dC/C) + \frac{\gamma(\gamma+1)}{2}(dC/C)^2 + \frac{\psi-\gamma}{1-\psi}(dl/l) + \frac{(\psi-\gamma)(2\psi-\gamma-1)}{2(1-\psi)^2}(dl/l)^2 \\ +r_t dt - \gamma\frac{\psi-\gamma}{1-\psi}(dC/C)(dl/l) + \frac{\psi-\gamma}{1-\psi} l^{-1} dt \end{array} \right]. \tag{22}$$

By (9) and (14), (22) can be simplified as

$$r_t = \tfrac{1-\gamma}{1-\psi}\delta + \gamma g_C^e - \tfrac{\gamma(\gamma+1)}{2}\sigma_C^2 - \tfrac{\psi-\gamma}{1-\psi} \times \left[\begin{array}{l} (\mu_S^e - \gamma\rho\sigma_C\sigma_S)(l'/l) + \frac{1}{2}\sigma_S^2(l''/l) \\ +\frac{2\psi-\gamma-1}{2(1-\psi)}\sigma_S^2(l'/l)^2 + l^{-1} \end{array} \right]. \tag{23}$$

When $\theta = 0$, $r = \delta + \psi g_C - \frac{\gamma(\psi+1)}{2}\sigma_C^2$.

B. Objective versus Subjective Expectations and Sharpe Ratios

At each point in time for a given sentiment level S_t, we can compute the subjective and objective expected log excess returns on the equity market as well as the subjective and objective Sharpe ratios.

[7] When $\theta = 0$, our model reduces to a fully rational benchmark and equations (19) and (20) give rise to

$$l = \left[\delta + (\psi-1)g_C - \tfrac{\gamma(\psi-1)}{2}\sigma_C^2\right]^{-1}, \quad f = \left[\delta + \psi g_C - g_D - \tfrac{\gamma(\psi+1)}{2}\sigma_C^2 + \gamma\rho\sigma_C\sigma_D\right]^{-1}.$$

Equations (6) and (7) imply that the log excess return from time t to $t+dt$ is

$$\begin{aligned} &\ell\mathrm{n}(P^D_{t+dt} + D_{t+dt}dt) - \ell\mathrm{n}(P^D_t) - r_t dt \\ \equiv &r^{D,e}_{t+dt}dt = [(1-\theta)g_D + \theta S_t + f^{-1} - \tfrac{1}{2}(\sigma^D_P)^2 - r_t]dt + \sigma^D_P d\omega^e_t. \end{aligned} \tag{24}$$

Therefore

$$\mathbb{E}^e_t[r^{D,e}_{t+dt}] = (1-\theta)g_D + \theta S_t + f^{-1} - \tfrac{1}{2}(\sigma^D_P)^2 - r_t. \tag{25}$$

The subjective Sharpe ratio is therefore $[(1-\theta)g_D + \theta S_t + f^{-1} - \frac{1}{2}(\sigma^D_P)^2 - r_t]/\sigma^D_P$.

By comparing (5) and (10)

$$d\omega^D_t = (g^e_D(S_t) - g_D)dt/\sigma_D + d\omega^e_t. \tag{26}$$

Therefore

$$r^{D,e}_{t+dt}dt = [(1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma^D_P(g^e_D - g_D) - \tfrac{1}{2}(\sigma^D_P)^2 - r_t]dt + \sigma^D_P d\omega^D_t. \tag{27}$$

The objective expectation of log excess returns is

$$\mathbb{E}_t[r^{D,e}_{t+dt}] = (1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma^D_P(g^e_D - g_D) - \tfrac{1}{2}(\sigma^D_P)^2 - r_t. \tag{28}$$

where $g^e_D(S)$ is derived in (12). The objective Sharpe ratio for P^D_t is $[(1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma^D_P(g^e_D - g_D) - \frac{1}{2}(\sigma^D_P)^2 - r_t]/\sigma^D_P$.

C. Steady-State Distribution for Sentiment

In order to evaluate the conditional and unconditional moments of the model, we need to compute the steady-state distribution for the sentiment variable S_t *objectively* measured by an outside econometrician. To do so, we substitute the change of measure from (26) into the subjective evolution of sentiment in (9) and obtain the objective evolution of sentiment as

$$dS_t = [\mu^e_S(S_t) + \sigma_D^{-1}\sigma_S(S_t)(g_D - g^e_D(S_t))]dt + \sigma_S(S_t)d\omega^D_t. \tag{29}$$

Compared to the subjective evolution of sentiment, the objective evolution is subject to a larger degree of mean reversion: the term $\sigma_D^{-1}\sigma_S(S_t)(g_D - g^e_D(S_t))$ in (29) is negative when sentiment S_t is high and positive when S_t is low.[8] Denote the objective steady-state distribution for sentiment

[8] With a lower degree of mean reversion for the subjective sentiment evolution, the steady-state distribution of S_t *perceived* by the representative agent sometimes become degenerate.

as $\xi(S)$. Its evolution follows the Kolmogorov forward equation (Fokker-Planck equation)

$$\begin{aligned} 0 =& \tfrac{1}{2}\tfrac{d^2}{dS^2}\left(\sigma_S^2(S)\xi(S)\right) - \tfrac{d}{dS}\left([\mu_S^e(S_t) + \sigma_D^{-1}\sigma_S(S_t)(g_D - g_D^e(S_t))]\xi(S)\right) \\ =& (\sigma_S')^2\xi + \sigma_S\sigma_S''\xi + 2\sigma_S\sigma_S'\xi' + \tfrac{1}{2}\sigma_S^2\xi'' \\ & -[(\mu_S^e)' + \sigma_D^{-1}\sigma_S'(g_D - g_D^e) - \sigma_D^{-1}\sigma_S(g_D^e)']\xi - [\mu_S^e + \sigma_D^{-1}\sigma_S(g_D - g_D^e)]\xi'. \end{aligned} \tag{30}$$

The expressions for g_D^e and σ_S are from (12) and (9), respectively, and the expressions for σ_S', σ_S'', $(\mu_S^e)'$ and $(g_D^e)'$ are included in the Appendix. The steady-state distribution must satisfy

$$\int_{\mu_L}^{\mu_H} \xi(S)dS = 1. \tag{31}$$

II. Model Implications

In this section, we examine the model implications. Overall, for parameters that have dispersed estimates of values in the literature, we rely on comparative statics in Section III to show how sensitive the model implications are with respect to changes in these parameter values.

First, we set the benchmark values for the model parameters. For the asset-specific parameters, we set $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$. These values are consistent with the values used in Campbell and Cochrane (1999), Barberis, Huang, and Santos (2001), Bansal and Yaron (2004), and Beeler and Campbell (2012). The parameter values for g_C and g_D are set such that both log consumption and log dividend grow at an annual rate of 1.84%; this rate is used in Barberis et al. (2001).

Next, we assign default values for our preference-specific parameters, γ, ψ, and δ. We set γ, the relative risk aversion parameter, to 10. As pointed out in Bansal et al. (2012) and Bansal and Yaron (2004), the long-run risks literature—a literature that depends significantly on the parameter values of Epstein-Zin preferences for its model implications—typically assigns a value of 10 or below for γ. Bansal and Yaron (2004), for instance, set γ to either 10 or 7.5. An estimate of 10 for γ is also the maximum magnitude that Mehra and Prescott (1985) find reasonable. For ψ, the reciprocal of the elasticity of intertemporal substitution, there exists a wide range of estimates in the asset pricing literature. The majority of previous work suggests that ψ should be lower than one, but several other papers argue the opposite.[9] Given these results, we set our benchmark value for ψ at 0.9, a value that is slightly below one. For δ, the subjective discount factor, we assign a value of 2%.

For the belief-specific parameters, we set $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. First recall that the parameter θ controls the extent to which the representative agent is behavioral: $\theta = 0$ means the agent is fully rational, whereas $\theta = 1$ means the agent is fully behavioral. Therefore, 0.5 seems to be a natural default value. As for χ, λ, μ_H, and μ_L, these parameters are not directly observable from the price and consumption data. However, they can

[9] See Bansal et al. (2012) for a discussion of this point.

be disciplined, to some extent, by survey data. Greenwood and Shleifer (2014) suggest that: 1) a significant portion of real-world investors extrapolates past returns when forecasting future returns; and 2) these investors only use the past 1-2 years of returns when forming expectations of returns over the next year. Given these observations, we discipline our choices of χ and λ such that our representative investor only uses the past couple of years of returns when forecasting future returns (see Table 3 for a detailed analysis). Lastly, we choose μ_H and μ_L that are approximately symmetric in magnitude.

We summarize our default parameter values in Table 1.

[Place Table 1 about here]

In the remaining part of this section, we first examine two building blocks for the model implications: the steady-state distribution of the sentiment and a set of important empirical quantities each as a function of sentiment. We then compute some key unconditional and conditional moments of the model and compare them with their corresponding empirical values. Next, we study the investor's belief structure by looking at the relationship between her subjective expectations about future returns and the current or past realized returns. Finally, we examine the asset pricing implications of the model by analyzing return predictability, autocorrelations of log excess returns and log price-dividend ratios, as well as the correlations between consumption growth and stock returns.

We begin our analysis of model implications by plotting the steady-state distribution of the sentiment variable in Figure 1. The steady-state distribution is slightly negatively skewed because at times when past returns are low, the concavity of the investor's utility function allows for a strong interaction between pessimistic beliefs and risk aversion. This interaction pushes the risky asset price further down and gives rise to a lower level of sentiment.

[Place Figure 1 about here]

Next we plot the price-dividend ratio f, the return volatility σ_P^D, the objective expectation of log excess return (equity premium) $\mathbb{E}[r^{D,e}]$, and the interest rate r, with each as a function of the sentiment variable S.

[Place Figure 2 about here]

Figure 2 shows that the model generates a strong procyclical pattern for the price-dividend ratio, the valuation measure for the aggregate equity market. With a sequence of high past returns on the risky asset, investors become more optimistic about future returns and hence push up the risky asset price. With a higher price, the current return gets higher, and therefore investors become even more optimistic about future returns, pushing up the risky asset price further. This two-way feedback loop not only serves as the main driver for the procyclicality of the price-dividend ratio, it is also the main source for excess volatility.

In addition, the model generates a strong countercyclical pattern for the equity premium. With a sequence of low past returns on the risky asset, investors become pessimistic about future returns and hence push down the risky asset price, causing future returns to rise on average. This effect is further amplified by the two-way feedback loop described above. At the same time, investors' pessimistic beliefs about future stock market returns partly translate into pessimistic beliefs about the consumption growth, which reduces the equilibrium interest rate. Both the higher (true) expected log return on the stock market and the lower interest rate help generate a higher equity premium during bad times, and the first effect is the dominant one: moving S_t from its mean to the bottom 25% percentile level causes a total increase of 8.2% for the equity premium, out of which 8.0% comes from the increase in the expected log return of the stock market.

Conversely, with a sequence of good past returns, investors become very optimistic about future returns; they push up the risky asset price and cause future returns to drop on average. At the same time, optimistic beliefs about stock market returns lead to optimistic beliefs about future consumption growth. This causes the interest rate to rise. In this case, both the low (true) expected log return on the stock market and the high interest rate help generate a low equity premium during good times, and again the first effect is the dominant one. Moving S_t from its mean to the top 25% percentile level causes a total decrease of 9.7% for the equity premium, out of which 9.5% comes from the decrease of the expected log return of the stock market.

Given the steady-state distribution (Figure 1) and the asset prices conditional on a given sentiment level (Figure 2), we provide some basic unconditional and conditional moments of the model in Table 2.

