Growth, Trade, and Inequality

Grossman and Helpman

Discussion by Jonathan Vogel for the 2014 Summer Institute

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Intro

- Combines trade and inequality/factor allocation...
  - Costinot and Vogel (2010)
- ...with trade and growth
  - Grossman and Helpman (1991)

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- Towards what purpose?
  - Assignment: supply & demand $\Rightarrow$ factor allocation & prices
    - but typically treat supply and demand as given...

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- Towards what purpose?
  - Assignment: supply & demand $\Rightarrow$ factor allocation & prices
    - but typically treat supply and demand as given...
  - GH: How do changes in...
    - ... arbitrary matrix of world trade costs
    - ... arbitrary matrix of world tariffs
    - ... arbitrary matrix of world spillover parameters
    - ... arbitrary vector of subsidy parameters...
  - shape factor supply to production and, through assignment, factor allocation and prices
- Takes a step in a very important direction

Intro

- Understand their (static) “assignment” model...
  - ... setup: role of assumptions
  - ... results: the key mechanism
    - use Costinot and Vogel (2010) and (*ARE* Forthcoming)
    - simplifies GH: exogenous labor supply to production

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- Understand their (static) "assignment" model...
  - ... setup: role of assumptions
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    - use Costinot and Vogel (2010) and (*ARE* Forthcoming)
    - simplifies GH: exogenous labor supply to production
- The mechanism emphasized here...
  - ... can it be related to data?
  - ... is closely related to an apparently different debate
  - ... and other mechanisms that operate through growth

Setup

- A set of goods with skill-intensity $\varphi \in \Phi \equiv \left[\underline{\varphi}, \bar{\varphi}\right]$
- A set of workers with ability $a \in A \equiv [\underline{a}, \overline{a}]$
- $L(a) > 0$ is the inelastic supply of workers with ability a
- Good and labor markets are perfectly competitive

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- Good and labor markets are perfectly competitive
- Workers are perfect substitutes in the production of each good

$$Y(\varphi) = \int_A \psi(a,\varphi) L(a,\varphi)\, da$$

- $\psi(a,\varphi) > 0$ is strictly log-supermodular (*strictly log-spm*)
- Output of the final good is given by a CES aggregator

$$Y = \left\{ \int_\Phi B(\varphi)^{\frac{1}{\sigma}} Y(\varphi)^{\frac{\sigma-1}{\sigma}} d\varphi \right\}^{\frac{\sigma}{\sigma-1}}$$

Production function implications

- **Linear production function** + cost minimization ⇒

$$c(\varphi) = \min_{a \in A} \{w(a) / \psi(a, \varphi)\}$$

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- ψ **strictly log-spm** ⇒ $m(\varphi)$ increasing in φ
- **Continuum of** φ ⇒ $m(\varphi)$ is a function
  - I'll refer to $M(a)$, which is $M(a) = m^{-1}(a)$

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- ψ strictly log-spm:
  - if M increases in some interval,
  - so does $w(a') / w(a)$ for all $a' > a$ in this interval

Market clearing implications

- **Market clearing**

$$\int_{\underline{\varphi}}^{M(a)} \frac{D(\varphi)}{\psi(M^{-1}(\varphi), \varphi)} d\varphi = \int_{\underline{a}}^{a} L(a)\, da$$

- Differentiating wrt a

$$\frac{dM(a)}{da} = \frac{L(a)}{D(M(a))/\psi(a, M(a))}$$

- Matching function equates factor supply with demand

Preferences + market structure implications

- Finally, CES utility $\Rightarrow$

$$D(M(a)) = Z \times B(M(a)) \times p(M(a))^{-\sigma}$$

- + perfect competition (PC) $\Rightarrow$

$$\begin{aligned} D(M(a)) &= Z \times B(M(a)) \times c(M(a))^{-\sigma} \\ &= Z \times B(M(a)) \times \left(\frac{w(a)}{\psi(a, M(a))} \right)^{-\sigma} \end{aligned}$$

