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Using Financial Assets to Hedge Labor Income Risks: Estimating the Benefits *

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Abstract

We develop a life-cycle framework with incomplete markets that delivers analytical solutions for asset prices, consumption and portfolio allocations in the face of hedgable and unhedgable labor income shocks. To implement the framework, we estimate the covariance structure between financial asset returns and labor income shocks using annual U.S. data from 1965 to 1994 for synthetic persons defined in terms of birth cohort, educational attainment and sex. We find that the correlation between earnings shocks and asset returns differs sharply among sex-education groups and varies systematically over the life cycle for some groups. For example, the correlation between returns on the S&P 500 and earnings shocks exceeds .4 for older, college-educated women, ranges from .1 to .3 over most of the life cycle for college-educated men and is roughly -.25 for men who did not finish high school.

We calibrate the theoretical framework to the estimated covariance structure and other data in order to calculate the welfare effects of using financial assets to hedge group-level earnings risk. This portion of the paper is still in process, but our preliminary results show tiny welfare gains from trading the S&P 500 under reasonable assumptions about risk aversion. However, trade in a broad-based equity index enables individuals to hedge only a small portion of group-level earnings risk. Future drafts will quantify the welfare effects of trading larger asset menus that enable individuals to hedge much or all of group-level earnings risk.

1 Introduction

New assets create new risk-sharing opportunities, and so do financial innovations that lead to better use of existing assets. Given the important economic risks that confront households, new risk-sharing opportunities offer the potential to greatly improve welfare. How much improvement in practice? And who benefits from specific innovations? We develop and implement a dynamic equilibrium framework to address these sorts of questions. We focus our analysis on labor income risk.

Theory tells us that the welfare effects of a new asset or other financial innovation depend on the particulars of the economic environment. Even in a pure exchange economy, new financial assets can reduce welfare for some or, more surprisingly, all agents (Hart, 1975). As a simple example, consider an individual who faces no risk, but who borrows against future income to finance current consumption. If a new asset improves risk-sharing opportunities for others and thereby reduces precautionary savings, the riskless interest rate rises. As a consequence, welfare falls for individuals who face little risk, but who borrow to finance current consumption. [1] More generally, because a new asset can alter the equilibrium prices on existing assets, it may harm some individuals while benefiting others.

These remarks highlight the need for an equilibrium framework that can be used to analyze the welfare effects of specific financial innovations for particular individuals and demographic groups. We develop an incomplete markets framework that can be (i) tailored to particular economies and financial innovations, (ii) implemented using available data, (iii) readily solved to compute equilibrium outcomes, and (iv) applied to important risks that confront individuals, families, groups and countries. The framework is a dynamic equilibrium exchange economy with incomplete markets, normal random variables and exponential preferences. Willen (1999) shows how to calculate equilibrium responses to the introduction of new financial assets in the exponential-normal framework. We extend his analysis to incorporate a flexible life-cycle structure for earnings and consumption. This generalization makes the framework especially suitable for studying the use of financial assets to hedge labor income risks.

We implement the framework using U.S. data from 1965 to 1994 on security returns and on labor income for synthetic persons defined in terms of birth cohort, educational attainment and sex. We quantify the risk-sharing benefits and costs

[1] Hart's (1975) example is more subtle and involves preferences that are not additively separable over time. He introduces a security at date 1 that opens up new opportunities for transferring resources at date 2 but has the adverse effect of eliminating gains from trade at date 3. If consumers value date-3 consumption sufficiently, they are made worse off by the new security.

associated with two types of financial innovation. First, in the spirit of Shiller (1993), we consider the introduction of new financial assets with payoffs tied to measures of aggregate outcomes. Second, we consider mutual funds constructed from existing securities in such a way as to mirror the industry distribution of employment for each synthetic person at each point in time. If the industry-level components of returns to equity and human capital are correlated, these mutual funds expand risk-sharing opportunities relative to portfolios defined over a riskless asset and a broad equity index only.

As an input to our analysis of financial innovation and an interesting topic in its own right, we characterize the covariance between returns on financial assets and returns on human capital. Rather surprisingly, few previous studies investigate this covariance despite its obvious relevance to individual portfolio optimization, mutual fund structuring, pension fund management, savings behavior and asset pricing. [2] This neglect is also surprising in another respect: Shifts in relative earnings among individuals and demographic groups have been the focus of extraordinary research and policy attention in recent years. Levy and Murnane (1992) and Katz and Autor (1998) review an extensive body of work in this area. Neither review devotes any attention to the idea that financial assets might serve to hedge income risks.

For each sex-education group, we estimate the covariance between financial asset returns and labor income shocks. We find that the covariance between stock market returns and labor income shocks differs among education groups and varies systematically over the life cycle for some groups. For example, the correlation between the S&P 500 return and labor income shocks is about -.25 over most of the working life for men who did not finish high school but mostly ranges from .1 to .3 for college-educated men. For college-educated women the correlation rises sharply after age 35, eventually exceeding .4 in their middle and late 50s.

Given the stochastic properties of labor income, including its covariance structure with financial asset returns, we calculate equilibrium asset prices and optimal

[2]Labor income risk plays an important role in work on precautionary savings (e.g., Carroll, 1997) and in some recent work on asset pricing (e.g., Heaton and Lucas, 1996). Precautionary theories of consumption emphasize the role of borrowing and lending, which is also important in our analysis, but sidestep the pure hedging function of financial assets. Asset-pricing studies that consider a role for labor income risk typically restrict attention to highly aggregated labor income measures or characterize individual income uncertainty in ways that preclude cross-sectional variation in the hedging portfolio. Heaton and Lucas (1997) consider how equity returns covary with labor income and proprietary business income. They report a small negative correlation of equity returns with aggregate wage income and a larger, but still modest, positive correlation with aggregate proprietary business income. They pursue the implications of this covariance structure for cross-sectional variation in portfolio choice and for asset pricing.

consumption and portfolio allocations for each (synthetic) individual. These calculations require an assumption about the asset choice menu over which individuals optimize. The available evidence suggests that neither individuals nor pension fund managers employ sophisticated portfolio strategies to hedge earnings risk. [3] Based on this evidence, we proceed under the assumption that individuals (or their portfolio managers) allocate financial wealth between a riskless asset and a broad-based equity portfolio. [4] This assumption yields a benchmark dynamic equilibrium, against which we evaluate the effects of financial innovations.

Relative to the benchmark equilibrium, new financial assets based on measures of aggregate output, consumption and employment outcomes yield tiny welfare gains. A new financial asset based on the gross rate of job destruction in the manufacturing sector generates larger, but still quite modest, welfare gains. The job destruction rate spikes sharply during recessions and is highly correlated with labor income shocks for some demographic groups.[5]

Our welfare calculations understate the potential gains from using financial assets to trade systematic components of labor income risk for at least four reasons. First, the assets that we have considered thus far enable individuals to hedge only a small portion of their group-level earnings risk. In future drafts, we will quantify the welfare effects of trading larger asset menus that enable individuals to hedge much or all of group-level earnings risk. Second, we ignore international risk-sharing possibilities in this paper, a potentially important source of welfare gains (e.g., Lewis, 1996, and Baxter and Jermann, 1997). Third, data limitations prevent us from examining the gains from sharing systematic risks within our broad demographic groups. Fourth, our characterization of the group-level earnings process probably understates the extent of low-frequency stochastic earnings variation, perhaps dramatically so. Using

[3]Most U.S. households have small financial asset holdings and even smaller holdings of equity securities. See, for example, Poterba and Samwick (1997) and Flavin and Yamashita (1998). Empirical work on portfolio allocation behavior devotes little attention to hedging labor income risk. See, in addition, Poterba et al. (1996), Papke (1998) and Sunden and Surette (1998). Professional financial advisors place little weight on hedging labor income risk (Canner, Mankiw and Weil, 1998).

[4]Our theoretical framework easily accommodates other assumptions about portfolio choice behavior, including choice over a large asset menu, but we require some assumption or estimate regarding portfolio choice behavior in order to calculate the initial equilibrium and the equilibrium response to financial innovation.

[5]The job destruction rate is calculated by summing establishment-level employment declines, as described in Davis, Haltiwanger and Schuh (1996). Many researchers and government agencies throughout the world have begun to construct and report statistics of this sort. See Davis and Haltiwanger (1998) for a review of this work. The U.S. Bureau of Labor Statistics plans to begin publishing official job destruction statistics of this type for most sectors of the U.S. economy in the near future (Spletzer, 1997).

a different approach that focuses on low-frequency variation, Attanasio and Davis (1996) find large potential gains to better risk sharing among groups defined by sex, education and birth cohort. [6]

The paper proceeds as follows. Section 2 constructs and examines labor income data for synthetic persons. We characterize systematic and stochastic aspects of labor income variation for each synthetic person, and we calculate the size of shocks to the present value of lifetime earnings (as a function of age, education and sex) implied by our characterization of the labor income process. Section 3 describes the data on financial asset returns and our procedures for constructing new financial assets and group-specific mutual funds. Section 4 investigates the covariance between human capital returns and financial asset returns. Section 5 sets forth a dynamic equilibrium life-cycle framework with incomplete markets. We show how to calculate asset prices, consumption paths and portfolio allocations. We also explain how to calculate the welfare effects of new risk-sharing opportunities in terms of the equilibrium response to financial innovation. Sections 6 and 7 empirically implement the theoretical framework. Section 6 draws on our characterization of labor income behavior, portfolio choice menus and other data to estimate the initial dynamic equilibrium. Section 7 then quantifies the equilibrium effects of financial innovations that take the form of new financial assets or expanded portfolio choice menus. Section 8 concludes.

2 Labor Income

2.1 Synthetic Panel Data

The Annual Demographic Files of the March Current Population Survey (CPS) contain individual data on pre-tax labor earnings in the previous calendar year. Using the CPS, we construct panel data on mean annual earnings and mean log earnings from 1963 to 1994 for groups of individuals defined by sex, educational attainment and birth cohort. By following particular groups over time as they age, we obtain longitudinal data on synthetic persons.

For each sex, we group persons into four educational attainment categories: less

[6] Attanasio and Davis (1996) show that changes in the cohort-education structure of pre-tax hourly wages among men drove large changes in the between-group distribution of household consumption during the 1980s. They also calculate large, but unrealized, welfare gains from sharing this type of group-level earnings risk. We assess the extent to which financial assets can be used to achieve some of the potential between-group risk-sharing gains uncovered by Attanasio and Davis. However, the methodology we develop in this paper is more general; it can be applied to panel data on countries, groups or individuals.

than high school, high school, some college but no degree, and a college education (including persons with post-graduate schooling). [7] Within each sex-education group, we consider three-year cohorts indexed by the birth year of the middle cohort members. For example, the 1956 cohort contains persons born between 1955 and 1957. [8] We also measure cohort age based on midpoints, so that the 1956 cohort is 34 years old in 1990. We exclude observations for which any cohort members are less than 24 or more than 59 years old. We also exclude cohorts with fewer than seven annual observations.

After imposing these restrictions and using two lags in the specification of the labor income process, our sample runs from calendar years 1965 to 1994 for cohorts born from 1911 to 1962. The panel contains the current and two lags of earnings for synthetic persons who are between 27 and 58 years old. The sample selection criteria and our procedures for grouping by sex, education and birth cohort yield panel data for $2 \times 4 \times 18 = 144$ synthetic persons. [9] After using two observations to compute lags, there are between 5 and 30 consecutive annual observations for each synthetic person.

To compute mean earnings and mean log earnings for synthetic persons, we use CPS data on private and public sector wage and salary workers. We exclude unincorporated self-employed persons from the earnings calculations, but we include self-employment and farm income for persons who are mainly wage and salary workers. We also exclude persons who were students or in the military at least part year. [10] We express earnings in 1982 dollars using the GDP deflator for personal consumption expenditures. Finally, we compute an employment count that includes the self employed and military personnel, which we use to aggregate over synthetic individuals in the calculation of equilibrium outcomes in the theoretical model.

Table 2 reports information about the sample sizes that underlie the synthetic panel data. Mean sample sizes for the earnings measures typically number in the hundreds, and no cell has fewer than 55 observations. Sample sizes are larger for the employment counts. [11]

[7] We construct time-consistent categories across the 1991-92 break in the CPS education codes following the recommendations of Jaeger (1997).

[8] More precisely, because the CPS records age at the March survey date, the 1956 cohort contains persons born between March of 1955 and February of 1958.

[9] In any given calendar year, 10 or 11 (9 in 1994) birth cohorts in each sex-education group satisfy the age restriction, which corresponds to 80 or 88 (72 in 1994) synthetic persons.

[10] In addition, we exclude persons who report an hourly wage less than 75 percent of the federal minimum. We handle top-coded earnings observations in the same manner as Katz and Murphy (1992).

[11] The employment counts listed in Table 1 are too large, because we mistakenly included persons

2.2 Income Processes for Synthetic Persons

To estimate the risk-sharing potential of financial assets, we must first identify and characterize the stochastic component of earnings variation for each (synthetic) person. Given a model for labor income, we can identify labor income shocks, calculate their effects on the present value of lifetime earnings, and investigate how the earnings shocks covary with asset returns. We characterize the labor income process presently and take up the covariance between earnings shocks and asset returns in section 4.

We model labor income as an autoregressive process, augmented by a polynomial in age to capture systematic life-cycle variation. We allow the autoregressive and life-cycle structure of the income process to vary freely across sex-education groups. For each sex-education group, we pool over birth cohorts, include cohort fixed effects in the specification, and estimate the earnings regression by least squares.

Preliminary investigation led to a third-order age polynomial in the levels specification, a second-order age polynomial in the log specification, and a second-order autoregressive process in both specifications. Thus, the log specification for a particular sex-education group is

$$y_t^h = \phi^h + \rho_1^h y_{t-1}^h + \rho_2^h y_{t-2}^h + \alpha_1^h \text{age}_t^h + \alpha_2^h (\text{age}^2)_t^h + \eta_t^h,$$

where y_t^h denotes mean log earnings for cohort h at t, ϕ^h denotes a vector of cohort-specific intercept terms, ρ and α denote coefficients on the autoregressive and age variables, and η is an error term. The levels specification is the same except for the addition of a cubic term in age.

Table 3 reports the estimated autoregressive parameters and selected other statistics. The autoregressive structure of earnings clearly differs among sex-education groups. Earnings shocks show greater persistence for men than for women at each education level. Men with a high school education show the most persistent response to earnings shocks, and women with some college or less than a high school education show the least persistent responses. We provide a fuller characterization of the dynamic response to earnings shocks shortly.

Figures 1 and 2 display the expected life-cycle earnings profiles and the evolution of cohort fixed effects for the estimated specifications. We generate the life-cycle profiles by initializing age-27 and age-28 earnings to sample mean values and then iterating forward on the estimated earnings equations with the regression error (η) set to zero.

The life-cycle profiles for men show steep earnings growth in their late twenties and early thirties, followed by a flatter trajectory and an eventual decline at older

with zero hours. We will fix this in the next draft.

ages. Earnings rise with educational attainment at each age for men, as does the age of peak earnings. For women, three of four education groups show marked earnings declines during their late twenties or early thirties, a pattern that presumably reflects child-bearing and child-rearing activity. Following this dip, women's earnings rise steadily with age for each education group. The age of peak earnings occurs later in life for women than for comparably educated men. Women earn less than comparably educated men throughout the life cycle.

Expected earnings have generally drifted downward for more recent cohorts of men with the exception of college-educated men born since 1950. For men with less than a high school education, expected earnings show a dramatic decline (38 log points) between the 1911 and 1962 birth cohorts. Expected earnings also show a large decline (20 log points) between the 1911 and 1962 birth cohorts for women with less than a high school education. In contrast, for women with some college or a college degree, expected earnings have risen sharply for cohorts born after 1929. Of course, these inferences about the evolution of cohort fixed effects rest on the accuracy of the price deflator applied to nominal earnings. In assessing the time path of these fixed effects, one should also bear in mind that later cohorts have sharply higher educational attainment than earlier cohorts.

2.3 Shocks to the Present Value of Lifetime Earnings

The potential welfare gains from improved risk sharing depend partly on the size of labor income shocks. To address this matter, table 3 reports how shocks to current labor income affect the expected present value of remaining lifetime earnings. The "present value multiplier" in the table equals the discounted cumulative response to a unit labor income shock at age 35 for a person who retires at age 58.

