

# Risk, Monetary Policy and The Exchange Rate\*

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## Abstract

In this research, we provide new empirical evidence on the importance of time-varying uncertainty for the exchange rate and the excess return in currency markets. Following an increase in monetary policy uncertainty, the dollar exchange rate appreciates in the medium run, while an increase in the volatility of productivity leads to a dollar depreciation. We propose a general-equilibrium theory of exchange rate determination based on the interaction between monetary policy and time-varying uncertainty aimed at understanding these regularities. In the model, the behaviour of the exchange rate following nominal and real volatility shocks is consistent with the empirical evidence. Furthermore we show that risk factors and interest-rate smoothing are important in accounting for the negative coefficient in the UIP regression.

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# 1 Introduction

One of the main features of the recent financial crisis has been the increase in financial and macroeconomic volatility. Currency markets have not been an exception: foreign exchange rate volatility has surged and large swings in the dollar exchange rate have occurred. How do changes in volatility affect exchange rates? Is the source of volatility, nominal or real, relevant for determining exchange rate fluctuations?

These are the questions that we address in this research by providing an empirical and theoretical analysis on the link between uncertainty and the exchange rate. The focus on uncertainty belongs to the tradition in international finance that has emphasized how variations in risk over time are essential for understanding the exchange rate. In fact, the large biases in the foreign-exchange forward premium that have been documented since Bilson (1981) and Fama (1984) constitute a compelling evidence of variations in risk premia as a rational-expectations explanation of the link between exchange rates and interest rates. Evidence of a time-varying risk component of the excess return in foreign-exchange market is further documented by the recent work of Menkhoff, Sarno, Schmeling and Schrimpf (2010), who show that deviations from Uncovered Interest-rate Parity (UIP) can be accounted for in terms of compensation for risk.<sup>1</sup> They identify global foreign exchange volatility as a key factor.

We propose a theory of exchange rate determination based on exogenous risk factors in which the link between risk and the nominal exchange rate is guided by monetary policy through interest-rate rules.<sup>2</sup> The aim is to understand the role of exogenous risk factors in explaining the main regularities that we observe in international finance. To this purpose, we depart from most of the existing models of exchange rate determination, which study the impact of the first moments of exogenous variables on the nominal exchange rate, and examine the exchange rate's response to changes in the volatility of nominal and real shocks.<sup>3</sup> Moreover, the structure upon which we build our analysis between risk factors and exchange rates is a theory of nominal exchange rate determination based on interest rate rules (Benigno and Benigno, 2008).<sup>4</sup>

This research contributes to the literature from an empirical and a theoretical perspective. In our empirical analysis we provide new evidence that justifies our focus on risk factors: the novelty of our contribution is to examine the role of nominal and real stochastic volatilities for the behavior of exchange rates in an otherwise standard open-economy VAR. We find that volatility shocks do matter for the equilibrium level of interest and exchange rates and that the exchange rate tends to appreciate in response to an increase in nominal volatility (both of the discretionary shock to monetary policy and of the inflation target) and to depreciate following

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<sup>1</sup>Engel (2010) provides further evidence on this even by looking at the real version of the UIP and shows that the expected appreciation of high-yield currencies is combined with a relatively stronger currency.

<sup>2</sup>In what follows we refer to risk, uncertainty and stochastic volatility in an interchangeable way.

<sup>3</sup>Hodrick (1989) and Obstfeld and Rogoff (2001) in a flexible- and sticky-price environment, respectively, relate the nominal exchange rate to monetary uncertainty through alternative specifications of money demand.

<sup>4</sup>This implies that the parameters of the policy rules (as opposed to preferences to money demand) become crucial in shaping exchange rate dynamics and in determining to what extent nominal or real disturbances matter for the nominal (and real) exchange rate.

an increase in real volatility (of the productivity shock). Moreover, the stylized facts reported by Eichenbaum and Evans (1995) about the response of interest and exchange rates to a shock to the level of the monetary policy instrument are not affected by the explicit consideration of time-varying volatility elements in the VAR.

In our theoretical model, the key channel through which exchange rates and uncertainty are related is a simple hedging motive. An increase in uncertainty does not necessarily lead to a depreciation of the currency: what matters is whether the currency is relatively safer when there are bad news. In this respect, uncertainty may improve the hedging properties of the currency leading to an increase in its demand and thereby an appreciation.

We develop a two-country open-economy model along the lines of Benigno and Benigno (2008), extended in two dimensions. As in Benigno and Benigno (2008), we assume differentiated home and foreign produced goods, international market completeness, nominal price rigidities and interest rate rules; here, however, we allow for a more general specification of preferences as in Epstein and Zin (1989) and for stochastic volatility in the exogenous processes driving the economy.

In this direction, our contribution to the literature is to provide a general-equilibrium perspective on the ability of currently used models with stochastic volatility to explain international macro-finance facts.<sup>5</sup> From a modelling point of view, the general equilibrium analysis is crucial for examining the transmission mechanism of risk factors and generating a non-trivial interaction between shocks and the variables of interests. From an empirical point of view, the general equilibrium analysis allows us to compare the model's performance with the shocks and factors highlighted in the VAR.

The assumption of time-varying exogenous uncertainty entails non-trivial issues in the solution of the model. To this end, we apply a new method that we have recently developed for general dynamic stochastic models with time-varying uncertainty (Benigno, Benigno and Nisticò, 2010). The main result of our previous work is that a second-order approximation of the model is sufficient to account for a distinct and direct role of time-varying uncertainty on the endogenous variables, provided that the structural shocks are conditionally linear. In contrast, recent works have emphasized the need of relying on a third-order approximation (see Fernandez-Villaverde and Rubio-Ramirez, 2010).<sup>6</sup> Our method has several advantages: it simplifies the computational burden, it reduces the degree of freedom that a third-order approximation would generate when evaluating the model performance through a calibration exercise and, finally, it allows to evaluate time-varying risk premia, which in our case are second-order terms, through just a first-order approximation of the equilibrium conditions.

For a special case of our general model we are able to obtain analytical results. When purchasing power parity holds, prices are flexible and monetary policy is specified as a Taylor

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<sup>5</sup>As we will discuss below, most of the models that have been developed recently specify exogenous process for consumption and/or output.

<sup>6</sup>The key difference is that our first-order approximation still displays heteroskedasticity and is the best approximation in the class of conditionally-linear processes.

rule that reacts to either PPI or CPI inflation, we obtain that an increase in the domestic volatilities of the nominal shocks appreciate the nominal exchange rate consistently with our empirical findings. Theoretically the excess return on home currency bonds decreases with an increase in nominal risk factors.

While this simple model is partly successful in capturing the link between nominal risk factors and the exchange rate, it fails in replicating other key international finance regularities. In fact the implied slope coefficient from a UIP regression would still be positive. We then consider the case in which policy authorities smooth interest rates over time and find that, conditional on shocks to the monetary policy instrument, it is possible to obtain a negative coefficient in the UIP regression which becomes more negative as the smoothing coefficient increases.

From a theoretical point of view we then explore the role of Epstein-Zin preferences. First, in an open economy, cross-country surprises in utility influence the international distribution of wealth so that equilibrium quantities are also affected by the preference specification, unlike in the closed-economy case. Second, if we focus on the case in which the subjective discount factor is very close to the unitary value, then the surprises to utility depend, up to a first order, only on the stochastic trend in world productivity. The implication is that, in this case, nominal stochastic discount factors are highly correlated across countries, an aspect that is consistent with a global explanation for risk premia.

We then evaluate quantitatively the properties of our model by calibrating it following the recent empirical literature (see, e.g. Lubik and Schorfede, 2005). We focus on a small set of facts that are related to exchange rates. The response of exchange rates and excess returns to volatility shocks is consistent with our empirical findings. Moreover, we show that the specification of monetary policy and the presence of stochastic volatility terms is crucial for obtaining a negative coefficient in the UIP regression (as discussed in Backus, Gavazzoni, Telmer and Zin, 2010).

## Related Literature

This paper is related to different strands of literature. From an empirical point of view, we build on the early analysis of Clarida and Gali (1994) and Eichenbaum and Evans (1995), which have examined the effects of monetary shocks on the exchange rate. Our contribution is to assess the role of real and nominal uncertainty on the exchange rate whereas their focus is on the innovation in real and nominal shocks.

From a theoretical perspective there are two key elements in our analysis: stochastic volatility and monetary policy. The emphasis on exogenous risk factors is not novel in exchange rate economics: early contributions by Frankel and Meese (1987) in a partial equilibrium setting, and Hodrick (1989) in general equilibrium, have pointed out the role of uncertainty in explaining exchange rate determination. More recently Obstfeld and Rogoff (2002) have studied the role of risk factors in a general equilibrium model when nominal prices are sticky, focusing on money supply as the monetary-policy instrument. Our paper follows this tradition in international finance and it is also connected to a more recent macroeconomic literature that has

examined the role and the effects that risk or uncertainty have on macroeconomic variables (see for example Bloom, 2009, Bloom, Floetotto and Jaimovich, 2009 and Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe, 2010).

The importance of monetary policy using interest-rate rules in exchange rate determination has been analyzed in Benigno and Benigno (2008) while its role for the understanding of the uncovered interest rate parity puzzle has been first highlighted by McCallum (1994) and more recently by Backus, Gavazzoni, Telmer and Zin (2010). The latter authors have recasted McCallum’s insight in a microfounded setting endogenizing the currency risk premium that is exogenous in McCallum’s model.

Our work is also related to a fast-growing literature in international macro-finance that has developed models of the risk premium based on specifications of the stochastic discount factors derived from alternative preferences. Bansal and Shaliastovich (2009) relies on Epstein-Zin preferences combined with long-run risk, Backus et al. (2010) emphasizes the role of monetary policy for addressing the uncovered interest rate parity puzzle in nominal terms, Gavazzoni (2009) relies on Epstein-Zin preferences combined with stochastic volatility, Moore and Roche (2010) and Verdelhan (2010) propose models based on external habit with preferences *à la* Campbell and Cochrane (1999). While we share some of the features of these studies, our analysis follows a general equilibrium approach by combining macro and financial market equilibrium and builds upon a theory of nominal exchange rate determination based on interest rate rules. The latter aspect is important insofar as we want to address, from a model perspective, the UIP puzzle in nominal rather than in real terms as most of these models do.

## 2 Empirical evidence

In this section, we provide new empirical evidence on the importance of time-varying uncertainty in open economies through a simple VAR analysis, along the lines of Eichenbaum and Evans (1995), which we take as our empirical benchmark. We aim at providing a quantitative assessment on the effects that innovations to the *volatility* of underlying disturbances may have on the *level* of macro variables of interest. In particular we focus on the conditional time-varying volatilities of three specific shocks, that are going to play a relevant role in the theoretical model of the next sections: the conditional volatility of the monetary-policy shock, of the inflation-target shock and of the productivity shock.<sup>7</sup> Our focus will be mainly to study how these shocks affect the nominal (and real) exchange rate and the foreign currency risk premium which captures the deviations from UIP. However, we will also look at the responses of output, inflation and the yield curve. Moreover, we will evaluate whether the results of Eichenbaum and Evans (1995), and investigated by a large body of subsequent literature, are robust to the inclusion of time-varying volatility into the picture.

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<sup>7</sup>In our model, the monetary policy shock represents a shock to the systematic component of the interest rate rule. The inflation target is also part of the interest rate rule and represents the target with respect to which any deviation of actual inflation triggers the policy response.

We use monthly data for the G7 countries on the sample period ranging from March 1971 through September 2010, and estimate a VAR with six lags for each pair of countries that includes the US.<sup>8</sup> We consider a benchmark specification with seven macroeconomic variables, in the spirit of Eichenbaum and Evans (1995). To this set of macro “level” variables, we then add three time series describing the time-varying volatilities of the monetary-policy shock ( $u_{\xi,t}$ ), the inflation-target shock ( $u_{\pi,t}$ ) and the productivity shock ( $u_{a,t}$ ). The “level” variables that we consider are the US nominal Federal Funds Rate ( $i$ ) indicating the stance of monetary policy, the US and foreign Industrial Production Indexes ( $y$ ,  $y^*$ ) measuring the domestic and foreign real activity, the US CPI Index ( $p$ ) capturing the domestic price level, the foreign short-term nominal interest rate measured by the 3-month Treasury Bill rate ( $i^*$ ), the slope of the US term structure computed as the difference between the 10-year Treasury Constant Maturities rate and the 3-month Treasury Bill rate ( $i_{sl} \equiv i_{10y} - i_{3m}$ ) and the real exchange rate, defined as  $q \equiv s + p^* - p$  where  $s$  denotes the nominal exchange rate, expressed in terms of units of USD needed to buy one unit of foreign currency.<sup>9</sup> As such, an increase in  $q$  (or  $s$ ) denotes a US Dollar real (nominal) depreciation. All variables are in logs, except for the interest rates which are monthly percentage points.

## 2.1 Measuring Time-varying Volatility

We now explain how we build the three conditional volatilities of interest. For the conditional volatility of the monetary-policy shock we use daily data from the Federal Funds futures markets, following Kuttner (2001), among others.

In particular, denoting with  $f_{t,d}^0$  the spot-month futures rate on day  $d$  for a contract with delivery in month  $t$  (with day  $d$  belonging to month  $t$ ) we can interpret  $f_{t,d}^0$  as the conditional time- $d$  expectation of the average funds rate in month  $t$ , plus a stochastic risk premium  $\mu$ :

$$f_{t,d}^0 = E_d \frac{1}{m_t} \sum_{j \in t} i_{1,j} + \mu_{t,d}^0,$$

where  $m_t$  is the number of days in month  $t$  and  $i_1$  is the daily interest rate. To extract information about revisions in time- $d$  expectations about future monetary policy actions from data on  $f$ , Kuttner (2001) suggests to use the daily change in the futures rate, scaled up to account for the number of days in month  $t$  that are affected by the surprise:  $\frac{m_t}{m_t-d}(f_{t,d}^0 - f_{t,d-1}^0)$ . This measure seems particularly appealing because it reduces the distortions associated with the time variation in the risk premium  $\mu$ .

As to our case, we use data on 1-month futures rates rather than spot-month rates,  $f_{t,d}^1$ ,

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<sup>8</sup>Many specifications of our empirical analysis, like the lag-order of the main VAR, the use of a price-level index instead of inflation, and the choice to display one-standard deviation bands in the impulse-response analysis, are borrowed from our empirical benchmark, Eichenbaum and Evans (1995), to which we seek to relate our results.

<sup>9</sup>We depart from Eichenbaum and Evans (1995), besides by including three volatility measures, by considering the slope of the US yield curve  $i_{sl,t}$ , on the one hand, while disregarding the measure of non-borrowed reserves, on the other hand.

where day  $d$  belongs to month  $t - 1$  rather than  $t$ . As a consequence, any revision in policy expectations reflected in a daily change of the futures rate is related to the full month  $t$ , rather than a fraction of it. Therefore, in our case we can measure day- $d$  revisions in expectations about next-month monetary policy actions using the simple daily change in the futures rate:  $(f_{t,d}^1 - f_{t,d-1}^1)$ . In what follows we will denote with  $u_{\xi,t}^2$  the variance of the monetary policy surprise in month  $t + 1$  conditionally on information available in month  $t$  and we use, as an approximate measure of such conditional variance, the empirical second moment, within month  $t$ , of daily revisions in expectations of time- $t + 1$  monetary policy actions:

$$u_{\xi,t} \approx \sqrt{\frac{1}{m_t} \sum_{d=2}^{m_t} (f_{t,d}^1 - f_{t,d-1}^1)^2}.$$

Since data for the Fed funds futures rates are only available starting October 1988, we complete the time series with realized volatilities, within the month, computed using daily data on the effective federal funds rate – net of settlement Wednesdays – standardized to the mean and variance of the measure coming from the futures market, for the period where the two measures overlap (correlation over that period is about .6)

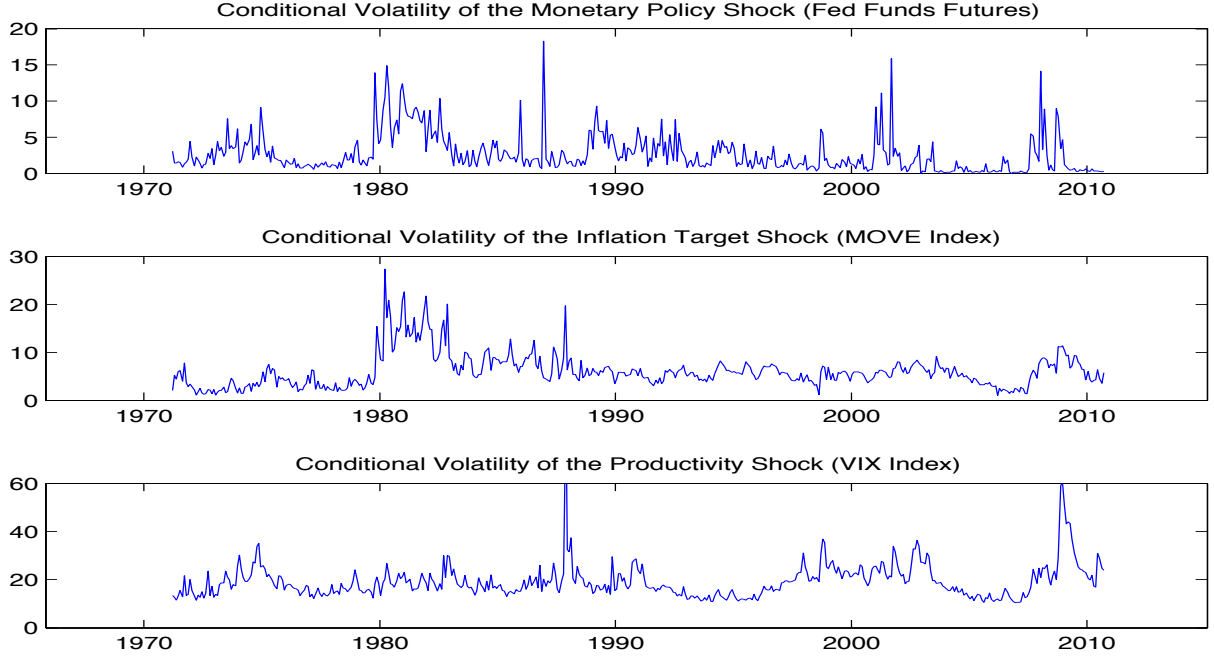
For the inflation-target shock, we measure the conditional volatility with the Merrill Lynch Option Volatility Estimate (MOVE). Movements in the inflation target can produce parallel shifts in the yield curve.<sup>10</sup> Indeed the MOVE Index can capture the volatility of this level factor since it is a yield curve weighted index of the normalized implied volatility on 1-month Treasury options, which are weighted on the 5, 10, and 30 year contracts. Since this index starts only in 1989, we complete the time series with the realized volatility, within the month, computed using daily data on US 10-year Treasury bonds; since the MOVE is an index, moreover, we standardize it to the mean and variance of the realized volatility, for the period where the two measures overlap (correlation over that period is about .8).

Finally, we build an approximate measure of the volatility of the productivity shock using the stock market option-based implied volatility, the VIX index (monthly averages of daily data). However, since data for the VIX are only available starting January 1990, we follow the approach of Bloom (2009) and complete the time series with within-month realized volatilities computed using daily returns on the S&P500, standardized to the mean and variance of the VIX, for the period where the two measures overlap (correlation over that period is about .9).

As a last step, since all above measures are based on implied and realized volatilities, we construct the *conditional* volatilities considering the fitted values of an AR(1) regression for each indicator, similarly to Bekaert and Engstrom (2009). Figure 1 displays the dynamic properties of the obtained indicators.

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<sup>10</sup>This is true in the theoretical model presented later in the paper but we leave the details for future work.



**Figure 1:** Time-varying conditional volatilities, standard deviations in percentage points. Note: the  $y$ -axis of the bottom panel has been truncated at 60 for the sake of readability; the value of the index around Black Monday is actually about 87.

