The Macroeconomics of Model T

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Abstract

We study a model of endogenous growth where firms invest both in product and process innovations. Product innovations (that open up completely new product lines) satisfy the luxurious wants of the rich. Subsequent process innovations (that decrease costs per unit of quality and/or drive down prices) transform the luxurious products of the rich into conveniences of the poor. A prototypical example for such a product cycle is the automobile. Initially an exclusive product for the very rich, the automobile became affordable to the middle class after the introduction of Ford's *Model T,* the car that "*put America on wheels*". We characterize long-run growth when new products follow such an innovation cycle and explore how economic inequality interacts with innovation incentives. An egalitarian society creates strong incentives for process innovations (such as the Model T) whereas an unequal society creates strong incentives for product innovations (new luxuries). Depending on which type of innovative activity drives technical progress, economic inequality is harmful or beneficial for long-run growth.

JEL classification: O15, O31, D30, D40, N32, N34

Keywords: inequality, technological change, growth, market structure, product cycle.

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"Consumer goods inventions that cut both cost and quality but reduce the former more than the latter, such as the Model T, have historically been an important means for transforming the luxuries of the rich into the conveniences of the poor."

Jacob Schmookler, *Invention and Economic Growth* (1966)

1 Introduction

This paper develops a model of endogenous growth based on a cycle of product and process innovations. Product inventions introduce new goods satisfying the luxurious wants of rich consumers. Process innovations lead to the adoption of new production processes that reduce the cost per unit of quality, making the product also affordable to the poorer classes in the society. As emphasized by Schmookler (1966), such a cycle of product and process innovations has historically been important to transform the luxuries of the rich into mass consumption markets.

The automobile, one of the most important durable goods in modern industrial societies, provides a prototypical example for such an innovation cycle. In the United States, the history of the commercial automobile production started with Charles and Frank Duryea who founded the Duryea Motor Wagon Company in 1893, the first American automobile manufacturing company; in 1902 and 1903 Oldsmobile (by Ransom E. Olds Company) and Cadillac (by Henry Ford Company) followed. At the time, the automobile was a luxury good consumed only by very rich households. Things started to change in 1908, when Ford introduced the *Model T,* the car that "*put America on wheels*". The concept was the use of assembly lines to produce a low-cost, low-quality car affordable to the middle class. Model T became a huge success and initiated the takeoff in car ownership in the U.S. Between 1908 and 1927 more than 15 million units of Model T were manufactured. The introduction of Model T contributed crucially to the fast diffusion of the automobile in the U.S.[1]

The auto industry provides a prototypical example for a product cycle where an initial innovation effort created a luxury goods market and major process innovations thereafter turned the good into a mass consumption good affordable to the large population of less wealthy households. Such a product cycle is not confined to the automobile but has been important for many other consumer durables, such as the refrigerator, the radio, the TV, the computer and many other durable consumer goods, showing a similar pattern of innovation cycles.

We develop a formal endogenous growth model where firms engage both in product and process innovations of indivisible consumption goods. Indivisibilities let the composition of

[1] Encyclopaedia Britannica

demand by rich consumer systematically differ from that of poorer households. The rich do not only purchase a larger variety of consumption goods, but also do consume these goods in higher quality. Poor households consume only a fraction of the available varieties and prefer lower qualities to higher ones.

Our analysis shows how the growth process and the associated mix of product and process innovations depend on the interaction between two major forces: the particular source of technical progress; and the extent of economic inequality in the society. If technical progress is mainly driven by product innovations, inequality is beneficial for long-run growth. Rising inequalities allow innovators to charge high prices both during the early period when the product is introduced as well as during the later period when the new product has generated mass markets but is still available as a differentiated high quality good that is purchased by the rich. In contrast, if technical progress is mainly driven by process innovations, the relationship between inequality and growth is turned upside down and inequality becomes harmful for long-run growth. When the large majority of households is extremely poor, there is little potential to open up mass consumption markets and hence investments in low-quality low-cost innovations are weak. Our analysis also highlights possible complementarities between process and product innovations. When an economy has invested relatively little in process innovation it is more likely to benefit more from process innovations and vice versa. In the presence of such complementarities, it turns out that both very high levels and very low levels of inequality are harmful for growth, and growth is maximized at an intermediate extent of economic inequality.

Our analysis contributes to the existing literature in at least three dimensions. *First*, our paper extends a small literature that has studied the impact of income inequality on technical progress. Matsuyama (2002) demonstrates the virtuous cycle between learning-by-doing and a large middle class, enabling the Flying Geese pattern discussed later in our paper. Foellmi and Zweimueller (2006) focus on product inventions and the scope of innovators' price setting power in the presence of a wealthy upper class. The present paper can be viewed as a synthesis of these classes of models. Our analysis highlights the conditions under which an unequal society suffers from lack of process innovations (and/or learning-by-doing) and from a small range of mass markets; and the conditions under which such a society benefits from large mark-ups and high incentives to open up completely new product lines.[2]

Focusing on capital accumulation, Galor and Moav (2004) have developed a model where consumers have non-homothetic preferences over consumption and bequests. More inequality may lead to higher growth at early stages of development, because the higher savings rate

[2]Furthermore, Murphy, Shleifer, and Vishny (1989) have studied the role of distribution of income in the adoption of modern technologies in a static context. And for an analysis of the effect of inequality on technical progress and long-run growth in quality ladder models, see for example Li (2003).

of the rich fosters physical capital accumulation. However, the positive impact reverses, as human capital emerges as a growth engine.

Second, our paper highlights the distinct role that product innovations and process innovations can play in the process of long-run growth. In this dimension our paper differs from the large literature on the determinants of the aggregate technological progress (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt, 1992, etc.). Aggregate models of product, process and quality innovations are often mathematically similar (Acemoglu, 2009), that is the source of technological change is not essential to answer the question of what factors influence economic growth. This is different in our framework where incentives for product inventions and process innovations are subject to systematic differences, in particular with respect to the extent of inequality in the society.

Third, our paper is related to the literature on directed technical change (Acemoglu, 1998 and 2002, Acemoglu and Zilibotti, 2001, and others). This literature analyzes the forces that generate biases in technical change towards one particular production factor. Similar to our paper, directed technical change models emphasize the tension between price and market size effects. However, the emphasis is on the relative demand for production factors, i.e. the supply/cost side of the economy. In contrast, our model focuses on demand/income effects. This channel generates an important role for the distribution of income across households, a mechanism that is absent in directed technical change models.

The paper is organized as follows: Section 2 analyzes empirical and historical evidence motivating the key assumptions and mechanisms of our models. Section 3 introduces our formal framework and derives the key conditions. Section 4 presents the solution of the balanced growth equilibrium, while section 5 discusses the relationship between inequality and growth. Section 6 introduces two adaptions to our framework, allowing us to study the product cycle in more detail. We conclude with a summary and a list of potential extensions to our framework.

2 Motivating Evidence

Casual observations and empirical evidence suggest that there is a strong impact of income on the number of varieties purchased by households, which is at odds with homothetic preferences. Jackson (1984) finds that the richest income class consumed twice as many different goods as the poorest class, using micro data from the Consumer Expenditure Survey of the Bureau of Labor Statistics. Falkinger and Zweimüller (1996) generate similar results using aggregate cross-country data from the International Comparison Project of the UN on per-capita expenditure levels on ninety-one different consumption categories. Figure 1 illustrates this point by exhibiting the shares of ownership of various consumer durables of urban Chinese households.

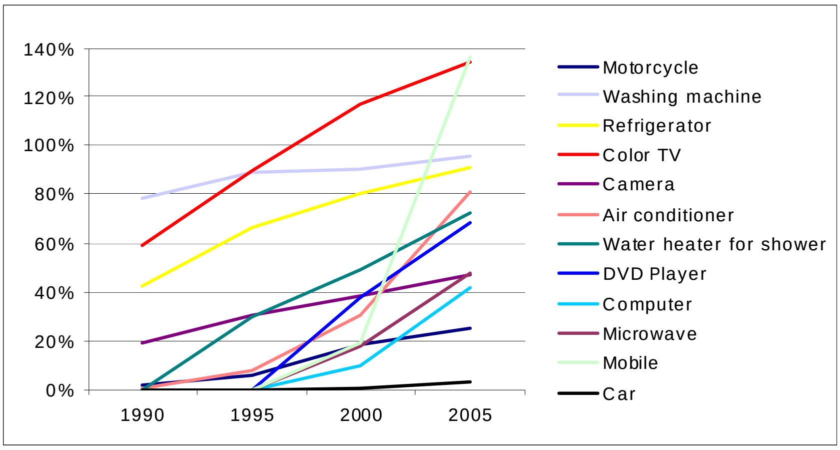

Figure 1: Ownership of consumer durables in Urban Chinese households (National Bureau of Statistics of China)

At any given point in time, most types of consumer durables are only consumed by a fraction of the households.

Figure 1 also shows that levels of penetration rise over time. This is what Matsuyama (2002) calls the Flying Geese pattern, in which a series of products takes off one after another, as productivity and income levels increase. Referring to Katona (1964), as penetration levels approach 100%, initial luxury goods, consumed only by a few, privileged households, have been transformed into necessities for most households (i.e. mass consumption goods). Many products such as cars, radios, television sets, washing machines and more recently computers have been through such product cycles in the developed world, and are presently going through similar cycles in developing countries. Besides plain income effects, a key element in the product cycle are manufacturing costs and process innovation. After a product has been invented, initial manufacturing costs are usually prohibitively high, and sales volumes linger, as the good can only be afforded by a few rich households. The takeoff and subsequent proliferation of the product is often ignited and enabled by a series of process innovations that reduce the manufacturing costs significantly.

