

Income Distribution Dynamics with Endogenous Fertility

By Michael Kremer and Daniel Chen*

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Abstract

Developing countries with highly unequal income distributions, such as Brazil or South Africa, face an uphill battle in reducing inequality. Educated workers in these countries have a much lower birthrate than uneducated workers. Assuming children of educated workers are more likely to become educated, this tends to increase the proportion of unskilled workers, reducing their wages, and thus their opportunity cost of having children, creating a vicious cycle. A model incorporating this effect generates multiple steady-state levels of inequality. Expanding access to educational opportunities reduces the basin of attraction of the unequal steady state and may even eliminate it. Empirical evidence suggests that the fertility differential between the educated and uneducated is greater in less equal countries, consistent with the model.

Guide to Notation

Y	output	α	share of output going to skilled labor
L_S	quantity of skilled labor	L_U	quantity of unskilled labor
w_S	wage of skilled labor	w_U	wage of unskilled labor
V	utility	X	consumption
n	number of children	ϕ	time commitment of each child
C_S	children of skilled labor	C_U	children of unskilled labor
θ	proportion of children with unskilled parents with low cost $\tilde{L}$ of education.		
$\tilde{L}$	units of time for children of skilled labor and a proportion θ of children of unskilled labor to become skilled		
$\tilde{H}$	units of time for a proportion $1 - \theta$ of children of unskilled labor to become skilled		
D	wage differential between skilled and unskilled labor		
R	ratio of skilled labor to unskilled labor		
γ	proportion of children of unskilled labor who become skilled		
R_{equal}^*	low-inequality stable steady state	R_{unequal}^*	high-inequality stable steady state
R_+^*	intermediate unstable steady state	R_-^*	intermediate stable steady state
θ_L	cut-off value of θ for which R_{equal}^* is admissible		
θ_H	cut-off value of θ for which R_{unequal}^* is admissible		
θ_{critical}	cut-off value of θ at which $R_+^* = R_-^*$		
T	value of L where θ_L attains its maximum, θ_{critical}		
G	Gini coefficient		

I. Introduction

In developing countries, fertility typically falls with education. For example, in Brazil, women with no education have three times as many children as women with ten or more years of education (United Nations, 1995). Since children of the uneducated are less likely to become educated themselves, this threefold difference in fertility creates a major demographic force increasing the proportion of unskilled workers.

One plausible hypothesis for why fertility declines with education is that because educated women command higher market wages, they face higher opportunity costs of time spent rearing children. If substitution effects outweigh income effects, then educated women have fewer children.

This paper examines the implications of combining three assumptions: (i) skilled and unskilled workers are complements in production; (ii) children of the unskilled are more likely to be unskilled; and (iii) higher wages reduce fertility because substitution effects outweigh income effects. A model incorporating these features implies that an initial increase in the proportion of unskilled workers will reduce wages of unskilled workers. Since lower wages decrease the opportunity cost of raising children, unskilled workers increase their fertility. Under the assumption that children of unskilled workers are more likely to be unskilled themselves, the proportion of unskilled workers in the next generation will therefore increase. An initial increase in the fraction of unskilled workers thus produces a multiplier effect in subsequent generations, suggesting that improving educational opportunities for even small numbers of children of unskilled workers could lead to large changes in the steady-state distribution of skill.

The model also generates multiple steady-state levels of inequality. If the initial proportion of skilled workers is great enough, wage and fertility differentials between skilled and unskilled workers will be small, allowing the economy to converge to a steady state with low inequality. However, if the initial proportion of skilled workers is too low, inequality will be self-reinforcing and the economy may approach a steady state with a low proportion of skilled workers and great inequality between the skilled and unskilled. Increasing the probability that children of unskilled parents will become skilled, for example, by expanding access to educational opportunities, reduces the basin of attraction of the unequal steady state and may even eliminate it. Some countries may face a brief window of opportunity in which small and temporary increases in the probability that children of unskilled parents become skilled can move them into the basin of attraction for the equal steady state. As time passes, however, and the economy approaches the unequal steady state, larger or longer-lasting increases in the probability that children of unskilled parents become skilled will be necessary to move to the more equal steady state.

We find empirical evidence that the fertility differential between educated and uneducated women is greater in countries with more income inequality. Using data on differential fertility by education from the World Fertility Surveys (United Nations, 1987; Jones, 1982), Demographic and Health Surveys (United Nations, 1995; Mboup and Saha, 1998), and microdata samples of US censuses (Retherford and Luther, 1996); data on income inequality (Deininger and Squire, 1996; Budd, 1967); data on returns to education (Bils and Klenow, 1998); and data on GDP (Summers and Heston, 1994; Maddison, 1995), we construct two data sets: 1) total fertility rates (TFR) by women's educational attainment and Gini coefficients of inequality

for 62 countries (88 country-years) from 1974 to 1994 and 2) TFR by women's educational attainment and Mincer coefficients of returns to education for 30 countries. We find the relationship between differential fertility and inequality to be statistically significant, positive, and economically sizeable for most specifications using both ordinary least squares regressions with country random effects and variance-weighted least squares regressions to correct for heteroskedasticity. However, since causality runs in both directions in the feedback model, this positive association between differential fertility and inequality is not definitive proof of the model; it only fails to reject a necessary condition for the model to be true.

Literature Review

A number of writers have explored one direction of the feedback mechanism—the impact of differential fertility on income inequality (David Lam, 1986; C. Y. Cyrus Chu and Hui-Wen Koo, 1990) or on the distribution of IQ or education (Samuel Preston and Cameron Campbell, 1993; Robert D. Mare, 1997). These papers use a Markovian framework in which fertility in each group (for example, skilled and unskilled) and the probability that a child born to parents in one group will transit to another group are both independent of the distribution of the population across groups. Many other papers discuss fertility and income inequality more generally (Robert Repetto, 1978; Oded Galor and Hyoungsoo Zang, 1997; Marc Nerlove, Assaf Razin, and Efraim Sadka, 1984; and Galor and Joseph Zeira, 1993). All these papers, however, do not capture the other direction of causality. As informally discussed by Nancy Birdsall (1988), an increase in income inequality, measured by a greater number of unskilled workers relative to skilled workers, is likely to suppress the wages of the unskilled. If fertility depends inversely on wages,

this increases the fertility of the unskilled and thus increases fertility differentials, creating a positive feedback. On the other hand, increasing income inequality implies higher wage differentials, which in turn, increase the incentive for children of the poor to become educated and transit to a higher income group. This may create a negative feedback that stabilizes the positive feedback between differential fertility and inequality.

Momi Dahan and Daniel Tsiddon (1998) incorporate this positive feedback in a paper modeling the demographic transition and Kuznets curve. Dahan and Tsiddon examine a non-Markovian context in which fertility and incentives for education depend on the wage structure, and thus on the fraction of skilled workers in the population. In this non-Markovian context, the positive and negative feedback discussed above balance each other at the steady state. Their paper focuses on transition dynamics. In their model, the poor initially do not invest in education and stay poor while the rich always invest in education and stay rich. During the first stage of their model, there is no intergenerational mobility between rich and poor. The proportion of poor increases, leading to greater inequality. Once income inequality reaches a certain threshold, wage differentials and incentives for education are great enough for the poor to obtain education. This is the second stage of their model. The number of uneducated people falls, which increases their wages and reduces income inequality. Due to the nature of their rational expectations model, inequality immediately moves to the steady-state level during the second stage.

Whereas Dahan and Tsiddon focus on transition dynamics in a model with a single steady state, we show that the positive feedback between fertility differentials and income inequality may lead to multiple steady states. We analyze the comparative statics and stability of the steady states and empirically examine the cross-country relationship between differential fertility and

income inequality. Technically, Dahan and Tsiddon obtain a single steady state because they assume all poor people are identical. We show that if people vary in their cost of education, there may be multiple steady states and multiplier effects.

The remainder of this paper is organized as follows: Section 2 presents the model, solves for the steady states, explores the transition path to the steady state from various initial conditions, and discusses how the dynamic system changes in response to changes in parameters. Section 3 describes the data, discusses the methodology, and presents the empirical results. Section 4 considers a refinement of the model. Section 5 argues that the basic results are robust to various generalizations and extensions of the model and concludes.

II. Theory

This section is organized as follows. Subsection A describes the model. Subsection B solves for the steady states. Subsection C discusses stability of the steady states and explores the transition path to the steady state from various initial conditions. Subsection D examines the admissibility of the steady states as a function of the underlying parameters. Subsection E explains how the basin of attraction and the wage differentials at the steady state change in response to changes in the parameters.

A. Model

Suppose the production technology is

$$Y = L_S^\alpha L_U^{1-\alpha} \quad (1)$$

where L_S and L_U are the number of skilled and unskilled workers respectively. Assuming competitive factor markets, the wages of skilled and unskilled workers will be

$$w_S = \alpha \left(\frac{L_U}{L_S} \right)^{1-\alpha} \text{ and } w_U = (1-\alpha) \left(\frac{L_S}{L_U} \right)^\alpha. \quad (2)$$

Utility is given by $V = \ln(n) + X$, where n is the number of children and X is consumption. Raising each child requires a time commitment of ϕ , and the total time endowment of an individual is 1. Thus the budget constraint is $X = w(1 - n\phi)$. Substituting X into the utility function yields $V = \ln(n) + w(1 - n\phi)$. The first order condition for optimal fertility implies $n = 1 / (w\phi)$. Under the assumed quasi-linear utility function, higher wages lead people to have fewer children. In this model, there are no marriages and each individual has n children so the number of children of unskilled and skilled workers is given by the number of adults in each group multiplied by the number of children per adult:

$$C_U = \frac{L_U}{\phi(1-\alpha) \left(\frac{L_S}{L_U} \right)^\alpha} \text{ and } C_S = \frac{L_S}{\phi \alpha \left(\frac{L_U}{L_S} \right)^{1-\alpha}}. \quad (3)$$

A property of this quasi-linear utility function is that all income is spent on children until the marginal utility of another child equals 1. All additional income is then spent on consumption, implying that for large enough incomes, there are no income effects to worry

about.¹

Fertility typically declines with education in developing countries. One hypothesis is that because educated women have higher wages, their opportunity cost of time spent raising children is higher. If substitution effects outweigh income effects, then educated women have fewer children. An alternative hypothesis is that parents substitute quality of children for quantity. Educated parents may have a lower shadow price of child quality, for instance, through easier access to schooling and health care; this raises demand for child quality and reduces demand for child quantity. Several papers give support for the first hypothesis, which we employ in our model. T. Paul Schultz (1981) points out a qualitative prediction that is confirmed by data. Since women usually bear most of the responsibility for childcare, the opportunity cost of having children for women is higher than the opportunity cost of having children for men. Assuming that education is a satisfactory proxy for lifetime wage rates, this implies that the fertility differential between educated and uneducated women should be higher than the fertility differential between educated and uneducated men. Schultz and Razzaz (1998) both cite evidence confirming that female education has a strong negative effect on fertility while male education has a smaller, less statistically significant, and sometimes positive effect on fertility (United Nations, 1987 also has corroborating evidence).

