

# General Equilibrium with Customer Relationships: A Dynamic Analysis of Rent-Seeking \*

Robert E. Hall  
Hoover Institution and Department of Economics  
Stanford University  
National Bureau of Economic Research  
rehall@stanford.edu    Website: Google "Robert Hall"

February 11, 2008

## Abstract

U.S. businesses spend almost \$300 billion a year on advertising. The goal of the spending is to attract loyal customers. The relationship capital associated with the resulting loyalty earns substantial rents for businesses and also some rents for customers. Businesses and households pursue those rents. I build a model of the development and valuation of customer relationships and of the resulting effects on the allocation of time within households. Shopping occupies about six hours per week for the average American adult. A small part of that time is search for new relationships with merchants and the bulk is buying goods and services and bringing them home. Influences on the profit margins of sellers—such as variations in productivity—have profound effects on the tightness of retail markets and on the allocation of time in the household. The model tracks data in the American Time Use Survey during the expansion from 2003 to 2006. The model implies that spending on advertising and spending on recruiting workers move in parallel over the business cycle. This implication is strikingly confirmed in data on advertising and the labor market.

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\*This paper is part of the Economic Fluctuations and Growth program of the National Bureau of Economic Research. I am grateful to members of the program's RSW group and to Alan Kackmeister for comments. Data, programs, and an extended version of this paper with derivations are available at my website.

# 1 Introduction

In retail markets, consumers form loyalties to stores and brands. Merchants and producers invest heavily in attracting customers and making them loyal. In 2006, spending on advertising in the U.S. was \$264 billion or \$2300 per household. If the customer turnover rate is 25 percent per year, the capital value of the sellers' share of the transaction surplus arising from advertising is about a trillion dollars. Producers invest heavily in attracting customers because producers receive a share of a large surplus. Customers pursue relationships as well because they also receive a share of the large surplus. This paper is about the dynamic general-equilibrium implications of these rent-seeking activities.

Using a model of customer search and seller recruiting, I investigate these topics by adapting some of the principles of search-and-matching models of the labor market. Retail markets are in equilibrium in the sense that the bilateral relationship between customer and seller offers no private opportunity for joint improvement. The model has a coherent notion of market tightness. The model also recognizes that shopping takes time—the average American adult spends more than an hour shopping for each five hours of work.

Notwithstanding the huge literature on pricing in markets with customer relationships, economists' understanding of retail price determination still appears to be limited. For example, economics has no accepted explanation for the well-documented failure of the prices of imported branded goods in the U.S., especially cars, to respond to the large changes in their dollar cost that occur as the exchange rate of the dollar fluctuates relative to the currency in which the costs are incurred. This paper does not make progress in that area. Rather, I investigate equilibrium prices without asking how the prices came to be. The strategy resembles that in Hall (2005) for wages. I focus on the magnitude and distribution of the rents associated with customer relationships and on the tightness of retail markets under alternative distributions of the rents.

Although this paper is about retail markets, I adopt the equilibrium view of the labor market, following my earlier paper. I study a simple economy with one product and one type of labor. The volume of advertising, the turnover rate of customers, the amounts of time that people spend shopping and working, and standard figures for the labor market determine the parameters of the model.

I find the equilibrium set for the price and the wage. A price-wage pair is in the equilibrium set if retail customers can trade voluntarily with merchants at the price and if workers

can trade voluntarily with employers at the wage. By voluntary trade I mean that both parties benefit from trade. The price is between the reservation price of the customer and the reservation price of the merchant, and the wage is between the reservation wage of the producer and the reservation wage of the worker. I start by describing the equilibrium set for a static version of the economy. It is a triangle where one leg is the minimum price where product trade can occur, a second is the maximum wage where employment can occur, and a third is the minimum real wage where households will participate in the two markets. For reasonable parameter values, the typical values of prices and wages are close to the first two bounds and far from the household participation bound.

Any analysis of retail prices with customer search encounters the Diamond (1971) paradox. Diamond considered a model of a product market with structure similar to the one considered here—customers visit sellers one at a time. He granted the seller all of the bargaining power in the bilateral relation between customer and seller. This assumption seems innocuous—after all, the typical merchant posts a price which amounts to a take-it-or-leave-it offer. The customer has the option to reject the offer and visit another store. But, as Diamond observed, the unique equilibrium in that setting is for every merchant to set the monopoly price. In any other candidate equilibrium, the merchant has an opportunity, derived from the customer’s search cost, to set a price above the customer’s best alternative price. The only way to prevent the merchant from acting on the opportunity is for the best alternative price to be the monopoly price. The literature contains numerous resolutions of the Diamond paradox, mostly ones that let customers work one seller against another. I do not adopt any particular model, for it would imply a rule for price formation, and, as noted above, I am skeptical that we have a realistic model of price formation. In the general-equilibrium model of this paper, the implication is the same for the retail market, but monopoly in that market turns out to imply that households derive no benefit at all from participation in the entire economy, including the labor market. The implications of the Diamond paradox are far more drastic than in a single market.

The model describes the policy menu in an economy where the retail price and the wage are predetermined but the government controls the flexible price of the intermediate product. In that economy, policy faces a tradeoff in the tightness of the retail and labor markets. By setting the intermediate product price at the low end of its permissible range, policy delivers a retail market that is tight—shoppers find it easy to form new retail relationships.

Households spend little time searching for available retail outlets and divide the extra time among searching for work, working, and buying goods. But that economy has a slack labor market. Employers have small incentives to create jobs, which are thus hard for job-seekers to find. Unemployment is high. Utility reaches its maximum with the optimal policy that puts the flexible price near the middle of its permissible range.

Under this optimal policy, fluctuations in productivity disturb both markets. A decline in productivity reduces the profit margin for both retailers and producer-employers. Incentives to recruit customers and workers decline. Advertising and labor-market recruiting effort both fall. The retail market slackens and households spend more time searching for merchants. The same thing happens in the labor market and unemployment rises.

Data from the U.S. economy on advertising and worker-recruiting spending show a pronounced correlation over the business cycle. Both decline dramatically in recessions and rise in booms. This finding confirms the basic idea of the paper. The cyclical relation between unemployment and shopping time pins down the distribution of the retail surplus. Shopping time remained constant during the expansion from 2003 to 2006 while unemployment fell substantially. According to the model, if households received more than about 18 percent of the retail surplus, their rent-seeking efforts, in pursuit of such high value, would cause them to assign a significant fraction of total family time to searching for valuable relationships with sellers. Further, the fraction would decline if productivity rose. The constancy of total shopping time is incompatible with such a response. Hence I conclude that search time is small and non-volatile, which requires that households receive relatively little of the retail surplus. This conclusion is consistent with the view that retailers commit to posted prices rather than bargaining with individual shoppers, though the effect is not as complete as Diamond hypothesized. Some of the resolutions to the Diamond paradox discussed in the literature, such as heterogeneity of consumers, seem to save the economy from the disturbing general-equilibrium implications of Diamond's analysis.

The model deals with two kinds of rents. Most of the discussion relates to rents associated with customer and employment relationships. Rents from these sources are a small but important part of total household income. Businesses earn no income from their shares of the rents, because they dissipate all of the rents in the form of customer-seeking spending (advertising) and worker-seeking spending (recruiting or vacancy costs). The bulk of household income is Ricardian rent, earned from the scarcity of the economy's only primary

factor, household time.

## 2 Model

The model embodies a retail market with search frictions. Because shoppers and merchants invest in search effort before locating each other, they enjoy a surplus from their relationship once it is formed. Any price that gives both the shopper and the merchant a non-negative share of the surplus is an equilibrium.

The model speaks coherently about tight and slack retail markets, a property of the real world that has not previously been incorporated in models of retail markets. The characterization of the product market follows Mortensen and Pissarides (1994) in important respects. The M-P model transformed macroeconomic thinking about the labor market by providing a rigorous and sensible model of market tightness.

The retail market is tight when a shopper finds a suitable merchant rapidly, thanks to enthusiastic customer-attracting efforts by merchants. In a tight market, it takes more resources than usual for a merchant to land a new customer. Markets are tight when the retail price is high relative to the wholesale price. A tight product market is analogous to a tight labor market, where a job seeker finds an opening rapidly, thanks to enthusiastic recruiting by employers. The labor market is tight when the wage is low relative to the wholesale value of the worker's marginal product.

In addition to the retail price and wage, the model contains two other prices. One is the wholesale price of the product—the price at which producers sell the product to retailers. I model this as a market price, but it could be an internal shadow price of a firm vertically integrated in production and retailing. The second is the shadow price of the product after it arrives in the household. This price, about 6 percent above the retail price, reflects the value added by the shopper.

In the body of the paper, I first develop the stationary version of the model. At the end I discuss dynamics. The Appendix contains the full dynamic version and many details relevant to the stationary version.

## 3 Households

A household has a continuum of members. Each member takes on one of four roles:

- $S$ : Searching for an appropriate retail merchant
- $B$ : Buying under an established relationship with a merchant
- $U$ : Unemployed—seeking work
- $N$ : Working at a job

I use the term *shopping* to refer to both searching and buying.

