Persistent Blessings of Luck*

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Abstract

Persistent fund performance in venture capital is often interpreted as evidence of differential abilities among managers. We present a dynamic model of venture investment with endogenous fund heterogeneity and deal flows that produces performance persistence without innate skill difference. Investors work with multiple funds and use tiered dynamic contracts to manage moral hazard. Recently successful funds receive continuation contracts that encourage greater innovation, and subsequently finance innovative projects through assortative matching. Initial luck, therefore, exerts an enduring impact on performance by altering funds' future investment opportunities. The model generates implications broadly consistent with empirical findings, such as that persistently outperforming funds encourage greater innovation and attract better entrepreneurs even with worse terms. The model further predicts "incumbent bias" in investing in funds, mean-reversion of long-term fund performance, backloading across contracts, and amplification of small skill differences.

JEL Classification: G10; L26; O31; G24

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1 Introduction

Financial economists have long debated whether or not investment managers differ in skills. Many studies of individual stocks, mutual funds, and other classes of funds generally find that investors do not consistently outperform passive benchmarks after-fee and outperformance does not persist (e.g., Wermers (2011)). An important exception is the private equity (PE) industry, most notably venture capital (VC) funds. Kaplan and Schoar (2005) show in their seminal study that VC firms typically manage sequences of funds, and the performance of one fund predicts the performance of the subsequent fund. Harris, Jenkinson, Kaplan, and Stucke (2014) confirm the phenomenon with more recent data. Korteweg and Sorensen (2017) find long-term persistence in expected net-of-fee return spread. Beyond the fund level, performance persistence also exists at the investment level (Nanda, Samila, and Sorenson (2017)) and the individual partner level (Ewens and Rhodes-Kropf (2015)). A widely-adopted interpretation of such performance persistence is that VC managers differ in their abilities, and the more skilled managers consistently outperform the others.

We present a new theory to challenge and complement this conventional wisdom. Our key argument is that a temporary and random shock may have an enduring impact on future investment opportunities. In the VC industry, the heterogeneity of funds in adding value is often endogenous (when compared to innate skills that are exogenous), as are the deal sourcing and flow of funds. To illustrate, our main setup focuses on differential investor-manager contracts as one manifestation of endogenous fund heterogeneity. Specifically, we recognize that not all capital are created equal: investors bring in expectations and non-monetary resources based on past performance, thus influencing managers' investment styles. This in turn leads to funds experiencing differential deal flows. If a manager is lucky in the current fund, he may find it easier to raise the next fund with more favorable funding terms, This in turn permits the manager to be more tolerant towards failure and experimentation in order to attract better deals that perpetuate his good performance. This positive reinforcement can lead to persistence in differential performance, both before- and after-fee, across managers even when they do not differ in skills.[1] More fundamentally, we demonstrate how luck

[1] In spirit, this paper is akin to Berk and Green (2004): they argue that the lack of persistence in

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could induce heterogeneity in fund characteristics and deal flows.

To be clear, our model does not contradict that in reality managers at VC and PE funds add value and probably possess differential skills. In fact, we demonstrate in an extension that our mechanism significantly augments exogenous skill heterogeneity under imperfect learning, and thus contributes to performance persistence. That said, our baseline setup considers homogeneous fund managers to underscore the point that besides the exogenous innate relative skills, the apparent differences can also be endogenous. Given that both persistent fund performance and entrepreneur's preference for offers from top funds are widely observed and interpreted as evidence of differential innate manager skills, we argue that this connection is more subtle: luck can both create and significantly amplify fund heterogeneity. Consequently, further empirical studies to carefully distinguish between luck and innate skills are called for.

Fund heterogeneity that endogenously arises in equilibrium due to luck can show up in other forms, such as in nurturing technologies, proprietary network formation, or size. Focus on contracting allows us to study the evolution of a portfolio of contracts, thus clarifying the role of inter-contract incentives. That said, our economic insights should apply more broadly to situations in which a lucky outcome gives a manager an advantage that is self-reinforcing and perpetuating, or amplifies real or perceived skill differentials under Bayesian learning. For example, an initial success IPO makes the entrepreneur more likely to become a future trusting investor and provide network support for the VC's future funds. In essence, our theory formalizes and extends the notion of the "snowball effect" or the "Matthew effect", in settings in which the initial heterogeneity could come from luck as well as innate differences.[2]

Although we believe our key insights apply more broadly to delegated investments such as buyout funds and the fund of funds, we cast our theory in the context of the VC industry for several reasons. First, entrepreneurship is the main engine for innovation and VC plays

returns does not necessarily mean differential ability across managers is non-existent or unrewarded; we argue that performance persistence and entrepreneur funding choices do not necessarily imply heterogeneity in managerial skills. In fact, luck could have an enduring impact on fund performance and managerial compensation due to the strong complementarity between endogenous capital and deal flows.

[2]The "Matthew effect" is a phenomenon sometimes summarized by the adage that "the rich get richer and the poor get poorer". See, for example, Azoulay, Stuart, and Wang (2013) and Simcoe and Waguespack (2011).

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an important role in supporting the formation of innovative enterprises. Its significance in motivating innovation warrants careful analysis. Yet despite the emerging empirical research, the theoretical underpinnings of fund performance are lacking. Second, endogenous deal flows and performance persistence are salient features in the VC industry (Kaplan and Schoar (2005); Phalippou and Gottschalg (2009); Robinson and Sensoy (2013); Harris, Jenkinson, Kaplan, and Stucke (2014)). Funds are not able to observe all potential deals due to limited attention and high search costs, and rely on the visibility of funds, investment style, network, reputation, track record, proximity to geographical focal point (Silicon Valley), and a presence on online platforms. Third, dynamic contracting is particularly relevant and interesting in the VC industry which is plagued by moral hazard and learning due to the early-stage nature of projects and serial fund raising. For instance, VC investors tend to be large and institutional and have considerable influence on the shaping of contracts. Moreover, the highly uncertain investment outcomes and long life cycles of VC funds makes it difficult for investors to commit capital to multiple sequential funds by a VC firm all at once. Contracts are thus written at the individual fund level, albeit with the understanding that investors and VC firms may form long-term relationships.

To be consistent with the aforementioned stylized facts about venture investment, we introduce a dynamic model with the following stylized features: (1) a mechanism allowing endogenous fund heterogeneity; (2) endogenous deal flow driven by the scarcity of truly innovative projects; (3) weaker competition among investors relative to fund competition in order to generate persistent net-of-fee returns; and (4) contractibility between investors and VCs limited to the most recent fund outcome in order to capture the relational nature of their interactions across multiple sequential funds.

Specifically, a group of entrepreneurs is born with projects in each period and seek financing and value-added services from VCs. A conventional project succeeds with some probability and pays off a mediocre amount, while innovative projects in limited supply benefit from experimentation and should be nurtured with innovative technology. What we call "technology" here includes the VC funds' network scale or tolerance towards initial failures (e.g., Tian and Wang (2014)). VCs, homogenous in our main setup, select projects and de-

cide on (1) whether to use an innovative or conventional nurturing strategy and (2) whether or not to exert effort to improve the chance of the project's success.[3] Investors invest in VCs, using both the explicitly specified share of the fund profit and implicit capital commitment for the next fund operated by the same manager in order to motivate effort.

We first establish that funding contracts and deals endogenously flow to fund managers who are successful due to luck. Investors implicitly commit future funding contracts to motivate managers' efforts, and improve contract terms so as to be more tolerant towards experimentation and innovation. This arrangement attracts entrepreneurs with innovative projects. The complementarity between contract and deal flows implies that under investors' equilibrium contracts, fund performance and investor returns are persistent and predictable, and entrepreneurs willingly accept offers from VC funds with more tolerant contracts. Unlike performance persistence caused by skill heterogeneity, better-performing managers do not increase fees to make after-fee returns unpredictable due to competition from other managers (none of them having superior skills compared with others). In addition, managers with lucky funds persistently receive higher compensation, which distinguishes our model from those based on information hold-up (e.g., Rajan (1992) and Hochberg, Ljungqvist, and Vissing-Jorgensen (2014)).

In equilibrium, investors offer two types of contracts; one encouraging VCs to use innovative nurturing technology, and the other using conventional nurturing. By using promotion from conventional contracts to innovative contracts, demotions from innovative contracts to conventional contracts, or the terminations of contracts, investors overcome the limitation in offering long-term contracts, and better incentivize managers to improve project success through tiered contracts. The set of incentive contracts used by investors exhibit either a hierarchical or parallel structure, depending on the discount rate and the managers' replacement costs. Each individual contract is shaped by the aggregate market environment such

[3]Evidence that VC funds differ in their tolerance for failure and nurturing technology abounds, see for example, Tian and Wang (2014). Many best performing funds have as high loss rates as average funds, if not higher, and take more innovative nurturing approach. A partner at Andreessen Horowitz, Alexander Rampell, aptly puts it, "You only score home runs if you swing HARD at pitches." Jo Tango from Kepha Partners blogged, "VC is not about minimizing losses" but about taking the risk to create real business. Fred Wilson, co-founder of Union Square Ventures that have invested in companies such as Twitter and Kickstarter, expressed a similar opinion.

as the supply of truly innovative projects, as well as the performance of other funds.

Our model predicts that VC firms with earlier successes are more likely to raise capital for subsequent funds that encourage innovative nurturing and greater risk-tolerance, or more broadly, larger capital or easier and more frequent access to capital (Gompers and Lerner (1999b); Kaplan and Schoar (2005); Tian and Wang (2014)). Expecting greater innovation under top-performing funds, entrepreneurs are willing to accept funding their offers even with less favorable contract terms (Hsu (2004)). Moreover, to the extent that project quality is private information for the fund managers or the entrepreneurs, matching up with a successful fund signals project quality through selection, and top-performing funds have an endogenous certification effect even though managers do not have differential skills.

Our findings not only show that it is *possible* to generate performance persistence with little or no innate skill heterogeneity, but also make unique predictions that are consistent with recent empirical studies, suggesting that our mechanism is a *likely* and *important* channel. Different from a pure skill difference or learning-based mechanism, our model implies that over the long horizon, investment performance are mean-reverting, because in the long run, the enduring impact of initial luck will be gradually offset by i.i.d. shocks in following periods. The model also predicts that performance persistence is mostly driven by deal flows, even when there is no skill difference or only *perceived* skill difference among managers. These model implications are consistent with Nanda, Samila, and Sorenson (2017), which documents several interesting patterns extant theories cannot fully explain: (1) the average performance of other VC funds investing in the same industry and location predicts the future investment performance of a fund in question, but there is no evidence that managers persistently pick the right industry or location; (2) and VC fund performance exhibits long-term mean-reversion; (3) deal flows explain the majority of performance persistence. Our model helps to rationalize these patterns.

Finally, our theory has several practical implications for investors, venture capitalists, and entrepreneurs. First, limited partners that are long-lived in the market and interacting with multiple VC firms should take advantage of the inter-contract incentives in managing agency rent and motivating effort from managers. Second, fund managers need to go be-

yond a simple track record to demonstrate superior skill and value-added to investors and entrepreneurs. Finally, entrepreneurs choosing which VC to work with should focus not only on the funds' status and past success, but also on the exact advantages of the funds or characteristics past successes endogenously generate.

Literature

Our paper contributes foremost to our understanding of managerial skills and fund performance. Berk and Green (2004) illustrate that the lack of return persistence in mutual funds does not necessarily imply the absence of skill difference. Garleanu and Pedersen (2015) show that search friction can lead to a persistent spread in net-of-fee returns. Hochberg, Ljungqvist, and Vissing-Jorgensen (2014) argue that incumbent investors with insider information can hold up managers and extract information rents. Marquez, Nanda, and Yavuz (2015) suggest that the excessive efforts of VCs to manipulate the entrepreneur's beliefs about his ability also leads to persistence. Acharya, Gottschalg, Hahn, and Kehoe (2013) analyze deal-level data and show that managers with disparate backgrounds add value in different deals. Gompers, Kovner, Lerner, and Scharfstein (2010) argue that performance persistence in entrepreneurship can be attributed to entrepreneurs' (perceived) skills. Unlike these papers, where results are often predicated on managers being inherently heterogeneous (for example, in skills), our paper shows that one period of luck could have the potential to contribute to performance persistence. Also offering an agnostic view on the innate skills of managers is the work of Glode and Green (2011), which shows that in the hedge fund industry, concerns about information spillover gives incumbent investors bargaining power and leads to persistence in excess returns. Our paper differs both in the mechanism and the application.

Among recent contributions to the literature, our paper is most closely related to and broadly consistent with two empirical studies. First, Nanda, Samila, and Sorenson (2017) document performance persistence at the investment level and provide suggestive evidence that performance persistence stems from early successes due to luck creating subsequently better deal flows, rather than as a result of managers' differential ability to select or govern

portfolio companies.[4] Our model complement the study by providing a framework to rationalize the long-term mean-reversion of performance and the impact of deal flow. Second, Sorensen (2007) finds that companies funded by more experienced VCs are more likely to go public, and structurally estimates that deal flows (sorting) are twice as important as direct value added by VCs (influence) in explaining the observation. Also related is Hochberg, Ljungqvist, and Lu (2007) who point out that a persistent VC network leads to a persistent fund and project out-performance, and provide initial evidence that an emerging track record of success improves a VC firm's network position over time. Venugopal and Yerramilli (2017) find similar effects of initial successes on networks dynamics in the case of angel investors, consistent with and corroborates our theory that fund heterogeneity (networks instead of funding contracts) can indeed arise endogenously due to initial luck and the provision of incentives for manager effort.