To complete the model, it is necessary to specify the process governing each individual's

¹While the inverse relationship between fertility and wages may be an acceptable approximation over moderate wage levels characteristic of middle-income countries, it is not likely to fit as well at very low or very high wages. At very low wages, wage increases may increase the number of surviving children, by reducing infant mortality and infertility due to disease and malnutrition. At very high wages, further wage increases are likely to reduce fertility only modestly. Section 4 examines this issue in further detail.

education decision. There needs to be a mechanism in which (i) educational decisions are responsive to the incentives provided by wage premia, and (ii) children of unskilled parents face higher costs of education than children of skilled parents, due to either differences in home environments or lack of access to capital markets. To capture these features, assume that all children of skilled parents, along with a proportion θ of children of unskilled parents, need $\tilde{L}$ units of time to become skilled, but that a proportion $1 - \theta$ of children of unskilled parents need $\tilde{H}$ units of time to become skilled, where $\tilde{L} < \tilde{H}$.² Children with the low time cost $\tilde{L}$ of obtaining education do so when the lifetime income from obtaining education is greater than the lifetime income from not obtaining education, that is, when $w_s (1 - \tilde{L}) > w_u$. This expression is equivalent to $1 / (1 - \tilde{L}) < w_s / w_u$. Let $D = w_s / w_u$, where D is the anticipated wage differential when they become adults. Analogously, children with the high time cost $\tilde{H}$ obtain education if $1 / (1 - \tilde{H}) < D$. To simplify the algebra below, let $L = 1 / (1 - \tilde{L})$ and $H = 1 / (1 - \tilde{H})$. Note that both $\tilde{L}$ and $\tilde{H}$ are bounded by the total time endowment, 1, and by 0, so L and H are unbounded above and bounded below by 1. Note also that D is similarly unbounded above and bounded below by 1 (skilled workers should make more than unskilled workers).

B. Steady States

²If children of skilled parents require $\epsilon < \tilde{L}$ units of time to obtain education, the steady state proportions of skilled workers and the steady state levels of inequality are identical. If a proportion of children of skilled parents also require $\tilde{H}$ units of time to become skilled, but the proportion is less than $1 - \theta$, the proportion of children of unskilled who require $\tilde{H}$ units of time, the basic story is similar but (5) becomes a cubic equation instead of a quadratic equation and we no longer get closed form solutions such as those of Proposition 1.

Define a steady state as a triplet (R^*, D^*, γ^*) , such that if the ratio of skilled to unskilled workers at time t is R^* , the wage differential would then be D^* . If fertility and education decisions are taken optimally, the proportion of children of unskilled workers who become skilled will be γ^* , and the ratio of skilled to unskilled workers in the next generation will remain at R^* .

To solve for steady states, we look for fixed points of $R_{t+1}(R_t)$. At time t , $L_{S,t}$ is the number of skilled workers and $L_{U,t}$ is the number of unskilled workers. Skilled and unskilled workers have $C_{S,t}$ and $C_{U,t}$ children respectively, who then choose whether or not to become educated at time $t + 1$ (Figure 1 diagrams the dynamics of these variables). If all the children of skilled workers and a fraction γ of children of unskilled workers become skilled workers, then

$$R_{t+1} = \frac{C_{S,t} + \gamma_t C_{U,t}}{(1 - \gamma_t) C_{U,t}} = \frac{1}{1 - \gamma_t} \left(\frac{1 - \alpha}{\alpha} \right) R_t^2 + \frac{\gamma_t}{1 - \gamma_t} . \quad (4)$$

Setting $R_{t+1} = R_t$ in (4) implies that any steady state must satisfy the following quadratic equation:

$$\frac{1}{1 - \gamma^*} \left(\frac{1 - \alpha}{\alpha} \right) R^{*2} - R^* + \frac{\gamma^*}{1 - \gamma^*} = 0 . \quad (5)$$

The assumption that all the children of skilled workers and a fraction γ of children of unskilled workers become skilled workers can be proved. In equation (4), a positive number of people are switching to become skilled in each generation. If, however, a non-positive number of people switch to become skilled, or equivalently, if all the children of unskilled workers and a proportion of children of skilled workers become unskilled (that is, if the γ -arrow in Figure 1

points in the opposite direction), then there cannot be a steady state. Consider the borderline case where each generation inherits the skill level of the previous generation. In this case, $\gamma = 0$.

Then $R_{t+1} = R_t^2 (1 - \alpha) / \alpha = R_t / D_t$ (using equations (4) and (2)). Since the wage differential, D_t , is always greater than 1, $R_{t+1} < R_t$ so the ratio of skilled workers to unskilled workers falls in the next time period. The ratio of skilled workers to unskilled workers must fall even further if some positive number of children of skilled workers become unskilled. Hence $\gamma > 0$ for any steady state and the number of children becoming skilled must be greater than $C_{s,t}$.³

Equation (5) has three possible types of solutions, each corresponding to a steady state.

(i) If the steady-state ratio of skilled to unskilled workers, R^* , induces a wage differential of exactly L , then among the children of unskilled workers, all those with a high cost of education will choose no education and some or all of those with a low cost of education will choose education. That is, the steady state fraction of children of unskilled workers who become skilled, $\gamma^* \leq \theta$, the proportion of children of unskilled parents with a low cost of education. (ii)

Similarly, if R^* induces a wage differential of H , then $\gamma^* \geq \theta$. (iii) Finally, if R^* induces a wage differential between L and H , then $\gamma^* = \theta$. All the children with a low cost of education will become educated and all the children with a high cost of education will not become educated.

No other solutions are possible since the wage differential can never be expected to be above H

³Note that some children of skilled workers can actually become unskilled as long as a greater number of children of unskilled workers become skilled and the net number of children becoming skilled is greater than $C_{s,t}$ (that is, there can be an additional arrow in Figure 1 that points in the opposite direction of the γ -arrow as long as the net movement is from unskilled to skilled). This indeterminacy about exactly who is getting educated does not affect any of the results. If children of skilled workers have some cost less than $\tilde{L}$ of obtaining education, the children of unskilled workers with low cost of education and the children of skilled workers would no longer be interchangeable.

or less than L . (Otherwise everyone or no one would become skilled, which would not be consistent with rational expectations.) Note that the quadratic equation in (5) yields more than two solutions because while there is a maximum of two solutions for a given γ^* , there may be more than one γ^* , which depends on the steady state as well.

An example when all three steady states are admissible is given in Figure 2, where R_{equal}^* , R_{unequal}^* , and R_+^* are the steady states corresponding to (i), (ii), and (iii) respectively. (For notational convenience, we will generally refer to a steady state by its ratio of skilled to unskilled workers, R^* . We will also henceforth refer to R_{equal}^* as the equal steady state and R_{unequal}^* as the unequal steady state.) We consider each of these three types of steady states in turn.

At the equal steady state, the unskilled have more children than the skilled, but enough children of the unskilled become skilled in each period to maintain the skilled–unskilled ratio at R_{equal}^* and the wage differential at L . Given $R_t = R_{\text{equal}}^*$, let γ_{equal}^* be the proportion of children of unskilled workers who become skilled at $t + 1$. The proportion γ_{equal}^* cannot be greater than the proportion θ of children of unskilled workers that have a low cost of education since the wage differential is L . If fewer than γ_{equal}^* children of unskilled become skilled, the wage differential at $t + 1$ would be greater than L , and hence all θ children of the unskilled with a low cost of education would have become skilled. On the other hand, if more than γ_{equal}^* children of unskilled become skilled, the wage differential at $t + 1$ would be less than L , and hence no one would become skilled. Neither of these is an equilibrium.

For the equal steady state to be admissible, however, the proportion of children of unskilled who have a low cost of education cannot be too small relative to the fertility differential between skilled and unskilled. If the proportion of children of unskilled who have a low cost of

education is too small, then too few children of unskilled will become skilled and the proportion of skilled adults in the population will shrink. Thus, in this situation there cannot be a steady state where the equilibrium wage differential is L . If the proportion of children of unskilled who have a low cost of education is large enough, then a sufficient number of children of unskilled can become skilled to maintain the equilibrium wage differential at L . This implies

PROPOSITION 1: R_{equal}^* is an admissible steady state where

$$[R_{equal}^*, D_{equal}^*, \gamma_{equal}^*] = \left[\frac{\alpha}{(1 - \alpha)L}, L, \frac{L\alpha - \alpha}{L^2 - \alpha L^2 + \alpha L} \right], \quad (6)$$

if and only if

$$\theta \geq \frac{L\alpha - \alpha}{L^2 - \alpha L^2 + \alpha L} \equiv \theta_L. \quad (7)$$

PROOF:

Equation (2) implies $D_t = \alpha / [(1 - \alpha) R_t]$, which implies $D^* = L \Leftrightarrow R^* = \alpha / [(1 - \alpha) L]$.

Substituting R_{equal}^* into (5) yields the expression for γ_{equal}^* in (6). The condition $D^* = L$ is equivalent to saying that the children of unskilled workers with a low cost of education are indifferent to obtaining education. Since these children are represented by the fraction θ , this last statement, in turn, is equivalent to saying $\gamma_{equal}^* \leq \theta$, which is the inequality in (7).

■

At the unequal steady state, the wage differential is H , and hence all children who have a

low cost of education become skilled and some children with a high cost of education may do so as well. Let $\gamma_{\text{unequal}}^*$ be the proportion of children of the unskilled who obtain education at $t + 1$ given $R_t = R_{\text{unequal}}^*$. The proportion $\gamma_{\text{unequal}}^*$ must be at least as large as the proportion θ of children of unskilled workers that have a low cost of education since the wage differential is H . If more than $\gamma_{\text{unequal}}^*$ children of unskilled become skilled, the wage differential at $t + 1$ would be less than H , and hence only the θ children of the unskilled with a low cost of education would have become skilled. If less than $\gamma_{\text{unequal}}^*$ children of unskilled become skilled, the wage differential at $t + 1$ would be greater than H , and hence everyone would become skilled. Neither of these is in equilibrium.

The unequal steady state is admissible if the proportion of children of unskilled workers with a low cost of education is low enough relative to the fertility differential between skilled and unskilled. In this case, even if all θ children become skilled, the ratio of skilled to unskilled workers in the next period does not increase and the wage differential does not fall below H .

PROPOSITION 2: R_{unequal}^* is an admissible steady state where

$$[R_{\text{unequal}}^*, D_{\text{unequal}}^*, \gamma_{\text{unequal}}^*] = \left[\frac{\alpha}{(1 - \alpha)H}, H, \frac{H\alpha - \alpha}{H^2 - \alpha H^2 + \alpha H} \right], \quad (8)$$

if and only if

$$\theta \leq \frac{H\alpha - \alpha}{H^2 - \alpha H^2 + \alpha H} \equiv \theta_H. \quad (9)$$

PROOF: The proof is similar to that of Proposition 1 except H replaces L .