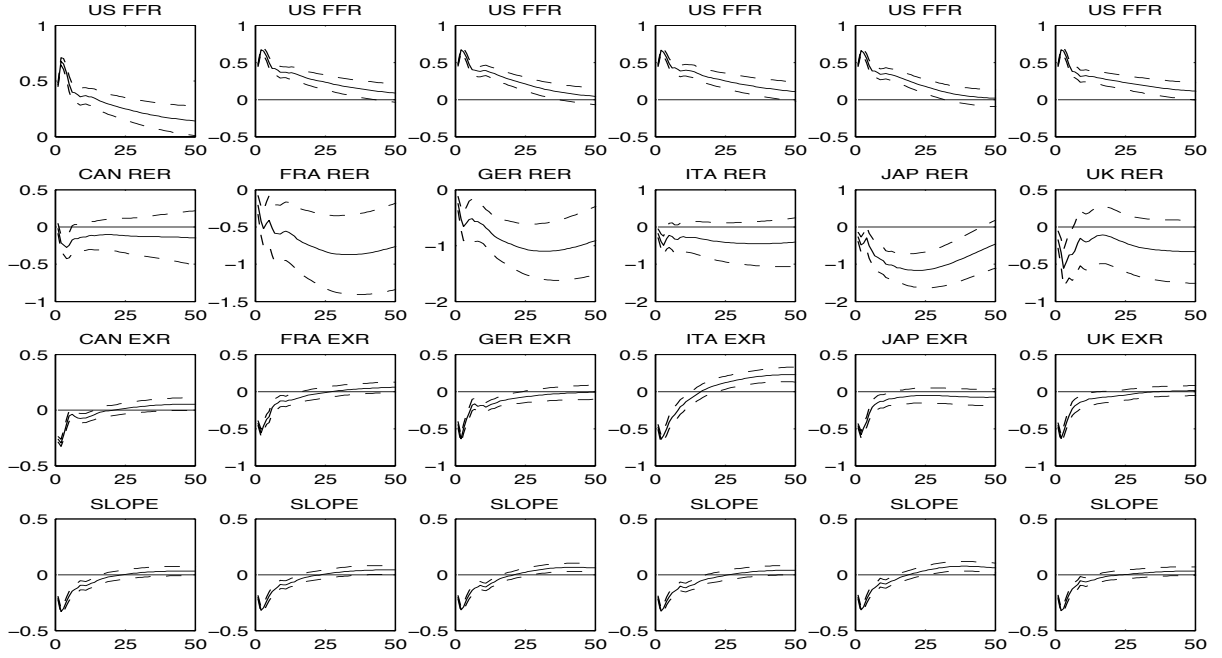
## 2.2 VAR Analysis

For each pair of the G7 countries that includes the US, we then estimate the following VAR( $p$ ) model

$$\mathbf{y}_t = \mathbf{A}(L)\mathbf{y}_{t-1} + \mathbf{e}_t, \quad (1)$$

where the data vector is defined as  $\mathbf{y}_t \equiv [u_{\xi,t}, u_{a,t}, u_{\pi,t}, y_t, p_t, i_t, i_{sl,t}, i_t^*, q_t, y_t^*]'$ , and the lag-order is six. This ordering allows for a contemporaneous response of the interest rate to domestic output and the price level, consistently with a Taylor-type monetary policy rule and with our empirical benchmark (see Eichenbaum and Evans, 1995). As to the order in which the volatility measures enter the VAR, our choice is driven by how volatility is modelled in the theoretical framework of the next sections. Indeed, we build a model in which volatility shocks are allowed to have contemporaneous effects on the endogenous variables; however, in order to apply the approximation methods developed by Benigno, Benigno and Nisticò (2010), we restrict our attention to conditionally-linear stochastic processes for the underlying structural disturbances of our theoretical model, implying contemporaneous orthogonality between “level” shocks and volatility measures. In order to be consistent with these features of our theoretical approach, therefore, in the VAR we place the volatility measures before all the other variables. As to the volatility measures, since the MOVE index might also be affected by the volatility of monetary policy or productivity shocks, we place it last among the three volatility indexes. On the other hand, since the monetary-policy volatility measure is built directly from data on the Federal





**Figure 2:** Dynamic responses to an orthogonalized innovation to the Federal Funds Rate. Each column reports, for each country pair, the responses of the US Federal Funds rate ( $i$ ), the Real Exchange Rate ( $q$ ), the foreign currency risk premium ( $exr$ ), the slope of the US term structure ( $i_{sl}$ ).  $x$ -axes: months,  $y$ -axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

Funds Market, we see it as very tightly related to monetary policy: we therefore assume that it is not affected contemporaneously by any other volatility measure and thus place it first.

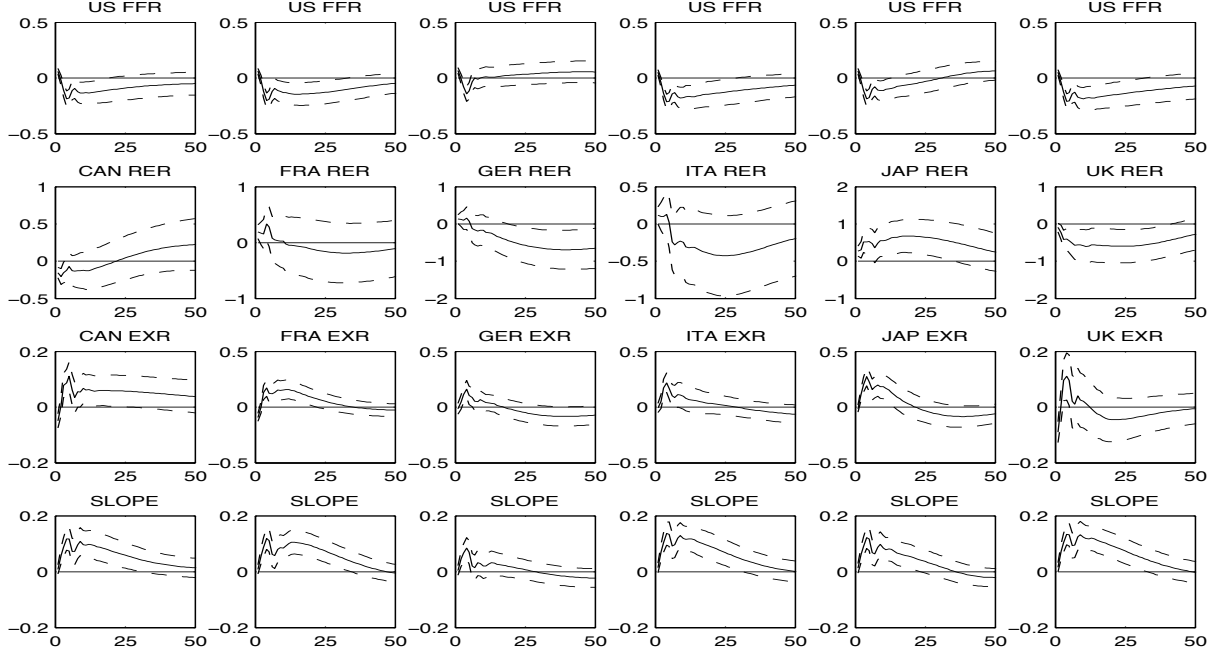
Figures 2 through 5 display the dynamic response of selected variables to, respectively, a “classic” monetary-policy shock (the orthogonalized innovation to the level of the Federal funds rate), an innovation to the volatility of the monetary-policy shock, an innovation to the volatility of the shock to the inflation target and an innovation to the volatility of the productivity shock. Each panel reports the point estimate of the impulse response function – the solid line – and the associated one-standard-deviation confidence intervals – the dashed lines. In each figure, the first row displays the dynamic response of the US Federal Funds Rate, the second row the response of the real exchange rate, the third the response of the excess return on foreign currency, and the last one shows the response of the slope of the yield curve.<sup>11</sup> In particular, the excess return on foreign currency is defined as

$$exr_t \equiv i_{1,t}^* - i_{1,t} + E_t \Delta s_{t+1},$$

and measures deviations from the UIP condition.

Figure 2 addresses the robustness of the findings of Eichenbaum and Evans (1995). The responses to a contractionary shock to monetary policy seem virtually unaffected by the explicit

<sup>11</sup>We do not report the responses of the nominal exchange rate, since they are very similar to those of the real exchange rate.

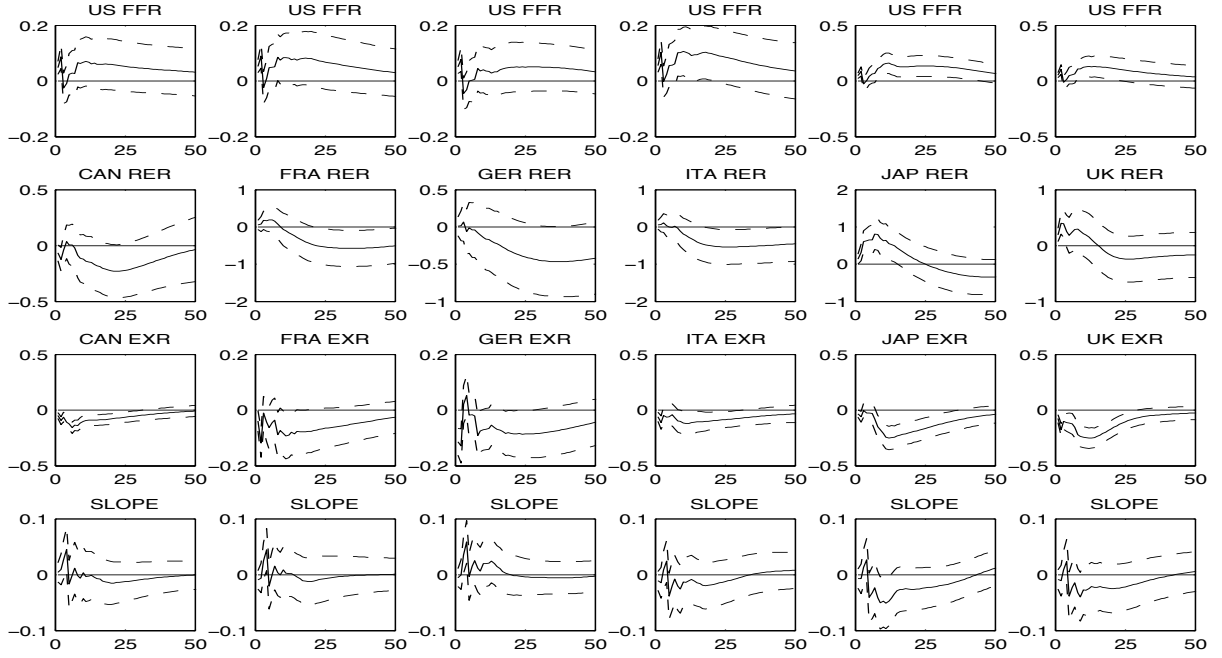


**Figure 3:** Dynamic responses to an orthogonalized innovation to the volatility of the monetary-policy shock. Each column reports, for each country pair, the responses of the US Federal Funds Rate ( $i$ ), the Real Exchange Rate ( $q$ ), the foreign currency risk premium ( $exr$ ), the slope of the US term structure ( $i_{sl}$ ).  $x$ -axes: months,  $y$ -axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

consideration of the interplay between time-varying volatility and the “level” variables. In particular, a positive innovation to the Federal Funds Rate implies a significant appreciation of the USD, on impact. Moreover, the exchange rate keeps appreciating also in the transition, and does not start depreciating but in the medium run. Second, the spread between foreign and domestic short-term interest rates decreases gradually through the implied, less-than-proportional, increase in the foreign one (not shown). Finally, the two results above drive the persistent deviations from UIP shown in the third row, in the form of positive excess returns on US securities. Additionally, Figure 2 also shows the negative response of the slope of the yield curve.

Figures 3, 4 and 5 present our new evidence on the importance of volatility shocks. The first result, common to all three figures, is that shocks to volatility indeed do affect the level of the other macro variables, although with different magnitudes and significance across variables and shocks. Hence volatility does have a *distinct and direct* effect which will be important in characterizing our theoretical model.

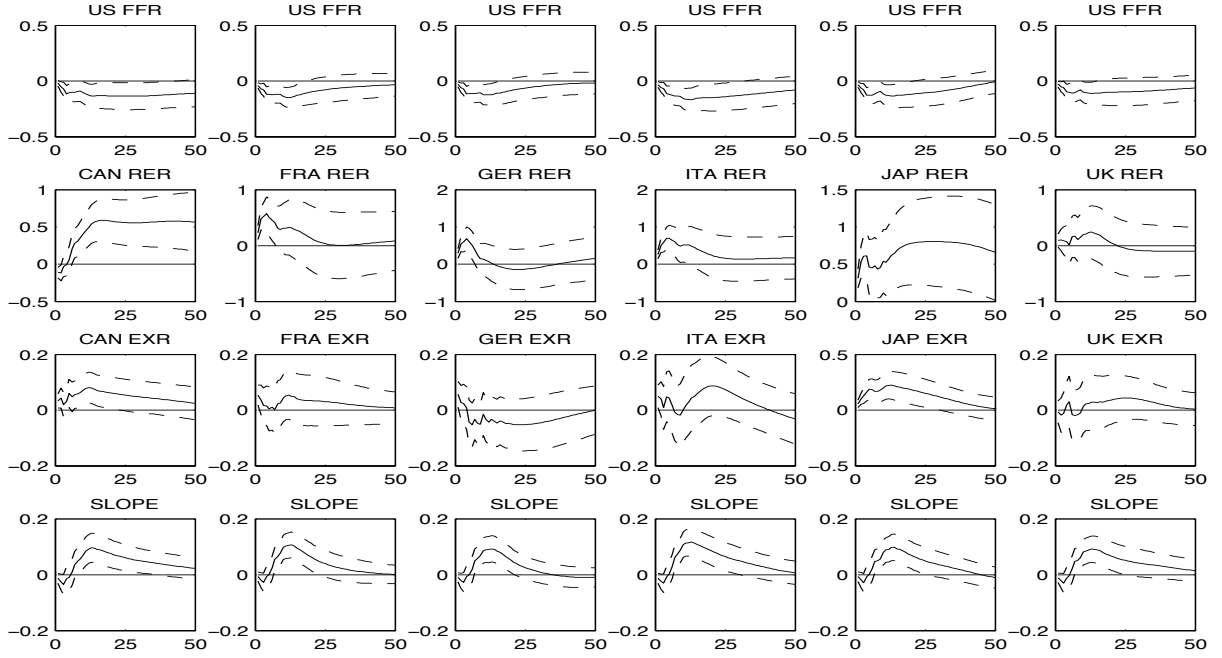
In particular, Figure 3 shows the responses to an orthogonalized innovation to the volatility of the monetary-policy shock. The response of the exchange rate (second row) is ambiguous. The point estimate indicates that an increase in the volatility of the monetary-policy shock strengthens the US dollar. However, this is not particularly significant (except for the case of the UK and, marginally, Germany). Later, we are going to evaluate if results change by exploiting the panel dimension of our dataset. The third row shows that an increase in the



**Figure 4:** Dynamic responses to an orthogonalized innovation to the volatility of the inflation-target shock. Each column reports, for each country pair, the responses of the US Federal Funds Rate ( $i$ ), the Real Exchange Rate ( $q$ ), the foreign currency risk premium ( $exr$ ), the slope of the US term structure ( $i_{sl}$ ).  $x$ -axes: months,  $y$ -axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

volatility of the monetary-policy shock induces significant and persistent deviations from UIP, in the form of positive excess returns on foreign securities. This result is mainly driven by the response of the spread in the short-term interest rate: the domestic rate falls significantly, and proportionately more than the foreign one, implying an increase in the spread by a magnitude of 5-10 basis points (not shown). The estimated response of the slope of the US yield curve, is positive on impact and keeps rising for a few months before reverting back to mean; it remains, however, significantly above the steady-state level for quite some time, regardless of the pair considered (except for the case of Germany, for which the effect dies out within six months).

Figure 4 shows the response to an orthogonalized innovation to the volatility of the inflation-target shock. Here, the implications for the exchange rate are very interesting. Indeed, while the response is weak on impact and not always significant, the point estimates indicate that an increase in the volatility of the inflation-target shock tends to appreciate the exchange rate (with the notable exception of Japan) in the medium run. This is a particularly appealing result considering the specific nature of the shock, which is indeed related to the medium-run target level for the inflation rate. This pattern is also reflected in the dynamic response of the foreign-currency risk premium: while the short-term response is ambiguous, the estimated impulse-response functions indicate that in the medium run a higher volatility of the inflation-target shock produces a lower foreign currency risk premium, consistently with the appreciation of the domestic currency. Finally, the estimated responses of the nominal interest rate and the



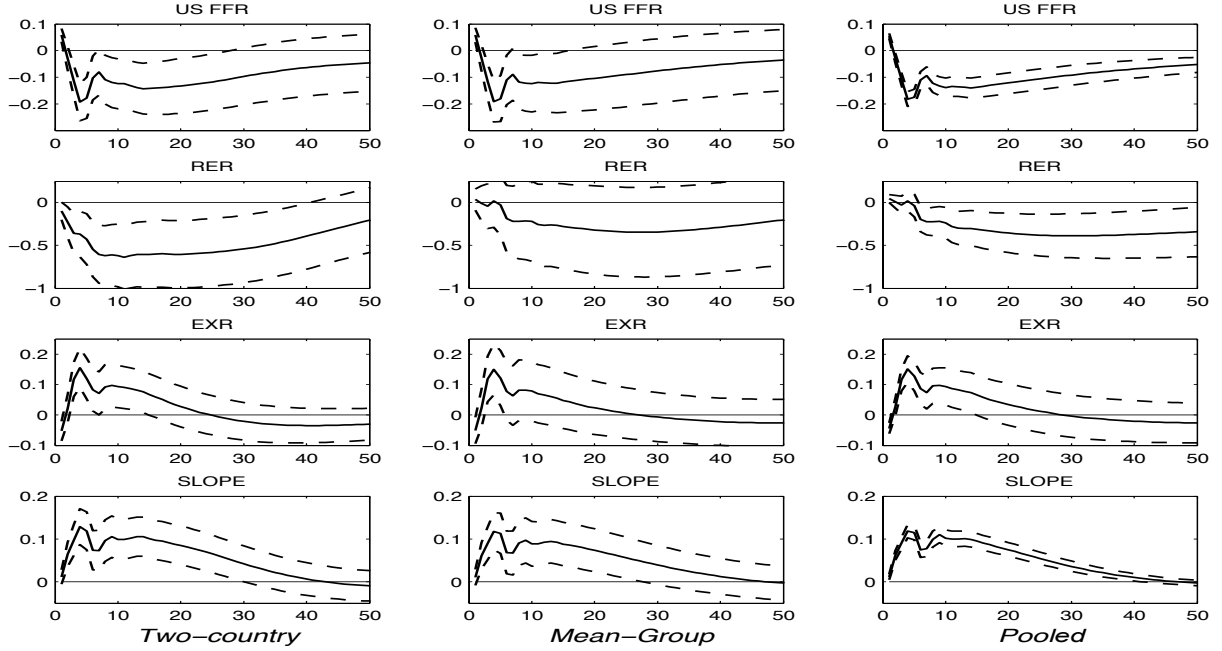
**Figure 5:** Dynamic responses to an orthogonalized innovation to the volatility of productivity shocks. Each column reports, for each country pair, the responses of the US Federal Funds Rate ( $i$ ), the Real Exchange Rate ( $q$ ), the foreign currency risk premium ( $exr$ ), the slope of the US term structure ( $i_{st}$ ).  $x$ -axes: months,  $y$ -axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

slope of the yield curve are not very precise. However, the point estimates suggest a positive response of both the domestic short-term interest rate and the term spread. Again, we will look further into this evidence by exploring the panel dimension of the data.

Figure 5 studies the responses to the conditional volatility of the productivity shock. In particular, the response of the exchange rate is quite clear: although with different timing and magnitude across country pairs, an increase in the volatility of the productivity shock tends to depreciate the exchange rate, mostly on impact. No significant deviation from UIP arises with the notable exception of Japan. Finally, the estimated response of the slope of the yield curve is muted on impact, but it becomes substantially and significantly positive after about six months and stays significantly positive until about two years after the shock, peaking at about 10-15 basis points after about one year. This response is virtually identical across all considered country pairs.

