

# INTERNATIONAL PORTFOLIOS, CAPITAL ACCUMULATION AND FOREIGN ASSETS DYNAMICS

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London Business School and CEPR                8:30 am

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June 12 2009

## Abstract

Despite the liberalization of capital flows among OECD countries, equity home bias remains sizable. We depart from the two familiar explanations of equity home bias: transaction costs that impede international diversification, and terms of trade responses to supply shocks that provide risk sharing, so that there is little incentive to hold diversified portfolios. We show that the interaction of the following ingredients generates a realistic equity home bias: capital accumulation and international trade in stocks *and* bonds. In our model, domestic stocks are used to hedge fluctuations in local wage income. Terms of trade risk is hedged using bonds denominated in local goods and in foreign goods. In contrast to related models, the low level of international diversification does not depend on strongly counter-cyclical terms of trade. The model also reproduces the cyclical dynamics of foreign asset positions and of international capital flows.

JEL classification: F2, F3, G1.

Keywords: capital accumulation;international equity and bond portfolios; capital flows; current account;valuation effects; terms of trade.

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We thank Mick Devereux and two anonymous referees for helpful comments and advice. We also thank Kosuke Aoki, Matthias Doepke, Stéphane Guibaud, Viktoria Hnatkovska, Mathias Hoffmann and Giovanni Lombardo for detailed and constructive discussions. For comments, we are also grateful to Luca Dedola, Pierre-Olivier Gourinchas, Harald Hau, Jonathan Heathcote, Akito Matsumoto, Werner Roeger, Alan Sutherland and Pedro Teles, and to workshop participants at IMF-JIE conference "International Macro-Finance", AEA 2009, RES 2008, SED 2008, Bundesbank Spring conference, ESSIM (CEPR), EU Commission (DG ECFIN), Bank of Portugal, Bank of Italy, San Francisco Fed, Dallas Fed, EUI (Florence), Birkbeck, City University (London) and at the Universities of Zurich, Leicester and Ghent. R. Kollmann thanks ECARES, the National Bank of Belgium and the EU Commission (DG ECFIN) for financial support. Philippe Martin thanks the Institut Universitaire de France for financial help.

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## 1 Introduction

Cross-country capital flows have increased greatly, since the liberalization of international capital markets two decades ago (e.g., Lane and Milesi-Feretti (2003, 2005, 2006)). Equity home bias, while less severe than in earlier decades, remains sizable and is observed in all industrialized countries (see French and Poterba (1991) for early evidence and Sercu and Vanpée (2007) for a recent survey). There are broadly two classes of explanations for the persisting equity home bias. The first one centers on transaction costs and informational barriers in cross-border financial transactions and suggests that international risk sharing is insufficient.<sup>1</sup> The second one focuses on the possibility that terms of trade changes in response to supply shocks may provide international insurance against these shocks, so that even a portfolio with home bias delivers efficient international risk sharing (Cole and Obstfeld (1991), Helpman and Razin (1978)).

Both types of explanations are helpful but are not without problems. Several authors have argued that frictions would have to be large to fully explain the equity home bias (French and Poterba (1991), Tesar and Werner (1995), Warnock (2002)). In order to interpret terms of trade changes as providing insurance (rather than a source of risk), the terms of trade would have to improve strongly after a negative supply shock. However, empirically the terms of trade are only weakly correlated with output (e.g., Backus, Kehoe and Kydland (1994)).

Using a two-country general equilibrium model with fully integrated financial markets, this paper shows that the interaction of the following ingredients is key for generating realistic equity home bias, without requiring strongly countercyclical terms of trade: capital accumulation and international trade in stocks **and** bonds denominated in local and foreign goods.<sup>2</sup>

By contrast, other recent general equilibrium models of international equity holdings (see Devereux and Sutherland (2006a,b) for references) have mostly assumed *endowment economies*, i.e. economies without production or capital accumulation – Heathcote and Perri (2007) is a notable exception discussed below. In such economies, households trade in international financial markets solely for consumption smoothing and risk sharing purposes so that the equity portfolio is structured to sustain net imports

<sup>1</sup>See, e.g., Heathcote and Perri (2002, 2004), Martin and Rey (2004), Coeurdacier and Guibaud (2008), Tille and van Wincoop (2007), and Van Nieuwerburgh and Veldkamp (2007) for recent studies on the role of frictions in international financial markets.

<sup>2</sup>Pavlova and Rigobon (2007a and b), Engel and Matsumoto (2006) and Coeurdacier, Kollmann and Martin (2007) have previously analyzed equity portfolio choice in general equilibrium models with trade in bonds.

in states of nature where local production is low; this leads to local equity bias if relative Home equity returns rise (compared to Foreign returns) when the Home terms of trade improve and the Home real exchange rate appreciates, in response to a drop in the Home output.<sup>3</sup> This condition however is not met in the data: empirically, the correlation between relative equity returns and the real exchange rate is low (van Wincoop and Warnock (2006)).

We consider a model with capital accumulation because, as discussed by Obstfeld and Rogoff (1996), one of the key functions of international financial markets is to finance physical investment; empirically, productive investment is a key driver of fluctuations in net imports (Sachs (1981), Backus, Kehoe and Kydland (1994)). With two stocks and two bonds, and two types of (Home and Foreign) technology shocks, markets are effectively complete, up to a first order (linear) approximation of the model. In addition to standard TFP (total factor productivity) shocks, the model here assumes shock to investment efficiency (as in Greenwood, Hercowitz and Krusell (1997, 2000), Fisher (2002, 2006)), because recent empirical research suggests that those shocks are an important source of fluctuations in real activity (Justiniano and Primiceri (2006), Justiniano et al. (2007)).

The equilibrium portfolio is structured to optimally hedge fluctuations in the real exchange rate and in labor incomes.<sup>4</sup> Specifically, *bonds* are used for real exchange rate hedging, since the difference between the pay-offs of bonds denominated in Home and Foreign goods is correlated with the real exchange rate. Fluctuations in labor incomes are hedged through the equity portfolio. The key mechanism here is that fluctuations in investment generate a negative co-movement between Home capital income (net of investment) and Home labor income (relative to their Foreign counterparts). A Home investment boom lowers Home payments to shareholders (to finance investment) and raises Home output and wage incomes (relative to foreign wages), under the realistic assumption (made here) that there is a local bias in investment spending. Thus, local equity offers a good hedge against movements in local labor incomes associated with investment fluctuations – which explains why equilibrium equity portfolios exhibit home bias. The predicted equity home bias only depends on the degree of home bias in investment spending, and on the labor share. In particular, it is independent of preference parameters.<sup>5</sup> Importantly, the

<sup>3</sup>See Uppal (1993), Coeurdacier (2009), Kollmann (2006b).

<sup>4</sup>See Adler and Dumas (1983) for early work that stresses exchange rate hedging as a determinant of portfolio choice. Baxter and Jermann (1997), Heathcote and Perri (2007), Engel and Matsumoto (2006), Bottazzi, Pesenti and van Wincoop (1996), and Julliard (2002 and 2004), among others, discuss the hedging of labor income risk.

<sup>5</sup>Coeurdacier and Gourinchas (2008) provide a general discussion of the conditions under which equity portfolios are

optimal portfolio does not hinge on the presence of investment efficiency shocks. These shocks help to explain the countercyclical nature of the trade balance and the acyclicity of the terms of trade but our portfolio results would also hold in a model with TFP shocks and a range of other (domestic and foreign) shocks to output and/or investment.

The closest paper to ours is Heathcote and Perri (2007) [HP henceforth] who were the first to investigate the importance of physical investment for equity portfolios. Trade in bonds, and the shocks to investment efficiency assumed here are the main difference between our model and HP. The HP model only generates realistic equity home bias if the terms of trade respond strongly to TFP shocks (or, equivalently, if preferences are "close enough" to log-separability between the two goods, as in a Cole and Obstfeld (1991) economy). Our model does not require strong terms of trade effects of productivity shocks—nevertheless, there is sizable equity home bias. This is important since the empirical evidence concerning the response of the terms of trade to technology shocks is mixed.<sup>6</sup>

Another paper close to ours is Engel and Matsumoto (2006) who analyze international equity portfolio choices in a model with money, sticky prices and trade in bonds, but without capital accumulation. Under price stickiness, the short run level of output is demand determined, so that a positive productivity shock leads to a fall in employment and labor income, but an increase in profits. Ownership of local equity is thus an effective hedge against labor income risk. In our model, local equity has a similar hedging property—but that property is driven by physical investment shocks (without requiring price stickiness).

