Global Value Chains and Inequality with Endogenous Labor Supply*

Eunhee Lee
University of Maryland

Kei-Mu Yi
University of Houston and NBER

June 2, 2017
[Preliminary and incomplete]

Abstract

We assess the role of global value chains transmitting global integration shocks to aggregate trade as well as distributional outcomes. We develop a multi-country general equilibrium trade model that features multi-stage production, with different stages having different productivities and using factors (occupations) with different intensities. The model also features a Roy mechanism, in which heterogeneous workers endogenously choose their production stage and occupation. Country- and worker-level comparative advantages interact. A reduction in trade costs, for example, leads to more workers shifting to the comparative advantage stage of production; such workers will also shift to their comparative advantage occupation. We show that the intensity of the global value chain magnifies the aggregate effects of trade liberalization, but it has a non-monotonic effect on the skill premia.

1 Introduction

One of the most significant economic developments over the past half-century is the increased fragmentation of production across borders. Goods are produced in sequential stages that traverse multiple countries – a global value chain. Countries specialize in particular stages of a good's production process. This increase in vertical specialization has

*The views expressed here are those of the authors and are not necessarily reflective of views of the Federal Reserve Banks of Dallas, Minneapolis or the Federal Reserve System. We thank Pol Antras, Alonso de Gortari, and Chinhui Juhn for very helpful discussions. All errors are our own.

occurred under a backdrop of a broad increase in international trade, one of the defining features of globalization across the world during this period.

Partly because of this backdrop, most of the research examining the effects of global integration on wages, employment, and other variables has focused on total trade. Autor et al. (2013) is a recent example. This has also been exemplified in the factor content studies that trade and labor economists have conducted since the 1990s. The purpose of our paper is to assess the role of global value chains as a propagation mechanism transmitting global integration shocks, such as China joining the WTO, to aggregate trade outcomes, as well as distributional outcomes, such as the skill premia.

Our approach is to build a model of global value chains and international trade and then to calibrate it and use it to study global integration shocks. We introduce global value chains following the work of Antràs and de Gortari (2017) and de Gortari (2017). They develop a tractable framework for incorporating multi-stage production in an international trade model that generalizes and extends previous research on this subject. In addition, a key feature of our model is to include for a labor supply channel. In particular, following Lee (2017), we include Roy selection effects, in which heterogeneous workers choose occupations and sectors based on their individual productivities in these occupations and sectors, as well as on prevailing prices. Lee (2017) and others have shown that these channels enhance our understanding of how trade affects inequality and are quantitatively important in explaining the increase in inequality.

The core elements of our model revolve around the production of a final good and the worker's decision of sector and occupation. A final good is made in a sequence of stages. Each stage involves labor, a composite intermediate, and output from the previous stage. There are several labor inputs, which we call "occupations". Different stages use these occupations with different intensities. The presence of the composite intermediate and the previous stage's output helps generate both "roundabout" and "snake" features in production. The final goods have two uses, consumption and input into the composite intermediate. On the worker's side, each worker is of multiple exogenous types. Within each type, a worker draws occupation and production stage specific productivities. Based on these productivities, as well as on prevailing prices, workers choose their optimal production stage and occupation. Our individual goods and workers are embedded in a multi-country general equilibrium framework. This framework features both Heckscher-Ohlin forces and Roy forces.

For our analysis, we first introduce a general version of the model and then study a simplified version of the model in order to develop intuition. We study a "2 x 2 x 2 x 2" version of the model in terms of countries, worker types, stages, and occupations.

We examine the response of GVCs, skill premia, and other variables to lower trade costs. The lower trade costs are mediated through several demand and supply forces before ultimately affecting GVCs and skill premia. The lower trade costs facilitate specialization in particular production stages. In turn, this changing specialization pattern shifts the relative demand for stage-specific occupations. This affects occupation-by-stage prices, which then affects workers' choices of occupations and stage of production. Ultimately, the skill premia are affected.

The intensity of a GVC is captured by the importance of the first-stage output used as an input in second stage production. The greater the importance of the first-stage output, the higher the GVC intensity. Our numerical exercises show that GVC intensity has a non-monotonic effect on the skill premia response to a decline in trade costs. That is, starting from a low GVC intensity, when trade costs decline, the skill premia rises by little in the country with the comparative advantage in the high-skilled-occupation-intensive stages. The reason is that with a low GVC intensity, because the first-stage output is not important, the relative occupation intensity of stage 1 is also not important. Hence, Heckscher-Ohlin forces are muted, which shows up as a small change in the skill premia following a decline in trade costs. As the GVC intensity increases, the importance of first-stage output increases, and Heckscher-Ohlin forces become more important. The skill premia changes by more in response to a decline in trade costs. However, as the GVC intensity approaches one, the change in skill premia becomes small again. This is because the importance of stage two becomes sufficiently small that the model again effectively resembles a one-sector moder, i.e., Heckscher-Ohlin forces are again muted. Our results show how the GVC and Roy mechanisms interact with both Ricardian and Heckscher-Ohlin comparative advantage.

1.1 Related literature

Our research is connected to several strands of research. One strand is the trade and wages research that sought to examine the effects of increased U.S. imports from developing countries on the skill premia. This research was especially active in the mid-1990s, and includes Katz and Murphy (1992), Lawrence and Slaughter (1993), Krugman (1995), and Feenstra and Hanson (1999) among others. All of these papers essentially employed a Heckscher-Ohlin type (HO) framework with its Stolper-Samuelson and factor content of trade implications. The main findings tended to be that the effect of trade was not large. However, the survey article by Goldberg and Pavcnik (2007) showed that the predictions of a simple Heckscher-Ohlin (HO) framework do not hold up in the data. In particular,

skill premia tended to rise in both developed and developing countries following trade liberalizations. Krugman (2008) revisits the trade and wage issues from the mid-1990s with the benefit of 15 years of additional data. In addition, Krugman argues that increased vertical specialization can generate inequality via Stolper-Samuelson effects. To our knowledge, Krugman's paper is the only paper that makes a case for examining the consequences of vertical specialization for inequality.

In recent years, there has been a new wave of interest on the employment and wage effects of increased trade. This is not surprising, because the emergence of China as a significant global economic force has only come about in the past 10-15 years. Autor et al. (2013), Pierce and Schott (2016), and many other papers in the literature document significant effects of China on labor markets of major partner countries such as the U.S. With the new interest has come an expanded set of methodologies. One new approach involves applying models with numbers, i.e., quantitative theory. Burstein and Vogel (2016) combine an HO framework in a model that features heterogeneous firms and skill-biased productivity. Our framework is in this vein.

A second strand of research is on documenting the extent of global value chains, vertical specialization, value-added exports, and related concepts, as well as on building models of these concepts. Contributions on the documentation side include Hummels et al. (2001), Johnson and Noguera (2012); Antràs and Chor (2013), and Koopman et al. (2014). Contributions on the modeling side include Yi (2003; 2010), Johnson and Moxnes (2016), and most recently, Antràs and de Gortari (2017) and de Gortari (2017). The latter two papers develop a general framework for GVCs and show how to map special cases of this framework into the Eaton and Kortum (2002) framework. Our modeling of GVCs draws from Antràs and de Gortari (2017); it combines that paper with Lee (2017).

To investigate the link between trade and labor market outcomes, such as wage inequality and labor reallocation, increasingly, papers focus on heterogeneous workers. While traditional trade models such as the HO model and the specific factors model assume that workers are all homogeneous in their productivities conditional on observable characteristics, this assumption misses the fact that workers differ in their productivities in reality. This worker-level heterogeneity is important especially when we study the effect of trade on labor market outcomes, because workers with same observable characteristics may respond to trade shocks differently depending on their idiosyncratic productivities. In recent years, trade models bring the idea of the Roy (1951) model to introduce worker heterogeneity under the setting of assignment models: e.g., Teulings (2005), Ohnsorge and Trefler (2007), and Costinot and Vogel (2010; 2015).

Our paper is also closely related to a recent strand of literature on quantitative models

with the Roy-based assignment structure. One of the key assumptions in this literature is that workers' idiosyncratic productivity is randomly drawn from a type-II extreme value distribution, a Frechet distribution. Hsieh et al. (2013), Burstein et al. (2015), Lagakos and Waugh (2013), Galle et al. (2015), and Lee (2017) use this assumption to investigate the role of worker heterogeneity in disentangling labor market outcomes in one country from labor demand shocks such as technological change or trade liberalization. Our paper also relies on this distributional assumption when we characterize workers' heterogeneous productivities. We introduce this Roy-based assignment framework into a multi-stage GVC model, where each stage of production is formulated based on the EK model. We can thus investigate the general equilibrium relation between workers' endogenous labor supply and trade through GVC in our model. To numerically solve our multi-country GVC model for a general equilibrium, we base our work on the iterative algorithm provided by Alvarez and Lucas (2007).

More papers in the literature recently focus on the occupational dimension as an important channel through which trade shocks are disseminated across workers--e.g., Autor et al. (2015), Ebenstein et al. (2014), Traiberman (2016), Harrigan et al. (2016). Our framework also allows workers to endogenously choose occupations in response to trade shocks under the GVC setting. We show that worker heterogeneity plays also a significant role for occupation-level labor reallocation. With this occupation-level analysis, we can also analyze the effect of generic trade shocks or GVC shocks on job polarization which is well-documented in the data for many developed economies.

The next section lays out our baseline model. This is followed by a description of a simpler version of our model with just two stages of production, two countries, two occupations, and two labor types. We solve the simpler version of the model and conduct several numerical exercises to illustrate how the model works. Section 4 describes our calibration, and our future research.

