Risk, Monetary Policy and The Exchange Rate*

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1 Introduction

Variation in risk over time is essential for understanding exchange rates. The large biases in the foreign exchange forward premium that have been documented since Bilson (1981) and Fama (1984) constitutes a compelling evidence for variation in risk premia as a rational-expectations explanation on the link between exchange rates and interest rates. Indeed, while uncovered interest parity predicts that high interest rate currencies should depreciate compared to low interest rate ones, a large body of empirical evidence has shown that among developed economies high interest rate currencies tend to appreciate demanding a higher expected return on their short term assets. [1]The time-varying risk interpretation on the existence of the excess return in foreign exchange market is further documented by a recent work Menkhoff, Sarno, Schmeling and Schrimpf (2010): these authors show that deviations from uncovered interest rate parity can be accounted as a compensation for risk and they identify global foreign exchange volatility as a key risk factor.

Our research proposes a theory of exchange rate determination based on exogenous risk factors in which the link between risk factors and the nominal exchange rate is guided by interest rates (i.e. monetary policy).[2] The aim of this work is study the role of exogenous risk factors in addressing the main regularities that we observe in international finance. To this purpose, we depart from most of the existing models of exchange rate determination that study the impact of the first moments of exogenous variables on the nominal exchange rate and examine the exchange rate's response to an increase in the uncertainty of monetary policy, its target and the rate of technological change.[3] Moreover, the structure upon which we build our analysis between risk factors and exchange rates is a theory of nominal exchange rate determination based on interest rate rules (Benigno and Benigno, 2008).[4].

This research contributes to the literature from an empirical and a theoretical perspective. In our empirical analysis we provide evidence that justifies our focus on risk factors: the novelty of our contribution is to examine the role of nominal and real stochastic volatilities for the behavior of exchange rates in an otherwise standard open-economy VAR. We find that volatility shocks do matter for the equilibrium level of interest and exchange rates and that the exchange rate tends to appreciate in response to an increase in nominal volatility (both of the discretionary shock to monetary policy and of the inflation target) and decreases following an increase in real volatility (of the productivity shock). Importantly the stylized facts reported by Eichenbaum and Evans (1995) about the response of interest and exchange rates to a shock to the level of

[1]Engel (2010) provides further evidence on this and by looking at the real version of uncovered interest rate parity shows that this expected appreciation of high-yield currencies is combined with a relatively stronger currency.

[2]In what follows we refer to risk, uncertainty and stochastic volatility in an interchangeable way.

[3]Hodrick (1989) and Obstfeld and Rogoff (2001) in a flexible and sticky prices environment respectively relate the nominal exchange rate to monetary uncertainty through alternative specification of money demand.

[4]This implies that the parameters of the policy rules (as opposed to preferences to money demand) becomes crucial in shaping exchange rate dynamics and in determining to what extent nominal or real disturbances matter for the nominal exchange rate.

the monetary policy instrument are not affected by the explicit consideration of time-varying volatility elements in the VAR.

From a theoretical perspective, we address these results and the link between time-varying risk and exchange rates in a two-country open-economy model along the lines of Benigno and Benigno (2008) extended in two dimensions. In Benigno and Benigno (2008), we assume differentiated home and foreign produced goods, international market completeness, nominal price rigidities and interest rate feedback rules: here we allow for a more general specification of preferences as in Epstein and Zin (1989) and for stochastic volatility in the exogenous processes driving the economy.

In this direction, our contribution to the literature is to provide a general-equilibrium perspective on the ability of currently used models with stochastic volatility in explaining international macro finance facts. [5] From a modelling point of view, the general equilibrium perspective is crucial for examining the transmission mechanism of risk factors and generating a non-trivial interaction between shocks and variable of interests. From an empirical perspective, the general equilibrium analysis allow us to compare the model performance with the shocks and factors highlighted in the VAR.

The assumption of time-varying exogenous uncertainty entails non-trivial issues in the solution of the model. To this end, we apply a new method that we have recently developed for general dynamic stochastic models with time-varying uncertainty (Benigno, Benigno and Nistico, 2010). By considering conditionally linear stochastic processes for our exogenous disturbances we can approximate the equilibrium conditions of the model up to second-order so as to retrieve a distinct and direct effect of risk factors on the endogenous variables. Recent works have emphasized the need of relying on a third-order approximation in order to capture a relevant role for uncertainty (see Fernandez-Villaverde and Rubio-Ramirez, 2010). Our method has several advantages: it simplifies the computational burden, it reduces the degree of freedom that a third-order approximation would generate when evaluating the model performance through a calibration exercise and finally allows to evaluate time varying risk-premia using a first-order approximation of our equilibrium conditions.

For a special case of our general model we are able to obtain analytical results. When purchasing power parity holds, prices are flexible and monetary policy is specified as Taylor-rules that reacts to CPI inflation, we obtain that an increase in the domestic volatilities of the monetary policy and inflation target shocks appreciate the nominal exchange rate consistently with our empirical findings. Theoretically the excess return on home currency bonds decreases with an increase in both nominal risk factors as the foreign currency becomes relatively less good for hedging against domestic nominal risks.

While this simple model is partly successful in capturing the link between nominal risk factors and the exchange rate, it fails in replicating key international finance regularities. In fact the

[5] As we will discuss below, most of the models that have been developed recently generally specify exogenous process for consumption and/or output.

implied slope coefficient from a UIP regression would still be positive. We then consider the case in which policy authorities smooth interest rates over time and find that, conditional on shocks to the monetary policy instrument, it is possible to obtain a negative coefficient in the UIP regression which becomes more negative as the smoothing coefficient increases. Moreover, the model is able to produce an hump-shaped appreciation of the exchange rates as found in the data.

From a theoretical point of view we then explore the role of Epstein-Zin: in general departing from time-additive preferences is needed once we want to explain asset prices facts. Here, this assumption has several interesting implications. First, in open economy, cross-country surprises in utility influence the international distribution of wealth so that equilibrium quantities are affected by the preference specification. Secondly, if we focus on the case in which the subjective discount factor is very close to the unitary value, then the surprises to utility depend, up to a first order, only on the stochastic trend in world productivity. The implications is that, in this case, nominal stochastic discount factor are highly correlated across countries, an aspect that is consistent with a global explanation for risk premia. Despite the high correlation of the nominal stochastic discount factor, the Epstein-Zin preferences also implies that consumption is not systematically correlated across countries once measured in common unit (this is usually referred as the Backus-Smith-Kollman puzzle). Potentially, surprises in utility across countries could be affected by stochastic volatility term and generate a cross-country correlation between consumptions and the real exchange rate close to zero as we observe in the data.

We then offer a first pass at evaluating quantitatively the properties of our model by calibrating it following Lubik and Schorfeide (2005). We focus on a small set of facts that are related to exchange rates. The response of exchange rates and excess returns to volatility shocks is consistent with our empirical findings. The specification of monetary policy and the presence of stochastic volatility terms is crucial for obtaining a negative coefficient in the UIP regression while. Sticky prices help in generating an hump-shaped response of the real exchange rate following a monetary policy shock (in level) while as nominal rigidities increases the slope in the UIP coefficient becomes less negative. The Epstein-Zin preferences specification produce a negative relationships between cross-country consumption differential and real exchange rate similar to what is observed in the data.

Related Literature

This paper is related to different strands of literature. From an empirical point of view, we build on the early analysis of Clarida and Gali (1994) and Eichenbaum and Evans (1995) that have examined the effects of monetary shocks on the exchange rate. Our contribution assesses the role of real and nominal uncertainty on the exchange rate while their focus is on the innovation in real and nominal shocks.

From a theoretical perspectives there are two key elements in our analysis: stochastic volatility and monetary policy. The emphasis on exogenous risk factors is not novel in exchange rate

economics: early contributions by Frankel and Meese (1987) in a partial equilibrium setting, and Hodrick (1989) in a general equilibrium one, have pointed out the role of uncertainty in explaining exchange rate determination. More recently Obstfeld and Rogoff (2002) have studied the role of risk factors in a general equilibrium setting when nominal prices are sticky. Our paper follows this tradition in international finance and it is also connected to a more recent macroeconomic literature that has examined the role and the effects that risk or uncertainty have on macroeconomic variables (see for example Bloom (2009), Bloom, Floetotto and Jaimovich (2009) and Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2010)).

The importance of monetary policy in exchange rate determination has been analyzed in Benigno and Benigno (2008) while its role for the understanding of the uncovered interest rate parity has been first highlighted by McCallum (1994) and more recently by Backus, Gavazzoni, Telmer and Zin (2010). The latter authors have recasted McCallum's insight in a microfounded setting endogenizing the currency risk premium that is exogenous in McCallum's model.

Our work is also related to a fast-growing literature in international macro-finance that has developed models of the risk premium based on specifications of the stochastic discount factors derived from alternative preferences. Bansal and Shaliastovich (2009) relies on Epstein-Zin preferences combined with long-run risk, Backus et al. (2010) emphasizes the role of monetary policy for addressing the uncovered interest rate parity in nominal terms, Gavazzoni (2009) relies on Epstein-Zin preferences combined with stochastic volatility, Moore and Roche (2010) and Verdelhan (2010) propose models based on external habit with preferences a la Campbell and Cochrane (1999). While we share some of the features of the these studies, our analysis follows a general equilibrium approach by combining macro and financial market equilibrium and builds upon a theory of nominal exchange rate determination based on interest rate rules. The latter aspect is important insofar we want to address, from a model perspective, the uncovered interest rate parity puzzle in nominal rather then in real terms as most of these models do.

2 Empirical evidence

In this section, we provide empirical evidence on the importance of time-varying uncertainty in open economies. These new results complement the findings of the literature on the deviations from Uncovered Interest Parity (UIP) which already stresses the relevance of time-varying risk premia, of which time-varying uncertainty can be a source.

We conduct our analysis in the context of a simple VAR model, along the lines of Eichenbaum and Evans (1995). We aim at providing a quantitative assessment on the effects that innovations to the *volatility* of underlying disturbances may have on the *level* of macro variables of interest. In particular we focus on the conditional time-varying volatilities of three specific shocks, that are going to play a relevant role in the theoretical model of the next sections: the conditional volatility of the monetary-policy shock, of the inflation-target shock and of the productivity

shock.[6] Our focus will be to study how these shocks affect the nominal (and real exchange rate), the foreign currency risk premium which captures the deviations from UIP, the slope of the yield curve. Moreover, we will evaluate whether the results of Eichenbaum and Evans (1995), and investigated by a large body of subsequent literature, are robust to the inclusion of time-varying volatility into the picture.

We use monthly data for the G7 countries on the sample period ranging from March 1971 through September 2010, and estimate a VAR with six lags for each pair of countries that includes the US. We consider a benchmark specification with seven macroeconomic variables, in the spirit of Eichenbaum and Evans (1995). To this set of macro "level" variables, we then add three time series describing the time-varying volatilities of the monetary-policy shock ($u_{\xi,t}$), the inflation-target shock ($u_{\pi,t}$) and the productivity shock ($u_{a,t}$). The "level" variables we consider are the US nominal Federal Funds Rate (i) indicating the stance of monetary policy, the US and foreign Industrial Production Indexes (y, y^*) measuring the domestic and foreign real activity, the US CPI Index (p) capturing the domestic price level, the cross-country differential in three-month short-term nominal interest rates measured as the difference in Treasury Bill rates ($i_r \equiv i_{3m}^* - i_{3m}$), the slope of the US term structure computed as the difference between the 10-year Treasury Constant Maturities rate and the 3-month Treasury Bill rate ($i_{sl} \equiv i_{10y} - i_{3m}$) and the real exchange rate, defined as $q \equiv s + p^* - p$ where s denotes the nominal exchange rate, expressed in terms of units of USD needed to buy one unit of foreign currency. As such, an increase in q (or s) denotes a US Dollar real (nominal) depreciation. All variables are in logs, except the interest rates which are monthly percentage points.

We now explain how we build the three conditional volatilities of interest. For the conditional volatility of the monetary policy shock we use daily data from the Federal Funds futures markets, following Kuttner (2001) among others.

In particular, denoting with $f_{t,d}^0$ the spot-month futures rate on day d for a contract with delivery in month t (with day d belonging to month t) we can interpret $f_{t,d}^0$ as the conditional time$-d$ expectation of the average funds rate in month t, plus a stochastic risk premium μ:

$$f_{t,d}^0 = E_d \frac{1}{m_t} \sum_{j \in t} i_{1,j} + \mu_{t,d}^0,$$

where m_t is the number of days in month t and i_1 is the daily interest rate. To extract information about revisions in time$-d$ expectations about future monetary policy actions from data on f, Kuttner (2001) suggests to use the daily change in the futures rate, scaled up to account for the number of days in month t that are affected by the surprise: $\frac{m_t}{m_t - d}(f_{t,d}^0 - f_{t,d-1}^0)$. This measure seems particularly appealing because it reduces the distortions associated with the time variation in the risk premium μ.

As to our case, we use data on 1-month futures rates rather than spot-month rates, $f_{t,d}^1$

[6] In our model, the monetary policy shock represents a shock to the systematic component of the interest rate rule. The inflation target is also part of the interest rate rule and represents the target to which interest rate reacts when actual inflation deviates from the target.

where day d belongs to month $t-1$ rather than t. As a consequence, any revision in policy expectations reflected in a daily change of the futures rate is related to the full month t, rather than a fraction of it. Therefore, in our case we can measure day$-d$ revisions in expectations about next-month monetary policy actions using the simple daily change in the futures rate: $(f^1_{t,d} - f^1_{t,d-1})$. In what follows we will denote with $u^2_{\xi,t}$ the variance of the monetary policy surprise in month $t+1$ conditionally on information available in month t and we use, as an approximate measure of such conditional variance, the empirical second moment, within month t, of daily revisions in expectations of time-$t+1$ monetary policy actions:

$$u_{\xi,t} \approx \sqrt{\frac{1}{m_t}\sum_{d=2}^{m_t}(f^1_{t,d} - f^1_{t,d-1})^2}.$$

Since data for the Fed funds futures rates are only available starting October 1988, we follow the approach of Bloom (2009) and complete the time series with realized volatilities, within the month, computed using daily data on the effective federal funds rate – net of settlement Wednesdays – standardized by the mean and variance of the measure coming from the futures market, for the period where the two measures overlap (correlation over that period is about .6)

For the inflation-target shock, we measure the conditional volatility with the Merrill Lynch Option Volatility Estimate (MOVE). As we will learn from the theoretical analysis, movements in the inflation target can produce parallel shifts in the yield curve. Indeed the MOVE Index can capture the volatility of this level factor since it is a yield curve weighted index of the normalized implied volatility on 1-month Treasury options, which are weighted on the 5, 10, and 30 year contracts. Since this index starts only in 1989, we complete the time series with the realized volatility, within the month, computed using daily data on US 10-year Treasury bonds, standardized to the mean and variance of the MOVE index, for the period where the two measures overlap (correlation over that period is about .8). Moreover, considering that other shocks beside the inflation target might imply parallel shifts of the yield curve, in the analysis we extract the component orthogonal to the other two volatility measures.

Finally, we build an approximate measure of the volatility of the productivity shock using the stock market option-based implied volatility, the VIX index (monthly averages of daily data). However, since data for the VIX are only available starting January 1990, we follow the approach of Bloom (2009) and complete the time series with within-month realized volatilities computed using daily returns on the S&P500, standardized to the mean and variance of the VIX, for the period where the two measures overlap (correlation over that period is about .9).

As a last step, since all above measures are based on implied and realized volatilities, we construct the *conditional* volatilities considering the fitted values of an AR(1) regression for each indicator, similarly to Bekaert and Engstrom (2009). Figure 1 displays the dynamic properties of the obtained indicators.

For each pair of the G7 countries that includes the US, we then estimate the following

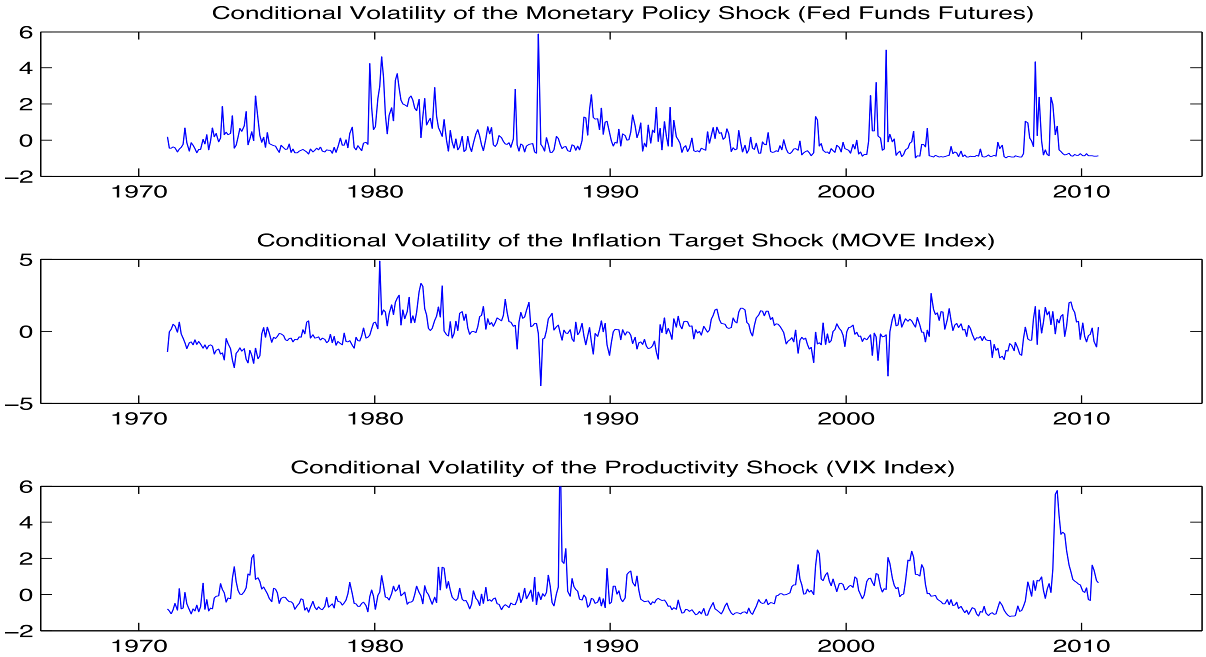

Figure 1: Time-varying Conditional Volatilities, standardized series. Note: the y-axis of the bottom panel has been truncated to 6 for the sake of readability; the value of the index around Black Monday is actually about 9.5.

VAR(p) model

$$\mathbf{y}_t = \mathbf{A}(L)\mathbf{y}_{t-1} + \mathbf{e}_t, \tag{1}$$

where the data vector is defined as $\mathbf{y}_t \equiv [u_{\xi,t},\ u_{\pi,t},\ u_{a,t},\ i_t,\ i_{r,t},\ i_{sl,t},\ q_t,\ p_t,\ y_t,\ y_t^*,\ ]'$, and the lag-order is six. This ordering implies that the relevant shocks we want to study – which are the orthogonalized innovations to the first four variables in $\mathbf{y}_t$ – affect first prices and then quantities.

Figures 2 through 5 display the dynamic response of selected variables to, respectively, a "classic" monetary policy shock (the orthogonalized innovation to the level of the Federal funds rate), an innovation to the volatility of the monetary-policy shock, an innovation to the volatility of the shock to the inflation target and an innovation to the volatility of the productivity shock. Each panel reports the point estimate of the impulse response function – the solid line – and the associated one-standard-deviation confidence intervals – the dashed lines. In each figure, the first row displays the dynamic response of the US Federal funds rate, the second row the response of the slope of the US term structure, the third the response of the real exchange rate,[7] and the last one shows the dynamic response of the excess return on the foreign currency. The latter is defined as

$$exr_t \equiv i_{1,t}^* - i_{1,t} + E_t \Delta s_{t+1},$$

which measures deviations from the UIP condition.

