

# Financial Innovation, the Discovery of Risk, and the U.S. Credit Crisis\*

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## Abstract

Financial innovation and uncertainty about the true riskiness of new financial products were important factors behind the U.S. financial crisis. We show that a sharp boom-bust cycle in credit and macroeconomic aggregates characterizes the dynamics that follow the introduction of debt securitization, in a setup in which agents learn about the true riskiness of the new financial environment over time. Early positive outcomes that feature high ability to securitize debt lead agents to turn overly optimistic about the probability of persistence of the regime with high securitization. In response, they borrow and increase leverage more than they would under rational expectations based on knowledge of the true probabilities of switching across high and low securitization regimes. When agents first observe that their ability to securitize debt can actually decline, they respond with sharp downward adjustments in credit and consumption. Over-borrowing generated by optimism accounts for roughly one quarter of the secular decline in net credit assets of U.S. nonfinancial sectors observed between 1983-2006.

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# 1 Introduction

A key factor behind the U.S. financial crisis was the large expansion of credit and leverage fuelled by new financial instruments that allowed the securitization of payment streams generated by a wide variety of assets. This process started with the gradual introduction of collateralized debt obligations (CDOs) in the 1980s, but became significantly more important during the past decade with the introduction of collateralized mortgage obligations (CMOs) and insurance contracts on the payments of CDOs and CMOs known as credit default swaps (CDSs). In addition, “synthetic securitization” allowed third parties to trade these securities as bets on the corresponding income streams, without being a party in the actual underlying loan contracts. By the end of 2007, the market of CDSs was worth about \$45 trillion (or 3 times U.S. GDP).

Two factors that are widely believed to have played important roles in the securitization boom and the mispricing of risk that it reflected are (a) the lack of data on the default and performance records of the new financial instruments; and (b) the “layering of risk” that justified the belief that the instruments were so well diversified that they were virtually risk free. The latter was presumably being attained by using portfolio models to combine top tranches of AAA-rated assets with tranches containing more risky assets. As Drew (2008) described it: *“The computer modelers gushed about the tranches. The layers spread out the risk. Only a catastrophic failure would bring the structure crashing down, and the models said that wouldn’t happen.”*

We examine in this paper the macroeconomic implications of a process of financial innovation affected by the above factors. In particular, we propose a model in which the true riskiness of a new financial environment can only be discovered with time, and financial innovation begins with a period of optimism that leads agents to underestimate the possibility of a financial crash. In this model, agents face a credit constraint that limits their debt not to exceed a fraction of income. Financial innovation is modeled as a structural reform that increases this fraction, thus moving the economy to a “high securitization” state. Agents know that two financial regimes are possible: one in which high securitization continues, and one in which there is a switch to a regime in which their debt is limited to a lower fraction of income (i.e. “low securitization”). They do not know the true riskiness of the financial environment, because they do not know the true regime-switching probabilities across high and low securitization states. They are Bayesian learners, however, and so they learn over time as they observe regime realizations, and in the long run their beliefs converge to the true regime-switching probabilities. Hence, in the long run the model converges to the rational

expectations (RE, henceforth) solution, but in the short run optimal plans deviate sharply from the rational expectations equilibrium, and these deviations have important quantitative effects on macroeconomic aggregates.

The priors that agents start with when financial innovation occurs, and the outcomes they observe in the first periods that follow, matter significantly for macroeconomic dynamics. In particular, if the priors are not overly pessimistic and the early realizations are consistent with a regime of high securitization, agents become overly optimistic. In this “optimistic phase,” they borrow and increase their leverage (i.e. their debt/income ratios) significantly above what the RE equilibrium predicts. That is, the economy experiences a credit boom. Conversely, when agents observe the first realization of the low securitization regime after the optimistic phase, they respond with sharp downward adjustments in credit and consumption. Thus, the process of discovery of true financial risk produces a credit boom followed by a bust.

In this setup, the degree of optimism generated in the optimistic phase is at its highest just before agents observe the first realization of the low securitization regime. This occurs because, when the new financial environment with high and low securitization regimes is first introduced, agents cannot rule out the possibility of the high securitization regime being absorbent until they experience the first realization of the low securitization state.

The credit constraint used in our model to represent debt securitization is similar to those widely examined in the macro literature on financial frictions and in the open-economy macro literature on Sudden Stops. When these constraints are used in stochastic environments with rational expectations, precautionary savings reduce significantly the probability of long-run equilibrium outcomes in which the constraints are binding (see Mendoza (2008) and Durdu, Mendoza and Terrones (2009)). In our learning model, however, agents have significantly weaker incentives for building precautionary savings than under rational expectations, until they attain the long run in which they have learned the true riskiness of the financial environment. In particular, since agents borrow too much during the optimistic phase described above, the economy is vulnerable to suffer a large credit crunch when the first switch to a regime with low securitization occurs.

Our modeling of imperfect information follows the approach proposed by Cogley and Sargent (2008). They offer an explanation of the equity premium puzzle by modeling a period of persistent pessimism caused by the Great Depression. Their model assumes high and low states for consumption growth, with the true transition probabilities across these states unknown. Agents learn the true probabilities over time as they observe (without noise) the realizations of consumption

growth. Similarly, in our setup, the true probabilities of switching across securitization regimes are unknown, and agents learn about them over time.

This paper is also related to two other recent strands of the macro-finance literature. One strand is the literature exploring the macroeconomic implications of learning. Most of this literature focuses on learning from noisy signals (see Blanchard et al. (2008), Boz (2009), Boz, Daude and Durdu (2008), Edge et al. (2007), Lorenzoni (2009), Nieuwerburgh and Veldkamp (2006) among others). The informational friction in these models typically stems from the imperfection of information about the decomposition of signals into a persistent component and a noise component. Our informational friction is fundamentally different in that we assume imperfect information regarding the true distribution of shocks, based on the observation that financial innovations are inherently new (thus implying lack of long time-series data from which to infer the true regime switching process of the securitization states).

The second strand of macro-finance literature that our work is related to is the literature on credit booms and over-borrowing. The stylized facts documented by Mendoza and Terrones (2008) show that credit booms have well-defined cyclical patterns, with the peak of credit booms preceded by periods of expansion in credit and economic activity followed by sharp contractions. In this regard, our model aims to explain both the boom and the bust phase of credit cycles, in contrast with the usual emphasis on credit frictions as a mechanism that amplifies downturns (as in most of the Sudden Stops literature). In this literature, another related study is Rancière and Tornell (2009) who explore a feedback effect between credit and prices in the context of the recent U.S. crisis.

Theoretical studies on over-borrowing like those by Uribe (2006), Korinek (2008) and Bianchi (2009) explore whether externalities induced by credit constraints can generate excessive borrowing in decentralized equilibria relative to the social optimum. Uribe (2006) shows examples of environments in which over-borrowing does not occur, and Korinek (2008) and Bianchi (2009) provide examples in which it does. Bianchi (2009) shows quantitatively that over-borrowing can range from moderate to negligible depending on the elasticity of substitution of demand for the goods relevant for determining the price at which income used as collateral is valued, relative to the unit of denomination of debt contracts. Our model generates sizable over-borrowing without externalities. The over-borrowing results from the overly optimistic expectations of agents during the optimistic phase of the learning dynamics.

The remainder of this paper proceeds as follows: Section 2 describes the model and the learning

process. Section 3 examines the model’s quantitative implications. Section 4 concludes.

## 2 A Model of Financial Innovation with Learning

We study a representative agent economy in which risk-averse individuals formulate optimal consumption and savings plans facing exogenous income volatility. The risk of income fluctuations cannot be fully diversified because asset markets are incomplete (individuals only have access to a single non-state contingent bond). The credit market is also imperfect because individuals’ ability to borrow is limited not to exceed a fraction  $\kappa$  of income.

The main feature that differentiates our model from other incomplete markets models with credit frictions is the assumption that agents have imperfect information about the regime-switching probabilities that drive fluctuations in  $\kappa$ .<sup>1</sup> We consider a situation in which financial innovation starts with an initial shift from a regime without debt securitization (or, more generally, a low  $\kappa$  value denoted  $\kappa^l$ ) to a regime with securitization (a high  $\kappa$  value  $\kappa^h$ ). Agents do not know the true regime-switching probabilities between  $\kappa^l$  and  $\kappa^h$  in this new financial environment. They are Bayesian learners, and hence in the long run they learn these true probabilities. In the short run, however, they form their expectations with the posteriors they construct as they observe actual realizations of  $\kappa$ . Hence, they “discover” the true riskiness of the new financial environment only after they have observed a sufficiently large sample of realizations of the two regimes.

We assume throughout that the risk-free interest rate is exogenous in order to keep the interaction between financial innovation and learning tractable. At the aggregate level, this assumption corresponds to an economy that is small and open with respect to world capital markets. This is in line with recent evidence suggesting that in the era of financial globalization even the U.S. risk-free rate has been significantly influenced by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998 (see Warnock and Warnock (2006), Bernanke (2005), Durdu et al. (2009), Mendoza et al. (2009)). Moreover, post-war data also show that, while pre-1980s the United States was in virtual financial autarky, because the fraction of net credit of U.S. nonfinancial sectors financed by the rest of the world was close to zero, about 1/2 of the surge in net credit since the mid-1980s was financed by the rest of the world (see Mendoza and Quadrini (2009)). Alternatively, our setup can be viewed

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<sup>1</sup>In work in progress we introduce also a debt-deflation mechanism that amplifies the effects of the learning dynamics. With this feature, the distortions produced by the learning process in the aftermath of financial innovation have an additional source to generate amplification by affecting the prices that determine agents’ incomes, and thereby feeding back on their ability to lever their incomes into debt.

as a partial equilibrium model of the U.S. economy that studies the effects of financial innovation taking the risk-free rate as given, as in Corbae and Quintin (2009).