Households have linear preferences. They value the time spent searching, buying, and seeking work at zero and the time spent working at  $-z$  units of consumption. See Hall and Milgrom (forthcoming) for a derivation of  $z$  from underlying preferences. Thus household utility is  $c - zn$ , where  $c$  is consumption per member and  $n$  is the fraction of household members holding jobs. A fraction  $s$  of household members are searchers looking for merchants and a fraction  $u$  are job-seekers:

$$s + b + u + n = 1. \tag{1}$$

A searcher has an endogenous probability  $\phi_S$  of finding a store and forming a buying relationship each period and a job-seeker has a probability  $\phi_U$  of finding a job each period. A buyer has a fixed probability  $\sigma_B$  of terminating the relationship with a seller and a worker has a fixed probability  $\sigma_N$  of terminating the current job. A buyer purchases  $\alpha$  units of consumption goods each period from the chosen supplier. Households and firms have a discount rate of  $r$ . Buyers pay  $p$  for each unit at the store. Workers receive a wage of  $w$  per period. These and other value-related variables are measured in the economy's unit of value.

Households have no opportunity to save, so they spend their earnings:

$$pc = wn \text{ or } \alpha pb = wn. \tag{2}$$

The stationary laws of motion of the number of buyers,  $b$ , and the number of workers,  $n$ , are

$$b = (1 - \sigma_B)b + \phi_S s \tag{3}$$

and

$$n = (1 - \sigma_N)n + \phi_U u. \tag{4}$$

The Appendix describes the household's dynamic program. As in the M-P model, it is useful to affiliate values with the four roles that family members may play. These values

satisfy the stationary recursions,

$$H_S = \frac{1}{1+r}(H_S + \phi_S H_B) \quad (5)$$

$$H_B = \alpha(\tilde{p} - p) + \frac{1}{1+r}(1 - \sigma_B - \phi_S)H_B \quad (6)$$

$$H_U = \frac{1}{1+r}(H_U + \phi_U H_N) \quad (7)$$

$$H_N = w - z\tilde{p} + \frac{1}{1+r}(1 - \sigma_N - \phi_U)H_N \quad (8)$$

These values are the natural generalizations of the values assigned to job-seeking and employment in the M-P model. The variable  $\tilde{p}$  is the shadow value of goods at home delivered by the buyers. The flow  $\tilde{p} - p$  is the value added by a buyer, the difference between the shadow value and the purchase price from the retailer. The household equates the values of the two functions that it assigns to newly idled members:

$$H_S = H_U. \quad (9)$$

Equations (5) through (9) comprise a system of five linear equations in the four role values and the shadow price  $\tilde{p}$ .

## 4 Firms and Markets

The economy has two types of firms: Producers, who hire workers and make goods, and retailers, who buy goods from the producers and sell them to households. Output sells for price  $y$  in the intermediate market, which is friction-free and competitive.

### 4.1 Retailers and the shopping market

In the retail market,  $F$  shoppers are looking for retailers, where  $F$  is the number of families in the economy. In the aggregate, the retailers spend  $A$  units of output on advertising and other efforts to attract customers. A function

$$m(s, A) = \frac{\gamma A F s}{F s + \gamma A} \quad (10)$$

gives the flow of new customer relationships formed each period. It is increasing and concave in  $s$  and  $A$  and has constant returns to scale. I define an index of market tightness,

$$\theta_S = \frac{\gamma A}{F_S}, \quad (11)$$

so

$$m(s, A) = F_S \frac{\theta_S}{1 + \theta_S}. \quad (12)$$

A searcher has a probability

$$\phi_S = \frac{m(s, A)}{F_S} = \frac{\theta_S}{1 + \theta_S} \quad (13)$$

of finding a buying relationship each period. The flow of newly attracted customers during a period per unit of advertising is

$$\frac{m(s, A)}{A} = \frac{\gamma}{1 + \theta_S}. \quad (14)$$

When a seller and a searcher meet, they form a customer relationship, denoted  $B$ . The firm assigns value  $F_B$  to customer relationships. The stationary condition for  $F_B$  is

$$F_B = \alpha(p - y) + \frac{1}{1 + r} (1 - \sigma_B) F_B. \quad (15)$$

Each customer buys  $\alpha$  units of output per period at price  $p$ , generating a profit margin of  $p - y$ . The relationship survives into the next period with probability  $1 - \sigma_B$ .  $F_B$  is the present value of the margin to be earned from the customer relationship.

I assume free entry into the retailing business. The corresponding zero-profit condition equates the flow benefit from a unit of advertising resources to the price of the resources:

$$\frac{\gamma}{1 + \theta_S} \frac{1}{1 + r} F_B = y. \quad (16)$$

The total surplus from retailing is  $H_B + F_B$ , the sum of the value gained by the household from forming a particular buying relationship,  $H_B$ , and the gain to the retailer,  $F_B$ . The retail market is in equilibrium whenever both of these are non-negative, so the parties are trading voluntarily. I define the endogenous variable  $\beta_S$  as the fraction of the joint surplus accruing to the household:

$$\beta_S = \frac{H_B}{H_B + F_B}. \quad (17)$$



## 4.2 Producers and the labor market

Here the model follows M-P closely, but without the Nash wage bargain. Producers post vacancies to recruit workers. The index of labor-market tightness is the vacancy/unemployment ratio,  $\theta_U$ . I assume that the job-finding rate of the unemployed is

$$\phi_U = \frac{\theta_U}{1 + \theta_U} \quad (18)$$

and the vacancy-filling rate is

$$\frac{1}{1 + \theta_U}. \quad (19)$$

One worker produces  $\eta$  units of output. The margin earned from production is  $y\eta - w$ . The producer assigns a value  $F_N$  to a filled job. This value has the stationary condition

$$F_N = y\eta - w + \frac{1}{1 + r} (1 - \sigma_N) F_N. \quad (20)$$

It costs a producer  $k$  units of output to maintain a vacancy for one period. Free entry to vacancy-creation implies that the expected benefit from posting a vacancy is equal to the cost:

$$\frac{1}{1 + \theta_U} \frac{1}{1 + r} F_N = yk. \quad (21)$$

The total surplus from employment is  $H_N + F_N$ , the sum of the value gained by the household when a member takes a job,  $H_N$ , and the gain to the employer,  $F_N$ . The labor market is in equilibrium whenever both of these are non-negative, so the parties are trading within their bargaining set. I define the endogenous variable  $\beta_U$  as the fraction of the joint surplus accruing to the household:

$$\beta_U = \frac{H_N}{H_N + F_N}. \quad (22)$$

## 4.3 Intermediate product market

Under the assumptions above, the intermediate product market is perfectly competitive—producers have perfectly elastic supply at the price  $y$  because they would expand infinitely at any higher price. Similarly, sellers have perfectly elastic demand at that price. Thus the market clears at the price  $y$ .

## 5 The Class of Models that Replicate Known Features of the Economy

In this section I define a class of models that replicate the known features of the economy, including the unemployment rate, turnover rates for customer relationships and jobs, time spent working and shopping goods, the volume of advertising, and total employment. These appear as parameters and endogenous variables in the model. Members of the class differ along a single dimension, the fraction of the surplus accruing to consumers in retail transactions. At the end of this section, I pick one member of the class to serve as a base case. Later I rationalize the base case from the dynamic behavior of shopping time and unemployment.

I take the unit of time to be a week. I build and solve a model that describes the stationary equilibrium of the model (the model is spelled out in detail in the extended version of the paper on my website). The solution of the model provides stationary values of the endogenous variables along with values of certain parameters. In addition to the equations derived already, the model has some auxiliary equations to describe its stationary equilibrium. Some of these provide values for variables corresponding to their observed historical values. Others describe relations among variables that hold in the base case.

The members of the class of models differ by the share of the retail surplus accruing to the consumer. Models with low surplus correspond to the economy of the Diamond paradox, where merchants capture all or almost all of the retail surplus. I find, however, that the model does not deliver the central feature of the paradox—a high retail price. In fact the retail price is the same for all models in the class.

The discussion in this section compares different models with different parameters and does not describe the response of any given model to a change in driving forces. I will discuss that topic later.

### 5.1 Retail market

FMI (1994) reports from a survey that 24 to 27 percent of shoppers changed supermarkets each year. Although the marketing literature contains many studies of customer turnover, especially among bank and cellphone customers, none of the studies I have found to date actually reports a turnover rate. I use a value  $\sigma_B = 0.55$  percent per week, corresponding to an annual rate of 25 percent.

To determine  $\alpha$ , the number of units of output purchased each period by a buyer, I use

the ratio of work hours to hours spent shopping, that is, purchasing goods and services, reported in the Bureau of Labor Statistics' American Time Use Survey for 2005, which is  $\bar{\alpha} = 4.6$ .<sup>1</sup> The ATUS does not distinguish time spent looking for merchants (what I call searching) from time spent buying from merchants within established relations (what I call buying). I incorporate the auxiliary equation,

$$\bar{\alpha} = \frac{n}{s + b} \quad (23)$$

and then solve for the implied value of  $\alpha$ .