System characterizing equilibrium

- Combining system of ODE with $D(M(a))$ solution

$$\frac{d \log w(a)}{da} = \frac{\partial \log \psi(a, M(a))}{\partial a}$$

$$\frac{dM(a)}{da} = \frac{1}{Z} \frac{L(a)\, \psi(a, M(a))}{B(M(a))} \left(\frac{\psi(a, M(a))}{w(a)} \right)^{\sigma}$$

characterizes eqm, together with boundary conditions

$$M(\underline{a}) = \underline{\varphi} \text{ and } M(\bar{a}) = \bar{\varphi}$$

Comparative statics (L)

$$\frac{d\log w(a;c)}{da} = \frac{\partial \log \psi(a, M(a;c))}{\partial a}$$

$$\frac{dM(a;c)}{da} = \frac{1}{Z(c)} \frac{L(a;c)\,\psi(a, M(a;c))}{B(M(a;c))} \left(\frac{\psi(a, M(a;c))}{w(a;c)}\right)^{\sigma}$$

- Countries indexed by c, differ only in $L(a;c)$
- $L(a';c')\,L(a;c) \geq L(a;c')\,L(a';c)$ for all $a' \geq a$ and $c' \geq c$

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- $L(a';c')\,L(a;c) \geq L(a;c')\,L(a';c)$ for all $a' \geq a$ and $c' \geq c$
    - strong set order dominance

$$\underline{a}(c) \leq \underline{a}(c')$$
$$\bar{a}(c) \leq \bar{a}(c')$$

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$$\underline{a}(c) \leq \underline{a}(c')$$
$$\bar{a}(c) \leq \bar{a}(c')$$

  - And, if $\underline{a}(c') \leq a \leq a' \leq \bar{a}(c)$

$$\frac{L(a';c')}{L(a;c')} \geq \frac{L(a';c)}{L(a;c)}$$

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- Proof: if $\exists M(a;c') > M(a;c)$, $\exists \underline{a}(c') \leq a_1 < a < a_2 \leq \bar{a}(c)$
    - $\varphi_1 = M(a_1;c) = M(a_1;c')$ and $\varphi_2 = M(a_2;c) = M(a_2;c')$
    - $\frac{dM(a_1;c')}{da} \geq \frac{dM(a_1;c)}{da}$ and $\frac{dM(a_2;c')}{da} \geq \frac{dM(a_2;c)}{da}$
    - $M(a_0;c') > M(a_0;c)$ for all $a_1 < a_0 < a_2$
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- $M(a;c') \leq M(a;c)$ for all a
- $w(a';c')/w(a;c') \leq w(a';c)/w(a;c)$ for all $a' \geq a$

Mechanism shaping inequality in GH

- Domestic & foreign shocks $\Rightarrow$ inequality through $a_R(c)$
  - In GH, inequality in c determined fully by (endogenous) $a_R(c)$
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- Are changes in $a_R(c)$ over time large?
- Is the share above $a_R(c)$ large?
- What is $a_R(c)$ in the data?
  - Share of U.S. industrial employment in R&D?
  - Share of self-employed?
  - Scientists?

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- Autor, Katz, and Kearny (2008): A problem for this argument — much of the $\uparrow$ in inequality occurred in the upper half of the distribution
- Teulings (2003): A fall in minimum wage $\Rightarrow \downarrow \underline{a} \Rightarrow M(a) \uparrow \Rightarrow$ wage inequality rises throughout distribution
- Through the lens of the model, a key (unanswered, but answerable) question is:
  - What % of workers moved to employment due to $\downarrow$ in minimum wage?

Mechanism shaping inequality in GH

- Directed technical change ⇒ number of firms of each φ?
  - Acemoglu (....)
- R&D workers with higher a have a comparative advantage creating high φ firms?
  - nothing I'm aware of

Conclusion

- An ambitious paper determining the effects of changes in
  - ... arbitrary matrix of world trade costs
  - ... arbitrary matrix of world tariffs
  - ... arbitrary matrix of world spillover parameters
  - ... arbitrary vector of subsidy parameters...
- on inequality
- Surprisingly successful theoretical exercise
- Two possible ways forward:
  - Can mechanism be linked to data?
  - Given the success focusing on one mechanism, can this approach be generalized allowing for alternative mechanisms?