The most famous historical example for this pattern is the Ford Model T, that set 1908 the historic year when the automobile came into popular usage in the U.S. It is generally regarded as the first affordable automobile, the car that "put America on wheels"; some of this was because of Ford's innovations, including assembly line production instead of individual hand crafting, as well as the concept of paying the workers a wage proportionate to the cost of the car, so that they would provide a ready made market. In total, Ford manufactured more than 15 million Model T's from 1908 through to 1927, which contributed critically to the fast diffusion of the automobile. Figure 2 shows automobile and truck registrations in the US from

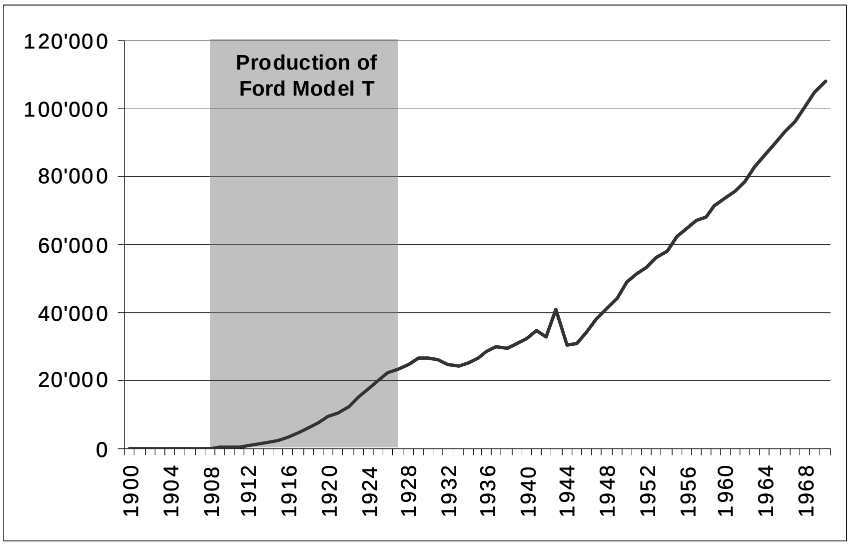

Figure 2: Automobile and truck registrations in the US in 1'000 units (US Census)

1900 to 1970. The number of registrations took off in the period of the Model T, and reached 23 million in 1927. Whereas 1% of households in the U.S. owned a car in 1908, the hour of birth of the Model T, penetration reached 50% in 1924.[3]

The auto industry is a prototypical example for the type of innovation and product cycle that our model aims to capture. However, other consumer goods experience similar patterns of innovation and market expansion. Two centuries after artificial refrigeration was pioneered by Dr. William Cullen, a GE home *refrigerator* still cost around 700$ in 1922, compared to 450$ for a 1922 Ford Model T. Penetration barely reached 1% in the U.S. in 1925. The introduction of freon expanded the refrigerator market during the 1930s, with penetration reaching 50% in the by 1938. Refrigerators went into mass production after WWII, reaching a penetration of 75% in 1948.[4] The history of *television* started with first experimental transmissions made by Charles Jenkins in 1923. Television usage in the U.S. exploded after WWII. Having reached penetration of 1% in 1948, it only took 5 years to reach 50%, and 2 more years to reach 75%. The rapid diffusion was enabled by the lifting of the manufacturing freeze, war-related technological advances, the expansion of the television networks, the drop in television prices enabled by mass production and additional disposable income.[5] Similarly for *computers.* Spurred by calculation requirements for ballistics during WWII, the first electronic digital computers were developed between 1940-1945. The development explosion of the microprocessor led to

[3] See Model T Facts on media.ford.com, Encyclopaedia Britannica, and Bowden and Offer (1994) for penetration levels.

[4] Association of Home Appliance Manufacturers, "The Story of the Refrigerator;" Bowden and Offer (1994)

[5] Steven Schoenherr, "History of Television," History Server of University of San Diego; Bowden and Offer (1994)

the proliferation of the personal computer after about 1975. Mass market pre-assembled computers allowed a wider range of people to use computers, and penetration reached 1% in the U.S. around 1980. Component prices continued to fall since then, leading to continuous price declines. Penetration reached 50% around 2000. The emergence of Netbooks in 2007, a new market segment of small, energy-efficient ultra low-cost devices, is likely to advance penetration significantly, especially in developing countries.[6]

Moreover, similar product cycles have been observed not only in the U.S. but also in other parts of the world. Coming back to the automobile, most of the large European economies had their own Model T to bring the car to the people. In Germany, a "people's car" - Volkswagen - was introduced.[7] Citroën[8], Fiat and Austin 7 brought the car to the people of France, Italy and the UK, respectively. In Asia, Tata has recently announced to produce the world's cheapest car mainly for the Indian market. The following table provides an overview of the world's major "Model T's":

Country	**Model**	**Year of introduction**
US	Ford Model T	1908
UK	Austin 7	1922
Italy	Fiat 500 Topolino & Nouva	1936
Germany	VW Käfer (Beetle)	1938
France	Citroën 2CV	1949
India	Tata Nano	2009

These examples demonstrate how closely process innovation and mass consumption markets are intertwined: Process innovation is a crucial element in tapping mass consumption markets, while mass production, in turn, facilitates process innovation. R&D investments in process innovation are more attractive for larger markets (compared to exclusive luxury markets). And mass production facilitates process innovation by increasing learning-by-doing and specialization benefits ("the division of labour is limited by the extend of the market", Adam Smith, *The Wealth of Nations, 1776*).

[6] Jeffrey Shallit, "A Very Brief History of Computer Science," University of Waterloo; W. Warner, "Great Moments in Microprocessor History," Technical Library IBM; "Computer Use and Ownership," U.S. Census, and authors' estimates

[7] Volkswagen was founded under Hitler in 1937 to produce an affordable car for the people of Germany.

[8] Citroën director Pierre-Jules Boulanger's early design brief for the 2CV supposedly asked for "a vehicle capable of transporting two peasants in boots, 100 pounds of potatoes or a barrel of wine, at a maximum speed of 40 mph, [...] Its price should be well below the one of our Traction Avant and, finally, its appearance is of little importance." (Translation, Technologie SCEREN - CNDP no. 138, 2005)

We have established a close connection between inequality, mass markets and process innovation. Higher inequality lowers the number of mass markets as less wealthy households consume a lower range of goods. And less mass consumption markets lowers process innovation in the economy. Comparing Japan to the U.S. over the last decades produces evidence in this direction: Income concentration in Japan has remained relatively low in Japan after WWII in contrast to the U.S., where concentration has risen sharply since 1970 (Moriguchi and Saez, 2005). In the same period of time, Japan has made itself a name as country of lean production and just-in-time management, i.e. process innovation. A very recent study by Nagaoka and Walsh (2009) provides formal evidence, using data from the RIETI-Georgia Tech inventor survey. The data on business objectives of research and patents show that R&D in Japan is more biased to process innovation compared to the U.S. which is relatively more directed to product innovation.

3 Model

Endowments, Preferences and Technology The economy is populated by a continuum of L infinitely-lived households. There are two homogeneous groups of households: rich, R, and poor, P. There are βL poor households and $(1-\beta)L$ rich. A poor household is endowed with $\theta_P = \theta < 1$ units of labor l, a rich with $\theta_R = (1-\theta\beta)/(1-\beta) > 1$ units. Figure 3 depicts the resulting Lorenz curve. Inequality is decreasing in θ and increasing in β. The aggregate endowment of labor is L, and we assume that labor supply is inelastic.

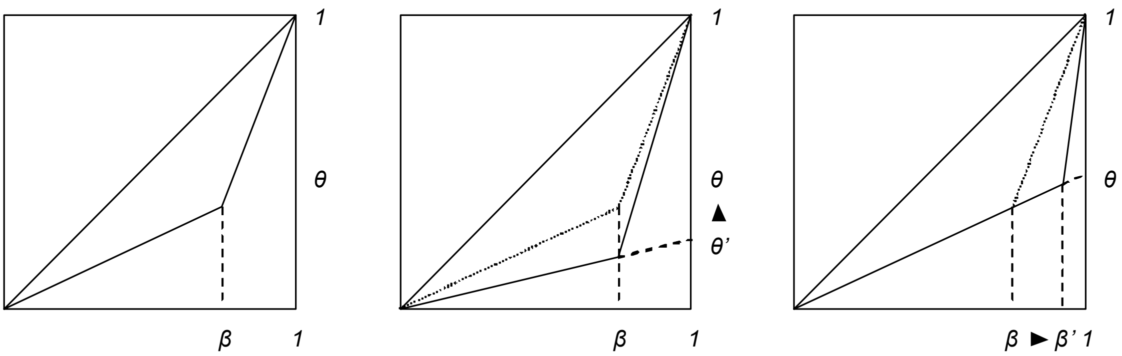

Figure 3: Lorenz curve

Households choose consumption from a continuum of $N(t)$ *indivisible* goods, indexed by j. Only the first unit of a particular good yields positive utility; no additional utility is derived from consuming more than one unit ("0/1 preferences"). Consumption indivisibilities provide a simple and tractable way to model non-homothetic preferences that generate differences in the composition of demand across households.

Households can invest $F(t)$ units of labor to invent a new good and set up a company, increasing the variety of available goods $N(t)$ in the economy. They obtain infinite patent rights, generating monopolistic power and positive profit streams, which makes their research efforts worthwhile. The production of one unit of a good requires $a_h(t)$ units of labor (constant returns to scale). We assume each poor household owns a share of θ/L in aggregate profit streams, proportional to its labor share. A rich household thus owns $(1-\theta\beta)/(1-\beta)/L$.

After the foundation, firms can invest $G(t)$ units of labor in process innovation that enables them to produce the good with less labor, $a_l(t) < a_h(t)$. As a side effect, the new process produces goods in a lower quality $q_l < q_h$ than the original process. The high quality version corresponds to the Cadillac, and the low quality version to the Model T.