Using Propositions 1 and 2, we can show

LEMMA 1: *The Gini coefficient for the equal steady state is less than the Gini coefficient for the unequal steady state.*

PROOF⁴:

Algebraic manipulation of the definition of the Gini coefficient leads to

$$G_{equal}^* = \alpha \left(\frac{L(1 - \alpha) + \alpha}{L(1 - \alpha) + 2\alpha} \right) \left(\frac{L - 1}{L + \alpha} \right). \quad (10)$$

The only difference between $G_{unequal}^*$ and G_{equal}^* is that L is replaced by H in equation (14).

Since the derivative of G_{equal}^* with respect to L is positive, $G_{unequal}^* > G_{equal}^*$.

■

The last case to consider is when the wage differential is between the low and high cost of education, in which case children obtain education if and only if they have a low cost of doing so.

PROPOSITION 3: R_+^* is a steady state if and only if D_+^* is between L and H and R_-^* is a steady state if and only if D_-^* is between L and H , where

$$R_{\pm}^* = \frac{1 \pm \sqrt{1 - \frac{4(1 - \alpha)\theta}{\alpha(1 - \theta)^2}}}{2(1 - \alpha)}, \quad (11)$$

⁴Due to space limitations, intermediate steps in this proof and many of the subsequent proofs are omitted; the reader can refer to Chen (1999) for more detail.

and the wage differential at R_-^* is at least as high as the wage differential at R_+^* .

PROOF:

$D_{\pm}^*$ is between L and H if and only if $\gamma^* = \theta$. Equation (11) follows immediately from solving for the roots of (5) and substituting $\gamma^* = \theta$. $D_-^* \geq D_+^*$ follows from (2) and the fact that $R_+^* \geq R_-^*$.

■

Proposition 3 leads immediately to the following lemma, where we derive the value of θ for which $R_-^* = R_+^*$.

LEMMA 2: $R_-^* = R_+^*$ if and only if $\theta = \theta_{critical}$ where

$$\theta_{critical} = \frac{2 - \alpha - 2\sqrt{1 - \alpha}}{\alpha}. \quad (12)$$

If $\theta > \theta_{critical}$, R_+^* and R_-^* do not exist. If $\theta < \theta_{critical}$, R_+^* exist and R_-^* does not equal R_+^* .

PROOF:

$R_-^* = R_+^*$ if and only if the discriminant of (11) is zero, which simplifies to a quadratic equation in θ and can be solved. The positive root is too large while the negative root is admissible, thus the negative root is $\theta_{critical}$, the value of θ for which $R_-^* = R_+^*$. If $\theta > \theta_{critical}$, the term under the square root in (11) is negative. Hence neither R_-^* nor R_+^* exist. If $\theta < \theta_{critical}$, the

term under the square root in (11) is positive, so R_-^* and R_+^* must be different.

■

C. Admissibility

Here, we discuss how changes in the underlying parameters affect the admissibility of R_{equal}^* , R_{unequal}^* , R_+^* , and R_-^* . Figure 3 shows how the dynamics of $R_{t+1}(R_t)$ depend on the values of θ , L , and H (It will be helpful to refer to Figure 3 frequently). On the two X-axes, θ varies from 0 to 1. On the Y-axis, L and H vary from 1 to infinity. The axes are discussed below. First note the following lemma.

LEMMA 3: (i) θ_L and θ_H are between 0 and 1.

(ii) θ_L and θ_H attain their maximum, θ_{critical} , when L and $H = T$, where

$$T = 1 + [1 / (1 - \alpha)]^{0.5}. \quad \theta_L \text{ and } \theta_H \text{ have no other local maxima.}$$

(iii) $\theta_L < \theta_H$ implies $L < T$. $\theta_H < \theta_L$ implies $H > T$.

PROOF: The proof follows from straightforward algebraic manipulation and is omitted.

Figure 4 shows $\theta_L(L)$ and is helpful to understand the axes in Figure 3. On Row 1 in Figure 3, $L < H < T$ so $\theta_L < \theta_H$. On Row 2, $L < T < H$ but H is sufficiently close to T such that $\theta_L < \theta_H$. On Row 3, $L < T < H$ but this time L is sufficiently close to T such that $\theta_H < \theta_L$. On Row 4, $T < L < H$ so $\theta_H < \theta_L$.

Since it seems plausible that improving the education system or subsidizing education could increase θ or decrease L and H , it is useful to discuss how the dynamics depend on θ , L , and H . The intuition is clearest for the lowest and highest values of θ (Columns A and D). If the proportion of children of unskilled with a low cost of education is very low, the economy converges to R_{unequal}^* (proved in Proposition 4 below). Recall the argument that for R_{equal}^* to be admissible, the proportion of children of unskilled who have a low cost of education cannot be too small relative to the fertility differential between skilled and unskilled. If the proportion of children of unskilled who have a low cost of education is too small, then too few children of unskilled will become skilled and the proportion of skilled adults in the population will shrink. Thus, there cannot be a steady state where the equilibrium wage differential is L .

For very high proportions of children of unskilled with a low cost of education, the economy converges to R_{equal}^* . R_{unequal}^* is inadmissible when the proportion of children of unskilled who have a low cost of education is too high relative to the fertility differential between skilled and unskilled. If θ is greater than θ_{critical} , then sufficiently many children of unskilled will become skilled so that the proportion of skilled adults in the population will rise.

Figure 3 broadly confirms the intuition that lower costs of education and a higher proportion children of unskilled with a low cost of education may help reduce inequality. As the cost of education falls, the economy moves up along the Y-axis towards Row 1. As the proportion of children of unskilled parents with low cost of education increases, the economy moves right along the X-axis. As both the cost of education falls and the proportion of children of unskilled with low cost of education increases, the economy goes from a single unequal steady state to multiple steady states to a single equal steady state. The lowest values of L and H (Row

1) have the largest region of θ over which only the equal steady state is admissible (Columns C and D). As H increases beyond T (Row 2), this region shrinks (Column D). For even larger values of L and H (Row 3), the region over which only an unequal steady state is admissible increases (Columns A and B). The largest values of L and H (Row 4) have one steady state for all values of θ but the wage differentials at all the steady states are high.

To prove the relationships shown in Figure 3, we need the following lemma:

- LEMMA 4: (i) If $\theta < \theta_H$, then $R_-^* < R_{unequal}^*$.⁵
- (ii) If $\theta < \theta_H$, then $R_+^* > R_{unequal}^*$.
- (iii) If $\theta_H < \theta < \theta_{critical}$ and $H > T$, then $R_-^* > R_{unequal}^*$.
- (iv) If $\theta_H < \theta < \theta_{critical}$ and $H < T$, then $R_-^* < R_{unequal}^*$.⁶
- (v) If $\theta < \theta_L$, then $R_-^* < R_{equal}^*$.
- (vi) If $\theta < \theta_L$, then $R_+^* > R_{equal}^*$.⁷
- (vii) If $\theta_L < \theta < \theta_{critical}$ and $L > T$, then $R_-^* > R_{equal}^*$.⁸
- (viii) If $\theta_L < \theta < \theta_{critical}$ and $L < T$, then $R_+^* < R_{equal}^*$.

⁵An equivalent necessary and sufficient condition for admissibility of $R_{\pm}^*$ in Proposition 3 is $R_{unequal}^* \leq R_{\pm}^* \leq R_{equal}^*$. This is because equation (2) and the expressions in Propositions 1 and 2 imply $D_{\pm}^*$ between L and $H \Leftrightarrow R_{\pm}^*$ between $R_{unequal}^*$ and R_{equal}^* . In other words, for a steady state to be admissible, the skilled-to-unskilled ratio must be between the skilled-to-unskilled ratios at the equal and unequal steady states. Thus Lemma 4 can be interpreted as saying if $R_{unequal}^*$ is admissible, then R_-^* is inadmissible.

⁶Hence, neither R_+^* nor R_-^* are admissible.

⁷In other words, if R_{equal}^* is inadmissible, then so is R_+^* .

⁸Hence neither R_+^* nor R_-^* are admissible.

PROOF:

Re-expressing $\theta < \theta_H$ yields

$$1 - \frac{4}{H(1 - \theta)} + \frac{4}{H^2(1 - \theta)^2} < 1 - \frac{4(1 - \alpha)\theta}{\alpha(1 - \theta)^2} . \quad (13)$$

Since both sides of this inequality are positive, taking the square root (note that either the positive or negative root of the left-hand side can be chosen) leads to the statements in (i) and (ii).

If $\theta_H < \theta < \theta_{\text{critical}}$, “>” replaces “<” in (13). The right-hand side is the term under the square root for R_* and is positive since $\theta < \theta_{\text{critical}}$ (recall the proof of Lemma 2). Taking the square root implies

$$\left| 1 - \frac{1}{H} \frac{2}{1 - \theta} \right| > \sqrt{1 - \frac{4(1 - \alpha)\theta}{\alpha(1 - \theta)^2}} . \quad (14)$$

If $H > T$, then $1 - 2 / [H(1 - \theta)]$ is positive, and (iii) follows. If $H < T$, then $1 - 2 / [H(1 - \theta)]$ is negative, and (iv) follows. The proofs of (v) - (viii) are similar to those of (i) - (iv), respectively, where L replaces H .

■

Proving the relationships in Figure 3 is a straightforward application of Propositions 1 and 2 and Lemmas 2-4.

PROPOSITION 4:

- (Column A) *If θ is less than both θ_H and θ_L , only $R_{unequal}^*$ is admissible.*
- (Column D) *If $\theta > \theta_{critical}$ only R_{equal}^* is admissible.*
- (1B and 2B) *If $\theta_L < \theta < \theta_H$, then only R_{equal}^* , R_-^* , and $R_{unequal}^*$ are admissible.*
- (3B and 4B) *If $\theta_H < \theta < \theta_L$, then only R_-^* is admissible.*
- (1C) *If $L < H < T$ and $\theta_H < \theta < \theta_{critical}$ then only R_{equal}^* is admissible.*
- (2C and 3C) *If $L < T < H$ and $\theta_L < \theta_H < \theta$, then only R_{equal}^* , R_-^* , and R_+^* are admissible. Or, if $L < T < H$ and $\theta_H < \theta_L < \theta$, then only R_{equal}^* , R_-^* , and R_-^* are admissible.*
- (4C) *If $T < L < H$ and $\theta_L < \theta < \theta_{critical}$ then only R_{equal}^* is admissible.*

PROOF:

- (Column A) Proposition 1 implies $R_{unequal}^*$ is admissible. Proposition 2 implies R_{equal}^* is inadmissible. Lemma 4.i implies R_-^* is inadmissible. Lemma 4.vi implies R_+^* is inadmissible.
- (1B and 2B) Propositions 1 and 2 imply R_{equal}^* and $R_{unequal}^*$ are admissible. Lemma 4.i implies R_-^* is inadmissible. Lemma 5 implies $R_+^* > R_{unequal}^*$. Lemma 3 implies $L < T$. Lemma 4.viii implies $R_+^* < R_{equal}^*$. Hence $R_{unequal}^* < R_+^* < R_{equal}^*$ so R_+^* is admissible.