To spell out the details of the multiplier calculation, let $m(j,a)$ denote the impact of a unit shock to labor income at age a on expected future earnings at age j. [12] For the levels specification, we compute the present value multiplier at age a as

$$\text{PVM}(a) = \sum_{j=a}^{58} m(j,a)(1+r)^{a-j}.$$

For the log specification, we compute the present value multiplier in a way that reflects the multiplicative nature of earnings shocks:

$$\text{PVM}(a) = \sum_{j=a}^{58} m(j,a)\exp\left[\hat{y}(j) - \hat{y}(a)\right](1+r)^{a-j},$$

[12] Setting $m(a,a) = 1$ and $m(a-1,a) = 0$, the dynamic response function satisfies $m(j+1,a) = \rho_1 m(j,a) + \rho_2 m(j-1,a)$ for $j > a$.

where $\hat{y}(a)$ denotes expected log earnings at age a.

Table 3 reports PVM(35) for annual discount rates of .01 and .04. The log and levels specifications deliver similar multipliers for each sex-education group, although the log specification tends to give somewhat larger multipliers for groups with a sharply rising trajectory of expected earnings near age 35 (e.g., college-educated men). The present value multipliers typically range from 2 to 3, except for much smaller multipliers in the neighborhood of 1.3 for women with some college or less than a high school education. At each educational attainment level, the present value multipliers are considerably larger for men than for women.

Figure 3 displays the impact of unit standard deviation earnings shocks on the present value of lifetime earnings as a function of sex, education and age. Letting $\sigma(\eta)$ denote the standard error of the earnings regressions reported in table 3, we calculate the present value function at age a as $\text{PVM}(a)\sigma(\eta)$ for the levels specification and $\text{PVM}(a)\sigma(\eta)\exp\left[\hat{y}(a)\right]$ for the log specification (evaluated at the 1911 birth cohort). These present value functions reflect the magnitude and persistence of labor income shocks, the discount rate and retirement horizon and, for the log specification, the shape of the expected life-cycle earnings profile.

According to Figure 3, a unit standard deviation shock to the group-level component of labor income affects the expected present value of lifetime earnings by roughly 3-4 thousand 1982 dollars for college-educated men, by roughly 2 thousand dollars for less educated men and college-educated women, and by one thousand dollars or less for less educated women. The present value response to earnings shocks tapers off sharply after age 50, which reflects our assumption of retirement at age 58. We consider alternative, and less extreme, assumptions regarding retirement behavior when we estimate the benefits of using financial assets to hedge labor income risks.

We stress that the present value calculations in table 3 and figure 3 do not reflect the labor income variation embedded in the cohort fixed effects. Figures 1 and 2 make clear that cohort-level earnings shocks are quite large, and that they evolve in a highly persistent manner over time. In principle, we could investigate the potential for sharing this aspect of earnings risk through financial assets, but it is unclear how private-sector institutions would perform this risk-sharing role. In contrast, pension funds and appropriately tailored mutual funds are natural vehicles for hedging the earnings risk reflected in figure 3.

2.4 Life-Cycle Heteroscedasticity in the Earnings Shocks

The size of labor income shocks may vary systematically over the life cycle. To investigate this matter, we regress the absolute value of residuals from the earnings

regressions on simple functions of age. Figure 4 displays the estimated profiles of the absolute earnings residuals when fit to a quadratic polynomial in age.

Many groups show little age-related heteroscedasticity in earnings, but the magnitude of labor income shocks rises with age in the levels specifications for more educated men and women. The magnitude of earnings shocks also rises with age in the log specifications among more educated women and, after their late 30s, more educated men. Among college-educated men, the estimated magnitude of earnings shocks in their late 50s is more than twice as large as in their middle 30s. We use the fitted heteroscedasticity profiles in figure 4 to evaluate the risk-sharing benefits of financial assets.

2.5 The Covariance Structure of Earnings Shocks

Full risk sharing in our theoretical framework is achieved by introducing a tradable asset for each (synthetic) person's human capital. We can calculate the welfare gains from full between-group risk sharing – given our characterization of earnings uncertainty – by computing the equilibrium response to the introduction of these assets. The *distribution* of welfare gains from completing markets in this manner depends on the covariances among contemporanous earnings shocks, because they determine asset prices and the cost of hedging. There are 144 synthetic persons and 30 periods in our analysis, which implies a potentially very large number of covariance parameters to estimate. We are looking for a sensible approach that will enable us to characterize the covariance matrix as a flexible but parsimonious function of sex, education and age.

In the meantime, Table 2.5 provides some information about the structure of these covariances. The contemporaneous correlation between average sex-education shocks is positive in almost all cases, and it shows a strong tendency to decline with the distance between groups as measured by sex or education. Hence, groups with more extreme characteristics have lower average shock correlations. For example, college-educated persons and men who did not finish high school have the lowest mean correlations. Men and women in the middle education groups have the highest mean correlations.

3 Financial Asset Returns

3.1 Equity Returns and Artificial Securities

To measure returns on a broad equity index, we use the annual rate of return on the S&P 500, as reported by Datastream. We also construct returns for several artificial securities that reflect movements in output and other economic aggregates. For output, we use real GDP in 1992 dollars from the National Income and Product Accounts (NIPA). It is convenient for our purposes to orthogonalize "returns" on the GDP asset with respect to returns on the S&P 500 index. The orthogonalized GDP asset pays off the residual in an OLS regression of GDP on the rate of return on the S&P 500 index.

We construct (orthogonalized) returns for other artificial securities in the same manner using the following data:

- Consumption: real personal consumption expenditures in 1992 dollars.
- Employment: civilian employment among persons 16 years and older.
- Unemployment: the civilian unemployment rate.
- Job Creation and Destruction: the gross annual rates of job creation and job destruction in the manufacturing sector, as described in Davis, Haltiwanger and Schuh (1996). We update the DHS data through 1993 and extend them back to 1965 using the methodology described in the appendix to Davis and Haltiwanger (1999).

3.2 Group-Specific Mutual Funds

We construct returns on group-specific mutual funds in three steps. First, we acquired monthly data on firm-level equity returns (inclusive of dividends and other distributions) from the Center for Research on Security Prices. From these data, we compute annual value-weighted equity returns for approximately 60 industries that cover most of the U.S. economy. Appendix A lists the industries. Second, using the CPS data described in section 2.1, we calculate the industry shares of earnings and hours for each synthetic person in each year from 1967 to 1994. The earnings-weighted calculations draw on data for wage and salary workers only, whereas the hours-weighted calculations include self-employed persons. Third, we compute the weighted mean equity return for each synthetic person in each year using the industry-level returns

from step one and the industry weights from step two. Depending on sex and education, we can assign an industry-level equity return to about 80-95 percent of observed hours and earnings. In a preliminary analysis, we obtained highly similar results for the hours-weighted and earnings-weighted mutual fund returns. The reported results make use of the hours-weighted returns.

4 Covariation between Labor Income Shocks and Asset Returns

In this section, we characterize the covariance structure between labor income shocks and various asset returns. We first characterize the covariance for the S&P 500 as a function of sex, education and age. We then consider artificial financial assets based on aggregate outcomes. We turn last to the covariance between group-level labor income shocks and group-specific mutual funds.

For risk-sharing purposes, we care about two aspects of the covariance structure. First, we seek assets with non-zero covariances for at least some labor income shocks, so that individuals can hedge by adopting a long or short position. Second, we seek assets with heterogeneity across people in the covariances. Absent such heterogeneity, an asset in zero net supply does not enable the sharing of income risk among persons with equal risk tolerances. Of course, such an asset is still useful for allocating labor income risk toward persons with greater risk tolerance.

4.1 The Covariance Structure for a Broad Equity Index

To estimate the covariance structure for the equity index, we fit regressions of the following form for each sex-education group:

$$\eta_t = \beta_0 x_t + \beta_1 \text{age}_t x_t + \beta_2 \text{age}_t^2 x_t + \beta_3 \text{age}_t^3 x_t,$$

where η_t is the labor income shock estimated in Table 3, and x_t is the rate of return on the S&P 500 in year t. We estimate the β parameters by least squares to characterize the covariance structure as a smooth function of age. As in Table 3, the regression for each group contains 315 annual observations on 18 synthetic individuals for the 1965-1994 period.

Panel A in Table 4 reports the regression R^2 values based on the levels and log specifications of the labor income process. The R^2 values are small for all groups; the largest values are in the neighborhood of .04-.05 for the least-educated men and, in the levels specification, .08 for college-educated women. Panel B reports p-values for

a test of the null hypothesis of no systematic age variation ($\beta_1 = \beta_2 = \beta_3 = 0$) in the covariance structure. The null hypothesis is rejected for the least-educated men and for college-educated women (levels specification).

Figure 5 displays the covariance as a function of sex, education and age based on the levels specification of labor income. The top panels pertain to the S&P 500. We construct these figures as follows. First, before running the regressions, we transform the asset return to have unit variance. Second, we evaluate the regression function at each age and divide through by the standard deviation of the labor income shock at that age to obtain a correlation coefficient. For this purpose, we let the standard deviation of the income shock vary with age as shown in Figure 4.

We see in Figure 5 that the S&P 500 is a good candidate for risk sharing in the sense that its correlation differs alot across people. The correlation with labor income shocks is around -.25 at all ages for men who did not finish high school. In contrast, the correlation is positive for college-educated workers, ranging from .1 to .3 over most of the working life for men and reaching even higher values for women in their middle and late 50s. So, even though returns on the S&P 500 account for a very modest portion of variation in labor income shocks, the heterogeneity in correlations across types of people suggests some potential as a risk-sharing tool.

The covariance structure for aggregate equity returns fits well with a large body of research on the demand for labor. Empirical studies of labor demand in the modern economy consistently find that more skilled (i.e., educated) workers are relatively complementary to physical capital, the use of advanced technologies and research and development activity. [13] In light of this body of evidence, and to the extent that equity value derives from residual claims on firms' physical capital, intellectual property and technological know-how, one might anticipate that the correlation between aggregate equity returns and labor income shocks rises with education. Figure 5 supports that view.

4.2 New Financial Assets

Table 3 also reports regression results for artificial financial assets based on aggregate outcomes, as described in section 3.1. For each new asset and each group, we fit regressions of the form:

$$\eta_t = \beta_0 x_t + \beta_1 \text{age}_t x_t + \beta_2 \text{age}_t^2 x_t + \beta_3 \text{age}_t^3 x_t + \beta_4 z_t + \beta_5 \text{age}_t z_t + \beta_6 \text{age}_t^2 z_t + \beta_7 \text{age}_t^3 z_t,$$

[13] See Chapter 3 in Hamermesh (1993) for a review of evidence on static labor demand relationships. Goldin and Katz (1996) discuss recent work and historical evidence on capital-skill and technology-skill complementarity.

where z_t denotes the return on the new asset. As before, we transform z to have unit variance. We also orthogonalize the vector $[z, \text{age}z, \text{age}^2 z, \text{age}^3 z]$ with respect to $[x, \text{age}x, \text{age}^2 x, \text{age}^3 x]$.

Panel A of Table 3 shows that the employment and job destruction assets outperform the other assets according to an R^2 metric, especially for less educated men. Indeed, the fraction of labor income risk accounted for by the S&P 500 plus the job destruction asset ranges from 18-28% for less educated men. Panel B shows that the null hypothesis of no systematic life-cycle variation in the covariance structure is rejected in at least half of the sex-education groups for both the employment and job destruction assets.

The middle panels in Figure 5 display the scaled covariance functions for the job destruction asset. Increases in manufacturing job destruction are bad news for everyone, but much worse news for people with low education. The correlation is well below -.5 over much of the life cycle for less educated men, as compared to about -.1 to 0 for college-educated men over 35. This sharp heterogeneity in the correlations and the high explanatory power of the job destruction asset (for some persons) make it a promising tool for risk-sharing puposes.

Figure 6 displays the regression R^2 for men as a function of age and education. We calculate the age-specific R^2 value as the ratio of the squared predicted variation to the variance of the dependent variable, both evaluated in an age-specific manner. The solid curves in Figure 6 show results for a regression on the S&P 500 only, and the other curves show results for the S&P 500 plus the indicated asset. The job destruction asset greatly improves the regression fit for less educated men throughout the life cycle. The destruction asset and the S&P 500 together account for 20-40% or more of group-level earnings risk for men who did not finish high school and for high-school educated men in their 20s, 30s and 50s.

4.3 Group-Specific Mutual Funds

Based on a sample that begins in 1967 rather than 1965, Table 3 also reports results for group-specific mutual funds. Except for college educated men and (in the levels specification) high school educated women, the mutual funds increase the goodness of fit in the income shock regressions by four-to-eight percentage points. For the most part, this improvement in fit holds whether or not the job destruction asset enters the regression. A portfolio of three risky assets – S&P 500, group-specific mutual fund, and job destruction asset – accounts for about one-third of group-level income risk for less-educated men and 12-32 percent of group-level risk for other men with no college degree and for college-educated women. Panel B of Table 3 indicates that

most groups exhibit statistically significant life-cycle variation in the mutual fund covariance.

The bottom panels in Figure 5 display scaled covariance functions for the group-specific mutual funds. (We remind the reader that the mutual fund returns are orthogonalized with respect to S&P 500 returns.) To our great surprise, mutual fund returns display a strong negative correlation with shocks to the present value of labor income for all men with less than a college education. A similar, but more muted, pattern holds for women. In other words, a *long* equity position in the worker's own industry acts to hedge group-level income risk.

This surprising finding runs directly counter to the view that industry-specific fluctuations in equity returns are mainly driven by factor-neutral demand and technology disturbances. That view implies a positive within-industry correlation between equity returns and human capital returns. The negative correlation in the data might arise because movements in equity returns are dominated by rent-shifting between firms and their workers, or because factor-biased technology shocks cause capital and labor returns to move in opposite directions.

Figure 7 shows the fraction of stochastic variability in group-level income risk jointly accounted for by the S&P 500, group-specific mutual funds and the job destruction asset. For men with less than a college education, these three risky assets account for roughly one-quarter of group-level income risk over most of the life cycle. For college-educated men and for women with at least a high school education, these assets account for modest fractions of group-level income risk, typically no more than 10 percent.

5 A life-cycle framework with incomplete markets

We adopt a dynamic exponential-normal model to evaluate the effects of financial innovation on equilibrium risk sharing and other outcomes. The model delivers closed-form solutions under fairly general specifications of endowment and asset price processes. Most other dynamic equilibrium models with incomplete markets rely heavily on numerical solution methods. See, for example, Telmer (1993), Heaton and Lucas (1996), Constantinides, Donaldson and Mehra (1998), Storesletten, Telmer and Yaron (1998) and Judd, Kubler and Schmedders (1998). These models typically require a very small number of agent types or other restrictions. In contrast, our framework remains tractable with an essentially arbitrary number of agents and assets, overlapping generations, and parameters that vary with age, cohort and time.

5.1 Description of the Model and Assumptions

The economy operates for $T+1$ periods, $t = 0, ..., T$, and contains a single consumption good at each date. There are H agents indexed by h. An agent enters the workforce at T_0^h, retires at T_1^h and lives for at most $T_2^h > T_1^h$ periods. The probability of being alive at date t, conditional on surviving until $t-1$, is s_t^h. Individual h has a pure time-discount factor, δ^h, and an effective time discount factor, $\delta_t^h \equiv \delta^h \mathsf{s}_t^h$. T^h denotes the lifetime of agent h.

A *consumption path* is a random vector $C^h = \left(\tilde{c}_t^h\right)_{t \in T^h}$. Agents have von Neumann-Morgenstern utility functions defined over consumption paths:

$$U^h\left(C^h\right) = \mathrm{E}\left[-\sum_{t \in T^h} \left(\delta_t^h\right)^t \frac{1}{A^h} \exp\left(-A^h \tilde{c}_t^h\right)\right]$$

where A^h is abolute risk aversion. Agents maximize U^h by choice of asset portfolios $\omega^h = \left(\omega_t^h\right)_{t \in T^h}$ with no restrictions on short sales. $\omega_{t,j}^h$ is the dollar amount invested in asset j at time t by person h.

Let H_t be the set of individuals who trade assets (i.e., adopt long or short positions) at t. In other words, H_t includes all workers and retirees alive at t except for persons known to be in their last period of life (age=T_2^h). Let H_t' be the union of H_t and persons known to be in the last period of life. Everyone in H_t satisfies an Euler equation that relates consumption in periods t and $t+1$, whereas the additional persons in H_t' simply consume their remaining wealth at t. In a slight abuse of notation, it will also be convenient to let H_t (H_t') denote the number of persons in the set H_t (H_t').