Lastly economic growth is enabled by spillovers from product and process innovation on a (non-excludable, non-rival) aggregate stock of knowledge $A(t) = A(N(t), P(t))$, where $P(t)$ denotes the number of firms that have implemented the process innovation. Accumulation of $A(t)$ increases the efficiency of labor in the economy:

$$F(t) = F/A(t),\ a_h(t) = a_h/A(t),\ G(t) = G/A(t), \text{ and } a_l(t) = a_l/A(t). \tag{1}$$

Consumption A household consumes either one unit in the high quality, one unit in the low quality, or nothing of a particular good from the variety $N(t)$. The intertemporal utility function of a household i is

$$\begin{aligned} U_i(0) &= \int_0^\infty \frac{1}{1-\sigma} c_i(t)^{1-\sigma} \exp(-\rho t) dt \\ \text{where } c_i(t) &= \int_0^{N(t)} x_i(j,t) q_i(j,k) dj. \end{aligned}$$

j is the index for goods, $k \in \{h, l\}$ the index for quality levels, and t the index for time. $x(j,t)$ is an indicator function with $x(j,t) = 1$ if good j is consumed, and $x(j,t) = 0$ if not. σ measures the willingness to shift consumption across time, which is constant (CES preferences), and ρ denotes the discount rate.

Households maximize utility subject to the flow budget constraint for wealth $v_i(t)$,

$$\begin{aligned} \dot{v}_i(t) &= r(t)v_i(t) + w(t)l_i - e_i(t) \\ \text{where } e_i(t) &= \int_0^{N(t)} x_i(j,t) p(j,k,t) dj. \end{aligned}$$

The variables $r(t)$ and $w(t)$ are the interest and wage rate, respectively, and $p(j,k,t)$ denotes the price of good j with quality k at date t. Further, the no bubble condition holds

$$\lim_{t\to\infty} \left[v_i(t) \exp\left(-\int_0^t r(s) ds \right) \right] \geq 0.$$

To derive the remaining necessary conditions for an optimum, we construct the current-value Hamiltonian,

$$\hat{H}(t) = \frac{1}{1-\sigma} c_i(t)^{1-\sigma} + \lambda_i(t) \left[r(t) v_i(t) + w(t) l_i - e_i(t) \right].$$

The first-order condition (FOC) for the consumption of good j reads

$$\{x_i(j,t), q_i(j,k)\} = \begin{cases} \{1,h\} & \text{if} \quad c_i(t)^{-\sigma} q_h - \lambda_i(t) p(j,h,t) \geq \max\left[0, c_i(t)^{-\sigma} q_l - \lambda_i(t) p(j,l,t)\right], \\ \{1,l\} & \text{if} \quad c_i(t)^{-\sigma} q_l - \lambda_i(t) p(j,l,t) \geq \max\left[0, c_i(t)^{-\sigma} q_h - \lambda_i(t) p(j,h,t)\right], \\ \{0,\cdot\} & \text{otherwise.} \end{cases} \tag{2}$$

The household consumes good j in the high quality h if the "consumer rent" is positive, i.e. $c_i(t)^{-\sigma} q_h \geq \lambda_i(t) p(j,h,t)$, and greater than the rent of the low quality l. If the rent of the low quality is greater and positive, the household consumes the low quality. Otherwise, the household does not consume good j at all.

The first-order condition for wealth accumulation is

$$\lambda_i(t) r(t) = \rho \lambda_i(t) - \dot{\lambda}_i(t), \tag{3}$$

and the transversality condition

$$\lim_{t \to \infty} \left[\exp(-\rho t) \lambda_i(t) v_i(t) \right] = 0.$$

The necessary conditions are sufficient given the quasiconcave nature of the problem.

Production The (continuum of) $N(t)$ monopolistic firms maximize their profits based on aggregate demand derived from the consumption FOC. We can separate instantaneous profit maximization from R&D decisions, since the control variables do not affect any relevant state variable.

Consider first a firm that has not yet invested in process innovation, in that case it can only manufacture the high quality of the good. There are two groups of potential customers, the rich and the poor households. Hence, the firm sets the price $p(j,h,t)$ to maximize profits

$$\pi(j,h,t) = \begin{cases} L\left[p(j,h,t) - a_h(t) w(t)\right] & \text{if} \quad q_h \mu_P(t) \geq p(j,h,t), \\ (1-\beta) L\left[p(j,h,t) - a_h(t) w(t)\right] & \text{if} \quad q_h \mu_R(t) \geq p(j,h,t) > q_h \mu_P(t), \\ 0 & \text{otherwise,} \end{cases}$$

where $\mu_i(t) = c_i(t)^{-\sigma}/\lambda_i(t)$ for $i = P, R$ denotes the willingness-to-pay for a good per quality unit. Note that rich households have a higher willingness-to-pay than poor households, $\mu_R(t) > \mu_P(t)$.[9] If a firm charges a price below the willingness-to-pay of the poor, it sells to all L

[9] If the willingness-to-pay of the rich were equal or smaller, the poor would have the same or higher expenditures than the rich, which cannot be the case in equilibrium.

households. And if the firm charges a price above the willingness-to-pay of the poor, but below the willingness-to-pay of the rich, only the $(1-\beta)L$ rich households buy the good. Hence, the firm can pursue either a *mass market strategy* m, selling to all households, or an *exclusive strategy* e, selling only to the rich. Profit maximization yields the following profit flows:

$$\begin{aligned} \pi_m(h,t) &= L\left[q_h\mu_P(t)-a_h\right], \\ \pi_e(h,t) &= (1-\beta)L\left[q_h\mu_R(t)-a_h\right]. \end{aligned} \tag{4}$$

where we set $W(t)=A(t)$ (numéraire) and use the fact that $a_h(t)=a_h/A(t)$. An exclusive producer sets prices equal to the willingness-to-pay of the rich, and a mass producer set prices equal to the willingness-to-pay of the poor.

In the other case, if the firm has already invested in process innovation, it can sell the low quality either only to rich or to all households, and generate

$$\begin{aligned} \pi_m(l,t) &= L\left[q_l\mu_P(t)-a_l\right], \\ \pi_e(l,t) &= (1-\beta)L\left[q_l\mu_R(t)-a_l\right]. \end{aligned}$$

However, there exists a fifth option. The firm can sell the high quality to the rich, and the low quality to the poor. Given the higher willingness-to-pay of the rich, it might be attractive for a firm to sell the higher quality to them, still using the original, more laborious process. The firm, however, cannot set the price of the high quality equal to the willingness-to-pay of the rich, and the price of the low quality equal to the one of the poor, simultaneously. In that case the rich households would prefer the low quality which would leave them a strictly positive consumer rent (given $\mu_P<\mu_R$). The firm needs to set incentives to *separate* rich from poor households. The maximization problem thus has to take into account these incentive constraints in addition to the willingness-to-pay:

$$\begin{aligned} \max_{p(j,l,t),p(j,h,t)} \pi_m(j,s,t) &= L\beta\left[p(j,l,t)-a_l\right]+L(1-\beta)\left[p(j,h,t)-a_h\right] \\ \text{s.t. } q_h\mu_R(t)-p(j,h,t) &\geq \max\left[0,q_l\mu_R(t)-p(j,l,t)\right], \\ q_l\mu_P(t)-p(j,l,t) &\geq \max\left[0,q_h\mu_P(t)-p(j,h,t)\right], \end{aligned}$$

where s denotes the separating strategy. The second expression in the max-operator of the constraints warrants that the rich prefers the high quality and the poor the low. The problem can be simplified since two of the constraints are not binding while the other two are binding, which is proofed in Appendix A. Hence, first-order conditions are

$$\begin{aligned} p(j,l,t) &= q_l\mu_P(t), \\ q_h\mu_R(t)-p(j,h,t) &= q_l\mu_R(t)-p(j,l,t). \end{aligned}$$

Note rich households receive an informational rent as they have to be incentivized to purchase the high quality, $p(j,h,t) = (q_h - q_l)\mu_R(t) + q_l\mu_P(t) < q_h\mu_R(t)$ given $\mu_P(t) < \mu_R(t)$. Otherwise the rich would be better off "pretending to be a poor household" and pick the lower quality. The maximized profit flow from this *mass separating strategy* thus is:

$$\pi_m(s,t) = L\beta(q_l\mu_P(t) - a_l) + L(1-\beta)((q_h - q_l)\mu_R(t) + q_l\mu_P(t) - a_h). \tag{5}$$

To sum up, if the firm has not yet invested in process innovation, it may sell the high quality exclusively to the rich or the mass market. After investment into process innovation, it may in addition sell the low quality to the rich or to all, or at the same time sell the high quality to the rich and the low quality to the poor.

R&D: New goods and process innovations Inventing a new good and setting up a firm is attractive if the present value of future cash flows $PV(j,t)$ exceeds initial R&D costs, $PV(j,t) \geq w(t)F(t) = F$ (using the numéraire $w(t) = A(t)$ and $F(t) = F/A(t)$). In a second step, firms have to decide whether and when to invest $w(t)G(t) = G$ and implement the process innovation. Hence, entry is attractive if

$$\max_{\Delta t^*}\left[\int_t^{t+\Delta t^*}\pi(t)\exp(-R(s,t))ds + \int_{t+\Delta t^*}^{\infty}\pi(t)\exp(-R(s,t))ds - G\exp(-R(t+\Delta t^*,t))\right] \geq F,$$

with

$$R(s,t) = \int_t^s r(\tau)d\tau,$$

where Δt^* denotes the optimum time period between the product and process innovation. Firms choose $\Delta t(j)$ to maximize the present value. We will analyze this entry and timing decision in more detail for the balanced growth equilibrium.

Resource Constraint Aggregate labor supply is equal to L as each household inelastically supplies one unit of labor. Aggregate labor demand is the sum of market demands to manufacture goods and pursue R&D. The resource constraint of the economy can then be written as

$$L = Y_h(t)a_h(t) + Y_l(t)a_l(t) + \dot{N}(t)F(t) + \dot{Q}(t)G(t),$$

where $Y_k(t)$ denotes total production of goods in quality level k, and $P(t)$ the number of firms that have implemented the process innovation.

4 Solution of Balanced Growth Equilibrium

We will focus on the balanced growth path of the economy in the main part. An extension to analyze transitional dynamics will be outlined in the Conclusion.

