# Interest-Setting and Central Bank Transparency

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**Abstract**

This paper extends the recent literature on central bank transparency by noting that most central banks currently publish their interest rates. As a consequence, central banks cannot hide all their information, they must reveal some of it as they set and publish the interest rate. We study the role of transparency in the presence of the distortionary common knowledge effect emphasized by Morris and Shin (2002). Publishing the interest rate implies that the central bank anyways becomes the source of such a distortion. The question, then, is whether additional information is desirable. The answer depends not just on the relative precision of central bank information but also on what is known of this precision. We find that the interest rate can be used by the central bank to offset the common knowledge effect. Transparency, which undermines the information content of the interest rate, tends to undermine its strategic use. We also find that uncertainty about private sector information precision leads to monetary policy mistakes that become the source of price volatility. Transparency helps reduce this volatility.

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# 1 Introduction

As central banks become increasingly transparent, the issue naturally arises of just how transparent they should be. Some central banks strive to reveal just about everything that is relevant; this is the case of the Reserve Bank of New Zealand, of the Bank of Norway and of Sweden's Riksbank. Others are more circumspect; they consider that there may be too much transparency.[1]

Likewise, the academic literature is divided about the welfare case for full transparency. Blinder (1998) argues that central banks should be as transparent as possible. As further elaborated by Woodford (2005), the case for transparency rests on the dominant role played by expectations of private agents when they make decisions on prices, spending and production. When the main channels of monetary policy operate through expected inflation, long-term interest rates, asset prices and exchange rates, central banks are most effective when the private sector fully understands their intentions.

The case for limiting transparency starts with the "constructive ambiguity" argument initially advanced by Cukierman and Meltzer (1986). The argument rests on two assumptions: 1) only unanticipated money matters (Kydland and Prescott, 1977) and 2) the central bank preferences are not precisely known by the public (Vickers, 1986). Under these combined assumptions, some degree of opacity enhances monetary policy effectiveness because a fully transparent central bank cannot create surprises.[2] These assumptions have become less appealing. The "only unanticipated money matter" view, convincingly criticized by McCallum (1995) and Blinder (1998), is further undermined by current central bank practice. Far from concealing their preferences, today's central banks clearly specify their objectives, as is the case with the increasingly popular inflation targeting strategy.

A different case for limited transparency has been developed in Amato, Morris and Shin (2002, 2003) and Morris and Shin (2002, 2005) which argue that central banks should not reveal all the information at their disposal.[3] M&S do not appeal to the assumptions of the constructive ambiguity literature. Their argument rests instead on three different assumptions: 1) the information available to both the central bank and the private sector is noisy; 2) the central bank's signals are seen by everyone in the private sector; and 3) private sector agent forecasts must not just be as precise as possible, they must be as close as possible to the consensus forecast (a case of strategic complementarity). The last assumption, which goes back to Keynes' celebrated beauty contest effect, is meant to capture the basic principle that it is relative prices that matter in competitive markets. An implication of the beauty contest assumption is that everyone knows that everyone else observes the same central bank signals. A consequence is the common knowledge effect: relative to private information, central bank signals receive undue attention in the sense that their impact will not just reflect their quality. It follows that when the quality of central bank signals is not good enough to overcome the weight that they receive, it may be desirable for the central bank to withhold releasing its information.

This influential result has been shown not to be robust. Svensson (2005) observes that, in practice, the quality of central bank signals is unlikely to be sufficiently poor to justify withholding information. Woodford (2005) observes that the result occurs because M&S use a welfare function that ignores the strategic complementarity that drives individual private preferences. This general observation is further developed in Hellwig (2005) and Roca (2006). In our setup, the M&S result is found to be robust to the choice of the

[1] An important reason for transparency is democratic accountability.

[2] For review of this literature, see Geraats (2002).

[3] These papers share a common model and differ only by their emphasis on particular issues. In what follows we refer to all of them when we quote Morris and Shin (2002), denoted as M&S.

welfare criterion.

The present paper revisits the common knowledge effect from a different angle, one that does not seem to have been examined so far. In the models of M&S and their critics, the central bank faces the choice of releasing either no information at all or all its information. We argue that it is unrealistic that a central bank can withhold *all* of its information. If only by setting the interest rate, the central bank is forced to reveal *some* information. The question then is not black-and-white - reveal or not reveal - but whether to reveal more than the interest rate. Because revealing the interest rare is not an option, any central bank anyhow faces a common knowledge effect, as the intense scrutiny of interest decisions readily confirms. Revealing more than the interest rate, therefore does not create a common knowledge effect, it merely adds another one. In second best fashion, it is no longer clear what is the welfare effect of more transparency.

The question now becomes: given that the central bank anyway releases some noisy information, the interest rate, should it also reveal all the detailed information that was used in its decision? The answer is that it is generally desirable for the central bank to release some information in addition to setting the interest rate. We find many cases where, in fact, it is desirable for the central bank to be fully transparent and to release all of its information.

Since the bank's preferences are assumed to be known, if only because the central bank's welfare objective is the same as that of the citizens, in the absence of time inconsistency the central bank has no interest in misrepresenting its information. In our static model, therefore, transparency means that the central bank truthfully reveals some or all of its information in addition to publishing the interest rate.

The reason why providing more information may raise welfare is quite intuitive. Any information provided by the central bank is a source of a welfare-reducing common knowledge effect. The interest rate is a choice parameter while the rest presumably is information about the economy or the bank's analysis - its "model". Thus, by suitably choosing the interest rate, the central bank may offset, partly at least, the common knowledge effect of the information that it releases, including the publication of the interest rate. Put differently, the interest rate becomes the antidote to the common knowledge effect. Transparency lowers welfare if it undermines the information content of the interest rate, and it raises welfare when the additionally provided information enhances the ability of the central bank to use the interest signal as an effective antidote.

