

What Do International Asset Returns Imply About Consumption Risk-Sharing?*

(Preliminary and Incomplete)

KAREN K. LEWIS[†]

EDITH X. LIU[‡]

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Abstract

An extensive literature has examined the potential risk-sharing gains from international diversification by focusing on models and data based upon consumption relationships across countries. These consumption-based studies have largely ignored the implications of the models for asset pricing moments, leading to counterfactual asset pricing relationships such as low equity premia, high risk free rates, and low volatility of asset returns. These counterfactual predictions in asset returns cast doubt on the ability of the literature to accurately measure gains from risk-sharing. In this paper, we begin to bridge this disconnect in the literature. We first show how the use of key preference parameters affect both asset return moments and risk-sharing measures. We then use asset return moments to discipline our parameter estimates. Based upon these estimates, we re-examine the gains from international consumption risk-sharing.

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[†]University of Pennsylvania - The Wharton School and NBER. E-mail: lewisk@wharton.upenn.edu

[‡]University of Pennsylvania - The Wharton School. E-mail: kkliu@wharton.upenn.edu

1 Introduction

An extensive literature has examined the potential risk-sharing gains from international diversification by focusing on models and data based upon consumption relationships across countries. These consumption-based studies have largely ignored the implications of the models for asset pricing moments, leading to counterfactual asset pricing relationships such as low equity premia, high risk free rates, and low volatility of asset returns. In particular, standard models assume constant relative risk aversion utility with risk aversion parameters less than 10, even though Mehra and Prescott (1985) have shown that the equity premium cannot be explained with parameters in this range. Even if the utility function is generalized to recursive utility, the standard assumptions about utility parameters generate risk-free rates that are too high as shown by Weil (1989) and others. Finally, the combination of assumptions about state variables and preferences typically imply low or even constant variability in risk-free rates.¹ These counterfactual predictions in asset returns cast doubt on the ability of the consumption-based literature to accurately measure gains from international risk-sharing.²

Despite these implications of standard consumption-based models, new approaches that reconsider consumption behavior have achieved better success in matching basic asset pricing moments in US returns. Bansal and Yaron (2004) show that the US equity premium, risk free rate, and variability of returns can be explained by a small, but persistent, component in consumption growth they term "long run risk." Campbell and Cochrane (1999) show that the asset return behavior can be explained by an independently and identically distributed consumption growth process, when a slow-moving external habit to the standard power utility function is added. Lettau and Ludvigson (2001a,b,2005) show that US cross-sectional and time series equity returns can be explained by a consumption-based model using consumption-wealth ratios as a proxy for the stochastic discount factor. Despite these successes, the empirical literature has focused upon the closed economy setting.

In this paper, we begin to bridge the disconnect between international risk-sharing and the empirical implications from consumption-based asset pricing. We develop the optimal risk-sharing model in a general decentralized economy in which countries can decide to remain in autarky or enter the risk-sharing arrangement. Using this framework, we show how welfare gains from international risk-sharing can be calculated from the social-planners problem which in turn depends upon the

¹See for example, the discussion in Campbell and Cochrane (1999) and Abel (1990).

²For a survey of these counterfactual predictions and the implications for risk-sharing gains, see Lewis (2000).

utility parameters and the state process distributions. Similar to the results from Lettau and Ludvigson (2001,2005), we show that the asset returns and optimal consumption plans depend upon the consumption-wealth ratios.

To consider the implications of international asset return behavior on consumption risk-sharing, we begin with the framework in Bansal and Yaron (2004).³ As in that framework, we develop an international consumption-based asset pricing model assuming a small, but persistent component in consumption growth. Using consumption and asset return data from seven countries, we use Simulated Method of Moments (SMM) to match the parameters of our consumption-based model with moments from asset returns. We then use these parameter estimates that are disciplined by our asset return data to re-examine international risk-sharing. In this way, our estimates of international consumption risk sharing gains are consistent with the implications of asset return behavior.

Our paper is also the first to match the basic moments of an international consumption-based asset pricing model.⁴ In our base model, returns are driven by a common persistent consumption risk component across countries. However, we also estimate the model allowing these components to differ by country. Our results show that a great deal of heterogeneity across countries in consumption processes is required to match asset returns.

The structure of the paper is as follows. Section 2 reviews the behavior of asset returns and consumption-based models in a closed economy setting. Section 3 then develops the relationship between these consumption-based asset pricing models and a standard international consumption-based asset pricing economy. Section 4 shows how the features of asset returns and the international model can be combined to evaluate international risk-sharing gains. Using a simple two-country version of the model, we show how the standard welfare gains are affected by the presence of long run risk and the preference parameters that have been found to match US data. In Section 5, we describe our empirical estimates for seven countries obtained by matching asset return moments to each country's consumption data. We then show how these estimates discipline the range of implied international risk-sharing gains. Concluding remarks follow in Section 6.

³We leave the framework of Campbell and Cochrane (1999) for the next version of our paper.

⁴In a complementary research agenda, Colce and Colacito (2005,2008) examine the effects of long run risk on real exchange rates. However, our paper focuses upon a asset return behavior and a common consumption good.

2 Asset Returns and Consumption-Based Models

A large literature has considered the implications of consumption behavior on international risk-sharing. Backus, Kehoe, and Kydland (1991) observed that consumption correlations are lower than output correlations which clearly violates the implications of perfect risk-sharing arising from complete markets. To understand this behavior, a large literature has considered the effects of deviations from the standard model. These deviations include incomplete markets, transactions costs, and country-specific non-tradeable risks such as immobile labor and non-tradeable goods.⁵

A natural, but important question that arises when assessing this large literature is: What is the economic cost of rejecting perfect international risk-sharing? If these costs are minor, then even though the low correlation of consumption technically implies a failure of perfect risk-sharing, this failure is economically insignificant. On the other hand, large foregone gains to risk-sharing would imply the contrary. Unfortunately, the literature has reported a wide range of foregone gains to risk-sharing. Some studies have found the gains to be exceedingly small and on the order of less than one-thousands of a percent of permanent consumption while others have found these gains to be in excess of 100% of permanent consumption.⁶

While all of these studies are based upon international consumption behavior, they differ in how well they relate this behavior to asset returns. A well known feature of standard consumption models is that the implied variability of consumption cannot explain asset returns behavior. In particular, as Mehra and Prescott (1985) pointed out, the standard consumption-based model generates an equity premium puzzle since the model predicts a lower premium than observed in the data. Therefore, some international risk-sharing models have attempted to match this feature of the data by using a risk-aversion coefficient that is sufficiently high to match the equity premium.⁷ While the equity premium can be matched with the correct choice of risk aversion coefficient, other features of asset returns are not. For example, Weil (1989) showed that the standard consumption-based model continues to imply a risk-free rate puzzle because the model generates a higher risk-free

⁵For a very partial list, see Baxter and Crucini (1995) on incomplete markets, Baxter and Jermann (1997) and Heathcote and Perri (2008) on labor risk, Stockman and Tesar (1995) on non-tradeable goods, and Tesar and Werner (1995) and Warnock (2002) on transactions costs. For a more detailed but dated survey, see Lewis (1999).

⁶While the assumptions underlying these gain calculations differ dramatically, these numbers were taken to emphasize the wide range in the literature. For a study implying tiny gains from international risk sharing, see Cole and Obstfeld (1991) and for large gains see Obstfeld (1994b). Tesar (1995) and van Wincoop (1994) provide surveys that consider the impact of various effects such as habit persistence and the presence of non-traded goods.

⁷See for example Obstfeld (1994b) and the discussion in Lewis (2000).

rate than the data. Moreover, high risk aversion can not resolve the high volatility of asset returns in the data, compared to the low, sometimes zero, volatility in the model.⁸

While the behavior of asset returns is only one way to discipline an international model of consumption, for questions concerning risk-sharing gains, asset return behavior is arguably the most important. Trade in international capital markets is often viewed as the most efficient or even primary mechanism in which risks can be shared globally. As such, the prices of assets in these markets reflect equilibrium views toward risk. For this reason, we take these asset returns as the standard on which to discipline our models of international consumption and implied risk parameters. To provide a general framework, we first review the standard consumption model and the new insights gained from "long run risks." Below, we will imbed this model into an international context to begin to match returns with consumption data.

2.1 Closed Economy Consumption Processes With and Without Long Run Risk

We begin by examining a standard consumption model in the closed economy using a standard Mehra-Prescott approach as well as the Bansal-Yaron "long run risk" model. Since the closed economy can also be viewed as representing an autarkic equilibrium, this model will provide an important benchmark for our gains from risk-sharing. Thus, we describe the model in terms of a representative agent in each country.

Each country j has a continuum of identical consumer-investors. Under standard iid consumption growth, the log consumption growth rate processes of each of these agents $g_{c,t}^j$ is determined by a mean growth rate μ^j , and variance to the innovation given by σ^j .

$$g_{c,t+1}^j = \mu^j + \sigma^j \eta_{t+1}^j$$

where $\eta_{t+1}^j \sim N.i.i.d.(0, 1)$. If further the agent in country j views his consumption profile as subject to long run risks as in Bansal and Yaron (2004) he will have a persistent stochastic component in the conditional mean as given by x_t^j in the following equation.

$$\begin{aligned} g_{c,t+1}^j &= \mu^j + x_t^j + \sigma^j \eta_{t+1}^j \\ x_{t+1}^j &= \rho^j x_t^j + \sigma \varphi_e^j e_{t+1}^j \end{aligned} \tag{1}$$

⁸See the discussion in Campbell and Cochrane (1999) and Lewis (2000).

where $e_{t+1}^j \sim N.i.i.d.(0, 1)$. Thus, the "long run risk" process, x^j induces a persistent deviation in the conditional mean of consumption away from its long run growth rate, μ^j . Bansal and Yaron (BY) (2004) argue that this deviation is difficult to detect because the difference in variance between the temporary deviation from the growth rate, η_{t+1}^j , and the variance of the persistent component, e_{t+1}^j , is large. In other words, φ_e^j is very tiny and close to 0.0003 in US data. BY also consider the effects of stochastic volatility such that σ^j is time-varying. In the present version of our paper, we do not include stochastic volatility, but will include these results in the next version.

In order to match asset return behavior, BY fit the behavior of dividends and consumption growth rates to the implied estimates of asset return moments. As such, they use moments of equity returns and the risk-free rate to estimate the parameters in equation (1) and the parameters in growth rate of dividend process $g_{d,t}^j$ given by:

$$g_{d,t+1}^j = \mu_d^j + \phi^j \bar{x}_t + \varphi_d^j \sigma^j u_{t+1}^j \quad (2)$$

where $u_{t+1}^j \sim N.i.i.d.(0, 1)$, μ_d^j is the growth rate of dividends, ϕ^j , is the loading of long run risk on the growth rate of dividends, and φ_d^j is the ratio of conditional variance in dividend growth to the transitory variance in consumption.

2.2 Closed Economy Asset Returns and Utility

We require a utility function to understand the relationship between consumption/dividend processes and asset returns and, ultimately, the welfare gains on risk-sharing. We further need a utility function that allows different risk aversion and intertemporal substitution for two reasons. First, as demonstrated by Obstfeld (1994a), the effects of gains from sharing differing growth rates and from reducing variability around these growth rates are confounded if relative risk aversion and intertemporal substitution are governed by the same parameter as in the constant relative risk aversion utility. Second, as the asset pricing literature has shown, constant relative risk aversion utility cannot jointly match the moments of equity and the risk-free rate.

For both of these reasons, we assume agents in each closed economy country has recursive preferences following Epstein and Zin (1989) and Weil (1989). Further, in our open economy model below, we will assume that all countries have the same utility function parameters. Specifically, using the index j to refer to each country, utility at time t can be written:

$$U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) = \left\{ (1 - \delta)C^j(S_t)^{\frac{1-\gamma}{\theta}} + \delta (E_t [U^j(C^j(S_{t+1}), E_t[U^j(C^j(S_{t+1}))])^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (3)$$

where $0 < \delta < 1$ is the time discount rate, so that $(\frac{1}{\delta} - 1)$ is the rate of time preference, where $\gamma \geq 0$ is the risk-aversion parameter, where $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 0$, the intertemporal elasticity of substitution, and where $E_t(\cdot)$ is the expectation operator conditional on the information set at time t $I_t \equiv \{S_t, S_{t-1}, \dots\}$ for S_t , the realization of the state process, at time t . As described by Epstein and Zin (1989), this utility function specializes to standard time-additive constant-relative risk aversion preferences when $\gamma = \frac{1}{\psi}$. In this case, the utility function becomes:

$$U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) = \left\{ (1 - \delta) E_t \sum_{\tau=0}^{\infty} \delta^\tau C^j(S_{t+\tau})^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}$$

Since the risk aversion coefficient and the inverse of the intertemporal elasticity of substitution are no longer constrained to move together under Epstein-Zin-Weil preferences, lifetime utility can be unbounded for some combinations of the utility parameters, δ , γ , and ψ , and the growth rate of consumption.⁹ Intuitively, the time discount rate, governed by δ and the intertemporal elasticity of substitution in consumption, measured by ψ , parameterize the sensitivity of utility to future consumption. If these parameters are sufficiently high, then certainty-equivalent consumption growth rates as measured by the growth rate of consumption adjusted by risk-aversion can induce current utility to become unbounded. For this reason, we will also require the condition that utility is bounded:

$$U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) = \left\{ (1 - \delta) C^j(S_t)^{\frac{1-\gamma}{\theta}} + \delta (E_t [U^j(C^j(S_{t+1}))], E[U^j(C^j(S_{t+1}))|I_{t+1}])^{1-\gamma} \right\}^{\frac{1}{\theta}} < \infty$$

The unboundedness in utility becomes more likely the higher is the time discount rate and the intertemporal elasticity of substitution in consumption.

Epstein and Zin (1991) derive the first-order condition in this environment as:

$$E_t \left\{ \delta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{jP})^{(\theta-1)} R_{t+1}^\ell \right\} = 1 \quad (4)$$

where R_{t+1}^{jP} is the gross return on the market portfolio of agent j and R_{t+1}^ℓ is the gross return on any asset ℓ available in country j . As we show in the appendix, these first order conditions can be used to derive solutions for the asset returns in terms of the utility parameters and the parameters in the processes of consumption and dividends. To show how the effects of long run risk compare to iid consumption in asset returns, we consider results based upon the US alone in the next section.

⁹Among others, this point has been described in Lewis (2000).

2.3 Asset Returns with and without Long Run Risk in US Data

In this section, we estimate the parameters for US consumption processes by matching the asset pricing moments in US data. Below we present our results using international data for seven countries that can trade claims on a common consumption good. We begin by examining the US data alone for two main reasons.

First, we follow much of the risk-sharing literature based on a common consumption good, by analyzing consumption data adjusted for deviations in purchasing power parity from the Penn World Tables. By contrast, the domestic asset pricing literature has used US real consumption data. Moreover the US data has been analyzed over a longer time period than we have available for the other countries. Therefore, we first present our results for the US alone to verify whether our data provide estimates that are similar to those in the domestic literature.

Second, we provide the US results to consider whether long run risk is needed to explain asset returns. If iid consumption is sufficient, we do not need to examine other models to allow asset returns to guide us in choosing the appropriate parameters for international risk-sharing gains.

According to Bansal and Yaron (2004), consumption decisions are made at a higher frequency than annual data. If the model is specified at a monthly level, then all decision parameters from the implicit income and dividend processes defined at the monthly frequency.¹⁰ As such, all estimates of model parameters must be time-aggregated to match annual data.