2 Model

In this section, we describe our model framework. Our model draws from the general global value chain (GVC, hereafter) model developed by Antràs and de Gortari (2017). Each final good is produced through a specific global value chain encompassing multiple stages of production that can potentially cross multiple countries. Production in the GVC features multiple factors of production, which we call occupations. Different stages of production have different factor intensities, and countries have different relative factor endowments. The standard *Heckscher-Ohlin (HO) channel* thus shows up across

production stages. In addition, workers in our model are heterogeneous in their stage- and occupation-specific productivities. Workers endogenously choose their occupation and the production stage to work in based on their productivities: the *Roy channel*. Introducing the Roy framework into a general equilibrium trade model is based on Lee (2017). Our model will deliver interaction between these two channels and GVCs.

2.1 Basic Framework

Our model features N countries, T worker types, value chains of fixed length J, and O occupations. Each worker is exogenously endowed with a specific type, which can be empirically defined by observable worker characteristics such as education. Labor supply of each type is exogenously given and different across countries. Each country is indexed by $i = 1, \ldots, N$, and each worker type is denoted by $t = 1, \ldots, T$. The number of type t workers in country i is given by $\bar{L}_{i,t}$.

There is a continuum of final goods over a unit interval $[0, 1]$. Each final good $\omega \in [0, 1]$ is produced following a specific value chain of length J. The optimal value chain for a final good ω is a J-dimensional vector of countries where each stage j of production takes place. In other words, products at an intermediate stage can cross multiple borders along the value chain. Occupations are used as production factors, and different production stages have different occupation intensities.

Preferences Consumers have common CES preference over final goods

$$U_i = \left(\int_0^1 (C_i^F(\omega))^{(\sigma-1)/\sigma} d\omega \right)^{\sigma/(\sigma-1)},$$

where $C_i^F(\omega)$ is consumption of a final good ω in country i, and $\sigma > 0$ is the elasticity of substitution between products.

Production As outlined above, each final good ω is produced from a specific value chain of length J during production, and this value chain is potentially spread over multiple countries. We denote the sequence of producing countries for a product ω by $l(\omega) = (l^1(\omega), \ldots, l^J(\omega))$. At each stage j of the value chain for a product ω, firms use domestic labor, the stage $j-1$ good for ω, and a composite intermediate. The composite intermdiate is a CES aggregate of the final goods, and it captures the "roundabout" structure of production, as in Caliendo and Parro (2015). In addition, using the immediately preceding stage as an input for the current stage of production captures the "snake" structure of production, as in Yi (2003). The goods comprising the composite intermedi-

ate, as well as the preceding stage goods can be sourced from other countries, while labor is assumed to be immobile across countries.

Countries can produce in any intermediate stage of production from $j = 1$ to $j = J - 1$ and also in final assembly of stage J. The production function for stage j of good ω is given by

$$f_i^j(L_i^{j,1}(\omega), \ldots, L_i^{j,O}(\omega), m_i^{j-1}(\omega)) = z_i^j(\omega)((x_i^j)^{1-\sum_o \beta^{j,o}} \prod_o (L_i^{j,o}(\omega))^{\beta^{j,o}})^{\gamma^j} (m_i^{j-1}(\omega))^{1-\gamma^j}.$$

Three categories of production factors are combined with this Cobb-Douglas production structure: 1) the composite intermediate good (x_i^j), which, as noted before, is a CES aggregate of the final goods; 2) occupational inputs from each of O occupations ($L_i^{j,o}(\omega)$); and 3) the stage $j-1$ good for ω ($m_i^{j-1}(\omega)$). The relative importance of the roundabout structure and the snake structure for each stage j is governed by parameter $\gamma^j \in [0,1]$. As $\gamma^j \to 1$, the roundabout production structure dominates the value chain effect for stage j, and vice versa for $\gamma^j \to 0$.

The HO channel operates through parameters $\beta^{j,o}$, intensity of occupation o in stage j. Different stages use occupations with different intensities. For example, a design stage would use more designers or engineers, while an assembly stage would employ relatively more production workers. Combining γ^j and $\beta^{j,o}$, the total value-added share from stage j is $\gamma^j \sum_o \beta^{j,o}$. We also assume that the initial stage 1 is produced using only occupations and composite intermediates by assuming $\gamma^1 \equiv 1$.

Factor-neutral productivity for stage j of product ω in country i is denoted by $z_i^j(\omega)$. We assume the productivity follows a Fréchet distribution from Eaton and Kortum (hereafter, EK, 2002). Productivity $z_i^j(\omega)$ is randomly drawn from

$$F_i^j(z) = \exp(-A_i^j z^{-\nu\tilde{\gamma}^j}),$$

where $\tilde{\gamma}^j \equiv \prod_{j'=j+1}^{N}(1-\gamma^{j'}) \in [0,1]$. We further assume that productivity draws are independent across stages. A_i^j governs the scale of stage j productivity in country i, and $\nu\tilde{\gamma}^j$ captures the dispersion of stage j productivities. ν is the standard Fréchet shape parameter, and governs the common variance of stage j productivity. The effective variance of stage j is stage-specific and is based on $\tilde{\gamma}^j$.

The stage-specific shape parameter $\nu\tilde{\gamma}^j$ has two advantages. First, as argued in Antràs and de Gortari (2017), this probability distribution makes a sequential sourcing decision equivalent to the case where a lead firm chooses the entire sourcing path from the beginning. This feature provides great analytic tractibility, which we will discuss in more

detail in the next subsection. Second, we can conveniently characterize the magnification effect of GVC as discussed in Yi (2003). At the equilibrium, the effective trade elasticity $\nu\tilde{\gamma}^j$ is larger in downstream production stages, as $\tilde{\gamma}^j$ is monotonically increasing in j. The magnification effect of GVC is thus active through $\tilde{\gamma}^j$. (In addition, $\sum_j \gamma^j \tilde{\gamma}^j = 1$ by the definition of $\tilde{\gamma}^j$, and we assume $\tilde{\gamma}^J \equiv 1$.)

Workers Workers are heterogeneous in their productivities for each production stage and occupation pair (j, o). A characterization of worker heterogeneity is based on Lee (2017). Each worker supplies one unit of time. Workers vary in their efficiency units of that time. The number of efficiency units $\epsilon^{j,o}$ that each individual worker of type t can supply for a specific (j, o) is randomly drawn from the following Fréchet distribution:

$$\epsilon^{j,o} \sim G_t^{j,o}(\epsilon) = \exp(-T_t^{j,o}\epsilon^{-\theta_t}).$$

Two types of stochastic comparative advantage arise from this probabilistic assumption. First, between-worker-type comparative advantage is governed by the relative magnitude of parameters $T_t^{j,o}$. For example, if $\frac{T_t^{j,o}}{T_t^{j',o'}} > \frac{T_{t'}^{j,o}}{T_{t'}^{j',o'}}$ holds, then it is more likely that a type t worker has comparative advantage for stage j and occupation o compared to another worker of type t' and for another pair (j', o'). Second, within-worker-type comparative advantage depends on the shape parameter θ_t. If workers' productivities are more dispersed within a type–i.e., lower θ_t–, then effects from the within-worker-type comparative advantage will be stronger than in the case of a larger θ_t. We further assume that draws of idiosyncratic productivity for each (j, o) are independent, which gives us the following joint distribution for a vector of worker productivity $\epsilon = (\epsilon^{1,1}, \ldots, \epsilon^{j,o}, \ldots, \epsilon^{J,O})$:

$$\epsilon \sim G_t(\epsilon) = \exp(-\sum_{j',o'} T_t^{j',o'}\epsilon^{-\theta_t}).$$

This framework for the labor supply side is an important channel which has not been widely studied in the literature. While changes in trade costs operate as one of labor demand shocks along the GVC, workers potentially respond to these shocks differently based on their own comparative advantage.

2.2 Equilibrium Sourcing Decision

In the above model, a final producer for ω chooses the entire path of $l(\omega) = (l^1(\omega), \ldots, l^J(\omega))$ by minimizing the total cost of production across all J stages. However, this approach makes solving the model challenging, because we can no longer take advantage of the

convenient characteristics of the Fréchet distribution. To deal with this issue, Antràs and de Gortari (hereafter, AG, 2017) introduce two alternative approaches. The first is a "sequential" approach in which each stage j producer chooses an optimal source for the $j-1$ stage by minimizing only its stage-specific production cost. The key assumption that they introduce is that stage j producers know the exact productivity draw of the stage $j-1$ producers, but do not know that of stage $1,\ldots,j-2$ producers. Instead, stage j producers know only the distribution of upstream producers up to stage $j-2$; thus, they take the expectation of productivity up to stage $j-2$ as given when they minimize the production cost for stage j. Thus, this is a limited information approach. The second approach is a "lead-firm" approach in which the assumption of a country-stage-specific Fréchet productivity parameter is replaced by a single Fréchet productivity parameter for an entire GVC. So, in a world with N countries and J stages, there are N^J possible GVCs, each with its own Fréchet productivity parameter. AG show that these two approaches are equivalent at the equilibrium under the probabilistic assumption of $z_i^j(\omega)$ as previously described. Our model draws from their result and, hereafter, we apply the sequential approach.

Another key assumption for the sourcing problem of this model is that each stage's sourcing decision is independent. Combining this assumption and the assumption of limited information on upstream productivities, we can derive analytic solution for the equilibrium GVC probability. We assume perfect competition for final goods and intermediate inputs, so each country sources from the lowest-cost supplier around the world. Given per-unit wages $w_i^{j,o}$ for each country, production stage, and occupation, and a CES price index for final goods P_i, the unit cost for the input bundle excluding materials from the previous stage is given by $c_i^j \equiv \alpha^j (P_i)^{1-\sum_o \beta^{j,o}} \prod_o (w_i^o)^{\beta^{j,o}}$, where $\alpha^j \equiv (1-\sum_o \beta^{j,o})^{-(1-\sum_o \beta^{j,o})} \prod_o (\beta^{j,o})^{-\beta^{j,o}}$ is a Cobb-Douglas constant.