### 2.3 Exploring the Panel Dimension of the Dataset

The empirical evidence of the last section points toward an interesting and non-trivial role of stochastic volatility for macro-financial variables, both domestic (like the short-term interest rate or the term spread) and international (like the exchange rate and deviations from the UIP). Although the point estimates in the pair-wise analysis suggest clear trends in the impulse responses of key variables, such trends are sometimes polluted by sampling uncertainty. In order



**Figure 6:** Dynamic responses to an orthogonalized innovation to the volatility of the monetary-policy shock. Each column reports, for each empirical approach, the responses of the US Federal Funds Rate ( $i$ ), the Real Exchange Rate ( $q$ ), the foreign currency risk premium ( $exr$ ), the slope of the US term structure ( $i_{st}$ ).  $x$ -axes: months,  $y$ -axes: annual percentage points. “Two-country”: single VAR, US-vs-G6 countries (Japan excluded); “Mean-Group”: average statistics from pair-specific VARs (from section 2.2); “Pooled”: single Pooled Panel VAR estimation.

to isolate more effectively the common components across countries, here we exploit the panel dimension of our dataset using three methods.<sup>12</sup>

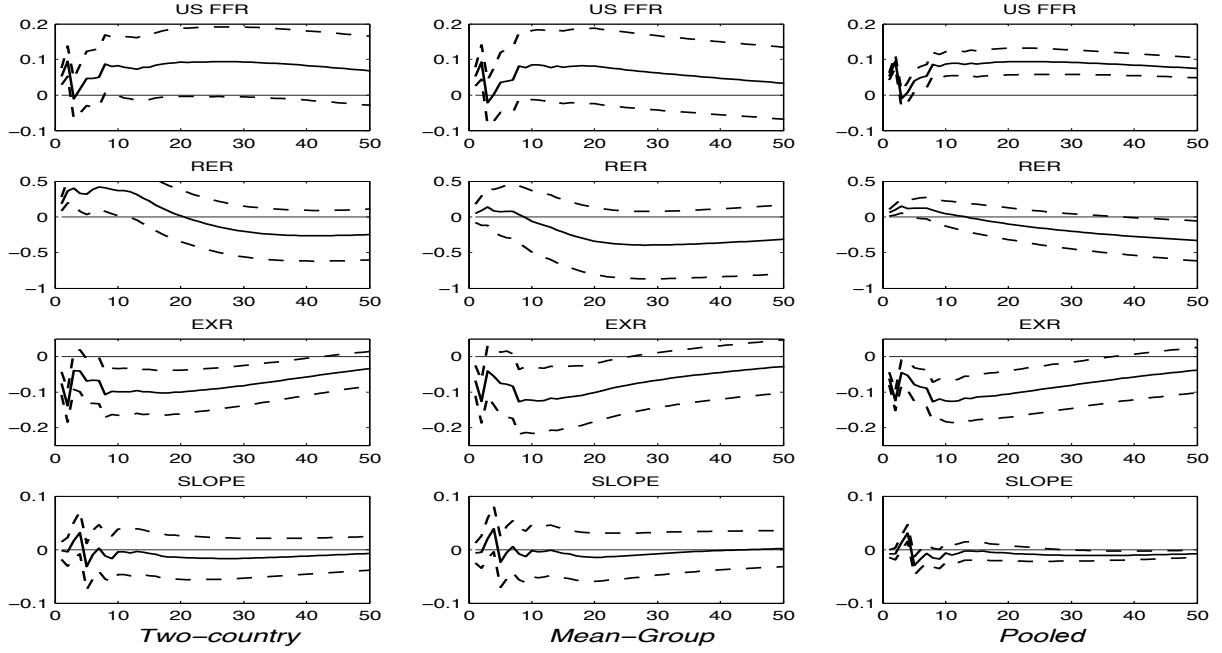
The first approach is to define a Two-Country version of our empirical model. We take the home country as describing the US economy, while the foreign country is a GDP-weighted average of the other G7 countries, Japan excluded.<sup>13</sup> The relevant exchange rate is therefore a multilateral exchange rate, while the foreign currency risk premium is actually the expected excess return on a portfolio of several foreign currencies, with the portfolio share of each currency being proportional to the respective country size. The dynamic responses of the four variables of interest are displayed in the first column of Figures 6 through 8, labeled “Two-Country”.<sup>14</sup>

The second approach is a Panel VAR Mean-Group estimation, in the spirit of Pesaran and Smith (1995): we estimate a separate VAR model for each country-pair and then evaluate the

<sup>12</sup>For a thorough discussion of these and other empirical approaches for the analysis of dynamic macro panels, see Canova (2007, Ch. 8).

<sup>13</sup>As shown in the previous section, Japan is often an outlier with respect to the dynamic responses of the exchange rate and the foreign currency risk premium. These differences suggest that the Japanese currency behaves in a somewhat peculiar way vis-à-vis the USD, for which the enormous and persistent positions in the yen carry-trade strategy might possibly play a key role. For this reason, we disregard Japan for the remaining of the section.

<sup>14</sup>This corresponds to the Aggregate Time Series estimator, as defined by Canova (2007), which we slightly modify by aggregating the time series using a GDP-weighted average (rather than a simple average) as in Benigno and Nisticò (2011), among others.

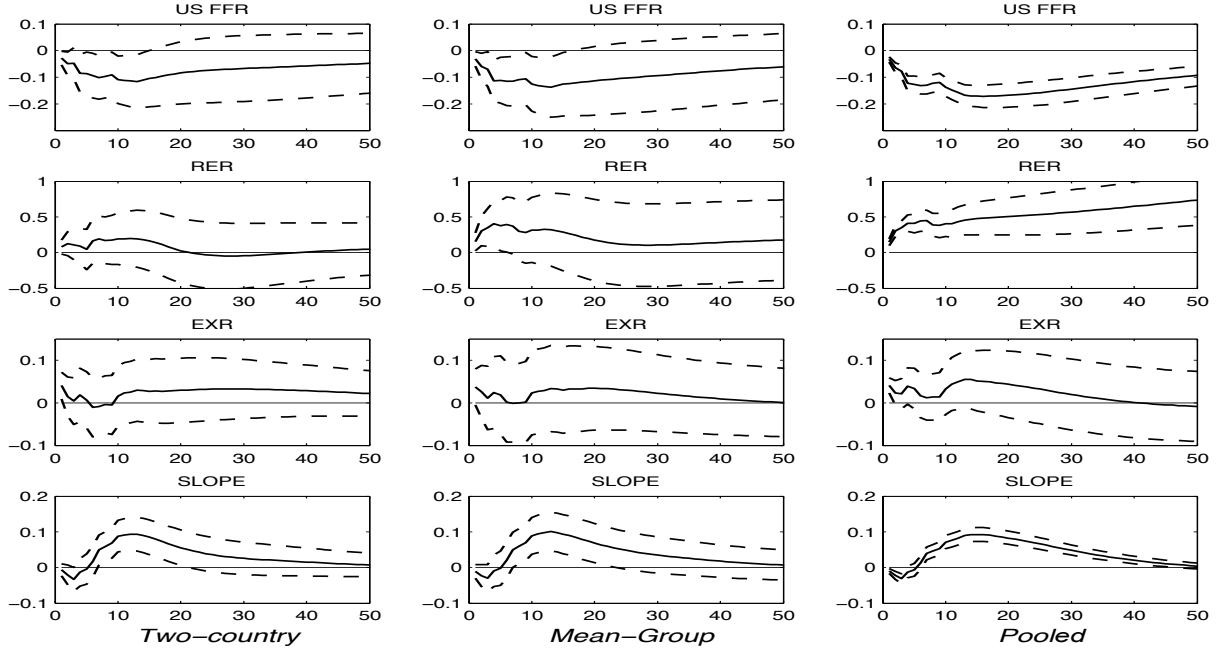


**Figure 7:** Dynamic responses to an orthogonalized innovation to the volatility of the inflation-target shock. Each column reports, for each empirical approach, the responses of the US Federal Funds Rate ( $i$ ), the Real Exchange Rate ( $q$ ), the foreign currency risk premium ( $exr$ ), the slope of the US term structure ( $i_{sl}$ ).  $x$ -axes: months,  $y$ -axes: annual percentage points. “Two-country”: single VAR, US-vs-G6 countries (Japan excluded); “Mean-Group”: average statistics from pair-specific VARs (from section 2.2); “Pooled”: single Pooled Panel VAR estimation.

mean of the estimated statistics of interest (namely the impulse-response function) across groups. Our dataset is sufficiently long, along the time-series dimension, to support consistency of the Mean-Group estimator. The impulse-responses of interest are displayed in the second column of Figures 6–8. In this case, the exchange-rate response measures the average response that the dollar bilateral exchange rate displays after a (domestic) level or volatility shock. Similarly, the third panel of the column shows the average response of the foreign currency risk premium with respect to the US dollar.

The third and final approach that we consider is the traditional Panel VAR Pooled estimation: we pool cross section and (demeaned) time series, and estimate and analyze a VAR( $p$ ) using the pooled series. This estimator, by construction, imposes the same dynamic structure to all countries, vis-à-vis the US. Accordingly, also in this case the impulse-response functions show an “average” response, capturing the common component across countries, of the bilateral USD exchange rate and foreign risk premium.

Figures 6 through 8 display the dynamic response of selected variables to the three volatility shocks that we analyze, for each method used. The variables are the US Federal Funds Rate ( $i$ ), the US Dollar Real Exchange Rate ( $q$ ), the expected excess return on foreign currency with respect to the US Dollar ( $exr$ ) and the slope of the US yield curve ( $i_{sl}$ ). In particular, the “Pooled” approach, displayed in the third column of each figure, seems quite useful in order to



**Figure 8:** Dynamic responses to an orthogonalized innovation to the volatility of productivity shocks. Each column reports, for each empirical approach, the responses of the US Federal Funds Rate ( $i$ ), the Real Exchange Rate ( $q$ ), the foreign currency risk premium ( $exr$ ), the slope of the US term structure ( $i_{sl}$ ).  $x$ -axes: months,  $y$ -axes: annual percentage points. “Two-country”: single VAR, US-vs-G6 countries (Japan excluded); “Mean-Group”: average statistics from pair-specific VARs (from section 2.2); “Pooled”: single Pooled Panel VAR estimation.

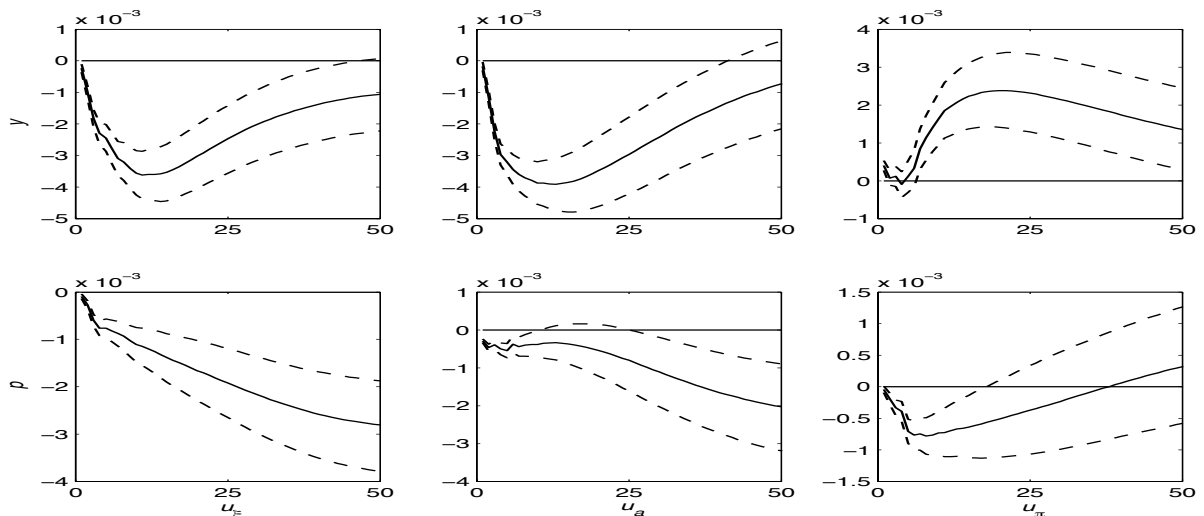
derive more precise impulse responses.

By looking at Figures 6 through 8, the overall picture shows that the main results, made in the previous section, are in fact reinforced considering the panel dimension of our data. In particular, in response to an unexpected increase in the volatility of the monetary-policy shock, Figure 6 shows that the US Dollar tends to appreciate while the foreign-currency risk premium increases. The latter result is driven by the domestic short-term interest rate falling more than the foreign one (not shown), which more than offsets the negative effect coming from the appreciation of the exchange rate. The yield curve, moreover, becomes significantly steeper.

Following an increase in the volatility of the inflation-target shock, the real exchange rate tends to appreciate in the medium term while the currency premium decreases, mainly as a result of the significant increase in the Federal Funds Rate. The slope of the yield curve, as also implied by the pair-wise analysis, does not seem to display any systematic response.

Finally, and again consistently with the evidence suggested by the pair-wise analysis of the previous section, an increase in the volatility of the productivity shock depreciates the US dollar and makes the yield curve significantly steeper. No clear effect is displayed by the foreign currency risk premium, regardless of the significant decrease in the domestic interest rate.

The important conclusion that we can draw from this analysis is that indeed volatility does matter. And it does matter also for traditional macro variables like real activity and the price level, as Figure 9 shows. In response to an increase in volatility of the monetary-policy shock



**Figure 9:** Dynamic responses of the domestic industrial production index (first row) and the domestic CPI (second row) to, respectively, a shock to the volatility of the monetary-policy instrument ( $u_\epsilon$ ) (first column), of the productivity shock ( $u_a$ ) (second column) and of the inflation-target shock ( $u_\pi$ ) (third column).

or to productivity, real activity substantially contracts and the price level falls, while a rise in volatility of the inflation-target shock tends to bring CPI inflation to a permanently higher level in the long-run and implies a temporary increase in real output. Finally, not displayed, the impulse responses to an increase in the level of the monetary-policy shock are standard as in the literature, with output falling and the prices rising in the short-run, consistently with the standard “price puzzle”.<sup>15</sup>

### 3 International finance regularities

In the previous section, we have provided evidence that volatility shocks have important effects on open-economy macro variables. In the next section, we are going to build a model in which indeed time-varying uncertainty plays a role. To nail down the desiderata that our model should meet, here we summarize the implications of our findings and report other empirical regularities along which we would like our model to perform well. The sense in which we refer to these facts (or puzzles) as international finance regularities relates to our focus on the joint behavior of interest rates and exchange rates.

The empirical evidence on the importance of volatility shocks can be summarized along two facts related respectively to the effects that volatility shocks have on the nominal (and real) exchange rate and the deviations from UIP.

*Fact 1:* An increase in the volatilities of the US monetary-policy and inflation-target shocks appreciates the dollar exchange rate, especially in the medium run. On the other hand, an increase in the volatility of the productivity shock depreciates the dollar exchange rate.

<sup>15</sup>See Figure 18 for the complete set of impulse-response functions for the Pooled Panel VAR.



*Fact 2:* An increase in the volatilities of both the monetary-policy and the inflation-target shocks generates significant and persistent deviations from UIP; in particular an increase in the excess return of foreign-versus-domestic short-term bonds in the case of the monetary-policy volatility shock and a decrease in the case of the inflation-target volatility shock.

The next fact is in common between our empirical analysis and the evidence reported by Eichenbaum and Evans (1995).

*Fact 3:* A positive innovation to the level of the monetary-policy shock (contractionary policy shock) produces a persistent appreciation in both the real and nominal exchange rates and a persistent deviations from the UIP in the form of positive excess returns on US securities.

To this list, we add another well-know fact, or puzzle, that we would like to address. While our previous facts are conditional statements about how excess returns and exchange rate co-move following different innovations (level or volatility shocks), another relevant empirical regularity is related to the joint behavior of exchange and interest rates as captured by the negative regression coefficient that arises from the UIP regression.

*Fact 4:* The regression coefficient between exchange rate changes and the nominal interest rate differential (UIP regression) is negative.

Related to the UIP puzzle, there is another one, recently discussed by Engel (2010), who documents that high real interest rate countries tend to have currencies that are strong in real terms and stronger than what can be explained by the real UIP. In particular, the puzzle comes from the fact that while current international-finance models struggle to account for the negative covariance between interest-rate differentials and exchange rate *changes*, those that succeed invariably miss the negative covariance between the interest-rate differential and the *level* of the exchange rate. Moreover, while the real interest-rate differential is negatively correlated with the real one-step-ahead excess return on foreign-versus-domestic currency, such correlation turns positive if we instead consider the “prospective excess return”, i.e. the expected cumulative excess return over the infinite future.

Finally, we would like our model to be also consistent with the responses of output and prices to the volatility shocks documented in Figure 9.

## 4 A two-country open economy model

To study the relationships between time-varying volatility and the exchange rate, we present a two-country open-economy model along the lines of Benigno and Benigno (2008). In particular we consider two extensions, whose relevance will be discussed later, which are important for the model to be able to match the empirical facts discussed above: *i*) we allow for more general recursive preferences as in the work of Epstein and Zin (1989, 1991) and Weil (1990) and *ii*) we consider stochastic volatility for the exogenous processes driving the economy. The latter addition, in particular, implies a careful treatment of the solution. To this end, we expound the method developed by Benigno, Benigno and Nisticò (2010) to show how we can handle in

a relatively easy way approximations of dynamic general equilibrium models with time-varying uncertainty and at the same time characterize the effect of uncertainty on the variables of interest.

#### 4.1 Households

The world economy consists of two countries, Home and Foreign, and is populated by a continuum of agents of measure one: Home households lie on the interval  $[0, n]$ , while Foreign households on  $(n, 1]$  where  $n \in (0, 1)$ . The population size is set equal to the range of goods produced so that Home firms produce goods on  $[0, n]$ , Foreign firms produce on  $(n, 1]$ . Home households are indexed by  $j$ , Foreign households by  $i$ .  $C_t^j$  denotes the level of consumption for household  $j$  in period  $t$  and  $L_t^j$  denotes its supply of working hours.

Preferences are recursive as in the framework of Epstein and Zin(1989, 1991) and Weil (1990). In particular, we assume that for a generic household of type  $j$  recursive utility can be written as

$$V_t^j = \left( U \left( C_t^j, L_t^j \right)^{1-\rho} + \beta \left( E_t(V_{t+1}^j)^{1-\gamma} \right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}} \quad (2)$$

where  $\rho$  is a measure of the inverse of the intertemporal elasticity of substitution over the utility flow,  $U(\cdot)$ ,  $\gamma$  represents the risk aversion towards static wealth gambles, and  $\beta \in (0, 1)$  is the household's subjective discount factor. The classical expected utility model is nested under the assumption  $\rho = \gamma$ .

The utility flow is a Cobb-Douglas index of aggregate consumption,  $C$ , and leisure,  $1 - L$

$$U \left( C_t^j, L_t^j \right) = \left( C_t^j \right)^\psi (1 - L_t^j)^{1-\psi} \quad (3)$$

where  $\psi \in (0, 1)$  reflects the preference for consumption versus leisure. As it is well known, this specification of preferences allows to disentangle the elasticity of substitution,  $1/\rho$ , from the risk-aversion coefficient.<sup>16</sup>

The aggregate consumption index  $C$  is a composite consumption good

$$C = \left[ v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 0 \quad (4)$$

where  $C_H$  and  $C_F$  are the two consumption sub-indexes that refer, respectively, to the consumption of Home-produced and Foreign-produced goods;  $\theta$ , with  $\theta > 0$ , is the elasticity of intratemporal substitution and  $v \in (0, 1)$  represents the weight given to home-produced goods in the aggregator  $C$ . Home bias in consumption arises when the weight given to Home goods is higher than the size of the country, i.e. when  $v > n$ .

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<sup>16</sup>See Swanson (2010) for how to compute risk-aversion toward consumption with Epstein-Zin preferences.

In the Foreign country, preferences have the same structure as in (2)

$$V_t^{*i} = \left( U(C_t^{*i}, L_t^{*i})^{1-\rho} + \beta (E_t(V_{t+1}^{*i})^{1-\gamma})^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}} \quad (5)$$

where the aggregate consumption bundle is given by

$$C^* = \left[ v^{\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (6)$$

for a different weight  $v^* \in (0, 1)$ .