To extract estimates of the parameters in the consumption and dividend process, we rely on the Campbell-Shiller decomposition that expresses returns as functions of the price-to-asset-payout ratio:

$$r_{t+1}^\ell = k_0^\ell + k_1^\ell z_{t+1}^\ell - z_t^\ell + g_{t+1}^\ell \quad (5)$$

where r_t^ℓ is the net return on asset ℓ , z_t^ℓ is the logarithm of the price-payout ratio, and g_t^ℓ is the growth rate of the payouts, either dividends in the case of equity or consumption in the case of the market portfolio. Finally, k_0^ℓ and k_1^ℓ are approximating constants that capture the long run return mean and the price-payout ratio, respectively.¹¹

Using this approximation along with the Euler equations in equation (4) and the utility parameters from Bansal and Yaron (2004) we estimate the monthly parameters of μ^j , σ^j , ρ^j , φ_e^j , μ_d^j , ϕ^j , and φ_d^j to match annual consumption, dividends, and asset pricing moments. In particu-

¹⁰See Bansal and Yaron (2004) for a more complete articulation of this argument.

¹¹Approximation constants are defined to be $k_1^j = \frac{\exp(\bar{z}^j)}{1+\exp(\bar{z}^j)}$ and $k_0^j = \log(1 + \exp(\bar{z}^j)) - k_1^j \bar{z}^j$, where \bar{z}^j is the steady state log price to consumption ratio in the close economy.

lar, the moments we match are the standard deviation of log consumption growth, the first order auto-correlation of consumption growth, the standard deviation of log dividend growth, the mean equity premium, the mean risk free rate, the standard deviation of the market return, and standard deviation of risk free rate. The appendix and Section 5 below describe this procedure in more detail.

To examine the US alone, Table 1 examine three different sets of consumption and asset return data. As the first data set, Mehra and Prescott (1985) use the Kuznet-Kendrik-USNIA measure of per capita real consumption of non-durables and services. For asset returns, they use the S&P composite stock price and dividend series. Both series span the period from 1889 to 1978. Second, we obtained the consumption data from NIPA following BY from 1929 - 1998, as well as dividend and return series from the value-weighted CRSP data. Third, for our study that will be applied in an international context, our sample period was necessarily shorter. In particular, we constructed consumption from the Penn World Tables spanning only 1950-2000..We then used the internationally consistent dividend growth rate data from Campbell (2003) for the US.

The first two columns of Table 1 report the first and second moments of returns for the Mehra and Prescott (1985) model assuming iid consumption growth. Using constant-relative risk aversion utility and a risk aversion coefficient of 10, Mehra and Prescott find that with an annualized consumption growth rate of 1.7% and the standard deviation of 3.6%, the model can generate an equity premium of only 1.42%, even though the equity premium in the data is 6.18%. Moreover, the risk-free rate from the model is too high at 12.71% while it is less than 1% in the data. Mehra and Prescott also assume that equity pays off the consumption growth rate.

The third and fourth columns of Table 1 show the effects of two main differences used in literature since Mehra and Prescott's seminal paper. First, using Epstein-Zin-Weil utility, the risk aversion and intertemporal elasticity of substitution parameters are allowed to differ. In particular, we use the estimates obtained by BY of 10 for risk aversion and 1.5 for the intertemporal elasticity of substitution. Second, following a number of papers, we treat equity as a payment on dividends instead of consumption. As the table shows, with slightly lower variability of consumption at 2.93%, the model generally generates the standard deviation and first-order autocorrelation of consumption and dividend growth. It also produces some variability in equity returns but at around 12%, it is lower than the 19% in the data. However, the model generates constant risk-free rates such that the variance is counterfactually equal to zero. Moreover, the iid model fails on the means of asset returns. The implied equity premium is negative at around -0.9% and the risk free rate is still too high at 2%.

The fifth column of Table 1 shows the effects of including the long run risk term. For this analysis, we use Bansal-Yaron NIPA data and time period to obtain the estimates of: $\mu^j = .15\%$, $\sigma^j = .78\%$, $\rho^j = .979$, $\varphi_e^j = .044$, $\mu_d^j = .15\%$, $\phi^j = 3$, and $\varphi_d^j = 4.5$. Not surprisingly, these numbers are similar to those found by Bansal and Yaron. When the model is combined with these parameters, the equity premium becomes positive and around 4.4%, the variability of equity returns are 16.7%, the risk-free rate declines to 1.7%, and the variability of the risk-free rate increases closer to the data.

The last three columns show the implications for asset pricing moments using our PWT consumption data and model estimates. Since we have a shorter time period, our consumption data exhibit lower variability reflecting the "Great Moderation" in the US.¹² In addition, the variability of dividend growth and first order auto-correlations in annual consumption and dividend growth are all lower over this time period. Despite these differences, our model generates a similar pattern to those obtained by Bansal and Yaron (2004). Our parameter estimates imply: $\mu^j = .19\%$, $\sigma^j = .79\%$, $\rho^j = .976$, $\varphi_e^j = .044$, $\mu_d^j = .12\%$, $\phi^j = 3.95$, and $\varphi_d^j = 1.4$. When we assume i.i.d. consumption, the equity risk premium is negative, the variability of the market returns is too low and that of the risk-free rate is zero. The risk-free rate is too high, albeit only slightly so since the risk free rate over our sample is higher.

When we add long run risk in the last column, our model matches the returns better. The equity premium rises to 3.5%, the risk-free rate declines closer to the data and the variability of the risk-free rate and the equity premium increase. Once we include the effects of stochastic volatility in the next version of our paper, we anticipate improving the fit of volatility even more.

Overall, then, the estimates in Table 1 show that iid consumption cannot generate plausible asset pricing implications, particularly concerning the variability in returns. Moreover, it is important to allow for differences between risk aversion and IES in utility. In the next section, we consider the impact of these features in assessing international risk-sharing gains.

3 The International Consumption-Based Economy

We now develop a standard international consumption-based model that can nest autarky within an optimal risk-sharing arrangement. Our goal is to provide a general framework that encompasses many of the existing international asset pricing models as surveyed in Lewis (2000). These models

¹²See the discussion in Stock and Watson (2002).

vary along several dimensions. First, they make different assumptions about their underlying state processes. Some models assume temporary deviations from a long run mean. Other models explicitly assume growth rates in country outputs¹³ As demonstrated by Obstfeld (1994a), the effects of gains from sharing growth and from reducing variability are confounded unless recursive utility is assumed that allows the separation between relative risk aversion and intertemporal substitution in consumption. For this reason, we continue to assume recursive utility below. Second, models differ in whether they allow for capital accumulation or assume an endowment output process. On one hand, the international real business cycle literature pioneered by Backus, Kehoe, and Kydland (1992) allows for capital accumulation. The asset pricing effects of capital accumulation can potentially be important. For example, Jermann (1998) shows that capital accumulation together with habit persistence can explain the equity premium and the risk-free rate in the US economy. On the other hand, much of the domestic consumption-based asset pricing literature has abstracted from production and taken the consumption process as given in analyzing returns. Moreover, the international asset pricing literature that focuses upon risk-sharing gains has often taken consumption as de facto exogenous.¹⁴ In order to be consistent with the consumption data-based literature, we follow the tradition of taking the consumption process as given while staying agnostic about the production process to the greatest degree possible, and specify places where we must assume an endowment economy. Given these considerations and the importance of features of asset pricing moments found above, we consider a canonical international economy model that can include these features. We describe this model next.

There are representative consumer-investors in J countries, indexed by j . Each country produces an output Y^j that depends upon a state process S_t , at time t . The state process spans the space of all J country production processes. The agent in each country has recursive preferences following Epstein and Zin (1989) and Weil (1989) given in equation (3). Above, we considered the closed economy version of this framework which we can be viewed as a solution to the economy under autarky.. We now consider the implications of this framework when agents consider an equilibrium full integration. We begin with the social planner's optimal allocation of resources before examining the decentralized closed and open economy equilibria. Later, we consider the welfare gains of moving from the autarky equilibrium to the full integration equilibrium.

¹³See in particular Obstfeld (1994). As described in Lewis (2000), many of the asset-returns based studies assume growth in dividends and/or output.

¹⁴See for example, Obstfeld (1994b) and the literature cited in Lewis (1999).

3.1 Social Planner's Problem

We now consider the social planner's problem faced with J output processes and agent's preferences given above. The social planner maximizes an objective function that values lifetime utility across each country's representative agent with weights, λ^j . At time 0, the planner maximizes utility over all states and dates given the output processes in each state.

$$\begin{aligned} & \underset{\{C^j(S_t)\}}{\text{Max}} \quad \sum_{j=1}^J \lambda^j U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) & (6) \\ & \forall j = \{1, \dots, J\} \\ & \forall S_t \in S \\ & \forall t \in \mathbb{N}^+ \end{aligned}$$

$$\text{s.t.} \quad \sum_{j=1}^J C^j(S_t) \leq \sum_{j=1}^J Y^j(S_t), \quad \forall S_t \in S, \quad \forall t \in \mathbb{N}^+ \quad (7)$$

Note that the planner maximizes a weighted average of the utility of the representative agent in each country in every state and date subject to the constraint that aggregate consumption does not exceed aggregate output. This constraint arises directly when production is given by an endowment process. Alternatively, optimization can be seen as the allocation of consumption after production decisions have been made. Rewriting the planner's problem using the recursive utility formulation above implies:

$$\begin{aligned} & \underset{\{C^j(S_t)\}}{\text{Max}} \quad \sum_{j=1}^J \lambda^j U^j(C(S_t), E[U^j(C^j(S_{t+1})|I_t)]) \quad \text{s.t.} \quad \sum_{j=1}^J C^j(S_t) \leq \sum_{j=1}^J Y^j(S_t) & (8) \\ & \forall j = \{1, \dots, J\} \\ & \forall S_t \in S \\ & \forall t \in \mathbb{N}^+ \end{aligned}$$

Assuming that the consumption good is non-durable, the resource constraint will hold with equality. In this case, the social planner's first order conditions give the familiar condition that marginal utilities are equalized across all states. We describe these first order conditions in more detail in the appendix.

3.2 Decentralized Closed Economy

We now examine the decentralized economy by focusing first on an international economy in which all countries are in autarky. The representative agent in each country j is originally endowed with

the ownership rights on the productivity stream of output from his country: $Y^j(S_t) \forall S_t \in S$. We further restrict this output to be generated by an exogenous endowment stream and, where there is no possibility of confusion, we adopt the convention that $X_t \equiv X(S_t)$ for any variable X that is a function of the state. Given dividend payments from the representative agent's endowment, Y_t^j , he consumes and then buys claims on the endowment process for the following period at price P_t^j . Defining the claims on country ℓ 's endowment process held by country j as $\varpi_t^{j\ell}$, the agent's optimization problem in autarky is given by:

$$\begin{aligned} \underset{\{C_t, \varpi_t^{jj}\}}{\text{Max}} U_t^j \quad \text{where } U_t^j &= \left\{ (1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta \left(E_t \left[U_t^j \right]^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ \text{s.t. } C_t^j + P_t^j \varpi_t^{jj} &\leq W_t^j \\ \left(Y_{t+1}^j + P_{t+1}^j \right) \varpi_t^{jj} &= W_{t+1}^j \end{aligned} \quad (9)$$

In autarky, the agent in country j consumes his own output and shares on this process are not sold internationally. Since the number of shares is time invariant, we normalize the number of outstanding shares to one so that the agent in country j holds his own country's shares $\varpi_t^{jj} = 1$ and is restricted from holding any shares in the other countries, $\varpi_t^{ji} = 0, \forall i \neq j$. Therefore, country j agent's problem can be written more succinctly as the Bellman equation:

$$V_t(C_t^j, W_t^j) = \underset{\{C_t^j\}}{\text{Max}} \left[(1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta E_t \left[V_{t+1}^j(C_{t+1}^j, W_{t+1}^j)^{1-\gamma} \right]^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (10)$$

$$\text{s.t. } W_{t+1}^j = (W_t^j - C_t^j) * R_{t+1}^j \quad (11)$$

Where $R_{t+1}^j = \frac{P_{t+1}^j + Y_{t+1}^j}{P_t^j}$ is the gross return on the claim on the home output and equation (11) rewrites the budget constraint using this definition and the restriction that $\varpi_t^{jj} = 1$.

As shown by Campbell (1993) and Obstfeld (1994a), the solution to this Bellman equation is

$$V_t(C_t^j, W_t^j) = (1 - \delta)^{-\left(\frac{\psi}{1-\psi}\right)} (C_t^j)^{\left(\frac{1}{1-\psi}\right)} (W_t^j)^{-\left(\frac{\psi}{1-\psi}\right)} \quad (12)$$

Applying the first-order condition in equation (4) to the return on the endowment process in autarky, the condition becomes:

$$E_t \left\{ \delta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^j)^\theta \right\} = 1 \quad (13)$$

¹⁵Bansal and Yaron (2001) and Lewis (2000) use the value function solutions in Campbell (1993) and Obstfeld (1994a), respectively, to analyze welfare gains of risk reduction.

Below we use this Euler equation to determine the equilibrium price of equity in home markets under closed economies, P_t^j . In this case, the home equity is priced only by its own representative agent. Since agent in country j consumes his own output alone, then his marginal utility uniquely determines the price of the asset that pays dividends Y_t^j . Moreover, the optimal consumption path depends only upon the home output process, a restriction which clearly violates the social planner's first order conditions given above. We will use this equilibrium as a benchmark for evaluating welfare gains below.

Another way to see the relationship between the decentralized closed economy and the social planner's problem is to directly use the solution to the value function above. Using the fact that by the envelope theorem, $(\partial V_t / \partial Y_t) = (\partial U^j(C_t, I_t) / \partial C_t)$ along the optimal consumption path, we show in the appendix that substituting the solution from the decentralized economy into the planner's first order conditions implies the following requirement for optimality:

$$\ln(\lambda^\ell) + \left(\frac{\psi}{1-\psi} \right) \left\{ \ln[C_t^\ell / W_t^\ell] \right\} = \ln(\lambda^j) + \left(\frac{\psi}{1-\psi} \right) \left\{ \ln[C_t^j / W_t^j] \right\} \quad (14)$$

Given that country j agent wants to smooth consumption over time according to his consumption-wealth ratio, $\ln[C_t^\ell / W(Y_t^\ell)]$, depending upon his intertemporal elasticity of substitution in consumption, ψ , the social planner would choose to allocate consumption across countries in proportion to their consumption-wealth ratio. This relationship is consistent with the approach taken in Lettau and Ludvigson (2001) who specify the stochastic discount rate to depend upon this ratio.

If we further assume that the planner's invariant weights are equal across countries so that $\lambda^\ell = \lambda^j$ then the optimality condition can be further be simplified to:

$$\ln[C_t^\ell / W_t^\ell] = \ln[C_t^j / W_t^j] \quad (15)$$

Along the social planner's optimal allocation of consumption across countries consumption-wealth ratios would be equalized. Since the two economies have different output processes, $\ln[C_t^\ell / W_t^\ell] \neq \ln[C_t^j / W_t^j]$ for all $\ell \neq j$ with probability one, under autarky the social planner's first order condition cannot hold with probability one.