The remainder of the proof is similar.

■

Note that the relationships at the cut-off values along the X- and Y-axes in Figure 3

follow immediately from the fact that Lemma 4 can also be proved with non-strict inequality.

D. Stability

When R_{equal}^* , R_{unequal}^* , R_-^* are admissible, they are generically stable as well. When R_+^* is admissible, it is generically unstable. For certain proportions of children of unskilled with low cost of education, some of the steady states become saddle points; but this occurs with measure zero. The intuition is as follows.

Consider the case in which the initial ratio of unskilled to skilled, and hence the initial fertility differential between the unskilled and the skilled, is sufficiently great so that even if a proportion θ children of unskilled workers obtain education the proportion of unskilled workers will rise in the next generation. In this case, the unskilled wage in the next generation will fall. The proportion of unskilled will keep growing until the wage differential between the groups becomes H . On the other hand, if the initial ratio of unskilled to skilled, and hence the between the unskilled and the skilled is small enough, then the proportion of unskilled in the next generation will fall, and the economy will approach a steady state in which the wage differential is L . R_+^* is the critical population ratio that is on the borderline between these cases and is an unstable steady state. For certain values of the underlying parameters, R_-^* replaces R_{unequal}^* as the unequal, stable steady state. This implies the following two propositions.

PROPOSITION 5: *If R_+^* and R_-^* are admissible then they are generically unstable and stable, respectively. When $\theta = \theta_{\text{critical}}$ R_+^* and R_-^* are saddle points.*

PROOF:

Taking the derivative of (5) and substituting $R_t = R_+^*$ and $\gamma^* = \theta$ yields

$$\frac{\delta R_{t+1}}{\delta R_t} = 1 + \sqrt{1 - \frac{4(1-\alpha)\theta}{\alpha(1-\theta)^2}} \geq 1, \quad (15)$$

which means R_+^* is unstable or a saddle point. R_+^* is a saddle point when the term under the square root is zero. Since this is the discriminant of equation (11), this is the same as saying $\theta = \theta_{\text{critical}}$ (recall the proof of Lemma 2), which occurs with measure zero.

Substituting R_-^* instead of R_+^* leads to the same expression as (15) except “+” becomes “-” and “ $\geq$ ” becomes “ $\leq$ ”. Thus R_-^* is generically a stable steady-state and a saddle point only when $\theta = \theta_{\text{critical}}$.

■

PROPOSITION 6: *If R_{equal}^* and R_{unequal}^* are admissible then they are stable as well. If $\theta = \theta_L$ and $L < T$, then R_{equal}^* is a saddle point. If $\theta = \theta_H$ and $H < T$, then R_{unequal}^* is a saddle point.*

PROOF:

Consider R_{equal}^* . Suppose the ratio of skilled workers to unskilled workers is perturbed to $R_{\text{equal}}^* + \epsilon$ at time t . At time $t + 1$, the ratio of skilled to unskilled must fall or else the wage differential is less than L , which is inconsistent with rational expectations since no one would have become educated in the first place.

Suppose the ratio of skilled to unskilled is perturbed to $R_{\text{equal}}^* - \epsilon$ at time t . Assume

towards contradiction that the ratio of skilled to unskilled does not rise at time $t + 1$, in which case the wage differential will be greater than L . Then all θ children of unskilled workers with low cost to education would have become educated. But for R_{equal}^* to be admissible, $\theta \geq \gamma_{\text{equal}}^*$, which means at least as many children become skilled at $R_{\text{equal}}^* - \epsilon$ as at R_{equal}^* . For ϵ small enough, if more people are becoming educated at $R_{\text{equal}}^* - \epsilon$ than at R_{equal}^* , this means the ratio of skilled to unskilled workers actually increases at $R_{\text{equal}}^* - \epsilon$, which is our contradiction.

If, however, the same proportion of people are becoming educated at $R_{\text{equal}}^* - \epsilon$ as at R_{equal}^* and $L < T$, then the ratio of skilled to unskilled falls at $R_{\text{equal}}^* - \epsilon$. $L < T$ and the fact that $\gamma_{\text{equal}}^* = \theta_L$ imply $R_{\text{equal}}^* = R_+^*$ (using Lemma 4.vi and viii with non-strict inequality), which is generically unstable. Hence R_{equal}^* is a saddle point. If, however, $\theta = \gamma_{\text{equal}}^*$ and $L \geq T$, then $R_{\text{equal}}^* = R_-^*$, which is also stable. The bottom line is that R_{equal}^* is stable or a saddle point that occurs with measure zero. The proof for R_{unequal}^* is similar.

■

E. Comparative Statics

In general, as the proportion of children with low cost of education increases, as the cost of education falls, or as the proportion of output that goes to skilled workers decreases, wage differentials fall and the basin of attraction to the unequal steady state shrinks, possibly becoming eliminated. In this section, we first consider how the basin of attraction of the stable steady states and the wage differentials at those steady states are affected by changes in the underlying parameters θ , L , H , and α ; that is, we consider how R_{equal}^* , R_{unequal}^* , R_+^* , and R_-^* themselves

change as the underlying parameters change. We then discuss how changes in the same parameters affect the cutoff values for the admissibility of R_{equal}^* , R_{unequal}^* , R_+^* , and R_-^* .

When there are multiple steady states as in Figure 2, the system converges to the unequal steady state if R_0 , the initial ratio of skilled to unskilled workers, is less than R_+^* , and to the equal steady state if R_0 is greater than R_+^* . The only parameters that effect R_+^* and thus the basin of attraction from which the system approaches either the unequal steady state or the equal steady state are θ and α . Increases in θ , the proportion of children of unskilled parents who have a low cost of education, reduce R_+^* , and expand the basin of attraction of the equal steady state. To see this, note that

$$\begin{aligned} \frac{\delta R_+^*}{\delta \theta} = & \frac{-\alpha}{2(1-\alpha)} \left(1 + \sqrt{1 - \frac{4(1-\alpha)\theta}{\alpha(1-\theta)^2}} \right) + \\ & \frac{(1-\theta)\alpha}{2(1-\alpha)} \frac{1}{2\sqrt{1 - \frac{4(1-\alpha)\theta}{\alpha(1-\theta)^2}}} \frac{-4(1-\alpha)}{\alpha} [(1-\theta)^{-2} + 2\theta(1-\theta)^{-3}] , \end{aligned} \quad (16)$$

which is negative since θ and α are less than 1. Since R_+^* exists, the term under the square root is positive. Decreases in α , the share of output going to skilled workers, reduce R_+^* , and also expand the basin of attraction from which the system approaches the equal steady state.

Intuitively, if the economy gives unskilled workers a greater proportion of output, wage differentials are likely to be reduced and it becomes easier to approach the steady state with low inequality. To see this, note that R_+^* can be written as

$$R_+^* = \frac{\frac{\alpha}{1-\alpha} + \sqrt{\frac{\alpha}{1-\alpha} \left(\frac{\alpha}{1-\alpha} - \frac{4\theta}{(1-\theta)^2} \right)}}{\frac{2}{1-\theta}} . \quad (17)$$

Since $\alpha / (1 - \alpha)$ increases as α increases, R_+^* also increases as α increases.

The model thus suggests that countries with R_0 just under R_+^* may face a brief window of opportunity in which small and temporary increases in θ or small and temporary decreases in α can move them into the basin of attraction for the equal steady state. As time passes, and R falls, larger or longer-lasting increases in θ or decreases in α would be necessary to move to the more equal steady state.

If a country wishes to reduce the steady-state wage differential at its current steady state, the model suggests reducing $\tilde{L}$ or $\tilde{H}$, the cost of education for different segments of the population, suffices to reduce the corresponding steady-state wage differential, L and H , and the equal and unequal steady states. For some parameter values, however, R_-^* replaces R_{unequal}^* as the unequal steady state, in which case changes in α and θ may also affect the steady-state wage differential. Decreases in α reduce the wage differential at R_-^* . Using equation (2), the wage differential D_-^* can be written as

$$D_-^* = \frac{\frac{2}{1-\theta}}{1 - \sqrt{1 - \frac{1-\alpha}{\alpha} \frac{4\theta}{(1-\theta)^2}}} . \quad (18)$$

As α falls, $(1 - \alpha) / \alpha$ increases, so the denominator of D_-^* increases and D_-^* itself falls.

Moreover, increasing θ , the proportion of children of unskilled with low cost of education, results in a lower wage differential D_-^* . To see this, refer back to the quadratic equation in (5).

Note that increases in θ increase the coefficient of R^2 and the constant term. This means the upwards-pointing parabola narrows and moves strictly upwards and does not cross the original parabola. Thus if R_+^* , one root of the quadratic equation, decreases as θ increases, as shown in equation (16), then the other root, R_-^* , must increase, which implies θ and D_-^* are inversely related.

The comparative statics for the cutoff values, T , θ_L , θ_H , and $\theta_{critical}$, in Figure 3 are less straightforward. For low values of L , increases in L can eliminate the equal steady state if increasing L causes θ_L to increase beyond θ . The intuition behind this is that higher L means that it becomes more difficult for people to become skilled and thus it becomes more difficult to maintain the equal steady state. Analogously, reductions in H can eliminate the unequal steady states, $R_{unequal}^*$ and R_-^* , if H is reduced to below T and θ_H falls below θ . Lower H means it becomes easier for some people to become skilled and thus unequal steady states become more difficult to maintain.⁹

Reductions in α , the share of output going to skilled workers, reduce $\theta_{critical}$ and thus can eliminate the unequal steady state. To see this, note that the following relationship can be

⁹For large values of L , however, increases in L can eliminate R_-^* and expand the region of θ over which only the equal steady state is admissible, if increasing L causes θ_L to decrease below θ . This is because $\theta_L(L)$ is increasing in L until $L = T$ and then decreasing thereafter (Lemma 3). This may seem counter-intuitive. However, recall that increasing L and H increase the wage differential at the equal and unequal steady states, respectively. For example, it may seem odd that increasing L from 3C to 4C in Figure 3 removes the unequal steady states. Or it may seem odd that increasing L so that θ_L falls below θ may move the system from 4B to 4C, from R_-^* to R_{equal}^* . But for R_-^* to be admissible in the first place, its wage differential must have been between L and H . Thus, increasing L increases the lower bound of admissibility for R_-^* . When increases in L cause R_-^* to become inadmissible, the economy then shifts to R_{equal}^* but the wage differential, L , at R_{equal}^* is now greater than it was at R_-^* . A similar phenomenon occurs when increases in H cause the system to shift from 4A to 4B by lowering θ_H below θ .

derived:

$$\frac{\delta \theta_{critical}}{\delta \alpha} = \frac{2 - \alpha - 2\sqrt{1 - \alpha}}{\alpha^2 \sqrt{1 - \alpha}}, \quad (19)$$

which is positive since this is just $\theta_{critical}/(1-\alpha)^{0.5}$. Reductions in α also reduce θ_L and θ_H and thus moves the system towards greater equality and to the right on the X-axis. To see this, note that θ_L can be rewritten as

$$\theta_L = \frac{\alpha(L - 1)}{L^2 + \alpha(1 - L)L}. \quad (20)$$

Since the numerator increases with α and the denominator decreases with α , θ_L is positively related with α . On the other hand, reductions in α move us down on the Y-axis since T is positively related to α . Note that $T = 1 + 1/(1-\alpha)^{0.5}$ decreases as α decreases. Except for this last case, reductions in α is associated with greater equality, as measured by wage differentials.