[7]We do not report the responses of the nominal exchange rate, since they are very similar to those of the real exchange rate.

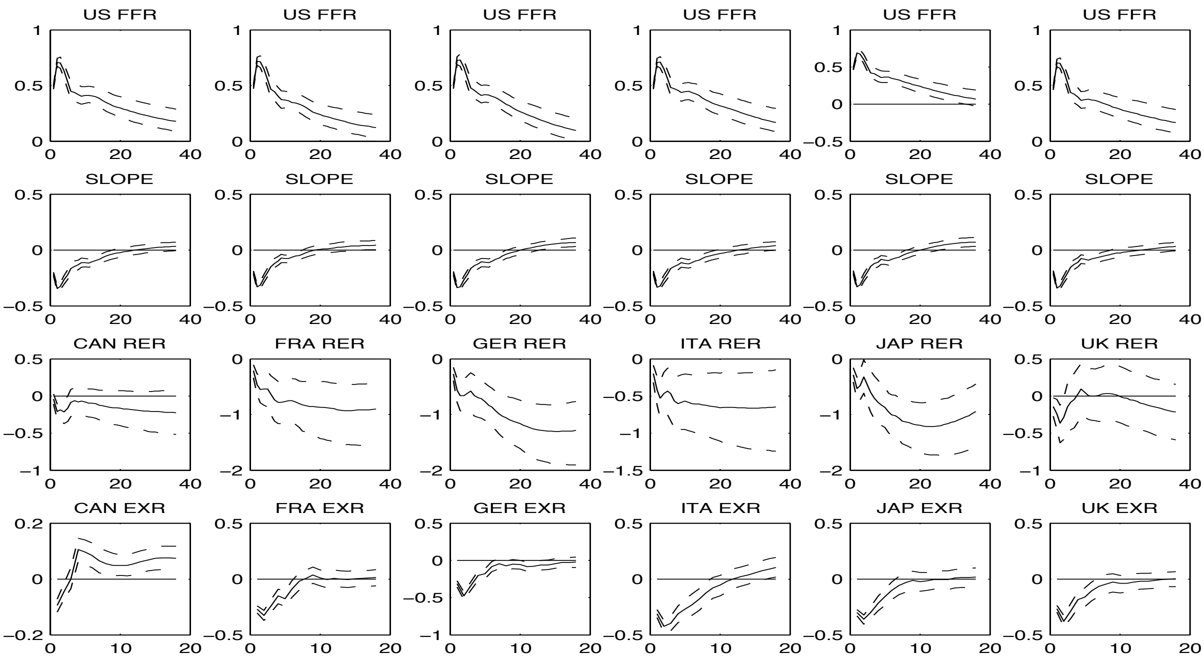

Figure 2: Dynamic responses to an orthogonalized innovation to the Federal Funds Rate. Each column reports, for each country pair, the responses of the US Federal Funds rate (i), the slope of the US term structure (i_{sl}), the Real Exchange Rate (q), the foreign currency risk premium (exr). x-axes: months, y-axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

Figure 2 addresses the robustness of the findings of Eichenbaum and Evans (1995). The responses to a contraction shock to monetary policy seem virtually unaffected by the explicit consideration of the interplay between time-varying volatility and the "level" variables. In particular, a positive innovation to the federal funds rate implies a significant appreciation of the USD, on impact. Moreover, the exchange rate keeps appreciating also in the transition, and does not start depreciating but in the medium run. Second, the spread between foreign and domestic short-term interest rates decreases gradually through the implied increase in the foreign one (not shown). Finally, the two results above drive the persistent deviations from UIP implied by the last row, in the form of positive excess returns on US securities. Additionally, Figure 2 also shows the negative response of the slope of the yield curve, consistently with a gradual transmission of the monetary policy shock along long-maturity bonds.

Figures 3, 4 and 5 present our new evidence on the importance of volatility shocks. The first result, common to all three figures, is that shocks to volatility indeed do affect the level of the other macro variables, although with different magnitudes and significance across variables and shocks. Hence volatility does have a *distinct and direct* effect which will be important in characterizing our theoretical model.

In particular, Figure 3 shows the responses to an orthogonalized innovation to the volatility of the monetary policy shock. The estimated response of the slope of the US yield curve, is positive on impact and keeps rising for a few months before reverting back to mean; it remains, however, significantly above the steady-state level for quite some time, regardless of the pair

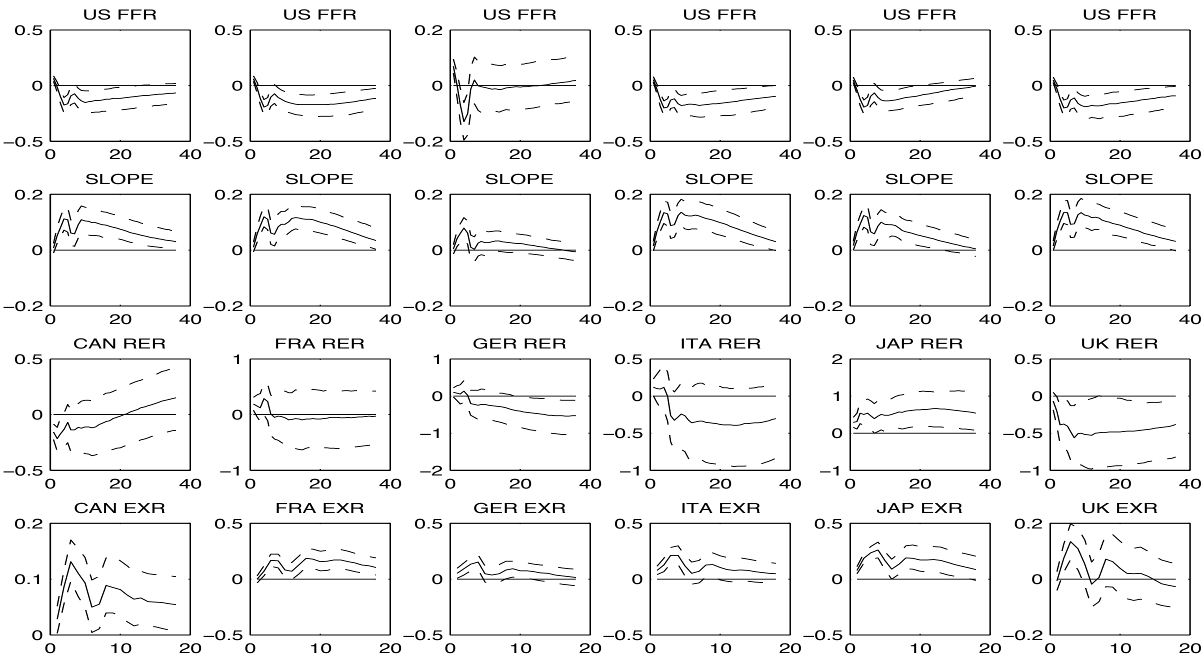

Figure 3: Dynamic responses to an orthogonalized innovation to the volatility of shocks to the monetary policy *instrument*. Each column reports, for each country pair, the responses of the US Federal Funds rate (i), the slope of the US term structure (i_{sl}), the Real Exchange Rate (q), the foreign currency risk premium (exr). x-axes: months, y-axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

considered (except for the case of Germany, for which the effects dies out within six months). An increase in the volatility of the monetary policy shock also induces a significant and persistent deviation from UIP, in the form of a positive excess return on foreign securities. This result is mainly driven by the response of the spread in the short-term interest rate, which increases by a magnitude of 5-10 basis points (not shown). The response of the exchange rate (third row) is more ambiguous. The point estimate indicates that an increase in the volatility of the monetary-policy shock strengthens the US dollar. However, this is not particularly significant (expect for the case of the UK and, marginally, Germany).

Figure 4 shows the response to an orthogonalized innovation to the volatility of the inflation-target shock. Here, the estimated responses of the first two variables are not very precise. The implications for the exchange rate, instead, are very interesting. Indeed, while the response is weak on impact and not always significant, the point estimates indicate that an increase in the volatility of the inflation-target shock tends to appreciate the exchange rate (with the notable exception of Japan) in the medium run. This is particularly interesting considering the specific nature of the shock, which is indeed related to the medium-run target level for the inflation rate. This pattern is also reflected in the dynamic response of the foreign currency risk premium: while the short term response is ambiguous, the estimated impulse-response functions indicate that in the medium run a higher volatility of the inflation-target shock produces a lower foreign currency risk premium, consistently with the appreciation of the domestic currency.

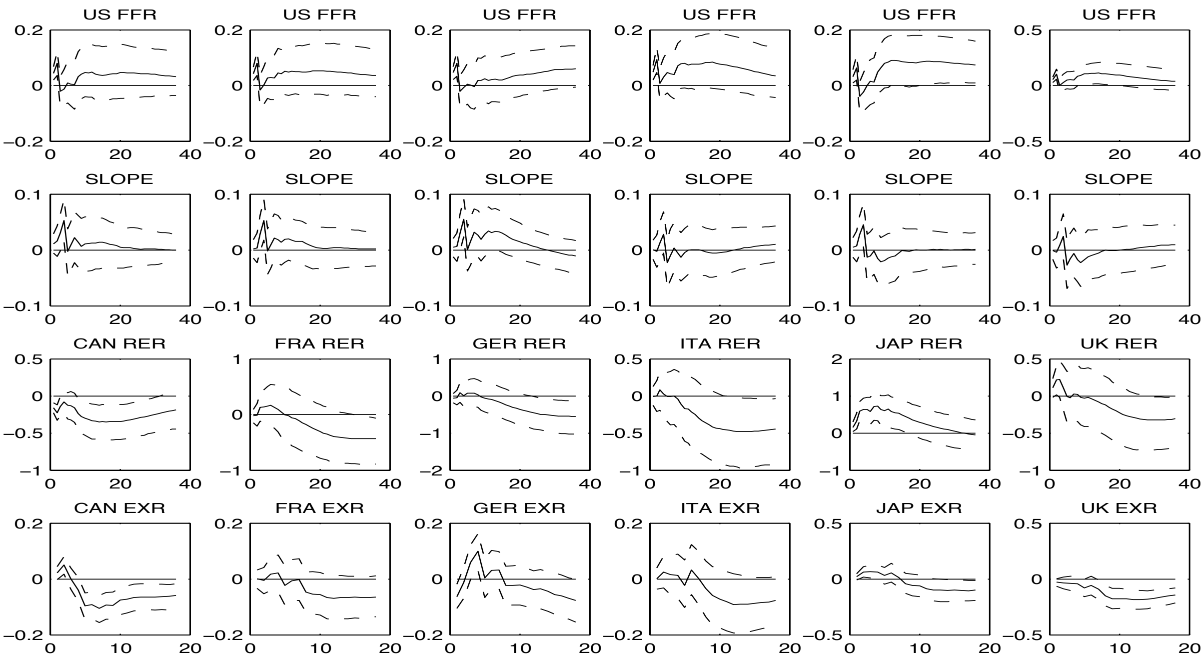

Figure 4: Dynamic responses to an orthogonalized innovation to the volatility of shocks to the monetary policy *target*. Each column reports, for each country pair, the responses of the US Federal Funds rate (i), the slope of the US term structure (i_{sl}), the Real Exchange Rate (q), the foreign currency risk premium (exr). x-axes: months, y-axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

Figure 5 studies the responses to the conditional volatility of the productivity shock. Following an increase in this volatility the estimated response of the slope of the yield curve is muted on impact, but it becomes substantially and significantly positive after about six months and stays significantly positive until about two years after the shock, peaking at about 10-15 basis points after about one year. This response is virtually identical across all the considered country pairs. Moreover, the response of the exchange rate is also quite clear: although with different timings and magnitudes across country pairs, an increase in the volatility of the productivity shock tends to depreciate the exchange rate, mostly on impact. No significant deviation from UIP arises with the notable exception of Japan.

To provide further evidence on the importance of volatility shock, we analyze the Forward-Error Variance Decomposition (FEVD) to assess the relative importance of level and volatility shocks for the three main variables of interest to us: term-structure spreads, nominal exchange rate and deviations from UIP. For these variables, respectively, Figures 6 through 8 show the resulting decomposition, for different horizons. The first implication is, again, that volatility does matter. Indeed, although with different magnitudes across variables and country pairs, innovations to the three volatility shocks considered seem to explain a non-negligible part of the variance of the level variables. Moreover, most of this explanatory power is related to longer horizon, as opposed to short ones.

Figure 6 shows that this result is not particularly robust for the nominal exchange rate, across

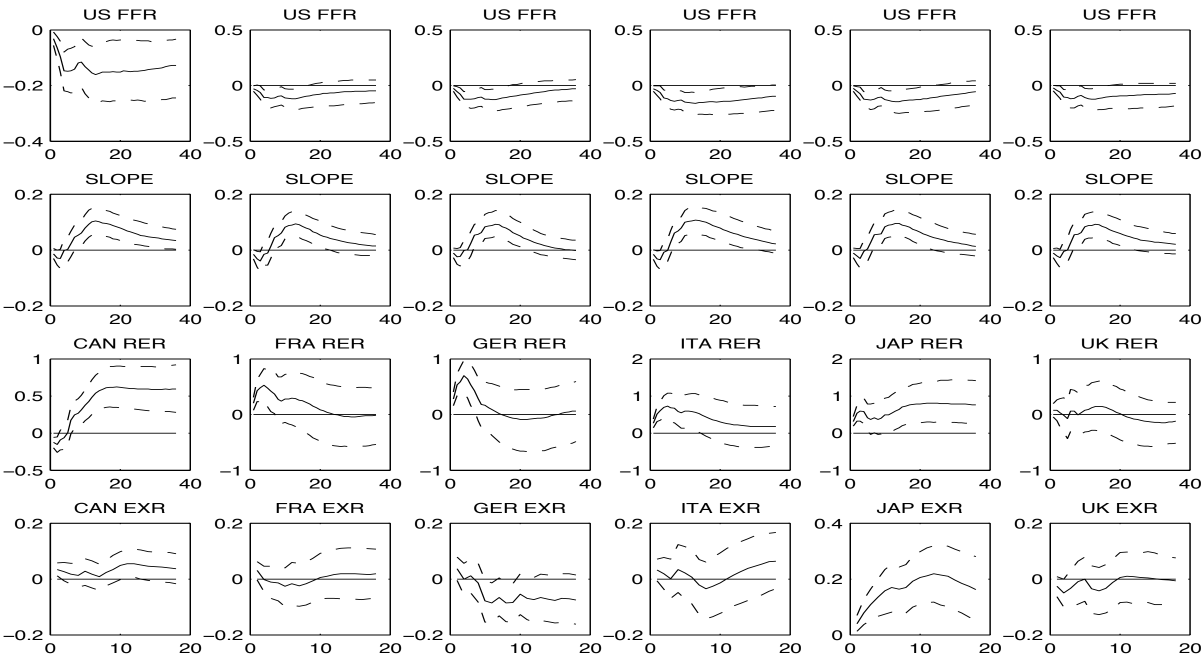

Figure 5: Dynamic responses to an orthogonalized innovation to the volatility of productivity shocks. Each column reports, for each country pair, the responses of the US Federal Funds rate (i), the slope of the US term structure (i_{sl}), the Real Exchange Rate (q), the foreign currency risk premium (exr). x-axes: months, y-axes: annual percentage points. Country pairs are, respectively, US-Canada, US-France, US-Germany, US-Italy, US-Japan, US-UK.

country pairs, both in terms of magnitude and relative composition. We go from the case of the US-Canada pair, in which innovations to volatility explain up to 20% of total variance, mainly due to the volatility of productivity shocks, to the US-Japan pair, in which they explain about the same share but with an equal split between the volatility of productivity and monetary policy shocks, to the US-France pair, where the overall share of variance explained by innovations in volatility is only about 5%.

More robust are the implications for the foreign currency risk premium (exr) and the US term spread (i_{sl}), respectively Figures 7 and 8. Innovations to volatility explain between 10 and 20 percent of total variance of the foreign currency risk premium, mainly due to the volatility of the monetary-policy shock and the inflation-target shock. While for the US term spread, volatility shocks are able to explain up to 25% of the total variance, equally split between shocks to the volatility of the monetary-policy shock and those to productivity, and basically no role for the volatility of the inflation-target shock.

3 International finance regularities

In the previous section, we have provided evidence that volatility shocks have important effects on the level of open-economy macro variables. In the next section, we are going to build a model in which indeed time-varying uncertainty plays a role. To nail down the desiderata that our

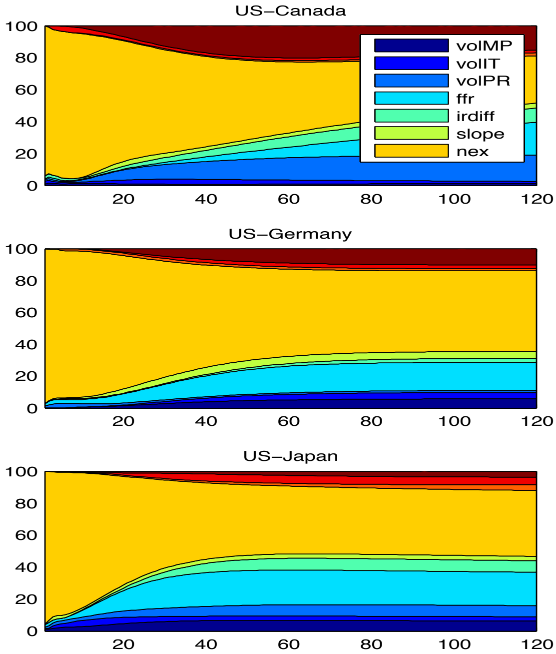

Figure 6: Forecast Error Variance Decomposition, Nominal Exchange Rate (s). Each panel reports, for each country-pair, the percentage of total FEV explained by the variance of all underlying orthogonalized innovations (x-axes: months). Legend only reports the components related to innovations to the first seven variables in the VAR: volatility of MP instrument (volMP); volatility of MP target (volIT); volatility of productivity shock (volPR); Federal Funds Rate (ffr); short-term interest-rate spread (irdiff); term spread (slope); nominal exchange rate (nex).

model should meet, we summarize the implications of our findings and report other empirical regularities along which we would like our model to perform well. The sense in which we refer to these facts (or puzzles) as international finance regularities relates to our focus on the joint behavior of interest rates and exchange rates.

The empirical evidence on the importance of volatility shocks can be summarized along three facts related respectively to the effects that volatility shocks have on the nominal (and real) exchange rate, the deviations from UIP and the slope of the yield curve.

Fact 1: An increase in the volatilities of the US monetary-policy and inflation-target shocks appreciates the dollar exchange rate especially in the medium run. This effect is stronger for the inflation-target volatility shock. On the other hand, an increase in the volatility of the productivity shock depreciates the dollar exchange rate.

Fact 2: An increase in the volatilities of both the monetary-policy and the inflation-target shocks generates significant and persistent deviations from UIP and in particular an increase in the excess returns of foreign short-term bonds.

Fact 3: An increase in volatility, regardless of the specific underlying disturbance, raises the slope of the term structure especially in the cases of productivity and monetary-policy shock.

The next fact is in common between our empirical analysis and the evidence reported by Eichenbaum and Evans (1995).

