Securitization, Ratings, and Credit Supply *

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Abstract

We illustrate a novel interaction between credit ratings and (endogenous) securitization and explore the implications for banks' lending standards and the supply of credit. In the model, banks first privately screen and originate loans and then issue securities that are backed by the loans cash flows. Issued securities are rated and sold to investors. We show that credit ratings increase the allocative efficiency of cash flows by reducing costly retention, but reduce lending standards and can lead too an oversupply of credit relative to first best. These findings are in contrast to regulators view of credit ratings as a disciplining device. We use the model to explore several commonly proposed policies and consider extensions to allow for rating shopping and manipulation. Provided investors are fully rational, both shopping and manipulation have effects similar to reducing the informativeness of ratings.

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1 Introduction

Asset-backed securitization is an important driver of credit supply (Loutskina and Strahan (2009) and Shivdasani and Wang (2011)). In recent years, we have witnessed growth in the securitization of many asset classes including mortgages, student loans, commercial loans, auto loans and credit card debt. In particular, the practice financed between 30% and 75% of these consumer lending markets, funding approximately 25% of outstanding US consumer credit (Gorton and Metrick (2012)). The expansion of securitized products has been facilitated by credit rating agencies (CRAs), which have allowed securitizers access to large pools of institutional investors that likely would not have participated in markets for unrated securities (Pagano and Volpin (2010)).

In the aftermath of the recent financial crisis, the role of both asset-backed securitization and rating agencies has been under intense scrutiny.[1] A variety of regulations have been proposed as an attempt to discipline these markets, some of which have already been implemented. For example, the Dodd Frank Act imposed a mandatory "skin in the game" rule on securitizers of assets, and it established disclosure requirements on rating agencies. Clearly, there are important interactions between the information content of ratings and banks' decisions of which loans to originate and securitize. Yet, surprisingly, the academic literature has little to say about these interactions. In this paper, we propose a stylized model to jointly study how securitization and ratings affect loan originators incentives at the loan screening stage. We explore the implications for lending standards and aggregate credit supply.

The model features a continuum of banks and a set of competitive investors. Each bank has access to a loan pool, and uses a screening technology to acquire private information about the quality of its pool. With this information, each bank decides whether to fund the loans – the *origination stage*. After origination, banks have an incentive to reallocate the cash flow rights from their loan pools to investors (e.g., due to capital constraints) and do so by selling securities backed by the loan pool in the secondary market – the *securitization stage*. At the latter stage, the bank has private information that hinders the efficient allocation of cash flow rights, which in turn distorts its incentives during the origination stage.

We explore two channels through which information can be conveyed to investors to mitigate these distortions. First, as in Leland and Pyle (1977), the bank can retain some fraction of the loan pool in order to signal its quality to investors. Second, information about the pool of loans underlying each security can be conveyed to investors through a public signal, which we interpret as a *rating*. In order to understand the role of each channel, we begin by analyzing a benchmark with neither as well as benchmarks with ratings and retention separately.

[1] See Dell'Ariccia et al. (2009), Keys et al. (2010), Jaffee et al. (2009), Mian and Sufi (2009), Agarwal et al. (2012) for how securitization negatively affected lending standards; and Pagano and Volpin (2010) and Benmelech and Dlugosz (2010) for the role, and failures, of CRAs in the securitization process.

We start by analyzing an originate-to-distribute model in which neither channel is present. That is, banks sell 100% of the loan pools that they originate without obtaining a rating. In this case, the market price for a loan pool in the secondary market is independent of loan quality, which (combined with zero retention) provides no incentive for banks to screen loans during the origination stage. Rather, banks are motivated purely by "volume lending." In the originate-to-distribute equilibrium, a banks originates its loan pool if its expected quality it at least equal to the secondary market price. Since the market price reflects average quality, the average NPV of loan pools originated in the economy is zero! Thus, the originate-to-distribute benchmark involves lending standards that are too low and an oversupply of credit. Moreover, since the secondary price for loans is low, the bank has an incentive to retain good loan pools on its balance sheet.

This motivates our analysis of a second benchmark where the bank optimally chooses how much of the loan pool cash flows to retain. In this second benchmark, the (least-cost) separating equilibrium is the unique outcome of the securitization stage. Banks retain a positive fraction if they originated a good pool and sell 100% of originated bad pools. By doing so, investors learn the quality of each loan sold on the secondary market and prices fully reflect all available information. However, because retention is costly, the bank does not realize the full social value of good loan pools, which leads to inefficiently high lending standards and an undersupply of credit.

We then introduce ratings to the model. After the retention decision, but prior to the sale of a security, a rating is publicly announced. We ask how the presence of ratings affect what loans are originated. One natural intuition is that informative ratings will lead to tighter lending standards and increase the quality of loans made. We confirm this intuition is correct in the originate-to-distribute environment (where the retention channel is not present). If banks securitize and sell all of the loans they originate regardless of loan quality or rating accuracy, then introducing ratings (or increasing their accuracy) improves the efficiency of loan origination.

The effect of ratings on lending standards and credit supply are more nuanced when the bank optimally chooses its retention level. When ratings are sufficiently informative, the bank relies on them for conveying information to investors and relies less on retention.[2] Since retention is costly and inefficient, ratings improves efficiency in the securitization stage, but because less is being retained, the introduction of ratings actually increases incentives to originate lower quality loans and may induce an *oversupply of credit*.

We highlight a novel and somewhat perverse interaction between ratings and the precision

[2]This result is consistent with empirical evidence that finds that increased third party certification, such as ratings or number of analysts, increases a firm's debt issuances, and sometimes equity issuances Faulkender and Petersen (2006), Sufi (2007), Derrien and Kecskés (2013).

of banks' screening technology. Without ratings, as the banks' screening technology becomes arbitrarily precise, the lending standard and supply of credit converge to first-best levels. With ratings, however, the lending standard collapses (to zero) as the banks' screening technology becomes arbitrarily precise and a non-negligible mass of bad loans are (deliberately) originated by banks.

We use our model to evaluate several different regulations. An intuitive and often proposed regulation is to require banks to retain a fraction of all originated loans. Proponents argue this will provide incentives for banks to make good loans by ensuring that they have some "skin in the game". Critics argue that such regulation may reduce the availability of financing. This trade-off is nicely captured within our framework. In addition, our model suggests a more subtle consideration in the evaluation of "skin in the game" regulation, which goes as follows. If banks use retention as a way to signal to investors then mandated retention will either reduce the informational content of retention or exacerbate its use as a signal of quality. Our model predicts that the latter case obtains and hence "skin in the game" leads to tighter lending standards and a reduction in credit supply. Productive efficiency may improve but ex-ante efficiency is strictly lower with a "skin in the game" requirement. We also investigate policies related to disclosure requirements, both for securitizers and for CRAs. These policies can be interpreted as attempts to increase rating informativeness. We show that such policies need not improve lending standards and can lead to an oversupply of credit, but do improve ex-ante efficiency.

A caveat to our findings pertaining to ex-ante efficiency is that our model does not include negative externalities associated with making bad loans (e.g., due to an increase in systemic risk). Bad loans have negative NPV, but otherwise impose no social costs to society. Therefore, all of our results pertaining to ex-ante efficiency hold provided any such negative externalities are sufficiently small. Perhaps more importantly, an argument for or against certain policies should hinge on the magnitude of these social costs relative to our measure.

There is an extensive literature that studies the strategic nature of CRAs and their incentives to provide unbiased information.[3] Inspired by the CRA models in Skreta and Veldkamp (2009), Sangiorgi and Spatt (2012), Bolton et al. (2012), and Opp et al. (2013), we consider two extensions of the model: rating shopping and rating manipulation.[4] First, we allow for rating shopping by assuming that the bank can pay for a given rating if it desires, or can choose to stay unrated. Second, we consider the possibility of rating manipulation by allowing the bank to make a

[3]Important considerations include Mathis et al. (2009); Bar-Isaac and Shapiro (2013); Fulghieri et al. (2014); Kashyap and Kovrijnykh (2016) who focus on the role of CRA reputation and moral hazard, Boot et al. (2006); Manso (2013) on feedback effects and ratings as coordination devices, and Opp et al. (2013); Josephson and Shapiro (2015) on the implications of rating-contingent regulation.

[4]These extensions are in line with empirical studies on rating shopping and manipulation: Ashcraft et al. (2011), Griffin and Tang (2011), Griffin et al. (2013), Becker and Milbourn (2011), He et al. (2011), Kraft (2015)

side-payment to the CRA to inflate its rating. In both cases, the information content of the rating becomes endogenous. We show that these frictions effectively reduce the information content of ratings and thus, have a similar effect to the comparative static of a reduction in the informativeness of (exogenously generated) ratings.

Several papers have highlighted the trade-off between productive and allocative efficiency studied in this paper. Parlour and Plantin (2008) study the effect of loan sales on banks' origination and on borrowers' capital structure decisions; while Malherbe (2012) explores the relation between risk-sharing post-origination and market discipline. Chemla and Hennessy (2014) explore a setting in which there is a moral hazard problem followed by a securitization decision. Absent regulation, they show that the incentive to exert effort is too low and an optimal policy to promote effort is forced retention. There is also a rich literature that focuses on optimal contracting with loan sales and moral hazard (Gorton and Pennacchi (1995), Hartman-Glaser et al. (2012), Vanasco (2017)). In these papers, investors do not have access to information about the assets being traded.

The approach adopted in this paper builds on our previous work. Daley and Green (2014) consider a signaling model in which receivers observe both the sender's costly signal as well as a stochastic grade that is correlated with the sender's type. We enrich this framework by incorporating an ex-ante (investment) stage where assets are originated, as in Vanasco (2017).

The remainder of the paper is organized as follows. In the next section, we introduce the model, our solution concept, and characterize the first-best benchmark. In Section 3, we present two benchmarks: an originate-to-distribute model, and a model where the bank optimally chooses what to sell in secondary markets in the absence of ratings. We present the complete model in Section 4 where we explore how the interaction of securitization decisions and ratings affects loan origination. In Section 5 we use our model to analyze the effect of several policies on retention levels and lending standards. Finally, in Section 6, we endogenize the information content of ratings by allowing for ratings shopping and manipulation. Section 7 concludes. All proofs are relegated to the Appendix.

2 The Model

There is a unit mass of loan originators, which we refer to as *banks*, and a competitive market of outside investors. There are two periods. In the first period, each bank makes two decisions: whether to originate a given pool of loans (the *Origination* Stage) and, if originated, what fraction of the loan pool to securitize and sell to the outside investors (the *Securitization* Stage)—what is not sold remains on the bank's balance sheet. In the second period, the state of the economy and the cash flows from the originated loans are realized. All agents are risk neutral.

Origination Stage. Each bank has access to one potential pool of loans. A loan pool requires one unit of capital to originate and generates a random future cash flow Y that depends on the state of economy, $\omega \in \{\mathbf{S}trong, \mathbf{W}eak\}$, and the pool's type, $t \in \{\mathbf{g}ood, \mathbf{b}ad\}$, which are independent. A *good* loan pool is expected to repay $1+\rho$ in both states of nature. In contrast, a *bad* loan pool is expected to repay $1+\rho$ in a *strong* economy, but only $\lambda\nu+(1-\lambda)(1+\rho) < 1$ if the economy is *weak*. One can interpret $\lambda \in (0,1)$ as the fraction of loans in a bad pool that default in a weak economy and $\nu < 1+\rho$ as the expected recovery given default. Let $\xi \in (0,1)$ denote the proportion of good pools in the economy, $\pi \in (0,1)$ be the probability that the economy is strong, and v_t be the expected repayment of a loan pool of type t. We assume $v_b < 1 < v_g$, meaning only good loan pools create value.

Prior to making origination decisions, banks acquire information about loan pools using their screening technology.[5] The screening technology is a pair of probability density functions, $\{\psi_b, \psi_g\}$, with common support. If a loan pool is of type t, then a bank observes a random variable drawn from ψ_t. Suppose that screening results in a realization s, then the bank's appraisal about its loan pool, denoted by p, is given by:

$$p = \Pr(t = good|s) = \frac{\xi\psi_g(s)}{\xi\psi_g(s) + (1-\xi)\psi_b(s)}. \tag{1}$$

As can be seen from above, the information content of s is fully captured by its likelihood ratio $L(s) = \psi_g(s)/\psi_b(s)$. We assume that L is a continuous random variable with support $[0,\infty)$.[6] Therefore, across the population of banks, appraisals p are distributed according to a cdf H, with density h, which is positive almost everywhere on $[0,1]$. Since there is a one-to-one match between banks and loan pools, each bank is indexed by its loan pool appraisal $p \in [0,1]$. That is, bank p refers to a bank who observes signal s satisfying (1) when it screens its loan pool. Define $A(p) \equiv \mathbb{E}[p'|p' \geq p]$, which is the average quality of loan pools whose appraisal is at least p.[7]

After observing the realization from the screening technology, each bank decides whether or not to originate the loans in its pool. If the bank chooses not to originate, it has no further actions and earns a payoff of 0. If the bank originates its loans, it has the opportunity to securitize the cash flows from the pool and sell them to the outside investors, which we turn to now.

Securitization Stage. Each originating bank has an incentive to raise cash through securitization of the cash flows from its loan pool. This need could arise for a variety of reasons (e.g.,

[5]Evidence of banks having the ability to acquire private information about borrowers can be found in Mikkelson and Partch (1986), Lummer and McConnell (1989), Slovin, Sushka, Polonchek (1993), Plantin (2009), Botsch and Vanasco (2016), among others.

[6]This assumption obtains if, for example, ψ_t is a normal pdf with mean m_t, $m_g \neq m_b$, and variance σ^2.

[7]If convenient, one could begin with the distribution of appraisals, H. From Kamenica and Gentzkow (2011), there always exists a screening technology that, paired with $\xi = E^H[p]$, endows this distribution of appraisals.

credit constraints, binding capital requirements, credit market imperfections combined with profitable investment opportunities). As in DeMarzo and Duffie (1999), we model this incentive in reduced form by assuming that banks discount second-period cash flows by a factor $\delta < 1$, while investors' discount factor is normalized to 1. Because banks are less patient than investors, fixing the origination decisions, the efficient allocation is for all loan cash flow rights to be transferred to investors.

During the securitization process, banks uncover additional information about the quality of its loan pool, which we capture as the bank learning the loan pool type t. For convenience, we focus on a simple securitization structure where banks choose the fraction of the cash flow rights to sell and retain the remaining fraction. Thus, if a bank chooses to sell a fraction $1-x$ then for any realization of the cash flow y, $(1-x)y$ and xy are the amounts distributed to investors and to the bank respectively in the second period. Choosing a higher x should therefore be interpreted as the bank retaining more, which can serve as a (costly) signal to investors about the quality of the underlying loans (as in Leland and Pyle (1977)).

Remark 1. *In principle, each bank could design and sell a security that is an arbitrary function of the cash flow Y. We study the relevant security design game in Daley et al. (2016b). Using the results therein, we demonstrate that the main insights of the present paper remain unchanged when we allow banks to design and sell arbitrary securities in Appendix B.*

Ratings. In addition to observing the level of costly retention (x), we consider a second channel through which information may be conveyed to investors, which we refer to as a *rating*. We start by modeling the rating as an exogenous public signal about the quality of the loan pool backing the security. That is, a rating is a publicly observable random variable R with type-dependent density function f_t on $\mathbb{R}$. In Section 6, we endogenize the information content of ratings by incorporating several additional features of the credit-rating industry.[8]

The informativeness of a rating realization, r, is captured by the likelihood ratio: $\Gamma(r) \equiv \frac{f_b(r)}{f_g(r)}$.[9] Without loss, order the ratings such that Γ is weakly decreasing. A higher rating therefore corresponds to a "better" signal about the quality of the underlying. We assume that ratings are informative, $E[\Gamma(R)|b] > E[\Gamma(R)|g]$, but boundedly so: $\inf_r \Gamma(r) > 0$ and $\sup_r \Gamma(r) < \infty$. To fix ideas, we will sometimes refer to a binary-symmetric rating system in which there are two ratings, G and B, with $\gamma = \Pr(G|g) = \Pr(B|b) \in (\frac{1}{2}, 1)$, where higher γ corresponds to more informative ratings.

[8] To encompasses a situation with a countable set of ratings $\{y_1, y_2, \dots\}$, with probabilities $q_t(y_n)$, let $f_t(r) = q_t(y_n)$ for $r \in [n, n+1)$ and $f_t(r) = 0$ for all other $r \in \mathbb{R}$.

[9] If $f_g(r) = f_b(r) = 0$, we adopt the convention that $\Gamma(r) = 1$.

2.1 Preliminaries

It useful to cover some preliminary features that must hold in any Perfect Bayesian Equilibrium (PBE) of the model. As is typical, analysis starts in the second (i.e., Securitization) stage and works backward.

At the beginning of the Securitization stage, investors have a (common) prior belief μ_0 about the quality of the loan pool backing each security. Investors then update their belief about a given security based on observing both the bank's retention level x and the rating r to some final belief $\mu_f(x, r)$. This updating can be decomposed into a first update (based on x) and a second update (based on r). The first update results in an *interim belief*, $\mu(x)$. Along the equilibrium path, the interim belief must be consistent with the retention strategy of banks.[10]

The second update is purely statistical; investors update from their interim belief to a final belief based on the rating according to Bayes rule:

$$\mu_f(x, r) = \frac{\mu(x) f_g(r)}{\mu(x) f_g(r) + (1 - \mu(x)) f_b(r)} = \frac{\mu(x)}{\mu(x) + (1 - \mu(x))\Gamma(r)}. \tag{2}$$

Let $P(x, r)$ denote the price of a security as a function of the retention level chosen by the bank and rating. Since investors are risk-neutral and competitive, the price equals the expected value of the cash flows generated by the security given the final investor belief:

$$P(x, r) = \mathbb{E}[(1 - x)Y | x, r] = (1 - x)\big(\mu_f(x, r) v_g + (1 - \mu_f(x, r)) v_b\big). \tag{3}$$

Given a schedule of interim beliefs, the expected payoff of a bank that has originated a type-t pool and then chooses retention level x is $u_t(x, \mu(x)) \equiv \mathbb{E}_R[P(x, R)|t] + \delta x v_t$. Equilibrium requires that banks select a retention level that maximizes u_t taking the belief schedule as given. Let u_t^* denote the equilibrium payoff of type t in the continuation game starting from the Securitization stage.

