Optimal Policy with Occasionally Binding Credit Constraints *

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Abstract

We study optimal policy in a small-open economy in which a foreign borrowing constraint binds occasionally. In the model that we set up scope for policy arises because of the pecuniary externality stemming from the presence of a key relative price in the borrowing constraint. The objective of the paper is to characterize an optimal policy rule for both crisis periods when the borrowing constraints binds (i.e., sudden stops), and for periods of relative tranquility in which the constraint does not bind (i.e., normal times). That is, should the government intervene before the constraint actually binds? Or does the optimal policy have a precautionary component? In the model, the policy instrument is a distortionary tax on consumption of non-tradable goods that affects their relative price and can be interpreted as an intervention to affect the real exchange rate. We find that the optimal policy is non-linear. If the constraint is not binding, the optimal tax rate is zero, as in an economy without credit constraint, and hence there is no precautionary component in the optimal policy or intervention on the real exchange rate in normal times. If the constraint is binding, the optimal policy is to intervene aggressively by subsidizing the consumption of non-tradable goods. The decrease in precautionary saving induced by such a policy is quantitatively small, but this does not necessarily imply a small welfare impact of the optimal policy.

1 Introduction

Emerging market countries have experienced periodic crises that cause significant economic dislocation. These episodes, labeled "sudden stops" by Calvo (1998), are characterized by a sharp reversal in private capital flows, large drops in output and consumption, coupled with large declines in asset prices and the real exchange rate. Progress has been made in understanding optimal policy responses in models in which the economy is in a sudden stop.[1] In this paper we address the complementary issue of optimal policy for an economy that might face a sudden stop. That is how much of a precautionary component should there be in the optimal policy in normal times? At what point before a possible sudden stop should the government intervene? Should the government wait until the sudden stop strikes, or should it intervene as the conditional probability of the crisis rises? In addressing these questions, we solve for optimal policy both in and away from the crisis period. We thus address key issues related to the design of optimal policy with financial frictions.

We model sudden stops as a situation in which an international borrowing constraint is binding. The constraint in our model binds endogenously, depending on agents' choices as well as the state of the economy. When the constraint does not bind the model economy exhibits normal business cycle fluctuations. The presence of the borrowing constraint, though, leaves the economy vulnerable to the possibility that a samll negative shocks pushes it into the binding region, for certain levels of foreign indebtedness. When this happens, the economy enters a crisis period and suffers the economic dislocation typically associated with a sudden stop episode.

To solve for optimal policy in this model we develop a global solution method. That is, we solve for a policy rule across both states of the world, when the constraint binds and when it does not. Such an approach enforces that the rule away from the crisis periods is designed with full knowledge of what the rule will be when the economy enters the sudden stop. This is true for both the policy maker and the agents in the economy. This solution method, while

[1] See Braggion, Christiano, Roldos (2007), Caballero and Panageas (2007), Christiano, Gust, and Roldos (2004), Cúrdia (2007), Caballero and Krishnamurthy (2005), Hevia (2008) on the optimal (monetary) policy response to these crisis periods.

computationally costly, is critical for understanding the interaction between precautionary behavior on the part of the private sector with precautionary behavior on the part of the policy maker. The technical challenge in solving such a model is that the constraint binds only occasionally and changes location in the state space of the model depending on the state of the economy.

To our knowledge, there are no contributions in the literature on the analysis of optimal policy in an environment in which a borrowing constraint both binds occasionally and is endogenous to the decisions in the model. The most closely related works to ours includes Durdu and Mendoza (2005) who analyze fixed alternative policy rules in a similar environment, but do not characterize an optimal decision rule for a government instrument as we do. Adams and Billi (2006a and b) study optimal monetary policy in a closed economy, new Keynesian model in which there is zero lower bound on interest rates. Their zero-bound constraint is fixed and does not evolve endogenously. Bordo and Jeanne (2002) and Devereaux and Poon (2004) investigate precautionary components of optimal monetary policy responses to asset prices and sudden stops, respectively, but not in the context of a fully specified DSGE models.

Our endogenous borrowing credit constraint is embedded in a standard two-sector (tradable and non-tradable good) small open economy (e.g., Obstfeld and Rogoff, 1996, Chapter 4) in which financial markets are not only incomplete but also imperfect, as in Mendoza (2002, 2008). The asset menu is restricted to a one period risk-free bond paying off the exogenously given foreign interest rate. In addition, we assume that access to foreign financing is constrained to a fraction of households' total income. Foreign borrowing is denominated in units of the tradable good but it is leveraged on income generated at different relative prices (i.e. the relative price of non-tradeable good), a specification of the borrowing constraint that captures "liability dollarization" a key feature of emerging market capital structure (e.g., Krugman (2002)).[2]

Within this framework, a scope for policy arises because of a pecuniary externality stemming from the presence of a relative price in the borrowing constraint (see also Bianchi

[2] The latest wave of crises in emerging Europe is striking evidence of the importance of such feature.

(2009), Korinek (2008) and Lorenzoni (2008) who analyze the same externality). In representative agent models, the individual takes prices as given. But, in our model, the borrowing constraint is at the individual level, so the agent knows that work effort affects income and hence collateral for borrowing, and borrowing decisions affect the probability of hitting the constraint in the future. In a sudden stop when the constraint binds, however, tradeable consumption and the relative price of the nontradedable fall, and the agent does not internalize the fact that her consumtion and labor choices to cope with the constraint individually further lower this price.[3] By internalizing this externality, a Ramsey-planner will use its policy tools to relax the borrowing constraint. Specifically, in our policy setting, a distortionary subsidy on non-traded consumption (interpretable in terms of exchange rate policy) financed through lump-sum transfers is used to support the relative price of nontradeable goods.

We find that the optimal policy is highly non-linear. If the credit constraint is not binding, optimal policy would mimic the one that would arise in an economy without a credit constraint (a zero tax rate in our model). Therefore, there is no precautionary component to the optimal policy and no intervention on the real exchange rate in normal times. If instead the credit constraint is binding, the optimal policy is to intervene aggressively to subsidize the price of non-tradable consumption (i.e. to defend the real exchange rate). This subsidy increases demand for nontradable goods. The worker then receives a higher wage, which increases labor supply and by extension the supply of nontraded goods. The induced change in the composition of consumption and the increase in income that serves as collateral for the foreign borrowing are the mechanisms through which policy operates as to relax the borrowing constraint.

The commitment to implement the optimal policy has a small impact on precautionary saving, but may have significant welfare consequences. In this class of models, agents self insure against the low-probability but high cost possibility of a sudden stop generated by the occasionally binding credit constraint. This is accomplished through by accumulation of foreign assets or reducing foreign liabilities. When comparing the solution of the model with

[3]In the competitive equilibrium of our model, consumption of non-tradable goods and labor effort can be too low or too high in a sudden stop relative to a planning equilibrium depending on the assumptions on functional forms and parameter values for household preferences.

and without the optimal policy, we find that there is only slightly less precautionary saving with the optimal policy. The additional foreign borrowing allows agents to increase average consumption and work less to support the higher level of borrowing. As our preliminary welfare calculations show, this small difference in precautionary saving can have large welfare consequences because the welfare gains for increasing consumption and decreasing labor efforts in constrained states of the work are much larger than in unconstrained states of the world.

This paper is related to three different strands of literature. The first focuses on financial frictions that may help replicate the main features of the business cycle in emerging market economies (for example, Gertler, Gilchrist and Natalucci (2007) Mendoza (1991, 2002, and 2008), and Neumeyer and Perri (2005)). Note here that, while Gertler et al. (2007) focus on a similar amplification mechanism, their financial accelerator framework would require the existence of large shocks to generate quantitatively realistic the sudden stops nested within regular business cycles. But such large shocks would affect the models ability to describe the business cycle fluctuations satisfactorily. Differently from Mendoza (2008) and Neumeyer and Perri (2005), mainly for simplicity, we abstract from working capital financing considerations. The second strand of literature (Korinek (2008), Lorenzoni (2008), and Bianchi (2009)) study the same pecuniary externality that creates scope for policy intervention in our model, but focuses on the consequences of such externality for private sector over or under-borrowing and do not study optimal government policy in such an environment (see also Uribe (2006) on this). Finally, the third strand focuses on the analysis of optimal fiscal and monetary policy in dynamic general equilibrium models (for example, Chari and Kehoe, 1999; Schmitt-Grohé and Uribe, 2004). While studies of emerging market business cycles can provide a realistic description of the economic environment in which these economies operate, the question of how policy should be set in such environments, particularly outside of the crisis period, is very much open. The optimal fiscal and monetary policies developed in standard open economy models may not be appropriate to provide insight on how policy should be set in the environment faced by emerging market economies.

The rest of the paper is organized as follows. Section 2 presents analytic results from a simple deterministic two-period model to illustrate the source and the nature of policy problem. Section 3 describes the fully speficied DSGE model we use. Section 4 discusses its calibration, solution, and basic properties. Section 5 contains the main results of the paper, characterizing the optimal policy and discussing its working. Section 6 concludes. Technical details, including the numerical algorithm we use to solve the model, are in appendix.

2 Lessons from a deterministic, two-period economy

Before turning to the full model we first study a simple two-period deterministic economy that can be solved analytically. In order to understand the nature of the policy problem we will consider two cases: a one good economy and a two-good economy. The objective is to illustrate the core of our optimal policy problem by comparing the solution of the competitive economy with the social planner's one in both cases, and then examine the implications for the Ramsey problem.

2.1 The one-sector case

We start with a two-period, deterministic small open economy framework. Production takes place only in the first period and agents are subject to an endogenous borrowing constraint. The household maximizes the two-period utility flow:

$$u(c_1, c_2, h_1) = \log c_1 - \frac{h_1^\delta}{\delta} + \beta \log c_2 \tag{1}$$

where c_1 and c_2 are consumption in period 1 and 2, and h_1 is period 1 labor. The household is subject to the following period-specific budget constraints:

$$w_1 h_1 + \pi + b_1 - T = (1 - \tau) c_1 + b_2$$

$$c_2 = b_2 (1 + i) + Y_2$$

The wage rate in period 1 is w_1, b_i is the net foreign asset position in period i, π is the total profit from owning the firm, T are lump-sum taxes, i is the net real interest rate which is taken as given, $\tau > 0$ (< 0) is the distortionary subsidy (or tax) on period 1 consumption, and Y_2 is the period 2 endowment of the tradeable good. The agents' ability to borrow in the world market for one period bonds is limited by a fraction of their income flow:

$$b_2 \geq -\frac{1-\phi}{\phi}\left(w_1 h_1 + \pi\right),$$

where ϕ is a parameter that governs the tightness of the borrowing constraint. The household solves the usual lifetime utility maximization problem subject to the two-period budget constraints and the borrowing constraint.

On the production side we assume that the firm is owned by the domestic household and produces the tradeable good, Y, only in the first period:

$$Y_1 = l_1^{\alpha}$$

where l_1 is the labor input. The firms' profit maximization problem is static. The firm maximizes its profits, π, by choosing the amount of labor input:

$$\max \pi = l_1^{\alpha} - w_1 l_1.$$

We close the model by specifying the government behavior. We assume that the government runs a balanced budget rule so that the distortionary tax (subsidy) on consumption is offset with lump sum taxes (i.e. $T = \tau c_1$).