Crain Communications (2006) reports that total advertising spending in the U.S. in 2006 was \$264 billion. It also reports that 29 percent of advertising spending among the top 25 advertisers was by retailers. I take the product of these two numbers as a rough estimate of retail advertising spending:  $A = 0.29 \times 265 \div 52 = \$1.5$  billion per week.

I take the share of the retail surplus accruing to consumers to be

$$\beta_S = 0.13. \quad (24)$$

## 5.2 Labor market

In the Current Population Survey, the total separation rate, including job-to-job transitions, is about 6 percent per month, while in the JOLTS survey, it is about 3 percent. Davis, Faberman, Haltiwanger and Rucker (2007) correct an important bias arising from non-reporting by shrinking establishments and show that the rate is about 5 percent per month or 1.4 percent per week. Thus I take the  $\sigma_N = 0.014$ . The unemployment rate,  $u/(u + n)$  is  $\sigma_N/(\sigma_N + \phi_U)$ . The long-run average of unemployment is 5.5 percent. The corresponding value of the job-finding rate is  $\phi_U = 24$  percent per week. The corresponding value of the tightness variable is

$$\theta_U = 0.32 \quad (25)$$

This auxiliary equation pins down the value of the vacancy cost,  $k$ .

To determine the number of families,  $F$ , I use  $N = 144$  million, employment from the Current Population Survey in 2006, and the auxiliary equation

$$n = \frac{N}{F}. \quad (26)$$

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<sup>1</sup>[www.bls.gov/news.release/atus.t01.htm](http://www.bls.gov/news.release/atus.t01.htm)

<i>Equation</i>	<i>Variable or parameter</i>	<i>Value</i>
(5)	$H_S$	229
(6)	$H_B$	1.37
(7)	$H_U$	229
(8)	$H_N$	0.91
(9)	$\tilde{p}$	1.06
(15)	$F_B$	9.07
(20)	$F_N$	0.91
(16)	$\theta_S$	0.19
(25)	$\theta_U$	0.32
(13)	$\phi_S$	0.160
(18)	$\phi_U$	0.691
(24)	$p$	1.01
(27)	$w$	0.99
(2)	$s$	0.006
(3)	$b$	0.164
(1)	$u$	0.828
(4)	$n$	0.002
(17)	$\beta_S$	0.13
(22)	$\beta_U$	0.5
(23)	$\alpha$	4.65
(11)	$\gamma$	0.13
(26)	$F$	184
(21)	$k$	0.69

Table 1: Base Case

I use a value of  $z$ , the disamenity of work, of  $z = 0.71$ , following Hall and Milgrom (forthcoming). I take the weekly discount to be  $r = 0.05/52$ . I normalize productivity at  $\eta = 1$  and the intermediate product price at  $y = 1$ .

I assume equal sharing of the employment surplus,

$$\beta_U = 0.5. \tag{27}$$

See Hall and Milgrom (forthcoming) for a discussion of data that support approximately equal sharing of the employment surplus.

### 5.3 The base case

Table 1 shows the equations and the parameters or endogenous variables I associate with the equations.

In the base case, households enjoy most of the fruits of production as Ricardian rents. Producers do not use a large fraction of output for recruiting and sellers do not use a large fraction for attracting customers. The real wage  $w/p = 0.974$  is close to its maximum feasible value of one.

Recall that the unit of value in Table 1 is a week's worth of output from one worker. A searcher generates a flow of  $\alpha(\tilde{p} - p) = 0.228$  units of value per week and a worker  $w - z\tilde{p} = 0.71$ . The present value of the future flow from a household member currently searching for a merchant or a job is 229 units. This member faces a career of jobs and spells as a buyer, interspersed with periods of search. Only  $s = 0.57$  percent of the member's time will be spent searching for a supplier. Another  $u = 0.046$  percent will be spent looking for work.

The shadow value of goods at home is  $\tilde{p} = 1.062$ , so the efforts of the buyers in the household add about 5 percent to the value of goods over the purchase price at the store of  $p = 1.013$

The most interesting finding within the household is that the incremental value of converting a shopper to a buyer is  $H_B = 1.37$  units, while the incremental value of converting a job-seeker to a worker is only  $H_N = 0.91$  units. The reason is that spending on advertising—the observable flow of customer-recruiting effort—is so much higher than is spending on worker recruitment. A spending flow of one unit on advertising generates  $\gamma/(1 + \theta_S) = 0.110$  new customers, so it costs  $1/0.110 = 9.1$  units to land one new customer. To sustain one customer, it takes a flow of  $\sigma_B = 0.55$  percent per week, so the flow advertising cost of sustaining a customer is  $9.1 \times 0.0055 = 0.050$  units per week. The corresponding logic for a worker is: A spending flow of  $k = Qk$  units of recruiting cost generates  $1/(1 + \theta_U) = 0.76$  hires, so it costs  $0.69/0.76 = 0.91$  units to land one new worker (this figure is somewhat below the figure of 14 percent of a quarter's pay cited in Hall and Milgrom (forthcoming)). To sustain one worker, it takes a flow of  $\sigma_N = 1.4$  percent per week, so the flow cost of sustaining a worker is  $0.91 \times 0.014 = 0.013$  units per week. Resources put into maintaining the customer base are four times higher, per customer, than are resources put into maintaining the work force, per worker. Because there are more than four times as many workers as customers, overall resources put into the two efforts are about the same.

The difference between the capital values of a customer relationship and an employment relationship is even stronger on the side of the firms. The value of a customer relationship

to a merchant is  $F_B = 9.1$  units while the value of an employment relation to a producer is  $F_N = 0.91$  units—because of the zero-profit conditions, these are the same as the costs of recruiting derived above. The difference arises from my assumption that the retail surplus is split 1387 between merchants and shoppers while the employment surplus is divided evenly.

In the retail market, searchers have a weekly probability of  $\phi_S = 16$  percent of finding a merchant, while in the labor market the weekly job-finding rate is  $\phi_U = 24$  percent per week. The lower success rate in the retail market is directly related to the higher gain from success. The gain is the avoided period of uncompensated search upon finding a merchant. The expected duration of that period of search is  $1/\phi_S = 6.3$  weeks.

## 5.4 The class of models

The models in the class of replicating models have non-negative values of the market-tightness variable  $\theta_S$ . These models cannot reach either of the extreme values of the consumer retail surplus share  $\beta_S$ , but only those in the interval from 0.03 to 0.80. Thus the class includes neither the Diamond paradox,  $\beta_S = 0$ , nor the opposite case of  $\beta_S = 1$  where merchants receive none of the surplus, but comes close to the paradox case.

Figure 1 shows how the allocation of time within the household differs among members of the class of models that replicate known features of the economy. At the left is the economy that is close to the Diamond paradox, where consumers receive only 3 percent of the retail surplus. Retailers are easy to find, so the household allocates essentially all of the members involved in obtaining goods to the buying role rather than the searching role. The fraction in one role or the other is fixed among all models in the class by the known fraction of those in the labor force who are unemployed and the known ratio of shoppers to workers.

The only difference in the figure when consumers receive a high fraction of the surplus is that merchants are hard to find, so most of the fixed fraction of household members involved in shopping are searching, leaving the buying to a small fraction. The model hypothesizes a high value for the unknown parameter  $\alpha$ , the volume of goods that one buyer purchases each week.

A number of key endogenous variables are essentially invariant in the class. First and foremost is the retail price—contrary to the intuition of Diamond’s analysis, a small consumer share of the retail surplus does not lead to a high price, because shoppers find it easier to wait for the next buying opportunity to appear by almost exactly enough to yield the same

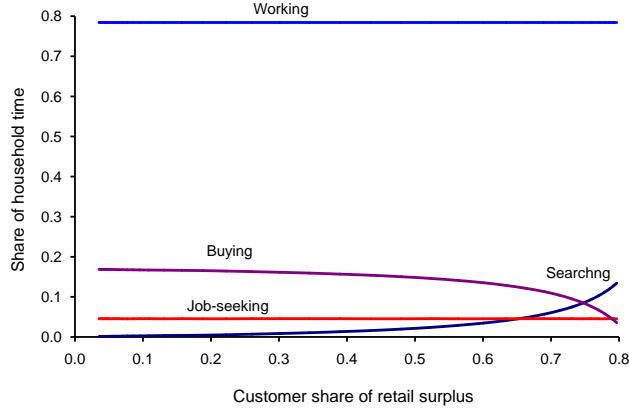


Figure 1: Time Allocation as a Function of the Surplus Share  $\beta_S$ , in the Class of Replicating Models

equilibrium price. The retail price  $p$  is the same in all members of the class. In the situation considered by Diamond, a low consumer retail fraction of the surplus corresponds to a high retail price, but here the price is independent of the seller's market power, as measured by  $\beta_S$ . The reason that the price-cost margin  $p - y$  is the same is that the model hypothesizes a high value for the unknown tightness  $\theta_S$  and thus for the unknown merchant-finding rate,  $\phi_S$ . Consumers are not at a disadvantage in the economy where they receive a low fraction of the retail surplus because their ability to locate another competing merchant is enough higher to raise their reservation price. Second, because the allocation of time to the labor market is taken from the time-use survey and thus is the same for all models in the class, the allocation of time to shopping is the same in all models in the class. Search time is higher and buying time is lower when the consumer share of the retail surplus is higher, but the sum is invariant. The wage,  $w$ , the household participation value,  $H_S = H_U$ , the increment from finding a job,  $H_N$ , the shadow value of goods at home,  $\tilde{p}$ , and the producing firm's value of an employment relationship,  $H_N$ , are essentially invariant as well.