[Place Table 2 about here]

Table 2 suggests that with the default parameter values, the model can generate a sizable equity premium, a significant amount of excess volatility, and a relatively low and stable interest rate, all consistent with the data.[10] As discussed earlier, return extrapolation generates a two-way feedback effect that serves as the main source of excess volatility. For the equity premium, three channels contribute to its large magnitude. First and most intuitively, excess volatility together with risk aversion causes the equity premium to rise. Second, with return extrapolation, the representative agent perceives that both the aggregate dividend process and the aggregate consumption process are persistent, even though the true processes are not persistent. This *perceived* persistence interacts with Epstein-Zin preferences and induces a larger equity premium.[11] Lastly, the separation of elasticity of intertemporal substitution and relative risk aversion keeps the interest rate low for the majority of sentiment levels and hence helps to keep the equity premium high. And the perceived consumption process does not differ significantly from the true consumption process, keeping the volatility of the interest rate low.

[10] Excess volatility is defined as the ratio of the average volatility of return over fundamental volatility. From Table 2, this ratio is 2.74.

[11] More precisely, the investor is averse to persistent shocks when $\gamma > \psi$.

The model can generate a large and *negative* equity premium during high-sentiment periods, consistent with the recent empirical findings of Greenwood and Hanson (2013), Baron and Xiong (2015), Cassella and Gulen (2015), and Yang and Zhang (2016). A rational expectations model—for instance, long-run risks models and habit formation models—does not generate a negative equity premium.[12] In our model, the subjective expectations and objective expectations differ significantly during high-sentiment or low-sentiment periods: When the sentiment level is very high, investors expect high equity returns moving forward, but precisely because of their incorrect beliefs, future equity returns will be low on average and the interest rate rises. Both contribute to the negative equity premium.

Variation of sentiment levels leads to variation in log price-dividend ratios, as observed in the data. The majority of the variation in log price-dividend ratios occurs with intermediate levels of sentiment; during high-sentiment or low-sentiment periods, the regime-switching structure of learning reduces excess volatility and thereby reduces the standard deviation of log price-dividend ratios.

Table 2 also suggests that the subjective return expectations are negatively correlated with model-implied expected returns. This result is consistent with the empirical findings of Greenwood and Shleifer (2014) and cannot be obtained from rational expectations models. Furthermore, it is important to recognize that the largest contribution to this negative correlation comes during high- and low-sentiment periods. After a sequence of high (low) past returns, the sentiment level becomes very high (low), pushing down (up) the log dividend-price ratio. Precisely because of the high (low) sentiment level, subsequent returns are low (high). So one testable prediction of the model is that the correlation between the model-implied expected returns and the difference between subjective and objective expectations should be negative and significant; in the model, $\mathbb{C}\text{orr}_\xi(\mathbb{E}^e[R^D] - \mathbb{E}[R^D], \ell\text{n}(D/P))$ is –0.986.

We now begin to examine the model implications on investor beliefs more closely. First, we regress the representative agent's subjective expectation about future returns on either the past twelve-month cumulative raw return or the current log price-dividend ratio, over a sample of 15 years or 50 years. We report in Table 3 the regression coefficient and its t-statistic, the intercept, as well as the adjusted R-squared. Specifically, we examine *four* expectations measures of returns: the subjective expectation of $R^D_{t+dt} = (dP^D_t + D_t dt)/(P^D_t dt)$, the percentage returns on the equity market, the subjective expectation of $R^{D,e}_{t+dt} = (dP^D_t + D_t dt)/(P^D_t dt) - r_t$, the percentage returns on the equity market in excess of the interest rate, the subjective expectation of $dP^D_t/(P^D_t dt)$, the capital gain on the equity market, and the subjective expectation of $dP^D_t/(P^D_t dt) - r_t$, the capital gain on the equity market in excess of the interest rate. The latter two measures are included based on the observation that investors may not actively think about dividend yields when answering survey questions.[13] Each reported value—for instance, the regression coefficient—is averaged over

[12] Strictly speaking, it is still possible for a rational expectations model to generate a negative equity premium if the stock market negatively correlates with some other factors and can therefore serve as a strong diversification device.

[13] When calibrating model parameters to survey expectations, Barberis et al. (2015) take this interpretation.

100 trials, with each trial being a regression based on simulated data of the model with a monthly frequency.

Table 3 suggests that in the model, the magnitude of investors' extrapolative beliefs about future returns matches the empirical values suggested by Greenwood and Shleifer (2014). Specifically, regressing $\mathbb{E}_t^e[R_{t+dt}^D]$ ($\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$) on past twelve-month cumulative raw returns for a 15-year long sample, the regression coefficient is 2.2% (4.0%) with a Newey-West adjusted t-statistic of 8.2 (8.4). Running the same regression for a 50-year long sample, the regression coefficient is 2.3% (4.0%) with a Newey-West adjusted t-statistic of 11.7 (12.1).[14] For comparison, with a 5-year long sample of sparse data from the Michigan survey, the regression coefficient is 3.9% with a t-statistic of 1.68; and with a 15-year long sample of data from the Gallup survey, the regression coefficient, after some conversion, is about 8% with a t-statistic of 8.81. Also consistent with the data, survey expectations are positively correlated with log price-dividend ratios. Since log price-dividend ratios negatively predict subsequent returns, survey expectations are also negatively correlated with realized returns. In Section IV, we compare in detail our model implications on investor expectations with those from rational expectations models of Campbell and Cochrane (1999), Bansal and Yaron (2004), and Bansal et al. (2012).

[Place Table 3 about here]

We next study belief formation of our representative investor by examining her memory structure. Specifically, we run non-linear least squares regressions of the form

$$\text{Expectation}_t = a + b\sum_{j=1}^{n} w_j R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)} + \varepsilon_t, \tag{32}$$

where $w_j = e^{-\phi(j-1)\Delta t}/\sum_{l=1}^{n} e^{-\phi(l-1)\Delta t}$. We also call (32) the investor's memory structure.

In Table 4 we report the regression coefficient a, the intercept b, the adjusted R-squared, and most importantly, the estimated ϕ. As before, we examine four expectations measures, $\mathbb{E}_t^e[R_{t+dt}^D]$, $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$, $\mathbb{E}_t^e[R_{t+dt}^{D,e}]$, and $\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$. Each reported value is averaged over 100 trials, with each trial being a regression based on simulated data of the model with a monthly frequency over either 15 years or 50 years. Also we set $\Delta t = 1/12$ and $n = 600$.

[Place Table 4 about here]

Table 4 shows that with our default parameters, the estimation of ϕ is stable across different expectations specifications, and it is about 0.42; this means that a monthly return three years ago is weighted about 25% as much as the most recent return. For comparison, Barberis et al. (2015) also obtain an estimate of about 0.44 for ϕ, an estimate that is very close to the model-implied value. This is not a coincidence. We assign a value of 0.18 to both λ and χ so that our representative investor has a short memory span when using past returns to forecast future returns. As we will

[14] Notice that subtracting the interest rate from expectations of returns does not significantly affect the regression results due to the low interest rate volatility in our model.

see later from our comparative statics analysis, a lower value for λ and χ—that is, a more "sticky" belief structure—implies a lower degree of informativeness of recent returns, and hence a longer memory span for investors.

The current behavioral literature lacks consensus on investors' memory span. While Greenwood and Shleifer (2014) and Kuchler and Zafar (2016) find that investor expectations only depend strongly on most recent returns from the past few years, Malmendier and Nagel (2011, 2013) and Vanasco et al. (2015) suggest that distant past events may also play an important role when investors form beliefs. Although reconciling this discrepancy is beyond the scope of the paper, one possible explanation is that when forming beliefs, investors adopt two separate belief formation processes: one with a short memory span for frequent but less salient events such as moderate stock market fluctuations on a daily basis, and the other with a long memory span for infrequent but salient events such as a market crash.

The remaining part of the section looks at the model implications on asset prices. First, we examine return predictability. As documented by Campbell and Shiller (1988) and Fama and French (1988), regressing subsequent log excess returns over certain time horizon on the current log price-dividend ratio leads to a negative regression coefficient. Moreover, the predictive power becomes greater at longer time horizons.

Table 5 reports the regression coefficient β_j and the corresponding adjusted R-squared when regressing the log excess return of the stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\mathrm{n}(P_t^D/D_t)$

$$r_{t\to t+j}^{D,e} = \alpha_j + \beta_j \ell\mathrm{n}(P_t^D / D_t) + \varepsilon_{j,t} \tag{33}$$

over various horizons j. Regressions are based on simulated data of the model with a total length of 10,000 years and with a monthly frequency. Consistent with the data, both β_j and the adjusted R-squared generally increase as the time horizon j increases. However, over longer horizons (beyond three years), the adjusted R-squared tends to decrease slightly as j increases further. To understand this, notice that in the model, the predictability of price-dividend ratios is driven by the mean reversion of sentiment, which, in turn, is driven by the fact that, from the representative investor's perspective, recent returns carry more information than distant returns when forecasting future returns. With a short memory span, mean reversion of sentiment occurs over a relatively short time horizon. Over longer horizons, no extra mean reversion in beliefs contributes to return predictability.

[Place Table 5 about here]

One analysis closely related to return predictability is the approximate decomposition of the

log dividend-price ratio proposed in Campbell and Shiller (1988)

$$\ell\mathrm{n}(D_t/P_t^D) \approx \sum_{j=0}^{\infty} \rho^j \left(r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)} - \Delta d^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)} \right) + (\ell\mathrm{n}(\rho) + (1-\rho)\delta)/(1-\rho), \tag{34}$$

where δ is the in-sample average of the annual log dividend price ratio, $\rho = e^{-\delta}/(\Delta t + e^{-\delta})$, $r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ is the log gross return from time $t+j\Delta t$ to $t+(j+1)\Delta t$, and $\Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ is the log dividend growth over the same time period. Equation (34) suggests that for regressing the weighted future log gross return $\sum_{j=0}^{\infty} \rho^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ and the weighted future log dividend growth $\sum_{j=0}^{\infty} \rho^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ on the current dividend-price ratio, the first regression coefficient minus the second regression coefficient is approximately equal to one. Empirically, the first coefficient is approximately one and the second coefficient is approximately zero. As Cochrane (2011) summarizes, all the stock price movements correspond to discount rate news instead of cash flow news.

[Place Table 6 about here]

We report, in Table 6, the two regression coefficients, one for the weighted future log gross return and one for the weighted future log dividend growth, as well as their corresponding adjusted R-squared. These regressions are based on simulated data of the model over 10,000 years and with a monthly frequency, and they are run by outside econometricians under the objective probability. Table 6 shows that consistent with the empirical literature on asset pricing, the variation of price-dividend ratio comes primarily from variation in discount rates. However, our belief-based behavioral model takes a step further and presents one important source of discount rate variation. To see this, notice that Table 6 also reports the two regression coefficients under the subjective probability measure. From the representative agent's perspective, the coefficient for the weighted future log gross return is close to zero, and the coefficient for the weighted future log dividend growth is close to -1. In other words, the behavioral agent *thinks* that the stock price movements primarily correspond to cash flow news rather than discount rate news. But precisely because of this, discount rate news, under rational expectations, drive stock price movements.[15]

We now look at the autocorrelations of asset prices in the model. Empirically, log excess returns exhibit negative autocorrelations at longer lags (Poterba and Summers (1988)), and price-dividend ratios are highly persistent at short lags. Table 7 below suggests that the model captures both facts. The negative autocorrelations of log excess returns are generated by mean reversion of the sentiment level. The persistence of price-dividend ratios, on the other hand, is driven by the persistence of sentiment levels.

Following our discussion of return predictability, a short memory span for investors implies a time series of log price-dividend ratios that is much less persistent than what the data suggests. A careful reconciliation between investors' memory span and the persistence levels of asset prices

[15] We obtain regression coefficients similar to those in Table 6 when using the VAR approach proposed in Cochrane (2011).

is left for future research. Here we only provide one possible hypothesis. Lopez-Salido, Stein, and Zakrajsek (2016) show that in the data, various types of financial frictions are more persistent than belief-based market sentiment. One consequence is that the interaction between frictions and sentiment can further prolong the impact of extrapolative beliefs on asset prices. Therefore, the absence of frictions in the model may prevent it from generating longer-term effects of market sentiment.

[Place Table 7 about here]

Hansen and Singleton (1982, 1983) document a low correlation of stock returns and consumption growth. Most consumption-based asset pricing models, however, generate a very high, if not perfect, correlation between realized stock returns and consumption growth. In Table 8 we examine our model's implications for correlations of this type.