This paper further contributes to the growing literature on venture capital regarding the role and behavior of intermediaries. Starting with Gompers (1996), a number of studies examine grandstanding by VC funds. Megginson and Weiss (1991) discuss the certification role of VC funds in IPOs to mitigate information asymmetry. Tian and Wang (2014) find that firms backed by more failure-tolerant VC investors are significantly more innovative. Landier (2005) documents that entrepreneurial activity varies substantially across regions and sectors, and appears related to the stigma of failure. Hsu (2004) finds that firms are more likely to accept an offer – even if the terms are less attractive – from a VC with good past performance. Our model provides theoretical foundations for these phenomena without necessarily resorting to innate manager heterogeneity. Instead, we emphasize endogenous deal flows.

Venture investments also play an important role in motivating innovation. Hellmann and Puri (2000) document that VCs are associated with significant reductions in commercialization time for a product, especially for innovators; Kortum and Lerner (2000) find that increased VC activity in an industry leads to significantly more innovation; Nanda and Rhodes-Kropf (2013) and Nanda and Rhodes-Kropf (2016) show that aggregate hot markets

[4]Korteweg and Sorensen (2017) also conclude that VC performance is mostly driven by luck.

facilitate experimentation (which is important for the diffusion of new technologies), and VC investments facilitate riskier and more innovative start-ups. Our paper complements the studies by demonstrating how past successes lead to heterogeneity across VC funds in facilitating innovation, a la Manso (2011) who argues that the optimal way to motivate innovation is to show tolerance for failure. More importantly, we highlight the complementarity of the luck-induced allocation of contracts and deal flows, which is new to the literature.

Our discussion on how such contracts between the investors and fund managers create fund heterogeneity is also related to delegated investment and dynamic agency. Bolton and Scharfstein (1990) use funding termination to mitigate managerial incentive problems. They also show that withholding future funding can play a role similar to that of demanding repayment in forcing liquidation. Stiglitz and Weiss (1983) provide conditions for and characterize equilibrium contingency contracts with the potential termination of a relationship. Chung, Sensoy, Stern, and Weisbach (2012) show that future fund-raising creates incentives for private equity funds to perform well, and these indirect incentives are approximately as large as direct incentives from carried interest. Our paper extends the discussion to contracting between investors and fund managers, which is important but largely unexplored (Rin, Hellmann, and Puri (2013)).

Furthermore, our discussion on contract allocation and hierarchy also relates to the literature on tournament and employee compensation (e.g., Lazear and Rosen (1981)), and contracting with externalities (e.g., Segal (1999)). Our innovation lies in analyzing the incentives channel generated by the limited supply of innovative projects (thus innovative contracts) in fund manager markets rather than within firms. Moreover, optimal dynamic contracting (e.g., Bolton and Dewatripont (2005); Sannikov (2008)) often entails backloading agency rent within a contract. We add to this by illustrating that principals who are constrained in backloading agency rent within a contract, such as investors in relational contracts with VC firms, can optimally backload it across contracts. Consequently, inter-contract incentives link an individual's contract and dynamic moral hazard to aggregate market conditions (such as the total supply of I-Projects or the performance of other funds).

Finally and more broadly, this paper adds to an emerging literature on the long-term

impact of random initial differences. Oreopoulos, Von Wachter, and Heisz (2012) and Kahn (2010) discuss how initial (random) shocks can lead to divergent career trajectories. Schoar and Zuo (2016) show that the career start of a CEO has a long-lasting impact on management and investment styles. Oyer (2008) finds evidence that investment bankers with high pay are largely "made" by circumstance rather than through their innate ability. Most of these studies are empirical in nature, and our paper complements them by providing plausible theoretical channels through which the persistent effect of initial luck may operate.

The rest of the paper is organized as follows: Section 2 lays out the basic framework and illustrates key mechanisms; Section 3 solves the model and characterizes the dynamic equilibrium; Section 4 presents model implications; and Section 5 further discusses and extends the model. Finally, Section 6 concludes.

2 Illustration and Model Setup

Before any formal analysis, we first describe a simple example to illustrate our key economic intuition. In the appendix we solve in detail a two-period model corresponding to our verbal illustration.

2.1 A Verbal Illustration

Consider a simple multiple-period economy. In each period, a representative investor invests in a group of funds. Each fund is operated by a venture capitalist, who screens projects, chooses the way he nurtures project, and decides on unobservable effort to increase the probability of success. There are two different types of projects, innovative and conventional. Innovative projects are of high quality and generate higher expected profit but are of limited supply.

Suppose there are two homogeneous venture capitalists, A and B, each running a fund. Since they are identical, they are assigned the same contract to start with and face the same deal flow. As shown latter, in equilibrium the principal motivates unobservable effort using both performance-based bonus and more favorable future contract terms conditional

on current fund success.

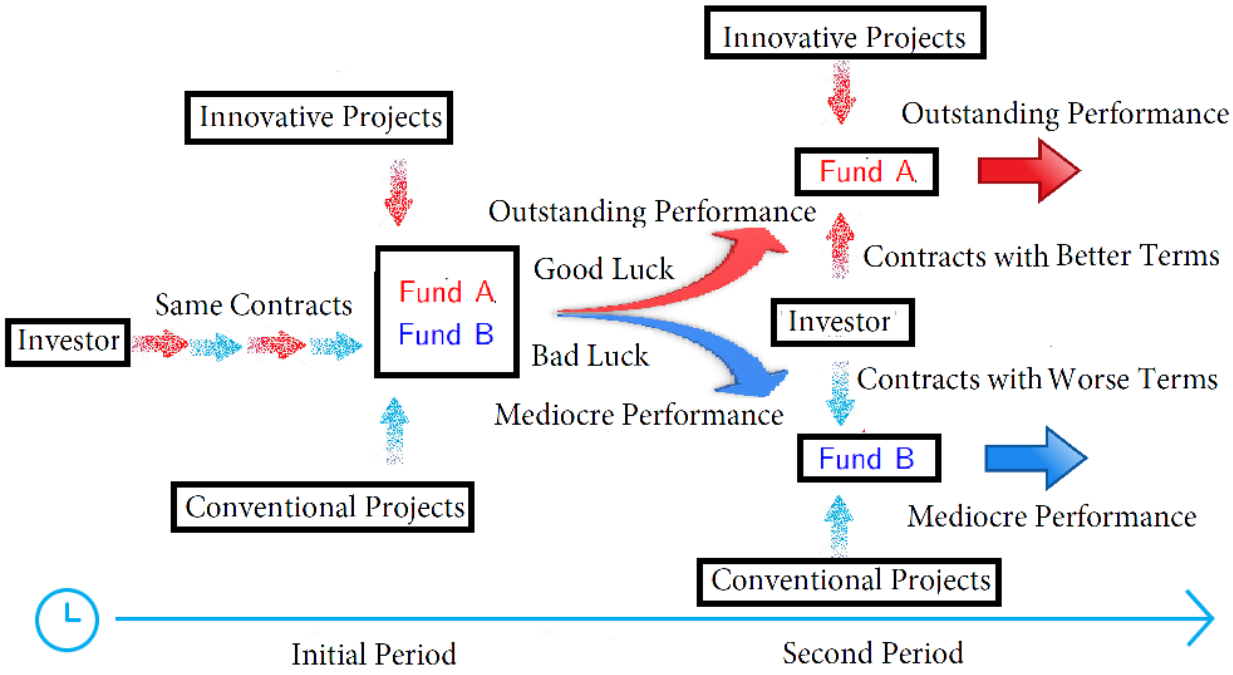


Figure 1: Illustration of Mechanism

Due to random luck, suppose A succeeds while B fails. Then A receives more favorable contract terms for the fund he operates in the second period. In other words, an idiosyncratic shock translates into endogenous fund heterogeneity in terms of contract terms. Instead of contract terms, the endogenous heterogeneity could also be business network acquired from the success, or additional funding liquidity. Notice that contract terms alone do not necessarily lead to persistence in gross performance if A and B still face the same deal flow in the second period. However oftentimes there is complementarity between contract terms and deal flow: the better contract might allow A to be more tolerant towards experimentation and failure, which could truly benefit innovative projects. Taking that into account, entrepreneurs endowed with innovative projects will always choose to approach A first, resulting in an assortative matching and endogenous heterogeneity in deal flows. Agent A thus generates a higher expected return in the second period.

Intuitively, the principal also finds it optimal to promise different future contracts based

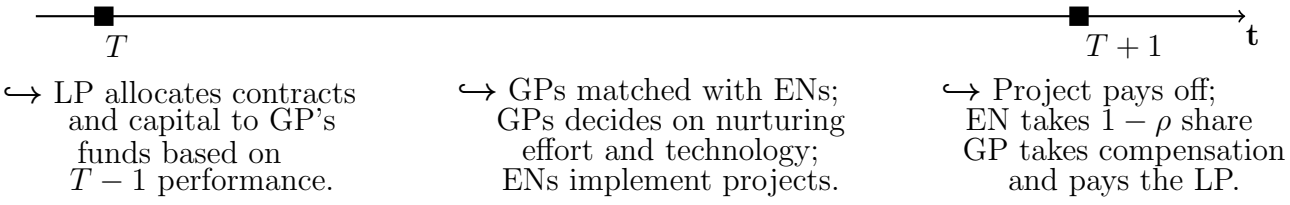
on current fund performance, because given the limited supply of innovative fund, the principal always face a deal flow hierarchy within the group of funds she invests. She may as well use heterogeneous contracts (which essentially allocates projects of different qualities) to reward or punish A and B, as a cheap way to motivate their efforts. Figure 1 illustrates this mechanism.

2.2 Model Setup

Now we formally describe the setup, and analyze role of capital (contract) and deal flows as well as the main economic intuitions. To clearly demonstrate the importance of luck in performance persistence, our baseline specification focuses on the venture capitalists' moral hazard of effort provision without an innate skill difference. In reality, the agency issues between investors and delegated managers involve both unobservable actions and learning or asymmetric information in regard to the managers' abilities. We discuss the latter case in Section 5 in which the mechanism amplifies innate skill difference.

Time is discrete and infinite, and is labeled by $t = 1, 2, 3, \cdots, \infty$. There are three groups of risk-neutral players in the economy that we introduce next. Figure 2 provides the timeline, and the sequence of actions that we elaborate below. For simplicity, all players share the same time discount rate $\beta \in (0, 1)$ for future cash flows.

Figure 2: Timeline within a period.



First, a unit measure of entrepreneurs (EN) is born in each period. A fraction ϕ of them are endowed with innovative projects (I-Projects, think of them as unicorns), and the rest with conventional projects (C-Projects). Let the set of all time-t entrepreneurs be $\mathcal{E}_t$, the ones with I-Projects $\mathcal{E}_t^I$, and these with C-Projects $\mathcal{E}_t^C$. Each project requires an investment K and nurturing by venture capitalists, and pays off X_C with probability

$p_C \in (0,1)$ under conventional technology (C-Technology). Innovative nurturing technology (I-Technology) costs investors an additional z and increases the payoff for I-Projects to $X_I > X_C$ with success probability p_I, but does not affect the payoff or success probability of the conventional project.[5] A project yields 0 if it fails. Each entrepreneur lives for one period and then permanently exits the market.[6]

Second, there is an infinite supply of venture capitalists aspiring to become general partners of VC funds (GPs). In each period, after successful fundraising, a GP $g \in G_t$ screens the type of projects, ascertains project types, and for simplicity is assumed to contract with one entrepreneur.[7] GPs choose either I- or C-Technology based on their contracts with the investors.[8] They also decide whether or not to incur effort cost $e > 0$ to augment the project's success probability by $\Delta \in \left(0, \frac{1-p_C}{p_C}\right)$.

In the next period, the fund's investment outcome is realized. The GP pays the entrepreneur and investors, raises capital for the next fund and closes the previous fund. If a GP fails to raise capital for a new fund, he permanently exits the market of venture capital. For simplicity, we assume the GPs contract with entrepreneurs to share the project payoff

[5]This assumption is not crucial, but simplifies our discussion. We could alternatively restrict our attention to some range of success probabilities which leads to a cost of using I-Technology when the project is conventional, as is typically done in the contracting literature. Moreover, the assumption is well-motivated, for example, by how capital and funding constraints affect VC failure tolerance (Tian and Wang (2014)), or by other considerations such as career concerns (Hirshleifer (1992); Goel, Nanda, and Narayanan (2004)). We treat z as an additional operation expense. Alternatively, we can view it as a reduced-form representation of waiting cost in a dynamic environment with grandstanding, where GPs could either myopically rush to exit, or wait longer for better exits. Though z could be alternatively formulated as an increase in fund size, we de-emphasize such an interpretation because GPs have limited attention for projects, and VC funds for a given investment stage are not very scalable (Metrick and Yasuda (2010) and Quindlen (2000)).

[6]In other words, each entrepreneur only interacts with venture capitalists once, which captures reality in a reduced-form. While serial entrepreneurs exist, they are rare and their contracts with financiers are based on individual projects; startups not immediately funded are often out-competed by rivals.