Moving the analysis back to the beginning of the game, there are two critical links between the two stages. First, given continuation payoffs u_g^*, u_b^*, banks optimally choose whether to originate their loan pools given their appraisal, where origination yields an expected profit of $p u_g^* + (1 - p) u_b^* - 1$ compared to 0 for not originating. Let O^* be the set of loan pools originated. Second, investors' prior belief in the Securitization stage must be consistent with the banks decisions to originate a pool with a given appraisal level p at the Origination Stage. Since investors are not privy to the initial p for each bank, their belief about the likelihood that any originated loan pool is good is simply $\mu_0 = E[p | p \in O^*]$.

[10] A pure strategy for a bank is a type-dependent retention level, and a mixed strategy is a type-dependent probability distribution over retention levels.

The Lending Standard. Intuitively, because good pools generate higher returns and better ratings, $u_g^* > u_b^*$ in any PBE. This implies that the origination decision takes a cutoff form, where bank p originates if and only if $p \geq p^*$. We refer to p^* as the equilibrium *lending standard*. To avoid the technicalities associated with corner solutions, the following assumption guarantees that the lending standard is always interior (as documented in Lemma 1 below).

Assumption 1. *We assume that* $\xi v_g + (1-\xi)v_b < 1 < \delta v_g$.

Substantively, the first inequality says that banks have ample access to low quality loans in the aggregate. Hence, if all loan pools were originated, their aggregate NPV would be negative. While banks have a higher discount rate than investors, the second inequality says that banks are patient enough that holding a good loan generates positive NPV for them, $\delta v_g > 1$.

Lemma 1. *In any PBE, the set of originated loan pools is a truncation,* $O^* = [p^*, 1]$*, where*

$$p^* = \frac{1 - u_b^*}{u_g^* - u_b^*} \in (0, 1). \tag{4}$$

An immediate corollary is that investors' prior beliefs in the Securitization stage simply condition on the loan pool appraisal p being above the lending standard p^*. That is, $\mu_0 = A(p^*)$.

Collecting these preliminaries, we have the following explicit connection between equilibrium behavior and beliefs across the two stages.

Corollary 1. *Any PBE of the model is characterized by the following.*

1. *In the Securitization stage: Given* μ_0*, for each originated loan pool, bank retention strategies, investor beliefs, and security prices comprise a PBE of the signaling game.*

2. *In the Origination stage: Given the continuation payoffs implied by the Securitization stage,* (u_g^*, u_b^*)*, the lending standard is* p^* *as given by (4).*

3. *Belief Consistency:* $\mu_0 = A(p^*)$.

Finally, as is typical in signaling games, the Securitization stage has multiple PBEs due to the flexibility of beliefs off the equilibrium path. To handle this multiplicity, we employ the D1 refinement (Banks and Sobel, 1987; Cho and Kreps, 1987). Roughly, D1 requires investors to attribute the offer of an off-path retention level to the type who is more likely to gain from this offer relative to equilibrium payoffs. See Appendix A.1 for a formal definition. Hereafter, we use *equilibrium* to refer to a PBE that satisfies D1 in the Securitization stage.

3 Benchmarks

3.1 Full-Information/First-Best

If type were publicly observable in the Securitization stage, there would be no need for banks to retain any of their cash flow rights, and full *allocative* efficiency would be achieved: $x_b^{FB} = x_g^{FB} = 0$. In addition, prices would perfectly reflect underlying value, so $u_t^* = v_t$. Moving back to the Origination stage, *productive* efficiency is also achieved as loan pools are originated if and only if they generate positive NPV, that is, $pv_g + (1-p)v_b - 1 \geq 0$. Hence, the first-best lending standard is

$$p^{FB} = \frac{1 - v_b}{v_g - v_b} \in (0, 1),$$

and the first-best total *supply of credit* is therefore $H(p^{FB})$.

Remark 2. *This measure of the first-best lending standard, p^{FB}, implicitly assumes that banks capture all the surplus from originated loans. If there is positive surplus to loan recipients, then the socially optimal lending standard, p^{SO}, would be lower than p^{FB}. Assuming bad loan pools still generate a negative expected social NPV, it is straightforward to adapt our analysis to compare to this benchmark. In particular, the settings under which there is an oversupply of credit relative to the social optimal are qualitatively similar to those with an oversupply relative to the full-information benchmark (see Proposition 4).*[11]

3.2 Originate-to-Distribute (OTD)

Suppose that banks are forced to sell 100% of the loan pools they originate. In this case, and perhaps in line with the popular intuition, (i) the lending standard is too lax compared to the first-best benchmark, leading to an oversupply of credit relative to first-best, and (ii) more informative ratings work to ameliorate (i).

To illustrate these findings, notice that without any retention decision, the price in the Securitization stage is based only on the rating-updated investor belief, $P(r) = \mu_f(r)v_g + (1-\mu_f(r))v_b$, and therefore continuation payoffs are

$$u_t^* = \mathbb{E}_R[\mu_f(R)v_g + (1-\mu_f(R))v_b|t] = \mathbb{E}_R[\mu_f(R)|t](v_g - v_b) + v_b. \tag{5}$$

Now, if we take the limit to uninformative ratings (e.g., as $\gamma \to \frac{1}{2}$ for binary-symmetric ratings), then $\mathbb{E}_R[\mu_f(R)|t] \to \mu_0 = A(p^*)$ for either type. Hence, any funded loan pool garners a price

[11] If even bad loans generate positive social value, then it is socially optimal to originate all loans, and oversupply is, trivially, impossible.

reflecting the average cash flow of all funded loans. In equilibrium, loan pools will be funded up until the average gross return is equal to the cost of funding:

$$A(p^*)v_g + (1 - A(p^*))v_b - 1 = 0.$$

Thus, in a OTD setting without ratings, the lending standard is set such that the *average* funded loan pool generates zero NPV, whereas efficiency requires the *marginal* funded loan pool to generate zero NPV. For any lending standard, p^*, the average qualifying loan pool is of higher quality than the marginal one, hence the lending standard is too lax:

$$A(p^*)v_g + (1 - A(p^*))v_b = 1 \implies p^*v_g + (1 - p^*)v_b < 1 \implies p^* < p^{FB}.$$

As the informativeness of ratings increase (e.g., as γ increases for binary/symmetric ratings), u_g^* increases and u_b^* decreases, leading to an increase in the lending standard, p^*, and a decrease in credit supply. As ratings become perfectly informative (e.g., as $\gamma \to 1$), $u_g^* \to v_g$ and $u_b^* \to v_b$, as they are in the first-best benchmark. Hence, in the limit, $p^* \to p^{FB}$, but there is always an oversupply of credit if ratings are short of perfectly informative.

3.3 Strategic Model without Ratings

Consider now the model as described in Section 2, but without informative ratings.[12] In this case, originators of good pools inefficiently retain a portion of their cash flows to signal their quality. This misallocation depresses the value of origination, leading to a lending standard that is too stringent compared to the first-best benchmark, resulting in an undersupply of credit relative to the first-best.

To illustrate, define x^{LC} as the unique solution to

$$\underbrace{u_b(0,0)}_{v_b} \quad = \quad \underbrace{u_b(x^{LC},1).}_{(1-x^{LC})v_g+\delta x^{LC}v_b} \tag{6}$$

That is, the originator of a b-pool is indifferent between efficiently selling all of its cash flow rights at price v_b, and retaining fraction x^{LC} if doing so leads to a price of v_g for the complementary fraction it sells. Therefore, x^{LC} is the minimum amount the g-type must retain to separate from the b-type in the Securitization stage. As seen in similar signaling games, D1 selects this "least-cost separating" equilibrium (LCSE).

[12] That is, $\Gamma(r) = 1$ for all $r \in \mathbb{R}$.

Proposition 1. *Without informative ratings, in any equilibrium, retention levels in the Securitization stage are* $x_b = 0$ *and* $x_g = x^{LC}$*. Hence,* $u_b^* = v_b$ *and* $u_g^* = (1 - x^{LC})v_g + \delta x^{LC} v_g < v_g$*.*

Having established that $u_b^* = v_b$ and $u_g^* < v_g$, it immediately follows from Lemma 1 that without ratings the equilibrium lending standard is higher than in the first-best benchmark:

$$p^* = \frac{1 - u_b^*}{u_g^* - u_b^*} = \frac{1 - v_b}{u_g^* - v_b} > \frac{1 - v_b}{v_g - v_b} = p^{FB}$$

Hence, there are socially positive NPV loans that are not being funded in this economy.

Corollary 2. *Without informative ratings, the equilibrium lending standard is higher than in the first-best benchmark.*

A final point to highlight is that without ratings, banks perfectly convey the quality of their underlying loans to investors through their retention decisions. Thus, there is a perfect correlation between retention levels, prices, and underlying loan pool quality. We will see that this feature does not hold with informative ratings.

4 Equilibrium

We now turn to the equilibrium of the full model in which banks strategically decide on retention/securitization and their issued securities receive an informative rating, modeled as the random variable R.[13] Again, we first characterize the equilibrium of the Securitization stage for any investor belief, μ_0 (Section 4.1), and then characterize banks' lending standard in the Origination stage along with the consistent investor belief (Section 4.2). The section concludes with comparative statics on the precision of the banks screening technology and the rating informativeness. Section 5 investigates policy implications.

4.1 Securitization Stage

Investors can potentially learn about the quality of a bank's pool from both the bank's securitization decision as well as from its rating. Intuitively, an originator of a g-pool would like to use both channels optimally. To this end, consider the following maximization problem:

$$\max_{x,\mu} \quad u_g(x, \mu) \quad s.t. \quad u_b(x, \mu) = v_b. \tag{7}$$

[13] In Section 6, we endogenize the information content of ratings by allowing for ratings shopping and manipulation.

That is, given the ratings systems, among all interim-belief/retention-level pairs that deliver the b-type its full-information expected payoff, which one delivers the g-type its highest expected payoff? In Appendix A.4, Lemma 6, we show that this problem has a unique solution, which we denote $(\tilde{x}, \tilde{\mu})$ and can be thought of as a bank with a g-pool making optimal use of the two channels at its disposal, subject to giving the bank with a b-pool its full information payoff. This optimality is a critical part of the equilibrium characterization (and where the D1 refinement plays its role), as Proposition 2 formalizes. However, we first characterize when the solution to (7) is interior.

Without ratings, the solution to (7) is $(\tilde{x}, \tilde{\mu}) = (x^{LC}, 1)$. That is, if there are no ratings to rely on, the g-type uses retention to fully establish the superior quality of its cash flows. Add now informative ratings. If the retention-level/interim-belief is $(x^{LC}, 1)$, then this addition has no effect because investors are completely convinced that $t = g$ even without the rating. Hence, for a g-type to rely on the rating at all, it must have an interim belief below 1. Banks will choose to rely on the rating when the latter are sufficiently informative, as precisely captured by the following lemma.

Lemma 2. *In the solution to* (7)*,* $(\tilde{x}, \tilde{\mu}) < (x^{LC}, 1)$ *if and only if* $E[\Gamma(R)|b] > \frac{v_g - \delta v_b}{(1-\delta)v_g}$.

Recall that the informativeness of a rating *realization*, r, is captured by its likelihood ratio: $\Gamma(r) = \frac{f_b(r)}{f_g(r)}$. $E[\Gamma(R)|b]$ is a measure of the informativeness of the rating *system*, $\{f_g, f_b\}$.[14] The right-hand side of the inequality in Lemma 2 is the ratio of the marginal cost of retaining cash flows for a bank with a b-type pool relative to that of a bank with a g-type pool, in a separating equilibrium. Thus, the solution to (7) has $(\tilde{x}, \tilde{\mu})$ below their LSCE values if and only if ratings are informative enough relative to the cost advantage of the g-type to use retention as a signaling device.

Given Lemma 2, it is perhaps not surprising that if $E[\Gamma(R)|b] \leq \frac{v_g - \delta v_b}{(1-\delta)v_g}$, then ratings are simply too noisy to alter the prediction from the no-ratings benchmark studied in Section 3.3. For the remainder, we analyze the model in which ratings are informative enough to impact the equilibrium outcome: that is, henceforth we assume $E[\Gamma(R)|b] > \frac{v_g - \delta v_b}{(1-\delta)v_g}$, unless otherwise stated. The equilibrium is then characterized as follows.

Proposition 2. *For any* $\mu_0 \neq \tilde{\mu}$*, there is a unique equilibrium of the Securitization stage. In it*

- *If* $\mu_0 < \tilde{\mu}$*, there is partial pooling at* $\tilde{x} < x^{LC}$*. That is, all banks with g-type pools retain* $x = \tilde{x}$*, a fraction* $\frac{\mu_0(1-\tilde{\mu})}{(1-\mu_0)\tilde{\mu}}$ *of banks with b-type pools retain* $x = \tilde{x}$*, and a fraction* $\frac{\tilde{\mu}-\mu_0}{(1-\mu_0)\tilde{\mu}}$ *retain* $x = 0$*. Hence, the interim belief for* $x = \tilde{x}$ *is* $\mu(\tilde{x}) = \tilde{\mu}$*.*

[14] The more informative the rating system, the higher is $E[\Gamma(r)|b]$. This measure is consistent with the notion of informativeness introduced by Blackwell (1951): if one rating system is Blackwell more informative than another, then $E[\Gamma(R)|b]$ is higher under the more informative system. Note that $E[\Gamma(r)|b] \geq E[\Gamma(r)|g] = 1$ for any rating system.

- *If $\mu_0 > \tilde{\mu}$, there is full pooling at $x = 0$. That is, all banks retain zero, regardless of type.*

For $\mu_0 = \tilde{\mu}$, there is full pooling in equilibrium, but it can be at any $x \in [0, \tilde{x}]$.

The proposition shows that, with informative ratings, banks with g-pools need not signal as vigorously to convey the quality of their security. Instead, they rely (to some extent) on the rating to convey information to investors. When investors are sufficiently optimistic ($\mu_0 > \tilde{\mu}$), there is full reliance on the rating. Otherwise, when $\mu_0 < \tilde{\mu}$, banks rely partially on retention and partially on the rating. That is, banks retain enough of g-backed pools to induce an interim belief of $\tilde{\mu}$ and rely on the rating beyond that.

An important implication of Proposition 2 is that ratings increase allocative efficiency by reducing the inefficient retention of g-type pools, since $\tilde{x} < x^{LC}$. Since without ratings the equilibrium is separating and fully reveals the type of loan pool backing each security, the increase in allocative efficiency occurs at the expense of reducing the total amount of information revealed to investors.

4.2 Origination Stage

Having characterized the Securitization stage, we now analyze the Origination stage. This analysis has two components: (i) optimality of the banks' lending standard to originate loan pools given investor beliefs and (ii) consistency of the investor beliefs with banks' origination decisions.

Optimal Origination. Recall that given expected payoffs in the Securitization stage of u_g^*, u_b^*, a bank (weakly) prefers to originate if and only if $pu_g^* + (1-p)u_b^* - 1 \geq 0$, or equivalently $p \geq \frac{1-u_b^*}{u_g^*-u_b^*}$. From Proposition 2, u_g^*, u_b^* vary with the investors' belief μ_0 when informative ratings are present—in contrast to the first-best benchmark (Section 3.1) and the model without ratings (Section 3.3). It is therefore useful to define the banks' reaction function as the marginal loan pool a bank is willing to originate (i.e., the lending standard) given investors' beliefs μ_0:

Definition 1. $\Psi(\mu_0) \equiv \left\{ \max\left\{ \frac{1-u_b^*}{u_g^*-u_b^*}, 0 \right\} \;\middle|\; u_g^*, u_b^* \text{ are equilibrium payoffs given } \mu_0 \right\}$.

The max operator in Ψ accounts for the fact that if $\frac{1-u_b^*}{u_g^*-u_b^*} < 0$, then banks will originate all loan pools, which is equivalent to setting the lending standard to 0. Next, from Proposition 2 we have that Ψ is single-valued for all $\mu_0 \neq \tilde{\mu}$. In more detail:

Corollary 3. *For given investor belief μ_0, the equilibrium lending standard with ratings satisfies*

$$p^* \in \Psi(\mu_0) = \begin{cases} \frac{1-v_b}{u_g(\tilde{x},\tilde{\mu})-v_b} & \mu_0 < \tilde{\mu} \\ \left\{ \frac{1-u_b(x,\tilde{\mu})}{u_g(x,\tilde{\mu})-u_b(x,\tilde{\mu})} \mid x \in [0,\tilde{x}] \right\} & \mu_0 = \tilde{\mu} \\ \max\left\{ \frac{1-u_b(0,\mu_0)}{u_g(0,\mu_0)-u_b(0,\mu_0)}, 0 \right\} & \mu_0 > \tilde{\mu}. \end{cases}$$

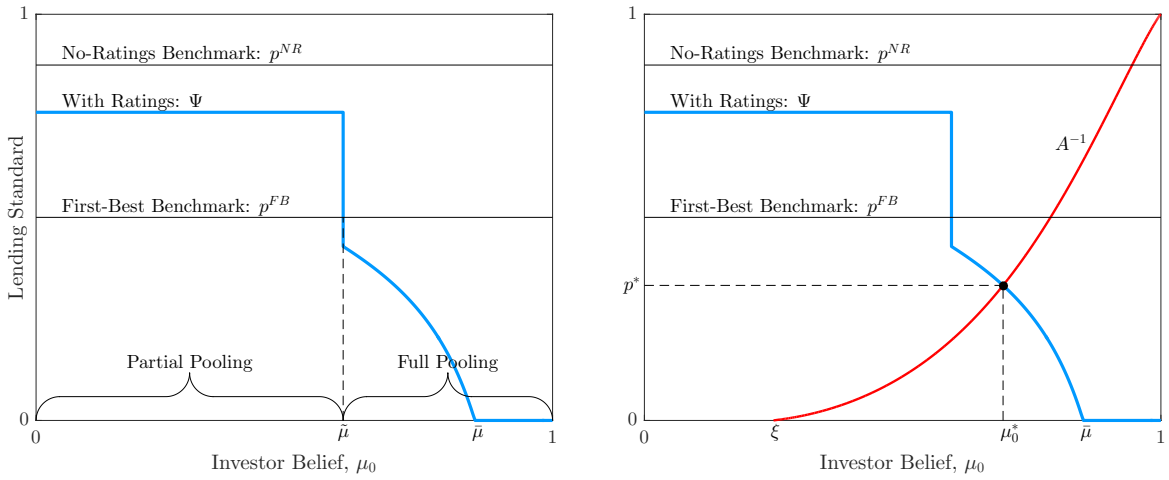


FIGURE 1: Panel (a) illustrates bank's lending as a function of investors' beliefs (Ψ), as well as the lending standard in the First-Best (p^{FB}) and No-Ratings (p^{NR}) benchmarks. Panel (b) incorporates the belief consistency line (A^{-1}), and illustrates the equilibrium lending standard and investor belief (p^*, μ_0^*).