3.3 Decentralized Open Economy

We now consider the decentralized open economy in which the representative agents in each country sell off the rights to their own output streams. We define the price of claims for country j output

payouts in world markets at time t as P_t^{*j} . In this case, the agent's optimization problem becomes:

$$\underset{\{C_t, \underline{\varpi}_t^j\}}{\text{Max}} U_t^j \quad \text{where } U_t^j = \left\{ (1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta \left(E_t \left[U_{t+1}^j \right]^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (16)$$

$$\begin{aligned} \text{s.t. } C_t^j + \underline{P}_t^* \underline{\varpi}_t^j &\leq W_t^{j*} \\ (\underline{Y}_{t+1} + \underline{P}_{t+1}^*) \underline{\varpi}_t^j &= W_{t+1}^{j*} \end{aligned}$$

where $\underline{\varpi}_t^j = \{\varpi_t^{j1}, \varpi_t^{j2}, \dots, \varpi_t^{jJ}\}$ is the vector of claims held by country j investors on each of the country outputs, \underline{Y}_t is the $J \times 1$ vector of the output realizations, \underline{P}_t^* is the price vector of these claims, and W_t^{j*} is the wealth of country j at world prices. Since the utility function is homogeneous in consumption and wealth, all agents will hold the same portfolio shares in a world mutual fund. If we define the portfolio share of country ℓ in country j 's wealth as $h_t^{j\ell} = (P_t^{*\ell} \varpi_t^{j\ell} / W_t^{j*})$, the mutual fund theorem implies that the vector of portfolio shares in the wealth portfolio, i.e., $\underline{h}_t^j = \{h_t^{j1}, h_t^{j2}, \dots, h_t^{jJ}\}$, is equalized for each element across countries: $\underline{h}_t^j = \underline{h}_t^\ell, \forall j, \ell$. The state of the economy is driven by the endowment processes of all J countries, \underline{Y}_t , so that country j agent's problem can then be rewritten as:

$$V_t(W_t^{j*}(\underline{Y}_t), \underline{Y}_t) = \underset{\{C_t, \underline{h}_t^j\}}{\text{Max}} \left[(1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta \left(E_t \left[V_{t+1}(W_{t+1}^{j*}(\underline{Y}_{t+1}), \underline{Y}_{t+1})^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (17)$$

$$\text{s.t. } W_{t+1}^{j*} = (W_t^{j*} - C_t^j) \underline{h}_t^{j'} \underline{R}_{t+1}^* \quad (18)$$

where \underline{R}_{t+1}^* is the $J \times 1$ return vector whose j -th component is $R_{t+1}^{*j} = (Y_{t+1}^j + P_{t+1}^{*j}) / P_t^{*j}$.

Defining the return on country j 's wealth portfolio as $R_{t+1}^{jP} \equiv \underline{h}_t^{j'} \underline{R}_{t+1}^*$, the first-order intertemporal optimization problem for the agent in country j must then satisfy the Euler equation:¹⁶

$$E_t \left\{ \delta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{jP})^{(\theta-1)} R_{t+1}^{*\ell} \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{jP})^{(\theta-1)} R_{t+1}^{*\ell} \right\} = 1 \quad (19)$$

where $G_{c,t+1}^j \equiv C_{t+1}^j / C_t^j$. While this Euler equation holds for each individual country's agent, all countries face the same asset market and thereby view the same return vector. Moreover, since all countries have the same utility function and this function is iso-elastic, we show in the appendix that all countries will choose to hold identical shares in a world mutual fund of claims on the output

¹⁶See Epstein and Zin (1991).

of all participating countries. Therefore, $\underline{h}_t^{j'} = \underline{h}_t' \forall j$. Furthermore, in equilibrium, the number of shares in each country are normalized to one so that the world mutual fund returns are given by: $R_{t+1}^{w*} \equiv \{(\underline{Y}_{t+1} + \underline{P}_{t+1}^*)\iota\} / \{(\underline{P}_t^*)\iota\}$ for ι a J-dimensional unit vector. Defining ϖ^j as the claims on the world mutual fund held by country j we can rewrite the value function more succinctly as:

$$V_t(W_t^{j*}(\underline{Y}_t), \underline{Y}_t) = \underset{\{C_t^j, \varpi_t^j\}}{\text{Max}} \left[(1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta \left(E_t \left[V_{t+1}(W_{t+1}^{j*}(\underline{Y}_{t+1}), \underline{Y}_{t+1})^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (20)$$

$$\text{s.t. } W_{t+1}^{j*} = (W_t^{j*} - C_t^j)R_{t+1}^{w*} \quad (21)$$

In this case, the Euler equation of all participants in world markets will be given by:

$$E_t \left\{ \delta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = 1 \quad (22)$$

This relationship holds as long as all countries hold a constant share of the same mutual fund.¹⁷. In this case, the aggregate resource constraint together with the budget constraint for each country implies further that: $C_t^j = \varpi^j(\underline{Y}_t \iota)$ or $(C_{t+1}^j / C_t^j) = (C_{t+1}^\ell / C_t^\ell), \forall j, \ell$. Then, the return on this common world consumption growth rate prices all countries and can also be priced itself by the Euler equation:

$$E_t \left\{ \delta^\theta (C_{t+1}^w / C_t^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = 1 \quad (23)$$

$$E_t \left\{ \delta^\theta (C_{t+1}^w / C_t^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^\theta \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^\theta \right\} = 1 \quad (24)$$

Below, we will use these Euler equations to solve for the equity prices in world markets. Since the number of shares in each country is normalized to one, the price of the share of the mutual fund is $\underline{P}_t^* \iota = P_t^{w*}$. We also solve for the individual equities on world markets using to the first order condition for individual securities above.

Before proceeding, we relate this decentralized decision-making to the planner's problem using the value function for each country:

$$V_t(W_t^{j*}(S_t), S_t) = (1 - \delta)^{-\left(\frac{\psi}{1-\psi}\right)} (C_t^j)^{\left(\frac{1}{1-\psi}\right)} (W_t^{j*}(S_t))^{-\left(\frac{\psi}{1-\psi}\right)} \quad (25)$$

As above, we use the envelope theorem result that $\partial V / \partial W = \partial U / \partial C$ and substitute the result into the planner's first order condition. Taking logs implies that

¹⁷ We present a justification for this assumption below

$$\ln(\lambda^\ell) + \left(\frac{\psi}{1-\psi}\right) \{\ln[C_t^\ell/W_t^{\ell*}]\} = \ln(\lambda^j) + \left(\frac{\psi}{1-\psi}\right) \{\ln[C_t^j/W_t^{j*}]\}$$

A necessary condition for the planner's problem to hold is that wealth depends upon the total state vector and that consumption-wealth ratios are equalized. In the appendix, we show that these conditions hold. Note, however, that given the initial consumption weights, the planner's problem only requires the consumption-wealth ratios to be equalized across countries in each state. If welfare gains are to be generated by opening up markets, each country's agent will have to decide whether to participate. These participation constraints can lead to different allocations of the welfare gains as we describe next.

3.4 The Decision to Open Markets

Above we described an open economy equilibrium when all countries are open and willing to participate. However, we have not shown that all countries would want to participate by selling off their claims on their home output and holding diversified shares of the world economy instead. This equilibrium requires that no country would prefer to deviate and close their markets. While this deviation could potentially dominate an open market in any date, we consider an equilibrium in which there are complete contingent claims in all future periods. For this equilibriums, we require only that no country would prefer autarky ex ante.¹⁸

To see the possibility for autarky to dominate, consider the timing of markets within the initial period. Agents enter the period with the perpetual claim on their home output and receive the initial endowment on this claim. They then sell off this claim in world markets and in turn buy shares in the world mutual fund. Thus, in equilibrium, an investor in country j faces the constraint.

$$C_0^j + P_0^{w*} \varpi_0^{jw} \leq (Y_0^j + P_0^{j*}) \quad (26)$$

With this timing, the agent consumes his endowment in the first period which implies: $\varpi_0^{jw} = (P_0^{j*}/P_0^{w*})$. Thereafter, the portfolio constraint tracks the evolution of wealth of country j as:

$$C_t^j + P_t^{w*} \varpi_t^{jw} \leq (Y_t^w + P_t^{w*}) \varpi_{t-1}^{jw} \quad (27)$$

where Y_t^w is the per capita endowment of the world, \underline{Y}_t^w , and P_t^{w*} is the price of a claim that pays off this world endowment.

Does an agent at time 0 choose to integrate with the rest of the world in all future periods? In this period the agent decides whether to sell claims in the open economy or whether to stay

¹⁸Other possible deviations are explained below.

in autarky. In making this decision, we assume that agents can fully commit to staying in the integrated world market once they have agreed to participate. The ex-ante participation constraint requires that for all countries j in the risk sharing equilibrium the expected lifetime utility is higher than in autarky. In other words, each country will participate in open markets only if:

$$V_0^j(C_0^{j*}, W_0^{j*}) > V_0^j(C_0^{jA}, W_0^{jA}) \quad (28)$$

where C_0^{j*} and C_0^{jA} are the initial consumption levels of country j agents under the open economy and autarky, respectively. In particular, the initial consumption levels of the country j agents implied by the decentralized economy above would imply the constraint is:

$$V_0^j(\varpi_0^j \underline{Y}, W_0^{j*}) > V_0^j(Y_0^j, W_0^j) \quad (29)$$

or alternatively,

$$(\varpi_0^j Y_0^w)^{\left(\frac{1}{1-\psi}\right)} (W_0^{j*})^{-\left(\frac{\psi}{1-\psi}\right)} > (Y_0^j)^{\left(\frac{1}{1-\psi}\right)} (W_0^j)^{-\left(\frac{\psi}{1-\psi}\right)} \quad (30)$$

Thus an agent will find it optimal to commit to engage in the integrated world market only if his utility is not higher under autarky. Therefore, his value function is the higher of the two expected utility paths. Using A to indicate "autarky", the decision can be written as:

$$V_0^j(C_0^j, W_0^j) = \text{Max}\{V_0^j(C_0^{jA}, W_0^{jA}), V_0^j(C_0^{j*}, W_0^{j*})\} \quad (31)$$

This decision implies a further restriction on the social planner's problem above. Define the set of countries that choose risk-sharing as \hat{J}^* and the complementary set as \hat{J} so that $\hat{J}^* \cup \hat{J} = J$. Then the set \hat{J}^* is defined as:

$$\hat{J}^* = \{j : V_0^j(C_0^{jA}, W_0^{jA}) < V_0^j(C_0^{j*}(\hat{J}^*), W_0^{j*}(\hat{J}^*))\}. \quad (32)$$

Note that since wealth and consumption under risk-sharing depend upon the set of countries choosing to be open, pinning down these countries requires solving for the fixed point set of countries with higher utility under open markets and the countries who indeed choose to be open. This set restricts the planner's problem above to a smaller set of open markets as follows:

$$\begin{aligned}
& \underset{\{C^j(S_t)\}}{\text{Max}} && \sum_{j=1}^J \lambda^j U^j(C(S_t), E[U^j(C^j(S_{t+1})|I_t)]) && (33) \\
& \forall j = \{1, \dots, J\} \\
& \forall S_t \in S \\
& \forall t \in \mathbb{N}^+
\end{aligned}$$

$$\text{s.t. } \sum_{j \in \hat{J}^*} C^j(S_t) \leq \sum_{j \in \hat{J}^*} Y^j(S_t), \quad (34)$$

$$C^j(S_t) \leq Y^j(S_t) \forall j \in \hat{J} \quad (35)$$

The presence of the participation constraints implies that some countries may choose to stay out of the integrated market which will in turn affect the wealth of other countries. If participation constraints are met, countries enter into an agreement where each country j forgoes Y_t^j in exchange for $h_0^j \underline{Y}_t^w$ in consumption. By market clearing condition for the Social Planner problem in equation (7), then feasibility requires that $\sum_{j \in \hat{J}^*} \varpi^j = 1$.

As described above, we assume that countries can fully commit to share the realization of their output each period once they have sold their equity shares. Alternatively, there may be some periods when individual countries have a large realization of their own output and would prefer to revert to autarky for the period rather than share in the world output. If countries were to effectively default on dividend payments, the risk of this default would affect asset pricing relationships and is beyond the scope of our paper. Given our assumption that countries can fully commit, countries initially sell off all rights to their own output and hence claims on world output, so that ϖ_t^{jw} are time-invariant. As a result, all countries with open markets share the same stochastic discount factor in pricing relationships. We use this property in our analysis below.

4 Evaluating International Risk-Sharing Gains

There are at least two measures for these welfare gains that have been calculated in the literature. First, the welfare gains can be calculated as the percentage increase in initial permanent consumption under autarky that would make the country indifferent between opening markets or leaving them closed. This approach followed by Obstfeld (1994a,b) and Lewis (2000) requires solving for Δ and equating the value functions under autarky and integration. Therefore, subsuming the

country superscript and the dependence of wealth on the set of risk-sharing countries \hat{J} for clarity, calculating welfare gains in this case requires solving for Δ in the following equation:

$$V_0((1 + \Delta)C_0^A, W_0^A) = V_0(C_0^*, W_0^*) \quad (36)$$

Where (C_0^A, W_0^A) and (C_0^*, W_0^*) are respectively the autarky and open economy consumption and wealth. Using the solution for the value function, this welfare gain has the form:

$$(1 + \Delta) = \left\{ \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} \right\}^{(1-\psi)} = \left\{ \frac{C_0^*/W_0^*}{C_0^A/W_0^A} \right\}^\psi \left(\frac{C_0^*}{C_0^A} \right)^{(1-\psi)} \quad (37)$$

Welfare gains depend upon both current consumption and the consumption-wealth ratio, reflecting future certainty-equivalent consumption. Welfare gains increase directly with higher current period consumption under risk sharing relative to autarky consumption, C_0^*/C_0^A , depending on whether the intertemporal elasticity of consumption is greater or less than one. If $\psi < 1$, intertemporal substitution is inelastic and countries prefer substitution into current period consumption and welfare gains increase with relatively higher current consumption under risk-sharing. On the other hand, welfare gains also depend upon the consumption-wealth ratio reflecting the expected stochastic discount rate in future periods. If $\psi < 1$, a higher consumption-wealth ratio under risk-sharing will have a smaller effect on welfare gains.

The second measure of welfare gains calculates the gains from simultaneously increasing permanent consumption and wealth. This approach is followed by Bansal and Yaron (2001). Since the value function is homogeneous of degree one in consumption and wealth, we can alternatively define welfare gains as the proportion of current consumption and wealth, that would make agents indifferent between autarky and risk sharing. In other words, the gains Δ are defined such that:

$$V_0((1 + \Delta)C_0^A, (1 + \Delta)W_0^A) = (1 + \Delta)V_0(C_0^A, W_0^A) = V_0(C_0^*, W_0^*) \quad (38)$$

So we can re-write welfare gains as

$$(1 + \Delta) = \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} = \left\{ \frac{C_0^*/W_0^*}{C_0^A/W_0^A} \right\}^{\left(\frac{-\psi}{\psi-1}\right)} \left(\frac{C_0^*}{C_0^A} \right) \quad (39)$$

Note that this interpretation of welfare gains affects consumption and wealth in the same proportion and leave the consumption-wealth ratio undistorted.