We impose three main conditions on labor income and asset returns:

Condition 1 *Labor incomes $\left(y_t^h\right)_{t \in T^h}$ follow ARMA processes, which may differ among individuals. Each individual's stochastic income process has a moving average representation.*

Some additional notation will be useful in describing the labor income processes. Let $\tilde{\eta}_t^h$ be the income innovation for individual h at time t. Let ψ_i be the i^{th} MA coefficient in the moving average representation; that is, $\mathrm{E}_t\left(\tilde{y}_{t+i}^h\right) - \mathrm{E}_{t-1}\left(\tilde{y}_{t+i}^h\right) = \psi_i^h \eta_t^h$. Define $\tilde{y}_t \equiv \frac{1}{H_t} \sum_{h \in H_t} \tilde{y}_t^h$ as labor income per capita among persons who trade assets at t.

Condition 2 *There are $J+1$ assets: (1) A riskless asset with certain but possibly time-varying return $R_{0,t}$ for $t = 1, ..., T$; and (2) J risky assets with random returns $\underset{J \times 1}{\tilde{R}_t}$, $t = 1, ..., T$ which are independent over time. Risky assets have the same distribution of payoffs in every period.*

We need make no strong assumptions about the form of these assets; they may be long-lived with ARMA dividend streams or short lived. The assumption that risky assets have the same distribution of payoffs in each period is a convenience, especially in the empirical implementation of the model, but it is inessential for preserving the tractability of our solution method.

Let the vector of errors be

$$\Phi = \begin{bmatrix} (\widetilde{\eta}_1)_{t=1}^T \\ \vdots \\ (\widetilde{\eta}_H)_{t=1}^T \\ \widetilde{R}_1 \\ \vdots \\ \widetilde{R}_J \end{bmatrix}$$

Condition 3 *The error terms are distributed jointly normal*

$$\Phi \sim N(0, \Sigma)$$

We can assume without loss of generality that the assets are uncorrelated with one another (i.e. $\text{cov}\left(\widetilde{R}_{i,t}, \widetilde{R}_{j,t}\right) = 0$). $\sigma_{j,t}^h$ is the covariance of individual h's income innovation in period t with asset j. So the covariance matrix is:

$$\Sigma = \left[\begin{array}{cccccccc|ccccc} & & & & \multicolumn{2}{c}{\Sigma_{11}} & & & & & & & \\ \hline \sigma_{1,1}^1 & \cdots & \sigma_{1,T}^1 & \cdots & \cdots & & \cdots & \sigma_{1,1}^H & \sigma_{1,T}^H & \sigma_{11} & \cdots & 0 & \cdots & 0 \\ \vdots & & \ddots & & & & & \vdots & & \vdots & \ddots & & & \vdots \\ \vdots & & & & \sigma_{j,1}^h & \sigma_{j,T}^h & & \vdots & & 0 & \cdots & \sigma_{jj} & \cdots & 0 \\ \vdots & & & & & & \ddots & \vdots & & \vdots & & & \ddots & \vdots \\ \sigma_{J,1}^1 & \cdots & \sigma_{J,T}^1 & \cdots & \cdots & & \cdots & \sigma_{J,1}^H & \sigma_{J,T}^H & 0 & \cdots & 0 & \cdots & \sigma_{JJ} \end{array}\right]$$

Define $\beta_{j,t}^h = \frac{\sigma_{j,t}^h}{\sigma_{jj}}$, which the coefficient on a regression of income innovations on asset returns. We can then introduce which is analogous to the R^2 of a regression. It tells us how much of the variance of individual income shocks an individual can hedge away using risky assets.

Now consider

$$R_{h,t}^2 = \frac{\text{var}\left(\widetilde{\eta}_t^h\right) - \text{var}\left(\widetilde{\eta}_t^h - \sum_{j=1}^J \beta_{j,t}^h \widetilde{R}_j\right)}{\text{var}\left(\widetilde{\eta}_t^h\right)}$$

which is simply the R^2 from an OLS regression of labor income errors on asset payoffs. This measure tells us how much of the variance of individual income shocks an

earnings	y_t^h
earnings innovations	η_t^h
t^{th} MA coefficient	ψ_t^h
Marginal impact of a shock to income on wealth	Ψ_t^h
Gross return of j^{th} asset at time t	$R_{j,t}$
$\frac{\text{cov}\left(\widetilde{R}_{j,t},\widetilde{\eta}_{h,t}\right)}{\text{var}\left(\widetilde{R}_{j,t}\right)}$	$\beta_{j,t}^h$
consumption	c_t^h
asset holdings	$\omega_{j,t}^h$
income	I_t^h
wealth	W_t^h
MPC out of wealth	a_t
constant	b_t^h

Table 1: Summary of notation

individual can hedge away using risky assets. It is the theoretical counterpart to the regression R^2 values shown in Table 4 and Figures 6 and 7.

Table 1 summarizes notation.

5.2 Individual optimization

Our first proposition provides analytical solutions for portfolio allocations and consumption as functions of asset prices, parameters and realizations of the labor income process, the covariance between income innovations and asset payoffs, and preference parameters.

To conserve on notation define the present discounted value operator $\text{PDV}_t(z)$ which gives the present discounted value of all expected future (i.e. from time $t+1$ forward) values of the stream z. I.e.

$$\text{PDV}_t(z) = \sum_{s=t+1}^{T} \frac{1}{\Pi_{i=t+1}^{s} R_{0,i}} \dot{\text{E}}(\tilde{z}_s)$$

In this vein, let $\text{PDV}_t\left(\mathbf{1}^h\right)$ be the present discounted value of an annuity paying off a dollar for the rest of individual h's life. And let $\text{PDV}_t\left(\eta_t^h\right)$ be the present discounted value of changes in expected future income due to a unitary shock to income today. I.e.

$$\text{PDV}_t\left(\eta_t^h\right) = \sum_{s=t+1}^{T^h} \frac{1}{\Pi_{i=t+1}^{s} R_{0,i}} \psi_{s-t}^h$$

Let $I_t^h = \left(\widetilde{y}_t^h + \sum_{j=1}^J \widetilde{R}_{j,t}\omega_{j,t-1}^h + \omega_{0,t-1}^h R_{0,t}\right)$, which we will call *financial wealth.* Then let $W_t^h = I_t^h + \text{PDV}_t\left(y^h\right)$ which we will call *total wealth.* Finally, let $V_t^h = \text{var}\left(c_t^h\right)$ which is the conditional variance of consumption one period ahead of time.

We assume a perfectly competitive economy in which the consumption goods prices are normalized to one at each date, and the asset returns are denoted by $\mathbf{R}$, a $J \times T-1$ dimensional random matrix. The *budget set* for individual h is

$$B^h(\mathbf{R}) = \left\{ \left(c_t^h, \omega_t^h\right)_{t\in T^h} \ni c_t^h + \sum_{j=0}^J \omega_{j,t}^h = y_t^h + \sum_{j=0}^J \omega_{j,t-1}^h \tilde{R}_{j,t} \right\},$$

Proposition 1: Assume that the price of the riskless asset is nonstochastic. Under conditions 1, 2, 3:

(1) $\omega_{j,t}^h = \frac{1}{a_{t+1}A^h}\left[\frac{\text{E}\left(\tilde{R}_{j,t+1}\right)-R_{0,t+1}}{\text{var}\left(\tilde{R}_{j,t+1}\right)}\right] - \Psi_{t+1}^h \beta_{j,t+1}^h$, for $j = 1, 2, \ldots, J$

(2) $\omega_{0,t}^h = \frac{1}{1+a_{t+1}R_{0,t+1}}\left(\begin{array}{c} I_t^h - a_{t+1}^h R_{0,t+1}\,\text{PDV}_t\left(\tilde{y}^h\right) - \sum_{j=1}^J \omega_{j,t}^h\left(1 + a_{t+1}^h\,\text{E}\left(\tilde{R}_{j,t+1}\right)\right) \\ +b_{t+1}^h + \frac{1}{2}A^h V_{t+1}^h + \frac{1}{A^h}\ln\delta^h R_{0,t+1} \end{array}\right)$

(3) $c_t^h = a_t^h W_t - b_t^h$

where:

(i) $\Psi_{t+1}^h = 1 + \text{PDV}_{t+1}\left(\eta_t^h\right)$ is the present discounted value of a unit labor income innovation at time t, the theoretical counterpart to the values plotted in figure 3.

(ii) $a_t^h = \frac{a_{t+1}^h R_{0,t+1}}{1+a_{t+1}^h R_{0,t+1}} = \frac{1}{1+\text{PDV}_t\left(1^h\right)}$ is the marginal propensity to consume out of wealth at t, which depends on the current and future prices of the one-period riskless asset, and which varies among persons because of differences in the end-of-life horizon T_2.

(iii) $b_t^h = \frac{1}{1+a_{t+1}^h R_{0,t+1}}\left(\sum_{j=1}^J\left(\text{E}\left(\tilde{R}_{j,t+1}\right) - R_{0,t+1}\right)\omega_{j,t}^h + \frac{A^h}{2}V_{t+1}^h + \frac{1}{A^h}\ln\delta^h R_{0,t+1} + b_{t+1}^h\right)$, for $t < T_2^h$, reflects precautionary behavior and time discounting.

(iv) $\text{var}_{t-1}\left(c_t^h\right) = \left(1 - R_{h,t}^2\right)\text{var}\left(a_t \Psi_t^h \tilde{\eta}_t^h\right) + \text{var}\left(\frac{1}{A^h}\sum_{j=1}^J\left[\frac{\text{E}\left(\tilde{R}_{j,t+1}\right)-R_{0,t+1}}{\text{var}\left(\tilde{R}_{j,t+1}\right)}\right]\tilde{R}_{j,t+1}\right)$ is the variance of time-t consumption conditional on information at time $t-1$.

Proof: See Appendix.

Formulas (i) and (ii) give expressions for a_t^h and Ψ_t^h. Formulas (iii) and (iv) applied recursively from the last period of life yield values for b_t^h, $\text{var}_t\left(c_{t+1}^h\right)$ and the risky asset allocations $\omega_{j,t}^h$. (The conditional variance of consumption is nonstochastic under our assumptions, although it can vary across persons and time, so we henceforth refer to it by $\text{var}\left(c_t^h\right)$.) Given these values, the time paths for financial wealth, human capital and the riskless asset allocation are solved forward from $t = 1$ using (2). The realized path of wealth and the riskless asset depend on the realizations of risky labor income and asset payoffs. The time path for consumption follows from (3).

5.3 Understanding consumption and portfolio choice

To acquire some insight into the workings of the model and the constructive solution method for the individual's problem, consider an example with $T = 4$. The individual is born at $t = 1$, retires after $t = 3$ and dies after $t = 4$. Assume that the return on the riskless asset is one (so the interest rate is zero percent). Assume that there is one risky asset with expected gross return of one (so the risk premium is zero) and that the variance of return of one. Labor income obeys an AR(1) process:

$$\begin{aligned} y_t &= 100 \quad \text{for } t = 1, \\ y_t &= 100 + \widetilde{\varepsilon}_t, \quad \text{for } t = 2, 3, \quad \text{and} \\ y_t &= 0, \quad \text{for } t = 4, \end{aligned}$$

where

$$\widetilde{\varepsilon}_t = \rho\widetilde{\varepsilon}_{t-1} + \eta_t, \quad \eta_t \sim N\left(0, \sigma_t^2\right), \text{ and } \varepsilon_1 = 0,$$

and we suppress the notation for indexing agents.

Table 5 shows selected outcomes and calculations for this example: Rows (a)-(d) show expectations of future income at different points in the life cycle. Row (e) shows the present discounted value of a unit labor income shock, and row (f) shows a_t, the marginal propensity to consume out of wealth. Row (g) shows how much of the risky asset the individual acquires. Because it offers no premium in this example, the individual's only motive for holding the risky asset is to hedge labor income. Rows (h) and (i) show the consumption variance and the deterministic parameter b_t. Row (j) shows the innovation in consumption as a function of labor income shocks and returns on the risky asset. Row (k) shows total wealth as a function of the riskless asset holdings and the realized values of labor income and the risky asset payoffs. Finally, row (l) shows the sum of consumption and b_t as a function of asset allocations, asset payoffs and labor income realizations.

5.3.1 Consumption smoothing

Suppose for the moment that $\text{cov}\left(\widetilde{R}_1, \widetilde{\eta}_t\right) = \beta_1 = 0$, so that the risky asset offers no hedging possibilities. Since the risky asset also offers no return premium in this example, the $\beta_1 = 0$ case is equivalent to restricting the individual to trade only the riskless asset.

Consider the consumption decision at $t = 2$ in this case. The consumption outcome depends on the realized labor income shock, η_2, as indicated by rows (j) and (l) in Table 4. This income shock is magnified by $1 + \rho$, its effect on wealth, as shown in row (k). But the marginal propensity to consume out of wealth is less than one, so consumption responds less than one for one in the shock to wealth. At $t = 2$, the marginal propensity to consume out of wealth is $\frac{1}{3}$, so the effect of the labor income shock on consumption is $\frac{1}{3}(1 + \rho)\eta_2$.

Notice that retirement plays an important role in smoothing consumption. Without a retirement period, the marginal propensity to consume out of wealth and the sensitivity of consumption to income shocks would be higher in each period. Thus, retirement leads to greater consumption smoothing.

The persistence of shocks also plays an important role in consumption smoothing. When there are no (useful) risky assets, Table 4 tells us that the variance of consumption at $t = 2$ becomes

$$\text{var}\,(c_2) = \frac{1}{9}\,(1 + \rho)^2\,\sigma_2^2,$$

where $\sigma_2^2 \equiv \text{var}(\eta_2)$ is the variance of labor income shocks at $t = 2$. As this expression indicates, the variance of consumption increases with the persistence of labor income shocks. Table 6 illustrates this effect. The columns labeled $\beta_1 = 0$, which correspond to the case of no risky asset, show that the variance of consumption at $t = 2$ is four times larger for $\rho = 1$ than for $\rho = 0$. Table 5 shows that the variance of consumption can rise or fall as a cohort ages depending on the persistence of labor income shocks.

5.3.2 Hedging Labor Income with Risky assets

Now suppose that the risky asset exhibits a nonzero covariance with labor income shocks. We consider two cases: $\beta_{1,2} = \beta_{1,3} = 5$, which affords partial hedging of risky labor income; and $\beta_{1,2} = \beta_{1,3} = 10$, which affords complete hedging.

The individual's Euler equation for the risky asset can be written

$$\text{E}\left(\widetilde{R}_{1,t+1}\right) - R_{0,t+1} = A\,\text{cov}_t\left(\widetilde{c}_{t+1}^h, \tilde{R}_{j,t+1}\right) = A\,\text{cov}_t\left(a_{t+1}\widetilde{W}_{t+1} - b_{t+1}, \tilde{R}_{j,t+1}\right)$$

(See Appendix B.) Consider the portfolio choice decision at $t = 1$. Using Table 4, the period-1 Euler Equation in the example becomes

$$\mathrm{E}\left(\widetilde{R}_{1,2}\right) - R_{0,2} = A\,\mathrm{cov}_1\left(\frac{1}{3}\left(200 + (1+\rho)\,\tilde{\eta}_2 + R_{0,2}\omega_{0,1} + \tilde{R}_{1,2}\omega_{1,1}\right), \tilde{R}_{1,2}\right)$$

which simplifies to

$$\mathrm{E}\left(\widetilde{R}_{1,2}\right) - R_{0,2} = \mathrm{cov}_1\left(\frac{1}{3}(1+\rho)\,\tilde{\eta}_2 + \frac{1}{3}\tilde{R}_{1,2}\omega_{1,1}, \tilde{R}_{1,2}\right).$$

Solving for $\omega_{1,1}$, the risky asset allocation at $t = 1$, we get

$$\omega_{1,1} = \frac{3}{A}\left(\mathrm{E}\left(\widetilde{R}_{1,2}\right) - R_{0,2}\right) - (1+\rho)\,\mathrm{cov}_1\left(\tilde{\eta}_2, \tilde{R}_{1,2}\right).$$

This equation illustrates how the individual's risky asset allocation is composed of two parts: The first part rises with the premium on the risky asset, $\left(\mathrm{E}\left(\widetilde{R}_{1,2}\right) - R_{0,2}\right)$. The coefficient on this term rises with risk tolerance, so that persons with greater absolute risk aversion take a smaller position in the risky asset, aside from the hedging motive.