In order to make our point as clearly as possible, we start from the same model as M&S. We accept its three basic assumptions - noisy information, common knowledge of central bank information publication and strategic complementarity among private agents - and we also use the same welfare criterion. We only add the short-term interest instrument whose role is not even macroeconomic, it just conveys some of the central bank information.

The next section presents our model, an extension of M&S to the case where there are two fundamentals. This extension is needed as we assume throughout that the central bank optimally sets and publishes the interest rate. Section 3 considers the case when the precision of the central bank and private sector information is perfectly known to both the central bank and the private sector. In this case, we find that it is never desirable for the central bank to be transparent because transparency fully undermines the signal content of the interest rate - there is no information left to communicate through the interest rate - and therefore the central bank's ability to use the interest rate as an antidote to the common knowledge effect.

Section 4 introduces one more degree of possible transparency by allowing for the precision of the private sector signals to be unknown to the central bank. Since the central bank must set the interest rate, and is assumed to do so by minimizing its expectation of

the private sector loss function, it must form a forecast of the private sector information precision. In this situation, the central bank may simply publish its chosen interest rate, or release its forecast of the private sector information precision, or be fully transparent and also release its information about the economy. We find that it is never desirable for the central bank to only publish the interest rate. The reason is that, mistaken private beliefs about the central bank's appraisal of private sector information precision become the source of a systematic bias as the private sector cannot appropriately interpret the interest rate decision of the central bank. Transparency, whether partial or full, erases this bias. Full information welfare-dominates partial information when it further allows the private sector to better interpret the central bank's interest rate decision.

Most of these results remain valid when private sector information precision is also unknown to the private sector itself, as is briefly indicated in Section 5. The last section concludes.

# 2 Central Banking with an Interest Rate Instrument

## 2.1 Monetary policy

Since we assume that the central bank publishes a signal by setting the interest rate, we need to extend the model of M&S to allow for more than one fundamental. If there were only one fundamental, the interest rate decision would be fully revealing, unless the central bank were intentionally introducing noise in its monetary policy decisions in order to manipulate private sector expectations, for example by creating surprises or by concealing its true preferences. We do not deal with these issues here as we assume that the central bank maximizes the social welfare, i.e. that its preferences are the same as those of the private sector and thus perfectly well known. To keep matters as simple as possible, we assume that there are two fundamentals that both the central bank and the private sector must forecast.[4]

As M&S, we consider the Lucas-Phelps island parable. There is one good produced by an infinite number of private producers $i$ spread over the unit interval. Each private producer lives on one island and has limited information on the situation in the archipelago. Producer $i$'s supply function is:

$$q_S^i = b(p^i - E^i(\overline{p}))$$

where $E^i$ is the expectation conditional on information available in island $i$ and $\overline{p} = \int_{i=0}^{1} p^i di$ is the aggregate price index. The demand in island $i$ is:

$$q_D^i = E^i(A\theta) - p_i$$

where $\theta = (\theta_1, \theta_2)'$ is a vector of two fundamentals and $A = (A_1, A_2)$ is a vector of structural parameters. Without any loss of generality, we set $(A_1, A_2) = (1, 1)$. This merely implies normalizing the two fundamentals. The market equilibrium condition yields:

$$p^i = (1 - r)E^i(\theta) + rE^i(\overline{p}) \tag{1}$$

with $r = b/(1 + b) \in [0, 1]$.[5]

[4]The model can be generalized to more than two fundamentals. It does not seem that this generalization would affect our results.

[5]There are other possible interpretations of 1. For example,Woodford (2003) derives a similar price setting for monopolistically competive firms.

M&S interpret their single fundamental as the money supply. Yet, in their framework the money supply is not controlled by the central bank whose sole function is to forecast this variable, possibly with less precision than the private sector. This characterization of monetary policy is unsatisfactory and, as it turns out, deeply affects the issue of transparency. Indeed, in this setup, there would be no issue of transparency if the central bank were to directly set the money supply. Presumably, the central bank sets the interest rate, which in turn affects the money supply but with significant noise. Assuming that the private sector observes the interest rate, both the central bank and the private sector infer the likely level of the money supply. In that case, the choice of the interest rate must be related to the central bank's forecast of the fundamental.

Short of fully specifying how monetary policy operates (channels of monetary policy effects, the instrument and the targets), we still need to recognize that the central bank uses one instrument to affect some intermediate target(s). To that effect, we depart from M&S in two ways. First, we allow for two fundamentals, which are meant to represent exogenous variables or intermediate targets that affect demand. Among the first category, one can think of foreign demand, fiscal policy or exogenous shocks. The intermediate targets can be the money supply, long-term interest rates, asset prices or the exchange rate. Second, we assume that the instrument is the short-term interest rate $R$, which is set by the central bank and observed by the private sector.

Note that the interest rate has no macroeconomic effect; it does not affect prices directly, the only economic variables in this model. It only serves as an information vehicle that the central bank *must* reveal. This is an extreme characterization of the observation by Woodford (2005) that "the current level of the overnight interest rates *as such* is of negligible importance for economic decisionmaking". This assumption is not innocuous. Its merit is to concentrate attention to the signaling role of interest rate decisions. Of course, it is unrealistic that the interest rate plays no other role, and the effect of this limitation will soon become clear.