[7]The essence of this assumption is that each fund only operates a limited number of projects, reflecting anecdotal evidence that VCs' scarce resource is time and deals (whether evaluation or nurturing) requiring approximately equal amounts of time. Quindlen (2000) and Kaplan and Strömberg (2004) provide more details.

[8]Though we show persistent dispersion in performance can arise when GPs do not differ in their relative skill, the nurturing and effort provision discussed next are all consistent with the literature documenting how VCs add value to entrepreneurial companies (Hellmann and Puri (2002); Cong, Howell, and Zhang (2017); Bernstein, Giroud, and Townsend (2015); Kortum and Lerner (2000)). Nurturing can be interpreted as experimentation jointly with the entrepreneurs. GPs in general can influence firm operations, team building, and experimentation styles by setting different contract terms, taking hidden actions, showing different attitudes towards failures, etc.

in fixed proportions $\rho : 1-\rho$, $\rho \in (0,1)$.[9] Denote g's decision on nurturing technology by $n \in \{C, I\}$. Then a GP's strategy has $\Lambda_g = (\mathbb{I}_e, n)$ where $\mathbb{I}_e$ is the effort indicator. Our baseline model recognizes that GPs provide value-added services because they screen, improve and nurture projects, but they do not have differential skills.

To account for payoff skewness in entrepreneurial endeavors (Scherer and Harhoff (2000)), and the scarcity of truly innovative projects, we assume:

Assumption 1

$$p_C \geq p_I \quad \textit{and} \quad p_C X_C < p_I X_I \qquad \textit{(Payoff Skewness)}, \tag{1}$$

and:

Assumption 2

$$\phi \leq \frac{p_C(1+\Delta)}{1+(p_C-p_I)(1+\Delta)} \qquad \textit{(Unicorn Scarcity)}, \tag{2}$$

which are non-restrictive. For example, these assumptions hold when innovative projects pay off more upon success and are scarce enough that the measure of failed I-Projects is still less than that of successful C-Projects (an alternative interpretation of Assumption 2 is that in any period there are more recently successful funds than innovative projects).

Third, there is one representative investor who can invest in multiple funds each period as a limited partner (LP). Readers can think of this investor as a university endowment, a pension fund, or a family office, etc, with deep pockets to finance all potential projects. We extend the discussion to multiple LPs in Section 5.4 and show LP competition does not qualitatively alter our results. In each period t, she invests in a unit measure of funds, and the set of time-t funds is denoted by F_t. She decides her investment plan $A_t(f)$, $f \in F_t$. Then the time-t set of GPs that are successful at fundraising is $G_t = \{g | \exists f \in F_t,\ s.t.\ g = A_t(f)\}$. For each fund she works with, she offers a contract explicit as to the nurturing technology and the ownership shares of the GPs (carry), and either explicit or implicit as to whether or

[9] Endogenizing the contract terms between GP and EN in a Nash bargaining game would not alter the main results.

not and how to continue working with the GP contingent on the current fund performance.[10] Using continuation as an incentive is consistent with findings in Chung, Sensoy, Stern, and Weisbach (2012), who find that 40% of manager pay is for indirect compensation from future fund raising.

We are essentially assuming that the LP can only contract on short histories of GP performance for profit split and contract extension. This assumption is consistent with the specifications and findings in aforementioned empirical studies (e.g. Kaplan and Schoar (2005)), and is well-motivated by real-life observations. First, compared to the long-lived LP such as university endowments, pension funds, family offices, etc., GPs have relatively short career horizons with a finite life-cycle of VCs and a high career turnover, therefore the performance history of GPs cannot be very long. Second, the contract between the LP and GP across sequential funds is mostly relational in real life, and it is hard to commit to contracting on long history or the future performance. If it were otherwise, contracts would entail strong initial back-loading using only the continuation value without cash payments (e.g., DeMarzo and Fishman (2007) and Sannikov (2008)), which is inconsistent with real-life observations that GPs receive cash payments for each fund they operate.

The LP thus offers contracts of the form $\Phi_g = \{\alpha, V_f, V_s\}$, where α is GP g's total share of the fund's profit, and V_S, $S = \{s, f\}$ is g's promised continuation value given the fund's project outcome and all agents' equilibrium strategies.[11] By committing the operational cost z, the LP can dictate whether a GP uses I-Technology (I-Contract) or C-Technology (C-Contract).[12] The LP's strategy is consistent because for each $g \in G_{t-1}$, his time t

[10]The specification of technology and renewal policies can be broadly interpreted as corresponding to the nebulous concept of fund "brand". Alexander Rampell, founder of Trial-Pay and a partner at Andreessen Horowitz explains, "The best firms attract the best entrepreneurs/deals, which provide the best returns, which increases the brand recognition, which attracts the best entrepreneurs/deals." Tolerance towards failure is one aspect of branding; network extensiveness is another. To some extent, GPs at successful funds develop networks with successful entrepreneurs who are likely to be future investors and startup buyers. Endogenous network strength can be similarly captured as nurturing technology in the current model.

[11]α corresponds more to GPs' carried interest. We could introduce a fixed fee, but it does not alter our results qualitatively. We leave it out because VCs' expected revenue depends significantly less than buyout funds on management fees, which also tends to vary little across a sequence of funds because of limited scalability (Metrick and Yasuda (2010)). Moreover, management fees instead of carried interest do not help to motivate effort, and rather prompt the greater use of continuation value, which will only strengthen our results.

[12]Besides the potential different cash flows for the LP. The LP can induce different nurturing strategies

expected utility equals the implicit promised value in his time $t-1$ contract given his time $t-1$ fund performance. Without loss of generality, at any time t if g has not started a fund yet or if he fails to raise a consecutive fund, the offer is denoted by $\Phi_g = 0$.

While in our baseline model funds offer the same ρ to entrepreneurs, the entrepreneur's endogenous fund choice is crucial because GPs use different technologies given contracts $\{\Phi_g\}$. For each entrepreneur $i \in \mathcal{E}_t$, her funding offer choice is denoted by $\Psi_i = g$, $g \in G_t$. We model project deal flows by two-sided stable matching in each period between projects and the GPs with successful fundraising. As in the Deferred-Acceptance Algorithm (Gale and Shapley (1962)), GPs observe entrepreneurs' types and simultaneously make offers to their top choices. In the offers, funds can either credibly post the nurturing strategy, or the valuation of the project. Entrepreneurs reject all but their top choices of funds, and break indifference by randomizing among the funds they equally prefer. Rejected funds then make the next round of offers, and the remaining entrepreneurs again reject all but their top choices. The process goes on until all the funds have projects or all the projects have VC backing.

We further assume that:

Assumption 3

$$\beta\Delta\rho p_C X_C > \frac{1+\Delta}{\Delta}e \qquad \textit{(Worthy Effort)}, \tag{3}$$

Assumption 4

$$\beta\rho\phi(1+\Delta)(p_I X_I - p_C X_C) < z < \beta\rho(1+\Delta)(p_I X_I - p_C X_C) \qquad \textit{(Technology Selection)}, \tag{4}$$

Assumption 5

$$\beta(1+\Delta)\rho p_C X_C - \frac{1+\Delta}{\Delta}e - K \geq 0 \qquad \textit{(Worthy Investment)}. \tag{5}$$

through contract terms. The I-Contracts penalize outcomes X_C to induce the GPs to use I-Technology; similarly, C-Contracts penalize outcomes X_I to induce C-Technology. When project payoffs are not contractible but project successes or failures are, the LP needs to incentivize the technology choice. However, our key intuition goes through this and we can show, under mild parameter restrictions, that the contracts are the same. See detailed discussions in Section 4.4.

Assumption 3 implies that the GP's effort improves the probability of success sufficiently much that the LP opts to incentivize effort in a single period regardless of the nurturing technology (a standard assumption in agency models). Assumption 4 states that relative to C-Technology, I-Technology is sufficiently costly so that the LP does not use it indiscriminately, but that it is sufficiently cheap to be used on funds that have a high possibility of getting I-Projects, a point that becomes clear once we introduce the matching of funds and projects. Together with Assumption 5, they represent LP's participation constraints in investing in a single period.[13] Only Assumptions 2 and 4 are key in our model and they allow heterogeneity in funds' characteristics and opportunity sets.

2.3 Equilibrium Definition

Following Levin (2003), we focus on dynamic equilibria that are stationary. Our two-period model in the Appendix discusses short-term transitions.

Definition 1

An equilibrium consists of LP's strategy $\Xi^* \equiv \{\{\Phi_f^*\}_{f\in F_t}, A_t\}_{t=1,2...}$*, GPs' strategies* $\{\{\Lambda_g^*\}_{g\in G_t}\}_{t=1,2...}$ *and entrepreneurs' strategies* $\{\{\Psi_i^*\}_{i\in I_t}\}_{t=1,2...}$ *such that:*

1. *For each GP* $g \in G_t$*, conditional on entrepreneurs' funding offer choices* $\{\Psi_i^*\}_{i\in i_t}$*, LP's contract* Φ_g^* *and other GPs' strategies* $\{\Lambda_{g'}^*\}_{g'\in G_t\setminus g}$*,* Λ_g^* *satisfies:*

$$\Lambda_g^* \in argmax_{\Lambda_g}\ E^{\Lambda_g}\{\alpha\rho R_g + \beta V_S\}; \tag{6}$$

   where R_g *is the GP* g*'s fund return, and* V_S *is the promised value in state* $S \in s, f$*;*

2. *For each EN* $i \in \mathcal{E}_t$*, conditional on GPs' strategies* $\{\Lambda_g^*\}_{g\in G_t}$*, her funding offer choice* Ψ_i^* *satisfies:*

$$\Psi_i^* \in argmax_{\Psi_i}\ E^{\Psi_i}\{(1-\rho)R_g\}; \tag{7}$$

   and

[13] In the dynamic equilibrium we derive later, the IC constraints for both participation and effort provision are actually less stringent than these sufficient conditions.

3. *Conditional on GPs' strategies* $\{\{\Lambda_g^*\}_{g\in G_t}\}_{t=1,2...}$ *and entrepreneurs' strategies* $\{\{\Psi_i^*\}_{i\in I_t}\}_{t=1,2...}$, Ξ^* *maximizes LP's discounted expected investment profit:*

$$\Xi^* \in argmax_{\Xi}\ E^{\Xi}\{\sum_{t=1}^{\infty}\beta^t[\int_{G_t}(1-\alpha)\rho R_f - z\mathbb{I}_{f=I}df - K]\}; \tag{8}$$

where $\mathbb{I}_{f=I}$ *is the indicator function that equals* 1 *if fund* f *is with type* I *nurturing technology.*

A Stationary Equilibrium of Delegated Investment is an equilibrium such that:

1. *The set of fund contracts* $\{\Phi_f^*\}$ *are time-invariant and non-random;*

2. *Let* $M_t(\Phi, \Phi')$ *be the time* t *measure of funds whose GPs were offered contract* Φ *in the last period and receive contract* Φ' *in the current period, then* $M_t(\Phi, \Phi')$ *is time-invariant for all* $\Phi, \Phi' \in \{\Phi_f^*\}$.

In the stationary equilibrium, the aggregate distribution of funds is time-invariant and deterministic. For the LP, the stationary equilibrium is essentially static. She will finance constant measures of contracts, finance the same measure of projects with funds that apply innovative nurturing technology over time and receive time-invariant investment returns. From the GP's perspective, the equilibrium is stationary in the sense that the aggregate measures of GPs accepting different contracts are time-invariant and deterministic.

3 Dynamic Equilibrium

While the verbal illustration provides the basic intuition of the mechanism of endogenous fund heterogeneity and deal flow, it is silent on GP entry and exit, and on the dynamic evolution of performance and compensation. Moreover, as shown shortly, examining the dynamic equilibrium also allows us to better understand inter-contract incentives.

Assortative Matching of Funds and Projects

Under I-Technology, an entrepreneur with I-Project gets $(1-\rho)(1+\Delta)p_I X_I$, which is greater than $(1-\rho)(1+\Delta)p_C X_C$ under C-Technology. Hence she strictly prefers funds

under I-Contracts. Moreover, accepting offers from I-Contracts funds is a weakly dominating strategy even when entrepreneurs are not as good at learning the type of their projects as the GPs. Similarly, funds with I-Contracts prefer I-Projects. Therefore, we have a unique matching equilibrium that is positive assortative, in the sense that more innovative projects are matched with more innovative funds.

Moral Hazard and Incentive Contracts

We are interested in the case in which the LP wants to motivate effort from the GPs, and decides on the technology choice in equilibrium by offering either an I-Contract or a C-Contract.