The competitive equilibrium in this economy is characterized by the first order conditions of the household and the firm and the government budget constraint. The equilibrium conditions for this model are straightforward to derive. (See the Appendix for a full derivation of the solution). The key equilibrium condition governing the competitive equilibrium allocations can be expressed as:

$$h_1^{\delta-1} = \left[\frac{1}{c_1(1-\tau)} + \frac{1-\phi}{\phi}\left(\frac{1}{c_1(1-\tau)} - \frac{1}{c_2}\beta(1+i)\right)\right]\alpha h_1^{\alpha-1} \tag{2}$$

We note here that the equilibrium allocation of labor is affected by the liquidity constraint: indeed, the marginal benefit of supplying more labor is higher when the constraint is binding as supplying more labor increases agents' income and their ability to borrow. On the other hand, an increase in distortionary taxation has a negative effect on labor supply by reducing the marginal utility of consumption and the marginal benefit of offering more labor when the constraint is binding.

We now consider the constrained-social planner problem.[4] The social planner problem consists in maximizing the household's utility (1) subject to the aggregate resource constraints and the international borrowing constraint. The period budget constraints are:

$$h_1^\alpha + b_1 = c_1 + b_2,$$

$$c_2 = b_2(1+i) + Y_2,$$

while the borrowing constraint can be expressed as:

$$b_2 \geq -\frac{1-\phi}{\phi} h_1^\alpha.$$

The solution of the planner problem (see the Appendix for full derivation) is characterized by the following equilibrium condition:

$$h_1^{\delta-1} = \left[\frac{1}{c_1} - \frac{1-\phi}{\phi}\left(\frac{1}{c_1} - \frac{1}{c_2}\beta(1+i)\right)\right]\alpha h_1^{\alpha-1} \tag{3}$$

We compare (2) with (3) : from inspection it is easy to see that the two allocations will be equivalent if and only if $\tau = 0$, regardless of whether the borrowing constraint binds or not.

The Ramsey planner that maximizes household utility subject to the resource constraint (as the social planner) and the first order conditions of the competitive allocation will also set $\tau = 0$ in every state in this case. That is, in this case the presence of distortionary taxation creates a inefficient wedge between marginal product of labor and marginal disutility of labor but it doesn't have any bearing on the international borrowing constraint: the distortion imposed by the borrowing constraint cannot be undone as the Ramsey planner is unable

[4]The planner problem is constrained by the international credit limit. The key difference between the competitive equilibrium and the planners problem is that the planner chooses quantities directly, rather than quantities being driven by price movements as in the competitive equilibrium.

to relax the borrowing constraint with the fiscal distortion we are considering.[5] Thus, the competitive allocation is constrained-efficient in this case (i.e., constrained by the existence of the borrowing constraint) and in this one-good economy then there is no role for distortionary taxation.[6]

2.2 The two-sector case

We now consider a two-sector small open economy framework. To keep the analysis simple, we assume that non-tradeable goods are produced and consumed only in the first period, while tradeable goods are endowed and consumed in both periods. Thus, the household maximizes the following utility flow:

$$u(c_1^T, c_1^N, c_2^T, h_1) = \omega \log c_1^T + (1-\omega) \log c_1^N - \frac{h_1^\delta}{\delta} + \beta \log c_2^T \tag{4}$$

where c_1^T and c_1^N are the consumption of the tradeable and non-tradeable goods in period 1, while c_2^T is period 2 consumption of tradeable goods. The household is subject to the following period budget constraints:

$$w_1 h_1 + \pi + b_1 - T = (1-\tau) p_1^N c_1^N + c_1^T + b_2$$

$$c_2^T = b_2(1+i) + Y_2,$$

in which p_1^N denotes the relative price of non-tradeable in terms of tradeable, Y_1 is the endowment of tradeable goods in period 1, and distortionary taxation applies to non-tradeable consumption. The international borrowing constraint is similar to before as agents can borrow only a fraction of the current income flow:

$$b_2 \geq -\frac{1-\phi}{\phi} (w_1 h_1 + \pi).$$

[5] In this simple two-period example it is possible to show that the same conclusion will hold also in the case in which distortionary taxation is on the wage rate rather than consumption.

[6] In the context of a one good economy it would be possible for the Ramsey allocation to differ from the social planner's one when the policy tool affect directly the collateral constraint. This might arise for example when net income rather than gross income is used as a collateral in the case described in the text. At $\tau = 0$ the social planner and the Ramsey allocation will coincide but the optimal τ will differ in the two cases.

The first order conditions of the household can be summarized as:

$$\frac{(1-\omega)\, c_1^T}{\omega c_1^N} = (1-\tau) p_1^N,$$

$$-\frac{\omega}{c_1^T} + \frac{\beta}{c_2^T}(1+i) + \lambda = 0,$$

$$h_1^{d-1} = \left(\frac{\omega}{c_1^T} + \frac{1-\phi}{\phi}\lambda\right) w_1$$

where λ is the multiplier associated with the international borrowing constraint. From the last equation we can see that the effects of the liquidity constraint and distortionary taxation. When the liquidity constraint binds the marginal benefit of supplying one unit of labor is higher as in the previous case. But now there is no direct effect of distortionary taxation on the labor choice, and changes in taxation might generate an indirect effect through changes in the relative prices of non-tradable goods, thereby also affecting the intratemporal allocation between tradeable and non-tradeable consumption.

The firm's problem is also similar to the one sector case, the only difference being that now production choices are related to the non-tradeable sector. The firm maximizes profits,

$$\pi = Y_1 + p_1^N h_1^\alpha - w_1 h_1,$$

by choosing only the labour input, and its first order condition is

$$w_1 = p_1^N \alpha h_1^{\alpha-1}.$$

As before, the government follows a balanced budget rule such that:

$$T = \tau p_1^N c_1^N.$$

The competitive allocation of the two-good economy is then characterized by the first order conditions of the household and the firm, the government budget constraint, and the equilibrium conditions in the tradeable and non-tradeable goods and labor markets. By combining the first order conditions and substituting out the multipliers we obtain the following equation

$$\frac{(1-\omega)}{\omega}\frac{c_1^T}{c_1^N} = \frac{(1-\tau) h^{\delta-\alpha}}{\alpha\left(\frac{\omega}{c_1^T} + \frac{1-\phi}{\phi}\left(\frac{\omega}{c_1^T} - \frac{\beta(1+i)}{c_2}\right)\right)} \tag{5}$$

We now examine the corresponding constrained social planner problem. The social planner maximizes the household's utility (4); subject to the aggregate resource constraint, and the borrowing constraint

$$c_1^T + b_2 = Y_1 + b_1$$

$$c_1^N = h_1^\alpha,$$

$$c_2^T = Y_2 + b_2(1+i).$$

Here by combining the household's borrowing constraint with firms' profit we can express the borrowing constraint from the planner's perspective as:

$$b_2 \geq -\frac{1-\phi}{\phi}\left(Y_1 + p_1^N \left(l_1^N\right)^\alpha\right),$$

in which we substitute $\frac{(1-\omega)}{\omega}\left(\frac{c^T}{c^N}\right)\frac{1}{(1-\tau)} = p_1^N$, so that we can rewrite it in terms of quantities as:

$$b_2 \geq -\frac{1-\phi}{\phi}\left(Y_1 + \frac{(1-\omega)}{\omega}\frac{c_1^T}{(1-\tau)}\right). \tag{6}$$

Once we derive the first order conditions of this problem, it is possible to express the social planner equilibrium allocation in terms of the following condition:

$$\frac{(1-\omega)}{\omega}\frac{c_1^T}{c_1^N} = \frac{h^{\delta-\alpha}}{\left(\frac{\omega}{c_1^T}\right)\alpha}. \tag{7}$$

By comparing (5) and (7) we can look for a τ such that the competitive allocation replicates the social planner's one. When the constraint does not bind, the two allocations will coincide if and only if τ is equal to zero. On the other hand, when the constraint does bind, τ must be set so that $1-\tau = 1 + \frac{1-\phi}{\phi}\left(\frac{\omega}{c_1^T} - \frac{\beta(1+i)}{c_2}\right)$, where the term in parentheses is the multiplier of the credit constraint associated with the social planner allocation, for the competitive allocation to replicate the social planner's one.[7] Thus, if the $\tau = 0$ in the binding region, the two allocations would differ.

[7]Indeed, we note that consumption here is evaluated at the social planner allocation.

In this simple two-good small open economy there is a scope for policy that stems from the presence of a relative price in the borrowing constraint. This gives rise to the same pecuniary externality analyzed by Bianchi (2009), Korinek (2008) and Lorenzoni (2008). The key difference between the competitive and the social planner allocation is that the social planner internalizes the effects of changes in the relative price on the borrowing constraint when this binds. Indeed, as we shall see, in the Ramsey allocation in which the planner chooses the optimal τ as to maximizes household utility subject to the competitive equilibrium conditions, the planner will act by varying τ as to attempt to relax the borrowing constraint.[8]

Figure 1 characterizes our policy problem in the case of a simple numerical example of our two-period economy.[9] It plots welfare as a function of the value of the planner's tax instrument for the competitive allocation and the social planner's one. The maximum in terms of welfare of the competitive equilibrium allocation is given by the Ramsey allocation (i.e. the one in which the planner chooses τ as to maximize agents' utility subject to the resource constraint and the competitive equilibrium conditions). In general, this is different from the maximum attainable by the social planner (i.e. the one in which the planner chooses τ as to maximize agents' utility subject to the resource constraints only). From the Figure, we can see that the optimal τ for the social planner is $\tau = 0.4$ while the Ramsey planner would set $\tau = 0.34$.(i.e. the social planner will set a higher subsidy and achieve a higher welfare that the Ramsey planner). In contrast, in the one good economy, we would have that both welfare-lines would achieve their maximum for $\tau = 0$.

3 Model

This section describes a standard two-sector (tradable and non-tradable good) small open economy in which financial markets are not only incomplete but also imperfect, as in Mendoza (2002, 2008). Key features of Mendoza's model that we retain include an occasionally binding credit constraint and production of goods with a variable labor input. We simplify

[8]We note here that this pecuniary externality would arise also in a framework in which future income rather than current income is used as a collateral in the borrowing decision by agents (as in Korinek (2008)).

[9]In this numerical example we set $Y_1 = 1$, $Y_1 = 3, \gamma = 0.9, \alpha = 0.5, d = 1.5, b_1 = 0.2$ and $\beta = 1$, $i = 0$.

his model by considering only one disturbance, a shock to a tradable good endownment. The specification of endogenous discounting is also simplified by assuming that the agents' discount rate depends on aggregate consumption as opposed to the individual one, as in Schmitt-Grohé and Uribe (2003).[10]

3.1 Households

There is a continuum of households $j \in [0,1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \exp\left(-\theta_t\right) \frac{1}{1-\rho} \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{1-\rho} \right\}, \tag{8}$$

with C_j denoting the individual consumption basket and H_j the individual supply of labor. For simplicity we omit the j subscript for the remainder of this section, it is understood that all choices are made at the individual level. The elasticity of labor supply is δ, while ρ is the coefficient of relative risk aversion. In (8) the preference specification follows from Greenwood, Hercowitz and Huffman (1988): in the context of a one-good economy this specification eliminates the wealth effect from the labor supply choice. Here, in multi-good economy the sectoral allocation of consumption will affect the labor supply decision through relative prices. The consumption basket, C_t, is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} \left(C_t^T \right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N \right)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \tag{9}$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of the two goods in the utility function. The discount factor, θ_t, is endogenous and evolves as:

$$\theta_t = \theta_{t-1} + \beta \ln \left(1 + C_t - \frac{H_{j,t}^{\delta}}{\delta} \right)$$

$$\theta_0 = 1.$$

[10] Our formulation corresponds to what Schmitt-Grohé and Uribe (2003) call "endogenous discount factor without internalization". Endogenous discounting pins down a well defined net foreign asset position in the deterministic steady state of the model.