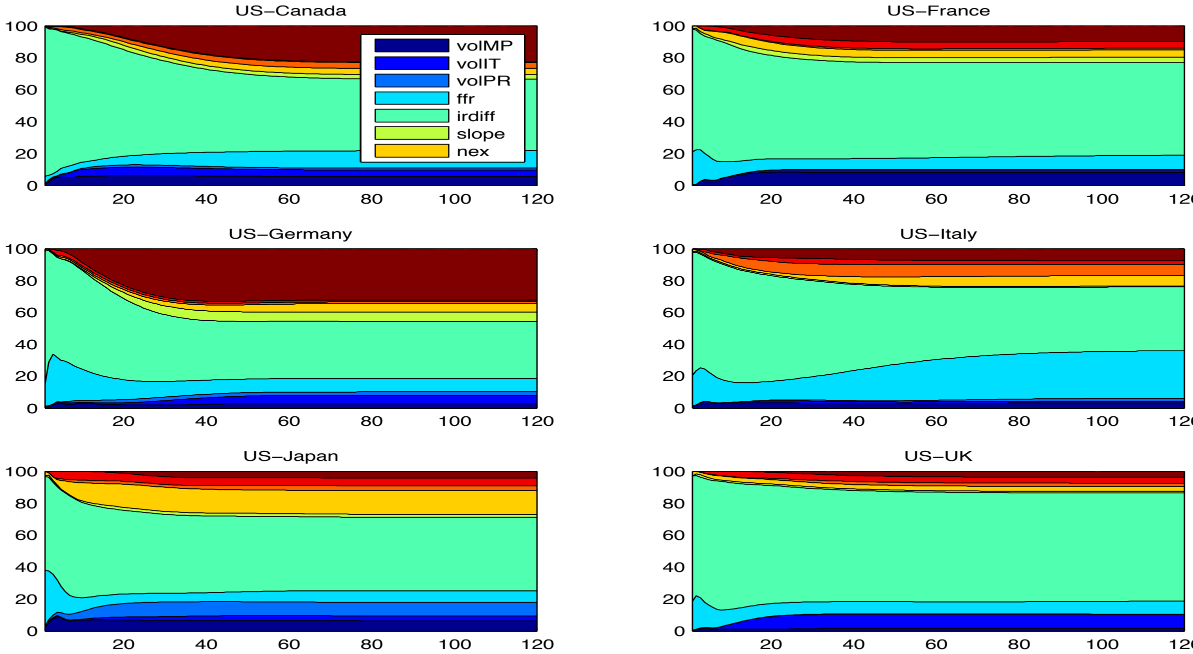

Figure 7: Forecast Error Variance Decomposition, Foreign Currency Excess Return (exr). Each panel reports, for each country-pair, the percentage of total FEV explained by the variance of all underlying orthogonalized innovations (x-axes: months). Legend only reports the components related to innovations to the first seven variables in the VAR: volatility of MP target (volIT); volatility of productivity shock (volPR); Federal Funds Rate (ffr); short-term interest-rate spread (irdiff); term spread (slope); nominal exchange rate (nex).

Fact 4: A positive innovation to the level of the monetary-policy shock (contractionary policy shock) produces a persistent appreciation in both the real and nominal exchange rates and a persistent deviations from the UIP in the form of positive excess returns on US securities.

To this list, we add other well-know facts or puzzles that we would like to address. While our previous facts are conditional statements about how excess returns and exchange rate comove following different innovations (level or volatility shocks), another relevant empirical regularity is related to the behaviour of exchange rates and interest rates as captured by the negative regression coefficient that arises from the UIP regression.

Fact 5: The coefficient in the regression of exchange rate changes on nominal interest rate differential (UIP regression) is negative.

However risk premia are relevant also along several other dimensions on financial data. Since, we have focused on the importance of volatility shock in shifting the slope of the yield curve, we also add as a fact the evidence that, on average, the yield curve is upward sloping, which is a result robust internationally (Wright, 2009).

Fact 6: On average the yield curve is upward sloping.

Finally, we add another regularity, which apparently seems more related to the real side of the economy but is strongly connected, as we will discuss later, to the international financial side since it depends on the specification of the stochastic discount factor used to evaluate asset prices.

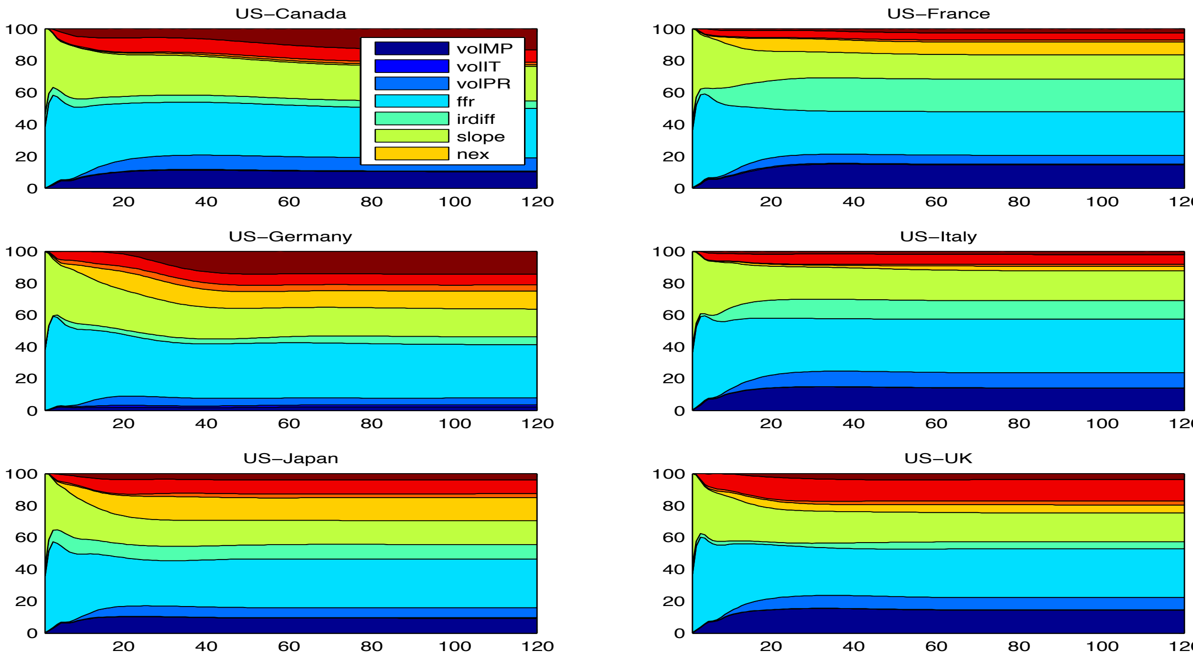

Figure 8: Forecast Error Variance Decomposition, Term Spread (i_{sl}). Each panel reports, for each country-pair, the percentage of total FEV explained by the variance of all underlying orthogonalized innovations (x-axes: months). Legend only reports the components related to innovations to the first seven variables in the VAR: volatility of MP target (volIT); volatility of productivity shock (volPR); Federal Funds Rate (ffr); short-term interest-rate spread (irdiff); term spread (slope); nominal exchange rate (nex).

Fact 7: While cross-country consumption correlation is low in the data and the consumption differential across countries does not move in any systematic pattern with the real exchange rate (see Backus-Smith, 1993, and Kollman, 1995), stochastic discount factors should be highly correlated across countries given the observed volatility of real exchange rates (Brandt, Cochrane and Santa Clara, 2006).

4 A two-country open economy model

To study the relationships between time-varying volatility and the exchange rate, we present a two-country open-economy model along the lines of Benigno and Benigno (2008). In particular we consider two extensions, whose relevance will be discussed later, which are important for the model to be able to match the empirical facts discussed above: *i*) we allow for more general recursive preferences as in the work of Epstein and Zin (1989, 1991) and Weil (1990) and *ii*) we consider stochastic volatility for the exogenous processes driving the economy. The latter addition, in particular, implies a careful treatment of the solution. To this end, we expound the method developed by Benigno, Benigno and Nisticò (2010) to show how we can handle in a relative easy way approximations of dynamic general equilibrium with time-varying uncertainty and at the same time characterize the effect of uncertainty on the variables of interest.

4.1 Households

The world economy is composed by two countries, Home and Foreign, and populated by a continuum of agents of measure one: Home households lie on the interval $[0, n]$, while Foreign households on $(n, 1]$ where $n \in (0, 1)$. The population size is set equal to the range of goods produced so that Home firms produce goods on $[0, n]$, Foreign firms produce on $(n, 1]$. Home households are indexed by j, Foreign households by i. C_t^j denotes the level of consumption for household j in period t and L_t^j denotes its supply of working hours.

Preferences are recursive as in the framework of Epstein and Zin(1989, 1991) and Weil (1990). In particular, we assume that for a generic household of type j recursive utility can be written as

$$V_t^j = \left(U\left(C_t^j, L_t\right)^{1-\rho} + \beta \left(E_t(V_{t+1}^j)^{1-\gamma}\right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}} \tag{2}$$

where ρ is a measure of the inverse of the intertemporal elasticity of substitution over the utility flow, $U(\cdot)$, γ represents the risk aversion towards static wealth gambles, and $\beta \in (0, 1)$ is the household's subjective discount factor. The classical expected utility model is nested under the assumption $\rho = \gamma$.

The utility flow is a Cobb-Douglas index of aggregate consumption, C, and leisure, $1 - L$

$$U\left(C_t^j, L_t^j\right) = \left(C_t^j\right)^{\psi} (1 - L_t^j)^{1-\psi} \tag{3}$$

where $\psi \in (0, 1)$ reflects the preference for consumption versus leisure. As it is well known, this specification of preferences allows to disentangle the elasticity of substitution, $1/\rho$, from the risk-aversion coefficient.[8]

The aggregate consumption index C is a composite consumption good

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1 - v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \ \theta > 0 \tag{4}$$

where C_H and C_F are the two consumption sub-indexes that refer, respectively, to the consumption of Home-produced and Foreign-produced goods; θ, with $\theta > 0$, is the elasticity of intratemporal substitution and $v \in (0, 1)$ represents the weight given to home-produced goods in the aggregator C. Home bias in consumption arises when the weight given to Home goods is higher than the size of the country, i.e. when $v > n$.

In the Foreign country, preferences have the same structure as in (2)

$$V_t^{*i} = \left(U\left(C_t^{*i}, L_t^{*i}\right)^{1-\rho} + \beta \left(E_t(V_{t+1}^{*i})^{1-\gamma}\right)^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}} \tag{5}$$

[8] See Swanson (2010) for how to compute risk-aversion toward consumption with Epstein-Zin preferences.

where the aggregate consumption bundle is given by

$$C^{*}=\left[v^{*\frac{1}{\theta}}C_{H}^{*\frac{\theta-1}{\theta}}+(1-v^{*})^{\frac{1}{\theta}}C_{F}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{6}$$

for a different weight $v^{*}\in(0,1)$.

We introduce home bias in consumption following Benigno and De Paoli (2010). Specifically, denoting with $\lambda\in(0,1)$ the (common) degree of openness of the two countries, the weights in the consumption bundle are related to the country sizes through:

$$1-v=(1-n)\lambda,$$

$$v^{*}=n\lambda.$$

The consumption bundles $C_H, C_F, C_H^{*}, C_F^{*}$, are in turn Dixit-Stiglitz aggregators of the goods produced in the two countries and are given by

$$C_{H}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}}\int_{0}^{n}c\left(h\right)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{\sigma-1}}\qquad C_{F}=\left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_{n}^{1}c\left(f\right)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

$$C_{H}^{*}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}}\int_{0}^{n}c^{*}\left(h\right)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{\sigma-1}}\qquad C_{F}^{*}=\left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_{n}^{1}c^{*}\left(f\right)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

where σ, with $\sigma>1$, is the elasticity of substitution across the consumption goods produced within a country. The appropriate consumption-based price indexes associated with C and C^{*} are given respectively by

$$P=\left[vP_{H}^{1-\theta}+(1-v)\left(P_{F}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{9}$$

$$P^{*}=\left[v^{*}P_{H}^{*1-\theta}+(1-v^{*})\left(P_{F}^{*}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{10}$$

where P_H (P_H^{*}) is the price sub-index for Home-produced goods expressed in the Home (Foreign) currency and P_F (P_H^{*}) is the price sub-index for Foreign-produced goods expressed in the Home (Foreign) currency. Moreover

$$P_{H}=\left[\left(\frac{1}{n}\right)\int_{0}^{n}p\left(h\right)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}}\qquad P_{F}=\left[\left(\frac{1}{1-n}\right)\int_{n}^{1}p\left(f\right)^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}, \tag{11}$$

$$P_{H}^{*}=\left[\left(\frac{1}{n}\right)\int_{0}^{n}p^{*}\left(h\right)^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}\qquad P_{F}^{*}=\left[\left(\frac{1}{1-n}\right)\int_{n}^{1}p^{*}\left(f\right)^{1-\sigma}dz\right]^{\frac{1}{1-\sigma}}, \tag{12}$$

where $p(h)$ and $p^{*}(h)$ are the prices of the generic good h produced by the Home country in the currencies of the Home and Foreign country respectively; while $p(f)$ and $p^{*}(f)$ are the prices of

the generic good f produced by the Foreign country in the currencies of the Home and Foreign country respectively. The law of one price holds across all individual goods: $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$, where S is the nominal exchange rate (the price of foreign currency in terms of domestic currency). Therefore, equations (11) and (12), imply that $P_H = SP_H^*$ and $P_F = SP_F^*$. However, equations (9) and (10) show that, since Home and Foreign agents' preferences are not necessarily identical, there can be deviations from purchasing power parity unless $v = v^*$, that is, $P \neq SP^*$. Appropriately we measure the deviations from PPP through the real exchange rate given by $Q \equiv SP^*/P$. We also define the terms of trade in the Home country as $T \equiv P_F/P_H$. Notice the following useful relationships between relative prices, the real exchange rate and terms of trade

$$1 = \left[v \left(\frac{P_H}{P} \right)^{1-\theta} + (1-v) \left(\frac{P_F}{P} \right)^{1-\theta} \right], \tag{13}$$

$$Q = \frac{\left[v^* + (1-v^*) (T)^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[v + (1-v) (T)^{1-\theta} \right]^{\frac{1}{1-\theta}}}, \tag{14}$$

$$T = \frac{P_F}{P} \frac{P}{P_H}. \tag{15}$$

Given the above-specified preferences, we can derive total demands of the generic good h, produced in country H, and of the good f, produced in country F:

$$y^d(h) = \left(\frac{p(h)}{P_H} \right)^{-\sigma} Y_H \qquad\qquad y^d(f) = \left(\frac{p(f)}{P_F} \right)^{-\sigma} Y_F \tag{16}$$

where output aggregators Y_H and Y_F are appropriately defined

$$Y_H = \left(\frac{P_H}{P} \right)^{-\theta} \left(vC + \frac{v^*(1-n)}{n} Q^\theta C^* \right), \tag{17}$$

$$Y_F = \left(\frac{P_F}{P} \right)^{-\theta} \left(\frac{(1-v)n}{1-n} C + (1-v^*) Q^\theta C^* \right). \tag{18}$$

We assume that asset markets are complete both at the domestic and international levels. In particular households can trade in a set of state-contingent nominal securities denominated in the Home currency which span all the uncertainty from one period to another.[9] Each of these securities pays respectively only in one of the possible states of nature in the next period. Let B_{t+1}^j the state-contingent payoff at time $t+1$ of the portfolio of state-contingent nominal securities held by household in the Home country at the end of period t. The value of this portfolio can be written as $E_t[M_{t,t+1} B_{t+1}^j]$ where $M_{t,t+1}$ represents the nominal stochastic discount factor for discounting units of Home-currency wealth from a state of nature at time $t+1$ back to time t. This stochastic discount factor is unique, because of the complete-market assumption,

[9] See Chari et al. (1998).

and equivalent to the price of a state-contingent security standardized by the time-t conditional probability of occurrence of the state of nature at time $t+1$ in which the security pays. We can write the flow budget constraint that the Home households face as

$$E_t[M_{t,t+1}B^j_{t+1}] \leq B^j_t + W_t L^j_t + D^j_t - P_t C^j_t,$$

for each j, where W_t is the nominal wage in the Home country, determined in a common labor market; D^j_t are nominal profits. Each household holds equal shares of all firms (domestic firms are located on the interval $[0, n]$ and the size of the Home population is normalized to n); there is no trade in firms' shares. Households are subject to a standard limit on their borrowing possibilities.

Households in the Foreign country can also trade in the state-contingent securities denominated in the currency of country H. Let B^i_{t+1} the state-contingent payoff at time $t+1$ of the portfolio of state-contingent nominal securities held by Foreign household at the end of period t. Since B^i_{t+1} is denominated in units of Home currency, the payoff in Foreign currency is given by $B^{*i}_{t+1} = B^i_{t+1}/S_{t+1}$ and the value of the portfolio in Foreign currency is simply $E_t[M_{t,t+1}B^i_{t+1}]/S_t = E_t[M_{t,t+1}B^{*i}_{t+1}S_{t+1}]/S_t$. We can appropriately define the nominal stochastic discount factor for discounting units of Foreign-currency wealth across time

$$M^*_{t,t+1} = \frac{S_{t+1}}{S_t} M_{t,t+1} \tag{19}$$

which is uniquely defined given that $M_{t,t+1}$ is unique. Therefore, the flow budget constraint for the Foreign households can be written as

$$E_t[M^*_{t,t+1}B^{*i}_{t+1}] \leq B^{*i}_t + W^*_t L^{*i}_t + D^{*i}_t - P^*_t C^{*i}_t,$$

for each i where the definition of the variables follows from before with the appropriate modifications. A standard borrowing-limit condition applies also here.

Households maximize utility under the sequence of the flow budget constraints and the borrowing-limit constraints by choosing aggregate consumption, labor and asset holdings in terms of the state contingent securities.

At optimum the marginal rate of substitution between labor and consumption is equal to the real wage

$$\frac{W_t}{P_t} = \frac{1-\psi}{\psi} \frac{C^j_t}{1-L^j_t} \tag{20}$$

$$\frac{W^*_t}{P^*_t} = \frac{1-\psi}{\psi} \frac{C^{i*}_t}{1-L^{*i}_t} \tag{21}$$

for each j and i in the respective country.

Optimality conditions with respect to the holdings of the state-contingent securities for the

Home household imply

$$\frac{\partial (V_t^j)}{\partial C_t^j}\frac{(V_t^j)^{-\rho}}{P_t}M_{t,t+1} = \beta\left(E_t(V_{t+1}^j)^{1-\gamma}\right)^{\frac{\gamma-\rho}{1-\gamma}}\frac{(V_{t+1}^j)^{-\gamma}}{P_{t+1}}\frac{\partial V_{t+1}^j}{\partial C_{t+1}^j},$$

for each contingency at time $t+1$ where the marginal utility of consumption is given by

$$\frac{\partial V_t^j}{\partial C_t^j} = \psi\frac{U\left(C_t^j, L_t^j\right)^{1-\rho}}{C_t^j}(V_t^j)^{\rho}.$$

Combining the above two equations, we obtain that the nominal stochastic discount factor in the Home country is

$$M_{t,t+1} = \beta\left(\frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}}\right)^{\frac{\rho-\gamma}{1-\gamma}}\left(\frac{U\left(C_{t+1}, L_{t+1}\right)}{U\left(C_t, L_t\right)}\right)^{1-\rho}\frac{C_t}{C_{t+1}}\frac{1}{\Pi_{t+1}} \tag{22}$$

where we have also neglected the index j from V, C, L.[10] Moreover, we have defined the gross CPI inflation rate as

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} = \Pi_{H,t}\frac{\left[v + (1-v)\left(T_t\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[v + (1-v)\left(T_{t-1}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}}, \tag{23}$$

where $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$.