In order to optimally set the interest rate, the central bank makes the best possible use of its information about the fundamentals. Denoting $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ the information that the central bank has regarding the two fundamentals, we assume that it uses a linear rule of the type:

$$R = \mu\widetilde{\theta}_1 + \nu\widetilde{\theta}_2 \tag{2}$$

Without loss of generality, we impose the restriction $\mu + \nu = 1$, which allows us to characterize policy as the choice of $\lambda = \mu/\nu$. This restriction amounts to normalizing the interest rate $R$. In accordance with private sector preferences, the central bank chooses the short-term interest rate that minimizes the social loss function:

$$L = E^{CB} \int \left(p^i - A\theta\right)^2$$

based on its own information set. The central bank observes the fundamentals with additive noise

$$\widetilde{\theta}_k = \theta_k + \varepsilon_k \qquad k = 1, 2$$

where the noise terms $\varepsilon_1$ and $\varepsilon_2$ are identically and independently distributed with zero mean and variance $V_\epsilon$.[6] Obviously, in choosing $\mu$ and $\nu$ the central bank must take into

[6]In line with the literature, we treat $V_\varepsilon$ as exogenous. Obviously, central bank signals are based on variables that include private sector actions and, therefore, private sector signals. Ignoring this dependence is subject to a Lucas critique since the precision of central bank signals may vary with the policy regime and, in particular, on central bank transparency. We thank Hyun Song Shin for attracting our attention to this limitation of our paper.

account how the private sector sets its prices.

## 2.2 The Private sector

Each island producer $i$ sets its price according to (1), using available private information on the fundamentals. We assume that producer $i$ receives a noisy signal for each fundamental:

$$x_k^i = \theta_k + \eta_k^i \qquad k = 1, 2$$

where the noise terms $\eta_k^i$ are independently distributed with zero mean and variance $V_{\eta,k}$. While the signals on each of the two fundamentals are identically distributed across all producers, their perceived variance may differ across the fundamentals (we may have $V_{\eta,1} \neq V_{\eta,2}$) and the variance perceived by the producers may differ from the variance perceived by the central bank (i.e. we may have $V_{\eta,k} \neq V_\varepsilon$). The perceived variances reflect the quality of information, which plays a central role in the analysis. Assuming $V(\varepsilon_1) = V(\varepsilon_2)$ and $V(\eta_1) \neq V(\eta_2)$ is the simplest way of introducing two meaningfully distinct fundamentals. Imposing $V(\varepsilon_1) = V(\varepsilon_2)$ does not restrict the generality of the model, it merely implies a normalization of $\tilde{\theta}_1$ and $\tilde{\theta}_2$. If, in addition we were to allow for $V(\eta_1) = V(\eta_2)$, we would have a symmetric case where the two signals are seen by both the central bank and the private sector as drawn from the same distribution but with different accuracy. The resulting asymmetry of treatment of the private sector and the central bank is meant to reflect private sector heterogeneity.

The beauty contest parameter $r$ reflects the reactivity of producers to each other's expectations. It implies that each producer must guess what all the other producers will guess, while they themselves guess everyone else's guess. Since each producer receives her own idiosyncratic information $\eta_k^i$, this is no trivial task. Iterating (1) infinitely, and denoting $\bar{E}^n$ the $n^{th}$ order expectation, the optimal pricing decision is:

$$p^i = (1 - r) \sum_{n=0}^{\infty} r^n E^i \left( \bar{E}^n (A\theta) \right) \tag{3}$$

which exists when $0 < r < 1$.

Thus each producer will set her own price in order to stay as close as possible to her estimate of the fundamental combination $A\theta$ that drives the aggregate price index $\bar{p}$. Her loss will be therefore $L_i = (p_i - A\theta)^2$, which justifies the central bank loss function in this one-shot game.[7] Since all producers are identical save for the white noise terms $\eta_k^i$ that they receive individually, the central bank objective can be treated as $E^{CB}(p^i - A\theta)^2$.

We now look for the equilibrium outcome, where the central bank optimally chooses $\mu$ and $\nu$ to set the short-term interest rate $R$ which is then observed by the private sector. The central bank decision involves forecasting $p^i$, which requires guessing the private sector price setting behavior (3). The question is whether the central bank should, in addition to making $R$ known, reveal its own information about the fundamentals $\tilde{\theta}_1$ and $\tilde{\theta}_2$? In order to deal with that question, we must specify carefully what is known by the central bank and by the private sector about the distribution of noises $\varepsilon_k$ and $\eta_k^i$, for $k = 1, 2$. We assume that everyone knows that the means are zero but what about the variances $V_\varepsilon$ and $V_{\eta,k}$? We consider three cases: when the variances are known to all, when private sector variances are only known by the private sector, and when private sector variances are unknown to all. Thus we always assume that central bank variances are common knowledge, which should favor the M&S case.

[7] For a criticism of this aggregate loss function, see Woodford (2005).

# 3 Information precision is known

We start with the simplest case, when the variances of the noise terms $V_\varepsilon$ and $V_{\eta,k}$ are known by everyone, both the central bank and the private sector. We first consider that the central bank does not reveal its private information $\tilde{\theta}_1$ and $\tilde{\theta}_2$; it only announces the interest rate $R$, which the sector knows is set optimally as a linear function of $\tilde{\theta}_1$ and $\tilde{\theta}_2$. We then ask whether welfare improves when the central bank releases its information $\tilde{\theta}_1$ and $\tilde{\theta}_2$.

It will prove helpful to simplify the notations as follows. Let $\delta = V_{\eta,1}/V_{\eta,2}$ be the relative variance of private sector signals on the two fundamentals and denote $V_\eta = V_{\eta,2}$ so that:

$$V_{\eta,1} = \delta V_{\eta,2} = \delta V_\eta$$

We will refer to $\delta = 1$ as the symmetric case when there is no discrepancy between the central bank and the private sector assessment of relative variances. Finally, we define the precision of the central bank and of public sector as $\alpha$ and $\beta$, respectively as:

$$\begin{aligned} \alpha &= \frac{1}{V_\varepsilon} \\ \beta &= \frac{1}{V_\eta} \end{aligned}$$

and we denote $z = \alpha/\beta$ the relative precision of the central bank information.