The second part of the risky asset allocation expression is central to our analysis. It says that an individual takes a short (long) position in the risky asset equal to (minus) its covariance with labor income shocks times the present value multiplier on these shocks. For a given covariance, the individual assumes a larger position when labor income shocks are more persistent, as reflected by a higher value of ρ in the example. This part of the optimal portfolio strategy aims to minimize the variance of wealth.

In short, the optimal portfolio strategy is two-fold: the hedging portfolio minimizes the variance of wealth, and the other portfolio exploits the return premium on the risky asset. The weight placed on the premium exploitation portfolio declines with absolute risk aversion. The relative importance of the hedging portfolio rises with the persistence of labor income shocks and with the magnitude of the covariance between labor income shocks and risky asset returns. As the covariance structure and the persistence of labor income shocks vary over the life cycle, so does the optimal hedging portfolio.

To illustrate how the risky asset affects hedging opportunities and consumption uncertainty, consider the period-2 variance of consumption:

$$\begin{aligned}
\mathrm{var}\left(\tilde{c}_2\right) &= \mathrm{var}\left(\frac{1}{3}\left((1+\rho)\,\tilde{\eta}_2 + \omega_{1,1}\tilde{R}_{1,2}\right)\right) \\
&= \frac{1}{9}\,\mathrm{var}\begin{pmatrix} (1+\rho)\,\tilde{\eta}_2 - (1+\rho)\,\mathrm{cov}\left(\tilde{\eta}_2, \tilde{R}_{1,2}\right)\tilde{R}_{1,2} \\ +\frac{3}{A}\left(\mathrm{E}\left(\widetilde{R}_{1,2}\right) - R_{0,2}\right)\tilde{R}_{1,2} \end{pmatrix}
\end{aligned}$$

But $\eta_2 - \text{cov}\left(\tilde{\eta}_2, \tilde{R}_{1,2}\right)\tilde{R}_{1,2}$ is the residual in an OLS regression of income shocks on asset returns and is therefore orthogonal to $\tilde{R}_{1,2}$. Thus

$$\text{var}\left(\tilde{c}_2\right) = \frac{(1+\rho)^2}{9}\text{var}\left(\tilde{\eta}_2 - \text{cov}\left(\tilde{\eta}_2, \tilde{R}_{1,2}\right)\tilde{R}_{1,2}\right) + A^{-2}\,\text{var}\left(\left(\text{E}\left(\widetilde{R}_{1,2}\right) - R_{0,2}\right)\tilde{R}_{1,2}\right).$$

Let R_2^2 be the R^2 in the regression of the period-2 income shock on the risky asset return, and rewrite the variance of consumption as

$$\text{var}\left(\tilde{c}_2\right) = \left(1 - R_2^2\right)\frac{(1+\rho)^2}{9}\text{var}\left(\tilde{\eta}_2\right) + A^{-2}\,\text{var}\left(\left(\text{E}\left(\widetilde{R}_{1,2}\right) - R_{0,2}\right)\tilde{R}_{1,2}\right).$$

According to the first term on the right side of this equation, the variance of consumption depends on the covariance of the income shock with assets, R_2^2, the marginal propensity to consume out of income, $(1+\rho)^2/9$, and the variance of the income shock, $\text{var}(\tilde{\eta}_2)$. Table 6 shows the variance of consumption for different assumptions about the covariance and the persistence of labor income shocks. To focus on the hedging portfolio, we assume that the premium on the risky asset is zero, so the premium exploitation portfolio vanishes. When $\beta_{1,t} = 5$, $R_2^2 = .25$, and the risky asset enables the individual to substantially reduce consumption risk. When $\beta_{1,t} = 10$, $R_2^2 = 1$, and the risky asset affords full hedging and complete elimination of consumption risk.

5.3.3 Comparative statics

We conclude our discussion of the individual's optimization problem by stating several comparative static results:

1. A change in the price of the risky asset alters holdings of the risky asset through the premium exploitation portfolio but does not affect the hedging portfolio.

2. An increase in the variance of income shocks, holding fixed the covariance with risky assets, has no effect on risky asset holdings but increases the variance and expected growth of consumption.

3. Consider a new risky asset, or new opportunity to trade risky assets, that increases R_t^2 for the individual. If the new asset is orthogonal to existing ones, the hedging portfolio for the pre-existing risky assets at $t-1$ does not change. If the new asset has non-zero covariance with labor income shocks, it reduces the variance of hedged income at t. However, it increases the variance of time-t income to the extent that the agent holds the new asset in his premium exploitation portfolio. Hence, the net effect of a new asset on the variance of consumption is ambiguous.

5.4 Equilibrium Asset Prices and Portfolio Allocations

With suitable restrictions on growth in the aggregate income endowment, it is possible to solve for the full general equilibrium of this model while preserving the analytical tractability of our theoretical framework. For details, see Willen (1999). However, in analyzing the introduction of new assets, it is often useful to consider situations in which some or all prices of existing assets are held fixed. Specifically, we will always assume that the interest rate is exogenous.

We therefore define our equilibrium concept which we call "semi-equilibrium"[14] . Assume that there is some subset of the set of assets $J' \subset J$ for which prices are exogenously determined.

An *semi-equilibrium* $\left(\mathbf{R}, \left(C^h, \omega^h\right)_{h \in H}\right)$ is an asset return matrix plus consumption paths and portfolio allocations for all h such that:

1. Each agent optimally chooses in his budget set: $\left(C^h, \omega^h\right) \in B^h\left(\mathbf{R}\right)$ and $\left(C^{h\prime}, \omega^{h\prime}\right) \in B^h\left(\mathbf{R}\right) \Rightarrow U\left(C^{h\prime}\right) \leq U\left(C^h\right)$.

2. Asset prices are fixed for assets $j \in J'$.

3. Asset markets clear for assets $j \notin J'$: $\sum_{h \in H_t} \omega_{j,t}^h = 0$ for all t.

Equilibrium condition (2) constrains assets to be in zero net supply. Although we do not pursue the matter here, our framework can also be adapted to analyze the risk-sharing effects of changes in net asset supply.

Note that only when $J' = \varnothing$ will goods markets automatically clear. In general, goods markets will not clear.

We now state a proposition that describes equilibrium outcomes. Some additional notation is useful. Let $\delta_t^* = \left(\Pi_{h \in H_t} \left(\delta_t^h\right)^{\frac{1}{A^h}}\right)^{\frac{1}{H_t}}$ be the "aggregate" time discount factor, and let $A_t^* = \left(\frac{1}{H_t} \sum_{h \in H_t} \frac{1}{a_t^h A^h}\right)^{-1}$ be the "aggregate" level of absolute risk aversion. Let $\widetilde{m}_{t+1} = \frac{1}{H_t} \sum_{h \in H_{t+1}} \Psi_{t+1}^h \widetilde{\eta}_{t+1}^h$ be the average shock to wealth at $t+1$. As this expression indicates, the average marginal propensity to consume depends on the distribution of income shocks and the persistence of the individual-level shocks. As before, $R_{0,t}$ denotes the (exogenous) time-t return on the riskless asset.

Proposition 2: Assume that the returns on the riskless asset and possibly other assets are exogenously determined. Under conditions 1, 2 and 3, the semi-equilibrium

[14] In an abuse of terminology, we will refer to our semi-equilibria as equilibria in the text from here on.

returns ($\tilde{R}_{j,t}$) and portfolio allocations ($\omega^h_{j,t}$) for a risky asset j for which the asset market clearing condition holds are given by:

1. $\mathrm{E}\left(\tilde{R}_{j,t+1}\right) - R_{0,t+1} = A^*_t \operatorname{cov}\left(\widetilde{m}_{t+1}, \widetilde{R}_{j,t+1}\right)$
2. $\omega^h_{j,t} = \frac{A^*_t}{a^h_{t+1} A^h} \operatorname{cov}\left(\widetilde{m}_{t+1}, \widetilde{R}_{j,t+1}\right) - \Psi^h_{t+1}\beta^h_{t+1}$

Proof: See Appendix.

5.5 Financial innovations and welfare

We define a *financial innovation* to be an expansion in asset-choice menus that improves risk-sharing opportunities. We quantify the welfare effects of better risk-sharing opportunities by calculating the equilibrium response to innovation in the theoretical model. In particular, define the *uniform equivalent variation* as the amount of extra consumption at each state and date in the pre-innovation equilibrium that makes an individual as well off as in the post-innovation equilibrium. Formally, the uniform equivalent variation for h is the value of θ^h that solves $U\left(C^h + \theta^h\right) = U\left(C^{h*}\right)$, where C^h is the pre-innovation consumption path for h and C^{h*} is the post-innovation path. The *uniform compensating variation* is the value θ^h that solves $U\left(C^h\right) = U\left(C^{h*} + \theta^h\right)$. For exponential preferences, these two welfare measures are identical.

We decompose the uniform variation generated by the introduction of a new financial asset into four pieces:

Direct Hedging Benefit: If the asset return and the earnings shock have nonzero covariance, the individual can reduce the overall variance of income and consumption by adding the asset to his hedging portfolio. This effect is always nonnegative.

Hedging Cost: Adopting a short (long) hedge position in the new asset reduces (increases) expected returns, if the new asset exhibits a positive (negative) risk premium.

Benefits of Risk Premium: If the new asset has a positive risk premium, and the individual has some tolerance for risk, he will add the new asset to his premium exploitation portfolio and thereby increase the portfolio expected return.

Cost of Risk Premium: The higher expected return on the premium exploitation portfolio comes at the cost of a riskier consumption stream.

Note that none of these effects involve changes in the equilibrium prices on other assets. The absence of equilibrium price responses for risky assets can be seen from expression 1 in Proposition 2: Given an unchanged path of prices for the riskless asset, $\tilde{m}_{t+1}$ is unaffected by the introduction of new assets in zero net supply. If we were to relax Condition 4 (exogenous risk-free rate), or consider changes in the net supply of risky assets, then the equilibrium prices on existing assets would respond, and the decomposition of welfare changes would involve additional effects.

A simple two-period, two-person example illustrates the welfare decomposition. For simplicity, assume that both individuals have a discount rate equal to one over riskless rate of return ($\frac{1}{R_{0,t}} = \delta$). Endowments and preferences are described in the following table:

Person	Period-1 Endowment	Period-2 Endowment	Risk Aversion
A	$\overline{y}$	$\tilde{y} = \overline{y} + \tilde{\epsilon}_A$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$	1
B	$\overline{y}$	$\tilde{y} = \overline{y} + \tilde{\epsilon}_B$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$	B
Average	$\overline{y}$	$\frac{1}{2}\overline{y} + \frac{1}{2}\tilde{y}$	$\frac{B}{B+1}$

Starting from a situation with no risky assets, consider the introduction of a new asset with returns $\tilde{R}_{1,2}$ and covariance structure, $\frac{\text{cov}\left(\tilde{\epsilon}_A, \tilde{R}_{1,2}\right)}{\text{var}\left(\tilde{R}_{1,2}\right)} = \gamma$ and $\frac{\text{cov}\left(\tilde{\epsilon}_B, \tilde{R}_{1,2}\right)}{\text{var}\left(\tilde{R}_{1,2}\right)}$ In other words, the asset payoff is correlated with A's earnings shock but not B's. Under exponential preferences and an exogenous path for the risk-free rate, utility is proportional to first-period consumption, so we can evaluate the welfare effects of the new asset in terms of first-period consumption allocations.

By Proposition 2, we have

$$\begin{aligned} \omega_1^A &= \text{E}\left(\tilde{R}_{j,t+1}\right) - R_{0,t+1} - \gamma \\ \omega_1^B &= -\frac{1}{B}\text{E}\left(\tilde{R}_{j,t+1}\right) - R_{0,t+1} \\ \text{E}\left(\tilde{R}_{j,t+1}\right) - R_{0,t+1} &= -\frac{B}{B+1}\gamma \end{aligned}$$

By Proposition 1, we can write A's first-period consumption as

$$c_1^A = \overline{y} - \frac{1}{1 + R_{0,t+1}}\left(\frac{1}{2}\text{var}\left(\tilde{c}_2^A\right) + \left[\text{E}\left(\tilde{R}_{j,t+1}\right) - R_{0,t+1}\right]\omega_1^A\right)$$

This equation tells us that consumption (and utility) falls with consumption risk and rises with the expected return on the financial asset.

Agents can reduce risk through hedging and increase it by exploiting the risk

premium. In this example,

$$\begin{aligned}
\text{var}\left(\widetilde{c}_2^A\right) &= \underbrace{\sigma^2 - \gamma^2}_{\text{idiosyncratic risk}} + \underbrace{\frac{B^2}{(B+1)^2}\gamma^2}_{\text{premium risk}} \\
\text{var}\left(\widetilde{c}_2^B\right) &= \underbrace{\sigma^2}_{\text{idiosyncratic risk}} + \underbrace{\frac{1}{(B+1)^2}\gamma^2}_{\text{premium risk}}
\end{aligned}$$

As the formulas indicate, idiosyncratic risk declines if the new asset covaries with the person's labor income shock. Here, person A's idiosyncratic risk component drops by γ^2 because of improved hedging opportunities. The second component shows that the consumption variance rises to the extent that an individual acquires the risky asset to exploit the risk premium.

Collecting this information, and substituting into the expression for A's first-period consumption allocation, we obtain

$$c_1^A = \overline{y} - \frac{1}{1+R_0}\left(\begin{array}{c} \frac{1}{2}\left(\underbrace{\sigma^2 - \gamma^2}_{\text{hedging benefit}} + \underbrace{\frac{B^2}{(B+1)^2}\gamma^2}_{\text{cost of premium}}\right) \\ - \underbrace{\frac{1}{(B+1)}\gamma^2}_{\text{cost of hedging}} - \underbrace{\frac{B^2}{(B+1)^2}\gamma^2}_{\text{risk premium}} \end{array}\right)$$

6 Empirical Implementation

We need several data inputs to compute equilibrium outcomes:

1. **Labor Income Processes for Synthetic Persons:** Section 2 estimates the life-cycle earnings process for each synthetic person. We use the fitted levels specifications reported in Table 2 and Figure 1. We allow the heteroscedasticity of earnings shocks to vary with age as shown in Figure 4.

2. **Covariance Structure:** Section 4 estimates the covariance between labor income shocks and asset returns as a function of sex, education and age. We use the fitted levels specifications reported in Table 3 and Figures 5, 6 and 7.

3. **Asset Choice Menus:** We consider two pre-innovation benchmark equilibria that impose different assumptions regarding the menu of financial assets over which persons optimize. One benchmark equilibrium is based on the assumption

that persons freely trade the riskless asset (i.e., borrow and lend at the riskless rate) but do not trade risky assets. A second benchmark equilibrium is based on the assumption that persons freely trade the riskless asset and a broad equity portfolio (the S&P 500 index). We evaluate the equilibrium effects of financial innovations relative to these two bechmarks.

4. **Working Life, Retirement Period, and Lifespan:** We assume that persons enter the world at age 25, earn labor income through age 65, and live through age 74 with certainty.[15] We assume that labor income declines linearly to zero between ages 58 and 65.

5. **Time Preference:** We set $\delta = .96$ for all persons, which corresponds to an annual discount rate of 4 percent.

6. **Risk Aversion:** In our baseline specification, we set A^h so that each person's relative risk aversion is roughly 3. In particular, we set A^h equal to 3 times the reciprocal of average annual labor income. Average annual labor income equals expected labor income summed over the working life divided by 50, the number of years in life. We also report welfare results that correspond to smaller and larger levels of relative risk aversion.

7. **Demographic Structure:** We assume that the sex-education-age structure of the population evolves over time in line with the CPS employment shares described in section 2. We assume that working-age adults (persons covered by our sample) account for two-thirds of aggregate income and consumption.

8. **Riskless Interest Rate:** We fix the riskless interest rate at 4 percent.

7 The Risk-Sharing Benefits of Financial Assets

This section is in progress. Preliminary results appear in Tables 6-10. In calculating the welfare effects of trading the S&P 500, we assume that agents trade only the riskless asset in the benchmark equilibrium. In all other cases, we assume that agents trade the riskless asset and the S&P 500 in the benchmark.

[15] In future drafts, the empirical implementation will incorporate bequests and stochastic lifespans fit to mortality tables.

7.1 Full Between-Group Risk Sharing

The "Full Menu" case shows the effects of an asset menu that allows full hedging of group-level earnings shocks. This case represents the maximal potential impact of financial assets in so far as they facilitate risk sharing among sex-education-cohort groups.