We introduce home bias in consumption following Benigno and De Paoli (2010). Specifically, denoting with  $\lambda \in (0, 1)$  the (common) degree of openness of the two countries, the weights in the consumption bundle are related to the country sizes through:

$$1 - v = (1 - n)\lambda,$$

$$v^* = n\lambda.$$

The consumption bundles  $C_H, C_F, C_H^*, C_F^*$ , are in turn Dixit-Stiglitz aggregators of the goods produced in the two countries and are given by

$$C_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \quad C_F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}, \quad (7)$$

$$C_H^* = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c^*(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \quad C_F^* = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^*(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}, \quad (8)$$

where  $\sigma$ , with  $\sigma > 1$ , is the elasticity of substitution across the consumption goods produced within a country. The appropriate consumption-based price indexes associated with  $C$  and  $C^*$  are given respectively by

$$P = \left[ v P_H^{1-\theta} + (1-v) (P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (9)$$

$$P^* = \left[ v^* P_H^{*1-\theta} + (1-v^*) (P_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (10)$$

where  $P_H$  ( $P_H^*$ ) is the price sub-index for Home-produced goods expressed in the Home (Foreign) currency and  $P_F$  ( $P_F^*$ ) is the price sub-index for Foreign-produced goods expressed in the Home (Foreign) currency. Moreover

$$P_H = \left[ \left( \frac{1}{n} \right) \int_0^n p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}} \quad P_F = \left[ \left( \frac{1}{1-n} \right) \int_n^1 p(f)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad (11)$$

$$P_H^* = \left[ \left( \frac{1}{n} \right) \int_0^n p^*(h)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \quad P_F^* = \left[ \left( \frac{1}{1-n} \right) \int_n^1 p^*(f)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad (12)$$

where  $p(h)$  and  $p^*(h)$  are the prices of the generic good  $h$  produced by the Home country in the currencies of the Home and Foreign country respectively; while  $p(f)$  and  $p^*(f)$  are the prices of the generic good  $f$  produced by the Foreign country in the currencies of the Home and Foreign country respectively. The law of one price holds across all individual goods:  $p(h) = Sp^*(h)$  and  $p(f) = Sp^*(f)$ , where  $S$  is the nominal exchange rate (the price of foreign currency in terms of domestic currency). Therefore, equations (11) and (12), imply that  $P_H = SP_H^*$  and  $P_F = SP_F^*$ . However, equations (9) and (10) show that, since Home and Foreign agents' preferences are not necessarily identical, there can be deviations from purchasing power parity unless  $v = v^*$ , that is,  $P \neq SP^*$ . Appropriately we measure the deviations from PPP through the real exchange rate given by  $Q \equiv SP^*/P$ . We also define the terms of trade in the Home country as  $T \equiv P_F/P_H$ . Notice the following useful relationships between relative prices, the real exchange rate and terms of trade

$$1 = \left[ v \left( \frac{P_H}{P} \right)^{1-\theta} + (1-v) \left( \frac{P_F}{P} \right)^{1-\theta} \right], \quad (13)$$

$$Q = \frac{\left[ v^* + (1-v^*) (T)^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[ v + (1-v) (T)^{1-\theta} \right]^{\frac{1}{1-\theta}}}, \quad (14)$$

$$T = \frac{P_F}{P} \frac{P}{P_H}. \quad (15)$$

Given the above-specified preferences, we can derive total demands of the generic good  $h$ , produced in country H, and of the good  $f$ , produced in country F:

$$y^d(h) = \left( \frac{p(h)}{P_H} \right)^{-\sigma} Y_H \quad y^d(f) = \left( \frac{p(f)}{P_F} \right)^{-\sigma} Y_F \quad (16)$$

where output aggregators  $Y_H$  and  $Y_F$  are appropriately defined

$$Y_H = \left( \frac{P_H}{P} \right)^{-\theta} \left( vC + \frac{v^*(1-n)}{n} Q^\theta C^* \right), \quad (17)$$

$$Y_F = \left( \frac{P_F}{P} \right)^{-\theta} \left( \frac{(1-v)n}{1-n} C + (1-v^*) Q^\theta C^* \right). \quad (18)$$

We assume that asset markets are complete both at the domestic and international levels. In particular households can trade in a set of state-contingent nominal securities denominated in the Home currency which span all the uncertainty from one period to another.<sup>17</sup> Each

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<sup>17</sup>See Chari et al. (1998).

of these securities pays respectively only in one of the possible states of nature in the next period. Let  $B_{t+1}^j$  the state-contingent payoff at time  $t + 1$  of the portfolio of state-contingent nominal securities held by household in the Home country at the end of period  $t$ . The value of this portfolio can be written as  $E_t[M_{t,t+1}B_{t+1}^j]$  where  $M_{t,t+1}$  represents the nominal stochastic discount factor for discounting units of Home-currency wealth from a state of nature at time  $t + 1$  back to time  $t$ . This stochastic discount factor is unique, because of the complete-market assumption, and equivalent to the price of a state-contingent security standardized by the time- $t$  conditional probability of occurrence of the state of nature at time  $t + 1$  in which the security pays. We can write the flow budget constraint that the Home households face as

$$E_t[M_{t,t+1}B_{t+1}^j] \leq B_t^j + W_tL_t^j + D_t^j - P_tC_t^j,$$

for each  $j$ , where  $W_t$  is the nominal wage in the Home country, determined in a common labor market;  $D_t^j$  are nominal profits. Each household holds equal shares of all firms (domestic firms are located on the interval  $[0, n]$  and the size of the Home population is normalized to  $n$ ); there is no trade in firms' shares. Households are subject to a standard limit on their borrowing possibilities.

Households in the Foreign country can also trade in the state-contingent securities denominated in the currency of country  $H$ . Let  $B_{t+1}^i$  the state-contingent payoff at time  $t + 1$  of the portfolio of state-contingent nominal securities held by Foreign household at the end of period  $t$ . Since  $B_{t+1}^i$  is denominated in units of Home currency, the payoff in Foreign currency is given by  $B_{t+1}^{*i} = B_{t+1}^i/S_{t+1}$  and the value of the portfolio in Foreign currency is simply  $E_t[M_{t,t+1}B_{t+1}^i]/S_t = E_t[M_{t,t+1}B_{t+1}^{*i}S_{t+1}]/S_t$ . We can appropriately define the nominal stochastic discount factor for discounting units of Foreign-currency wealth across time

$$M_{t,t+1}^* = \frac{S_{t+1}}{S_t} M_{t,t+1} \quad (19)$$

which is uniquely defined given that  $M_{t,t+1}$  is unique. Therefore, the flow budget constraint for the Foreign households can be written as

$$E_t[M_{t,t+1}^*B_{t+1}^{*i}] \leq B_t^{*i} + W_t^*L_t^{*i} + D_t^{*i} - P_t^*C_t^{*i},$$

for each  $i$  where the definition of the variables follows from before with the appropriate modifications. A standard borrowing-limit condition applies also here.

Households maximize utility subject to the sequence of the flow budget constraints and the borrowing-limit constraints by choosing aggregate consumption, labor and asset holdings in terms of the state contingent securities.

At optimum the marginal rate of substitution between labor and consumption is equal to the real wage

$$\frac{W_t}{P_t} = \frac{1 - \psi}{\psi} \frac{C_t^j}{1 - L_t^j} \quad (20)$$

$$\frac{W_t^*}{P_t^*} = \frac{1-\psi}{\psi} \frac{C_t^{i*}}{1-L_t^{*i}} \quad (21)$$

for each  $j$  and  $i$  in the respective country.

Optimality conditions with respect to the holdings of the state-contingent securities for the Home household imply

$$\frac{\partial(V_t^j)}{\partial C_t^j} \frac{(V_t^j)^{-\rho}}{P_t} M_{t,t+1} = \beta \left( E_t(V_{t+1}^j)^{1-\gamma} \right)^{\frac{\gamma-\rho}{1-\gamma}} \frac{(V_{t+1}^j)^{-\gamma}}{P_{t+1}} \frac{\partial V_{t+1}^j}{\partial C_{t+1}^j},$$

for each contingency at time  $t+1$  where the marginal utility of consumption is given by

$$\frac{\partial V_t^j}{\partial C_t^j} = \psi \frac{U(C_t^j, L_t^j)^{1-\rho}}{C_t^j} (V_t^j)^\rho.$$

Combining the above two equations, we obtain that the nominal stochastic discount factor in the Home country is

$$M_{t,t+1} = \beta \left( \frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{\frac{\rho-\gamma}{1-\gamma}} \left( \frac{U(C_{t+1}, L_{t+1})}{U(C_t, L_t)} \right)^{1-\rho} \frac{C_t}{C_{t+1}} \frac{1}{\Pi_{t+1}} \quad (22)$$

where we have also neglected the index  $j$  from  $V$ ,  $C$ ,  $L$ .<sup>18</sup> Moreover, we have defined the gross CPI inflation rate as

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} = \Pi_{H,t} \frac{\left[ v + (1-v)(T_t)^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[ v + (1-v)(T_{t-1})^{1-\theta} \right]^{\frac{1}{1-\theta}}}, \quad (23)$$

where  $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$ .

Similarly in the Foreign country we obtain

$$M_{t,t+1}^* = \beta \left( \frac{V_{t+1}^{*1-\gamma}}{E_t V_{t+1}^{*1-\gamma}} \right)^{\frac{\rho-\gamma}{1-\gamma}} \left( \frac{U(C_{t+1}^*, L_{t+1}^*)}{U(C_t^*, L_t^*)} \right)^{1-\rho} \frac{C_t^*}{C_{t+1}^*} \frac{1}{\Pi_{t+1}^*}, \quad (24)$$

where the Foreign gross CPI inflation rate is given by

$$\Pi_t^* = \Pi_t \frac{Q_t}{Q_{t-1}} \frac{S_{t-1}}{S_t}. \quad (25)$$

The above nominal discount factors correspond to those of the standard expected-utility model, under the assumption  $\rho = \gamma$ . In this case, they depend on the ratio between the marginal

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<sup>18</sup>Given the assumption that a common labor market exists in each country and that each firm employs all the workers, as it will be discussed later, we can impose symmetry in labor supply and set  $L^j = L$  for each  $j$ . It follows from (20) that  $C^j = C$  for each  $j$ . Therefore also  $V^j = V$ .

utilities of nominal income across the two periods. With Epstein-Zin preferences, there is an additional term reflecting the preference for an early, in the case  $\rho < \gamma$ , or late, in the case  $\rho > \gamma$ , resolution of intertemporal uncertainty. This intertemporal uncertainty is captured by the ratio of the utility at time  $t + 1$  with respect to its risk-adjusted expected value, where the risk-adjustment occurs through the factor  $1 - \gamma$ . When agents prefer an early resolution of uncertainty ( $\rho < \gamma$ ) bad realizations of the utility at time  $t + 1$  with respect to its risk-adjusted expected value increase the stochastic discount factor and therefore the appetite for state-contingent wealth in that state of nature.

The above nominal stochastic discount factor can be used to price any security in arbitrage-free markets and in particular they imply that the short-term nominal interest rates satisfy

$$\frac{1}{1 + i_t} = E_t M_{t+1}, \quad (26)$$

$$\frac{1}{1 + i_t^*} = E_t M_{t+1}^*, \quad (27)$$

where  $i_t$  and  $i_t^*$  are the one-period nominal interest rates in the Home and Foreign country, respectively.

Using (22) and (24) into (19), we can obtain

$$\left( \frac{V_{t+1}^{1-\gamma} (E_t V_{t+1}^{*1-\gamma})}{V_{t+1}^{*1-\gamma} (E_t V_{t+1}^{1-\gamma})} \right)^{\frac{\rho-\gamma}{1-\gamma}} \left( \frac{U(C_{t+1}, L_{t+1})}{U(C_{t+1}^*, L_{t+1}^*)} \right)^{1-\rho} \frac{C_{t+1}^*}{C_{t+1}} Q_{t+1} = \left( \frac{U(C_t, L_t)}{U(C_t^*, L_t^*)} \right)^{1-\rho} \frac{C_t^*}{C_t} Q_t. \quad (28)$$

To close the assumption of complete markets, we need to specify initial conditions for the holdings of the state-contingent securities. A standard assumption in the literature is to choose initial state-contingent wealth in a way to equalize the ratio between the marginal utilities of nominal income across countries, converted in the same currency. Let  $G_t$  denote this ratio at time  $t$ , it follows that we can write it as

$$G_t = \frac{\frac{\partial V_t^{1-\rho}}{\partial C_t} \frac{1}{P_t}}{\frac{\partial V_t^{*1-\rho}}{\partial C_t^*} \frac{1}{S_t P_t^*}} = \left( \frac{U(C_t, L_t)}{U(C_t^*, L_t^*)} \right)^{1-\rho} \frac{C_t^*}{C_t} Q_t \quad (29)$$

where we have rescaled utility as  $V_t^{1-\rho}$  in order to make a direct comparison with the expected-utility model.<sup>19</sup> Combining (28) and (29) we obtain the following law of motion for  $G_t$

$$G_{t+1} = G_t \left( \frac{V_{t+1}^{1-\gamma} (E_t V_{t+1}^{*1-\gamma})}{V_{t+1}^{*1-\gamma} (E_t V_{t+1}^{1-\gamma})} \right)^{\frac{\gamma-\rho}{1-\gamma}}. \quad (30)$$

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<sup>19</sup>When  $\gamma = \rho$ , utility (2) coincides with the expected utility model where indeed intertemporal utility is defined as  $V_t^{1-\rho}$ .

We set  $G_{t_0} = 1$  and therefore assume that initial state-contingent wealth equalizes the ratio of the marginal utilities of nominal income across countries in the initial period. Notice that, under the expected-utility model ( $\gamma = \rho$ ), this assumption implies equalization of the ratio at all times and contingencies. With Epstein-Zin preferences, instead, this ratio evolves over time depending on cross-country realizations of utility with respect to their risk-adjusted expected values.

## 4.2 Firms

The Home country produces goods on the interval  $[0, n]$  while the Foreign country on  $(n, 1]$ . At first pass we abstract from investment and capital accumulation.<sup>20</sup> A generic firm  $h$  producing in the Home country uses the following technology

$$y_t(h) = A_t(L_t(h))^\varphi \quad (31)$$

where  $A_t$  is a productivity shifter common to all the firms in the Home country,  $\varphi$  with  $\varphi \in (0, 1]$  measures decreasing return to scale in the labor input  $L_t(h)$ , which is a composite of all the differentiated labor supplied by households  $j$  according to

$$L_t(h) = \frac{1}{n} \int_0^n L_t^j(h) dj$$

where  $L_t^j(h)$  denotes the demand of household  $j$ 's labor by firm  $h$ .

We assume that there are frictions in the price adjustment. In particular, we model price rigidity as in the Calvo's (1983) model, but with indexation. In each period, in the Home country, only a fraction  $(1 - \alpha)$  of firms, with  $0 \leq \alpha < 1$ , can reset their prices independently of the last time they had reset them. In this case, the price is chosen to maximize the expected discounted value of the profits under the circumstances that the price, appropriately indexed, still applies. These firms choose prices to maximize the following objective

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \{p_{t,T}(h) y_{t,T}(h) - W_T L_T(h)\}$$

where total demand is:

$$y_{t,T}(h) = \left( \frac{p_{t,T}(h)}{P_{H,T}} \right)^{-\sigma} Y_{H,T}$$

and moreover  $p_{t,T}(h) = \tilde{p}_t(h) \bar{P}_{H,T} / \bar{P}_{H,t}$  where  $\tilde{p}_t(h)$  is the price chosen at time  $t$  and  $\bar{P}_{H,T} / \bar{P}_{H,t}$  is the gross inflation target from  $t$  to  $T$  to which all prices are automatically adjusted. The

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<sup>20</sup>Otherwise we can assume that each firm is endowed with a fixed amount of non-depreciating capital.



optimal price  $\tilde{p}_t(h)$  is chosen to satisfy the following first-order condition:

$$\tilde{p}_t(h) = \mu \frac{E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} W_T \left( \frac{y_{t,T}(h)}{A_T} \right)^{\frac{1}{\varphi}}}{E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left( \frac{\bar{P}_{H,T}}{\bar{P}_{H,t}} \right) y_{t,T}(h)},$$

where the overall mark-up has been defined as  $\mu = \sigma/(\varphi(\sigma - 1))$ . Using (20) and (22), we can write the above equation as

$$\left( \frac{\tilde{p}_t(h)}{\bar{P}_{H,t}} \right)^{1-\sigma+\frac{\sigma}{\varphi}} = \mu \frac{1-\psi}{\psi} \frac{E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} N_{t,T} U(C_T, L_T)^{1-\rho} \frac{1}{1-L_T} \left( \frac{P_{H,T}}{\bar{P}_{H,t}} \frac{\bar{P}_{H,t}}{\bar{P}_{H,T}} \right)^{\frac{\sigma}{\varphi}} \left( \frac{Y_{H,T}}{A_T} \right)^{\frac{1}{\varphi}}}{E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} N_{t,T} U(C_T, L_T)^{1-\rho} C_T^{-1} \left( \frac{P_{H,T}}{\bar{P}_{H,t}} \frac{\bar{P}_{H,t}}{\bar{P}_{H,T}} \right)^{\sigma-1} \frac{P_{H,T}}{\bar{P}_T} Y_{H,T}}, \quad (32)$$

where we have defined

$$N_{t,T} = \left( \frac{V_{t+1}^{1-\gamma} V_{t+2}^{1-\gamma} \dots V_T^{1-\gamma}}{E_t V_{t+1}^{1-\gamma} E_{t+1} V_{t+2}^{1-\gamma} \dots E_{T-1} V_T^{1-\gamma}} \right)^{\frac{\rho-\gamma}{1-\gamma}},$$

with  $N_{t,t} = 1$ .

The remaining fraction of firms, of measure  $\alpha$  can change their prices only by indexing them to the current inflation index, which does not necessarily coincide with actual inflation. Therefore, we note that the Calvo's model implies the following law of motion for the aggregate price index  $P_{H,t}$

$$P_{H,t}^{1-\sigma} = \alpha \bar{\Pi}_{H,t}^{1-\sigma} P_{H,t-1}^{1-\sigma} + (1-\alpha) \tilde{p}_t(h)^{1-\sigma}, \quad (33)$$

where  $\bar{\Pi}_{H,t} \equiv \bar{P}_{H,t}/\bar{P}_{H,t-1}$ . Using (33), we can write (32) as

$$\left( \frac{1-\alpha \left( \frac{\Pi_{H,t}}{\bar{\Pi}_{H,t}} \right)^{\sigma-1}}{1-\alpha} \right)^{\frac{1}{1-\sigma}} = \left( \frac{F_t}{K_t} \right)^{\frac{\varphi}{\varphi-\sigma\varphi+\sigma}} \quad (34)$$

where  $F_t$  and  $K_t$  can be written recursively as

$$F_t = \mu \frac{1-\psi}{\psi} \frac{U(C_t, L_t)^{1-\rho}}{1-L_t} \left( \frac{Y_{H,t}}{A_t} \right)^{\frac{1}{\varphi}} + \alpha\beta E_t \left\{ \left( \frac{\Pi_{H,t+1}}{\bar{\Pi}_{H,t+1}} \right)^{\frac{\sigma}{\varphi}} \left( \frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{\frac{\rho-\gamma}{1-\gamma}} F_{t+1} \right\}, \quad (35)$$

$$K_t = U(C_t, L_t)^{1-\rho} \frac{P_{H,t} Y_{H,t}}{P_t C_t} + \alpha\beta E_t \left\{ \left( \frac{\Pi_{H,t+1}}{\bar{\Pi}_{H,t+1}} \right)^{\sigma-1} \left( \frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{\frac{\rho-\gamma}{1-\gamma}} K_{t+1} \right\}. \quad (36)$$

Notice that equilibrium in the labor market requires

$$L_t = \frac{1}{n} \int_0^n L_t(h) dh = \frac{1}{n} \int_0^n \left( \frac{y_t(h)}{A_t} \right)^{\frac{1}{\varphi}} dh = \Delta_t \left( \frac{Y_{H,t}}{A_t} \right)^{\frac{1}{\varphi}} \quad (37)$$

where the index of price dispersion  $\Delta_t$  can be written recursively as

$$\Delta_t \equiv \frac{1}{n} \int_0^n \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\frac{\sigma}{\varphi}} dh = \alpha \Delta_{t-1} \left( \frac{\Pi_{H,t}}{\bar{\Pi}_{H,t}} \right)^{\frac{\sigma}{\varphi}} + (1 - \alpha) \left( \frac{1 - \alpha \left( \frac{\Pi_{H,t}}{\bar{\Pi}_{H,t}} \right)^{\sigma-1}}{1 - \alpha} \right)^{-\frac{\sigma}{\varphi(1-\sigma)}}. \quad (38)$$

The price-setting mechanism is similar in the Foreign country, where now  $(1 - \alpha^*)$  represents the mass of firms, with  $0 \leq \alpha^* < 1$ , that can reset their prices each period. Following similar steps, the Foreign country's aggregate-supply equation can be written as

$$\left( \frac{1 - \alpha^* \left( \frac{\Pi_{F,t}^*}{\bar{\Pi}_{F,t}^*} \right)^{\sigma-1}}{1 - \alpha^*} \right)^{\frac{1}{1-\sigma}} = \left( \frac{F_t^*}{K_t^*} \right)^{\frac{\varphi}{\varphi - \sigma\varphi + \sigma}}, \quad (39)$$

with

$$F_t^* = \mu \frac{1 - \psi}{\psi} \frac{U(C_t^*, L_t^*)^{1-\rho}}{1 - L_t^*} \left( \frac{Y_{F,t}^*}{A_t^*} \right)^{\frac{1}{\varphi}} + \alpha^* \beta E_t \left\{ \left( \frac{\Pi_{F,t+1}^*}{\bar{\Pi}_{F,t+1}^*} \right)^{\frac{\sigma}{\varphi}} \left( \frac{V_{t+1}^{*1-\gamma}}{E_t V_{t+1}^{*1-\gamma}} \right)^{\frac{\rho-\gamma}{1-\gamma}} F_{t+1}^* \right\}, \quad (40)$$

$$K_t^* = U(C_t^*, L_t^*)^{1-\rho} \frac{P_{F,t} Y_{F,t}^*}{P_t Q_t C_t^*} + \alpha^* \beta E_t \left\{ \left( \frac{\Pi_{F,t+1}^*}{\bar{\Pi}_{F,t+1}^*} \right)^{\sigma-1} \left( \frac{V_{t+1}^{*1-\gamma}}{E_t V_{t+1}^{*1-\gamma}} \right)^{\frac{\rho-\gamma}{1-\gamma}} K_{t+1}^* \right\}, \quad (41)$$

where  $\Pi_{F,t}^* = P_{F,t}^*/P_{F,t-1}^*$  and  $\bar{\Pi}_{F,t}^*$  is the gross inflation target to which foreign prices adjust each period. Equilibrium in the Foreign labor market implies

$$L_t^* = \frac{1}{1-n} \int_n^1 L_t^*(f) df = \frac{1}{1-n} \int_n^1 \left( \frac{y_t^*(f)}{A_t^*} \right)^{\frac{1}{\varphi}} df = \Delta_t^* \left( \frac{Y_{F,t}^*}{A_t^*} \right)^{\frac{1}{\varphi}} \quad (42)$$

where now  $\Delta_t^*$  is given by

$$\Delta_t^* \equiv \frac{1}{1-n} \int_n^1 \left( \frac{p_t^*(f)}{P_{F,t}^*} \right)^{-\frac{\sigma}{\varphi}} df = \alpha^* \Delta_{t-1}^* \left( \frac{\Pi_{F,t}^*}{\bar{\Pi}_{F,t}^*} \right)^{\frac{\sigma}{\varphi}} + (1 - \alpha^*) \left( \frac{1 - \alpha^* \left( \frac{\Pi_{F,t}^*}{\bar{\Pi}_{F,t}^*} \right)^{\sigma-1}}{1 - \alpha^*} \right)^{-\frac{\sigma}{\varphi(1-\sigma)}}. \quad (43)$$

Finally we note the following relationship between the terms of trade and producer-price inflation

rates

$$T_t = T_{t-1} \frac{S_t}{S_{t-1}} \frac{\Pi_{F,t}^*}{\Pi_{H,t}}. \quad (44)$$

### 4.3 Monetary policy rules

We close the model by specifying the monetary policy rules. A broad class of policy rules that we consider can be written as

$$(1 + i_{1,t}) = (1 + i_{1,t-1})^{\phi_i} \left( \frac{\bar{\Pi}_t}{\tilde{\beta}} \right)^{1-\phi_i} \left( \frac{\Pi_{H,t}}{\bar{\Pi}_t} \right)^{(1-\phi_i)\phi_\pi} \left( \frac{\tilde{Y}_{H,t}}{\tilde{Y}_{H,t-1}} \right)^{(1-\phi_i)\phi_y} \left( \frac{S_t}{S_{t-1}} \right)^{(1-\phi_i)\phi_s} e^{\xi_t} \quad (45)$$

for the Home monetary policymaker where the short-term interest rate reacts to its past value, to the deviation of the gross producer inflation from a target, to domestic output growth and to the changes in the exchange rate;<sup>21</sup>  $\phi_i, \phi_\pi, \phi_y, \phi_s$  are non-negative parameters,  $\tilde{\beta}$  is an appropriately-defined parameter,  $\xi_t$  is the policy shock and  $\bar{\Pi}_t$  represent the inflation target followed by the Home monetary policymaker which is generally different from the target to which prices are indexed. The link between the two inflation targets could be expressed as

$$\bar{\Pi}_{H,t} = \bar{\Pi}_t^\kappa \Pi_{H,t-1}^{1-\kappa},$$

with a weight  $\kappa \in [0, 1]$  which can be interpreted as a measure of the credibility of monetary policy in the Home country. When  $\kappa = 1$  producer prices are indexed to the inflation target used by the monetary policymaker, otherwise prices are indexed to a weighted average of past realized producer inflation and the current policy target.