In summary, the model indicates that countries can either reduce inequality by reducing the wage differential at their current steady state or by inducing a shift into the basin of attraction of the equal steady state. A small, permanent decrease in the cost of education is sufficient to create a small reduction of the steady-state wage differential. For some parameter values, a small, permanent increase in the probability that children of unskilled parents become skilled is also sufficient. However, if a country is stuck at an unequal steady state and wishes to move to a more equal one, only a big, but temporary, push to increase the probability that children of unskilled parents become skilled will move it to a more equal steady state. This push must last

long enough for the country to move near the equal steady state so that when the push ends, the country is still in the basin of attraction of the equal steady state. For some parameter values, a large, temporary decrease in the cost of education for children with a high cost of education is also sufficient.

III. Empirical Evidence

We find some evidence that the fertility differential between educated and uneducated women is greater in countries with more inequality. However, since causality runs in both directions in the feedback model, the evidence is not conclusive; if only one direction of the causality exists, there would still be a positive association between differential fertility and inequality. Thus, the data only fails to reject a necessary condition for the model to be true.

The model predicts that there should be a positive relationship between differential fertility and Gini coefficients (recall Lemma 1) as well as a positive relationship between differential fertility and wage differentials, measured by the return to education. In two separate cross-country samples, we find that this is the case.

We test the model by creating two data sets¹⁰: 1) 88 observations in a sample of 62 countries from 1974 to 1994 for which observations on total fertility rates (TFR) by women's educational attainment and Gini coefficients of inequality could be obtained and 2) 30

¹⁰In an attempt to use data as consistent as possible across countries and across time, we avoid using individual country censuses, which presumably would vary significantly in definitions and measurement. To reduce measurement error, we use data sources, such as the World Fertility Survey and Demographic and Health Surveys, that have already standardized definitions and measurements across many countries.

observations of 30 developed and less-developed countries for which TFR by women's educational attainment and Mincer coefficients of returns to education could be obtained.

The remainder of the section is organized as follows. Subsection A discusses the sources for the data (Table 1 summarizes the available data).¹¹ Subsection B describes the methodology and empirical models. Subsection C reports the results.

A. Data

To measure differential fertility, we use data on total fertility rates (TFR) by women's educational attainment from four comparative studies (United Nations 1987, Jones 1982, United Nations 1995, and Mboup and Saha 1998). The four comparative studies calculate TFR by summing the age-specific fertility rates for 5-year age groups and multiplying by 5. They calculate age-specific fertility rates by dividing the number of births to women in a specific age group, during a specific period, by the number of woman-years of exposure during the same period. Thus, we can interpret the total fertility rate as the expected number of children a woman would have if she lives until the end of her reproductive years and experiences the age-specific fertility schedule for each 5-year period.¹² Data on income inequality comes from Deininger and Squire (1996). Since almost every country-year observation had only one high quality Gini coefficient available, we do not take into account the Gini coefficient's manner of calculation. In

¹¹For an in-depth discussion of the process by which data sets were put together, see Chen (1999).

¹²Theoretically, the model calls for having data on the number of surviving children, however in practice, this data is hard to get.

returns to education, for 52 countries. Mincer coefficients are coefficients from semi-log OLS regressions of log earnings on years of schooling with potential years of labor market experience and its square as additional independent variables.¹³ GDP data comes from Summers and Heston (1994), who report real GDP per capita in constant dollars (Chain index) expressed in international prices, base 1985.

B. Methodology

To get an actual measure for differential fertility¹⁴ that can be compared with inequality, we use a weighted ordinary least squares regression to regress TFR on a proxy representing years of education, where the weights are the percentage distribution of women in each education category¹⁵. Weights are used in order to take into consideration the fact that there may be very few women with extreme levels of educational attainment, which may increase the measurement noise of fertility at these levels. The negative of the coefficient from the regression is reported,

¹³To check if the return to education is a good measure of inequality, we found that the relationship between Mincer coefficients and Gini coefficients is positive and significant at the 1% level using an OLS regression.

¹⁴In the model, differential fertility is measured as the ratio of the number of children of the skilled to the number of children of the unskilled. Here, differential fertility is measured as an absolute difference through a regression of fertility across education levels.

¹⁵ Note that the percentage distribution of women is only available for data from United Nations (1987) and United Nations (1995).

so a large, positive number corresponds to a large fertility differential between educated and uneducated women. The proxy for the number of years of education is typically the average of the range of years in an education category.

Empirical Model

As explained earlier, since the causality runs in both directions in the model between differential fertility and inequality, the empirical evidence can only test a necessary condition for the model to be true: is there a strong, positive relationship between income inequality or returns to education on the one hand and fertility differential between educated and uneducated women on the other?

To answer this question, we use two econometric models¹⁶. The first is an error components model with country random effects. Because the data comes from several surveys at three different points in time (1970s, 1980s, and 1990s), the data set includes some countries up to three times. Moreover, since fertility differentials and income inequality are likely to be correlated for observations of the same country due to unobservable characteristics of the environment, we need to include country random effects. Failing to account for these correlations leads to underestimates of standard errors of the coefficients. More specifically, the model is:

$$F_{ct} = a + bG_{ct} + cX_{ct} + u_c + \epsilon_{ct}$$

where c indexes countries and t indexes time. F_{ct} is the fertility differential, which is

¹⁶Note that due to software limitations we could not simultaneously include both country random effects and use weights to correct for heteroskedasticity. Consequently, for each set of regressions, we report the results from both models where appropriate.

approximated as the coefficient from the weighted least squares regression described earlier, a is a constant, b reflects the relationship between the fertility differential and the Gini coefficient, G_{ct} is the Gini coefficient, X_{ct} is a vector of country and time variables such as $\ln(\text{GDP})$, a Latin America dummy variable, and the year of survey, and u_c is a country-specific random effect. ϵ_{ct} is the usual residual with the usual properties: mean 0, uncorrelated with itself, G_{ct} , X_{ct} , and u_c and homoskedastic.

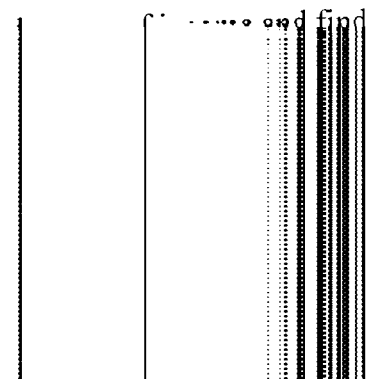
The second model takes into account the fact that ϵ_{ct} is actually heteroskedastic. The noise in my approximation of F_{ct} , the fertility differential, varies across observations. The earlier regressions that provide the measurements of differential fertility, however, also provide an estimate of their noise—the standard errors of the same regression. Using variance-weighted least squares, we estimate the model:

$$F_{ct} = a + bG_{ct} + cX_{ct} + \epsilon_{ct}$$

where the variables are defined as before. In effect, this regression puts more weight on observations with low variance.

C. Results

The empirical testing of the model indicates a strong relationship between differential fertility and Gini coefficients as well as between differential fertility and Mincer coefficients for the cross-country samples. This relationship is positive and significant at the 1% level for most



approximated as the coefficient from the weighted least squares regression described earlier, a is a constant, b reflects the relationship between the fertility differential and the Gini coefficient, G_{ct} is the Gini coefficient, X_{ct} is a vector of country and time variables such as $\ln(\text{GDP})$, a Latin America dummy variable, and the year of survey, and u_c is a country-specific random effect. ϵ_{ct} is the usual residual with the usual properties: mean 0, uncorrelated with itself, G_{ct} , X_{ct} , and u_c and homoskedastic.

The second model takes into account the fact that ϵ_{ct} is actually heteroskedastic. The noise in my approximation of F_{ct} , the fertility differential, varies across observations. The earlier regressions that provide the measurements of differential fertility, however, also provide an estimate of their noise—the standard errors of the same regression. Using variance-weighted least squares, we estimate the model:

$$F_{ct} = a + bG_{ct} + cX_{ct} + \epsilon_{ct}$$

where the variables are defined as before. In effect, this regression puts more weight on observations with low variance.

C. Results

The empirical testing of the model indicates a strong relationship between differential fertility and Gini coefficients as well as between differential fertility and Mincer coefficients for the cross-country samples. This relationship is positive and significant at the 1% level for most specifications. To see if these result are robust, we add income and square of income and find that the results still hold. Including a Latin America dummy and breaking the sample into sub-

samples by income drive out some of the results in the cross-country samples.

In the remainder of the section, we discuss the results from the two data sets. Table 2 and Chart 1 refer to the sample of countries for which observations on TFR by women's educational attainment and Gini coefficients of inequality could be obtained. Table 3 and Chart 2 refer to the sample of countries for which TFR by women's educational attainment and Mincer coefficients of returns to education could be obtained.

Table 2 -- Fertility Differential Regressed on Gini Coefficient, 1974-1995

Using country random effects, the coefficient of the fertility differential regressed on the Gini coefficient is 0.639 and significant at the 1% level for the specification in column (i) of the country random effects regressions. This indicates that going from a relatively equal country like Indonesia with a Gini coefficient of 0.320 in 1987 to a relatively unequal country like Brazil with a Gini coefficient of 0.545 in 1986 increases the fertility differential by 0.14. A fertility differential increase of 0.14 means, for example, the difference in expected number of children between a woman with 10 years of education and a woman with no schooling is 1.4 children greater in a country like Brazil than it is in a country like Indonesia. Another interpretation is that increasing by two standard deviations of the Gini coefficient (two standard deviations is 0.188) increases the fertility differential by 0.12. Or, going from the 25th percentile of the Gini coefficients, 0.349, to the 75th percentile, 0.490, increases the fertility differential by 0.09.

Since Gini coefficients may be higher among middle-income countries than among either low-income or high-income countries, we control for linear and nonlinear income effects by adding $\ln(\text{GDP})$ and $\ln(\text{GDP})^2$ as additional covariates. Controlling for income does not change

the results much. Since Latin American countries have higher income inequality, we also include a Latin America dummy as another covariate. The relationship between differential fertility and income inequality decreases in magnitude and significance when the Latin America dummy is added, indicating that some of the increase in the fertility differential could be due to other factors related to Latin America. The coefficient on the Latin America dummy is always statistically significant at the 1% level for the specifications listed in columns (iv)-(vi). However, the coefficient on the Gini coefficient stays positive and, for the variance-weighted least squares regressions, the coefficient stays significant at the 1% level as well. When the Latin America dummy is included, controlling for a non-linear relationship with GDP actually increases the magnitude of the relationship between the fertility differential and Gini coefficient.