We normalize the price of traded goods to 1. The relative price of the nontraded good is denoted P^N. The aggregate price index is then given by

$$P_t = \left[\omega + (1-\omega)\left(P_t^N\right)^{1-\kappa}\right]^{\frac{1}{1-\kappa}},$$

where we note that there is a one to one link between the aggregate price index P and the relative price P^N. Households maximize utility subject to their budget constraint, which is expressed in units of tradeable consumption. The constraint each household faces is:

$$C_t^T + \left(1+\tau_t^N\right) P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} - (1+i) B_t - P_t^N T^N, \tag{10}$$

where W_t is the real wage, B_{t+1} denotes the foreign lending at the end of period t with gross real return $1+i$, τ_t^N is a distortionary taxes on non-tradables consumption, and T^N is lump sum taxes of non-tradables. Households receive profits, π_t, from owning the representative firm. Their labor income is given by $W_t H_t$.

International financial markets are incomplete and access to them is also imperfect. The asset menu includes only a one period bond denominated in units of tradable consumption. This captures the effects of liability dollarization since foreign borrowing is denominated in units of tradables. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[\pi_t + W_t H_t\right]. \tag{11}$$

The constraint (11) depends endogenously on the current realization of profits and wage income. We don't derive explicitely the credit constraint as the outcome of an optimal credit contract between lenders and borrowers.[11] We could interpret this constraint as the outcome of the interaction between lenders and borrowers in which the lenders are not willing to permit borrowing beyond a certain limit. This limit depends on ϕ that measures the tightness of

[11] As emphasized in Mendoza (2002), this form of liquidity constraint shares some features, namely the endogeneity of the risk premium, that would be the outcome of the interaction between a risk-averse borrower and a risk-neutral lender in a contracting framework as in Eaton and Gersovitz (1981). It is also consistent with anecdotal evidence on lending criteria and guidelines used in mortgage and consumer financing.

the borrowing constraint and current gross income that could be used as a good proxy of future income. [12]

Households maximize (8) subject to (10) and (11) by choosing C_t^N, C_t^T, B_{t+1}, and H_t. The first order conditions of this problem are the following:[13]

$$\frac{C_{C_t^N}}{C_{C_t^T}} = \left(1 + \tau_t^N\right) P_t^N, \tag{12}$$

$$u_{C_t} C_{C_t^T} = \mu_t, \tag{13}$$

$$\mu_t + \lambda_t = \theta_t \left(1 + i\right) E_t \left[\mu_{t+1}\right], \tag{14}$$

and

$$z_H(H_t) = C_{C_t^T} W_t \left[1 + \frac{\lambda_t}{\mu_t} \frac{1 - \phi}{\phi}\right]. \tag{15}$$

where μ_t and λ_t are the multipliers on the budget and liquidity constraint. Equation (12) determines the optimal allocation of consumption across tradable and nontradable goods by equating the marginal rate of substitution between C_t^N and C_t^T with the relative price of non-tradable and distortionary taxation in the nontradable sector. The presence of the tax in this equation makes it clear that policy will be aimed at altering the housholds choice of consumption basket through its affect on relative prices. Equation (13) determines the

[12] A constraint expressed in terms of future income which could be the outcome of the interaction between lenders and borrowers in a limited commitment environment would introduce further computational difficulties that we are avoiding at first pass. Namely this type of constraint implies that a decision made tomorrow that is conditional on states not realized yet – alters the constraint set of choices today. As a result, the problem would become not recursive.

Moreover further complications might be introduces as the lender will condition his lending behavior on the worst possible state.

Despite this technical/computational issue as we discussed in the context of our two-period example the nature of the policy problem is very similar if we consider current or future income as a collateral for lending decisions.

[13] We denote with $C_{C_t^N}$ the partial derivative of the consumption index C with respect to non-tradable consumption. u_C denotes the partial derivative of the period utility function with respect to consumption and z_H denotes the derivative of labor disutility with respect to labor.

multiplier μ_t in (14) in terms of the marginal utility of tradable consumption. Equation (14) determines the optimal choices of foreign bonds and thus saving. Note that when the the credit constraint is binding ($\lambda_t > 0$), the Euler equation incorporates effects that can be interpreted as arising from a country-specific risk premium on external financing. The extent of this affect is governed by the degree of risk aversion. Moreover in this framework there is an intertemporal effect coming from the possibility that the constraint might be binding in the future: this effect is embedded in the term $E_t[\mu_{t+1}]$ and implies that consumption of tradeable goods would be lower compared to the unconstrained case. Finally, (15) determines the optimal supply of labor as a function of the relevant real wage and the multipliers. Again, it is important to note that a binding international credit constraint increases the marginal benefit of supplying one unit of labor since this improves households' borrowing capacity. Consumption, saving, labor effort, and output are therefore distorted by the presence of a binding credit constraint in a manner that depends on the interaction between the two effects highlighted above.[14]

3.2 Firms

The firms are endowed with a stochastic stream of tradable goods, $\exp(\varepsilon_t^T)Y^T$, where ε_t^T is a stochastic process, and produces non-tradable goods, Y^N. Unlike Mendoza (2002), we assume that ε^T follows an autoregressive process of the first order(AR(1) for accuracy). We abstract from other sources of macroeconomic uncertainty, such as shocks to the technology for producing non-tradables, the world interest rate, and the tax rate.

Firms produce non-tradables goods, Y_t^N, with a variable labor input and Cobb-Douglas technology

$$Y_t^N = AH_t^{1-\alpha},$$

where A is a scaling factor. The firm's problem is static and current-period profits (π_t) are:

$$\pi_t = \exp\left(\varepsilon_t^T\right) Y^T + P_t^N A H_t^{1-\alpha} - W_t H_t.$$

[14]The effect of the binding constraint on the labor supply choice is the one that we capture in the two-period version of our economy while the intertemporal effect that induce a reduction of tradeable consumption is absent as at least three periods would be required for this effect to be present.

The first order condition for labor demand is:

$$W_t = (1-\alpha) P_t^N A H_t^{-\alpha}, \tag{16}$$

so the value of the marginal product of labor is set equal to the real wage (W_t).

3.3 Government

Policy is implemented by means of a distortionary tax rate τ_t^N on private domestic nontradables consumption. The tax rate can be negative, in which case the government is subsidizing nontraded consumption. One of the reason for which we choose this as our policy tool is that it is possible to interpret this instrument in terms of exchange rate policy. The government runs a balanced budget in each period. Changes in the policy variable τ^N are financed by lump-sum taxes/transfers on nontradables, T_t^N. The government budget constraint is then given by

$$\tau_t^N P_t^N C_t^N = -P_t^N T_t^N.$$

Lump sum financing of the optimal policy is a simplifying assumption that allows us to focus on the implications of the occasionally binding constraint for the design of stabilization policy, abstracting from other distortions that may arise with alternative financing arrangements. Our interpretation of the lump sum taxation is that the government has accumulated reserves, and once accumulated, these reserves can be used for an intervention at no new cost.

3.4 Aggregation and equilibrium

Combining the household budget constraint, government budget constraint, and the firms' profits, we have the aggregate resource constraint:

$$C_t^T + P_t^N C_t^N + B_{t+1} = \exp\left(\varepsilon_t^T\right) Y^T + P_t^N Y_t^N + (1+i) B_t.$$

All goods produced in the nontraded sector are consumed, the price of the nontraded good adjusts to ensure that this happens in the competitive equilibrium. The output of the traded good is either consumed or used to pay off interest on debt:

$$C_t^T = Y_t^T - B_{t+1} + (1+i)\, B_t. \tag{17}$$

High levels of debt then, imply lower consumption of the traded good. Equation (17) shows the evolution of the net foreign asset position as if there were no international borrowing constraint. In this model though, using the definitions of firm profit and wages, the liquidity constraint implies that the amount that the country as a whole can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[\exp\left(\varepsilon_t^T\right) Y^T + P_t^N Y^N\right]. \tag{18}$$

4 Parameter values and solution

In this section we discuss the parameter values chosen and describe the global solution method that we use in the numerical computations. We then discuss the competitive equilibrium that we compute, show how a sudden stop episode occurs in the model, and compare a sudden stop in the model and the data.

4.1 Parameter values

The parameter values we use are reported in Table 1. These values are set following the work of Mendoza (2002, 2008) and Kim and Ruhl (2008) to the extent possible, but also to facilitate the convergence of the numerical algorithms we use to solve the model. It is important to note that we parameterize the model to match the available empirical evidence on the probability of sudden stop. As we shall see below, the model produces a sharp reversal in capital flows, a large drop in output and consumption, and a large real exchange rate depreciation (proxied by the fall in the relative price of non-tradable goods) that is typical of a sudden stop. In this sense, our model is quantitatively capturing the sudden stop phenomena we observe in the data. However, in our model, we allow for only one shock, which hits only one sector of the economy. This means that the model will not match well the business cycle dynamics in the data. Nonetheless, we know from the work of Mendoza

(2008), that the model has the potential to match also the business cycle dynamics in the data if we were to add more shocks and features to the baseline we use year (See for instance Mendoza, 2002). We do not do so here because of the difficulties of computing optimal policy that require a parsimonious model specification.

We set the world interest rate to $i = 0.0159$, which yields an annual real rate of interest of 6.5 percent; a value that is between the 5 percent of Kehoe and Ruhl (2008) and the 8.6 percent of Mendoza (2008). The elasticity of intratemporal substitution between tradables and nontradables follows from Ostry and Reinhart (1992) who estimates a value of $\kappa = 0.760$ for developing countries.[15] The value of δ is set to 1.193 to yield a steady state value for hours of 0.33 and implying a Frisch elasticity of labor of 2. For simplicity, the elasticity of intertemporal substitution is unitary ($\rho = 1$).

For simplicity, the labor share of production in the non-tradable sector is also assumed to be unitary ($\alpha = 1$). We then normalize steady-state tradable output to one (i.e., $Y_T = 1$) and set ω and A to obtain a steady-state ratio of tradable to non-tradables output of 0.75 (slightly higher than Mendoza, 2002) and a unitary relative price of non-tradables in steady state (i.e., $P^N = 1$).

The tax rate on non-tradable consumption is fixed at $\tau = 0$ to permit accurate welfare comparisons across policy regimes. Government spending and lump sum taxes are also set to zero in the steady state for simplicity.

We set $\beta = 0.029148$ to obtain a steady-state, annual foreign borrowing to GDP ratio of 43 percent in the deterministic steady state. This value corresponds to the average for Mexico during the period 1990-2006 in the Lane and Milesi- Ferretti data set (2007).

The value of the credit constraint parameter (ϕ) determines the probability of a sudden stop. We set this parameter to 0.7, which makes the constraint binding in the deterministic steady state and yields a realistic probability of sudden stop, as typically defined in the empirical literature.[16] For instance, with this parameter value, the model generates sudden

[15]There is considerable debate about the value of this parameter. The estimate we use is consistent with Kehoe and Ruhl (2005) who set this parameter to 0.5.

[16]Note that, when we compare an economy with borrowing constraint to an economy without constraint, we set the value of $\phi = 0.1$ so that the constraint never binds in the ergodic distribution of the model in this

stops according to the definition of Calvo, Izquierdo, and Meja (2008) in 2.1 percent of the quarters when we simulate the model 100,000 periods; a probability that is consistent with the one estimated by these authors. The model generates empirically defined sudden stops that occur with this realistic frequency with a much lower probability that the borrowing constraint binds strictly (i.e., 0.12 percent of the simulated quarterly periods), which is the model consistent definition of sudden stops.

Calvo et al. (2008) define a sudden stop as a capital flow reversal (i.e., the second difference of B_{t+1}) that is more than two standard deviations above the distribution average. To rule out certain episodes from their analysis, these authors also require that the capital flow reversal meets some secondary criteria on either the output drop or the country spread increase associated with it. As we work with simulated data, these additional criteria are not necessary in our case. Jeanne and Ranciere (2008) define a sudden stop empirically as a capital flow reversal in absolute rather than relative terms—i.e., larger than 5 percent of GDP—and estimate a quarterly unconditional probability of sudden stop ranging between 0.5 and 2.5 percent. We can match the probability of sudden stop according to only one of these two alternative empirical definitions of sudden stop. Working with simulated data, it is preferable to adopt the relative criterion (i.e., defining sudden stop relative to the distribution of the capital flow reversals).