A key contribution of the paper here is to explore the *quantitative* implications of the model regarding the *dynamics* of external asset positions and international capital flows. Gourinchas and Rey (2005), Tille (2005) and Lane and Milesi-Ferretti (2006) document empirically that fluctuations in the value of domestic and foreign assets induce external capital gains/losses that have a substantial effect on countries' net foreign asset positions (NFA). We show that the present model generates sizable international valuation effects. Here, fluctuations in a country's NFA are driven by asset price changes—NFA is thus predicted to have the time series properties of asset prices; in particular, the first difference of a country's NFA is predicted to be highly volatile and to have low serial correlation. We show that these predictions are independent of preference parameters.

<sup>6</sup>Corsetti, Dedola and Leduc (2006) argue that, empirically, a positive technology shocks triggers a terms of trade appreciation; Acemoglu and Ventura (2002) and Kollmann (2006c) provide evidence that higher productivity depreciates the terms-of-trade.

consistent with the data. When there is a positive TFP or investment efficiency shock, net imports are predicted to rise on impact (due to a strong short run rise in investment), and to fall thereafter. As NFA equals the present value of current and future net imports, the NFA drops, on impact. Thus, the change in NFA is predicted to be countercyclical, which is likewise consistent with the data. Finally, the model generates sizable asset trades.<sup>7</sup>

We also show that our model has several interesting business cycle features. The investment efficiency shocks assumed here generate net exports and real exchange rate volatility that is larger—and thus closer to the data—than the volatility induced by TFP shocks. In the model here, a positive shock to a country's TFP raises that country's output while worsening its terms of trade; a country-specific shock to investment efficiency likewise raises output, but (on impact) it *improves* the terms of trade (the shock raises investment spending which is biased towards local inputs; hence it raises the relative price of those inputs). As a result, with the combined two types of shock, the terms of trade (and the real exchange rate) are less cyclical than in standard RBC models that are driven just by TFP shocks (e.g., Backus, Kehoe and Kydland (1994)). The presence of investment efficiency shocks also generates a cross-country correlation of consumption that is lower, and closer to the low correlations seen in the data. However, investment efficiency shocks generate cross-country correlations of investment, and within country correlations between investment and consumption that are too small when compared to the data.<sup>8</sup>

The paper is structured as follows. In section 2, we present the model set-up. In section 3, we derive equilibrium equity and bond portfolios, and we provide empirical support for the key condition that drives equity home bias in the model. In section 4, we provide stylized facts on the dynamics of external asset positions in G7 countries; we present simulation results that show that the model quantitatively captures key dynamic stylized facts.

<sup>7</sup>For other related recent empirical and theoretical analysis of international portfolios and external valuation effects, see e.g. Lewis (1999), Hau and Rey (2004), Siourounis (2004), Kraay and Ventura (2005), Devereux and Saito (2005), Ghironi, Lee and Rebucci (2005), Obstfeld (2006), Kollmann (2006a), and Matsumoto (2007). Evans and Hnatkovska (2005,2007) and Hnatkovska (2005) also discuss a world with capital accumulation and portfolios; those papers do not analyze the hedging logic that is central to our paper, and have a different empirical focus. Cantor and Mark (1988) provided an early theoretical discussion of the role of equity price changes for current accounts, based on a one-good model with equities trade (their model predicts full portfolio diversification).

<sup>8</sup>In simultaneous and independent work, Raffo (2008) also studies the effect of investment efficiency shocks on international business cycles.

## 2 The model

There are two (ex-ante) symmetric countries, Home ( $H$ ) and Foreign ( $F$ ), each with a representative household. Country  $i = H, F$  produces one good using labor and capital. There is trade in goods and in financial assets (stocks and bonds). All markets are perfectly competitive.

### 2.1 Preferences

Country  $i$  is inhabited by a representative household who lives in periods  $t = 0, 1, 2, \dots$ . The household has the following life-time utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \frac{l_{i,t}^{1+\omega}}{1+\omega} \right), \quad (1)$$

with  $\omega > 0$ .  $C_{i,t}$  is  $i$ 's aggregate consumption in period  $t$  and  $l_{i,t}$  is labor effort. Like much of the macroeconomics and finance literature, we take the coefficient of relative risk aversion to be greater than unity:  $\sigma > 1$ .

$C_{i,t}$  is a composite good given by:

$$C_{i,t} = \left[ a^{1/\phi} (c_{i,t}^i)^{(\phi-1)/\phi} + (1-a)^{1/\phi} (c_{j,t}^i)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}, \text{ with } j \neq i, \quad (2)$$

where  $c_{j,t}^i$  is country  $i$ 's consumption of the good produced by country  $j$  at date  $t$ .  $\phi > 0$  is the elasticity of substitution between the two goods. In the (symmetric) deterministic steady state,  $a$  is the share of consumption spending devoted to the local good. We assume a preference bias for local goods,  $\frac{1}{2} < a < 1$ .

The welfare based consumer price index that corresponds to these preferences is:

$$P_{i,t} = \left[ a(p_{i,t})^{1-\phi} + (1-a)(p_{j,t})^{1-\phi} \right]^{1/(1-\phi)}, \quad j \neq i, \quad (3)$$

where  $p_{i,t}$  is the price of good  $i$ .

### 2.2 Technologies and firms

In period  $t$ , country  $i$  produces  $y_{i,t}$  units of good  $i$  according to the production function

$$y_{i,t} = \theta_{i,t}(k_{i,t})^\kappa(l_{i,t})^{1-\kappa}, \quad (4)$$

with  $0 < \kappa < 1$ .  $k_{i,t}$  is the country's stock of capital. Total factor productivity (TFP)  $\theta_{i,t} > 0$  is an exogenous random variable. The law of motion of the capital stock is:

$$k_{i,t+1} = (1 - \delta)k_{i,t} + \chi_{i,t}I_{i,t} \quad (5)$$

where  $0 < \delta < 1$  is the depreciation rate of capital.  $I_{i,t}$  is gross investment in country  $i$  at date  $t$ .  $\chi_{i,t} > 0$  is an exogenous shock to investment efficiency (see Fisher (2002, 2006), Greenwood, Hercowitz and Krusell (1997), Justiniano, Primiceri and Tambalotti (2007)). The stochastic properties of the exogenous shocks are symmetric across countries.

In both countries, gross investment is generated using Home and Foreign inputs:

$$I_{i,t} = [a_I^{1/\phi_I} (i_{i,t}^i)^{(\phi_I-1)/\phi_I} + (1 - a_I)^{1/\phi_I} (i_{j,t}^i)^{(\phi_I-1)/\phi_I}]^{\phi_I/(\phi_I-1)}, \quad j \neq i, \quad (6)$$

where  $i_{j,t}^i$  is the amount of good  $j$  used for investment in country  $i$ . We assume a local bias in investment spending,  $\frac{1}{2} < a_I < 1$ . Home bias and the substitution elasticity between domestic and imported inputs may be different for investment and consumption (i.e. we allow for the possibility that  $a_I \neq a, \phi_I \neq \phi$ ). The associated investment price index is:

$$P_{i,t}^I = [a_I (p_{i,t})^{1-\phi_I} + (1 - a_I) (p_{j,t})^{1-\phi_I}]^{1/(1-\phi_I)}, \quad j \neq i. \quad (7)$$

There is a (representative) firm in country  $i$  that hires local labor, accumulates physical capital and produces output, using the technology (4),(5); it maximizes the present value of dividend payments, taking prices and wage rates as given.

Due to the Cobb-Douglas technology, a share  $1 - \kappa$  of output is paid to workers. Thus, the country  $i$  wage bill is:

$$w_{i,t}l_{i,t} = (1 - \kappa)p_{i,t}y_{i,t}, \quad (8)$$

where  $p_{i,t}$  is the price of the country  $i$  good and  $w_{i,t}$  is the country  $i$  wage rate.

For simplicity, we consider a baseline model specification in which investment is financed out of retained earnings; a share  $\kappa$  of the country's output, net of physical investment spending is thus paid out as a dividend  $d_{i,t}$  to shareholders:

$$d_{i,t} = \kappa p_{i,t}y_{i,t} - P_{i,t}^I I_{i,t}. \quad (9)$$

Below, we also discuss a model variant in which firms issue debt to finance investment spending; the household's equilibrium equity portfolio in that variant is the same as in the baseline specification.