There is some evidence that fertility differentials may be greatest among middle-income countries. The coefficient on $\ln(\text{GDP})$ in column (iii) of Table 2 is positive and significant at the 1% level while the coefficient on $\ln(\text{GDP})^2$ is negative and significant at the 1% level; for the range of income levels in this data set, fertility differentials initially rise among low to middle-income countries and then fall among middle to high-income countries, all else equal.

Table 3 -- Fertility Differential Regressed on Returns to Education

In the second data set, since only one Mincer coefficient is available per country, we do not use country random effects and only use the variance-weighted least squares regressions to test the relationship between differential fertility and returns to education. The coefficient on the Mincer coefficient is positive and significant at the 1% level for two specifications. The number 0.597 in column (i) implies that an increase in two standard deviations of returns to education has

children of skilled workers and θ children of unskilled workers would invest in education. In between L and H, there is a set ratio of children of unskilled workers who become skilled. Consequently, between L and H, the demographic force described above dominates. Once the wage premium reaches L or H, the traditional, stabilizing force by which increasing wage differentials increase incentives for education dominates. With a more general, continuous distribution of cost of education, either force could locally dominate in different areas. We conjecture that, with a more general distribution of cost of education, the R_{t+1} (R_t) curve could cross the 45° line an arbitrary number of times, generating an arbitrary number of stable steady states.

Furthermore, with a continuous distribution of cost, the response of fertility differentials to wages should make R_{t+1} increase more steeply in R_t than if fertility were exogenous. The steady-state distribution of skill would therefore be more sensitive to changes in parameters with

women should be small. This appears to be true for the case of Taiwan. During the communist takeover of China, Taiwan had exogenous increases in the proportion of skilled workers due to the exodus of educated people fleeing China. Since then, Taiwan has had a fairly equal wage distribution. Moreover, differential fertility by education is very low, in fact, it is reversed. Women with nine or more years of education have twice as many children as women with less than six years of education (Taiwan-Fukien demographic fact book, 1996).

²¹The empirical investigation of the model can also be expanded but evidence is limited by the available data.

Since Gini coefficients may be higher among middle-income countries than among either low-income or high-income countries, we test to see if the results can be driven out by controlling for linear and nonlinear income effects. Adding $\ln(\text{GDP})$ and $\ln(\text{GDP})^2$ as additional covariates does not change the results much as columns (ii) and (iii) of Tables 2 and 3 show.

We also attempt to drive out the results by dividing the data set into sub-samples according to income. We find that the relationship increases in magnitude among middle-income countries but at extreme income levels, the relationship is no longer significant. Among low-income countries, there is a negative relationship while among the highest-income countries, there is neither a positive or negative relationship (these results are reported in Section 4). The cut-off between middle-income and low-income developing countries is \$950 GDP per capita while the cut-off between middle-income and high-income countries is \$9500 GDP per capita (note that this is in 1985 dollars but the high-income observations are from the 1970s, indicating that the high-income countries are very rich).

These results may indicate that differences between income groups rather than within groups explain the association between differential fertility and inequality. However, there are several reasons why these results may not invalidate the model. First of all, restrictions to sub-samples of the data tend to increase the noise level relative to the true association. Moreover, to drive out the significance among the high-income countries, we need to restrict the sample to the richest 6 countries in the sample of 88. Furthermore, as noted earlier, controlling for linear and nonlinear income effects among the entire sample do not significantly change the results. Thus, it is not clear if the relationship between differential fertility and income inequality is driven by

between fertility differential and income inequality is greatest among middle-income countries, negative among low-income countries, and weak among high-income countries.

To see this, suppose the production function includes an additional variable, A , for the productivity of available technology, which increases a country's income level and consequently reduces the fertility of that countries' skilled and unskilled labor. Suppose also that there is a maximum fertility level that may be binding for poor countries but not for rich ones and that maximum fertility declines for the very poor due to infertility or infant mortality.

Consider a country whose unskilled wages are sufficiently low so that the fertility restriction is binding among unskilled labor. Then, if wage differentials increase and unskilled labor's wages fall, their maximum fertility will also fall. If unskilled labor's fertility falls fast enough relative to the decrease in skilled labor's fertility, then differential fertility falls.

Among high-income countries, the fertility of both skilled and unskilled labor is small so the maximum fertility level is not binding. Since fertility levels, and hence differential fertility, are low, the variance in differential fertility across high-income countries is low. Hence, measurement noise may obscure the relationship between differential fertility and inequality.

For middle-income countries, fertility differentials, as measured in Section 3, should be the greatest relative to low and high-income countries. Consequently, the relationship between differential fertility and inequality should also be the strongest.

We find that the positive relationship between inequality and differential fertility is indeed primarily found in the sample consisting of middle-income countries while among low-income countries the relationship is negative and among the high-income countries the

relationship is insignificant (Table 5 presents these results¹⁸). The cut-off between middle-income and low-income developing countries is \$950 GDP per capita while the cut-off between middle-income and high-income countries is \$9500 GDP per capita.

Among middle-income countries, the association between differential fertility and the Gini coefficient has a larger magnitude than the coefficient for the sample of all countries and is statistically significant at the 1% level. The number 0.919 in column (i) can be interpreted as increasing the fertility differential by 0.20 when inequality increases from the 1987 Indonesian level (with a Gini coefficient of 0.320) to the 1986 Brazilian level (with a Gini coefficient of 0.545).

For low-income countries the association between differential fertility and Gini coefficient is large, negative, and significant at the 1% level for one specification with country random effects. The coefficient of -1.757 in column (ii) means that a relatively equal country like Ghana in 1992 with a Gini coefficient of 0.339 has a decrease in fertility differential of 0.36 when compared to a relatively unequal country like Kenya in 1992 with a Gini coefficient of 0.544. This indicates that the difference in expected number of children for a woman with 10 additional years of education is 3.6 children greater in a country like Ghana than in a country like Kenya.¹⁹

¹⁸Controlling for other variables such as $\ln(\text{GDP})$, $\ln(\text{GDP})^2$, and a Latin America dummy did not change the results much. These additional results are presented in Chen (1999).

¹⁹Controlling for $\ln(\text{GDP})$ halves the magnitude of the coefficient on the Gini. The coefficient on $\ln(\text{GDP})$ is large, positive, and significant at the 1% level. This is consistent with the hypothesis presented in Section 2.6: at very low wages, wage increases may increase the number of surviving children, by reducing infant mortality and infertility due to disease and malnutrition. Hence, rising GDP increases income for the poorest, all else equal, and may actually increase the number of children they have, which consequently increases the fertility

Among high-income countries, the association between differential fertility and Gini coefficients is close to 0. The coefficient on the Gini coefficient is not statistically significant at the 10% level for any specification.

differential. This strong income effect does appear in the other samples.

endogenous fertility. For example, suppose a program educates an additional 1,000,000 children in each generation. Then, unskilled wages will rise, raising the opportunity cost of childbearing among the unskilled and reducing their fertility, which will further increase wages among the unskilled, creating a multiplier effect on inequality. Figure 5 shows, in a local neighborhood of a steady state, that a permanent increase in the reaction function $R_{t+1}(R_t)$ leads to a larger steady-state proportion of skilled workers when there is endogenous fertility than when there is not. This is because endogenous fertility creates a positive feedback, which makes the reaction function steeper. Note that this does not apply in the simple model with a two-point distribution of cost. At the stable steady states with L and H as wage premiums, the supply of skilled labor is infinitely elastic. The traditional force of children choosing education when wage differentials are high has an infinite weight relative to the demographic force and hence the slope of the response function $R_{t+1}(R_t)$ is zero.

Another limitation of the analysis is that we approximate relative wages as depending on the skilled-to-unskilled ratio of the population, rather than on the skilled-to-unskilled ratio of market labor time, which also depends on the proportion of time each group spends on education and raising children. Dispensing with this approximation would make the problem more complicated. During the time when young people are in school learning rather than working, the supply of unskilled labor will be lower. Moreover, the greater child bearing of the unskilled will also reduce their time in the market. This will tend to increase the wage of the unskilled for every level of R . Formally, the problem becomes much more difficult, because the choice of whether to become educated depends on people's expectations of fertility of the skilled and unskilled when they become adults. Thus R_t would depend not only on R_{t-1} but also on R_{t+1} . R_{t+1}

takes into account the number of children who are in school at time t since these are the children that will become skilled adults at $t+1$. We conjecture that most of the intuition would go through in such a model.

We focus on a tractable case of quasilinear utility but conjecture that the results in this paper still hold for more general forms of utility as long as (i) skilled and unskilled workers are complements in production; (ii) children of the unskilled are more likely to be unskilled, and (iii) higher wages reduce fertility because substitution effects outweigh income effects²².

²²As discussed earlier, economists tend to propose two explanations for why fertility declines with education. One hypothesis is that parents substitute quality of children for quantity. This paper uses the second hypothesis—that substitution effects outweigh income effects. Several papers give support for this hypothesis and suggest that it is economically significant.

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Appendix

*.Kremer: Massachusetts Institute of Technology, Department of Economics, 50 Memorial Drive, E52-251C, Cambridge, MA 02142, Brookings Institution, Harvard Institute for International Development, and National Bureau of Economic Research; Chen: Harvard College, Cambridge, MA 02138. We thank George Akerlof and David Weil for helpful discussions, and Yu-chin Chen and Paras Mehta for comments. Kremer thanks the MacArthur Foundation for financial support.

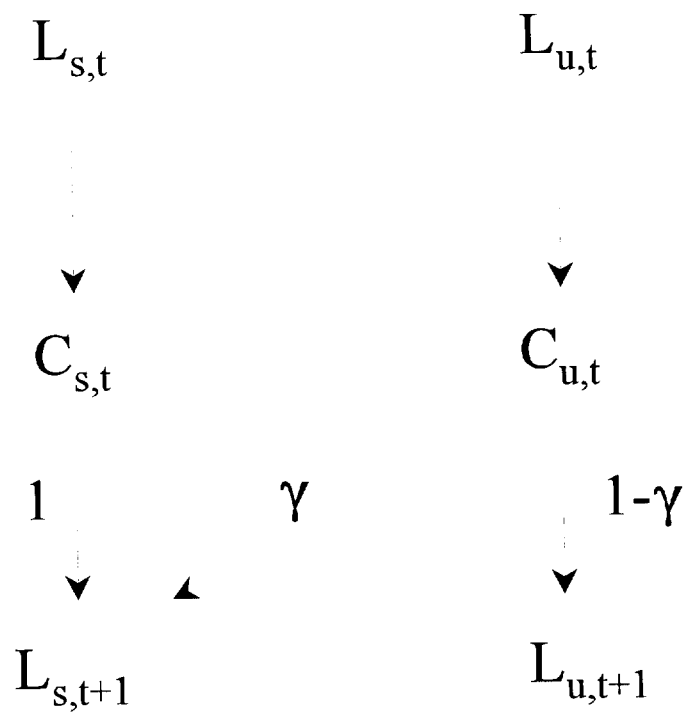


Figure 1 -- Dynamics of $C_{S,t}$, $C_{U,t}$, $L_{S,t}$, and $L_{U,t}$.