Similarly in the Foreign country we obtain

$$M_{t,t+1}^* = \beta\left(\frac{V_{t+1}^{*1-\gamma}}{E_t V_{t+1}^{*1-\gamma}}\right)^{\frac{\rho-\gamma}{1-\gamma}}\left(\frac{U\left(C_{t+1}^*, L_{t+1}^*\right)}{U\left(C_t^*, L_t^*\right)}\right)^{1-\rho}\frac{C_t^*}{C_{t+1}^*}\frac{1}{\Pi_{t+1}^*}, \tag{24}$$

where the Foreign gross CPI inflation rate is given by

$$\Pi_t^* = \Pi_t\frac{Q_t}{Q_{t-1}}\frac{S_{t-1}}{S_t}. \tag{25}$$

The above nominal discount factors correspond to those of the standard expected-utility model, under the assumption $\rho = \gamma$. In this case, they depend on the ratio between the marginal utilities of nominal income across the two periods. With Epstein-Zin preferences, there is an additional term reflecting the preference for an early, in the case $\rho < \gamma$, or late, in the case $\rho > \gamma$, resolution of intertemporal uncertainty. This intertemporal uncertainty is captured by the ratio of the utility at time $t+1$ with respect to its risk-adjusted expected value, where

[10] Given the assumption that a common labor market exists in each country and that each firm employs all the workers, as it will be discussed later, we can impose symmetry in labor supply and set $L^j = L$ for each j. It follows from (20) that $C^j = C$ for each j. Therefore also $V^j = V$.

the risk-adjustment occurs through the factor $1-\gamma$. When agents prefer an early resolution of uncertainty ($\rho < \gamma$) bad realizations of the utility at time $t+1$ with respect to its risk-adjusted expected value increase the stochastic discount factor and therefore the appetite for state-contingent wealth in that state of nature.

The above nominal stochastic discount factor can be used to price any security in arbitrage-free markets and in particular they imply that the short-term nominal interest rates satisfy

$$\frac{1}{1+i_{1,t}} = E_t M_{t+1}, \tag{26}$$

$$\frac{1}{1+i^*_{1,t}} = E_t M^*_{t+1}, \tag{27}$$

where $i_{1,t}$ and $i^*_{1,t}$ are the one-period nominal interest rates in the Home and Foreign country, respectively.

Using (22) and (24) into (19), we can obtain

$$\left(\frac{V_{t+1}^{1-\gamma}\left(E_t V_{t+1}^{*1-\gamma}\right)}{V_{t+1}^{*1-\gamma}\left(E_t V_{t+1}^{1-\gamma}\right)}\right)^{\frac{\rho-\gamma}{1-\gamma}} \left(\frac{U\left(C_{t+1}, L_{t+1}\right)}{U\left(C^*_{t+1}, L^*_{t+1}\right)}\right)^{1-\rho} \frac{C^*_{t+1}}{C_{t+1}} Q_{t+1} = \left(\frac{U\left(C_t, L_t\right)}{U\left(C^*_t, L^*_t\right)}\right)^{1-\rho} \frac{C^*_t}{C_t} Q_t. \tag{28}$$

To close the assumption of complete markets, we need to specify initial conditions for the holdings of the state-contingent securities. A standard assumption in the literature is to choose initial state-contingent wealth in a way to equalize the ratio between the marginal utilities of nominal income across countries, converted in the same currency. Let G_t denote this ratio at time t, it follows that we can write it as

$$G_t = \frac{\frac{\partial V_t^{1-\rho}}{\partial C_t}\frac{1}{P_t}}{\frac{\partial V_t^{*1-\rho}}{\partial C^*_t}\frac{1}{S_t P^*_t}} = \left(\frac{U\left(C_t, L_t\right)}{U\left(C^*_t, L^*_t\right)}\right)^{1-\rho} \frac{C^*_t}{C_t} Q_t \tag{29}$$

where we have rescaled utility as $V_t^{1-\rho}$ in order to make a direct comparison with the expected-utility model.[11] Combining (28) and (29) we obtain the following law of motion for G_t

$$G_{t+1} = G_t \left(\frac{V_{t+1}^{1-\gamma}\left(E_t V_{t+1}^{*1-\gamma}\right)}{V_{t+1}^{*1-\gamma}\left(E_t V_{t+1}^{1-\gamma}\right)}\right)^{\frac{\gamma-\rho}{1-\gamma}}. \tag{30}$$

We set $G_{t_0} = 1$ and therefore assume that initial state-contingent wealth equalizes the ratio of the marginal utilities of nominal income across countries in the initial period. Notice that, under the expected-utility model ($\gamma = \rho$), this assumption implies equalization of the ratio at all times and contingencies. With Epstein-Zin preferences, instead, this ratio evolves over time

[11] When $\gamma = \rho$, utility (2) coincides with the expected utility model where indeed intertemporal utility is defined as $V_t^{1-\rho}$.

depending on cross-country realizations of utility with respect to their risk-adjusted expected values.

4.2 Firms

The Home country produces goods on the interval $[0, n]$ while the Foreign country on $(n, 1]$. At first pass we abstract from investment and capital accumulation.[12] A generic firm h producing in the Home country uses the following production function

$$y_t(h) = A_t(L_t(h))^{\varphi} \tag{31}$$

where A_t is a productivity shifter common to all the firms in the Home country, φ with $\varphi \in (0, 1]$ measures decreasing return to scale in the labor input $L_t(h)$, which is a composite of all the differentiated labor supplied by households j according to

$$L_t(h) = \frac{1}{n}\int_0^n L_t^j(h)dj$$

where $L_t^j(h)$ denotes the demand of household j's labor by firm h.

We assume that there are frictions in the price adjustment. In particular, we model price rigidity as in the Calvo's (1983) model, but with indexation. In each period, in the Home country, only a fraction $(1-\alpha)$ of firms, with $0 \leq \alpha < 1$, can reset their prices independently of the last time they had reset them. In this case, the price is chosen to maximize the expected discounted value of the profits under the circumstances that the price, appropriately indexed, still applies. These firms choose prices to maximize the following objective

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left\{ p_{t,T}(h) y_{t,T}(h) - W_T L_T(h) \right\}$$

where total demand is:

$$y_{t,T}(h) = \left(\frac{p_{t,T}(h)}{P_{H,T}}\right)^{-\sigma} Y_{H,T}$$

and moreover $p_{t,T}(h) = \tilde{p}_t(h)\bar{P}_{H,T}/\bar{P}_{H,t}$ where $\tilde{p}_t(h)$ is the price chosen at time t and $\bar{P}_{H,T}/\bar{P}_{H,t}$ is the gross inflation target from t to T to which all prices are automatically adjusted. The optimal price $\tilde{p}_t(h)$ is chosen to satisfy the following first-order condition:

$$\tilde{p}_t(h) = \mu \frac{E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} W_T \left(\frac{y_{t,T}(h)}{A_T}\right)^{\frac{1}{\varphi}}}{E_t \sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left(\frac{\bar{P}_{H,T}}{\bar{P}_{H,t}}\right) y_{t,T}(h)},$$

where the overall mark-up has been defined as $\mu = \sigma/(\varphi(\sigma-1))$. Using (20) and (22), we can

[12] Otherwise we can assume that each firm is endowed with a fixed amoung of non-depreciating capital.

write the above equation as

$$\left(\frac{\tilde{p}_t(h)}{P_{H,t}}\right)^{1-\sigma+\frac{\sigma}{\varphi}} = \mu\frac{1-\psi}{\psi}\frac{E_t\sum\limits_{T=t}^{\infty}(\alpha\beta)^{T-t}N_{t,T}U\left(C_T, L_T\right)^{1-\rho}\frac{1}{1-L_T}\left(\frac{P_{H,T}}{P_{H,t}}\frac{\bar{P}_{H,t}}{\bar{P}_{H,T}}\right)^{\frac{\sigma}{\varphi}}\left(\frac{Y_{H,T}}{A_T}\right)^{\frac{1}{\varphi}}}{E_t\sum\limits_{T=t}^{\infty}(\alpha\beta)^{T-t}N_{t,T}U\left(C_T, L_T\right)^{1-\rho}C_T^{-1}\left(\frac{P_{H,T}}{P_{H,t}}\frac{\bar{P}_{H,t}}{\bar{P}_{H,T}}\right)^{\sigma-1}\frac{P_{H,T}}{P_T}Y_{H,T}}, \tag{32}$$

where we have defined

$$N_{t,T} = \left(\frac{V_{t+1}^{1-\gamma}V_{t+2}^{1-\gamma}...V_T^{1-\gamma}}{E_tV_{t+1}^{1-\gamma}E_{t+1}V_{t+2}^{1-\gamma}...E_{T-1}V_T^{1-\gamma}}\right)^{\frac{\rho-\gamma}{1-\gamma}},$$

with $N_{t,t} = 1$.

The remaining fraction of firms, of measure α can change their prices only by indexing them to the current inflation index, which does not necessarily coincide with actual inflation. Therefore, we note that the Calvo's model implies the following law of motion for the aggregate price index $P_{H,t}$

$$P_{H,t}^{1-\sigma} = \alpha\bar{\Pi}_{H,t}^{1-\sigma}P_{H,t-1}^{1-\sigma} + (1-\alpha)\tilde{p}_t(h)^{1-\sigma}, \tag{33}$$

where $\bar{\Pi}_{H,t} \equiv \bar{P}_{H,t}/\bar{P}_{H,t-1}$. Using (33), we can write (32) as

$$\left(\frac{1-\alpha\left(\frac{\Pi_{H,t}}{\bar{\Pi}_{H,t}}\right)^{\sigma-1}}{1-\alpha}\right)^{\frac{1}{1-\sigma}} = \left(\frac{F_t}{K_t}\right)^{\frac{\varphi}{\varphi-\sigma\varphi+\sigma}} \tag{34}$$

where F_t and K_t can be written recursively as

$$F_t = \mu\frac{1-\psi}{\psi}\frac{U\left(C_t, L_t\right)^{1-\rho}}{1-L_t}\left(\frac{Y_{H,t}}{A_t}\right)^{\frac{1}{\varphi}} + \alpha\beta E_t\left\{\left(\frac{\Pi_{H,t+1}}{\bar{\Pi}_{H,t+1}}\right)^{\frac{\sigma}{\varphi}}\left(\frac{V_{t+1}^{1-\gamma}}{E_tV_{t+1}^{1-\gamma}}\right)^{\frac{\rho-\gamma}{1-\gamma}}F_{t+1}\right\}, \tag{35}$$

$$K_t = U\left(C_t, L_t\right)^{1-\rho}\frac{P_{H,t}Y_{H,t}}{P_tC_t} + \alpha\beta E_t\left\{\left(\frac{\Pi_{H,t+1}}{\bar{\Pi}_{H,t+1}}\right)^{\sigma-1}\left(\frac{V_{t+1}^{1-\gamma}}{E_tV_{t+1}^{1-\gamma}}\right)^{\frac{\rho-\gamma}{1-\gamma}}K_{t+1}\right\}. \tag{36}$$

Notice that equilibrium in the labor market requires

$$L_t = \frac{1}{n}\int_0^n L_t(h)dh = \frac{1}{n}\int_0^n\left(\frac{y_t(h)}{A_t}\right)^{\frac{1}{\varphi}}dh = \Delta_t\left(\frac{Y_{H,t}}{A_t}\right)^{\frac{1}{\varphi}} \tag{37}$$

where the index of price dispersion Δ_t can be written recursively as

$$\Delta_t \equiv \frac{1}{n}\int_0^n\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\frac{\sigma}{\varphi}}dh = \alpha\Delta_{t-1}\left(\frac{\Pi_{H,t}}{\bar{\Pi}_{H,t}}\right)^{\frac{\sigma}{\varphi}} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_{H,t}}{\bar{\Pi}_{H,t}}\right)^{\sigma-1}}{1-\alpha}\right)^{-\frac{\sigma}{\varphi(1-\sigma)}}. \tag{38}$$

The price-setting mechanism is similar in the Foreign country, where now $(1-\alpha^*)$ represents the mass of firms, with $0 \leq \alpha^* < 1$, that can reset their prices each period. Following similar steps, the Foreign country's aggregate-supply equation can be written as

$$\left(\frac{1-\alpha^*\left(\frac{\Pi^*_{F,t}}{\bar{\Pi}^*_{F,t}}\right)^{\sigma-1}}{1-\alpha^*}\right)^{\frac{1}{1-\sigma}} = \left(\frac{F^*_t}{K^*_t}\right)^{\frac{\varphi}{\varphi-\sigma\varphi+\sigma}}, \tag{39}$$

with

$$F^*_t = \mu\frac{1-\psi}{\psi}\frac{U\left(C^*_t, L^*_t\right)^{1-\rho}}{1-L^*_t}\left(\frac{Y^*_{F,t}}{A^*_t}\right)^{\frac{1}{\varphi}} + \alpha^*\beta E_t\left\{\left(\frac{\Pi^*_{F,t+1}}{\bar{\Pi}^*_{F,t+1}}\right)^{\frac{\sigma}{\varphi}}\left(\frac{V^{*1-\gamma}_{t+1}}{E_t V^{*1-\gamma}_{t+1}}\right)^{\frac{\rho-\gamma}{1-\gamma}} F^*_{t+1}\right\}, \tag{40}$$

$$K^*_t = U\left(C^*_t, L^*_t\right)^{1-\rho}\frac{P_{F,t}Y^*_{F,t}}{P_t Q_t C^*_t} + \alpha^*\beta E_t\left\{\left(\frac{\Pi^*_{F,t+1}}{\bar{\Pi}^*_{F,t+1}}\right)^{\sigma-1}\left(\frac{V^{*1-\gamma}_{t+1}}{E_t V^{*1-\gamma}_{t+1}}\right)^{\frac{\rho-\gamma}{1-\gamma}} K^*_{t+1}\right\}, \tag{41}$$

where $\Pi^*_{F,t} = P^*_{F,t}/P^*_{F,t-1}$ and $\bar{\Pi}^*_{F,t}$ is the gross inflation target to which foreign prices adjust each period. Equilibrium in the Foreign labor market implies

$$L^*_t = \frac{1}{1-n}\int_n^1 L^*_t(f)df = \frac{1}{1-n}\int_n^1\left(\frac{y^*_t(f)}{A^*_t}\right)^{\frac{1}{\varphi}} df = \Delta^*_t\left(\frac{Y^*_{F,t}}{A^*_t}\right)^{\frac{1}{\varphi}} \tag{42}$$

where now Δ^*_t is given by

$$\Delta^*_t \equiv \frac{1}{1-n}\int_n^1\left(\frac{p^*_t(f)}{P^*_{F,t}}\right)^{-\frac{\sigma}{\varphi}} df = \alpha^*\Delta^*_{t-1}\left(\frac{\Pi^*_{F,t}}{\bar{\Pi}^*_{F,t}}\right)^{\frac{\sigma}{\varphi}} + (1-\alpha^*)\left(\frac{1-\alpha^*\left(\frac{\Pi^*_{F,t}}{\bar{\Pi}^*_{F,t}}\right)^{\sigma-1}}{1-\alpha^*}\right)^{-\frac{\sigma}{\varphi(1-\sigma)}}. \tag{43}$$

Finally we note the following relationship between the terms of trade and producer-price inflation rates

$$T_t = T_{t-1}\frac{S_t}{S_{t-1}}\frac{\Pi^*_{F,t}}{\Pi_{H,t}}. \tag{44}$$

4.3 Monetary policy rules

We close the model by specifying the monetary policy rules. A broad class of policy rules that we consider can be written as

$$(1+i_{1,t}) = (1+i_{1,t-1})^{\phi_i}\left(\frac{\bar{\Pi}_t}{\tilde{\beta}}\right)^{1-\phi_i}\left(\frac{\Pi_{H,t}}{\bar{\Pi}_t}\right)^{(1-\phi_i)\phi_\pi}\left(\frac{\tilde{Y}_{H,t}}{\tilde{Y}_{H,t-1}}\right)^{(1-\phi_i)\phi_y}\left(\frac{S_t}{S_{t-1}}\right)^{(1-\phi_i)\phi_s} e^{\xi_t} \tag{45}$$

for the Home monetary policymaker where the short-term interest rate reacts to its past value, to the deviation of the gross producer inflation from a target, to domestic output growth and to the changes in the exchange rate;[13] ϕ_i, ϕ_π, ϕ_y ϕ_s are non-negative parameters, $\tilde{\beta}$ is an appropriately-defined parameter, ξ_t is the policy shock and $\bar{\Pi}_t$ represent the inflation target followed by the Home monetary policymaker which is generally different from the target to which prices are indexed. The link between the two inflation targets could be expressed as

$$\bar{\Pi}_{H,t} = \bar{\Pi}_t^{\kappa} \Pi_{H,t-1}^{1-\kappa},$$

with a weight $\kappa \in [0, 1]$ which can be interpreted as a measure of the credibility of monetary policy in the Home country. When $\kappa = 1$ producer prices are indexed to the inflation target used by the monetary policymaker, otherwise prices are indexed to a weighted average of past realized producer inflation and the current policy target.

In a specular way we assume that in the Foreign country the short-term nominal interest rate follows

$$(1 + i_{1,t}^*) = (1 + i_{1,t-1}^*)^{\phi_i^*} \left(\frac{\bar{\Pi}_t^*}{\tilde{\beta}^*}\right)^{1-\phi_i^*} \left(\frac{\Pi_{F,t}^*}{\bar{\Pi}_t^*}\right)^{(1-\phi_i^*)\phi_\pi^*} \left(\frac{\tilde{Y}_{F,t}^*}{\tilde{Y}_{F,t-1}^*}\right)^{(1-\phi_i^*)\phi_y^*} \left(\frac{S_t}{S_{t-1}}\right)^{-(1-\phi_i^*)\phi_s^*} e^{\xi_t^*} \tag{46}$$

where ϕ_i^*, ϕ_π^*, ϕ_y^*, ϕ_s^* are non-negative parameters, $\tilde{\beta}^*$ is an appropriately-defined parameter, ξ_t^* is the policy shock and $\bar{\Pi}_t^*$ represents the inflation-target followed by the Foreign monetary policymaker where now

$$\bar{\Pi}_{F,t}^* = (\bar{\Pi}_t^*)^{\kappa^*} (\Pi_{F,t-1}^*)^{1-\kappa^*}$$

with a weight $\kappa^* \in [0, 1]$ measuring the credibility of Foreign monetary policy.

4.4 Equilibrium

We now define the equilibrium of the above model. Given processes for the exogenous state variables ($\ln A_t$, $\ln \xi_t$, $\ln \bar{\Pi}_{H,t}, \ln A_t^*$, $\ln \xi_t^*$, $\ln \bar{\Pi}_{F,t}^*$) an equilibrium is an allocation (V_t, V_t^*, C_t, C_t^*, L_t, L_t^*, $Y_{H,t}$, $Y_{F,t}^*$, $P_{H,t}/P_t$, $P_{F,t}/P_t$, S_t/S_{t-1}, Q_t, T_t, G_t, $\Pi_{H,t}$, $\Pi_{F,t}^*$, Π_t, Π_t^*, Δ_t, Δ_t^*, F_t, F_t^*, K_t, K_t^*, $M_{t,t+1}$, $M_{t,t+1}^*$, $i_{1,t}$, $i_{1,t}^*$) which satisfies the equations (2), (5), (13), (14), (15), (17), (18), (22), (23), (24), (25), (26), (27), (29), (30), (34), (35), (36), (37), (38), (39), (40), (41), (42), (43), (44) given the two policy rules (45) and (46) and the relationships between the inflation targets of the firms and of the monetary policymaker.

We assume that the vector of exogenous variables follow conditionally-linear processes with time-varying volatility. In particular we assume a general specification of the stochastic productivity processes to take into account the possibility of a trend in productivity. We model the productivity shock in country H as $A_t = A_{W,t}\tilde{A}_t$ and that in country F as $A_t^* = A_{W,t}\tilde{A}_t^*$where $A_{W,t}$ has a stochastic trend and can be interpreted as a global common productivity shock while

[13]We will also consider a target in terms of CPI inflation instead of PPI inflation.