[Place Table 8 about here]

Table 8 suggests that the contemporaneous correlations between changes in log consumption and log excess returns are around 0.2. These correlations are fairly close to the empirical value of 0.09; for comparison, the correlations in Campbell and Cochrane (1999) are 0.79, and 0.40 at a monthly and an annual frequency, respectively. To understand why our model generates low correlations, we note that return extrapolation, the primary force that drives systematic time variation in stock returns, is directly applied to the risky asset of dividend streams, but much less directly to the consumption stream. Empirically, consumption growth and dividend growth have a low correlation ($\rho = 0.2$). As a result, a large component of consumption movements does not affect the risky asset price.[16] For many rational expectations models in which asset pricing implications rely heavily on adjusting the consumption-based pricing kernels, the correlations between consumption growth and stock returns tend to become very high.

Consistent with the data, the model also generates a small and negative correlation between the current change in log consumption and the subsequent log excess returns. Because consumption growth and dividend growth are weakly and positively correlated, a period of high consumption growth tends to coincide with a period of high dividend growth. High dividend growth drives up the asset price and therefore pushes up the sentiment level. In subsequent periods, as sentiment mean-reverts, stock returns become low.

We conclude this section by discussing the role of rational arbitrageurs. In our model, the representative agent has biased beliefs. One natural question to ask is: if we introduce rational arbitrageurs to the model, to what extent can they counteract the mispricing caused by the behavioral agents and therefore attenuate the significance of our model implications? Having such a model extension is beyond the scope of the paper. However, three observations seem to suggest that

[16] Similarly, Barberis et al. (2001) apply prospect theory to preferences over financial assets, not preferences over consumption stream. As a result, the correlation between consumption growth and stock returns remains low in their model.

most of our model implications will remain intact after taking rational arbitrageurs into account. First, as discussed in Barberis et al. (2015), extrapolative expectations can be persistent over time. As a result, behavioral agents tend to have heightened and persistent demand on the stock market following good past returns. The persistence of extrapolator demand prevents stock market returns from becoming too low in the near future, hence preventing rational arbitrageurs from significantly counteracting the mispricing. Second, Borovicka (2016) shows that under recursive preferences, with a positive probability, behavioral agents may even end up dominating the economy in the long run. Lastly, although extrapolators may exit the market over time, in reality they may be replaced, at least in part, by a new group of behavioral agents.

III. Comparative Statics

In this section, we look at comparative statics of the model. We focus on parameters that are difficult to measure empirically, and examine the sensitivity of the model implications with respect to changes in these parameters.

[Place Figure 4 about here]

Figure 4 analyzes the impact of the degree-of-extrapolation parameter θ on asset prices. When $\theta = 0$, the model reduces to its rational benchmark. In this case, the equity premium reduces to 0.23%; the return volatility reduces to the fundamental volatility of 11%; the price-dividend ratio stays constant at 135.1; and the interest rate stays constant at 2.35%. With a higher θ, two effects arise. First, a higher degree of return extrapolation induces a stronger two-way feedback loop described earlier. This results in larger return volatility, a higher equity premium, and lower price-dividend ratios. Second, a higher degree of return extrapolation leads to higher persistent risks perceived by the investor. In conjunction with Epstein-Zin preferences, these higher perceived risks lead to an even higher equity premium.

Overall, a 1% increase in θ leads to an approximately 0.09% increase in the equity premium and an approximately 0.29% increase in return volatility. On the other hand, the interest rate is much less sensitive to changes in θ. This is because: 1) the separation of elasticity of intertemporal substitution and relative risk aversion keeps the interest rate low and stable; and 2) only a small fraction of incorrect beliefs about g_D^e gets transformed into incorrect beliefs about g_C^e due to the low correlation between consumption growth and dividend growth (see (15)), and therefore the perceived evolution of the consumption-based pricing kernel—the key process that drives the interest rate—is not much affected by extrapolation biases.

We next look at the impact of belief parameters θ, χ, and λ on investor memory and the persistence of asset prices. As suggested by Table 4, investor memory (span) is characterized by ϕ and it is estimated from the regression of (32): a higher ϕ means that the investor looks at a shorter history of past return realizations when forming her beliefs about future returns. For the persistence of asset prices, we look at the autocorrelation of log price-dividend ratios at a one-year

lag.

[Place Figure 5 about here]

Figure 5 suggests that increasing θ, χ, and λ all lead to a shorter memory span for the investor (a higher ϕ) and less persistent log price-dividend ratios. With higher intensities of regime switch—that is, a higher χ or λ for (8)—recent returns carry more information for the investor when forecasting future returns. This leads to a shorter memory span and hence faster mean reversion of investor sentiment, which in turn results in less persistent price-dividend ratios. Increasing θ generates a similar effect. With rational expectations, the investor uses *all* past returns to form an efficient estimate for the expected return of the equity market. These rational expectations effectively require a longer memory span. A higher θ leads to a higher degree of *over*-extrapolation, inducing the investor to pay more attention to recent returns and therefore shortening her memory span. Increasing θ from 0.2 to 0.5 increases ϕ from 0.38 to 0.43 and reduces the one-year autocorrelation of log price-dividend ratios from 0.52 to 0.33.

Furthermore, comparative statics of the model with respect to θ provides implications that are consistent with the recent empirical findings of Cassella and Gulen (2015). They show that during periods when investor expectations depend heavily on recent past returns, price-dividend ratios strongly predict returns over the next year. But during periods when investor expectations depend more on distant past returns, price-dividend ratios do not strongly predict returns over the next year. In our model, not only does a higher θ lead to a higher ϕ, it also leads to a larger regression coefficient (in magnitude) with a higher R-squared when regressing subsequent excess returns over a short horizon on the current price-dividend ratio: increasing θ from 0.05 to 0.5 changes β_1 of the regression (33) from –0.32 (with an adjusted R-squared of 0.001) to –0.72 (with an adjusted R-squared of 0.13). At the same time, with a lower θ and hence a lower ϕ, regressing subsequent excess returns over a *long* horizon on the current price-dividend ratio gives rise to a larger regression coefficient with a lower R-squared: reducing θ from 0.5 to 0.05 changes β_7 of the regression (33) from –1.1 (with an adjusted R-squared of 0.12) to –1.3 (with an adjusted R-squared of 0.003). Intuitively, a higher degree of over-extrapolation (a higher θ) implies a higher degree of mispricing and a shorter memory span for the investor, which, in turn, leads to stronger mean reversion of investor sentiment and causes the short-term predictability of price-dividend ratios to rise.

To conclude our discussion on comparative statics, we now look at the preference-based parameters. We find that 1) lowering γ from 10 to 7.5 reduces the equity premium from 4.88% to 4.01%; and 2) changing the reciprocal of the elasticity of intertemporal substitution ψ between 0.85 and 1.15 does not affect the moments of the model much. In other words, our model implications are quantitatively robust with respect to changes in ψ.

IV. Comparison with Rational Expectations Models

Modelling return extrapolation in a Lucas economy allows us to quantitatively compare our model implications with those derived from rational expectations models. In this section, we first reduce the behavioral model we laid out in Section I to a fully rational model and study its implications. We then examine some important differences between our behavioral model and the rational expectations models of Campbell and Cochrane (1999), Bansal and Yaron (2004), and Bansal et al. (2012).

The rational expectations model most comparable to our model is the one that assumes the regime-switching process characterized by equations (8) and (9) to be the *true* data generating process. Specifically, the true evolution of the stock market price, instead of (7), is

$$dP_t^D/P_t^D = [(1-\theta)g_D + \theta\tilde{\mu}_{S,t}]dt + \sigma_P^D(S_t)d\omega_t^P, \tag{35}$$

where ω_t^P is a standard Brownian motion, and the representative agent uses past prices to form a rational estimate of $\tilde{\mu}_{S,t}$; $S_t = \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^P]$. In this model, the true dividend process is (10) instead of (5), and the true consumption process is (14) instead of (4).[17] By construction, such a rational expectations model shares the same equilibrium prices with our behavioral model; the solutions to the differential equations of (19) and (20) also apply to this rational model. Nonetheless, these two models remain significantly different. One such difference lies in the model implication on return predictability.

[Place Table 9 about here]

In Table 9 we report the regression coefficient β_j and the corresponding adjusted R-squared for regressing the log excess return of the stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$ over various horizons j (one to seven years), now using the true regime-switching model described above. In comparison to the regression results documented in Table 5 for the behavioral model, with rational expectations, the current log price-dividend ratio does not predict subsequent stock market returns in any significant way: the adjusted R-squared is very close to zero, inconsistent with the empirical findings of Campbell and Shiller (1988) and Fama and French (1988).

To understand this model implication, first notice that in the behavioral model,

$$\mathbb{E}_t[d\omega_t^e] = \sigma_D^{-1}(g_D - g_D^e(S_t))dt, \tag{36}$$

which is significantly negative when S_t is high and significantly positive when S_t is low. On the

[17] This rational expectations model lies between a Lucas economy and a Merton economy, in the sense that, for the stock market price, its growth rate is exogenous but the volatility is endogenous. To be consistent with the survey evidence, here the regime-switching assumption (35) is imposed on price not dividend.

other hand, in the true regime-switching model,

$$\mathbb{E}_t[d\omega_t^e] = (\sigma_P^D)^{-1}\theta(\tilde{\mu}_{S,t} - S_t)dt, \tag{37}$$

which is positive when $\tilde{\mu}_{S,t}$ is high (equals to μ_H)—these are typically periods when S_t is high—and negative when $\tilde{\mu}_{S,t}$ is low (equals to μ_L)—these are typically periods when S_t is low. The difference in expectations of $d\omega_t^e$ between these two models implies that under the objective probability measure, the evolution of sentiment S_t is much *less* mean-reverting and more volatile in the true regime-switching model with rational expectations than in the behavioral model. Figure 3 plots the steady-state distribution of S_t under the true regime-switching model, and the distribution is indeed much more dispersed in comparison to the steady-state distribution of sentiment under the behavioral model as shown in Figure 1. Given that the evolution of excess returns of the stock market is driven by the evolution of sentiment in this model, sentiment being more volatile and less mean-reverting means that during high or low sentiment periods, excess returns tend to stay flat in subsequent periods, reducing the statistical significance of the return predictability regression. Furthermore, with rational expectations, sentiment S_t tends to be high when the true state of the economy $\tilde{\mu}_{S,t}$ is high. In such circumstances, the agent correctly anticipates high capital gain on the stock market dP_t^D/P_t^D. At the same time, a high value of $\tilde{\mu}_{S,t}$ leads to a low dividend yield and a high interest rate, altogether giving rise to flat excess returns of the stock market over subsequent periods.

[Place Figure 3 about here]

For the remaining part of this section, we compare our behavioral model with some leading asset pricing models about the stock market in the literature. Table 10 reports the regression coefficients and their t-statistics for regressing, in the model of Campbell and Cochrane (1999), *rational* expectations about future returns either on the past twelve-month return or on the current log price-dividend ratio. The regression coefficients on the past twelve-month return are negative and insignificant for simulated samples of either 15 years or 50 years. In their model, high past twelve-month returns are primarily driven by a sequence of positive shocks on dividend, and these shocks are positively correlated with shocks on consumption, suggesting that realized consumption growth has been high. As a result, the difference between the level of consumption and the level of habit increases, reducing the effective risk aversion and therefore lowering the expected returns moving forward. Because the correlation between dividend shocks and consumption shocks are low in the model, the correlation between past twelve-month returns and the expectations of (short-term) future returns is also low, giving rise to the low levels of t-statistics for the coefficients on past returns.[18] In contrast to the positive and significant regression coefficients we obtain in Table 3 for

[18]On the other hand, the regression coefficients on the current log price-dividend ratio are negative and significant; high levels of current log price-dividend ratio are primarily driven by a sequence of positive shocks on *consumption* instead of dividend, and these shocks directly reduce the effective risk aversion in the model and therefore lowering the expected returns moving forward.

our model, the negative regression coefficients in Table 10 are inconsistent with the extrapolative beliefs implied by survey data.