4.1 Measuring Gains with Asset Pricing Moments

In order to calculate welfare gains, we now use the Euler equations to calculate the prices under closed and open economies. In closed economy, we use the identity $W_0^A = C_0^A + P_0^A$, to rewrite the value function in terms of the price to consumption ratio¹⁹. When markets open, we allow the countries to sell shares of their own contingent claim for a share of the world contingent claim. Defining $Z_{c,t}^A$ as the price-consumption ratio under closed markets, $Z_{c,t}^A = P_t^A/C_t^A$, and $Z_{c,t}^*$ as the price-consumption ratio under open markets, $Z_{c,t}^* = P_t^*/C_t^*$, and using the budget constraint $W_t^* = C_t^* + P_t^*$, we can rewrite the value function at time 0 as

$$V_0(C_0^A, W_0^A) = (1 - \delta)^{\frac{-\psi}{1-\psi}} (1 + Z_{c,0}^A)^{\frac{\psi}{\psi-1}} C_0^A \quad (40)$$

$$V_0^*(C_0^*, W_0^*) = (1 - \delta)^{\frac{-\psi}{1-\psi}} (1 + Z_{c,0}^*)^{\frac{\psi}{\psi-1}} C_0^* \quad (41)$$

Using the above definition for the value function and solving for the definition of welfare gain, we have:

$$(1 + \Delta) = \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} = \left(\frac{1 + Z_{c,0}^*}{1 + Z_{c,0}^A} \right)^{\frac{\psi}{\psi-1}} \left(\frac{C_0^*}{C_0^A} \right) \quad (42)$$

We use these autarky and open economy consumption claim price measures to calculate the potential gains from consumption risk-sharing based upon three different allocation weights of initial consumption, $\left(\frac{C_0^*}{C_0^A} \right)$. In the first version, the "equally weighted" allocation, we simply set $C_0^{j*} = C_0^{jA}$. Although we show that in many cases this allocation cannot be an equilibrium, many studies ignore the reallocation of resources required to induce countries with more productive and better hedged endowment streams to participate. Our second version is the "price weighted" allocation based upon our decentralized economy above. In this case, $C_0^{j*} = \varpi_0^j Y^w$ where $\varpi_0^{jw} = (P_0^{j*}/P_0^{w*})$ or the share of world per capita endowment that country j can buy when selling of its endowment on world markets. Similarly, in this equilibrium, $C_0^{jA} = Y_0^{jA}$, country j's initial endowment. Thus, in this allocation, the gains can be measured according to:

$$(1 + \Delta) = \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} = \left(\frac{1 + Z_{c,0}^*}{1 + Z_{c,0}^A} \right)^{\frac{\psi}{\psi-1}} \left(\frac{\varpi_0^w Y_0^w}{Y_0^A} \right) \quad (43)$$

As we noted above, there may be some countries for whom the autarky path dominates sharing all productivity payments with the rest of the world. For this reason, we also consider a third version of allocation weights we term the "reservation" allocation. In this allocation, we ask what the original consumption level must be under open markets in order to make each country

¹⁹See Appendix A

indifferent between opening or not. This allocation is defined by setting Δ equal to zero in equation (42), setting $C_0^{jA} = Y_0^{jA}$, and then solving for C_0^* . In other words, the "reservation allocation" is determined by:

$$C_0^{*R} = \left(\frac{1 + Z_{c,0}^A}{1 + Z_{c,0}^*} \right)^{\frac{\psi}{\psi-1}} Y_0^A \quad (44)$$

We next use the data for the US to consider the impacts on welfare gains.

4.2 Measuring Gains: A Two Country Example

To understand the effects of matching asset return moments to international welfare gains, we first consider a two country example. For this example, we use the US data described above before considering truly international data in the next section.

Note that welfare gains for all three measures of initial consumption allocations depend upon the ratio of the price-to-consumption ratio, $Z_{c,0}$. Moreover in the equally weighted allocation, these prices determine the gains uniquely. To develop intuition about these prices, consider how these prices depend upon the autarky and world endowment processes.

For this purpose, consider the pricing of the equilibrium consumption process. First, using the Campbell-Shiller decomposition to express returns in terms of the price-to-consumption ratio, with approximating constants k_0^j and k_1^j we have:

$$r_{c,t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j \quad (45)$$

We then solve for the equilibrium log price to consumption ratio by conjecturing that the log price to consumption ratio is linear in the source of long run risk, $z_t^j = A_0^j + A_1^j \bar{x}_t + A_2^j x_t^j$. Using the Euler equation and applying the properties of log normality for consumption asset return, $r_{c,t+1}^j$, and consumption growth, $g_{c,t+1}^j$, implies:

$$E_t \left[\left(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j \right) \right] + \frac{1}{2} \text{Var}_t \left[\left(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j \right) \right] = 0 \quad (46)$$

Into this Euler equation we then substitute the stochastic processes for log consumption growth, $g_{c,t+1}^j$, log price to consumption ratios, z_{t+1}^j , and the long run risk terms, x_{t+1} . Taking conditional expectations and conditional variances²⁰, the left hand side becomes the sum of a constant term

²⁰Note conditional expectation of shocks are zero and conditional variance of shocks are equal to one

and x_t terms²¹. In the appendix, we show that these steps imply the following analytical form for the coefficients of the log price to consumption ratio when long run risk is shared across countries²²:

$$A_1^j = \frac{1 - \frac{1}{\psi}}{1 - k_1^j \rho} \quad (47)$$

$$A_0^j = \frac{\ln \delta + k_0^j + (1 - \frac{1}{\psi})\mu^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma^j \varphi_e^j)^2)}{1 - k_1^j} \quad (48)$$

Thus, the value function depends upon $1 + Z_{c,0}^j = 1 + \exp(z_{c,0}^j) = 1 + \exp(A_0^j)$. But what is A_0^j ? It measures in certainty equivalent terms the long run effect of the consumption processes on the price-to-consumption ratio. In particular, when long run risk is absent, A_0^j depends upon approximating constants, the time preference parameter, δ , and $(1 - \frac{1}{\psi})(\mu^j + \frac{1}{2}(1 - \gamma)(\sigma^j)^2)$

Figure 1 illustrates the trade-offs in certainty equivalent consumption implied by these parameters under autarky and the open economy for a base case of two countries parameterized to look like the US. In this example under autarky, the US mean growth rate μ^j and the standard deviation σ^j are given by our monthly parameter estimates of .19% and .79%, respectively. We consider two different assumption about risk aversion: $\gamma = 2, 10$. We then consider the effects upon the consumption growth path if two identical countries agree to share risk and growth rates so that $\mu^{j*} = \sum_i \frac{1}{2}\mu^{iA}$ and $\sigma^{j*} = \iota'\Omega\iota$ where Ω is the variance-covariance matrix across countries. For this figure, we assume that the correlation across countries is 0.5.

The figure depicts the trade-offs in growth rates and transitory variability over time. When risk aversion is equal to 10, each country faces a flatter consumption profile given by the solid blue line than under the world pink line. Since both countries face the same mean growth rates, the increase in consumption profile arises from the reduction in variability alone. Alternatively, when the risk aversion parameter is only 2, the improvement in consumption profile moving from the dotted green line to the world line is very minor. Finally, the figure shows the effects on long run risk when it is assumed that all countries face the same long run risk. First, note that since the variance of long run risk is small in the data (e.g., with a standard deviation of only 0.0003), the effects of this risk on the unconditional consumption profile is small. Second, for the case depicted in this figure, all countries share the same long run risk effects so there is no opportunity to share the risk. As such, the effects of long run risk on welfare have been shut down.

²¹For a detailed derivations of prices in autarky and the open economy see the appendix.

²²When long run risk differs across countries there is an addition effect arising from idiosyncratic long run risk. The solution is detailed in the appendix.

Figure 2 depicts a similar example of two countries except that now country 2 is assumed to have a standard deviation in monthly consumption that is three times as variable as country 1. For this picture, we subsume the long run risk effect and assume that risk aversion is equal to 10. We also continue to assume that the initial consumption allocation is the same for both countries and therefore would correspond to an "equal weight" allocation. However, the picture demonstrates that the effect upon welfare improvement would not be the same for both countries. In particular, country 2 has much higher variability resulting in a downward sloping consumption profile. By contrast, country 1 has an increasing consumption profile. While the reduction in variance still leads to an increase in the consumption profile in the world, country 1 obviously does not benefit as much as country 2.

Figure 3 shows an example in which the two countries have the same variance, but differing growth rates. In this case, country 2 has an autarky growth rate that is 3% higher than country 1. Now country 1 benefits more than country 2 by pooling claims to productivity.

While this example has illustrated the effects of mean growth rates and variance on consumption profiles and thereby $Z_{c,0}^j$, it does not address the effects of the intertemporal elasticity of substitution in consumption, ψ . To understand this effect, it is first useful to consider the effect of the welfare ratio $\left(\frac{1+Z_{c,0}^*}{1+Z_{c,0}^A}\right)^{\frac{\psi}{\psi-1}}$ holding constant the effects of ψ on $Z_{c,0}^j$. Differentiation implies that $\partial\Delta/\partial\psi = \frac{1}{(1-\psi)^2} \left(\frac{1+Z_{c,0}^*}{1+Z_{c,0}^A}\right)^{\frac{\psi}{1-\psi}} \log\left(\frac{1+Z_{c,0}^*}{1+Z_{c,0}^A}\right)$. Thus in the extreme when the autarky price of consumption stream is close to the open economy price of consumption stream, the log of the ratios of these price-consumption streams is zero and welfare gains are not affected by the intertemporal elasticity of consumption (IES). Alternatively when the price of the world to consumption ratio is significantly higher, the welfare gains will tend to increase with IES although here the effects of IES on Z become relevant.

We now use our two country example to consider the impacts on welfare gains assuming our three different sets of initial consumption allocations. We begin in Table 2 by assuming the two countries are symmetric but have a correlation of 0.5. Since the countries are symmetric, their price shares are equal and the equally weighted and price weighted gains are identical. We report the gains when risk aversion is equal to 2 and 10 and also when IES is equal to 0.5 and also 1.5, as BY argue. These gains are reported for three different assumptions about long run risk across countries. Panel A assumes that the countries all have the same long run risk \bar{x}_t so that $\sigma^j \varphi_e^j = \sigma_e, \forall j$. In this case, the gains to risk-sharing are lower because countries are not able to pool their long run risk, but can only pool their idiosyncratic risk. Panels B and C assume that

the two countries have different long run risk processes, x_t^j components. Panel B attributes the correlation between countries to arise solely from the correlation between the idiosyncratic risk. Thus, in Panel B, the correlation between monthly consumption growth rates would be determined as: $\text{Cov}(g_{c,t+1}^1, g_{c,t+1}^2) = \text{Cov}(\eta_{t+1}^1, \eta_{t+1}^2)$. Moreover, the long run risk components are assumed uncorrelated. In this case, risk-sharing allows the countries to diversify both the idiosyncratic risk and the long run risk component. As a result, the gains increase for given preference parameters. For example, when $\text{IES} = 1.5$, the gains increase from about 0.9% to 8.5% when risk aversion is 2 and from 5.4% to 66.7% when risk aversion is 10. Panel C shows the results assuming that the correlation between monthly consumption growth rates are solely determined by the long run risk components or that $\text{Cov}(g_{c,t+1}^1, g_{c,t+1}^2) = \frac{1}{1-\rho^2} \text{Cov}(e_{t+1}^1, e_{t+1}^2)$. In this case, since the correlation is one-half on long run risk and the idiosyncratic risk is uncorrelated, the gains from long run risk are smaller than in Panel B. For example, when $\text{IES} = 1.5$, the gains decline from about 8.5% to 5.5% when risk aversion is 2 and from 67% to 39% when risk aversion is 10. However, these gains are all larger than when all countries share the same long run risk as in Panel A.

Table 3 examines the gains for the individual long run risk case when correlations are based upon the transitory component. In this table, we show how gains are affected by the use of naive initial "Equal Consumption" allocations and the "Price Weighted" allocations based upon our decentralized economy. Panel A begins with the results for the symmetric case. These gains mirror the results in Table 2. The welfare gains increase in the risk aversion coefficient for a given IES. Moreover, higher IES leads to greater welfare gains for a given risk aversion. Panel A also shows the effects on gains when the correlation is negative. In this case, the two countries are better able to reduce their risk. As a result welfare gains increase as the correlations across countries decrease.

Panel B of Table 3 demonstrates the effects on gains when one country has higher variance than the other. In particular, we assume that country 2 has 1.1 times higher variance than country 1. In this case, the equally weighted gains show that country 2 would benefit more than country 1. The intuition behind this result is clear since country 2 had a higher variance in autarky. However, the table also shows the price effects. Since country 1 has a more valuable endowment claim in world markets, it commands a higher price and therefore a higher initial consumption share. As a result, the price-weighted gains show that country 1 captures more of the welfare gains in the equilibrium.

Panel C of Table 3 shows the same exercise except that now country 2 has a mean growth

rate that is 1.1 times that of country 1. In this case, equally weighted "gains" are negative for country 2 when risk aversion is 2. The reason is that the country 2 agent gives up a higher growth rate in return for a lower variability in consumption that with low risk aversion, he doesn't care as much about reducing volatility. Therefore, he clearly would prefer autarky to an equally weighted agreement. However, this example only illustrates that the equally weighted gains are not equilibria. The price weighted gains show once more that the high growth country 2 is compensated with a higher price in world markets and, hence, a greater welfare gain. In the case of risk aversion of 2 and IES of 0.5, country 2 goes from a welfare cost of 0.5% under equal weights to a gain of 7.8% under priced weights. Similarly, when risk aversion is 2 and IES is 1.5, the "gains" go from -0.7% to 31%.

Table 4 considers an alternative consumption allocation to our decentralized economy. We examine the allocation implied when the country with the highest reservation consumption level commands an "all-or-nothing" offer to the other countries. In particular, the country with the highest reservation consumption, defined as $j = H$ for "High", $C_0^{H,R}$ gives other countries the offer to set $\{C_0^{1,*}, C_0^{2,*}, \dots, C_0^{j,*}, \dots, C_0^{J,*}\} = \{C_0^{1,R}, C_0^{2,R}, \dots, C_0^{j,R}, \dots, C_0^{J,R}\}, j \neq H$ subject to the feasibility constraint that $\sum_{j=1}^J C_0^{j,R} \leq \sum_{j=1}^J Y_0^j$ when populations are equal.²³ We also show the other extreme if the low country is able to extract the rents.

Considering once again the individual long run risk on the transitory consumption component case, Table 4 reports the allocation weights and gains for the set of BY parameters given in the table. Panel A shows the allocations for equal weighted gains equal to 1 by construction. For these parameter estimates, the gains are 67%. In the columns with headings "Reservation 1," we report the allocation for country 1 implied by his agents reservation allocation. The allocation of 0.6 implies that he would be willing to participate in risk-sharing as long as his allocation is at least 60% of his autarky endowment. If he gives the other country a take it or leave it offer of the other country's reservation allocation, he offers only 0.6 by symmetry. The table reports that country 1's gains are then 233.4%. The information under "Reservation 2" reports the symmetric version of these results. Similarly, the price allocation gains in the symmetric case imply that both countries get the same gains since the prices of their equity are the same on world markets.

Panel B of Table 4 shows the results of these allocations when country 2 has a higher variance than country 1. In this case, the equally weighted gains will again show that the higher autarky variance country 2 receives more gains. The reservation allocations show that country 2 values

²³In the appendix, we describe the analysis when population weights differ.

opening markets more. In particular, the reservation allocation is 0.52 which is lower than country 1's reservation allocation of 0.64. If country one can extract all the gains from risk-sharing then it will enjoy a gain of almost 160%. The "Price-Weighted" allocations show how the value of the endowment stream on world markets affects the allocations. Since country 1 has a lower variance, the value of its equity is higher in world markets and it gets a higher initial allocation consumption allocation of 1.14. The extra 14% is paid for by country 2 which has to reduce its initial allocation to 86%.

Finally, Panel C of Table 4 shows the initial allocation effects on risk-sharing gains when mean growth rates differ. Again, country 2 has a higher growth rate so that country 1 clearly benefits more in the equally weighted allocation. Similarly, the reservation allocations are now switched between the two countries. Country 1 is now will to give up more initial consumption to 0.56 in order to participate in the higher growth rate, which country 2 agent's reservation allocation now increases to 0.63. Once again, the impact of prices on world markets tilt the initial allocations toward the high price country. Country 2 gets 8% more of the initial consumption while country 1 get 8% less.

Having now considered how the gains depend upon parameters and allocations depending upon asset returns, we reconsider these interactions next.