7.2 Risk Sharing Using Financial Assets

The job destruction asset is more useful as a hedging tool, especially for less-educated men.

None of the artificial assets considered here even remotely approaches the potential gains illustrated by the "Full Menu" case.

7.3 An Aggregate Welfare Calculation

In Tables 6-10, the portfolio choice menu remains unchanged along an equilibrium path. The welfare effects reflect comparisons between equilibrium paths that entail different portfolio choice menus. While the tables focus on particular birth cohorts, similar results hold for other cohorts in this type of comparison. The precise welfare effects vary among birth cohorts within sex-education groups because of varying cohort fixed effects, which influence the calibrated levels of absolute risk aversion.

We plan to consider the aggregate welfare effects of group-level risk sharing in a different sort of experiment. We imagine that the economy moves along a benchmark equilibrium path for some time before experiencing a one-time, unanticipated expansion in portfolio choice menus. Agents rebalance asset portfolios in light of the new hedging opportunities, and the economy then evolves along a different equilibrium path as a consequence. For each agent, we calculate the uniform variation by comparing the new equilibrium path to an alternative path with no expansion of portfolio choice menus. We compute the present value of the uniform variation for each person, living and unborn, and sum over persons to obtain an aggregate figure.

7.4 Sensitivity Analysis

This section will treat the following issues:

- Standard errors for some of the welfare results that take account of uncertainty regarding the estimated covariance structure.
- Higher and lower levels of risk aversion.

- Portfolio choice menus that include risky bonds.
- Alternative assumptions about the labor income process near retirement.

8 Concluding Remarks

To be written.

Appendix A: Industry Classifications for Mutual Funds (Preliminary)

Description	*1987 SIC*
Agriculture	01, 02, 07
Forestry and Logging	08, 241
Fishing and Hunting	09
Mining	10, 12, 13, 14
Food and Beverages	20
Tobacco	21
Textiles	22 31
Apparel	23
Leather	31
Lumber and Wood Products	24 (ex. 241)
Furniture	25
Pulp and Paper Products	26
Printing and Publishing	27
Chemicals	28
Petroleum and Coal Products	29
Rubber and Plastics	30
Stone, Clay and Glass Products	32
Basic (Primary) Metals	33
Fabricated Metals	34
Nonelectrical Machinery	35
Electrical Machinery	36
Automobiles	371
Aircraft and Missiles	372, 376
Other Transport Equipment	373, 374, 375, 379
Professional and Scientific Insturments	38
Miscellaneous Manufacturing	39
Electricity, Gas, Steam and Water Works	46, 49 (ex. 495)
Construction	15, 16, 17
Wholesale Trade	50, 51
Retail Trade	52, 53, 54, 55, 56, 57, 59
Eating and Drinking Establishments	58
Hotels and Other Lodging	70

Railway Transport	40, 474
Local and Interurban Passenger Transport	41, 4725
Freight Transport by Road	42, 472 (ex. 4725), 473, 478
Parking Services	752
Vehicle Rental Services	751
Water Transport	44
Air Transport	45
Postal Services	43 (U.S. Postal Service)
Telecommunications	48
Banking	60, 61
Other Financial Institutions	62, 67
Insurance	63, 64
Real Estate	65, 7349
Legal Services	81
Accounting and Auditing	872
Computer and Data Processing	737 (ex. 7377)
Engineering, Architectural and Technical Services	871
Advertising Services	731, 7331
Typing, Duplicating, Copying Services	7334, 7338
Business Management and Consulting Services	874
Business Services nec	732, 736, 7383, 7389, 89
Business Machinery and Equipment Rental and Leasing	735, 7377
Sanitary Services	495, 7342
Education Services	82 (ex. 823), 833
Research and Scientific Institutes	873
Medical, Dental, Health Services	80
Welfare Services	83 (ex. 833)
Business, Professional and Labor Associations	861, 862, 863
Recreational and Cultural Services	78, 79, 823, 84
Repair Services nec	753, 754, 76
Laundries and Cleaning Services	721
Other Personal Services	72 (ex. 721), 7335, 7384

Appendix B: Proofs of Theoretical Propositions

The theoretical model is based on Willen (1998), which contains a fuller discussion and more detailed proofs. Here we provide a somewhat abbreviated treatment.

Two basic results underlie our propositions: first, that consumption is affine in wealth $\left(c_t^h = a_t^h W_t^h - b_t^h\right)$, and, second, that the Euler equations have the form derived in the lemma below. The first result is well known (e.g., Caballero, 1990), although its derivation under the conditions of this model is somewhat tedious. The interested reader is referred to Willen (1998).

We first prove the lemma that gives the Euler equations governing consumption and portfolio allocations. We then show that this lemma, when combined with the result that consumption is affine in wealth, allows us to prove Proposition 1 on individual optimization and Proposition 2 on equilibrium asset prices.

Lemma: Suppose that $\tilde{c}_{t+1}^h$ and $(\tilde{x}_j)_{j=1}^J$ are jointly normally distributed. Then, under the assumptions specified in section 5.1,the Euler equations between t and $t+1$ can be written:

$$\text{(B.1)} \quad \mathrm{E}\left(\tilde{R}_{j,t+1}\right) - R_{0,t+1} = A^h \operatorname{cov}\left(\tilde{c}_{t+1}^h, \tilde{R}_{j,t+1}\right) \quad j = 1, ..., J$$

$$\text{(B.2)} \quad \mathrm{E}\left(\tilde{c}_{t+1}^h\right) - c_t^h = \frac{A^h}{2} \operatorname{var}\left(\tilde{c}_{t+1}^h\right) + \frac{1}{A^h} \ln R_{0,t+1}\delta^h$$

Proof: Necessary conditions for optimal portfolio allocations at t include

$$u_C^h\left(c_t^h\right) = R_{0,t+1}\delta^h \,\mathrm{E}\left(u_C^h\left(\tilde{c}_{t+1}^h\right)\right)$$
$$u_C^h\left(c_t^h\right) = \delta^h \,\mathrm{E}\left(u_C^h\left(\tilde{c}_{t+1}^h\right)\tilde{R}_{j,t+1}\right) \quad j = 1, ..., J$$

By definition of covariance and Stein's lemma:

$$u_C^h\left(c_t^h\right) = \delta^h \,\mathrm{E}\, u_C^h\left(\tilde{c}_{t+1}^h\right) \mathrm{E}\left(\tilde{R}_{j,t+1}\right) + \delta^h \,\mathrm{E}\, u_{CC}^h\left(\tilde{c}_{t+1}^h\right) \operatorname{cov}\left(\tilde{c}_{t+1}^h, \tilde{R}_{j,t+1}\right) \quad j = 1, ..., J$$

Dividing through by $u_C^h\left(c_t^h\right) = R_{0,t+1}\delta^h \,\mathrm{E}\left(u_C^h\left(\tilde{c}_{t+1}^h\right)\right)$ and noting that $-\frac{\mathrm{E}\, u_{CC}^h\left(\tilde{c}_{t+1}^h\right)}{\mathrm{E}\, u_C^h\left(\tilde{c}_{t+1}^h\right)} = A^h$ we get

$$1 = \frac{\mathrm{E}\left(\tilde{R}_{j,t+1}\right)}{R_{0,t+1}} - \frac{A^h \operatorname{cov}\left(\tilde{c}_{t+1}^h, \tilde{R}_{j,t+1}\right)}{R_{0,t+1}} \quad j = 1, ..., J$$

which we solve to get our risky asset pricing equation. Because $\tilde{c}_{t+1}^h$ is normal

$$\exp\left(-A^h c_t^h\right) = R_{0,t+1}\delta^h \exp\left(-A^h \,\mathrm{E}\left(\tilde{c}_{t+1}^h\right) + \frac{\left(A^h\right)^2}{2} \operatorname{var}\left(\tilde{c}_{t+1}^h\right)\right)$$

$$A^h = c_t^h - \mathrm{E}\left(\tilde{c}_{t+1}^h\right) + \frac{A^h}{2} \operatorname{var}\left(\tilde{c}_{t+1}^h\right) + \frac{\ln R_{0,t+1}\delta^h}{A^h}$$

Proof of Proposition 1 (sketch): Recall that $c_t^h = a_h^t W_h^t - b_h^t$, which allows us to rewrite (B.1) as

$$\mathrm{E}\left(\widetilde{R}_{j,t+1}\right) - R_{0,t+1} = A^h \operatorname{cov}\left(a_h^t \widetilde{W}_{t+1}^h - b_h^t, \tilde{R}_{j,t+1}\right) \quad \Rightarrow$$

$$(1/A^h)\left[\mathrm{E}\left(\widetilde{R}_{j,t+1}\right) - R_{0,t+1}\right] = \operatorname{cov}\left(a_h^t\left[\mathrm{E}\left(\widetilde{W}_{t+1}^h\right) - \widetilde{W}_{t+1}^h\right], \widetilde{R}_{j,t+1}\right), \quad j = 1, ..., J.$$

In other words, an individual sets a^h times the covariance between wealth shocks and the returns on a particular asset to the product of his risk tolerance and the asset's expected return premium. By the definition of wealth,

$$W_h^t = \widetilde{y}_t^h + \sum_{j=1}^{J} \widetilde{R}_{j,t}\omega_{j,t-1}^h + \omega_{0,t-1}^h R_{0,t} + \mathrm{PDV}_t\left(y^h\right)$$

From the MA representation for y_h^t,

$$\mathrm{E}_{t+1}\left(\widetilde{y}_s^h\right) - \mathrm{E}_t\left(\widetilde{y}_s^h\right) = \psi_s^h \widetilde{\eta}_{t+1}^h,$$

we get

$$\left[\mathrm{E}\left(\widetilde{W}_{t+1}^h\right) - \widetilde{W}_{t+1}^h\right] = \begin{pmatrix} \sum_{j=1}^{J} \omega_{j,t-1}^h\left(\widetilde{R}_{j,t+1} - \mathrm{E}\left(\widetilde{R}_{j,t+1}\right)\right) + \\ \eta_t^h\left(1 + \mathrm{PDV}_{t+1}\left(\eta_t^h\right)\right) \end{pmatrix}$$

Notice that the only objects over which the agent has control are the $\omega_{j,t}^h$'s. So the agent chooses these values to solve equation (B.1). Solving from here is straightforward.

Proof of Proposition 2:

Combining condition (B.1) and equation 8 gives us:

$$\frac{1}{a_{t+1}^h A^h}\mathrm{E}\left(\widetilde{R}_{j,t+1}\right) - R_{0,t+1} = \operatorname{cov}\left(\begin{array}{c} \sum_{j=1}^{J} \omega_{j,t-1}^h\left(\widetilde{R}_{j,t+1} - \mathrm{E}\left(\widetilde{R}_{j,t+1}\right)\right) + \\ \eta_t^h\left(1 + \mathrm{PDV}_{t+1}\left(\eta^h\right)\right) \end{array}, \tilde{R}_{j,t+1}\right)$$

Taking averages we get:

$$\left(\mathrm{E}\left(\widetilde{R}_{j,t+1}\right) - R_{0,t+1}\right)\sum_{h\in H_t}\frac{1}{a_{t+1}^h A^h} = \operatorname{cov}\left(\sum_{j=1}^{J}\sum_{h\in H_t}\omega_{j,t-1}^h \widetilde{R}_{j,t+1} + \sum_{j=1}^{J}\Psi_{t+1}^h \eta_{t+1}^h, \tilde{R}_{j,t+1}\right)$$

But market clearing assures that $\sum_{h\in H_t}\omega_{j,t-1}^h = 0$. One can then solve for risky asset returns. To get asset holdings one can then substitute asset returns into the individual risky asset demand equation.

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Table 2: CPS Sample Sizes for Sex-Education-Cohort-Year Cells, 1965-94

Sex	Education Level	*Earnings Measures* Mean (Minimum)	*Employment Count* Mean (Minimum)
Men	< High School	477 (188)	636 (318)
Men	High School	688 (222)	851 (281)
Men	Some College	403 (86)	497 (116)
Men	≥ College	496 (90)	596 (120)
Women	< High School	277 (136)	558 (258)
Women	High School	650 (185)	1008 (233)
Women	Some College	328 (55)	482 (75)
Women	≥ College	310 (57)	414 (74)

Table 3: Regression Estimates of the Labor Income Processes

	Men				*Women*			
	<HS	High School	Some College	College	<HS	High School	Some College	College
Levels								
AR(1)	.55	.71	.48	.45	.19	.38	.27	.46
AR(2)	-.02	-.05	.13	.18	.06	.09	-.04	.07
R^2	.903	.908	.873	.898	.703	.819	.736	.779
st. error	694	681	957	1327	435	381	701	1066
PVM (r=.01)	2.11	2.89	2.52	2.63	1.32	1.87	1.29	2.08
PVM (r=.04)	2.05	2.75	2.39	2.48	1.31	1.81	1.28	2.01
Logs								
AR(1)	.52	.70	.49	.47	.17	.40	.26	.46
AR(2)	-.02	-.05	.12	.16	.04	.11	-.00	.06
R^2	.909	.918	.888	.916	.715	.815	.736	.772
st. error	.052	.035	.042	.043	.072	.048	.075	.080
PVM (r=.01)	1.99	2.86	2.53	2.78	1.27	2.09	1.35	2.11
PVM (r=.04)	1.94	2.72	2.41	2.62	1.26	2.01	1.33	2.03

Notes:

i. Each column corresponds to a separate regression. The dependent variable is annual earnings in the levels specifications and log annual earnings in the log specifications.

ii. Each regression contains 315 annual observations on 18 synthetic individuals for the 1965 to 1994 period.

iii. All regressions contain cohort fixed effects. The levels (log) specification includes a cubic (quadratic) polynomial in age.

iv. The standard errors for the AR(1) and AR(2) coefficients lie between .05 and .06 in all regressions.

v. "PVM" reports the present value multiplier – the estimated effect of a unit labor income shock on the present value of lifetime earnings under the following assumptions: current age of 35, retirement at age 58, and annual discount rate of r. The text describes the present value multiplier calculations in detail.

Table 4: Regressions of Labor Income Shocks on Asset Returns

A. Regression R^2 Values

	Men				*Women*			
Specification and Asset	<HS	High School	Some College	College	<HS	High School	Some College	College
Levels								
S&P 500	.041	.001	.003	.022	.015	.021	.015	.076
Consumption	.048	.037	.022	.055	.036	.059	.027	.091
Employment	**.127**	**.117**	.049	.065	.029	.038	.030	.080
Unemployment	.081	.029	.010	.030	.016	.023	.021	.091
GDP	.073	.070	.020	.034	.024	.031	.035	.082
Job Creation	.058	.004	.015	.035	.047	.026	.065	.078
Job Destruction	**.269**	**.183**	.075	.027	.053	.049	.022	.083
Mutual Funds	.097	.053	.082	.020	.051	.031	.056	**.131**
MF + Job Dest.	**.337**	**.220**	**.134**	.028	.076	.064	.063	**.131**
Logs								
S&P 500	.053	.012	.003	.021	.007	.019	.017	.020
Consumption	.055	.029	.014	.058	.019	.044	.027	.034
Employment	**.136**	**.120**	.046	.057	.032	.045	.020	.056
Unemployment	.099	.045	.010	.031	.009	.026	.034	.036
GDP	.097	.091	.026	.034	.009	.032	.029	.031
Job Creation	.062	.025	.015	.031	.031	.036	.051	.023
Job Destruction	**.281**	**.278**	.091	.045	.031	.063	.028	.042
Mutual Funds	**.111**	.067	.071	.023	.053	.073	.080	.097
MF + Job Dest.	**.344**	**.319**	**.142**	.050	.063	**.117**	.091	**.122**

R^2 values that exceed .10 are in bold font.