Finally, in our analysis, we focus on the behavior of the economy subject to only one stochastic shock, to the endowed tradeable output, which we model as an AR(1) process. Specifically, the shock process for tradable GDP is,

$$\varepsilon_t^T = \rho_\varepsilon \varepsilon_{t-1}^T + v_t, \tag{19}$$

where v_t is an iid N(0,σ_ε^2) innovation. The parameters of this process are set to $\rho_\varepsilon = 0.647$and $\sigma_\varepsilon = 0.02734$, which are the the first autocorrelation and the standard deviation of total GDP reported by Mendoza (2008), respectively.[17]

case.

[17]Results obtained using $\rho_\varepsilon = 0.533$ and $\sigma_\varepsilon = 0.0337$ for tradable GDP as reported by Mendoza (2002), or using $\rho_\varepsilon = 0.95$ and $\sigma_\varepsilon = 0.01$ are similar.

4.2 Solution

The equilibrium of the model can be represented as a recursive dynamic programming problem summarized by the following Bellman equation:

$$V\left(b_t, B_t, \varepsilon_t^T, \tau\right) = \max_{B_{t+1}} \left\{ \begin{array}{c} u\left(C_t - z\left(H_t\right)\right) + \\ \exp\left(-\theta_t\right) E\left[V\left(b_{t+1}, B_{t+1}, \varepsilon_{t+1}^T, \tau\right) |\varepsilon_t^T\right] \end{array} \right\}. \quad (20)$$

The value function, $V\left(b_t, B_t, \varepsilon_t^T, \tau\right)$, depends on three state variables: individual borrowing (b), aggregate borrowing (B), and the stochastic shock to the tradable endowment (ε_t^T), as well as the value the policy instrument (τ). In equilibrium, individual and aggregate borrowing must coincide, but from the perspective of the representative agent of our model the borrowing constraint is imposed at the individual level, taking relative prices as given. Our solution explicitly accounts for this feature of the model specification by treating aggregate and individual debt separately in the value function. Note that the value function also depends on the tax rate, τ, but this is not an active policy instrument in the decentralized equilibrium, but rather a fixed value for the instrument of the government policy. As we noted already, in the competitive equilibrium, we set $\tau = 0$.

A solution for the decentralized equilibrium defined above will be given by (i) a value function $V\left(B_t, \varepsilon_t^T, \tau\right)$ and (ii) a set of laws of motion (hereafter, also called decision rules or policy functions) for aggregate borrowing ($B = G_B^n\left(B, \varepsilon^T, \tau\right)$), aggregate employment ($H = G_H^n\left(B, \varepsilon_T, \tau\right)$), and the relative price of the non-tradable good basket ($P^N = G_{P^N}^n\left(B, \varepsilon_T, \tau\right)$), and function for government transfers ($T = G_T^n\left(B, \varepsilon_T, \tau\right)$) that satisfy the Bellman equation above. Note that while the value function depends on both individual and aggregate borrowing, these decision rules only depend on aggregate borrowing. Since we set $\tau = 0$ the transfer function is also zero in the decentralized equilibirum.[18]

The appendix provides a detailed description of the equations and method we use to solve for the competitive equilibrium. The key ingredient in the algorithm is a transformation of the system of Kuhn-Tucker conditions into a standard system of nonlinear equations that is

[18]For simplicity we will suppress the tax notation in describing the solution to the competitive equilibrium.

due to Garcia and Zangwill (1981). The transformed system can be solved using standard nonlinear equation solution methods.

To solve for optimal policy we add to this solution procedure an optimization step over the instrument of government policy τ, which thus becomes a policy function for the government. The optimal policy function for τ is the tax plan that maximizes the value function:

$$\tau(B, \varepsilon_T) = argmax_\tau \{V(B, \varepsilon_T, \tau)\}. \tag{21}$$

The solution of the optimal policy problem is an iterative procedure. First, given a tax rate, we solve for the competitive equilibrium as described above. Second, given the solution for the competitive equilibrium, we compute the optimization step above selecting a tax plan which, by construction, yields a improvement in welfare. Then we recompute the decentralized equilibrium as a function of the new tax plan, and so on until the procedure converges to a fix point in the decision rule for τ. This descision rule is the optimal value of τ for each pair of B and ε_T. Note that, as the procedure finds a policy function for the dynamic programming problem, finding a fixed point in τ is the same as finding a fixed point in the value function. The government thus uses dynamic programming to solve for τ as a function of the state variables of the economy.

This global solution method is computationally expensive and in practice can be difficult to implement. Nonetheless, this approach is essential to address the question we ask in the paper. To motivate our approach, consider the alternative of combining more conventional perturbation methods with a penalty function following Kim, Kim and Kollman (2008). This alternative method adds a penalty to the utility function that imposes an increasing cost as the economy nears the constraint. At the constraint, however, the penalty is infinite, so the constraint is never actually reached. The model thus becomes differentiable and can be solved with more conventional approximation methods.

There are two key advantages of a global solution method over an approximated solution such as the one by Kim et al. (2008) for the specific questions we ask in the paper. First, with a global method we can match the shape of the optimal policy function accurately, while with an approximated solution it would not be possible. For instance, with a 3rd order approximation and a penalty function, when the constraint is not binding, we would infer the presence of a small precautionary component to optimal policy which is not present in the global solution. This is due to the fact that the 3rd order approximation forces a polynomial function on a decision rule that, in this case, is constant over a significant part of the support before displaying a kink. The polynomial also smooths the kink and displays a departure from the constant in the global solution that cannot be distinguished from a "precautionary component" away from the binding point of the borrowing constraint. With the approximated solution, therefore, we would not know whether there is a precautionary component to policy or just an approximation error as the economy nears the constraint.[19] Second, the moments and the shapes of the ergodic distributions of the endogenous variables are very different under the two solution procedures. This is due to the fact that the third order solution is solved around a deterministic steady state, but in order to approximate accurately the economy behavior at the constraint it must approximate it at the point where the constraint binds. This results in a ergodic distribution for debt that is too concentrated either around the steady state or where the constraint binds, and too symmetric around those points relative to the true distribution. In contrast, in the global solution, the variables spend much time far away from the constraint or the deterministic steady state. This misplacment of the ergodic distributions, and in particular the mean of consumption, implies that welfare calculations based on the third order approximation are not as reliable as those from the global solution. Since analyzing welfare is a critical part of a policy exercise we conduct, this is a fundamental limitation of the approximate solution.

4.3 Competitive Equilibrium

Before turning to our optimal policy results, it is useful to discuss the competitive equilibrium of the model and some of its properties and characteristics. We begin with the policy function for B_t, which is plotted in Figure 2. In this figure, each solid line depicts the policy function for B_t conditional on a particular state of the tradable shock. For illustrative purposes, we

[19] More generally, the global solution method we use spans a larger set of functional forms for the decision rules.

report the decision rule for the worst state (State 1), and progressively better ones (State 4, State 10, etc.), together with the 45 degree line (dashed black line) and the values of B_t that satisfy the constraint exactly for each state (dashed colored lines). In the figure, we can see that the constraint can bind strictly in the model if certain states are received perpetually or if the economy switches from a given state to a worse one. For instance, if the economy perpetually receives state 1, it would end up intersecting the 45 degree line at a point in which the constraint binds strictly, and then remains at this fix point thereafter. Suppose instead that the economy is currently at the intersection between the decision rule for state 15 and the 45-degree line; a point in which the constraint is not binding. If the economy continues to receive this state, the constraint would never bind. However, if a worse state such as 10 realizes, the constraint would bind strictly on impact as the economy jumps to the corresponding new decision rule. Then, if State 10 continues to realize, the economy would travel back to the fix point on this rule with a falling foreign debt value and a constraint that continues to bind indefinitely.

Figure 3 illustrates the behavior of other endogenous variables for the worst value of the exogenous state ε_t. Specifically, we report the equilibrium decision rules for the relative price of non tradable goods (P_t^N), tradable and non-tradable consumption ($C_{N,t}$ and $C_{T,t}$, respectively), and hours (H_t), in the model with and without the borrowing constraint. The vertical line in the figure is the first point on the grid in which the constraint binds strictly (i.e., with a positive multiplier). To the left of this line, the economy travels along the decision rule driven by the binding constraint and a positive multiplier. To the right of this point, the economy behavior is driven by the necessary conditions for household and firm optimization with a zero multiplier.

The first point to note is that the policy functions for the two models are similar and nearly linear in the region of the state space in which the constraint does not bind. However, the borrowing constraint distorts the equilibrium behavior of the economy well before the constraint is reached. As the economy travels along the decision rule toward the constraint, the increased probability of hitting it in future periods induces households to reduce their tradeable consumption and save more (or borrow less) on a precautionary basis. As goods are complements, this also lead to a decrease in non-tradeable consumption. As non-tradeable consumption decreases more than its production, the relative price of non-tradeable declines reducing the value of the collateral, and hence further exacerbating and amplifying the mechanism; an amplification that resembles the debt-deflation narrative and it is at the core of the sudden stop dynamics in the model.

Once the economy hits the constraint, its behavior is driven by the necessity to meet it. The constraint forces tradable consumption and the relative price of non tradable (i.e., the inverse of the relative price of tradable goods) to plunge (to soar), while non-tradable consumption and hours now increase sharply because of substitution between the tradables for non tradables in consumption forced by the constraint and higher supply of non-tradables to accommodate it. This is because, for the functional forms and the parameter values assumed, the fall in the labor demand associated with a falling P_t^N is dominated by the increase in labor supply with high level of foreign debt.

To illustrate the sudden stop dynamics produced by the model with borrowing constraint in the time dimension, in Figure 4 we plot the average value of foreign debt, the relative price of nontraded goods, traded and non-traded consumption, hours and total GDP, before and after the beginning of a sudden stop episode. In Figure 4, we plot the average value of these variables four periods before and four periods after the beginning of a sudden stop episode (i.e., the first simulated quarter in which the constraint binds strictly, denoted t). As the economy approaches the sudden stop period, the amount foreign borrowing increases to smooth bad shocks. But once the constraint binds, borrowing decreases suddenly and sharply. Associated with the sudden stop the mode produces a dramatic fall in both GDP and the relative price of non-tradable goods. Note however that the GDP decline is driven by the fall in relative price of non-tradable in the model that more than offsets the non-tradable output and consumption increase driven by the labor supply increase when the constraint binds. The expansionary effect of the sudden stop in the non-tradable sector of the economy is a clearly countefactual prediction of the model under the functional form and parameter value assumption made.

Figure 5 shows that our model with borrowing constraint quantitatively fits relatively well the type of sudden stops that we see in the data. The figure compares actual data around Mexico's "tequila" crisis in the fourth quarter of 1994 with the averages of the simulated data from the model. In the model output is equivalent to consumption because there is no investment and government expenditure is zero. So the figure compares simulated and actual data for total private consumption. For consumption and relative prices the figure plots quarterly rates of growth. For changes in capital flows (the second difference of B_{t+1} in the model), the figure plots the share of annual GDP. As we can see from the figure, the model generates a capital flow reversal comparable to that in the data—although slightly smaller and less persistent than in the data. Total consumption and the relative price of non-traded goods fall in the model slightly less and slightly more than 20 percent, respectively, in the ballpark of what we see in the data. In the data in fact consumption falls about 13 percent, the relative price of non-tradable goods falls by about 2 percent,[20] while the real effective exchange rate falls more than 40 percent. Consumption and relative prices takes longer to recover in the data than in the model.