To induce effort, a C-Contract must satisfy:

$$\beta[(1+\Delta)p_C(V_s^C+\rho X_C\alpha^C)+(1-p_C(1+\Delta))V_f^C]-e \geq \beta[p_C(V_s^C+\rho X_C\alpha^C)+(1-p_C)V_f^C], \quad (9)$$

where α^C is the share given to the GP. Therefore, fixing V_s^C and V_f^C, the cheapest contract from the LP's perspective satisfies:

$$\alpha^C = \frac{e-\beta\Delta p_C(V_s^C-V_f^C)}{\beta\Delta p_C\rho X_C}. \quad (10)$$

The payoff (agency rent) for a GP under C-Contract is then:

$$V_{GP}^C = \beta[(1+\Delta)p_C(V_s^C+\alpha^C\rho X_C)+(1-(1+\Delta)p_C)V_f^C]-e = \frac{e}{\Delta}+\beta V_f^C. \quad (11)$$

Intuitively, for the GP the value of operating a C-Contract fund consists of two components. $V_f^C \geq 0$ represents the future payoff in the worst case scenario under limited liability, while $\frac{e}{\Delta}$ describes the GP's minimal agency rent. Similarly, the incentive-compatibility for exerting effort in an I-Contract requires:

$$\alpha^I = \frac{e-\beta\Delta p_I(V_s^I-V_f^I)}{\beta\Delta p_I\rho X_I}, \quad (12)$$

and the GP's rent under I-Contract is then:

$$V_{GP}^I = \beta[(1+\Delta)p_I(V_s^I + \alpha^I \rho X_I) + (1-(1+\Delta)p_I)V_f^I] - e = \frac{e}{\Delta} + \beta V_f^I. \tag{13}$$

Equilibrium with Fixed Technology

Let us first examine the benchmark scenario where there is only one technology available. Without loss of generality, we focus on C-Technology.

Notice that in an equilibrium with only C-Contracts, the present value to the LP is:

$$V_{LP}^C = \frac{1}{1-\beta}(\beta(1+\Delta)p_C(1-\alpha)\rho X_C - K). \tag{14}$$

To maximize the LP's profit, the optimal contract minimizes α, which can be derived from:

$$\begin{aligned} \frac{\beta}{1-\beta}(1+\Delta)p_C\alpha\rho X_C = &\left[\frac{(1+\Delta)e}{\Delta} + \beta V_f^C\right] \\ &- \frac{\beta}{1-\beta}P(V_s^C, V_f^C)\left[\frac{(1+\Delta)e}{\Delta} + \beta V_f^C\right]. \end{aligned} \tag{15}$$

The LHS is NPV of future cash payments to the GPs operating the fund. The first term on the RHS is the expected total value paid to the current GP to motivate effort, and the second term is the net present value of total payment to future GPs, where $P(V_s^C, V_f^C)$ is the replacement probability given the contract. Now the LP's profit is:

$$\begin{aligned} V_{LP}^C = &\frac{1}{1-\beta}(\beta(1+\Delta)p_C\rho X_C - K) - \left[\frac{(1+\Delta)e}{\Delta} + \beta V_f^C\right] \\ &- \frac{\beta}{1-\beta}P(V_s^C, V_f^C)\left[\frac{(1+\Delta)e}{\Delta} + \beta V_f^C\right]. \end{aligned} \tag{16}$$

The first term is NPV of future cash flows from the projects, the second term is the expected total compensation to the current GP to motivate effort, and the third term is the net present value of compensation to future GPs. Replacements are costly because we are giving rent to a new GP without using the rent to motivate his effort in the previous period. Let us now consider two extreme cases.

Example 1 (Temporary Worker Contract)

The LP commits to terminate the incumbent GP and paying a new GP in each period. Then $V_s^C = V_f^C = 0$.

Given the contract, $V_{GP}^C = \frac{e}{\Delta} + \beta \times 0 = \frac{e}{\Delta}$. The LP's expected profit is:

$$\begin{aligned} V_{LP}^C =& \frac{1}{1-\beta}(\beta(1+\Delta)p_C\rho X_C - K) - \frac{1}{1-\beta}\frac{(1+\Delta)e}{\Delta} \\ =& \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - \frac{1}{1-\beta}\left[K + \frac{(1+\Delta)e}{\Delta}\right]. \end{aligned} \tag{17}$$

Under the single period contract, the low continuation value $V_s^C = V_f^C = 0$ suggests a low agency cost $\frac{(1+\Delta)e}{\Delta}$ for the incumbent GP but a high replacement rate and associated high replacement agency costs.

Example 2 (Job-for-Life Contract)

The LP commits to renew the contract with the incumbent GP in each period. Then $V_s^C = V_f^C = V_{GP}^C$.

Given the contract, $V_{GP}^C = \frac{e}{\Delta} + \beta V_{GP}^C = \frac{1}{1-\beta}\frac{e}{\Delta}$, the LP's expected profit is:

$$\begin{aligned} V_{LP}^C =& \frac{1}{1-\beta}(\beta(1+\Delta)p_C\rho X_C - K) - \frac{1}{1-\beta}\left(\frac{e}{\Delta} + \beta V_f^C\right) - 0 \\ =& \frac{\beta}{1-\beta}(1+\Delta)p_C\rho X_C - \frac{1}{1-\beta}\left[K + \frac{(1+\Delta)e}{\Delta}\right]. \end{aligned} \tag{18}$$

Under the perpetual contract, the high continuation value $V_s^C = V_f^C = V_{GP}^C$ suggests a high agency cost $\frac{1}{1-\beta}\frac{(1+\Delta)e}{\Delta}$ for the incumbent GP but a low replacement rate and associated low costs for paying future GPs (in fact 0). The following proposition states the optimal contract that balances the trade off between incumbent GP and replacement and future GP costs.

Proposition 1 (**Equilibrium with Fixed Technology**)

With a fixed single technology n, $n \in \{I, C\}$, *there is an essentially unique equilibrium. The LP offers a measure* p_n *of* n*-contracts to GPs that are recently successful, and a measure* $1 - p_n$ *to new GPs, all with terms:*

1. $\alpha^n = \frac{(1-\beta p_n)e}{\beta \Delta p_n \rho X_n}$;

2. *Renewal of the same contract upon project success;*

3. *Permanent termination of the current GP.*

The equilibrium involves termination upon failure and contract renewal upon success. Mathematically, $V_f^C = 0$, and $V_s^C = V_{GP}^C$. Under the optimal contract, the present value to the LP is:

$$V_{LP}^C = \frac{1}{1-\beta}(\beta(1+\Delta)p_C \rho X_C - K) - \frac{(1+\Delta)e}{\Delta} - \frac{\beta}{1-\beta}(1 - p_C(1+\Delta))\frac{(1+\Delta)e}{\Delta}. \quad (19)$$

Given Assumption 3, $V_{LP}^C \geq 0$, and therefore all projects are funded.

Equilibrium with Two Technologies

When both technologies are available, the LP can use both I- and C-Contracts. The optimal contract problem is complicated because the LP may promise a mixture of different types of contracts as the continuation value, and the associated contract type transition rates must guarantee the existence of the steady state. Instead of deriving the optimal contract directly, we first postulate certain characteristics of an equilibrium (if it exists). We then argue that for any steady state that does not possess these characteristics, the LP can deviate to propose different contracts or allocation strategies A_t to induce another steady state and to be strictly better off.

Lemma 1

In the stationary equilibrium, let the measure of I-Contracts be m_I, then:

1. *The LP offers only one type of C-Contract and one type of I-Contract;*

2. $m_I = \phi$;

3. $V_f^C = 0$;

4. $V_f^I = V_{GP}^C$ *if* $\beta \geq p_I(1+\Delta)$, *and* $V_f^I = 0$ *otherwise. In either case* $V_s^I = V_{GP}^I$.

The first result simplifies our analysis in the sense that we only need to focus on a single contract for each type of project. Result 2 confirms that every I-Project will be matched with an I-Contract fund in the equilibrium.

Since the agency rent V_{GP}^C and V_{GP}^I depend on V_f^C and V_f^I, our analysis focuses on the continuation values of GPs upon project failures. Result 3 suggests that in the optimal contract, the LP will terminate the C-Contract for certain after a failure. As illustrated in the fixed technology case, the termination of a contract is costly in the sense that LP needs to grant agency rent to more new GPs. However, LP still finds it optimal to do so because the termination strategy lowers the value of agency rent promised to each new GP $V_{GP}^C = \frac{e}{\Delta} \leq V_{GP}^I$. On the other hand, for the I-Contract case, termination lowers the agency rent for the current I-Contract GPs, but increases the number of new GPs in each period. When the LP is patient $\beta \geq p_I(1+\Delta)$, the LP cares more about future agency costs and will not terminate I-Contracts but will downgrade them to C-Contracts. When she is impatient, termination becomes the optimal strategy.

We now derive the stationary equilibrium. Funds with C-Contracts would make offers to all ENs, but ENs with I-Projects always prefer funds with I-Contracts. Therefore, given a stationary deal sources distribution in each period, the law of large numbers and LP rationality imply that a measure ϕ of I-Projects are financed by funds with I-Contracts, and that the remaining projects are financed by funds with C-Contracts.

The promised continuation value comes from future cash flows, which are determined by the steady state α_{ss}^I, α_{ss}^C, and the amount of I-Contracts and C-Contracts. Therefore, the LP solves:

$$\max_{\{\alpha_{ss}^I, \alpha_{ss}^C\}} \frac{\beta}{1-\beta} \left\{(1-\phi)p_C(1-\alpha_{ss}^C)\rho X_C + \phi p_I(1-\alpha_{ss}^I)\rho X_I\right\} - \frac{1}{1-\beta}(K+\phi z). \tag{20}$$

To solve this, we first observe that in any period in the steady state, the total future cash payments to all GPs that manage a fund now or later can be written as:

$$\frac{\beta}{1-\beta}[(1-\phi)p_C\alpha_{ss}^C\rho X_C + \phi p_I\alpha_{ss}^I\rho X_I]. \tag{21}$$

Therefore to maximize the LP's payoff is equivalent to minimizing this total payment to GPs. On the other hand, since all promised value must be paid in future, this total payment to GPs can be written alternatively as:

$$\mathcal{C} \equiv \frac{e}{1-\beta} + \frac{\beta}{1-\beta} r_{new} V_{GP}^{new} + V_{GP}^{total}, \tag{22}$$

where r_{new} is the steady state replacement rate for GPs. The first term is the total effort expense incurred from now on; the second term is the present value of future payoffs given to the future new entrant GPs; and the third term represents the present value to the operating funds in this period:

$$V_{GP}^{total} = \phi V_{GP}^{I} + (1-\phi) V_{GP}^{C} = \phi \left(\frac{e}{\Delta} + \beta V_f^I \right) + (1-\phi) \left(\frac{e}{\Delta} + \beta V_f^C \right). \tag{23}$$

Proposition 2 (**Equilibrium with Hierarchical Contracts**)
When the LP is patient, that is, $\beta \geq p_I(1+\Delta)$, the LP offers a measure ϕ of I-Contracts to GPs who are recently successful with terms:

1. $\alpha^I = \frac{(1-\beta^2 p_I)e}{\beta \Delta p_I \rho X_I}$;
2. *Renewal of the same contract upon project success and payoff X_I;*
3. *Continued funding under a C-Contract upon project failure.*

The LP offers a measure $1-\phi$ of C-Contracts. To be more specific, she offers a measure $(1-\phi)(1-(1+\Delta)p_C)$ of C-Contracts to new GPs, $(1-\phi)(1-\lambda)(1+\Delta)p_C$ to GPs who are recently successful under C-Contracts, and $\phi(1-(1+\Delta)p_I)$ to GPs who recently failed under I-Contracts, all with terms:

1. $\alpha^C = \frac{(1-\beta(1+\lambda\beta)p_C)e}{\beta \Delta p_C \rho X_C}$;
2. *Upon project success and payoff X_C, continued funding with an I-Contract with probability λ and renewal of the current C-Contract with probability $1-\lambda$;*
3. *Permanent termination of the current GP upon project failure;*

where $\lambda > 0$ *solves* $\lambda(1+\Delta)p_C(1-\phi) = [1-(1+\Delta)p_I]\phi$.

With hierarchical contracts, new GPs are offered C-Contracts with low agency rent and will be promoted to I-Contracts with high agency rent upon their success. GPs with I-Contract will be demoted to C-Contract when they fail, and will be terminated if they fail under the C-Contract. Intuitively, when the discount rate is high $\beta \geq p_I(1+\Delta)$, the LP cares about the replacement and future GP costs. To save the I-Contract replacement costs, instead of replacing those failed GPs, the LP downgrades them to operate C-Contracts, making I-Contracts more attractive to GPs. Promoting successful C-Contract GPs to the more attractive I-Contracts increases their promised continuation value upon success. Given no I-Contract GPs will be kicked out: this promotion and demotion feature redistributes the continuation values among GPs, providing extra incentive for GPs to exert effort.

Equilibrium with hierarchical contracts are the plausible because in reality, the probability for innovative projects to succeed under managerial effort is still very small, whereas the discount rate is unlikely to be lower than that. Otherwise, in the unlikely scenario where $p_I(1+\Delta) > \beta$ (unicorns are commonplace), we have a complete separation of contracts:

Proposition 3 (**Equilibrium with Parallel Contracts**)
When the LP is impatient, that is, $p_I(1+\Delta) > \beta$*, the LP offers a measure* $\phi(1+\Delta)p_I$ *of I-Contracts to GPs who are recently successful under I-Contracts and a measure* $\phi(1-(1+\Delta)p_I)$ *to new GPs, all with terms:*

1. $\alpha^I = \frac{(1-\beta p_I)e}{\beta\Delta p_I \rho X_I}$;
2. *Renewal of the same contract upon project success and payoff* X_I;
3. *Permanent termination of the current GP upon project failure.*

The LP offers a measure $1-\phi$ *of C-Contracts. To be more specific, she offers a measure* $(1-\phi)(1+\Delta)p_C$ *to GPs who are recently successful under C-Contracts and a measure* $(1-\phi)(1-(1+\Delta)p_C)$ *to new GPs, all with terms:*

1. $\alpha^C = \frac{(1-\beta p_C)e}{\beta\Delta p_C \rho X_C}$;

2. *Renewal of the same contract upon project success and payoff X_C;*

3. *Permanent termination of the current GP upon project failure.*

Intuitively, when the discount rate is relatively low $\beta < p_I(1+\Delta)$, the future replacement is not very costly for the LP. She prefers to replace the incumbent GPs regardless, suggesting that GPs are indifferent between I- and C-Contracts. In the equilibrium, I- and C-Contracts can be viewed as independent contracting problems and the optimal solution is similar to the fixed technology case.