Table 3 (continued): Regressions of Labor Income Shocks on Asset Returns

B. F-test p-Values for Joint Significance of Interacted Age Polynomial

	Men				*Women*			
Specification and Asset	<HS	High School	Some College	College	<HS	High School	Some College	College
Levels								
S&P 500	.00	.93	.85	.08	.19	.09	.21	.00
Consumption	.58	.01	.10	.01	.08	.01	.27	.15
Employment	.00	.00	.00	.00	.22	.14	.19	.67
Unemployment	.00	.03	.50	.45	.97	.87	.58	.17
GDP	.02	.00	.14	.28	.44	.36	.09	.53
Job Creation	.15	.82	.27	.24	.02	.62	.00	.89
Job Destruction	.00	.00	.00	.67	.01	.03	.54	.47
Mutual Funds	.00	.00	.00	.96	.02	.20	.00	.00
Logs								
S&P 500	.00	.28	.82	.08	.54	.12	.16	.14
Consumption	.91	.16	.35	.01	.28	.04	.38	.26
Employment	.00	.00	.00	.01	.05	.04	.78	.02
Unemployment	.00	.02	.52	.37	.87	.51	.14	.20
GDP	.00	.00	.07	.24	.87	.24	.29	.37
Job Creation	.42	.28	.29	.36	.06	.14	.01	.81
Job Destruction	.00	.00	.00	.06	.06	.00	.31	.10
Mutual Funds	.01	.00	.00	.64	.00	.00	.00	.00

Notes:

i. Each column corresponds to a separate regression. The dependent variable is the labor income shock from the corresponding column and specification (levels versus logs) in Table 2.

ii. The specification for the "S&P 500" row contains four regressors: the return on the S&P 500 plus a third-order polynomial in age interacted with the S&P 500. The specification for the other rows includes, in addition, the return on the indicated asset(s) plus a third-order age polynomial interacted with the asset return(s).

iii. Each regression contains 315 annual observations on 18 synthetic individuals for the 1965 to 1994 period. (For regressions with mutual funds, the sample begins in 1967.)

iv. The regressions are estimated by OLS. The p-values in Panel B report the marginal significance level in an F-test for the joint significance of the interacted age polynomial for the indicated asset.

Table 5: Consumption and Portfolio Choice: An Example

		$t=1$	$t=2$	$t=3$	$t=4$
(a)	y_1	100	–	–	–
(b)	$\mathrm{E}_t(y_2)$	100	$100+\eta_2$	–	–
(c)	$\mathrm{E}_t(y_3)$	100	$100+\rho\eta_2$	$100+\rho\eta_2+\eta_3$	–
(d)	$\mathrm{E}_t(y_4)$	0	0	0	0
(e)	Ψ_t	–	$1+\rho$	1	–
(f)	a_t	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1
(g)	$\omega_{1,t}$	$-(1+\rho)\beta_{1,2}$	$-\beta_{1,3}$	0	0
(h)	$\mathrm{var}(c_t)$	0	$(1-R_2^2)(1+\rho)^2\sigma_2^2/9$	$(1-R_3^2)\sigma_3^2/4$	0
(i)	b_t	Note (i)	$(1/12)A(1-R_3^2)\sigma_3^2$ $+A^{-1}\ln(\delta)$	$(1/2)A^{-1}\ln(\delta)$	0
(j)	$c_t-\mathrm{E}_{t-1}(c_t)$	0	$\frac{1}{3}\left((1+\rho)\eta_2+\omega_{1,1}\widetilde{R}^*_{1,2}\right)$	$\frac{1}{2}\left(\eta_3+\omega_{1,2}\widetilde{R}^*_{1,3}\right)$	$\omega_{1,3}\widetilde{R}_{1,4}$
(k)	W_t	300	$200+(1+\rho)\eta_2$ $+\omega_{0,1}R_{0,2}+\omega_{1,1}\widetilde{R}^*_{1,2}$	$100+\rho\eta_2+\eta_3$ $+\omega_{0,2}R_{0,3}+\omega_{1,2}\widetilde{R}^*_{1,3}$	$\omega_{0,3}R_{0,4}$ $+\omega_{1,3}\widetilde{R}^*_{1,4}$
(l)	c_t+b_t	$\frac{1}{4}(300)$	$\frac{1}{3}\left(\begin{array}{c}200+(1+\rho)\eta_2\\+\omega_{0,1}R_{0,2}+\omega_{1,1}\widetilde{R}^*_{1,2}\end{array}\right)$	$\frac{1}{2}\left(\begin{array}{c}100+\rho\eta_2+\eta_3\\+\omega_{0,2}R_{0,3}+\omega_{1,3}\widetilde{R}^*_{1,3}\end{array}\right)$	$\omega_{0,3}R_{0,4}$ $+\omega_{1,2}\widetilde{R}^*_{1,4}$

Notes:

(i) $b_1=(4/5)\left[(A/18)(1-R_2^2)(1+\rho)^2\sigma_2^2+A^{-1}\ln(\delta)+b_2\right]$

(ii) $\tilde{R}^*_{j,t}=\tilde{R}_{j,t}-\mathrm{E}\left(\tilde{R}_{j,t}\right)$

Table 6: The Variance of Consumption: Alternative Assumptions about the Persistence of Income Shocks and their Covariance with Financial Assets

	$\rho = 0$			$\rho = 1$		
	$\beta_1 = 0$	$\beta_1 = 5$	$\beta_1 = 10$	$\beta_1 = 0$	$\beta_1 = 5$	$\beta_1 = 10$
$t = 2$	11.11	8.33	0	44.44	33.33	0
$t = 3$	25	18.75	0	25	18.75	0
$t = 4$	0	0	0	0	0	0

Notes:

(i) All table entries assume that the variance of labor income equals 100 in the second and third periods of life; that is, $\sigma_2^2 = \sigma_3^2 = 100$.

(ii) Given our assumption that there is no return premium on the risky asset, the columns labelled $\beta_1 = 0$ are equivalent to the case of no risky assets.

Table 7: Welfare Effects (Uniform Variation) of Hedging with Financial Assets, Present Value Benefits Per Capita for Selected Groups

sex	birth coh	educ	S&P 500	Job destruction	GDP	Mutual funds	Full menu
0	36	1	34.17	88.55	11.52		659.57
0	36	2	2.46	69.73	36.60		554.27
0	36	3	2.57	22.24	30.14		1311.38
0	36	4	53.15	13.06	44.37		2310.78
1	36	1	2.80	11.84	4.47		144.10
1	36	2	9.03	1.41	0.87		184.95
1	36	3	4.46	14.40	14.54		307.19
1	36	4	126.24	3.59	21.01		1374.12
0	39	1	33.74	81.15	12.88		671.60
0	39	2	1.36	62.75	37.05		679.82
0	39	3	2.68	22.00	14.05		791.50
0	39	4	28.67	11.19	27.51		1212.24
1	39	1	3.68	1.72	0.66		205.08
1	39	2	2.96	1.28	1.78		132.81
1	39	3	3.59	10.02	12.39		222.80
1	39	4	58.60	6.22	13.26		1163.61

Notes:

(i) The relative risk aversion for each person is set to approximately 3 as explained in the text.

(ii) For the "S&P 500" column, agents trade only the riskless asset in the benchmark equilibrium. For all other columns, agents trade the riskless asset and the S&P 500 in the benchmark equilibrium.

(iii) The calculations take into account the general equilibrium response of risky asset prices to the change in the set of traded assets.

(iv) All welfare measures are stated in 1998 dollars.

(v) The "Full Menu" case shows the welfare effects of introducing assets that allow for full hedging of group-level labor income shocks.

sex	birth coh	education	Hedging benefit	Cost of hedging	Cost of equity risk	Benefit of equity premium	Total
0	36	1	30.62	3.35	-0.20	0.39	34.17
0	36	2	1.59	0.60	-0.28	0.55	2.46
0	36	3	3.52	-1.26	-0.33	0.64	2.57
0	36	4	59.96	-7.22	-0.44	0.86	53.15
1	36	1	2.43	0.29	-0.08	0.16	2.80
1	36	2	9.05	-0.12	-0.11	0.21	9.03
1	36	3	5.76	-1.42	-0.12	0.24	4.46
1	36	4	134.23	-8.15	-0.17	0.34	126.24
0	39	1	29.98	3.54	-0.23	0.45	33.74
0	39	2	0.66	0.39	-0.33	0.64	1.36
0	39	3	3.22	-0.90	-0.38	0.74	2.68
0	39	4	34.37	-6.19	-0.51	0.99	28.67
1	39	1	3.07	0.52	-0.09	0.19	3.68
1	39	2	3.60	-0.76	-0.12	0.24	2.96
1	39	3	5.02	-1.57	-0.14	0.28	3.59
1	39	4	64.67	-6.26	-0.20	0.39	58.60

Table 8: Welfare Decomposition for S&P 500

sex	birth coh	education	Hedging benefit	Cost of hedging	Cost of equity risk	Benefit of equity premium	Total
0	36	1	187.28	-116.42	-21.33	39.03	88.55
0	36	2	174.51	-129.70	-30.05	54.98	69.73
0	36	3	95.57	-102.32	-34.95	63.94	22.24
0	36	4	8.51	-34.26	-46.81	85.63	13.06
1	36	1	20.87	-16.28	-8.74	15.98	11.84
1	36	2	7.37	-15.31	-11.28	20.63	1.41
1	36	3	7.07	-3.51	-13.07	23.90	14.40
1	36	4	11.15	-22.69	-18.25	33.38	3.59
0	39	1	178.15	-115.60	-22.32	40.92	81.15
0	39	2	172.24	-135.69	-31.45	57.65	62.75
0	39	3	98.17	-106.64	-36.57	67.04	22.00
0	39	4	13.75	-43.37	-48.98	89.79	11.19
1	39	1	6.73	-12.62	-9.14	16.76	1.72
1	39	2	6.02	-14.56	-11.80	21.63	1.28
1	39	3	2.55	-3.93	-13.67	25.06	10.02
1	39	4	9.22	-18.91	-19.09	35.00	6.22

Table 9: Welfare Decomposition for Job destruction asset

sex	birth coh	education	low variance		high variance	
			RRA=5	RRA=10	RRA=3	RRA=10
0	36	1	51.25	93.96	170.84	341.67
0	36	2	3.69	6.77	12.32	24.63
0	36	3	3.86	7.08	12.87	25.74
0	36	4	79.73	146.17	265.77	531.54
1	36	1	4.20	7.70	14.01	28.01
1	36	2	13.54	24.82	45.13	90.25
1	36	3	6.70	12.27	22.32	44.64
1	36	4	189.36	347.16	631.20	1262.40
0	39	1	50.61	92.79	168.71	337.42
0	39	2	2.03	3.73	6.78	13.55
0	39	3	4.02	7.37	13.40	26.80
0	39	4	43.00	78.83	143.33	286.65
1	39	1	5.52	10.12	18.40	36.81
1	39	2	4.44	8.14	14.79	29.58
1	39	3	5.39	9.87	17.95	35.90
1	39	4	87.90	161.14	292.99	585.97

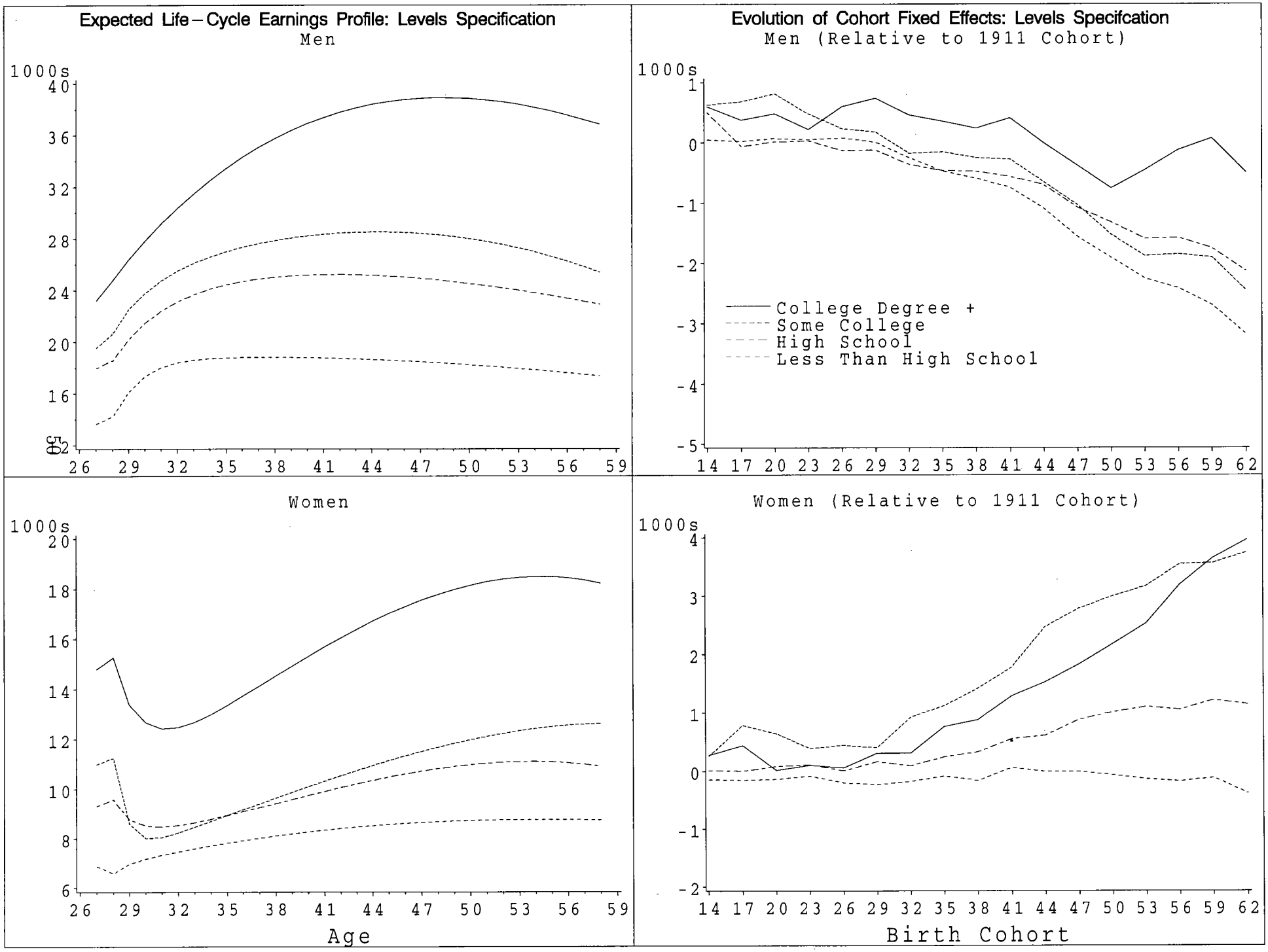
Table 10: Sensitivity Analysis for S&P 500