The firm chooses  $I_{i,t}$  to equate the expected future marginal gain of investment to the marginal cost. This implies the following first-order condition:

$$1 = E_t \varrho_{i,t+1}^i \frac{\chi_{i,t}}{P_{i,t}^I} [p_{i,t+1} \theta_{i,t+1} \kappa k_{i,t+1}^{\kappa-1} l_{i,t+1}^{1-\kappa} + (1 - \delta) \frac{P_{i,t+1}^I}{\chi_{i,t+1}}], \quad (10)$$

where  $\varrho_{i,t+1}^i \equiv \beta(C_{i,t+1}/C_{i,t})^{-\sigma} (P_{i,t}/P_{i,t+1})$  is a pricing kernel used at date  $t$  to value date  $t+1$  payoffs. Note that we assume that  $\varrho_{i,t+1}^i$  equals the intertemporal marginal rate of substitution of the country  $i$  household.<sup>9</sup> The firm chooses the Home and Foreign investment inputs  $i_{H,t}^i, i_{F,t}^i$  that minimize the cost of generating  $I_{i,t}$ . That cost minimization problem has the following first-order conditions:

$$i_{i,t}^i = a_I \left( \frac{p_{i,t}}{P_{i,t}^I} \right)^{-\phi_I} I_{i,t}, \quad i_{j,t}^i = (1 - a_I) \left( \frac{p_{j,t}}{P_{i,t}^I} \right)^{-\phi_I} I_{i,t}, \quad j \neq i. \quad (11)$$

## 2.3 Financial markets, household decisions, market clearing

There is international trade in stocks and bonds. The country  $i$  firm issues a stock that represents a claim to its stream of cash-flows  $\{d_{i,t}\}$ . The supply of each share is normalized at unity. There is a bond denominated in the Home good, and a bond denominated in the Foreign good; buying one unit of the Home (Foreign) bond in period  $t$  gives one unit of the Home (Foreign) good in all future periods. Both bonds are in zero net supply. Each household fully owns the local stock, at birth, and has zero initial foreign assets.<sup>10</sup> Let  $S_{j,t+1}^i$  denote the number of shares of stock  $j$  held by the country  $i$  household at the end of period  $t$ , while  $b_{j,t+1}^i$  represents claims held by  $i$  (at the end of  $t$ ) to future unconditional payments of good  $j$ . At date  $t$ , the country  $i$  household faces the following budget constraint:

$$\begin{aligned} & P_{i,t} C_{i,t} + p_{i,t}^S S_{i,t+1}^i + p_{j,t}^S S_{j,t+1}^i + p_{i,t}^b b_{i,t+1}^i + p_{j,t}^b b_{j,t+1}^i \\ &= w_{i,t} l_{i,t} + (p_{i,t}^S + d_{i,t}) S_{i,t}^i + (p_{j,t}^S + d_{j,t}) S_{j,t}^i + (p_{i,t}^b + p_{i,t}) b_{i,t}^i + (p_{j,t}^b + p_{j,t}) b_{j,t}^i, \quad j \neq i, \end{aligned} \quad (12)$$

<sup>9</sup>When the Home and Foreign households' Euler equations for Home/Foreign stocks shown below hold (see (14)), then (10) holds also for a pricing kernel that equals the intertemporal marginal rate of substitution of the country  $j \neq i$  household.

<sup>10</sup>We also assume that initial capital stocks and productivities are identical across countries:  $K_{H,0} = K_{F,0}$ ,  $\theta_{H,0} = \theta_{F,0}$ ,  $\chi_{H,0} = \chi_{F,0}$ . This ensures that both countries have equal wealth at birth, and preserves the (ex ante) symmetry of the two countries.

where  $p_{i,t}^S$  is the price of stock  $i$  and  $p_{i,t}^b$  is the price of the good- $i$  bond.

Each household selects portfolios, consumptions and labor supplies that maximize her life-time utility

(1) subject to her budget constraint (12) for  $t \geq 0$ . Ruling out Ponzi-schemes, the following equations are first-order conditions of household  $i$ 's decision problem:

$$c_{i,t}^i = a \left( \frac{p_{i,t}}{P_{i,t}} \right)^{-\phi} C_{i,t}, \quad c_{j,t}^i = (1-a) \left( \frac{p_{j,t}}{P_{i,t}} \right)^{-\phi} C_{i,t}, \quad l_{i,t}^\omega = \left( \frac{w_{i,t}}{P_{i,t}} \right) C_{i,t}^{-\sigma} \quad (13)$$

$$1 = E_t \varrho_{t,t+1}^i R_{j,t+1}^S, \quad 1 = E_t \varrho_{t,t+1}^i R_{j,t+1}^b \quad \text{for } j = H, F, \quad (14)$$

$$\text{with } R_{j,t+1}^S \equiv \frac{p_{j,t+1}^S + d_{j,t+1}}{p_{j,t}^S}, \quad R_{j,t+1}^b \equiv \frac{p_{j,t+1}^b + p_{j,t+1}}{p_{j,t}^b}. \quad (15)$$

$R_{j,t+1}^S$  and  $R_{j,t+1}^b$  are the gross returns of stock  $j$ , and of the good- $j$  bond, respectively (between periods  $t$  and  $t+1$ ). (13) represents the optimal allocation of consumption spending across goods, and the labor supply decision. (14) shows Euler equations with respect to the two stocks and the Home and Foreign good bonds.

*Market-clearing* in goods and asset markets requires:

$$c_{H,t}^H + c_{H,t}^F + i_{H,t}^H + i_{H,t}^F = y_{H,t}, \quad c_{F,t}^F + c_{F,t}^H + i_{F,t}^F + i_{F,t}^H = y_{F,t}, \quad (16)$$

$$S_{H,t}^H + S_{H,t}^F = S_{F,t}^F + S_{F,t}^H = 1, \quad b_{H,t}^H + b_{H,t}^F = b_{F,t}^F + b_{F,t}^H = 0. \quad (17)$$

## 2.4 Relative consumption and investment demand

Subsequent discussions will use the following properties of consumption and investment demand. The first-order condition for consumption (13) implies:

$$c_{H,t}^H + c_{H,t}^F = p_{H,t}^{-\phi} \left[ a C_{H,t} P_{H,t}^\phi + (1-a) C_{F,t} P_{F,t}^\phi \right], \quad c_{F,t}^F + c_{F,t}^H = p_{F,t}^{-\phi} \left[ a C_{F,t} P_{F,t}^\phi + (1-a) C_{H,t} P_{H,t}^\phi \right]$$

Taking the ratio of these expressions gives:

$$y_{C,t} \equiv \frac{c_{H,t}^H + c_{H,t}^F}{c_{F,t}^F + c_{F,t}^H} = q_t^{-\phi} \Omega_a \left[ \left( \frac{P_{F,t}}{P_{H,t}} \right)^\phi \frac{C_{F,t}}{C_{H,t}} \right], \quad \text{with } \Omega_z(x) \equiv \frac{1+x(\frac{1-z}{z})}{x+(\frac{1-z}{z})}. \quad (18)$$

$y_{C,t}$  is the ratio of world consumption of Home goods over world consumption of Foreign goods, while  $q_t \equiv p_{H,t}/p_{F,t}$  denotes the country H terms of trade.

The ratio of world demand for Home vs. Foreign goods used for physical investment  $y_{I,t} \equiv \frac{i_{H,t}^H + i_{H,t}^F}{i_{F,t}^F + i_{F,t}^H}$  can similarly be expressed as:

$$y_{I,t} \equiv q_t^{-\phi_I} \Omega_{a_I} \left[ \left( \frac{P_{F,t}^I}{P_{H,t}^I} \right)^{\phi_I} \frac{I_{F,t}}{I_{H,t}} \right]. \quad (19)$$

## 3 Characterization of (steady state) equilibrium portfolios

Equilibrium portfolio holdings chosen at date  $t$  ( $S_{i,t+1}^i, S_{j,t+1}^i, b_{i,t+1}^i, b_{j,t+1}^i$ ) are functions of predetermined state variables, and of exogenous shocks at  $t$ . Devereux and Sutherland (2006a,b) show how to compute Taylor expansion of the portfolio decision rules, in the neighborhood of the deterministic steady state. In this Section, we provide closed form solutions for the Home/Foreign 'zero-order portfolio' (denoted by variables without time subscripts)  $S_i^i, S_j^i, b_i^i, b_j^i$ , i.e. portfolio decision rules evaluated at steady state values of state variables. That portfolio can be determined by linearizing the model around its steady state.<sup>11</sup>

### 3.1 Linearization of the model

Henceforth, variables without a time subscript refer to the steady state.  $\hat{z}_t \equiv (z_t - z)/z$  denotes the relative deviation of a variable  $z_t$  from its steady state value  $z$ .