Definition 1 *A* balanced growth equilibrium *in our economy consists of a path where the stock of knowledge $A(t)$, the wage rate $w(t)$, total expenditures $E(t)$ and the variety $N(t)$ grow at a constant rate g, and hence the labor requirements $a_h(t)$, $a_l(t)$, $G(t)$ and $F(t)$ shrink at that rate g. On this path, interest rates r, product prices, instantaneous profit flows and the fraction of producers pursuing a given strategy is constant (as is the level of inequality). The balanced growth equilibrium can be reached and sustained in a decentral economy with households maximizing utility and firms maximizing profits, given product and factor prices, such that all markets clear, and given initial variety of firms $N(0) > 0$.*

Let us take a closer look at the market structure in equilibrium. There are $N(t)$ monopolistic firms (monopolistic competition). The consumption structure of the rich and poor differ in equilibrium. Rich households consume all goods, while the poor only a fraction m thereof. As a result, a fraction $(1-m)$ of the firms are *exclusive producers*, selling only to the rich, while the other fraction m are *mass producers*, selling to both rich and poor households.[10]

Firms invest in process innovation if it is sufficiently attractive. We make the following assumption regarding the attractiveness of process innovation:

Assumption 1 *The process innovation is sufficiently attractive, but not too attractive, i.e. $(q_h - q_l)\,\mu_R > (a_h - a_l)$ and $(1-\beta)\,(q_h\mu_R - a_h) > (q_h\mu_P - a_h)$, with μ_R and μ_P determined by*

$$\begin{aligned}
\mu_P &= (1-\beta)\left[\mu_R + (q_h\mu_R - a_h)\,G/F/q_l\right] + \beta a_l/q_l, \\
\mu_R &= \frac{(1-\theta)\left[a_l\beta/(1-\beta) - a_h G/F\right]}{q_h\theta/m - q_l - q_h(1-\theta)G/F}, \\
\frac{L\left[1-(1-\beta)a_h/m^{1-\psi} - \beta a_l m^{\psi}\right]}{F/m^{1-\psi} + Gm^{\psi}} &= \frac{1}{\sigma}\left(\frac{L}{F}\left[\frac{\beta a_l - (1-\beta)a_h G/F}{\theta/(1-\theta)/m - q_l/q_h/(1-\theta) - G/F} - (1-\beta)a_h\right] - \rho\right).
\end{aligned}$$

If the process innovation is not sufficiently attractive, i.e. the costs of process innovation G is high, and the low quality level q_l/q_h and market for the process innovation β sufficiently low, no firm will invest in process innovation. If G is low, and q_l/q_h and β sufficiently high, all firms will immediately invest in process innovation. Therefore, Assumption 1 allows us to focus on the most relevant case:

Proposition 1 *Given Assumption 1, exclusive producers have not yet innovated and sell the high quality to the rich, while mass producers have innovated and manufacture with both the*

[10] The existence of exclusive goods is driven by the higher willingness-to-pay of the rich. Mass producers have to leave the rich a positive consumer rent in order to also attract the poor, and to incentivize the rich to pick the high quality in the case of two qualities offered, as discussed above. A rich with a positive consumption rent in all products would, however, have an infinite willingness-to-pay for goods as the household would not exhaust its budget, thus attracting producers to go exclusive and charge arbitrarily high prices.

original and innovated process, selling the high quality to the rich and the low quality to the poor, in the balanced growth equilibrium.

Proof. *See Appendix B.* ■

If the assumption is violated, three other types of *regimes* may arise. Either no or all firms immediately invest in process innovation, which is less interesting, as the dimension of the endogenous growth model is reduced, making it equivalent to a more simple model of expanding variety. Or exclusive producers have not yet innovated, while mass producers have innovated and only use the innovated process selling the low quality to all households. That regime is very similar to the one we focus on. Note that exclusive producers never invest in process innovation if mass producers do not invest, as their market for the innovation is smaller. We will characterize these other regimes in Appendix C, and analyze and compare the conditions in more detail in Appendix D.

In the regime we focus on, the process innovation is the key step to transform the good into a convenience, and the mass consumption market is a key incentive for process innovation, in turn. This is in line with our motivating evidence as well as our introductory quote, stating that process innovations "that cut both cost and quality [...], such as the Model T, have historically been an important means for transforming the luxuries of the rich into the conveniences of the poor" (Schmookler, 1966). Furthermore, the Cadillac is still being produced for the rich households after the introduction of the Model T which both empirically and intuitively makes sense.

R&D and the Product Cycle In the balanced growth equilibrium, the process innovation timing problem can be simplified to:

$$\max_{\Delta t} \int_t^{t+\Delta t} \pi_e \exp(-r(s-t))ds + \int_{t+\Delta t}^{\infty} \pi_m \exp(-r(s-t))ds - G\exp(-r\Delta t),$$

We have used the fact that, in the regime we analyze, firms that have not yet innovated are exclusive producers generating profit flows π_e, whereas firms become mass producers following the process innovation with profit flows π_m, (4) and (5). Using the Leibniz rule we obtain

$$\Delta t = \begin{cases} 0 \text{ if } (\pi_m - \pi_e)/r > G, \\ [0,\infty) \text{ if } (\pi_m - \pi_e)/r = G, \\ \infty \text{ if } (\pi_m - \pi_e)/r < G. \end{cases}$$

The present value of the increased profit flow is compared to innovation costs. The first and third case can be ruled out on the balanced growth path in the regime we analyze, as either everybody would instantly innovate, or nobody would innovate at all. Hence, Δt is undetermined, and firms are indifferent whether and when to invest in process innovation.

The aggregate fraction of firms that have innovated, however, is determined which we will see later. This indifference is due to the symmetry of the model. Introducing asymmetry in preferences or innovation costs across products would pinpoint a cutoff, leading to a determined product cycle where firms initially are exclusive producers, and later, after process innovation, become mass producers. We will discuss the product cycle in more detail in section 6.

Hence, the following no-arbitrage conditions must hold:

$$\begin{aligned} \pi_e/r &= F \\ (\pi_m - \pi_e)/r &= G \end{aligned} \tag{6}$$

The present value of the exclusive strategy must be equal to initial R&D costs; otherwise there would either be arbitrage opportunities or no growth. And the present value of the increased profit flow from the mass strategy must be equal to process innovation costs.

Consumption Growth and Technological Progress In the balanced growth equilibrium, expenditures grow at rate g and prices are constant. Hence, consumption growth of poor and rich households follows the standard Euler equation:

$$r = \sigma g + \rho, \tag{7}$$

where we have used the wealth and a binding consumption FOC, (3) and (2).

Remember that growth is enabled by spillovers from R&D activities on productivity in our economy. Productivity rises proportionally with the cumulative experience in R&D, the stock of knowledge $A(t) = A(N(t), P(t))$. Sustained, non-explosive growth is only possible if $A(\cdot,\cdot)$ is homogenous of degree one (knife-edge condition of endogenous growth models). For analytical convenience, we initially assume that $A(\cdot,\cdot)$ is Cobb-Douglas:

$$A(t) = N(t)^{\psi} P(t)^{1-\psi} \tag{8}$$

with $\psi = [0, 1]$.

Solving for Exclusion and Growth We can characterize and analyze the balanced growth equilibrium by constructing two equations, a *no-arbitrage curve* and a *resource curve*. Using the profit flow equations (4) and (5), and the no arbitrage conditions (6), we can express the price of the low quality as:

$$q_l \mu_P = (1-\beta)\left[q_l \mu_R + (q_h \mu_R - a_h)\, G/F\right] + \beta a_l. \tag{9}$$

Next, we need the budget constraint of the households. Note that a rich household receives an income stream that is $(1-\beta\theta)/(1-\beta)/\theta$ times as large as the one of a poor since both labor and capital income streams are $(1-\beta\theta)/(1-\beta)/\theta$ times as large. Due to the CES-form of intertemporal preferences, the flow of expenditures on goods of a rich household compared to a poor on the balanced growth path need to be $(1-\beta\theta)/(1-\beta)/\theta$ times as large, as well. We can thus work with the budget constraint of the rich relative to the poor,

$$\frac{1-\beta\theta}{(1-\beta)\theta} = \frac{m\left[(q_h - q_l)\,\mu_R + q_l\mu_P\right] + (1-m)q_h\mu_R}{mq_l\mu_P}, \tag{10}$$

where we have used the fraction of mass goods m, which is constant on the balanced growth path, and the prices for the mass good in high quality $p_h = (q_h - q_l)\,\mu_R + q_l\mu_P$, the mass good in low quality $p_l = q_l\mu_P$, and the exclusive good (only in high quality) $p_e = q_h\mu_R$. Combining the two equations above, we can express the price of the exclusive good as

$$q_h\mu_R = \frac{(1-\theta)\left[a_l\beta/(1-\beta) - a_hG/F\right]}{\theta/m - q_l/q_h - (1-\theta)G/F}. \tag{11}$$

Plugging this equation into the no-arbitrage condition of the exclusive producer and using the Euler equation (7) yields a no-arbitrage curve:

$$g = \frac{1}{\sigma}\left(\frac{L}{F}\left[\frac{\beta a_l - (1-\beta)a_hG/F}{\theta/(1-\theta)/m - q_l/q_h/(1-\theta) - G/F} - (1-\beta)a_h\right] - \rho\right) \tag{12}$$

which expresses the growth rate g in terms of the fraction of mass goods m. Note that the no-arbitrage curve is upward sloping in the case of $F\beta a_l > G(1-\beta)a_h$, and downward sloping if this inequality is reverse. More importantly, keeping g constant, the fraction of mass producers m rises in θ and falls in β.[11] Reducing inequality lowers exclusion and raises the fraction of mass goods in the economy, ceteris paribus.