## 2.1 Agents' Optimization Problem and Imperfect Information

Agents formulate optimal plans for consumption,  $c_t$ , so as to maximize the following intertemporal utility function:

$$E_0^s \left[ \sum_{t=0}^{\infty} \exp \left( - \sum_{\tau=0}^{t-1} \beta L n(1 + c_{\tau}) \right) \frac{c_t^{1-\sigma}}{1-\sigma} \right] \quad (1)$$

This utility function is in the Uzawa-Epstein class of preferences that support a well-defined long-run distribution of asset holdings under incomplete markets because the rate of time preference is endogenous. The period utility function has the standard constant-relative-risk-aversion (CRRA) form. The discount factor that applies to date- $t$  consumption is  $\exp \left( - \sum_{\tau=0}^{t-1} \beta L n(1 + c_{\tau}) \right)$ , where  $\beta$  is the semi-elasticity of the rate of time preference with respect to consumption. It is critical to note that  $E_t^s$  represents the expectations operator conditional on the representative agent's beliefs formulated with the information available up to and including date  $t$ . As we explain below, these beliefs will differ in general from the rational expectations formulated with perfect information about the stochastic processes of the shocks hitting the economy, which are denoted  $E_t^a$ .

Optimal plans must satisfy a standard budget constraint:

$$c_t = y_t - \frac{b_{t+1}}{R_t} + b_t. \quad (2)$$

The economy faces exogenous Markov shocks that can hit income  $y_t$  and the risk-free rate on one-period discount bonds  $R_t$  (i.e. the inverse of the gross risk-free interest rate). For these shocks, we can assume that agents know their true joint Markov process without informational frictions, or alternatively we can assume they are also affected by imperfect information. If agents know their true process, they know the Markov transition matrix  $\pi(z_{t+1}|z_t)$  of all pairs  $z = (y, R)$  and they know the corresponding vectors of realizations.

Credit markets are imperfect, so agents' optimal plans must satisfy the following credit constraint:

$$b_{t+1} \geq -\kappa_t y_t \quad (3)$$

where  $\kappa_t$  is a random variable that follows a "true" Markov process characterized by a standard two-point regime-switching process with regimes  $\kappa^h$  and  $\kappa^l$ , with  $\kappa^h > \kappa^l$ , and transition probabilities

given by  $F^a = p^a(\kappa_{t+1}|\kappa_t)$ . The continuation transition probabilities are denoted  $F_{hh}^a \equiv p^a(\kappa_{t+1} = \kappa^h|\kappa_t = \kappa^h)$  and  $F_{ll}^a \equiv p^a(\kappa_{t+1} = \kappa^l|\kappa_t = \kappa^l)$ , and hence the switching probabilities are  $F_{hl}^a = 1 - F_{hh}^a$  and  $F_{lh}^a = 1 - F_{ll}^a$ . The long run probability of the high securitization regime is  $\Pi^h = F_{lh}^a/(F_{lh}^a + F_{hl}^a)$  and that of the low securitization regime is  $\Pi^l = F_{hl}^a/(F_{lh}^a + F_{hl}^a)$ . The mean durations are  $1/F_{hl}^a$  and  $1/F_{lh}^a$  for the high and low securitization regimes respectively. The long-run unconditional moments of  $\kappa$  are the following:

$$E^a[\kappa] = (F_{lh}^a \kappa^h + F_{hl}^a \kappa^l)/(F_{lh}^a + F_{hl}^a) \quad (4)$$

$$VAR(\kappa) = \Pi^h(\kappa^h)^2 + \Pi^l(\kappa^l)^2 - (E[\kappa])^2 \quad (5)$$

$$AUTOCORR(\kappa) = F_{ll}^a - F_{hl}^a = F_{hh}^a - F_{lh}^a \quad (6)$$

Agents do not know  $p^a(\kappa_{t+1}|\kappa_t)$ , and instead make decisions based on their individual beliefs  $F^s = p_t^s(\kappa_{t+1}|\kappa_t)$ , which evolve over time. In the more general case in which a similar informational friction affects also  $z$ , the arguments of  $F^a$  and  $F^s$  are the triplets  $\varepsilon_{t+1} = (\kappa_{t+1}, y_{t+1}, R_{t+1})$  and  $\varepsilon_t = (\kappa_t, y_t, R_t)$ . In the remainder of this Section we use this notation. We also assume that there are a total of  $N$  triplets that represent all the possible combinations of the three shocks that can be observed.

Following Cogley and Sargent's (2008), our learning setup features two-point passive learning without noise, so that  $p_t^s(\varepsilon_{t+1}|\varepsilon_t) \rightarrow p^a(\varepsilon_{t+1}|\varepsilon_t)$  for sufficiently large  $t$ . With this setup, agents learn about the transition probability matrix only as they observe realizations of the shocks. In particular, they learn about the true regime-switching probabilities of  $\kappa$  only after observing a sufficiently long set of realizations of  $\kappa^h$  and  $\kappa^l$ .

This learning setup fits nicely our goal of studying a situation in which financial innovation (i.e. the introduction of debt securitization) represents an initial condition with a state  $\kappa^h$  but with imperfect information about the true riskiness of this new environment. Agents are ignorant about the true transition distribution of  $\kappa$ , since there is no data history to infer it from, and so they start with arbitrary priors. Over time, if they observe a sequence of realizations of  $\kappa^h$  for a few periods, they build optimism by assigning a probability to the possibility of transiting to the state  $\kappa^l$  that is lower than the true value. We refer to this situation as the "optimistic phase." Such optimism by itself is a source of vulnerability, because it is quickly overturned with the first few realizations of  $\kappa^l$  that hit the economy. In addition, during the optimistic phase, the incentives to build precautionary savings against the risk of a shift in the ability to securitize debt are weaker

than in the long-run RE equilibrium. This increases the agents' risk of being caught "off-guard" (i.e. with too much debt) when the first shift to the low securitization regime occurs.

Modeling imperfect information in this fashion implies a deviation from rational expectations, but there is bounded rationality because agents use Bayes' rule to update their beliefs about the transition probabilities. The RE equilibrium is reached only after sufficient data have been gathered to correctly calculate the transition probabilities. This is a key feature of our model, because it highlights the role of the short history of a new financial regime in hampering the ability of agents to correctly assess risk. This approach seems better suited to study the role of the U.S. financial innovation process in causing the financial crisis, as opposed to a standard signal extraction problem with noisy signals and rational expectations. In our setup, agents observe variables perfectly, without noise. Their problem is that when the new regime starts, they do not know what are the true probabilistic processes driving risk, until sufficient data is available. Note also that time alone does not determine how fast agents learn about these processes. The sequence in which realizations and switches between realizations occur also matters.

The passive learning structure facilitates significantly the analysis and numerical solution of the model. In particular, it allows us to split the analysis into two parts. The first part uses Bayesian learning to generate the agents' sequence of posterior density functions  $\{f(F^s|\varepsilon^t)\}_{t=1}^T$ . Each of these density functions (one for each date  $t$ ) is a probability distribution over possible Markov transition matrices  $F^s$ . Since agents do not know the true transition matrix  $F^a$ , the density function changes with the sequence of realizations observed up to date  $t$  (i.e.  $\{\varepsilon^t, \varepsilon^{t-1}, \dots, \varepsilon^1\}$  where  $\varepsilon^t = (\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_1)$ ) and with the initial date-0 prior, as we explain later. Notice that if  $T$  is high enough, the sequence  $\{f(F^s|\varepsilon^t)\}_{t=1}^T$  converges to a distribution with all its mass in  $F^a$ . In other words, beliefs converge to the true values, so that in the long run the model converges to the RE equilibrium.

The second part of the analysis characterizes the model's equilibrium by combining the sequences of posterior densities obtained in the first part,  $\{f(F^s|\varepsilon^t)\}_{t=1}^T$ , with chained sequential solutions from a set of "conditional" dynamic programming problems. These problems are conditional on the posterior density function of  $F^s$  that agents form with the history of realizations up to each date  $t$ . This approach takes advantage of the fact that the variables included in  $\varepsilon$  are exogenous, agents do not benefit from trying to improve their inference about the regime switching probabilities by "experimenting" using their optimization problems. As a result, the evolution of beliefs can be analyzed separately from the agents' optimal consumption and savings plans. The

remainder of this Section examines in more detail the Bayesian learning setup and the construction of the model's equilibrium.

## 2.2 Learning and the Sequence of Beliefs

The learning framework takes as given an observed history of realizations of  $T$  periods of the exogenous shock, denoted by  $\varepsilon^T = (\varepsilon^T, \varepsilon^{t-1}, \dots, \varepsilon^1)$ , and a prior of  $F^s$  for date  $t = 0$ ,  $p(F^s)$ , and it yields a sequence of posteriors over  $F^s$  for every date  $t$ ,  $\{f(F^s|\varepsilon^t)\}_{t=1}^T$ .<sup>2</sup> At every date  $t$ , the information set of the agent includes  $\varepsilon^t$  as well as the possible values that  $\varepsilon$  can take (including  $\kappa^h$  and  $\kappa^l$ ).

Agents form posteriors from priors using a beta-binomial probability model. Since agents know the realization vectors of the discrete Markov processes, imperfect information reduces to imperfect information about the Markov transition matrix only. In particular, since we assume that the process driving  $\kappa$  is independent of the joint Markov process for the variables in  $z$ , imperfect information about the transition probability matrix of  $\kappa$  boils down to imperfect information about two parameters, namely the ‘‘continuation’’ probabilities  $F_{hh}^a$  and  $F_{ll}^a$ . The other two elements of the transition matrix of  $\kappa$  are recovered using  $F_{ii}^a + F_{ij}^a = 1$  where  $F_{ij}^a$  denotes the probability of switching from state  $i$  to state  $j$ .