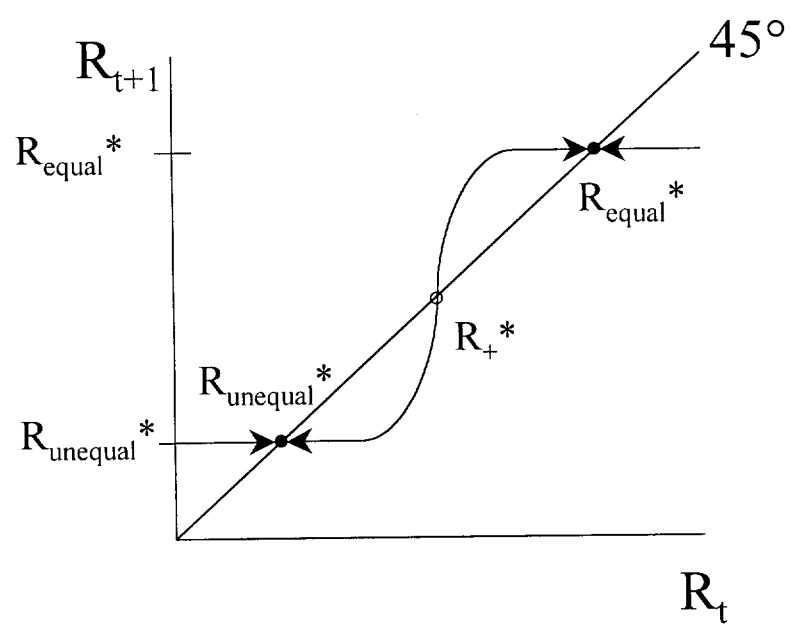


Figure 2: Multiple steady states

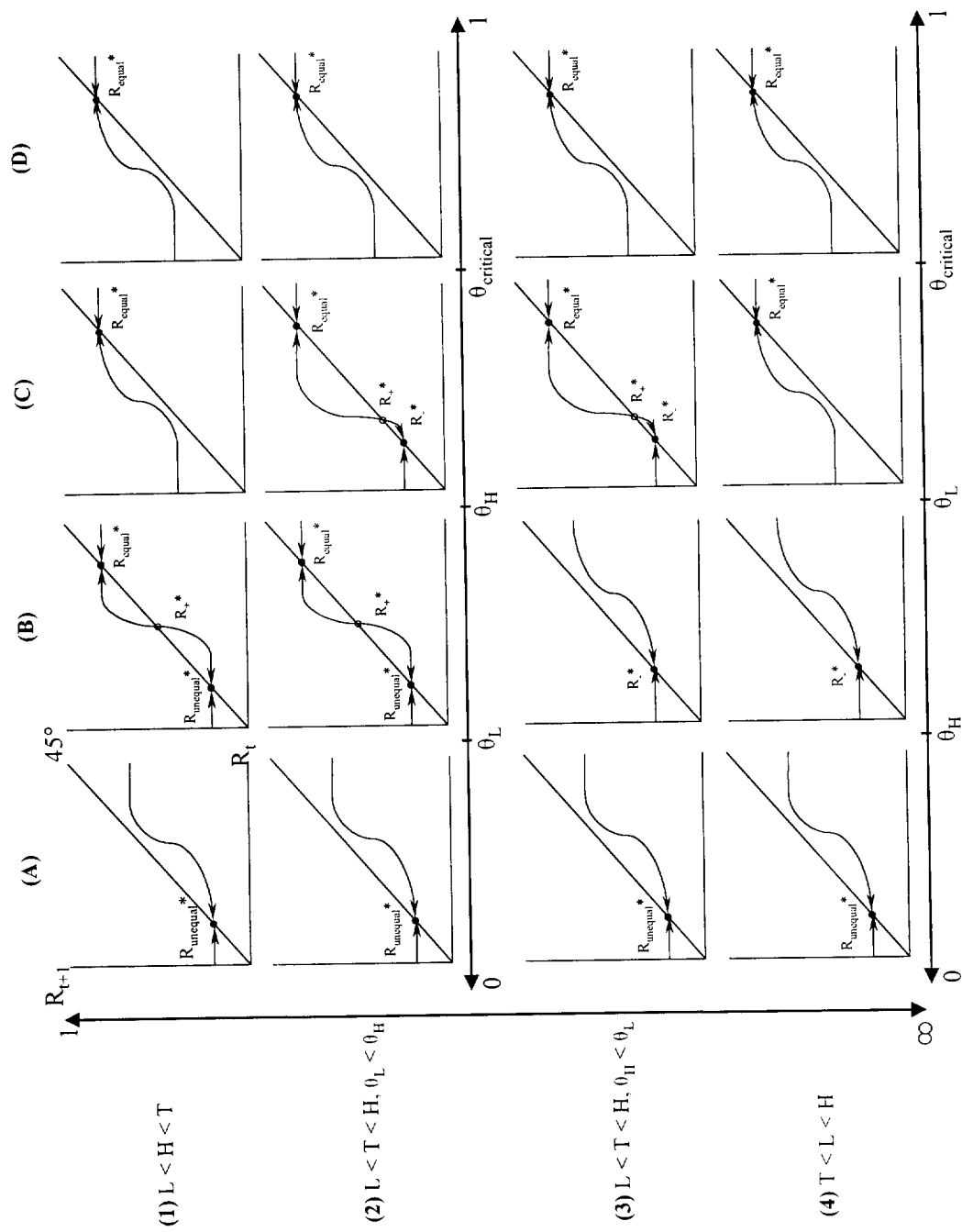


Figure 3 -- Dynamics of $R_{t+1}(R_t)$ as θ , L , and H vary

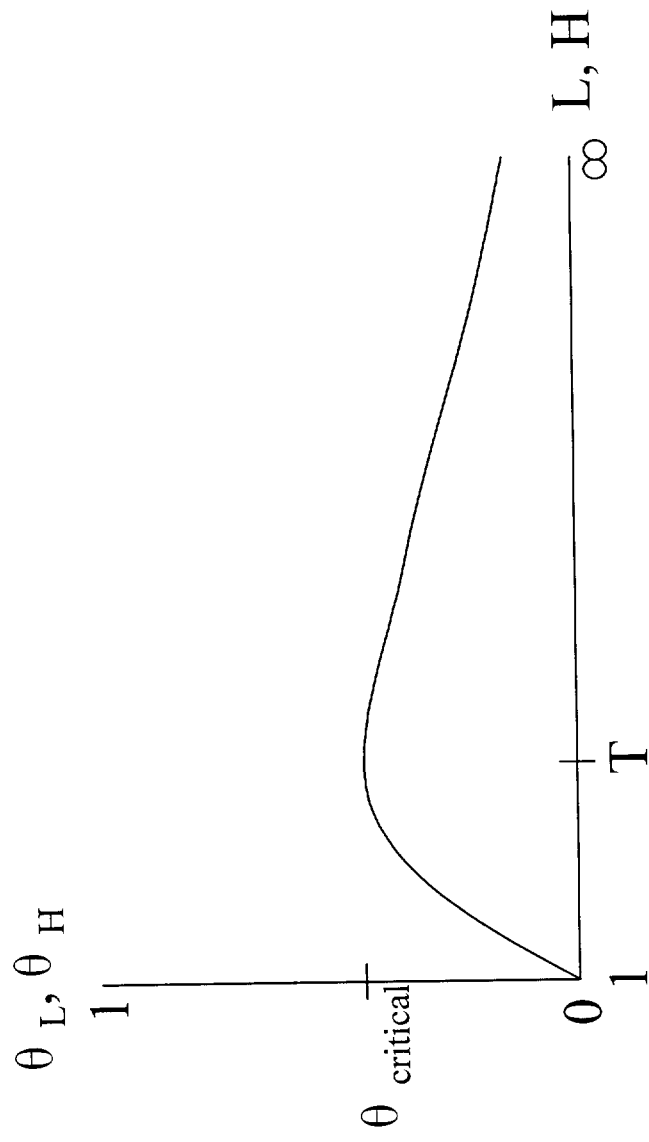


Figure 4: $\theta_L(L)$ and $\theta_H(H)$

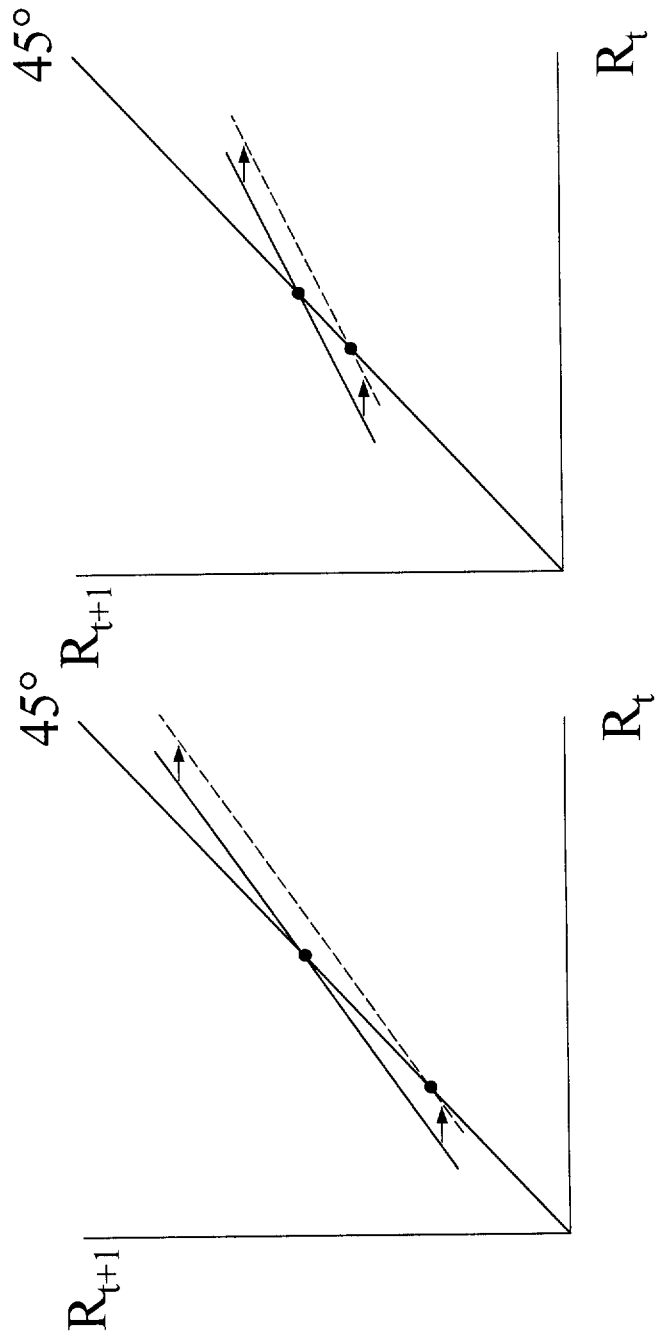


Figure 5: Multiplier effect in a local neighborhood of a steady state
with and without endogenous fertility