Finally, Figure 6 reports the ergodic distribution of foreign borrowing with and without borrowing constraint. Precautionary saving induced by the occasionally binding credit constraint is quantitatively very large in the model, given the small shocks that hit the economy and the unitary elasticity of intertemporal substitution assumed. This can be seen by the fact that the two distributions are very far from each other (average $B = -4.006$ and $B = -0.517$ respectively). To put these numbers in perspective, they can be compared to average annual GDP in the two economies. The average foreign borrowing position in the ergodic distribution of the economy with no borrowing constraint is 44 percent of average GDP, about the same as in the deterministic steady state of the model. In contrast, in the economy with the constraint, the ratio of the average debt to the average GDP is only about 5 percent. It is also notable that the distribution without the constraint is more symmetric.[21]

A large precautionary saving effect does not necessarily translate into a large welfare cost

[20]The relative price on non-tradable goods is measured as the ratio of the consumer price index to the producer price index, which is a rough measure of this relative price.

[21]Preliminary welfare calculations indicate that The calculation of the welfare implications of this precautionary saving is still work in progress.

of the borrowing constraint. The calculation of the welfare implications of this precautionary saving can be done by computing the percentage change in average lifetime consumption, at every date and state, that would leave the stand-in household indifferent between the economy with and without the borrowing constraint. One such calculation based on a slightly different model calibration and solution method indicates that the cost of the borrowing constraint is significant, at about 0.5 percent of average lifetime consumption.

5 Optimal Policy

We turn now to the central questions of the paper using our stochastic small open economy with the occasionally binding borrowing constraint. We ask three questions about stabilization policy in this environment. First, does the optimal policy have any precautionary motive? Second, what does the policy do in a crisis period when the constraint binds? Third, how does optimal policy affect private agents' behavior in different state of the world?

The decision rule for τ_t^N, as well as the implied lump sum transfer (scaled by GDP, $P_t^N T_t^T / Y_t$) that now adjusts in order to balance the government budget, are plotted in Figure 7. The optimal policy schedule for τ_t^N is highly non-linear. In states of the world in which the constraint is not binding the optimal policy is "passive", i.e., $\tau^N = 0$. In states of the world in which the constraint is binding, the optimal policy is "active" with a subsidy to non-tradable consumption, i.e. $\tau^N < 0$.

Thus, in states of the world in which the constraint is not binding the optimal policy is "no policy action" or $\tau^N = 0$. This result means that there is no precautionary motive for the optimal policy related to the presence of the borrowing constraint. When the constraint is not binding, policy is set to minimize the distortion associated with the use of the policy instrument so that $\tau^N = 0$, as in the unconstrained economy.

One interpretation of this result is that the Ramsey planner can intervene ex-post, once the constraint is met, at no cost. The Ramsey planner has therefore no incentive to intervene on a precautionary basis at the cost of distorting the economy with its tax instrument when the constrain is not binding. Note in fact that in our benchmark model, the financing of the optimal policy is non-distortionary through lump-sum transfers. Note also that, while the size of the subsidy call for by optimal policy when the constraint binds is limited (Figure 7), its budgetary implications are not negligible compared to the size of the capital flow reversal in the data reported in Figure 5. Hence, a large subsidy can be applied right when it is needed because the constraint binds. We interpret this cost-free financing of the optimal policy as loosely capturing a policy framework in which official reserves have been accumulated slowly over time at some small distortionary cost and can then be used in time of crisis at no additional costs.[22]

As Figure 7 illustrates, when the constraint is binding, optimal policy calls for subsidizing non-tradable consumption (i.e., $\tau^N < 0$). In this case, there is a trade-off between efficiency and the need to mitigate the effects of the credit constraint or attempt to relax it. The Ramsey planner subsidizes non-tradable consumption to support its relative price and thereby relax the collateral constraint on foreign borrowing that permits higher tradable consumption and lower labor effort.

Figure 8 plots the decision rule for foreign borrowing along with the value of B_t at which the constraint binds under optimal policy for the worst state of the shock to the tradable endowment. Figure 9 reports the same decision rules as in Figure 3, which govern the evolution of the economy with the constraint, with and without optimal policy. Figure 8 shows that optimal policy allows for a much smoother change in foreign borrowing when the constraint binds, and hence for a less disruptive adjustment process to the sudden stop. Figure 9 shows how this policy works to alleviate the binding constraint. With the government subsidy, demand for non-tradable goods increases which supports their price. This directly raises the value of the collateral and allows agents to increase their consumption of tradeable goods compared to the competitive equilibrium allocation. Also we observe an increase in the consumption of non-tradeable but less so than in the competitive equilibrium allocation. This is because, as tradeable consumption increases, the decrease in the marginal utility of income has a negative effect on labor supply so that in the Ramsey equilibrium labor and

[22]Indeed, solving a model with tradable production and distortionary taxation that fiances the optimal policy is an important extension of our analysis to verify this conjecture.

consumption of non-tradeable are lower than in the competitive equilibrium case.

Finally, Figure 10 reports the ergodic distribution of the net foreign asset position for the economy with borrowing constraint, with and without optimal policy. Consistent with Figure 9, the difference between these two distributions is quantitatively small. With optimal policy, in the neighborhood of the borrowing constraint, there is more borrowing that permits a reduction in precautionary saving and labor effort on the part of agents in the model.

Note however that a small difference in precautionary saving does not necessarily imply a small welfare difference. For instance, a calculation based on a slightly different model calibration and solution method indicates that the gain attached to optimal policy may be significant at about 40 percent of the gain from the complete elimination of the constraint despite finding very small differences in the ergodic distribution of the two model economies. This is because a small increase in tradable consumption or decrease in labor effort in states of the work in which the valuation is distorted by the borrowing constraint multiplier are much more valuable than in more normal state of the world.

6 Conclusions

In this paper we study an optimal stabilization problem in a small open economy subject to an occasionally binding borrowing constraint. We found that optimal policy is non-linear. When the constraint is not binding, the tax rate is set to zero. Thus, in our benchmark economy, optimal policy does not exhibit any precautionary motive. An important implication of this result is that stabilization policy in an economy with an occasionally binding financial friction should be set as if the friction were not present when this is not binding, even though the constraint does distort private sector behavior even in the non-binding state.

When the credit constraint is binding, optimal policy is to intervene to subsidize the price of non-tradable consumption. This subsidy increases demand for nontradable goods and the relative price of non-tradable goods. The increase in income increases collateral and alleviates the effects of the binding borrowing constraint.

We also find that the commitment to optimal policy has a small quantitative effect on private agents' behavior, and particularly precautionary saving. This however does not imply that the optimal policy we computed has small welfare effects.

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A Appendix: Solution Algorithm

This appendix provides the details on our solution method. We first describe the general methodology to solve for a competitive equilibrium in the model given a fixed $\tau = 0$. We then add a second optimization to the problem to solve the optimal policy problem.

A.1 Competitive Equilibrium

To solve for the competitive equilibrium we start with a function $\lambda_{t+1} = G_{\lambda}^{0}(b_{t+1}, \epsilon_{t+1})$. Given that function the following set of equations describe a competitive equilibrium. These equations are solved for the endogenous variables $(b_{t+1}, h_t, \mu_t, \lambda_t, c_{T,t}, c_{N,t}, p_{N,t})$ as functions of (b_t, ϵ_t). For the competitive equilibrium we take $\tau_t = G_{\tau}(b_t, \epsilon_t)$ as exogenous and fixed. The set of equations we solve is:

$$
\begin{aligned}
\frac{\omega c_{T,t}^{\kappa-1}}{\omega c_{T,t}^{\kappa} + (1-\omega)\, c_{N,t}^{\kappa}} &= \lambda_t \\
\frac{(1-\omega)\, c_{N,t}^{\kappa-1}}{\omega c_{T,t}^{\kappa} + (1-\omega)\, c_{N,t}^{\kappa}} &= \lambda_t p_{N,t} (1+\tau_t) \\
\frac{\delta}{1-h_t} &= \lambda_t p_{N,t} (1-\alpha)\, A h_t^{-\alpha} + \max\{0, \mu_t\}^2 \frac{1-\phi}{\phi} p_{N,t} (1-\alpha)\, A h_t^{-\alpha} \\
\lambda_t &= \left(1 + \left(\omega c_{T,t}^{\kappa} + (1-\omega)\, c_{N,t}^{\kappa}\right)^{\frac{1}{\kappa}} (1-h_t)^{\delta}\right)^{-\beta} (1+r)\, E_{\epsilon_{t+1}}\left[G_{\lambda}^{0}(b_{t+1}, \epsilon_{t+1})\, | \epsilon_t\right] + \max\{0 \\
c_{T,t} &= (1+r)\, b_t + \epsilon_t - b_{t+1} \\
c_{N,t} &= A h_t^{1-\alpha} \\
\max\{0, -\mu_t\}^2 &= b_{t+1} + \frac{1-\phi}{\phi}\left(\epsilon_t + p_{N,t} (1-\alpha)\, A h_t^{1-\alpha}\right).
\end{aligned}
$$

Here, the multiplier on the borrowing constraint is the quantity

$$\max\{0, \mu_t\}^2 .$$

This transformation follows Garcia and Zangwill (1981) and allows us to turn a system of Kuhn-Tucker conditions into a standard system of nonlinear equations. This system can then be solved using standard methods. In particular, we use a hybrid version of Powell's method. If that method fails we use a homotopy algorithm. Note that

$$\max\{0, \mu_t\}^2 \max\{0, -\mu_t\}^2 = \max\{0, \mu_t\}^2 \left(b_{t+1} + \frac{1-\phi}{\phi}\left(\epsilon_t + p_{N,t} (1-\alpha)\, A h_t^{1-\alpha}\right)\right) = 0,$$

which is just the complementary slackness condition, and that

$$
\begin{aligned}
\max\{0, \mu_t\}^2 &\geq 0 \\
b_{t+1} + \frac{1-\phi}{\phi}\left(\epsilon_t + p_{N,t} (1-\alpha)\, A h_t^{1-\alpha}\right) &\geq 0
\end{aligned}
$$

are both guaranteed to hold. Therefore, the sign for both the multiplier and the constraint are both consistent with the Kuhn-Tucker conditions.

The system above can be reduced to three equations in three unknowns, namely (b_{t+1}, h_t, μ_t). The function $G_\tau(b_t, \epsilon_t)$ is then updated using the solution for λ_t, and this process is repeated to convergence.

The value function can be computed as the solution to the functional equation

$$v(b_t, \epsilon_t) = \frac{1}{1-\rho}\left(\left(\omega c_{T,t}^{\kappa} + (1-\omega)\, c_{N,t}^{\kappa}\right)^{\frac{1}{\kappa}} (1-h_t)^{\delta}\right)^{1-\rho} +$$
$$\left(1 + \left(\omega c_{T,t}^{\kappa} + (1-\omega)\, c_{N,t}^{\kappa}\right)^{\frac{1}{\kappa}} (1-h_t)^{\delta}\right)^{-\beta} E_{\epsilon_{t+1}}\left[v(b_{t+1}, \epsilon_{t+1})\, |\epsilon_t\right],$$

which has a unique solution due to the contraction mapping theorem.