## 3.1 Opacity: the central bank only reveals its interest rate

Assuming that the central bank sets the interest rate $R$ according to (2), producer $i$ forms her expectations of the two fundamentals, using Bayes theorem:

$$\begin{aligned} E^i(\theta_1) &= \gamma_1 \left( \frac{R - \nu x_2^i}{\mu} \right) + (1 - \gamma_1) x_1^i \\ E^i(\theta_2) &= \gamma_2 \left( \frac{R - \mu x_1^i}{\nu} \right) + (1 - \gamma_2) x_2^i \end{aligned}$$

with

$$\gamma_1 = \frac{\lambda^2 \delta z}{(1 + \lambda^2 \delta) z + \left(1 + \lambda^2\right)} \tag{5a}$$

$$\gamma_2 = \frac{z}{(1 + \lambda^2 \delta) z + \left(1 + \lambda^2\right)} \tag{5b}$$

Agent $i$'s expectation of the combined fundamentals, which enters her pricing decision (1), is:

$$E_i(A\theta) = \frac{1 + \lambda}{\lambda} \left(\gamma_1 + \lambda \gamma_2\right) R + \left[(1 - \gamma_1) - \gamma_2 \lambda\right] x_1^i + \frac{\left(1 - \gamma_2\right) \lambda - \gamma_1}{\lambda} x_2^i$$

The Appendix shows how to iterate this relation and find agent's $i$ pricing decision :

$$p^i = \frac{1 + \lambda}{\lambda} \left(\varphi_1 + \lambda \varphi_2\right) R + \left[(1 - \varphi_1) - \varphi_2 \lambda\right] x_1^i + \frac{\left(1 - \varphi_2\right) \lambda - \varphi_1}{\lambda} x_2^i \tag{6}$$

where $\varphi_1$ and $\varphi_2$ are defined as:

$$\varphi_k = \frac{\gamma_k}{1 - r[1 - (\gamma_1 + \gamma_2)]} \qquad k = 1, 2 \tag{7}$$

Much as in M&S, the interest rate, the central bank signal, receives more weight than it should if based only on the precision of available information, as given by (5). This can be seen by considering the case where $r = 0$, when agents do not care about each other's expectations. In that case $\varphi_k = \gamma_k$ and the weights on $R$ and the private signals $x_k^i$ in the pricing equation (6) are exactly those that correspond to Bayesian signal extraction. When $r > 0$ instead, $\varphi_k > \gamma_k$ and the weight on $R$ in (6) increases with $r$ while the weights on $x_1^i$ and $x_2^i$ decline.

In contrast to M&S, however, the interest rate is a manipulable signal. It provides the central bank with a tool to mitigate the common knowledge problem. Knowing that the interest rate $R$ excessively influences the private sector, like a benevolent flute-payer sure to be followed by the crowds, the central bank can use its power to steer private expectations in a socially desirable direction. Even though the interest rate does not have any macroeconomic effect, it can play a useful signaling role.[8] Formally, the central bank sets $\lambda$ to minimize the loss function $E^{CB}(p_i - A\theta)^2$:

$$\begin{aligned} E^{CB}(p_i - A\theta)^2 &= L^{op}(\delta, \lambda) \\ &= \frac{1}{\beta}\frac{\left(1+\lambda^2\right)(1+\delta\lambda)^2 z + \delta[(1-\lambda)z + (1-r)\left(1+\lambda^2\right)]^2}{\left[\left(1+\delta\lambda^2\right)z + (1-r)\left(1+\lambda^2\right)\right]^2} \\ &\quad + \frac{1}{\beta}\frac{[\lambda(\lambda-1)\delta z + (1-r)\left(1+\lambda^2\right)]^2}{\left[\left(1+\delta\lambda^2\right)z + (1-r)\left(1+\lambda^2\right)\right]^2} \end{aligned} \tag{8}$$

and the optimum policy outcome delivers a loss $L^{op}(\delta) = Arg\min L^{op}(\delta, \lambda)$. The first order condition $\partial L^{op}(\delta, \lambda)/\partial\lambda = 0$ is a fourth-order equation that cannot be explicitly solved for an arbitrary value of $\delta$, but the following observations allow us to characterize the central bank choice when $\delta$ lies in the neighborhood of one. We need to distinguish two cases.

### Relatively high central bank precision and weak beauty contest effect $(1 + z - 2r \geq 0)$

The first case describes a situation where the relative precision $z$ of central bank information is high and the strategic interaction coefficient $r$ , which determine the size of the common knowledge effect, is relatively low.[9]

When $\delta = 1$ - the symmetry case - the optimal choice is $\lambda = 1$, so $\mu = \nu = 1/2$.[10] Weighting equally the two signals is natural since the two fundamentals are drawn from the same distribution for both the central bank and the private sector. When $\delta$ lies in the neighborhood of 1, the unique solution is described by the following first-order Taylor approximation around $\delta = 1$ and $\lambda = 1$:

$$\lambda - 1 = \frac{(1 - 3r + 2r^2) + z}{(1 - 3r + 2r^2) + (2 - 3r)z + z^2}(\delta - 1) \tag{9}$$

[8] Could the central bank eliminate the externality? This would require that $\varphi_k = \gamma_k \ \forall r$. This occurs when $\gamma_1 + \gamma_2 = \dfrac{(1+\lambda^2\delta)\alpha}{(1+\lambda^2\delta)\alpha + (1+\lambda^2)\beta} = 1$, which is impossible unless $\beta = 0$, the limit case where the private sector signal precision is nil.

[9] The condition $z \geq 2r - 1$ is the second-order condition in the neighboorhood of $\delta = 1$ since, when $\delta = 1$ : $\dfrac{\partial^2 L_1(1,\lambda)}{\partial\lambda^2} = \frac{1}{2}\alpha z\frac{z+(1-2r)}{((1-r)+z)^2}$.