Whenever stage j materials in country i are shipped to another country n to be used in stage $j+1$ production, there is iceberg trade cost $\tau_{in} \geq 1$. We adopt standard assumptions for iceberg trade costs: $\tau_{ii} = 1$ and $\tau_{in} \geq \tau_{ik}\tau_{kn}$. Given these assumptions, stage 2 producers in country i choose the optimal source $l_i^1(\omega)$ for stage 1 materials of product ω by solving the following problem:

$$l_i^1(\omega) = \arg\min_l [(p_l^1(\omega)\tau_{li})^{1-\gamma^2}] = \arg\min_l [(\frac{c_l^1}{z_l^1(\omega)}\tau_{li})^{1-\gamma^2}],$$

where $c_i^1 = \alpha^1 (P_i)^{1-\sum_o \beta^{1,o}} \prod_o (w_i^o)^{\beta^{1,o}}$.

Before we derive the sourcing decision for stage $j+1$ producers, we define the follow-

ing expectation variable as introduced by AG using the law of iterated expectations:

$$\begin{aligned}\Theta_i^j &\equiv E_j[(p^j_{l_i^j(\omega)}(\omega)\tau_{l_i^j(\omega)i})^\alpha] \\ &= E_j[(\frac{(c^j_{l_i^j(\omega)})^{\gamma^j}}{z^j_{l_i^j(\omega)}(\omega)})^\alpha \times \Theta^{j-1,\alpha(1-\gamma^j)}_{l_i^j(\omega)} \times (\tau_{l_i^j(\omega)i})^\alpha].\end{aligned}$$

We denote the optimal source for stage j materials of product ω for stage $j+1$ producers in country i by $l_i^j(\omega)$. Then, this expectation variable Θ_i^j describes the expected price of stage j materials of product ω to the power of some constant α, if they are shipped from the optimal source country $l_i^j(\omega)$ to country i. The sourcing decision for stage $j+1$ producers in country i can be written using this expectation variable.

$$l_i^j(\omega) = \arg\min_l\{(\frac{(c_l^j)^{\gamma^j}}{z_l^j(\omega)})^{1-\gamma^{j+1}} \times \Theta_l^{j-1,(1-\gamma^{j+1})(1-\gamma^j)} \times (\tau_{li})^{1-\gamma^{j+1}}\}$$

Similarly, final good consumers in country i buy ω from $l_i^J(\omega)$ which solves

$$l_i^J(\omega) = \arg\min_l\{\frac{(c_l^J)^{\gamma^J}}{z_l^J(\omega)} \times \Theta_l^{J-1,(1-\gamma^J)} \times \tau_{li}\}.$$

Probability of GVC The probability that stage $j+1$ producers in country i source stage j materials from another country n is

$$\begin{aligned}\Pr(l_i^j(\omega)=n) &= \Pr[(\frac{(c_n^j)^{\gamma^j}}{z_n^j(\omega)})^{1-\gamma^{j+1}} \times \Theta_n^{j-1,(1-\gamma^{j+1})(1-\gamma^j)} \times (\tau_{ni})^{1-\gamma^{j+1}} \\ &\quad \leq \min_{j'}(\frac{(c_{n'}^j)^{\gamma^j}}{z_{n'}^j(\omega)})^{1-\gamma^{j+1}} \times \Theta_{n'}^{j-1,(1-\gamma^{j+1})(1-\gamma^j)} \times (\tau_{n'i})^{1-\gamma^{j+1}}].\end{aligned}$$

For notational simplicity, we define $B_{ni}^j \equiv (c_n^j)^{\gamma^j(1-\gamma^{j+1})} \times \Theta_n^{j-1,(1-\gamma^{j+1})(1-\gamma^j)} \times (\tau_{ni})^{1-\gamma^{j+1}}$ for $j = 1,\ldots,J-1$, and $B_{ni}^J \equiv (c_n^J)^{\gamma^J} \times \Theta_n^{J-1,1-\gamma^J} \times \tau_{ni}$. Using the Fréchet distribution of product-specific productivity for each stage and each country, the equilibrium probability of the sourcing decision by stage $j+1$ producers in country i can be written as

$$\Pr(l_i^j(\omega)=n) = \frac{A_n^j(B_{ni}^j)^{-\nu\tilde{\gamma}^j/(1-\gamma^{j+1})}}{\sum_{n'} A_{n'}^j(B_{n'i}^j)^{-\nu\tilde{\gamma}^j/(1-\gamma^{j+1})}}$$

for $j = 1, \ldots, J-1$. Similar to the EK model, this probability is equal to the share of stage j goods that are produced in country n and used for stage $j+1$ production in country i.

This GVC probability clearly shows the magnification effect of hierarchical production as we go downstream. Because $\tilde{\gamma}^j$ is increasing in j, the effective elasticity of bilateral trade flows $\nu\tilde{\gamma}^j$ is increasing in j. Therefore, the effect of changes in trade costs between two countries is magnified in downstream production compared to upstream production. As different production stages use occupations with different intensities, the demand for occupations will depend on this magnification effect.

Using the GVC probability result and the independence assumption for sourcing decisions, we derive the equilibrium probability of an entire GVC path. The probability that a final good ω consumed in country i has followed a specific GVC path $l = (l^1, \ldots, l^J)$ is

$$\begin{aligned}\lambda_{l,i} &= \Pr(l_i^J(\omega) = l^J | l_{lJ}^{J-1}(\omega) = l^{J-1}) \times \Pr(l_{lJ}^{J-1}(\omega) = l^{J-1} | l_{lJ-1}^{J-2}(\omega) = l^{J-2}) \times \ldots \times \Pr(l_{l2}^1(\omega) = l^1) \\ &= \frac{\prod_{j=1}^J A_{lj}^j [(c_{lj}^j)^{\gamma^j} (\tau_{ljlj+1})]^{-\nu\tilde{\gamma}^j}}{\sum_{l' \in \mathbf{N}^J} \prod_{j=1}^J A_{l'j}^j [(c_{l'j}^j)^{\gamma^j} (\tau_{l'jl'j+1})]^{-\nu\tilde{\gamma}^j}},\end{aligned}$$

where $\mathbf{N}^J$ is the set of all possible sequences of N countries along J stages. The derivation of this probability again uses the law of iterated expectation and characteristics of the Fréchet distribution.

The expression for bilateral trade flows of final goods from the location of final assembly n to country i is derived similarly:

$$\Pr(l_i^J(\omega) = n) = \frac{A_n^J (B_{ni}^J)^{-\nu}}{\sum_{n'} A_{n'}^J (B_{n'i}^J)^{-\nu}}.$$

The exact price index of final goods is also derived in a similar way to EK:

$$P_i = \Gamma\left(\frac{\nu + 1 - \sigma}{\nu}\right) \left(\sum_{l' \in \mathbf{N}^J} \prod_{j=1}^J A_{l'j}^j [(c_{l'j}^j)^{\gamma^j} (\tau_{l'jl'j+1})]^{-\nu\tilde{\gamma}^j} \right)^{-1/\nu}.$$

We assume $\sigma < \nu + 1$ so that the gamma function in the price index is well-defined.

2.3 Equilibrium Labor Supply

Workers' labor supply response à la Roy model is based on Lee (2017). We assume that every worker inelastically supplies all of their time for working. Hence, the worker's labor supply decision is only about allocating that time to a production stage, occupation

pair. Each worker chooses a pair of production stage j and occupation o to maximize her potential labor income conditional on her $(J \times O)$-dimensional productivity matrix ϵ. In other words, worker's problem can be written as

$$\max_{j,o} w_i^{j,o} \epsilon^{j,o},$$

where $w_i^{j,o}$ is per-unit wage for workers in production stage j of country i with occupation o. Since ϵ is randomly drawn from a joint Fréchet distribution $G_t(\epsilon)$, the equilibrium labor supply decision for workers of type t for production stage j and occupation o is

$$\pi_{i,t}^{j,o} = \frac{T_t^{j,o}(w_i^{j,o})^{\theta_t}}{\sum_{j',o'} T_t^{j',o'}(w_i^{j',o'})^{\theta_t}}.$$

The shape parameter θ_t for type t workers' productivity distribution is thus the labor supply elasticity of type t workers at the production stage and occupation level. Different worker types are allowed to potentially have different labor supply elasticity in this model. Conditional on the optimal labor supply decision, the equilibrium average wage of type t workers can be derived as

$$\bar{w}_{i,t} = [\sum_{j',o'} T_t^{j',o'}(w_i^{j',o'})^{\theta_t}]^{1/\theta_t} \Gamma(1 - \frac{1}{\theta_t}).$$

If we define worker types based on educational attainment, the relative $\bar{w}_{i,t}$ of high-skilled workers over low-skilled workers will be a model counterpart of the skill premium, which will be one of our core objects of interest in the quantitative exercises.