The agents' posteriors about  $F_{hh}^s$  and  $F_{ll}^s$  have Beta distributions as well, and the parameters that define them are determined by the number of regime switches observed in a particular history  $\varepsilon^t$ . We assume that the priors for  $F_{hh}^s$  and  $F_{ll}^s$  included in  $p(F^s)$  are independent and determined by the number of transitions assumed to have been observed prior to date  $t = 1$ .<sup>3</sup> More formally,

$$p(F_{ii}^s) \propto (F_{ii}^s)^{n_0^{ii}-1} (1 - F_{ii}^s)^{n_0^{ij}-1} \quad (7)$$

where  $n_0^{ij}$  denotes the number of transitions from state  $i$  to state  $j$  assumed to have been observed prior to date 1.

The likelihood function of  $\varepsilon^t$  conditional on  $F_{hh}^s, F_{ll}^s$  is obtained by multiplying the densities of  $F_{hh}^s$  and  $F_{ll}^s$ :

$$f(\varepsilon^t | F_{hh}^s, F_{ll}^s) \propto (F_{hh}^s)^{(n_t^{hh} - n_0^{hh})} (1 - F_{hh}^s)^{(n_t^{hl} - n_0^{hl})} (1 - F_{ll}^s)^{(n_t^{lh} - n_0^{lh})} (F_{ll}^s)^{(n_t^{ll} - n_0^{ll})}. \quad (8)$$

<sup>2</sup>In describing the learning problem, we follow the presentation format in Cogley and Sargent (2008).

<sup>3</sup>This assumption of independent priors follows also Cogley and Sargent (2008), but it can be relaxed.

Multiplying the likelihood function by the priors delivers the posterior kernel:

$$k(F^s|\varepsilon^t) \propto (F_{hh}^s)^{(n_t^{hh}-1)}(1 - F_{hh}^s)^{(n_t^{hl}-1)}(1 - F_{ll}^s)^{(n_t^{lh}-1)}F_{ll}^s(n_t^{ll}-1), \quad (9)$$

and dividing the kernel using the normalizing constant  $M(\varepsilon^t)$  yields the posterior density:

$$f(F^s|\varepsilon^t) = k(F^s|\varepsilon^t)/M(\varepsilon^t) \quad (10)$$

where

$$M(\varepsilon^t) = \iint (F_{hh}^s)^{(n_t^{hh}-1)}(1 - F_{hh}^s)^{(n_t^{hl}-1)}(1 - F_{ll}^s)^{(n_t^{lh}-1)}(F_{ll}^s)^{(n_t^{ll}-1)} dF_{hh}^s dF_{ll}^s.$$

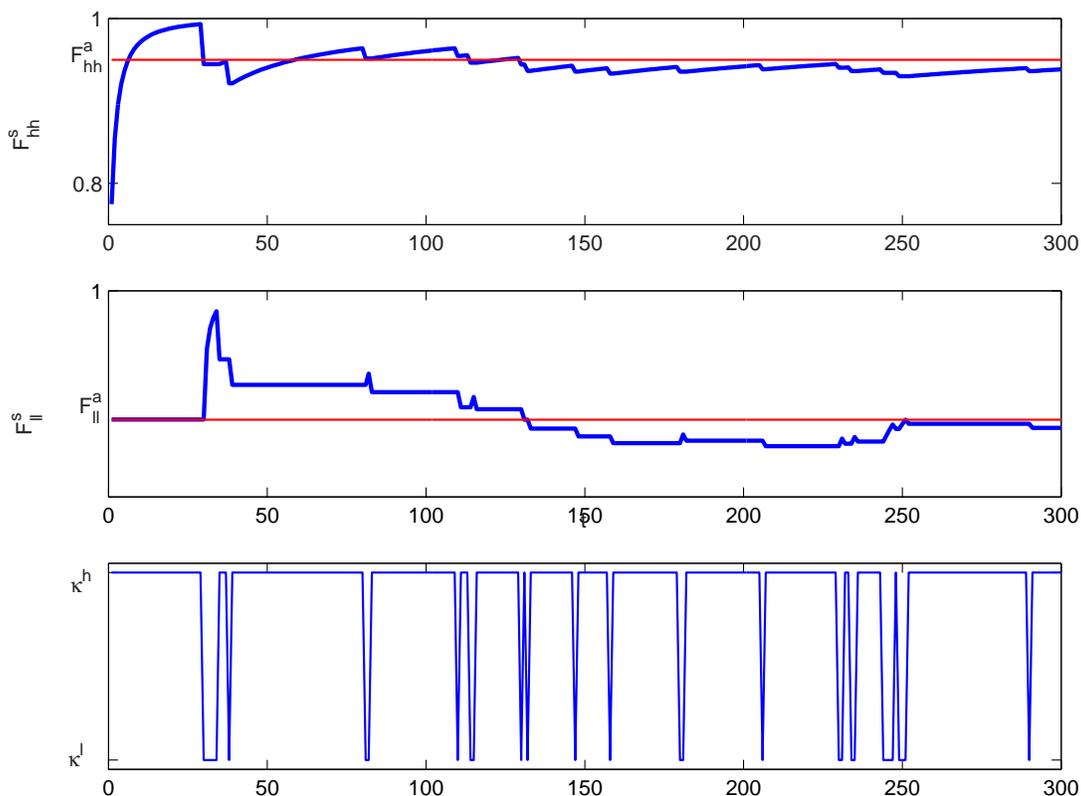
The number of transitions across regimes is updated as follows:

$$n_{t+1}^{ij} = \begin{cases} n_t^{ij} + 1, & \text{if } \kappa_{t+1} = \kappa^j \text{ and } \kappa_t = \kappa^i, \\ n_t^{ij}, & \text{otherwise.} \end{cases}$$

Note that the posteriors are of the form  $F_{hh}^s \propto \text{Beta}(n_t^{hh}, n_t^{hl})$  and  $F_{ll}^s \propto \text{Beta}(n_t^{lh}, n_t^{ll})$ . That is, the posteriors for  $\kappa^h$  only depend on  $n_t^{hh}$  and  $n_t^{hl}$  and not on the other two counters,  $n_t^{lh}$  and  $n_t^{ll}$ , and the posteriors for  $\kappa^l$  only depend on  $n_t^{lh}$  and  $n_t^{ll}$ . This is important because it implies that the posteriors of  $F_{hh}^s$  change only as  $n_t^{hh}$  and  $n_t^{hl}$  change, and this only happens when the date- $t$  realization is  $\kappa^h$ . If, for example, the economy experiences realizations  $\kappa = \kappa^h$  for several periods, agents learn only about the persistence of the high securitization regime. They learn nothing about the persistence of the low securitization regime. To learn about this, they need to observe realizations of  $\kappa^l$ . Since in a two-point regime-switching setup persistence parameters also determine mean durations, it follows that both the persistence and the mean durations of securitization regimes can be learned only as the economy actually experiences  $\kappa^l$  and  $\kappa^h$ .

We illustrate the learning dynamics of this setup by means of a simple numerical example. We choose a set of values for  $F_{hh}^a, F_{ll}^a$ , and initial priors, and then simulate the learning process for 300 quarters (75 years). The results are plotted in Figure 1. The true regime-switching probabilities are set to  $F_{hh}^a = 0.95$  and  $F_{ll}^a = 0.5$ . These values are used only for illustration purposes (they are not calibrated to actual data as in the solution of the full model in Section 3). In addition, the initial priors are set to  $F_{hh}^s \sim \text{Beta}(0.7, 0.7)$  and  $F_{ll}^s \sim \text{Beta}(0.7, 0.7)$ . With these priors the agents set their beliefs about the persistence of the high securitization regime to around 0.78 after observing the first period's realization.

Figure 1: Evolution of Beliefs



Notes: This figure plots the evolution of beliefs about  $F_{hh}^s$  (top panel),  $F_{ll}^s$  (middle panel), and the associated realizations of  $\kappa$  (lower panel). The horizontal red lines in the upper two panels mark the true values of the corresponding variables.

The most striking result evident in Figure 1 is that financial innovations, when “untested,” can lead to significant underestimation of risk. In other words, the initial sequence of realizations of  $\kappa^h$  observed until just before the first realization of  $\kappa^l$  (the “optimistic phase”) generates substantial optimism. In this example, this optimistic phase covers the first 30 periods. The degree of optimism produced during this phase can be measured by the difference between the conditional expectation based on the date- $t$  beliefs,  $F_{hh}^s$ , and the corresponding rational expectations value,  $F_{hh}^a$  (horizontal line). As the Figure shows, this difference is much larger during the optimistic phase than in any of the subsequent periods. For example, even though the economy remains in  $\kappa^h$  from around date 40 to date 80, the magnitude of the optimism that this period generates is smaller than in the initial optimistic phase. This is because it is only after  $\kappa$  observing at least once that a switch to  $\kappa^l$  is possible that agents rule out the possibility of  $\kappa^h$  being an absorbent state. As a result,  $F_{hh}^s$  cannot surge as high as it did during the initial boom. Notice also that the first realizations of  $\kappa^l$  generate a “pessimistic phase,” in which  $F_{ll}^s$  is significantly higher than  $F_{ll}^a$ , so the period of overoptimistic

expectations of high securitization is followed by a period of overpessimistic expectations.

Figure 1 also reflects the fact that the beliefs about the average duration of  $\kappa^h$  ( $\kappa^l$ ) are updated only when the economy is in the high (low) state. This is evident, for example, in the initial optimistic phase (the first 30 periods), when  $F_{ll}^s$  does not change at all. As explained above, for the agents to learn about the duration of the high (low) securitization regime, the economy needs to actually be in that regime. This feature of the learning dynamics also explains why in this example  $F_{hh}^s$  converges to  $F_{hh}^a$  faster than  $F_{ll}^s$ . Given that the low securitization regime is visited much less frequently, it takes longer for the agents to learn about its persistence, and therefore  $F_{ll}^s$  takes longer to converge to  $F_{ll}^a$ .