[10] $\mu = \dfrac{\lambda}{1+\lambda}$ and $\nu = \dfrac{1}{1+\lambda}$.

The weight $\lambda$ put by the central bank on its signal $\tilde{\theta}_1$ of the first fundamental $\theta_1$ rises when the corresponding private signal becomes less precise - when $\delta$ rises. In order to understand this result, note that the two fundamentals are normalized to have the same effect on prices; in brief, they are "equivalent". As $\delta$ increases, the private sector puts less weight on signals $x_1^i$ relative to signals $x_2^i$; it underweighs the noisy signal. By raising the weight of its signal $\tilde{\theta}_1$, the central bank shrewdly exploits the common knowledge effect to correct the private sector underweighting of its first signal. This is efficient when the central bank precision is sufficiently high and as long as the private sector is not too reactive.

The result is further illuminated by considering two limit cases. When $z \to \infty$, $\lambda \to 1$: when its information is infinitely more precise than that of the private sector, which is understood by everyone, the best action of the central bank is to treat its own signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$, which are of equal precision ($V(\varepsilon_1) = V(\varepsilon_2)$) equally since the two fundamentals are equivalent. In the second polar case, as $z \to 0$, $\lambda \to \delta$; since the private sector is infinitely better informed, the central bank weighs the signals according to the relative precision of the private sector forecasts. For intermediate values of $z$, the central bank chooses $\lambda$ to optimally shape private sector forecasts of $\theta_1$ and $\theta_2$. In this sense the central bank manipulates private sector expectations to mitigate the common knowledge effect.

#### Relatively low central bank precision and strong beauty contest effect ($1 + z - 2r < 0$)

We now look at the case when the central bank is relatively poorly informed and/or the beauty contest effect is strong, which implies a powerful common knowledge effect. When $\delta$ is in the neighborhhod of unity, the optimal policy decision is to set $\lambda = -1/\delta$.[11] When $\delta$ is close to 1, $\lambda$ is close to -1 and $\mu$ and $\nu$ become infinitely large in absolute value. A poorly-informed central bank that faces a highly reactive private sector would rather not issue *any* signal. This option is closed, however, because by assumption the central bank must set and reveal the interest rate. The best policy, then, is to produce a highly volatile interest rate, which leads the private sector to put an even higher weight on its own signals.

This result may be surprising. It is partly a consequence of our assumption that the interest rate plays no direct macroeconomic role. In a more realistic setting, the central bank would weigh the two objectives of affecting the macroeconomy and sending an appropriate signal. The first one would call for an interest rate set to adequately offset the disturbances, the second would push to intentionally make it volatile to reduce its information content. It remains that interest volatility, possibly less extreme, may be a valid policy objective for a central bank that is relatively poorly informed when the private sector is subject to a strong beauty contest effect.[12]

### 3.2 Transparency: the central bank reveals the interest rate and its signals

In our model, transparency is achieved when the central bank fully reveals its information, i.e. $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$. In that case, the interest rate is not informative anymore. Producer $i$ now receives two signals for each of the two fundamentals: her own signals $x_k$.and the central bank signals $\widetilde{\theta}_k$ for $k = 1, 2$. Optimal Bayesian signal extraction leads to:

[11] In fact, $\lambda = -1/\delta$ is always a solution. It delivers a global optimum when $z\delta(1+\delta) + \beta\left(1+\delta^2\right)(1-2r) < 0$, which reduces to $z < 2r - 1$ when $\delta = 1$. Elsewhere, it is only a local optimum.

[12] This result is remindful of the suggestion that central banks might encourage exchange rate or asset price volatility to tame jittery financial markets.

$$E^i(\theta_1) = \overline{\gamma}_1\widetilde{\theta}_1 + (1-\overline{\gamma}_1)x_1^i \quad (10a)$$
$$E^i(\theta_2) = \overline{\gamma}_2\widetilde{\theta}_2 + (1-\overline{\gamma}_2)x_2^i \quad (10b)$$

with :

$$\overline{\gamma}_1 = \frac{\delta z}{1+\delta z} \quad (11a)$$
$$\overline{\gamma}_2 = \frac{z}{1+z} \quad (11b)$$

Since individual noises are white noise, we have $\int \bar{E}(x_k^i)di = \theta_k$ and therefore:

$$\bar{E}(A\theta) = \overline{\gamma}_1\widetilde{\theta}_1 + \overline{\gamma}_2\widetilde{\theta}_2 + (1-\overline{\gamma}_1)\theta_1 + (1-\overline{\gamma}_2)\theta_2$$

Iterating this relation to compute $p^i = (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(A\theta)\right)$ we have:

$$p_i = \overline{\varphi}_1\tilde{\theta}_1 + \overline{\varphi}_2\tilde{\theta}_2 + (1-\overline{\varphi}_1)\,x_1^i + (1-\overline{\varphi}_2)\,x_2^i \quad (12)$$

where:

$$\overline{\varphi}_1 = (1-r)\,\overline{\gamma}_1 + r\frac{(1-r)\overline{\gamma}_1 + \delta z}{(1-r)+\delta z} \text{ and } \overline{\varphi}_2 = (1-r)\,\overline{\gamma}_2 + r\frac{(1-r)\overline{\gamma}_2 + z}{(1-r)+z}$$

This is exactly the same result as in M&S except that we allow for two signals with different variances for the private sector. We find the familiar result that, because $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ are common knowledge, they receive excessive weight in (12). To see this, note that when $r = 0$, i.e. when individual producers have no incentive to guess what the others expect, $\overline{\varphi}_k = \overline{\gamma}_k$ for $k = 1, 2$ and the weights on $\widetilde{\theta}_k$ and $x_k^i$ in (12) exactly reflect the precision of the respective signals as given in (11). When $r > 0$, $\overline{\varphi}_k > \overline{\gamma}_k$ and the weights on $\widetilde{\theta}_k$ are increased at the expense of the weights on $x_k^i$.

