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Heterogeneous**

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Models of Production Treating Capital as Heterogeneous

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"I only want to urge that research not get tied to any one particular picture of the way the economy functions "

- Robert Solow [1998]

Abstract

This paper describes the production function of a "machine" (representing any individual capital asset). The likely shape of this function is explored. The model is used to elucidate a number of concepts such as rent seeking behavior, the retirement of assets, deterioration, embodied technical change, obsolescence and depreciation. The paper then develops a vintage model using a specification chosen to reflect the fact that aggregate vintage capital is a collection of individual assets. The model is estimated, using highly aggregate data and simple techniques, in order to illustrate the potential usefulness of vintage models. The results are consistent with a world in which labor productivity and rents decline as assets age, and in which new investments bring on higher productivity. Rates of embodied technical change and deterioration are estimated, as are indexes corresponding to the "age/efficiency profiles" used in capital stock measurement.

I. Introduction

Robert Solow proposed two alternative approaches to explaining the roles of technology and capital in aggregate economic behavior. Solow [1957] first proposed the use of an aggregate capital stock and microeconomic analysis to show how increased capital intensity can explain some growth in labor productivity. By assuming a production function, f , of the form $Y = f(L, K, t)$ (Y is output, L is labor, K is aggregate capital, and t is time), differentiating totally with respect to time, assuming constant returns to scale and that inputs are paid the values of their marginal products, Solow showed that:

$$(y-l) = s_k (k-l) + a , \quad (1)$$

where y , l , and k represent the growth rates of output, labor and capital stock, respectively, and where s_k is the share of capital input in the value of output. The remainder, a , which is often called the "Solow residual", has come to be regarded as the best measure of productivity. The "Solow equation", expression (1), can be used to explain a portion of labor productivity growth.

But it was widely recognized that the conditions for the existence of a homogeneous aggregate capital stock were unlikely to obtain. Capital had to be like "jelly" --- the ratios of marginal products of different assets could not vary with demand. There is evidence that this is unrealistic. For example, older electric power plants are often used only to help meet peak demand. In response to this type of problem, Solow [1959] proposed a vintage capital model. In this model, each vintage, v , of capital has its own production function, $f_{t,v}$, describing how much output can be made with any given amount of surviving capital, $K_{t,v}$, and labor, $L_{t,v}$, in any period subsequent to the year of an initial investment, I_v :

$$Y_{t,v} = f_{t,v}(K_{t,v}, L_{t,v}) \quad . \quad . \quad (2)$$

This allowed for technical progress to be "embodied" in capital goods. A key assumption is that labor is deployed to work with specific vintages. Solow [1959] used a Cobb-

Douglas¹ specification for expression (2), and assumed that the marginal product of the labor deployed to each vintage is the same at any point in time. He further assumed that there is no simultaneous "disembodied" technical change (like the residual, a , in expression (1)). From this model he obtained empirical estimates of an assumed constant rate of "embodied" technical progress, λ . Later, in an effort to reconcile the two models, Intriligator [1965] used a more general specification to jointly estimate rates of embodied and disembodied change.

However, the residual model soon crowded out the vintage model in the literature. We can identify at least three reasons for this. First, early efforts to empirically identify vintages models relied on specifications which limited their power to explain behavior. Second, Denison [1964] argued that it made little empirical difference to the sources of long-term growth whether technology was embodied or not. Jorgenson [1966] demonstrated this point formally by showing that models of embodied and disembodied technical change are equivalent when there is long run exponential growth in all variables. Finally, Fisher [1965] showed that, if machines of different vintages were not effectively interchangeable, then one could not consistently aggregate capital investments of different vintages. Thus vintage models cannot lead to an aggregate capital measure.

¹ Solow assumed $Y_{t,v} = B e^{\lambda v} (L_{t,v})^a (I_{t,v})^{(1-a)}$.

Distinguishing embodied from disembodied technical change may have contributed little to the explanation of long term growth. However, the vintage model has greater generality than the residual model, and this type of generality may be necessary if the maintained model is to account for a wider range of phenomena. For example, it is well-known that the Solow residual model does not account for the robust short-term (cyclical) fluctuations in productivity.² Cooper and Haltiwanger [1993] have pointed out that the "lumpy" nature of investment "naturally creates endogenous fluctuations in output which are positively correlated with productivity". Also, Caballero and Hammour [1994] used a vintage model to show how capital's heterogeneity could account for the surge in plant closings³ observed during recessions. Even when the relevant

² With the residual model, we assume that there are diminishing marginal returns to each input, one of which is labor. This leads some to the expectation of counter-cyclical movements in labor productivity. To explain the robust finding of pro-cyclical labor productivity, researchers have turned to explanations which posit some shortcoming in the residual model. Some examples are as follows. Nadiri and Rosen [1973] assumed a lag in the adjustment of quantities to a change in prices. Morrison [1988] assumed that unexpected price change results in adjustment costs. An idea favored by many is "labor hoarding", which can be traced back to Walter Oi's [1962] observation that fixed costs associated with labor may influence demand for this factor. If the supply of labor is inelastic in the short run, we can account for the propensity of output to rise faster than labor. Another approach, devised by Hall [1988], proposes that a preponderance of firms really do operate with market power and so there really are increasing returns to scale. The shares in the Solow equation, as usually implemented, would then be poor proxies for output elasticities. Yet another approach, developed by Berndt and Fuss [1986] and by Hulten [1986], emphasizes the Marshallian distinction between short run and long run costs. Berndt and Fuss define "temporary equilibrium" as the minimum of a short run cost function. They conclude that the Solow residual must be calculated using "ex post (realized) prices." They measure the ex post price of capital, c_k , as $(pY - wL)/K$ (residual property income divided by an aggregate capital stock).

variables are growing smoothly, as Jorgenson [1966] assumed, we may need to retreat to a vintage model to analyze the sources of capital accumulation itself. Hulten [1989 and 1996] has shown this to be the case if we wish to distinguish the aggregate implications of technological advances which occur autonomously from those produced by costly research and development.

In Section II of this paper we propose a general model of production which posits a collection of discrete capital assets. This "machine" model expands upon a key idea of Solow's vintage model --- that output is made by labor which has been deployed to specific capital assets. The model is general and detailed enough that it can allow for many of the realistic features of capital such as the abrupt retirement of assets. The model also provides a basis for considering many concepts related to differences in productivity such as deterioration, embodied technical change, obsolescence, and depreciation.