Below we find a zero-order portfolio such that the ratio of Home to Foreign marginal utilities of aggregate consumption,  $C_{H,t}^{-\sigma}/C_{F,t}^{-\sigma}$ , is equated to the consumption-based real exchange rate,  $RER_t \equiv \frac{P_{H,t}}{P_{F,t}}$ , up to first order:

$$-\sigma (\widehat{C_{H,t}} - \widehat{C_{F,t}}) = \widehat{RER_t}. \quad (20)$$

This is a linearized version of a risk sharing condition that holds under complete markets (Backus and Smith (1993), Kollmann (1991, 1995)). Up to first order, the asset structure here (four assets, in a

<sup>11</sup> Devereux and Sutherland (2006a,b) show that the zero-order equilibrium portfolio has to satisfy a second-order accurate approximation of household Euler equations, expressed in 'relative' form:  $0 = E_t \varrho_{t,t+1} r_{t+1}^X$ , where  $\varrho_{t,t+1} \equiv \varrho_{t,t+1}^H - \varrho_{t,t+1}^F$  is the 'relative' IMRS of the two households, while  $r_{t+1}^X \equiv (R_{H,t+1}^S - R_{H,t+1}^b, R_{F,t+1}^S - R_{F,t+1}^b, R_{F,t+1}^b - R_{H,t+1}^b)$  is a vector of excess returns. As  $\varrho = r^X = 0$  in steady state, a second-order accurate approximation is given by  $0 = E_t (\varrho_{t,t+1})^{(1)} (r_{t+1}^X)^{(1)}$ , where  $(\varrho_{t,t+1})^{(1)}$  and  $(r_{t+1}^X)^{(1)}$  are first-order accurate. The zero-order portfolio discussed below satisfies a linearized risk sharing condition (see (20)) that entails that  $(\varrho_{t,t+1})^{(1)} = 0$ ; thus, the zero-order portfolio discussed below ensures that  $0 = E_t \varrho_{t,t+1} r_{t+1}^X$  holds to second-order.

world with four exogenous shocks) is thus (effectively) complete.<sup>12</sup>

It follows from the definition of Home and Foreign CPI indices (see (3)) that

$$\widehat{RER}_t = \widehat{P_{H,t}} - \widehat{P_{F,t}} = (2a - 1)\widehat{q}_t. \quad (21)$$

Due to consumption home bias ( $a > \frac{1}{2}$ ), an improvement of the Home terms of trade generates an appreciation of the Home real exchange rate.

When (20) holds, then the relative world consumption demand for the Home good obeys (from (18)):

$$\widehat{y_{C,t}} = -\left[\phi\left(1 - (2a - 1)^2\right) + (2a - 1)^2 \frac{1}{\sigma}\right] \widehat{q}_t \equiv -\lambda \widehat{q}_t \quad (22)$$

where  $\lambda \equiv \phi(1 - (2a - 1)^2) + \frac{(2a - 1)^2}{\sigma}$ . Note that  $\lambda > 0$  (as  $1/2 < a < 1$  implies  $0 < 1 - (2a - 1)^2$ ). Thus, an improvement in the Home terms of trade lowers worldwide relative consumption of the Home good.

Linearization of (19) and (7) shows that relative world investment demand for the Home good,  $y_{I,t}$ , obeys:

$$\widehat{y_{I,t}} = -\phi_I\left(1 - (2a_I - 1)^2\right) \widehat{q}_t + (2a_I - 1)\widehat{I}_t, \quad (23)$$

where  $I_t \equiv I_{H,t}/I_{F,t}$  is relative aggregate investment. Holding constant the terms of trade, the relative demand for Home investment goods,  $y_{I,t}$ , increases with relative real investment in the Home country,  $I_t$ , since Home aggregate investment is biased towards the Home good ( $a_I > \frac{1}{2}$ ).

The market clearing condition for goods (16) implies:

$$(1 - \Lambda)\widehat{y_{C,t}} + \Lambda\widehat{y_{I,t}} = \widehat{y}_t, \quad (24)$$

where  $y_t \equiv Y_{H,t}/Y_{F,t}$  is relative Home output, while  $\Lambda \equiv \frac{P_H^I I_H}{p_H y_H} = \frac{P_F^I I_F}{p_F y_F}$  is the steady state investment/GDP ratio.<sup>13</sup>

Substituting (22) and (23) into (24) gives:

$$\widehat{y}_t = -\lambda^* \widehat{q}_t + \Lambda(2a_I - 1)\widehat{I}_t \quad (25)$$

<sup>12</sup>Using the apparatus of Devereux and Sutherland (2006a,b) we confirmed for the model calibration below (and for all of many other calibrations with which we experimented) that the zero-order equilibrium portfolio is unique; there is no zero-order equilibrium portfolio for which the risk sharing condition (20) does *not* hold, to first order.

<sup>13</sup>Note that, because of symmetry,  $P_H^I/p_H = P_F^I/p_F = 1$ ,  $I_H = I_F$ ,  $y_H = y_F$ .

where  $\lambda^* = (1 - \Lambda)\lambda + \Lambda\phi_I\left(1 - (2a_I - 1)^2\right) > 0$ .<sup>14</sup>

Not surprisingly, Home terms of trade worsen when the relative supply of Home goods increases, for a given amount of relative Home country investment. Home terms of trade improve when Home investment rises (due to local bias in investment spending), for a given value of the relative Home/Foreign output.

### 3.2 Zero-order portfolios

Ex-ante symmetry implies that the zero-order portfolios have to satisfy these conditions:  $S \equiv S_H^H = S_F^F = 1 - S_H^F = 1 - S_F^H$ ;  $b \equiv b_H^H = b_F^F = -b_H^F = -b_F^H$ . The pair  $(S, b)$  thus describes the (zero-order) equilibrium portfolio. Note that  $S$  denotes a country's holdings of local stock, while  $b$  denotes its holdings of bonds denominated in the local good. There is equity home bias when  $S > \frac{1}{2}$ .  $b > 0$  means that a country is long in local-good bonds (and short in foreign-good bonds).

We now show that there exists a unique portfolio  $(S, b)$  that satisfies the following 'static' budget constraint, for consumptions that are consistent with the linearized risk sharing condition (20):

$$P_{i,t} C_{i,t} = w_{i,t} l_{i,t} + S d_{i,t} + (1 - S) d_{j,t} + b(p_{i,t} - p_{j,t}), \text{ for } i = H, F. \quad (26)$$

According to this constraint, country  $i$  consumption spending at date  $t$  equals date  $t$  wage income,  $w_{i,t} l_{i,t}$ , plus the financial income generated by the zero-order portfolio  $(S, b)$ . We show in Appendix A.1 that when this 'static' budget constraint holds, then the period-by-period budget constraint (12) is likewise satisfied, up to first-order. We here focus on the 'static' budget constraint, as it greatly simplifies the analysis.

Subtracting the 'static' budget constraint of country  $F$  from that of country  $H$  gives:

$$P_{H,t} C_{H,t} - P_{F,t} C_{F,t} = (w_{H,t} l_{H,t} - w_{F,t} l_{F,t}) + (2S - 1)(d_{H,t} - d_{F,t}) + 2b(p_{H,t} - p_{F,t}) \quad (27)$$

Linearizing this yields:

$$(1 - \Lambda)(\widehat{P_{H,t} C_{H,t}} - \widehat{P_{F,t} C_{F,t}}) = (1 - \Lambda)\underbrace{\left(1 - \frac{1}{\sigma}\right)(2a - 1)}_{\widehat{RER}_t} \widehat{q}_t = (1 - \kappa)\widehat{w_l l_t} + (2S - 1)(\kappa - \Lambda)\widehat{d}_t + 2\tilde{b}\widehat{q}_t, \quad \tilde{b} \equiv b/y_H, \quad (28)$$

where  $\widehat{w_l l_t} \equiv \widehat{w_{H,t} l_{H,t}} - \widehat{w_{F,t} l_{F,t}}$  denotes relative Home labor income, while  $\widehat{d}_t \equiv \widehat{d_{H,t}} - \widehat{d_{F,t}}$  is the relative Home dividend, and  $\tilde{b}$  represents holdings of local-good bonds, divided by steady state GDP.

<sup>14</sup>When  $\phi_I = \phi$  and  $a_I = a$  then  $\lambda^* = \phi(1 - (2a - 1)^2) + \frac{1 - \Lambda}{\sigma}(2a - 1)^2$ .

The first equality in (28) follows from the linearized risk-sharing condition (20); it shows the efficient reaction of relative consumption spending to a change of the welfare based real exchange rate. This reaction depends on the coefficient of relative risk aversion. A shock that appreciates the real exchange rate of country  $H$ , induces an increase in country  $H$  relative consumption spending when  $\sigma > 1$  (as assumed here). (20) shows that when the Home real exchange rate appreciates by 1%, then relative aggregate country  $H$  consumption  $\left(\frac{C_H}{C_F}\right)$  decreases by  $1/\sigma$  %. Hence, relative country  $H$  consumption spending  $(\frac{P_H C_H}{P_F C_F})$  increases by  $(1 - \frac{1}{\sigma})\%$ .

The expression to the right in (28) shows the change in country  $H$  income (relative to the income of  $F$ ) necessary to finance the efficient consumption (up to first order). Given  $\sigma > 1$ , the efficient portfolio has to be such that a real appreciation is associated with an increase in relative Home income.