### 2.3 Recursive Equilibrium

We define the model's recursive equilibrium using a recursive strategy to characterize the agents' optimal intertemporal plans. Consider first the date-1 problem of the agents. We pull  $f(F^s|\varepsilon^1)$  from the sequence of posterior density functions defined in the previous subsection. This is the first density function in the sequence  $\{f(F^s|\varepsilon^t)\}_{t=1}^T$ , and it represents the first posterior about the distribution of  $F^s$  that agents form, given that they have observed  $\varepsilon_1$  and given their initial prior. At this point, agents solve the following date-1 conditional dynamic programming problem (CDPP):

$$\begin{aligned} V_1(b_1, \varepsilon_1) &= \max_{b_2 \in B} \left\{ u(c_1) + \exp(-v(c_1)) \left[ \int E^S[V_1(b_2, \varepsilon_2)|f(\varepsilon_2|\varepsilon^1, F^s)]f(F^s|\varepsilon^1)dF^s \right] \right\} \quad (11) \\ \text{s.t. } c_1 &= y_1 - \frac{b_2}{R_1} + b_1, \\ b_2 &\geq -\kappa_1 y_1 \end{aligned}$$

where:

$$E^S[V_1(b_2, \varepsilon_2)|f(\varepsilon_2|\varepsilon^1, F^s)] \equiv \sum_{n=1}^N \Pr(\varepsilon_2 = n|\varepsilon^1, F^s)V_1(b_2, \varepsilon_2)$$

Note that  $E^S[V_1(b_2, \varepsilon_2)|f(\varepsilon_2|\varepsilon^1, F^s)]$  is calculated in the same way as the expectation in the right-hand-side of a standard Bellman equation for a conventional rational expectations model with a discrete Markov chain of  $N$  realizations and transition function  $F^s$ . Since agents in our model do not know  $F^a$ , however, the relevant expectation in the right-hand-side of (11) integrates over  $f(F^s|\varepsilon^1)$ . The time subscripts that index the value functions in both sides of (11) indicate the date of the most recent observation of  $\varepsilon$  (which is date 1 in this case). It is also critical to note that,

conditional on  $f(F^s|\varepsilon^1)$ , this dynamic optimization problem remains recursive because the law of iterated expectations holds for Bayesian updating with passive learning (see Appendix B in Cogley and Sargent (2008)).

The solution to the date-1 CDPP is given by a policy function  $\hat{b}_2(b_1, \varepsilon_1)$  and an associated value function  $\hat{V}_1(b_1, \varepsilon_1)$ , both conditional on  $f(F^s|\varepsilon^1)$ . Given  $\hat{b}_2(b_1, \varepsilon_1)$ , we can solve for the decision rules for consumption, savings ( $b_2/R_1 - b_1$ ) and the credit flow ( $b_2 - b_1$ ).

Generalizing the date-1 problem we can define the CDPPs for all subsequent dates  $t = 2, \dots, T$  by resetting time subscripts as needed and using the corresponding density function  $f(F^s|\varepsilon^t)$  for each date  $t$  in the sequence of posteriors solved for earlier. This is crucial because  $f(F^s|\varepsilon^t)$  changes as time passes, reflecting the passive Bayesian learning, which implies that the value and policy functions that solve each CDPP also change. This justifies keeping the time subscripts in each Bellman equation, as opposed to the standard practice of dropping them and using primes to denote  $t+1$  variables. Thus, in fact the “full solution” of the model is not recursive because history matters since different trajectories  $\varepsilon^t$  imply different densities  $f(F^s|\varepsilon^t)$ , and hence different value and policy functions. If at any two dates  $t$  and  $t+j$  we give agents the same values for the states  $(b, \varepsilon)$ , they, in general, will not choose the same bond holdings for the following period because  $f(F^s|\varepsilon^t)$  and  $f(F^s|\varepsilon^{t+j})$  will differ.

The date- $t$  CDPP is:

$$\begin{aligned} V_t(b_t, \varepsilon_t) &= \max_{b_{t+1} \in B} \left\{ u(c_t) + \exp(-v(c_t)) \left[ \int E^S[V_t(b_{t+1}, \varepsilon_{t+1}) | f(\varepsilon_{t+1}|\varepsilon^t, F^s)] f(F^s|\varepsilon^t) dF^s \right] \right\} \\ \text{s.t. } c_t &= y_t - \frac{b_{t+1}}{R_t} + b_t, \\ b_{t+1} &\geq -\kappa_t y_t \end{aligned}$$

where:

$$E^S[V_t(b_{t+1}, \varepsilon_{t+1}) | f(\varepsilon_{t+1}|\varepsilon^t, F^s)] \equiv \sum_{n=1}^N \Pr(\varepsilon_{t+1} = n | \varepsilon^t, F^s) V_t(b_{t+1}, \varepsilon_{t+1})$$

The solution of this problem is characterized by a policy function  $\hat{b}_{t+1}(b_t, \varepsilon_t)$  and an associated value function  $\hat{V}_t(b_t, \varepsilon_t)$ , both conditional on  $f(F^s|\varepsilon^t)$ , and hence conditional on the particular history  $\varepsilon^t$  that drives the posterior density of  $F^s$ .

Given the recursive structure of each of the date- $t$  CDPPs, we can define the model’s recursive equilibrium as follows:

**Definition** Given a  $T$ -period history of realizations  $\varepsilon^T = (\varepsilon^T, \varepsilon^{T-1}, \dots, \varepsilon^1)$ , a recursive equilibrium for the economy is given by a sequence of functions  $\left[ \hat{V}_t(b_t, \varepsilon_t), \hat{b}_{t+1}(b_t, \varepsilon_t), f(F^s | \varepsilon^t) \right]_{t=1}^T$  such that:

1.  $\hat{V}_t(b_t, \varepsilon_t)$  and  $\hat{b}_{t+1}(b_t, \varepsilon_t)$  solve the date- $t$  CDPP (12) conditional on  $f(F^s | \varepsilon^t)$ .
2.  $f(F^s | \varepsilon^t)$  is the date- $t$  posterior density of  $F^s$  determined by the Bayesian passive learning process summarized in Equation (10)

Intuitively, conditions 1. and 2. imply that the equilibrium is formed by combining the solutions for each of the  $T$  date- $t$  problems. That is, the equilibrium dynamics for a particular trajectory  $\varepsilon^T$  (i.e. a given sequence of realized histories  $\varepsilon^T, \varepsilon^{T-1}, \dots, \varepsilon^1$  that agents observe at each date and use to update their beliefs) are given by the sequences  $\left[ \hat{V}_t(b_t, \varepsilon_t), \hat{b}_{t+1}(b_t, \varepsilon_t), f(F^s | \varepsilon^t) \right]_{t=1}^T$ . In particular, the sequence of bond decision rules determining equilibrium bond holdings for dates  $t = 2, \dots, T + 1$  would be:  $b_2 = \hat{b}_1(b_1, \varepsilon_1)$ ,  $b_3 = \hat{b}_2(b_2, \varepsilon_2)$ , ...,  $b_{T+1} = \hat{b}_T(b_T, \varepsilon_T)$ .

The following features of the recursive equilibrium are worth noting, because they emphasize important implications of the passive Bayesian learning that are useful for the numerical solutions of the next Section.

- (1) If  $f(F^s | \varepsilon^{t+j}) = f(F^s | \varepsilon^t)$ , the solutions for the date  $t + j$  and the date  $t$  CDPPs are the same—note that posteriors that satisfy this condition are quite possible as Figure 1 showed. This does not mean the equilibrium dynamics are the same. It only means that the value *function* and policy *function* are the same. As long as in a time series simulation of the equilibrium dynamics  $(b_{t+j}, \varepsilon_{t+j})$  are not the same as  $(b_t, \varepsilon_t)$ , the actual values for  $\hat{V}_{t+j}(b_{t+j}, \varepsilon_{t+j})$ ,  $\hat{b}_{t+j+1}(b_{t+j}, \varepsilon_{t+j})$  will differ from  $\hat{V}_t(b_t, \varepsilon_t)$ ,  $\hat{b}_{t+1}(b_t, \varepsilon_t)$ .
- (2) The above suggests that, for a particular CDPP at some date  $t + j$ , we can speed convergence of the numerical solutions if, whenever  $\|f(F^s | \varepsilon^{t+j}) - f(F^s | \varepsilon^t)\|$  is small enough under some metric, we use for the date  $t + j$  problem initial guesses of  $\hat{V}_{t+j}(b_{t+j}, \varepsilon_{t+j})$ ,  $\hat{b}_{t+j+1}(b_{t+j}, \varepsilon_{t+j})$  given by the date- $t$  value function  $\hat{V}_t(b_t, \varepsilon_t)$  and policy function  $\hat{b}_{t+1}(b_t, \varepsilon_t)$ .
- (3) The solutions to each date- $t$  CDPP are not functionally related (i.e. solving a particular date- $t$  problem does not require knowing anything about the solution for any other date). Thus, the model can be solved by solving each date- $t$  problem independently.<sup>4</sup>
- (4) If  $j \leq T$  is large enough for  $f(F^s | \varepsilon^{t+j})$  to converge to  $F^a$  (for some converge criterion defined over  $\|f(F^s | \varepsilon^{t+j}) - f(F^a | \varepsilon^t)\|$ ), the solutions for all dates  $t \geq j$  collapse to a standard Bellman

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<sup>4</sup>This fact can be used to develop a strategy to speed up the full solution different from the one suggested in (2), because in a computer with  $x$  number of cores, we can solve  $x$  problems for  $x$  different dates at the same time.

equation that solves the recursive rational RE equilibrium using the true Markov process  $F^a$ . This RE solution satisfies the following Bellman equation:

$$\begin{aligned}
V(b, \varepsilon) &= \max_{b' \in B} \{u(c) + \exp(-v(c))E^a[V^a(b', \varepsilon')]\} \\
s.t. \quad c &= y - qb' + b, \\
b' &\geq -\kappa y
\end{aligned} \tag{13}$$

where  $E^a$  is formed using  $F^a$ . The solution to this functional equation yields a value function  $V^a(b, \varepsilon)$ , a policy function  $\hat{b}'^a(b, \varepsilon)$ , and an ergodic distribution  $\Gamma^a(b, \varepsilon)$ , none of which requires time subscripts.