In Section III we estimate a vintage model for the U.S. private nonfarm business sector. We draw on Section II in determining the specification. From the vintage model we recover "deterioration rates", "rates of embodied technical change", and "age/efficiency" profiles from data on output, labor input, property income, and "real" investment.

³ Powers [1985] measured these fluctuations by comparing book value data

II. A Model of Production with Individual Assets

The Solow Vintage Model

Solow [1959] analyzed and estimated a vintage model using a Cobb-Douglas specification of each vintage's production function. At the outset of this paper, Solow described the model in very general terms, as in expression (2). He assumed each vintage of capital had its own production function. To achieve this vintage separation of production functions, Solow imposed the "ground rule" that firms must apply labor to specific vintages with no joint effects. Thus the observed totals for labor hours and output are the sums of vintage specific contributions:

$$L_t = \sum_v L_{t,v} \text{ and } Y_t = \sum_v Y_{t,v} . \quad (3)$$

The idea was to allow for *heterogeneity* among the production processes used by capital assets of different vintages. Two old machines might cost the same as one new one and therefore represent the same "amount" of capital. However, that does not imply that they are perfect substitutes in production. Nor does it imply that they are affected the same by changing demand for the industry's products. Solow's vintage model provides a simple vehicle for addressing their differences.

on capital with perpetual inventory measures.

Now, suppose that the amount of a investment in the assets of each vintage is fixed in history. Expressing the general function describing the vintage model (equation 2) in terms output per unit of capital:

$$Y_{t,v} / K_{t,v} = g_{t,v}(L_{t,v} / K_{t,v}) \quad . \quad (4)$$

Now, if we think of the unit of capital as "one machine", we can think of a collection of individual "machines", i , and we can write a production function for each machine:

$$Y_{t,v,i} = f_{t,v,i}(L_{t,v,i}) \quad \forall i \in \mathbb{K}_{t,v} \quad , \quad (5)$$

where $\mathbb{K}_{t,v}$ is the set of all machines of vintage, v , available in time, t . To be consistent with expression (4), the machines, i , described in expression (5) must all be identical. However, if they were really different, we could extend Solow's logic, in proposing the vintage model, to propose a "model of production with individual assets". In doing so, we would extend Solow's "ground rule" to "individual" machines so that:

$$L_{t,v} = \sum_i L_{t,v,i} \text{ and } Y_{t,v} = \sum_v Y_{t,v,i} \quad . \quad (6)$$

In order to work with this model empirically, we would have to determine a reasonable specification for the functions defined in expression (5). In this section we will only take the first step of considering what properties we would expect these functions to have.

Assume that each machine has an owner who decides how much labor to hire for it, and who sells the resulting output. Figure 1 (at the back of the paper) contains a diagram of the first quadrant of a plane where the domain is labor hours and the range is units of output. Were we to have plot a Cobb-Douglas specification on this plane, the function would emerge from the origin and exhibit diminishing marginal returns to labor at every point along the function.

But we haven't plotted a Cobb-Douglas because this may be unrealistic for individual assets. The nature capital is that it is found in large discrete items, which do not resemble jelly. Because of this, "individual" plants or machines generally require some minimum amount of labor required in order to make any output. For example, an automobile plant can't make any autos with just a few workers. On a smaller scale, a truck can't haul a load unless it first picks it up. It is essential to take account of this important characteristic of capital, in order to account for asset retirement behavior. Thus figure 1 illustrates production functions shaped to reflect minimum labor requirements. The functions are drawn to illustrate diminishing marginal returns to labor at all points where output is generated, as microeconomic theory would predict. Notice that average returns to labor are

increasing over a portion of the domain where output first begins to be produced. Diminishing average returns set in later. For the functions illustrated, the technically efficient options are either to operate somewhere in the range where output is nonzero, or to "shut down" the machine entirely, using no labor and making no output.

Figure 1 depicts functions for three individual assets. While labeled as belonging to three successive vintages, the idea could be applied to any three "individual" assets. While the curves are depicted as "nested" (not intersecting), they could, in general, intersect.

Solow showed that a firm owning capital of several vintages, and achieving the maximum output from a given total amount of labor, will be allocating labor such that the marginal products of labor are equal among vintages. This result can be extended to the "individual" machines:

$$\partial f_{t,v,i} / \partial L_{t,v,i} = c_t \quad \forall v, i . \quad (7)$$

In figure 1, this condition means that the machines will be operated where a set of parallel lines are tangent to the production functions, as depicted. Note that the labor productivity of each machine can be represented as the slope of the line segment from the origin to the corresponding tangency point. Labor productivity will clearly differ

among machines. Labor productivity will change as c_t changes, and it need not vary proportionally.

A Model of the Extraction of Rents from Discrete Assets

We will now assume that "individual" assets (be they plants or machines) are operated by relatively small, competitive producers, so that the price of output, p , and the wage, w , are perceived to be given. In the competitive model labor is paid the value of its marginal product and so we can rewrite expression (7) as:

$$\partial f_{t,v,i} / \partial L_{t,v,i} = w_t / p_t \quad \forall v, i . \quad (8)$$

Let us define property income, $\Psi_{t,v,i}$, as the difference between revenues and variable costs:

$$\Psi_{t,v,i} = R_{t,v,i} - VC_{t,v,i} = p_t Y_{t,v,i} - w_t L_{t,v,i} . \quad (9)$$

As Berndt and Fuss [1986] stressed, in the short run firms can be expected to behave as if capital costs are fixed and sunk, and so they go about the business of maximizing the rate at which they accrue property income, $\Psi_{t,v,i}$. If each firm is competitive, it will *extract as much rent as possible* from each machine it owns.

By focusing on firms which are price-takers, we can gain some additional insights into how firms "should" use their capital assets. For any given wage/price combination,

we can represent the input and output combinations in terms of costs and revenues. That is, we can write a relationship describing how much revenue can be earned from a given expenditure on variable costs:

$$p_t Y_{t,v,i} = f_{t,v,i}^*(w_t L_{t,v,i}) , \quad (10)$$

where $\partial f_{t,v,i}^* / \partial L_{t,v,i} = w_t \partial f_{t,v,i} / \partial L_{t,v,i} \quad \forall i \text{ and } v$. Given this, in any year, t , we can relabel the axes of figure 1 and use the same curves to depict the production process in terms of nominal revenues and compensation (variable costs), subject to the given output price and wage. We have done this in figure 2.