The only parameters that differ by more than a tiny amount within the class of models are, first,  $\gamma$ , the efficiency parameter for the matching function in the retail market, which is a negatively sloped function of  $\beta_S$ , and, second,  $\alpha$ , the volume of goods that one buyer acquires each week, which is positively sloped. The high matching efficiency in markets with low retail consumer surplus shares offsets the high price consumers would otherwise face, as in Diamond's analysis. The high buying efficiency in markets where merchants are hard to find offsets the larger fraction of household effort devoted to search in models where consumers receive a high fraction of the surplus.

The endogenous variables that differ more than a tiny amount among models in the class are, first, the value gained by converting a shopper into a buyer,  $H_B$ , which is an increasing function of  $\beta_S$ , the tightness of the retail market,  $\theta_S$ , which is decreasing, the merchant-finding rate,  $\phi_S$ , which is also decreasing, the fraction of household members who are searchers, which is increasing, and the fraction who are buyers, which is decreasing.

## 6 Equilibrium Price and Wage

In this section I examine the stationary equilibria for different values of the prices, using the values of the parameters in the base case.

### 6.1 Stationary equilibrium

Given a stationary price  $p$ , wage  $w$ , and intermediate product price  $y$ , a stationary equilibrium of the model is a vector

$$[H_B, H_S, H_U, H_N, \tilde{p}, F_B, F_N, \theta_S, \theta_U, \phi_S, \phi_U, s, b, u, n, \beta_S, \beta_U] \quad (28)$$

satisfying equations (5) through (9), (15), (20), (16), (21), (13), (18), (1) through (3), (17), and (22).

### 6.2 The stationary equilibrium set

Here I derive the set of stationary equilibrium prices of the economy—the prices for which the model can be in equilibrium, with positive amounts traded voluntarily by all agents.

I consider the three prices  $p$  (retail price paid to merchants),  $y$  (wholesale price of output), and  $w$  (compensation to a worker). The fourth price,  $\tilde{p}$ , is a function of the other three, so I do not include it explicitly. The prices have the usual homogeneity property that if  $(p, y, w)$  is an equilibrium, so is  $(\lambda p, \lambda y, \lambda w)$  for any positive  $\lambda$ . Accordingly, I normalize the equilibrium prices in the form  $(p/y, w/y)$ .

Figure 2 shows the stationary equilibrium set for the economy. The left side shows the entire set and the right side magnifies the relevant portion, containing the lowest prices and the highest wages. Household equilibrium requires that the  $H$  values be non-negative. At the boundary where they are all zero,  $w = zp$ —the wage equals only the value of time,  $zp$ . If the wage is at least this high, the shadow value  $\tilde{p}$  can take on a value such that  $\tilde{p} \geq p$  and  $w \geq z\tilde{p}$ , in which case equations (5) through (8) imply that  $H_B \geq H_S$  and  $H_N \geq H_U$ .



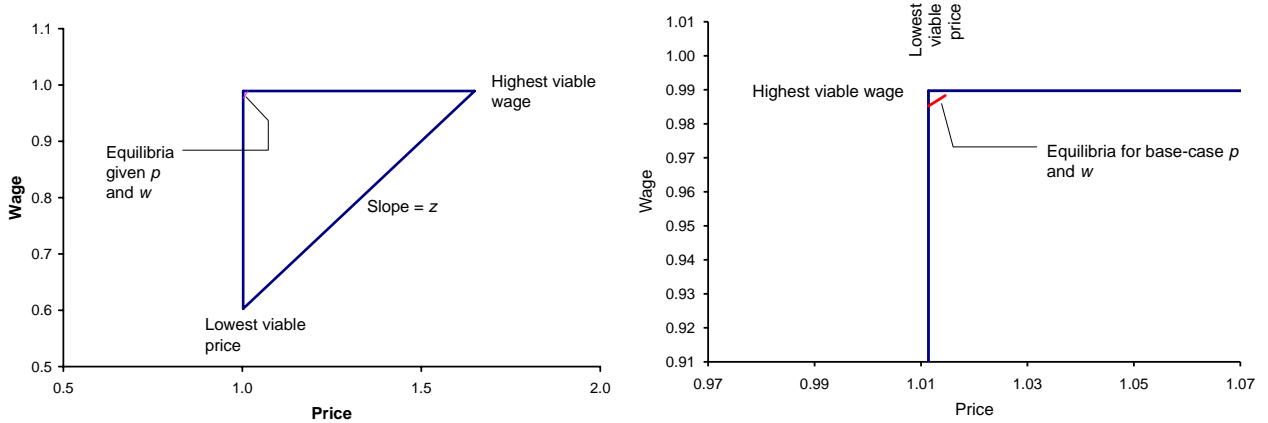


Figure 2: Static Equilibria

The household is engaging in voluntary trade in the sense that having a relationship with a supplier or having a job is at least as valuable as not having the relationship or job. Below the slanting line, the real wage  $w/p$  is too low to support voluntary trade.

The left edge of the equilibrium set is determined by the seller's willingness to engage in sales to customers. Further to the left, the price is too low to induce supply. At the critical price, the customer-finding rate,  $\frac{\gamma}{1+\theta_S}$ , is at its maximal value of  $\gamma$ . Solving the stationary versions of equations (15) and (16), I find

$$\frac{p}{y} = 1 + \frac{r + \sigma_B}{\alpha\gamma}. \quad (29)$$

Stated as a ratio to the intermediate product cost,  $y$ , the price needs to be at least one plus the annuity cost of forming the customer relationship.

The top edge of the set is determined by the producer's willingness to hire workers. Above the horizontal line in Figure 2, the wage is too high to permit hiring. The critical wage, where the recruiting rate is one, is, from equations (20) and (21),

$$\frac{w}{y} = \eta - (r + \sigma_N)k. \quad (30)$$

This is the worker's productivity  $\eta$  less the annuity cost of hiring.

This establishes

**Proposition 1** *The stationary equilibrium set is the triangle,*

$$z \leq \frac{w}{y} \leq \eta - (r + \sigma_N)k \quad (31)$$

$$1 + \frac{r + \sigma_B}{\alpha\gamma} \leq \frac{p}{y} \leq \frac{w}{yz}. \quad (32)$$

### 6.3 Equilibrium set for the real wage

Figure 2 and Proposition 1 places bounds on the real wage:

**Proposition 2** *The equilibrium set for the real wage is:*

$$z \leq \frac{w}{p} \leq \frac{\eta - (r + \sigma_N)k}{1 + \frac{r + \sigma_B}{\alpha\gamma}} \quad (33)$$

If the real wage is too high, there is no value of the intermediate product price  $y$  under which both producers and sellers are viable—the household is receiving an untenably high share of the benefits of production and distribution. If the real wage is too low, households are unwilling to participate in the market. As I noted earlier, the relevant bound is the upper one, because households capture most of the value of production as Ricardian rents.

## 7 The Equilibrium Set for a Given Wage and Intermediate Product Price

Here I explore the equilibria that lie on a horizontal line in Figure 2. These equilibria have different values of the retail price  $p$ , with  $y = 1$  and the wage  $w$  at its base-case value. At the left end of the line, the price is at the minimum value that can sustain retail selling, given the intermediate product price. I call this the *Soviet* region, as the low retail price results in an equilibrium similar to the account in Smith (1984) of ordinary life in Soviet Russia. At the right end of the line, the price hits the point where the real wage is at the minimum level to induce household participation in the market economy. I call this the *monopoly capitalism* region. The Diamond paradox occurs at the right end of the line, where retailers take all of the surplus from retail transactions.

Figure 3 shows the allocation of time among family members as a function of the retail price  $p$ . Notice the highly nonlinear horizontal axis, which is needed to bring out the differences in allocations from tiny price differences on the left or Soviet side of the figure. For a retail price that gives retailers only a tiny incentive to attract customers, the family



Figure 3: Allocation of Time as a Function of  $p$

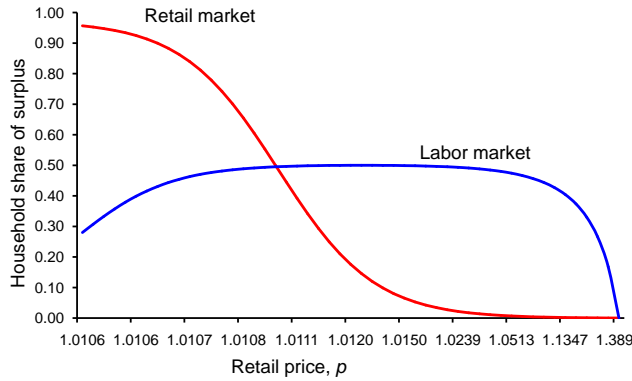


Figure 4: Division of Retail and Labor-Market Surpluses as Functions of  $p$

allocates more than half its total effort to trying to find merchants. Most remaining time is spent working and the rest buying. The economy at this point suffers from the reverse of the Diamond paradox—essentially all of the retail surplus goes to the customer, leaving too little for retailers to cover the cost of attracting customers. Retailers are almost impossible to find. Figure 4 shows how division of the surplus as a function of the retail price.