- (5) The full equilibrium solution of the intertemporal sequence of optimal plans from dates 0 to  $T$  is obtained by chaining the solutions of all  $T$  date- $t$  problems, and this would need to be done for each different trajectory  $\varepsilon^T = (\varepsilon^T, \varepsilon^{T-1}, \dots, \varepsilon^1)$  that one assumes to generate a sequence of posterior densities  $f(F^s | \varepsilon^t)_{t=1}^T$ . This suggests two solution strategies. One is to take a stance on a particular  $\varepsilon^T$  based on actual realizations for a particular time period of actual data or a particular episode. Another would be to generate a set of  $M$  trajectories  $[\varepsilon_i^T]_{i=1}^M$  using the true Markov process  $F^a$ , and then solve the model for each of them, and take averages across these different solutions at each date  $t$ .

### 3 Quantitative Analysis

In this Section we study the model's quantitative predictions for our financial innovation experiment. The experiment assumes that learning takes place over  $T$  periods. At  $t = 1$  financial innovation begins with the first realization of  $\kappa^h$ , followed by an optimistic phase in which the same regime continues for  $J$  periods ( $\kappa_t = \kappa^h$  for  $t = 1, \dots, J$ ) and the arrival of the first realization of  $\kappa^l$  at date  $J + 1$  ( $\kappa_{J+1} = \kappa^l$ ). After this, the economy remains in state  $\kappa^l$  from dates  $J + 1$  to  $T$ . We start with a baseline scenario in which, for simplicity, the learning process only involves the securitization regime, because the Markov process of  $y$  is known and there are no interest rate shocks.

### 3.1 Calibration

The calibration requires setting parameter values for  $\beta, \sigma, R$ , the Markov process for  $y$ , the securitization regimes and the parameters of the learning setup ( $\kappa^h, \kappa^l$ , the initial priors  $p(F^s)$ , and the length of the optimistic phase  $J$ ). We calibrate the model to a quarterly frequency using U.S. data. The calibration of the preference parameters, the interest rate and the income process is fairly standard. The calibration of the learning parameters and the securitization regimes is set to match the U.S. credit boom of the period between the mid 1980s and the mid 2000s, as we explain below. Since the actual values of these parameters are more uncertain than the others, we will conduct extensive sensitivity analysis to evaluate the robustness of our results to the assumptions in the baseline calibration.

We set  $\sigma = 2.0$ , the standard value in quantitative DSGE models. The real interest rate is set to 2.8 percent annually, which is the ex-post average real interest rate on U.S. three-month T-bills during the period 1980Q1-2005Q2. The Markov process for  $y$  is set to approximate an AR(1) process ( $y_t = \rho y_{t-1} + e_t$ ) fitted to U.S. HP-filtered real GDP per capita using data for the period 1949Q2-2007Q4. The estimation yields  $\rho = 0.8327$  and  $\sigma_e = 0.0087$ , which imply a standard deviation of income of  $\sigma_y = 1.651$  percent. We use Tauchen and Hussey's (1991) quadrature method to construct the Markov approximation to this process assuming a vector of 9 realizations. The transition probability matrix and realization vector are reported in an Appendix available on request.

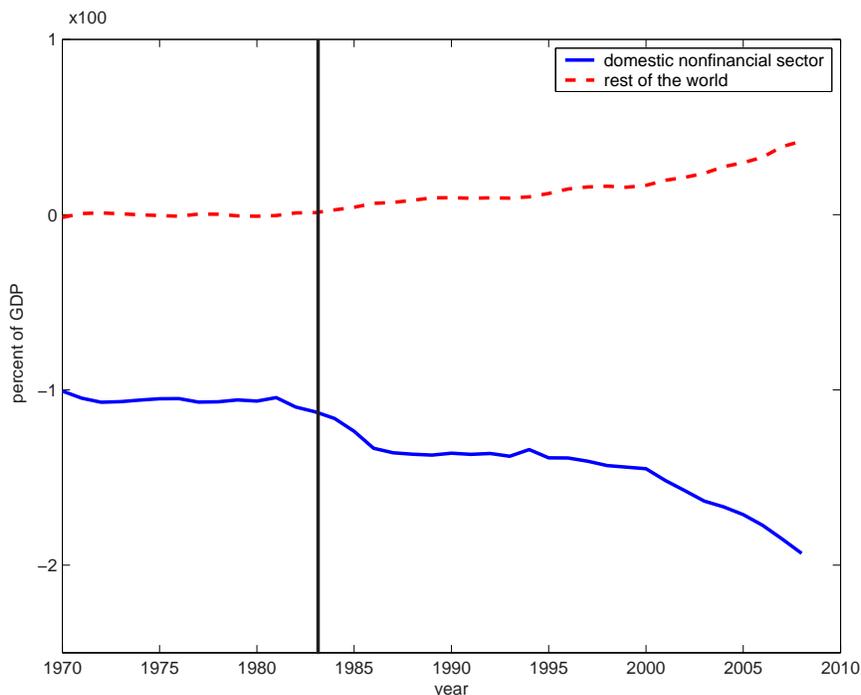
We normalize mean output to one and set the model's parameters so that the resource constraint at the deterministic steady state (or at the average of the stochastic stationary state) matches the ratios of consumption and net credit market assets of the domestic nonfinancial sector to GDP observed in U.S. data.<sup>5</sup> The average ratio of household consumption expenditures (including nonperishables) to GDP is relatively stable in the data with a slight upward trend (it has a mean of 0.68 with a standard deviation of 0.019 over the 1985Q1-2008Q3 period). We take the 2007 value, which is 0.7052, and since we normalized mean output, consumption at the deterministic steady state is  $c = 0.7052$ .

Pinning down a proxy in the data for the long-run average of net credit market assets of the domestic nonfinancial sectors is more difficult because, as Figure 2 shows, the ratio of these assets to GDP has been declining secularly for most of the last twenty years. This is a measure of the

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<sup>5</sup>Consumption and GDP data are from the *International Financial Statistics* of the IMF. Net credit market assets of the nonfinancial sectors are from the Federal Reserve's *Flow of Funds Accounts of the United States*.

Figure 2: U.S.: Net Credit Market Assets



Notes: This figure plots the net credit market assets to GDP ratio for the domestic nonfinancial sector and the rest of the world. Source: Flow of Funds Accounts of the U.S. provided by the Board of Governors of the Federal Reserve System.

large expansion in net credit that the U.S. economy experienced (an unprecedented doubling from 100 percent of GDP in the early 1980s, and on average since 1945, to nearly 200 percent of GDP in 2008!). Since this ratio is non-stationary, we calibrate  $b$  to the most recent observation, which is for 2008 and is equal to -1.93. Hence, at the deterministic steady state  $b = -1.93$ .

Investment and government expenditures are not explicitly modeled. Hence, to close the resource constraint we introduce a fixed, exogenous amount of autonomous spending  $A$ . Thus, the normalized steady-state resource constraint is  $1 = c + A - br$ , and given  $b$  and  $c$  we calculate  $A$  as a residual  $A = y - c + br = 0.281$ .

The value of the time preference semi-elasticity parameter is set using the deterministic steady state equilibrium condition equating the endogenous discount factor with the gross interest rate. Given the values of  $c$  and  $R$ , we obtain  $\beta = \ln(R)/\ln(1 + c) = 0.0129$ .

We date the introduction of financial innovation and the first realization of  $\kappa^h$  as of 1984Q1 (this is date 1 in our experiment). This is consistent with the observations that CDOs were first sold in the early 1980s, and that 1984 is the year in which the net credit assets-GDP ratio shown in Figure 2 started on its declining trend. We date the start of the financial crisis with the early

Table 1: Parameters

$\beta$	Rate of time preference elasticity	0.013
$\sigma$	Risk aversion coefficient	2.000
$b$	Net credit GDP ratio	-1.933
$c$	Consumption GDP ratio	0.705
$A$	Lump-sum absorption	0.281
$r$	Interest rate (annualized)	2.800
$\rho$	Persistence of endowment shocks	0.839
$\sigma_e$	Standard deviation of endowment shocks	0.009
$\kappa^h$	Value of $\kappa$ in the high state	2.400
$\kappa^l$	Value of $\kappa$ in the low state	1.695
$F_{hh}^a$	True persistence of $\kappa^h$	0.989
$F_{ll}^a$	True persistence of $\kappa^l$	0.500
$n_0^{hh}, n_0^{hl}$	Priors	0.010

stages of the subprime mortgage crisis in late 2006. Hence, we assume that agents observe  $\kappa^h$  until 2006Q4. This implies an optimistic phase of  $J = 92$  quarters (23 years). We consider a learning period with a total length of  $T = 100$  quarters, in which the first 92 realizations of  $\kappa$  are  $\kappa^h$  while the remaining 8 are  $\kappa^l$ .

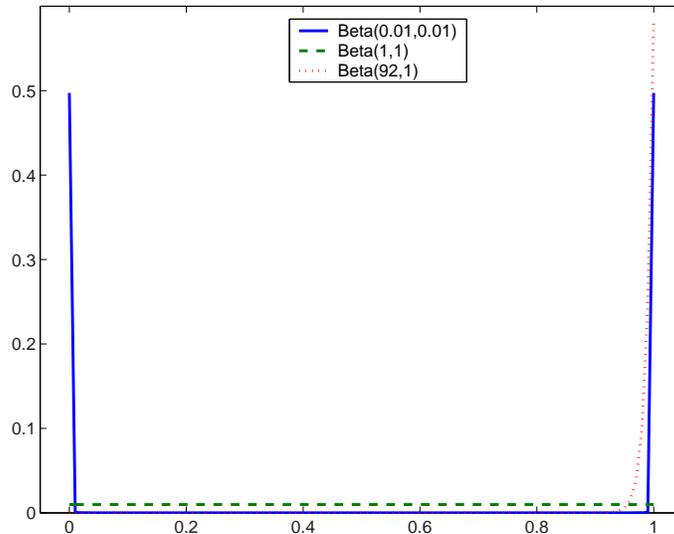
Next we calibrate the initial priors and the values of  $\kappa^h$  and  $\kappa^l$ . We assume that  $\kappa^l$  represents the level of securitization that the U.S. economy was able to support before the financial innovation of the 1980s. Hence, to calibrate  $\kappa^l$  we first calculate the mean ratio of net credit market assets of the domestic nonfinancial sector to GDP in 1970-1983, which is -1.063 (a very similar average of 0.98 is obtained using the data starting in the earliest observation available, which is for 1945). Then we set  $\kappa^l$  such that the unconditional long-run average of bond holdings in a rational expectations version of our model with a constant  $\kappa$  is -1.063. This exercise yields  $\kappa^l = 1.695$ .