A.2 Optimal Policy

To solve for optimal policy we add to this solution procedure an optimization step over the instrument of government policy τ, which thus becomes a policy function for the government. The optimal policy function for τ is the tax plan that maximizes the value function:

$$\tau(B, \varepsilon_t) = argmax_\tau \{V(B, \varepsilon_t, \tau)\}. \tag{22}$$

The solution of the optimal policy problem is an iterative procedure. Specifically the algorithm is as follows:

Given G_λ^0, v^0, and τ^0 first solve

$$\begin{aligned}
c_{T,t}(b_t, \varepsilon_t) + b_{t+1}(b_t, \varepsilon_t) &= (1+r)\, b_t + \varepsilon_t \\
c_{N,t}(b_t, \varepsilon_t) &= A h_t(b_t, \varepsilon_t) \\
\frac{\delta}{1-h_t} &= p_{N,t}(b_t, \varepsilon_t)\, A\left(\lambda(b_t, \varepsilon_t) + \frac{1-\phi}{\phi}\max\{0, \mu(b_t, \varepsilon_t)\}^2\right) \\
b_{t+1}(b_t, \varepsilon_t) + \frac{1-\phi}{\phi}\left(\varepsilon_t + p_{N,t}(b_t, \varepsilon_t)\, A h_t(b_t, \varepsilon_t)\right) &= \max\{0, -\mu(b_t, \varepsilon_t)\}^2 \\
p_{N,t}(b_t, \varepsilon_t) &= \frac{1-\omega}{\omega}\left(\frac{c_{N,t}(b_t, \varepsilon_t)}{c_{T,t}(b_t, \varepsilon_t)}\right)^{\kappa-1} \frac{1}{1+\tau} \\
\lambda &= \frac{\omega c_T(b_t, \varepsilon_t)^{\kappa-1}}{\omega c_{T,t}(b_t, \varepsilon_t)^{\kappa} + (1-\omega)\, c_{N,t}(b_t, \varepsilon_t)^{\kappa}} \\
\lambda(b_t, \varepsilon_t) &= \beta E\left[G_\lambda^0\left(b_{t+1}, \varepsilon_{t+1}, \tau^0(b_{t+1}, \varepsilon_{t+1})\right)|\varepsilon_t\right]
\end{aligned}$$

and set

$$G_\lambda^1(b_t, \varepsilon_t) = \lambda(b_t, \varepsilon_t).$$

Then calculate

$$V(b_t, \varepsilon_t, \tau) = \frac{1}{\kappa}\log\left(\omega c_{T,t}^{\kappa} + (1-\omega)\, c_{N,t}^{\kappa}\right) + \delta\log(1-h_t) + \left(1 + \left(\omega c_{T,t}^{\kappa} + (1-\omega)\, c_{N,t}^{\kappa}\right)^{\frac{1}{\kappa}} (1-h_t)^{\delta}\right)^{-\beta} E\left[v^0(b_t\right.$$

Finally, set

$$\begin{aligned} v^1(b_t, \varepsilon_t) &= \max_\tau \{V(b_t, \varepsilon_t, \tau)\} \\ \tau^1(b_t, \varepsilon_t) &= argmax\{V(b_t, \varepsilon_t, \tau)\} \end{aligned}$$

and repeat until convergence.

Table 1. Parameters and steady state values

Structural parameters	**Values**
Elasticity of substitution between tradable and non-tradable goods	$\kappa = 0.7578$
Intertemporal substitution and risk aversion	$\rho = 1$
Labor supply elasticity	$\delta = 1.193$
Credit constraint parameter	$\phi = 0.7$
Labor share in production	$\alpha = 1$
Relative weight of tradable and non-tradable goods	$\omega = 0.48568$
Discount factor	$\beta = 0.029148$
Exogenous variables	**Values**
Production factor	$AK^{(1-\alpha)} = 4.0404$
Tax rate on non-tradable consumption	$\tau = 0.00$
World real interest rate	$i = 0.0159$
Per capita tradable GDP	$Y_T = 1$
Relative price of non-tradable	$P_N = 1$
Tradable government consumption	$\exp(G_T) = 1$
Nontradable government consumption	$\exp(G_N) = 1$
Endogenous variables	**Steady state values**
Per capita NFA	$B = -4.0133$
Per capita tradable consumption	$C_T = 0.93619$
Per capita non-tradable consumption	$C_N = 1.33333$
Per capita consumption	$C = 2.26952$
Per capita non-tradable GDP	$Y_N = 1.33333$
Per capita GDP	$Y = 2.333$
Hours	$H = 0.33$
Productivity process	
Persistence	$\rho_{\varepsilon^T} = 0.647$
Volatility	$\sigma_{\varepsilon^T} = 0.02734$

Figure 1: RAMSEY PLANNER VERSUS SOCIAL PLANNER

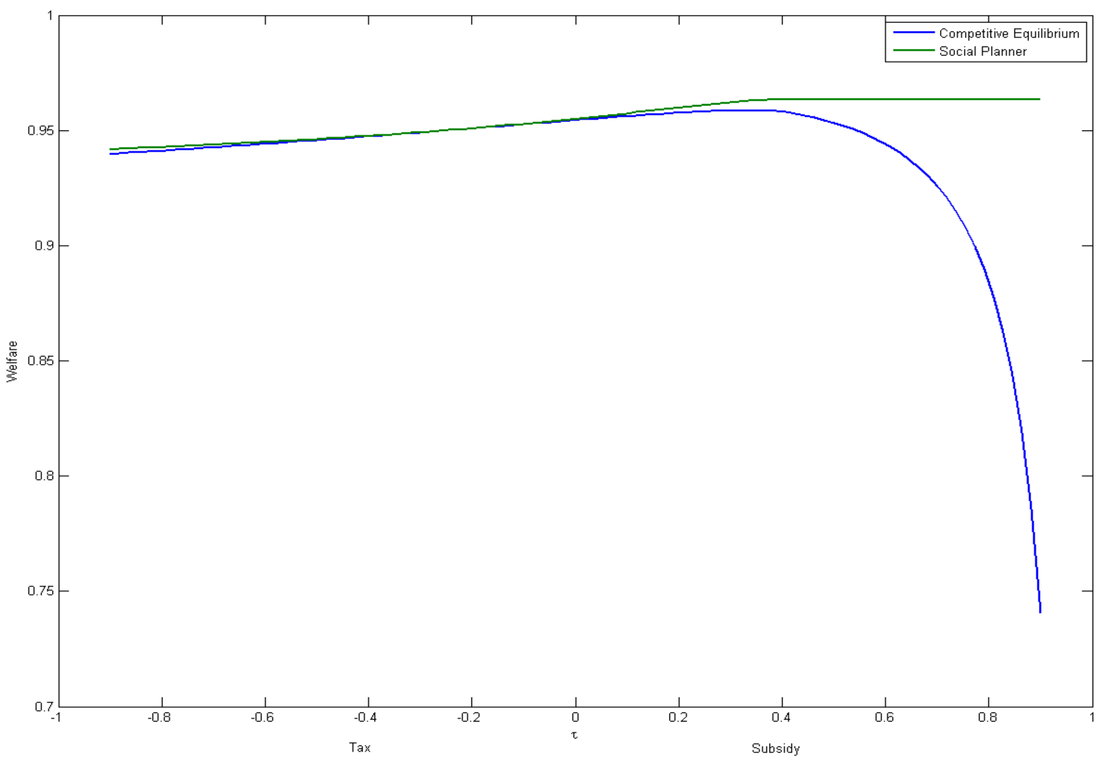

Figure 2: Decision Rule For Foreign Borrowing (Competitive Equilibrium)

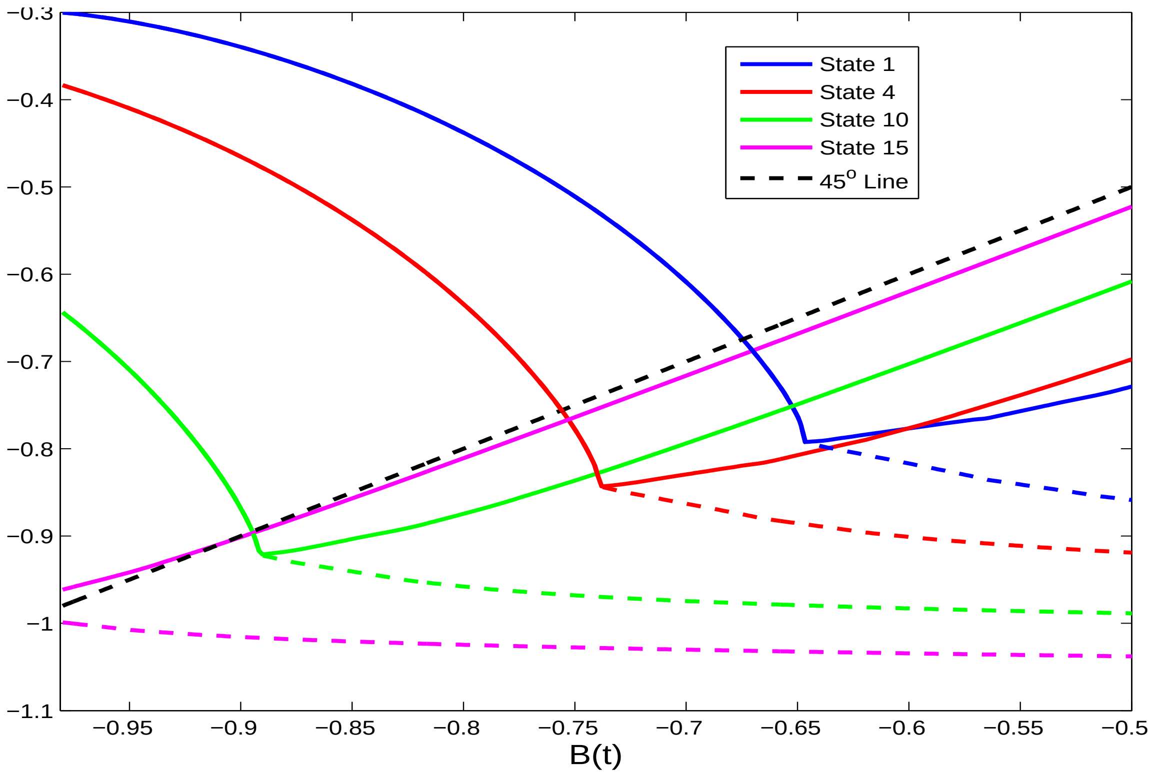

Figure 3: Decision Rules For Relative Prices, Consumption, Labor (Competitive Equilibrium)