With transparency, the individual loss of agent $i$ is:

$$E\left(p^i - A\theta\right)^2 = L^{tr}(\delta) = \frac{1}{\beta}\left[\frac{z+(1-r)^2}{[z+(1-r)]^2} + \delta\frac{\delta z+(1-r)^2}{[\delta z+(1-r)]^2}\right] \quad (13)$$

The central bank does not have any optimization to do since the interest rate does not affect the economy other than through its signal content, which is uninformative with full transparency.

## 3.3 Welfare comparisons

We find that opacity always dominates transparency. This can be verified by noting that when it sets the interest rate with $\lambda = \delta\dfrac{1+z-r}{1+\delta z-r}$ under opacity, the central bank reproduces the transparency outcome. Since in general this particular value of $\lambda$ is not the optimal choice, it follows that the central bank can always do at least as well by not revealing its information and generally better. In fact, the result is very general as stated in the following proposition which is established in the Appendix:

**Proposition 1** *When the precision of central bank and private sector information is known, it is desirable for the central bank not to be transparent. This result holds for any loss function (which preserves the price setting) and any number of fundamentals.*

The reason is that the publication of the interest rate creates a common knowledge effect, which is distortionary because of information heterogeneity. The central bank can

partially correct this distortion by using the interest rate to manipulate private sector expectations. If the central fully reveals its information, it still creates a distortionary common knowledge effect but it now loses its ability to strategically set the interest rate.

The conclusion differs quite sharply from the previous results reached in the transparency literature. The reason is that the interest rate is both a signal and a policy instrument. It can be used as an antidote to its own common knowledge effect. It is useful, therefore, not to undermine its information content.

The result is valid independently of the relative precision of central bank information and of the strength of the beauty contest effect. It applies for any number of fundamentals, provided that there are more fundamentals than compulsory signals (here we have one compulsory signal, the interest rate, and two fundamentals). The relative precision of central bank information and the strength of the beauty contest effects affect the way that the central bank uses the interest rate to offset the common knowledge effect, not the welfare conclusion.[13]

It is also worth emphasizing that this conclusion is reached independently of any particular welfare criterion. Hellwig (2005), Roca (2006) and Woodford (2005) have noted that the conclusion reached by M&S critically depends on their choice of a particular loss function. In our model, where the central bank must reveal some information by announcing its optimally-chosen interest rate, we find that it is never desirable for the central bank to be fully transparent.[14]

# 4 The private sector precision is not known

So far, uncertainty only concerned the first moments of the fundamentals. The second moments, the precision of information received by the central banks and the private sector, were assumed to be well-known. We now introduce a new source of uncertainty by considering that the precision may not be known.

This can be done in a number of ways. It could be that the central bank and the private sector know their own precision but not that of the other sector. It could also be that no precision is known for sure. There can be an asymmetry whereby one sector knows the other's precision but not the other way round. In addition, we could allow for gradual learning of information precision.

Given these many possibilities, we consider a simple example. We assume that neither the central bank nor the private sector know $V_\eta$. On the other hand, both the central bank and the private sector know $V_\varepsilon$, the variance of the central bank signal. This postulated asymmetry is logical as we focus on the role of common knowledge. Since the central bank actions are closely watched by a wide group of observers, it is plausible that much more is known about central bank signal precision than about private signal precision. We go the extreme case when there is no uncertainty about the precision of central bank information.

We also need to specify who knows what. We assume that both the central bank and the private sector mistakenly believe that their own estimates of private signal variance, $E^{CB}V_\eta$ and $E^{pr}V_\eta$, respectively, are accurate. This may be seen as a symptom of overconfidence, which is a common assumption in the finance literature. We further assume that the private sector realizes that the central bank is mistaken, i.e. $E^{pr}E^{CB}V_\eta \neq E^{pr}V_\eta$. This assumption, again, is meant to capture intense scrutiny of central bank actions and

[13] With more than two fundamentals, we could look at cases of partial transparency, when some but not all signals are revealed. In a way, this is what is done in the next section.

[14] On the other hand, our result assumes that the interest rate does not play any direct macroeconomic role. If it did, the central bank might not be able to adequately manipulate the interest rate as it faces a trade-off between its macroeconomic and communication objectives.

communication. The many agents that make up the private sector each detect some discrepancy between what the central bank does and what they would expect it to do given their own estimates but the central bank itself does not follow as closely each and every private agent's own forecasts and actions.

As before, we define the two private sector signal as $x_k^i = \theta_k + \eta_k^i$ and we assume that the (unknown) signal quality is the same among all producers:

$$Var\left(\eta_k^i\right) = V_{\eta,k} \qquad \forall i \qquad k = 1, 2$$

Note that we still allow for the private sector signal variances of $\eta_1$ and $\eta_2$ to differ (i.e. $V_{\eta,1} \neq V_{\eta,2}$). To keep matters simple, we assume that the estimates of these variances are exogenous and differ between the central bank and the private sector:

$$\begin{aligned} V_{\eta,k}^{pr} &\equiv E^{pr} V_{\eta,k} \\ V_{\eta,k}^{CB} &\equiv E^{CB} V_{\eta,k} \end{aligned}$$

We start with the simplest case: we assume that the private sector in fact knows its own precision, i.e. $V_{\eta,k}^{pr} = V_{\eta,k}$. Since the private sector knows that $V_{\eta,k}^{CB} \neq V_{\eta,k}$, it must forecast the central bank's estimate. Again, we assume that this forecast is exogenous:

$$\tilde{V}_{\eta,k}^{pr} \equiv E^{pr} E^{CB} V_{\eta,k} = E^{pr} V_{\eta,k}^{CB}$$