We set the high securitization regime to  $\kappa^h = 2.40$ . This represents a non-binding upper bound on debt in our numerical solutions. As we show in the next subsection, we find that even at the peak of the optimistic phase, agents never choose bond holdings that are lower than -2.28, because otherwise they are exposed to the risk of non-positive consumption (which would produce infinite marginal utility given CRRA utility). Therefore, any  $\kappa^h$  that satisfies  $\kappa^h > 2.28$  would guarantee

that the upper bound on debt does not bind, and our results are invariant to it.

The beta-distribution parameters that determine initial priors are calibrated as follows. A natural choice for the priors of  $F_{hh}^s$  and  $F_{ll}^s$  would be  $Beta(1, 1)$ , but this implicitly assumes that the agents have already observed one transition from  $\kappa^h$  to  $\kappa^l$  and one in the opposite direction. This is not desirable in our setting because we are assuming financial innovations for which the time series data is “too short” or non-existent. A  $Beta(n_0^{ii}, n_0^{ij})$  distribution requires  $0 < n_0^{ii}$  and  $0 < n_0^{ij}$ . The lower those values (or the less the agents know about the innovation), the more optimistic agents turn about the persistence of the high securitization state after the first observation of  $\kappa^h$ , and remain more optimistic throughout the learning period. We set  $F_{hh}^s \sim Beta(0.01, 0.01)$  and  $F_{ll}^s \sim Beta(0.01, 0.01)$  and discuss the economic interpretation and implications of this calibration below. Reducing  $n_0^{ii}$  and  $n_0^{ij}$  further did not result in any significant change in the magnitude of over-borrowing generated by the model.

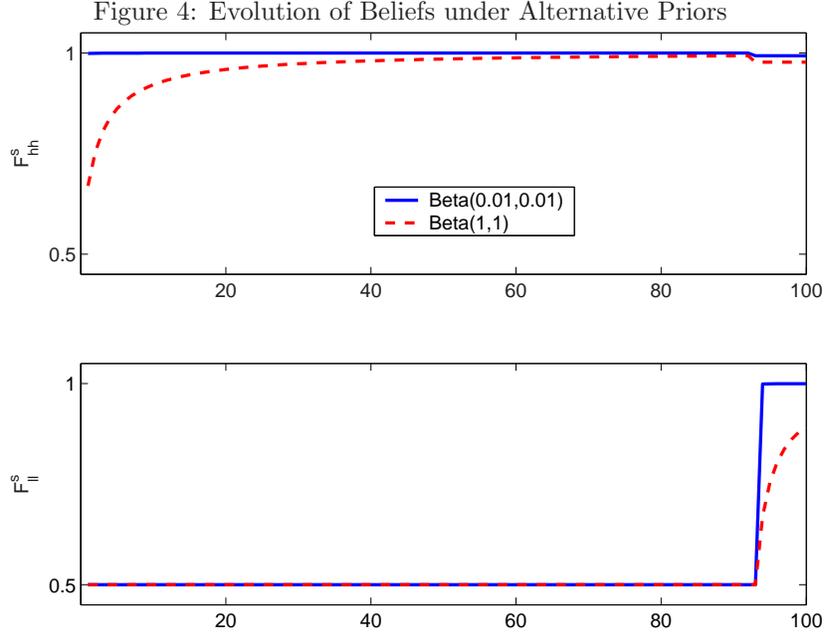
Figure 3: Beta Distribution



Notes: This figure plots the probability density function of the *Beta* distribution with different  $n_0^{ii}$  and  $n_0^{ij}$  where  $Beta(n_0^{ii}, n_0^{ij})$ .

In order to further elaborate the economic meaning of choosing low values for  $n_0^{ii}$  and  $n_0^{ij}$ , we plot the probability density functions for the Beta distribution with three different parameter sets in Figure 3.  $Beta(0.01, 0.01)$  corresponds to our baseline calibration. Note that this parameter choice, the Beta distribution is U-shaped with almost all of the mass concentrated around 0 and 1. Given  $n_0^{ii} = n_0^{ij}$ , this distribution is symmetric with a mean of 0.5 and a variance of 0.245. Similarly,  $Beta(1, 1)$  case also has a mean of 0.5 however, its variance is 0.129, half that of  $Beta(0.01, 0.01)$ .<sup>6</sup>

<sup>6</sup> $Beta(1, 1)$  is the same as a continuous uniform distribution,  $U(0, 1)$ .



Notes:  $Beta(0.01, 0.01)$  corresponds to our baseline calibration while  $Beta(1, 1)$  shows beliefs if the priors were uniformly distributed.

Figure 3 also plots the probability density function of  $Beta(92, 1)$  in order to illustrate the beliefs,  $F_{hh}^s$ , in period 93. At this stage, the agent would have observed 92 transitions from high to high and only one transition from high to low state. With the observation of a long duration of the high state, this distribution is highly skewed to the right with all of the mass concentrated around 1.

Intuitively, assuming priors of the form  $Beta(0.01, 0.01)$  for both  $F_{hh}^s$  and  $F_{ll}^s$  implies that the agents conjecture that there four likely scenarios: *a)* Both high and low securitization regimes are extremely persistent, i.e.  $F_{hh}^s \approx 1$  and  $F_{ll}^s \approx 1$ , *b)* The high securitization regime is extremely persistent and the economy switches to the low securitization regime rarely and for a short time, i.e.  $F_{hh}^s \approx 1$  and  $F_{ll}^s \approx 0$ , *c)* The low securitization regime is extremely persistent and high securitization regime occurs rarely and has a short duration, i.e.  $F_{hh}^s \approx 0$  and  $F_{ll}^s \approx 1$ , *d)* Neither regime is persistent and the economy constantly transits between the two, i.e.  $F_{hh}^s \approx 0$  and  $F_{ll}^s \approx 0$ .

After observing a few realizations of  $\kappa^h$ , the agents realize the high persistence of the high securitization state and rule out the third and fourth scenarios. This is evident in Figure 4 where we plot the evolution of  $F_{hh}^s$  and  $F_{ll}^s$  assuming  $Beta(0.01, 0.01)$  and  $Beta(1, 1)$  priors. As discussed before, unless the economy switches to the low securitization state, the agents cannot learn about the persistence of that state. Compared to the evolution of  $F_{hh}^s$  under  $Beta(1, 1)$  priors plotted in the same figure, our baseline calibration with  $Beta(0.01, 0.01)$  priors leads the agents to turn

optimistic right from the start. It is important to note that, unlike Cogley and Sargent (2008) who inject an initial pessimism to the agents' priors, we do not build in an initial optimism. In our case, agents were not optimistic prior to period 1 because with  $Beta(0.01, 0.01)$  priors in period 0,  $F_{hh}^s = F_{ll}^s = 0.5$  which is the mean of this Beta distribution as plotted in Figure 3. By leaving the agents with the four plausible scenarios mentioned above, all we assume is that agents believe that either the switches in the securitization regimes will not be frequent (scenarios 1-3) or that there will be a switch every period (scenario 4).

At this point we have calibrated all of the parameters that are needed for solving the equilibrium with learning under our financial innovation experiment. Notice in particular that this does not require knowing the true transition probability matrix of  $\kappa$  ( $F^a$ ). Solving the CDPPs of the agents requires the sequence of beliefs about the transition probability matrix ( $f(F^s|\varepsilon^t)_{t=0}^T$ ), which is determined with the parameters we already set. Still, calibrating the true transition probability matrix is necessary in order to evaluate the macroeconomic effects of the imperfect information friction by comparing against the standard RE solution. In particular, we will compare the dynamics of the optimistic phase with those of the RE model to quantify the magnitude of over-borrowing and the credit and consumption booms and busts.

We calibrate  $F_{hh}^a$  in the true regime-switching process of  $\kappa$  so that  $F_{hh}^a = 92/93 = 0.989$ . This calculation assumes that under rational expectations the true continuation probability of the high securitization regime matches the frequency of continuation “observed” in our financial innovation experiment (out of 93 periods in which the economy starts with  $\kappa = \kappa^h$ , it stays in the same state in 92 of them). With this calibration of  $F_{hh}^a$  and conditional on observing  $\kappa^h$  at date 1, the probability of observing  $\kappa^h$  the following 91 periods is 0.374. We assume  $F_{ll}^a = 0.5$  based on the estimates of Cogley and Sargent (2008) implying an average duration of 2 quarters for the  $\kappa^l$  state, which can be viewed as a somewhat short duration.