$\tilde{A}_t$ and $\tilde{A}_t^*$ are log-stationary processes that are country-specific.[14]

The stochastic processes of the shocks are:

$$\ln A_{W,t+1} = \ln a + \ln A_{W,t} + u_{aw,t}\varepsilon_{aw,t+1}$$

$$\ln \tilde{A}_{t+1} = \delta_a \ln \tilde{A}_t + u_{a,t}\varepsilon_{a,t+1}$$

$$\ln \bar{\Pi}_{t+1} = \ln \bar{\Pi}_t + u_{\pi,t}\varepsilon_{\pi,t+1}$$

$$\xi_{t+1} = u_{\xi,t}\varepsilon_{\xi,t+1}$$

where a is a parameter measuring the deterministic trend in productivity growth and $0 \leq \delta_a \leq 1$. In what follows, all the ε shocks are *iid* white-noise processes. Moreover $0 \leq \chi_a, \chi_\pi \leq 1$.

Time-varying volatility is modelled through linear processes for the variances:

$$u_{aw,t+1}^2 = (1-\rho_{aw})\sigma_u^2 + \rho_{aw}u_{aw,t}^2 + \sigma_\zeta^2\zeta_{aw,t+1}$$

$$u_{a,t+1}^2 = (1-\rho_a)\sigma_u^2 + \rho_a u_{a,t}^2 + \sigma_\zeta^2\zeta_{a,t+1}$$

$$u_{\pi,t+1}^2 = (1-\rho_\pi)\sigma_u^2 + \rho_\pi u_{\pi,t}^2 + \sigma_\zeta^2\zeta_{\pi,t+1}$$

$$u_{\xi,t+1}^2 = (1-\rho_\xi)\sigma_u^2 + \rho_\xi u_{\xi,t}^2 + \sigma_\zeta^2\zeta_{\xi,t+1}$$

in which all the ζ are *iid* white-noise processes and $0 \leq \rho_{aw}, \rho_a, \rho_\pi, \rho_\xi, \leq 1$ with $\sigma_u^2, \sigma_\zeta^2 > 0$. The processes for the stochastic disturbances hitting the Foreign economy behave similarly:

$$u_{a^*,t+1}^2 = (1-\rho_{a^*})\sigma_u^2 + \rho_{a^*}u_{a^*,t}^2 + \sigma_\zeta^2\zeta_{a^*,t+1}$$

$$u_{\pi^*,t+1}^2 = (1-\rho_{\pi^*})\sigma_u^2 + \rho_{\pi^*}u_{\pi^*,t}^2 + \sigma_\zeta^2\zeta_{\pi^*,t+1}$$

$$u_{\xi^*,t+1}^2 = (1-\rho_{\xi^*})\sigma_u^2 + \rho_{\xi^*}u_{\xi^*,t}^2 + \sigma_\zeta^2\zeta_{\xi^*,t+1}$$

In what follows we will refer to the shocks to the inflation target and the shock to the policy instruments as *monetary* or *nominal* shocks while the productivity shock will be the *real* shock.

4.5 Solution

Given the above specification for the processes of the exogenous state variable, we can write them more compactly as

$$z_{t+1} = \Lambda_z z_t + \eta_{t+1} \tag{47}$$

[14]In this way our model will allow for a balanced-growth path. As we will show in the next section, the stochastic trend is in particular important for the relevance of the Epstein-Zin assumption.

where the vector z_t is defined as $z_t \equiv (\ln A_{W,t}, \ln \tilde{A}_t,\ \xi_t,\ \ln \bar{\Pi}_{H,t}, \ln \tilde{A}_t^*,\ \xi_t^*,\ \ln \bar{\Pi}_{F,t}^*)$ and Λ_z is an appropriately-defined square matrix. The vector η_{t+1} is given by

$$\eta_{t+1} = U_t \varepsilon_{z,t+1} \tag{48}$$

where $\varepsilon_{z,t+1}$ collects the innovations, which are assumed to have a bounded support and to be independently and identically distributed with mean zero and variance/covariance matrix I_z, where I_z is an identity matrix of the same dimension of the vector z; U_t is a diagonal matrix whose elements on the diagonal are collected into a vector u_t. In particular u_t follows the exogenous stochastic linear process given by

$$u_{t+1}^2 = \sigma_u^2 (I_z - \Lambda_u)\bar{u}^2 + \Lambda_u u_t^2 + \sigma_\zeta^2 Z \zeta_{u,t+1}. \tag{49}$$

Each element of u_t^2 is the corresponding squared value of each element of u_t, which still corresponds to the diagonal of matrix U_t as in (48); $\bar{u}^2$ is a vector of steady-state variances, Z and Λ_u are appropriately defined square matrices; $\zeta_{u,t+1}$ is a vector of innovation collecting the above ζ which are assumed to have a bounded support and to be independently and identically distributed with mean zero and variance/covariance matrix I_z; σ_u and σ_ζ are scalars with $\sigma_u, \sigma_\zeta \geq 0$.

Noticing that (47) with (48) and (49) defines a conditionally-linear process, we can write the set of equilibrium conditions of the model together with the conditional expectation of (47) in a more compact form

$$E_t \{ f(y_{t+1}, x_{t+1}, y_t, x_t) \} = \mathbf{0}, \tag{50}$$

for an appropriately defined vector of function $f(\cdot)$ where y_t identifies the non-predetermined variables while the vector x_t of state variables contains also the vector of exogenous predetermined variables z_t. Given the processes (47), with (48) and (49), an equilibrium of our model is a sequence for the vector of endogenous non-predetermined variables y_t and for the state variables x_t that satisfies (50), given the initial conditions.

Benigno et al. (2010) characterize the solution of the above model and show that a first-order approximation of the solution can be written as

$$\tilde{y}_t = \bar{g}_x \tilde{x}_t,$$

$$\tilde{x}_{t+1} = \bar{h}_x \tilde{x}_t + \bar{h}_\eta \eta_{t+1},$$

for appropriately-defined matrices $\bar{g}_x$, $\bar{h}_x$ and $\bar{h}_\eta$. This approximation does not correspond to a fully linear solution since η_{t+1}, defined, in (48) is non-linear. However, it is the best conditionally linear approximation and, in particular, the matrices $\bar{g}_x$ and $\bar{h}_x$ coincide with those of a fully linear approximation. Time-varying volatility does not play a distinct role in this first-order approximation since the impulse response of the endogenous variables with respect to the shock

to volatility, $\zeta_{u,t+1}$, is always zero. The advantage of performing a conditionally-linear approximation instead of a fully-linear approximation, in which η_{t+1} is also linearized, is clear when we look at a second-order approximation of the solution. Benigno et al. (2010) show that this takes the form

$$\tilde{y}_t = \bar{g}_x \tilde{x}_t + \frac{1}{2}(I_y \otimes \tilde{x}_t')\bar{g}_{xx}\tilde{x}_t + \frac{1}{2}\bar{g}_{uu}u_t^2 + \frac{1}{2}\bar{g}_{zz}\sigma_u^2, \tag{51}$$

$$\tilde{x}_{t+1} = \bar{h}_x \tilde{x}_t + \frac{1}{2}(I_x \otimes \tilde{x}_t')\bar{h}_{xx}\tilde{x}_t + \frac{1}{2}\bar{h}_{uu}u_t^2 + \frac{1}{2}\bar{h}_{zz}\sigma_u^2 + \bar{h}_\eta \eta_{t+1}, \tag{52}$$

for appropriately defined matrices $\bar{g}_{xx}$, $\bar{g}_{zz}$, $\bar{g}_{uu}$ and $\bar{h}_{xx}$, $\bar{h}_{zz}$, $\bar{h}_{uu}$. In this second-order approximation, the volatility of the exogenous state variables now plays a distinct and direct role through the matrices $\bar{g}_{uu}$ and $\bar{h}_{uu}$. Indeed the endogenous variables are now in a linear relationship with the vector of volatilities, u_t^2. Other methods discussed in the literature, as Fernandez-Villaverde et al. (2010), need instead to rely to at least a third-order approximation to get such a distinct role for volatilities in influencing the endogenous variables.

The second advantage of our conditionally-linear approximation is that risk-premia, evaluated using a first-order approximation of the model, will be also time-varying. This feature enables the model to characterize some stylized facts on the role of volatility on international data in a simple way.

5 Exchange rates and risk: a simple example

In this section, before we turn to the solution of our general model, we present a simplified framework to study whether we can already account for some of the facts that we have underlined in the empirical analysis. The framework of this section, with its analytical solutions, will be also helpful to explain how our solution method works and represent a useful benchmark through which we can later evaluate the effects of relaxing the assumptions of this section. The simplifying assumptions are: 1) monetary policy in each country is modeled through Taylor rules reacting only to the domestic CPI inflation rate with the same coefficients across countries, later in the section we allow for interest-rate smoothing; 2) purchasing power parity holds ($v = v^* = n$) 3) flexible prices ($\alpha = \alpha^* = 0$) and constant real rates which make real shocks irrelevant for the analysis of this section. Therefore, we will abstract completely from productivity shocks and give just a monetary explanation of the facts related to the nominal exchange rate, the UIP deviations and the slope of the yield curve. Clearly this framework cannot aim at matching the Backus-Smith puzzle.

The starting points are the standard arbitrage-free conditions (26) and (27). As discussed more generally in Benigno et al. (2010), we rely on approximation methods to solve our model. In particular we show that it is sufficient to use a second-order approximation of the model to characterize how risk influences the variables of interest and in particular the exchange rate.

By taking a second-order approximation of (26) and (27), we obtain

$$\hat{\imath}_{1,t} = -E_t \hat{M}_{t+1} - \frac{1}{2} Var_t \hat{M}_{t+1} \tag{53}$$

$$\hat{\imath}^*_{1,t} = -E_t \hat{M}^*_{t+1} - \frac{1}{2} Var_t \hat{M}^*_{t+1} \tag{54}$$

where hats denote log-deviations with respect to the steady state, in which we assume $i_1 = i_1^* = 1/\beta - 1$, and E_t and Var_t are expectation and variance operators, respectively.[15] In logs, the complete-market assumption (19) implies

$$\hat{M}_{t+1} = \hat{M}^*_{t+1} - \Delta s_{t+1}. \tag{55}$$

We can combine (53), (54) and (55) to write the short-term excess return of investing in the currency of country F with respect to investing in country H

$$\hat{\imath}^*_{1,t} + E_t \Delta s_{t+1} - \hat{\imath}_{1,t} = \frac{\vartheta^*_t}{2} - \frac{\vartheta_t}{2} \tag{56}$$

where

$$\vartheta_t = cov_t(\hat{M}_{t+1}, \Delta s_{t+1}) \qquad \vartheta^*_t = cov_t(\hat{M}^*_{t+1}, -\Delta s_{t+1}). \tag{57}$$

As it is standard in finance models, the stochastic discount factors measure the agents' appetites for state contingent wealth. When $\hat{M}_{t+1}$ and $\hat{M}^*_{t+1}$ are high in some contingencies, the appetites for wealth of the Home and Foreign agents are also high in those contingencies. If the currency of country H depreciates (the nominal exchange rate depreciates, i.e. $\Delta s_{t+1} > 0$) then having invested in the currency of country F is a good strategy since it delivers more money when it is really needed. In this case ϑ_t is positive and ϑ^*_t is negative. The expected short-term excess return of investing in the currency of country H with respect to that of investing in the currency of country F is positive because simply the currency of country H is not a good hedge with respect to the appetite for wealth of both agents. In general, to have a positive expected excess return for the Home currency it is not necessary that ϑ_t should be positive and ϑ^*_t negative, but just $\vartheta_t > \vartheta^*_t$. It is worth stressing that the right-hand side of equation (56) captures the deviations from uncovered interest parity in any model in which no-arbitrage restrictions apply. Indeed, so far, none of the simplifying assumptions 1), 2) and 3) has been used.

[15]Notice that (53) and (54) do not hold exactly but up to residuals which are of third-order in an appropriate norm on the stochastic disturbances. Under the assumption of log-normality, as in Backus et al. (2010), they would hold exactly. However, our analysis will be a local analysis and their will be a global analysis. Therefore, their approach is limited to the possibility of getting a closed-form solution. The two frameworks will also deliver subtle differences in terms of the conditions needed for the determinancy of the equilibrium.

5.1 Simple Taylor Rules

By making assumption 1) ($\phi_i = \phi_i^* = \phi_y = \phi_y^* = \phi_s = \phi_s^* = 0$, $\phi_\pi = \phi_\pi^*$ with interest rate reacting to CPI inflation into (45) and (46)), we can further use (56) to determine the equilibrium nominal exchange rate. In particular, the short-term nominal interest rates follow simple Taylor rules in which

$$\hat{\imath}_{1,t} = \bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t) + \xi_t \tag{58}$$

$$\hat{\imath}_{1,t}^* = \bar{\pi}_t^* + \phi_\pi(\pi_t^* - \bar{\pi}_t^*) + \xi_t^* \tag{59}$$

where $\bar{\pi}_t$ and $\bar{\pi}_t^*$ represent the logs of Home and Foreign inflation-target shocks and ξ_t and ξ_t^* are the Home and Foreign policy shocks as in (45) and (46).

We now use the simplifying assumption 2), that there is no home bias in consumption implying that purchasing power parity holds, i.e. $\pi_t = \pi_t^* + \Delta s_t$.

5.1.1 Exchange rate determination

Using PPP and rules (58) and (59) into (56) we obtain a first-order stochastic difference equation in Δs_t

$$E_t \Delta s_{t+1} = \phi_\pi \Delta s_t + (1 - \phi_\pi)(\bar{\pi}_t - \bar{\pi}_t^*) + (\xi_t - \xi_t^*) - \frac{1}{2}(\vartheta_t - \vartheta_t^*)$$

which can be solved forward to deliver a unique bounded solution for the nominal exchange rate of the form

$$\Delta s_t = E_t \sum_{T=t}^{\infty} \left(\frac{1}{\phi_\pi}\right)^{T+1-t} \left[(\phi_\pi - 1)(\bar{\pi}_T - \bar{\pi}_T^*) - (\xi_T - \xi_T^*) + \frac{1}{2}(\vartheta_T - \vartheta_T^*)\right], \tag{60}$$

under the requirement, for determinacy, that the Taylor's principle holds, i.e. $\phi_\pi > 1$.[16]

There are several implications of the above simple model for nominal exchange rate determination. First, the design of the monetary policy rules is important. Indeed, equation (60) holds only under the special policy rules (58) and (59).[17] Within this class of rules, variation in the policy parameter ϕ_π can also change in an important way the relationship between exchange rate and fundamentals. But, which are the fundamentals for exchange rate determination under this simple model? Shocks and risk. Given that $\phi_\pi > 1$ is needed for equilibrium determinacy, a shock that increases the inflation target in a country depreciates its currency, whereas a contractionary policy shock in a country appreciate its own currency (the sign of the response to the policy shock is consistent with the empirical findings that we reported in section 2). In particular, a (temporary) contractionary policy shock appreciates permanently the exchange rate, but without producing the hump-shaped curve found in the data.

[16] Necessary and sufficient conditions for the local determinacy of equilibrium are discussed more extensively in Benigno and Benigno (2008), for two-country open-economy models.

[17] Hodrick (1989) and Obstfeld and Rogoff (2001) restrict their attention to special money-supply rules in which the equilibrium in the money market becomes also relevant for the determination of the exchange rate.

Current and future shocks matter, but also current and future risk premia. If the currency of country F has relatively good hedge properties with respect to the currency of country H ($\vartheta_t > \vartheta_t^*$) then currency F strengthens and current nominal exchange rate s_t rises.

Equation (60) represents a second-order approximation for the solution of the equilibrium nominal exchange rate which depends on first-order terms $\{\bar{\pi}_t, \bar{\pi}_t^*, \xi_t, \xi_t^*\}$ and second-order terms $\{\vartheta_t, \vartheta_t^*\}$. However, to get an explicit solution for the exchange rate in terms of the state variables, we need to solve the second-order terms. The simplification comes by observing that these second-order terms can be just evaluated using a first-order approximation.[18] In particular, given (57), to evaluate ϑ_t and ϑ_t^* we need a first-order approximation of the stochastic discount factors $\hat{M}_{t+1}$ and $\hat{M}_{t+1}^*$ and first-order approximation of Δs_t, which we already have in (60). In the general model of the previous section, the stochastic discount factors $\hat{M}_{t+1}$ and $\hat{M}_{t+1}^*$ are complex linear function, in a first-order approximation, of the shocks of the model. In our simple illustrative example, we assume flexible prices and constant real interest rate (assumption 3). In this case, the stochastic discount factors are just exact linear functions of the inflation rates

$$\hat{M}_{t+1} = -\pi_{t+1} \qquad\qquad \hat{M}_{t+1}^* = -\pi_{t+1}^*.$$

Moreover the inflation target shocks behave as random walks with stochastic volatility

$$\bar{\pi}_t = \bar{\pi}_{t-1} + u_{\pi,t-1}\varepsilon_{\pi,t}$$

$$\bar{\pi}_t^* = \bar{\pi}_{t-1}^* + u_{\pi,t-1}^*\varepsilon_{\pi,t}^*,$$

where $\varepsilon_{\pi,t}$ and $\varepsilon_{\pi,t}^*$ are *iid* white-noise processes. The random-walk assumption will be particularly relevant to capture a level effect affecting the yield curve. For the policy shocks we assume

$$\xi_t = u_{\xi,t-1}\varepsilon_{\xi,t}$$

$$\xi_t^* = u_{\xi,t-1}^*\varepsilon_{\xi,t}^*$$

where $\varepsilon_{\xi,t}$ and $\varepsilon_{\xi,t}^*$ are *iid* white-noise processes.[19] The variances of the above processes are all time varying following the linear stochastic processes

$$u_{\pi,t}^2 = \sigma_u^2 + \rho_\pi(u_{\pi,t-1}^2 - \sigma_u^2) + \sigma_\zeta^2\zeta_{\pi,t}$$

$$u_{\pi,t}^{*2} = \sigma_u^2 + \rho_\pi(u_{\pi,t-1}^{*2} - \sigma_u^2) + \sigma_\zeta^2\zeta_{\pi,t}^*$$

$$u_{\xi,t}^2 = \sigma_u^2 + \rho_\xi(u_{\xi,t-1}^2 - \sigma_u^2) + \sigma_\zeta^2\zeta_{\xi,t}$$

$$u_{\xi,t}^{*2} = \sigma_u^2 + \rho_\xi(u_{\xi,t-1}^{*2} - \sigma_u^2) + \sigma_\zeta^2\zeta_{\xi,t}^*,$$

[18] See also Lombardo and Sutherland (2007).

[19] We could surely generalize to autoregressive process for the policy shock, but the most common assumption in the literature is that of white-noise processes.

where $0 \leq \rho_\pi, \rho_\xi \leq 1$ and all the zetas are *iid* white-noise processes while σ_u^2 and σ_ζ^2 are non-negative parameters.

Given the above defined processes, and up to a first-order approximation, equation (60) implies

$$\Delta s_t = (\bar{\pi}_t - \bar{\pi}_t^*) - \frac{1}{\phi_\pi}(\xi_t - \xi_t^*) \tag{61}$$

where movements in the inflation-target shocks move one-to-one the nominal exchange rate, while the response of the nominal exchange rate to policy shocks depends on the parameter of the Taylor rules. Using (53) and (58), and (54) and (59) respectively, we can determine the domestic and foreign inflation rates as

$$\pi_t = \bar{\pi}_t - \frac{1}{\phi_\pi}\xi_t \tag{62}$$

$$\pi_t^* = \bar{\pi}_t^* - \frac{1}{\phi_\pi}\xi_t^* \tag{63}$$

which in this simple example only reflect the influence of their own monetary shocks. We can use (61), (62) and (63) to evaluate the risk-premia component in (57)

$$\vartheta_t = cov_t(\hat{M}_{t+1}, \Delta s_{t+1}) = -u_{\pi,t}^2 - \frac{1}{\phi_\pi^2}u_{\xi,t}^2$$

$$\vartheta_t^* = cov_t(\hat{M}_{t+1}^*, -\Delta s_{t+1}) = -u_{\pi,t}^{*2} - \frac{1}{\phi_\pi^2}u_{\xi,t}^{*2}$$

which can be plugged into (60) to obtain the equilibrium exchange rate

$$\begin{aligned} \Delta s_t &= (\bar{\pi}_t - \bar{\pi}_t^*) - \frac{1}{\phi_\pi}(\xi_t - \xi_t^*) - \frac{1}{2}\frac{1}{\phi_\pi - \rho_\pi}(u_{\pi,t}^2 - u_{\pi,t}^{*2}) \\ &\quad -\frac{1}{2}\frac{1}{\phi_\pi - \rho_\xi}\frac{1}{\phi_\pi^2}(u_{\xi,t}^2 - u_{\xi,t}^{*2}). \end{aligned} \tag{64}$$

In this solution, the time-varying volatilities of the monetary shocks matter for the determination of the nominal exchange rate.[20] This is the important consequence of the solution method proposed by Benigno et al. (2010) in which a second-order approximation of the model is sufficient to get a distinct role for time-varying uncertainty in affecting the determination of variables of interest. In (64), the higher the variance of the inflation-target and of the policy shocks in country H, the stronger the currency of country H is. And specularly for the volatility of the monetary shocks in country F. These theoretical findings are in part consistent with the empirical results of Section 2: there, we reported that an increase in both volatilities leads to an appreciation of the currency (at least in the medium-run with the exception of the USD/Yen bilateral).