To derive a second equation in m and g, we need to consider the aggregate resource constraint in the economy,

$$L = (1-\beta)LN(t)a_h/A(t) + \beta LM(t)a_l/A(t) + \dot{N}(t)F/A(t) + \dot{M}(t)G/A(t)$$

where $M(t) = Q(t)$ is the number of mass producers. Simplifying and rearranging yields a resource curve,

$$g = \frac{L\left[1 - (1-\beta)a_h/m^{1-\psi} - \beta a_l m^{\psi}\right]}{F/m^{1-\psi} + Gm^{\psi}} \tag{13}$$

where we have used $m = M(t)/N(t) = P(t)/N(t)$ and $A(t) = N(t)^{\psi}P(t)^{1-\psi}$. The resource curve is upward or downward sloping, depending mainly on ψ and the level of m. The more important process innovation (the lower ψ), and the lower the level of cumulative process innovation in the economy m (as only the mass producers invest into the process innovation),

[11] It is straightforward to see that a decrease in β requires an increase in m to offset the effect on g. An increase in θ is offsetting an increase in m, as the relevant denominator in the no-arbitrage curve is strictly increasing in θ, since its derivative is $\partial\left[\theta/(1-\theta)/m - q_l/(1-\theta)/q_h - G/F\right]/\partial\theta = 1/(1-\theta)^2(1/m - q_l/q_h) > 0$.

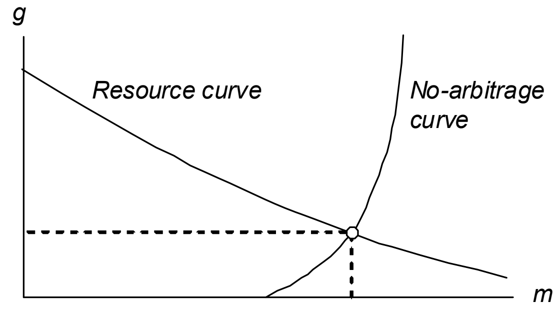

Figure 4: Equilibrium in the case of $A(t) = N(t)$

the more likely growth g is increasing in m. Note that θ does not enter the resource curve, and the resource curve may shift up or down in β.

The balanced growth path is determined by the intersection of the two curves (12) and (13), which allow us to study the content of the model.

5 Income Inequality, Technological Change and Growth

We will first analyze the equilibrium for the two polar cases $\psi = 1$ and $\psi = 0$, where spillovers are either generated only by product or only by process innovation, and then proceed to the general case.

Case 1: Product innovation as driver of productivity growth If product innovation is the only driver of productivity growth, $\psi = 1$, the resource constraint can be simplified to:

$$g = \frac{L\left[1 - (1-\beta)a_h - \beta a_l m\right]}{F + Gm} \tag{14}$$

whereas the no-arbitrage curve remains unchanged. The resource curve is downward sloping in m, since a larger share of mass producers requires more labor for manufacturing and process innovation, leaving less labor for product R&D, the driver of growth. Figure 4 displays the two curves and the equilibrium.[12]

We can show that in the case of $A(t) = N(t)$, inequality is beneficial for growth. Let us increase inequality by reducing the share of a poor household in income θ (the share of a rich rises given $\theta_R = (1-\theta\beta)/(1-\beta)$). While the resource curve is not affected by this change, the no-arbitrage curve shifts left, as depicted in the left-hand panel of Figure 5. A richer upper class has a higher willingness-to-pay for products, driving up profits. Product inventions become

[12] As discussed above, the no-arbitrage constraint is upwards sloping in the case of $F\beta a_l > G(1-\beta)a_h$, and downwards sloping if the inequality is reversed.

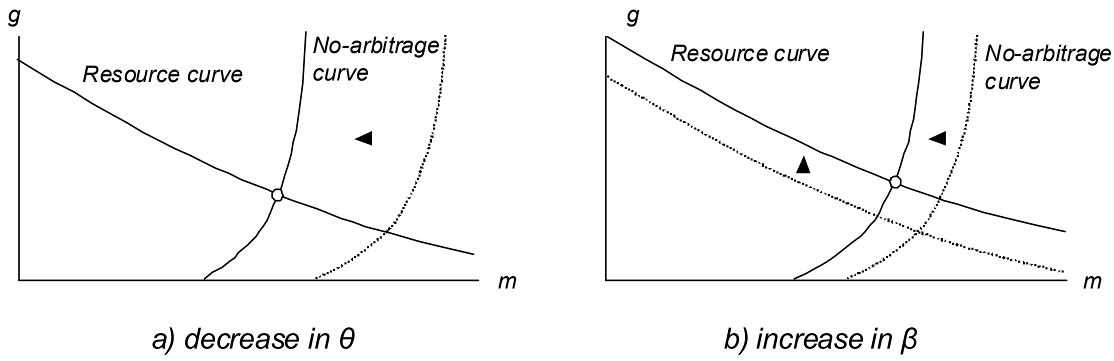

Figure 5: Impact of inequality in the case of $A(t) = N(t)$

more attractive, spurring technological progress and growth. From a resource point of view, redistributing wealth from the poor to the rich raises exclusion in the economy, setting free resources in production and process innovation who can be employed in product R&D, the driver of growth.

Increasing the size of the group of poor households β, while holding θ constant, raises inequality (see Figure 3). As can be seen in the right-hand panel of Figure 5, both the resource and the no-arbitrage curve shift up. Note that as a by-product of increasing β, the share in income of the rich households rises, given $\theta_R = (1 - \theta\beta)/(1 - \beta)$. There are less rich making the market for the exclusive producers smaller. But the remaining rich have a higher willingness-to-pay. It turns out that the latter effect dominates, and profit levels increase, spurring product R&D. Furthermore, more exclusion sets free manufacturing resources required for an equilibrium with increased product R&D. Increasing the size of poor households, hence, increases growth.

Case 2: Process innovation as driver of productivity growth The result that inequality benefits growth, critically depends on the driver of growth. If process innovation is the only driver of productivity, $A(t) = P(t)$, the relationship between inequality and growth reverses. In this case, inequality is harmful for growth.

The resource constraint can be rewritten as:

$$g = \frac{L\left[1 - (1-\beta)a_h/m - \beta a_l\right]}{F/m + G}. \tag{15}$$

The no-arbitrage curve remains unchanged, while the resource curve in this case is upward sloping. As process innovation is the key to become a mass producer, a higher share of mass producers is beneficial for growth from a resource point of view, given that more process

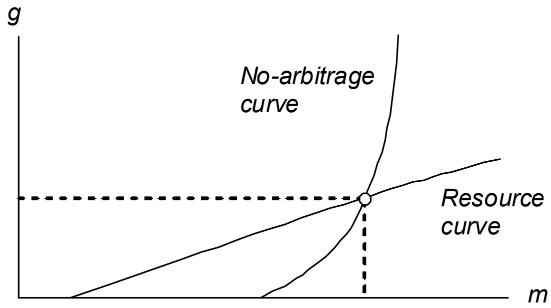

Figure 6: Equilibrium in the case of $A(t) = P(t)$

innovation raises aggregate productivity in the economy. Figure 6 illustrates the two curves and the equilibrium graphically.[13]

Increasing inequality, by reducing the share in income of the poor households θ, shifts the no-arbitrage curve to the left, as depicted graphically in the left-hand panel of Figure 7. Redistributing income from poor households to the rich raises exclusion, reducing mass consumption markets in the economy. Since mass consumption markets are the key driver of process innovation, process innovation in the economy drops. Inequality thus hurts growth by lowering process innovation, the driver of aggregate productivity.

Increasing the group size of the poor households β shifts both the no-arbitrage and the resource curve up, as shown in the right-hand panel of Figure 7. The effect on growth is ambiguous. Mass consumption in the economy is lowered, as discussed for the case of $A(t) = N(t)$. On the one hand, less mass consumption markets lower process innovation and hence productivity levels in the economy. On the other hand, higher exclusion sets free manufacturing resources for product R&D. Computation have shown, that both effects may dominate.

General case By having analyzed these two polar cases, we have demonstrated that inequality may be either beneficial or harmful for growth, depending on the source of technological progress and productivity growth in the economy. Inequality has an *effect on prices* and on the *size of markets*. On the one hand, a higher willingness-to-pay of the rich households raises prices and profit margins, spurring entry and thus product innovation. On the other hand, a high level of exclusion reduces mass consumption markets, and thus incentives for process innovation. Inequality fosters growth in the case of product innovation being the driver of productivity growth, and slows growth in the other case.

[13] In the case of an upward sloping no-arbitrage curve, multiple equilibria with a low and high positive growth state may arise. However, simulations have shown, that rather special parameter constellations are required.

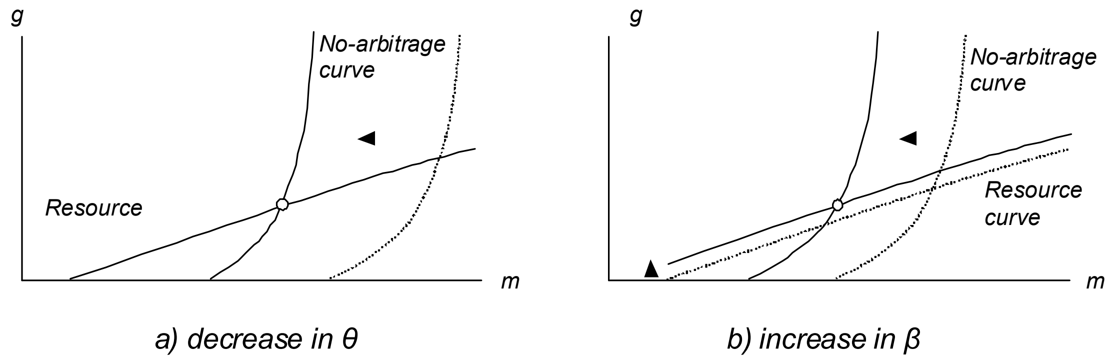

Figure 7: Impact of inequality in the case of $A(t) = P(t)$

For intermediate cases of ψ, as well as more general functional forms of $A(\cdot)$, where productivity growth is driven by both product and process innovation, the sign of the inequality growth relationship depends on the dominating source of technological change. The resource curve, equation 13, may be upward sloping or downward sloping. If product innovation has sufficient weight in $A(\cdot)$, the resource curve is downward sloping. Analogous to the case of $A(t) = N(t)$, inequality is beneficial for growth. If process innovation is more dominant, the resource curve is upward sloping, and the relationship between inequality and growth is reversed. The resource curve can also be hump-shaped as depicted in Figure 8. Rising inequality can foster growth, if inequality is rather low (and the fraction of mass producers m high), and it can slow growth, if inequality is already high.