[Place Table 10 about here]

In Table 11, we report the regression coefficients and their t-statistics for regressing, in the model of Bansal and Yaron (2004), four measures of rational expectations of returns—returns and excess returns, with and without the dividend yield—either on the past twelve-month return or on the current log price-dividend ratio. Interestingly, Bansal and Yaron (2004) generate, to a weaker extent than our model, extrapolative expectations when regressing expectations of raw returns (not excess returns) on past twelve-month returns: for a 15-year (50-year) long sample, the regression coefficient is about 2.5% (3.0%) with a Newey-West adjusted t-statistic of about 2.5 (4.0). In their model, consumption growth and dividend growth share a stochastic yet persistent component. High past twelve-month returns are typically caused by a sequence of positive shocks on this shared component, which, given its persistence, implies high dividend growth moving forward, pushing up the expectations of future raw returns. However, precisely because this persistent component is shared between consumption growth and dividend growth, high dividend growth often coincides with high consumption growth, and the latter implies a higher interest rate. As a result, the expectations of excess returns are not extrapolative: regressing these expectations on past twelve-month returns, the coefficients are not significantly different from zero.

These regression results highlight the fundamental difference between our model and the long-run risks model of Bansal and Yaron (2004). In our model, the investor becomes optimistic about future returns when past returns are high while her beliefs about future consumption growth remain roughly unchanged. This mechanism of backward-looking and asset-specific formation of beliefs allows our model to generate extrapolative expectations for both raw returns and excess returns. Furthermore, because these extrapolative beliefs are perceived and incorrect, subsequent returns are on average low, leading to a negative correlation between expectations of returns and realized returns. On the contrary, the model of Bansal and Yaron (2004) cannot robustly generate extrapolative expectations for excess returns, neither can it generate the negative correlation between expected returns and realized returns.

[Place Table 11 about here]

Table 12 repeats the regression analyses of Table 11 using the model of Bansal et al. (2012). Compared to the original model of Bansal and Yaron (2004), this model introduces additional time variation in the long-run risks that further reduces its ability to generate extrapolative expectations: even when regressing expectations of raw returns on past returns, the coefficients now become insignificant.

[Place Table 12 about here]

We complete our discussion for this section by noting that for long-run risks models, the persistence introduced in the consumption and dividend growths can sometimes lead to excess predictability, the notion that future consumption and dividend growths are excessively predicted by current variables such as the log price-dividend ratio (see Beeler and Campbell (2012) and Collin-Dufresne et al. (2016b) for detailed discussions). On the other hand, with return extrapolation, our model introduces perceived persistence in consumption and dividend growths while the true growths themselves are unpredictable. Therefore, our model does not generate any excess predictability.

V. Fundamental Extrapolation

Biased beliefs can be imposed on different quantities. Earlier work has focused on fundamental extrapolation, the notion that some investors in the economy hold extrapolative expectations about *fundamentals* such as dividend growth or GDP growth (e.g., Barberis et al. (1998), Fuster et al. (2011), Choi and Mertens (2013), Alti and Tetlock (2014), Hirshleifer et al. (2015)).

One key reason for imposing biased beliefs on fundamentals is that in this way, the evolution of beliefs can be fully specified without solving the equilibrium; this greatly simplifies many of these models. On the other hand, recent survey evidence (Vissing-Jorgensen (2004), Bacchetta et al. (2009), Amromin and Sharpe (2013), Greenwood and Shleifer (2014), Koijen et al. (2015), Kuchler and Zafar (2016)) strongly suggests that at the aggregate level, both individual and institutional investors form extrapolative expectations about future market *returns*.

In this section, we compare our return extrapolation model with a model with fundamental extrapolation. To facilitate the comparison, we keep the two models almost identical. The only difference is that, for the model with fundamental extrapolation, the sentiment variable is constructed based on past dividend growth instead of past returns. Below we first briefly lay out the fundamental extrapolation model. To avoid repetition, we keep the exposition to a minimum. We then compare the asset pricing implications of the two models.

For fundamental extrapolation, the key behavioral assumption, instead of (6) and (7), is that, from the representative agent's perspective,

$$dD_t/D_t = g_D^e(S_t)dt + \sigma_D d\omega_t^e, \tag{38}$$

where

$$g_D^e(S_t) = (1-\theta)g_D + \theta S_t. \tag{39}$$

Similar to the return extrapolation model, the construction of S_t is through learning about a hidden regime-switching model, but now based on past realizations of dividend growth. This gives rise to a sentiment process perceived by the representative agent as

$$\begin{aligned} dS_t =& (\lambda\mu_H + \chi\mu_L - (\lambda+\chi)S_t)dt + \sigma_D^{-1}\theta(\mu_H - S_t)(S_t - \mu_L)d\omega_t^e \\ \equiv& \mu_S^e(S_t)dt + \sigma_S(S_t)d\omega_t^e. \end{aligned} \tag{40}$$

The perceived evolution of P_t^D becomes

$$dP_t/P_t = \mu_P^{D,e}(S_t)dt + \sigma_P^D(S_t)d\omega_t^e, \tag{41}$$

where

$$\begin{aligned}\sigma_P^D(S) =&\sigma_D + (f'/f)\sigma_D^{-1}\theta(\mu_H - S)(S - \mu_L),\\ \mu_P^{D,e}(S) =&(f'/f)\mu_S^e + \tfrac{1}{2}(f''/f)\sigma_S^2 + (1-\theta)g_D + \theta S - \sigma_D^2 + \sigma_D\sigma_P^D(S).\end{aligned} \tag{42}$$

As before, the price-dividend ratio and the price-consumption ratio, f and l, are determined by simultaneously solving (19) and (20), except that g_D^e, μ_S, σ_S, $\mu_P^{D,e}$, and σ_P^D are now determined by (39), (40), and (42).

Two technical observations are worth making. First, different from the model with return extrapolation, $\sigma_P^D(S)$, the volatility of return that determines the speed of learning, does not directly enter the partial differential equations of (19) and (20) in the case of fundamental extrapolation. Second, different from the model with return extrapolation, the steady-state distribution of the sentiment variable can be now specified without solving equilibrium prices. These observations highlight a key distinction between return extrapolation and fundamental extrapolation. With fundamental extrapolation, high realized dividend growth generates optimistic beliefs about future dividend growth and this factors into prices, but higher prices do *not* create a feedback on beliefs about future dividends. By contrast, with return extrapolation, a two-way feedback loop on returns is formed: high realized returns generate optimistic beliefs about future returns and this leads to higher prices that further cause more optimistic beliefs about future returns. This two-way feedback loop provides an additional source of excess volatility.

[Place Table 13 about here]

In Table 13 we provide some basic unconditional and conditional moments of the fundamental extrapolation model using the same default parameter values as in the return extrapolation model. By comparing Table 13 with Table 2, we observe that, with fundamental extrapolation, both the average equity premium and the average Sharpe ratio become much lower, and the average price-dividend ratio becomes much higher. As discussed in the previous paragraph, the same sequence of positive fundamental shocks tend to lead to *less* optimistic beliefs in the case of fundamental extrapolation; higher returns are only an outcome but not a source for higher perception of future dividend growth. As a result, risks in returns associated with fundamental extrapolation are less significant, reducing excess volatility and the equity premium. Furthermore, in the absence of a two-way feedback loop, risks associated with fundamental extrapolation are perceived to be less persistent. In conjunction with Epstein-Zin preferences, this further lowers the equity premium. Overall, with our parameter values, fundamental extrapolation generates 37.9% of the equity premium, 74.4% of excess volatility, and 42.6% of the Sharpe ratio that return extrapolation generates.

One needs to interpret these comparisons with caution. Table 13 does not serve as a rejection

of fundamental extrapolation models. Instead, it suggests that with a similar learning structure and with the same degree of extrapolation, fundamental extrapolation generates a smaller equity premium. This does not prohibit the view that in reality, the degree of fundamental extrapolation is indeed larger and hence sufficient to generate a sizable equity premium.

Next we examine investor beliefs in the fundamental extrapolation model. Similar to the regressions we run and report in Table 3, we regress subjective expectations about future returns, measured by $\mathbb{E}_t^e[R_{t+dt}^D]$, $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$, $\mathbb{E}_t^e[R_{t+dt}^{D,e}]$, or $\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$, on either past twelve-month cumulative raw returns or on the current log price-dividend ratios, over a sample of 15 years or 50 years. Table 14 reports the regression coefficient and its t-statistic, the intercept, as well as the adjusted R-squared.

[Place Table 14 about here]

In contrast to the extrapolative expectations results presented in Table 3, with fundamental extrapolation, investors' expectations about future returns typically depend *insignificantly* on past returns; the t-statistics are quite small and the regression coefficients are close to zero. In this model, good realizations of dividend growth push up both equity returns and the price-dividend ratios. With fundamental extrapolation, investors expect higher dividends moving forward, but not higher *returns*. Subjective expectations of returns are determined by investors' perceptions of risks, and without significant changes in risks, the regression coefficients stay close to zero.

These regressions results suggest that a representative agent model with fundamental extrapolation may face greater challenges in matching the empirical findings of Greenwood and Shleifer (2014), the findings that subjective return expectations are positively correlated with past returns and negatively correlated with model-implied returns. However, a heterogeneous agent model with fundamental extrapolation—for instance, a model with both fundamental extrapolators and fully rational investors—may generate return extrapolation.[19]

VI. Conclusion

In this paper, we develop a representative agent general equilibrium model with recursive preferences and with return extrapolation. The model matches many empirical regularities documented in the literature, including the degree of return extrapolation and investors' memory structure derived from survey data. A two-way feedback loop endogenously arises with return extrapolation: asset returns are driven by investor beliefs but at the same time they also affect these beliefs. A comparison with a fundamental extrapolation model shows that this two-way feedback loop serves as a significant amplification mechanism.

Our analysis has left several important issues to future work. First, we have discussed the discrepancy between investors' short memory span of a year or two documented by Greenwood and

[19] For instance, investor heterogeneity allows the model of Ehling et al. (2015) to generate extrapolative expectations about returns.

Shleifer (2014) and the more persistent levels of price-dividend ratios suggested by price data. A reconciliation between these two sets of facts calls for more conceptual and theoretical development.

Second, our representative agent approach is a simplifying approach. It highlights the role of investor beliefs and allows the model to be directly compared to the rational asset pricing models in the literature. However, at the same time, it neglects an important channel that can affect asset prices, namely, the time-varying fraction of wealth held by behavioral agents. The interaction between this time-varying wealth of behavioral agents and their beliefs can further amplify business cycle fluctuations and yield additional implications.

Lastly, there exists a close connection between long-run risks models and extrapolation models with Epstein-Zin preferences. In long-run risks models, Epstein-Zin preferences interact with a true aggregate consumption process that has a long-run and persistent component in it. This interaction gives rise to a large equity premium, excess equity volatility, and predictability of equity returns, among other things. With extrapolation-based models, the true aggregate consumption process does not have a persistent component, but some investors *perceive* such a component for both the consumption process and the dividend process. In conjunction with Epstein-Zin preferences, these perceptions also lead to the same empirical predictions that the long-run risks models try to explain, but at the same time generate very different predictions about investor expectations. A closer comparison between these two types of models may provide additional insight into the nature of asset price fluctuations.