The explanation for these effects according to the model runs as follow: the higher the

[20] Notice that the terms in σ_u^2 cancel out because of the symmetry assumed.

volatility of the monetary shocks in country H, the lower (more negative) is the risk component ϑ_t, which means that foreign currency is not a good instrument for hedging. Therefore, demand for the currency of country H increases and the exchange rate appreciates.

The magnitude of the effects on the exchange rate depends obviously on the magnitude of the shock but also on the persistence. The higher the persistence the higher the response. It is further influenced by the policy parameter of the Taylor rule, the higher ϕ_π the muted the response of the exchange rate. In this symmetric example, as for the primitive shocks, what matters for the determination of the equilibrium exchange rate is the relative strength between the volatilities of the monetary shocks across countries. However, while a positive inflation-target shock and a positive policy shock produce responses of opposite sign on the equilibrium nominal exchange rate, an increase in the volatility of the inflation-target shock or of the policy shock impact in the same direction.

5.1.2 UIP implications

$$\hat{\imath}^*_{1,t} + E_t \Delta s_{t+1} - \hat{\imath}_{1,t} = \frac{1}{2}\left(u^2_{\pi,t} - u^{*2}_{\pi,t}\right) + \frac{1}{2\phi^2_\pi}\left(u^2_{\xi,t} - u^{*2}_{\xi,t}\right)$$

The expected excess return of investing in the currency of country F with respect to that of country H rises with the increase in the volatilities of the monetary shocks in country H. While the response of the foreign excess return to an increase in volatility of shock to the policy instrument is, at first pass, consistent with the empirical findings in Section 2, an increase in the volatility of shock to the target goes in the opposite direction with what we found in the data.

This is not the only counterfactual result of this section. As discussed in Backus et al. (2010), this stylized framework cannot account for the negative slope coefficient in the UIP regression: the regression of the one-period changes in the nominal exchange rate on the interest rate differential. Using (64), (65) and (66) the coefficient of the UIP regression implied by our model would be

$$\hat{\beta}^{uip} = \frac{Cov(\Delta s_{t+1}, \hat{\imath}_{1,t} - \hat{\imath}^*_{1,t})}{Var(\hat{\imath}_{1,t} - \hat{\imath}^*_{1,t})}$$

$$\hat{\beta}^{uip} = \frac{var(\bar{\pi}_t - \bar{\pi}^*_t) + a^2_{1,u_\pi}\frac{\rho_\pi}{\phi_\pi} var(u^2_{\pi,t} - u^{*2}_{\pi,t}) + a^2_{1,u_\xi}\frac{\rho_\xi}{\phi_\pi} var(u^2_{\xi,t} - u^{*2}_{\xi,t})}{var(\bar{\pi}_t - \bar{\pi}^*_t) + a^2_{1,u_\pi} var(u^2_{\pi,t} - u^{*2}_{\pi,t}) + a^2_{1,u_\xi} var(u^2_{\xi,t} - u^{*2}_{\xi,t})}$$

where the assumption of a unit-root processes for the inflation-target shocks blows up numerator and denominator, in large samples, to produce a unitary coefficient. However, abstracting from this issue or focusing on small samples, the only possibility $\hat{\beta}^{uip}$ to be negative is that ρ_ξ/ϕ_π be negative, as shown in Backus et al. (2010). Since assuming $\rho_\xi < 0$ is not plausible, then in our simplified framework $\hat{\beta}^{uip}$ is positive and decreasing with ϕ_π, the coefficient to inflation in the Taylor rule.

5.1.3 Term Structure and Term Premia Implications

Our model has also interesting implications for the effects of uncertainty on the term structure of interest rates and in particular on the spreads between interest rates at different maturities. To build an arbitrage-free term structure we need to determine the short-term nominal interest rates. Using equations (53) and (58) and our solution methods, we can get

$$\hat{\imath}_{1,t} = \bar{\pi}_t - a_{1,u_\pi} u^2_{\pi,t} - a_{1,u_\xi} u^2_{\xi,t} - a_{1,\sigma}\sigma^2_u \tag{65}$$

$$\hat{\imath}^*_{1,t} = \bar{\pi}^*_t - a_{1,u_\pi} u^{*2}_{\pi,t} - a_{1,u_\xi} u^{*2}_{\xi,t} - a_{1,\sigma}\sigma^2_u \tag{66}$$

where

$$a_{1,u_\pi} \equiv \frac{1}{2}\frac{\phi_\pi}{\phi_\pi - \rho_\pi}$$

and

$$a_{1,u_\xi} \equiv \frac{1}{2}\frac{1}{\phi_\pi(\phi_\pi - \rho_\xi)} \qquad\qquad a_{1,\sigma} \equiv \frac{1}{2}\left[1 + \frac{1}{\phi^2_\pi}\right]\frac{(1-\rho_u)}{\phi_\pi(\phi_\pi - 1)}.$$

Given the simplifying assumptions of this section, interest rates in each country depend only on domestic nominal factors, shocks and volatilities. An educated guess is that all the yields at different maturities will be function of the same factors

$$\hat{\imath}_{n,t} = \bar{\pi}_t - a_{n,u_\pi} u^2_{\pi,t} - a_{n,u_\xi} u^2_{\xi,t} - a_{n,\sigma}\sigma^2_u$$

$$\hat{\imath}^*_{n,t} = \bar{\pi}^*_t - a_{n,u_\pi} u^{*2}_{\pi,t} - a_{n,u_\xi} u^{*2}_{\xi,t} - a_{n,\sigma}\sigma^2_u$$

where indeed $\hat{\imath}_{n,t}$ denotes the yield with maturity n periods. In particular, the coefficients which relate yields with factors can be identified using the no-arbitrage equation

$$\left(\frac{1}{1+i_{n,t}}\right)^n = E_t(M_{t+1}M_{t+2}...M_{t+n}),$$

which can be written as

$$\left(\frac{1}{1+i_{n,t}}\right)^n = E_t\left(M_{t+1}\left(\frac{1}{1+i_{n-1,t+1}}\right)^{n-1}\right). \tag{67}$$

A second-order approximation of (67) delivers the following relationship

$$\hat{\imath}_{n,t} = \frac{1}{n}\hat{\imath}_{1,t} + \left(\frac{n-1}{n}\right)E_t\hat{\imath}_{n-1,t+1} - \frac{1}{2}\frac{(n-1)^2}{n}Var_t(\hat{\imath}_{n-1,t+1}) + \frac{n-1}{n}Cov_t(\hat{\imath}_{n-1,t+1},\hat{M}_{t+1}) \tag{68}$$

in which we can substitute the solution for $\hat{\imath}_{1,t}$ and $\hat{M}_{t+1}$, the guessed solutions for $\hat{\imath}_{n,t}$ and $\hat{\imath}_{n-1,t+1}$, the processes for the shocks and variances to obtain the following recursive identifica-

tions for the unknowns coefficients a_{n,u_π}, a_{n,u_ξ}, $a_{n,\sigma}$

$$a_{n,u_\pi} = \frac{1}{n}a_{1,u_\pi} + \left(\frac{n-1}{n}\right)\rho_\pi a_{n-1,u_\pi} - \frac{1}{2}\frac{(n-1)(n+1)}{n},$$

$$a_{n,u_\xi} = \frac{1}{n}a_{1,u_\xi} + \left(\frac{n-1}{n}\right)\rho_\xi a_{n-1,u_\xi}$$

$$a_{n,\sigma} = a_{1,\sigma}.$$

There are some interesting features of the term-structure model behind the simple framework of this section. First, each yield curve reacts only to domestic shocks. This is a consequence of the special assumptions of symmetry that have been imposed. The inflation-target shock acts as a level factor since it shifts in a parallel way all the yields at all maturities. We also note that the policy shock itself has no influence on short and long yields because of our assumption of no autocorrelation. The other factors are instead all slope factors since a_{n,u_π}, a_{n,u_ξ} all decrease with maturity. In particular, when the volatility of the inflation-target shocks increases, the short-term interest rate falls, but long-term yields fall less, and it might be the case that longer-term yields actually increase, since a_{n,u_π} can turn negative for large n. This in line with the empirical evidence where an increase in the volatility of the inflation target raises the slope of the yield curve. Consistently with the empirical evidence also an increase in the volatility of the policy shock implies a steep slope since it shifts down more short rather than long-term yields.

Notice that the one-period returns on a $n-$maturity bonds satisfy the following no-arbitrage conditions

$$E_t(M_{t+1}R_{n,t+1}) = 1,$$

$$E_t(M^*_{t+1}R^*_{n,t+1}) = 1.$$

By taking a second-order approximation of the above equations and using (53) and (54) we obtain

$$E_t\hat{R}_{n,t+1} - \hat{\imath}_{1,t} + \frac{1}{2}Var_t\hat{R}_{n,t+1} = -Cov_t(\hat{M}_{t+1}, \hat{R}_{n,t+1}) \tag{69}$$

which is nothing more than a way to rewrite (68), since $\hat{R}_{n,t+1} = n\hat{\imath}_{n,t} - (n-1)\hat{\imath}_{n-1,t+1}$. Equation (69) tells us about the expected excess one-period returns for investing in long-term bonds with respect to investing in the risk-free asset, corrected for Jensen's inequalities terms. Long-term bonds command a premium if their return is negatively correlated with nominal stochastic discount factor, since they pay well when it is less needed. In the example of this section, we can write

$$Cov_t(\hat{M}_{t+1}, \hat{R}_{n,t+1}) = -(n-1)Cov_t(\hat{\imath}_{n-1,t+1}, \hat{M}_{t+1}) = (n-1)u^2_{\pi,t}$$

which shows instead that long-term bonds are a good hedge with respect to the risk captured by the stochastic discount factor and therefore they require a negative premium. Moreover, expected excess returns will depend inversely on the variability of the domestic inflation-target shock. The higher the variance of this shock the lower the expected excess return. This result

shows that the simplified framework of this section cannot aim to match the average upward sloping yield curve of the data.

5.2 Taylor rules with interest rate smoothing

One natural extension to the previous setting is to consider a model in which the interest rate set by the policy authority moves gradually (interest rates are smoothed over time as in McCallum, 1994, and Backus et al., 2010) so that interest rates depend also on their past value. The modified Taylor's rules take the form

$$\hat{\imath}_{1,t} = \phi_i \hat{\imath}_{1,t-1} + (1-\phi_i)[\bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t)] + \xi_t,$$

$$\hat{\imath}^*_{1,t} = \phi_i \hat{\imath}^*_{1,t-1} + (1-\phi_i)[\bar{\pi}^*_t + \phi_\pi(\pi^*_t - \bar{\pi}^*_t)] + \xi^*_t,$$

to replace (58) and (59). Following the same steps as before, it is possible to show that the equilibrium exchange rate is given by

$$\begin{aligned} \Delta s_t \quad = \quad & -\frac{\phi_i}{\lambda-\phi_i}(\hat{\imath}_{1,t-1} - \hat{\imath}^*_{1,t-1}) + \frac{\lambda}{\lambda-\phi_i}(\bar{\pi}_t - \bar{\pi}^*_t) - \frac{1}{\lambda}(\xi_t - \xi^*_t) \\ & -\frac{1}{2}\frac{1}{\lambda-\rho_u}\left(\frac{\lambda}{\lambda-\phi_i}\right)^2 (u^2_{\pi,t} - u^{*2}_{\pi,t}) - \frac{1}{2}\frac{1}{\lambda-\rho_u}\frac{1}{\lambda^2}(u^2_{\xi,t} - u^{*2}_{\xi,t}). \end{aligned} \tag{70}$$

where we are restricting ϕ_i to be $0 < \phi_i < 1$ and where $\lambda \equiv \phi_\pi(1-\phi_i) + \phi_i$ with the requirement $\lambda > 1$ for equilibrium determinacy implying again $\phi_\pi > 1$. In general allowing for interest-rate smoothing changes also the short-run responses to the shocks and the volatilities but does not change the sign of the response. Responses are obviously changed at longer horizons given the lagged reaction to the interest rate.

There are two important contributions of assuming interest-rate smoothing. First, a contractionary policy shock can now produce an hump-shaped appreciation of the exchange rate as in the data. Second, the negative dependence on lagged interest rates can be such to reduce the coefficient of the UIP regression and eventually to turn it negative, as discussed in Backus et al. (2010) but it does not change the sign of the responses of the expected excess return on foreign-versus-domestic currency to the volatilities of the monetary shocks.

On the contrary, this simple modification of the benchmark model does not produce significant differences for the shape of the yield curve and the responses of the slope and interest rates to the shocks and their volatility. Indeed, among other results, we report that the expected excess return is still negative and given by

$$E_t\hat{R}_{n,t+1} - \hat{\imath}_{1,t} + \frac{1}{2}Var_t\hat{R}_{n,t+1} = -\frac{\lambda(n-1)}{\lambda-\phi_i}u^2_{\pi,t}.$$

6 Exchange rates and risk: the general case (preliminary)

We now turn to the implications of the more general framework with sticky prices presented in Section 4. First, we investigate the properties of the nominal stochastic discount factor which, as shown in the previous section, is critical to understand the relationship between exchange rate and risk, and to evaluate the risk-premia embedded in asset prices.

In our general framework the stochastic discount factor depends on the Epstein-Zin preference specification. Our first result shows a peculiarity of Epstein-Zin preferences in an international context. In closed economy, a standard finding is the irrelevance of Epstein-Zin preferences for quantities and the importance for asset pricing.[21] The irrelevance result can be understood by observing that up to a first-order approximation, Epstein-Zin preferences do not matter for the equilibrium allocation. In contrast, we will show that Epstein-Zin preferences are also important for quantities, in our two-country open-economy model, since, as shown in equation (30), the cross-country surprises in utility affect the international distribution of wealth. Indeed in a first-order approximation, we obtain

$$\hat{G}_{t+1} = \hat{G}_t + (\gamma - \rho)[(\hat{V}_{t+1} - E_t\hat{V}_{t+1}) - (\hat{V}^*_{t+1} - E_t\hat{V}^*_{t+1})].$$

where hats denote log-deviations with respect to the steady-state. Under expected utility, $\rho = \gamma$, and $\hat{G}_t$ will be constant across time, implying the standard risk-sharing condition which links marginal utilities of nominal income across countries. Instead, with the Epstein-Zin preferences the cross-country differences in the realization of utility matter for the distribution of wealth. This might have interesting consequences for the ability of the model to explain the Backus-Smith anomaly and in general for the equilibrium allocation of quantities.[22]

However, the general-equilibrium flavor of our analysis makes it difficult to keep track of all the effects through analytical solutions. To get further insights and to study the contribution of the Epstein-Zin preferences to the evaluation of risk premia, we now discuss more deeply the properties of the stochastic discount factor. In a first order approximation of (22), the Home-country nominal discount factor can be written as[23]

$$\hat{M}_{t,t+1} = -(\gamma-\rho)(\hat{V}_{t+1} - E_t\hat{V}_{t+1}) + (1-\rho)(1-\psi)\Delta\hat{L}_{t+1} - [1-\psi(1-\rho)](\Delta\hat{C}_{t+1} + \Delta\hat{A}_{W,t+1}) - \pi_{t+1}, \tag{71}$$

where we have defined $\hat{L}_t = \ln(1-L_t)/\ln(1-L)$ while $\hat{C}_t$ denotes the deviations of detrended consumption with respect to the steady state, $\hat{C}_t \equiv \ln C_t/A_{W,t} - \ln(C/A_W)$.[24] Under the expected-utility model, $\gamma = \rho$, the stochastic discount factor is a function of consumption growth, which

[21] See among others Rudebush and Swanson (2009).

[22] Notice however that in this first-order approximation $E_t\hat{G}_{t+1} = \hat{G}_t$ and therefore $\hat{G}_t$ is a local martingale.

[23] A first-order approximation of the stochastic discount factor is all that is needed to evaluate the risk-premia of the model. In particular, through this first-order approximation, we can evaluate the expected excess returns on long-term bonds, still given by (69), and the deviations from uncovered interest parity, still captured by (56) and (57).