Since labor income is a constant share of output (see (8)), relative labor income ( $\widehat{w_t l_t}$ ) is given by:  $\widehat{w_t l_t} = \widehat{q_t} + \widehat{y_t}$ . (9) and (7) imply that the relative dividend ( $\widehat{d}_t$ ) is given by:

$$(\kappa - \Lambda)\widehat{d}_t = \kappa(\widehat{q}_t + \widehat{y}_t) - \Lambda(\widehat{P}_{H,t}^I \widehat{I}_{H,t} - \widehat{P}_{F,t}^I \widehat{I}_{F,t}) = \kappa(\widehat{q}_t + \widehat{y}_t) - \Lambda((2a_I - 1)\widehat{q}_t + \widehat{I}_t). \quad (29)$$

Substituting (29) into (28) gives:

$$(1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1)\widehat{q}_t = (1 - \kappa)(\widehat{q}_t + \widehat{y}_t) + (2S - 1)\{\kappa(\widehat{q}_t + \widehat{y}_t) - \Lambda((2a_I - 1)\widehat{q}_t + \widehat{I}_t)\} + 2\tilde{b}\widehat{q}_t \quad (30)$$

Using (25), we can express (30) as:

$$(1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1)\widehat{q}_t = [(1 - \kappa) + \kappa(2S - 1)][(1 - \lambda^*)\widehat{q}_t + \Lambda(2a_I - 1)\widehat{I}_t] - \Lambda(2S - 1)[(2a_I - 1)\widehat{q}_t + \widehat{I}_t] + 2\tilde{b}\widehat{q}_t \quad (31)$$

The asset structure supports the full risk sharing condition (20), up to first-order, if (31) holds for all realizations of the two (relative) exogenous shocks  $(\widehat{\theta}_t, \widehat{\chi}_t)$ . The following portfolio  $(S, \tilde{b})$  ensures that (31) holds for arbitrary realizations of  $(\widehat{q}_t, \widehat{I}_t)$ :

$$S = \frac{1}{2} \left[ 1 + \frac{(2a_I - 1)(1 - \kappa)}{1 - (2a_I - 1)\kappa} \right] > \frac{1}{2}, \quad (32)$$

$$\tilde{b} = \frac{1}{2} \left[ (1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1) + \frac{(1 - \kappa)[\lambda^* - 1 + \Lambda(2a_I - 1)^2]}{1 - (2a_I - 1)\kappa} \right] \quad (33)$$

Thus, the model generates equity home bias:  $1/2 < S < 1$ . Interestingly, the equity portfolio is independent of preference parameters; in particular,  $S$  is independent of the substitution elasticity between

Home and Foreign goods, and thus of the strength of the response of the terms of trade to shocks.<sup>15</sup> The equity portfolio solely depends on the local bias in investment spending ( $a_I$ ) and on the capital share ( $\kappa$ ); equity home bias is increasing in the local spending bias—this prediction is strongly supported by the data (see Heathcote and Perri (2007) and Collard, Dellas, Diba and Stockman (2007)).

The persistence of shocks and their correlation do not matter for the (zero-order) equilibrium portfolio (as long as the shocks are not perfectly correlated). Note that, to solve for the equilibrium portfolio, we do not have to solve for output and investment, as a unique pair of terms of trade and relative real investment  $(\widehat{q}_t, \widehat{I}_t)$  is associated with each realizations of  $(\widehat{\theta}_t, \widehat{\chi}_t)$ . In fact, any other combinations of *two* types of relative (Home vs. Foreign) shocks that only affect the (linearized) relative budget constraint through their effect on the terms of trade and relative investment generates the same equilibrium portfolio—other potential shocks that would generate the same portfolio are e.g. labor supply shocks, "news" shocks regarding future TFP or investment efficiency, and shocks to the depreciation rate of capital.

Note also that, in contrast to the setting here (with trade in stocks and bonds), general equilibrium models with just trade in stocks (no bonds) predict that the equity portfolio exhibits strong sensitivity with respect to the substitution elasticity between local and imported goods (e.g., Kollmann (2006b), Coeurdacier (2009) and Heathcote and Perri (2007)).<sup>16</sup>

In the model here, the *bond* portfolio does depend on the substitution elasticities  $\phi, \phi_I$  (via  $\lambda^*$ ) and on risk aversion ( $\sigma$ ); however this dependence is 'smooth': in particular, the net local-good bond position  $\tilde{b}$  is a *linear* function of  $\phi$  and  $\phi_I$ .<sup>17</sup> Depending on preference parameters, the model can generate a positive or negative value of  $\tilde{b}$ . The country will go short in the local-good bond ( $\tilde{b} < 0$ ) when  $\phi$  is sufficiently low (roughly below unity). When  $\phi$  is low, then the terms of trade respond strongly to shocks; an improvement in the Home terms of trade (induced by a fall in Home TFP and/or an increase in Home investment efficiency) increases Home relative wage plus dividend income (due to the strong terms of trade change); risk sharing requires to compensate this relative income effect by shorting the local good bond (when  $\tilde{b} < 0$ , a terms of trade improvement lowers the net bond income received by Home). By

<sup>15</sup>However it is necessary that good are imperfect substitutes so that the terms of trade show a non-zero response to shocks.

<sup>16</sup>In those models, the asset structure cannot support the efficient allocation when Home and Foreign dividends are co-linear, which occurs for a value of the substitution elasticity roughly located between 1 and 2; for substitution elasticities just below or above the critical value, the local equity share takes extremely large positive or negative values.

<sup>17</sup>See Coeurdacier, Kollmann and Martin (2007) and Coeurdacier and Gourinchas (2008) for a similar result.

contrast, the country goes long in the local good bond, and short in the foreign good bond, when  $\phi$  is (roughly) greater than unity.

Recent empirical research (Lane and Shambaugh (2007, 2008)) shows that, on average, the advanced countries have negative net foreign-currency debt positions—which is consistent with our model for values of  $\phi$  above unity. However, there is a great deal of cross-country heterogeneity in net foreign/domestic currency debt positions. While net debt positions are small for most advanced countries, some major countries have large negative net domestic-currency debt positions; most notably this is the case for the US. In our theoretical framework, negative net local-good debt positions obtain for a wider range of parameters, if one allows for corporate debt (see below). Finally, one should note that countries can easily alter the effective currency composition of their debt portfolio by taking net positions in the forward currency market; this further complicates the comparison of our theoretical bond positions with their empirical counterparts.

### Debt and Equity Financing

We have assumed so far that firms are fully financed through equity, and that investment is fully financed through retained earnings. In Appendix A.3, we discuss a model variant in which firms are partly financed through debt. Since the Modigliani-Miller theorem applies in the structure here, corporate debt does not affect the value of firms, physical investment and the equilibrium consumption allocation. As shown in the Appendix, the equilibrium equity portfolio is likewise unaffected by the presence of corporate bonds (i.e.  $S$  continues to be given by (32)). The country  $i$  household holds a fraction  $S$  of the corporate debt issued by the local firm, in order to offset the implicit debt position entailed by the household's local equity position; thus households exhibit home bias for corporate debt, in the same proportion as for stocks. If firms mainly issue local-good debt, this lowers the countries' overall (household+corporate) net local-good bond position: when the local firm issues one unit of debt denominated in the local good, then the overall net local debt position changes by  $S - 1 < 0$  units, as a share  $S$  of the new debt will be purchased by the local household, while a share  $1 - S$  will be bought by the foreign household.

### The hedging roles of bonds and stocks

We now show that the bond portfolio hedges terms of trade (real exchange rate) risk—as preference parameters affect the response of relative consumption to terms of trade changes, bond holdings depend on

those preference parameters. Equities are used to hedge fluctuations in relative wages and dividends that are orthogonal to the terms of trade. The comovement of relative wages and dividends, at constant terms of trade, depends on  $a_I$  and  $\kappa$ , but not on preference parameters—which explains why the equilibrium value of  $S$  is a function of  $a_I$  and  $\kappa$ .<sup>18</sup>

Assume a combination of exogenous shocks  $(\hat{\theta}_t, \hat{\chi}_t)$  that raises relative country  $H$  real investment spending, without altering the terms of trade:  $\hat{I}_t > 0, \hat{q}_t = 0$ . From (25), we know that this combination of shocks raises  $H$  relative output  $\hat{y}_t$ , due to local bias in investment spending ( $a_I > 1/2$ ):  $\hat{y}_t = \Lambda(2a_I - 1)\hat{I}_t > 0$ , when  $\hat{q}_t = 0$ . As the real exchange rate is unaffected when  $\hat{q}_t = 0$  (see (21)), efficient risk sharing requires that countries' relative consumption spending remains unchanged. Hence, the efficient portfolio has to be such that the countries' relative income too is unaffected. From (30) it can be seen that this requires that:

$$0 = (1 - \kappa)\hat{y}_t + (2S - 1)\{\kappa\hat{y}_t - \Lambda\hat{I}_t\}. \quad (34)$$