There certainly are complementarities between product and process innovation in an economy. When an economy has invested relatively little in process innovation, it is more likely to benefit more from process innovations and vice versa.. Therefore, in a very unequal economy with few mass consumption markets, lowering inequality is likely to increase growth. The expansion of mass consumption markets spurs process innovation and thus growth. However, in a very egalitarian economy, ceteris paribus, the relationship may be reversed, as the economy profits more from a better funded upper class, spurring the introduction of new goods, i.e. product innovation. Hence, in the presence of such complementarities, both very high levels and very low levels of inequality are harmful for growth, and growth is maximized at an intermediate extent of economic inequality.

Our theoretical framework is able to replicate the intuitions and evidence we have developed in the first part of our paper. We have established a channel through which inequality becomes a key determinant of the nature of technological change and ultimately economic growth. Inequality shapes the structure of product markets, that is size of mass consumption and

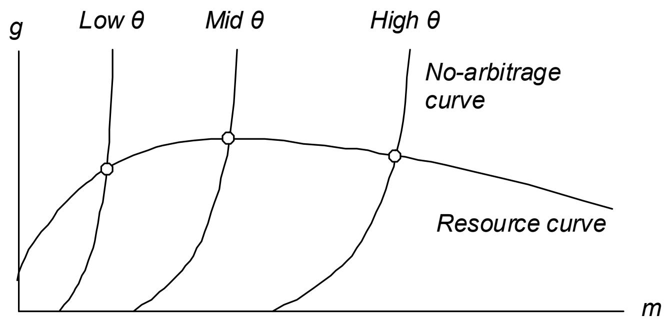

Figure 8: Impact of inequality in the general case

exclusive markets. This in turn has implications for the relative attractiveness of product and process innovation, key sources of long-term economic growth.

6 Product cycle

In our model we have assumed symmetry in preferences and technology across products in order to simplify the analysis. An implication of this strong form of symmetry is that firms are indifferent about the timing of the process innovation and becoming a mass producer (as briefly discussed in the section on R&D). The individual product cycle is indeterminate.

This strong form of symmetry is not critical for our main results, as the aggregated fraction of mass producers is determined. In fact, weakening these assumptions generates product cycles that closely replicate empirically observed patterns. In the section on empirical motivation and the economics of the Model T, we have discussed the product cycle, where goods initially are only consumed by an upper class. Over time, the penetration levels rise, as more and more households can afford particular goods, critically accelerated by process innovation. The exclusive good is transformed into a mass consumption good over time. Figure 9 shows the product cycle for our economy. The first $mN(t)$ goods are consumed by the entire population, whereas the latest $(1-m)N(t)$ inventions are only consumed by the rich group, $(1-\beta)L$.

Penetration levels of particular goods rise over time, driven by general increases in welfare levels, expanding the range of *necessities* in an economy, and by process innovations accelerating this transformation. There are two natural adjustments to our model, either to preferences or to technology, breaking the strong symmetry and thus replicating the empirically observed product cycles.

Hierarchic preferences We have assumed, that households are indifferent between goods. Intuitively and empirically, it makes sense that there is a *hierarchy of needs*, i.e. that certain

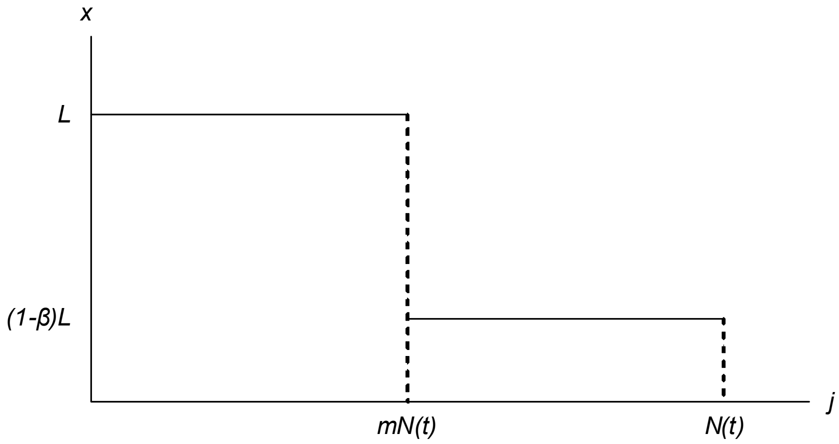

Figure 9: Product cycle

more *basic* goods have priority. We can reflect such a hierarchy in the instantaneous utility function of the households as follows:

$$u(t) = \int_0^{N(t)} \xi(j)x(j,t)q(j,k)dj,$$

where we have added a *hierarchy weight* $\xi(j)$ which is strictly monotonically decreasing in j. Hence low-j goods get a higher weight than high-j goods, and households are no longer indifferent between the products. Households have a higher willingness-to-pay for low-j than for high-j goods. Product innovation R&D would thus focus on the lowest-j goods not yet invented. For balanced growth, the hierarchy weight needs to be a power function, $\xi(j) = \epsilon j^{-\eta}$ (see Bertola, Foellmi, and Zweimueller, 2006, Chapter 12). The process innovation and strategy switch timing problem thus becomes:

$$\begin{aligned}
\max_{\Delta t} PV(j,t) &= \int_t^{t+\Delta t} \pi_e(j,s)\exp(-r(s-t))ds + \int_{t+\Delta t}^{\infty} \pi_m(j,s)\exp(-r(s-t))ds - G\exp(-r\Delta t), \\
\text{where } \pi_e(j,s) &= L(1-\beta)\left[\epsilon j^{-\eta} q_h \mu_R(s) - a_h\right], \\
\text{and } \pi_m(j,s) &= L\left[\beta\left(\epsilon j^{-\eta} q_l \mu_P(s) - a_l\right) + (1-\beta)\left(\epsilon j^{-\eta}\left((q_h - q_l)\mu_R(s) + q_l\mu_P(s)\right) - a_h\right)\right].
\end{aligned}$$

Profit flows depend on the hierarchy level and on time, as $\mu_R(t)$ and $\mu_P(t)$ are increasing over time at the rate ηg.[14] Therefore, the difference between the profit flows from the mass strategy $\pi_m(j,s)$ and the exclusive strategy $\pi_e(j,s)$ grows over time.[15] In equilibrium, firms start out

[14] In order that the no-arbitrage condition holds, the initial present value of every newly set up firm must equal F. Hence the hierarchy-independent part of the willingness-to-pay $\mu_i(t)$ must rise at $-\partial/\partial t\left(\epsilon j^{-\eta}\right) = \eta g$ over time, in order that the overall willingness-to-pay for a product only depends on the time span since inception, and not on time.

[15] The revenues of the mass strategy must be larger in equilibrium. Otherwise, firms would never switch to the mass strategy, since it has higher manufacturing and process innovation costs. Note also that both revenue streams grow at the same rate. It follows that $\pi_m(j,s) - \pi_e(j,s)$ grows over time.

being exclusive producers. As the difference narrows to $\pi_m(j,s) - \pi_e(j,s) = G$, Δt units of time after the inception date, it becomes optimal for firms to switch to the mass strategy (use Leibniz rule to see this). The size makes low-j goods more attractive to sell in mass consumption markets than high-j goods.

Hence, we have established a product cycle in our framework, where a firm initially sells its good exclusively to rich households, who have a high willingness-to-pay for new goods, even if they are very low on their priority list. After a certain period of time elapses, the firm invests in process innovation to tap the mass consumption market, as the good has climbed the *relative* hierarchic ladder of available goods in the economy. Firms start out as exclusive producers to eventually become mass producers, as their good is transformed from a *relative* luxury into a *relative* necessity.

We can adapt all the other equations of our model along these lines. Prices now vary across products, even if of the same quality and type of strategy. However, the main conclusions of our model remain unchanged. Inequality hurts growth, when process innovation is the dominant driver of product growth, and accelerates growth, otherwise.

Learning-by-doing The required investment expenditure for process innovation $G(t)$ is identical for all firms in our model. Intuitively, it makes more sense that $G(t)$ is idiosyncratic across firms, and correlates negatively with the cumulative manufacturing experience of a firm. Experience in manufacturing should facilitate process innovation. Hence learning-by-doing, the cumulative experience of a firm in manufacturing, should be a key input into the process innovation, i.e. lower $G_j(t)$. In fact, instead of modelling process innovation as an intentional investment effort of $G_j(t)$ depending on manufacturing experience, it is instructive to analyze the case of process innovation as a pure (passive) by-product of manufacturing:

$$a(j,t) = (1 - \Lambda(j,t))a/N(t), \text{ with } \Lambda(j,t) = \int_{-\infty}^{t} \delta x(j,s) \exp(-\delta(t-s)) ds,$$

where δ is the speed of learning as well as the depreciation rate of learning capital, and $a(j,t)$ and $x(j,s)$ productivity and production level of firm j. Further let us assume that there is only one quality level $q = 1$. Individual productivity of a firm increases due to individual cumulative manufacturing experience, as well as through spillovers from product innovation. The more experience a firm has in manufacturing, the higher is the productivity level of the firm. In equilibrium, the mass consumption market is more attractive for higher productivity levels, due to the market size effect. This gives rise to the same product cycle as in the case of hierarchic preferences. Firms start out being exclusive producers, and eventually become mass producers, after a determined time interval Δt:

$$\max_{\Delta t} \int_0^{\Delta t} (1-\beta)L\left[p_h - (1-\Lambda(j,t))\right]\exp(-rt)dt + \int_{\Delta t}^{\infty} L\left[p_l - (1-\Lambda(j,t))\right]\exp(-rt)dt = F/a, \tag{16}$$

where p_h is the price charged by exclusive producers, and p_l by mass producers, and we set $w(t) = N(t)/a$ as numéraire. The maximized present value needs to equal the set-up costs $F(t)w(t) = F/a$ (given spillovers $F(t) = F/N(t)$), generating the no-arbitrage condition. The optimal period of time Δt for being an exclusive producer is determined by:[16]

$$p_l = (1-\beta)p_h + \beta\left[1 - L(1-\beta)\left[1-\exp(-\delta\Delta t)\right] - \delta L/(r+\delta)\right], \tag{17}$$

and the fraction of mass producers by Δt:

$$m = 1 - \int_{-\Delta t}^{0} gN(0)\exp(gt)dt/N(0) = \exp(-g\Delta t). \tag{18}$$