2.4 General Equilibrium

The equilibrium per-unit wages $w_i^{j,o}$ and the price variable P_i are solved at the general equilibrium from market clearing conditions. We have occupation market clearing conditions for each country, production stage, and occupation:

$$\sum_t \bar{w}_{i,t} \pi_{i,t}^{j,o} \bar{L}_{i,t} = \gamma^j \tilde{\gamma}^j \frac{\beta^{j,o}}{\sum_o \beta^{j,o}} \sum_{n=1}^{N} \sum_{l \in \Lambda_i^j} \lambda_{l,n} (\sum_t \bar{w}_{n,t} \bar{L}_{i,t}),$$

where we define

$$\Lambda_i^j \equiv \{l = (l^1, \ldots, l^J) \in \mathbf{N}^J | l^j = i\}$$

as the set of GVC that produces j-th stage in country i. The left-hand side of this occupation market clearing condition is the total labor income earned by workers in production stage j of country i with occupation o. This term should be equal to the right-hand side, which is the total payment for those specific workers. The final goods market clearing condition is embedded in the total income term $\sum_{n=1}^{N}\sum_{l\in\Lambda_i^j}\lambda_{l,n}(\sum_t \bar{w}_{n,t}\bar{L}_{i,t})$ on the right-hand side.

We normalize wage to be $\sum_{i,j,o} w_i^{j,o} = 1$. With this normalization, occupation market clearing conditions, and the exact price index as derived above, we can solve the model for the equilibrium $w_i^{j,o}$ and P_i using the Alvarez and Lucas (2007) algorithm.

2.5 Discussion

The core mechanism of our model is the interaction between the HO channel and the Roy channel along the GVC. To understand how these interactions work, it will be useful to start with a special case in which only the HO channel is operative. The following assumptions deliver only the HO channel. 1. $\theta_t \to \infty$ for all worker types. In this case, workers have the same productivity conditional on their type and their choice of production stage and occupation. 2. Occupations and worker types are identical, i.e., $T = O$. 3. $T_t^{j,o} = 0$ for every $o \neq t$ and $T_t^{j,t} = 1$ for all j and t. Under these assumptions, type t workers will always choose occupation t, because the variance of the productivity distribution goes to 0 as $\theta_t \to \infty$. Also, the assignment of worker types into production stages is entirely determined by relative wages as in standard trade models without endogenous labor supply.

The nature of the sourcing decision does not change when we shut down the Roy channel. Because we have one-to-one mapping between worker types and occupations and workers of same type have the same productivity for different production stages, type-level wages are equalized across production stages. Thus, we have $N \times T = N \times O$ wage variables w_i^t and the same number of labor market clearing conditions:

$$w_i^t \bar{L}_i^t = \sum_{j=1}^{J} \gamma^j \tilde{\gamma}^j \frac{\beta^{j,o}}{\sum_o \beta^{j,o}} \sum_{n=1}^{N} \sum_{l\in\Lambda_i^j} \lambda_{l,n} \sum_o w_n^t \bar{L}_n^t.$$

Without the Roy channel, exogenous labor endowments and stage-specific factor intensities entirely determine relative wages across different worker types in a similar way as the standard HO model. In addition, this standard model is not able to capture the channel that workers of the same type may move to the same production stage, but choose

different occupations within that stage. If we have an exact one-to-one mapping between worker types and occupations and assume that these are exogenously fixed as in standard trade models, then workers are not allowed to switch their occupation as they switch their production stage. This new channel is in fact quantitatively relevant, as we will show later in this paper.

In order to further study the magnification effect of GVC, we need to focus on the role of γ^j. γ^j captures the relative importance of the "roundabout" structure over the "snake" structure. Since $(1-\gamma^j)$ denotes the share of stage $j-1$ used for production of stage j, the sequential structure of production through GVC becomes less important as $\gamma^j \to 1$. In the extreme case where $\gamma^j = 1$ for all $j = 1, \ldots, J$, only stage J production remains active using only domestic labor inputs and composite intermediates through the roundabout structure. Our baseline model is then equivalent to the EK model with intermediate inputs.

As discussed in many papers in the literature including Yi (2003; 2010) and Johnson and Noguera (2012), introducing GVCs into standard trade models can yield magnified effects of changes in trade costs on aggregate outcomes such as bilateral and aggregate trade flows and prices. Our result is in line with the prediction on the magnification effect from the literature. As $\gamma^j \to 0$, production stages become more inter-dependent, and the effective trade elasticity $\nu\tilde{\gamma}^j$ becomes larger. Thus, aggregate effects from trade liberalization are increasingly more magnified with GVC intensity.

In contrast, the distributional effect of reduction in trade cost is not monotonic in GVC intensity. It is straightforward that the HO channel diminishes as $\gamma^j \to 1$, where the baseline model becomes an one-sector EK model. Production-stage-specific factor intensity generates differential gains from trade across different worker types as in the HO model. As $\gamma^j \to 0$, the difference in factor intensity across production stages becomes irrelevant, since previous-stage materials are almost not used in immediately following stages. As a consequence, a reduction in trade costs has very small effects on between-worker-type inequality–e.g., the skill premium–, if a trade model features either the full roundabout structure ($\gamma^j \to 1$) or the full snake structure ($\gamma^j \to 0$). We will discuss interaction of distributional effects of trade with the GVC intensity based on a simple 2-stage version of our model.

3 A Simple 2-Stage Model

In this section, we simplify our baseline model for 2 production stages, 2 countries, 2 worker types, and 2 occupations in order to show the underlying mechanism intuitively.

We then perform numerical exercises with this simple 2-stage model.

3.1 Model Setup and the Equilibrium

We denote countries by $i = 1, 2$, worker types by $t = H, L$ and production stages by $j = 1, 2$. As in the baseline model, worker types are exogenously given from workers' perspective, while workers are allowed to endogenously choose production stage and occupation $o = H, L$. Labor supply at the worker type level is thus exogenously fixed: $\bar{L}_{1,H}$, $\bar{L}_{1,L}$, $\bar{L}_{2,H}$, and $\bar{L}_{2,L}$. In this simple 2-stage case, production functions for each stage are given by:

$$f_i^1(L_i^{1,H}(\omega), L_i^{1,L}(\omega)) = z_i^1(\omega)(L_i^{1,H}(\omega))^{\beta^{1,H}}(L_i^{1,L}(\omega))^{\beta^{1,L}}(x_i^1)^{1-\beta^{1,H}-\beta^{1,L}}$$

$$f_i^2(L_i^{2,H}(\omega), L_i^{2,L}(\omega), m_i^1(\omega)) = z_i^2(\omega)[(L_i^{2,H}(\omega))^{\beta^{2,H}}(L_i^{2,L}(\omega))^{\beta^{2,L}}(x_i^2)^{1-\beta^{2,H}-\beta^{2,L}}]^{\gamma}(m_i^1(\omega))^{1-\gamma}.$$

The occupation intensity parameters β's, along with the relative type-level labor endowments, generate the HO mechanism. If $\frac{\beta^{1,H}}{\beta^{1,L}} > \frac{\beta^{2,H}}{\beta^{2,L}}$, for example, stage 1 production uses H occupations more intensively than does stage 2 production. Stage 2 producers of product ω use domestic labor, the composite intermediate (from the roundabout structure), and stage 1 material of product ω to produce final goods. The overall GVC intensity decreases in γ, the value-added share at stage 2. Netting out final goods used to make the composite intermediate through the roundabout production structure, the effective value-added share of stage 2 is $\gamma(\beta^{2,H} + \beta^{2,L})$. The stage 1 good that stage 2 producers in country i use is denoted by $m_i^1(\omega)$. Stage 1 good can be sourced either domestically or from another country $i' \neq i$ through GVC.

The stage-specific factor-neutral productivity again follows the Fréchet distribution: for each country $i = 1, 2$,

$$z_i^1(\omega) \sim F_i^1(z) = \exp(-A_i^1 z^{-\nu(1-\gamma)})$$

$$z_i^2(\omega) \sim F_i^2(z) = \exp(-A_i^2 z^{-\nu}).$$

Similarly to our baseline model, the dispersion parameter of stage 1 is weakly smaller than that of stage 2. This assumption will again capture the magnification effect of GVC, as we go to the downstream stages. We also keep the sequential independence assumption that final good consumers in this 2-stage setup do not know the exact productivity draw of stage 1 producers.

The choice set for workers consists of four pairs of production stage and occupation.

Given per-unit wage variables $w_i^{j,o}$, workers choose one among $(j,o) = (1,H),(1,L),(2,H),(2,L)$ to maximize their potential labor income conditional on their idiosyncratic productivities randomly drawn from a type-specific distribution. The equilibrium solution of the worker's problem yields the same expressions for $\pi_{i,t}^{j,o}$ and $\bar{w}_{i,t}$ as in the baseline model.

The sourcing decision for stage 2 producers in country i can be written as

$$l_i^1(\omega) = \arg\min_l [(p_l^1(\omega)\tau_{li})^{1-\gamma}] = \arg\min_l [(\frac{c_l^1}{z_l^1(\omega)}\tau_{li})^{1-\gamma}],$$

where $c_i^1 \equiv \alpha^1 (P_i)^{1-\beta^{1,H}-\beta^{1,L}} (w_i^{1,H})^{\beta^{1,H}} (w_i^{1,L})^{\beta^{1,L}}$ and α^1 is a Cobb-Douglas constant. Similarly, the sourcing decision for final good consumers in country i is

$$l_i^2(\omega) = \arg\min_l [\frac{(c_l^2)^\gamma}{z_l^2(\omega)} \times \Theta_l \times \tau_{li}],$$

where $\Theta_l \equiv E[(p^1_{l_l^1(\omega)}(\omega)\tau_{l_l^1(\omega),l})^{1-\gamma}]$, and $l_l^1(\omega)$ is the optimal source of stage 1 of product ω for stage 2 producers in country l. The unit cost for stage 2 is given by $c_i^2 \equiv \alpha^2 (P_i)^{1-\beta^{2,H}-\beta^{2,L}} (w_i^{2,H})^{\beta^{2,H}} (w_i^{2,L})^{\beta^{2,L}}$ for a stage-2 Cobb-Douglas constant α^2.