$(1 - \kappa)\hat{y}_t$  and  $\{\kappa\hat{y}_t - \Lambda\hat{I}_t\}$  respectively represent relative labor income of country  $H$  and the relative dividend of stock  $H$ , for  $\hat{q}_t = 0$ . Note that  $\kappa\hat{y}_t - \Lambda\hat{I}_t = [\kappa(2a_I - 1) - 1]\Lambda\hat{I}_t$  when  $\hat{y}_t = \Lambda(2a_I - 1)\hat{I}_t$ . Thus,  $\kappa\hat{y}_t - \Lambda\hat{I}_t < 0$  when  $\hat{I}_t > 0, \hat{q}_t = 0$ . In other terms, a combination of shocks that raises  $H$  relative investment without affecting the terms of trade induces a rise in  $H$ 's relative wage income, and a fall in the relative dividend of stock  $H$ . This makes holding local equity attractive:  $S > 1/2$  is needed to ensure that (34) holds.<sup>19</sup>

Once shocks that do not affect the terms of trade have been hedged by holding local equity, the remaining risk (changes in output/investment that are associated with terms of trade changes) can be hedged (up to a linear approximation) using the bond portfolio; this is so because terms of trade movements are perfectly correlated with the difference between the pay-offs of Home and Foreign good bonds.

### Comparison with Heathcote and Perri (2007)

Our equity portfolio (32) corresponds to that obtained by Heathcote and Perri (2007) [HP], for a special case of their model with a unit risk aversion coefficient ( $\sigma = 1$ ) and a unit elasticity of substitution

<sup>18</sup>Coeurdacier and Gourinchas (2008) provide a general discussion of conditions under which international equity portfolios are independent of preferences; they show that an important condition is that bonds exist whose pay-offs perfectly track real exchange rate movements.

<sup>19</sup>To derive the value of  $S$  shown in (32), one can substitute  $\hat{y}_t = \Lambda(2a_I - 1)\hat{I}_t$  into (34); the only value of  $S$  for which the resulting expression holds for arbitrary  $\hat{I}_t$  is given by (32).

between domestic and foreign good ( $\phi = 1$ ). HP assume a two-country world with capital accumulation, with just trade in stocks (no bonds), and just TFP shocks. In their model, the equity portfolio is sensitive to slight changes in  $\sigma$  and  $\phi$ ; extreme home or foreign equity bias occurs for values of  $\sigma$  and  $\phi$  in a plausible range above unity.<sup>20</sup>

Here we have shown that this sensitivity of portfolio choices disappears once we allow for trade in bonds, and an additional source of uncertainty on the production side (here shock to investment efficiency). This robustness is due to the fact that, in our model, terms of trade risk is hedged by the bond portfolio. This result is important, as there is considerable uncertainty regarding the value of the substitution elasticity between domestic and foreign goods: estimates from aggregate macro data are scattered around unity, but estimates from sectoral trade data are above 4 (see Imbs and Méjean (2008) for a detailed discussion).

The reason why the HP model delivers equity home bias when  $\sigma = \phi = 1$  is that, for that parameterization, the two countries' efficient relative consumption spending is constant, while a country's relative wage income is (perfectly) negatively correlated with the relative dividend of the stock issued by the country ( $\text{Corr}(\widehat{w_t l_t}, \widehat{d_t}) < 0$ ), which implies that local equity is a good hedge for labor income risk. As documented below, the correlation between relative wage income ( $\widehat{w_t l_t}$ ) and the relative dividend ( $\widehat{d_t}$ ) is positive, for G7 countries. Thus, the key mechanism that generates equity home bias in the HP model is rejected empirically.

### 3.3 The role of the correlation between relative wage incomes and relative 'dividends'

In our model, the unconditional correlation  $\text{Corr}(\widehat{w_t l_t}, \widehat{d_t})$  per se is irrelevant for the equilibrium equity portfolio. What matters is the correlation between the components of  $\widehat{w_t l_t}$  and  $\widehat{d_t}$  that are *orthogonal* to the terms of trade,  $\widehat{q}_t$ : there is equity home bias when that correlation is negative. To see this, project equation (28) on  $\widehat{q}_t$ . This gives:

$$(1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1)\widehat{q}_t = (1 - \kappa)P[\widehat{w_t l_t}|\widehat{q}_t] + (2S - 1)(\kappa - \Lambda)P[\widehat{d_t}|\widehat{q}_t] + 2\widetilde{b}\widehat{q}_t, \quad (35)$$

<sup>20</sup>Castello (2007) considers a model of portfolio choice with capital accumulation close to HP; in her model too, equity portfolios are highly sensitive to preference parameters.

where  $P[\widehat{w_t l_t}|\widehat{q}_t]$  is the (linear) projection of  $\widehat{w_t l_t}$  on  $\widehat{q}_t$ . (NB  $\widehat{q}_t = P[\widehat{q}_t|\widehat{q}_t]$ .) Subtracting (35) from (28) gives:

$$0 = (1 - \kappa)\{\widehat{w_t l_t} - P[\widehat{w_t l_t}|\widehat{q}_t]\} + (2S - 1)(\kappa - \Lambda)\{\widehat{d_t} - P[\widehat{d_t}|\widehat{q}_t]\}. \quad (36)$$

Thus, the equity portfolio has to hedge the components of  $\widehat{w_t l_t}$  and  $\widehat{d_t}$  that are orthogonal to the terms of trade  $\widehat{q}_t$ . (36) implies that

$$S = \frac{1}{2} - \frac{1}{2} \frac{1 - \kappa}{\kappa - \Lambda} \frac{\text{Cov}_{\widehat{q}}(\widehat{w_t l_t}, \widehat{d_t})}{\text{Var}_{\widehat{q}}(\widehat{d_t})}, \quad (37)$$

with  $\text{Cov}_{\widehat{q}}(\widehat{w_t l_t}, \widehat{d_t}) \equiv E\{\widehat{w_t l_t} - P[\widehat{w_t l_t}|\widehat{q}_t]\}\{\widehat{d_t} - P[\widehat{d_t}|\widehat{q}_t]\}$ ,  $\text{Var}_{\widehat{q}}(\widehat{d_t}) \equiv E\{\widehat{d_t} - P[\widehat{d_t}|\widehat{q}_t]\}^2$ .<sup>21</sup> Hence there is equity home bias if and only if  $\text{Cov}_{\widehat{q}}(\widehat{w_t l_t}, \widehat{d_t}) < 0$ .<sup>22</sup> In the model here,  $\text{Cov}_{\widehat{q}}(\widehat{w_t l_t}, \widehat{d_t}) = (\kappa - \Lambda)(2a_I - 1)/[(\kappa(2a_I - 1) - 1)] < 0$ . Empirically,  $\text{Cov}_{\widehat{q}}(\widehat{w_t l_t}, \widehat{d_t}) < 0$ , for G7 countries, as documented below.

A similar condition is derived by Engel and Matsumoto (2006) who show, in a model with trade in equity and nominal forward currency contracts, that the equilibrium equity position depends on the conditional covariance between wage income and dividends, conditional on the nominal exchange rate.

Note also that  $(1 - \kappa)P[\widehat{w_t l_t}|\widehat{q}_t] + (2S - 1)(\kappa - \Lambda)P[\widehat{d_t}|\widehat{q}_t] = \gamma\widehat{q}_t$  for some coefficient  $\gamma$ . Hence, (35) can be expressed as:  $(1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1)\widehat{q}_t = \gamma\widehat{q}_t + 2\widetilde{b}\widehat{q}_t$ . The bond position is set at the value for which this condition holds for any realization of  $\widehat{q}_t$ :  $\widetilde{b} = \frac{1}{2}[(1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1) - \gamma]$ . Thus, the optimal bond position ensures that terms of trade fluctuations induce movements in the two countries' relative incomes (given the optimal equity portfolio) that track optimal relative consumption spending. For this to be the case, relative bond payments need to track the real exchange rate. In the data, domestic-versus foreign-currency bond return differentials are tightly linked to real exchange rate changes (see Coeurdacier and Gourinchas (2009) and van Wincoop and Warnock (2006)).

#### Equilibrium equity portfolios for countries of different size

In order to permit empirical analysis of the determinants of equity home bias, we now briefly consider a two-country model with countries of unequal size, due to different steady state TFP (and/or population). Assume that all preference and technology parameters are the same across countries. Then  $S_i^i$  is given

<sup>21</sup>To see this, multiply (36) by  $\{\widehat{d_t} - P[\widehat{d_t}|\widehat{q}_t]\}$  and take expectations; solving the resulting equation for  $S$  gives (37).