5 Matching the International Asset Return and Consumption Moments

So far, we have shown two main points. First, US asset return and PPP-adjusted consumption moments can be explained by an asset pricing framework that includes "long run risk."²⁴ Second, we have used a two country example of the standard international consumption risk-sharing model to show that the gains from this diversification depend strongly upon this long run risk factor. In this section, we combine these two points in an international data set. In particular, we use our international framework to analyze the implications of an international data set of consumption, dividends, and asset returns. As with the US data, we match the implied consumption processes to asset return means and variances. We then use these parameter estimates that are disciplined by asset returns to calculate the implied risk-sharing gains.

²⁴In the next version of our paper, we plan to do the same for the "habit-persistence" asset pricing framework of Campbell and Cochrane (1999).

In this section, we begin by describing our data set in more detail. We then detail our methodology for estimating the model parameters. This methodology uses a simulated method of moments (SMM) procedure to find a set of country parameters that best replicates the data moments. We then describe the implied risk-sharing gains based upon these parameters.

5.1 Data Description

Our data set is comprised of two main data sources broken down by asset returns and consumption. For consumption, we use PPP-adjusted per capita consumption measures from Penn World Tables National Accounts. As we described in Table 1, we also checked our results for the US against the existing literature based upon the National Product and Income Accounts (NIPA). Note that our analytical framework above is based upon growth rates except for the initial consumption allocations. Therefore, we use the US as a numeraire real consumption measure in growth rates. However, our results are not sensitive to this assumption. Details about constructing the consumption series are provided in the appendix.

For dividend and return data, we use the data from Campbell (2003). To be consistent with the annual consumption data in PWT, we aggregate the quarterly data in Campbell using the same deflator series from the Penn World Tables to form real annual equity returns, risk free rates, and dividend growth rates. The equity return data span the sample period 1970-1999. To maintain consistency, we use the real risk free rate for the same period. Details about the aggregation of the asset return and dividend series are provided in the appendix.

The set of countries we examine are restricted to a group for which we have a consistent set of asset returns and consumption. These countries are Australia, Canada, France, Germany, Japan, UK, and the US. All the data moments are presented in Table 5. The top panel shows the mean and standard deviations of equity returns and risk-free rates for the seven countries since 1970. The mean equity premium ranges from a low of about 1.5% for Australia to a high of 6.6% for the UK. The risk free rate shows a tighter range of 1.2% for Japan to 2.7% for Canada. The standard deviation of equity is high for all these countries. Moreover, the standard deviation of the risk-free rate is comparable to the size of the mean risk-free rate ranging from 1.7% to 3% per annum. As noted above, typical models based upon i.i.d. consumption imply that the variance of the risk-free rate is equal to zero.

The middle panel shows basic statistics for consumption. The mean growth rates are reported first along with their standard errors. Canada has the lowest growth rate at 1.9%. But the

highest growth rate by far is Japan at 4.9%. Another outlier is Australia which exhibits a small but negative first order auto-correlation coefficient. The standard deviations show a significant amount of variability in annual consumption growth.

The bottom panel reports summary moments for dividend growth. The mean growth rate in the US is positive, generating higher anticipated pay-offs in the dividend-paying asset. However, for Canada and France, the dividend growth rate is marginally negative and for Japan it is significantly negative. However, the standard errors on these mean growth rates show that the mean dividend growth rates are typically insignificantly different from one. Fortunately, as equations (1) and (2) show, the key variable from the dividend process needed to determine the consumption process depends on the variance and not the mean of dividend growth.²⁵

Given these summary statistics, we next discuss the methodology used to solve for the parameters in the model.

5.2 Solution Method

To discipline our model, we require the parameter values for the processes of consumption and dividends to generate the asset return moments we observe in the data. To generate the parameters values, we first use the annual means of consumption growth and dividend growth to calibrate the monthly rates μ and μ_d . For this purpose, we calculate the mean annual growth rates from the data and divide by 12. In trial runs of the SMM procedure described below, we find that this change makes little difference in the estimation of the remaining parameters and greatly decreases the computation time.

Next we use a reduced (first-pass) SMM to estimate the parameters for each country. Implementing this procedure involves the following step. For every set of parameter values, we first solve the model using the analytical solutions for returns in the closed economy. We then compute a weighted difference between a targeted set of model generated moments and the data moments using a weighting matrix.²⁶ The set of parameter values that minimizes this difference is the SMM estimate.

We choose the following set of data moments to target for each country: the standard deviation

²⁵The mean of dividend growth will be important for measuring the welfare gains when we consider the incomplete markets version of the model, however. We are currently investigating longer sources of dividends that may provide a better estimate of long term dividend growth.

²⁶In weighting the target moments, we implement the reduced SMM procedure using both the identity matrix and a diagonal matrix with typical components equal to the sample variance.

of log consumption growth ($\sigma(g_c)$), the first order auto-correlation of log consumption growth ($\rho_1(g_c)$), the standard deviation of log dividend growth ($\sigma(g_d)$), the mean equity premium ($E(r_m - r_f)$), the mean risk free rate ($E(r_f)$), the standard deviation of the market return ($\sigma(r_m)$), and standard deviation of risk free rate ($\sigma(r_f)$). Using these seven moments per country, we estimate the 5 parameters in the model for each country given in equations (1) and (2). These are: the variance of the transitory component of consumption, σ^j , the ratio of this variance to the long run risk variance, φ_e^j , and to the dividend variance, φ_d^j , the autocorrelation of the long run risk component, ρ^j , and the sensitivity of dividends to long run risk, ϕ^j . The set of targeted moments were chosen to best represent both consumption and asset pricing data²⁷.

Below we also consider two restricted cases. In one case, we assume all countries have the same autocorrelation coefficient on long run risk; $\rho^j = \rho$ for all j. In this case, the autocorrelation is set equal to the mean across countries since the parameter estimates are relatively close to each other. In the other restricted case, we assume that all countries have a common long run risk component. This long run risk component is assumed to be the same as the estimate from the US.

Our estimation requires a set of preference parameters. For this purpose, we use parameter estimates that have been found to fit asset returns best in the US. We therefore take the parameters from Bansal and Yaron (2004) of IES = 1.5, $\gamma = 10$, and $\beta = .998$. As is standard in the literature and required from our model, these parameters are the same across all countries.

As stated earlier, the model is written and estimated at the monthly level and therefore the simulated data from the model must be time-aggregated to match the annual data moments²⁸. Therefore to match our annual consumption, dividend growth and asset return moments, we time-aggregate the model-generated data from monthly to annual frequency. Parameter estimates and simulated model moments are the averages of 500 simulations, each with 840 time-aggregated monthly observations.

Table (6) shows the resulting SMM generate parameters of $(\rho^j, \sigma^j, \phi^j, \varphi_e^j, \varphi_d^j)$ for each country. Panel A shows the results of the monthly calibrated means of consumption and dividends. These numbers mirror the pattern in the annual data described in Table 5. Panel B reports the estimation results assuming a common long run risk component. With this constraint, the autocorrelations ρ , and the variance of the long run component $\sigma\varphi_e$ are the same across countries. In this case, only

²⁷All the usual criticisms of moment selection apply, see Gallant and Tauchen for discussion on efficient method of moments

²⁸By time-aggregate, we compute the growth between the levels at t+12 and t, given the realizations of 12 monthly growth rates. In comparison, by annualize, we mean monthly growth rate times 12

the idiosyncratic variance σ^j , the effect of long run risk on dividends, ϕ^j , and the ratio of dividend variance to long run risk, φ_d^j , differ across countries. Panel C shows the same parameter estimates but without requiring the long run risk to be common across countries. As the estimates show, the variances on the idiosyncratic and long run components are comparable across both versions of the model. The main difference is that the long run risk variance is 2 to 3 times higher for the US, UK, and Japan in the individual long run risk case compared to the common long run risk case.

Table 7 compares the model moments from simulated data with actual data moments for both versions of the model. For convenience, Panel A repeats the targeted data moments from Table 5. These are the standard deviation and first order autocorrelation of consumption growth, the standard deviation of dividend growth, the mean of the equity premium, the standard deviation of equity returns, the mean of the risk-free rate, and the standard deviation of the risk free rate. Panels B and C show the counterparts generated by the model for the case in which there is a common long run risk and individual long run risk component, respectively. Generally, the model does a reasonable job at matching the standard deviation patterns of consumption and dividend growth, although it has a harder time matching the first order autocorrelations across countries. The model also matches the general pattern of mean equity premia, the risk-free rate and the variance of equity returns. However, the standard deviation of the risk free rate in the model is lower than the data. This feature is likely to improve once we include stochastic volatility in the next version of our paper.

Overall, therefore, our consumption processes match the basic features of our asset return model. We next use these estimates to reconsider the implications for international risk-sharing gains.

5.3 Welfare Gains Implications

We now use the consumption, dividend and preference parameters to generate the implied international risk-sharing gains.²⁹ For each country, we follow the same steps as we did for the two country example above. In particular, we compute the autarky and open economy log price to consumption ratios using with the same preference parameters (ψ, γ, β) and set of consumption and dividend parameters $(\mu^j, \mu_d^j, \rho^j, \sigma^j, \phi^j, \varphi_e^j, \varphi_d^j)$ that match the moments. Also, as above, we

²⁹As above, all of our gains assume that markets are complete and that equity is a redundant asset. Therefore, the country's price in world markets is the value of the consumption-paying, not dividend-paying, asset. In the incomplete markets version of our model (in progress), risk-sharing is based upon the dividend-paying asset and the consumption-paying asset is not tradeable.

assume that the initial long run risk value is at its long run mean of zero: $x_0^j = E(x_t^j)$. Moreover, we assume that population weights are constant over time and equal across countries.

In the two-country example above, we showed that risk-sharing gains depend critically upon the degree to which long run risk is common or idiosyncratic across countries. When the long run risk is common across countries, countries can only pool their idiosyncratic risk and the risk-sharing gains are attenuated. However, when long run risk is idiosyncratic across countries, gains can be much greater because countries can pool this persistent source of risk. We therefore identify the correlation of endowment processes across countries using the variance-covariance matrix of consumption growth rates in the data.

Table 8 shows the correlation matrix of consumption. The final two columns show the correlation of each country with the implied world mutual fund, first assuming all countries have equal weights and lastly adjusting for population.. For the equal weight case, Germany has the lowest correlation at 0.28 while France has the highest at 0.64. For the population-weighted correlations, Australia has the lowest correlation of 0.1 and the US has the highest correlation near 0.8. In the analysis below, we assume that the correlation of consumption reflects the transitory component only: $\text{Cov}(g_{c,t+1}^1, g_{c,t+1}^2) = \text{Cov}(\eta_{t+1}^1, \eta_{t+1}^2)$. As above, we consider two extreme cases. In one case, long run risk is common across countries. In the other case, long run risk is uncorrelated, thereby generating much higher gains.

As in our simple two-country example above, we consider three different assumptions about initial consumption allocations. In the first naive "equally weighted allocation", initial open economy consumption is the same as the autarky, $C_0^* = C_0^A$. As we showed above, this allocation may not be a feasible equilibrium because it imply negative risk-sharing "gains" to some countries with high mean growth rates or low variances under autarky.

Our second set of gains is based upon the "reservation allocation" in which we compute the minimum level of initial open economy consumption that would leave the country willing to participate in the risk-sharing arrangement. As we showed in equation (44), this initial consumption allocation is the defined as C_0^{*R} such that the welfare gains Δ are equal to zero. If $(C_0^{*R}/C_0^A) < 1$, that country's agent would be willing to give up some of his initial consumption allocation in autarky in order to participate in future risk-sharing. However, if $(C_0^{*R}/C_0^A) > 1$. the country will not participate in the risk-sharing agreement without initial compensation from other countries. In our reported results, the country with the highest reservation consumption allocation extracts all the risk-sharing rents by offering the other countries their reservation allocations. In other words,

we assume that all the gains go to country ℓ implied by:

$$\underset{\ell}{\text{ArgMax}}\{C_0^{*R,\ell}/C_0^{A,\ell}\} \quad (49)$$

Since the country with the highest reservation allocation would be most likely to defect from the risk-sharing agreement, we consider this country to have the most bargaining power.

Our third set of gains is the "price weighted allocation" implied by the decentralized equilibrium we described above. Each country's consumer-investor sells off the rights to his endowment stream and purchases shares in the world mutual fund. In this case, the initial consumption allocation is determined by the market value of these shares. Thus, the allocation for each country is given by: $(C_0^{j*}/C_0^{jA}) = (P_0^{j*}/P_0^{w*})(Y_0^w/Y_0^{jA})$. At this point, we are still computing the open economy prices for the multi-country case and will include these in the next version of the paper. In the meantime, the naive "equally weighted" and winner-takes-all "reservation" gains provide a set of extremes.

Table 9 shows the results of this analysis. Panel A reports the gains and consumption allocations when all countries are assume to share the same long run risk component. As the analysis shows, Japan would choose not to participate without further inducement since the implied costs of participating would be 46% of permanent consumption and wealth. Among countries with positive equally weighted gains, France gains the least at 1.3%. The second, third, and fourth columns provide information about reservation consumption allocations. Under "Reservation Share," we report (C_0^{*R}/C_0^A) for each country. The numbers demonstrate why Japan would lose utility without compensation, since the reservation share significantly exceeds 1 at 1.86. Moreover, France's reservation allocation is only marginally less than one at 0.99 which shows that the country is close to indifferent between participating as is reflected by its relatively low implied "equal gains." Under "Ranking," we report the rank of reservation shares $C_0^{*R,\ell}/C_0^{A,\ell}$ from highest of 1 to lowest of 7. Clearly, Japan has the highest rank. In a winner-takes-all equilibrium, the Japanese consumer-investor would set the other countries at their reservation allocations and extract all the risk-sharing rents. This set of reservation weights is reported under "Reservation Gains." Japan has gains of 226% of permanent consumption and wealth while the rest of the countries get zero.

The first two sets of gains provide extreme assumptions about which countries extract the gains from international risk-sharing. Under equal gains, the countries with the highest reservation consumption allocations get the least gains and potentially may not choose to participate. Under the "reservation" allocation gains, the country with the highest reservation consumption allocation

is assumed to take all the gains leaving none for the other countries. In our equilibrium priced shares, these allocations are the outcome of the prices of endowments in world markets. <TO BE COMPLETED>

Panel B of Table 9 reports these same results for the case when all countries have idiosyncratic long run risk. In this case, the gains are much greater as we have shown above. In autarky, long run risk reduces expected utility but open markets allows this risk to be diversified. As a result the gains increase significantly, especially for the countries with higher variance to their long run risk. As Table 6 shows, these countries are Japan, the UK, and the US. As a result, even with equally weighted allocation shares, Japan is now willing to participate without compensation. Indeed, the reservation shares now show that the US has the lowest ranking and would have be willing to participate in the risk-sharing agreement even if the initial consumption allocation were as low as 1% of its initial autarky endowment level. The country with the highest reservation share is Canada. Recall from Table 6 in estimating the parameters of the individual long run risk, Canada has the the lowest variances of long run risk. If Canada is able to extract all the risk-sharing gains with an all-or-nothing offer, the Canadian investor gains a hefty 1843% of permanent consumption.

Once again, however, these gains represent different extremes about initial consumption allocations. For equilibrium-determined allocations, we report the "priced shares." <TO BE COMPLETED>

6 Conclusion

In this paper we have examined the implications of asset return moments for consumption-based models of international risk-sharing. Relative to standard consumption models, recent asset pricing models have acheived better success at matching these moments by either assuming a small but persistent "long run risk" component in consumption (Bansal and Yaron (2004)) or by assuming a persistence to "habit" in utility (Campbell and Cochrane (1999)).³⁰ We developed a framework that nests many models of international financial models and provides a benchmark for calculating welfare gains. By incorporating "long run risk" into our framework, we showed key features of the model with a two country example. In particular, we showed that risk-sharing gains depend upon the degree to which this risk component is common or idiosyncratic across countries.