Appendices

A. *Derivation of the Differential Equations System of (19) and (20)*

For the risky asset of the dividend stream (10), the subjective Euler equation (2) can be written as

$$\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)\Delta t/(1-\psi)}\left(\frac{\tilde{C}_{t+\Delta t}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)}\left(\frac{\tilde{P}^C_{t+\Delta t}+\tilde{C}_t\Delta t}{P^C_t}\right)^{(\psi-\gamma)/(1-\psi)}\frac{\tilde{P}^D_{t+\Delta t}+\tilde{D}_{t+\Delta t}\Delta t}{P^D_t}\right]=1. \tag{A.1}$$

Letting Δt go to zero and omitting terms with an order of $o(dt)$, (A.1) becomes

$$\begin{aligned}&\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)dt/(1-\psi)}\left(\tilde{C}_{t+dt}\right)^{-\psi(1-\gamma)/(1-\psi)}(\tilde{P}^C_{t+dt})^{(\psi-\gamma)/(1-\psi)}\tilde{P}^D_{t+dt}\left(1+\tfrac{\psi-\gamma}{1-\psi}\tfrac{\tilde{C}_{t+dt}}{\tilde{P}^C_{t+dt}}dt+\tfrac{\tilde{D}_{t+dt}}{\tilde{P}^D_{t+dt}}dt\right)\right]\\&=C_t^{-\psi(1-\gamma)/(1-\psi)}(P^C_t)^{(\psi-\gamma)/(1-\psi)}P^D_t.\end{aligned} \tag{A.2}$$

Rearranging terms gives

$$0=\mathbb{E}_t^e\left[\begin{array}{l}d(\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}Df)+\frac{\psi-\gamma}{1-\psi}\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(2\psi-\gamma-1)/(1-\psi)}Dfdt\\+\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}Ddt\end{array}\right], \tag{A.3}$$

where $\Theta(C,t)\equiv e^{-\delta(1-\gamma)t/(1-\psi)}C^{-\psi(1-\gamma)/(1-\psi)}$. By Ito's lemma, (A.3) leads to

$$0=\mathbb{E}_t^e\left[\begin{array}{l}-\frac{\delta(1-\gamma)}{1-\psi}dt-\gamma(dC/C)+(dD/D)+(df/f)+\frac{\psi-\gamma}{1-\psi}(dl/l)+\frac{\gamma(\gamma+1)}{2}(dC/C)^2\\+\frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(dl/l)^2-\frac{\gamma(\psi-\gamma)}{1-\psi}(dC/C)(dl/l)-\gamma(dC/C)(dD/D)-\gamma(dC/C)(df/f)\\+\frac{\psi-\gamma}{1-\psi}(dl/l)(dD/D)+\frac{\psi-\gamma}{1-\psi}(dl/l)(df/f)+(df/f)(dD/D)+\frac{\psi-\gamma}{1-\psi}l^{-1}dt+f^{-1}dt\end{array}\right]. \tag{A.4}$$

Using (9), (10),and (14) to further simplify (A.4) then gives (19).

For the optimal portfolio itself, the Euler equation (2) becomes

$$\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)\Delta t/(1-\psi)}\left(\frac{\tilde{C}_{t+\Delta t}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)}\left(\frac{\tilde{P}^C_{t+\Delta t}+\tilde{C}_t\Delta t}{P^C_t}\right)^{(1-\gamma)/(1-\psi)}\right]=1. \tag{A.5}$$

Its continuous-time limit is

$$0=\mathbb{E}_t^e\left[d(\Theta C^{(1-\gamma)/(1-\psi)}l^{(1-\gamma)/(1-\psi)})+\frac{1-\gamma}{1-\psi}\Theta C^{(1-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}dt\right]. \tag{A.6}$$

By Ito's lemma, (A.6) leads to

$$0 = \mathbb{E}_t^e \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta dt - (\gamma-1)(dC/C) + \frac{\gamma(\gamma-1)}{2}(dC/C)^2 + \frac{1-\gamma}{1-\psi}(dl/l) \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(dl/l)^2 + \frac{(1-\gamma)^2}{1-\psi}(dC/C)(dl/l) + \frac{1-\gamma}{1-\psi}l^{-1}dt \end{array} \right]. \tag{A.7}$$

Using (9) and (14) to further simplify (A.7) gives (20). ■

B. *Steady-State Distribution for Sentiment*

Below we derive all the terms necessary for solving the Kolmogorov forward equation (30). From the expression of σ_S in (9)

$$\begin{aligned} \sigma_S' =& \frac{\theta\sigma_P^D(\mu_H+\mu_L-2S) - \theta(\mu_H-S)(S-\mu_L)(\sigma_P^D)'}{(\sigma_P^D)^2}, \\ \sigma_S'' =& \frac{\theta(\mu_H-S)(S-\mu_L)\{2[(\sigma_P^D)']^2 - \sigma_P^D(\sigma_P^D)''\}}{(\sigma_P^D)^3} - 2\theta\frac{\sigma_P^D(\sigma_P^D)'(\mu_H+\mu_L-2S)+(\sigma_P^D)^2}{(\sigma_P^D)^3}. \end{aligned} \tag{B.1}$$

For the expression of μ_S^e in (9) and the expression of g_D^e in (12)

$$\begin{aligned} (\mu_S^e)' =& -(\lambda+\chi), \\ (g_D^e)' =& \theta - \sigma_D(\sigma_P^D)' - \mu'_S(f'/f) - \mu_S[f''/f - (f')^2/f^2] \\ & - \sigma_S\sigma_S'(f''/f) - \tfrac{1}{2}\sigma_S^2[f'''/f - f'f''/f^2], \end{aligned} \tag{B.2}$$

where σ_P^D is from (12), and $(\sigma_P^D)'$ and $(\sigma_P^D)''$ are

$$\begin{aligned} (\sigma_P^D)' =& \frac{\theta(\mu_H+\mu_L-2S)(f'/f) + \theta(\mu_H-S)(S-\mu_L)[f''/f-(f')^2/f^2]}{\sqrt{\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)}}, \\ (\sigma_P^D)'' =& -\frac{2\{\theta(\mu_H+\mu_L-2S)(f'/f) + \theta(\mu_H-S)(S-\mu_L)[f''/f-(f')^2/f^2]\}^2}{[\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)]^{3/2}} \\ & + \frac{-2\theta f'/f + 2\theta(\mu_H+\mu_L-2S)[f''/f-(f')^2/f^2]}{\sqrt{\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)}} \\ & + \frac{\theta(\mu_H-S)(S-\mu_L)[f'''/f - 3(f'f'')/f^2 + 2(f')^3/f^3]}{\sqrt{\sigma_D^2 + 4\theta(\mu_H-S)(S-\mu_L)(f'/f)}}. \end{aligned} \tag{B.3}$$

■

C. *Numerical Procedure for Solving the Equilibrium*

We use a projection method with Chebyshev polynomials to jointly solve the two differential equations (19) and (20). The value of the sentiment variable S ranges from μ_L to μ_H whereas the domain for Chebyshev polynomials is [–1, 1]. So we transform S to a new state variable z

$$z \equiv aS + b, \qquad \text{where} \quad a = \frac{2}{\mu_H - \mu_L}, \; b = -\frac{\mu_H + \mu_L}{\mu_H - \mu_L}, \tag{C.1}$$

and define $h(z) \equiv f(S(z))$ and $j(z) \equiv l(S(z))$. Equations (19) and (20) can be rewritten as

$$0 = \left[\begin{array}{l} -\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(h'/h) + \frac{\psi-\gamma}{1-\psi}(j'/j)]a\mu_S^e + \frac{1}{2}[(h''/h) + \frac{\psi-\gamma}{1-\psi}(j''/j)]a^2\sigma_S^2 \\ +\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(aj'/j)^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(aj'/j) - \gamma\rho\sigma_C\sigma_D - \gamma\rho\sigma_C\sigma_S(ah'/h) \\ +\frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(aj'/j) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2 a^2(j'/j)(h'/h) + \sigma_D\sigma_S(ah'/h) + \frac{\psi-\gamma}{1-\psi}j^{-1} + h^{-1} \end{array} \right] \tag{C.2}$$

and

$$0 = \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(aj'/j)\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(a^2j''/j)\sigma_S^2 \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(aj'/j)^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(aj'/j) + \frac{1-\gamma}{1-\psi}j^{-1} \end{array} \right]. \tag{C.3}$$

We approximate h and j by

$$\hat{h}(z) = \sum_{r=0}^{n} a_r T_r(z), \qquad \hat{l}(z) = \sum_{r=0}^{m} b_r T_r(z), \tag{C.4}$$

where $T_r(z)$ is the r^{th} degree Chebyshev polynomial of the first kind.[20] The projection method chooses the coefficients $\{a_r\}_{r=0}^n$ and $\{b_r\}_{r=0}^m$ so that the differential equations are *approximately* satisfied. One criterion for a good approximation is a minimum weighted sum of squared errors

$$\sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l} -\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(\hat{h}'/\hat{h}) + \frac{\psi-\gamma}{1-\psi}(\hat{j}'/\hat{j})]a\mu_S^e + \frac{1}{2}[(\hat{h}''/\hat{h}) + \frac{\psi-\gamma}{1-\psi}(\hat{j}''/\hat{j})]a^2\sigma_S^2 \\ +\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(a\hat{j}'/\hat{j})^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(a\hat{j}'/\hat{j}) - \gamma\rho\sigma_C\sigma_D \\ -\gamma\rho\sigma_C\sigma_S(a\hat{h}'/\hat{h}) + \frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(a\hat{j}'/\hat{j}) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2 a^2(\hat{j}'/\hat{j})(\hat{h}'/\hat{h}) + \sigma_D\sigma_S(a\hat{h}'/\hat{h}) \\ +\frac{\psi-\gamma}{1-\psi}\hat{j}^{-1} + \hat{h}^{-1} \end{array} \right]^2_{z=z_i}$$
$$+ \sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(a\hat{j}'/\hat{j})\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(a^2\hat{j}''/\hat{j})\sigma_S^2 \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(a\hat{j}'/\hat{j})^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(a\hat{j}'/\hat{j}) + \frac{1-\gamma}{1-\psi}\hat{j}^{-1} \end{array} \right]^2_{z=z_i}, \tag{C.5}$$

where $\{z_i\}_{i=0}^N$ are the N zeros of $T_N(z)$. By the Chebyshev interpolation theorem, if N is sufficiently larger than n and m, and if the sum of weighted square in (C.5) is sufficiently small, the

[20] See Mason and Handscomb (2003) for detailed discussion of the properties of Chebyshev polynomials.

approximated functions $\hat{h}(z)$ and $\hat{l}(z)$ are very close to the true solutions.

For the numerical results in the main text, we set $m = 40$, $n = 40$, $N = 400$. We then apply the Levenberg-Marquardt algorithm and obtain a minimized sum of squared errors less than 10^{-11}. The small size of errors indicates a good convergence of the numerical solution. The solution is also insensitive to the choice of n, m, and N. These findings indicate that the numerical solutions are sufficient approximations for the true h and j functions.