In figure 2, we draw a ray, OA, through points in the first quadrant where revenues equal variable costs. A machine earns positive rents at any point above this ray. The rents earned at any given point are proportional to the vertical distance of the point from OA. In figure 2, the firm operating the asset labeled "vintage 3" earns rents EC, and a firm operating the asset of vintage 2 earns DB. We have depicted the machine labeled vintage 1 as a marginal asset earning zero rents. Any asset whose production function fails to reach OA would presumably be "shut down". The "shape" properties we have assumed give the model the ability to account for the abrupt shutdown of plants. Note that, had we started with a Cobb-Douglas specification, and neglected the fixed labor requirement, we would not be able

to account for plant shutdowns with this model. The production functions would emanate from the origin and so the model would predict that old assets don't die, but just gradually fade away. Note that the slopes of the curves at points E, D and A are all w_t/p_t , as is the slope of OA. Also note that w_t and p_t are the exogenous variables in the short run. Given a set of machine production functions, the position of the ray OA among these curves drives the short run decisions as to how much to produce with each machine.

It is interesting to note, for a given production function, labor productivity is maximized at the point where a ray from the origin is tangent to the function. But a firm will only operate at this point when rents are zero --- when the asset is marginal. In all other circumstances, the firm will operate *to the right* of this tangency point. It follows that an increase in labor (and hence output) will always lead to a decrease in labor productivity. One would conjecture, from this, that labor productivity would be counter-cyclical. Of course it isn't. It is likely that we need to look at capital from a more dynamic perspective to account for the pro-cyclical behavior of productivity, as Cooper and Haltiwanger [1993] have suggested. We will next describe several types of temporal changes that can affect figures 1 and 2.

Deterioration of Assets

As time goes by, assets age and they may deteriorate. Assume that this manifests itself as an asset producing less output from a given amount of labor. This may be due to increased maintenance requirements, increased downtime, or simply less efficiency. In figure 1, we can represent deterioration as a downward and rightward shift in a given machine's function over time. We define the *deterioration rate of a machine's production function* as $\phi_{t,v,i} = \partial \ln f_{t,v,i} / \partial t$. We note that $\phi_{t,v,i}$ is a function. Even if deterioration occurs as a fixed, predetermined process, the rate at which deterioration occurs will in general depend on where on the function the firm is operating. Note that deterioration occurs independently of prices and wages. So if prices and wages stay constant from one time period to the next, rents will decline due to deterioration as the production function slides south in figure 2. Also note that deterioration involves contemporaneous decreases in output and labor productivity, contributing to the appearance of pro-cyclical aggregate labor productivity.

Embodied Technical Change

Now we turn to a more complex phenomenon. As time goes by, the new assets acquired often "embody" efficiency improvements. In developing his vintage model, Solow [1959] assumed that all technical change was embodied, while noting

that a mixture of embodied and disembodied technical change was possible. In adopting a Cobb-Douglas production function, Solow also assumed that embodied technical change occurs at a constant rate, λ .

Care must be taken in defining embodied technical change. It involves an improvement in new assets relative to old. Therefore it is not a property of specific machines like deterioration is. In terms of figure 1, we might be tempted to depict a process by which new functions tend to "pop up" further above and to the left of where the functions for assets for earlier vintages had "popped up" when they were new. However, it is not quite so simple. We need to be careful to distinguish embodied technical change from the use of a more capital intensive process. For example, without any technology change, a truck driver could haul more dirt by using a bigger truck, as distinct from a newer truck. So in order to define embodied technical change, we must somehow control for the *amount* of investment the assets represent. The normal way to deal with this, and the one we are forced to adopt in Section III, is to use a price index to "deflate" investments from different vintages. If we restrict ourselves to a class of machines, j , which "cost the same", we can define a *rate of embodied technical change*, for machines of class j , $\lambda_{t,j} = \partial \ln f_{t,t,j} / \partial t$. Then λ can represent the tendency of functions for new

machines, *costing the same in real terms*, to "pop up" above where earlier functions "popped up". Having defined λ in terms of this "normal" procedure, we would like to note that problems may lurk in the deflation process, and we will return to this shortly.

Finally note that investments in improved assets will cause contemporaneous increases in outputs and productivity. Thus embodied technical change, like deterioration, can contribute to the observed pro-cyclical behavior of productivity.

Changes in the Relative Prices of Output and Labor

Consider what happens to figure 2 if the price of output decreases. If we assume that the production functions themselves are unaffected, we can simply rescale the vertical axis so that revenues correspond to the new price. The change will cause us to redraw the ray OA, and we can think of it as having "rotated" upward in the first quadrant, decreasing rents, Ψ , for each machine and also shifting the equilibrium points to lower levels of production, and raising labor productivity. (Since there is a downward rescaling of the revenues along the vertical axis, we need to be cautious in comparing rents before and after the ray rotates). Similarly, as the price of labor increases, the ray will appear to rotate upward past the

production functions as we redraw the diagram (due here to a rescaling of the costs along horizontal axis).

This dynamic view can be used to illustrate how a wage increase or output price decrease would tend to force marginal assets to "shut down". If a shutdown is expected to be permanent, firms would presumably retire the relevant machines. Again, the assumption of a minimum labor requirement was necessary to predict the discrete transition involved in retiring a plant.

Capital Goods Prices

In competitive equilibrium, producers are expected to acquire capital up to the point where the value of any asset equals the discounted value of its expected future earnings. In terms of our notation, the value of any asset, $q_{t,v,i}$, is given by:

$$\begin{aligned} q_{t,v,i} &= \exp_t \{ \sum_{\tau=0}^{\infty} \Psi_{t+\tau,ti} (1-r)^{\tau} \} = \\ &= \exp_t \{ \sum_{\tau=0}^{\infty} (p_{t+\tau} Y_{t+\tau,ti} - w_{t+\tau} L_{t+\tau,ti}) (1-r)^{\tau} \} \quad . \quad (11) \end{aligned}$$

This expression is complex, and this highlights a difficulty in using price indexes in capital measurement. The observed price "relatives" reflect the perceived relative values of new goods throughout the future. If we attempt to aggregate vintage investments deflated with these prices, we run the risk of overstating the differential in rent-generating ability in the initial period. This is

because the *percentage* differential in rents, attributable to an improvement, is likely to grow as a given pair of goods ages. For example, in figure 2, the differential could approach infinity as the older item in the pair becomes marginal. In general, it may not be possible for price index numbers, for durable goods or construction outputs, to correct for differences in marginal products, as these differences are subject to variation.⁴

Expression (11) does help to clarify which factors do influence capital goods prices. Uncertainty is one such factor, and this might be handled by assuming that the discount rate reflects the degree of uncertainty. Asset prices can adjust abruptly when interest rates change, even while all the variables in figures 1 and 2 remain unchanged. As Evans, Honkapohja, and Romer [1998] observe, these effects can be destabilizing to asset prices and can potentially account for the volatile nature of investment.