Because we assume that productivities are drawn from country- and stage-specific Fréchet distributions, the above sourcing problems imply the following bilateral GVC probabilities:

$$\Pr(l_i^1(\omega) = n) = \frac{A_{l1}^1 (c_n^1 \tau_{ni})^{-\nu(1-\gamma)}}{\sum_{n'} A_{l'1}^1 (c_{n'}^1 \tau_{n'i})^{-\nu(1-\gamma)}}$$

$$\Pr(l_i^2(\omega) = n) = \frac{A_n^2 ((c_n^2)^\gamma \Theta_n \tau_{ni})^{-\nu}}{\sum_{n'} A_{n'}^2 ((c_{n'}^2)^\gamma \Theta_{n'} \tau_{n'i})^{-\nu}}.$$

Using the independence assumption for stage-specific productivity draws, the sourcing decision, and limited information across stages, the equilibrium probability that a product consumed by country i consumers follows a specific GVC $l = (l^1, l^2)$ is given by:

$$\lambda_{li} = \frac{A_{l^1}^1 (c_{l^1}^1 \tau_{l^1 l^2})^{-\nu(1-\gamma)} \times A_{l^2}^2 [(c_{l^2}^2)^\gamma \tau_{l^2 i}]^{-\nu}}{\sum_{l' \in \mathbf{N}^2} A_{l'^1}^1 (c_{l'^1}^1 \tau_{l'^1 l'^2})^{-\nu(1-\gamma)} \times A_{l'^2}^2 [(c_{l'^2}^2)^\gamma \tau_{l'^2 i}]^{-\nu}},$$

where $\mathbf{N}^2 = \{(l_1, l_2) : (1,1),(1,2),(2,1),(2,2)\}$. As in the baseline model, the effective trade elasticity varies by production stage. We have a larger effective trade elasticity for final goods than for stage 1 goods.

In general equilibrium, all markets clear. We have eight labor market clearing conditions for the $2 \times 2 \times 2$ countries, production stages, and occupations. For each country

$i = 1, 2$, the labor market clearing conditions are

$$\sum_t \bar{w}_{i,t} \pi_{i,t}^{1,H} \bar{L}_{i,t} = (1-\gamma) \frac{\beta^{1,H}}{\beta^{1,H} + \beta^{1,L}} \sum_{n=1}^{N} \sum_{l \in \Lambda_i^1} \lambda_{l,n} (\sum_t \bar{w}_{n,t} \bar{L}_{i,t})$$

$$\sum_t \bar{w}_{i,t} \pi_{i,t}^{1,L} \bar{L}_{i,t} = (1-\gamma) \frac{\beta^{1,L}}{\beta^{1,H} + \beta^{1,L}} \sum_{n=1}^{N} \sum_{l \in \Lambda_i^1} \lambda_{l,n} (\sum_t \bar{w}_{n,t} \bar{L}_{i,t})$$

$$\sum_t \bar{w}_{i,t} \pi_{i,t}^{2,H} \bar{L}_{i,t} = \gamma \frac{\beta^{2,H}}{\beta^{2,H} + \beta^{2,L}} \sum_{n=1}^{N} \sum_{l \in \Lambda_i^2} \lambda_{l,n} (\sum_t \bar{w}_{n,t} \bar{L}_{i,t})$$

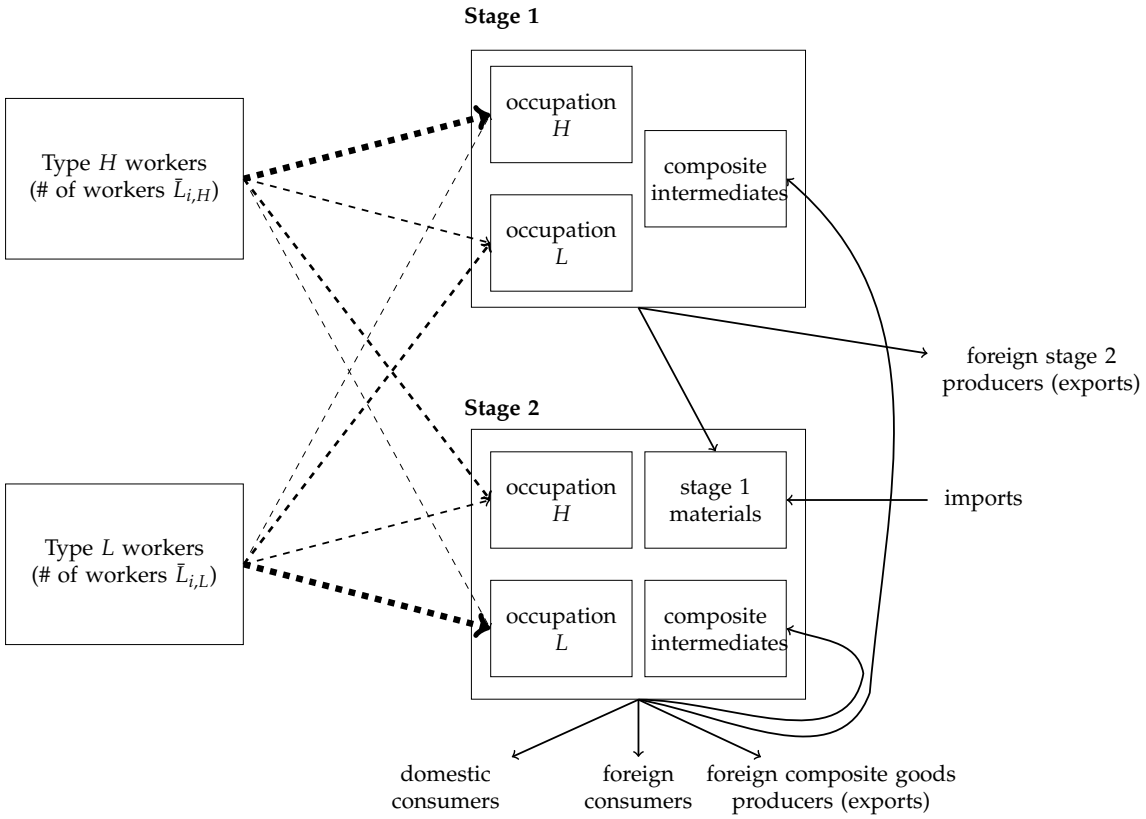
$$\sum_t \bar{w}_{i,t} \pi_{i,t}^{2,L} \bar{L}_{i,t} = \gamma \frac{\beta^{2,L}}{\beta^{2,H} + \beta^{2,L}} \sum_{n=1}^{N} \sum_{l \in \Lambda_i^2} \lambda_{l,n} (\sum_t \bar{w}_{n,t} \bar{L}_{i,t}),$$

where $\Lambda_i^1, \Lambda_i^2 \in \mathbf{N}^2$ are defined as in the baseline model.

Figure 1 describes the structure of this 2-stage model. For country i, there are $\bar{L}_{i,H}$ type H workers and $\bar{L}_{i,L}$ type L workers. These workers endogenously choose a production stage and an occupation to maximize their labor income based on their productivity draws; this choice is described by the dashed arrows in this diagram. Depending on the workers' comparative advantage, the within-type labor allocation pattern should differ across worker types. Combining the labor supplied from workers in the form of occupations H and L with the composite intermediate (again, they are CES aggregates of final goods), stage 1 producers in country i produce the stage 1 good. These stage 1 goods are either used by stage 2 producers in the same country i or exported to foreign stage 2 producers. Stage 2 producers then combine labor input in the form of occupations H and L, the composite intermediate, and stage 1 materials to produce final goods. Five agents demand these final goods: domestic consumers, foreign consumers, foreign composite goods producers, domestic stage 1 producers, and domestic stage 2 producers.

If trade costs change in this economy, the relative demands for country i's stage 1 goods and final goods change. This change will lead to changes in the relative demand for occupations, which should depend on the relative occupation intensity of each production stage. Stage- and occupation-specific per-unit wages respond to this labor demand shock. Workers observe changes in per-unit wages across production stages and occupations and then re-optimize their choice of production stage and occupation. Even though workers observe the same change in wages for each production stage and each occupation, the individual worker's response should be different depending on their idiosyncratic productivity.

Figure 1: A 2-stage GVC Model with Endogenous Labor Supply



As can be easily seen from the labor market clearing conditions, both the occupation intensities β and the value-added share parameter γ affect workers' relative wages. The standard HO channel operates through the relative magnitude of β. If $\frac{\beta^{1,H}}{\beta^{1,L}} > \frac{\beta^{2,H}}{\beta^{2,L}}$, then labor demand is relatively larger for H occupation (L occupation) from stage 1 (stage 2) producers. The relative factor intensity across production stages is linked directly to countries' comparative advantage based on their relative endowment of type-level labor as long as the Roy mechanism has stochastic log-supermodularity: i.e., type H workers have higher productivities for H occupations on average ($\frac{T_H^{j,H}}{T_H^{j,L}} > \frac{T_L^{j,H}}{T_L^{j,L}}$ for every j), and country 1 is relatively more abundant in type H workers.

While specialization across production stages in our model seems similar to the between-sector specialization in the HO model, the degree of specialization across stages is governed by the GVC intensity parameter γ. As we can see from the set of labor market clearing conditions above, extreme values of γ (i.e., close either to 0 or to 1) make this 2-stage model essentially a one-stage model in terms of distributional effects of trade. If

γ is close to 1, then stage 2 producers do not use the stage 1 good for final goods production. Thus, the relative factor intensity of stage 1 does not generate distributional effects from a reduction in trade costs. If γ is close to 0, on the other hand, the relative factor intensity of stage 2 becomes irrelevant, because labor has almost no value-added in stage 2 production.