Figure 1(a) illustrates Ψ, and compares it to the lending standard in the first-best and no-ratings benchmarks, labeled p^{FB} and p^{NR}, respectively. In these two benchmarks, payoffs in the Securitization stage do not depend on investors' prior beliefs, so the lending standards are independent of μ_0. Furthermore, $p^{FB} < p^{NR}$, as documented in Corollary 2.

With ratings, the lending standard adopted by banks depends on the investor belief. When investors are pessimistic about loan pool quality, $\mu_0 < \tilde{\mu}$, the bad type earns it full-information payoff ($u_b^* = v_b$, just as in the two benchmarks), and a g-type optimally relies on both retention and the rating to earn a payoff higher than in the LCSE but still below its full-information payoff ($u_g(x^{LC}, 1) < u_g^* < v_g$). Hence, the lending standard with ratings falls in between the two benchmarks ($p^{FB} < \Psi(\mu_0) < p^{NR}$, for $\mu_0 < \tilde{\mu}$).

However, when investors are optimistic about loan pool quality, $\mu_0 > \tilde{\mu}$, banks eschew inefficient retention, which increases the payoff of both types. Hence, origination is more attractive, and the lending standard drops at $\mu_0 = \tilde{\mu}$. It continues to decrease as μ_0 further increases, as a higher investor belief translates directly into higher security prices for both types. Eventually, a b-type's expected payoff in the Securitization stage reaches 1, the cost of origination. We denote this belief level as $\bar{\mu}$ (i.e., $u_b(0, \bar{\mu}) = 1$). Hence, for all investor beliefs $\mu_0 > \bar{\mu}$, banks are willing to originate all loan pools, regardless of their appraisals, since even the pools that turn out to be bad will earn a positive return. Consequently, $\Psi(\mu_0) = 0$ for all $\mu_0 \geq \bar{\mu}$, as seen in the figure.

Investor Belief Consistency. Finally, in equilibrium, investors' belief that a given loan pool

is of high quality must be consistent with the banks loan appraisal at origination surpassing the lending standard: $\mu_0 = A(p^*)$. Combining this condition with the banks' optimal origination condition, $p^* \in \Psi(\mu_0)$, we having the following.

Proposition 3. *There exists a unique equilibrium. Its lending standard is given by the unique* p^* *satisfying* $p^* = A^{-1}(\mu_0) \in \Psi(\mu_0)$.

Figure 1(b) illustrates how the bank-origination-optimality and investor-belief-consistency conditions pin down the equilibrium lending standard, p^*, and investor's beliefs, μ_0^*, as the strictly increasing function A^{-1} intersects the decreasing function Ψ exactly once. The figure depicts an example with a lending standard, p^*, that is lower than the first-best benchmark (i.e., an oversupply of credit). However, there are also examples in which the intersection of Ψ and A^{-1} lead to an equilibrium lending standard above the first-best level.

Corollary 4. *With ratings, the lending standard can be either above or below the first-best benchmark.*

To understand the determinants of an over-supply of credit relative to the first-best benchmark, the following thought experiment is useful. Imagine starting with the first-best lending standard p^{FB}. This lending standard implies a consistent investor belief of $A(p^{FB})$. What are the banks' incentives to originate in this scenario? If $\Psi(A(p^{FB})) < p^{FB}$, then by definition of Ψ, banks have an incentive to originate some additional loans with $p < p^{FB}$, lowering equilibrium the lending standard below the first-best level. In what follows, we study how changes in the banks' screening technology and/or in the rating informativeness impact banks' origination decisions, and under which scenarios an over-supply of credit arises.

4.3 Determinants of Credit Supply

Precision of Banks' Screen Technology

A more precise screening technology means that, overall, banks become more certain whether their individual loan opportunities are bad or good before their origination decisions. Analytically, this is captured by mass in the distribution of appraisals shifting toward the extreme values of 0 or 1, which then has implications for the $A(\cdot)$ function that we just saw is used to pin down the equilibrium lending standard (Section 4.2).

Figure 2 illustrates how the precision of banks screening technology affects origination. For this example, the screening technology, $\{\psi_g, \psi_b\}$, are Normal density functions with means $m_g > m_b$ and common standard deviation σ. As σ decreases, the screening technology becomes more precise and $A^{-1}(\mu)$ decreases for all $\mu \in (\xi, 1)$. This is because, for any $p \in (0, 1)$, if the loan

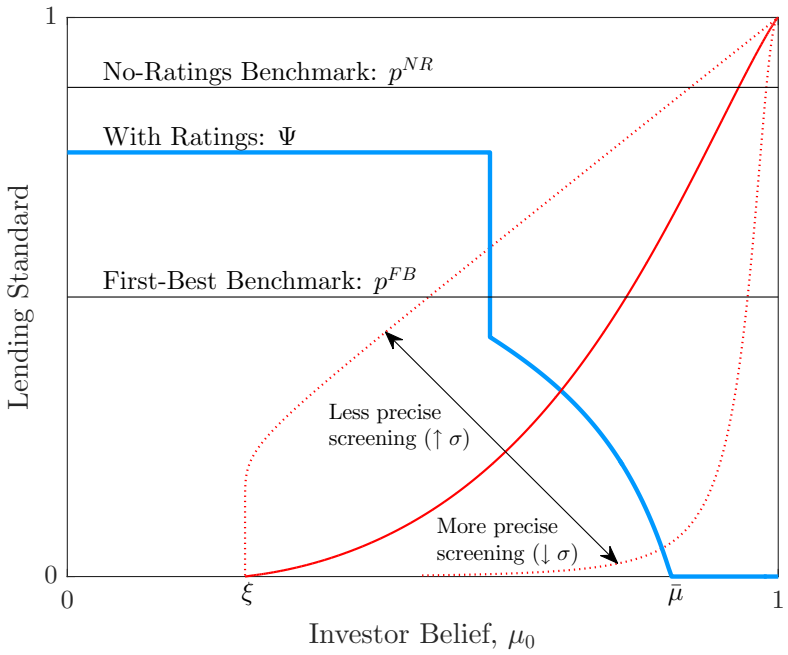


FIGURE 2: This figure illustrates how the precision of the screening technology affects the equilibrium lending standard and investor belief.

pool is bad (good) it is becoming more likely that it would have generated an appraisal below (above) p. Consequently, as σ decreases, the equilibrium lending standard falls and the supply of credit increases.

The figure suggests that as σ goes to zero, $A^{-1}(\mu)$ tends to zero for all $\mu \in (\xi, 1)$, meaning the equilibrium lending standard p^* would fall to 0. Proposition 4 shows that this result is indeed true and holds more generally for any screening technology that becomes arbitrarily precise as defined below.

Definition 2. *A sequence of screening technologies* $\{\psi_b^n, \psi_g^n\}_{n=1}^{\infty}$ ***limits to perfect screening*** *if* $\lim_{n\to\infty} \Pr(L^n(s) \in (a,b)) = 0$ *for all* $0 < a < b < \infty$.

That is, the screening technology of banks becomes perfect when there is essentially no chance of receiving a signal that does not indicate the pool's underlying quality with arbitrary precision. However, just because banks *can* discern loan quality with arbitrary accuracy does not mean they will only originate good loans.

Proposition 4. *With ratings, if* $\{\psi_b^n, \psi_g^n\}_{n=1}^{\infty}$ *limits to perfect screening, then as* $n \to \infty$,

1. *The equilibrium lending standard,* p^* *limits to zero.*
2. *The equilibrium supply of credit limits to* $\dfrac{\xi}{\bar{\mu}} > \xi$.

Hence, when banks are very good at appraising which loan opportunities are good or bad, they fund (virtually) all good loan pools and a (non-vanishing) amount of loan pools that they are (virtually) certain are bad. This is because there is an incentive to originate until the average quality, and therefore equilibrium investor belief, is driven down to $\bar{\mu}$—the investor belief level at which origination of a bad pool is expected to exactly break even.[15]

It is worth noting that (informative) ratings are critical for this result. In the no-ratings benchmark, the lending standard is p^{NR} regardless of the screening technology. Further, since bad pools are sold for $v_b < 1$, it is not profitable to originate pools with low appraisals. Hence, without ratings, if $\{\psi_b^n, \psi_g^n\}_{n=1}^{\infty}$ limits to perfect screening then only good loan pools will be originated in the limit (i.e., the supply of credit tends to ξ—the mass of good loan opportunities).

Informativeness of Ratings

We next analyze how changes in rating informativeness affect origination and securitization decisions. To sharpen our predictions, we focus on the binary-symmetric rating system (introduced in Section 2): $P(R = G|g) = P(R = B|b) = \gamma \in (\frac{1}{2}, 1)$, where higher γ implies a more informative ratings. To begin, we examine how an increasing in rating informativeness effects the Securitization stage, and consequently, its effect on banks' reaction function for origination, Ψ.

Lemma 3. *As the informativeness of ratings (γ) increases,*

1. *$\tilde{\mu}$ and $\tilde{x}$ both decrease, implying lower retention levels for all μ_0.*

2. *Letting $\hat{\mu} \equiv \max\{\tilde{\mu}, p^{FB}\}$, Ψ decreases for $\mu_0 < \hat{\mu}$ and Ψ increases for $\mu_0 > \hat{\mu}$.*

From statement (2) of the lemma, it is not surprising that more informative ratings can increase or decrease the lending standard/credit supply (as illustrated in Figure 3). There is however structure to these possibilities.

Proposition 5. *If the equilibrium lending standard is at least as high as the first-best benchmark ($p_\gamma^* \geq p^{FB}$), then the lending standard is strictly decreasing in rating informativeness (γ).*

Hence, starting from no-ratings/completely uninformative ratings (where $p^* = p^{NR} > p^{FB}$), increasing informativeness decreases the lending standard and increases the supply of credit. Will further increases in informativeness eventually lead to $p^* < p^{FB}$? In general the answer

[15] We can note that the screening technology affects the equilibrium lending standard/credit supply by affecting A^{-1}. The only other ingredient that determines A^{-1} is the proportion of good loans, ξ. Increasing ξ shifts A^{-1} to the right, leading to a decrease in the lending standard. At some point, ξ becomes sufficiently large that Assumption 1 is violated (the aggregate loan opportunity pool becomes positive NPV) and the lending standard falls to 0.

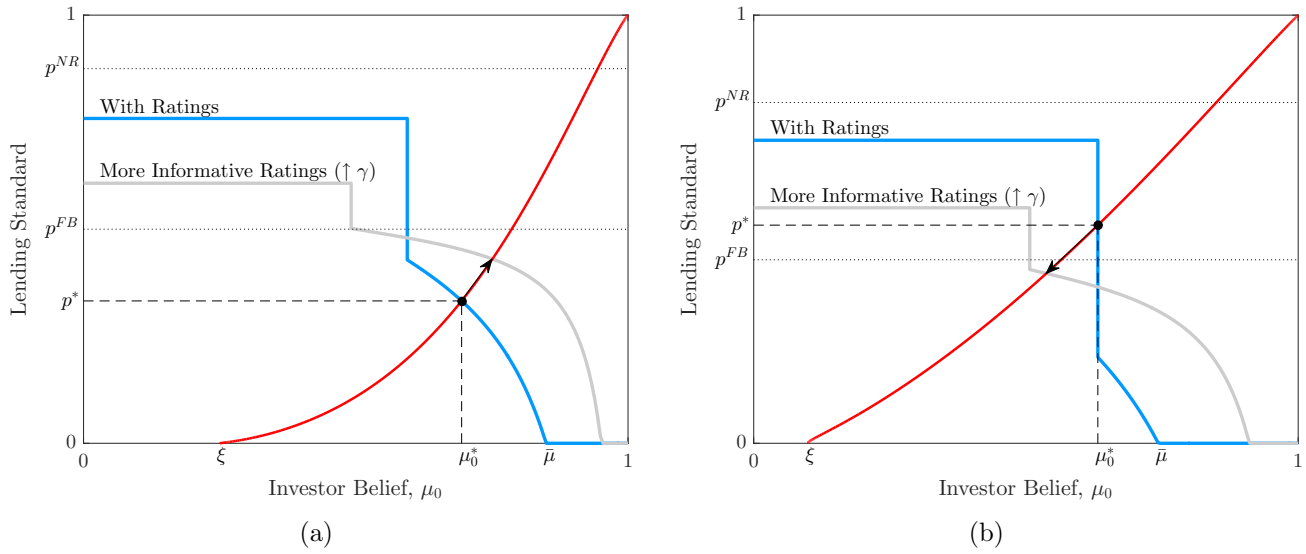


FIGURE 3: This figure illustrates how the informativeness of the rating technology (γ) affects the equilibrium lending standard and investor belief. In panel (a), an increase in rating informativeness leads to a higher lending standard, whereas in panel (b) the lending standard decreases.

may depend on the distribution of appraisals. However, an unambiguous result can be obtained if overall loan opportunities are not too valuable.

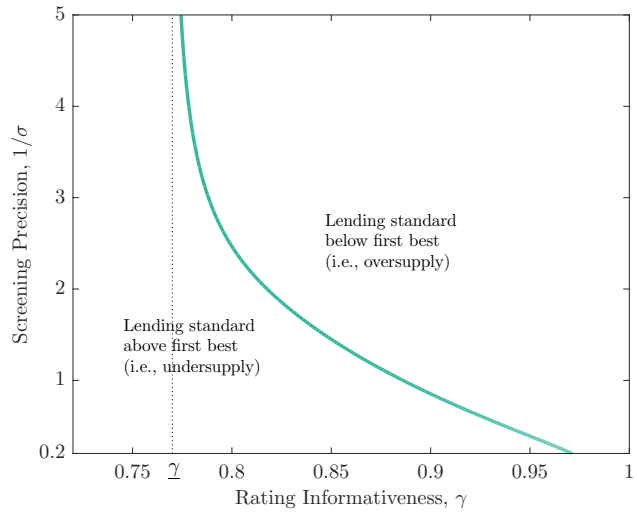
Proposition 6. *If $v_g \cdot v_b \leq 1$, then there exists $\hat{\gamma} \in (\frac{1}{2}, 1)$ such that the equilibrium lending standard is below the first-best benchmark ($p^*_\gamma < p^{FB}$) if and only if $\gamma \in (\hat{\gamma}, 1)$.*

Notice that the condition $v_g \cdot v_b \leq 1$ is *not* independent of Assumption 1, as both restrict how valuable loan opportunities are in the aggregate. For example, if $\xi \leq \frac{1}{2}$, then Assumption 1 implies $v_g \cdot v_b \leq 1$, and sufficiently informative ratings always lead to an oversupply of credit. Intuitively, the more valuable are loan opportunities, the more pools banks should originate, that is, the lower is the first-best lending standard. When that value (measured by $v_g \cdot v_b$) is large, the first-best lending standard is so low that it is no longer guaranteed that informative enough ratings can push the equilibrium lending standard below the first-best one.

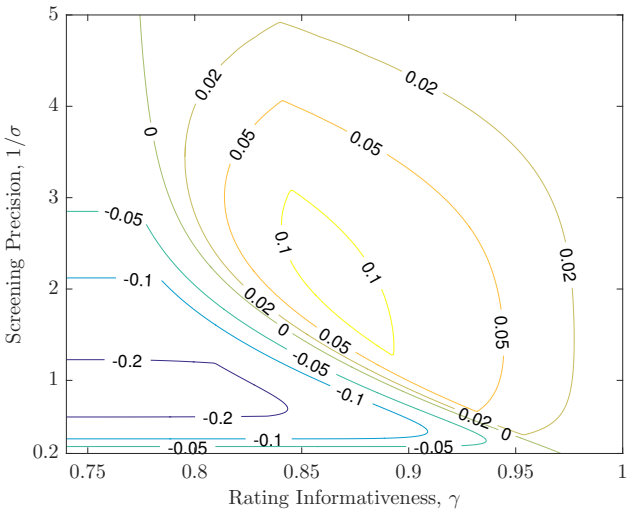
Finally, and perhaps unsurprisingly, as the rating becomes perfectly informative, any mismatch between equilibrium and first-best origination (be it over or undersupply) disappears.

Proposition 7. *As ratings become perfectly informative ($\gamma \to 1$), the equilibrium lending standard tends to the first-best benchmark ($p^*_\gamma \to p^{FB}$).*

Having analyzed the effects of the precision of banks' screening technology and the informativeness of ratings, Figure 4 depicts the two in conjunction. Panel (a) illustrates when the equilibrium lending standard is above, equal to, or below the first-best benchmark. Recall from Lemma 2 that there is an minimum level of ratings informativeness, labeled $\underline{\gamma}$ in the figure, required to



(a) Lending standard



(b) Contour plot of origination quantity minus the first best level of origination

FIGURE 4: This figure illustrates the interaction between the precision of screening technology and rating informativeness on the equilibrium lending standard and quantity of origination.

alter the equilibrium predictions from the no-ratings model in which the lending standard is $p^{NR} > p^{FB}$. Hence, if $\gamma < \underline{\gamma}$ there is an undersupply of credit, regardless of the screening precision. In contrast, for ratings informativeness above $\underline{\gamma}$, there is a strictly decreasing threshold for screening precision above which the lending standard is below the first-best benchmark (in accordance with Proposition 4).[16] As ratings become more informative, less screening precision is required for the equilibrium to exhibit oversupply.

While Figure 4(a) shows the under/oversupply regions, Figure 4(b) shows the quantity under/oversupplied in equilibrium to give a sense of when the mismatch is most pronounced (i.e., the "0.1"-contour implies there is an oversupply of credit equal in size to 10% of all loan opportunities).[17] Notice that the "0"-contour is identical to the single under/oversupply threshold in Figure 4(a). In accordance with Proposition 7, the supply of credit tends to the first-best level as rating become perfectly informative ($\gamma \to 1$). The configurations with the largest supply of credit occur for intermediate levels of both γ and σ (roughly around $(\gamma, \frac{1}{\sigma}) \approx (0.875, 2)$), highlighting the potential non-monotonicity of credit supply in both parameters.

[16]This threshold asymptotes to ∞ as $\gamma \to \underline{\gamma}$.

[17]The quantity of origination above first best is calculated as $(1 - H(p^*)) - (1 - H(p^{FB}) = H(p^{FB}) - H(p^*))$.