Figure 3 shows that a slightly higher retail price, such as base-case value of 1.0126, rather than the 1.0106 of the Soviet economy, results in dramatically better performance of the economy. Time spent searching is small and the productive activities of buying, seeking work, and working are correspondingly higher.

The allocation of time is not terribly different under monopoly capitalism on the right side. Time spent buying is lower because the quantity of goods purchased is lower at the higher price and the same wage. Time spent working rises to fill the gap. Retailers are extremely easy to find and time spent searching for them is tiny. Figure 5 shows that

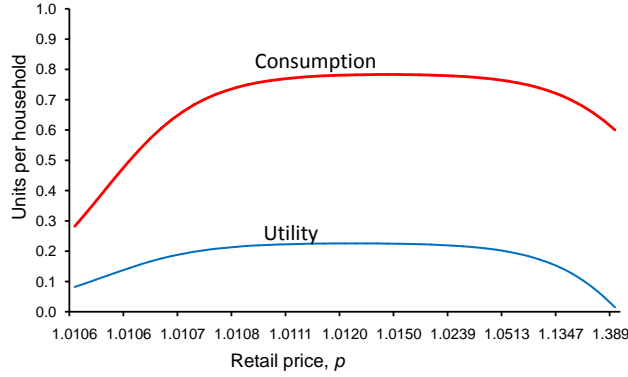


Figure 5: Family Consumption and Utility as Functions of  $p$

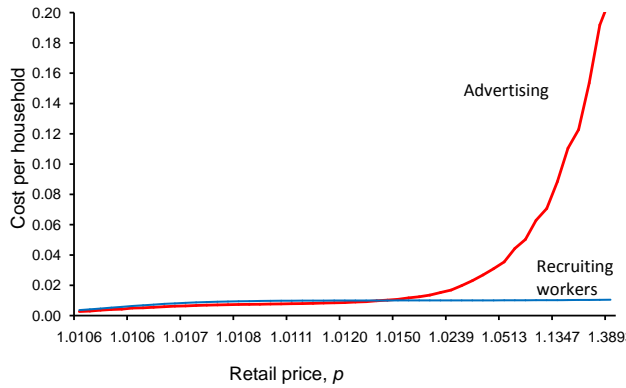


Figure 6: Advertising and Worker Recruiting Costs as Functions of  $p$

consumption is low on the Soviet side because families spend so much time shopping and is also low on the monopoly capitalism side because the real wage is low. Consumption is highest at a retail price near the base-case value of 1.0126. The figure also shows utility  $c - zn$ , which is zero at the Soviet extreme where all family members are searchers and at the monopoly capitalism extreme where the real wage is  $z$ .

The wedge between the retail price  $p$  and the wage  $w$  finances advertising by retailers and labor recruiting cost by producers. Figure 6 shows these costs per household as a function of the retail price. Labor recruiting (vacancy) cost is at about the same low level for all values of  $p$ , because the producer margin  $y - w$  is held constant in the figure. Advertising is zero on the Soviet side, is low at the retail price of the base case in the middle of the figure, and rises to a high level on the monopoly capitalism side. Monopoly capitalism returns no monopoly profit to the capitalists, but rather dissipates a high retail gross margin in costly efforts to attract customers.

Figure 7 shows the three distinct household role values as functions of the retail price.

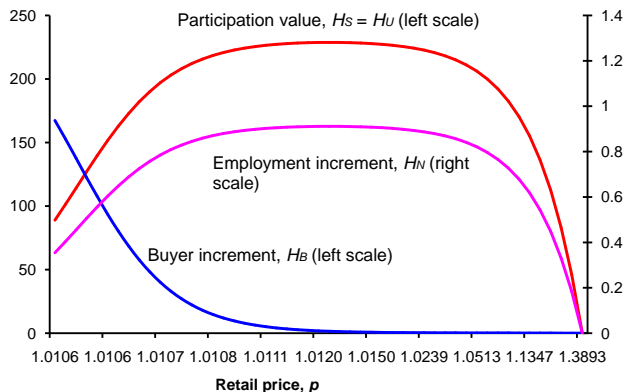


Figure 7: Household Role Values as Functions of  $p$

The participation value  $H_S = H_U$  is zero in the pure Soviet economy where merchants are impossible to find and zero in the monopoly capitalism economy, where goods are priced so high that merchants appropriate all of the consumer surplus. The increment to household value when a buyer succeeds in finding a merchant,  $H_B$ , is extremely high in the Soviet economy, is at a low level in the base case, and is zero in the monopoly-capitalism economy. The increment to household value when a job-seeker finds a job,  $H_N$ , is always modest and is zero in the two extreme economies.

## 8 The Equilibrium Set for Given Price and Wage

If the values of  $p$  and  $w$  are given but  $y$  is flexible, the equilibria of the economy lie along the part of the line  $\{(p/y, w/y) | y \geq 0\}$  that is inside the equilibrium set of Proposition 1. This line is shown as the slanting dashed line in Figure 2. The line is interesting because it is the set of policy alternatives in an economy where policy cannot change  $p$  or  $w$ , the sticky price and wage, but can affect the flexible, competitive intermediate product price  $y$ . I discuss the policy implications shortly.

Along the flexible- $y$ , sticky- $p$ -and- $w$  line, the product market changes in ways similar to those discussed in the earlier section with fixed  $y$  and  $w$ . The new element here is the similar but opposite changes in the labor market. Tightnesses of the two markets move in opposite directions along the line. At the lower left end of the line, where it touches the vertical line, the product market is in the Soviet condition. The labor market is in the equivalent of monopoly capitalism, where employers appropriate all of the employment surplus. At the upper right, the product market suffers from monopoly capitalism and the labor market

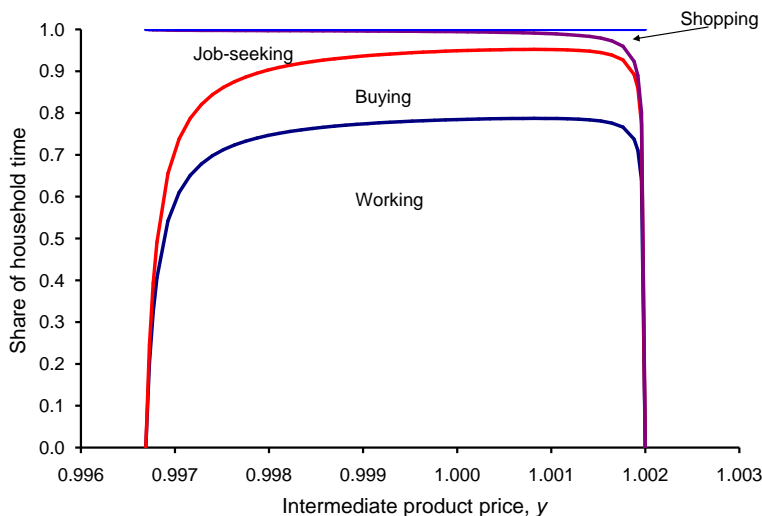


Figure 8: Allocations of Household Time as Functions of the Intermediate-Product Price, for Given Price and Wage

from the Soviet condition, where employers have no interest in hiring workers because their price-cost margin is so thin.

Figure 8 shows the allocations of household time as a function of  $y$  along the line defined by the base-case values of  $p$  and  $w$ . The left side of the figure corresponds to the upper-right end of the dashed line in Figure 1—it is the point where the price and wage in relation to the intermediate price,  $p/y$  and  $w/y$ , are at their maximum feasible levels. The labor market is slack, with a low job-finding rate. Households devote a large fraction of time to job-seeking. As  $y$  rises and  $w/y$  falls in proportion, the fraction working and the fraction buying rise rapidly. Consumption rises in proportion to the buying fraction  $b$ . This process continues as  $y$  rises further until slightly below the maximum feasible value, where the economy suddenly collapses because sellers lose the incentive to find customers. At the left, the retail market is in the state of monopoly capitalism, with heavy advertising spending and almost no household members searching. As  $y$  rises, the price  $p/y$  falls and the retail margin falls. Merchants put less effort into attracting customers, the success rate of searchers falls, and the fraction of household members searching rises. Ultimately the retail market enters the Soviet condition. The fraction of time that households spend searching for available sellers rises dramatically and eventually consumes all available time, as the product market ceases functioning. Output collapses for lack of work effort.

The pronounced asymmetry in Figures 8 arises from an asymmetry in the model. People place a substantial disamenity  $z$  on working relative to searching, buying, and job-seeking.