[24] The balance growth path of the model is defined with respect to the common trend in productivity, A_W.

can be decomposed in the growth of detrended consumption and in the growth of world productivity, a function of the CPI inflation rate and of the growth in hours worked. An increase in consumption reduces the stochastic discount factor and the appetite for wealth, as long as the intertemporal elasticity of substitution, ρ, is close to the unitary value. The impact of the growth in hours worked depends on $\rho \lesseqgtr 1$ while an increase in the inflation rate reduces instead unambiguously the appetite for wealth. On top of affecting the equilibrium allocation and therefore the allocation of consumption and labor, as discussed above, Epstein-Zin preferences bring the novelty that also surprises in the indirect utility matter through the term $(\hat{V}_{t+1} - E_t\hat{V}_{t+1})$. To get further insights on this component, we take a first-order approximation of the indirect utility (2) and show that we can relate it to the present discounted value of the surprises in consumption and labor

$$\hat{V}_{t+1} - E_t\hat{V}_{t+1} = (1-\beta)\sum_{T=t+1}^{\infty}\beta^{T-t-1}[\Delta E_{t+1}(\psi(\hat{C}_T + \hat{A}_{W,T}) + (1-\psi)\hat{L}_T)].$$

where we have defined $\Delta E_{t+1}(\cdot) = E_{t+1}(\cdot) - E_t(\cdot)$. In general equilibrium, interaction terms will be quite complex. However, at the cost of losing generality, we can get further retrieve insights by looking at a limiting case in which the discount factor, β, is close to the unitary value. In this case, indeed, Epstein-Zin preferences do not matter for the equilibrium allocation of quantities, up to a first-order approximation. Under the assumption $\beta \longrightarrow 1$ we can write

$$\hat{V}_{t+1} - E_t\hat{V}_{t+1} \approx \Delta E_{t+1}(\psi(\hat{C}_\infty + \hat{A}_{W,\infty}) + (1-\psi)\hat{L}_\infty),$$

which shows that only the stochastic trend in the respective variables influences the current surprises in utility. However, since $\hat{C}$ and $\hat{L}$ are respectively a detrended and a stationary variable, their stochastic trends are zero. The surprises to indirect utility will therefore only depend on the stochastic trend in world productivity

$$\hat{V}_{t+1} - E_t\hat{V}_{t+1} \approx \psi\Delta E_{t+1}(\hat{A}_{W,\infty}) = \psi u_{a,t}\varepsilon_{a,t+1},$$

which displays also time-varying risk. The importance of this factor in (71) will be higher, the larger the difference between γ and ρ. Under this particular case, the ability of Epstein-Zin preferences to explain risk-premia and the average upward sloping yield curve hinges upon the comovement between returns and the nominal stochastic discount factor. In particular when agents have a preference for an early resolution of uncertainty, i.e. $\gamma > \rho$, a negative shock to world productivity $\varepsilon_{a,t}$ implies bad news with respect to long-run consumption which are reflected in bad news on utility. In this case, the stochastic discount factor increases and the appetite for state contingent wealth too. Long-term bonds, of a generic maturity n, will require a premia when their return is low at the same time in which the stochastic discount factor is high as shown in (69). This means that $\hat{\imath}_{n-1,t+1}$ should go up following a bad productivity shock. To have a positively-sloped yield curve, we should therefore find that yields go up when there are

bad news on world productivity. This mechanism would apply also to the country F. Indeed, it is also true that the surprise in the utility of the foreign country depends on the shifts in the long-run component of world productivity

$$\hat{V}^*_{t+1} - E_t\hat{V}^*_{t+1} \approx \psi\Delta E_{t+1}(\hat{A}_{W,\infty}) = u_{a,t}\varepsilon_{a,t+1}.$$

Under the case $\beta \longrightarrow 1$, Epstein-Zin preferences might therefore contribute to imply highly correlated discount factors across countries and deliver a global explanation for the risk-premia, which will be time-varying and driven by the shocks to the common technological process. The consequence of this result, is indeed that, up to a first-order approximation, general equilibrium effects will be shut down. Since surprises in the utility of the Home and Foreign country are highly correlated then, using (29) and (30), G_t is approximately constant over time[25]

$$\begin{aligned}
\hat{G}_{t+1} &= \hat{G}_t + (\gamma-\rho)[(\hat{V}_{t+1} - E_t\hat{V}_{t+1}) - (\hat{V}^*_{t+1} - E_t\hat{V}^*_{t+1})] \\
&\approx \hat{G}_t.
\end{aligned}$$

But this result might impair the ability of the model to address the Backus-Smith anomaly. This is true up to a first-order approximation, but not in a second-order approximation. In a first-order approximation, when G_t is constant over time, we have indeed

$$[1-\psi(1-\rho)](\Delta\hat{C}_{t+1} - \Delta\hat{C}^*_{t+1} + \Delta\hat{A}_{t+1} - \Delta\hat{A}^*_{t+1}) \approx -(1-\rho)(1-\psi)(\Delta\hat{L}_{t+1} - \Delta\hat{L}^*_{t+1}) + \Delta Q_{t+1}$$

showing a strong relationship between $\Delta\hat{C}_{t+1} - \Delta\hat{C}^*_{t+1}$ and ΔQ_{t+1}. This relationship is perturbed by other factors like the growth in hours worked, but a common result in the literature is that a positive correlation arises following standard assumptions on the parameters ψ and ρ. However when we approximate the equilibrium conditions of the model up to second-order, the assumption of Epstein Zin preferences could still help in explaining the Backus-Smith anomaly under the particular case $\beta \longrightarrow 1$. Indeed (30) becomes

$$\begin{aligned}
\hat{G}_{t+1} &= \hat{G}_t + (\gamma-\rho)[(\hat{V}_{t+1} - E_t\hat{V}_{t+1}) - (\hat{V}^*_{t+1} - E_t\hat{V}^*_{t+1})] - \frac{1}{2}(\gamma-\rho)(\gamma-1)(var_t\hat{V}_{t+1} - var_t\hat{V}^*_{t+1}) \\
&\approx \hat{G}_t + (\gamma-\rho)[(\hat{V}_{t+1} - E_t\hat{V}_{t+1}) - (\hat{V}^*_{t+1} - E_t\hat{V}^*_{t+1})] \qquad (72)
\end{aligned}$$

where now it is clear that the last two terms in the variances will cancel each other out since they are evaluated using a first-order approximation of the model and we have already shown that, up to a first-order approximation, surprises in utility are perfectly correlated across countries –when β is close to the unitary value. However, now the second term in the square brackets of the right hand side of the above expression is no longer zero since it needs to be evaluated using a second-order approximation of the model. To this purpose a second-order approximation of

[25]The statement is true under the assumption that β is close to the unitary value, up to a first-order approximation and independently of the values assumed by the parameters γ and ρ.

the utility (2) can be written as

$$\hat{V}_t = (1-\beta)\hat{U}_t + \beta E_t \hat{V}_{t+1} + \frac{\beta}{2}(1-\gamma) var_t \hat{V}_{t+1} + \frac{\beta(1-\beta)}{2}(1-\rho)(\hat{U}_t - E_t \hat{V}_{t+1})^2$$

where $\hat{U}_t$ represents the log approximation of the utility flow U given in.(3). Once again, under the assumption that β is close to the unitary value, we can write

$$[(\hat{V}_{t+1} - E_t \hat{V}_{t+1}) - (\hat{V}^*_{t+1} - E_t \hat{V}^*_{t+1})] \approx \frac{1}{2}(1-\rho)[\Delta E_{t+1}(\hat{U}_\infty - E_\infty \hat{V}_\infty)^2 - \Delta E_{t+1}(\hat{U}^*_\infty - E_\infty \hat{V}^*_\infty)^2]$$

which might not be necessarily zero when the intertemporal elasticity of substitution is different from the unitary value and can depend, in an important way, on time-varying volatility. The term above represents an important additional factor that can help to disentangle cross-country consumption from the real exchange rate. Indeed (72) implies that

$$\begin{aligned} [1-\psi(1-\rho)](\Delta \hat{C}_{t+1} - \Delta \hat{C}^*_{t+1} + \Delta \hat{A}_{t+1} - \Delta \hat{A}^*_{t+1}) \quad &\approx \quad -(1-\rho)(1-\psi)(\Delta \hat{L}_{t+1} - \Delta \hat{L}^*_{t+1}) + \Delta Q_{t+1} \\ &\quad -(\gamma-\rho)[(\hat{V}_{t+1} - E_t \hat{V}_{t+1}) - (\hat{V}^*_{t+1} - E_t \hat{V}^*_{t+1})] \end{aligned}$$

showing that EZ preferences can help to explain the Backus-Smith anomaly through second-order effects.

6.1 Quantitative Evaluation

We now move to a quantitative evaluation of the model implications.

In particular, a second-order approximation of the model will be relevant to study the relationship between risk and the exchange rate in the model, and to provide a quantitative assessment of such links. This will be implicit in the general solution of the nominal and real exchange rate

$$\begin{aligned} \Delta \hat{S}_t &= \bar{g}^s_x \tilde{x}_t + \frac{1}{2}\tilde{x}'_t \bar{g}^s_{xx} \tilde{x}_t + \frac{1}{2}\bar{g}^s_{uu} u_t^2 + \frac{1}{2}\bar{g}^s_{zz}\sigma_u^2 \\ \hat{Q}_t &= \bar{g}^q_x \tilde{x}_t + \frac{1}{2}\tilde{x}'_t \bar{g}^q_{xx} \tilde{x}_t + \frac{1}{2}\bar{g}^q_{uu} u_t^2 + \frac{1}{2}\bar{g}^q_{zz}\sigma_u^2 \end{aligned}$$

where the index $i = s, q$ selects appropriate elements of the respective vector or matrices. In this solution, time-varying uncertainty for the stochastic disturbances of the model affects linearly the nominal and real exchange rates through the factors $\bar{g}^i_{uu}$. Yield curve can also been written in a similar form

$$i_{n,t} = c_{n,x}\tilde{x}_t + \frac{1}{2}\tilde{x}'_t c_{n,xx}\tilde{x}_t + \frac{1}{2}c_{n,uu}u_t^2 + \frac{1}{2}c_{n,zz}\sigma_u^2$$

where the vectors and matrices,$c_{n,x}, c_{n,xx},\ c_{n,uu},$,$c_{n,zz}$ satisfy appropriate recursive restrictions as shown in Benigno and Woodford (2005) for the case of no variation in uncertainty.

6.1.1 Calibration

In this section we describe our baseline calibration for the general model. The strategy that we adopt for the calibration exercise is to rely as much as possible on a standard values for our choice of parameters and conduct a sensitivity analysis on those for which there are divergences in the literature. We assume that the home and foreign economy are of equal size and are calibrated in a symmetric fashion. In this calibration section we think about our two-country world as U.S. versus the Euro area abstracting then for asymmetries that might be important for understanding some empirical regularities when it comes to small open economies.[26]

In choosing the parameters of utility function, we set β to match a 4% annual discount rate. We set the inverse of the intertemporal elasticity of substitution ρ to 2, implying an intertemporal elasticity of substitution (IES) in consumption of 0.5, which is consistent with estimates in the micro literature (e.g., Vissing-Jorgensen, 2002) and used also in the international real business cycle literature as in Stockman and Tesar (1995). We set the coefficient of relative risk aversion γ to 5 as Backus et al. (2010). We set ψ the share of consumption in utility to 0.3 as in Cooley and Prescott (1995) as to imply that in steady state agents devote one-third of their time to work.

We calibrate the parameters pertaining to the consumption basket in the following way. We set the share of home goods in tradable consumption, ν, is 0.72. We assume an elasticity of substitution between home and foreign traded goods, θ, of 2.5 which is in the range of the plausible value.

We set the firms'output elasticity with respect to labor, φ, to 2/3, and the elasticity of substitution among differentiated goods, σ, to 6 (implying a steady state markup of 20%) and we set $\alpha = 0.66$ and $\alpha^* = 0.75$ implying an average length of price contracts equal to 3 and 4 quarters, respectively all of which are standard in the literature and consistent with the posterior estimates for the U.S. and Euro area by Lubik and Schorfheide (2005).

Regarding the policy rules we set $\phi_i = 0.76$, $\phi_\pi = 1.41$, $\phi_s = 0.03$ and $\phi_y = .66$ for the U.S. economy and $\phi_i^* = 0.84$, $\phi_\pi^* = 1.37$, $\phi_s^* = 0.03$ and $\phi_y^* = 1.27$ for the Euro area that corresponds to the posterior estimates that Lubik and Schorfeide (2005) have found for the U.S. and Euro area respectively.

We now turn to the calibration of the stochastic processes. For the productivity shocks we use the posterior estimates of Lubik and Schorfeide (2005) for the U.S. and Euro-area: $\delta_A = 0.83$, $\delta_A^* = 0.85$ with $\sigma_A = 1.66$ and $\sigma_A^* = 2.71$ as the values through which we scale the individual standard deviation for the home and foreign shocks respectively. We assume no persistence for the policy shocks and we scale its standard deviation by $\sigma_\xi = 0.18$ for both countries based on the estimates of Lubik and Schorfeide (2005). For the inflation target shocks we follow Ireland (2007) in scaling by $\sigma_\pi = 0.1$. As to the persistence of the volatility shocks, we calibrate the autocorrelation coefficients at the values implied by fitting an AR(1) process for each of the three

[26] Relevant asymmetries could be in terms of a policy rule that reacts to exchange rate for small open economies or different degree of openess that affect critically the international transmission mechanism of shocks.

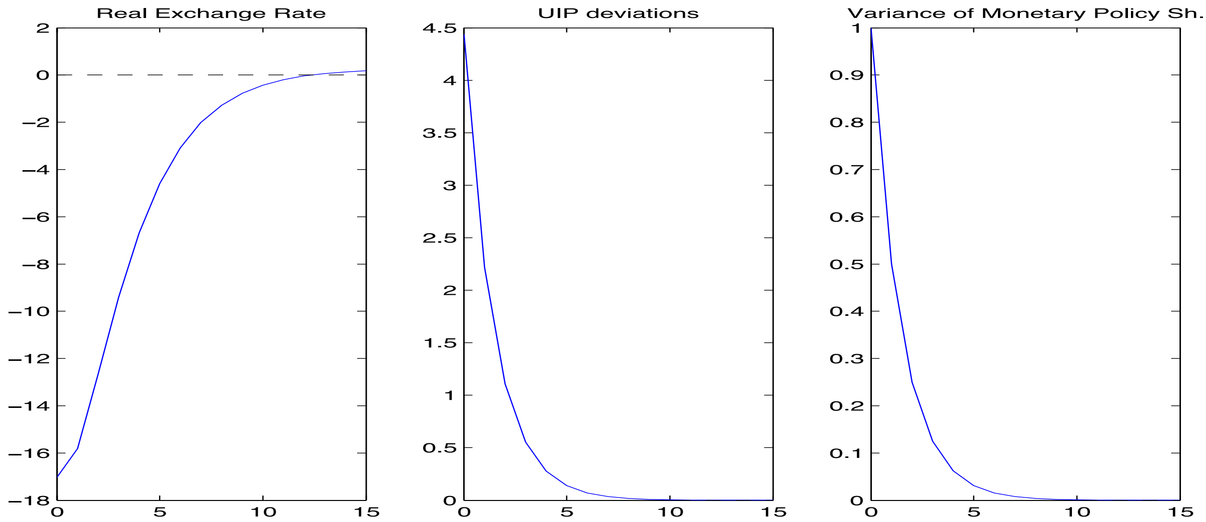

Figure 9: Dynamic responses to a monetary-policy volatility shock (innovation to the *volatility* of the monetary policy *instrument*).

time-series employed in the empirical part. As a consequence, we set $\rho_{aw} = \rho_a = \rho_{a^*} = .71$, $\rho_\pi = \rho_{\pi^*} = .67$ and $\rho_\xi = \rho_{\xi^*} = .53$.

6.1.2 Impulse-Response Analysis

In this Section we evaluate to what extent our two-country model with recursive preferences and stochastic volatilities can replicate the dynamic properties of the data that we identified in Sections 2 and 3.

In particular, we identified two main regularities with respect to the links between exchange rate and risk: (i) an increase in the volatility of both monetary-policy and inflation-target shocks appreciates the exchange rate, while an increase in the volatility of the productivity shock induces an exchange rate depreciation; (ii) an increase in the volatility of both the monetary-policy and the inflation-target shocks induces significant and persistent deviations from UIP, in the form of an increase in the excess returns on foreign-versus-domestic currency.

An additional regularity, originally documented by Eichenbaum and Evans (1995) and that we confirm also controlling for the effects of time-varying volatility, is that a contractionary monetary-policy shock produces a persistent appreciation of the exchange rate and persistent deviations from the UIP in the form of positive excess returns on domestic securities.

Figures 9 and 10 display the dynamic response of the Real Exchange Rate and the excess return on foreign-versus-domestic currency to volatility shocks hitting the monetary policy instrument and target, respectively. The figures show that the model is indeed able to imply significant appreciations of the real exchange rate and deviations from the UIP in the form of positive excess returns from investing in foreign-currency denominated bonds, consistently with our empirical findings.

Figure 11, similarly shows the dynamic response of Real Exchange Rate and the deviation

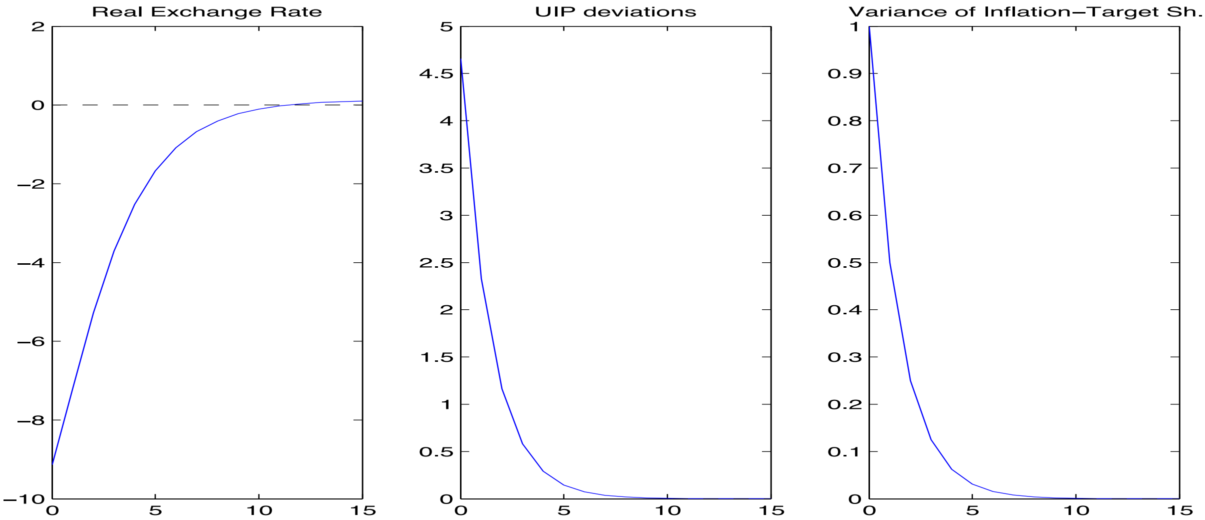

Figure 10: Dynamic responses to an inflation-target volatility shock (innovation to the *volatility* of the monetary policy *target*).

from UIP to a volatility shock hitting global productivity. The asymmetries in the degrees of price stickiness and the response coefficients of the policy rules imply that innovations in the level and/or volatility of the global productivity shocks are able to induce a non-zero response in international variables. In particular, in response to an increase in the volatility of the global productivity shock, the RER depreciates and we observe positive deviations from the UIP.

All these results are qualitatively consistent with the empirical regularities (i) and (ii), and also show that our approximation method is indeed able to make time-varying volatility matter for endogenous variables like the exchange rate.

As to the third regularity, related to the effects of monetary policy *level* shocks on the exchange rate and UIP deviations, Figure 12 shows the implication of our theoretical model. A contractionary monetary policy shock indeed implies an appreciation of the RER but it is unable to imply the hump-shaped response that we observe in the data, nor deviations from the UIP.

In order to look deeper into this result, we next explore what element of the theoretical model is responsible for this behavior and whether a different calibration would be able to induce the persistent appreciation that we observe in the data. In doing this, we can find some hints in the theoretical implications of the simplified version of Section 5. In particular, there we showed that an interest rate rule that smooths interest rates may induce hump-shaped dynamics of the exchange rate in response to monetary-policy shocks and larger deviations from the UIP induced by higher volatilities of monetary-policy and inflation-target shocks. The natural thing to do, then, is to test this predictions also in the context of the general case with sticky prices, home bias in consumption and stochastic real rates.

Figures 13 and 14 perform this task by displaying the dynamic responses of the variables of interest to a monetary policy shock and volatility shocks, respectively, and for different degrees

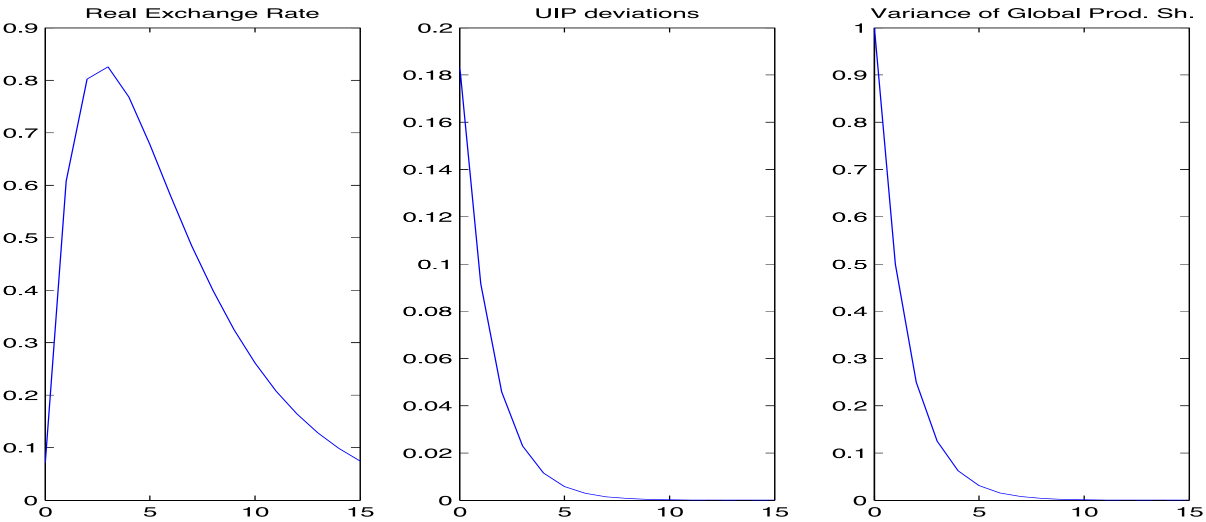

Figure 11: Dynamic responses to an innovation to the volatility of the productivity shock.

of monetary policy inertia, as measured by the smoothing parameter ϕ_i in equation (45).