Aside from uncertainty, the value of an asset is more or less determined by the projected future paths of output prices and wages. These factors, together with the future locations of the asset's production function, presumably determine the quantities of labor and output.

⁴ If we were to approach this problem with a Solow residual model and a capital stock assumed to decay geometrically, this problem would vanish.

In projecting output prices and wages, a firm might regard wages as determined by the labor market as a whole, while considering how factors affecting its specific industry influence the future price of its product. The factors likely to be most important are changes in product demand and technical change embodied in the capital goods that the industry will be able get in the future. If demand is falling, product prices will tend to be lower, hurting rents. If demand is growing, increases in product prices can be expected which will help maintain future rents. The influences of future improvements in capital goods are also important.

Obsolescence

These improvements will likely stimulate investments by future competitors, increasing supply and lowering the future product price. The lowering of product price leads to an upward rotation of the ray OA, lowering rents on existing assets (figure 2). This is the phenomenon of **obsolescence**. This phenomenon is complex. The rents earned by an asset will be lowered only *to the extent that new assets are introduced which embody the technological improvements*.

Rents will decline over time due to either obsolescence or deterioration (or both). It is difficult to separately

identify the two⁵. Jorgenson, Gollop, and Fraumeni [1987] treated all differences in vintage productivity as if they were deterioration. However, for computers obsolescence may dominate deterioration as a source of these price differences and so Oliner [1993] and Jorgenson and Stiroh [1995] treat differences in vintage productivity as obsolescence. While no one has yet done so, it ultimately may prove possible to identify deterioration and obsolescence separately, because the rate of obsolescence depends on the amount of new investment while the rate of deterioration, which we have presumed to be an intrinsic property of the machines, does not.

Depreciation

In capital measurement work, it is common to estimate the rate of economic depreciation, δ . This is customarily defined as the rate of decline in price due to deterioration. In the machine model, deterioration is assumed to be the pure shift in the production function. If we assume, as is customary among capital specialists, that deterioration is a function of age, then,

$\delta_{t,v,i} = -\partial \ln q_{t,v,i} / \partial (t-v)$. Ignoring uncertainty, substituting expression (9) into expression (11) and using the definition of deterioration, we find that:

$$\delta_{t,v,i} = -\sum_{\tau=0}^{\infty} p_t \phi_{t,v} (1-r)^{\tau} / q_{t,v,i} . \quad (12)$$

⁵ It is impossible to do so without a vintage model.

Some discussion of this result is in order. First, the economic depreciation rate, δ , is driven primarily by future deterioration, ϕ , but the two are distinct. Secondly, even if the future deterioration schedule is forecast with certainty, the depreciation rate depends on the expected future industry product prices and the interest rate. This is important because, using our definition, these other factors *must be held constant* in order to evaluate economic depreciation.

Studies such as Hulten and Wykoff [1981] measure economic depreciation using market prices of used assets. From equation (11) we would expect that used asset prices will be influenced by interest rates and expectations about future product prices. To the extent that product price forecasts change or interest rates change, measured declines in market prices may reflect these changes and may differ from economic depreciation. Thus market prices may be appropriate for estimating wealth (the total value of the assets), but they may be inappropriate for identifying economic depreciation as we have defined it.

One might avoid these complications by simply defining the depreciation rate in terms of the actual decline in market prices of used assets, rather than as the decline due to deterioration. However, this would weaken the link

between depreciation and deterioration. This can be illustrated by the effects of obsolescence. While investors presumably account for the effects of obsolescence in forecasting product prices, unexpected news of an innovation can cause an instantaneous drop in the prices of existing capital goods. Such news might have no immediate effect on figure 2, but the decline in prices occurs because forecasts of how figure 2 will look in the future change --- investment in the improved goods will gradually cause an upward rotation in ray OA. Hence the decline in price reflects neither advancing age of the goods, nor deterioration, nor declines in the current product price. Under such circumstances it would be impossible to infer the deterioration rate, ϕ , from data on used asset prices.

III. Demonstration of a Vintage Model Using Aggregate Data

In this section, an exercise is presented to illustrate the potential for vintage models to play a role in estimating some of the fundamental characteristics of capital such as the rate of deterioration. This work is strongly motivated by the analysis of the preceding section. The exercise might be legitimately criticized for being done with data which are too highly aggregated or for the overly simple econometrics used. But it is hoped that it will persuade people that observations of labor productivity and property income could be useful in identifying the role capital is playing in economic growth and fluctuations.

Specification

For this exercise, we sought a specification capable of letting expression (4) take on the general "shape" we proposed for the function described in expression (5). By selecting this shape we are recognizing that investment is a collection of discrete capital goods, and not a blob of jelly. We are allowing the production function to "look like" the production function for any representative machine. By working with aggregate investment data, we are assuming that all goods of a given vintage are identical. With a semilog form we can get the hypothesized "shape" and so we propose the following specification for a production function:

$$Y_{t,v} = I_v \{a_Y + B_Y t + g_Y(t-v) + [a_L + B_L t + g_L(t-v)] \log (L_{t,v} / I_v)\}. \quad (13)$$

where $a_Y, B_Y, g_Y, a_L, B_L,$ and g_L are parameters and $(t-v)$ is age. In addition to taking on the proper "shape", this allows the relationship to vary flexibly by age and time.

Next, we note that in the machine model of section II, labor and output are jointly determined. The levels selected are determined when we impose competitive market clearing conditions (expression 8). When we impose this on (13), we find that $L_{t,v} = I_v [a_L + B_L t + g_L(t-v)] p_t / w_t$.