5 Policy Analysis

In this section, we discuss some the main policies implemented in the US and in Europe after the 2008/09 financial crisis. Several reforms were made in the securitization and rating industries. These regulatory responses conceptually fell into four categories: increasing information disclosure, requiring risk-retention, reforming rating agencies, and imposing capital requirements.[18] In addition, central banks have intervened in a variety of ways to affect banks liquidity needs, both before and after the crisis. We discuss the effect of these policies on both securitization and origination decisions through the lenses of our model. First, we analyze the effect of imposing a skin-in-the-game (risk-retention) rule on securitizers of assets. Second, we examine policies that aim to improve information disclosure, and we use these results to discuss the potential effects of policies that address frictions in the ratings industry. Finally, we study the effect of policies that relax banks liquidity needs.

5.1 Skin-in-the-Game Rules

In October, 2014, the US passed the skin-in-the-game rule that requires sponsors of securitization transactions to retain risk in those transactions, as stated in the risk retention requirements in the Dodd-Frank Wall Street Reform and Consumer Protection Act. More specifically, the rule requires sponsors of asset-backed securities to retain not less than five percent of the credit risk of the assets collateralizing the issuance. The rule also sets forth prohibitions on transferring or hedging the credit risk that the sponsor is required to retain. This rule aims to align incentives between the originators of assets and the investors who end up holding these assets. In Europe, the Capital Requirements Regulation (CRR) requires the originator/sponsor/original lender to explicitly disclose that it will retain, on an ongoing basis, a material net economic interest in the securitization of at least five percent for the life of the transaction.[19]

We study the impact of retention rules by considering a policy in which banks are forced to retain an exposure to their loan pool of at least $\bar{x}$. As in practice, the retention requirements is not contingent on the choice of security, the rating, nor on other measures of quality of the underlying cash flows.[20] We show that risk-retention rules hinder banks' ability to signal (through retention) the quality of their underlying loans to investors. In turn, this tightens lending standards and

[18] See Schwarcz (2015) for an analysis of the regulatory changes in securitization in response to the financial crisis, both in the US and in Europe.

[19] An interesting difference between both regulations is that while in the US the responsibility falls on the originator/sponsor, in Europe it falls on the investor banks (regulated banks), who need to establish mechanisms to monitor the exposure of the originator/sponsor to ensure a lower risk-weight on these assets.

[20] The present regulation does make exceptions for particular asset classes. However, for a given asset class, retention rules are equal for all asset qualities.

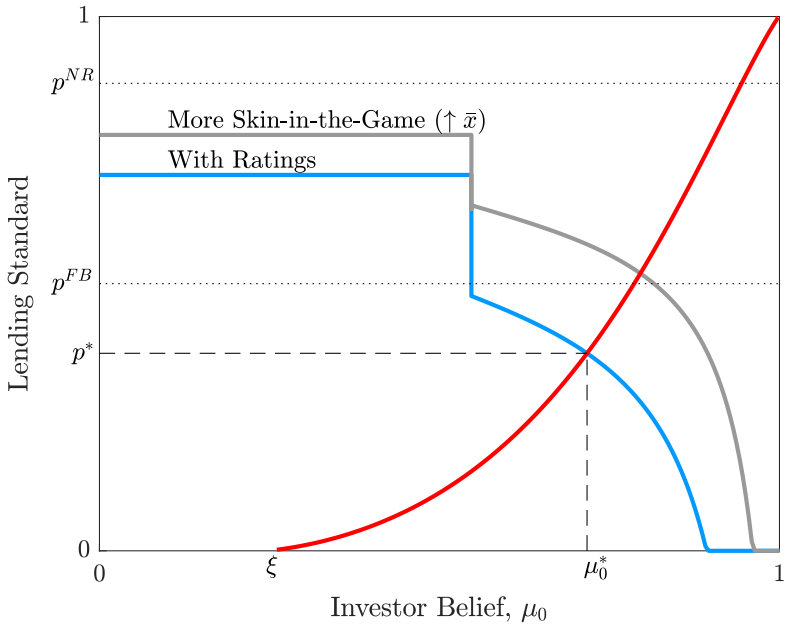


FIGURE 5: The Effect of a Skin-in-the-Game Rule.

decreases aggregate credit supply. These results are summarized in the following proposition.

Proposition 8. *Imposing a retention requirement increases the equilibrium lending standard and retention levels, but it reduces aggregate credit supply and results in a decrease in ex-ante efficiency.*

To understand the proposition, notice that forced cash flow retention worsen the outside option of banks holding b-type loan pools, i.e. their full information payoff falls. This, in turn, makes it more costly for banks with g-type pools to signal their quality, i.e., retention levels have to increase to signal successfully. As a result, $\tilde{x}$ increases and securitization payoffs weakly decrease for all banks. In turn, the reduction in the expected outcomes from securitization worsens banks incentives to originate, increasing lending standards and decreasing aggregate credit supply. Figure 5 illustrates the effect of a skin-in-the-game rule on (i) banks lending standard as a function of investors' beliefs, $\Psi(\mu)$ and (ii) equilibrium lending standards, p^*.

5.2 Disclosure Requirements and CRA Regulation

Rules have been adopted in both the US and Europe to improve the disclosure, reporting, and offering process of securitized products. The rules require the disclosure of standardized, detailed, loan-level information and of the risk-models used to analyze them. They also increase the time given to investors to process and analyze this information. In addition, to increase the reliability of ratings issued by CRAs in the US, the Dodd-Frank Act mandated the creation of the Office

of Credit Ratings (OCR) to conduct oversight of the "nationally recognized statistical rating organizations" (NRSROs). The role of the OCR is to monitor and report on the NRSROs internal control structures, rating methodologies and models, conflicts of interest, quality of information disclosure, etc.[21] Europe followed with the creation of the European Securities and Markets Authority (ESMA) in charge of supervising CRAs in the European Union. The goal of these policies is to protect investors by increasing the quality of information available to investors.[22]

In our model, a rating is a public signal that is made available to investors at the time a security is issued. To the extent that there is noise in the information being disclosed by securitizers and CRAs, this information operates as a public signal to investors, and thus plays the same role as ratings in our framework. Policies that aim at addressing frictions in the ratings industry can also be interpreted as policies that aim to improve the quality of the public information investor's receive (we show why this may be the case in Section 6). As a result, we interpret policies that improve information disclosure in markets and that address frictions in the ratings industry as policies that increases the informativeness of the ratings in our model.

We analyzed the effect of rating informativeness on origination and securitization decisions in Section 4.3. Improvements in rating informativeness increase ex-ante efficiency, but they may result in a reduction of lending standards, and potentially an over-supply of credit (as illustrated in Figure 3(b)). Therefore, such policies may not be desirable, particularly if there are negative externalities of originating bad loans that are not internalized by banks (e.g., an increase in systematic risk or default costs).

5.3 Liquidity Needs

Central banks often undertake policies aimed at easing credit constraints of distressed financial institutions, in attempt to stimulate the economy by inducing banks to lend more.[23] Within our model, such policies can be interpreted as increasing δ. The following proposition summarizes the comparative static effects of such a policy.

Proposition 9. *As liquidity needs are reduced (δ increases), $\tilde{\mu}$ and $\tilde{x}$ increase. A reduction in liquidity needs increases retention levels, but their effect on lending standards and aggregate credit supply is ambiguous.*

[21] For a more detailed description of the OCR mandate, see the *2016 Summary Report of Commission Staff's Examinations of Each Nationally Recognized Statistical Rating Organization* prepared by the SEC.

[22] In Section 6, we show that frictions in the ratings industry (such as rating shopping and manipulation), reduce rating informativeness. As a result, policies that address industry frictions would increase rating informativeness.

[23] For example, in March 2008, the Federal Reserve announced the Term Securities Lending Facility that enabled banks to use MBS as collateral for short-term loans, which naturally reduced their need to sell such securities. Later, during quantitative easing, the Federal Reserve purchased outright billions of dollars in MBS. The European Central Banks adopted similar policy measures during the European Financial Crisis.

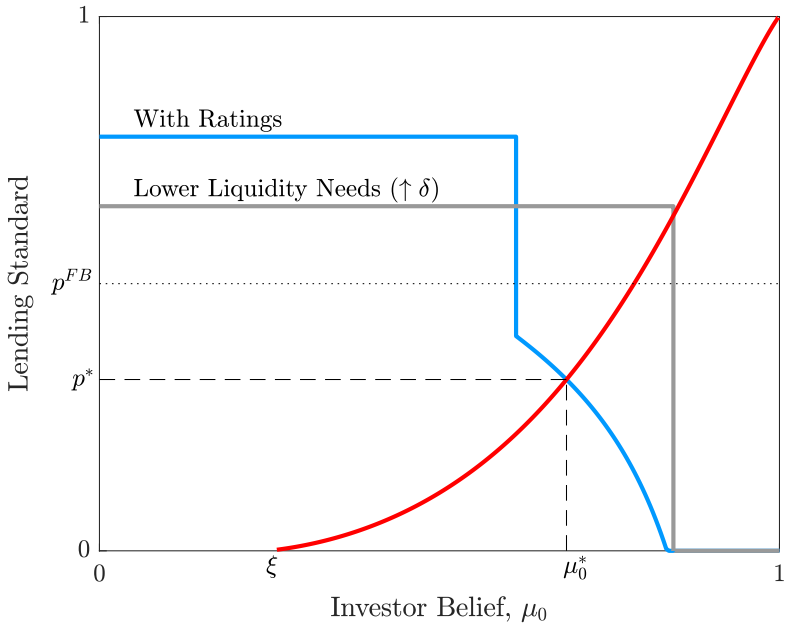


FIGURE 6: The Effect of a Decrease in Liquidity Needs.

As liquidity needs decrease, banks have less incentives to securitize their loan pools, so they retain more. As can be seen in Figure 6 , the response of lending standards and aggregate credit supply to changes in banks' liquidity needs depend on initial equilibrium beliefs, determined by the intersection $\Psi(\mu_0^*) = A^{-1}(\mu_0^*)$. As illustrated in Figure 6, when $\mu_0^* > \tilde{\mu}$, lower liquidity needs tighten lending standards, reducing aggregate credit supply in the economy. In contrast, when $\mu_0^* < \tilde{\mu}$, lower liquidity needs reduce lending standards and increase aggregate credit supply.

6 Endogenous Ratings

In this section, we consider two extensions to the model to investigate different features of the CRA industry. First, we incorporate rating shopping by supposing that banks have to pay for a rating if they wish to publish it. Second, we allow for rating manipulation by supposing that banks can inflate their rating by giving a side payment to the CRA. Our main result is that, provided investors are rational, rating shopping and manipulation act mainly to reduce rating informativeness. Hence, the effect of these considerations is largely captured by the comparative static results in Section 4.3 (see Propositions 5-7). When investors are not aware of the possibility of rating shopping or manipulation, the consequences are more severe (see Section.

6.1 Ratings Shopping

We extend the model as follows. After choosing the fraction of the loan pool to securitize, a bank approaches the CRA to get a rating for its issuance. The CRA generates rating R with type-dependent pdf f_t, as in the baseline model. The CRA reports this rating to the bank, which then chooses whether or not to pay the CRA to have this rating become public.

We assume that the fee charged by the CRA is proportional to the cash flows being issued, $1-x$, and we normalize it by the difference in expected payoffs from a good vs a bad pool, which is without loss of generality. That is, given a CRA fee $\phi \geq 0$, a bank that issues fraction $1-x$ has to pay $\frac{\phi}{v_g - v_b}(1-x)$ to have its rating published. We denote by $\mu_f(\mu, r)$ the posterior belief after rating r is observed (as in the baseline model), and by $\mu_n(\mu, \phi)$ the posterior belief associated with an unrated issuance, which we show in equilibrium depends on the rating fee.

Shopping for Ratings. We begin by studying the decision of a bank with a t-type loan pool that has chosen to sell fraction $1-x(t)$ and that has been proposed rating r by the CRA. The payoff from paying the CRA to have rating r published is

$$u_t(\mu, r, \phi) = (1 - x(t)) \{(\mu_f(\mu, r) - \phi)(v_g - v_b) + v_b\} + \delta x(t) v_t \tag{8}$$

while the payoff from staying unrated is:

$$u_t(\mu, n, \phi) = (1 - x(t)) \{\mu_n(\mu, \phi)(v_g - v_b) + v_b\} + \delta x(t) v_t \tag{9}$$

The bank chooses to hire the CRA if:

$$u_t(\mu, r, \phi) \geq u_t(\mu, n, \phi) \iff \mu_f(\mu, r) - \phi \geq \mu_n(\mu, \phi) \tag{10}$$

Off-Equilibrium Beliefs. From (10), it follows that the decision to get rated, conditional on the rating r, is not type-dependent. As a result, we characterize equilibria in which investors assign beliefs $\mu_f(\mu, r)$, as defined in (2), for all observed ratings, even if they are not on the equilibrium path. That is, reporting an off-equilibrium path rating does not give more or less information to investors about pool type than the reported rating. This is because any observed rating r must have been drawn from type-contingent pdf, f_t, and thus contains relevant information. It is worth noting that these off-equilibrium beliefs are consistent with the D1-refinement.

The following Lemma characterizes a bank's decision to get rated.

Lemma 4. *For any prior belief $\mu_0 \in (0,1)$, banks pay the CRA to publish rating r if and only if*

$r \geq \bar{r}(\mu_0, \phi)$, where threshold $\bar{r}(\mu_0, \phi)$ is given by the banks' indifference condition in (10)

$$\mu_f(\mu_0, \bar{r}) - \mu_n(\mu_0, \bar{r}) = \phi \tag{11}$$

Furthermore, the set of ratings for which banks choose not to get rated, $\{r : r < \bar{r}(\mu_0, \phi)\}$, has positive measure iff $\phi > 0$.

From Lemma 4, a bank's decision to not get rated indicates that $r < \bar{r}(\mu_0, \phi)$. Thus, in any equilibrium, the belief assigned to an unrated issuance is

$$\mu_n(\mu_0, \phi) = \frac{\mu_0 F_g(\bar{r}(\mu_0, \phi))}{\mu_0 F_g(\bar{r}(\mu_0, \phi)) + (1 - \mu_0) F_b(\bar{r}(\mu_0, \phi))}. \tag{12}$$

Lemma 4 yields the first implication of rating shopping. When ratings are costly ($\phi > 0$) and banks can decide whether or not to publish a given rating, the ratings observed by investors are truncated below. As a result, the average rating observed by investors is "inflated" relative to the baseline model.

Securitization Decisions. Given the previous characterization of banks' decision to get rated, we proceed to the securitization stage, where banks choose the fraction of cash flows to securitize. The expected payoff of selling fraction $1 - x$ of a t-type loan pool given investor's beliefs μ_0 is

$$u_t(\mu_0, x, \phi) = (1 - x)\left(\tilde{\alpha}_t(\mu_0, \phi)(v_g - v_b) + v_b - \phi\right) + \delta x v_t \tag{13}$$

where $\tilde{\alpha}_t(\mu, \phi) \equiv E_R[\max\{\mu_f(\mu, \bar{r}); \mu_f(\mu, r)\} | t]$ is the new expected posterior, which incorporates the truncated nature of ratings. The expression is obtained after incorporating the indifference condition from (11).

From Lemma 4, when $\phi = 0$ all ratings are published, in which case $\tilde{\alpha}_t(\mu, 0) = E_R[\mu_f(\mu, r)|t] = \alpha_t(\mu)$. That is, when $\phi = 0$, equilibrium allocations are as those described in the baseline model of Section 4. In what follows, we analyze how rating shopping affects equilibrium allocations when $\phi > 0$. In Appendix A.3, we show that any equilibria with $\phi > 0$ must have the same properties of the equilibrium of the baseline model as described in Proposition 2. We also provide sufficient conditions for the existence and uniqueness of such an equilibrium.

To understand the effect of rating shopping on equilibrium allocations we compare the expected payoff from originating a t-type pool in the model with $\phi > 0$, given by (13), with that obtained when $\phi = 0$, in (5). There are two differences: first, banks have to pay ϕ if they get rated and this reduces the value to originate loans for both types. Second, expected posterior beliefs are different: $E_R[\tilde{\alpha}_t(\cdot, \phi)|t]$ vs. $E_R[\alpha_t(\cdot)|t]$, since the distribution of ratings is truncated below for both types with rating shopping. Therefore, banks expect ratings to be less informative, and this

reduces the expected payoff of originating a g-type pool. This is shown formally in the following Lemma, that states that as the CRA fee increases, the difference between expected posteriors between a g-type and a b-type decreases with the rating fee.

Lemma 5. *For all* $\mu \in (0,1)$, $\tilde{\alpha}_g(\mu,\phi) - \tilde{\alpha}_b(\mu,\phi) \geq 0$ *and decreasing in* ϕ.

Therefore, rating shopping acts mainly by reducing the informativeness of ratings. The effect of reducing the informativeness of ratings on equilibrium outcomes was described in Section 4.

6.2 Manipulable Ratings

In this extension, we suppose that banks can manipulate ratings upward by paying m to the CRA. This can be interpreted as a side payment a bank gives to the CRA to obscure the value of the underlying loan pool. To keep this extension simple, we assume that when a bank with a b-type loan pool pays the CRA, it obtains a rating from the distribution corresponding to a g-type loan pool with probability σ_m. There is no rating shopping in this model, so after the payment m is given to the CRA, the model is as the one described in Section 4. The main difference is that now the CRA may report a manipulated rating.

Naive Investors. Assume investors are not aware of the ability of the bank to manipulate ratings. Let (x^*, μ_0^*) be the pooling action/belief absent manipulation from Proposition 2. If investors are not aware of manipulation, the expected gain from paying to manipulate for banks with b-type pools is given by:

$$\Delta = \sigma_m(E_R[\mu_f(x^*,r)|g] - E_R[\mu_f(x^*,r)|b])(1-x^*)(v_g - v_b) \tag{14}$$

Thus, manipulation takes place if $m < \Delta$. As a result, the expected payoff associated with originating a b-type loan pool, $u_b(\mu_0)$, increases, while that of originating a g-type loan remains unchanged, $u_g(\mu_0)$. When investors are not aware of manipulation, they transfer value to the bank selling b-backed security. As a result, banks increase their willingness to originate loans. With manipulation and naive investors, lending standards fall, credit supply increases, efficiency decreases, and investors make losses. In this situation, policies such as a skin-in-the-game or improvements in rating informativeness may be beneficial.

Sophisticated Investors. Now we suppose that investors are aware of banks' ability to manipulate their ratings. Then the belief updating of investors is adjusted as follows:

$$\mu_f(\mu_0, r) = \frac{\mu_0 f_g(r)}{(\mu_0 + \sigma_m(1-\mu_0))f_g(r) + (1-\sigma_m)(1-\mu_0)f_b(r)} \tag{15}$$

Thus, when investors are aware of manipulation, the latter only acts by reducing rating informativeness, with the consequences for securitization, lending standards, and credit supply discussed in Section 4.