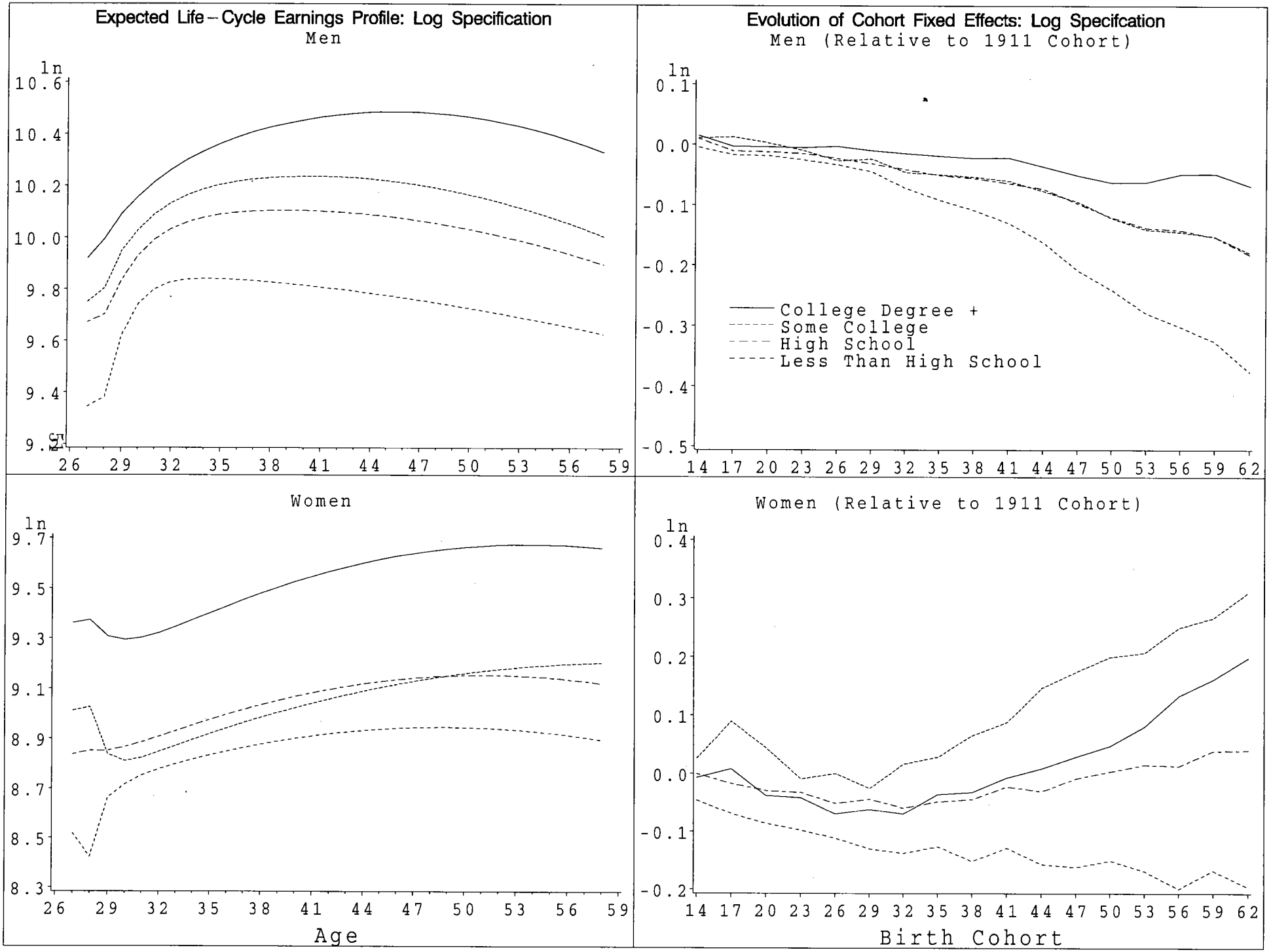
sex	birth	education	low variance		high variance	
sex	coh		RRA=5	RRA=10	RRA=3	RRA=10
0	36	1	132.83	243.52	442.77	885.54
0	36	2	104.60	191.76	348.66	697.32
0	36	3	33.36	61.17	111.22	222.43
0	36	4	19.59	35.92	65.31	130.62
1	36	1	17.76	32.56	59.19	118.39
1	36	2	2.12	3.88	7.06	14.12
1	36	3	21.59	39.59	71.98	143.96
1	36	4	5.39	9.88	17.97	35.94
0	39	1	121.73	223.18	405.77	811.55
0	39	2	94.13	172.57	313.76	627.52
0	39	3	33.00	60.50	110.00	220.00
0	39	4	16.79	30.77	55.95	111.91
1	39	1	2.58	4.74	8.62	17.23
1	39	2	1.93	3.53	6.42	12.84
1	39	3	15.03	27.55	50.09	100.18
1	39	4	9.33	17.10	31.09	62.18

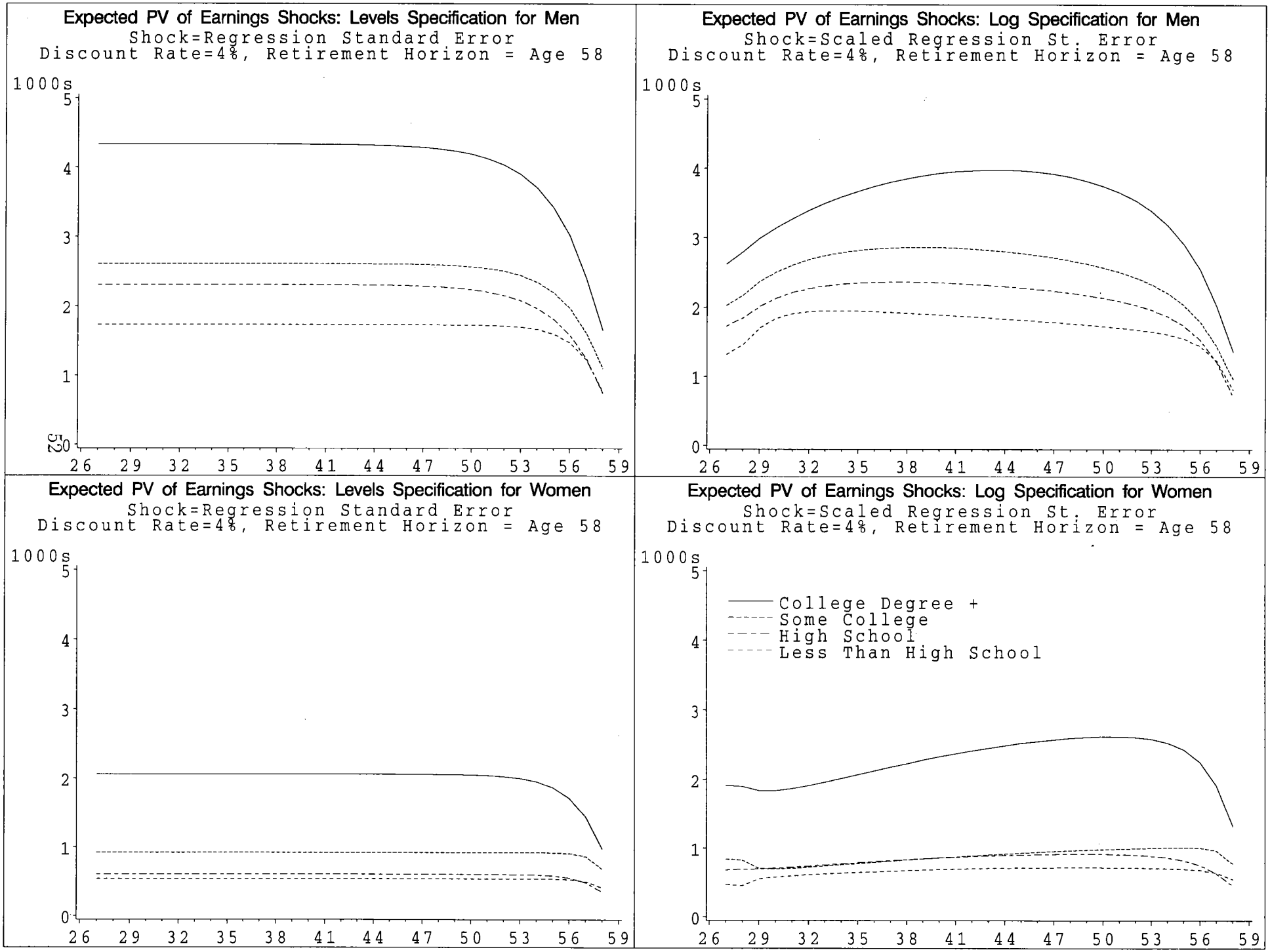
Table 11: Sensitivity Analysis for Job destruction asset

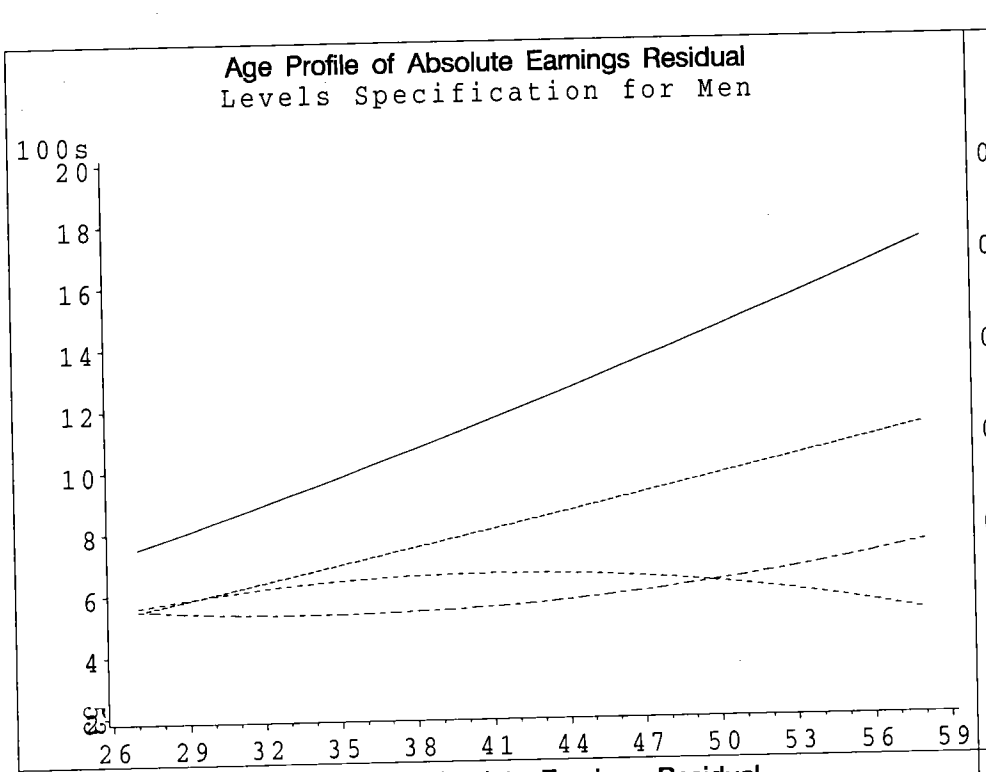
sex	birth	education	low variance		high variance	
sex	coh		RRA=5	RRA=10	RRA=3	RRA=10
0	36	1	1332.11	2442.19	3552.28	9768.78
0	36	2	1119.45	2052.32	2985.20	8209.30
0	36	3	2635.76	4832.22	7028.69	19328.90
0	36	4	4680.20	8580.36	12480.53	34321.45
1	36	1	286.62	525.47	764.32	2101.87
1	36	2	368.70	675.96	983.21	2703.83
1	36	3	613.74	1125.19	1636.65	4500.78
1	36	4	2751.01	5043.52	7336.03	20174.07
0	39	1	1324.21	2427.72	3531.23	9710.88
0	39	2	1373.00	2517.17	3661.34	10068.69
0	39	3	1588.75	2912.70	4236.66	11650.82
0	39	4	2452.85	4496.89	6540.93	17987.56
1	39	1	409.16	750.12	1091.08	3000.47
1	39	2	263.25	482.63	702.01	1930.53
1	39	3	444.19	814.35	1184.51	3257.39
1	39	4	2327.67	4267.40	6207.12	17069.59

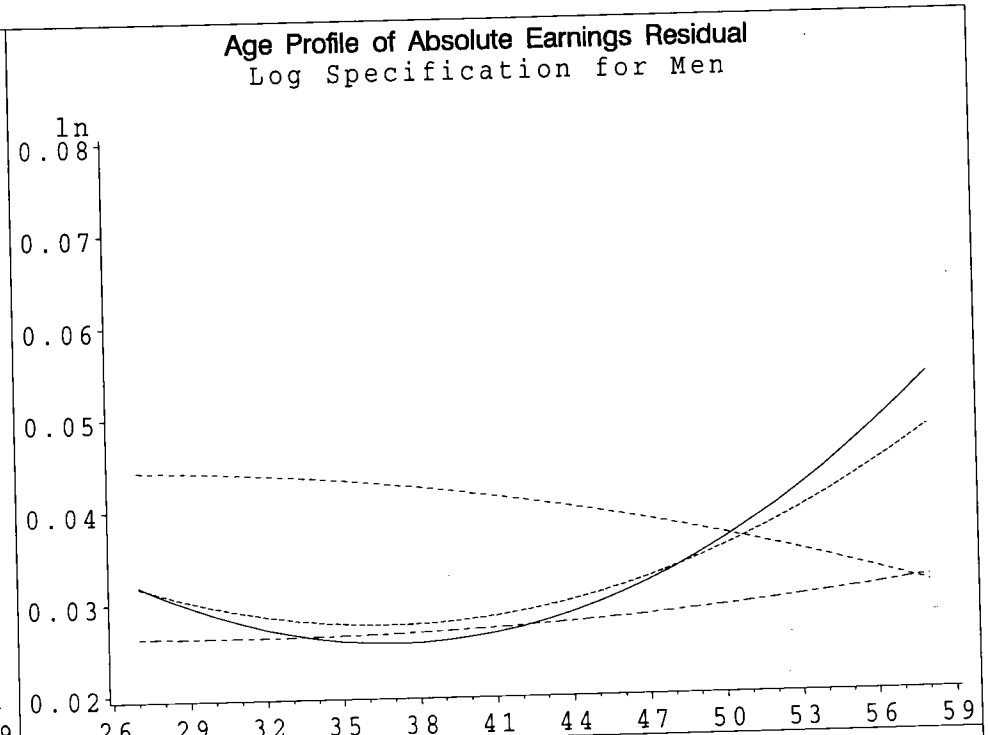
Table 12: Sensitivity Analysis for Full Menu Case

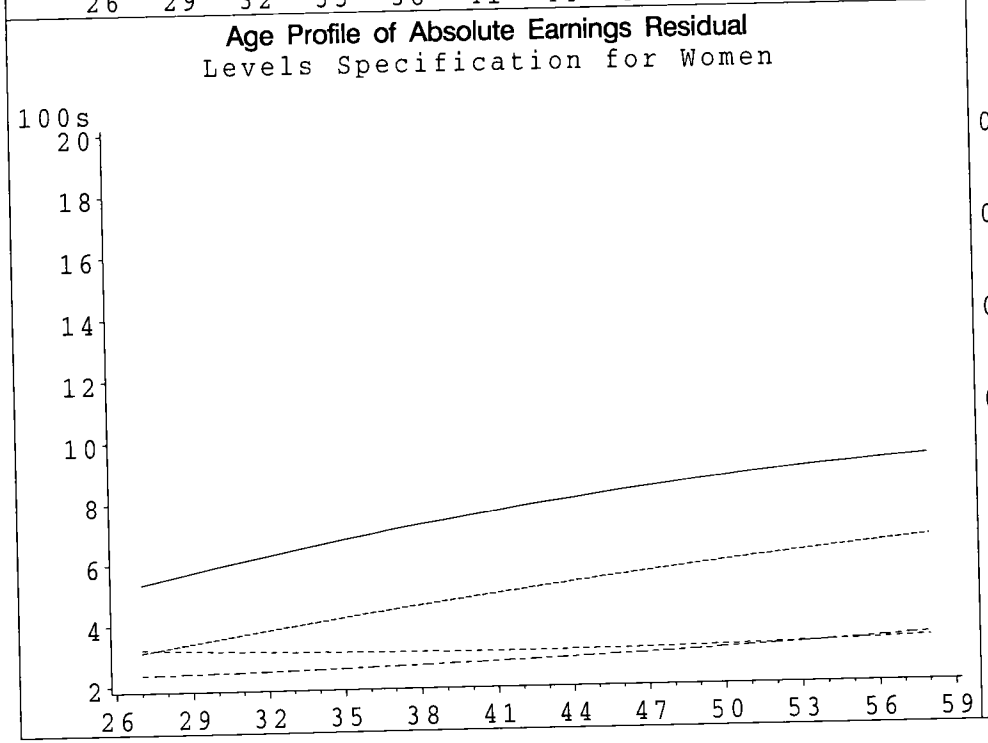

Expected Life – Cycle Earnings Profile: Levels Specification
Men
1000s
40
36
32
28
24
20
16
12
26 29 32 35 38 41 44 47 50 53 56 59
Evolution of Cohort Fixed Effects: Levels Specifcation
Men (Relative to 1911 Cohort)
1000s
1
0
-1
-2
-3
-4
-5
14 17 20 23 26 29 32 35 38 41 44 47 50 53 56 59 62
College Degree +
Some College
High School
Less Than High School
Women
1000s
20
18
16
14
12
10
8
6
26 29 32 35 38 41 44 47 50 53 56 59
Age
Women (Relative to 1911 Cohort)
1000s
4
3
2
1
0
-1
-2
14 17 20 23 26 29 32 35 38 41 44 47 50 53 56 59 62
Birth Cohort


Expected Life-Cycle Earnings Profile: Log Specification
Men
ln
10.6
10.4
10.2
10.0
9.8
9.6
9.4
9.2
26 29 32 35 38 41 44 47 50 53 56 59
Evolution of Cohort Fixed Effects: Log Specifcation
Men (Relative to 1911 Cohort)
ln
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
14 17 20 23 26 29 32 35 38 41 44 47 50 53 56 59 62
College Degree +
Some College
High School
Less Than High School
Women
ln
9.7
9.5
9.3
9.1
8.9
8.7
8.5
8.3
26 29 32 35 38 41 44 47 50 53 56 59
Age
Women (Relative to 1911 Cohort)
ln
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
14 17 20 23 26 29 32 35 38 41 44 47 50 53 56 59 62
Birth Cohort