The same numerical procedure is applied for solving the Kolmogorov forward equation (30). ■

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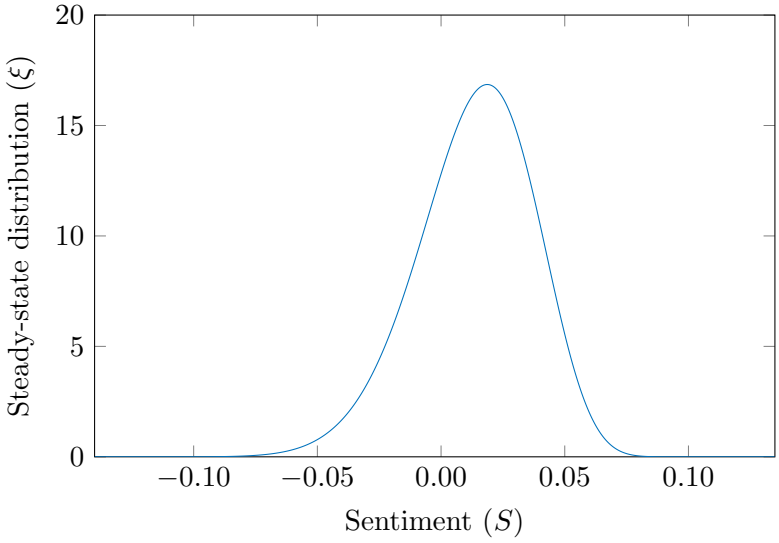


Figure 1. Objectively Measured Steady-State Distribution of Sentiment. The figure plots the objective steady-state distribution of sentiment ξ as a function of the sentiment variable S. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

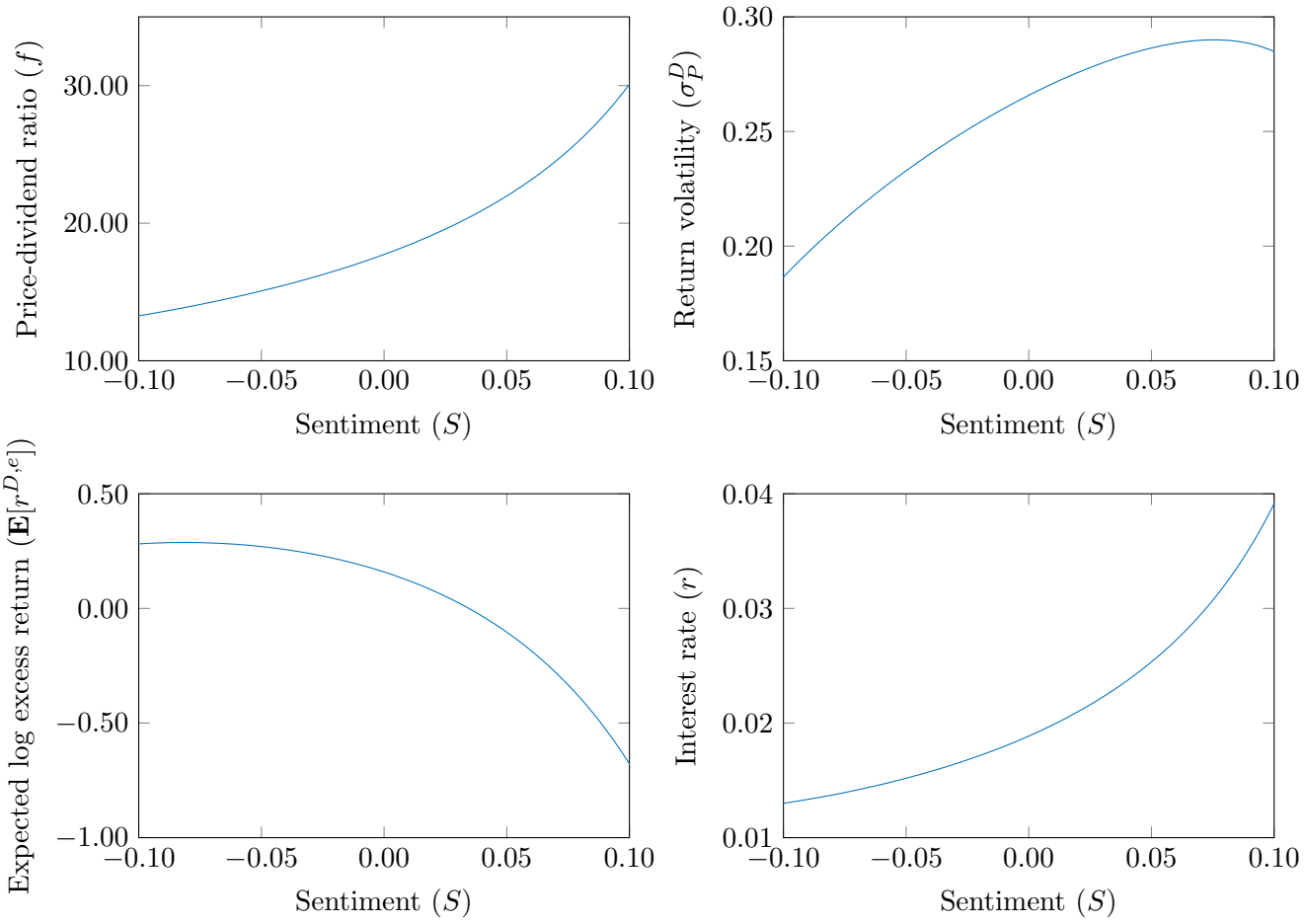


Figure 2. State-Dependent Model Implications. The figure plots the price-dividend ratio f, the return volatility σ_P^D, the objective expectation of log excess return $\mathbb{E}[r^{D,e}]$ and the interest rate r, with each as a function of the sentiment variable S. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

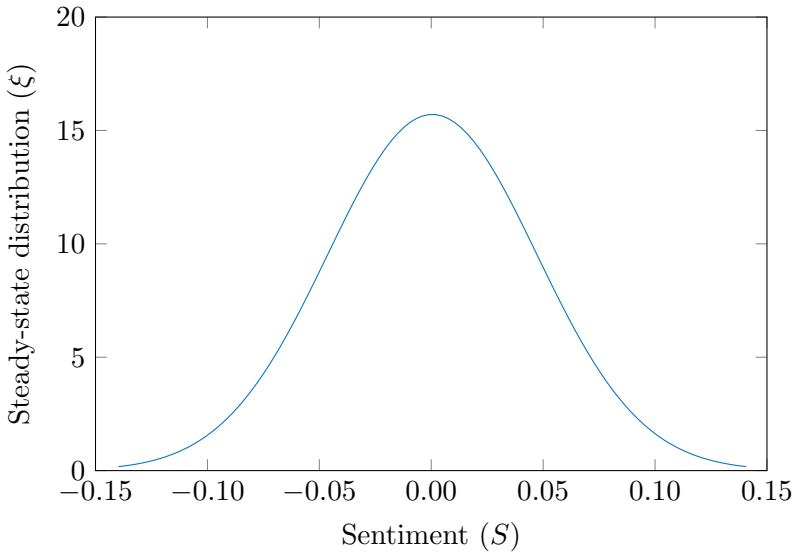


Figure 3. Steady-State Distribution of Sentiment for the True Regime-switching Model with Rational Expectations. The figure plots the steady-state distribution of sentiment ξ as a function of the sentiment variable S for the true regime-switching model with rational expectations. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

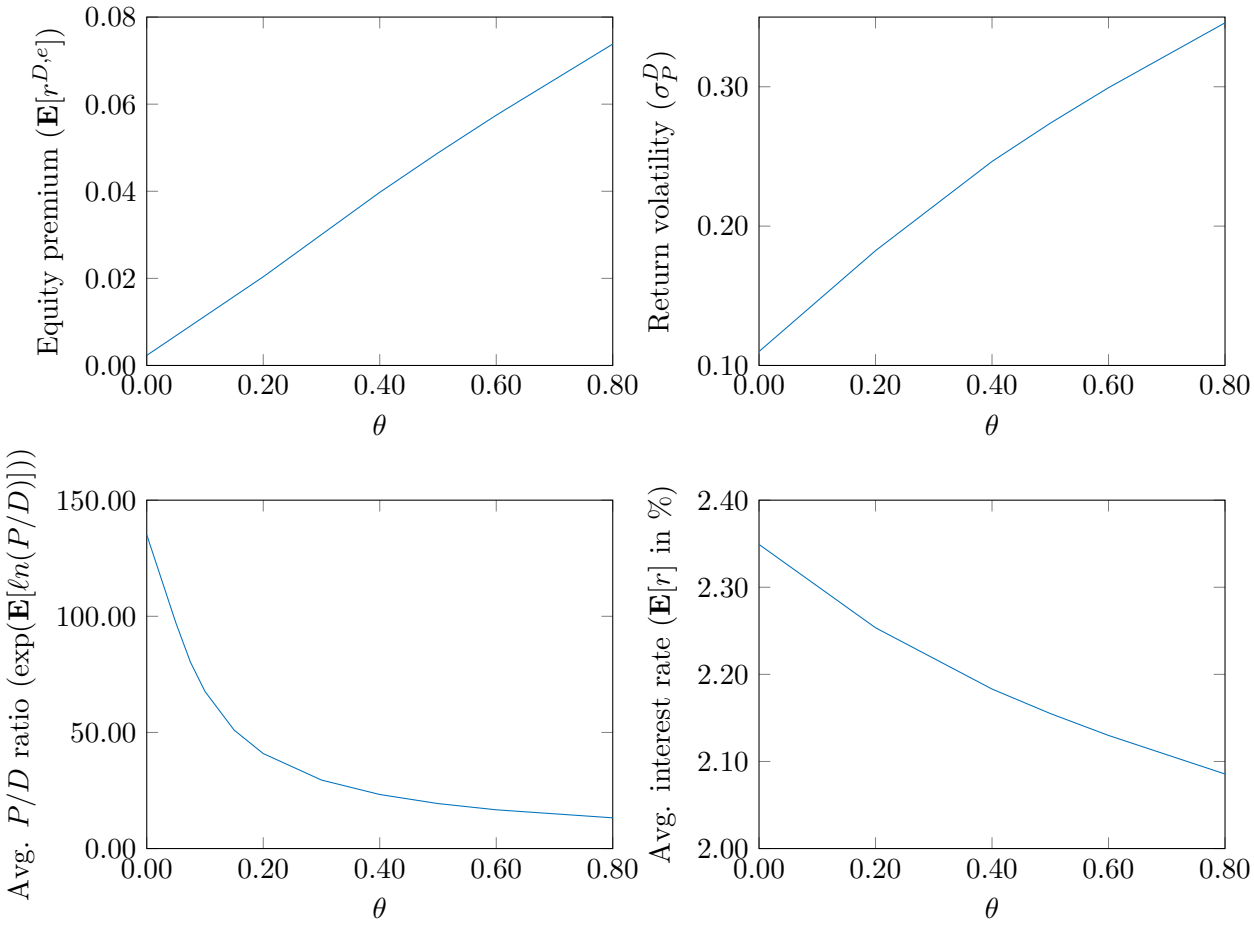


Figure 4. Impact of θ on Asset Prices. The figure plots the annual expectation of log excess return $\mathbb{E}[r^{D,e}]$, the annual volatility of log excess return $\sigma(r^{D,e})$, the average price-dividend ratio $\exp(\mathbb{E}[\ell\text{n}(P/D)])$, and the average interest rate $\mathbb{E}[r]$ (in percentage), each as a function of the degree-of-extrapolation parameter θ. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