In a similar way we assume that in the Foreign country the short-term nominal interest rate follows

$$(1 + i_{1,t}^*) = (1 + i_{1,t-1}^*)^{\phi_i^*} \left( \frac{\bar{\Pi}_t^*}{\tilde{\beta}^*} \right)^{1-\phi_i^*} \left( \frac{\Pi_{F,t}^*}{\bar{\Pi}_t^*} \right)^{(1-\phi_i^*)\phi_\pi^*} \left( \frac{\tilde{Y}_{F,t}^*}{\tilde{Y}_{F,t-1}^*} \right)^{(1-\phi_i^*)\phi_y^*} \left( \frac{S_t}{S_{t-1}} \right)^{-(1-\phi_i^*)\phi_s^*} e^{\xi_t^*} \quad (46)$$

where  $\phi_i^*, \phi_\pi^*, \phi_y^*, \phi_s^*$  are non-negative parameters,  $\tilde{\beta}^*$  is an appropriately-defined parameter,  $\xi_t^*$  is the policy shock and  $\bar{\Pi}_t^*$  represents the inflation target followed by the Foreign monetary policymaker where now

$$\bar{\Pi}_{F,t}^* = (\bar{\Pi}_t^*)^{\kappa^*} (\Pi_{F,t-1}^*)^{1-\kappa^*}$$

with a weight  $\kappa^* \in [0, 1]$  measuring the credibility of Foreign monetary policy.

### 4.4 Equilibrium

We now define the equilibrium of the above model. Given processes for the exogenous state variables  $(\ln A_t, \ln \xi_t, \ln \bar{\Pi}_{H,t}, \ln A_t^*, \ln \xi_t^*, \ln \bar{\Pi}_{F,t}^*)$  an equilibrium is an allocation  $(V_t, V_t^*, C_t,$

<sup>21</sup>We will also consider a target in terms of CPI inflation instead of PPI inflation.

$C_t^*, L_t, L_t^*, Y_{H,t}, Y_{F,t}^*, P_{H,t}/P_t, P_{F,t}/P_t, S_t/S_{t-1}, Q_t, T_t, G_t, \Pi_{H,t}, \Pi_{F,t}^*, \Pi_t, \Pi_t^*, \Delta_t, \Delta_t^*, F_t, F_t^*, K_t, K_t^*, M_{t,t+1}, M_{t,t+1}^*, i_{1,t}, i_{1,t}^*)$  which satisfies the equations (2), (5), (13), (14), (15), (17), (18), (22), (23), (24), (25), (26), (27), (29), (30), (34), (35), (36), (37), (38), (39), (40), (41), (42), (43), (44) given the two policy rules (45) and (46) and the relationships between the inflation targets of the firms and of the monetary policymaker.

We assume that the vector of exogenous variables follows conditionally-linear processes with time-varying volatility. In particular we assume a general specification of the stochastic productivity processes to take into account the possibility of a trend in productivity. We model the productivity shock in country  $H$  as  $A_t = A_{W,t} \tilde{A}_t$  and that in country  $F$  as  $A_t^* = A_{W,t} \tilde{A}_t^*$  where  $A_{W,t}$  has a stochastic trend and can be interpreted as a global common productivity shock while  $\tilde{A}_t$  and  $\tilde{A}_t^*$  are log-stationary processes that are country-specific.<sup>22</sup>

The stochastic processes of the shocks are:

$$\ln A_{W,t+1} = \ln a + \ln A_{W,t} + u_{aw,t} \varepsilon_{aw,t+1}$$

$$\ln \tilde{A}_{t+1} = \delta_a \ln \tilde{A}_t + u_{a,t} \varepsilon_{a,t+1}$$

$$\ln \bar{\Pi}_{t+1} = \ln \bar{\Pi}_t + u_{\pi,t} \varepsilon_{\pi,t+1}$$

$$\xi_{t+1} = u_{\xi,t} \varepsilon_{\xi,t+1}$$

where  $a$  is a parameter measuring the deterministic trend in productivity growth and  $0 \leq \delta_a \leq 1$ . In what follows, all the  $\varepsilon$  shocks are *iid* white-noise processes.

Time-varying volatility is modelled through linear processes for the variances:

$$u_{aw,t+1}^2 = (1 - \rho_{aw}) \sigma_u^2 + \rho_{aw} u_{aw,t}^2 + \sigma_\zeta^2 \zeta_{aw,t+1}$$

$$u_{a,t+1}^2 = (1 - \rho_a) \sigma_u^2 + \rho_a u_{a,t}^2 + \sigma_\zeta^2 \zeta_{a,t+1}$$

$$u_{\pi,t+1}^2 = (1 - \rho_\pi) \sigma_u^2 + \rho_\pi u_{\pi,t}^2 + \sigma_\zeta^2 \zeta_{\pi,t+1}$$

$$u_{\xi,t+1}^2 = (1 - \rho_\xi) \sigma_u^2 + \rho_\xi u_{\xi,t}^2 + \sigma_\zeta^2 \zeta_{\xi,t+1}$$

in which all the  $\zeta$  are *iid* white-noise processes and  $0 \leq \rho_{aw}, \rho_a, \rho_\pi, \rho_\xi \leq 1$  with  $\sigma_u^2, \sigma_\zeta^2 > 0$ . The processes for the stochastic disturbances hitting the Foreign economy behave similarly:

$$u_{a^*,t+1}^2 = (1 - \rho_{a^*}) \sigma_u^2 + \rho_{a^*} u_{a^*,t}^2 + \sigma_\zeta^2 \zeta_{a^*,t+1}$$

$$u_{\pi^*,t+1}^2 = (1 - \rho_{\pi^*}) \sigma_u^2 + \rho_{\pi^*} u_{\pi^*,t}^2 + \sigma_\zeta^2 \zeta_{\pi^*,t+1}$$

$$u_{\xi^*,t+1}^2 = (1 - \rho_{\xi^*}) \sigma_u^2 + \rho_{\xi^*} u_{\xi^*,t}^2 + \sigma_\zeta^2 \zeta_{\xi^*,t+1}$$

In what follows we will refer to the shocks to the inflation target and the shock to the policy

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<sup>22</sup>In this way our model will allow for a balanced-growth path. As we will show in the next section, the stochastic trend is in particular important for the relevance of the Epstein-Zin assumption.

instruments as *monetary* or *nominal* shocks while the productivity shock will be the *real* shock.

#### 4.5 Solution

Given the above specification for the processes of the exogenous state variable, we can write them more compactly as

$$z_{t+1} = \Lambda_z z_t + \eta_{t+1} \quad (47)$$

where the vector  $z_t$  is defined as  $z_t \equiv [(\Delta \ln A_{W,t} - \ln a), \ln \tilde{A}_t, \xi_t, \ln \bar{\Pi}_{H,t}, \ln \tilde{A}_t^*, \xi_t^*, \ln \bar{\Pi}_{F,t}^*]'$  and  $\Lambda_z$  is an appropriately-defined square matrix. The vector  $\eta_{t+1}$  is given by

$$\eta_{t+1} = U_t \varepsilon_{z,t+1} \quad (48)$$

where  $\varepsilon_{z,t+1}$  collects the innovations, which are assumed to have a bounded support and to be independently and identically distributed with mean zero and variance/covariance matrix  $I_z$ , where  $I_z$  is an identity matrix of the same dimension of the vector  $z$ ;  $U_t$  is a diagonal matrix whose elements on the diagonal are collected into a vector  $u_t$ . In particular  $u_t$  follows the exogenous stochastic linear process given by

$$u_{t+1}^2 = \sigma_u^2 (I_z - \Lambda_u) \bar{u}^2 + \Lambda_u u_t^2 + \sigma_\zeta^2 Z \zeta_{u,t+1}. \quad (49)$$

Each element of  $u_t^2$  is the corresponding squared value of each element of  $u_t$ , which still corresponds to the diagonal of matrix  $U_t$  as in (48);  $\bar{u}^2$  is a vector of steady-state variances,  $Z$  and  $\Lambda_u$  are appropriately defined square matrices;  $\zeta_{u,t+1}$  is a vector of innovation collecting the above  $\zeta$  which are assumed to have a bounded support and to be independently and identically distributed with mean zero and variance/covariance matrix  $I_z$ ;  $\sigma_u$  and  $\sigma_\zeta$  are scalars with  $\sigma_u, \sigma_\zeta \geq 0$ .

Noticing that (47) with (48) and (49) defines a conditionally-linear process, we can write the set of equilibrium conditions of the model together with the conditional expectation of (47) in a more compact form

$$E_t \{f(y_{t+1}, x_{t+1}, y_t, x_t)\} = \mathbf{0}, \quad (50)$$

for an appropriately defined vector of function  $f(\cdot)$  where  $y_t$  identifies the non-predetermined variables while the vector  $x_t$  of state variables contains also the vector of exogenous predetermined variables  $z_t$ . Given the processes (47), with (48) and (49), an equilibrium of our model is a sequence for the vector of endogenous non-predetermined variables  $y_t$  and for the state variables  $x_t$  that satisfies (50), given the initial conditions.

Benigno et al. (2010) characterize the solution of the above model and show that a first-order approximation of the solution can be written as

$$\tilde{y}_t = \bar{g}_x \tilde{x}_t,$$

$$\tilde{x}_{t+1} = \bar{h}_x \tilde{x}_t + \bar{h}_\eta \eta_{t+1},$$

for appropriately-defined matrices  $\bar{g}_x$ ,  $\bar{h}_x$  and  $\bar{h}_\eta$ . This approximation does not correspond to a fully linear solution since  $\eta_{t+1}$ , defined, in (48) is non-linear. However, it is the best conditionally linear approximation and, in particular, the matrices  $\bar{g}_x$  and  $\bar{h}_x$  coincide with those of a fully linear approximation. Our first-order approximation maintains heteroskedastic shocks but time-varying volatility does not play a distinct role, meaning that the impulse response of the endogenous variables with respect to the shock to volatility,  $\zeta_{u,t+1}$ , is always zero. The advantage of performing a conditionally-linear approximation instead of a fully-linear approximation, in which  $\eta_{t+1}$  is also linearized, is clear when we look at a second-order approximation of the solution. Benigno et al. (2010) show that this takes the form

$$\tilde{y}_t = \bar{g}_x \tilde{x}_t + \frac{1}{2}(I_y \otimes \tilde{x}'_t) \bar{g}_{xx} \tilde{x}_t + \frac{1}{2} \bar{g}_{uu} u_t^2 + \frac{1}{2} \bar{g}_{zz} \sigma_u^2, \quad (51)$$

$$\tilde{x}_{t+1} = \bar{h}_x \tilde{x}_t + \frac{1}{2}(I_x \otimes \tilde{x}'_t) \bar{h}_{xx} \tilde{x}_t + \frac{1}{2} \bar{h}_{uu} u_t^2 + \frac{1}{2} \bar{h}_{zz} \sigma_u^2 + \bar{h}_\eta \eta_{t+1}, \quad (52)$$

for appropriately defined matrices  $\bar{g}_{xx}$ ,  $\bar{g}_{zz}$ ,  $\bar{g}_{uu}$  and  $\bar{h}_{xx}$ ,  $\bar{h}_{zz}$ ,  $\bar{h}_{uu}$ . In this second-order approximation, the volatility of the exogenous state variables now plays a distinct and direct role through the matrices  $\bar{g}_{uu}$  and  $\bar{h}_{uu}$ . Indeed the endogenous variables are now in a linear relationship with the vector of volatilities,  $u_t^2$ . Other methods discussed in the literature, as Fernandez-Villaverde et al. (2010), need instead to rely at least on a third-order approximation to get such a distinct role for volatilities in influencing the endogenous variables.

The second advantage of our conditionally-linear approximation is that risk premia, evaluated using a first-order approximation of the model, will be also time-varying. This feature enables the model to characterize some stylized facts on the role of volatility on international data in a simple way.

## 5 Exchange rates and risk: a simple example

In this section, before we turn to the solution of our general model, we present a simplified framework to study whether we can already account for some of the facts that we have underlined in the empirical analysis. The framework of this section, with its analytical solutions, will be also helpful to explain how our solution method works and represent a useful benchmark through which we can later evaluate the effects of relaxing the assumptions of this section. The simplifying assumptions are: 1) monetary policy in each country is modeled through Taylor rules reacting only to the domestic CPI inflation rate with the same coefficients across countries, later in the section we allow for interest-rate smoothing; 2) purchasing power parity holds ( $v = v^* = n$ ) 3) flexible prices ( $\alpha = \alpha^* = 0$ ) and constant real rates which make real shocks irrelevant for the analysis of this section. Therefore, we will abstract completely from productivity shocks and give just a monetary explanation of the facts related to the nominal exchange

rate and the UIP deviations.

The starting points are the standard arbitrage-free conditions (26) and (27). As discussed more generally in Benigno et al. (2010), we rely on approximation methods to solve our model. In particular we show that it is sufficient to use a second-order approximation of the model to characterize how risk influences the variables of interest and in particular the exchange rate.

By taking a second-order approximation of (26) and (27), we obtain

$$\hat{i}_t = -E_t \hat{M}_{t+1} - \frac{1}{2} Var_t \hat{M}_{t+1} \quad (53)$$

$$\hat{i}_t^* = -E_t \hat{M}_{t+1}^* - \frac{1}{2} Var_t \hat{M}_{t+1}^* \quad (54)$$

where hats denote log-deviations with respect to the steady state, in which we assume  $i = i^* = 1/\beta - 1$ , and  $E_t$  and  $Var_t$  are conditional expectation and variance operators, respectively.<sup>23</sup> In logs, the complete-market assumption (19) implies

$$\hat{M}_{t+1} = \hat{M}_{t+1}^* - \Delta s_{t+1}. \quad (55)$$

We can combine (53), (54) and (55) to write the short-term excess return of investing in the currency of country  $F$  with respect to investing in the currency of country  $H$  as

$$\hat{i}_t^* + E_t \Delta s_{t+1} - \hat{i}_t = \frac{\vartheta_t^*}{2} - \frac{\vartheta_t}{2} \quad (56)$$

where

$$\vartheta_t = cov_t(\hat{M}_{t+1}, \Delta s_{t+1}) \quad \vartheta_t^* = cov_t(\hat{M}_{t+1}^*, -\Delta s_{t+1}). \quad (57)$$

The intuition for why there can be or cannot be an excess return on foreign currency with respect to domestic currency depends on whether foreign currency is or is not a bad hedge with respect to risk relatively to domestic currency. The standard principle is that an asset is "risky" when it does not pay well when money is really needed. In this case, investors command a premium to hold it which shows up in an excess return relatively to other assets. In our context, the stochastic discount factors measure the agents' appetites for state contingent wealth and therefore when money is needed or not. When  $\hat{M}_{t+1}$  and  $\hat{M}_{t+1}^*$  are high in some contingencies, the appetites for wealth of the Home and Foreign agents are also high in those contingencies. An asset that pays well under this case is a good asset and represents a good hedge with respect to risk. If, for example, the currency of country  $H$  depreciates (the nominal exchange rate depreciates, i.e.  $\Delta s_{t+1} > 0$ ) then having invested in the currency of country  $F$  is indeed a good

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<sup>23</sup>Notice that (53) and (54) do not hold exactly but up to residuals which are of third-order in an appropriate norm on the stochastic disturbances. Under the assumption of log-normality, as in Backus et al. (2010), they would hold exactly. Our analysis is a local analysis and there is a global analysis. Therefore, their approach is limited to the possibility of a closed-form solution. Moreover, the two frameworks will also deliver subtle differences in terms of the conditions needed for the determinacy of the equilibrium.

investment since it delivers more money when it is really needed. In this case  $\vartheta_t$  is positive and  $\vartheta_t^*$  is negative. In general, the expected short-term excess return of investing in the currency of country  $F$  with respect to that of investing in the currency of country  $H$  is negative because simply the currency of country  $H$  is not a good hedge with respect to the appetite for wealth of both agents. In general, to have a negative expected excess return on the Foreign-versus-Home currency it is not necessary that  $\vartheta_t$  should be positive and  $\vartheta_t^*$  negative, but just  $\vartheta_t > \vartheta_t^*$ .

Finally, it is worth stressing that the right-hand side of equation (56) captures the deviations from uncovered interest parity in any model in which no-arbitrage restrictions apply. Indeed, so far, none of the simplifying assumptions 1), 2) and 3) has been used.

## 5.1 Simple Taylor Rules

By making assumption 1) ( $\phi_i = \phi_i^* = \phi_y = \phi_y^* = \phi_s = \phi_s^* = 0$ ,  $\phi_\pi = \phi_\pi^*$  with interest rate reacting to CPI inflation into (45) and (46)), we can further use (56) to determine the equilibrium nominal exchange rate. In particular, the short-term nominal interest rates follow simple Taylor rules in which

$$\hat{i}_t = \bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t) + \xi_t \quad (58)$$

$$\hat{i}_t^* = \bar{\pi}_t^* + \phi_\pi(\pi_t^* - \bar{\pi}_t^*) + \xi_t^* \quad (59)$$

where  $\bar{\pi}_t$  and  $\bar{\pi}_t^*$  represent the logs of Home and Foreign inflation-target shocks and  $\xi_t$  and  $\xi_t^*$  are the Home and Foreign policy shocks as in (45) and (46).

We now use the simplifying assumption 2), that there is no home bias in consumption implying that purchasing power parity holds, i.e.  $\pi_t = \pi_t^* + \Delta s_t$ .

### 5.1.1 Exchange rate determination

Using PPP and rules (58) and (59) into (56) we obtain a first-order stochastic difference equation in  $\Delta s_t$

$$E_t \Delta s_{t+1} = \phi_\pi \Delta s_t + (1 - \phi_\pi)(\bar{\pi}_t - \bar{\pi}_t^*) + (\xi_t - \xi_t^*) - \frac{1}{2}(\vartheta_t - \vartheta_t^*)$$

which can be solved forward to deliver a unique bounded solution for the nominal exchange rate of the form

$$\Delta s_t = E_t \sum_{T=t}^{\infty} \left( \frac{1}{\phi_\pi} \right)^{T+1-t} \left[ (\phi_\pi - 1)(\bar{\pi}_T - \bar{\pi}_T^*) - (\xi_T - \xi_T^*) + \frac{1}{2}(\vartheta_T - \vartheta_T^*) \right], \quad (60)$$

under the requirement, for determinacy, that the Taylor's principle holds, i.e.  $\phi_\pi > 1$ .<sup>24</sup>

There are several implications of the above simple model for nominal exchange rate determination. First, the design of the monetary policy rules is important. Indeed, equation (60) holds

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<sup>24</sup>Necessary and sufficient conditions for the local determinacy of equilibrium are discussed more extensively in Benigno and Benigno (2008), for two-country open-economy models.



only under the special policy rules (58) and (59).<sup>25</sup> Within this class of rules, variation in the policy parameter  $\phi_\pi$  can also change in an important way the relationship between exchange rate and fundamentals. But, which are the fundamentals for exchange rate determination under this simple model? Shocks and risk. Given that  $\phi_\pi > 1$  is needed for equilibrium determinacy, a shock that increases the inflation target in a country depreciates its currency, whereas a contractionary policy shock in a country appreciates its own currency (the sign of the response to the policy shock is consistent with the empirical findings that we reported in section 2). In particular, a (temporary) contractionary policy shock appreciates permanently the exchange rate, but without producing the hump-shaped curve found in the data.