This result is in sharp contrast with aggregate effects of reduction in trade cost, which are increasingly magnified as we have a smaller value for γ. When $\gamma \to 0$, our model has only the pure snake channel of trade in intermediates. Thus, the effects of a reduction in trade costs on aggregate trade flows have the maximum magnification. On the other hand, from the perspective of relative gains for different worker types, γ close to 0 rather reduces the effect of trade. The relative factor intensity of stage 2 does not play a role with γ close to 0; thus, effectively only the factor intensity of stage 1 and workers' labor supply determine the relative gains from trade. If we also shut down the Roy channel and rely only on exogenous labor supply, then the exogenous relative labor supply and β's for stage 1 entirely determine relative wages. Thus, changes in underlying trade environment have little effect on inequality with γ close to 0.

A simple 2-stage case of our model reiterates the importance of both the roundabout structure and the snake structure when we investigate the effect of GVC on inequality. Even though aggregate effects are monotonically magnified with a larger GVC intensity, distributional effects of GVC are largest with an intermediate value of γ, when the HO channel remains active through production stages.

3.2 Numerical Analysis

In this section, we solve the simple 2-stage case of our baseline model numerically. We assign parameter values so that both the Ricardian and the HO comparative advantages operate. In other words, countries have different stage-specific scale parameters for the distribution of factor-neutral productivity and different endowments of type-level labor. In order to lay out the core mechanism more clearly, we focus on the case where the Ricardian comparative advantage coincides with the HO comparative advantage.

Thus, the numerical example we consider satisfies: 1) the relative factor-neutral productivity of stage 1 to stage 2 is larger in country 1 on average– i.e., $\frac{A_1^1}{A_1^2} > \frac{A_2^1}{A_2^2}$; 2) country 1 is relatively more abundant in type H workers– i.e., $\frac{\bar{L}_{1,H}}{\bar{L}_{1,L}} > \frac{\bar{L}_{2,H}}{\bar{L}_{2,L}}$; 3) stage 1 uses H occupations relatively more intensively than does stage 2– i.e., $\frac{\beta^{1,H}}{\beta^{1,L}} > \frac{\beta^{2,H}}{\beta^{2,L}}$; 4) type H workers have comparative advantage in stage 1 and in H occupations– i.e., $\frac{T_H^{1,o}}{T_H^{2,o}} > \frac{T_L^{1,o}}{T_L^{2,o}}$ for every o

and $\frac{T_H^{j,H}}{T_H^{j,L}} > \frac{T_L^{j,H}}{T_L^{j,L}}$ for every j; and 5) type H workers on average have absolute advantage over all pairs of production stage and occupation– i.e., $T_H^{j,o} > T_L^{j,o}$ for all (j,o). The first condition implies Ricardian comparative advantage between two countries. The second and third conditions imply HO comparative advantage. The fourth condition describes the Roy channel of our model, and it assumes that the Roy channel does not reverse the HO comparative advantage. Combining the fourth and the last conditions, we consider a case where type H workers have an absolute advantage for all (j,o) over L workers but have a comparative advantage in stage 1 and in H occupations.

Based on these conditions, we assign the following values for our benchmark numerical exercise: 1) Ricardian comparative advantage parameter $(A_1^1, A_1^2, A_2^1, A_2^2) = (10, 5, 5, 10)$; 2) type-level labor supply $(\bar{L}_{1,H}, \bar{L}_{1,L}, \bar{L}_{2,H}, \bar{L}_{2,L}) = (0.7, 0.3, 0.3, 0.7)$; 3) occupation intensity for each productions stage $(\beta^{1,H}, \beta^{1,L}, \beta^{2,H}, \beta^{2,L}) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2})$; and 4) the scale parameter of workers' productivity distribution $(T_H^{1,H}, T_H^{1,L}, T_H^{2,H}, T_H^{2,L}) = (4, 2, 3, 1)$ and $(T_L^{1,H}, T_L^{1,L}, T_L^{2,H}, T_L^{2,L}) = (\frac{1}{4}, \frac{3}{4}, \frac{1}{2}, 1)$.

We set $\gamma = 1/3$ for the benchmark simulation. Because the GVC intensity parameter γ plays an important role in our model for both aggregate and distributional effects of reduction in trade costs, we will experiment with different values of γ later in this section. For the elasticity parameters, we set $\nu = 5.5$ (a common component of trade elasticity across productions stages), $\sigma = 2$ (elasticity of substitution across products in preference), and $\theta_t = 2$ for all t (labor supply elasticity.) The labor supply elasticity parameter is allowed to be different across worker types in a general version of our baseline model. While we consider a case where two worker types share the same labor supply elasticity in this numerical exercise, we will estimate this parameter separately for different worker types in the full quantification of our model.

We compare the case with $\tau_{in} = 2$ for $i \neq n$ and $\tau_{ii} = 1$ ("high trade cost case") to a "free trade case" with $\tau_{in} = 1$ for all i, n. Given trade costs and parameter values, we solve the occupation market clearing conditions and the exact price indices for the equilibrium $w_i^{j,o}$ and P_i following the Alvarez and Lucas (2007) algorithm. We also normalize wages to satisfy $\sum_{i,j,o} w_i^{j,o} = 1$.

Benchmark results Once we solve for the equilibrium $w_i^{j,o}$ and P_i, we then calculate type-level average wages $\bar{w}_{i,t}$, within-worker-type labor allocation across production stages and occupations $\pi_{i,t}^{j,o}$, unit cost for stage-specific input bundles c_i^j, and the GVC flows λ_{li} as functions of $w_i^{j,o}$ and P_i. Distributional effects of reduction in trade costs are measured by changes in the skill premium from the high trade cost case to the free trade case. We define the skill premium by $SP_i = \bar{w}_{i,H}/\bar{w}_{i,L}$.

A reduction in trade costs enables producers to source intermediates from the lowest cost supplier and also enables consumers to purchase final goods from the lowest cost producer. Conseqently, not surprisingly, as trade costs fall to zero, the aggregate price index decreases in each country by 37.5% in country 1 and by 27.2% in country 2. In free trade, both countries' aggregate price index are identical, so another way of characterizing our result is that country 1 has the higher price level under high trade costs. To understand this result, it is useful to look at Table 1, which presents the GVC probability λ_{li} for both trade cost cases. Table 1 also shows the changes in λ_{l1} and λ_{l2}, as we move from $\tau_{in} = 2$ for $i \neq n$ to $\tau_{in} = 1$ for all i and n.

Table 1: Changes in λ_{li} from the Benchmark Simulation

		(1) $\tau_{in} = 2$		(2) $\tau_{in} = 1$		(3) change (pp)	
		$l_2 = 1$	$l_2 = 2$	$l_2 = 1$	$l_2 = 2$	$l_2 = 1$	$l_2 = 2$
λ_{l1}	$l_1 = 1$	0.920	0.005	0.255	0.387	-66.580	38.240
	$l_1 = 2$	0.041	0.034	0.142	0.216	10.080	18.240
λ_{l2}	$l_1 = 1$	0.012	0.120	0.255	0.387	24.300	26.710
	$l_1 = 2$	0.001	0.868	0.142	0.216	14.160	-65.170

In the high trade cost case described in the first panel of the table, both countries mostly source stage 1 materials and final goods domestically. About 92% of final goods consumed in country 1 follow a GVC $(l_1, l_2) = (1, 1)$, and 87% of final goods consumed in country 2 follow $(l_1, l_2) = (2, 2)$. Even though country 1 has a comparative advantage in stage 1 in terms of both the Ricardian and the HO mechanisms, high trade costs keep countries from specializing in their comparative advantage stages.

When trade costs are uniformly reduced to zero, i.e., $\tau_{in} = 1$ for all i and n, then it is most likely that a final good follows the GVC of country-level comparative advantage, $(l_1, l_2) = (1, 2)$. About 39% of final goods follow the GVC $(l_1, l_2) = (1, 2)$ regardless of the location of final goods consumers, because there is no trade cost. We do not have a complete specialization due to heterogeneity in the Ricardian productivity, but the allocation within $\lambda_{l1} = \lambda_{l2}$ is exactly consistent with the stochastic comparative advantage we imposed based on the relative magnitude of A_i^j. In percentage points, the largest decline of the GVC share is from the domestic sourcing case–$(l_1, l_2) = (1, 1)$ for final goods consumers in country 1 and $(l_1, l_2) = (2, 2)$ for final goods consumers in country 2–when we move from the high trade cost case to the free trade case. On the other hand, the GVC share for comparative-advantage stages $(l_1, l_2) = (1, 2)$ increases most.

Note that under free trade, the most prevalent GVC is the one in which stage 1 is made

in country 1 and stage 2 is made in country 2. The stage 2 goods are then shipped back to country 1 for its consumers. But, under high trade costs, this GVC becomes especially costly, because it involves two border crossings. Consequently, this GVC falls almost to a zero share. Country 1 has to substitute high cost alternatives as a result. Essentially, the upstream nature of country 1's specialization makes it more costly to engage in a GVC under high trade costs. Hence, country 1's price index is high relative to country 2's under such costs.

The reduction in trade cost changes the skill premium in the direction predicted by the Stolper-Samuelson theorem. Our benchmark parameter values assume that: 1) country 1 has a comparative advantage in stage 1; and 2) type H workers have comparative advantage in stage 1 and in occupation H. Therefore, as trade costs go down, the relative demand for type H workers increases in country 1 and decreases in country 2. Our model predicts that the skill premium increases by 7.5% in country 1, and decreases by 8.2% in country 2, as we move from the high trade cost case to free trade. This numerical exercise is set up to highlight the Stolper-Samuelson effect through GVC by assigning appropriate parameter values. Based on a general setup, our model allows for non-Stolper-Samuelson effects as well depending on different values for key parameters $\beta^{j,o}$, $\bar{L}_{i,t}$, $T_t^{j,o}$, and θ_t.