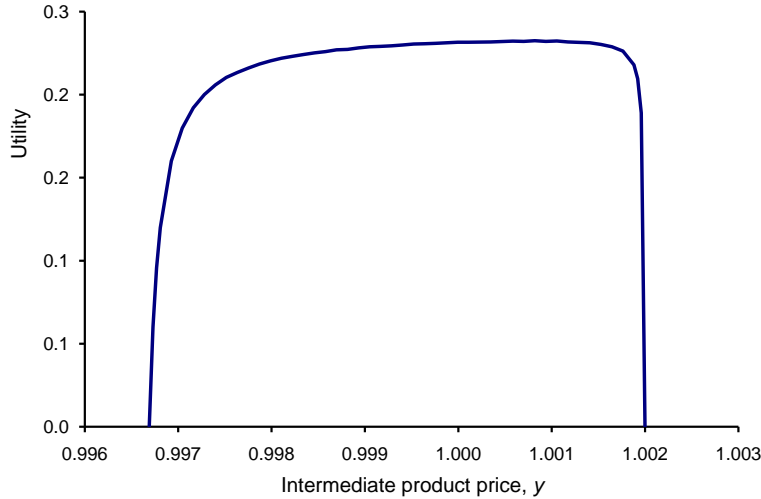


Figure 9: Utility as a Function of the Intermediate-Product Price, for Given Price and Wage

The figure would be symmetric if the disamenity also applied to buying and if the base case had the same tightness in the product and labor markets.

Figure 9 shows the household’s flow utility as a function of  $y$ . Utility is low on the left side because job-seeking displaces work, so the production of goods for consumption is low. Utility is low on the right side, in the Soviet economy, because searching displaces work in the Soviet economy. Utility reaches its maximum near the value of  $y = 1$  assumed in the base case.

## 9 Treating $y$ as a Policy Instrument

Next I consider an economy where the wage  $w$  and price  $p$  are given, but the government has the power to set the intermediate product price  $y$ . I take this as a stylized version of an economy where the central bank influences flexible competitive prices (here,  $y$ ) immediately but where the retail price and wage are state variables unaffected by central-bank policy except with a lag. As customary in modern price-level economics, I do not describe how the bank’s manipulation of its immediate instruments—its portfolio and the money supply— affect flexible prices. Rather, I assume that the bank uses its instruments as needed to shift  $y$  to its desired level.

Figures 8 and 9 now take the role of a policy menu. Policy can set a low value of  $y$  close to  $w$  and achieve a slack labor market with low utility or a high value of  $y$  close to  $p$  and a Soviet retail market and low utility. Or policy can pick an intermediate value of  $y$

and achieve the maximum value of utility, where both the retail and labor markets function normally. The notion that policy has a choice about conditions in the labor market—specifically unemployment—has been present in macroeconomic theory since Keynes. The notion that policy has a choice, in the sense of a tradeoff, about conditions in product markets has escaped formal analysis in the past, I believe.

Next I consider economies with productivity  $\eta$  at levels that depart from the value assigned in the base case of 1. Recall that  $\eta$  enters the model in only one place—it is a determinant of the production margin,  $y - \eta w$ . The effects of  $\eta$  on the economy depend on the policy rule for  $y$ . I will consider a simple example. The bank sets  $y$  to keep the two key margins in the economy, the retail and production margins, at the same ratio as in the base case:

$$p - y = \mu(\eta y - w), \tag{34}$$

where  $\mu$  is the ratio of  $p - y$  to  $\eta y - w$  in the base case. The policy rule is

$$y = \frac{p + \mu w}{1 + \mu \eta}. \tag{35}$$

This rule places  $y$  midway between  $w$  and  $p$  in the base case, where  $\eta = 1$ , and raises  $y$  if productivity is below 1 and lowers  $y$  if above. If  $y$  did not change with productivity, a decline in productivity would leave the retail margin unchanged and thus conditions in the retail market not much affected, but would substantially raise unemployment because of the decline in the production margin. The policy shares the effects of changes in productivity between the two markets.

The common value of the retail and production margins under the policy rule is

$$\mu \frac{\eta p - w}{1 + \mu \eta}. \tag{36}$$

Around the base-case value  $\eta = 1$ ,  $\mu = 0.93$ , and  $\eta p - w = 0.026$ , the derivative of the margin with respect to  $\eta$  is a little less than 0.5. If  $\eta$  falls by one percent, both margins fall by about half a percent. Both markets become slacker. Advertising falls, consumers are less able to find merchants, and search time rises. In the labor market, recruiting activity falls, job-seekers are less able to find openings, and unemployment rises. A recession occurs.

Figure 10 shows how the allocation of time within the household varies with productivity, with the retail price  $p$  and the wage  $w$  held fixed and the policy rule in effect for  $y$ . Time spent searching for merchants is quite sensitive to productivity—it varies from 0.99



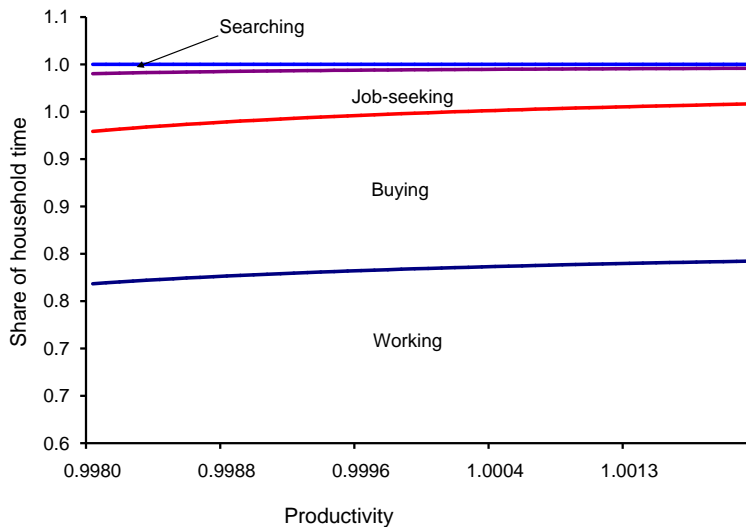


Figure 10: Household Time Allocations as Functions of Productivity

percent of total household time when productivity is 0.2 percent below normal to 0.42 percent when productivity is 0.2 percent above normal. Time spent looking for work varies from 6.1 percent at low productivity to 3.8 percent at high productivity. The sensitivity of unemployment is comparable to the findings of earlier work in the MP framework with a fixed wage. Buying goods,  $b$ , and working,  $n$ , account for the remaining household time. These move in proportion because the household satisfies the budget constraint  $\alpha pb = wn$ .

## 9.1 Advertising and recruiting in the U.S. economy

One cannot check the main implication of the model for household time allocation, the sensitivity of time spent searching for merchants, because no survey measures search separately from buying. As Figure 10 shows, shopping time,  $s + b$ , is not sensitive to productivity. This non-response is confirmed in the ATUS. The first year of the ATUS, 2003, had fairly high unemployment, 6.0 percent, while the most recently available year, 2006, had fairly low unemployment, 4.6 percent. In both years, time spent shopping was identical at 0.81 hours per adult per day. The agreement of model and data is not a coincidence—it is the way I chose the base-case retail customer surplus share, as I will explain shortly.

On the other hand, the model has an unambiguous, verifiable implication about the sensitivity of advertising spending to productivity, as shown in Figure 11. Spending is  $s\theta_S/\gamma$ . Over the tight range of productivity variation shown in the figure, advertising spending rises from 0.0074 units per person per week to 0.0090 units. Recall that a unit is a week of

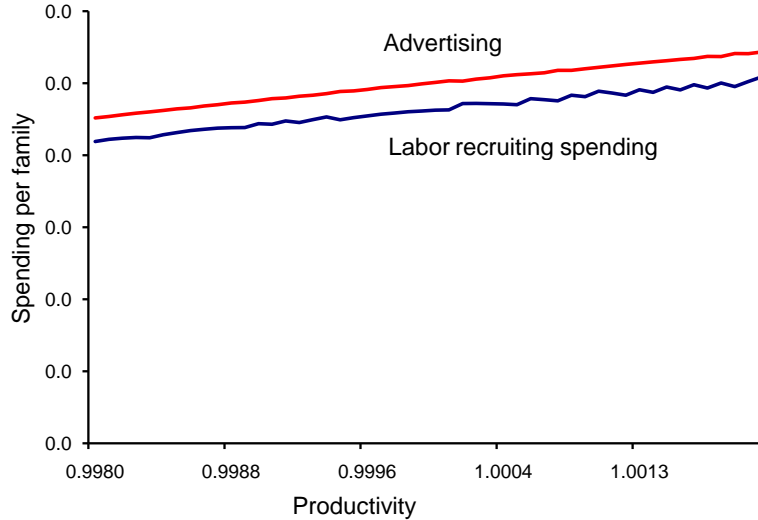


Figure 11: Advertising Spending as a Function of Productivity

productivity of a worker.

The figure also shows labor-recruiting spending,  $u\theta_U k$ . The model implies that advertising and recruiting spending should change in proportion as productivity changes, in the setting just discussed where  $p$  and  $w$  are fixed and  $y$  follows the policy rule of preserving the ratio of the two margins. The ratio of advertising to recruiting spending is

$$\frac{\sigma_B}{\sigma_N} \frac{w}{\alpha p} \frac{r + \sigma_N}{r + \sigma_B} \mu, \quad (37)$$

a constant.