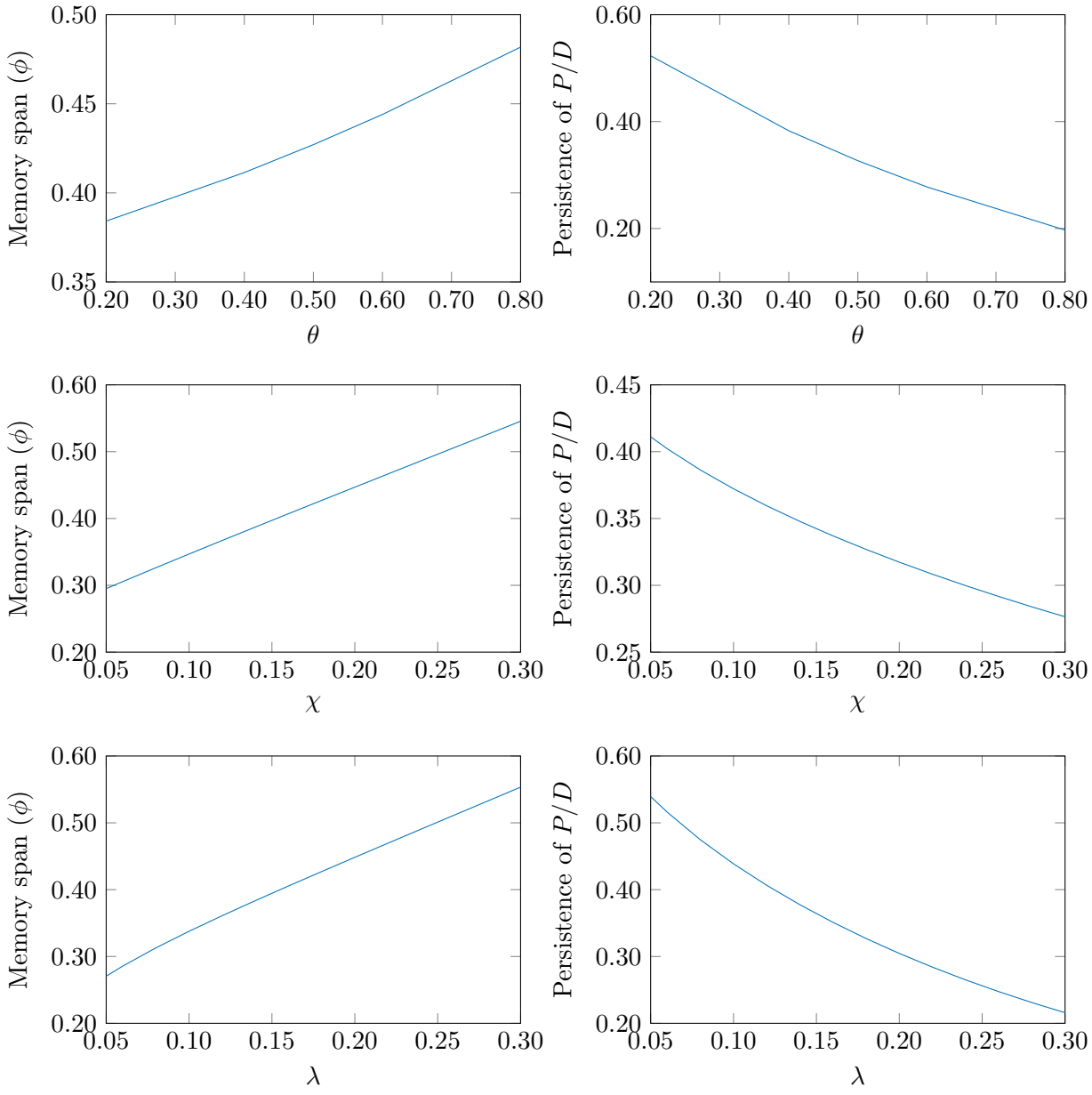


Figure 5. Impact of Belief Parameters on Investor Memory and Persistence of Asset Prices. The figure plots ϕ (investors' memory span estimated from the regression in (32) using $\mathbb{E}^e[R_t^D]$ as the expectation measure over a sample of 15 years) and the one-year autocorrelation of log price-dividend ratios, both as a function of θ, χ, and λ. The default values for θ, χ, or λ are 0.5, 0.18, and –0.18, respectively. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 1. Parameter Values.

Parameter	Variable	Value
Asset parameter:		
Expected consumption growth	g_C	1.91%
Standard deviation of consumption growth	σ_C	3.80%
Expected dividend growth	g_D	2.45%
Standard deviation of dividend growth	σ_D	11%
Correlation between dD and dC	ρ	0.2
Utility parameter:		
Relative risk aversion	γ	10
Reciprocal of EIS	ψ	0.9
Subjective discount factor	δ	0.02
Belief parameter:		
Degree of extrapolation	θ	0.5
Transition intensity from H to L	χ	0.18
Transition intensity from L to H	λ	0.18
Return in state H	μ_H	0.15
Return in state L	μ_L	–0.15

Table 2. Basic Unconditional and Conditional Moments.

Statistic	Overall	Bottom quartile	Top quartile	Empirical value (overall)		
				CC(1999)	C(2003)	BC(2012)
$\mathbb{E}[r^{D,e}]$	4.88%	20.05%	–13.05%	3.90%	6.69%	4.91%
$\sigma(r^{D,e})$	27.4%	25.6%	28.7%	18.0%	15.7%	-
$\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$	0.20	0.79	–0.45	0.22	0.43	-
$\mathbb{E}[r^{D}]$	7.04%	21.81%	–10.46%	-	-	5.47%
$\sigma(r^{D})$	27.4%	25.6%	28.7%	-	-	20.17%
$\mathbb{E}[r]$	2.16%	1.76%	2.60%	2.92%	0.90%	0.56%
$\sigma(r)$	0.33%	0.12%	0.20%	-	$\leq 1.75\%$	2.89%
$\exp(\mathbb{E}[\ell\mathrm{n}(P/D)])$	19.4	16.8	22.3	21.1	24.7	28.8
$\sigma(\ell\mathrm{n}(P/D))$	0.11	0.05	0.05	0.27	0.30	0.45
$\mathbb{C}\mathrm{orr}_{\xi}(\mathbb{E}^{e}[R^{D}], \mathbb{E}[R^{D,e}])$	–0.95	–0.97	–0.99		–0.57	

The table reports the annual expectation of log excess return $\mathbb{E}[r^{D,e}]$, the annual volatility of log excess return $\sigma(r^{D,e})$, the Sharpe ratio $\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$, the annual expectation of log return $\mathbb{E}[r^{D}]$, the annual volatility of log return $\sigma(r^{D})$, the average interest rate $\mathbb{E}[r]$, the annual volatility of interest rate $\sigma(r)$, the average price-dividend ratio $\exp(\mathbb{E}[\ell\mathrm{n}(P/D)])$, the volatility of log price-dividend ratio $\sigma(\ell\mathrm{n}(P/D))$, and the correlation between subjective return expectations and model-implied expectations $\mathbb{C}\mathrm{orr}_{\xi}(\mathbb{E}^{e}[R^{D}], \mathbb{E}[R^{D,e}])$. The averages are taken over the entire steady-state distribution of S, the bottom quartile of S (bad times), and the top quartile of S (good times), for the first, second, and third column, respectively. The last three columns provide empirical value for each quantity: the first nine values under the column "CC(1999)" are based on the long sample (1871–1993) used in Campbell and Cochrane (1999); the first nine values under the column "C(2003)" are based on the post-war samples (1947–1995 or 1947–1998) used in Campbell (2003); the first nine values under the column "BC(2012)" are based on the annual dataset (1930–2008) used in Beeler and Campbell (2012); and the very last value (–0.57) is based on the Gallup investor survey data used in Greenwood and Shleifer (2014). The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 3. Extrapolative Expectations.

	$\mathbb{E}_t^e[R_{t+dt}^D]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$			
$R_{t-12\to t}^D$	0.022	0.023			0.040	0.040		
	(8.2)	(11.7)			(8.4)	(12.1)		
$\ell\mathrm{n}(P/D)$			0.068	0.069			0.120	0.121
			(29.5)	(39.2)			(36.8)	(48.9)
Constant	0.07	0.07	–0.13	–0.13	0.02	0.02	–0.33	–0.34
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.57	0.54	0.98	0.98	0.58	0.55	0.99	0.99

	$\mathbb{E}_t^e[R_{t+dt}^{D,e}]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$			
$R_{t-12\to t}^D$	0.013	0.013			0.030	0.030		
	(6.6)	(9.0)			(7.8)	(11.1)		
$\ell\mathrm{n}(P/D)$			0.039	0.039			0.091	0.091
			(12.4)	(16.6)			(22.3)	(28.7)
Constant	0.05	0.05	–0.06	–0.06	–0.002	–0.002	–0.27	–0.27
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.51	0.47	0.92	0.91	0.56	0.53	0.97	0.97

The table reports the regression coefficient and its t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing subjective expectations about future returns either on the past twelve-month cumulative raw returns $R_{t-12\to t}^D$ or on the current log price-dividend ratios $\ell\mathrm{n}(P_t/D_t)$ over a sample of 15 years or 50 years. In the top panel, the subjective expectation used for the first four columns is $\mathbb{E}_t^e[R_{t+dt}^D]$, and the subjective expectation used for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the subjective expectation used for the first four columns is $\mathbb{E}_t^e[R_{t+dt}^{D,e}]$, and the subjective expectation used for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$. Each reported value is averaged over 100 trials, and each trial represents a regression based on simulated data of the model with a monthly frequency. The t-statistics are based on a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 4. Memory Structure of Investor Expectations.

	$\mathbb{E}_t^e[R_{t+dt}^D]$		$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$	
ϕ	0.432	0.417	0.428	0.420
a	0.064	0.064	0.004	0.004
b	1.15	1.18	2.04	2.07
Sample size	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.98	0.98	0.99	0.98
	$\mathbb{E}_t^e[R_{t+dt}^{D,e}]$		$\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$	
ϕ	0.414	0.408	0.418	0.420
a	0.047	0.047	–0.013	-0.010
b	0.70	0.67	1.58	1.55
Sample size	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.92	0.90	0.97	0.96

The table reports the memory decay parameter ϕ, the intercept a, the regression coefficient b, and the adjusted R-squared, for running the non-linear least squares regression

$$\text{Expectation}_t = a + b \sum\nolimits_{j=1}^{n} w_j R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)} + \varepsilon_t,$$

over a sample of 15 years or 50 years, where $w_j = e^{-\phi(j-1)\Delta t}/\Sigma_{l=1}^{n} e^{-\phi(l-1)\Delta t}$. Here $\Delta t = 1/12$ and $n = 600$. In the top panel, the subjective expectation used for the first two columns is $\mathbb{E}_t^e[R_{t+dt}^D]$ and the subjective expectation used for the last two columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the subjective expectation used for the first two columns is $\mathbb{E}_t^e[R_{t+dt}^{D,e}]$ and the subjective expectation used for the last two columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$. Each reported value is averaged over 100 trials, and each trial represents a regression based on simulated data of the model with a monthly frequency. The t-statistics are based on a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

Table 5. Return Predictability Regressions.

Horizon (years)	Model		Empirical value	
	10× coefficient	Adjusted R-squared	10× coefficient	Adjusted R-squared
1	–7.2	0.13	–1.3	0.04
2	–9.5	0.16	–2.8	0.08
3	–10.1	0.15	–3.5	0.09
5	–10.6	0.13	–6.0	0.18
7	–11.0	0.12	–7.5	0.23

The table reports the regression coefficient β_j for regressing log excess return of stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$ (first column) and the corresponding adjusted R-squared (second column) for $j = 1$, 2, 3, 5, and 7 (years). Regressions are based on simulated data of the model with a total length of 10,000 years and with a monthly frequency. The last two columns provide the empirical values based on the long sample (1871–1993) used in Campbell and Cochrane (1999). The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 6. Campbell-Shiller Decomposition (Long Sample).

Dependent variable	Objective probability measure		Subjective probability measure	
	Coefficient	Adjusted R-squared	Coefficient	Adjusted R-squared
$\sum_{j=0}^{\infty} \rho^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$	0.96	0.09	–0.14	8.6×10^{-4}
$\sum_{j=0}^{\infty} \rho^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$	–0.04	1.8×10^{-4}	–1.14	0.06

The table reports the regression coefficient and the corresponding adjusted R-squared for regressing either the weighted future log gross return $\sum_{j=0}^{\infty} \rho^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ (first row) or the weighted future log dividend growth $\sum_{j=0}^{\infty} \rho^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ (second row) on the current log price-dividend ratio $\ell\text{n}(D_t/P^D_t)$, under either the objective probability measure (first two columns) or the subjective probability measure (last two columns). Regressions are based on simulated data of the model with a total length of 10,000 years and with a monthly frequency. Consistent with Campbell and Shiller (1988), $\rho = e^{-\delta}/(\Delta t + e^{-\delta})$, where δ is the in-sample average of the annual log dividend-price ratio, and $\Delta t = 1/12$. At each point in time t, $\sum_{j=0}^{\infty} \rho^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ and $\sum_{j=0}^{\infty} \rho^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ are approximated by looking forward for 50 years. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. Under the objective probability measure, $\rho = 0.9957$, and under the subjective probability measure, $\rho = 0.9954$.

Table 7. Autocorrelations of Log Excess Returns and Log Price-dividend Ratios.