### 3.2 Quantitative Findings

Figure 5 plots “conjectured” ergodic distributions of bond holdings for dates 1 and 92 in the learning model and the true ergodic distribution for the RE model. It is critical to note a key difference between these distributions: The ergodic distribution of the RE model represents the stochastic stationary state of the economy both under rational expectations and in the learning model (since in the long run agents learn the true regime-switching process  $F^a$ , and thus the long-run equilibrium is the same as under rational expectations). The “conjectured” ergodic distributions for dates 1

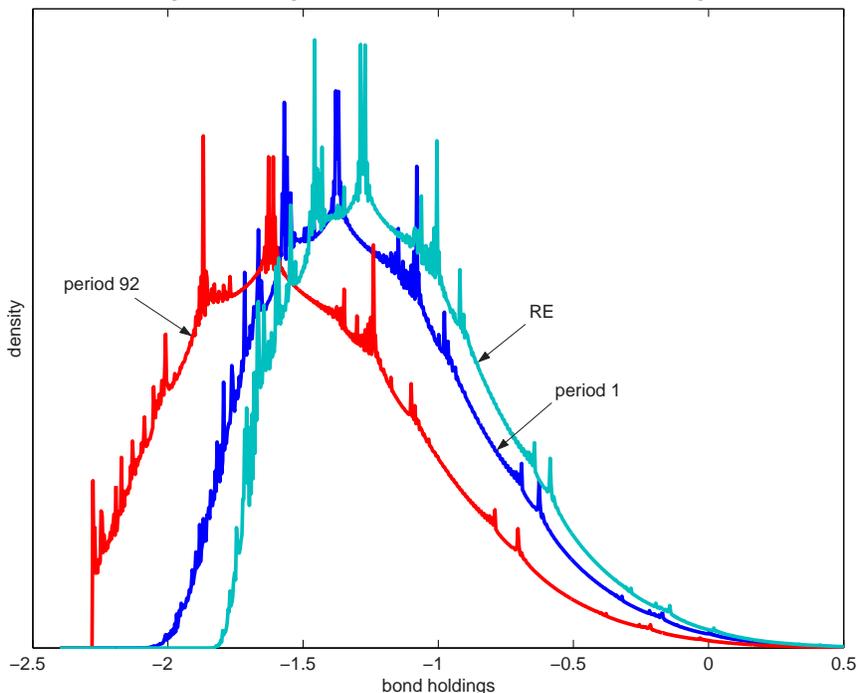
and 92 in the learning model do not represent the model’s stochastic stationary state. Instead, they are the agents’ projections, or conjectures, of what the long-run equilibrium would look like if they forecast the dynamics of  $F^a$  using the date 1 and date 92 beliefs respectively (i.e. these distributions assume that the corresponding period’s beliefs about the transition probability matrix of  $\kappa$  are correct).

This assumption is clearly not valid in the model’s equilibrium dynamics, because in the short run beliefs can change substantially from one period to another and deviate sharply from the true transition probability matrix. Plotting the conjectured and RE long-run distributions is useful, however, for illustrating the build-up of optimism during the optimistic phase of the learning model’s dynamics. Note in particular that bond holdings in the interval  $[-2.27, -1.75]$  are never a long-run outcome in the RE distribution, but they would be projected to be using the agents’ date-1 or date-92 beliefs as correct. It takes observing only the first realization of  $\kappa^h$  for agents to turn overly optimistic with respect to rational expectations, and conjecture that long-run debt positions in the  $[-2, -1.75]$  range are probable long-run equilibria, while in the RE distribution they have zero long-run probability. As optimism builds, the highest debt conjectured to have positive long-run probability of occurring rises, and the mass of probability assigned to debt levels larger than the largest debt under rational expectations also rises. This process peaks at the peak of the optimistic phase in date 92.

Figure 5 shows that even at the peak of optimism agents do not choose bond holdings below  $-2.28$ . This is because bond holdings lower than this amount leave the agents exposed to the risk of negative consumption after a non-zero-probability sequence of income and securitization shocks. To demonstrate this point, consider the “worst case scenario” in which output remains at its lowest realization in the Markov chain,  $\min(y)$ , and  $\kappa = \kappa^l$ . If agents borrow as much as the credit constraint allows,  $b = -\kappa^l \min(y)$ , and we plug in this debt and the values of  $\min(y)$  and  $\kappa^l$  in the budget constraint (2), we find that  $b > -2.28$  must hold for the agents to be able to rule out negative consumption in all states of nature. This debt limit is a variant of Aiyagari’s “natural debt limit” in incomplete markets economies.

We study next the learning model’s equilibrium dynamics. We defined in the calibration a trajectory of realizations  $\kappa^T$  with  $T=100$ , the first 92 are equal to  $\kappa^h$  and the last 8 are equal to  $\kappa^l$ , and we used the calibrated values of the learning process to compute the sequence of beliefs  $[f(F^s|\varepsilon^t)]_{t=1}^{100}$ . We can then solve the corresponding CDPPs to obtain  $[\hat{V}_t(b_t, \varepsilon_t), \hat{b}_{t+1}(b_t, \varepsilon_t)]_{t=1}^{100}$ . Given these bond decision rules, we can determine the equilibrium bond dynamics by chaining

Figure 5: Ergodic Distributions of Bond Holdings



Notes: This figure plots the ergodic distribution of bond holdings implied by the learning model in periods 1 (initial period) and 92 (peak of optimism) as well that of the rational expectations model marked by “RE.”

the decision rules that correspond to each period’s beliefs. For dates  $t = 1, \dots, 92$ , the equilibrium bond dynamics are given by  $b_2 = \hat{b}_1(b_1, \kappa^h, y)$ ,  $b_3 = \hat{b}_2(b_2, \kappa^h, y), \dots, b_{92} = \hat{b}_{91}(b_{91}, \kappa^h, y)$ , and those for dates  $t = 93, \dots, 100$  are given by  $b_{93} = \hat{b}_{92}(b_{92}, \kappa^l, y), \dots, b_{100} = \hat{b}_{99}(b_{99}, \kappa^l, y)$ . Notice we have solutions for this sequence for all initial conditions  $b_1$  defined in the discrete grid of  $b$  over which we solved the model, and for all values of  $y$  in the discrete Markov process of output. Hence, in analyzing the dynamics we set  $b_1 = -1.125$  and  $y_1 = 1$ , which aim to match observations from U.S. data for 1983, and study forecast functions to average across the exogenous output dynamics over time (full details are provided in the Appendix).

Figure 6 plots the forecast functions for bond holdings, consumption, and the saving flow as percent deviations from their long run means (which are the same as the long-run means of the RE scenario). The solid (blue) lines correspond to the learning model while the dashed (red) lines are for the RE model. Note that even the RE model generates some dynamics in this exercise, because  $b_1$  is not the mean of the RE scenario and also because we are using a particular time-series of realizations of  $\kappa$  (instead of using the true Markov process of  $\kappa$  to build the forecast functions of  $\kappa$  conditional on  $\kappa_1 = \kappa^h$ ).

Agents consistently borrow and consume more, and save less, than under rational expectations

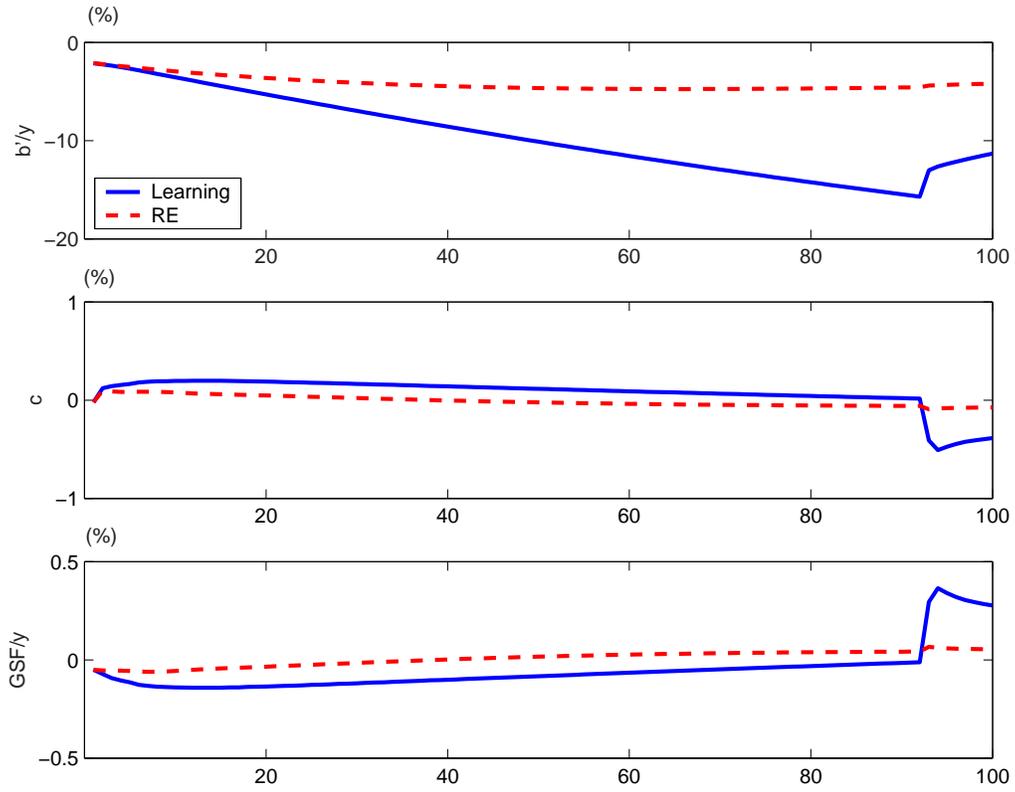
during all of the optimistic phase. Bond holdings as a share of output decline in nearly linear fashion by about as much as 16 percentage points below the long-run average, and then bounce back about 3 percentage points as optimism starts disappearing in period 93. These dynamics are in line with the downward trend in net credit market assets observed in the data. Since bond holdings reach -126 percent of output at date 92 in our forecast functions, we can conclude that the informational friction accounts for about 21 percent (or 14 percentage points) of the 65 percentage point build up of debt observed in the data during 1984-2006 (from 113 to 177 percent of GDP ).

In line with the debt dynamics, the forecast functions show low saving rates coupled with overconsumption (with respect to rational expectations) during the optimistic phase, followed by sharp corrections at date 93. The rise in consumption and the drop in saving are very persistent but small relative to long-run averages. Recall, that output is set to its mean and  $b_1 = -1.125$  is near the RE long-run average, so the changes in consumption and savings observed in 6 are mainly the result of the optimism produced by the informational friction, in the presence of a negligible deviation from long-run average in bond holdings. The gap between rational expectations and learning scenarios with regards to consumption shrinks during the optimism phase as the interest payments to the rest of the world increase due to gradual accumulation of debt in this phase. In other words, as the debt builds, agents end up using the additional net borrowing to repay the interest on the existing debt rather than financing higher consumption.