Specifically, Figure 13 displays the dynamic response of the economy to a monetary-policy *level* shock, which raises the interest rate differential, and it shows that for high enough degrees of interest-rate smoothing, the model is indeed able to imply a substantial degree of persistence in the real appreciation induced by monetary policy shocks. On the other hand, increasing the monetary policy inertia does not seem to imply significant deviations from the UIP. This result, however, is common to any rational-expectations open-economy model with no financial frictions, where the Uncovered Interest Rate parity holds up to a first-order approximation. As a consequence, the increase in the interest rate differential implied by the domestic monetary policy shock is sterilized by the nominal depreciation which follows the initial appreciation: UIP holds and the model fails to reproduce the hump-shaped response of the nominal exchange rate.

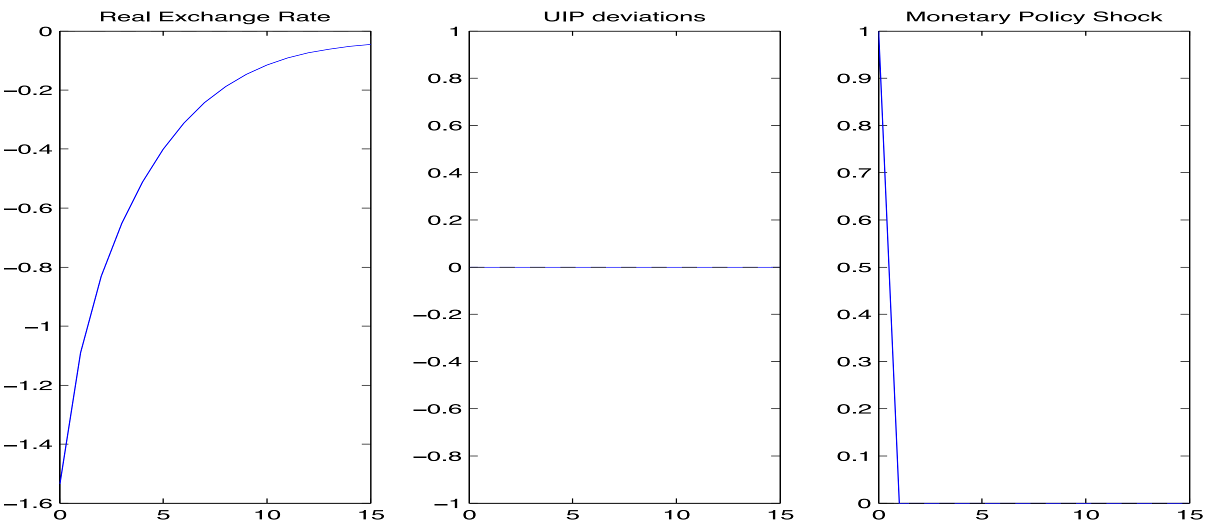

Figure 12: Dynamic responses to a monetary policy shock.

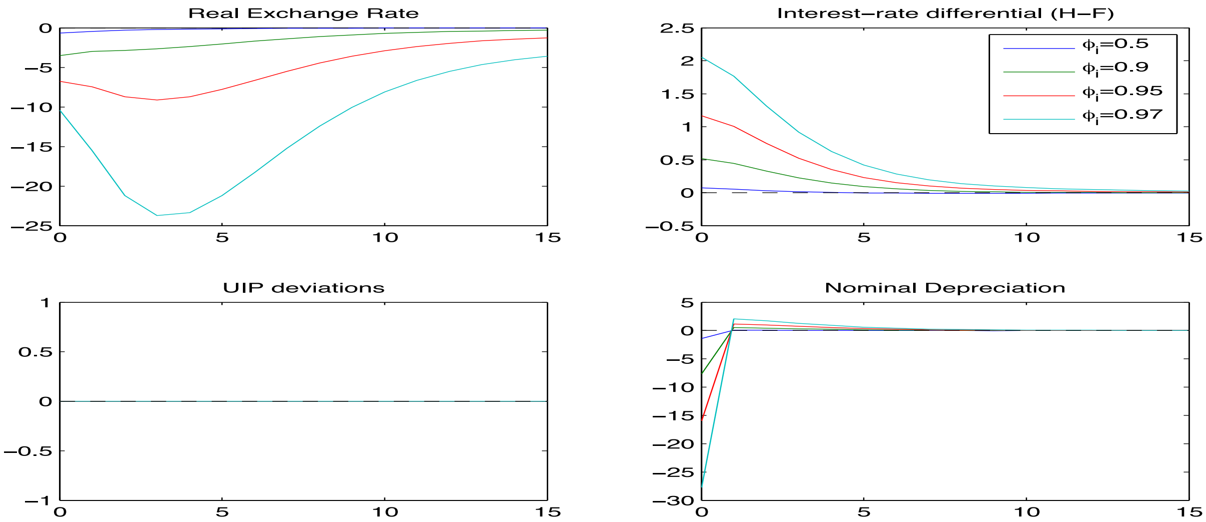

Figure 13: Dynamic responses to a monetary policy shock: the role of interest-rate smoothing.

With respect to this latter point, however, we know that in our model deviations from the UIP can be implied by second-order terms and in particular by stochastic volatility, as shown analytically in the simple case of Section 5. Figure 14, then, shows the role of interest-rate smoothing in shaping the response of deviations from the UIP following volatility shocks on the monetary policy instrument and target. As the graphs clearly document, for both cases of volatility shocks, the response of the excess return on foreign-versus-domestic currency monotonically increases with the coefficient of interest-rate smoothing, as expected. A higher policy inertia, moreover, is also able to exacerbate the exchange rate appreciation (not shown).

An additional test for our model would hence be to see how it performs in terms of other two well-documented puzzles of international finance: the negative slope of the UIP regression

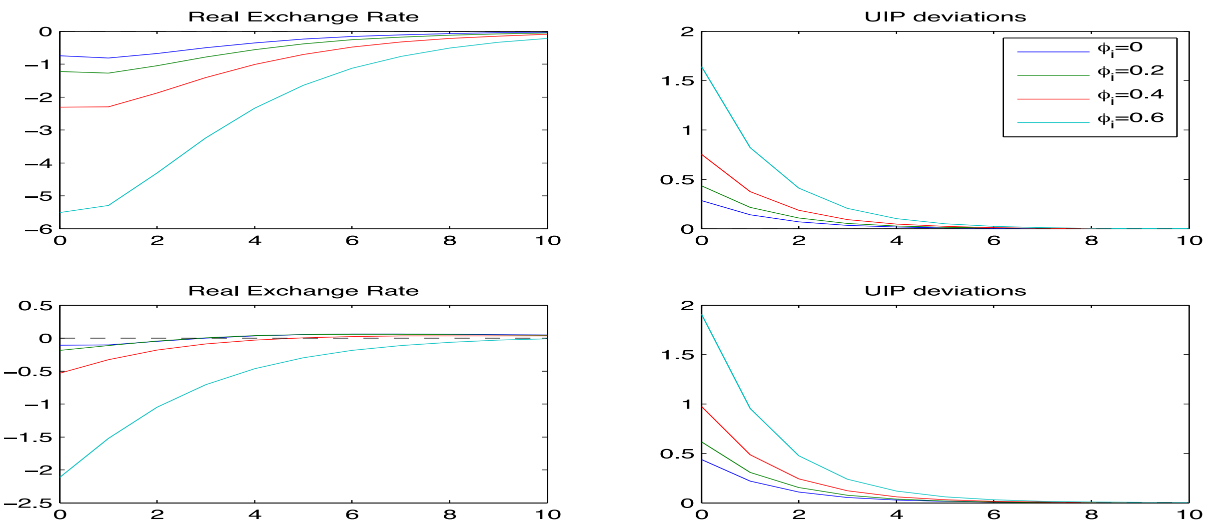

Figure 14: Dynamic responses to a volatility shock on the monetary policy instrument (u_{ξ}^2, top panels) and inflation target (u_{π}^2, bottom panels): the role of interest-rate smoothing.

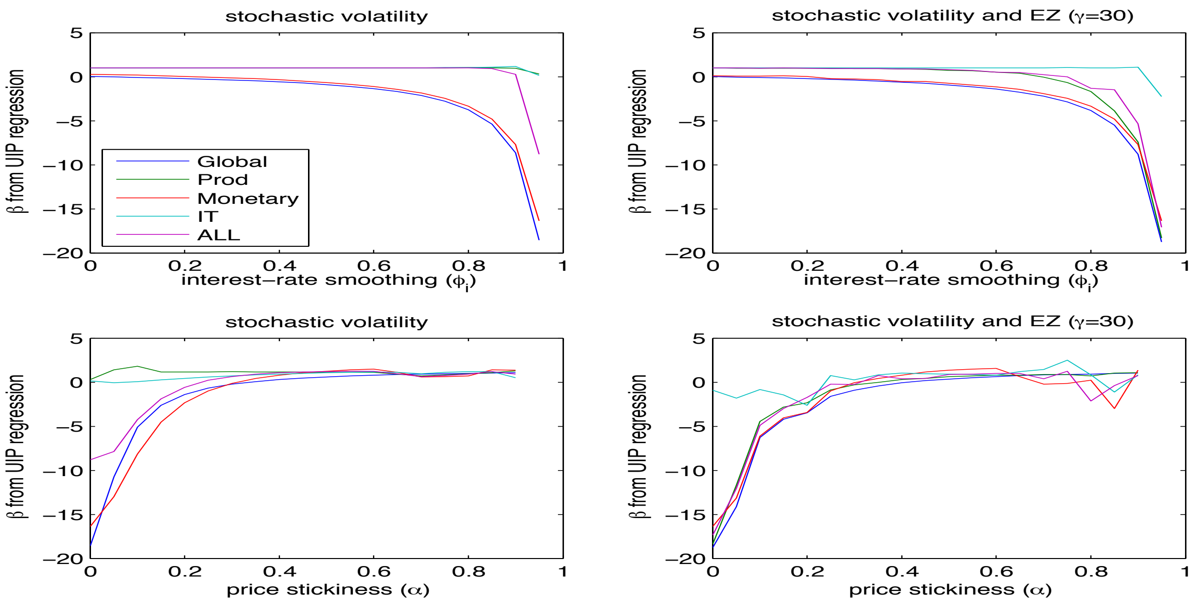

Figure 15: The slope of the UIP regression: the role of interest-rate smoothing (top panels) and price stickiness (bottom panels).

that projects the nominal-exchange-rate depreciation onto the interest-rate differential, and the absence of any systematic correlation between relative consumption growth and the real exchange rate (Kollman-Backus-Smith anomaly).

The first issue is related to the point of Figure 14, which shows that the interaction between interest-rate smoothing and stochastic volatility is able to produce persistent deviations from the UIP. The natural next step is to see to what extent such deviations are consistent with a negative slope in the UIP regression, and what are the theoretical factors that, within the model, can have an effect on it. We tackle this issue by simulating the theoretical model and computing the moments of interest from the simulated time series.

Figure 15 then shows the implications for slope of the UIP regression of the interaction among a number of key factors that characterize our theoretical framework. The top panels display the implications of the interaction among stochastic volatility, interest-rate smoothing and Epstein-Zin preferences, in a flexible-price economy. The bottom panels instead studies the role of price stickiness and its interaction with stochastic volatility and Epstein-Zin preferences, for a given degree of interest-rate smoothing ($\phi_i = 0.95$). As to the residual structural and policy parameters, they are calibrated as discussed earlier.

Three main implications stem from Figure 15. *i)* the interaction between stochastic volatility and interest-rate smoothing helps in driving the negative covariance between nominal exchange rate depreciations and interest-rate differential that is observed in the data. For this result to arise, stochastic volatility is a necessary ingredient of the model. The effect of interest-rate smoothing on the slope of the UIP regression, however, can vary quite a bit depending on the specific kind of shock to which we condition the simulation of the model: in particular,

the effects of raising monetary-policy inertia on the covariance between nominal-exchange-rate depreciations and interest-rate differentials are stronger conditional on monetary policy and global shocks, while a smaller impact is implied by conditioning on inflation-target shocks or idiosyncratic productivity shocks. The result related to monetary-policy and inflation-target shocks is qualitatively consistent with the simple case discussed in section 5: the unit root in the process for the inflation target tends to drive the slope toward unity in large samples, regardless of the degree of interest-rate smoothing, while the latter is indeed able to imply a negative correlation between nominal depreciation and interest-rate differentials. The result related to the global shock is similarly related to monetary factors: the asymmetric calibration of the policy rules makes a global shock have implications for international relative variables and the degree of interest-smoothing in those policy rule can play again a key role in driving the slope of the UIP regression.

ii) high degrees of price stickiness, on the contrary, tend to drive the slope of the UIP regression toward the unitary value, even for a high degree of interest-rate smoothing (calibrated at 0.95). Moderate degrees of price stickiness, however, are still consistent with a negative covariance between nominal depreciation rates and interest-rate differential, provided that the degree of monetary policy inertia is sufficiently strong. This result holds conditionally on monetary and global shocks, and fades instead away if we condition on inflation-target and idiosyncratic productivity shocks, consistently with implication *i*).

iii) deviating from the Expected Utility paradigm has little but beneficial effects on both respects: the effect of monetary-policy inertia becomes stronger also conditionally on idiosyncratic productivity shocks and even on inflation-target shocks, while moderate degrees of price stickiness are now consistent with a negative slope in the UIP regression also conditionally on country-specific productivity shocks.

It is worth noticing, that none of the above results would arise in a model without stochastic volatility, where the slope of the UIP would always be one: stochastic volatility is therefore a necessary ingredient to understand these regularities.

Finally, Figure 16 shows the implications of our model with stochastic volatility and recursive preferences for the correlation between the real exchange rate and the relative consumption growth rate. As the figure shows, the interaction among low degrees of IES, stochastic volatility and moderate degrees of monetary policy inertia can break the link between the real exchange rate and relative consumption growth, with no major role played by recursive preferences.

7 Conclusion

Time-variation in uncertainty and risk can be an important source of fluctuations for macroeconomic variables and in particular for the exchange rate. Using a standard open-economy VAR, we have provided new evidence on the importance of both real and nominal volatility shocks for the behavior of the nominal and real exchange rate. These findings complement the well-known

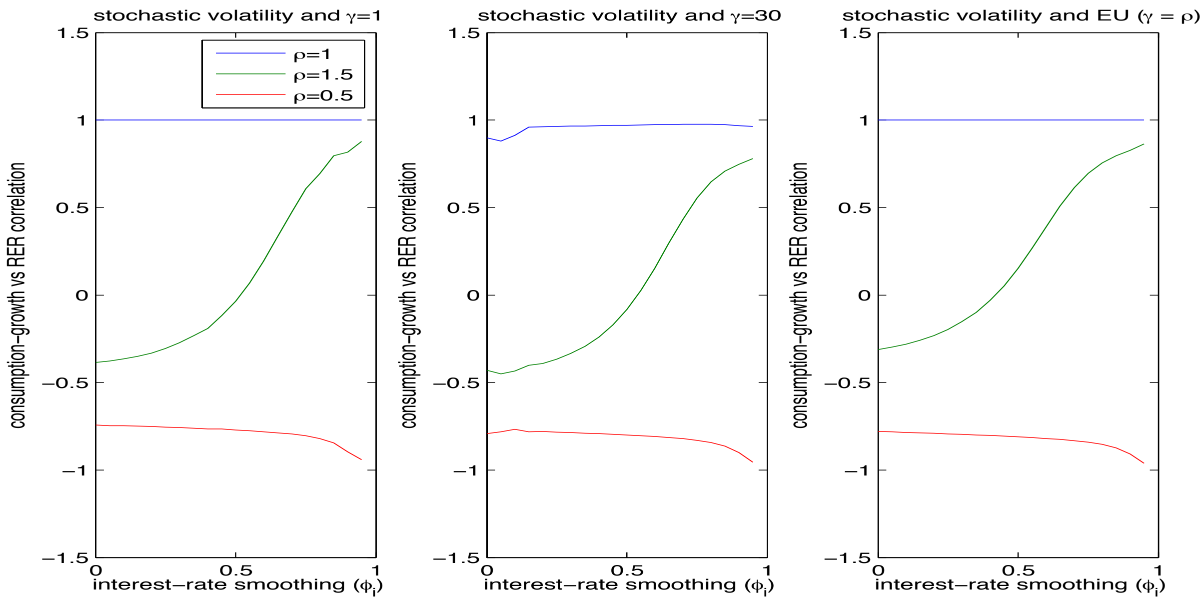

Figure 16: The Kollman-Backus-Smith anomaly: the role of interest-rate smoothing and recursive preferences.

evidence, documented by several studies, based on the UIP regression. Under rational expectations, the negative regression coefficients found in these works can be interpreted as variation over time in risk premia. Time-variation in the uncertainty can be also an important source of the variation over time in risk-premia.

Our VAR analysis shows that a rise in the volatilities of the nominal shocks appreciates the dollar exchange rate especially in the medium run. On the other hand, an increase in the volatility of the real shock (productivity) has the opposite effect. Moreover, a rise in the volatilities of the nominal shocks generates significant and persistent deviations from UIP and in particular an increase in the excess returns of foreign short-term bonds. Finally, we retrieve the evidence reported by Eichenbaum and Evans (1995) that a positive innovation to the level of the monetary-policy shock (contractionary policy shock) produces a persistent appreciation in both the real and nominal exchange rates and a persistent deviations from the UIP in the form of positive excess returns on US securities.

We propose a New-Keynesian open-economy as a unifying framework for reconciling these findings in a general equilibrium model with time-varying exogenous uncertainty.

Our model is successful along some dimensions. The key element is the specification of monetary policy through interest rate rules and in particular the smoothing coefficient relating current to past interest rates in the rule. The smoothing coefficient together with price stickiness is important to produce an hump-shaped response of the real exchange rate to the level interest-rate shock and combined with time-varying uncertainty can capture the negative coefficient that we observe in the UIP regression. Among the other factors that affect critically the coefficient in the UIP regression, higher nominal rigidities do not help while an increase in risk aversion improves the results. In this sense, allowing for Epstein-Zin preferences that

disentangle intertemporal elasticity of substitution and risk aversion is an important feature of our framework. Our model can also produce negative relationships between cross-country consumption differential and real exchange rate and therefore address the Backus-Smith-Kollman anomaly. Here again the assumption of Epstein-Zin preferences is important together with a non-unitary intertemporal elasticity of substitution.

We consider this work as a primal approach for the analysis of variation in uncertainty in open economies for the methodology that we use in the solution and the general features that we allow in the model. However, there are several limitations. First, our model, as any framework in which UIP holds up to a first-order approximation, cannot produce an hump-shaped response of the nominal exchange rate to a policy shock, but only of the real exchange rate. Directions to explore could be in the form of financial frictions or departure from rational expectations. Second, there are several tensions between the parameter values in their success to match one fact or another. We cannot claim a complete success on all directions simultaneously nor we did analyze a full match of the model with the data. Finally, related to the latter point, we have calibrated the parameters of our model based on empirical studies building on first-order approximations of the model. This is in contrast with the message of our work that second-order terms are important. Therefore, the estimation of the model is really needed to evaluate its fit. To this purpose, an appropriate methodology should be elaborated to handle the features of our general second-order approximated solutions. We leave this research for future work.

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