Figure 12 shows television advertising and labor recruiting spending for the U.S. since 1972. The television data are from Universal-McCann and are deflated by the GDP deflator, detrended by regressing on a quadratic in time, and stated as an index that is 100 in the first year. The labor recruiting figures are based on the model together with time-series data on unemployment from the BLS. The model implies that spending per household is  $k\theta_U u$ , the product of the vacancy cost  $k$ , vacancies per unemployed person,  $\theta_U$ , and the number of unemployed members per household,  $u$ . I assume that the fraction of household members in the labor force,  $f = u + n$ , is constant at the base-case value of 83 percent. Then equations (3) and (18) imply that vacancy cost per household is

$$\frac{\sigma_N(1 - \tilde{u})}{(1 + \sigma_N)\tilde{u} - \sigma_N} k f \tilde{u}. \quad (38)$$

Here  $\tilde{u}$  is the unemployment rate on the BLS basis as a fraction of the labor force ( $\tilde{u} = u/f$ ). I convert the result to an index that is 100 in the first year.

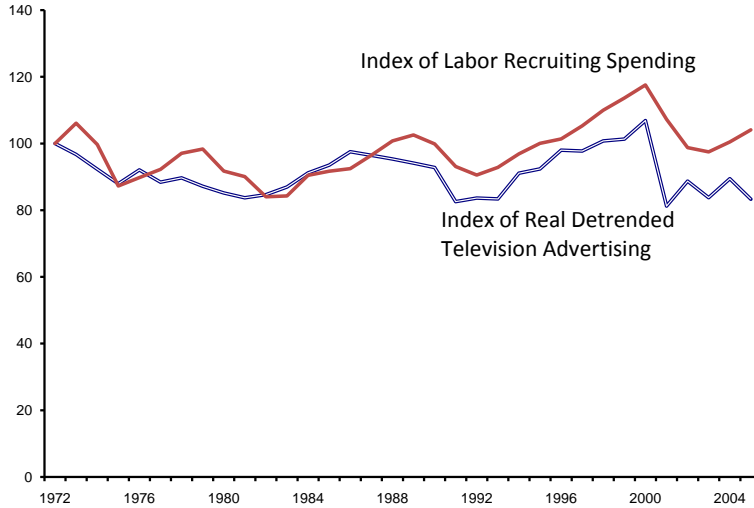


Figure 12: Real Detrended Television Advertising Spending and Inferred Labor Recruiting Spending, 1972-2005

Both series track the business cycle and each other fairly closely, with about the same volatility in advertising as in recruiting spending. Both are vastly more volatile than most categories of spending. The results confirm the prediction of the model when coupled with sticky prices and wages and the policy rule under which the Fed keeps flexible prices at an intermediate point that causes the two margins to move in proportion over the cycle.

I am not aware of any other research on the cyclical behavior of advertising.

## 9.2 Measuring the retail consumer surplus share from the movements of time allocations

In the ATUS, as I noted earlier, time spent shopping (searching and buying) did not change during the expansion from 2003 to 2006, at the same time that the unemployment rate was falling substantially. This evidence supports a derivative of  $s + b$  with respect to  $\tilde{u}$  of about zero. Hall (2007) provides supporting evidence—the labor-force participation rate falls by about 0.2 percentage points for each percentage point increase in unemployment. The model would interpret this finding as a derivative of  $-0.2$ . Figure 13 shows the relation between the derivative and the consumer surplus share. If the share were much higher than 0.15, slackening of the labor market as indicated by a rise in unemployment would cause a large slackening of the retail market, in which search  $s$  rose much more than buying  $b$  fell. The finding of stability in shopping time  $s + b$  fits the model only if the consumer retail surplus share is small. Thus I take the share to be 0.13 in the base case, corresponding to the ATUS

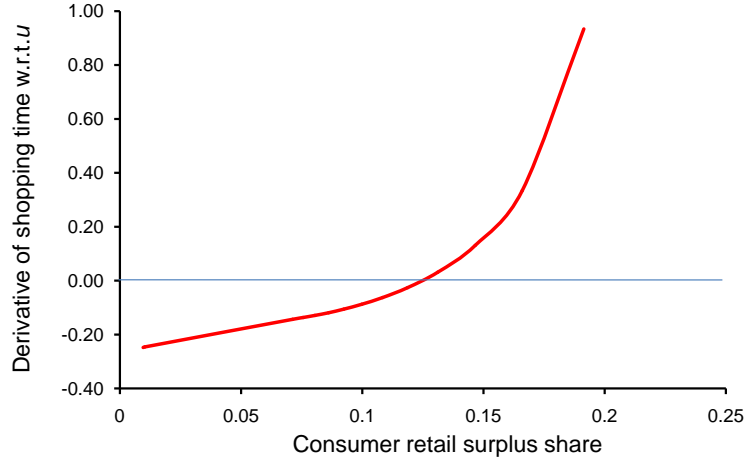


Figure 13: Derivative of  $s+b$  with Respect to  $\tilde{u}$  as a Function of the Consumer Retail Surplus Share

finding of no change during the expansion.

## 10 Dynamic Model

Any complete dynamic model would need to take a stand on the central issue, sidestepped in this paper, of how prices and wages are determined. Without taking that major step, however, it is interesting to consider the dynamics of allocations conditional on the values of  $p$  and  $w$  and the value of  $y$  dictated by the policy rule.

The Appendix derives the full dynamic model. Conditional upon  $p$ ,  $w$ , and  $y$ , the behavior of market tightness depends on only a small part of the total model:

$$F_{B,t} = \alpha(p_t - y_t) + \frac{1}{1+r} (1 - \sigma_B) F_{B,t+1}. \quad (39)$$

$$\frac{\gamma}{1 + \theta_{S,t}} \frac{1}{1+r} F_{B,t} = y_t. \quad (40)$$

$$F_{N,t} = y_t \eta_t - w_t + \frac{1}{1+r} (1 - \sigma_N) F_{N,t+1}. \quad (41)$$

$$\frac{1}{1 + \theta_{N,t}} \frac{1}{1+r} F_{N,t} = y_t k. \quad (42)$$

To find the values of the two tightness variables, form  $F_{B,t}$  and  $F_{N,t}$  as the present values of the future margins  $\alpha(p_t - y_t)$  and  $y_t \eta_t - w_t$  and then solve the zero-profit conditions of equations (40) and (42) for  $\theta_S$  and  $\theta_U$ .

The numbers of buyers  $b_t$  and workers  $n_t$  are state variables of the model, as discussed in the Appendix. From initial conditions for the two variables, their laws of motion, equations (45) and (46), the allocation constraint, equation (1), and the budget constraint, equation (44), applied one period ahead, one can solve forward to find the paths of the four role allocation variables. These four equations comprise a linear system in the unknowns  $s_t$ ,  $u_t$ ,  $b_{t+1}$ , and  $n_{t+1}$ .

I study a productivity shock that follows the path,

$$\eta = 1 - \delta e^{-\rho t} \tag{43}$$

I take  $\rho = 0.005$  at a weekly frequency, so that the shock is moderately persistent,  $\delta = 0.0030$  for a small shock, and  $\delta = 0.0043$  for a large shock.

Figure 14 shows the responses of the allocation of family members' time to the two shocks. In the upper panel, where the shock is small, the labor market allocations respond in about the same way as in the MP model with a fixed wage. Job-seeking rises quite rapidly at first, then gradually declines as the adverse productivity shock wears off. Except for the first few months, the path of unemployment tracks productivity. The MP model makes little contribution to the dynamic path; the path of the labor market is close to what it would be under a static model. The path of retail searching is similar, but with higher amplitude. The responses of time allocations and other variables in the dynamic model, apart from the initial lag, is quite similar to the responses shown in Figure 10.

In the lower panel of the figure, the responses are much larger. The model is highly nonlinear, for the reason shown in Figure 8. As the retail margin declines, the retail search role absorbs large numbers of people, whereas in normal times it absorbs hardly any. The larger shock pushes the economy closer to the Soviet region where hunting for available goods is an important part of the time budget. The dynamics of the model become quite different when retail search takes people out of employment. During the period when employment is declining, job-seeking does not rise, even though the labor market has become slack in the sense that the job-finding rate is low. The depletion of employment causes the household to assign newly idle members to retail search rather than to job-seeking. In the labor market, the result is an extension of the time when unemployment rises. Unlike the MP model, this model has a propagation mechanism; its responses have different dynamics from the productivity driving force, if the impulse pushes the retail market close to the Soviet point.