The equilibrium can be analyzed most conveniently, as above, by combining these equations with the Euler equation (7) and the relative budget constraint, which in this case is $(1-\beta\theta)/(1-\beta)/\theta = ((1-m)p_h + mp_l)/mp_l$, to form a no-arbitrage curve in m and g. The resource curve is determined by the resource constraint:

$$L = gF + \frac{aL}{N(t)}\left[\int_0^{mN(t)}(1-\Lambda(j,t))dj + (1-\beta)\int_{mN(t)}^{N(t)}(1-\Lambda(j,t))dj\right]. \tag{19}$$

Our main conclusions remain unchanged. Computations show that the resource curve may be rising or falling in m, depending on the strength of learning-by-doing (LBD). Rising inequality, through a fall in θ, raises prices and decreases mass consumption markets m, which tends to reduce resources required in manufacturing. However, by lowering aggregate manufacturing, LBD in the economy is reduced. Either effect may dominate. Inequality and exclusion lowers mass production and LBD. If LBD is the dominant driver of productivity growth in the economy, inequality hurts growth.

Comparing the learning-by-doing approach with the intentional process innovation R&D approach, developed in the main section, highlights the close linkage of process innovation and mass consumption markets. Process innovation is a key step, *ex-ante,* to tap the mass market, and is facilitated *ex-post* by mass production (learning-by-doing). Tapping the mass consumption market creates incentives to invest in process innovation. And increased learning-by-doing, in turn, creates incentives to pursue a mass market strategy.

[16] Maximizing equation (16), using Leibniz rules and the fact that $\Lambda(j,t) = L(1-\beta)\left[1-\exp(-\delta t)\right]$ if $t \leq \Delta t$, and $\Lambda(j,t) = L\left[1-\exp(-\delta t)\right] - \beta L\left[\exp(-\delta(t-\Delta t) - \exp(-\delta t)\right]$ if $t > \Delta t$.

7 Conclusion

In this paper we presented an endogenous growth model where firms invest both in product and process innovations. Product innovations (that open up completely new product lines) satisfy the luxurious wants of the rich. Subsequent process innovations (that decrease costs per unit of quality and/or drive down prices) transform the luxurious products of the rich into conveniences of the poor. A prototypical example for such a product cycle is the automobile. Initially an exclusive product for the very rich, the automobile became affordable to the middle class after the introduction of Ford's *Model T,* the car that "*put America on wheels*".

Our analysis shows that the extent of economic inequality in a society generates substantially different incentives for the direction of technical change. An egalitarian society creates strong incentives for process innovations (such as the Model T) whereas an unequal society creates strong incentives for product innovations (new luxuries). Depending on which type of innovative activity drives technical progress, economic inequality is harmful or beneficial for long-run growth. This distinct role of product and process innovations goes in an important way beyond standard R&D based growth models, in which process/quality innovations and product inventions are often mathematically very similar (Acemoglu, 2009). To generate a role of income inequality we assume indivisibilities in consumption. This implies that the wealthy upper class consumes both more and better goods than the large majority of poor households.

For the sake of simplicity and tractability, our model reduced the income distribution to two groups of households. A more general income distribution would smooth the product cycle with penetration levels following logistic Engel curves in the aggregate (rather than a jump as in the stylized case of two groups of consumers). After inception, the producer would start out serving only the richest households, and step-by-step or continuously (in the case of a continuous income distribution) expand the market size, by lowering prices. There would be a "cut-off" where the producer would invest into the process innovation. However, apart from generating more realistic dynamics of product penetration, such a generalization – while substantially complicating the formal analysis – would add little additional economic insight to the model.

Another interesting extension, which we are currently working on, is to analyze the transitional dynamics of our framework. Consider an economy in a stagnant/low-growth state with high inequality, where the process innovation is too expensive ($G(t)$ too high), all the firms are owned by the upper class, and the lower classes are crowded in subsistence. If a positive productivity shock lowers G sufficiently, the economy would experience a takeoff transforming an initial exclusive society into a mass consumptions society with higher growth and lower exclusion.

Finally, as a further extension, our model could be set in an international trade context. For example, it could be combined with the international technological change framework initiated by Krugman (1979). Besides adding to the theoretical trade literature, the wealth of trade data would allow us to test some of the key premises of our basic framework empirically.

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Appendix

A. Proof of Optimal Separating Strategy A mass producer that has invested in process innovation cannot separate the rich into the low quality and the poor into the high, since the incentive constraints $q_l\mu_R - p_l \geq q_h\mu_R - p_h$ and $q_h\mu_P - p_h \geq q_l\mu_P - p_l$ would imply $(q_h - q_l)\mu_P \geq p_h - p_l \geq (q_h - q_l)\mu_R$, which cannot hold due to $\mu_R > \mu_P$ (willingness-to-pay of rich is higher). Hence, the profit maximization problem of the mass producers is:

$$\begin{aligned} \max \pi_m &= L\beta(p_l - a_l) + L(1-\beta)(p_h - a_h) \\ \text{s.t. (i) } p_h &\leq q_h\mu_R, \text{ (ii) } p_l \leq q_l\mu_P, \text{ (iii) } q_h\mu_R - p_h \geq q_l\mu_R - p_l, \text{ and (iv) } q_l\mu_P - p_l \geq q_h\mu_P - p_h, \end{aligned}$$

Constraint (iii) and the fact that $\mu_R > \mu_P$ implies $q_h\mu_R - p_h \geq q_l\mu_R - p_l > q_l\mu_P - p_l$. This shows that if the constraint (ii) were inactive, so would be (i). But then the firm could increase both prices by the same amount without violating (iii) and (iv). Hence, constraint (ii) must be active: $q_h\mu_R - p_h \geq q_l\mu_R - p_l > q_l\mu_P - p_l = 0$. But this implies that the constraint (iii) needs to active, as well. Otherwise the firm could increase the price of the high quality without violating constraints (iii) and (i). Constraint (iii) is active: $q_h\mu_R - p_h = q_l\mu_R - p_l > q_l\mu_P - p_l = 0$, showing the (i) is not active. Rewritting active (iii), $p_h - p_l = q_h\mu_R - q_l\mu_R > q_h\mu_P - q_l\mu_P$ shows that constraint (iv) is not active as well. The active constraints are the first-order conditions. QED

B. Proof of Proposition 1 In a first step we proof the following statement about possible regimes:

Lemma 1 *In equilibrium, at least two strategies - an exclusive and a mass strategy - are pursued by (different) firms.*[17] *Abstracting from border cases, four regimes are possible in equilibrium: (i) exclusive and mass producers never innovate, (ii) both exclusive and mass producers only use the innovated process, (iii) exclusive producers have not yet innovated, while mass producers have innovated and only use the innovated process, and (iv) exclusive producers have not yet innovated, while mass producers have innovated and use both the original and innovated process, selling the high quality to the rich and the low quality to the poor.*

[17] Three or at most four strategies may co-exist for special border parameter values.

Proof. There are five possible optimal strategies in equilibrium,

$$\begin{aligned}
\pi_{eh} &= L(1-\beta)(q_h\mu_R - a_h), \\
\pi_{mh} &= L(q_h\mu_P - a_h), \\
\pi_{el} &= L(1-\beta)(q_l\mu_R - a_l), \\
\pi_{ml} &= L(q_l\mu_P - a_l), \\
\pi_{ms} &= L\beta(q_l\mu_P - a_l) + L(1-\beta)((q_h - q_l)\mu_R + q_l\mu_P - a_h),
\end{aligned}$$

where the first subscript distinguishes between mass m and exclusive strategy e and the second between the high quality h, low quality l and the separating strategy s. There must be at least one mass and one exclusive strategy in equilibrium. Otherwise, either the poor would consume nothing, or the rich would have an infinite willingness-to-pay for products. This leaves us with six combinations of two strategies. Two of these combinations can be ruled out.

Compare profit flows:

$$\begin{aligned}
(q_h - q_l)\mu_P &> a_h - a_l \Longrightarrow \pi_{mh} > \pi_{ml}, \\
(q_h - q_l)\mu_R &> a_h - a_l \Longrightarrow \pi_{eh} > \pi_{el}, \\
(q_h - q_l)\mu_R &> a_h - a_l \Longrightarrow \pi_{ms} > \pi_{ml}.
\end{aligned}$$

Since $\mu_R > \mu_P$, we can deduce the following relations:

$$\begin{aligned}
\pi_{mh} &\geq \pi_{ml} \Longrightarrow \pi_{eh} > \pi_{el}, \qquad (20)\\
\pi_{el} &\geq \pi_{eh} \Longrightarrow \pi_{ml} > \pi_{mh}, \\
\pi_{eh} &> \pi_{el} \Longleftrightarrow \pi_{ms} > \pi_{ml}.
\end{aligned}$$

If the mass separating strategy is equally or more attractive than the mass low strategies in terms of profit flows, the exclusive producers make more or equal profits producing the high quality instead of the low. The exclusive producers thus would not invest in process innovation. In equilibrium, both mass separating and exclusive low strategies cannot co-exist.