³⁰So far, we have focused upon the former explanation, but intend to include the latter in the next version of the paper.

We then used asset pricing and consumption data for seven countries to estimate key distributional assumptions. Our estimation disciplined our model by requiring the parameters to match standard moments of asset returns with consumption data. The results also indicate where domestic-based estimation of asset pricing models fall short in the international arena.

We then took these estimated parameters and applied them to our model to calculate the gains from risk-sharing. While our full equilibrium price results are still forthcoming, our analysis so far shows a range of gains that depend critically upon the degree of co-movement in long run risk. One way to further discipline the measure of co-movement is to look at the co-movement in equity returns. For this purpose, we are developing an incomplete markets version of the market in which claims on equity are traded, but claims on consumption are not tradeable. This assumption will allow us to identify the degree to which long run risk co-moves across countries.

Despite these caveats, we believe that the analysis in the current paper provides an important step forward as a first attempt to use asset returns to provide insights into the consumer's views toward risk across countries.

Table 1: Model Comparison for US Consumption and Asset Pricing Moments (In Annual Percent)

	Mehra/Prescott ^a		Bansal/Yaron ^b			Lewis/Liu ^c		
	Data ^d	Model	Data ^e	IID	LRR	Data ^f	IID	LRR
$\sigma(g_c)$	3.6	n/a	2.93	2.19	2.85	1.89	2.71	2.89
$\rho 1(g_c)$	-0.14	n/a	0.49	0.23	0.47	0.19	0.24	0.30
$\sigma(g_d)$	n/a	n/a	11.49	9.89	11.28	5.47	4.61	5.94
$\rho 1(g_d)$	n/a	n/a	0.21	0.22	0.37	0.08	0.22	0.44
$E(r_m - r_f)$	6.18	1.42	6.33	-0.89	4.37	5.47	-0.17	3.50
$\sigma(r_m)$	n/a	n/a	19.42	12.17	16.67	17.56	5.66	10.77
$E(r_f)$	0.80	12.71	0.86	2.18	1.74	1.46	1.88	1.78
$\sigma(r_f)$	n/a	n/a	0.97	0.00	0.63	1.53	0.00	0.42

^aCRRA Utility with Parameters: $\beta = 0.99$, $\gamma = 10$

^bEpstein-Zin-Weil Utility with Parameters: $\beta = 0.987$, $\psi = 1.5$, $\gamma = 10$, $\mu = 0.15$, $\sigma = 0.78$, $\phi_e = 0.044$, $\rho = 0.979$, $\phi = 3.0$, $\phi_d = 4.5$

^cEpstein-Zin-Weil Utility with Parameters: $\beta = 0.987$, $\psi = 1.5$, $\gamma = 10$, $\mu = 0.19$, $\sigma = 0.64$, $\phi_e = 0.044$, $\rho = 0.979$, $\phi = 3.4$, $\phi_d = 1.7$

^dConsumption: Kuznet-Kendrik-USNIA Non-durable and Services for 1889-1978, Asset Data: S&P Composite

^eConsumption: NIPA Non-durable and Services for 1929-1998, Asset Data: CRSP 1929-1998

^fConsumption: PWT Total Consumption for 1950-2000, Asset Data: Campbell Handbook for 1970-1998

Table 2: Symmetric Two Country Welfare Gains (in Annual percent)

		$\psi = 0.5$		$\psi = 1.5$	
		$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
A. Common LRR: $\text{corr}(\eta_t^i, \eta_t^j) = 0.5$					
Gain		0.281	5.013	0.895	5.434
B. Individual LRR: $\text{corr}(\eta_t^i, \eta_t^j) = 0.5, \text{corr}(e_t^i, e_t^j) = 0$					
Gain		2.193	41.001	8.478	66.706
C. Individual LRR: $\text{corr}(\eta_t^i, \eta_t^j) = 0, \text{corr}(e_t^i, e_t^j) = 0.5$					
Gain		1.523	28.345	5.523	39.031

^aModel parameters: $\beta = 0.987, \mu_1 = \mu_2 = 0.15, \sigma_1 = \sigma_2 = 0.78, \phi_e = 0.044, \rho = 0.979$

Table 3: Two Country Welfare Gains with Individual LRR (in Annual percent)

$\psi = 0.5$					$\psi = 1.5$			
$\gamma = 2$		$\gamma = 10$		$\gamma = 2$		$\gamma = 10$		
Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd	
A. Symmetric: $\mu_1 = \mu_2, \sigma_1 = \sigma_2, corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$								
Country 1	2.19	2.19	41.00	41.00	8.48	8.48	66.71	66.71
Country 2	2.19	2.19	41.00	41.00	8.48	8.48	66.71	66.71
A. Symmetric: $\mu_1 = \mu_2, \sigma_1 = \sigma_2, corr(\eta_t^i, \eta_t^j) = -0.5, corr(e_t^i, e_t^j) = 0$								
Country 1	2.75	2.75	50.37	50.37	10.46	10.46	89.75	89.75
Country 2	2.75	2.75	50.37	50.37	10.46	10.46	89.75	89.75
B. Different σ: $\sigma_2 = 1.10 * \sigma_1, corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$								
Country 1	1.91	2.46	35.95	45.85	7.36	9.67	55.22	77.18
Country 2	2.99	1.35	72.01	26.04	11.42	5.04	90.93	33.26
C. Different μ: $\mu_2 = 1.10 * \mu_1, corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$								
Country 1	4.93	2.03	46.14	40.68	18.80	6.61	78.83	64.99
Country 2	-0.54	7.83	31.70	51.59	-0.65	30.99	59.08	92.67

^aModel parameters: $\beta = 0.987, \mu_1 = 0.15, \sigma_1 = 0.78, \phi_e = 0.044, \rho = 0.979$

Table 4: Two Country Welfare Gains with Individual LRR (in Annual percent)

	Equal Wgt		Reserve 1		Reserve 2		Price Wgt	
	Gain	Alloc	Gain	Alloc	Gain	Alloc	Gain	Alloc
A: Symmetric: $corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$								
Country 1	66.7%	1.00	0.0%	0.60	233.4%	1.40	66.7%	1.00
Country 2	66.7%	1.00	233.4%	1.40	0.0%	0.60	66.7%	1.00
B: Different σ: $\sigma_2 = 1.10 * \sigma_1, corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$								
Country 1	55.2%	1.00	0%	0.64	129.1%	1.48	77.2%	1.14
Country 2	90.9%	1.00	158.9%	1.36	0%	0.52	33.3%	0.86
C: Different μ: $\mu_2 = 1.10 * \mu_1, corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$								
Country 1	78.8%	1.00	0%	0.56	145.2%	1.37	65.0%	0.92
Country 2	59.1%	1.00	129.2%	1.44	0%	0.63	92.7%	1.08

^aModel parameters: $\beta = 0.987, \psi = 1.5, \gamma = 10, \mu_1 = 0.15, \sigma_1 = 0.78, \phi_e = 0.044, \rho = 0.979$

Table 5: Summary Statistics: in Annual percent

	AUS	CAN	FRA	GER	JAP	UK	US
Asset Pricing Data^a:							
$E(r_m)$	3.55	5.43	8.73	7.73	4.96	7.92	6.93
$\sigma(r_m)$	22.60	17.28	22.51	19.81	21.77	21.14	17.56
$E(r_f)$	2.06	2.69	2.42	2.61	1.24	1.28	1.46
$\sigma(r_f)$	2.49	1.77	1.69	1.32	2.17	2.92	1.53
$E(r_m - r_f)$	1.49	2.74	6.31	5.12	3.72	6.65	5.47
$E(g_d)$	0.637	-0.416	-0.429	0.250	-2.29	0.739	1.49
$\sigma(g_d)$	13.68	8.16	11.43	9.37	5.43	8.28	5.47
$\rho 1(g_d)$	0.181	0.397	0.514	0.490	0.545	0.137	0.076
	AUS	CAN	FRA	GER	JAP	UK	US
Log Consumption Growth Data^b:							
$E(g_c)$	2.17	1.90	3.12	2.85	4.90	2.17	2.29
$\sigma(g_c)$	3.51	2.05	3.28	3.86	3.35	1.86	1.89
$\rho 1(g_c)$	-0.074	0.236	0.110	0.164	0.552	0.323	0.188

^aSource: Campbell (1970-1999)

^bSource: Penn World Tables (1950-2000)

Table 6: Estimated Monthly Parameters by Country - W/O Stochastic Volatility

	AUS	CAN	FRA	GER	JAP	UK	US	BY-US
A: Calibrated								
μ	0.181	0.158	0.26	0.238	0.408	0.181	0.191	0.150
μ_g	0.053	-0.035	-0.036	0.021	-0.191	0.062	0.124	0.150
B: Common LRR (w/BY $\sigma = 0.78$, $\phi_e = 0.044$, $\rho = 0.979$)								
σ_j	0.860	0.260	0.930	1.330	1.710	0.770	0.640	
ρ	0.979	0.979	0.979	0.979	0.979	0.979	0.979	
ϕ_e	0.044	0.044	0.044	0.044	0.044	0.044	0.044	
$\phi_e * \sigma$	0.034	0.034	0.034	0.034	0.034	0.034	0.034	
ϕ	3.14	3.11	3.82	3.29	3.43	3.86	3.42	
ϕ_d	7.2	7.7	3.6	2.2	1.0	2.5	1.7	
C: Individual LRR								
σ_j	1.050	0.570	0.970	1.430	1.600	0.720	0.490	0.780
ρ	0.979	0.979	0.979	0.979	0.979	0.979	0.979	0.979
ϕ_e	0.034	0.051	0.051	0.021	0.045	0.119	0.200	0.044
$\phi_e * \sigma_j$	0.035	0.029	0.049	0.030	0.073	0.086	0.098	0.034
ϕ	3.43	3.49	3.50	3.50	3.50	3.50	3.50	3.00
ϕ_d	5.0	5.8	4.5	2.9	1.6	4.5	3.8	4.5

^aPreference parameters: $\beta = 0.987$, $\psi = 1.5$, $\gamma = 10$

^bAll μ 's, σ_j 's, and $(\phi_e * \sigma_j)$'s are in percent

Table 7: **Data Moments and Simulated Model Moments**

	AUS	CAN	FRA	GER	JAP	UK	US
A. Data Moments:							
$\sigma(g_c)$	3.51	2.05	3.28	3.86	3.35	1.86	1.12
$\sigma(g_d)$	13.68	8.16	11.43	9.37	5.43	8.28	5.47
$E(r_m - r_f)$	1.49	2.74	6.31	5.12	3.72	6.65	5.47
$E(r_f)$	2.06	2.69	2.42	2.61	1.24	1.28	1.46
$\sigma(r_m)$	22.60	17.28	22.51	19.81	21.77	21.14	17.56
$\sigma(r_f)$	2.48	1.77	1.69	1.32	2.17	2.92	1.53
B. Model Moments: Common LRR							
$\sigma(g_c)$	3.08	1.98	3.18	4.17	5.16	2.83	2.60
$\sigma(g_d)$	18.97	8.85	11.65	10.27	7.72	8.71	6.42
$E(r_m - r_f)$	1.95	4.07	5.34	4.66	4.99	5.82	5.38
$E(r_f)$	1.73	2.29	2.34	1.28	1.54	1.94	2.18
$\sigma(r_m)$	25.22	14.02	17.99	16.10	13.06	15.76	12.91
$\sigma(r_f)$	0.62	0.62	0.62	0.62	0.62	0.62	0.62
C. Model Moments: Individual LRR							
$\sigma(g_c)$	3.35	2.11	3.43	4.31	4.90	2.88	2.21
$\sigma(g_d)$	15.35	10.07	13.77	12.38	10.14	11.14	7.83
$E(r_m - r_f)$	2.14	3.48	6.77	4.64	7.50	7.23	6.40
$E(r_f)$	1.61	2.17	1.98	0.90	1.45	1.58	2.18
$\sigma(r_m)$	21.22	15.53	20.31	19.05	16.58	18.30	14.77
$\sigma(r_f)$	0.49	0.47	0.71	0.53	0.73	0.68	0.56

^aData moments as previously shown in Table 5 and SMM procedure described in Solution Method

Table 8: Log Consumption Growth Correlations

	AUS	CAN	FRA	GER	JAP	UK	US	World Eq ^a	World Pop ^b
AUS	1.000	0.165	-0.027	-0.111	0.055	0.089	0.046	0.388	0.102
CAN	0.165	1.000	0.147	-0.256	0.007	0.530	0.608	0.484	0.536
FRA	-0.027	0.147	1.000	0.050	0.159	0.252	0.216	0.642	0.532
GER	-0.111	-0.256	0.050	1.000	0.058	-0.259	-0.188	0.285	0.171
JPN	0.055	0.007	0.159	0.058	1.000	0.117	-0.145	0.415	0.316
UK	0.089	0.530	0.252	-0.259	0.117	1.000	0.593	0.530	0.620
US	0.046	0.608	0.216	-0.188	-0.145	0.593	1.000	0.460	0.790

^aWorld is Equally weighted

^bWorld is Population weighted, where Population data is from PWT

^cSource: PWT 1950-2000

Table 9: Multi-Country Welfare Gains

A: Common LRR						
	Eq Wtd Gains	Reserve Share	Ranking	Gains	Pr Wtd Share ^d	Pr Wtd Gains ^d
AUS	98.1%	0.50	7	0.0%	n/a	n/a
CAN	86.7%	0.54	5	0.0%	n/a	n/a
FRA	1.3%	0.99	2	0.0%	n/a	n/a
GER	81.3%	0.55	4	0.0%	n/a	n/a
JPN	-46.2%	1.86	1	226.4%	n/a	n/a
UK	87.6%	0.53	6	0.0%	n/a	n/a
US	60.5%	0.62	3	0.0%	n/a	n/a
B: Individual LRR						
	Eq Wtd Gains	Reserve Share	Ranking	Gains	Pr Wtd Share ^d	Pr Wtd Gains ^d
AUS	364.8%	0.22	3	0.0%	n/a	n/a
CAN	211.0%	0.32	1	1843.4%	n/a	n/a
FRA	440.1%	0.19	4	0.0%	n/a	n/a
GER	214.4%	0.32	2	0.0%	n/a	n/a
JPN	1382.9%	0.07	5	0.0%	n/a	n/a
UK	4646.1%	0.02	6	0.0%	n/a	n/a
US	7039.1%	0.01	7	0.0%	n/a	n/a

^aPreference parameters: $\beta = 0.987$, $\psi = 1.5$, $\gamma = 10$

^bModel parameters: Estimated model parameters for Individual LR in Table 6

^cCorrelation matrix from Table 8 on η_t^j and identity correlation matrix on e_t^j