In response to the reduction in trade costs, the Roy channel of our model reallocates workers across production stages and occupations within their type. Workers may stay in the same production stage, but move to a different occupation, an ouctome not present in standard trade models with homogeneous workers. The within-type labor reallocation is measured by changes in $\pi_{i,t}^{j,o}$ as we move from high trade costs to free trade. In addition, when trade costs decrease, country 1 specializes more in stage 1 production, and country 2 specializes more in final goods production, i.e., the usual HO mechanism.

Table 2: Changes in $\pi_{i,t}^{j,o}$ from the Benchmark Simulation

		(1) $\tau_{in}=2$		(2) $\tau_{in}=1$		(3) change (pp)	
$\pi_{i,t}^{j,o}$		$o=H$	$o=L$	$o=H$	$o=L$	$o=H$	$o=L$
Country 1, Type H	$j=1$	0.526	0.212	0.584	0.232	5.746	1.979
	$j=2$	0.116	0.146	0.083	0.101	-3.303	-4.423
Country 1, Type L	$j=1$	0.119	0.287	0.153	0.364	3.432	7.763
	$j=2$	0.070	0.525	0.058	0.425	-1.174	-10.022
Country 2, Type H	$j=1$	0.657	0.146	0.582	0.134	-7.457	-1.205
	$j=2$	0.124	0.074	0.176	0.108	5.222	3.441
Country 2, Type L	$j=1$	0.216	0.288	0.162	0.224	-5.353	-6.374
	$j=2$	0.109	0.388	0.131	0.483	2.240	9.488

Table 2 first shows within-type labor allocation pattern predicted by our baseline model. First, the simulation results show that workers are more likely to work in their comparative advantage production stage and with a comparative advantage occupation in both the high trade cost case and the free trade cost case. Second, even though a certain worker type has a comparative advantage in a certain production stage and a certain occupation on average, the within-type labor allocation does not involve complete specialization owing to within-type heterogeneity in productivity. For example, about 10% of type H workers in country 1 work in production stage 2 in occupation L, while on average, type H workers have a comparative advantage in stage 1 and occupation H.

Table 2 also shows a different picture from predictions of standard trade models regarding worker's labor reallocation pattern in response to a reduction in trade costs. First, as countries further specialize in their comparative advantage production stage in response to the reduction in trade costs, workers of both types move between stages accordingly. In other words, as we move from high trade costs to free trade, both type H and L workers in country 1 work more in stage 1, while workers in country 2 work more in stage 2. Second, workers choose different occupations when they move between production stages in response to trade shocks. This pattern does not depend on the occupation intensity of each stage but depends on worker-level comparative advantage for occupations. For example, while some of both type H and L workers move from stage 2 to stage 1, type H workers in H occupations increase by 5.75 percentage points, and type H workers in L occupations increase by 1.98 percentage points. By contrast, type L workers in L occupations increase by 7.76 percentage points, and those type workers in H occupations increase by 3.43 percetnage points, in stage 1. There is a similar pattern for country 2, where workers of both types are likely to move from stage 1 to stage 2, but with different occupations depending on their comparative advantage. Workers' endogenous labor supply choice at the production stage and occupation level generates another important layer of labor reallocation that is not captured by standard trade models.

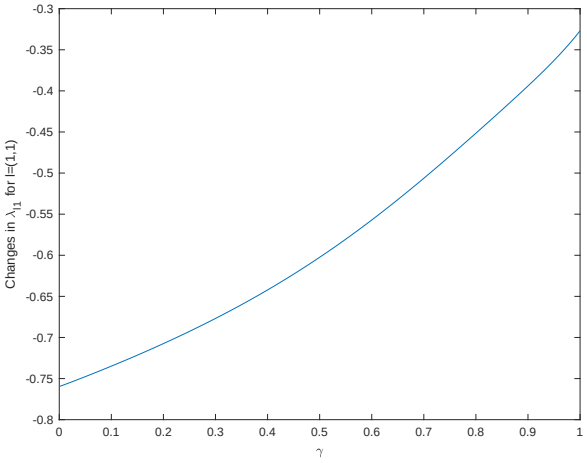
The role of GVC intensity The key parameter of our 2-stage model which governs the relative importance of the roundabout structure of intermediate trade to the snake structure is γ. The effective GVC intensity, as well as the effective trade elasticity, decrease in γ, as stage 1 and stage 2 are less inter-dependent with a larger γ. In an extreme case $\gamma = 1$, the effective trade elasticity for stage 1 is zero. In other words, the effects of the same change in trade costs will be different depending on the GVC intensity. If the GVC intensity is large–i.e., small γ–, effects of a reduction in trade costs are further magnified as noted in Yi (2003; 2010) and Johnson and Noguera (2012).

Figure 2 shows changes in $\lambda_{(1,1),1}$ and $\lambda_{(2,2),2}$ when we move from the high trade cost

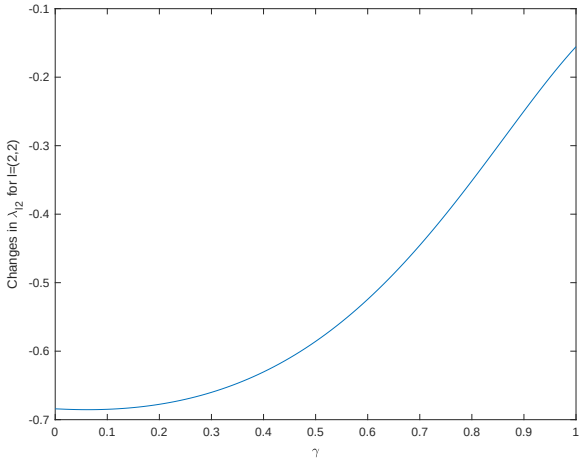
case to the free trade cost case with different values of γ. The GVC shares $\lambda_{(1,1),1}$ and $\lambda_{(2,2),2}$ are counterparts of domestic absorption share in standard trade models. As we show in Table 1, these two shares are about 90% in the high trade cost case but decrease to 22-25% in the free trade cost case with the benchmark parameter value $\gamma = 1/3$. If the hierarchical production structure through GVC magnifies the effect of reduction in trade costs on trade flows, we should have larger decreases of $\lambda_{(1,1),1}$ and $\lambda_{(2,2),2}$ with lower values of γ. Figure 2 clearly shows this magnification effect through GVC. While $\lambda_{(1,1),1}$ and $\lambda_{(2,2),2}$ decrease by about 10-30 percentage points in response to the trade shock with γ close to 1, the decreases are about 70 percentage points with γ close to 0. Therefore, multi-stage production and intermediate trade through GVC magnify the effect of the reduction in trade costs, and the magnification effect is monotonically increasing in the GVC intensity. Changes of all other aggregate variables from the model such as price index and other entries of the λ_{li} matrices are also monotonic in γ: i.e., larger changes with lower γ.

Figure 2: Changes in $\lambda_{(1,1),1}$ and $\lambda_{(2,2),2}$ with Different Values of γ (%)

(a) $\lambda_{(1,1),1}$



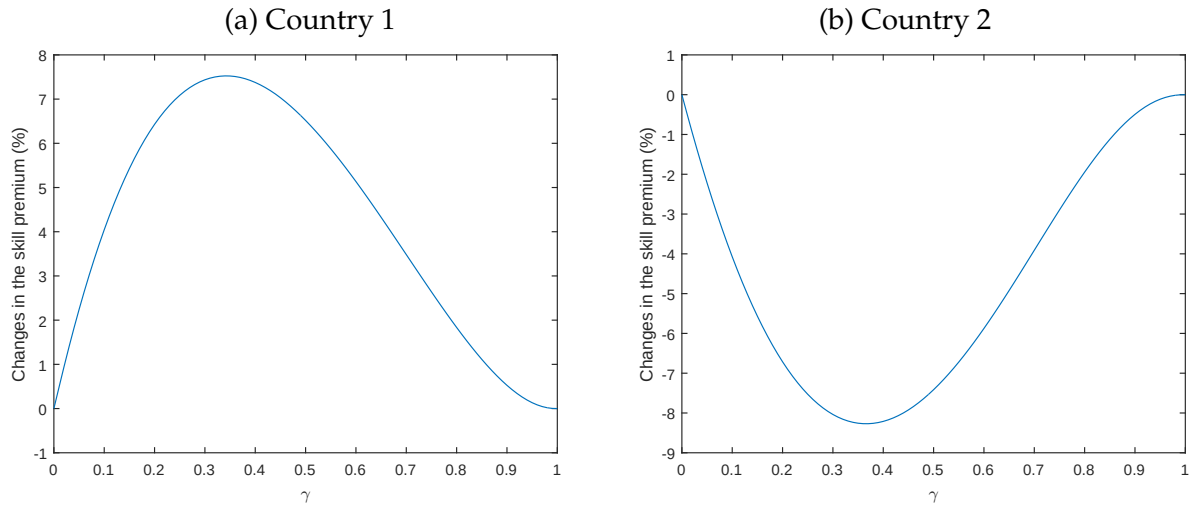
(b) $\lambda_{(2,2),2}$



On the other hand, the distributional effect of a reduction in trade costs is not monotonic in the GVC intensity as predicted from our baseline model. Figure 3 shows changes in the skill premium in each country with different values of γ as we go from the high trade cost case to the free trade case. In this simple 2-stage case with both Ricardian and HO comparative advantages in line with each other, the skill premium increases in country 1 and decreases in country 2 as a result of the reduction in trade costs for all values of γ. However, the magnitude of the effect of reduction in trade costs is not monotonic in the GVC intensity but largest with an intermediate value of γ around 0.35. This result

shows that both the snake mechanism and the roundabout mechanism are important to measure the effect of trade through GVC on the skill premium. If $\gamma \to 1$ thus the snake mechanism is shut down, then the relative occupation intensity of stage 1 becomes irrelevant, because no stage 2 producers demand stage 1 materials. Therefore, changes in trade costs do not affect the skill premium of either country. In this case, aggregate effects of a reduction in trade cost are also smallest, because there is no magnification effect through GVC. On the other hand, if $\gamma \to 0$ thus we only have the snake mechanism but no roundabout mechanism for intermediate trade, then our model again effectively resembles a one-sector model where the relative factor intensity of stage 2 becomes irrelevant. This is because there is no value added from labor in stage 2 production. Therefore, a reduction in trade costs does not change the skill premium in either country.