Another combination can be ruled out: mass high and exclusive low. For this combination, we would have to have $\pi_{el} - \pi_{eh} \geq \pi_{ml} - \pi_{mh}$ which implies $(q_h - q_l)\mu_P\,[1/\beta - (1-\beta)\mu_P/\beta/\mu_R] \geq a_h - a_l$. But from above we know: $\pi_{el} \geq \pi_{eh} \Longrightarrow \pi_{ml} > \pi_{mh}$, implying $(q_h - q_l)\mu_P < a_h - a_l$, which contradicts the inequality in the previous sentence, since the term in the square bracket is smaller than one ($\mu_R > \mu_P$, and $\beta < 1$). Hence, we proofed that mass high and exclusive low never co-exist in an equilibrium. No more combinations can be ruled out.

Hence, at most four strategies may co-exist. Three and four strategies only co-exist for special parameter values (borders between different regimes). Abstracting from these special border cases, four regimes may arise: (i) eh and mh, (ii) el and ml, (iii) eh and ml, and (iv) eh and ms. ■

Let us now derive the conditions for the fourth regime, the one we analyze in the main text. The mass producers, having invested in process innovation, prefer still selling the high quality to the rich if

$$\pi_{ms} > \pi_{ml} \Longleftrightarrow (q_h - q_l)\mu_R > a_h - a_l.$$

Given (20), the exclusive producers generate higher profit flows from selling the high quality, and thus do not invest in the process innovation. Lastly, the mass strategy selling the high quality is not attractive if

$$(1-\beta)(q_h\mu_R - a_h) > (q_h\mu_P - a_h),$$

i.e. the exclusive high strategy yields higher profit flows. QED

Obtaining conditions containing only exogenous parameters, using (9), (11), (12) and (13), is rather cumbersome, yielding no extra insights. We will analyze these conditions graphically in more details in Appendix D, using computations, after analyzing the other regimes briefly.

C. Characterization of Other Regimes If Assumption 1 is violated, other regimes govern:

Mass high and exclusive high regime If the process innovation is too unattractive, firms do not invest in process innovation. Pursuing mass and exclusive strategies must yield the same profit flow, $\pi_{eh} = \pi_{mh}$. We can derive a no-arbitrage curve in g and m, proceeding in the same way as for the regime analyzed in the main part of the paper:

$$g = \frac{1}{\sigma}\left[\frac{a_h L}{F}\left(\frac{\beta\theta(1-m)}{\theta - m} - 1\right) - \rho\right].$$

Note that if $\psi < 1$ in $A(t) = N(t)^{\psi}Q(t)^{1-\psi}$, growth would eventually halt, since there is no process innovation in this regime. In the case of $\psi = 1$, i.e. setting $A(t) = N(t)$, the resource curve in g and m in this regime is

$$g = \frac{L}{F}\left[1 - a_h(\beta m + 1 - \beta)\right].$$

The resource curve is falling in m and the no-arbitrage curve rising. Increasing inequality in this case is beneficial for growth: Reducing θ shifts up the no-arbitrage curve which increases g. As process innovation does not matter, redistributing income to the rich rises their willingness-to-pay and thus profits, spurring entry and growth.

This regime arises if the process innovation is not sufficiently attractive, i.e.

$$
\begin{aligned}
G &> \frac{L(1-\beta)(q_l\mu_R - a_l)}{\sigma g + \rho} - F, \\
G &> \frac{L(q_l\mu_P - a_l)}{\sigma g + \rho} - F, \\
G &> \frac{L\beta(q_l\mu_P - a_l) + L(1-\beta)((q_h - q_l)\mu_R + q_l\mu_P - a_h)}{\sigma g + \rho} - F.
\end{aligned}
$$

We will analyze these conditions graphically in Appendix D.

Mass low and exclusive high regime The mass producers innovate, but then only use the new process, selling the lower quality to both poor and rich households. Along the lines of the main text, we can derive the no-arbitrage curve

$$
g = \frac{1}{\sigma}\left[\frac{L}{F}\left(\frac{a_l - (1-\beta)a_h(1+G/F)}{\theta/(1-\theta)/m - 1/(1-\theta) - G/F} - (1-\beta)a_h\right) - \rho\right],
$$

and the resource curve

$$
g = \frac{L\left[1 - (1-\beta)a_h(1-m)/m^{1-\psi} - a_l m^{\psi}\right]}{F/m^{1-\psi} + g m^{\psi}}.
$$

This regime yields similar results to the mass separating and exclusive high regime. However, there is one crucial difference: Even in the case of $A(t) = N(t)$, i.e. $\psi = 1$, inequality may be harmful for growth. The resource curve in this case is $g = L\left[1 - (1-\beta)a_h - m(a_l - (1-\beta)a_h)\right] / \left[F + mG\right]$. For a sufficient small a_l, the resource constraint may be increasing in m. An increase in the fraction of mass producers m may set free resources for product R&D, as the mass producers only use the less laborious process. Even though more goods are produced, less production labor is needed. This is another channel through which more equality is beneficial for growth. If a_l is not sufficiently small, the analysis and results are analogous to the mass separating and exclusive high regime.

Conditions for this regime are

$$
\begin{aligned}
(1-\beta)(q_h\mu_R - a_h) &> (q_h\mu_P - a_h), \\
a_h - a_l &> (q_h - q_l)\mu_R, \\
(q_l\mu_P - a_l) &> (1-\beta)(q_l\mu_R - a_l).
\end{aligned}
$$

i.e. that the other strategies - mass high, mass separating and exclusive low - are not attractive to pursue.

Mass low and exclusive low regime This regime is equivalent to the mass high and exclusive high regime (substituting $F + G$, a_l, q_l for F, a_h, q_h), with one difference: There is sustainable growth for every ψ, since $A(t) = A(t) = N(t)^{\psi}Q(t)^{1-\psi} = Q(t) = N(t)$. Inequality in this case is beneficial for growth as in the mass high and exclusive high regime.

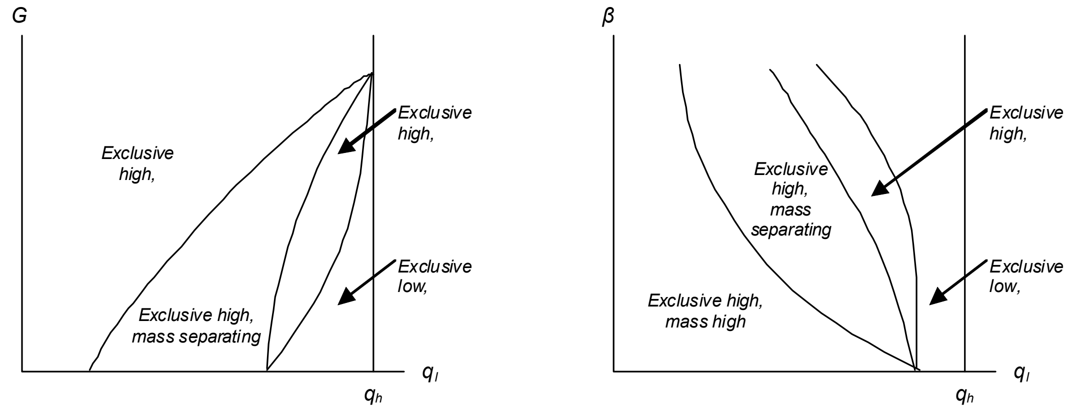

Figure 10: Conditions for the different regimes

Conditions for this regime are

$$\begin{aligned} F &> \frac{L(1-\beta)(q_h\mu_R - a_h)}{\sigma g + \rho}, \\ F &> \frac{L(q_h\mu_P - a_h)}{\sigma g + \rho}, \\ a_h - a_l &> (q_h - q_l)\mu_R, \end{aligned}$$

that is exlusive high, mass high and mass separating strategies are not attractive in equilibrium.

D. Conditions for Regimes The conditions for the different regimes to arise can be best analyzed graphically. Let us first assume the one polar case $A(t) = N(t)$. Figure 10 maps out parameter regions for the different regimes. On the horizontal axis, the quality level of the low quality is varied, whereas on the vertical axis the costs of the process innovation and the size of the group of poor households, respectively, are varied.

The left panel shows, as the ratio of the costs of the process innovation G to the low quality q_l increases, firms are less likely to invest in process innovation. If G is prohibitively high, or the quality discount $(q_h - q_l)$ compared to the cost reduction $(a_h - a_l)$ to high, no firms invest in process innovation. If G and the quality discount are not too high, the mass producers invest, but use the original process for the rich, who have a higher willingness-to-pay for the better quality. If the quality discount is reduced further, the mass producers stop using the original process, and if it is sufficiently low, the exclusive producers also invest in the process innovation. Note that if $q_h = q_l$, either all or no producers invest in process innovation (it is easy to check that the conditions for the exclusive high and mass low cannot be fullfilled simultaneously in this case).

On the right hand side, one can see that if the group of poor β is large, even at a relatively high quality discounts the process innovation can be attractive for the mass producers.

The larger the customer group for the innovated process, the more attractive is the process innovation.

Note that computations reveal that the share of income of the poor θ has limited influence on the type of regime to arise in equilibrium. θ mainly drives the fraction of mass producers m (and g).

For the other polar case $A(t) = Q(t)$, the analysis is similar, but we have to account for the fact that not all regimes generate sustainable growth. Similar to above, the higher the low quality q_l and the higher the group of poor β, the more likely the mass producers are to invest into the process innovation. And in this case, the more likely the economy experiences sustained growth. However, in this case, θ interacts more strongly with the type of regime. The share of income of the poor households θ influences the share of mass producers m, and thus the level of process innovation, which is the growth driver in the economy. As higher growth requires higher interest rates, $R = \sigma g + \rho$, the hurdle for investments rise, and the mass producers are less likely to invest in process innovation. Initially, redistributing income from the rich to the poor increases growth, but if too much is redistributed, it may halt growth (as firms stop investing in process innovation).

The general case of $A(t) = N(t)^{\psi} Q(t)^{1-\psi}$ falls between the two polar cases.