7 Conclusion

We have studied the effect of ratings on loan origination and securitization decisions. Sufficiently informative ratings increase market liquidity and improve allocative efficiency; but reduce lending standards and may lead to an oversupply of credit relative to the first-best benchmark. In particular, we show that if banks cannot adjust their securitization decisions in response to the presence or informativeness of ratings, the latter discipline markets and improve loan origination, increasing the standards of loan origination. However, when banks can adjust their securitization decisions in response to ratings, their disciplining role is weakened. We show that informative ratings can induce an oversupply of credit relative to the first-best by improving incentives to originate negative NPV loans. Moreover, with ratings, the lending standard collapses to zero as the banks' screening technology becomes arbitrarily precise. Our results are robust to extensions that incorporate some features of the rating industry, such as shopping and manipulation; where these market frictions act mainly by reducing rating informativeness. We then use our model to discuss the implications of policies, such as the "skin-in-the-game" rule for securitizers or the information disclosure requirements for CRAs.

References

Agarwal, S., Y. Chang, and A. Yavas (2012): "Adverse selection in mortgage securitization," *Journal of Financial Economics*, 105, 640–660.

Ashcraft, A., P. Goldsmith-Pinkham, P. Hull, and J. Vickery (2011): "Credit ratings and security prices in the subprime MBS market," *The American Economic Review*, 101, 115–119.

Banks, J. and J. Sobel (1987): "Equilibrium Selection in Signaling Games," *Econometrica*, 55, 647–662.

Bar-Isaac, H. and J. Shapiro (2013): "Ratings quality over the business cycle," *Journal of Financial Economics*, 108, 62–78.

Becker, B. and T. Milbourn (2011): "How did increased competition affect credit ratings?" *Journal of Financial Economics*, 101, 493–514.

Benmelech, E. and J. Dlugosz (2010): "The credit rating crisis," *NBER Macroeconomics Annual*, 24, 161–208.

Blackwell, D. (1951): "Comparison of Experiments," *Proceedings of the Second Berkeley Symposium on Mathematics, Statistics and Probability, (1950)*, 93–102.

Bolton, P., X. Freixas, and J. Shapiro (2012): "The credit ratings game," *The Journal of Finance*, 67, 85–111.

Boot, A. W., T. T. Milbourn, and A. Schmeits (2006): "Credit ratings as coordination mechanisms," *Review of Financial Studies*, 19, 81–118.

Chemla, G. and C. Hennessy (2014): "Skin in the game and moral hazard," *The Journal of Finance*.

Cho, I.-K. and D. M. Kreps (1987): "Signaling games and stable equilibria," *The Quarterly Journal of Economics*, 179–221.

Daley, B., B. Greeen, and V. Vanasco (2016b): "Security Design with Ratings," .

Daley, B. and B. Green (2014): "Market signaling with grades," *Journal of Economic Theory*, 151, 114–145.

Dell'Ariccia, G., D. Igan, and L. a. Laeven (2009): "Credit Booms and Lending Standards: Evidence from the Subprime Mortgage Market," *SSRN Electronic Journal*.

DeMarzo, P. M. (2005): "The pooling and tranching of securities: A model of informed intermediation," *Review of Financial Studies*, 18, 1–35.

DeMarzo, P. M. and D. Duffie (1999): "A Liquidity-Based Model of Security Design," *Econometrica*, 67, 65–99.

Derrien, F. and A. Kecskés (2013): "The real effects of financial shocks: Evidence from exogenous changes in analyst coverage," *The Journal of Finance*, 68, 1407–1440.

Faulkender, M. and M. A. Petersen (2006): "Does the source of capital affect capital structure?" *Review of financial studies*, 19, 45–79.

Fulghieri, P., G. Strobl, and H. Xia (2014): "The Economics of Solicited and Unsolicited Credit Ratings," *Review of Financial Studies*, 27, 484–518.

Gorton, G. and A. Metrick (2012): "Securitization," Tech. rep., National Bureau of Economic Research.

Gorton, G. B. and G. G. Pennacchi (1995): "Banks and Loan Sales Marketing Nonmarketable Assets," *Journal of Monetary Economics*, 35, 389–411.

Griffin, J. M., J. Nickerson, and D. Y. Tang (2013): "Rating shopping or catering? An examination of the response to competitive pressure for CDO credit ratings," *Review of Financial Studies*, 26, 2270–2310.

Griffin, J. M. and D. Y. Tang (2011): "Did credit rating agencies make unbiased assumptions on CDOs?" *The American Economic Review*, 101, 125–130.

Hartman-Glaser, B., T. Piskorski, and A. Tchistyi (2012): "Optimal securitization with moral hazard," *Journal of Financial Economics*, 104, 186–202.

He, J., J. Qian, and P. E. Strahan (2011): "Credit ratings and the evolution of the mortgage-backed securities market," *The American Economic Review*, 101, 131–135.

Jaffee, D., A. Lynch, M. Richardson, and S. Van Nieuwerburgh (2009): "Mortgage Origination and Securitization in the Financial Crisis," in *Restoring Financial Stability*, ed. by V. Acharya and M. Richardson, chap. 1.

Josephson, J. and J. Shapiro (2015): "Credit Ratings and Structured Finance," *Working Paper*.

Kamenica, E. and M. Gentzkow (2011): "Bayesian Persuasion," *American Economic Review*, 101, 2590–2615.

Kashyap, A. K. and N. Kovrijnykh (2016): "Who Should Pay for Credit Ratings and How?" *Review of Financial Studies*, 29, 420–456.

Keys, B. J., T. Mukherjee, A. Seru, and V. Vikrant (2010): "Did securitization lead to lax screening? Evidence from Subprime Loans," *The Quarterly Journal of Economics*.

Kraft, P. (2015): "Do rating agencies cater? Evidence from rating-based contracts," *Journal of Accounting and Economics*, 59, 264–283.

Leland, H. E. and D. H. Pyle (1977): "Informational Asymmetries, Financial Structure, and Financial Intermediation," *The Journal of Finance*, 32, 371–387.

Loutskina, E. and P. E. Strahan (2009): "Securitization and the Declining Impact of Bank Finance on Loan Supply: Evidence from Mortgage Originations," *The Journal of Finance*, 64, 861–889.

Malherbe, F. (2012): "Market Discipline and Securitization," .

Manso, G. (2013): "The Feedback Effects of Credit Ratings," *Journal of Financial Economics*, 109, 535–548.

Mathis, J., J. McAndrews, and J.-C. Rochet (2009): "Rating the raters: Are reputation concerns powerful enough to discipline rating agencies?" *Journal of Monetary Economics*, 56, 657–674.

Mian, A. and A. Sufi (2009): "The Consequences of Morgage Credit Expantion : Evidence from the U.S. Mortage Default Crisis," .

Opp, C. C., M. M. Opp, and M. Harris (2013): "Rating agencies in the face of regulation," *Journal of Financial Economics*, 108, 46–61.

Pagano, M. and P. Volpin (2010): "Credit ratings failures and policy options," *Economic Policy*, 25, 401–431.

Parlour, C. A. and G. Plantin (2008): "Loan Sales and Relationship Banking," *The Journal of Finance*, LXIII, 1291–1314.

Sangiorgi, F. and C. Spatt (2012): "Opacity, credit rating shopping and bias," *Unpublished working paper. Stockholm School of Economics and Carnegie Mellon*.

Schwarcz, S. L. (2015): "Securitization and post-crisis financial regulation," *Cornell L. Rev. Online*, 101, 115.

Shivdasani, A. and Y. Wang (2011): "Did Structured Credit Fuel the LBO Boom?" *The Journal of Finance*, 66, 1291–1328.

Skreta, V. and L. Veldkamp (2009): "Ratings shopping and asset complexity: A theory of ratings inflation," *Journal of Monetary Economics*, 56, 678–695.

Sufi, A. (2007): "Information Asymmetry and Financing Arrangements: Evidence fromSyndicated Loans," *The Journal of Finance*, LXII, 629–668.

Vanasco, V. (2017): "The Downside of Asset Screening for Market Liquidity," *forthcoming, Journal of Finance*.

A Appendix

A.1 Preliminaries and Definitions

Let $\alpha_t(\mu) \equiv E_R[\mu_f(\mu, r)|t]$ be the expected posterior belief for a t-type bank given investor's interim belief μ. The following claims are rudimentary or, in the case of Fact 1(3), have been established previously.

Fact 1. *For any $t \in \{b, g\}$,*

1. *$\alpha_t(\mu)$ is strictly increasing in μ for any $x \in (0, 1]$.*
2. *$\alpha_g(\mu) - \alpha_b(\mu)$ is concave and achieves a unique maximum at $\hat{\mu} \in (0, 1)$.*
3. *$\frac{\partial}{\partial \mu}\left(\frac{\alpha_g'(\mu)}{\alpha_b'(\mu)}\right) < 0$ for all $\mu \in (0, 1)$ (shown in Lemma A.1. of Daley and Green (2014)).*
4. *$E[(1-x)Y|g] > E[(1-x)Y|b]$ for any $x \in (0, 1]$*
5. *$u_t(x, \mu)$ is strictly increasing in μ for any $x \in (0, 1]$.*
6. *$u_b(x, \mu)$ is strictly decreasing in x for any $\mu \in [0, 1]$.*

Fact 2. *In any PBE, $u_t \in [v_b, v_g)$ for any $t \in \{b, g\}$.*

The D1 Refinement

Definition 3. *We define $b_t(x, v)$ as the belief necessary to provide the t-type utility v if retention is x; that is, $u_t(b_t(x, v), x) = v$, and by $B_t(x, v) = (b_t(x, v), 1]$ the set of beliefs for which the t-type obtains strictly higher utility than v when retention is x.*

Fix $k \in [v_g, v_g)$ and $x \in [0, 1]$, and consider the belief $b_t(x, k)$ as defined in Definition 3. By Fact 1(5), there exists at most one $b_t(x, k)$ such that $u_t(b_t(x, k), x) = k$. Furthermore, the connection between b_t and B_t is immediate: if $b_t(x, k)$ exists, then $B_t(x, k) = (b_t(x, k), 1]$. If $b_t(x, k)$ fails to exist, then either $B_t(x, k) = [0, 1]$ or $B_t(x, k) = \emptyset$.

In our model, the D1 refinement can be stated as follows. Fix an equilibrium endowing expected payoffs $\{u_b, u_g\}$. Consider a retention choice x that is not in the support of either type's strategy. If $B_L(x, u_b) \subset B_H(x, u_g)$, then D1 requires that $\mu(x) = 1$ (where $\subset$ denotes strict inclusion). If $B_H(x, u_g) \subset B_L(x, u_b)$, then D1 requires that $\mu(x) = 0$.

A.2 Proofs of Section 2

Proof of Lemma 1. An originating bank always has the option to retain its loan pool. So, in any PBE, $u_t^* \geq \delta v_t$. Let $\bar{p}$ satisfy, $\bar{p}\delta v_g + (1 - \bar{p})\delta v_b = 1$. Hence, any bank with loan opportunity $p \in (\bar{p}, 1]$ originates in any PBE (and $\bar{p} < 1$ follows from Assumption 1). So the set of originated pools, O^*, has positive measure under H, and $\mu_0 = E[p|p \in O^*] \in (0, 1)$, by the belief consistency condition of PBE.

Next, given any $\mu_0 \in (0,1)$, in any PBE of the Securitization stage, $u_g^* > u_b^*$. To see, consider first any separating equilibrium: then $u_b^* = v_b < \delta v_g \leq u_g^*$, as no portion of a bad loan is retained, whereas the originator of a good loan must earn at least its full-retention payoff. If instead, the equilibrium has any degree of pooling on some retention level x, then

$$u_t^* = u_t(x, \mu(x)) = \mathbb{E}_R[P(x,R)|t] + \delta x v_t,$$

where both $\mathbb{E}_R[P(x,R)|g] > \mathbb{E}_R[P(x,R)|b]$ and $\delta x v_g > \delta x v_b$. The second inequality is immediate, and the first inequality follows from $\mu(x)$ being non-degenerate since there is (some) pooling on x, and the rating being informative.

Given that $u_g^* > u_b^*$, a bank's expected payoff from origination, $pu_g^* + (1-p)u_b^*$, is strictly increasing in p, implying that there is a cutoff lending standard, $p^* \leq \bar{p} < 1$. Finally, to see that $p^* > 0$, suppose not. Then $\mu_0^* = A(0) = \xi$. We already have that in any PBE, $u_b^* < u_g^*$. Hence, $u_b^* < \xi u_g^* + (1-\xi)u_b^* \leq \xi v_g + (1-\xi)v_b < 1$ since $\xi v_g + (1-\xi)v_b$ is the highest possible average bank payoff in the Securitization stage given $\mu_0^* = \xi$, and the last inequality is Assumption 1. But if $u_b^* < 1$, then a bank with p sufficiently close to 0 earns negative expected profit by originating, contradicting $p^* = 0$. □

A.3 Proofs of Section 3.3

Proof of Proposition 1. To check that this is a PBE, we need to check that neither type wishes to deviate at any stage. [Securitization Stage] First, an originator of a bad loan pool (b-type) does not profit from deviating since the retention of those holding good loan pools (g-types), x_g, is chosen so that the incentive compatibility (IC) for b-type binds. Second, a binding IC for the b-type implies a slack IC for the g-type since $v_g > v_b$ and thus $v_b < x_g v_g + \delta(1-x_g)v_g$. The following off-equilibrium beliefs: $\mu(x) = 0$ for all $x < x_g$ and $\mu(x) = 1$ for all $x \geq x_g$ satisfy D1 and support this equilibrium. Single-crossing ensures that the LCSE is the unique equilibrium that satisfies D1 (see DeMarzo (2005)).

[Origination Stage] From the previous results, the payoffs associated with originating a g- and a b-type loan pool, respectively, are:

$$u_g^{LC} = (1-x_g)v_g + \delta x_g v_g \tag{16}$$
$$u_b^{LC} = v_b \tag{17}$$

where $u_b^{LC} < 1 < u_g^{LC}$. Most importantly, these payoffs are independent of the actual lending standard chosen by banks at the Origination Stage, p^*. Since there is a continuum of banks and lending standards are not observable by investors, deviations in individual lending standards do not impact Securitization Stage payoffs. As a result, banks choose lending standard p^* to maximize their $t = 0$ value, and there are no profitable deviations at the Origination Stage either. □

A.4 Proofs of Section 4

Lemma 6. *The solution to the following $\mathcal{M}(k)$ problem:*

$$\max_{\mu,x} \quad u_g(x,\mu) \quad s.t. \quad u_b(x,\mu) = k \tag{18}$$

denoted by $\{\mu(k), x(k)\}$ is unique and characterized by the problem's first-order conditions. In addition, $\mu(k)$ is independent of k and $x(k)$ is decreasing in k.

Proof. We write expected Securitization Stage payoffs as a function of retention level and investors beliefs as follows

$$u_t(x,\mu) = (1-x)(\alpha_t(\mu)(v_g - v_b) + v_b) + \delta x v_t \tag{19}$$

where $\alpha_t(\mu) \equiv E_R[\mu_f(\mu, r)|t]$. Let $\alpha(\mu) \equiv \alpha_g(\mu) - \alpha_b(\mu)$ be the difference between expected posteriors for prior beliefs μ. It will be useful to re-state the $\mathcal{M}(k)$ problem as follows:

$$\max_{\mu,x} \quad u_g(\mu,x) - k \quad s.t. \quad u_b(\mu,x) = k \tag{20}$$

By plugging in the corresponding expressions and the binding constraint, we obtain:

$$\max_{\mu,x} \quad (1-x)(\alpha(\mu) - \delta)(v_g - v_b) + \delta(v_g - v_b) \tag{21}$$

$$s.t. \quad k(1-x)(\alpha_b(\mu)(v_g - v_b) + v_b) + \delta x v_b = k \tag{22}$$

Let $\{\mu(k), x(k)\}$ satisfy the problem's first-order conditions

$$\alpha(\mu) - \delta - \frac{\alpha'(\mu)}{\alpha_b'(\mu)}\alpha_b(\mu) = \frac{\alpha'(\mu)}{\alpha_b'(\mu)}\frac{(1-\delta)v_b}{v_g - v_b} \tag{23}$$

$$(1-x)(\alpha_b(\mu)(v_g - v_b) + v_b) + \delta x v_b = k \tag{24}$$

First, $\mu(k)$ is given by (23) and is thus independent of k. To see that $\mu(k)$ is unique, we analyze the left-hand side (LHS) and right-hand side (RHS) of (23) separately. From Facts 1(3) and 1(2), we have that the RHS is strictly decreasing in μ, positive for $\mu < \hat{\mu}$, zero for $\mu = \hat{\mu}$, and negative otherwise. Also from Fact 1(3) we have that the LHS is strictly increasing in μ, and negative ($-\delta$) for $\mu = 0$. Therefore, if a solution to (23) exists, it is unique. Otherwise, the solution is given by the corner $\mu = 1$ (which by (24) implies a retention of x^{LC}). Finally, note that $x(k)$ is strictly decreasing in k and given by (24).