^dTo Be Completed

Figure 1: Certainty Equivalent Consumption - Symmetric

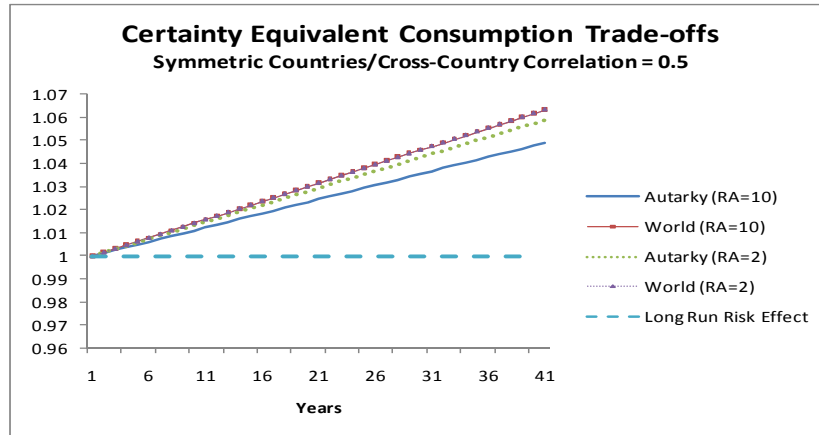


Figure 2: Certainty Equivalent Consumption - Different Sigma

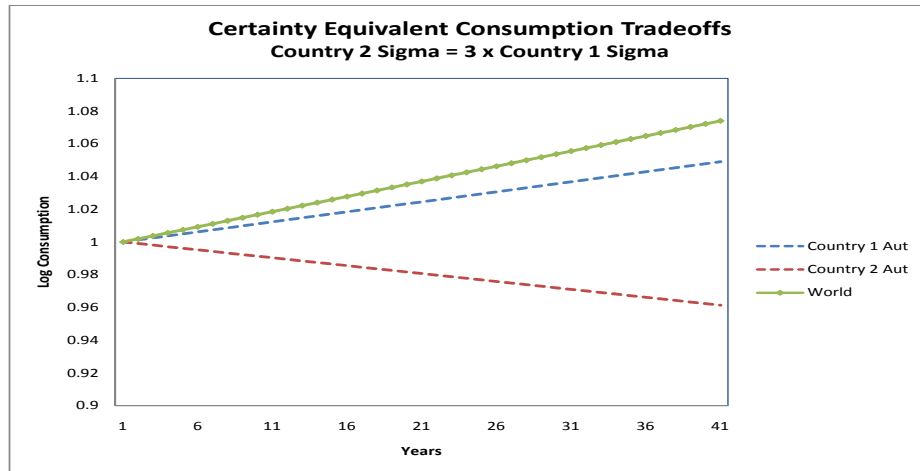
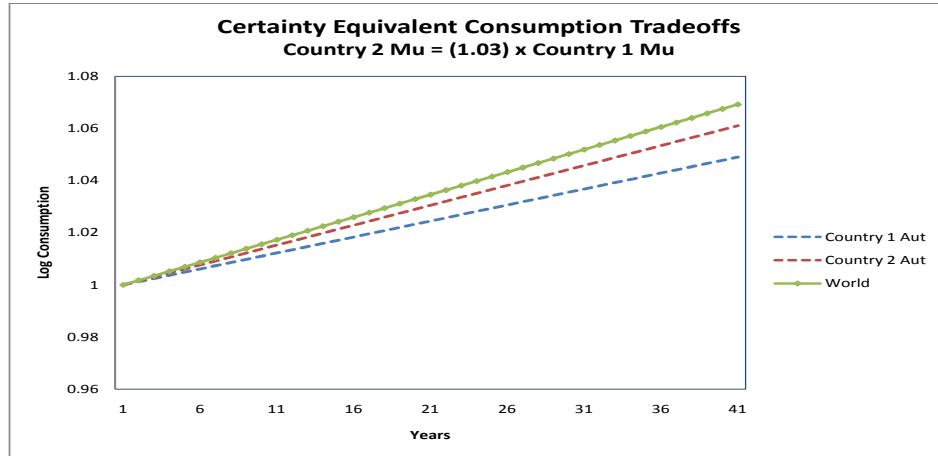


Figure 3: Certainty Equivalent Consumption - Different Mu



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A The Social Planner's Problem

Given the Epstein-Zin-Weil utility, the first order conditions of the planners problem are:

$$\begin{aligned} & \lambda^\ell \left\{ U_1^\ell(C^\ell(S_t), E[U^\ell(C^\ell(S_{t+1})|I_t)]) + U_2^\ell(C^\ell(S_t), E[U^\ell(C^\ell(S_{t+1})|I_t)]) \frac{\partial E[U^\ell(C^\ell(S_{t+1})|I_t)]}{\partial C^\ell(S_t)} \right\} \\ = & \lambda^i \left\{ U_1^i(C^i(S_t), E[U(C^i(S_{t+1})|I_t)]) + U_2^i(C^i(S_t), E[U^i(C^i(S_{t+1})|I_t)]) \frac{\partial E[U^i(C^i(S_{t+1})|I_t)]}{\partial C^i(S_t)} \right\} \end{aligned}$$

where U_n is the partial derivative of U with respect to the n th argument. Thus, the social planner equalizes marginal utilities across states including the effects of consumption on future expected utility.

Under CRRA preferences, utility is time separable. In this case, $U_2 = 0$ and the first order conditions become:

$$\lambda^\ell U_1^\ell(C^\ell(S_t), E[U^\ell(C^\ell(S_{t+1})|I_t)]) = \lambda^i U_1^i(C^i(S_t), E[U^i(C^i(S_{t+1})|I_t)]) \quad (50)$$

Or,

$$\lambda^\ell C^\ell(S_t)^{-\gamma} = \lambda^i C^i(S_t)^{-\gamma} \quad (51)$$

which does not depend upon the recursive next period utility. Thus, under CRRA, marginal period utility is equalized across states whereas under Epstein Zin Weil preferences, marginal period utility relative to marginal recursive next period utility is equalized across countries and states.

Note that if the country is in autarky, $C_t^j(S_t) = Y_t^j(S_t)$ for all countries j . In this case with CRRA preferences consumption clearly does not solve the social planner's problem unless $Y_t^i(S_t) = Y_t^j(S_t) \forall S_t, t$, which occurs with probability zero.

B Wealth to Consumption

By the identity $W_t = C_t + P_t$, we can write the wealth to consumption ratio in terms of the price to consumption ratio, $\frac{W_t}{C_t} = 1 + \frac{P_t}{C_t} = 1 + Z_{c,t}$ (note in the Campbell-Shiller approximation, we approximate for $z_{c,t} = \log(Z_{c,t})$). Then we can re-write the value function in terms of price to

consumption ratio, $\frac{C_t}{W_t} = (1 + Z_{c,t})^{-1}$:

$$\begin{aligned}
V_t(C_t, W_t) &= [(1 - \delta)^{-\psi} (\frac{C_t}{W_t})]^{\frac{1}{1-\psi}} W_t \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (C_t)^{\frac{1}{1-\psi}} (\frac{1}{W_t})^{\frac{\psi}{1-\psi}} \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (C_t)^{\frac{1}{1-\psi}} (\frac{1}{W_t})^{\frac{\psi}{1-\psi}} C_t^{-1} C_t \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (\frac{C_t}{W_t})^{\frac{\psi}{1-\psi}} C_t \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (1 + Z_{c,t})^{\frac{\psi}{\psi-1}} C_t
\end{aligned}$$

C Closed Economy Asset Prices with Long Run Risk

In the closed economy, each country j has a representative agent and is endowed with the stochastic consumption growth processes in equation (1). In this appendix, we detail how we calculate the price of consumption in the closed economy. For simplicity, we here consider the case when long run risk is common across countries and is given by \bar{x} .

First, we use the Campbell-Shiller decomposition to express returns in terms of price to consumption ratio, with approximating constants k_0^j and k_1^j ³¹:

$$r_{c,t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j \quad (52)$$

Following Bansal and Yaron 2004, we solve for equilibrium log price to consumption ratio by conjecturing that the log price to consumption ratio is linear in the source of long run risk, $z_t^j = A_0^j + A_1^j \bar{x}_t$. Given the Campbell-Shiller decomposition, it is sufficient solve for the coefficients A_0^j and A_1^j in order to solve for the equilibrium returns.

Using the first order condition derived earlier in equation (13) and applying the properties of log normality of return, $r_{c,t+1}^j$, and consumption growth, $g_{c,t+1}^j$, we have:

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] = 0 \quad (53)$$

Now we substitute in the return decomposition and the stochastic processes for log consumption growth, $g_{c,t+1}^j$, log price to consumption ratios, z_{t+1}^j , and the long run risk, \bar{x}_{t+1} , into the above

³¹ Approximation constants are defined to be $k_1^j = \frac{\exp(\bar{z}^j)}{1 + \exp(\bar{z}^j)}$ and $k_0^j = \log(1 + \exp(\bar{z}^j)) - k_1^j \bar{z}^j$, where \bar{z}^j is the steady state log price to consumption ratio in the close economy.

Euler condition. Then taking conditional expectation and conditional variances³², the left hand side of the equation (53) becomes the sum of a constant term and an \bar{x}_t term³³.

$$[\theta(\ln \delta + (1 - \frac{1}{\psi})\mu_c^j + k_0^j + (k_1^j - 1)A_0^j + \frac{1}{2}\theta((k_1 A_1 \varphi_e)^2 + (1 - \frac{1}{\psi})^2\sigma^2)] + [\theta(k_1^j A_1^j \rho - A_1^j + 1 - \frac{1}{\psi})\bar{x}_t] = 0 \quad (54)$$

As we can see from equation (54), the left hand side is the sum of a constant term and an \bar{x}_t term. Since the sum of the two terms must be zero, each of the terms should be zero. This requirement gives us a system of two equations and two unknowns. Solving the system of equations gives us the following analytical form for the coefficients of the log price to consumption ratio:

$$A_1^j = \frac{1 - \frac{1}{\psi}}{1 - k_1^j \rho} \quad (55)$$

$$A_0^j = \frac{\ln \delta + k_0^j + (1 - \frac{1}{\psi})\mu_c^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)}{1 - k_1^j} \quad (56)$$

We now detail the steps required to get these solutions. For this purpose, we rewrite the dynamics of the long run risk, \bar{x}_{t+1} , and country j's log consumption growth process, $g_{c,t+1}^j$ from equations (1) as follows:

$$\begin{aligned} g_{c,t+1}^j &= \mu^j + \bar{x}_t + \sigma^j \eta_{t+1}^j \\ \bar{x}_{t+1} &= \rho \bar{x}_t + \sigma \varphi_e e_{t+1} \end{aligned}$$

To solve for prices, we first conjecture that the closed economy log price to consumption ratio of the country j's consumption claim asset is linear in the common long run risk: $z_t^j = A_0^j + A_1^j \bar{x}_t$

Next, using the Campbell & Shiller approximation and substituting in the stochastic processes for z_t^j , z_{t+1}^j , and \bar{x}_{t+1} we have:

$$\begin{aligned} r_{t+1}^j &= k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j \\ &= k_0^j + k_1^j (A_0^j + A_1^j \bar{x}_{t+1}) - (A_0^j + A_1^j \bar{x}_t) + g_{c,t+1}^j \\ &= k_0^j + k_1^j A_0^j + k_1^j A_1^j (\rho \bar{x}_t + \sigma \varphi_e e_{t+1}) - A_0^j - A_1^j \bar{x}_t + g_{c,t+1}^j \\ &= [k_0^j + k_1^j A_0^j - A_0^j] + (k_1^j A_1^j \rho - A_1^j) \bar{x}_t + k_1^j A_1^j \sigma \varphi_e e_{t+1} + g_{c,t+1}^j \end{aligned}$$

From the Euler equation and exploiting the log-normality of the pricing kernel and the returns, we can have:

$$\exp[E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] = 1$$

³²Note conditional expectation of shocks are zero and conditional variance of shocks are equal to one

³³For a detailed derivations of closed economy prices see Appendix

or equivalently,

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] = 0 \quad (57)$$

Now substituting the return decomposition into equation (57), we have that:

$$\begin{aligned} & E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] \\ &= E_t[\theta \ln \delta + (\theta - \frac{\theta}{\psi}) g_{c,t+1}^j + \theta[k_0^j + k_1^j A_0^j - A_0^j] + \theta(k_1^j A_1^j \rho - A_1^j) \bar{x}_t + \theta k_1^j A_1^j \sigma \varphi_e e_{t+1}] \\ &= \theta \ln \delta + (\theta - \frac{\theta}{\psi}) E_t[g_{c,t+1}^j] + \theta[k_0^j + k_1^j A_0^j - A_0^j] + \theta(k_1^j A_1^j \rho - A_1^j) \bar{x}_t \\ &= \theta[\ln \delta + k_0^j + k_1^j A_0^j - A_0^j + (1 - \frac{1}{\psi}) \mu^j] + \theta[A_1^j(k_1^j \rho - 1) + (1 - \frac{1}{\psi})] \bar{x}_t \end{aligned}$$

Where we used the fact that $E_t[e_{t+1}] = E_t[\eta_{t+1}^j] = 0$

And,

$$\begin{aligned} & \frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] = \\ &= \frac{1}{2} Var_t[\theta(1 - \frac{1}{\psi}) g_{c,t+1}^j + \theta k_1^j A_1^j \sigma \varphi_e e_{t+1}] = \\ &= \frac{1}{2} Var_t[\theta(1 - \frac{1}{\psi}) \sigma^j \eta_{t+1}^j] + \frac{1}{2} Var_t[\theta k_1^j A_1^j \sigma \varphi_e e_{t+1}] = \\ &= \frac{1}{2} (\theta(1 - \frac{1}{\psi}) \sigma^j)^2 Var_t[\eta_{t+1}^j] + \frac{1}{2} (\theta k_1^j A_1^j \sigma \varphi_e)^2 Var_t[e_{t+1}] = \\ &= \frac{1}{2} [(\theta(1 - \frac{1}{\psi}) \sigma^j)^2 + (\theta k_1^j A_1^j \sigma \varphi_e)^2] = \\ &= \frac{1}{2} \theta^2 [(1 - \frac{1}{\psi})^2 (\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2] = \end{aligned}$$

Note, we used the fact that $Var_t(e_{t+1}) = Var_t(\eta_{t+1}^i) = 1$

Combining the above derivations, we can re-write the left hand side of the equation (57) in terms of a constant term and an \bar{x}_t term.

$$\begin{aligned} & E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] + \frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] = \\ &= \theta[\ln \delta + k_0^j + k_1^j A_0^j - A_0^j + (1 - \frac{1}{\psi}) \mu^j + \frac{1}{2} \theta((1 - \frac{1}{\psi})^2 (\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)] \\ &+ \theta[A_1^j(k_1^j \rho - 1) + (1 - \frac{1}{\psi})] \bar{x}_t \\ &= 0 \end{aligned}$$

Since the sum of the constant term and the \bar{x}_t is zero, then both terms must be zero. This gives us a system of two equations and two unknowns, A_0^j and A_1^j .

1. To solve for A_1^j , we set \bar{x}_t term to zero:

$$\begin{aligned} & \theta[A_1^j(k_1^j \rho - 1) + (1 - \frac{1}{\psi})] \bar{x}_t = 0 \\ & A_1^j = \frac{(1 - \frac{1}{\psi})}{1 - k_1^j \rho} \end{aligned}$$

2. From the constant term, we can solve for A_0^j :

$$\theta[\ln \delta + k_0^j + k_1^j A_0^j - A_0^j + (1 - \frac{1}{\psi})\mu^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)] = 0$$

$$A_0^j = \frac{\ln \delta + k_0^j + (1 - \frac{1}{\psi})\mu^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)}{1 - k_1^j}$$

D Pricing Open Economy Returns

In this appendix, we detail the open economy equilibrium.