In estimating this model, we used six time series associated with the published BLS [1983;1998a] measures of multifactor productivity for the U.S. private nonfarm business sector: total output (Y_t), total labor input⁶ (L_t), their prices (p_t and w_t), total property income ($\Psi_t = p_t Y_t - w_t L_t$), and historical real investment (I_v).

Other than investment, we do not observe any variables separately by vintage. Therefore, we make use of Solow's "ground rule" (expression 3) that vintage output and labor are additive components of the total. The production functions which we estimated were therefore of the form:

$$Y_t = \sum_v I_v \{ a_v + B_v t + g_v(t-v) + [a_v + B_v t + g_v(t-v)] \log [[a_v + B_v t + g_v(t-v)] p_t / w_t] \} + \varepsilon_v. \quad (14)$$

Note that labor vanishes in this expression, as a result of being fully determined by the level of output, vintage investments, and the equality of marginal products. In order to ensure that the amount of labor assigned to vintages is consistent with total observed labor, we simultaneously estimated:

$$L_t = \sum_v I_v \{ [a_v + B_v t + g_v(t-v)] p_t / w_t \} + \varepsilon_v. \quad (15)$$

⁶ The labor input measure used here is adjusted for labor composition, as described in BLS [1993]. Hence, to the extent that upgrades in the skills of labor have been identified, they have been allowed to augment the labor measure used in estimating this model. This adjustment arises from the logic of the Solow [1957] residual model. We think it makes

Finally, we estimated a property income equation:

$$\Psi_t = \sum_v \{ p_t Y_{t,v} - w_t L_{t,v} \} + \varepsilon_\Psi, \quad (16)$$

with, of course, the parameterizations of $Y_{t,v}$ and $L_{t,v}$ substituted in. In addition to helping identify the parameters, equation (16) allows the explicit calculation of the property income allocated to each vintage.

We estimated equations (14-16) using a maximum likelihood program for simultaneous nonlinear equations. We wanted a model in which a vintage's assets are "shut down" when their property income falls below zero.⁷ To allow for this dynamically would greatly complicate the model. For the present purposes, we fit various versions of the model, each carrying the summations over v back to some specific fixed age $(t-v)$. For each we examined the amount of property income allocated to each vintage. We selected a fixed "lifetime" for which property income was as close as possible to zero throughout the time series sample. In this way we allowed the data to help determine the lifetime.

Results

The ability of the models to converge to solutions which fit the dependent variables well and which yield

sense to use it in vintage models. In Section V we will discuss the reconciliation of the two models.

reasonable looking patterns of vintage labor and output seemed fairly robust. One model we examined was fitted to annual data for 1962 through 1996 which included terms for investments of the previous 14 years (to 1948 for the 1962 observation). In most years, investments older than 10 years were tagged with negative property income by this model, with the income substantially negative for the 13 and 14 year old assets. After examining various capital service lives, we settled on a model for 1962 through 1996 which specified investments going back 10 years. The property income assigned to 10 year old asset was near zero in most years (except for the first few in the sample). This is the model we selected to report in this section. The estimated coefficients were:

a_y	=	1.2597	a_L	=	0.3996
		(0.1772)			(0.1748)
B_y	=	0.00454	B_L	=	-0.00226
		(0.00062)			(0.00059)
g_y	=	-0.0443	g_L	=	0.0381
		(0.0325)			(0.0348)

The data on output, labor, and property income were fit reasonably well, with respective "R-squared" statistics of .983, .954 and .995.

⁷ Ideally, we would also allow the machines to be turned back on if conditions for earning rents became more favorable. However, this would

The model can be used to generate vintage allocations for output, hours, and property income in each year of the sample. This can be done by simply examining the individual terms of the respective summations in equations (14) through (16), using the estimated parameters. While the terms themselves are interesting, the patterns in output per unit of labor input, and property income per unit of nominal output (capital's share) are most revealing. These are presented in Table 1.

In the first panel we present indexes for assets of different ages (thus different vintages) for particular years. The patterns are largely in accord with what we might expect. *Labor productivity⁸ is highest for the newest, and presumably most efficient, vintages.* It is lower for older vintages which have deteriorated and which embody older technology. The patterns of decline are fairly smooth, reflecting the fact that we specified output and hours to be linear functions of age and time. Nonetheless, the estimates are somewhat free to vary over time. The model indicates that output per hour has declined more rapidly, as a function of age, in recent years.

On the right side of the first panel, we present indexes of property income, by age, relative to new assets. These may be of special interest to those estimating

greatly complicate the specification and estimation of the model.

traditional capital stocks. In neoclassical theory it is assumed that the ratios of the efficiencies of assets correspond to the ratios of rents they generate (Hall [1968, p. 37]). If one wanted to employ a perpetual inventory model, these estimates could be regarded as local estimates of the *age/efficiency schedules*. (The rub is that, if the vintage model is the right model, then the capital aggregate would not "exist" in the sense that it would not be useful in studying many short-run phenomena). Reasonable estimates were obtained from this particular model for 1973, 1984, and 1996, where efficiency declines appear to be roughly "straight line". This corresponds to the finding by Blades [1998] that the straight line form provides the best fit to age/rent profiles. While this may, at first, seem surprising, there may be a simple explanation. Regardless of the pattern of the underlying process of deterioration of the vintage production functions, firms will "truncate" property income when it reaches zero. So whether the underlying process of deterioration is convex (geometric) or concave (hyperbolic), the decline in property income toward zero is likely to be roughly linear.

The failure of this particular set of parameters to eliminate property income by the 10th year in 1962, and for other early years is interesting in itself. This would be consistent with longer service lives in the earlier years.

⁸ Again, labor hours are adjusted for changes in the skill composition

In this specification, we have treated all investment as if it were a single type of capital. We know that in reality, there has been a marked shift in the mix of investment from longer lived assets like structures to shorter lived assets like equipment. This trend has been accentuated by the growing importance of computers. One interesting extension of this work might be to try to specify and fit this model with several types of capital.

The second panel of Table 1 presents indexes of labor productivity and property income which track particular vintages as they age. It is striking how similar they are to the results in the top panel. The main difference is that, in the middle panel, each vintage is assumed to embody the fixed technology of the year it was built. The declines in the top panel reflect differences in technology between vintages. The model is finding relatively strong effects associated with deterioration and relatively weak effects associated with embodied technical change. We will return to this point shortly.