<sup>22</sup>The steady state investment/GDP ratio is  $\Lambda = \kappa/[(1/\delta)(1 - \beta)/\beta + 1]$ . Hence,  $\kappa > \Lambda$ . This ensures that dividends are strictly positive in steady state.

## • References

- [1] Acenoglu D. and J. Ventura, 2002, The World Income Distribution, Quarterly Journal of Economics, 117, 659-94.
- [2] Adler, M. and B. Dumas, 1983, International Portfolio Choice and Corporation Finance: a Synthesis, *Journal of Finance*, 38, 925-984.
- [3] Backus, D., P. Kehoe and F. Kydland, 1994, Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? *American Economic Review*, 84, 84-103.
- [5] Baxter, M. and U. Jermann, 1997, The International Portfolio Diversification is Worse Than You Think, *American Economic Review*, 87, 170-80.
- [6] Bottazzi, L., P. Pisenti and E. van Wincoop, 1996, Wages, Profits and the International Portfolio Puzzle, *European Economic Review*, 40, 219-254.
- [8] Castello M., 2007, A New Open Economy Macroeconomic Model with Endogenous Portfolio Diversification and Firm Entry, Working Paper, European University Institute.
- [9] Coeurdacier, N., 2009, Do Trade Costs in Goods Markets Lead to Home Bias in Equities? *Journal of International Economics*, 77, 86-100.
- [10] Coeurdacier, N. and S. Guibaud, 2008, A Dynamic Equilibrium Model of Imperfectly Integrated Financial Markets, Paris School of Economics, Working Paper 2005-24.
- [11] Coeurdacier, N., R. Kolmann and P. Martin, 2007, International Portfolios with Supply, Demand and Redistributive Shocks, NBER International Seminar on Macroeconomics 2007, 231-263.
- [12] Coeurdacier, N. and Gourinchas P-O, 2009, Home Bias in Goods and Assets: the Role of Bond Trading, Working Paper, University of California, Berkeley.
- 37
- [26] Ghironi, F., J. Lee and A. Rebucci, 2006, The Valuation Channel of External Adjustment, Working Paper, Boston College.
- [27] Gourinchas, P-O and H. Rey, 2005, International Financial Adjustment, National Bureau of Economic Research, Working Paper 11155.
- [28] Greenwood, J., Z. Hercowitz and P. Krusell, 1997, Long-Run Implications of Investment-Specific Technological Change, *American Economic Review*, American Economic Association, vol. 87(3), pages 342-62.
- [29] Greenwood, J., Z. Hercowitz and P. Krusell, 2000, The role of investment-specific technological change in the business cycle, *European Economic Review* 44, 91-115.
- [30] Greenwood, J., Z. Hercowitz and G. Huffman, 1988, Investment, Capacity Utilization and the Real Business Cycle, *American Economic Review* 78, 402-17.
- [31] Han, H. and H. Rey, 2004, Can Portfolio Rebalancing Explain the Dynamics of Equity Returns, *Equity Flows and Exchange Rates?* *American Economic Review P&P*, 94, 126-133.
- [32] Heathcote, J. and F. Perri, 2002, Financial Autarky and International Business Cycles, *Journal of Monetary Economics*, 49, 601-627.
- [33] Heathcote, J. and F. Perri, 2004, Financial Globalization and Real Regionalization, *Journal of Economic Theory*, 119, 207-243.
- [34] Heathcote, J. and F. Perri, 2007, The international diversification is not as bad as you think, National Bureau of Economic Research, Working Paper 13483.
- [35] Helpman, E. and A. Razin, 1978, A Theory of International Trade Under Uncertainty, New York, Academic Press.
- [36] Hnatkovska, V., 2005, Home Bias and High Turnover: Dynamic Portfolio Choice with Incomplete Markets, working paper, Georgetown University.
- [37] Imbs, J. and I. Méjean, 2008, Elasticity Optimism, Working Paper, University of Lausanne.
- [38] Jainovich, Nir and Sergio Rebelo, 2009, Can News about the Future Drive the Business Cycle?, forthcoming, *American Economic Review*.
- 38
- [39] Julliard, C., 2004, Human Capital and International Portfolio Choice, Working Paper, Princeton University.
- [40] Julliard, C., 2002, International Portfolio Diversification Is Not Worse Than You Think, Working Paper, Princeton University.
- [41] Justiniano, A. and G. Primiceri, 2006, The Time Varying Volatility of Macroeconomic Fluctuations, *American Economic Review*, forthcoming, National Bureau of Economic Research, Working Paper No. 12022
- [42] Justiniano, A., G. Primiceri and A. Tambalotti, 2007, Investment Shocks and Business Cycles, Working Paper, Northwestern University.
- [43] Kolmann, R., 1991, Essays on International Business Cycles, PhD Dissertation, Economics Department, University of Chicago.
- [44] Kolmann, R., 1995, Consumption, Real Exchange Rates and the Structure of International Asset Markets, *Journal of International Money and Finance*, 1995, 14, 191-192.
- [45] Kolmann, R., 1996, Incomplete Asset Markets and the Cross-Country Consumption Correlation Puzzle, *Journal of Economic Dynamics and Control*, 20, 945-962.
- [46] Kolmann, R., 1998, US Trade Balance Dynamics: The Role of Fiscal Policy and Productivity Shocks and of Financial Market Linkages, *Journal of International Money and Finance*, 17, 637-69.
- [47] Kolmann, R., 2006a, A Dynamic General Equilibrium Model of International Portfolio Holdings: *Comment, Econometrica*, 74, 269-273.
- [48] Kolmann, R., 2006b, International Portfolio Equilibrium and the Current Account, Centre for Economic Policy Research, Working Paper 5512.
- [49] Kolmann, R., 2006c, Comment on 'International Prices and Productivity: an Empirical Analysis of the Transmission among OECD Countries', NBER International Seminar on Macroeconomics 2006, 186-194.
- [50] Kray, A., N. Loayza, L. Servén and J. Ventura, 2005, Country Portfolios, *Journal of the European Economic Association*, 3 (4), 914-945.
- 39
- [13] Cole, H., M. Obstfeld, 1991, Commodity Trade and International Risk-Sharing: How Much do Financial Markets Matter? *Journal of Monetary Economics*, 28, 3-24.
- [14] Collard, F., H. Dellas, B. Diba and A. Stockman, 2007, Home Bias in Goods and Assets, Mimeo, Working Paper, IDEI-University of Toulouse.
- [15] Corsetti, G., L. Dedola and S. Leduc, 2007, International Risk-Sharing and the Transmission of Productivity Shocks, Working Paper, European Central Bank, forthcoming Review of Economic Studies.
- [16] Devereux, M. and M. Saito, 2005, A Portfolio Theory of International Capital Flows, Working Paper, University of British Columbia.
- [17] Devereux, M. and A. Sutherland, 2006a, Solving for Country Portfolios in Open Economy Macro Models, Working Paper, University of British Columbia and St. Andrews University.
- [18] Devereux, M. and A. Stutterland, 2006b, Country Portfolio Dynamics, Working Paper, University of British Columbia and St. Andrews University.
- [19] Engel, C. and A. Matsumoto, 2006, Portfolio Choice in a Monetary Open-Economy DSGE Model, National Bureau of Economic Research, Working Paper 12214.
- [20] Evans, M. and V. Hnatkovska, 2005, International Capital Flows Returns and World Financial Integration, National Bureau of Economic Research, Working Paper 11701.
- [21] Evans, M. and V. Hnatkovska, 2007, Solving General Equilibrium Models with Incomplete Markets and Many Financial Assets, Working Paper, Georgetown University.
- [22] Faruqi, H. and J. Lee, 2007, Global Dispersion of Current Accounts: Is the Universe Expanding? Working Paper, IMF.
- [23] Fisher, J., 2002, Technology Shocks Matter, Working Paper, Federal Reserve Bank of Chicago.
- [24] Fisher, J., 2006, The Dynamic Effects of Neutral and Investment-Specific Technology Shocks, *Journal of Political Economy*, 114, 413-52.
- [25] French, K. and J. Poterba, 1991, Investor Diversification and International Equity Markets, *American Economic Review*, 81, 222-26.

- [51] Lane, P. and G.M. Milesi-Ferretti, 2003, International Financial Integration, IMF Staff Papers, 50., 82-113.
- [52] Lane, P. and G.M. Milesi-Ferretti, 2005, A Global Perspective on External Positions, IHS Discussion Paper 79, Trinity College Dublin.
- [53] Lane, P. and G.M. Milesi-Ferretti, 2006, The External Wealth of Nations Mark II, IMF Working Paper 06-69.

- [64] Palacios-Huerta I., 2001, The Human Capital of Stockholders and the International Diversification Puzzle, Journal of International Economics, vol. 54(2), August, pp. 309-331.