D.1 Open Economy Asset Prices with Long Run Risk

When markets open, all assets are priced with a common pricing kernel. This stochastic discount factor is determined by the intertemporal optimization problem of the Representative Agent and is a function of the world log consumption growth, g_t^w . We define the world consumption growth as a weighted average of the individual consumption growths with weights, a^j , for each country.

$$g_{c,t}^w = \sum_{j=1}^J a_j * g_{c,t}^j$$

$$= \bar{\mu} + \bar{x}_t + \bar{u}_{t+1}$$

where $\bar{\mu}_{t+1} = \sum_{j=1}^J a_j \mu^j$, $\bar{u}_{t+1} = \sum_{j=1}^J a_j \sigma^j \eta_{t+1}^j$, $\sum_{j=1}^J a_j = 1$. While a_j should adjust for the population size of country j 's, for now, we will assume that all countries are equally weighted.

Similar to the steps used in the closed economy solution, we can approximate open economy returns in terms of open economy price to consumption ratio using the Campbell-Shiller decomposition. But now the approximating constants are defined by $k_1^w = \frac{\exp(\bar{z}^w)}{1 + \exp(\bar{z}^w)}$ and $k_0^w = \log(1 + \exp(\bar{z}^w)) - k_1^w \bar{z}_j^w$, where \bar{z}^w is the steady state open economy log price to world consumption ratio.

$$r_{c,t+1}^w = k_0^w + k_1^w z_{t+1}^w - z_t^w + g_{c,t+1}^w \quad (58)$$

As shown in the previous sections, in the open economy returns for each country j 's consumption asset will be priced with the first order condition in equation (23), and applying the normality of log returns and log consumption growth we have:

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j)] = 0 \quad (59)$$

In particular, the above equation holds for the return on the world portfolio in the open economy, $r_{c,t+1}^w$, which is needed for the pricing of all contingent claims. To price the return on the world portfolio, (59) becomes:

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] = 0 \quad (60)$$

Following similar steps as in the closed economy, we again conjecture that the world log price to consumption ratio is linear in the long run risk, $z_t^w = A_0^w + A_1^w \bar{x}_t$, and solve for the coefficients A_0^w and A_1^w . Like the closed economy solutions, after substituting the stochastic processes for $g_{c,t+1}^w$, \bar{x}_{t+1} , and r_{t+1}^w , the left hand side of equation (60) will have a constant term and an \bar{x}_t term. We again set the constant term to zero and the \bar{x}_t term to zero, and solve the system of two equations³⁴:

$$A_1^w = \frac{1 - \frac{1}{\psi}}{1 - k_1^w \rho} \quad (61)$$

$$A_0^w = \frac{\ln \delta + k_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2}{1 - k_1^w} \quad (62)$$

Having solved the return on the world portfolio, we can price the open economy return on country j's consumption claim. We approximate open economy returns with the price to consumption ratio, $\bar{r}_{c,t+1}^j$, using the Campbell-Shiller decomposition. But now the approximating constants, \bar{k}_0^j and \bar{k}_1^j , are defined by the open economy steady state log price to consumption ratio, \bar{z}^j .³⁵

$$\bar{r}_{c,t+1}^j = \bar{k}_0^j + \bar{k}_1^j \bar{z}_{t+1}^j - \bar{z}_t^j + g_{c,t+1}^j \quad (63)$$

We assume that country j's open economy log price to consumption ratio, \bar{z}_t^j , is linear in the long run risk $\bar{z}_t^j = \bar{A}_0^j + \bar{A}_1^j \bar{x}_t$. To solve for the coefficients, \bar{A}_0^j and \bar{A}_1^j , we substitute the defined processes for $g_{c,t+1}^j$, \bar{x}_{t+1} , and $g_{c,t+1}^j$ into equation (59). Then we simplify until the left hand side of (59) has only a constant term and an \bar{x}_t term, which again by earlier reasoning gives us a system of two equations for the two unknowns. As shown more completely in Appendix C, the coefficients \bar{A}_0^j and \bar{A}_1^j are as follows:

$$\bar{A}_1^j = \frac{1 - \frac{1}{\psi}}{1 - \bar{k}_1^j \rho} \quad (64)$$

$$\bar{A}_0^j = \frac{(\dots) + \frac{1}{2}[(\sigma^j)^2 + ((\theta - 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j)^2 \sigma^2 \varphi_e^2 + (\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}]]}{1 - \bar{k}_1^j} \quad (65)$$

where $(\dots) = \theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + \bar{k}_0^j + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j$ and $Var_t[\bar{u}_{t+1}] = Var_t[\sum_{i=1}^N a_i \sigma^i \eta_{t+1}^i]$, the variance-covariance matrix of the idiosyncratic component of log.

We now detail how the solutions to the \bar{A}_0^j, \bar{A}_1^j are obtained.

³⁴See detailed solution in appendix

³⁵Approximation constants are defined to be $\bar{k}_1^j = \frac{\exp(\bar{z}^j)}{1 + \exp(\bar{z}^j)}$ and $\bar{k}_0^j = \log(1 + \exp(\bar{z}^j)) - \bar{k}_1^j \bar{z}^j$

D.2 Price of World Consumption Claims In Open Market

To price return in the open economy, we begin with the pricing of the "world" consumption claim. We conjecture that the log price to consumption ratio of the world consumption claim asset is linear in the common long run risk: $z_t^w = A_0^w + A_1^w \bar{x}_t$

Again we approximate returns using Campbell-Shiller decomposition and substitute in the stochastic processes for z_t^w , z_{t+1}^w , and \bar{x}_{t+1} :

$$\begin{aligned}
r_{t+1}^w &= k_0^w + k_1^w z_{t+1}^w - z_t^w + g_{c,t+1}^w \\
&= k_0^w + k_1^w (A_0^w + A_1^w \bar{x}_{t+1}) - (A_0^w + A_1^w \bar{x}_t) + g_{c,t+1}^w \\
&= k_0^w + k_1^w A_0^w + k_1^w A_1^w \bar{x}_{t+1} - A_0^w - A_1^w \bar{x}_t + g_{c,t+1}^w \\
&= (k_0^w + k_1^w A_0^w - A_0^w) + k_1^w A_1^w (\rho \bar{x}_t + \sigma \varphi_e e_{t+1}) - A_1^w \bar{x}_t + g_{c,t+1}^w \\
&= (k_0^w + k_1^w A_0^w - A_0^w) + k_1^w A_1^w \rho \bar{x}_t - A_1^w \bar{x}_t + k_1^w A_1^w \sigma \varphi_e e_{t+1} + g_{c,t+1}^w
\end{aligned}$$

From the Euler equation and exploiting the log-normality of the pricing kernel and the returns, we have:

$$\exp[E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] = 1$$

or equivalently,

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] = 0 \quad (66)$$

Now substituting the return decomposition into equation (66), we have that:

$$\begin{aligned}
E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] &= \\
&= \theta \ln \delta + (\theta - \frac{\theta}{\psi}) E_t(g_{c,t+1}^w) + \theta (k_0^w + k_1^w A_0^w - A_0^w) + \theta A_1^w (k_1^w \rho - 1) \bar{x}_t + \theta k_1^w A_1^w \sigma \varphi_e E_t(e_{t+1}) \\
&= \theta (\ln \delta + k_0^w + k_1^w A_0^w - A_0^w + (1 - \frac{1}{\psi}) \bar{\mu}) + \theta (1 - \frac{1}{\psi} + A_1^w (k_1^w \rho - 1)) \bar{x}_t
\end{aligned}$$

And,

$$\begin{aligned}
\frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] &= \\
&= \frac{1}{2} Var_t[-\frac{\theta}{\psi} g_{c,t+1}^w + \theta ((k_0^w + k_1^w A_0^w - A_0^w) + k_1^w A_1^w \rho \bar{x}_t - A_1^w \bar{x}_t + k_1^w A_1^w \sigma \varphi_e e_{t+1}) + g_{c,t+1}^w] = \\
&= \frac{1}{2} Var_t[\theta (1 - \frac{1}{\psi}) g_{c,t+1}^w + \theta k_1^w A_1^w \sigma \varphi_e e_{t+1}] = \\
&= \frac{1}{2} Var_t[\theta (1 - \frac{1}{\psi}) (\bar{\mu} + \bar{x}_t + \bar{u}_{t+1}) + \theta k_1^w A_1^w \sigma \varphi_e e_{t+1}] = \\
&= \frac{1}{2} \theta^2 (1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2} \theta^2 (k_1^w A_1^w \sigma \varphi_e)^2
\end{aligned}$$

Note: $Var_t(e_{t+1}) = Var_t(\eta_{t+1}^i) = 1$ and $Var_t[\bar{u}_{t+1}] = Var_t[\sum_{i=1}^N a_i \sigma^i \eta_{t+1}^i]$

Combining the above derivations, we re-write the left hand side of the equation (66) in terms of a constant term and an \bar{x}_t term.

$$\begin{aligned}
& E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] + \frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] = \\
& = [\theta(\ln \delta + k_0^w + k_1^w A_0^w - A_0^w + (1 - \frac{1}{\psi})\bar{\mu}) + \frac{1}{2}\theta^2(1 - \frac{1}{\psi})^2 Var_t(\bar{u}_{t+1}) + \frac{1}{2}\theta^2(k_1^w A_1^w \sigma \varphi_e)^2] \\
& + [\theta(1 - \frac{1}{\psi} + A_1^w(k_1^w \rho - 1))]\bar{x}_t \\
& = 0
\end{aligned}$$

Since the sum of the constant term and the \bar{x}_t is zero, then both terms must be zero. This gives us a system of two equations and two unknowns, A_0^w and A_1^w .

1. To solve for A_1^w , we set \bar{x}_t term to zero:

$$\begin{aligned}
[1 - \frac{1}{\psi} + A_1^w(k_1^w \rho - 1)]\bar{x}_t &= 0 \\
A_1^w &= \frac{1 - \frac{1}{\psi}}{1 - k_1^w \rho}
\end{aligned}$$

2. From the constant term, we can solve for A_0^w :

$$\begin{aligned}
\ln \delta + k_0^w + k_1^w A_0^w - A_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2 &= 0 \\
\ln \delta + k_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2 &= (1 - k_1^w)A_0^w \\
A_0^w &= \frac{\ln \delta + k_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2}{1 - k_1^w}
\end{aligned}$$

D.3 Price of Country Contingent Claims In Open Market

Again, we conjecture that open economy log price to consumption ratio for country j is linear in the long run risk: $\bar{z}_t^j = \bar{A}_0^j + \bar{A}_1^j \bar{x}_t$

Then by Campbell & Shiller approximation:

$$\begin{aligned}
\bar{r}_{c,t+1}^j &= \bar{k}_0^j + \bar{k}_1^j \bar{z}_{t+1}^j - \bar{z}_t^j + g_{c,t+1}^j \\
&= \bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j + \bar{k}_1^j \bar{A}_1^j \bar{x}_{t+1} - \bar{A}_0^j - \bar{A}_1^j \bar{x}_t + g_{c,t+1}^j \\
&= (\bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j - \bar{A}_0^j) + g_{c,t+1}^j + \bar{A}_1^j (\bar{k}_1^j \rho - 1) \bar{x}_t + \bar{k}_1^j \bar{A}_1^j \sigma \varphi_e e_{t+1}
\end{aligned}$$

Using the Euler equation and exploiting the log-normality of the pricing kernel and the returns, we have:

$$\exp[E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j)] = 1$$

or equivalently,

$$E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) = 0 \quad (67)$$

Now substituting the return decomposition into equation (67), we have that:

$$\begin{aligned} E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j] \\ = [\theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + (\bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j - \bar{A}_0^j)] + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j + [(\theta - \frac{\theta}{\psi}) \\ + (\theta - 1)A_1^w(k_1^w \rho - 1) + \bar{A}_1^j(\bar{k}_1^j \rho - 1)]\bar{x}_t \end{aligned}$$

And:

$$\begin{aligned} \frac{1}{2}Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) = \\ = \frac{1}{2}Var_t[\sigma^j \eta_{t+1}^j] + \frac{1}{2}Var_t[(\theta - 1 - \frac{\theta}{\psi})\bar{u}_{t+1}] + \frac{1}{2}Var_t\{[(\theta - 1)k_1^w A_1^w + k_1^j \bar{A}_1^j]\sigma \varphi_e e_{t+1}\} = \\ = \frac{1}{2}(\sigma^j)^2 + \frac{1}{2}[(\theta - 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j]^2 \sigma^2 \varphi_e^2 + \frac{1}{2}(\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}] \end{aligned}$$

Following the same logic as before, we solve for the coefficients \bar{A}_0^j and \bar{A}_1^j by setting the \bar{x}_t term to zero and the constant term to zero

1. We solve \bar{A}_1^j by setting \bar{x}_t term to zero:

$$\begin{aligned} [(\theta - \frac{\theta}{\psi}) + (\theta - 1)A_1^w(k_1^w \rho - 1) + \bar{A}_1^j(\bar{k}_1^j \rho - 1)]\bar{x}_t = 0 \\ (\theta - \frac{\theta}{\psi}) + (\theta - 1)A_1^w(k_1^w \rho - 1) = \bar{A}_1^j(1 - \bar{k}_1^j \rho) \end{aligned}$$

But from above we have $(1 - k_1^w \rho)A_1^w = 1 - \frac{1}{\psi}$.

$$\bar{A}_1^j = \frac{1 - \frac{1}{\psi}}{1 - \bar{k}_1^j \rho}$$

2. Solve for \bar{A}_0^j by setting the constant term to zero:

$$\begin{aligned} \theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + (\bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j - \bar{A}_0^j) + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j + \frac{1}{2}(\sigma^j)^2 + \frac{1}{2}[(\theta - \\ 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j]^2 \sigma^2 \varphi_e^2 + \frac{1}{2}(\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}] = 0 \\ \bar{A}_0^j = \frac{\theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + \bar{k}_0^j + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j + \frac{1}{2}(\sigma^j)^2 + \frac{1}{2}[(\theta - 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j]^2 \sigma^2 \varphi_e^2 + \frac{1}{2}(\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}]}{1 - \bar{k}_1^j} \end{aligned}$$

Where A_0^w and A_1^w are defined above.

D.4 Data Description

Consumption: As described in the text, we are interested in matching the US PWT consumption data implications with those of Bansal-Yaron (2004) based upon the National Income and Product Account (NIPA) data from the Bureau of Economic Analysis. Therefore, for every country except the US, we construct real per-capita consumption by taking total consumption at 1996 constant prices (CKON) and dividing it by the population (POP). For the US, we compare the PWT estimates to those reported by the NIPA estimates. Since the NIPA data give a finer breakdown of personal expenditures, we construct real per-capita consumption by using personal consumption

on Non-Durables and Services chained to 2000 dollars from NIPA table 7.1 for the sample period 1950-2000.

Dividends: We obtain the data described in Campbell (2003) from John Campbell's website. In order to make real dividend growth consistent with real consumption growth, we use the same deflator series from the Penn World Tables to adjust nominal dividends into real terms. Because the Penn World Table only gives annual series, we summed the quarterly dividends from the Campbell data to construct annual nominal dividends, then deflate by the PWT annual consumption deflator. Note that there is a difference between our annualized moments compared to Table 3 of Campbell (2003). To be consistent with time-aggregation as emphasized in long run risk, we annualize by summing the quarterly dividends. By contrast, Campbell (2003) takes average quarterly moments and annualizes by multiplying means by 400 and standard deviations by 200.

Asset Returns: As in the case of dividends, we obtain the data from John Campbell's website. To form consistent real annual equity returns and risk free rates, we aggregate from quarterly to annual returns. All quarterly nominal rates of return are adjusted using quarterly CPI included in the Campbell data. For annual real returns in percentages, we follow the convention in Campbell (2003) and multiply means by 400 and standard deviations by 200. Our numbers closely match Campbell 1999, with only slight variations due to increased sample size. We compute quarterly equity premium and annualize in the same way as the other rates of return. Similarly we aggregate quarterly dividends and use the equity price for the fourth quarter of the year to construct annual P/D ratios.