The bottom panel of Table 1 presents indexes of labor productivity and property income for assets of a given age. Focusing on first year assets (columns 1 and 5) we note that labor productivity has increased substantially, but property income, as a share of the value of output, has increased

of the work force.

only slightly. Property income itself (rather than its share), rose dramatically, but this mainly reflects inflation. We chose to present it as a share in order to depict the relative earnings of new capital. One might expect this series to be relatively stable due to competitive pressures. It is really interesting to look at the property income share for the ten-year old assets. This declined dramatically between 1962 and 1973 (when it reached 82.0), perhaps reflecting the shortening of asset lives. While resuming a pattern of growth, this share increased sharply in trough years (1975, 1982 and 1991). The model is finding a relative increase in the earnings of older assets in these years. New investment is the explanatory variable in this model. As investment slacks off during recessions, earnings of existing assets may benefit, more than offsetting the deleterious effects of the depressed output price. In such a scenario the older assets would become increasingly vulnerable to retirement during the early stages of a recovery, as new investment surges. With the productivity of assets deteriorating rapidly in all years, any slowdown in investment lowers overall productivity by shifting production to older assets. This point is reinforced in 1991, when a surge in the share of property income associated with ten year old assets accompanies a slowdown in investment.

The results in Table 1 are derived from the estimated state of variables under observed (equilibrium) conditions. Thus they allow for shifts in the amount of labor allocated to each machine. It is interesting to examine some intrinsic properties of the estimated vintage production functions.

One way to approach this is to calculate the rates at which age and time are shifting the property income generated by the production functions, holding relative prices and labor fixed. Deterioration, $D_{t,v}$, can be calculated as the partial derivative of expression (16) with respect to age $(t-v)$, and embodied technical change, $E_{t,v}$, as the partial derivative with respect to time. The respective rates of deterioration, $d_{t,v}$, and embodiment, $e_{t,v}$, are calculated by dividing $D_{t,v}$ and $E_{t,v}$ by $\Psi_{t,v}$. Using the following expressions,

$$d_{t,v} = p_t [g_L \log L_{t,v} + g_I I_v] / \Psi_{t,v} , \quad (17)$$

$$\text{and } e_{t,v} = p_t [B_L \log L_{t,v} + B_I I_v] / \Psi_{t,v} , \quad (18)$$

we calculated rates for new assets, as presented in Table 2.

The signs are in the directions we would expect, but deterioration proceeds roughly 10 times more quickly than technical change. Real investment, the explanatory variable in this model, has been deflated using price

indexes which attempt to control for quality differences between vintages. Hence the embodiment rate of 0.7 percent will reflect only the growth in *unmeasured quality change* in the price index.⁹ The model may settle on a rapid rate of deterioration in large part because this helps it explain the *cyclical fluctuations* in productivity. It appears that investment booms are followed by productivity increases and investment droughts are followed by productivity slowdowns. The model is trying to account for these phenomena by finding rapid deterioration in the productivity of existing assets. We shall return to this point shortly.

It is interesting to examine the vintage production functions themselves, rather than merely considering the equilibrium points. These can be calculated by substituting various possible amounts of labor into the vintage production function, expression (13), using the estimated parameters. These were plotted for 1992 and appear in figure 3. The functions take the expected shape and also shift in the expected fashion.

Finally, it is possible to partially characterize any function by the maximum possible labor productivity that could be achieved with that function. This maximum would occur at the point where a ray from the origin is just tangent to the function, ie. where $df/dL = Y/L$. This may be

⁹ Gordon [1990] presented evidence that unmeasured quality change for

determined from the set of estimated parameters as follows.
Differentiating (13),

$$Y_{t,v}/L_{t,v} = df/dL = [a_L + B_L t + g_L(t-v)] / L_{t,v} \quad (19)$$

Cancelling the "L"s and then substituting for Y, again in equation (9):

$$[a_L + B_L t + g_L(t-v)] = a_Y + B_Y t + g_Y(t-v) + [a_L + B_L t + g_L(t-v)] \log(L_{t,v,i}). \quad (20)$$

Then solving for L at the maximum:

$$L_{t,v} = \exp\{1 - [a_Y + B_Y t + g_Y(t-v)] / [a_L + B_L t + g_L(t-v)]\} \quad (21)$$

In Table 3 we compare several variables for new assets, and for five year old assets, in 1996, at a) their observed (equilibrium) levels and at b) the points on the vintage production functions where labor productivity would be maximized.

From these comparisons we can conclude that the new and five year old investments are being operated in the regions of their production functions characterized by *diminishing returns to labor*. Firms owning these assets would earn positive rents if they attempted to maximize labor productivity. But they earn more rents by using more labor and producing more output. The ten year old investments have become marginal, and their owners must maximize labor productivity just to break even.

durable goods was on the order of 3 percent.

There is some confirmation in micro establishment data of the scenario in which potential productivity is much higher than actual. Jensen, McGuckin and Stiroh [1998] have detected that the labor productivity of some plants may actually rise with age. For plants with very slow deterioration, our model would actually predict an increase in labor productivity as p_t/w_t falls over time.

As mentioned earlier, actual labor productivity is markedly "pro-cyclical". The correlation coefficient between the rate of change of actual labor productivity with the rate of change of actual output is 0.67. The correlation coefficient between the rate of change of fitted labor productivity with the change in actual output is 0.32. Thus fitted labor productivity retains a pro-cyclical tendency in spite of the fact that it is consistent with a capital story of diminishing returns to labor. In the fitted model, productivity change is driven mainly by new investment, and this is sufficient to offset the pro-cyclical tendency. This is in line with Cooper and Haltiwanger [1993].

There is also confirmation in industry data of the scenario in which productivity growth is associated with growth in output. Using new BLS [1998b] data on output per hour by industry, we have estimated the following

relationship for the 1987-1996 trends associated with 499 manufacturing industries:

$$\begin{aligned} (\text{Trend in Productivity}) &= 0.878 + 0.455 (\text{Trend in Output}). \\ &\quad (0.100) \quad (0.028) \end{aligned}$$

Again, one possible explanation is investment. If new technology were embodied in investment, growth in an industry's output would be necessary to drive productivity growth.

It would be interesting to apply specifications like the one described in this section to some of these more detailed datasets, such as an industry or establishment dataset. It would also be interesting to explore whether vintage contributions of more than one type of capital can be identified with such a model. However, in keeping with tradition of Solow's studies, this paper has focused on aggregate growth.