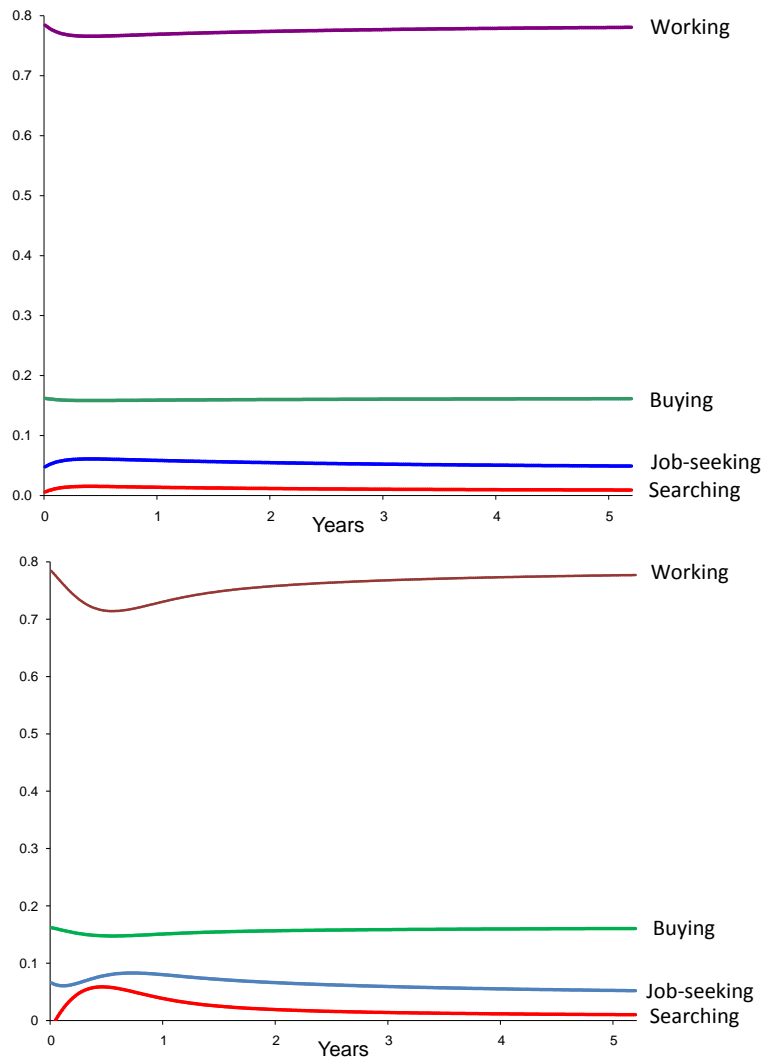


Figure 14: Responses to Productivity Shock

## 11 Concluding Remarks

Customer-merchant relationships have an important role in one of the most fundamental allocations in general equilibrium, that among the various activities that occupy household members. Shopping is an important use of time. Because advertising spending is a significant fraction of total income and because advertising is generally intended to capture the loyalty of customers, sellers have large surpluses associated with their customer bases. Within a search-and-matching model, I infer the amount of time that people spend searching for appropriate sources of goods and services. I find that the payoff to search must be small for customers relative to sellers, else fluctuations in search time would be large enough to see in the time-use survey. Instead, time spent searching and buying goods and services does not fluctuate along with unemployment. I infer that customers receive only about 13 percent of the surplus from their relations with sellers. This finding supports the notion that retail prices are determined by a mechanism not too different from a posted price, where a seller would extract all of the surplus from homogeneous customers. Although the economy is not at the point of the Diamond paradox, it is not too far away from that point.

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## A Appendix: Full Dynamic Model

A searcher has a probability  $\phi_{S,t}$  of finding a store and forming a buying relationship each period and a job-seeker has a probability  $\phi_{U,t}$  of finding a job each period. A buyer has a fixed lower bound  $\sigma_B$  on the probability of terminating the relationship with a seller and a worker has a fixed lower bound  $\sigma_N$  on the probability of terminating the current job. The household may choose higher values than these bounds to shift family members from one role to another faster than occurs from natural turnover. I let  $x_{B,t} \geq 0$  and  $x_{N,t} \geq 0$  be the extra members liberated from their duties as buyers or workers.

The budget constraint is:

$$p_t c_t \leq w_t n_t. \quad (44)$$

The laws of motion of the number of buyers,  $b_t$ , and the number of workers,  $n_t$ , are

$$b_{t+1} = (1 - \sigma_B)(b_t - x_{B,t}) + \phi_{S,t} s_t \quad (45)$$

and

$$n_{t+1} = (1 - \sigma_N)(n_t - x_{N,t}) + \phi_{U,t} u_t. \quad (46)$$

In period  $t$ , the household assigns newly idle members to search for retail relationships and jobs in the proportion that yields the ratio of spending to earnings mandated by equation (44) in period  $t+1$ , if possible. I describe the allocations of household members in a dynamic system with two state variables, the number of members serving in the buying function,  $b_t$ , and the number employed,  $n_t$ . Consumption is

$$c_t = \alpha b_t. \quad (47)$$

The allocations satisfy

$$s_t + b_t + u_t + n_t \leq 1. \quad (48)$$

The household solves the dynamic program,

$$V_t(b_t, n_t) = \max_{s_t, u_t, x_{B,t}, x_{N,t}} \left( c_t - z n_t + \frac{1}{1+r} V(b_{t+1}, n_{t+1}) \right). \quad (49)$$

subject to equations (44) through (48).

The household's problem is simple if variations in the real wage  $w/p$  are small enough so that the family can satisfy the budget constraint, equation (47), with equality while avoiding sacrificing the value of ongoing buying relationships and jobs—that is, with  $x_{B,t} = 0$  and

$x_{N,t} = 0$ . I assume that real wage variation is sufficiently mild to meet this condition. In that case, the budget constraint, applied in  $t$  and  $t + 1$ , the laws of motion, equations (45), (46), and the allocation constraint, equation (48), also taken as an equality, make up a linear system that can be solved for  $b_{t+1}$ ,  $n_t$ ,  $n_{t+1}$ ,  $s_t$ , and  $u_t$ , given only  $b_t$ . Thus the model in that case has only a single state variable, the number of buyers,  $b_t$ , with a linear law of motion and time-varying coefficients.

To affiliate values with the four roles that family members may play, it is more instructive to return to treating both the number of buyers,  $b_t$ , and the number of workers,  $n_t$ , as state variables and to associate a Lagrangian multiplier,  $1/\tilde{p}_t$ , with the budget constraint, equation (44);  $\tilde{p}_t$  is interpreted as the shadow price of a unit of utility and, equivalently, the shadow price of a unit of consumption. The Bellman equation becomes

$$V(b_t, n_t) = \max_{s_t} \left( \alpha b_t - z n_t + \frac{w_t n_t - p_t c_t}{\tilde{p}_t} + \frac{1}{1+r} V_{t+1}(b_{t+1}, n_{t+1}) \right). \quad (50)$$

The value function is linear. It assigns value  $\tilde{H}_{S,t}$  to the fraction of the family available to search for stores or to seek jobs,  $\tilde{H}_{B,t}$  to the incremental value of a buying relationship, and  $\tilde{H}_{N,t}$  to the incremental value of a job:

$$V(b_t, n_t) = \tilde{H}_{S,t} + \tilde{H}_{B,t} b_t + \tilde{H}_{N,t} n_t \quad (51)$$

I substitute the laws of motion, equations (45) and (46) and the allocation constraint, equation (1), into the linear form of the Bellman equation, (51). The condition for an interior solution for the maximization over  $s_t$  is

$$\phi_{S,t} \tilde{H}_{B,t} = \phi_{U,t} \tilde{H}_{N,t}. \quad (52)$$

I define

$$H_{S,t} = \tilde{p}_t \tilde{H}_{S,t} \quad (53)$$

and similarly for the other role values, now stated in terms of the unit of value.

Then I equate the constants and the coefficients on  $b_t$  and  $n_t$  to find recursions for the household role valuations:

$$H_{S,t} = \frac{1}{1+r} (H_{S,t+1} + \phi_{S,t} H_{B,t+1}) \quad (54)$$

$$H_{B,t} = \alpha(\tilde{p}_t - p_t) + \frac{1}{1+r} (1 - \sigma_{B,t} - \phi_{S,t}) H_{B,t+1} \quad (55)$$

$$H_{U,t} = \frac{1}{1+r}(H_{U,t+1} + \phi_{U,t}H_{N,t+1}) \quad (56)$$

$$H_{N,t} = w_t - z\tilde{p}_t + \frac{1}{1+r}(1 - \sigma_{N,t} - \phi_{U,t})H_{N,t+1} \quad (57)$$

These values are the natural generalizations of the values assigned to job-seeking and employment in the M-P model. The flow  $\tilde{p}_t - p_t$  is the value added by the buyer, the difference between the shadow value of goods at home and the purchase price from the retailer. Equation (52) implies that

$$H_{S,t} = H_{U,t}. \quad (58)$$

The household equates the values of the two roles that it assigns to newly idled members.

Equations (54) through (58) comprise a system of five linear equations in the four role values and the shadow price  $\tilde{p}_t$ , given the future values.

The dynamic model also includes equations (16), (21), (13), (18),(17), and (22) with time subscripts and equations (39) through (42).

Thus a dynamic equilibrium comprises a vector time series

$$[H_{N,t}, \tilde{p}_t, F_{B,t}, F_{N,t}, \theta_{S,t}, \theta_{U,t}, \phi_{S,t}, \phi_{U,t}, s_t, b_t, u_t, n_t, \beta_{S,t}, \beta_{U,t}] \quad (59)$$

satisfying equations (54) through (58), (39) through (42), (13), (18), (45) through (48), (17), and (22).