Current and future shocks matter, but also current and future risk premia. If the currency of country  $F$  has relatively good hedge properties with respect to the currency of country  $H$  ( $\vartheta_t > \vartheta_t^*$ ) then currency  $F$  strengthens and current nominal exchange rate  $s_t$  rises.

Equation (60) represents a second-order approximation for the solution of the equilibrium nominal exchange rate which depends on first-order terms  $\{\bar{\pi}_t, \bar{\pi}_t^*, \xi_t, \xi_t^*\}$  and second-order terms  $\{\vartheta_t, \vartheta_t^*\}$ . However, to get an explicit solution for the exchange rate in terms of the state variables, we need to solve the second-order terms. The simplification comes by observing that these second-order terms can be just evaluated using a first-order approximation.<sup>26</sup> In particular, given (57), to evaluate  $\vartheta_t$  and  $\vartheta_t^*$  we need a first-order approximation of the stochastic discount factors  $\hat{M}_{t+1}$  and  $\hat{M}_{t+1}^*$  and also a first-order approximation of  $\Delta s_t$ , which we already have in (60). In the general model of the previous section, the stochastic discount factors  $\hat{M}_{t+1}$  and  $\hat{M}_{t+1}^*$  are complex linear function, in a first-order approximation, of the shocks of the model. In our simple illustrative example, we assume flexible prices and constant real interest rate (assumption 3). In this case, the stochastic discount factors are just exact linear functions of the inflation rates

$$\hat{M}_{t+1} = -\pi_{t+1} \quad \hat{M}_{t+1}^* = -\pi_{t+1}^*.$$

Moreover we assume that the inflation-target shocks behave as random walks with stochastic volatility

$$\begin{aligned} \bar{\pi}_t &= \bar{\pi}_{t-1} + u_{\pi,t-1} \varepsilon_{\pi,t} \\ \bar{\pi}_t^* &= \bar{\pi}_{t-1}^* + u_{\pi,t-1}^* \varepsilon_{\pi,t}^*, \end{aligned}$$

where  $\varepsilon_{\pi,t}$  and  $\varepsilon_{\pi,t}^*$  are *iid* white-noise processes. For the policy shocks we assume

$$\begin{aligned} \xi_t &= u_{\xi,t-1} \varepsilon_{\xi,t} \\ \xi_t^* &= u_{\xi,t-1}^* \varepsilon_{\xi,t}^* \end{aligned}$$

where  $\varepsilon_{\xi,t}$  and  $\varepsilon_{\xi,t}^*$  are *iid* white-noise processes.<sup>27</sup> The variances of the above processes are all

<sup>25</sup>Hodrick (1989) and Obstfeld and Rogoff (2001) restrict their attention to special money-supply rules in which the equilibrium in the money market becomes also relevant for the determination of the exchange rate.

<sup>26</sup>See also Lombardo and Sutherland (2007).

<sup>27</sup>We could surely generalize to autoregressive process for the policy shock, but the most common assumption

time varying following the linear stochastic processes

$$\begin{aligned}
u_{\pi,t}^2 &= \sigma_u^2 + \rho_\pi(u_{\pi,t-1}^2 - \sigma_u^2) + \sigma_\zeta^2 \zeta_{\pi,t} \\
u_{\pi,t}^{*2} &= \sigma_u^2 + \rho_\pi(u_{\pi,t-1}^{*2} - \sigma_u^2) + \sigma_\zeta^2 \zeta_{\pi,t}^* \\
u_{\xi,t}^2 &= \sigma_u^2 + \rho_\xi(u_{\xi,t-1}^2 - \sigma_u^2) + \sigma_\zeta^2 \zeta_{\xi,t} \\
u_{\xi,t}^{*2} &= \sigma_u^2 + \rho_\xi(u_{\xi,t-1}^{*2} - \sigma_u^2) + \sigma_\zeta^2 \zeta_{\xi,t}^*,
\end{aligned}$$

where  $0 \leq \rho_\pi, \rho_\xi \leq 1$  and all the zetas are *iid* white-noise processes while  $\sigma_u^2$  and  $\sigma_\zeta^2$  are non-negative parameters.

Given the above defined processes, and up to a first-order approximation, equation (60) implies

$$\Delta s_t = (\bar{\pi}_t - \bar{\pi}_t^*) - \frac{1}{\phi_\pi}(\xi_t - \xi_t^*) \quad (61)$$

where movements in the inflation-target shocks move one-to-one the nominal exchange rate, while the response of the nominal exchange rate to policy shocks depends on the parameter of the Taylor rules. Using (53) and (58), and (54) and (59) respectively, we can determine the domestic and foreign inflation rates as

$$\pi_t = \bar{\pi}_t - \frac{1}{\phi_\pi} \xi_t \quad (62)$$

$$\pi_t^* = \bar{\pi}_t^* - \frac{1}{\phi_\pi} \xi_t^* \quad (63)$$

which in this simple example only reflect the influence of their own monetary shocks. We can use (61), (62) and (63) to evaluate the risk premia component in (57)

$$\vartheta_t = cov_t(\hat{M}_{t+1}, \Delta s_{t+1}) = -u_{\pi,t}^2 - \frac{1}{\phi_\pi^2} u_{\xi,t}^2$$

$$\vartheta_t^* = cov_t(\hat{M}_{t+1}^*, -\Delta s_{t+1}) = -u_{\pi,t}^{*2} - \frac{1}{\phi_\pi^2} u_{\xi,t}^{*2}$$

which can be plugged into (60) to obtain the equilibrium exchange rate

$$\begin{aligned}
\Delta s_t &= (\bar{\pi}_t - \bar{\pi}_t^*) - \frac{1}{\phi_\pi}(\xi_t - \xi_t^*) - \frac{1}{2} \frac{1}{\phi_\pi - \rho_\pi} (u_{\pi,t}^2 - u_{\pi,t}^{*2}) \\
&\quad - \frac{1}{2} \frac{1}{\phi_\pi - \rho_\xi} \frac{1}{\phi_\pi^2} (u_{\xi,t}^2 - u_{\xi,t}^{*2}).
\end{aligned} \quad (64)$$

In this solution, the time-varying volatilities of the monetary shocks matter for the determination of the nominal exchange rate.<sup>28</sup> This is the important consequence of the solution method

in the literature is that of white-noise processes.

<sup>28</sup>Notice that the terms in  $\sigma_u^2$  cancel out because of the symmetry assumed.

proposed by Benigno et al. (2010) in which a second-order approximation of the model is sufficient to get a distinct role for time-varying uncertainty in affecting the determination of variables of interest. In (64), the higher the variance of the inflation-target and of the policy shocks in country  $H$ , the stronger the currency of country  $H$  is. And specularly for the volatility of the monetary shocks in country  $F$ . These theoretical findings are in part consistent with the empirical results of Section 2: there, we reported that an increase in both volatilities leads to an appreciation of the currency (at least in the medium-run with the exception of the USD/Yen bilateral).

The model is then consistent with the view that more uncertainty can be good for the nominal exchange rate, meaning that the exchange rate can even appreciate when volatility rises. The intuition insists on the good or bad hedging properties of the currency. If a currency is a good hedge with respect to a particular risk and this risk increases, then there is more demand of the currency and its exchange rate appreciates. For example, when the Home inflation target shock falls the appetite for wealth for the Home consumers rises. At the same time the nominal exchange rate appreciates, therefore Home currency delivers more money when needed, relatively to foreign currency. This is good for hedging purposes. When the variance of the Home inflation-target shock rises, the good hedging properties of Home currency are enhanced and therefore the higher demand of Home currency leads to an appreciation.

The magnitude of the effects on the exchange rate depends obviously on the magnitude of the shock but also on the persistence. The higher the persistence the higher the response. It is further influenced by the policy parameter of the Taylor rule, the higher  $\phi_\pi$  the muted the response of the exchange rate. In this symmetric example, as for the primitive shocks, what matters for the determination of the equilibrium exchange rate is the relative strength between the volatilities of the monetary shocks across countries. However, while a positive inflation-target shock and a positive policy shock produce responses of opposite sign on the equilibrium nominal exchange rate, an increase in the volatility of the inflation-target shock or of the policy shock impacts in the same direction.

### 5.1.2 UIP implications

$$\hat{i}_t^* + E_t \Delta s_{t+1} - \hat{i}_t = \frac{1}{2} (u_{\pi,t}^2 - u_{\pi,t}^{*2}) + \frac{1}{2\phi_\pi^2} (u_{\xi,t}^2 - u_{\xi,t}^{*2})$$

The expected excess return of investing in the currency of country  $F$  with respect to that of country  $H$  rises with the increase in the volatilities of the monetary shocks in country  $H$ . Consistently with the discussion of the previous section, a rise in the volatility of both the monetary shocks in country  $H$  enhance the hedging properties of currency  $H$  and reduces those of currency  $F$ . Currency  $F$  requires a premium to be held. While the response of the foreign excess return to an increase in volatility of monetary-policy shock ( $u_{\xi,t}^2$ ) is, at first pass, consistent with the empirical findings in Section 2, an increase in the volatility of the inflation-target shock ( $u_{\pi,t}^2$ ) goes in the opposite direction with what we found in the data.

This is not the only counterfactual result of this section. As discussed in Backus et al. (2010), this stylized framework cannot account for the negative slope coefficient in the UIP regression: the regression of the one-period changes in the nominal exchange rate on the interest rate differential. Using (64), and analogous solutions for the interest rates in the two countries, the coefficient of the UIP regression implied by our model would be

$$\hat{\beta}^{uip} = \frac{Cov(\Delta s_{t+1}, \hat{i}_t - \hat{i}_t^*)}{Var(\hat{i}_t - \hat{i}_t^*)}$$

$$\hat{\beta}^{uip} = \frac{var(\bar{\pi}_t - \bar{\pi}_t^*) + a_{1,u_\pi}^2 \frac{\rho_\pi}{\phi_\pi} var(u_{\pi,t}^2 - u_{\pi,t}^{*2}) + a_{1,u_\xi}^2 \frac{\rho_\xi}{\phi_\pi} var(u_{\xi,t}^2 - u_{\xi,t}^{*2})}{var(\bar{\pi}_t - \bar{\pi}_t^*) + a_{1,u_\pi}^2 var(u_{\pi,t}^2 - u_{\pi,t}^{*2}) + a_{1,u_\xi}^2 var(u_{\xi,t}^2 - u_{\xi,t}^{*2})}$$

where the assumption of a unit-root processes for the inflation-target shocks blows up numerator and denominator, in large samples, to produce a unitary coefficient. However, abstracting from this issue or focusing on small samples, the only possibility for  $\hat{\beta}^{uip}$  to be negative is that  $\rho_\xi/\phi_\pi$  be negative, as shown in Backus et al. (2010). Since assuming  $\rho_\xi < 0$  is not plausible, then in our simplified framework  $\hat{\beta}^{uip}$  is positive and decreasing with  $\phi_\pi$ , the inflation's coefficient in the Taylor rule.

## 5.2 Taylor rules with interest rate smoothing

One natural extension to the previous setting is to consider a model in which the interest rate set by the policy authority moves gradually (interest rates are smoothed over time as in McCallum, 1994, and Backus et al., 2010) so that interest rates depend also on their past value. The modified Taylor's rules take the form

$$\hat{i}_t = \phi_i \hat{i}_{t-1} + (1 - \phi_i)[\bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t)] + \xi_t,$$

$$\hat{i}_t^* = \phi_i \hat{i}_{t-1}^* + (1 - \phi_i)[\bar{\pi}_t^* + \phi_\pi(\pi_t^* - \bar{\pi}_t^*)] + \xi_t^*,$$

to replace (58) and (59). Following the same steps as before, it is possible to show that the equilibrium exchange rate is given by

$$\begin{aligned} \Delta s_t = & -\frac{\phi_i}{\lambda - \phi_i}(\hat{i}_{t-1} - \hat{i}_{t-1}^*) + \frac{\lambda}{\lambda - \phi_i}(\bar{\pi}_t - \bar{\pi}_t^*) - \frac{1}{\lambda}(\xi_t - \xi_t^*) \\ & - \frac{1}{2} \frac{1}{\lambda - \rho_u} \left( \frac{\lambda}{\lambda - \phi_i} \right)^2 (u_{\pi,t}^2 - u_{\pi,t}^{*2}) - \frac{1}{2} \frac{1}{\lambda - \rho_u} \frac{1}{\lambda^2} (u_{\xi,t}^2 - u_{\xi,t}^{*2}). \end{aligned} \quad (65)$$

where we are restricting  $\phi_i$  to be  $0 < \phi_i < 1$  and where  $\lambda \equiv \phi_\pi(1 - \phi_i) + \phi_i$  with the requirement  $\lambda > 1$  for equilibrium determinacy implying again  $\phi_\pi > 1$ . In general allowing for interest-rate smoothing changes also the short-run responses to the shocks and the volatilities but does not change the sign of the response. Responses are obviously changed at longer horizons given the lagged reaction to the interest rate.

The important contribution of assuming interest-rate smoothing is that the negative depen-

dence on lagged interest rates can be such to reduce the coefficient of the UIP regression and eventually to turn it negative, as discussed in Backus et al. (2010). However, it does not change the sign of the responses of the expected excess return on foreign-versus-domestic currency to the volatilities of the monetary shocks.

## 6 Exchange rates and risk: the general case

We now turn to the implications of the more general framework with sticky prices presented in Section 4. First, we investigate the properties of the nominal stochastic discount factor which, as shown in the previous section, is critical to understand the relationship between exchange rate and risk, and to evaluate the risk premia embedded in asset prices.

In our general framework the stochastic discount factor depends on the Epstein-Zin preference specification. Our first result shows a peculiarity of Epstein-Zin preferences in an international context. In closed economy, a standard finding is the irrelevance of Epstein-Zin preferences for quantities and the importance for asset pricing.<sup>29</sup> The irrelevance result can be understood by observing that up to a first-order approximation, Epstein-Zin preferences do not matter for the equilibrium allocation. In contrast, we will show that Epstein-Zin preferences might be also important for quantities, in our two-country open-economy model, since, as shown in equation (30), the cross-country surprises in utility affect the international distribution of wealth. Indeed in a first-order approximation, we obtain

$$\hat{G}_{t+1} = \hat{G}_t + (\gamma - \rho)[(\hat{V}_{t+1} - E_t \hat{V}_{t+1}) - (\hat{V}_{t+1}^* - E_t \hat{V}_{t+1}^*)].$$

where hats denote log-deviations with respect to the steady state. Under expected utility,  $\rho = \gamma$ ,  $\hat{G}_t$  will be constant across time, implying the standard risk-sharing condition which links marginal utilities of nominal income across countries. Instead, with the Epstein-Zin preferences, the cross-country differences in the realization of utility matter for the distribution of wealth. This might have interesting consequences for the equilibrium allocation of quantities.<sup>30</sup>

However, the general-equilibrium flavor of our analysis makes it difficult to keep track of all the effects through analytical solutions. To get further insights and to study the contribution of the Epstein-Zin preferences to the evaluation of risk premia, we now discuss more deeply the properties of the stochastic discount factor. In a first order approximation of (22), the Home-country nominal discount factor can be written as

$$\hat{M}_{t,t+1} = -(\gamma - \rho)(\hat{V}_{t+1} - E_t \hat{V}_{t+1}) + (1 - \rho)(1 - \psi)\Delta \hat{L}_{t+1} - [1 - \psi(1 - \rho)](\Delta \hat{C}_{t+1} + \Delta \hat{A}_{W,t+1}) - \pi_{t+1}, \quad (66)$$

where we have defined  $\hat{L}_t = \ln(1 - L_t)/\ln(1 - L)$  while  $\hat{C}_t$  denotes the deviations of detrended con-

<sup>29</sup>See among others Rudebush and Swanson (2009).

<sup>30</sup>Notice however that in this first-order approximation  $E_t \hat{G}_{t+1} = \hat{G}_t$  and therefore  $\hat{G}_t$  is a local martingale.

sumption with respect to the steady state,  $\hat{C}_t \equiv \ln C_t/A_{W,t} - \ln(C/A_W)$ .<sup>31</sup> Under the expected-utility model,  $\gamma = \rho$ , the stochastic discount factor is a function of consumption growth, which can be decomposed in the growth of detrended consumption and in the growth of world productivity, a function of the CPI inflation rate and of the growth in hours worked. An increase in consumption lowers the stochastic discount factor and the appetite for wealth, for realistic values of the intertemporal elasticity of substitution,  $\rho$ . The impact of the growth in hours worked depends on  $\rho \leq 1$  while an increase in the inflation rate reduces instead unambiguously the appetite for wealth. On top of affecting the equilibrium allocation and therefore the allocation of consumption and labor, as discussed above, Epstein-Zin preferences bring the novelty that also surprises in the indirect utility matter through the term  $(\hat{V}_{t+1} - E_t \hat{V}_{t+1})$ . To get further insights on this component, we take a first-order approximation of the indirect utility (2) and show that we can relate it to the present discounted value of the surprises in consumption and labor

$$\hat{V}_{t+1} - E_t \hat{V}_{t+1} = (1 - \beta) \sum_{T=t+1}^{\infty} \beta^{T-t-1} [\Delta E_{t+1}(\psi(\hat{C}_T + \hat{A}_{W,T}) + (1 - \psi)\hat{L}_T)].$$

where we have defined  $\Delta E_{t+1}(\cdot) = E_{t+1}(\cdot) - E_t(\cdot)$ . In general equilibrium, interaction terms will be quite complex. However, at the cost of losing generality, we can get further insights by looking at a limiting case in which the discount factor,  $\beta$ , is close to the unitary value. In this case, indeed, we show that Epstein-Zin preferences do not matter for the equilibrium allocation of quantities, up to a first-order approximation. Under the assumption  $\beta \rightarrow 1$  we can write

$$\hat{V}_{t+1} - E_t \hat{V}_{t+1} \approx \Delta E_{t+1}(\psi(\hat{C}_{\infty} + \hat{A}_{W,\infty}) + (1 - \psi)\hat{L}_{\infty}),$$

which shows that only the stochastic trend in the respective variables influences the current surprises in utility. However, since  $\hat{C}$  and  $\hat{L}$  are respectively a detrended and a stationary variable, their stochastic trends are zero. The surprises to indirect utility will therefore only depend on the stochastic trend in world productivity

$$\hat{V}_{t+1} - E_t \hat{V}_{t+1} \approx \psi \Delta E_{t+1}(\hat{A}_{W,\infty}) = \psi u_{a,t} \varepsilon_{a,t+1},$$

which displays also time-varying risk. The importance of this factor in (66) will be higher, the larger the difference between  $\gamma$  and  $\rho$ . Under this particular case, the ability of Epstein-Zin preferences to explain risk premia hinges upon the comovements between returns and the nominal stochastic discount factor. In particular when agents have a preference for an early resolution of uncertainty, i.e.  $\gamma > \rho$ , a negative shock to world productivity  $\varepsilon_{a,t}$  implies bad news with respect to long-run consumption which are reflected in bad news on utility. In this case, the stochastic discount factor rises and the appetite for state contingent wealth too. This mechanism would apply also to the country  $F$ . Indeed, it is also true that the surprise in the utility of the foreign country depends on the shifts in the long-run component of world

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<sup>31</sup>The balance growth path of the model is defined with respect to the common trend in productivity,  $A_W$ .

productivity

$$\hat{V}_{t+1}^* - E_t \hat{V}_{t+1}^* \approx \psi \Delta E_{t+1}(\hat{A}_{W,\infty}) = u_{a,t} \varepsilon_{a,t+1}.$$

Under the case  $\beta \rightarrow 1$ , Epstein-Zin preferences might therefore contribute to imply highly correlated discount factors across countries and deliver a global explanation for the risk premia, which will be time-varying and driven by the shocks to the common technological process. The consequence of this result is indeed that, up to a first-order approximation, general equilibrium effects will be shut down. Since surprises in the utility of the Home and Foreign country are highly correlated then, using (29) and (30),  $G_t$  is approximately constant over time<sup>32</sup>

$$\begin{aligned} \hat{G}_{t+1} &= \hat{G}_t + (\gamma - \rho)[(\hat{V}_{t+1} - E_t \hat{V}_{t+1}) - (\hat{V}_{t+1}^* - E_t \hat{V}_{t+1}^*)] \\ &\approx \hat{G}_t. \end{aligned}$$

This is true up to a first-order approximation, but not in a second-order approximation where it might be possible that EZ preferences have sizeable effects also on quantities.