4 Model Implications

4.1 Fund Performance

We clarify here that our notion of persistence refers to persistent dispersion across fund performance, which naturally implies that there would be persistent out-performance or under-performance relative to the VC industry. However, it is agnostic on the performance relative to other asset classes, such as the public equity index.

Luck-induced Persistence

In our model, luck has an enduring impact on fund performance. The conventional interpretation of fund performance persistence as the evidence to support differential managerial skill stems from the idea that luck is independent over time. However, our model shows that even though luck is not persistent, in the optimal contract the LP implicitly rewards successful funds by giving a higher continuation value, which can be interpreted as a higher probability of investing in a rookie GP's future fund and possibly better contract terms. One temporary experience of luck can have a persistent impact through the promised continuation value channel as the optimal way to address the agency problem.

The contract allocation channel alone may not affect the persistence of a fund's gross performance. Suppose the GP finances the same type of projects regardless of the last period outcome, then, conditional on the GP exerting effort, the expected fund gross return

is a constant across different funds and there would be no gross performance persistence. The endogenous deal flow affects gross performance persistence through its complementarity with endogenous fund heterogeneity. Under more favorable contract terms, the GP is more tolerant towards failure and is willing to nurture projects with I-Technology. Taking that into account, entrepreneurs with I-Projects will take funding offers from I-Contract GPs. The complementarity between capital and deal flows suggest that GPs with more favorable contract terms generate an expected project payoff $\beta(1+\Delta)p_I X_I - z > \beta(1+\Delta)p_C X_C$, yielding higher gross returns.

By Assumption 4, the LP's profit exhibits $\beta(1+\Delta)p_I(1-\alpha_I)\rho X_I - z > \beta(1+\Delta)p_C(1-\alpha_C)\rho X_C$, which implies that the net-of-fee return with a recently successful manager on average is higher than that with a new manager or recently failed manager. Net-of-fee performance is therefore also persistent; a phenomenon that occurs in general when the benefit of innovation is sufficiently large (such as in Assumption 4).

One critique for skill heterogeneity as an explanation for persistent performance is that the more skilled GPs can charge a higher fee, thus eroding superior returns to the LP. Our theory survives this critique because GPs do not have differential skills, and if they do seek to extract all the surplus, the LP can simply replace them by someone from the pool of aspiring GPs. They can only extract the difference between the LP's payoff hiring an incumbent vs an outsider.

Performance Forecasts and Mean Reversion

If innate skill in choosing promising companies or nurturing them were the only source for performance persistence, then the average performance of *other* VC funds investing in similar sorts of deals (investing in the same industry, sharing the same location, etc.) should have no predictive power on the focal GP's future performance. However, Nanda, Samila, and Sorenson (2017) use *other* funds' performance (in the same industry) as an instrument, and still finds strong predictive power. Our model is consistent with their findings if one considers industry, location or year as sources of common shock across funds.

That said, GPs may still differ in their ability to pick the right region or sector based on

signals beyond those that are easily publicly observed. One potential channel to generate the predictive power of other related VC fund performance is that GPs may have different skills in spotting macro trends. However, Nanda, Samila, and Sorenson (2017) find no evidence for inherent differences in the ability to forecast the trends. Instead, they find that success rates decline with experience, and initially under-performing GPs do better over time while those that initially outperform decline in the long run.

This finding is again consistent with our model, which predicts a long term mean-reverting process of VC fund performance. To see this, let R_I be the expected performance of an I-Contract fund and R_C be the expected performance of a C-Contract fund. In the equilibrium with hierarchical contracts, $R_I > R_C$ because under I-Contracts GPs execute projects more efficiently and they are endogenously matched with high quality projects. When an I-Contract fund succeeds at time t, the GP can raise another round of I-Contract fund and his time $t+1$ expected performance is R_I. However, since he may fail at time $t+1$ and will be demoted to C-Contract, his time $t+2$ expected performance is $(1+\Delta)p_I R_I + (1-(1+\Delta)p_I)R_C < R_I$. This result comes from the fact that not only does the initial luck have a persistent effect, but shocks in other periods will also have a persistent effect. In the long run, the positive enduring impact of initial luck will be gradually offset by the persistent effect of *i.i.d.* shocks in following periods if the contract hierarchy is not infinite.

While it is true that if more skilled GPs run larger funds or invest in later rounds (which due to a diminishing return of scale generate a decline in excess return), empirically one does not see unsuccessful GPs gradually moving to smaller funds or investing in earlier rounds. This means heterogeneity in GP skill cannot fully explain why initially under-performing VC funds do better over time.

4.2 Manager Compensation

Although the GPs are homogenous in skill, it naturally follows from Propositions 2 that in equilibrium GPs in funds with I-Contracts are persistently better compensated, even though they do not have superior skills relative to those at funds with C-Contracts: $V_{GP}^C = \frac{e}{\Delta}$,

$V_{GP}^I = \frac{e}{\Delta}(1+\beta)$. Moreover, the LP may let the GP continue whereas she has zero tolerance for failures for GPs in funds with C-Contracts, that is, $V_f^I > V_f^C = 0$.

Because $\alpha^I X^I > \alpha^C X^C$ and $\alpha^I p^I X^I > \alpha^C p^C X^C$, the GP under I-Contract gets paid more upon success and in expectation. Another way to see this is that V_s and V_f for I-Contracts are both higher than those for C-Contracts. The compensation jump for successful funds moving up the VC ladder (perhaps for investment in later rounds) is reminiscent of the extensive literature documenting wage jumps at promotion (e.g., Baker, Gibbs, and Holmstrom (1994b,a)). This compensation differential motivates effort in I-Contracts (contemporaneous incentive), as well as in C-Contracts (continuation incentive).

The fact that recently successful managers or funds get weakly higher compensation distinguishes our channel from Hochberg, Ljungqvist, and Vissing-Jorgensen (2014). In their setup, because investors can hold up the managers, they extract greater rents from the managers after getting information, which means even recently successful managers can receive lower compensation.[14]

We note that this compensation differential is reflected in a combination of carried interest and fees. When $p_I X_I$ is close to $p_C X_C$, or effort is proportional to the expected project payoff (proportional to $p_i X_i$), one can show $\alpha^I > \alpha^C$, implying that recently successful VCs have greater performance sensitivity. This is intuitive because continuation value is less of an incentive for the effort of recently successful firms. To the extent that the recently successful VCs are more experienced, this prediction is consistent with Gompers and Lerner (1999a), though not necessarily driven by reputational concerns.

Finally, we note that when the discount rate is high $\beta \geq P_I(1+\Delta)$ such that hierarchical contracts are used in equilibrium, the offer of I-Contracts exhibit an "incumbent bias" because they are only offered to GPs that the LP is already working with. This is driven by the fact that the LP has to pay rents to new GPs which does not help to incentivize those who are newly hired. This is analogous to the "insider bias" in hiring in firms (Oyer (2007); Huson, Malatesta, and Parrino (2004); Ke, Li, and Powell (2014)). Our model also points out that such an "incumbent bias" is less prominent when the LP (employer firm)

[14]Another channel is that when managers' compensation is higher, they can invest more in their own funds, mitigating agency problems. This is outside our model and constitutes an interesting future project.

cares less about the replacement and future GP costs, that is, $\beta < P_I(1+\Delta)$ (under parallel contracts).

4.3 Endogenous Fund Heterogeneity and Deal Flows

To focus on contracting between the LP and GPs, in this paper we abstract away from fund size and the model has no explicit fund flow predictions. However, our model generates implicit fund flow predictions through the continuation value. In the equilibrium, the LP implicitly rewards successful funds by giving a higher continuation value, which can be interpreted as a higher probability of investing in the GP's next fund and a higher probability of offering better contract terms. Investing in the GP's next fund suggests no fund outflow, and in the real world one way to offer a more favorable contract to the GP is to increase the fund size. This continued capital for funds of recently successful GPs is consistent with empirical findings in Kaplan and Schoar (2005). Note that our emphasis on the nature of capital (contract) differs from other studies on capital flow, such as Berk and Green (2004) and Pástor and Stambaugh (2012), and is particularly relevant for delegated investments where risk-tolerant capital and quality deal are complements.

Our model also predicts that the more innovative projects naturally flow to more innovative funds because either the more innovative entrepreneurs choose funds with I-Contracts, or funds with I-Contracts can select innovative projects. This is apparent from the assortative matching. This result is not an artifact of our binary technology or project type. Suppose we allow type θ EN to have the probability of θ of having I-Project. On the one hand, a type-θ entrepreneur would get $(1-\rho)[(1+\Delta)p(\theta X_I + (1-\theta)X_C)]$ under innovative nurturing, which is more than $(1-\rho)(1+\Delta)pX_C$ under conventional nurturing. Hence she strictly prefers funds under I-Contracts. On the other hand, a fund with I-Contract gets $\rho[(1+\Delta)p(\theta X_I + (1-\theta)X_C)]$ and therefore strictly prefers higher θ. That said, if the LP expects the fund to get a project of expected quality $\theta < \underline{\theta} \equiv \frac{z}{\beta\rho(1+\Delta)p(X_I - X_C)}$, she would rather use C-Contract.

Therefore, if the measure of funds with I-Contracts m_I satisfies $m_I \leq 1$, we have a unique matching equilibrium that is positive assortative, in the sense that more innovative projects

are matched with more innovative funds. All I-Contracts are matched, and the remaining projects are matched to C-Contracts. The average project quality for funds with I-Contracts is $\theta_h \equiv \int_{F^{-1}(1-m_I)}^{\overline{\theta}} \theta dF(\theta)$. However, if the measure of I-Contracts $m_I > 1$, some I-Contracts are also left unmatched.

More generally, we can allow for various types of I-Technology, T_1, T_2, etc. Type T_i augments the payoff to $X_I = T_i X_C$ and costs $\frac{zX_C^2}{(X_I - X_C)^2}(T_i - 1)^2$. The payoff on type θ is then $\theta T_i X_C + (1-\theta)X_C - \frac{IX_C^2}{(X_I - X_C)^2}(T_i - 1)^2$, which is super-modular in T_i and θ. Therefore, in equilibrium, it has to be positive assortative matching: more innovative technology matched with the more innovative entrepreneur.

One critique for differential manager skills as an explanation for performance persistence is that if the fund size grows and investment has diminishing returns to scale, we would expect superior performance to be eroded. However, with the endogenous deal flow, this effect is mitigated and it is possible that even when the fund size grows, an initially lucky fund can outperform for a long time.

4.4 Motivating Innovation

In our model, what motivates innovation is the nurturing technology – a reduced-form representation of VCs' capital constraints, fund-raising concerns, or career concerns (Gompers (1996); Gompers and Lerner (2000); Lee and Wahal (2004)). Rather than implicitly treating the VC fund's tolerance for failure as its characteristic, we describe the GP's endogenous attitude towards failure using equilibrium contracts reflecting a different level of capital constraints (the additional z) and fund-raising concerns (renewal policy), and accordingly attract different projects.

We find that top-performing funds are more tolerant towards experimentation and encourage greater innovation. It also follows naturally that the GP's tolerance increases following recent investment successes or contract improvements. There are two channels to make successful GPs more tolerant towards failure. One reason is that in our model, higher agency rent is associated with higher continuation value given project failure ($V_f^I = \frac{e}{\Delta} > 0 = V_f^C$).

This implies that:

$$V_s^I - V_f^I = \beta \frac{e}{\Delta} < \frac{e}{\Delta} = V_s^C - V_f^C. \tag{24}$$

So the difference between success and failure continuation value shrinks, making GPs less sensitive to the current fund's project type when considering future fund-raising. Similar to Manso (2011), less punishment towards failure motivates innovation. Another reason lies in the fact that the more favorable I-Contract places more emphasis on the cash payment. Due to incentive provision needs and anticipated deal flows, the LP endogenously gives greater immediate payoff to GPs under I-Contracts to nurture I-Projects, making them more tolerant towards I-Projects:

$$\beta(1+\Delta)p_I\alpha_I X_I > \beta(1+\Delta)p_C\alpha_C X_C. \tag{25}$$

The high cash payment motives GPs to implement I-Projects with high expected returns on I-Technology.

These finding are consistent with Tian and Wang (2014), who convincingly show that firms backed by more failure tolerant VC investors are significantly more innovative, and this result is not driven by other VC characteristics. Tian and Wang (2014) also document that VC's failure tolerance indeed increases following recent investment success or capital infusion. In addition, in an earlier version of this paper, we solved an OLG model with finitely-lived GPs and derived model implications that are corroborated by Tian and Wang (2014). Less experienced VC firms are less failure tolerant, suffer greater career concerns (see also Chevalier and Ellison (1999)), and are more influenced by capital infusion and recent success.