The lower bond holdings become more likely to be chosen as optimism builds. Figure 7 plots the evolution of cumulative density functions of bond holdings for periods 1, 25, 92 and 100. From period 25 to 92, the distributions become significantly more skewed to the left. Note that the underlying distributions plotted here are different from those in Figure 5. Those plotted in Figure 5 were “conjectured” ergodic distributions as explained before while those in Figure 7 show the evolution of bonds given  $y_1 = 1$ ,  $b_1 = -1.125$  and the assumed path for  $\kappa$ . For example in period 1, since the agents know exactly the initial point, all of the mass is in  $b_1 = -1.125$ . The bond holdings shown in the top panel of Figure 6 are the means of the corresponding period’s distributions plotted in Figure 7.

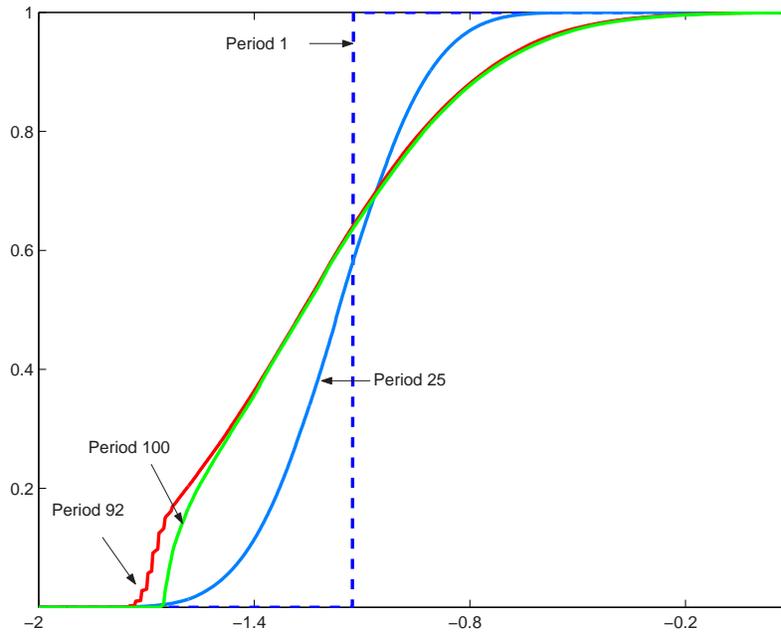
Tables 2 and 3 show the statistical moments of the macro variables for the data in the 1984-2008 period and for the learning and RE setups. We report the long-run moments of the RE model in Table 2, which are identical to those of the learning model, and short-run moments in Table 3 for each model are based on the 100-period sample of the financial innovation experiment (with the same initial conditions for bond holdings, output and  $\kappa$  used in the forecast function calculations).

Figure 6: Forecast Functions



Notes: This figure plots the forecast functions of bond holdings to output ratio, consumption, and gross saving flow to output ratio ( $GSF/y$ ) as percentage deviations from their long run means implied by the rational expectations scenario.  $GSF/y$  is calculated as  $(qb' - b)/y$ . Realizations of  $\kappa$  are as described in the text,  $\kappa^h$  in the first 92 periods and  $\kappa^l$  in the remaining 8. Bond holdings and output in period 0 are set to -1.125 and 1, respectively based on 1983 data.

Figure 7: Cumulative Density Function of Bond Holdings



Notes: This figure plots the evolution of cumulative density functions of bond holdings as optimism builds. Realizations of  $\kappa$  are set to the path described in the text,  $\kappa^h$  in the first 92 periods and  $\kappa^l$  in the remaining 8. Bond holdings and output in period 0 are set to -1.125 and 1, respectively based on 1983 data.

The short-run moments can be calculated following one of two approaches to track the evolution of  $\kappa$ . One approach uses the true regime-switching probability matrix for the financial regimes, and the other uses the pre-determined series of  $\kappa$  values. Under the first approach, the differences in moments between the learning and RE scenarios arise only from differences in the decision rules.

The most important result illustrated in 3 is that the learning model generates significantly more debt in the short run than the RE model. Table 3 shows that the change in bond holdings from period 1 to 92 is about 11 percentage points of GDP lower in the learning setup than in the RE model. Hence, financial innovation when agents are uncertain about the true nature of the new financial environment produces significant over-borrowing relative to what RE predicts.

## 4 Conclusion

In this paper we introduced informational frictions into a simple consumption-savings model to study the implications of financial innovation in an environment in which agents do not know the true regime-switching probabilities across high and low debt securitization regimes. Agents are Bayesian learners, however, so in the long run, after observing a sufficiently long history of

Table 2: Long-run Moments

	Data	L-R
$\sigma(c)$	0.698	0.931
$\sigma(y)$	1.645	1.616
$\sigma(GSF/y)$	4.078	1.460
$\rho(c, y)$	0.712	0.415
$\rho(GSF/y, y)$	0.138	0.909
$\rho(c, c_{-1})$	0.796	0.937
$\rho(y, y_{-1})$	0.838	0.835
$\rho(GSF/y, (GSF/y)_{-1})$	0.792	0.824

Note: Data column reports statistics using the data in 1984-2008. Consumption, and output are logged and HP filtered with smoothing parameter 1600. L-R column reports the long-run moments that are identical for both learning and RE scenarios.  $GSF/y$  is calculated as  $(qb' - b)/y$ .

Table 3: Short-run Moments

	Data	S-R w/ true $F$		S-R w/ pre-set $\kappa$	
		RE	Learning	RE	Learning
$E[GSF/y]$	0.0826	0.0078	0.0100	0.0080	0.0102
$E[b_{92} - b_1]$	-0.6099	-0.0217	-0.1272	-0.0245	-0.1359

Note: Data column reports statistics using the data in 1984-2008. Columns 2-3 report the moments calculated using the true transition probability matrix while columns 4-5 assume a given path for  $\kappa$ . Realizations of  $\kappa$  are set to the path described in the text,  $\kappa^h$  in the first 92 periods and  $\kappa^l$  in the remaining 8. Bond holdings and output in period 0 are set to -1.125 and 1, respectively based on 1983 data.  $GSF/y$  is calculated as  $(qb' - b)/y$ .

realizations of the financial regimes, they learn the true the transition probability matrix driving the Markov-switching process.

This imperfect information and learning structure is motivated by the fact that, when financial innovation emerged in the early 1980s, financial markets participants did not have available a long time series of data to correctly calculate the true transition probability matrix across financial regimes. Since they were imperfectly informed about the true transition probabilities across regimes, and therefore about the true mean duration of a newly introduced high securitization regime, it is reasonable to argue that they would turn overly optimistic because their prior knowledge about the true riskiness of the new financial environment was limited, and the period between the mid 1980s and the mid 2000s was characterized by a very high ability to securitize debt. Despite the simplicity of the model, the informational friction and learning generate a substantial amount of over-borrowing, which accounts for roughly one quarter of the massive surge in net credit assets of U.S. nonfinancial sectors observed between 1983-2006. Moreover, the model also predicts a credit crunch, a collapse in consumption and a surge in private savings when the economy experiences the first realization of the low securitization regime and in the periods that follow.

Our work leads us to ponder an important tradeoff the financial innovation process. By definition, the true riskiness of brand-new financial products cannot be correctly evaluated when these products are first introduced. Hence, exposure to the credit boom-bust process we studied in this paper comes along with the potential benefits of financial innovation. Strong regulation and supervision of financial intermediaries in the early stages of financial innovation are therefore critical. Capital requirements can be used to contain the over-borrowing mechanism we studied in this paper, but on the other hand they have to be used carefully, because tight regulatory constraints on securitization introduce additional distortions on financial intermediation and can undermine all the benefits of financial innovation.

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## Appendix

We calculate the forecast functions by taking  $y_1 = 1$ ,  $b_1 = -1.125$ , the distribution of  $y$ , and the assumed path for  $\kappa$  as given. We then evaluate the predicted path of the bond holdings:<sup>7</sup>

$$b_2 = \hat{b}'_1(b_1, \kappa^h, 1) \quad (\text{A-1})$$

$$E[b_3] = \sum_{i=1}^{NY} Pr(y_2 = y^i | y_1 = 1) \hat{b}'_2(b_2, \kappa^h, y^i) \quad (\text{A-2})$$

$$E[b_4] = \sum_{i=1}^{NY} \sum_{j=1}^{NY} \sum_{m=1}^{NB} Pr(y_3 = y^j | y_2 = y^i) Pr(y_2 = y^i | y_1 = 1) Pr(b_3 = b^m | b_2) \hat{b}'_3(b^m, \kappa^h, y^j) \quad (\text{A-3})$$

...

In some sense we construct a probability tree where the uncertainty is about the future realizations of  $y$ . For example, at time 2, as displayed in Equation (A-2), the economy can possibly face  $NY$  different realizations of  $y$  and therefore we compute the bond holdings that will be chosen under different contingencies about  $y$  using the time 2 decision rule and calculate the expected value. Similarly, at time 3, the economy can be in one of  $NY$  different states for  $y$  but it can go to a particular  $y_3 = y^j$  from  $NY$  different possible  $y_2 = y^i$ . In the case of the bond holdings, at time 3, the economy might start from  $NB$  different bond holdings,  $b_3 = b^m$ , and we need to keep track of this uncertainty as displayed in Equation (A-3).

Note that in the calculation of the forecast functions, we are essentially calculating the mean of a distribution at every point in time. Figure 7 plots the cumulative density functions of these distributions. Consistent with Equation (A-1) above, there is no uncertainty at time 1 and all the mass of the distribution at time 1 is concentrated around the value of  $b_2$ .

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<sup>7</sup>The paths for  $c$  and  $GSF/y = (b'/R - b)/y$  are evaluated in the same fashion.