It remains to verify the second order conditions. We verify that the determinant of the Bordered Hessian is negative at our interior candidate $\{\mu(k), x(k)\}$:

$$BH = \begin{bmatrix} 0 & \frac{\partial u_b(x,\mu)}{\partial x} & \frac{\partial u_b(x,\mu)}{\partial \mu} \\ \frac{\partial u_b(x,\mu)}{\partial x} & L_{xx} & L_{x\mu} \\ \frac{\partial u_b(x,\mu)}{\partial \mu} & L_{\mu x} & L_{\mu\mu} \end{bmatrix}$$

where $L(x,\mu) = u_g(x,\mu) - \lambda\left(u_b(x,\mu) - k\right)$ and λ is the Lagrange multiplier.

$$\begin{aligned}
L_{xx} &= 0 \\
L_{\mu\mu} &= \left(\alpha_g''(\mu) - \lambda^* \alpha_b''(\mu)\right)(1-x)(v_g - v_b) \\
L_{x\mu} &= L_{\mu x} = -(v_g - v_b)(\alpha_g'(\mu) - \lambda^* \alpha_b'(\mu)) = 0
\end{aligned}$$

A sufficient condition for our solution to be a local maximum is that the bordered Hessian is negative definite. That is, $|BH_1| < 0$ and $|BH_2| > 0$. It is easy to see that $|BH_1| = -(\frac{\partial u_b(x,\mu)}{\partial x})^2 < 0$ and that $|BH_2| = -\left(\frac{\partial u_b(x,\mu)}{\partial x}\right)^2 L_{\mu\mu} > 0$. Thus, SOC are satisfied since $L_{\mu\mu} < 0$ from $\frac{\partial}{\partial \mu}\left(\frac{\alpha_g'(\mu)}{\alpha_b'(\mu)}\right) < 0$. □

Proof of Lemma 2. First see Lemma 6 and its proof, where $\tilde{\mu} = \mu(v_b)$ and $\tilde{x} = x(v_b)$. The solution to (21) is interior with $\tilde{\mu} < 1$ (and thus $\tilde{x} < x^{LC}$) if and only if condition (23) holds for an interior μ, which requires that the RHS and the LHS of (23) intersect at $\mu < 1$. In the proof of Lemma 6 we show that the LHS is negative and the RHS positive at $\mu = 0$, and that the LHS is strictly increasing while the RHS is strictly decreasing in μ for all $\mu \in (0,1)$. Thus, a necessary and sufficient condition for $\tilde{\mu} < 1$ is that LHS>RHS at $\mu = 1$:

$$\alpha(1) - \delta - \frac{\alpha'(1)}{\alpha_b'(1)}\alpha_b(1) > \frac{\alpha'(1)}{\alpha_b'(1)}\frac{(1-\delta)v_b}{v_g - v_b} \tag{25}$$

$$-\delta - \frac{\alpha_g'(1) - \alpha_b'(\mu)}{\alpha_b'(1)} > \frac{\alpha_g'(1) - \alpha_b'(\mu)}{\alpha_b'(1)}\frac{(1-\delta)v_b}{v_g - v_b} \tag{26}$$

$$\iff \frac{\alpha_b'(1)}{\alpha_g'(1)} > \frac{v_g - \delta v_b}{(1-\delta)v_g} \tag{27}$$

Since $\alpha_b'(1) = E[\Gamma(R)|b]$ and $\alpha_g'(1) = E[\Gamma(R)|g] = 1$ the result follows. This condition is a statement about the slope of the indifference curves at the LCSE outcome $\{\mu = 1, x^{LC}\}$, and it states that an interior solution exists iff:

$$\left(\frac{\partial u_g(\mu,x)}{\partial x} \,/\, \frac{\partial u_g(\mu,x)}{\partial \mu}\right)|_{\mu=1,x^{LC}} > \left(\frac{\partial u_b(\mu,x)}{\partial x} \,/\, \frac{\partial u_b(\mu,x)}{\partial \mu}\right)|_{\mu=1,x^{LC}}$$

That is, if the slope of the indifference curve at the LCSE outcome is steeper for the g-type than it is for the b-type, breaking the single-crossing condition necessary for separation. □

Proof of Proposition 2. Let $\{\mu(k), x(k)\}$ be the solution to the constrained maximization problem $\mathcal{M}(k)$ in Lemma 6. From that same Lemma, this solution is unique, $\mu(k)$ is constant and $x(k)$ is continuous and decreasing in $k \in [v_g, v_g]$. Let $\tilde{\mu} = \mu(v_b)$ and $\tilde{x} = x(v_b)$. By Lemma 2, the equilibrium is not separating if ratings are sufficiently informative in the following sense: $E[\Gamma(R)|b] > \frac{v_g - \delta v_b}{v_g - v_b}$.

The next step is to show that the equilibrium proposed in Proposition 2 is a PBE that satisfies D1. Before doing so, we introduce the following definitions. Let S_t be the support of the t-type's

strategy. In the proposed unique equilibrium, the good type plays a pure strategy, denoted it x_g, so $S_g = \{x_g\}$, while the bad type could mix, and thus $S_b \subseteq \{0, x_g\}$. For completeness, we must specify the off-path beliefs: $\mu(x) = 0$ for all $x \neq x_g$.

Verifying that the proposed profile is a PBE is straightforward. To see that it satisfies D1, fix a μ_0 and consider the Proposition's unique equilibrium candidate. Denote the good type's equilibrium payoff u_g^e and the bad type's equilibrium payoff k, so $x_g = \tilde{x}(k)$. Let x be an arbitrary retention level in $[0,1]$ such that $x \neq \tilde{x}(k)$. First, if $B_b(x,k) = [0,1]$, then the low type could deviate to x and obtain a payoff strictly greater than k, regardless of $\mu(x)$, breaking the PBE. Hence, either $b_b(x,k) \in (0,1]$ exits or $u_b(x,1) < k$. If $b_b(x,k)$ exits, then since $\{\tilde{x}(k), \tilde{\mu}(k)\}$ is the unique solution to (18), $u_g(x, b_b(x,k)) < u_g(\tilde{x}(k), \tilde{\mu}(k)) = u_g^e$. By Fact 1(5) then, $b_g(x, u_g^e) > b_b(x,k)$ implying $B_g(x, u_g^e) \subseteq B_b(x,k)$. So, $\mu(x) = 0$ is consistent with D1. If instead $u_b(x,1) < k$ (so $B_b(x,k) = \emptyset$), then there exists a unique $\epsilon > 0$ such that $u_b(x-\epsilon, 1) = k$. Since $\{\tilde{x}(k), \tilde{\mu}(k)\}$ solves (18), $u_g(\tilde{x}(k), \tilde{\mu}(k)) \geq u_g(x-\epsilon, 1) > u_g(x,1)$. Hence, $B_g(x, u_h^e) = \emptyset$ as well, and D1 places no restriction on $\mu(x)$.

We now establish uniqueness. Fix an equilibrium with $u_g = u_g^e$ and $u_b = k$. Since banks with bad loan pools have the option to choose the same retention as banks with good loan pools, $u_b(x, \mu(x)) \leq k$ for all $x \in S_g$. Fix now $x \in S_g$ and suppose that $u_b(x, \mu(x)) < k$. Then $x \notin S_b$, so $\mu(x) = 1 = b_g(x, u_g^e)$ and $B_b(x,k) = \emptyset$. Further, it must be that $x \neq 0$ since $u_b(0,1) = v_g > k$. Then for $\epsilon > 0$ small enough $b_g(x-\epsilon, u_g^e) \in (0,1)$ and $B_b(x-\epsilon, k) = \emptyset$. Therefore, $x - \epsilon \notin S_b$ and $\mu(x-\epsilon) = 1$ (by belief consistency if $x - \epsilon \in S_g$, by D1 if not). Since $u_g(x-\epsilon, 1) > u_g(x,1) = u_g^e$, the high type would gain by deviating to $x - \epsilon$, breaking the equilibrium. Therefore, $u_b(x, \mu(x)) = k$, or equivalently $\mu(x) = b_b(x,k)$, for all $x \in S_g$.

Suppose now there exists $x \in S_g$ such that $x \neq \tilde{x}(k)$. Then

$$u_g(x, \mu(x)) = u_g(x, b_b(x,k)) < u_g(\tilde{x}(k), \tilde{\mu}(k)) = u_g(\tilde{x}(k), b_b(\tilde{x}(k), k)),$$

and thus $b_g(\tilde{x}(k), u_h^e) < \tilde{\mu}(k) = b_b(\tilde{x}(k), k)$. D1 then implies that $\mu(\tilde{x}(k)) = 1$, meaning that deviating to $\tilde{x}(k)$ is profitable for the high type and breaking the equilibrium. Hence, if the b-type's equilibrium payoff is k, then $S_g = \{\tilde{x}(k)\}$ and $\mu(\tilde{x}(k)) = \tilde{\mu}(k)$.

Further, if the b-type selects $x \notin S_g \cup \{0\}$, then $\mu(x) = 0$, and $u_b(x,0) < u_b(0,0) \leq u_b(0, \mu(0))$ for any value of $\mu(0)$. It could therefore profitably deviate to $x = 0$. Hence, $S_b \subseteq S_g \cup \{0\}$.

The final step is to characterize which values of $u_b = k$ are consistent with equilibrium, which depends on the prior, μ_0. Recall that $\tilde{\mu}$ does not vary with k. First, let $\mu_0 < \tilde{\mu}$, and let $u_b = k$. Therefore, $S_g = \{\tilde{x}(k)\}$ and $\mu(\tilde{x}(k)) = \tilde{\mu} > \mu_0$. For this belief to be consistent with seller strategies, $S_b \neq \{\tilde{x}(k)\}$. Hence, $S_b = \{\tilde{x}(k), 0\}$ and $k = v_b$. The precise mixing probabilities given in the proposition are required for the Bayesian consistency: $\mu(\tilde{x}(v_b)) = \tilde{\mu}(v_b)$. Second, let $\mu_0 \geq \tilde{\mu}$. Then $S_g = S_b = \{\tilde{x}(k)\}$ is consistent with $\tilde{\mu}(\tilde{x}(k)) = \tilde{\mu}(k) = \mu_0$, as stated in the Proposition.

□

Proof of Proposition 3. An equilibrium is pinned down by μ_0 such that $A^{-1}(\mu_0) \in \Psi(\mu_0)$. First, note that A^{-1} is continuous and increasing ($E[p'|p' \geq p]$ is increasing in p) with $A^{-1}(\xi) = 0$ and $A^{-1}(1) = 1$. Second, $\Psi(\mu_0)$ is constant for $\mu_0 < \tilde{\mu}$ at $\bar{p} \equiv \frac{1-v_b}{u_g(\tilde{x},\tilde{\mu})-v_b} > p^{FB}$. At $\tilde{\mu}$, $\Psi(\tilde{\mu}) = [\frac{1-u_b(0,\tilde{\mu})}{u_g(0,\tilde{\mu})-u_b(0,\tilde{\mu})}, \bar{p}]$. And for $\mu_0 > \tilde{\mu}$, $\Psi(\mu_0) = \max\{\tilde{\Psi}(\mu_0), 0\}$ where $\tilde{\Psi}(\mu_0) \equiv \frac{1-u_b(0,\mu_0)}{u_g(0,\mu_0)-u_b(0,\mu_0)}$ with

$\lim_{\mu_0 \to 1} \tilde{\Psi}(\mu_0) = -\infty$ (and thus $\lim_{\mu_0 \to 1} \Psi(\mu_0) = 0$). For existence and uniqueness, it suffices to show that $\tilde{\Psi}$ is strictly decreasing, since this guarantees that A^{-1} intersects Ψ at a unique μ_0. Let $G = \{\mu_0 : 0 \leq \Psi(\mu_0) \leq 1\}$. Let $\mu^* \in [0,1]$ be an equilibrium of the full game, then $\mu^* \in G$. In addition, for any $\mu \in G$, it must be that $u_g(0,\mu) \geq 1 \geq u_b(0,\mu)$. With this, and the fact that both $u_g(0,\cdot)$ and $u_b(0,\cdot)$ are increasing in μ_0, it is straightforward to check that $\tilde{\Psi}$ decreases in μ_0 for all $\mu_0 \in G$. □

A.5 Proofs for Section 4.3

Definition 4. *Let* $\tilde{\Psi}(\mu_0) \equiv \max\left\{\frac{1-u_b(0,\mu_0)}{u_g(0,\mu_0)-u_b(0,\mu_0)}, 0\right\}$.

Lemma 7. *Under any rating system,*

1. *If* $\tilde{\Psi}(\mu_0) \in (0,1) > 0$, *then* $\tilde{\Psi}'(\mu_0) < 0$.
2. $\tilde{\Psi}(p^{FB}) = p^{FB}$.
3. *With binary symmetric ratings, if* $\tilde{\Psi}(\mu_0) > 0$, *then* $\text{sign}\left(\frac{d\tilde{\Psi}(\mu_0)}{d\gamma}\right) = \text{sign}(\mu_0 - p^{FB})$.

Proof. For (1), see Proof of Proposition 3.

For (2), because investor earn zero expected profit, expected bank gross return in the Securitization stage must equal the expected value of their offering. Therefore, for any μ_0,

$$\begin{aligned} \mu_0 u_g(0,\mu_0) + (1-\mu_0)u_b(0,\mu_0) &= \mu_0 v_g + (1-\mu_0)v_b \\ u_g(0,\mu_0) &= v_g + \frac{1-\mu_0}{\mu_0}(v_b - u_b(0,\mu_0)) \end{aligned} \tag{28}$$

Substituting (28) into the definition for $\tilde{\Psi}$,

$$\tilde{\Psi}(\mu_0) = \max\left\{\frac{1-u_b(0,\mu_0)}{v_g + \frac{1-\mu_0}{\mu_0}(v_b - u_b(0,\mu_0)) - u_b(0,\mu_0)}, 0\right\}$$

Finally, using that $p^{FB} = \frac{1-v_b}{v_g - v_b}$,

$$\begin{aligned} \tilde{\Psi}(p^{FB}) &= \max\left\{\frac{1-u_b(0,p^{FB})}{v_g + \frac{1-p^{FB}}{p^{FB}}(v_b - u_b(0,p^{FB})) - u_b(0,p^{FB})}, 0\right\} \\ &= \max\left\{\frac{1-u_b(0,p^{FB})}{v_g + \frac{v_g-1}{1-v_b}(v_b - u_b(0,p^{FB})) - u_b(0,p^{FB})}, 0\right\} = p^{FB} > 0. \end{aligned}$$

For (3), we compute the derivative directly

$$\frac{\partial \tilde{\Psi}(\mu_0)}{\partial \gamma} = \frac{\mu_0 v_g + (1-\mu_0)v_b - 1}{(2\gamma-1)^3(1-\mu_0)\mu_0(v_g - v_b)} \tag{29}$$

The sign of this derivative is given by the sign of its numerator, $\mu_0 v_g + (1-\mu_0)v_b - 1$. Since the numerator is strictly increasing in μ_0 and it takes value zero when $\mu_0 = p^{FB}$, the result follows. □

Proof of Proposition 4. Let $\{\psi_b^n, \psi_g^n\}_{n=1}^{\infty}$ limit to perfect screening. From (1), p is a monotone, bijective function of the likelihood ratio $L(s)$. By Definition 2 then, as $n \to \infty$,

$$H^n(d) - H^n(c) = \Pr(p \in (c,d]) \to 0$$

for all $0 < c < d < 1$. Therefore, $H^n(p)$ limits to some constant K for all $p \in (0,1)$. For any screening technology, $E[p] = \xi$ by the Law of Iterated Expectation, implying K must be $1-\xi$.

Next, by definition, for any screening technology $A(1) = 1$. Fix $p \in (0,1)$, and $a \in (p,1)$.

$$\begin{aligned} A^n(p) = E[p'|p' \geq p] &\geq \Pr(p' \in [p,a)|p' \geq p) \cdot p + \Pr(p' \geq a|p' \geq p) \cdot a \\ &= \frac{H^n(a) - H^n(p)}{1 - H^n(p)} \cdot p + \frac{1 - H^n(a)}{1 - H^n(p)} \cdot a. \end{aligned} \tag{30}$$

Since $H^n(p) \to 1-\xi$ for all $p \in (0,1)$, the expression in (30) limits to a as $n \to \infty$. Therefore, as $n \to \infty$, $A^n(p)$ grows at least as large as any $a \in (p,1)$. Since, $A^n(p) \leq 1$ for all n, $A^n(p) \to 1$ for all $p > 0$. Equivalently, $(A^n)^{-1}(\mu_0) \to 0$ for all $\mu_0 \in (\xi, 1)$. Hence, the single intersection of $(A^n)^{-1}$ and Ψ, $(\mu_0^{*,n}, p^{*,n}) \to (\bar{\mu}, 0)$. Letting S^* denote the supply of credit in the limit, we have $\Pr(t = g|origination) = \frac{\xi}{S^*}$ which must equal $\bar{\mu}$. Hence, $S^* = \frac{\xi}{\bar{\mu}}$. □

Proof of Lemma 3. For statement (1) in the lemma, we analyze how $\{\tilde{x}, \tilde{\mu}\}$ from Proposition 2 change with rating informativeness γ. After some algebra, we have that the $\{\tilde{x}, \tilde{\mu}\}$ are given by the solution to:

$$\frac{(1-\delta)\, v_b}{v_g - v_b} = (\alpha(\mu^*) - \delta)\, \frac{\alpha_b'(\mu^*)}{\alpha'(\mu^*)} - \alpha_b(\tilde{\mu}) \tag{31}$$

$$x^* = \frac{\alpha_b(\tilde{\mu})\,(v_g - v_b)}{\alpha_b(\tilde{\mu})\,(v_g - v_b) + (1-\delta)v_b} \tag{32}$$

We proceed to characterize how this solution changes with γ. We have that:

$$\alpha_g(\mu) = \frac{\mu\gamma^2}{\mu\gamma + (1-\mu)(1-\gamma)} + \frac{\mu(1-\gamma)^2}{\mu(1-\gamma) + (1-\mu)\gamma} \tag{33}$$

$$\alpha_b(\mu) = \frac{\mu\gamma(1-\gamma)}{\mu\gamma + (1-\mu)(1-\gamma)} + \frac{\mu\gamma(1-\gamma)}{\mu(1-\gamma) + (1-\mu)\gamma} \tag{34}$$

$$\alpha(\mu) = \frac{(2\gamma-1)^2\mu(1-\mu)}{(\mu\gamma + (1-\mu)(1-\gamma))(\mu(1-\gamma) + (1-\mu)\gamma)} \tag{35}$$

After some algebra, from the RHS of condition (31) we have that:

$$\frac{\partial RHS}{\partial \gamma}|_{\mu=\tilde{\mu}} = \frac{\delta}{(2\gamma-1)^3(1-2\tilde{\mu})}$$
$$\frac{\partial RHS}{\partial \mu}|_{\mu=\tilde{\mu}} = -\frac{\alpha(\tilde{\mu})-\delta}{\alpha(\tilde{\mu})^2}\frac{\partial}{\partial \mu}\left(\frac{\alpha_g'(\tilde{\mu})}{\alpha_b'(\tilde{\mu})}\right)$$

If $\delta > \alpha(\mu)$ for all $\mu \in [0,1]$, then from FOC the solution requires $\alpha'(\tilde{\mu}) < 0 \iff \tilde{\mu} > \arg\max_\mu \alpha(\mu) = \frac{1}{2}$ (see Fact 2). As a result, $\frac{\partial RHS}{\partial \gamma} < 0$ and $\frac{\partial RHS}{\partial \tilde{\mu}} < 0$. Otherwise, $\max_\mu \alpha(\mu) > \delta$ and, as stated in the proof Proposition 2, the solution has to satisfy $\alpha(\tilde{\mu}) > \delta$, which from FOC requires $\alpha'(\tilde{\mu}) > 0$. Therefore, $\tilde{\mu} < \frac{1}{2}$, and thus $\frac{\partial RHS}{\partial \gamma} > 0$ with $\frac{\partial RHS}{\partial \tilde{\mu}} > 0$. If follows that as γ increases, $\tilde{\mu}$ has to decrease.