IV. Conclusion

The reigning paradigm of productivity measurement is based on the Solow [1957] residual model. The Solow [1959] vintage model was brushed aside largely because it complicated the picture without doing much to explain long-term growth. However, models of many other phenomena, such as cyclical fluctuations in productivity or the retirement of assets, include capital measures. To the extent that the

"lumpy" nature of capital influences these phenomena, it may be necessary to alter the specification of capital in these models.

Motivated by the vintage model, this paper has proposed a model of how firms decide on how much output to make with individual capital assets. The paper shows that small adjustments to this static model imply that productivity should be counter-cyclical. Deterioration of assets and investment in improved goods occur contemporaneously with changes in output, and so such effects may be responsible for the pro-cyclical behavior of productivity. The specification of production associated with capital is also considered. It may be necessary to postulate a minimum labor requirement in order to predict the abrupt shut down of plants which can no longer generate positive rents.

Finally, a vintage model is estimated using highly aggregate data. While rather simple, this data exercise illustrates the potential for a production model, with a "realistic" treatment of capital, to account for aggregate productivity trends and movements.

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Table 1. Indexes of output per unit of labor input and of property income per unit of nominal output

Output per unit of labor					Property Income			
<i>By age at a point in time</i>								
Age	1962	1973	1984	Point in time 1996	1962	1973	1984	1996
1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2	90.4	88.3	87.5	86.6	92.6	89.9	90.5	91.4
3	82.6	78.8	77.3	75.9	85.2	79.4	80.5	82.3
4	76.0	70.9	68.9	67.2	77.9	68.5	70.0	72.8
5	70.4	64.2	61.9	59.9	70.6	57.4	59.1	62.7
6	65.7	58.6	56.1	53.9	63.4	45.9	47.8	52.3
7	61.7	53.8	51.1	48.7	56.5	34.3	36.2	41.4
8	58.2	49.7	46.8	44.4	49.7	22.5	24.4	30.2
9	55.2	46.1	43.1	40.7	43.2	10.7	12.3	18.7
10	52.6	43.1	40.0	37.4	36.9	-1.1	0.0	6.9
<i>By age as a vintage ages</i>								
Age	1962	1973	1979	1986	Vintage 1962	1973	1979	1986
1	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2	92.4	88.9	89.2	87.7	92.3	91.0	90.1	91.2
3	86.6	79.8	79.6	77.8	83.6	81.6	80.3	81.9
4	80.7	73.6	71.3	69.8	75.2	69.1	71.1	72.1
5	76.3	67.2	64.5	63.1	64.8	58.0	61.3	61.5
6	71.7	61.9	59.3	57.2	55.1	45.3	49.3	51.6
7	67.9	57.0	54.5	52.6	43.6	32.5	37.7	38.5
8	63.9	52.7	50.5	48.2	33.6	21.0	24.7	27.8
9	60.5	48.9	46.9	44.4	22.1	11.7	8.2	18.9
10	57.3	45.7	43.3	41.2	9.9	1.6	-5.6	7.6
<i>By time for assets of a given age</i>								
Year	1	2	5	10	Age 1	2	5	10
1962	56.6	59.0	66.3	74.0	94.3	96.1	110.5	187.7
1963	57.9	60.3	67.4	74.9	94.1	95.8	109.5	181.8
1964	59.8	62.1	69.0	76.2	93.5	94.9	106.6	170.3
1973	76.4	77.8	81.6	85.2	90.8	89.9	86.8	82.0
1974	76.9	78.2	82.1	85.8	91.7	91.0	89.0	89.1
1975	77.5	78.8	82.7	86.3	92.4	91.8	90.7	94.1
1976	80.2	81.3	84.5	87.4	91.8	90.9	87.1	77.8
1977	81.3	82.4	85.4	88.2	92.3	91.4	87.7	78.2
1978	83.0	84.0	86.6	89.0	92.4	91.4	86.9	72.9
1979	84.5	85.4	87.7	89.8	92.8	91.7	86.9	71.1
1980	85.2	86.1	88.4	90.4	93.5	92.6	88.6	76.3
1981	85.3	86.2	88.6	90.9	94.6	94.1	92.0	88.5
1982	85.7	88.6	89.1	91.5	95.5	95.2	94.4	96.4
1983	87.5	88.2	90.4	92.4	95.7	95.3	94.0	92.6
1984	88.7	89.4	91.3	93.2	96.2	95.9	94.7	93.9
1990	96.9	97.1	97.7	98.2	99.1	99.0	98.8	98.9
1991	97.4	97.6	98.2	98.8	99.9	100.0	101.1	106.9
1992	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
1993	100.7	100.6	100.6	100.7	100.8	101.0	102.2	107.3
1994	100.5	100.5	100.7	101.1	101.9	102.4	105.6	120.1
1995	101.5	101.5	101.6	101.9	102.5	103.1	107.0	124.4
1996	103.2	103.1	102.9	102.9	102.9	103.6	107.5	124.5

**Table 2. Rates of change in the property income earned
a) by a new asset as it ages (deterioration) and b) by new
assets from one year to the next (embodiment), all other
things equal**

	Deterioration Rates	Embodiment Rates
1962	-0.0693	.0072
1973	-0.0811	.0081
1990	-0.0724	.0074
1996	-0.0679	.0069

**Table 3 Variables of Vintage Production Functions Compared
as Observed and at Point Where Labor Productivity Would be
Maximized at**

	Observed	Maximized Labor Productivity
<i>New assets in 1992</i>		
Output	508	191
Hours	191	36
Productivity	2.66	5.26
<i>Five year old assets in 1992</i>		
Output	458	286
Hours	286	157
Productivity	1.60	1.82
<i>Ten year old assets in 1992</i>		
Output	329	330
Hours	330	331
Productivity	1.00	1.00