## 6.1 Quantitative Evaluation

We now move to a quantitative evaluation of the model implications. In particular, a second-order approximation of the model will be relevant to study the relationship between risk and the exchange rate, and provide a quantitative assessment of such links. This will be implicit in the general solution of the nominal and real exchange rate

$$\begin{aligned} \Delta \hat{S}_t &= \bar{g}_x^s \tilde{x}_t + \frac{1}{2} \tilde{x}_t' \bar{g}_{xx}^s \tilde{x}_t + \frac{1}{2} \bar{g}_{uu}^s u_t^2 + \frac{1}{2} \bar{g}_{zz}^s \sigma_u^2 \\ \hat{Q}_t &= \bar{g}_x^q \tilde{x}_t + \frac{1}{2} \tilde{x}_t' \bar{g}_{xx}^q \tilde{x}_t + \frac{1}{2} \bar{g}_{uu}^q u_t^2 + \frac{1}{2} \bar{g}_{zz}^q \sigma_u^2 \end{aligned}$$

where the index  $i = s, q$  selects appropriate elements of the respective vector or matrices. In this solution, time-varying uncertainty for the stochastic disturbances of the model affects linearly the nominal and real exchange rates through the factors  $\bar{g}_{uu}^i$ .

### 6.1.1 Calibration

In this section we describe our baseline calibration for the general model. The strategy that we adopt for the calibration exercise is to rely as much as possible on a standard values for the parameters and conduct a sensitivity analysis on those for which there are divergences in the literature. We assume that the Home and Foreign economy are of equal size and are calibrated in a symmetric fashion. In this calibration section we think about our two-country world as U.S. versus the Euro area abstracting then from asymmetries that might be important for

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<sup>32</sup>The statement is true under the assumption that  $\beta$  is close to the unitary value, up to a first-order approximation and independently of the values assumed by the parameters  $\gamma$  and  $\rho$ .

understanding some empirical regularities when it comes to small open economies.<sup>33</sup>

In choosing the parameters of utility function, we set  $\beta$  to 0.994 consistent with other studies with Epstein-Zin preferences (e.g. Fernandez-Villaverde et al., 2010). We set the inverse of the intertemporal elasticity of substitution  $\rho$  to 2, implying an intertemporal elasticity of substitution (IES) in consumption of 0.5, which is consistent with estimates in the micro literature (e.g., Vissing-Jorgensen, 2002) and used also in the international real business cycle literature as in Stockman and Tesar (1995). We set the coefficient of relative risk aversion  $\gamma$  to 5 as in Backus et al. (2010). We set the share of consumption in the utility bundle,  $\psi$ , to 1/3 as in Cooley and Prescott (1995) in order to imply that in the steady state households devote one-third of their time to work.

We calibrate the parameters pertaining to the consumption basket in the following way. The share of home goods in tradable consumption,  $\nu$ , is set to 0.87. The elasticity of substitution between home and foreign traded goods,  $\theta$ , is assumed equal to 1.5 which is in the range of the plausible values.

We set the firms' output elasticity with respect to labor,  $\varphi$ , to 2/3, and the elasticity of substitution among differentiated goods,  $\sigma$ , to 6 (implying a steady state markup of 20%) and  $\alpha = 0.66$  and  $\alpha^* = 0.75$  implying an average length of price contracts equal to 3 and 4 quarters, respectively; all these are standard in the literature and consistent with the posterior estimates for the U.S. and Euro area by Lubik and Schorfeide (2005).

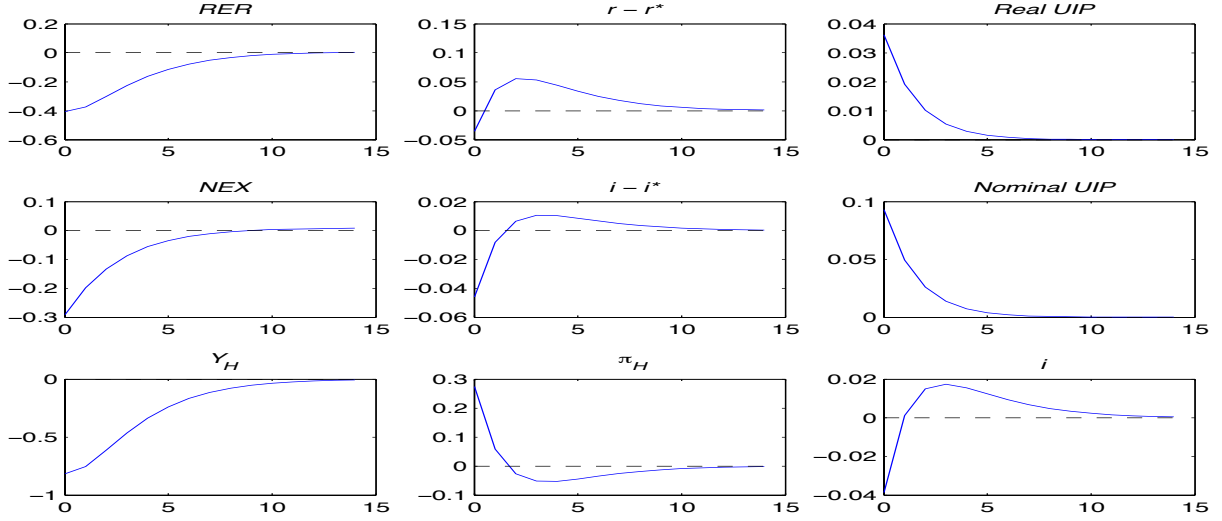
Regarding the policy rules we assume  $\phi_i = 0.76$ ,  $\phi_\pi = 1.41$ ,  $\phi_s = 0.03$  and  $\phi_y = .66$  for the U.S. economy and  $\phi_i^* = 0.84$ ,  $\phi_\pi^* = 1.37$ ,  $\phi_s^* = 0.03$  and  $\phi_y^* = 1.27$  for the Euro area that corresponds to the posterior estimates that Lubik and Schorfeide (2005) have found for the U.S. and Euro area respectively.<sup>34</sup>

We now turn to the calibration of the stochastic processes. For the productivity shocks we use the posterior estimates of Lubik and Schorfeide (2005) for the U.S. and Euro-area:  $\delta_A = 0.83$ ,  $\delta_A^* = 0.85$  with  $\sigma_A = 1.66$  and  $\sigma_A^* = 2.71$  as the values through which we scale the individual standard deviation for the home and foreign shocks respectively. We assume no persistence for the policy shocks and we scale its standard deviation by  $\sigma_\xi = 0.18$  for both countries based on the estimates of Lubik and Schorfeide (2005). For the inflation-target shocks we follow Ireland (2007) and set it to  $\sigma_\pi = 0.1$ . For the persistence of the volatility shocks, we calibrate the autocorrelation coefficients at the values implied by fitting an AR(1) process for each of the three time-series employed in the empirical part. As a consequence, we set  $\rho_{aw} = \rho_a = \rho_{a^*} = .71$ ,  $\rho_\pi = \rho_{\pi^*} = .67$  and  $\rho_\xi = \rho_{\xi^*} = .53$ .

<sup>33</sup>Relevant asymmetries could be in terms of a policy rule that reacts to exchange rate for small open economies or different degree of openness that affect critically the international transmission mechanism of shocks.

<sup>34</sup>Although we specified a theoretical model in which monetary-policy credibility might possibly play a role, we disregard this role in the present work, and accordingly parameterize  $\kappa = \kappa^* = 1$ .





**Figure 10:** Dynamic responses to a monetary-policy volatility shock (innovation to the *volatility* of the monetary policy *instrument*). The panels show: real exchange rate (RER), real interest rate differential ( $r - r^*$ ), deviations from Real UIP, nominal exchange rate (NEX), nominal interest rate differential ( $i - i^*$ ), deviations from Nominal UIP, domestic output ( $Y_H$ ), domestic inflation ( $\pi_H$ ), domestic short-term nominal interest rate ( $i$ ).

### 6.1.2 Results

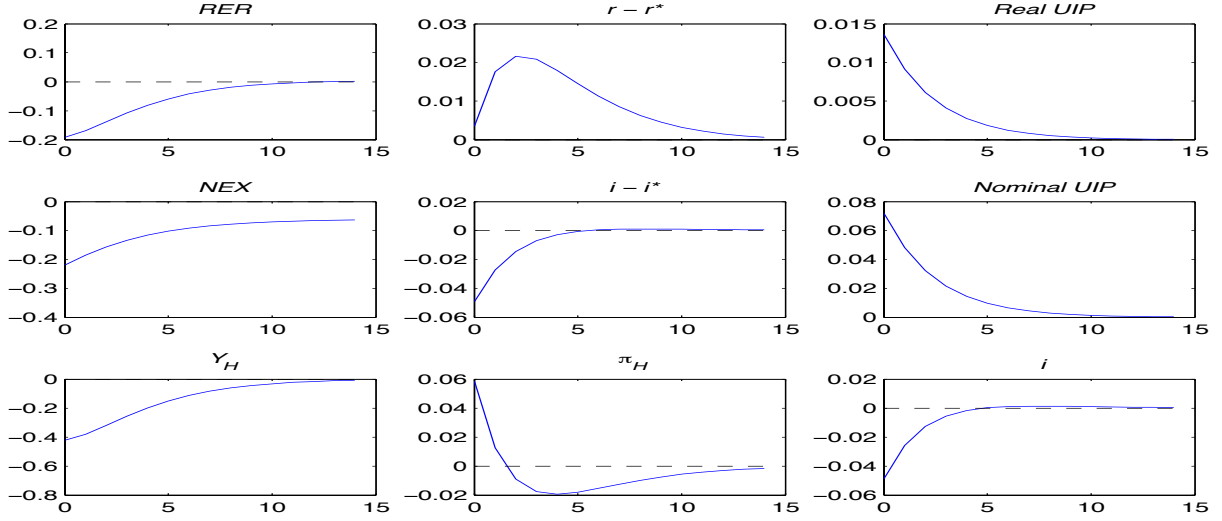
In this Section we evaluate to what extent our two-country model with recursive preferences and stochastic volatility can replicate the dynamic properties of the data found in Sections 2 and 3. Our analysis is mainly qualitative as we compare our model-based impulse response with the ones generated by the VAR. In what follows, we plot impulse response of the main variables of interest: real exchange rate (RER), real interest rate differential ( $r - r^*$ ), deviations from real uncovered interest-rate parity (Real UIP), nominal exchange rate (NEX), nominal interest rate differential ( $i - i^*$ ), nominal uncovered interest-rate parity (Nominal UIP), and domestic output ( $Y_H$ ), producer inflation ( $\pi_H$ ) and Home nominal interest rate ( $i$ ).<sup>35</sup>

In particular, we identified two main regularities on the relationship between exchange rate and risk: (i) an increase in the volatility of both monetary-policy and inflation-target shocks appreciates the exchange rate, while an increase in the volatility of the productivity shock induces an exchange rate depreciation; (ii) an increase in the volatility of the monetary-policy shock leads to deviations from UIP in the form of an increase in the excess return on foreign-versus-domestic currency while an increase in the volatility of the inflation-target shock leads to a fall in the excess return.

An additional regularity, originally documented by Eichenbaum and Evans (1995) and that we also confirm by controlling for the effects of time-varying volatility, is that a contractionary monetary-policy shock produces a persistent appreciation of the exchange rate and persistent deviations from the UIP in the form of positive excess returns on domestic securities.

Figures 10 and 11 display the dynamic response of our variables of interest to volatility

<sup>35</sup>For ease of comparison with the empirical analysis, we normalize the size of each shock to the one featured in the VAR analysis.

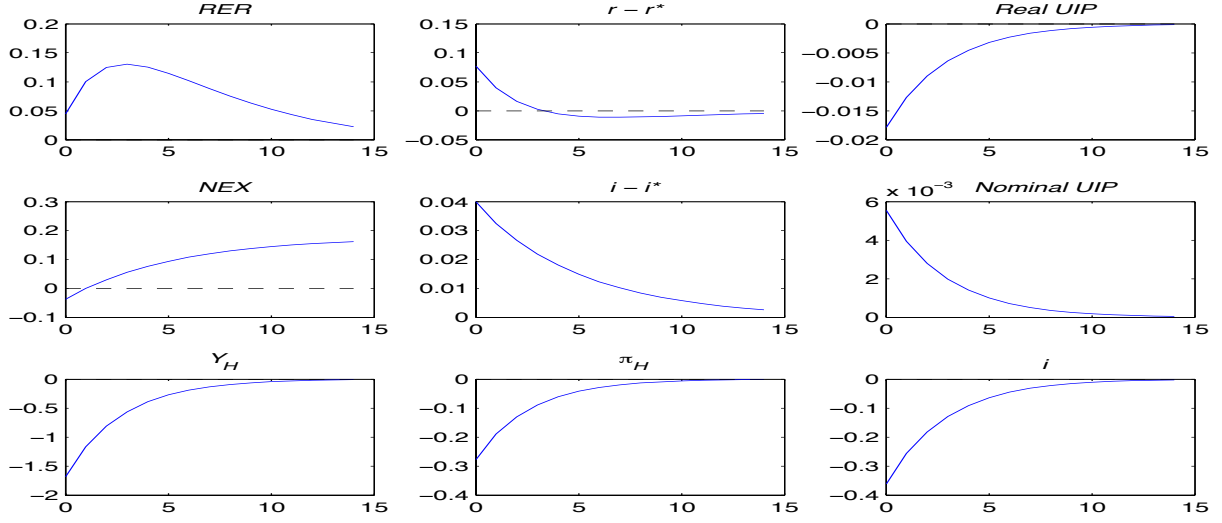


**Figure 11:** Dynamic responses to an inflation-target volatility shock (innovation to the *volatility* of the inflation target). The panels show: real exchange rate (RER), real interest rate differential ( $r - r^*$ ), deviations from Real UIP, nominal exchange rate (NEX), nominal interest rate differential ( $i - i^*$ ), deviations from Nominal UIP, domestic output ( $Y_H$ ), domestic inflation ( $\pi_H$ ), domestic short-term nominal interest rate ( $i$ ).

shocks hitting the monetary-policy instrument and the inflation target, respectively. The figures show that the model is indeed able to imply an appreciation of the real exchange rate and deviations from the UIP in the form of positive excess returns from investing in foreign-currency denominated bonds, consistently with our empirical findings both in real and in nominal terms. A rise in home nominal volatility tends also to reduce domestic output and increase domestic producer inflation while the domestic nominal interest rate declines, proportionately more than the foreign one. An interesting difference among the two nominal shocks arises in the response of the real interest rate differential. In the case of the shock to volatility of the inflation target, the real interest rate differential is positive on impact and increasing in the short run while it is negative on impact following a shock to the volatility of the monetary instrument. This difference arises because the volatility shock to the monetary instrument generates more inflation than the shock to the inflation target.

Figure 12, similarly shows the dynamic response of Real Exchange Rate and the deviation from UIP to a volatility shock hitting global productivity. The asymmetries in the degrees of price stickiness and the response coefficients of the policy rules imply that innovations in the level and/or volatility of the global productivity shocks are able to produce a non-zero response on international variables. In particular, in response to an increase in the volatility of the global productivity shock, the RER depreciates and we observe positive deviations from nominal UIP consistent with the sign of the response that we observe in our empirical findings. However, the nominal exchange rate appreciates on impact. Therefore, the movements in the real exchange rate are mainly driven by changes in domestic CPI inflation.

All these results are qualitatively consistent with the empirical regularities *Fact 1* and *2*, and also show that our approximation method is effective to study the link between time-varying

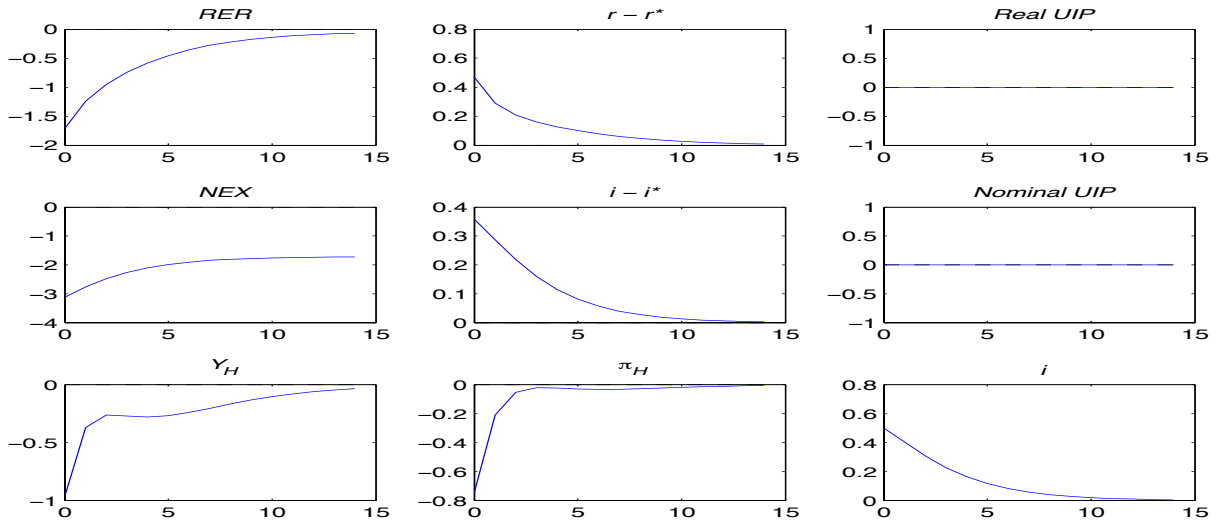


**Figure 12:** Dynamic responses to an innovation to the volatility of the productivity shock. The panels show: real exchange rate (RER), real interest rate differential ( $r - r^*$ ), deviations from Real UIP, nominal exchange rate (NEX), nominal interest rate differential ( $i - i^*$ ), deviations from Nominal UIP, domestic output ( $Y_H$ ), domestic inflation ( $\pi_H$ ), domestic short-term nominal interest rate ( $i$ ).

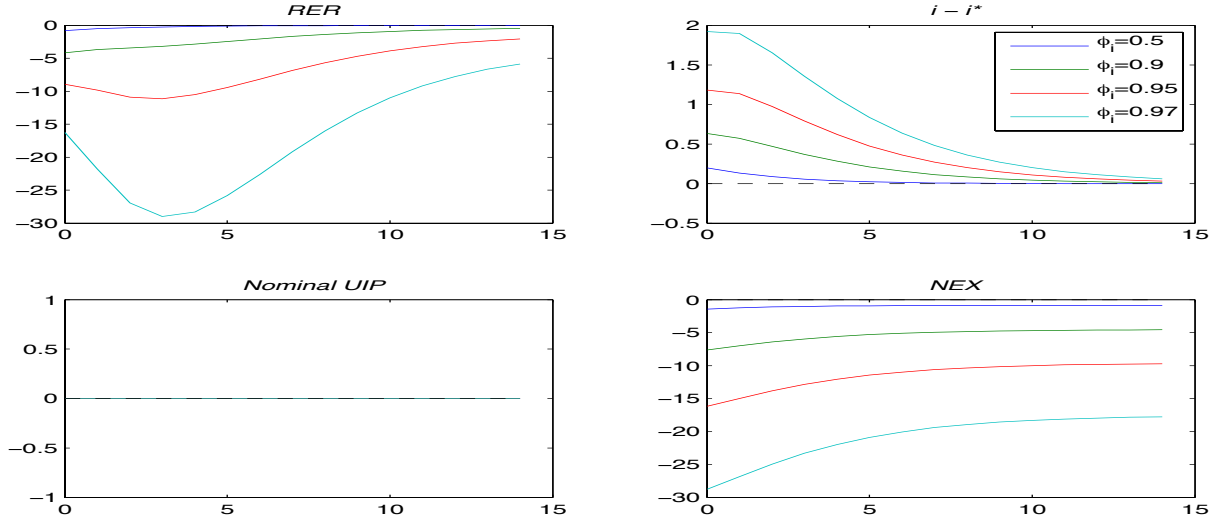
volatility and the endogenous variables, like the exchange rate.

As to *Fact 3*, related to the effects of monetary-policy *level* shocks on the exchange rate and UIP deviations, Figure 13 shows that a contractionary monetary-policy shock indeed implies an appreciation of the RER but it is unable to generate the hump-shaped response that we observe in the data, nor deviations from the UIP.

In order to look deeper into this result, we next explore which element of the theoretical model is responsible for this behavior and whether a different calibration would lead to the



**Figure 13:** Dynamic responses to a monetary policy shock. The panels show: real exchange rate (RER), real interest rate differential ( $r - r^*$ ), deviations from Real UIP, nominal exchange rate (NEX), nominal interest rate differential ( $i - i^*$ ), deviations from Nominal UIP, domestic output ( $Y_H$ ), domestic inflation ( $\pi_H$ ), domestic short-term nominal interest rate ( $i$ ).



**Figure 14:** Dynamic responses to a monetary policy shock (level): the role of interest-rate smoothing. The panels show: real exchange rate (RER), nominal interest rate differential ( $i - i^*$ ), deviations from Nominal UIP, nominal exchange rate (NEX).