Lag (years)	Model			Empirical value		
	$\ell n(P^D/D)$	$r^{D,e}$	$\Sigma_{i=1}^{j}\rho(r_t^{D,e}, r_{t-i}^{D,e})$	$\ell n(P^D/D)$	$r^{D,e}$	$\Sigma_{i=1}^{j}\rho(r_t^{D,e}, r_{t-i}^{D,e})$
1	0.33	–0.28	–0.28	0.78	0.05	0.05
2	0.11	–0.09	–0.37	0.57	–0.21	–0.16
3	0.05	–0.02	–0.39	0.50	–0.08	–0.09
5	0.00	–0.01	–0.40	0.32	–0.14	–0.28
7	–0.02	–0.01	–0.41	0.29	0.11	–0.15

The table reports autocorrelations of log price-dividend ratios, log excess returns, and the partial sum of autocorrelations of log excess returns for lag j of 1, 2, 3, 5, and 7 (years). The model values are based on simulated data with a total length of 10,000 years and with a monthly frequency; for each month, we compound subsequent monthly log excess returns over a year to obtain an annualized $r^{D,e}$. The empirical values are based on the long sample (1871–1993) used in Campbell and Cochrane (1999). The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 8. Correlation between Consumption Growth and Stock Returns.

Correlation	Model			Empirical value
	monthly	quarterly	annual	annual
$\mathbb{C}\text{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t-1}/C_{t-2}))$	–0.01	–0.02	–0.02	–0.05
$\mathbb{C}\text{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\text{n}(C_t/C_{t-1}))$	–0.01	–0.03	–0.06	–0.08
$\mathbb{C}\text{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+1}/C_t))$	0.20	0.20	0.19	0.09
$\mathbb{C}\text{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+2}/C_{t+1}))$	0.00	–0.00	0.01	0.49
$\mathbb{C}\text{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+3}/C_{t+2}))$	–0.00	–0.00	0.01	0.05

The table reports correlations between changes in log consumption and log excess returns, with changes in log consumption and log excess returns computed over a monthly, a quarterly, and an annual horizon. Correlations include both contemporaneous correlations and cross-correlations with a lead-lag structure (for instance, at a monthly frequency, $\mathbb{C}\text{orr}_\xi(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+2}/C_{t+1}))$ is the correlation between the current monthly log excess return and the subsequent month's change in log consumption. The model values are based on simulated data with a total length of 10,000 years and with a monthly frequency. The empirical values are based on the long sample (1871–1993 for Standard & Poors 500 stock returns and commercial paper returns; 1889–1992 for consumption data) used in Campbell and Cochrane (1999). The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 9. Return Predictability Regressions in the True Regime-switching Model with Rational Expectations.

Horizon (years)	Model		Empirical value	
	10× coefficient	10^3×Adjusted R-squared	10× coefficient	Adjusted R-squared
1	0.2	0.16	–1.3	0.04
2	0.4	0.31	–2.8	0.08
3	0.5	0.32	–3.5	0.09
5	0.8	0.54	–6.0	0.18
7	0.7	0.34	–7.5	0.23

The table reports the regression coefficient β_j for regressing log excess return of the stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$ (first column) and the corresponding adjusted R-squared (second column) for $j = 1$, 2, 3, 5, and 7 (years). Regressions are based on simulated data of the true regime-switching model with rational expectations described in Section IV. Each simulation is of a total length of 10,000 years and of a monthly frequency. The last two columns provide the empirical values based on the long sample (1871–1993) used in Campbell and Cochrane (1999). The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 10. Return Expectations in the Model of Campbell and Cochrane (1999).

	$\mathbb{E}_t[R^D_{t+dt}]$				$\mathbb{E}_t[dP^D_t/(P^D_t dt)]$			
$R^D_{t-12\to t}$	–0.002	–0.001			–0.013	–0.011		
	(–1.7)	(–1.2)			(–1.7)	(–1.4)		
$\ell\text{n}(P/D)$			–0.009	–0.008			–0.059	–0.062
			(–64.4)	(–68.4)			(–51.6)	(–38.7)
Constant	0.03	0.03	0.05	0.05	0.09	0.09	0.25	0.26
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.085	0.017	0.992	0.991	0.090	0.020	0.995	0.987

The table reports the regression coefficient and its t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing, in the model of Campbell and Cochrane (1999) rational expectations about future returns either on the past twelve-month cumulative raw returns $R^D_{t-12\to t}$ or on the current log price-dividend ratios $\ell\text{n}(P_t/D_t)$ over a sample of 15 years or 50 years. Each reported value is the estimator median over 1000 trials, and each trial represents a regression based on simulated data of the model with a monthly frequency. The t-statistics are based on a Newey-West estimator with twelve-month lags. The parameters take their default values from Table 1 of Campbell and Cochrane (1999). Given that the interest rate is constant in this model, the regression coefficients and their t-statistics are unchanged if $\mathbb{E}_t[R^D_{t+dt}]$ is replaced by $\mathbb{E}_t[R^{D,e}_{t+dt}]$ and if $\mathbb{E}_t[dP^D_t/(P^D_t dt)]$ is replaced by $\mathbb{E}_t[dP^D_t/(P^D_t dt) - r_t]$.

Table 11. Return Expectations in the Model of Bansal and Yaron (2004).

	Expected return				Expected return without dividend			
$R^D_{t-12\to t}$	0.025	0.030			0.025	0.031		
	(2.4)	(3.8)			(2.5)	(4.0)		
$\ell\mathrm{n}(P/D)$			0.068	0.067			0.069	0.067
			(5.6)	(7.6)			(5.8)	(7.6)
Constant	1.06	1.05	0.88	0.88	1.06	1.05	0.88	0.89
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.140	0.142	0.500	0.451	0.136	0.145	0.529	0.451

	Expected excess return				Expected excess return w/o divd.			
$R^D_{t-12\to t}$	–0.001	0.000			0.000	0.001		
	(–0.1)	(0.1)			(–0.1)	(0.2)		
$\ell\mathrm{n}(P/D)$			–0.008	–0.012			–0.010	–0.011
			(–0.8)	(–1.3)			(–0.8)	(–1.2)
Constant	0.06	0.06	0.08	0.10	0.06	0.06	0.09	0.09
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.026	0.008	0.087	0.054	0.026	0.008	0.093	0.053

The table reports the regression coefficient and its t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing, in the model of Bansal and Yaron (2004), four measures of rational expectations of returns—returns (in the top panel) and excess returns (in the bottom panel), with and without dividend yield—either on the past twelve-month return or on the current log price-dividend ratio. Conditional expectations of subsequent returns, the dependent variable of the regressions, are computed over a twelve-month horizon by averaging realized returns across simulations for each given state of the economy. Each reported value is the estimator median over 1000 trials, and each trial represents a regression based on simulated data of the model with a monthly frequency. The t-statistics are based on a Newey-West estimator with twelve-month lags. The parameters take their default values from Table II and IV of Bansal and Yaron (2004).

Table 12. Return Expectations in the Model of Bansal et al. (2012).

	Expected return				Expected return without dividend			
$R^D_{t-12\to t}$	0.006	0.010			0.006	0.011		
	(0.7)	(1.3)			(0.7)	(1.3)		
$\ell n(P/D)$			–0.006	–0.041			–0.006	–0.038
			(–0.3)	(–3.1)			(–0.2)	(–2.9)
Constant	1.08	1.08	1.11	1.22	1.08	1.08	1.11	1.21
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.031	0.021	0.129	0.147	0.033	0.020	0.139	0.136

	Expected excess return				Expected excess return w/o divd.			
$R^D_{t-12\to t}$	–0.003	0.000			–0.004	–0.001		
	(–0.3)	(0.1)			(–0.5)	(–0.1)		
$\ell n(P/D)$			–0.071	–0.095			–0.036	–0.054
			(–5.2)	(–10.7)			(–3.4)	(–5.2)
Constant	0.09	0.08	0.30	0.37	0.08	0.08	0.19	0.24
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.021	0.010	0.432	0.599	0.026	0.008	0.226	0.254

The table reports the regression coefficient and its t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing, in the model of Bansal et al. (2012), four measures of rational expectations of returns—returns (in the top panel) and excess returns (in the bottom panel), with and without dividend yield—either on the past twelve-month return or on the current log price-dividend ratio. Conditional expectations of subsequent returns, the dependent variable of the regressions, are computed over a twelve-month horizon by averaging realized returns across simulations for each given state of the economy. Each reported value is the estimator median over 1000 trials, and each trial represents a regression based on simulated data of the model with a monthly frequency. The t-statistics are based on a Newey-West estimator with twelve-month lags. The parameters take their default values from Table 1 of Bansal et al. (2012).

Table 13. Basic Unconditional and Conditional Moments for the Fundamental Extrapolation Model.

Statistic	Overall	Bottom quartile	Top quartile	Empirical value (overall)		
				CC(1999)	C(2003)	BC(2012)
$\mathbb{E}[r^{D,e}]$	1.85%	9.74%	–5.62%	3.90%	6.69%	4.91%
$\sigma(r^{D,e})$	20.4%	18.8%	17.5%	18.0%	15.7%	-
$\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$	0.08	0.53	–0.33	0.22	0.43	-
$\mathbb{E}[r]$	2.27%	1.97%	2.56%	2.92%	0.90%	0.56%
$\sigma(r)$	0.23%	0.06%	0.06%	-	$\leq 1.75\%$	2.89%
$\exp(\mathbb{E}[\ell\mathrm{n}(P/D)])$	44.4	39.2	49.3	21.1	24.7	28.8
$\sigma(\ell\mathrm{n}(P/D))$	0.09	0.03	0.02	0.27	0.30	0.45

The table reports, for the fundamental extrapolation model, the annual expectation of log excess return $\mathbb{E}[r^{D,e}]$, the annual volatility of log excess return $\sigma(r^{D,e})$, the Sharpe ratio $\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$, the average interest rate $\mathbb{E}[r]$, the annual volatility of interest rate $\sigma(r)$, the average price-dividend ratio $\exp(\mathbb{E}[\ell\mathrm{n}(P/D)])$, and the volatility of log price-dividend ratio $\sigma(\ell\mathrm{n}(P/D))$. The averages are taken over the entire steady-state distribution of S, the bottom quartile of S (bad times), and the top quartile of S (good times), for the first, second, and third column, respectively. The last three columns provide empirical value for each quantity: the fourth column under "CC(1999)" is based on the long sample (1871–1993) used in Campbell and Cochrane (1999); the fifth column under "C(2003)" is based on the post-war samples (1947–1995 or 1947–1998) used in Campbell (2003); and the last column under "BC(2012)" is based on the annual dataset (1930–2008) used in Beeler and Campbell (2012). The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 14. Return Expectations in the Fundamental Extrapolation Model.

	$\mathbb{E}_t^e[R_{t+dt}^D]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$			
$R_{t-12\to t}^D$	0.004	0.003			0.010	0.010		
	(0.21)	(0.63)			(1.15)	(2.04)		
$\ell\mathrm{n}(P/D)$			0.017	0.019			0.040	0.042
			(0.88)	(1.62)			(2.08)	(3.52)
Constant	0.05	0.05	–0.01	–0.02	0.03	0.03	–0.12	–0.13
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.14	0.04	0.24	0.10	0.17	0.08	0.31	0.22

	$\mathbb{E}_t^e[R_{t+dt}^{D,e}]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$			
$R_{t-12\to t}^D$	–0.004	–0.004			0.003	0.002		
	(–0.88)	(–1.13)			(0.07)	(0.42)		
$\ell\mathrm{n}(P/D)$			–0.009	–0.007			0.014	0.016
			(–0.49)	(–0.54)			(0.71)	(1.35)
Constant	0.03	0.03	0.06	0.05	0.006	0.006	–0.05	–0.05
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.15	0.05	0.22	0.06	0.14	0.04	0.23	0.08

The table reports, for the fundamental extrapolation model, the regression coefficient and its t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing subjective expectations about future returns on either the past twelve-month cumulative raw returns $R_{t-12\to t}^D$ or on the current log price-dividend ratios $\ell\mathrm{n}(P_t/D_t)$ over a sample of 15 years or 50 years. In the top panel, the subjective expectation used for the first four columns is $\mathbb{E}_t^e[R_{t+dt}^D]$, and the subjective expectation used for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the subjective expectation used for the first four columns is $\mathbb{E}_t^e[R_{t+dt}^{D,e}]$, and the subjective expectation used for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt) - r_t]$. Each reported value is averaged over 100 trials, and each trial represents a regression based on simulated data of the model with a monthly frequency. The t-statistics are based on a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.