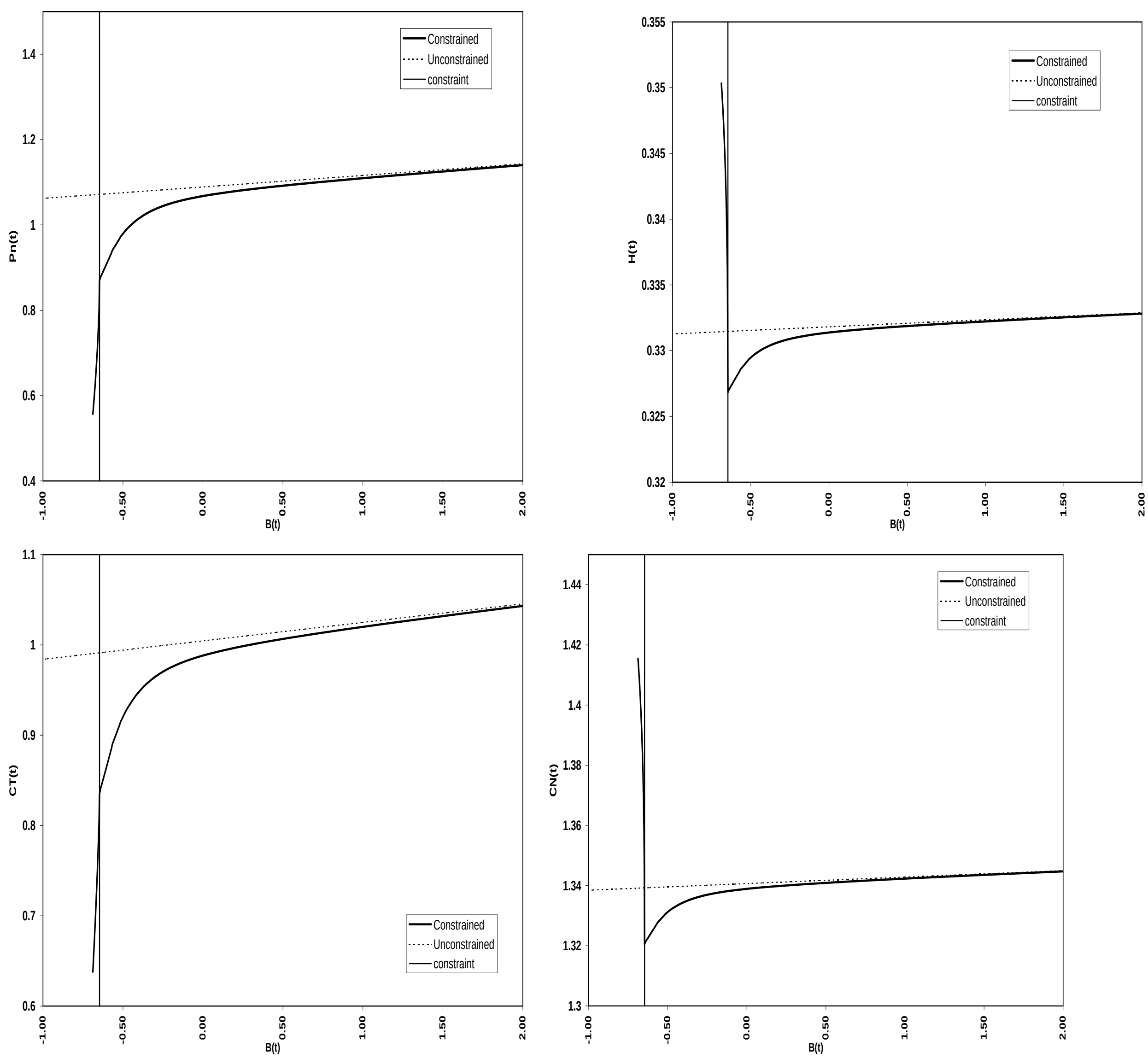

Figure 4: Simulated Data (Competitive Equilibrium)

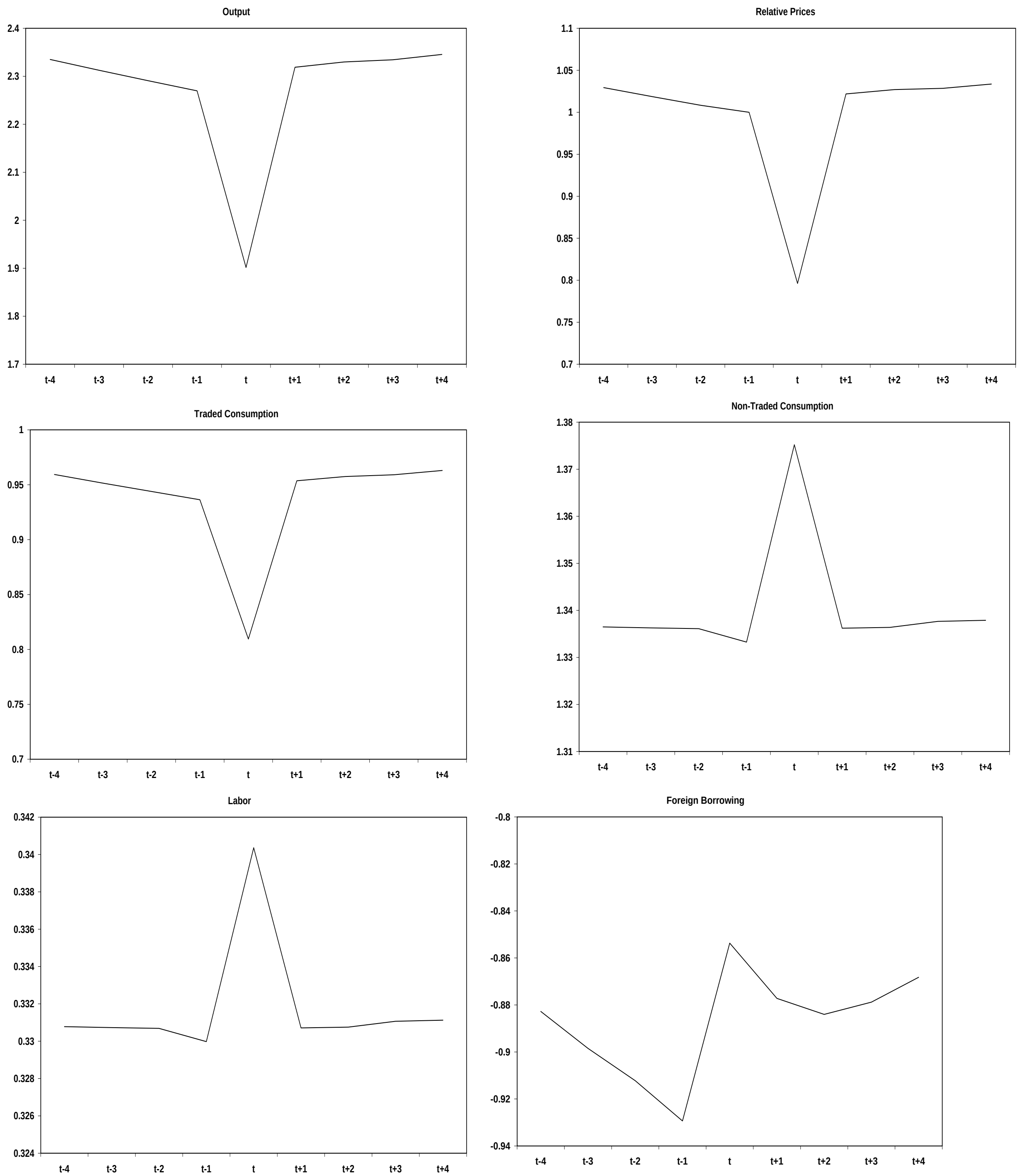

Figure 5: Simulated Data and Mexican Data

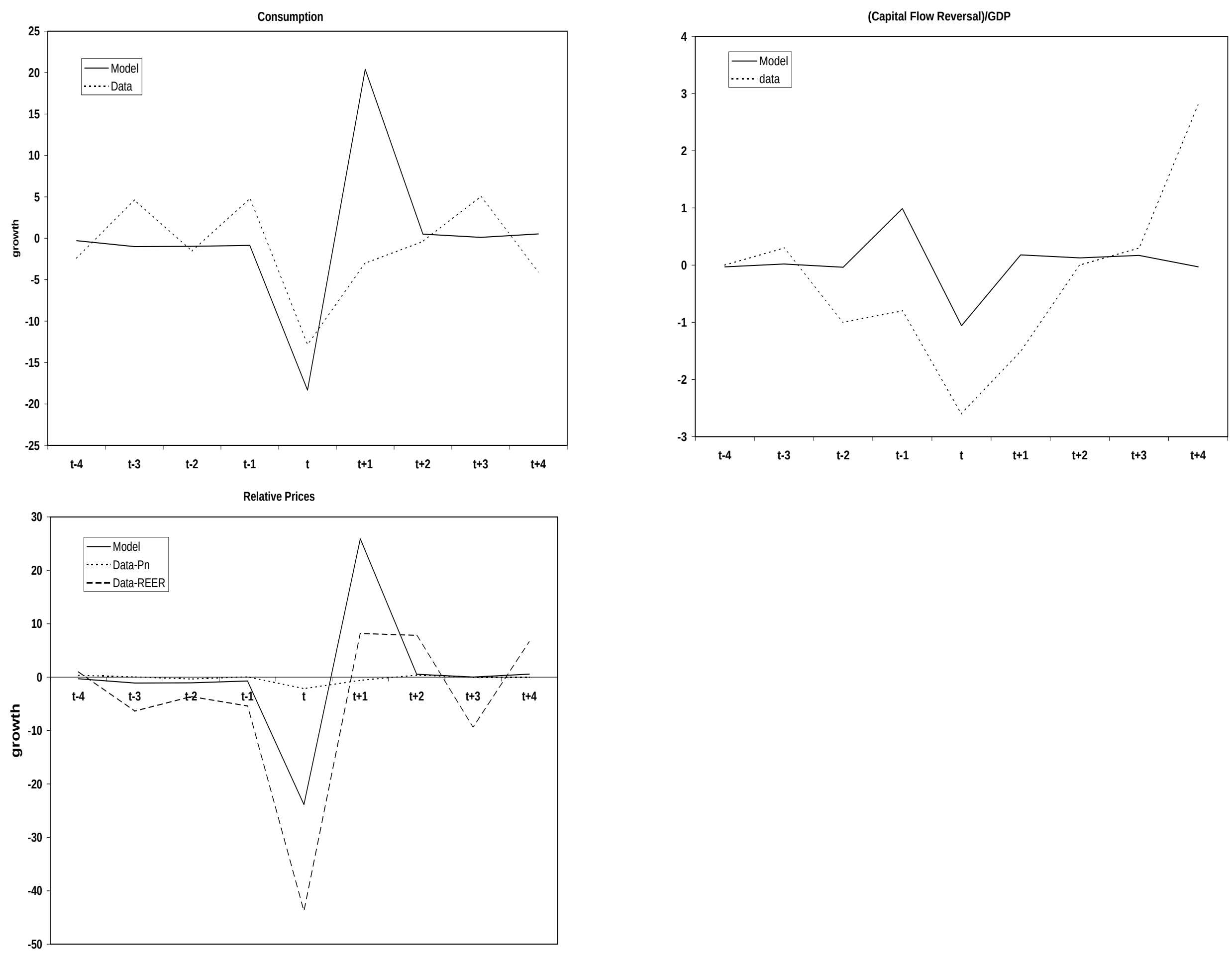

Figure 6: Ergodic Distribution For Foreign Borrowing (Constrained and Unconstrained)