4.5 Financing Choice and Certification Effect

Additional important evidence supporting differential managerial skill is the entrepreneur's funding offer choice. Hsu (2004) shows that entrepreneurs are more likely to accept funding offers from lead VC funds even with less favorable terms. Consistent with Hsu (2004), our model predicts the preference of entrepreneurs for top VC funds. Given the fixed $\rho : 1-\rho$ contracts, an entrepreneur with I-Projects always prefers to take offers from I-Contract GPs

because those funds will nurture their projects in the most efficient innovative manner. Moreover, they will still prefer to take offers from I-Contract GPs with less favorable terms $\rho_I > \rho$ if:

$$(1+\Delta)p_I(1-\rho_I)X_I > (1+\Delta)p_C(1-\rho)X_C. \tag{26}$$

This prediction suggests that entrepreneurs may be willing to accept a smaller share if the GP can efficiently nurture the project, generating larger than expected project returns. There could also be externalities in innovative or quality projects, if we allow each fund to finance multiple projects, which could further strengthen our results.

The analysis above implicitly assumes that entrepreneurs know the type of projects they have, and that entrepreneurs with C-Projects would be indifferent among GPs. However, the endogenous deal flow in the equilibrium is robust if project quality is private information to the managers. Even if entrepreneurs may not know the quality of their projects, always accepting funding offers from top VC funds is a weakly dominating strategy. Moreover, this extension predicts a VC certification effect. Matching up with a successful fund is a strong signal for project quality, and the entrepreneur and/or outsiders will update their beliefs about the project accordingly. Differing from standard certification stories based on high quality managers, in this case top-performing funds have an endogenous certification effect even though managers do not have differential skills.

5 Discussion

5.1 Endogenous Heterogeneity and Investment Opportunity Set

Although we have focused on VC investment, it is worth noting that the insight of this paper could more generally apply to delegated investments with endogenous deal flows. For example, initial luck can also play a long-lasting role in buyout funds. While investments in public markets typically do not involve deal flows, our theory potentially applies to the fund of funds (FoFs) where star funds are matched to FoFs. To the extent that a recently successful FoF is not under pressure to outperform in the short-term and can commit capital

over the intermediate and long-term horizons, it can be better matched with star underlying funds so as to perpetuate its success.

We have also focused on the heterogeneous continuation contracts GPs get based on disparate performance, which is only one form of endogenous fund heterogeneity, and is not necessarily the most important form. More broadly, it could also be the opportunity to invest in later stage firms or to syndicate investment or growth in networks (Nanda, Samila, and Sorenson (2017); Hochberg, Ljungqvist, and Lu (2007); Venugopal and Yerramilli (2017)). It not only applies at fund level but also at partner level. For example, if a GP helps an entrepreneur to succeed, the latter often becomes an investor for a subsequent fund or acquires the GP's other portfolio firms. This gives VC firms that have become known for their past successes better access to future sought-after deals.

Furthermore, deal flows are just one manifestation of funds' differential investment opportunity sets. The essence of our theory is the complementarity between fund heterogeneity that gives some funds privileged positions, and the resulting differential investment opportunities. In that sense, if a hedge fund or a mutual fund's recent success leads to a superior investment opportunity set, performance persistence can also emerge. For example, a public equity hedge fund may have more profitable strategies that involve higher short-term risks (for example, due to limits of arbitrage, as seen in the case of LTCM). To the extent that a recently successful hedge fund is not concerned with panic withdrawal by investors, their investment opportunity set is enlarged and may potentially exhibit persistent superior performance, even though other hedge funds know similar strategies.

Our emphasis is on the role of luck rather than skill for the endogenous heterogeneity among GPs, because in our example, the entrepreneur can still choose to work with another fund. One could alternatively model the phenomenon on how there is an implicit understanding between entrepreneurs and GPs that successful entrepreneurs would bring better networks to their GPs in the future, which in turn provides incentives for the GPs to exert effort. It is only the social norm and the implicit agreement that binds the entrepreneur to help the GP that has previously invested in her startup. This is to be distinguished from an innate differential ability of those firms to select and help startups.

5.2 Amplification of Skill Differentials

Thus far, we have focused on the effort provision of GPs, and we assume no skill differential to clearly illustrate how capital and deal flows can generate performance persistence. Of course, we believe in real life that there may very well be a dispersion in manager skills and learning in relation to the different types of manager. Using a two-period simplification of the general setup, we illustrate how endogenous heterogeneity and deal flows still matter in such settings.

In each period, we model endogenous funding contracts that the LP offers to the GPs, and deal flows as the matching of projects to the funds. If the deal flow is absent or all funds have the same nurturing technology, projects are randomly matched to the funds. Instead of having the GPs provide effort, we deviate to assume that GPs have heterogeneous skills. To simplify exposition without compromising the intuition and results qualitatively, we keep $p_I = p_C = p$, and take $\beta = 1$, $\rho = 1$, and $\alpha = 1$. We also focus on the case of symmetric learning in order to avoid complicating key intuitions with signaling by managers.

Heterogeneous Skills

For simplicity, GPs are of two types: high-type GPs that can augment the success probability of projects to $1 + S$ and low-type GPs that do not augment the probability, where $(1 + S)p \leq 1$. The probability that a GP is a high-type is π_0 and let $S_0 = \pi_0 S$. In the first period, all funds look the same, the projects are randomly matched to GPs and the LP offers the same contracts to GPs.

Because ϕ is sufficiently small, the payoff for using I-Technology is lower than for using C-Technology in period $t = 0$. All funds get C-Contract, and the successful ones pay off X_C. Moreover, there is a measure of p projects that are successful, and all corresponding GPs are perceived to be of high-type with probability $\pi_{1s} = \pi_0 \frac{1+S}{1+S_0}$ through Bayesian inference. Failed funds are perceived to be of high-type with probability $\pi_{1f} = \pi_0 \frac{1-p(1+S)}{1-p(1+S_0)}$. Let $S_{1s} = \pi_{1s} S$ and $S_{1f} = \pi_{1f} S$, and the sufficient condition for the above to hold is simply a modified

Assumption 4:

$$\beta p\phi(1+S_{1s})(X_I - X_C) < z < \beta(1+S_0)(X_I - X_C) \qquad \textit{(Technology Selection).} \tag{27}$$

Now in period $t = 1$, without endogenous capital and deal flows, the LP gives C-Contracts to GPs. The expected performance of the recently successful fund is higher than that of a recently failed fund by:

$$D_o = \beta p X_C \left[\phi(1+S_{1s}) + (1-\phi)\right] - \beta p X_C \left[\phi(1+S_{1f}) + (1-\phi)\right] = \beta p X_C \phi(S_{1s} - S_{1f}). \tag{28}$$

Now if we allow endogenous deal flow, I-Projects rationally seek recently successful funds because the posterior on their managers' skill is higher. By Assumption 2, the measure of successful funds by the Law of Large Numbers is $p > \phi$. Therefore, the probability of each previously successful fund getting an I-Project is $\frac{\phi}{p}$, while previously failed GPs are only matched with C-Projects. The expected performance of the recently successful fund is higher than that of a recently failed fund by:

$$D_d = \beta p X_C \left[\frac{\phi}{p}(1+S_{1s}) + \frac{p-\phi}{p}\right] - \beta p X_C = \beta \phi S_{1s} X_C. \tag{29}$$

Now if we only allow endogenous future contracts, the LP rationally gives C-Contracts to recently failed funds, but gives I-Contracts to recently successful funds if it is profitable in expectation:

$$\beta p\phi X_I(1+S_{1s}) + pX_C(1-\phi) - z > \beta p X_C \left[\phi(1+S_{1s}) + 1 - \phi\right]. \tag{30}$$

Otherwise, the LP still only offers C-Contract, and we return to Equation (28). However, if a recently successful fund receives an I-Contract, its expected performance is higher than that of a recently failed fund by:

$$\begin{aligned} D_c &= \beta p\phi X_I(1+S_{1s}) + \beta p X_C(1-\phi) - z - \beta p X_C \left[\phi(1+S_{1f}) + 1 - \phi\right] \\ &= \beta p\phi \left[X_I(1+S_{1s}) - X_C(1+S_{1f})\right] - z. \end{aligned} \tag{31}$$

We note that for S big enough, D_c could be positive, indicating performance persistence with endogenous capital and contract alone.

Finally, if we endogenize both capital and deal flows, the LP gives ϕ measure of I-Contracts to recently successful funds because of the new variant of *Technology Selection* assumption in (27). Then, the expected performance of the recently successful fund is higher than that of a recently failed fund by:

$$\begin{aligned} D_{c\&d} &= \beta p X_I (1 + S_{1s}) \tfrac{\phi}{p} + \beta p X_C \tfrac{p-\phi}{p} - \tfrac{\phi}{p} z - \beta p X_C \\ &= \beta \phi [X_I (1 + S_{1s}) - X_C] - \tfrac{\phi}{p} z. \end{aligned} \tag{32}$$

We first note that $D_d > D_o$ obviously, and $D_c > D_o$ by (27). Both cash and deal flows amplify persistent performance dispersion. There is also a compounding amplification when both cash and deal flows are endogenous because $D_{c\&d} > \max\{D_c, D_d\}$ by (27). As we take the limit $S \to 0$, we have $D_c \to 0$, $D_d \to 0$, D_c becomes negative, but $D_{c\&d} = \beta\phi(X_I - X_C) - \frac{\phi}{p} z > 0$. Therefore, a very small dispersion in skills can lead to significant persistence and dispersion in performance, and when both cash and deal flows are endogenous, the amplification is one magnitude higher than that with either endogenous cash flow only or endogenous deal flow only.

A Behavioral Interpretation

The above analysis also applies when there is no real skill differential, but only the perception of it.[15] Suppose $S = 0$, but both the LP and ENs believe $S > 0$, then we still observe $D_{c\&d} > 0$. Alternatively, if the LP believes $S > 0$, and ENs know $S = 0$ but understand the LP's belief, then the ENs would anticipate the contract evolution and would rationally opt for the recently successful funds if they have I-Projects. The case where ENs believe $S > 0$ and the LP is rational is similar. Basically even if there is no skill differential, but either the LP or the ENs perceive luck as a superior skill, the endogenous capital and deal

[15] Nanda, Samila, and Sorenson (2017) also suggest that persistent performance appears to stem from initial differences in success creating beliefs about ability that persist as investors, entrepreneurs and others act on those beliefs.

flows can still generate performance persistence and predictable dispersion. The conclusion shares the spirit of Gompers, Kovner, Lerner, and Scharfstein (2010), but concerns perceived fund manager skills rather than perceived entrepreneur skills. We therefore conclude that endogenous capital and deal flows amplify or perpetuate initial luck under rather general settings.

5.3 Inter-contract Incentive Provision

Suppose now that there are no I-Projects. If the LP offers the optimal contract conditional on only C-Projects being available and makes zero profit, then she should be indifferent between investing in funds or not. As shown in the model solution, when both conventional and innovative projects present, the LP is strictly better off by investing in a non-zero measure set of conventional deals. The benefit comes from the fact that in the optimal contract, the LP can save motivating cost by redistributing continuation values among different GPs. The existence of another type of projects enables the LP to redistribute continuation values among different type of contracts, creating extra incentives for GPs to exert efforts. In the case of conventional deals, the optimal contract without I-Projects features $V_s^C = V_{GP}^C = \frac{e}{\Delta}$, while the optimal contract with I-Projects features $V_s^C = V_{GP}^I > \frac{e}{\Delta}$, suggesting a lower α_C and strictly positive profit for the C-fund investment. Moreover, it is straightforward to see that even if the LP loses money in fund investment when only C-Projects are available, she is willing to invest in conventional deals when both type of deals are available and the loss is dominated by the benefit of cost savings on motivating effort.

The benefits of using inter-contract incentives are clearly seen in the dynamic setup. We note that the LP's incentive compatibility conditions for participation and motivating effort are relaxed from Assumptions (5) and (3) in a single period, to $\beta(1+\Delta)\rho p_C X_C - \frac{1+\Delta}{\Delta} e[1-\beta p_C(1+\Delta)] - K \geq 0$ and $\beta\Delta\rho p_C X_C > \frac{1+\Delta}{\Delta} e[1-\beta p_C(1+\Delta)]$ in a dynamic setup. That means that the LP is funding more projects on the extensive margin, as well as using dynamic promotions and demotions to save agency costs.

By analyzing the LP's portfolio choice of contracts, we also link individual contracts to one another and to the aggregate market conditions. For example, when ϕ is higher and

the economy is more innovative, promotional likelihood for C contracts goes up but profit share α goes down. The payoff and evolution of each contract also depend on other funds' performance and contracts, which sharply contrasts with exogenously given outside options in the contract literature that focuses on individual contracts, and with the literature on tournaments where rewards are exogenously given.

5.4 Multiple LPs and Investor Competition

In the real world, instead of having one LP, there could be multiple LPs competing for funds and potential deals. We now extend the model to such a case. To have an interior analytical solution, we generalize our baseline setup.