A more elaborate model would allow for endogenous expectation formations, but this would require a much heavier setup with little additional insight. Section 4 shows that removing the assumption $V_{\eta,k}^{pr} = V_{\eta,k}$ does not significantly alter the conclusions. As previously, we normalize the private sector variances around $\tilde{V}_{\eta,2}^{pr}$ and adopt the following notation:

$$\tilde{V}_{\eta} = \tilde{V}_{\eta,2}^{pr} \qquad \tilde{\delta} = \frac{\tilde{V}_{\eta,1}^{pr}}{\tilde{V}_{\eta,2}^{pr}} \qquad \alpha = \frac{1}{V_{\varepsilon}} \qquad \widetilde{\beta} = \frac{1}{\tilde{V}_{\eta}} \qquad \tilde{z} = \frac{\alpha}{\widetilde{\beta}}$$

The uncertainty about signal precision now changes the notion of central bank transparency. The private sector is interested in knowing not just the central bank signals $\tilde{\theta}_k$ but also the central bank forecasts $V_{\eta,k}^{CB}$ of the private sector signals. The latter only matters when the central bank does not reveal its signals. Thus there are now three degrees of transparency: 1) opacity, when the central bank only publishes the interest rate; 2) partial transparency, when the central bank reveals its estimates of private sector variances $V_{\eta,k}^{CB}$; and 3) full transparency, when the central bank additionally reveals its signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$.[15]

## 4.1 Opacity: the central bank only reveals the interest rate

When she observes the interest rate $R$ and try to figure out what is the central bank reaction function, producer $i$ must distinguish between her own (correct) estimates $V_{\eta,k}^{pr} = V_{\eta,k}$ of $V_{\eta,k}$ and those $\tilde{V}_{\eta,k}^{pr}$ that she believes are (erroneously) made by the central bank. To do so, each producer applies the usual Bayesian signal extraction method to interpret the interest rate on the basis of its own signals $x_k^i$. This allows producer $i$ to form her expectation $E^i(A\theta)$ of the fundamentals. But the producer also knows that the central bank reaction function is set on the basis of its own mistaken expectations $V_{\eta,k}^{CB}$ that the private sector estimates as $\tilde{V}_{\eta,k}^{pr}$. Producer $i$ must therefore also extract information from

[15] One could imagine a second case of partial transparency, when the central bank reveals its signals but not its estimates of private sector variances. As will become clear, this is not different from the case of full transparency.

the observed interest rate to infer the parameters of the central bank reaction function that enter $E^i(A\theta)$.

We now examine the outcome in four steps: 1) The private sector's inference from the observed interest rate: 2) Private producers's price decisions; 3)The central bank's optimal choice of the interest rate; 4) The resulting equilibrium.

### Private sector inference conditional on the central bank reaction function

As before, producer $i$ observes the interest rate, knowing that the central bank follows a linear reaction function $R = \widetilde{\mu}\tilde{\theta}_1 + \widetilde{\nu}\tilde{\theta}_2$, and receives the signals $x_k^i$, $k = 1, 2$, corresponding to the two fundamentals. Using Bayes theorem, her estimates of the two fundamentals is as in (5). The price decision of producer $i$ is given by (6).

Producer $i$ must estimate the central bank choice of $\lambda$. She knows that the central bank chooses $\lambda$ to minimize $E^{CB}(p^i - A\theta)^2$ but that $\tilde{V}_{\eta,k}^{pr} = E^{pr}E^{CB}V_{\eta,k} \neq V_{\eta,k}$. Accordingly, following (8), the private sector expects the central bank to choose $\widetilde{\lambda} = \dfrac{\widetilde{\mu}}{\widetilde{\nu}}$ to minimize:

$$\begin{aligned} E^{CB}\left(p^i - A\theta\right)^2 &= L^{op}\left(\widetilde{\delta}, \widetilde{\lambda}\right) \\ &= \frac{1}{\widetilde{\beta}} \frac{\left(1+\widetilde{\lambda}^2\right)(1+\widetilde{\delta}\widetilde{\lambda})^2\tilde{z} + \widetilde{\delta}[(1-\widetilde{\lambda})\tilde{z} + (1-r)\left(1+\widetilde{\lambda}^2\right)]^2}{[\left(1+\widetilde{\delta}\widetilde{\lambda}^2\right)\tilde{z} + (1-r)\left(1+\widetilde{\lambda}^2\right)]^2} \\ &\quad + \frac{1}{\widetilde{\beta}} \frac{[\widetilde{\lambda}(\widetilde{\lambda}-1)\widetilde{\delta}\tilde{z} + (1-r)\left(1+\widetilde{\lambda}^2\right)]^2}{\widetilde{\beta}[\left(1+\widetilde{\delta}\widetilde{\lambda}^2\right)\tilde{z} + (1-r)\left(1+\widetilde{\lambda}^2\right)]^2} \end{aligned} \tag{14}$$

In the neighborhood of $\widetilde{\delta} = 1$, the solution is in the neighborhood of $\widetilde{\lambda} = 1$ when the second order condition $1 + \tilde{z} - 2r \geq 0$ is satisfied. Section 4.4.2 deals with the alternative case. A first order approximation is:

$$\widetilde{\lambda} - 1 = \frac{(1 - 3r + 2r^2) + \tilde{z}}{(1 - 3r + 2r^2) + (2 - 3r)\tilde{z} + \tilde{z}^2}\left(\tilde{\delta} - 1\right) \tag{15}$$

This is similar to (9) and has the same interpretation.