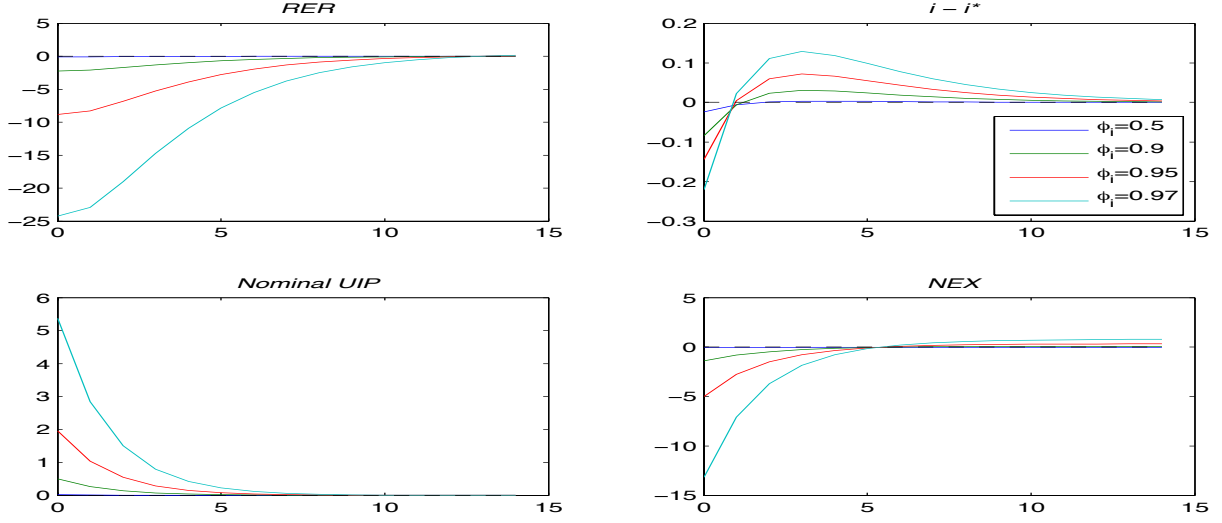
persistent appreciation observed in the data.

Figures 14 through 16 perform this task by displaying the dynamic responses of the variables of interest to a monetary-policy shock (level shock) and to volatility shocks, respectively, and for different degrees of monetary policy inertia, as measured by the smoothing parameter  $\phi_i$  in equation (45).

Specifically, Figure 14 displays the dynamic response of the economy to a monetary-policy *level* shock, which raises the interest rate differential, and it shows that for high enough degrees of interest-rate smoothing, the model is indeed able to imply a substantial degree of persistence in the real appreciation. On the other hand, increasing the inertia in the monetary-policy rules does not imply significant deviations from the UIP. This result, however, is not at all surprising, as it is common to any rational-expectations open-economy model with no financial frictions, where the Uncovered Interest Rate parity holds up to a first-order approximation. As a consequence, the increase in the interest rate differential implied by the domestic monetary-policy shock is offset by the nominal depreciation which follows the initial appreciation: UIP holds and the model fails to reproduce the hump-shaped response of the nominal exchange rate.

With respect to this latter point, however, we know that in our model deviations from the UIP can be implied by second-order terms and in particular by stochastic volatility, as shown analytically in the simple case of Section 5. Figures 15 and 16, then, show the role of interest-rate smoothing in shaping the response of deviations from the UIP following volatility shocks on the monetary policy instrument and target. As the graphs clearly document, for both cases of volatility shocks, the response of the excess return on foreign-versus-domestic currency monotonically increases with the coefficient of interest-rate smoothing, as expected. A higher policy inertia, moreover, is also able to amplify the nominal and real exchange rate appreciation.

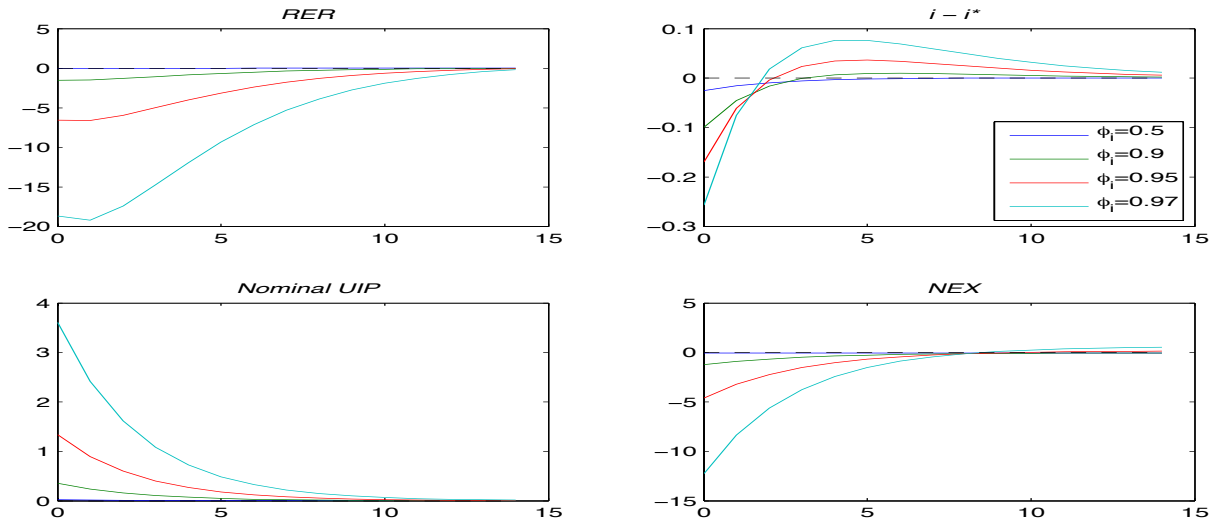
An additional test for our model would be to see how it performs in terms of the UIP puzzle:



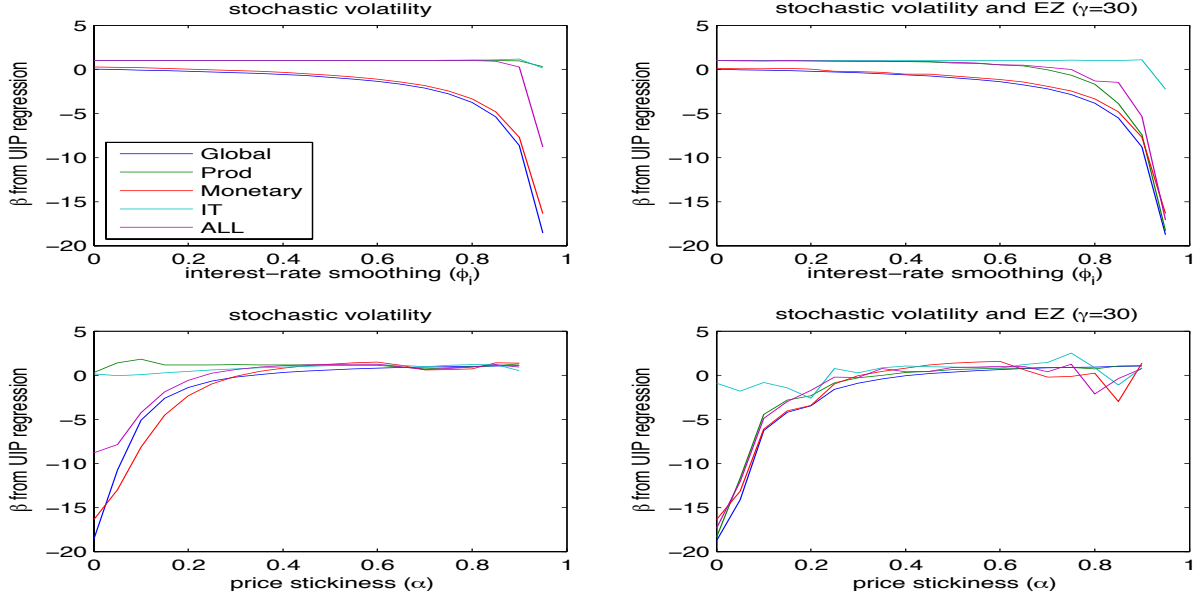
**Figure 15:** Dynamic responses to a shock to the monetary-policy volatility ( $u_\xi^2$ ): the role of interest-rate smoothing. The panels show: real exchange rate (RER), nominal interest rate differential ( $i - i^*$ ), deviations from Nominal UIP, nominal exchange rate (NEX).

the negative slope of the regression between nominal exchange-rate changes and the interest-rate differential.

In Figures 15–16, we show that the interaction between interest-rate smoothing and stochastic volatility is able to produce persistent deviations from the UIP. The natural next step is to see to what extent such deviations are consistent with a negative slope in the UIP regression, and what are the theoretical factors that, within the model, can have an effect on it. We study this issue by simulating the theoretical model and computing the moments of interest from the simulated time series.



**Figure 16:** Dynamic responses to a shock to the volatility of the inflation target ( $u_\pi^2$ ): the role of interest-rate smoothing. The panels show: real exchange rate (RER), nominal interest rate differential ( $i - i^*$ ), deviations from Nominal UIP, nominal exchange rate (NEX).



**Figure 17:** The slope of the UIP regression: the role of interest-rate smoothing (top panels) and price stickiness (bottom panels).

Figure 17 studies the slope of the UIP regression under different parametrizations. The top panels display the interaction among stochastic volatility, interest-rate smoothing and Epstein-Zin preferences, in a flexible-price economy. The bottom panels instead studies the role of price stickiness and its interaction with stochastic volatility and Epstein-Zin preferences, for a given degree of interest-rate smoothing ( $\phi_i = 0.95$ ). The other parameters are calibrated as discussed earlier.

Three main implications arise from Figure 17:

*i)* the interaction between stochastic volatility and interest-rate smoothing can drive the negative covariance between nominal exchange-rate changes and interest-rate differential that is observed in the data. For this result, stochastic volatility is a necessary ingredient of the model. The effect of interest-rate smoothing on the slope of the UIP regression, however, can vary quite a bit depending on the specific type of shock to which we condition the simulation of the model: in particular, the effects of raising monetary-policy inertia on the covariance between nominal-exchange-rate depreciations and interest-rate differentials are stronger conditional on monetary policy and global productivity shocks, while smaller impact is implied by conditioning on inflation-target shocks or idiosyncratic productivity shocks. The result on monetary-policy and inflation-target shocks are qualitatively consistent with the simple case discussed in section 5: the unit root in the process for the inflation target tends to drive the slope toward unity in large samples, regardless of the degree of interest-rate smoothing, while a negative correlation between exchange-rate changes and interest-rate differentials arises following monetary-policy shocks. The asymmetric calibration of the policy rules implies that even a global productivity shock can have implications for international relative variables. The degree of interest-rate

smoothing in the policy rules can again play a key role in driving the slope of the UIP regression.

*ii)* high degrees of price stickiness, on the contrary, tend to drive the slope of the UIP regression toward the unitary value, even for a high degree of interest-rate smoothing (calibrated at 0.95). Moderate degrees of price stickiness, however, are still consistent with a negative covariance between nominal exchange-rate changes and interest-rate differential, provided that the degree of monetary policy inertia is sufficiently strong. This result holds conditional on monetary and global productivity shocks, and fades instead away if we condition on inflation-target and idiosyncratic productivity shocks, consistently with implication *i*);

*iii)* deviating from Expected Utility has little but beneficial effects on both respects: the effect of monetary-policy inertia becomes stronger also conditional on idiosyncratic productivity shocks and even on inflation-target shocks, while moderate degrees of price stickiness are now consistent with a negative slope in the UIP regression also conditional on country-specific productivity shocks.

It is worth noticing, that none of the above results would arise in a model without stochastic volatility, in which case the slope of the UIP would always be one: stochastic volatility is therefore a necessary ingredient to understand these regularities.

Another relevant empirical regularity that is connected to the UIP puzzle has been recently pointed out by Engel (2010) and is related to the behavior of the *level* of the *real* exchange rate: Engel (2010) shows that when a country real interest rate is high (relative to the foreign one), then its currency tends to be stronger in real terms than what would be implied by the *real* uncovered interest rate parity. As discussed in Engel (2010) this observation poses a challenge for the models that have been designed to address the UIP puzzle in nominal terms. Indeed, while matching the empirical co-movement between real interest-rate differentials and real-exchange-rate expected one-period changes, most of the existing models fail to capture the sign of the covariance between real interest-rate differentials and the *level* of the real exchange rate.

We focus on the impulse response to volatility shocks, since these are the shocks that in our model can generate deviations from nominal or real UIP. Our impulse response analysis suggests that, conditionally on a shock to the volatility of the inflation target, the real exchange rate appreciates on impact while the real interest rate differential is positive (see Figure 11): this pattern is consistent with Engel's evidence. However, conditional on the same shock, the exchange rate would depreciate in its adjustment path, contradicting the evidence in Engel (2010). Moreover, the current real interest-rate differential, following a shock to the volatility of the inflation target, is positively related to both current and future deviations from UIP, while in Engel's findings it covaries negatively with the short-run deviations from UIP.

There are three main caveats that are important to keep in mind when looking at our model-based impulse responses to assess our model's ability to replicate Engel's findings. First, Engel's finding in terms of the behavior of the real exchange rate are based on a VAR-estimate of real interest rates, where the VAR model considers only a subset of the variables involved in our theoretical model ( $Q_t$ ,  $i_t - i_t^*$  and  $\pi_t - \pi_t^*$ ). Second, the estimates in Engel (2010) are based on

a *linear* projection while our approach emphasizes the importance of *second-order* moments for exchange rate determination. Third, and most important, the puzzle discussed in Engel (2010) is related to *unconditional* covariances – the linear-regression coefficients of the real exchange rate on the interest-rate differential at various time-horizons – while our model-based impulse-response functions only reflect co-movements *conditional* on specific shocks.

Therefore, while looking at the model-implied impulse-response functions is useful at first pass, a proper analysis of the evidence discussed by Engel would require to use our theoretical model to simulate the relevant time series, and then estimate the same VAR and construct the same statistics that are presented and discussed in Engel (2010), and that are at the heart of the puzzle. Also, it would be interesting to check the robustness of Engel’s findings from an empirical point of view by augmenting his VAR specification with volatility measures, to make it consistent with the implications of our approach to exchange rate determination. We plan on pursuing this research avenue in future works.

## 7 Conclusion

Time-variation in uncertainty and risk can be an important source of fluctuations for macroeconomic variables and in particular for the exchange rate. Using a standard open-economy VAR, we have provided new evidence on the importance of both real and nominal volatility shocks for the behavior of the nominal and real exchange rate. These findings complement the well-know evidence, documented by several studies, based on the UIP regression. Under rational expectations, the negative regression coefficients found in these works can be interpreted as variation over time in risk premia. Time-variation in uncertainty can be also an important source of the variation over time in risk premia.

Our VAR analysis shows that a rise in the volatilities of the nominal shocks appreciates the dollar exchange rate especially in the medium run. On the other hand, an increase in the volatility of the real shock (productivity) has the opposite effect. Moreover, a rise in the volatilities of the nominal shocks generates significant and persistent deviations from UIP and in particular an increase in the excess returns of foreign short-term bonds. We also investigate the response of the slope of the term structure to volatility shocks and find that both real and nominal shocks steepen the term structure. Finally, we also confirm the evidence reported by Eichenbaum and Evans (1995) that a positive innovation to the level of the monetary-policy shock (contractionary policy shock) produces a persistent appreciation in both the real and nominal exchange rates and persistent deviations from the UIP in the form of positive excess returns on US securities.

We propose a New-Keynesian open-economy model as a unifying framework for reconciling these findings in a general equilibrium model with time-varying uncertainty.

Our model is successful along some dimensions. The key element is the specification of monetary policy through interest rate rules and in particular the smoothing coefficient relating



current to past interest rates in the rule. The smoothing coefficient together with price stickiness is important to produce an hump-shaped response of the real exchange rate to the level interest-rate shock and combined with time-varying uncertainty can capture a negative coefficient in the UIP regression. Among the other factors that affect critically the coefficient in the UIP regression, higher nominal rigidities do not help while an increase in risk aversion improves the results. In this sense, allowing for Epstein-Zin preferences that disentangle intertemporal elasticity of substitution and risk aversion is an important feature of our framework. However, at a first look, it is not clear that Epstein-Zin preferences, in a general equilibrium, maintain their appeal to explain some puzzles in asset pricing as in other partial equilibrium analysis. This is an issue that needs further investigation.

We consider this work as a primal approach for the analysis of time-varying uncertainty in open economies because of the methodology that we use for its solution and the general features that we allow for in the model. However, there are several limitations. First, our model, as any framework in which UIP holds up to a first-order approximation, cannot produce an hump-shaped response of the nominal exchange rate to a policy shock, but only of the real exchange rate. Directions to explore could be in the form of financial frictions or departures from rational expectations. Second, there are several tensions between the parameter values of the model relevant to match one fact or another. We cannot claim a complete success on all directions simultaneously nor we did analyze a full match of the model with the data. Finally, related to the latter point, we have calibrated the parameters of our model based on empirical studies building on first-order approximations of the model. This is in contrast with the message of our work that second-order terms are important. Therefore, the estimation of the model is really needed to evaluate its fit. To this purpose, an appropriate methodology should be elaborated to handle the features of our general second-order approximated solutions. We leave this research for future work.

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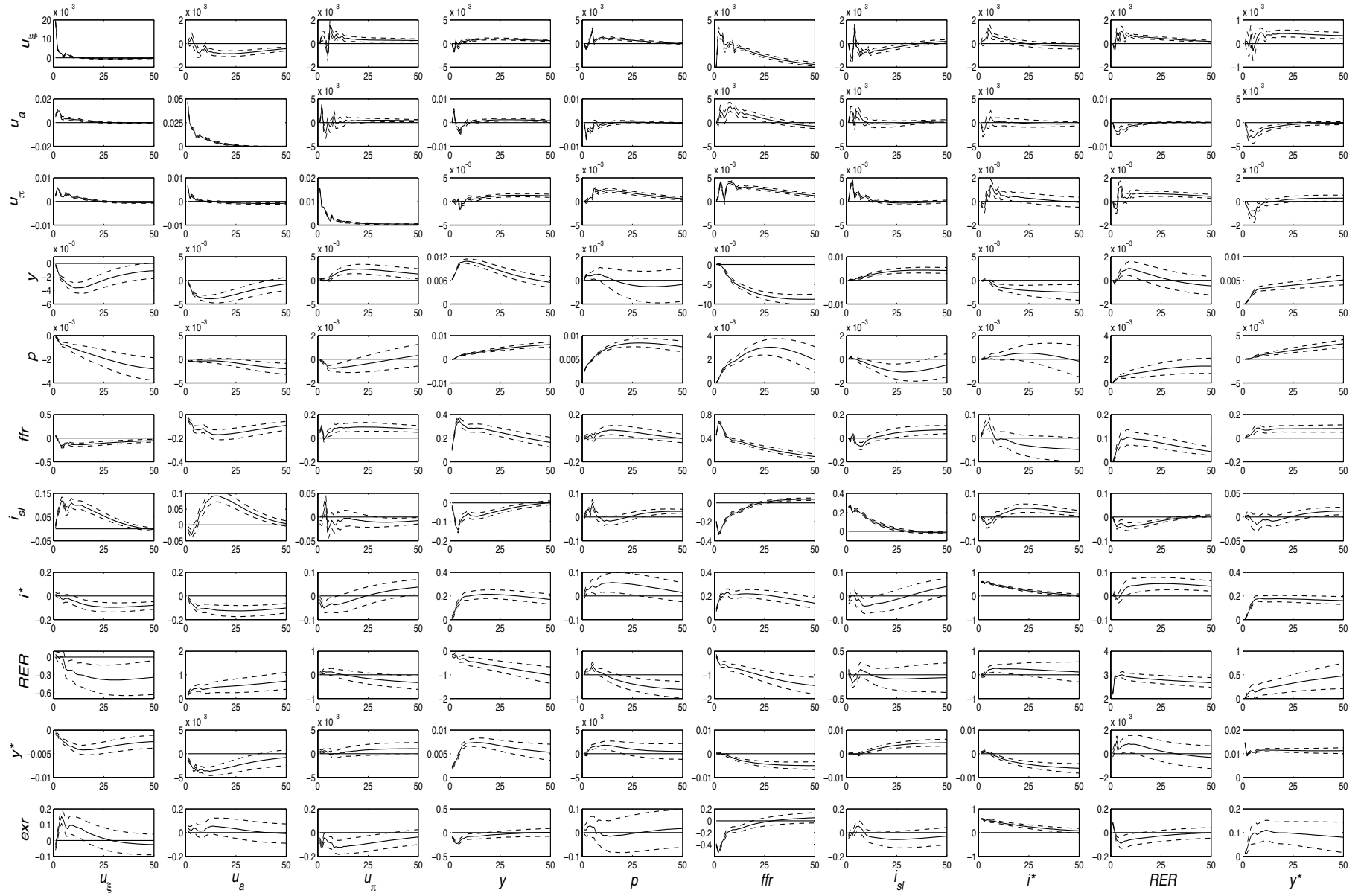


Figure 18: Impulse Response Functions: Pooled Panel VAR.