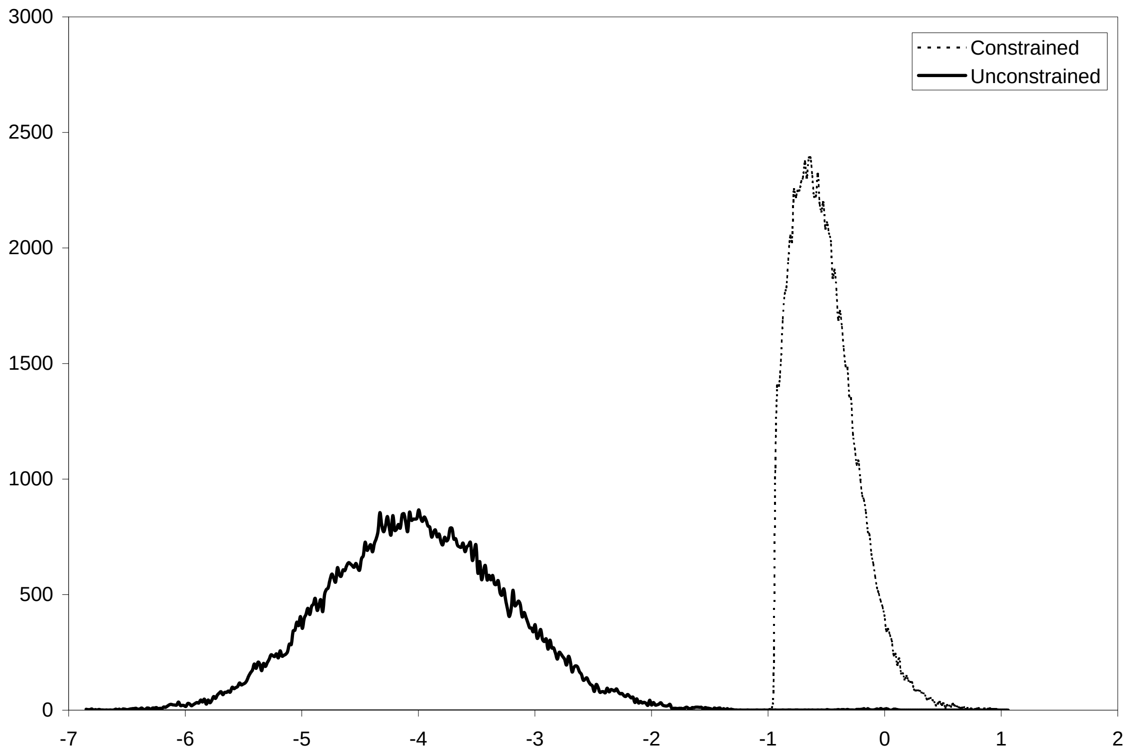

Figure 7: Optimal Tau and Transfers

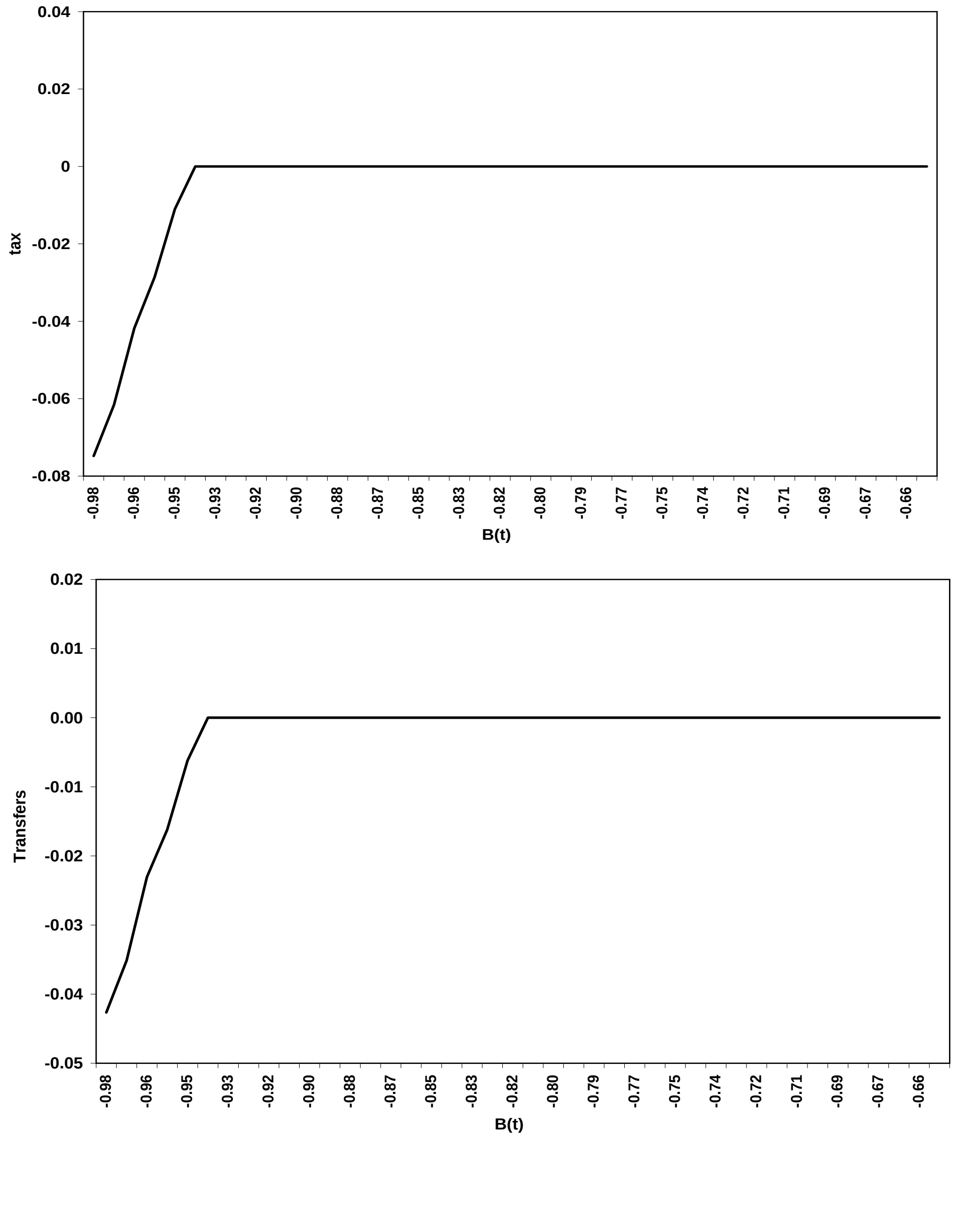

Figure 8: Decision Rule For Foreign Borrowing (Competitive Equilibrium and Optimal Policy)

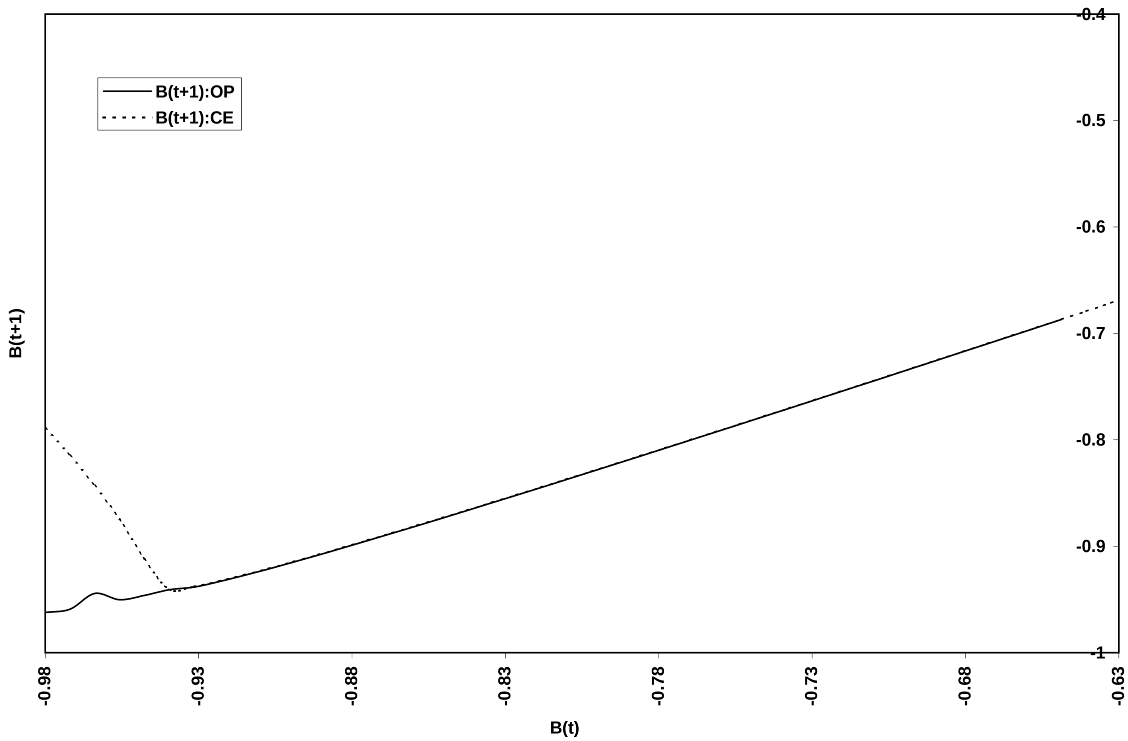

Figure 9: Decision Rules For Relative Prices, Consumption, Labor (Competitive Equilibrium and Optimal Policy)

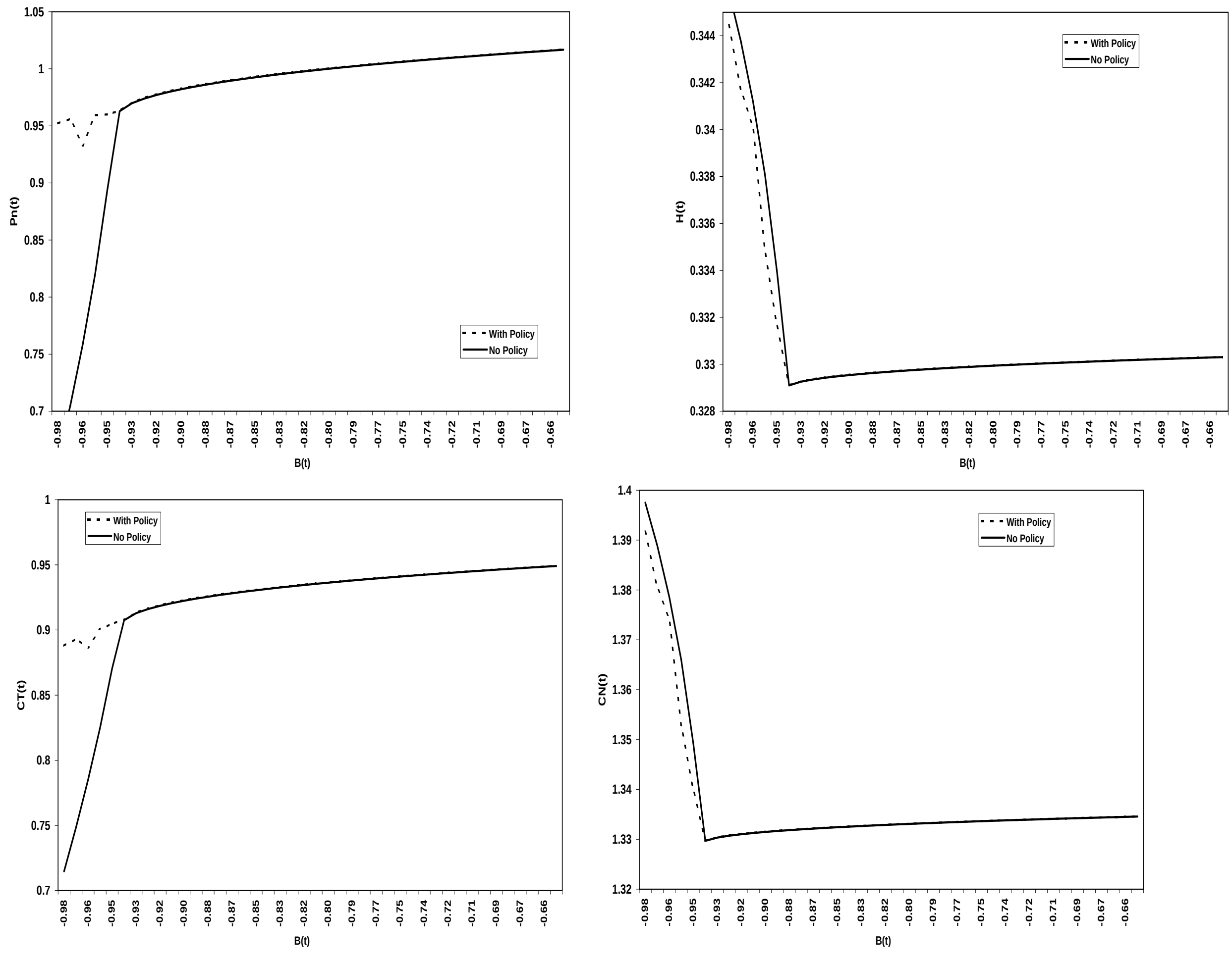

Figure 10: Ergodic Distribution For Foreign Borrowing (Competitive Equilibrium and Optimal Policy)