- [65] Pavlova, A. and R. Rigobon, 2007a, Asset Prices and Exchange Rates, Review of Financial Studies, 20(4), pp. 1139-1181.
- [66] Pavlova, A. and R. Rigobon, 2007b, An Asset-Pricing View of External Adjustment, LBS Working Paper.

- [67] Rafto, A., 2008, Technology Shocks: Novel Implications for International Business Cycles, Working Paper, FRB.

- [68] Sachs, J., 1981, The Current Account and Macroeconomic Adjustment in the 1970s, Brookings Papers on Economic Activity, Vol. 1981, No.1, 201-268.

- [69] Saren, P. and R. Vampé, 2007, Home Bias in International Equity Portfolios: A Review, Working Paper, Leuven School of Business and Economics.

- [70] Sims, C., 2000, Second Order Accurate Solution of Discrete Time Dynamic Equilibrium Models, Working Paper, Princeton University.

- [71] Souroounis, G., 2004, Capital Flows and Exchange Rates: an Empirical Analysis, Working Paper, London Business School.

- [72] Tesar, L. and I. Werner, 1995, Home Bias and High Turnover, Journal of International Money and Finance 14, 467-92.

- [73] Tille, C., 2005, Financial Integration and the Wealth Effect of Exchange Rate Fluctuations, Federal Reserve Bank of New York Staff Report 226.

- [74] Tille, C. and E. van Wincoop, 2007, International Capital Flows, National Bureau of Economic Research, Working Paper 12856.

- [75] Uppal, R., 1993, A General Equilibrium Model of International Portfolio Choice, Journal of Finance, 48, 529-553.

- [76] Warnock, F., 2002, Home Bias and High Turnover Reconsidered, Journal of International Money and Finance 21, 795-805.

[54] Lane, P. and J. C. Shambaugh, 2007, Financial Exchange Rates and International Currency Exposures, NBER Working Papers 13433, National Bureau of Economic Research.

- [55] Lane, P. and J. C. Shambaugh, 2008, The Long and Short of It: Determinants of Foreign Currency Exposure in External Balance Sheets, Working paper, Dartmouth College.
- [56] Lewis, K., 1999, Explaining Home Bias in Equities and Consumption, Journal of Economic Literature, 37, 571-608.

- [57] Martin, P. and H. Rey, 2004, Financial Super-Markets: Size Matters for Asset Trade, Journal of International Economics, 64, 335-361.

- [58] Matsunoto, A., 2007, The Role of Nonseparable Utility and Nontradables in International Business Cycle and Portfolio Choice, IMF working paper 7163.

- [59] McElroy, T., 2008, Exact formulas for the Hodrick-Prescott filter, Econometrics Journal, 11, 209-217.

- [60] Mendoza, E., 1995, The Terms of Trade, The Real Exchange Rate and Economic Fluctuations, International Economic Review 36, 101-137.

- [61] Obstfeld, M. and K. Rogoff, 1996, Foundations of International Macroeconomics, Cambridge, MA: MIT Press.

- [62] Obstfeld, M. and K. Rogoff, 2000, New Directions for Stochastic Open Economy Models, Journal of International Economics, 50, 117-153

- [63] Obstfeld, M., 2006, International Risk Sharing and the Costs of Trade, Ohlin Lecture, Stockholm School of Economics.

**Table 1. Correlations between relative labor incomes and dividends, and implied locally held equity shares**

	US	Japan	Germany	France	UK	Italy	Canada
<b>(a) Unconditional correlations</b>							
$\text{Corr}(\widehat{w}_t \widehat{l}_t, \widehat{d}_t)$	0.73	0.59	0.70	0.77	-0.24	0.89	0.28
	(.12)	(.08)	(.14)	(.12)	(.12)	(.02)	(.11)
<b>(b) Conditional correlations based on terms of trade measure given by relative GDP deflators</b>							
$\text{Corr}_{\widehat{q}_t}(\widehat{w}_t \widehat{l}_t, \widehat{d}_t)$	-0.17	-0.75	-0.69	-0.39	-0.71	-0.26	-0.22
	(.15)	(.05)	(.10)	(.07)	(.11)	(.19)	(.16)
<b>Implied locally held equity share <math>S_i^i</math></b>							
$S_i^i$	0.58	1.03	0.50	0.51	0.68	0.34	0.18
	(.17)	(.16)	(.09)	(.23)	(.13)	(.24)	(.14)
<b>(c) Conditional correlations based on terms of trade measure given by relative export prices</b>							
$\text{Corr}_{\widehat{q}_t}(\widehat{w}_t \widehat{l}_t, \widehat{d}_t)$	-0.27	-0.61	-0.22	-0.25	-0.74	0.87	-0.50
	(.18)	(.09)	(.22)	(.15)	(.08)	(.03)	(.13)
<b>Implied locally held equity share <math>S_i^i</math></b>							
$S_i^i$	0.84	1.22	0.41	0.50	2.58	-1.73	1.61
	(.31)	(.29)	(.30)	(.35)	(.51)	(.21)	(.59)

Notes

$\text{Corr}(\widehat{w}_t \widehat{l}_t, \widehat{d}_t)$ : correlation between relative labor income and the relative dividend income in a given country (compared to total labor income and total dividend in remaining G7 countries).

$\text{Corr}_{\widehat{q}_t}(\widehat{w}_t \widehat{l}_t, \widehat{d}_t)$ : correlation between components of relative labor income and the relative dividend income that are orthogonal to terms of trade.

The data are annual, 1984-2004. Figures in parentheses are standard errors (based on GMM). See text for further explanations.

**Table 3. Model predictions: dynamic properties**

	Shocks to:			
	$\theta_H \theta_F$ $\chi_H \chi_F$	$\theta_H \theta_F$	$\chi_H \chi_F$	Data (G7)
<b>Standard deviations (%)</b>	(1)	(2)	(3)	(4)
GDP	1.87	1.65	0.88	2.07
Consumption	0.76	0.49	0.57	1.54
Investment	8.26	4.73	6.77	6.89
Hours worked	1.31	0.63	1.14	1.89
Net exports	1.07	0.24	1.05	1.14
$\Delta(\text{Net foreign assets})$	2.21	1.39	1.72	3.23
$\Delta(\text{Net foreign bond assets})$	5.32	3.69	3.82	2.20
$\Delta(\text{Net foreign equity assets})$	3.13	2.31	2.11	2.97
Net bond purchases	3.23	2.39	2.18	1.71
Net equity purchases	3.23	2.39	2.18	1.38
Real exchange rate	1.38	0.60	1.24	8.38
<b>Correlations with domestic GDP</b>				
Consumption	0.38	0.81	-0.22	0.78
Investment	0.71	0.95	0.59	0.85
Hours worked	0.61	0.81	0.65	0.83
Net exports	-0.07	0.06	-0.17	-0.39
$\Delta(\text{Net foreign assets})$	-0.26	-0.32	-0.20	-0.22
$\Delta(\text{Net foreign bond assets})$	-0.24	-0.30	-0.17	-0.24
$\Delta(\text{Net foreign equity assets})$	0.23	0.28	0.15	-0.02
Net bond purchases	-0.27	-0.30	-0.22	-0.22
Net equity purchases	0.27	0.30	0.22	-0.03
Real exchange rate	-0.22	-0.52	-0.07	0.12
<b>Cross-country correlations</b>				
Output	0.17	0.30	-0.28	0.49
Consumption	0.58	0.81	0.42	0.46
Investment	-0.37	0.25	-0.67	0.27
Hours worked	0.18	0.15	0.19	0.43
<b>Autocorrelations</b>				
GDP	0.59	0.55	0.76	0.74
$\Delta(\text{Net foreign assets})$	0.12	-0.03	0.21	-0.01
$\Delta(\text{Net foreign bond assets})$	0.04	-0.07	0.15	0.14
$\Delta(\text{Net foreign equity assets})$	-0.00	-0.10	0.10	0.02
Net bond purchases	0.07	-0.06	0.23	0.32
Net equity purchases	0.07	-0.06	0.23	0.23
<b>Other correlations</b>				
$\Delta(\text{Net foreign equity assets})$ & $\Delta(\text{Net foreign bond assets})$	-0.99	-0.99	-0.99	-0.27
Net bond purchases & Net equity purchases	-1.00	-1.00	-1.00	-0.68

Notes--The following variables are normalized by domestic GDP: Net exports,  $\Delta(\text{Net foreign assets})$ ,  $\Delta(\text{Net foreign bond assets})$ ,  $\Delta(\text{Net foreign equity assets})$ , Current account, Net bond purchases, Net equity purchases. GDP, Investment and the Real exchange rate (CPI-based) are lagged. All variables are Hodrick-Prescott filtered. The empirical measure of consumption is non-durables plus services expenditures (from OECD National Accounts). The Real exchange rate measure is based on relative prices indices of non-durables and services consumption.