Suppose type θ EN has probability of θ of having I-Project, and ENs type follows the uniform distribution $U[0,1]$. There are N LPs, and because there is in total a unit measure of projects, each LP can finance a measure $\frac{1}{N}$ of funds in any symmetric equilibrium. In the first stage of the game, LPs compete for I-Projects by determining what fraction of funds they finance are with I-Contract. In the second stage, given the I-Contract fund ratio decision, each of them will choose the optimal contracting strategy Ξ_i^* as discussed in the baseline model. For simplicity, we assume that both LP and GPs learn the true type of each EN after the matching. Every I-Contract fund that finances a C-Project will change its contract to C-Contract. We focus on the symmetric equilibrium with tiered contracts.

We solve the model by backward induction. Let w_i, $i = 1, \dots N$ be the fraction of I-Contract funds each LP finances, then the measure of all I-Contract funds becomes $\frac{\sum_{i=1}^N w_i}{N}$. Since LPs can always choose to finance all projects with C-Contract funds, the cost and benefit of I-Contract funds are both measured as their cost and benefit increments compared with C-Contract funds. Given Proposition 2, in the first stage, for each LP the average cost increment to issue a larger fraction of I-Contract is:

$$C_N \equiv \frac{z}{1-\beta} + [\beta\frac{e}{\Delta} - \frac{\beta}{1-\beta}\frac{e}{\Delta}(1-(1+\Delta)p_C)](1-\frac{\sum_{i=1}^N w_i}{2N}), \tag{33}$$

where $\frac{z}{1-\beta}$ is the discounted value of future operational costs, $\beta\frac{e}{\Delta}$ is the higher agency rent

assigned to the current GP, $\frac{\beta}{1-\beta}\frac{e}{\Delta}(1-(1+\Delta)p_C)$ is the discounted value of saved replacement costs because I-Contract does not kick out GP after failure, and $1-\frac{\sum_{i=1}^{N} w_i}{2N}$ is the probability of an I-Contract fund being matched with an I-Project given the uniform distribution and the fact that all ENs with $\theta \geq \frac{\sum_{i=1}^{N} w_i}{N}$ receive I-Contracts.

In the assortative matching, ENs with higher θ will be matched with I-Contract funds. When the LPs finance a larger fraction of funds with I-Contracts, ENs with lower θ types will be matched with I-Contract funds and the marginal benefit of I-Contract funds is decreasing. The average benefit increment of I-Contract funds is:

$$P_N \equiv \frac{\beta}{1-\beta}(1+\Delta)\rho(p_I X_I - p_C X_C)(1-\frac{\sum_{i=1}^{N} w_i}{2N}), \tag{34}$$

where $(1+\Delta)\rho(p_I X_I - p_C X_C)$ is the net benefit of nurturing an I-Project with I-Technology (compared with C-Technology), and $1-\frac{\sum_{i=1}^{N} w_i}{2N}$ is the probability of an I-Contract fund being matched with an I-Project given the uniform distribution and the fact that all ENs with $\theta \geq \frac{\sum_{i=1}^{N} w_i}{N}$ receive I-Contracts.

Now LP's I-Contract ratio decision in the first stage becomes a standard Cournot competition problem. One can interpret the average benefit increment of I-Contract funds as the I-Contract funds sale price and the marginal cost to issue a larger fraction of I-Contract funds as the constant production cost. Since both price and cost are linear functions in w_i, as in standard Cournot competition models, each LP's optimal I-Contract ratio can be solved as:

$$w_i = [1 - \frac{\frac{z}{1-\beta}}{\frac{\beta}{1-\beta}(1+\Delta)\rho(p_I X_I - p_C X_C) - [\beta\frac{e}{\Delta} - \frac{\beta}{1-\beta}\frac{e}{\Delta}(1-(1+\Delta)p_C)]}]\frac{2N}{N+1}. \tag{35}$$

It is straightforward to see that $w_i > 0$ and as N increases, the provision of I-Contract funds increases, and ENs with lower θ will be matched with I-Contract funds. That is to say, competition among fund investors encourages more I-Contract funds and more innovation.

6 Conclusion

We present a dynamic model of delegated investment that produces performance persistence and predictability without manager skill difference. Funding with terms conducive to innovation and quality projects exhibits strong complementarity, and endogenously flows to recently successful managers due to assortative matching and the incentivization of managerial effort with continuation value. The main intuition applies to other forms of endogenous fund heterogeneity that affects future opportunity sets, and to situations where small innate skill differentials are amplified. Consistent with empirical findings, our model predicts that venture funds that persistently outperform encourage greater innovation and attract quality projects with seemingly less favorable contract terms. The model further predicts an "incumbent bias" in allocating capital to funds, mean reversion of funds' long-term performance, backloading across contracts, and amplification of small skill differences.

Appendix

A Two-period Illustration

Here, let us examine a simplified setting from the general setup, with (1) two periods, $t = 0, 1$, and (2) no entry or exit of GPs. These simplifications allow us to focus on how exactly the endogenous capital and deal flows impact fund performance. For simplicity, we further assume $p_I = p_C = p$ and $\beta = 1$. Assumption 2 then implies that there are fewer I-Projects than there are successful projects (innovative or mediocre).

First consider the case with no capital (contract) or deal flows. Because projects are randomly assigned to the funds, the LP offers identical contracts (either C-Contract or I-Contract) to the funds. Assumption 4 ensures ϕ is sufficiently small that the LP finds it suboptimal to offer I-Contracts at an additional cost z. Success in $t = 0$ then is not correlated with success in $t = 1$, leading to no persistent performance. Moreover, the IC constraint for effort provision under C-Contract gives $\alpha_0^C = \frac{e}{\Delta p \rho X_C}$, therefore the net-of-fee return to LP in each period is $(1 - \alpha_t^C) p \rho X_C (1 + \Delta)/K = \frac{(1+\Delta)(\Delta p \rho X_C - e)}{K\Delta}$.

Now let us allow capital (contract) and deal flows in $t = 1$. Suppose the LP offers C-Contracts in $t = 0$ with $\alpha_0^C = \frac{e}{\Delta p \rho X_C}$, and then offers I-Contracts in $t = 1$ with probability ϕ/p to funds successful in $t = 0$ with $\alpha_1^I = \frac{e}{\Delta \rho p X_I}$, and C-Contracts with $\alpha_1^C = \frac{e}{\Delta p \rho X_C}$ to all remaining funds.

First we show that the GP's incentive compatibility for effort provision is always satisfied. For $t = 1$, because all GPs are homogeneous, the LP understands if she gives a GP an I-Contract, the GP can attract an I-Project. Therefore she optimally offers ϕ measure of I-Contracts and $1 - \phi$ measure of C-Contracts. The IC-constraints for effort provision and technology choice require:

$$(1 + \Delta) p X_i \rho \alpha_1^i - e \geq p X_i \rho \alpha_1^i, \qquad where \quad i \in \{I, C\} \tag{36}$$

$$(1 + \Delta) p X_I \rho \alpha_1^I \geq (1 + \Delta) p X_C \rho \alpha_1^I. \tag{37}$$

Note that condition (37) automatically holds, and funds with C-Contracts never deviate to I-Technology because the LP gives no provision z (and in equilibrium only C-Projects go to them anyway). Going back to $t = 0$, the IC condition for exerting effort again is satisfied:

$$(1 + \Delta) p (X_C \alpha_0^C \rho + p V_s) + [1 - (1 + \Delta) p] V_f - e \geq p X_C \alpha_0^C \rho + p V_s + (1 - p) V_f, \tag{38}$$

where $V_s = V_f = \frac{e}{\Delta}$ are continuation values of GPs upon success or failure in $t = 0$.[16]

We note the LP's total portfolio return is also higher: he gets $\frac{1+\Delta}{K\Delta}(p\rho\Delta(\phi X_I + (1 - \phi) X_C) - e - \phi z)$ in $t = 1$, which is higher than the case without capital (contract) and deal flows, by Assumption 4. Therefore, the LP is happy to take advantage of endogenous deal flows in assigning contracts. In fact, one can show that in equilibrium, the LP exactly offers the aforementioned contracts. When a fund is successful in $t = 0$, quality capital (with I-Contracts) flows to them with probability $\frac{\phi}{p}$. An I-Project benefits more from an I-Contract, and would only flow to (accept offers from) funds with I-Contracts. The complementarity drives capital and deal flows.

[16] In the full dynamic model with GP entry and exits, $V_s \neq V_f$ in general and the LP's motive for using continuation value (contract at $t = 1$) to motivate effort at $t = 0$ also drives contract allocations.

Performance Persistence

Importantly, contract renewal and deal flows lead to persistence in performance. If a fund fails in $t = 0$, it gets C-Contract in $t = 1$ and the LP's return in $t = 1$ is $\frac{1+\Delta}{K\Delta}(\Delta p\rho X_C - e)$. If a fund succeeds, it gets I-Contract with probability $\frac{\phi}{p}$, in which case the LP's net-of-fee return in $t = 1$ increases to $\frac{1+\Delta}{K\Delta}(\Delta p\rho X_I - e - z)$.

It is then apparent that in terms of both fund gross return and returns to investors, initially successful funds are expected to continue performing well subsequently, and initially failed funds are expected to continue performing less well subsequently. Without quality capital (I-Contract) flowing to some recently successful funds, even if I-Projects choose which funds to be matched with, they cannot gain additional benefits from C-Technology, and there would be no deal flow. Conversely, we would not observe differential contracts and performance persistence without deal flows. To see this, suppose projects are again randomly matched to funds in $t = 1$. Then rather than giving a C-Contract, giving an I-Contract to a fund yields the LP an additional $\phi(1 + \Delta)p\rho(X_I - X_C) - z < 0$, by Assumption 4. Therefore, the LP would not favor recently successful funds and there is no persistent dispersion in performance.

B Derivations and Proofs

B.1 Proof of Proposition 1

Proof. We know that payment to the GP has value $V_{GP}^C = \frac{e}{\Delta} + \beta V_f^C$. Suppose in the equilibrium contract V_f^C involves positive probability of continuation with a renewed contract (potentially of different α and renewal policy but same technology.), the LP is better off offering a similar contract but with $V_f^C = 0$ (termination) and the corresponding α based on equation (11), and giving the same continuation contract that would be given to the current GP in the original equilibrium to a new GP from the aspirant pool. In terms of the total payments to GPs, this costs the LP less because in equilibrium α is lower, and the stream of payments to the LP is higher than in the original equilibrium because GPs are still motivated to exert effort, and the LP gets a bigger share. So the original contract form cannot be optimal. Therefore, $V_f^C = 0$.

Next, we notice that if V_s^C does not involve continuation for sure, the LP has a profitable deviation: in the states that she terminates the GP and offers a replacement contract to a new GP from the pool under the original equilibrium, she can deviate to allow the current GP to continue with the same terms as in the replacement contract, for which the current GP is happy to accept. The LP can further divate by lowering α based on equation (11) because V_s^C is higher. The stream of payments to the LP is now higher while the effort motivation cost is still the same in each period. Therefore an optimal contract form involves continuation upon success for sure.

It remains to show that in equilibrium, we do not need different contracts with the same technology. Suppose we have a distribution of different C contracts, we note that all the payoffs are linear in α, so we can simply use the mean α to be our equilibrium contract and the payoffs would be equivalent. In other words, we can use one single type of C-Contract.

The case of using I-Contract alone in equilibrium follows the same logic.

□

B.2 Proof of Lemma 1

Proof. Instead of solving the optimal contract directly, we begin our analysis with characteristics of the optimal contract in a special scenario, and then show that this solution is also the optimal contract in our model.

Unlimited Supply of Zero Profit C-Projects

We first consider the optimal contract problem in a hypothetical setup: A measure M_I of entrepreneurs (EN) are born with I-Projects in each period, there are unlimited supply of entrepreneurs with C-Projects. Suppose the C-Projects' payoff satisfies something more stringent than Assumption 3,

$$\beta\Delta p_C \rho X_C - \frac{\Delta}{1+\Delta}K = (1-\beta p_C(1+\Delta))e. \tag{39}$$

So the LP will be indifferent to finance or abandon C-Projects under the contract characterized in proposition 1.

Result 1 (Essentially Unique Contract for Each Technology)

Suppose in equilibrium, the LP offers a measure m_C^1 of C-Contract $C^1 = \{\alpha^1, V_f^{C,1}, V_s^{C,1}\}$ and a measure m_C^2 of C-Contract $C^2 = \{\alpha_C^2, V_f^{C,2}, V_s^{C,2}\}$. When $V_{GP}^{C,1} = V_{GP}^{C,2}$, then those two types of contracts can be considered as a measure $m_C^1 + m_C^2$ of weighted contract $C = \frac{m_C^1}{m_C^1+m_C^2}C^1 + \frac{m_C^2}{m_C^1+m_C^2}C^2$, where any promised future contracts C^1 and C^2 are replaced by C. When $V_{GP}^{C,1} \neq V_{GP}^{C,2}$, without loss of generality, assume $V_{GP}^{C,1} < V_{GP}^{C,2}$, then the LP is better off replacing C^2 with C^1 and changing any promised future contracts C^2 to C^1. By construction, this is still a steady state. Similarly, in equilibrium the LP offers only one type of I-Contracts.

Result 2 (I-Contracts for I-Projects)

Since $\beta\Delta p_C \rho X_C = (\frac{1}{\beta} - p_C(1+\Delta))e$, the LP is indifferent to finance or abandon C-Projects under the contract characterized in proposition 1. Assumption 4 further suggests that the LP earns strictly positive profit by offering I-Projects a contract similar to the one in proposition 1. Now suppose in equilibrium $m_I < \phi$, then the LP can always offer a measure $M_I - m_I$ of I-Contract similar to the one in proposition 1, a contradiction.