Table 1 -- Available Data

	Number of Observations	Mean	Standard Deviation
Fertility differentials vs. Gini coefficients			
Countries	88		
with fertility differentials			
Country-Years	96		
with fertility differentials			
Fertility differentials	96	0.254	0.157
Gini coefficients	88	0.423	0.094
GDP	96	2755	2747
Fertility differentials vs. Mincer coefficients			
Countries	52		
with Mincer coefficients			
Mincer coefficients	52	0.096	0.047
Gini coefficients	52	0.407	0.099
Fertility differentials	30	0.259	0.152
GDP	52	5510	4538

Table 2 -- Fertility Differentials Regressed on Gini Coefficient, 1974-1995

A. Country random effects						
Dependent variable: Weighted OLS coefficient of TFR on years of education						
Independent Variable	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Gini coefficient	0.639*** (0.179)	0.606*** (0.182)	0.535*** (0.173)	0.316** (0.158)	0.190 (0.156)	0.204 (0.154)
Year	-0.005** (0.002)	-0.006** (0.002)	-0.006*** (0.002)	-0.002 (0.002)	-0.004* (0.002)	-0.004** (0.002)
ln(GDP)		-0.023 (0.022)	1.006*** (0.308)		-0.056*** (0.019)	0.540* (0.276)
ln(GDP)^2			-0.066*** (0.020)			-0.038** (0.018)
Latin America dummy				0.183*** (0.032)	0.217*** (0.032)	0.190*** (0.034)
Constant	9.645** (4.138)	11.388** (4.520)	7.751* (4.419)	3.592 (3.987)	8.493** (4.289)	6.950* (4.197)
N	88	88	88	88	88	88

B. Variance-weighted least squares regressions						
Dependent variable: Weighted OLS coefficient of TFR on years of education						
Independent Variable	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Gini coefficient	0.739*** (0.030)	0.639*** (0.030)	0.795*** (0.033)	0.424*** (0.032)	0.372*** (0.032)	0.451*** (0.036)
Year	0.004*** (0.001)	-0.0004 (0.001)	-0.003*** (0.001)	0.006*** (0.001)	0.002*** (0.001)	0.001* (0.001)
ln(GDP)		-0.068*** (0.004)	0.959*** (0.075)		-0.053*** (0.004)	0.336*** (0.080)
ln(GDP)^2			-0.062*** (0.005)			-0.024*** (0.005)
Latin America dummy				0.218*** (0.008)	0.201*** (0.008)	0.186*** (0.009)
Constant	-8.210*** (1.009)	1.202 (1.136)	2.233** (1.139)	-12.022*** (1.019)	-4.421*** (1.148)	-3.602*** (1.170)
N	88	88	88	88	88	88

Notes:

GDP is real GDP per capita in constant dollars (Chain index) expressed in international prices, base 1985. Standard errors are in parentheses.

* Significant at 10% level

** Significant at 5% level

*** Significant at 1% level

Table 3 -- Fertility Differential Regressed on Returns to Education

Variance-weighted least squares regressions						
Dependent variable: Weighted OLS coefficient of TFR on years of education						
Independent Variable	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Mincer coefficient	0.597*** (0.086)	0.362*** (0.087)	0.353*** (0.100)	-0.385*** (0.096)	-0.361*** (0.096)	-0.041 (0.108)
Year	0.004*** (0.001)	0.003*** (0.001)	0.003*** (0.001)	-0.001** (0.001)	-0.001 (0.001)	0.0001 (0.001)
ln(GDP)		-0.106*** (0.006)	-0.140 (0.190)		-0.066*** (0.007)	-0.069*** (0.007)
ln(GDP) ²			0.002 (0.011)			
Latin America dummy				0.316*** (0.014)	0.269*** (0.015)	0.425*** (0.028)
Mincer*Latin America						-1.522*** (0.236)
Constant	-7.032*** (1.040)	-5.580*** (1.044)	-5.407*** (1.426)	2.588** (1.124)	2.046* (1.125)	0.553 (1.149)
N	30	30	30	30	30	30

Notes:

Mincer Coefficients are coefficients from semi-log OLS regression of ln(earnings) on years of schooling and potential years of labor market experience and its square as independent variables.

Standard errors are in parentheses.

* Significant at 10% level

** Significant at 5% level

*** Significant at 1% level

Table 4 -- US Fertility Differentials Regressed on Gini Coefficient, 1925-1989

Variance-weighted least squares regressions			
Dependent variable: OLS coefficient of TFR on years of education			
Independent Variable	(i)	(ii)	(iii)
Gini coefficient	0.390 (0.244)	0.517* (0.272)	-0.035 (0.553)
Year	-0.001*** (0.0003)	-0.003 (0.002)	-0.005* (0.002)
ln(GDP)		0.087 (0.083)	-1.984 (1.808)
ln(GDP)^2			0.117 (0.102)
Constant	1.693** (0.721)	4.437* (2.695)	17.487 (11.694)
N	13	13	13

Notes:

GDP after 1970 is real GDP per capita in constant dollars (Chain index) expressed in international prices, base 1985. GDP between 1959 and 1970 is spliced real GDP per capita using quinquennially changing benchmarks. GDP prior to 1959 is spliced real GDP per capita estimated by the Department of Commerce. Standard errors are in parentheses.

* Significant at 10% level

** Significant at 5% level

*** Significant at 1% level

Table 5 -- Fertility Differentials Regressed on Gini Coefficient, 1974-1995

Dependent variable: Weighted OLS coefficient of TFR on years of education			
Independent Variable	Middle (i)	Low (ii)	High (iii)
Gini coefficient	0.919*** (0.217)	-1.757*** (0.515)	0.086 (0.078)
Year	-0.007*** (0.003)	-0.003* (0.002)	0.006 (0.004)
Constant	14.455*** (4.995)	7.133** (3.510)	-11.768 (9.538)
N	64	18	6

Notes:

(i) Middle-Income countries (with GDP measure between 950 and 9500 US\$)

(ii) Low-Income countries (with GDP measure < 950 US\$)

(iii) High-Income countries (with GDP measure > 9500 US\$)

Columns (i) and (ii) use country random effects. Column (iii) uses variance-weighted least squares.

Countries in high-income sample appear only once, so country random effects could not be used.

GDP is real GDP per capita in constant dollars (Chain index) expressed in international prices, base 1985. Standard errors are in parentheses.

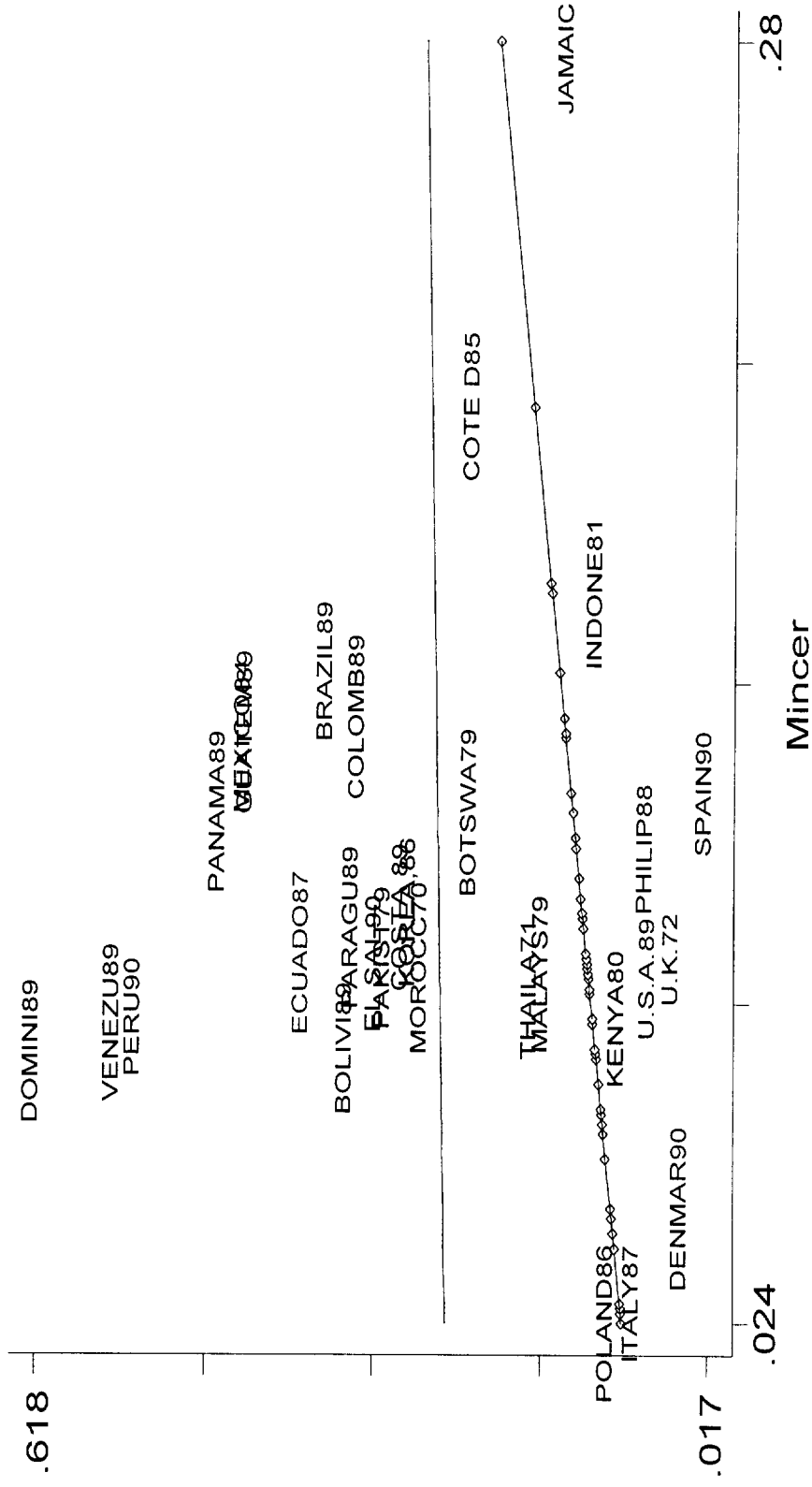
* Significant at 10% level

** Significant at 5% level

*** Significant at 1% level

Fertility Differential vs. Return to Education
Chart 2

Weighted OLS Coefficient of TFR on Educa



- line - line of best fit
- ◇ line - predicted values from variance-weighted least squares regression