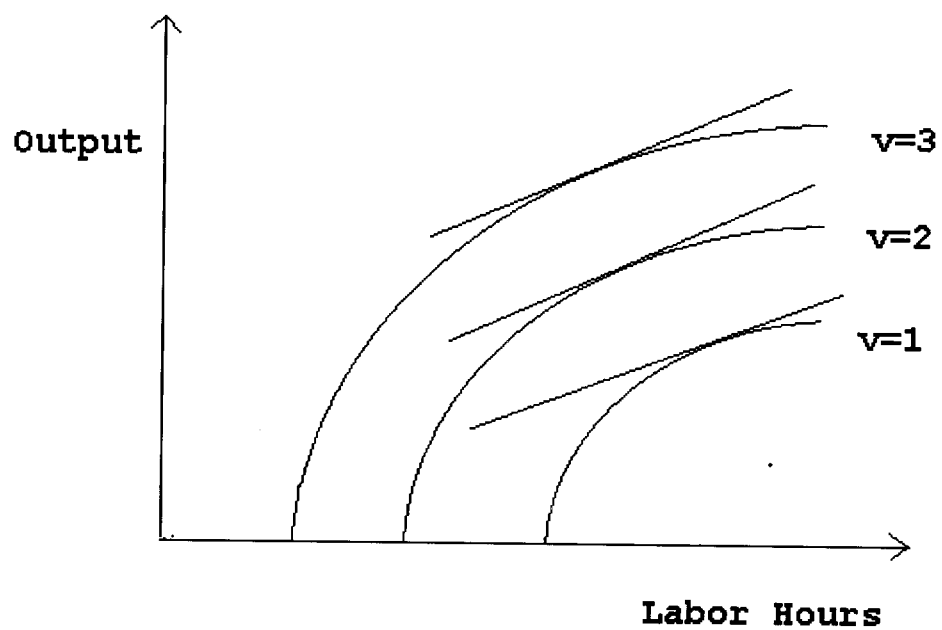


Figure 1. A Model of Production with Machines

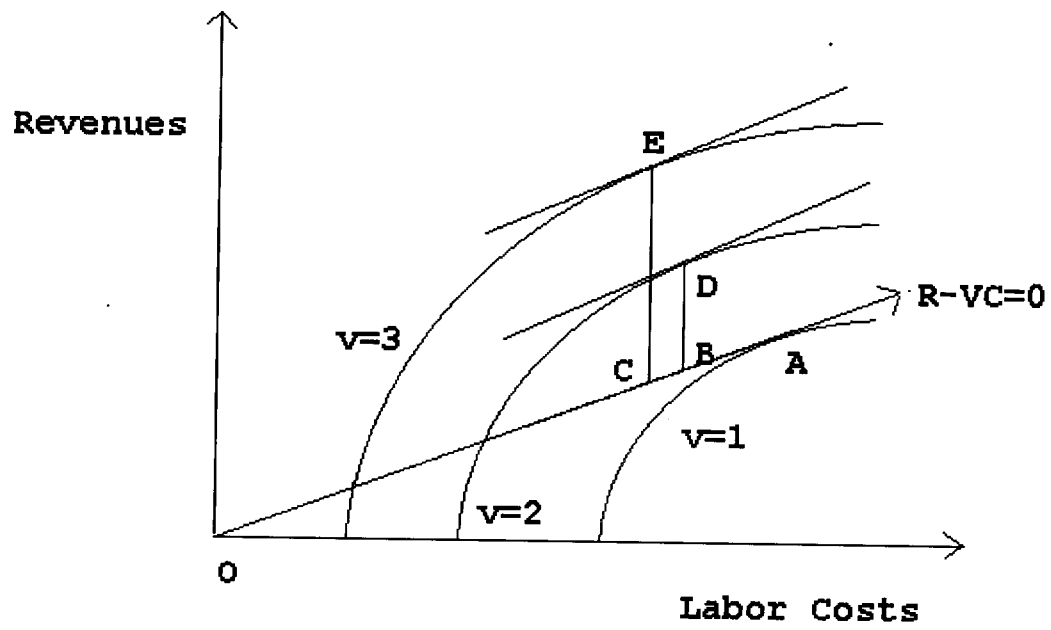


Figure 2. Extraction of Rent from Machines

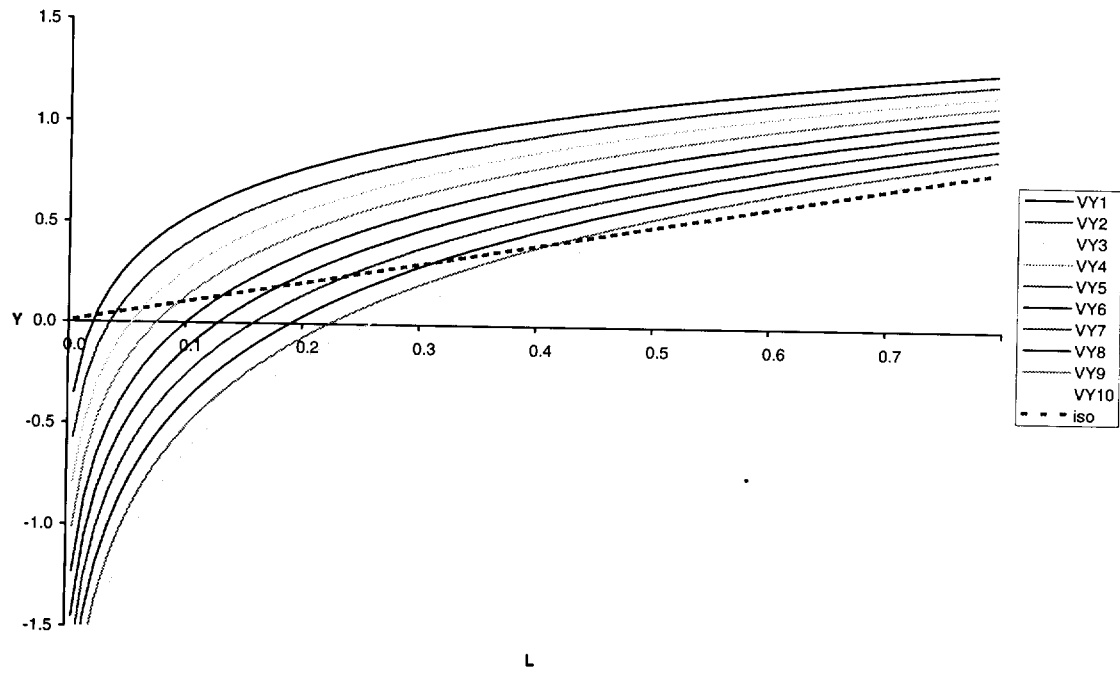


Figure 3. Estimates of Vintage Production Functions by Age (VY) for 1992
Output per unit of real investment plotted as functions of labor per unit of real investment for new assets (VY1) through marginal assets (VY10)