Expected PV of Earnings Shocks: Levels Specification for Men
Shock=Regression Standard Error
Discount Rate=4%, Retirement Horizon = Age 58
1000s
5
4
3
2
1
0
-.52
26 29 32 35 38 41 44 47 50 53 56 59
Expected PV of Earnings Shocks: Log Specification for Men
Shock=Scaled Regression St. Error
Discount Rate=4%, Retirement Horizon = Age 58
1000s
5
4
3
2
1
0
26 29 32 35 38 41 44 47 50 53 56 59
Expected PV of Earnings Shocks: Levels Specification for Women
Shock=Regression Standard Error
Discount Rate=4%, Retirement Horizon = Age 58
1000s
5
4
3
2
1
0
26 29 32 35 38 41 44 47 50 53 56 59
Expected PV of Earnings Shocks: Log Specification for Women
Shock=Scaled Regression St. Error
Discount Rate=4%, Retirement Horizon = Age 58
1000s
5
4
3
2
1
0
College Degree +
Some College
High School
Less Than High School
26 29 32 35 38 41 44 47 50 53 56 59


Age Profile of Absolute Earnings Residual
Levels Specification for Men
100s
20
18
16
14
12
10
8
6
4
26 29 32 35 38 41 44 47 50 53 56 59


Age Profile of Absolute Earnings Residual
Log Specification for Men
ln
0.08
0.07
0.06
0.05
0.04
0.03
0.02
26 29 32 35 38 41 44 47 50 53 56 59


Age Profile of Absolute Earnings Residual
Levels Specification for Women
100s
20
18
16
14
12
10
8
6
4
2
26 29 32 35 38 41 44 47 50 53 56 59

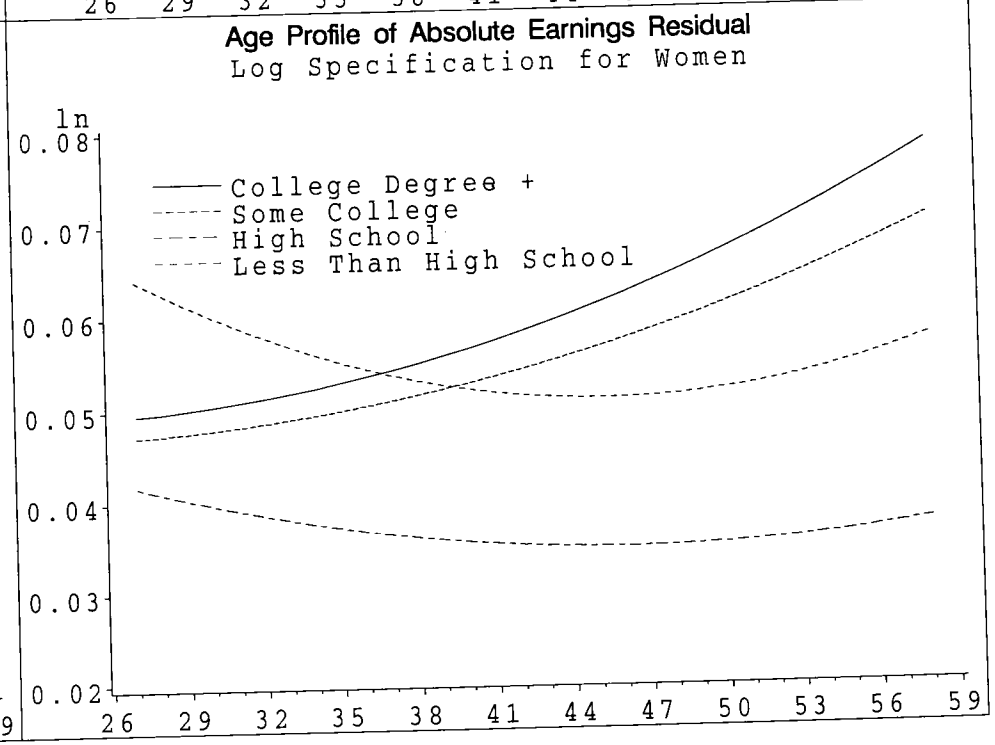

Age Profile of Absolute Earnings Residual
Log Specification for Women
ln
0.08
0.07
0.06
0.05
0.04
0.03
0.02
College Degree +
Some College
High School
Less Than High School
26 29 32 35 38 41 44 47 50 53 56 59

Figure 5:

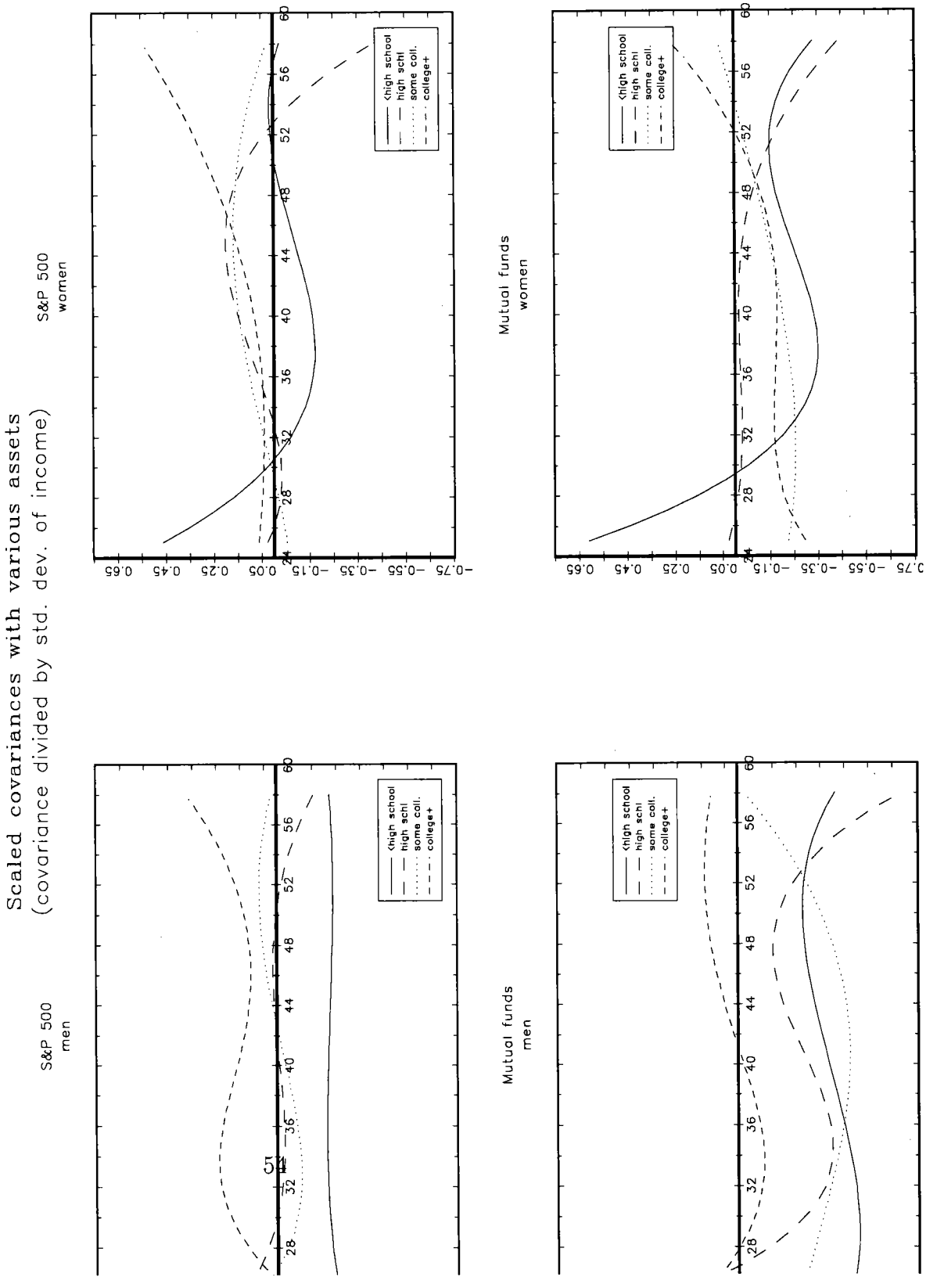

Scaled covariances with various assets
(covariance divided by std. dev. of income)
S&P 500
men
S&P 500
women
Mutual funds
men
Mutual funds
women
<high school
high schl
some coll.
college+
-0.75
-0.55
-0.35
-0.15
0.05
0.25
0.45
0.65
24
28
32
36
40
44
48
52
56
60

Figure 6:

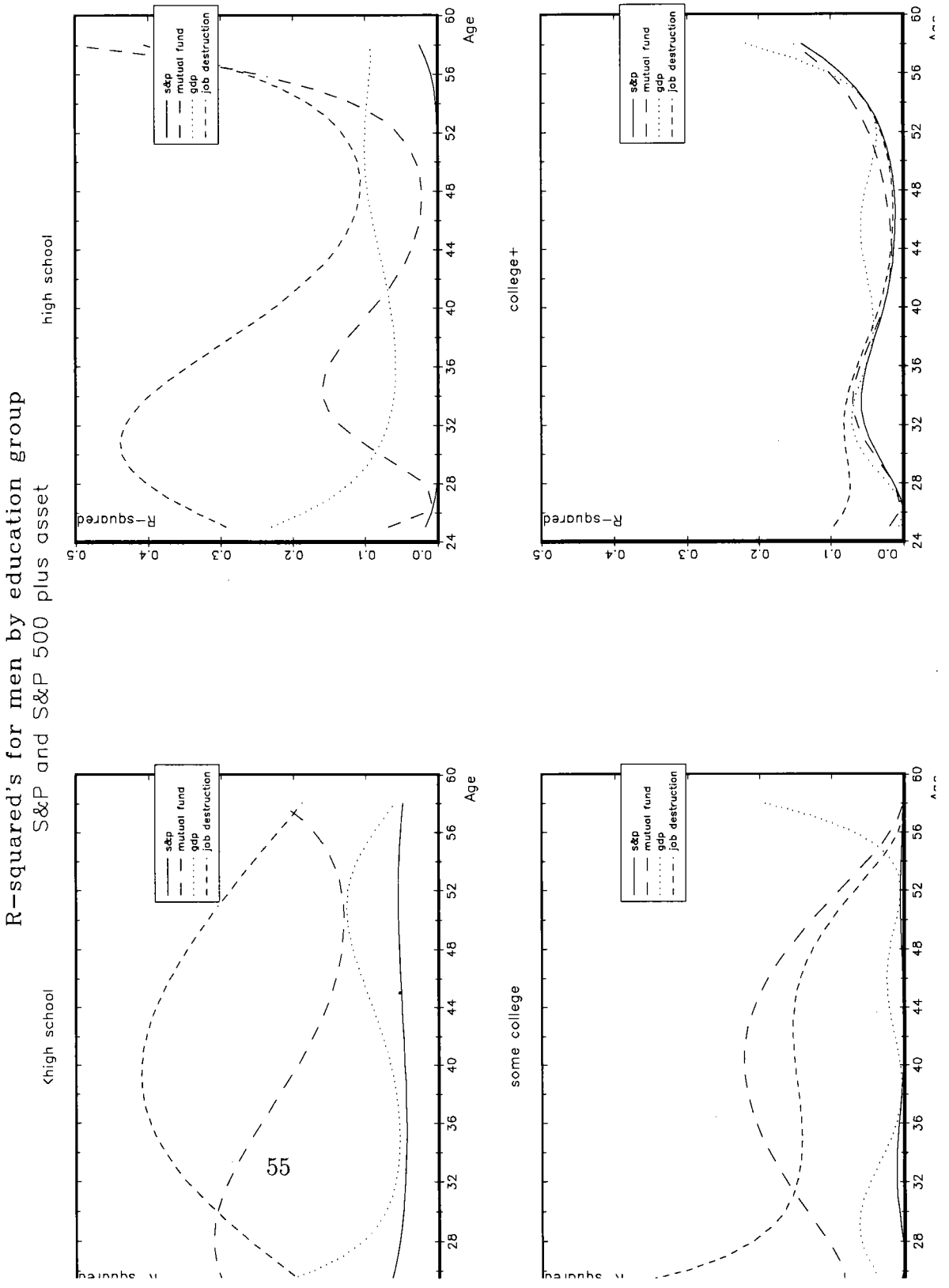
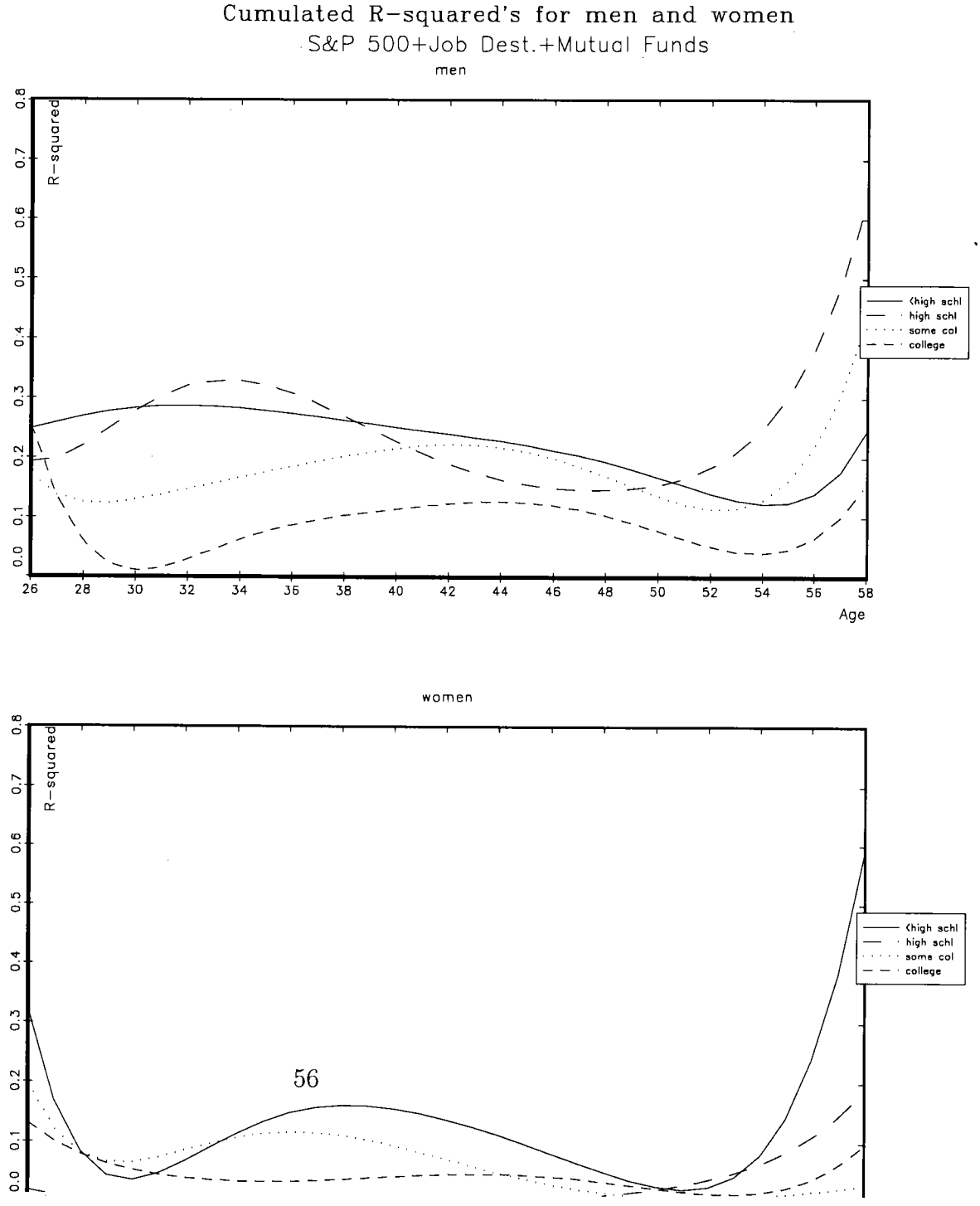

R-squared's for men by education group
S&P and S&P 500 plus asset
high school
some college
college+
s&p
mutual fund
gdp
job destruction
R-squared
Age

Figure 7:


Cumulated R-squared's for men and women
S&P 500+Job Dest.+Mutual Funds
men
R-squared
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
Age
<high schl
high schl
some col
college
women
R-squared
<high schl
high schl
some col
college