We have established that $\tilde{\mu}$ decreases in γ. It remains to characterize how $\tilde{x}$ changes in γ. Let RHS_c denote the left-hand side of the constraint (32). Then, we have that:

$$\frac{d\tilde{x}}{d\gamma} = \frac{\partial RHS_c}{\partial \tilde{\mu}}\frac{d\tilde{\mu}}{d\gamma} + \frac{\partial RHS_c}{\partial \gamma} < 0 \tag{36}$$

Where the results follows from (i) $\frac{\partial \alpha_b(\mu)}{\partial \gamma} < 0$ for all $\mu \in (0,1)$; (ii) $\alpha_b'(\cdot) > 0$ for all $\gamma \in (\frac{1}{2}, 1)$; and (iii) RHS_c being increasing in $\alpha_b(\mu)$ for all $\mu \in (0,1)$ since $v_b > 0$, which are all easy to check.

For statement (2) in the lemma, let $\tilde{\mu}'$ denote the $\tilde{\mu}$ after the increase in γ, which from (i) implies $\tilde{\mu}' < \tilde{\mu}$. First consider the case where $p^{FB} < \tilde{\mu}$. For all $\mu_0 > \tilde{\mu}, \Psi(\mu_0) = \tilde{\Psi}(\mu_0)$, which by Lemma 7 is increasing in γ since $\mu_0 > p^{FB}$. For $\mu_0 \in [\tilde{\mu}', \tilde{\mu}]$, $\Delta\Psi(\mu_0) = \tilde{\Psi}(\mu_0) - \frac{1-v_b}{u_g(\tilde{x},\tilde{\mu})-v_b} < 0$. For $\mu_0 < \tilde{\mu}'$, $\Psi(\mu_0) = \frac{1-v_b}{u_g(\tilde{x},\tilde{\mu})-v_b}$ which decreases in γ since $u_g(\tilde{x},\tilde{\mu})$ increases in γ. Now consider the case with $p^{FB} > \tilde{\mu}$. As before, for all $\mu_0 > p^{FB}$, we know that $\Psi(\mu_0) = \tilde{\Psi}(\mu_0)$ which increases in μ_0. For $\mu_0 < p^{FB}$, if $\mu_0 \in [\tilde{\mu}, p^{FB})$, $\Psi(\mu_0) = \tilde{\Psi}(\mu_0)$, which by Lemma 7 is now decreasing in γ since $\mu_0 - p^{FB} < 0$. For $\mu_0 < \tilde{\mu}$, we have already shown that Ψ decreases in γ.

□

Proof of Proposition 5. By definition, $A^{-1}(\mu_0) < \mu_0$ for all $\mu_0 \in [\xi, 1)$. Suppose that $p^* \geq p^{FB}$. By Lemma 7(1)-(2) then, $p^* \neq \tilde{\Psi}(\mu_0^*)$. From the structure of Ψ in Corollary 3 it then follows that $\tilde{\mu} > p^{FB}$ and $\mu_0^* \in (p^{FB}, \tilde{\mu}]$. An increase in γ implies a decrease in both $\tilde{\mu}$ and $\Psi(\mu_0)$ for $\mu_0 < \tilde{\mu}$ (Lemma 3), and therefore a decrease in the point of intersection of Ψ and the strictly increasing A^{-1} function, meaning a lower p^*. □

Proof of Proposition 6. Let μ_1 be the unique solution to $A^{-1}(\mu_1) = p^{FB}$. By definition, $A^{-1}(\mu_0) < \mu_0$ for all $\mu_0 \in [\xi, 1)$, meaning $\mu_1 > p^{FB}$. Suppose now that $\tilde{\mu} < \mu_1$. We claim that $p^* < p^{FB}$ is implied. To see this, recall that (μ_0^*, p^*) is the unique intersection of A^{-1} and Ψ. From Corollary 3 and A^{-1} strictly increasing, $\Psi(\mu_0) > p^{FB} > A^{-1}(\mu_0)$ for all $\mu_0 < \tilde{\mu} < \mu_1$. So $\mu_0^* \geq \tilde{\mu}$. If $\mu_0^* = \tilde{\mu}$, then $p^* = A^{-1}(\tilde{\mu}) < A^{-1}(\mu_1) = p^{FB}$. If, instead, $\mu_0^* > \tilde{\mu}$, then by from Corollary 3, $\Psi(\mu_0^*) = \tilde{\Psi}(\mu_0^*) < p^{FB}$, where the inequality follows from Lemma 7(1)-(2).

From Lemma 3, $\tilde{\mu}_\gamma$ is decreasing in γ. Hence, it is sufficient to show that $\lim_{\gamma\to 1} \tilde{\mu}_\gamma \leq p^{FB}$ if $v_g v_b \leq 1$. This is a matter of direct calculation. With binary-symmetric ratings, one can use

(31) to obtain the closed-form expression:

$$\tilde{\mu} = \frac{\frac{\sqrt{(\delta-1)(1-2\gamma)^2(\delta(\gamma v_b+(\gamma-1)v_g)((\gamma-1)v_b+\gamma v_g)-(1-2\gamma)^2 v_b v_g)}}{(\delta-1)(1-2\gamma)^2} + v_b}{v_b - v_g} \underbrace{\to \frac{v_b}{v_b+\sqrt{v_b v_g}}}_{\gamma\to 1} \tag{37}$$

Immediately, the limit value is no greater than $p^{FB} = \frac{1-v_b}{v_g-v_b}$ if $v_g v_b \leq 1$. □

Proof of Proposition 7. Restating (32):

$$\tilde{x} = \frac{\alpha_b(\tilde{\mu})(v_g - v_b)}{\alpha_b(\tilde{\mu})(v_g - v_b) + (1-\delta)v_b}$$

From (34), as $\gamma \to 1$, $\alpha_b(\mu) \to 0$ for all $\mu \in (0,1)$. Hence, $\tilde{x} \to 0$ as $\gamma \to 0$.

In addition, $u_t(0,\mu_0) = \mathbb{E}_R[P(0,R)|t] = v_b + \mathbb{E}_R[\mu_f(x,r)|t](v_g - v_b)$. Finally, as $\gamma \to 1$, $\mathbb{E}_R[\mu_f(x,r)|t] \to \mathbb{I}_{t=g}$, and $u_t(0,\mu_0) \to v_t$. Hence, for all μ_0, $u_t^*(\mu_0) \to v_t$ and $\Psi(\mu_0) \to p^{FB}$, as $\gamma \to 1$. Since (μ_0^*, p^*) is the unique intersection of Ψ and A^{-1}, $p^* \to p^{FB}$. □

A.6 Proofs for Section 5

Proof of Proposition 8. The skin-in-the-game rule requires all securitizers to retain at least a fraction $\bar{x}$. As a result, $\{\tilde{x}, \tilde{\mu}\}$ from Proposition 2 are now given by the solution to:

$$\begin{aligned} &\max_{\mu,x} (1-x)(\alpha_g(\mu)(v_g - v_b) + v_b) + \delta x v_g \\ &s.t. (1-x)(\alpha_g(\mu)(v_g - v_b) + v_b) + \delta x v_g = (1 - \bar{x} + \delta\bar{x})\, v_b \end{aligned} \tag{38}$$

where the only adjustment has been a change in the outside option (full information payoff) of the banks with b-type pools in the constraint. From Proposition 2, we know that the solution to (38) fully characterizes the PBE of the securitization stage with skin-in-the-game.

From the constraint, it follows that $\tilde{x} \geq \bar{x}$. Therefore, equilibrium retention levels satisfy: $x_t \in [\bar{x}, \tilde{x}]$. We have shown that $\{\tilde{x}, \tilde{\mu}\}$ are given by the problem's FOC. In particular, $\tilde{\mu}$ continues to be determined by condition (23), while $\tilde{x}$ is determined by the new constraint (38). As a result, $\tilde{\mu}$ is unaffected by the skin-in-the-game rule. In contrast, from the constraint, we know $\tilde{x}$ responds to the retention rule as follows. For $\mu > \tilde{\mu}$, equilibrium retention increases from 0 to $\bar{x}$. For $\mu = \tilde{\mu}$, retentions in the range $x \in [\tilde{x}, \bar{x}]$ can be D1-equilibria. Finally, for $\mu < \tilde{\mu}$, there is partial pooling at the new (higher) $\tilde{x}$, where banks with g-type pools retain $\tilde{x}$ and those with b-type pools mix between $\{\tilde{x}, \bar{x}\}$ sd described in Proposition 2. Since payoffs at the securitization stage, u_t^*, decrease with the skin-in-the-game rule, lending standards weakly increase, credit supply weakly decreases, and so does ex-ante efficiency. □

Proof of Proposition 9. We analyze how $\{\tilde{x}, \tilde{\mu}\}$ from Proposition 2 change with δ. We know that $\tilde{\mu}$ satisfies condition (31). Let $RHS(\tilde{\mu}, \delta)$ denote the right-hand side of this condition. We have

shown in the proof of Lemma 3 that $\frac{\partial RHS}{\partial \tilde{\mu}}$ takes the sign of $\delta - \alpha(\tilde{\mu})$. In addition, we have

$$\frac{\partial RHS}{\partial \delta} = \frac{1}{[...]^2} \frac{1}{\alpha'(\tilde{\mu})} \left[(1 - \alpha_g(\tilde{\mu})) \alpha_b'(\tilde{\mu}) + \alpha_g'(\tilde{\mu}) \alpha_b(\tilde{\mu}) \right]$$

which takes the sign of $\alpha'(\tilde{\mu})$. Since the left-hand-side of the constraint is positive, so has to be the the RHS, which requires $\alpha'(\tilde{\mu}) \times (\alpha(\tilde{\mu}) - \delta) \geq 0$. Thus, we have that $\frac{\partial \tilde{\mu}}{\partial \delta} \geq 0$. To see the effect on retention, we do a total differentiation of the constraint:

$$\begin{aligned} \left[-\alpha_b(\tilde{\mu})(v_g - v_b) - (1 - \delta) v_b \right] dx + \left[(1 - x) \alpha_b'(\tilde{\mu})(v_g - v_b) \frac{d\tilde{\mu}}{d\delta} + \tilde{x} v_b \right] d\delta &= 0 \\ \frac{d\tilde{x}}{d\delta} = \frac{\left[(1 - x) \alpha_b'(\tilde{\mu})(v_g - v_b) \frac{d\tilde{\mu}}{d\delta} + \tilde{x} v_b \right]}{\left[\alpha_b(\tilde{\mu})(v_g - v_b) + (1 - \delta) v_b \right]} &> 0 \end{aligned}$$

Finally, to study the effect of a change in δ on lending standards and credit supply, we characterize the changes in Ψ. We know that $\tilde{\mu}$ has increased, and since δ does not affect payoffs when retention is zero, $\tilde{\Psi}$ remains unaffected. For $\mu_0 < \tilde{\mu}$, we need to analyze the effect of δ on $u_g(\tilde{x}, \tilde{\mu})$:

$$\begin{aligned} u_g^* \equiv u_g(x^*, \mu^*) = \max_{\mu, x} & (1 - x) \left[\alpha_g(\mu)(v_g - v_b) + v_b \right] + \delta x v_g \\ s.t. & (1 - x) \left[\alpha_b(\mu)(v_g - v_b) + v_b \right] + \delta x v_b = v_b \end{aligned}$$

We have that

$$\frac{\partial u_g^*}{\partial \delta} = v_g - \frac{\alpha_g'(\tilde{\mu})}{\alpha_b'(\tilde{\mu})} v_b \tag{39}$$

since $\frac{\alpha_g'(\tilde{\mu})}{\alpha_b'(\tilde{\mu})}$ is the Lagrange multiplier of the constraint in this problem at $\{\tilde{\mu}, \tilde{x}\}$.

Therefore, to complete the proof it suffices to show that $\frac{\alpha_g'(\tilde{\mu})}{\alpha_b'(\tilde{\mu})} < \frac{v_g}{v_b}$. To see this, first rewrite the FOC of $\mathcal{M}(v_b)$ as

$$\frac{\alpha_g'(\tilde{\mu})}{\alpha_b'(\tilde{\mu})} = \frac{\alpha_g(\tilde{\mu}) v_g + (1 - \alpha_g(\tilde{\mu})) v_b - \delta v_g}{\alpha_b(\tilde{\mu}) v_g + (1 - \alpha_b(\tilde{\mu})) v_b - \delta v_b} \tag{40}$$

and observe that the numerator on the RHS is increasing in α_g, while the denominator is increasing in α_b, therefore

$$\frac{\alpha_g(\tilde{\mu}) v_g + (1 - \alpha_g(\tilde{\mu})) v_b - \delta v_g}{\alpha_b(\tilde{\mu}) v_g + (1 - \alpha_b(\tilde{\mu})) v_b - \delta v_b} < \frac{\alpha_g(1) v_g + (1 - \alpha_g(1)) v_b - \delta v_g}{\alpha_b(0) v_g + (1 - \alpha_b(0)) v_b - \delta v_b} = \frac{v_g}{v_b}.$$

□

A.7 Proofs of Section 6

Proof of Lemma 4. Let $\mathcal{R}$ be the set of ratings that the bank chooses to report in equilibrium, where $\mu_n(\mu,\phi)$ is the belief assigned to a bank not being rated. Define $\bar{r}$ be such that

$$\mu_f(\mu,\bar{r}) - \mu_n(\mu,\phi) = \phi,$$

where $\bar{r}$ exists since $\mu_f(\mu,r) = \frac{\mu}{\mu+(1-\mu)\Gamma(r)}$ is increasing and continuous in r for $\mu \in (0,1)$, and in any equilibrium it must be that $\mu_n(\mu,\phi) \in [\inf_r \mu_f(\mu,r), \sup_r \mu_f(\mu,r)]$. Suppose $r < \bar{r}$ and $r \in \mathcal{R}$. Since $\mu_f(\mu,r)$ is increasing in μ, $u_t(\mu,x,r) < u_t^n(\mu,n,r)$. Contradiction, since it violates bank's optimality. Now suppose there exists $r > \bar{r}$ such that $r \notin \mathcal{R}$. Then, if a bank receives rating r and chooses to pay and report it, investors assign beliefs $\mu_f(\mu,r)$ and the bank profits from this deviation. Contradiction. Thus, it must be that $\mathcal{R} = \{r : r \geq \bar{r}\}$.

Then, in any equilibrium, not being rated indicates that $r < \bar{r}(\mu,\phi)$. As a result:

$$\mu_n(\mu,\phi) = \frac{\mu}{\mu + (1-\mu)\frac{F_b(\bar{r}(\mu,\phi))}{F_g(\bar{r}(\mu,\phi)))}}.$$

It remains to show that the set $\mathcal{R}$ differs from the full set of ratings iff $\phi > 0$. ($\Leftarrow$) Let the set $\{r:\ r < \bar{r}\}$ has zero measure. Then, $\bar{r} = \inf r$ and $\mu_f(\mu,\bar{r}) - \mu_n(\mu,\bar{r}) = 0$ since by L'Hopital,

$$\lim_{r \to \inf r} \frac{F_b(r)}{F_g(r)} = \frac{f_b(r)}{f_g(r)}.$$

This is only consistent with with $\phi = 0$. ($\Rightarrow$) Now assume that $\phi = 0$. Then, $\bar{r}$ is given by: $\mu_f(\mu,\bar{r}) - \mu_n(\mu,\bar{r}) = 0$, which implies $\bar{r} = \inf r$ since $\mu_f(\mu,r) - \mu_n(\mu,r) > 0$ for $\mu \in (0,1)$ and all $r > \inf r$. Thus, $\{r:\ r < \bar{r}(\mu,\phi)\}$ has zero measure. □

Proof of Lemma 5. We have that:

$$\frac{\partial \alpha_t(\mu,\phi)}{\partial \phi} = F_t(\bar{r}(\mu,\phi)) \frac{\partial \mu_f(\mu,\bar{r})}{\partial \bar{r}} \frac{\partial \bar{r}(\mu,\phi)}{\partial \phi}|_{r=r(\mu,\phi)} \geq 0$$

and thus:

$$\frac{\partial \alpha(\mu,\phi)}{\partial \phi} = [F_g(\bar{r}(\mu,\phi)) - F_b(\bar{r}(\mu,\phi))] \left(\frac{\partial \mu_f(\mu,\bar{r})}{\partial \bar{r}} \frac{\partial \bar{r}(\mu,\phi)}{\partial \phi} \right) \leq 0$$

since it is easy to check that $\frac{\partial \mu_f(\mu,\bar{r})}{\partial \bar{r}} > 0$ and that $\frac{\partial \bar{r}(\mu,\phi)}{\partial \phi} > 0$. □

The following proposition characterizes the allocations at the securitization stage in the model with rating shopping. As we show below, the condition for an equilibrium in the securitizatin stage to exist and be unique is that $\frac{\partial}{\partial \mu}\left(\frac{\tilde{\alpha}_g'(\mu,\phi)}{\tilde{\alpha}_b'(\mu,\phi)}\right) < 0$, for all $\mu \in (0,1)$.

Proposition 10. *If $\frac{\partial}{\partial \mu}\left(\frac{\tilde{\alpha}_g'(\mu,\phi)}{\tilde{\alpha}_b'(\mu,\phi)}\right) < 0$, for all $\mu \in (0,1)$, and ratings that are sufficiently informative in the following sense: $\frac{\tilde{\alpha}_b'(1,\phi)}{\tilde{\alpha}_g'(1,\phi)} > \frac{v_g - \delta v_b - \phi}{(1-\delta)v_g - \phi}$, the unique equilibrium at the securitization stage features some degree of pooling. In particular, there exists $\{\tilde{\mu}, \tilde{x}\}$ with $u_b(\tilde{\mu},\tilde{x}) = v_b$ such that:*

- *For $\mu_0 < \tilde{\mu}$, there is partial pooling at $x = \tilde{x} < x^{LCSE}$*
- *For $\mu_0 = \tilde{\mu}$, full pooling at any level $x \in [0, \tilde{x}]$ is an equilibrium.*
- *For $\mu_0 > \tilde{\mu}$, there is full-pooling at $x = 0$*

When ratings are not sufficiently informative, the unique equilibrium is the LCSE described in Section 3.