Result 3

Suppose in equilibrium, there is a measure M_I of I-Contracts and a measure m_C^1 of C-Contracts C^1, then all I-Projects must be financed by I-Contracts, otherwise adding I-Contracts as in Proposition 1 would be a profitable deviation by the LP. Now we prove the lemma in two steps. We first argue that upon seeing failure (zero output) under C-Contract, the LP finds it suboptimal to let the GP continue with C-Contract. We then show that it would not be optimal to let the GP continue in with I-Contract either.

Suppose after a failure with C-Contract C^1, the probabilities of getting I- and C- contracts are $\pi_I \geq 0$ and $\pi_C > 0$. Then we have,

$$\begin{aligned} V_f^{C^1} &= (1-\pi_I-\pi_C)0 + \pi_I V_{GP}^I + \pi_C V_{GP}^{C^1} \\ V_{GP}^{C^1} &= \frac{e}{\Delta} + \beta V_f^{C^1} = \frac{e}{\Delta} + \pi_I \beta V_{GP}^I + \pi_C \beta V_{GP}^{C^1} \end{aligned}$$

which gives the total payment to GP under C-Contract when it is matched with a project:

$$V_{GP}^{C^1} + e = \frac{\frac{e}{\Delta} + \beta\pi_I V_{GP}^I}{1 - \pi_C\beta} + e > \frac{e}{\Delta} + e. \tag{40}$$

Consider an alternative scenario, where we add a measure $m_2^C > 0$ of C-Projects under the contract characterized in proposition 1, denoted by C^2. Given $\beta\Delta(p_C\rho X_C - \frac{K}{1+\Delta}) = (\frac{1}{\beta} - p_C(1+\Delta))e$ and proposition 1, this is still a steady state and the LP receives the same profit. Given result 1, we construct a measure $m_C^1 + m_C^2$ of weighted contract $C = \frac{m_C^1}{m_C^1+m_C^2}C^1 + \frac{m_C^2}{m_C^1+m_C^2}C^2$, where any promised future contracts C^1 and C^2 are replaced by C. By construction, this is still a steady state. Then we have,

$$\begin{aligned}
V_f^C &= (1 - \frac{m_C^1}{m_C^1+m_C^2}\pi_I - \frac{m_C^1}{m_C^1+m_C^2}\pi_C)0 + \frac{m_C^1}{m_C^1+m_C^2}\pi_I V_{GP}^I + \frac{m_C^1}{m_C^1+m_C^2}\pi_C V_{GP}^C \\
V_{GP}^C &= \frac{e}{\Delta} + \beta V_f^C = \frac{e}{\Delta} + \frac{m_C^1}{m_C^1+m_C^2}\pi_I\beta V_{GP}^I + \frac{m_C^1}{m_C^1+m_C^2}\pi_C\beta V_{GP}^C
\end{aligned}$$

which gives the total payment to GP under C-Contract when it is matched with a project:

$$\begin{aligned}
V_{GP}^C + e =& \frac{\frac{e}{\Delta} + \beta\frac{m_C^1}{m_C^1+m_C^2}\pi_I V_{GP}^I}{1 - \frac{m_C^1}{m_C^1+m_C^2}\pi_C\beta} + e \\
=& \frac{m_C^1}{m_C^1+m_C^2}(\frac{\frac{e}{\Delta} + \beta\pi_I V_{GP}^I}{1 - \frac{m_C^1}{m_C^1+m_C^2}\pi_C\beta} + e) + \frac{m_C^2}{m_C^1+m_C^2}(\frac{\frac{e}{\Delta}}{1 - \frac{m_C^1}{m_C^1+m_C^2}\pi_C\beta} + e) \\
<& \frac{m_C^1}{m_C^1+m_C^2}(\frac{\frac{e}{\Delta} + \beta\pi_I V_{GP}^I}{1 - \pi_C\beta} + e) + \frac{m_C^2}{m_C^1+m_C^2}(\frac{e}{\Delta} + e) \\
=& \frac{m_C^1}{m_C^1+m_C^2}(V_{GP}^{C^1} + e) + \frac{m_C^2}{m_C^1+m_C^2}(V_{GP}^{C^2} + e)
\end{aligned} \tag{41}$$

Also, since $V_{GP}^{C^2} = \frac{e}{\Delta} < V_{GP}^{C^1}$, and V_{GP}^I decreases given the contract V_{GP}^C. Thus compared to the contract C^1, the LP pays less agency costs under contract C and is strictly better off. The total cost paid to all the GPs under C contracts is

$$C = m_C\left[V_{GP}^C + e + \frac{\beta}{1-\beta}(1 - p_C(1+\Delta))(1 - \pi_C - \pi_I^{'})(V_{GP}^C + e)\right] \tag{42}$$

where $m_C = m_C^1 + m_C^2$, $\pi_I^{'} = \frac{m_C^1}{m_C^1+m_C^2}\pi_I$. Then $\frac{\partial C}{\partial \pi_C}$ is

$$\begin{aligned}
&\frac{\partial}{\partial \pi_C}\left[\frac{\frac{e}{\Delta}+\beta\pi_I^{'}V_{GP}^I}{1-\pi_C\beta}+\frac{\beta}{1-\beta}(1-p_C(1+\Delta))(1-\pi_C-\pi_I^{'})\left(\frac{\frac{e}{\Delta}+\beta\pi_I^{'}V_{GP}^I}{1-\pi_C\beta}\right)\right]\\
=&\beta\left(\frac{e}{\Delta}+\beta\pi_I^{'}V_{GP}^I\right)\left[\frac{1}{(1-\pi_C\beta)^2}+\frac{\beta}{1-\beta}(1-p_C(1+\Delta))\left(\frac{1-\pi_C-\pi_I^{'}}{(1-\pi_C\beta)^2}-\frac{1}{\beta(1-\pi_C\beta)}\right)\right]\\
=&\frac{\beta}{(1-\pi_C\beta)^2}\left(\frac{e}{\Delta}+\beta\pi_I^{'}V_{GP}^I\right)\left[1+\frac{\beta}{1-\beta}(1-p_C(1+\Delta))(1-\pi_C-\pi_I^{'}-(\frac{1}{\beta}-\pi_C))\right]\\
=&\frac{\beta}{(1-\pi_C\beta)^2}\left(\frac{e}{\Delta}+\beta\pi_I^{'}V_{GP}^I\right)\left[p_C(1+\Delta)-\frac{\beta}{1-\beta}\pi_I^{'}(1-p_C(1+\Delta))\right] \qquad (43)
\end{aligned}$$

which is positive for sufficiently small $\pi_I^{'}$. So unless $\pi_C = 0$, we can always find a large enough m_C^2 such that there is a better LP strategy. Therefore, $\pi_C = 0$ and $V_f^C = \pi_I V_{GP}^I$.

Now, suppose $\pi_I > 0$ in the stationary equilibrium, then there must be a corresponding measure $\pi_I m_C(1-(1+\Delta)p_C)$ of contemporaneous GPs under I-Contract who are not renewed with I-Contract. On the one hand, if these GPs are successful under I-Contract, consider the alternative I'- and $C^{'}$-contracts with all the terms intact except for $\pi_I' = \pi_I - \epsilon$ and V_s^I has $\frac{m_C}{M_I}\epsilon$ probability higher of renewing with I-Contracts, where $\epsilon > 0$ is an infinitesimal deviation. This would make the transition balanced and would still motivate efforts in both contracts yet reducing the motivation cost for C'-contracts. On the other hand, if successful GPs under I-Contracts are all renewed I-Contract, at most a measure $M_I(1-(1+\Delta)p_I)$ of GPs under I-Contract are not renewed I-Contract, because the LP always has the option to add more C-Projects, $M_I(1-(1+\Delta)p_I)$ can be less the measure of successful GPs under C-Contracts. This implies that the LP can use $C^{'}$-contracts with all the terms intact except for $\pi_I' = \pi_I - \epsilon$ and V_s^C has $\frac{\epsilon(1-\phi)}{\phi}$ probability higher of continuing with I-Contracts, which is again a profitable deviation. Therefore $\pi_I = 0$.

Result 4

Again, we prove it in two steps. First we show that upon failing under I-Contract, the agent would not be renewed with I-Contract. Now suppose V_f^I involves positive probability of continuing with I-Contracts, I-Projects are sufficiently scarce that we can instead let all GPs that should receive I-Contracts after their I-Projects failures change to C-Contracts, and since in equilibrium no I-Projects are financed by funds with C-Contracts, we can let some GPs who should be given C-Contracts after C-Project successes to receive I-Contracts instead. This swap is feasible given the fact that the LP can add sufficiently many C-Projects and $0 < \Delta < \frac{1-p_C}{p_C}$. The transition is still balanced, but we can still motivate efforts with reduced costs as V_f^I is reduced. Therefore, it cannot be the case that upon failure under I-Contract, the GP still continues with I-Contract.

Next, let $V_f^I = \pi_C V_{GP}^C$, we get $V_{GP}^I = (1+\beta\pi_C)\frac{e}{\Delta}$. Then $\frac{\partial C_I}{\partial \pi_C}$ can be written as

$$\begin{aligned}
&\beta\frac{e}{\Delta}+\frac{\beta}{1-\beta}\frac{\partial}{\partial \pi_C}\left[(1-p_I(1+\Delta))(1-\pi_C)\frac{e}{\Delta}\right]\\
=&\beta\frac{e}{\Delta}\left(1-\frac{1-(1+\Delta)p_I}{1-\beta}\right) \qquad (44)
\end{aligned}$$

which is positive if and only if $p_I(1+\Delta) > \beta$. When $p_I(1+\Delta) \leq \beta$, $\frac{\partial C_I}{\partial \pi_C} \leq 0$. In the equilibrium $\pi_C = 1$,

then $V_f^I = V_{GP}^C$ and $V_{GP}^I = (1+\beta)\frac{e}{\Delta}$. When $p_I(1+\Delta) > \beta$, $\frac{\partial C_I}{\partial \pi_C} > 0$. In the equilibrium $\pi_C = 0$, then $V_f^I = 0$ and $V_{GP}^I = \frac{e}{\Delta}$. It then follows $V_s^I = V_{GP}^I$.

In the equilibrium, the LP offers an arbitrary measure $m_C \geq m_C^* \equiv \frac{[1-(1+\Delta)p_I]M_I}{(1+\Delta)p_C}$ of C-Contracts. To be more specific, she offers a measure $m_C(1-(1+\Delta)p_C)$ of C-Contracts to new GPs, a measure of $m_C(1-\lambda)(1+\Delta)p_C$ of C-Contracts to GPs who are recently successful under C-Contracts, and a measure $m_I(1-(1+\Delta)p_I)$ of C-Contracts to GPs who recently failed under I-Contracts, where $\lambda = \frac{[1-(1+\Delta)p_I]M_I}{(1+\Delta)p_C m_C}$. The LP is indifferent for the choice of $m_C \geq m_C^* \equiv \frac{[1-(1+\Delta)p_I]M_I}{(1+\Delta)p_C}$ because for any m_C, the corresponding C-Contracts equal to a portfolio of a measure m_C^* of the above C-Contracts conditional on m_C^* and a measure $m_C - m_C^*$ of C-Contracts as in Proposition 1.

The General Case

Given the assumption 3, all project are profitable and the LP can at least finance projects with the contracts stated in proposition 1, suggesting that in any equilibrium all projects are financed. Conditional on all projects being financed, the optimal contracting problem is equivalent to a minimization problem of agency costs.

The optimal contract for each $m_C \geq m_C^*$ minimizes the associated agency costs given the financed project type ratio $\frac{m_C}{M_I}$, which is independent of the expected project payoff. Given assumption 2, for each ratio $\frac{1-\phi}{\phi}$, there is a corresponding $\frac{m_C}{M_I}$ and the equilibrium for the corresponding m_C in the unlimited supply of zero profit C-Projects case can be normalized to $m_C + M_I = 1$, which is the equilibrium for the project distribution ϕ. □

B.3 Proof of Proposition 2

Proof. Given assumption 2, lemma 1 determines the transition of GPs in equilibrium. Given the equilibrium transition of GPs, $V_f^C = 0$, so $V_{GP}^C = \frac{e}{\Delta} + \beta V_f^C = \frac{e}{\Delta}$. Similarly, $V_f^I = V_{GP}^C = \frac{e}{\Delta}$ and $V_{GP}^I = \frac{e}{\Delta} + \beta V_f^I = (1+\beta)\frac{e}{\Delta}$. Thus $V_s^I = V_{GP}^I$ and α^I is pinned down by equation 12. Similarly, α^C is determined by equation 10 and $V_s^C = (1-\lambda)V_{GP}^C + \lambda V_{GP}^I = (1+\lambda\beta)\frac{e}{\Delta}$. □

B.4 Proof of Proposition 3

Proof. Given lemma 1, the transition of GPs is straight forward and feasible. Given the equilibrium transition of GPs, $V_f^C = V_f^I = 0$, so $V_{GP}^C = \frac{e}{\Delta} + \beta V_f^C = \frac{e}{\Delta} = V_{GP}^I$. Thus $V_s^I = V_{GP}^I = V_s^C = V_{GP}^C$. α^C and α^I are determined by equation 10 and 12, respectively. □

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