Figure 3: Changes in the Skill Premium with Different Values of γ (%)



Our numerical exercise shows that both the roundabout mechanism and the snake mechanism are relevant for explaining the effect of trade through GVC on wage inequality within a country. A simple monotonic magnification effect does not apply for the distributional consequence of trade liberalization through GVC. The GVC intensity interacts with relative occupation intensity across production stages. The effect of reduction in trade costs is largest with some intermediate value of $\gamma \in [0, 1]$ with which difference in relative occupation intensity is most pronounced across productions stages.

In summary, the results from the numerical exercises show how the GVC mechanism and the Roy mechanism interact with standard Ricardian and HO comparative advantage. First, countries specialize in their comparative advantage production stage as trade costs go down, which shifts relative demand for occupations in our model. Second, the

Roy mechanism based on workers' heterogeneous productivities makes workers respond to trade shocks differently across production stages and occupations, even though workers are exposed to the same trade shock. Third, as predicted in existing papers in the literature, aggregate effects of reduction in trade costs on trade flows and prices are monotonically increasing in the GVC intensity. Lastly, distributional effects of the reduction in trade costs are not monotonic in the GVC intensity. The effects on the skill premium are maximized with an intermediate value of the GVC intensity parameter γ, while the trade shock has no effect on the skill premium with extreme values for γ, 0 or 1.

4 Calibration

Our numerical exercise shows the basic mechanism of our model in a simple 2-stage case with only 2 countries, 2 worker types, 2 production stages, and 2 occupations. We now take the general version of our model to data for a more complete calibration. Our goal is to assess the role of GVCs as a propagation mechanism transmitting global integration shocks, such as China joining the WTO, to aggregate trade outcomes, as well as distributional outcomes, such as the skill premia. In particular, we investigate the role of the length of the GVC.

We use $N = 28$ countries for the baseline calibration. Workers are classified by $T = 5$ types which are defined by educational attainment: 1) high school dropouts; 2) high school graduates; 3) workers with some college education; 4) college graduates; and 5) workers with advanced degrees. When we calculate the skill premium, we define skilled workers as those who have at least some college education. We define five occupation categories following Dorn (2009): 1) low-skilled service occupations and agricultural workers; 2) assemblers and machine operators; 3) precision production and crafts occupations; 4) administrative, clerical, and sales occupations; and 5) managers, professionals, and technicians. This categorization is based on both skill levels required by occupation and the routineness of occupation.

In addition we use the World Input-Output Database (WIOD) for 2000. We reduce the WIOD tables for that year into one with 27 countries plus the rest-of-the-world, and with four industries: agriculture, mining, manufacturing, and services. AG and de Gortari (2017) show how to map the GVC concepts into input-output flows. We will do this from our framework with two stages of production, as well as with three stages.

The GVC intensity parameter γ^j is calibrated using the WIOD as described above. We then use the ratio of GDP between countries to calibrate the scale parameter A_i^j of the distribution of Ricardian productivity. Occupation intensity parameter $\beta^{j,o}$ is calibrated

to the data of sector- and occupation-specific payment to labor from the International Labor Organization Statstics (ILOSTAT). Parameters for the distribution of worker's idiosyncratic productivity are estimated using individual wage profiles for U.S. workers as outlined in Lee (2017). After we pin down all the necessary parameters, we will perform counterfactual exercises quantifying aggregate and distributional effects of changes in trade cost τ_{in}.

5 Results and Conclusion

[To be added]

References

ALVAREZ, F. AND R. E. LUCAS (2007): "General Equilibrium Analysis of the Eaton–Kortum Model of International Trade," *Journal of Monetary Economics*, 54, 1726–1768.

ANTRÀS, P. AND D. CHOR (2013): "Organizing the Global Value Chain," *Econometrica*, 81, 2127–2204.

ANTRÀS, P. AND A. DE GORTARI (2017): "On the Geography of Global Value Chains," *NBER Working Paper Series*, 23456.

AUTOR, D. H., D. DORN, AND G. H. HANSON (2013): "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," *American Economic Review*, 103, 2121–68.

——— (2015): "Untangling Trade and Technology: Evidence from Local Labour Markets," *The Economic Journal*, 125, 621–646.

BURSTEIN, A., E. MORALES, AND J. VOGEL (2015): "Accounting for Changes in Between-Group Inequality," *NBER Working Paper Series*, 20855.

BURSTEIN, A. AND J. VOGEL (2016): "International Trade, Technology, and the Skill Premium," *Journal of Political Economy (forthcoming)*.

CALIENDO, L. AND F. PARRO (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 82, 1–44.

COSTINOT, A. AND J. VOGEL (2010): "Matching and Inequality in the World Economy," *Journal of Political Economy*, 118, 747–786.

——— (2015): "Beyond Ricardo: Assignment Models in International Trade," *Annual Review of Economics*, 7, 31–62.

DE GORTARI, A. (2017): "Disentangling Global Value Chains," *Working Paper*.

DORN, D. (2009): "Essays on Inequality, Spatial Interaction, and the Demand for Skills," Ph.D. thesis, University of St. Gallen.

EATON, J. AND S. KORTUM (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

EBENSTEIN, A., A. HARRISON, M. MCMILLAN, AND S. PHILLIPS (2014): "Estimating the Impact of Trade and Offshoring on American Workers Using the Current Population Surveys," *Review of Economics and Statistics*, 96, 581–595.

FEENSTRA, R. C. AND G. H. HANSON (1999): "The Impact of Outsourcing and High-Technology Capital on Wages: Estimates For the United States, 1979–1990," *The Quarterly Journal of Economics*, 114, 907–940.

GALLE, S., A. RODRÍGUEZ-CLARE, AND M. YI (2015): "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade," *Working Paper*.

GOLDBERG, P. K. AND N. PAVCNIK (2007): "Distributional Effects of Globalization in Developing Countries," *Journal of Economic Literature*, 45, 39–82.

HARRIGAN, J., A. RESHEF, AND F. TOUBAL (2016): "The March of the Techies: Technology, Trade, and Job Polarization in France, 1994—2007," *NBER Working Paper Series*, 22110.

HSIEH, C.-T., E. HURST, C. I. JONES, AND P. J. KLENOW (2013): "The Allocation of Talent and U.S. Economic Growth," *NBER Working Paper Series*, 18693.

HUMMELS, D., J. ISHII, AND K.-M. YI (2001): "The Nature and Growth of Vertical Specialization in World Trade," *Journal of International Economics*, 54, 75–96.

JOHNSON, R. C. AND A. MOXNES (2016): "Technology, Trade Costs, and the Pattern of Trade with Multi-Stage Production," *Working Paper*.

JOHNSON, R. C. AND G. NOGUERA (2012): "Accounting for Intermediates: Production Sharing and Trade in Value Added," *Journal of International Economics*, 86, 224–236.

KATZ, L. F. AND K. M. MURPHY (1992): "Changes in Relative Wages, 1963–1987: Supply and Demand Factors," *The Quarterly Journal of Economics*, 107, 35–78.

KOOPMAN, R., Z. WANG, AND S.-J. WEI (2014): "Tracing Value-Added and Double Counting in Gross Exports," *American Economic Review*, 104, 459–494.

KRUGMAN, P. (1995): "Growing World Trade: Causes and Consequences," *Brookings Papers on Economic Activity*, 1995, 327–377.

KRUGMAN, P. R. (2008): "Trade and Wages, Reconsidered," *Brookings Papers on Economic Activity*, 2008, 103–137.

LAGAKOS, D. AND M. E. WAUGH (2013): "Selection, Agriculture, and Cross-Country Productivity Differences," *American Economic Review*, 103, 948–80.

LAWRENCE, R. Z. AND M. J. SLAUGHTER (1993): "International Trade and American Wages in the 1980s: Giant Sucking Sound or Small Hiccup?" *Brookings Papers on Economic Activity. Microeconomics*, 1993, 161–226.

LEE, E. (2017): "Trade, Inequality, and the Endogenous Sorting of Heterogeneous Workers," *Working Paper*.

OHNSORGE, F. AND D. TREFLER (2007): "Sorting It Out: International Trade with Heterogeneous Workers," *Journal of Political Economy*, 115, 868–892.

PIERCE, J. R. AND P. K. SCHOTT (2016): "The Surprisingly Swift Decline of US Manufacturing Employment," *American Economic Review*, 106, 1632–1662.

ROY, A. D. (1951): "Some Thoughts on the Distribution of Earnings," *Oxford economic papers*, 3, 135–146.

TEULINGS, C. N. (2005): "Comparative advantage, relative wages, and the accumulation of human capital," *Journal of Political Economy*, 113, 425–461.

TRAIBERMAN, S. (2016): "Occupations and Import Competition Evidence from Danish Matched Employee-Employer Data," *Working Paper*.

YI, K.-M. (2003): "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy*, 111, 52–102.

——— (2010): "Can Multistage Production Explain the Home Bias in Trade?" *American Economic Review*, 100, 364–393.