Proof. Let $\{\mu(k), x(k)\}$ be given by the solution of the adjusted $\mathcal{M}(k)$ problem:

$$\begin{aligned} \max_{\mu,x} \quad & u_g(\mu, x, \phi) \\ s.t. \quad & u_b(\mu, x, \phi) = k \end{aligned} \tag{41}$$

where $\phi = 0$ indexes the star problem for the baseline ratings model of Section 4, and $\phi > 0$ the extension ratings shopping model of Section 6. When $\phi > 0$, the problem is:

$$\begin{aligned} \max_{\mu,x} \quad & (1-x)\left[\left(\tilde{\alpha}_g(\mu, \phi) - \phi\right)(v_g - v_b) + v_b\right] + \delta x v_g \\ s.t. \quad & (1-x)\left[\left(\tilde{\alpha}_b(\mu, \phi) - \phi\right)(v_g - v_b) + v_b\right] + \delta x v_b = k \end{aligned} \tag{42}$$

In an abuse of notation, let $\tilde{\alpha}'(\mu, \phi)$ denote $\frac{\partial \tilde{\alpha}(\mu,\phi)}{\partial \mu}$. From the first-order conditions of the $\mathcal{M}(k)$ problem, we obtain:

$$\frac{\tilde{\alpha}'_g(\tilde{\mu}, \phi)}{\tilde{\alpha}'_b(\tilde{\mu}, \phi)} = \frac{\left(\tilde{\alpha}_g(\tilde{\mu}, \phi) - \delta\right)(v_g - v_b) + (1-\delta)v_b - \phi}{\tilde{\alpha}_b(\tilde{\mu}, \phi)(v_g - v_b) + (1-\delta)v_b - \phi} \tag{43}$$

$$x^* = \frac{\left(\tilde{\alpha}_b(\tilde{\mu}, \phi) - \phi\right)(v_g - v_b) + (1-\delta)v_b - (k - \delta v_b)}{\left(\tilde{\alpha}_b(\tilde{\mu}, \phi) - \phi\right)(v_g - v_b) + (1-\delta)v_b} \tag{44}$$

Let $\tilde{\alpha}(\mu, \phi) \equiv \tilde{\alpha}_g(\mu, \phi) - \tilde{\alpha}_b(\mu, \phi)$. After some algebra, condition (43) becomes:

$$\left(\tilde{\alpha}(\mu^*, \phi) - \delta\right)\frac{\tilde{\alpha}'_b(\mu^*, \phi)}{\tilde{\alpha}'(\mu^*, \phi)} - \tilde{\alpha}_b(\tilde{\mu}, \phi) = \frac{(1-\delta)v_b - \phi}{v_g - v_b} \tag{45}$$

The RHS is constant and independent of μ. As for the LHS, note that:

$$\frac{\partial LHS(\mu, \phi)}{\partial \mu} = -\frac{\tilde{\alpha}(\mu, \phi) - \delta}{\tilde{\alpha}(\mu, \phi)^2}\frac{\partial}{\partial \mu}\left(\frac{\tilde{\alpha}'_g(\mu, \phi)}{\tilde{\alpha}'_b(\mu, \phi)}\right)$$

As shown in Proposition 2, if $\frac{\partial}{\partial \mu}\left(\frac{\tilde{\alpha}'_g(\mu,\phi)}{\tilde{\alpha}'_b(\mu,\phi)}\right) < 0$, then there is a unique $\tilde{\mu}$ that solves the maximization problem.

In addition, the solution to the above problem is interior with $\tilde{\mu} < 1$ (and thus $x < x^{LC}$) if and only if condition (45) hold for an interior μ. Since the LHS of the condition is positive at $\mu = 0$ and decreasing, and the RHS of the condition is negative at $\mu = 0$ and increasing, it follows that

an intersection between the RHS and the LHS does exists iff there exists $\bar{\mu}$ such that:

$$\begin{aligned}\frac{\tilde{\alpha}'_g(1,\phi)}{\tilde{\alpha}'_b(1,\phi)} &< \frac{(\tilde{\alpha}_g(1,\phi)-\delta)(v_g-v_b)+(1-\delta)v_b-\phi}{\tilde{\alpha}_b(1,\phi)(v_g-v_b)+(1-\delta)v_b-\phi} \\ &< \frac{(1-\delta)(v_g-v_b)+(1-\delta)v_b-\phi}{v_g-v_b+(1-\delta)v_b-\phi} = \frac{(1-\delta)v_g-\phi}{v_g-\delta v_b-\phi}\end{aligned}$$

This is an adjusted Γ-informativeness condition, which is now a function of ϕ, since the informativeness of ratings is a function of the cost of being rated.

It remains to verify the second order conditions of this problem. We verify that the determinant of the Bordered Hessian is negative at our interior critical point, where λ denotes the multiplier on the constraint of the problem:

$$BH = \begin{bmatrix} 0 & \frac{\partial u_b(\mu,x,\phi)}{\partial x} & \frac{\partial u_b(\mu,x,\phi)}{\partial \mu} \\ \frac{\partial u_b(\mu,x,\phi)}{\partial x} & L_{xx} & L_{x\mu} \\ \frac{\partial u_b(\mu,x,\phi)}{\partial \mu} & L_{\mu x} & L_{\mu\mu} \end{bmatrix}$$

where $L(x,\mu,\phi) = u_g(\mu,x,\phi) - \lambda\left(u_b(\mu,x,\phi)-k\right)$ and λ is the Lagrange multiplier.

$$\begin{aligned}L_{xx} &= 0 \\ L_{\mu\mu} &= \left(\tilde{\alpha}''_g(\tilde{\mu},\phi) - \lambda^*\tilde{\alpha}''_b(\tilde{\mu},\phi)\right)(1-\tilde{x})(v_g-v_b) \\ L_{x\mu} &= L_{\mu x} = -(v_g-v_b)(\tilde{\alpha}_g(\tilde{\mu},\phi) - \lambda^*\tilde{\alpha}_b(\tilde{\mu},\phi)) = 0\end{aligned}$$

A sufficient condition for our solution to be a local maximum is that the bordered Hessian is negative definite. That is, $|BH_1| < 0$ and $|BH_2| > 0$. It is easy to see that $|BH_1| = -(\frac{\partial u_b(\mu,x,\phi)}{\partial x})^2 < 0$ and that $|BH_2| = -\left(\frac{\partial u_b(\mu,x,\phi)}{\partial x}\right)^2 L_{\mu\mu} > 0$. Thus, SOC are satisfied if $L_{\mu\mu} < 0$, that is, if $\frac{\partial}{\partial \mu}\left(\frac{\tilde{\alpha}'_g(\mu,\phi)}{\tilde{\alpha}'_b(\mu,\phi)}\right)|_{\mu=\tilde{\mu}} < 0$.

Finally, the proof that this characterizes the solution of unique D1 PBE of the securitization stage is as the one in Proposition 2. □

Let $u_t(\mu,\phi)$ denote the outcomes of the securitization stage, as described in Proposition 10. Then, the decision to originate a given loan pool is as in the baseline model, and determined by $\Psi(\mu,\phi) = \frac{1-u_b(\mu,\phi)}{u_g(\mu,\phi)-u_b(\mu,\phi)}$. An equilibrium $\{\tilde{x},\mu_0,x(\mu_0)\}$ of the model with rating shopping exists if there exists $\tilde{x}$ such that $A(\tilde{x}) = \mu_0$ and $p(\tilde{x}) = \Psi(\mu_0)$.

B Security Design

Thus far, we have studied how much banks will retain taking the "class" of securities, $F = (1-x)Y$, as given. In this section, we demonstrate that the main results of the paper remain unchanged when banks can choose the design of the security. Our demonstration relies heavily on Daley et al. (2016b) (henceforth, DGV16) in which, we study the optimal security design in the presence of public information (e.g., ratings).

B.1 Summary

In DGV16, we characterize the equilibrium of the securitization stage where the securitizer can choose any *security*, $F = \psi(Y)$, to offer for sale. Specifically, for any realization of the cash flow y, $\psi(y)$ is the amount paid to the purchaser of the security and $y - \psi(y)$ is the amount retained by the securitizer, where $0 \leq \psi(y) \leq y$ for all y. As in much of the security design literature, we focus on securities for which both the amount paid and the amount retained must be nondecreasing in y. We retain the assumption that each pool of loans is either good or bad (i.e., $t \in \{b, g\}$). Further, a type-t pool delivers a cash flow distributed according to the cdf and pdf denoted by Π_t and π_t respectively on a common support $[0, \bar{y}]$, where $\frac{\pi_g(y)}{\pi_b(y)}$ is weakly increasing (i.e., MLRP holds). We refer to this setting as the *Security Design Game*.

We show that the form of the security that emerges in the equilibrium of the Security Design Game depends on a new measure of rating informativeness (denoted RI) and the cost of retention, δ.[24] In particular, if $RI < \delta$, then the securitizer issues debt and retains a levered-equity claim, while if $RI > \delta$, then the securitizer issues a levered-equity claim and retains debt (see DGV16, Theorem 1).[25] We also show that a result analogous to Proposition 2 holds in the Security Design Game. That is, when ratings are informative enough the unique equilibrium involves some degree of pooling (either partial or full) whereas when ratings are not sufficiently informative the unique equilibrium is separating (see DGV16, Theorem 2).

Below, we characterize the equilibrium payoffs of the security design game as a function of the prior belief, μ_0, about the type of the pool. Denote these payoffs by $u_\theta^{SDG}(\mu_0)$. Importantly, these payoff functions share similar characteristics to the ones derived earlier, where the bank is restricted to issuing (and retaining) equity.

Fact 3. *There exists a unique equilibrium of the Security Design Game. Moreover, the following statements are generically true.*

(i) $u_\theta^{SDG}(\mu_0)$ *is continuous and weakly increasing in* μ_0.

(ii) There exists a $\mu_1 \in (0, 1]$ *such that for all* $\mu_0 \leq \mu_1$, $u_g^{SDG}(\mu_0) \in (u_g^*(\mu_0), v_g)$ *and* $u_b^{SDG}(\mu_0) = v_b$.

Proof. See DGV16. □

B.2 Security Design: Lending Standards and Credit Supply

Let us now turn to the implications for lending standards and the supply of credit when the securitization stage is replaced by the security design game. Analogous to Corollary 3, the lending standard is given by

$$p^{SDG} \in \Psi^{SDG}(\mu_0) \equiv \max\left\{\frac{1 - u_b^{SDG}(\mu_0)}{u_g^{SDG}(\mu_0) - u_b^{SDG}(\mu_0)}, 0\right\} \tag{46}$$

From Fact 3, we know that $\Psi^{SDG}(\mu_0)$ is decreasing and continuous in the relevant range. Hence, there is a unique solution to (46) and a unique level of credit supply that is consistent with an

[24] The measure is defined as $RI \equiv \max_\mu \alpha_g(\mu) - \alpha_b(\mu)$.

[25] If $RI = \delta$, the security designed in equilibrium is not pinned down.

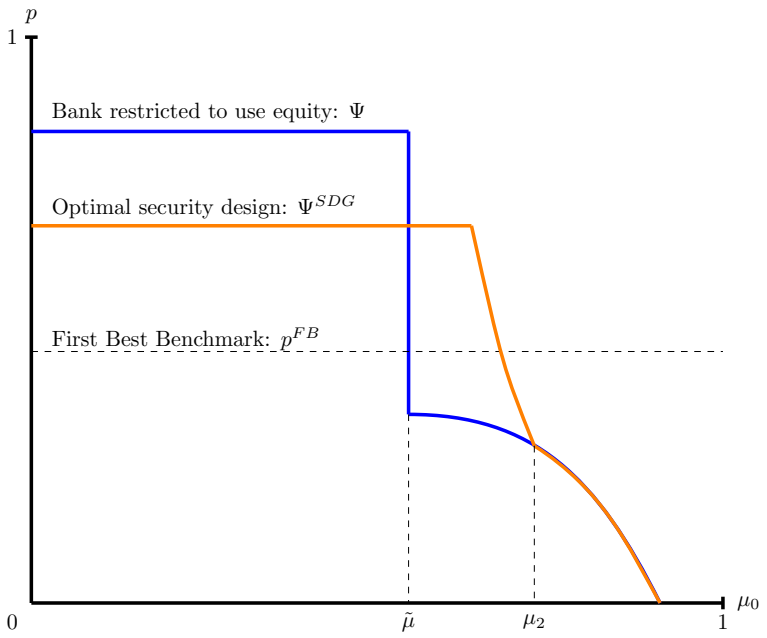


FIGURE 7: Security Design and Lending Standards.

equilibrium. We also know that $\Psi^{SDG}(\mu_0) \in \left(\frac{1-v_b}{v_g-v_b}, \Psi(\mu_0)\right)$ for $\mu_0 \leq \tilde{\mu}$. In this region, a bank with a bad pool of loans gets the full-information value, while the bank with a good pool of loans does strictly better by being able to choose the security design, which eases banks lending standards.

Provided the rating is Γ-informative, one can also show that Ψ^{SDG} lies weakly above Ψ for all priors above a threshold (and strictly above for at least some priors).[26] In the region where the inequality is strict, banks use retention to signal quality when they can design the security, but rely purely on ratings when they are restricted to equity. These properties are summarized in Figure 7 and formally stated in Lemma 8.

Lemma 8. *If the rating is Γ-informative, then the following statement is generically true.*[27]

(i) $\Psi^{SDG}(\mu_0) \in \left(\frac{1-v_b}{v_g-v_b}, \Psi(\mu_0)\right)$ *for all* $\mu_0 < \tilde{\mu}$.

(ii) $\Psi^{SDG}(\mu_0) \geq \Psi(\mu_0)$ *for all* $\mu_0 \geq \tilde{\mu}$, *where the inequality holds strictly for at least some* μ_0.

If the rating is not Γ-informative then $\Psi^{SDG} < \Psi$ for all μ_0.

The next proposition summarizes the implications of security design on the equilibrium lending standard.

Proposition 11. *If each bank can optimally design the security that they issue (i.e., if the securitization stage is replaced with the security design game) then:*

[26] Under some conditions (e.g., if the rating is not β-informative at $\bar{y}$), Ψ^{SDG} lies strictly above Ψ for *all* priors above a threshold.

[27] In the non-generic case that $\max_\mu \alpha(\mu) = \delta$, the security design game has the exact same payoffs as the securitization stage and hence $\Psi^{SDG} = \Psi$.

- *If $p(A^{-1}(\tilde{\mu})) > \Psi^{SDG}(\tilde{\mu})$: the lending standard decreases, which improves productive efficiency, though an undersupply of credit persists.*
- *If $p(A^{-1}(\tilde{\mu})) < \Psi^{SDG}(\tilde{\mu})$: the lending standard increases, which may improve or reduce productive efficiency.*
- *If $p(A^{-1}(\tilde{\mu})) < \Psi^{SDG}(\tilde{\mu})$, where $\tilde{\mu}$ is defined implicitly by $\Psi^{SDG}(\tilde{\mu}) = p(q^e)$: the lending standard increases, which improves productive efficiency, though oversupply of credit persists.*

In essence, Proposition 11 says that the ability to design the security improves productive efficiency when the distribution of pools is sufficiently strong or weak. For intermediate distributions, optimal security design results in tighter lending standards, which may improve or hinder productive efficiency depending on the distribution of appraisals.

Proof of Lemma 8. The proof of the case in which the rating is not Γ-informative is trivial. Regardless of the prior, in any equilibrium of the security design game, the b-payoff is weakly greater than v_b and the g-payoff is strictly greater than u_g^{LC}. Noting the Ψ is decreasing in both u_b and u_g (whenever it is non-negative) yields the result.

When the rating is Γ-informative. The statement in (i) follows immediately from Fact 3. To prove (ii), it will be useful to break the proof into three cases.

Case 1: Rating is not β-informative at $\bar{y}$. In this case, the equilibrium of the security design game does not converge to full-pooling with zero retention as $\mu_0 \to 1$. In particular, g retains a non-trivial levered equity claim and the low-type either pools are full separates with zero retention. In either case, $u_t^{SDG}(\mu_0) < u_t^*(\mu_0)$ for all $\mu_0 \geq \tilde{\mu}$, which implies the result.

Case 2: Rating is β-informative at $\bar{y}$, but not α-informative. In this case, there is full-pooling with zero retention for μ_0 large enough. Let μ_2 denote the smallest prior belief at which the zero-retention, full-pooling outcome obtains in the SDG. It suffices to show that $\mu_2 > \tilde{\mu}$. The FOC characterizing μ_2 is

$$g(\mu_2) = \frac{\pi_g(\bar{y})}{\pi_b(\bar{y})} - 1, \tag{47}$$

where $g(\mu) \equiv \frac{1-\delta}{(\alpha(\mu)-\delta)\frac{\alpha_b'(\mu)}{\alpha'(\mu)} - \alpha_b(\mu)}$. The FOC characterizing $\tilde{\mu}$ (see (31)) can be written as $g(\tilde{\mu}) = \frac{v_g}{v_b} - 1$. Note that $g(\mu) \leq 0$ for all $\mu < \hat{\mu} \equiv \arg\max_\mu \alpha(\mu)$, whereas the RHS of the FOC is strictly positive in both cases. Hence, it must be that both μ_2 and $\tilde{\mu}$ are above $\hat{\mu}$. Further, g is positive, strictly increasing, continuous, and tends to $+\infty$ on the interval $(\hat{\mu}, \bar{m})$ and g is negative above $\bar{m}$, where $\bar{m}$ is such that the denominator of g is zero. Finally, by MLRP $\frac{\pi_g(\bar{y})}{\pi_b(\bar{y})} > \frac{v_g}{v_b}$. Therefore $g(\mu_2) > g(\tilde{\mu})$ and thus $\mu_2 > \tilde{\mu}$.

Case 3: Rating is α-informative. The proof for this case is similar to case 2. Again, there is full-pooling with zero retention for μ_0 large enough in the SDG. Let μ_2 denote the smallest prior belief at which the zero-retention, full-pooling outcome obtains in the SDG. We will show that $\mu_2 > \tilde{\mu}$. The FOC characterizing μ_2 in this case is

$$g(\mu_2) = \frac{1 - \Pi_g(\underline{y})}{1 - \Pi_b(\underline{y})} - 1 = 0 \tag{48}$$

where $g(\mu)$ is defined in Case 2. When the rating is α-informative, g is strictly decreasing over the relevant domain and equal to zero only at $\hat{\mu}$.[28] Therefore, $\mu_2 = \hat{\mu}$, whereas $g(\tilde{\mu}) = \frac{v_g}{v_b} - 1 > 0 \implies \tilde{\mu} < \hat{\mu}$, which implies the desired result. □

Proof of Proposition 11. Follows immediately from Lemma 8. □

[28]The relevant domain includes all $\mu \in (\underline{\mu}, \bar{\mu})$, where any solution to either FOC must lie.