### Pricing

Once producer $i$ has determined the value of $\widetilde{\lambda}$ that she expects the central bank to choose in deciding on the interest rate, she estimates $E^i(\theta_k)$ from observing the interest rate and receiving the signals $x_k^i$ on the two fundamentals. This leads her to modify the coefficients $\gamma_k$ of her Bayesian inference (5):

$$\widetilde{\gamma}_1 = \frac{\delta\widetilde{\lambda}^2\alpha}{\left(1+\widetilde{\lambda}^2\right)\beta + \left(1+\widetilde{\lambda}^2\delta\right)\alpha} \tag{16a}$$

$$\widetilde{\gamma}_2 = \frac{\alpha}{\left(1+\widetilde{\lambda}^2\right)\beta + \left(1+\widetilde{\lambda}^2\delta\right)\alpha} \tag{16b}$$

Using these parameters in (6) delivers the pricing decision:

$$
\begin{aligned}
p^i &= (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(A\theta)\right) \\
&= \frac{1+\widetilde{\lambda}}{\widetilde{\lambda}}\left(\tilde{\varphi}_1+\widetilde{\lambda}\tilde{\varphi}_2\right)R+\left[(1-\tilde{\varphi}_1)-\tilde{\varphi}_2\widetilde{\lambda}\right]x_1^i+\left[(1-\tilde{\varphi}_2)\,\widetilde{\lambda}-\tilde{\varphi}_1\right]\frac{x_2^i}{\widetilde{\lambda}} \qquad (17)
\end{aligned}
$$

where:

$$\tilde{\varphi}_k = \frac{\widetilde{\gamma}_k}{1-[1-r(\widetilde{\gamma}_1+\widetilde{\gamma}_2)]}$$

These are exactly the same as in (6) and (7), the only difference being that $\widetilde{\lambda}$, the producers estimate of the central bank action, is based on $\tilde{V}_{\eta,k}^{pr} \equiv E^{pr}E^{CB}V_{\eta,k}$, their estimate of the central bank forecast instead of $V_{\eta,k}$ itself (even though producer $i$ knows $V_{\eta,k}$).

### The actual central bank reaction function

So far, we have computed the private sector estimate of central bank expectations (15), but the private sector does not know the later for sure (and knows that it does not know). What will be the central bank choice of $\lambda$? The central bank will first extract information from its own signals $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$, using its forecasts $V_{\eta,k}^{CB} \equiv E^{CB}V_{\eta,k}$. As before, we normalize on $V_{\eta,2}^{CB}$ and adopt the following notation:

$$V_\eta^{CB} = V_{\eta,2}^{CB} \qquad \delta' = \frac{V_{\eta,1}^{CB}}{V_{\eta,2}^{CB}} \qquad \alpha = \frac{1}{V_\varepsilon} \qquad \beta' = \frac{1}{V_\eta^{CB}} \qquad z' = \frac{\alpha}{\beta'}$$

The situation is exactly the same as in Section 3.1.

### Market equilibrium: the systematic bias

We now have a situation where the central bank chooses $\lambda'$ and sets the interest rate according to $R = \dfrac{\lambda_1'\theta+\theta_2}{1+\lambda'}$ while the private sector looks at it believing that it is $R = \dfrac{\tilde{\lambda}\tilde{\theta}_1+\tilde{\theta}_2}{1+\widetilde{\lambda}}$. Inserting the true value of the interest rate into (17) gives the actual market price equilibrium. As a deviation from its fundamentals, it is:

$$
\begin{aligned}
p^i - A\theta &= \frac{\tilde{\varphi}_1+\tilde{\varphi}_2\widetilde{\lambda}}{\widetilde{\lambda}(1+\lambda')}(\tilde{\theta}_1-\tilde{\theta}_2)(\lambda'-\widetilde{\lambda}) \\
&\quad +\frac{\tilde{\varphi}_1+\tilde{\varphi}_2\widetilde{\lambda}}{\widetilde{\lambda}}\left[\widetilde{\lambda}\left(\tilde{\theta}_1-\theta_1\right)+\left(\tilde{\theta}_2-\theta_2\right)\right] \\
&\quad +\left[(1-\tilde{\varphi}_1)-\tilde{\varphi}_2\widetilde{\lambda}\right]\left(x_1^i-\theta_1\right)+\frac{(1-\tilde{\varphi}_2)\,\widetilde{\lambda}-\tilde{\varphi}_1}{\widetilde{\lambda}}\left(x_2^i-\theta_2\right) \qquad (18)
\end{aligned}
$$

This expression includes three terms. The second and third ones involve the signal errors of the central bank and of the private sector, respectively. Such mispricing is unavoidable and contributes to price volatility but, in contrast with the first term, the last two terms are nil on average. Thus the first term introduces a systematic bias between prices and their fundamentals. This systematic bias arises because of the discrepancy between the private sector and the central bank information. The central bank's choice

$\lambda'$ given by (15) systematically differs from the corresponding private sector forecast $\widetilde{\lambda}$ shown in (20).[16]

**Proposition 2** *When the central bank and the private sector differ in their forecasts of the precision of private sector information, the interest rate becomes the source of a systematic bias between the price level and its fundamentals. The bias occurs irrespective of the relative precision of central bank and private sector information.*

The size of the systematic bias is proportional to $\lambda' - \widetilde{\lambda}$. Consider again the two polar limit cases in the neighborhood of 1 for $\widetilde{\delta}$, $\delta'$, $\widetilde{\lambda}$ and $\lambda'$. From (15) and (20), we know that there is no bias ($\lambda' \simeq \widetilde{\lambda} \simeq 1$) when $z \to \infty$, while $\lambda' - \widetilde{\lambda} \simeq \delta' - \widetilde{\delta}$ when $z \to 0$. When the central bank is well informed it relies on its own signals; with $V(\varepsilon_1) = V(\varepsilon_2)$ it weighs both signals equally. The private sector knows that, so the bias is eliminated. On the contrary, when the central bank is poorly informed, it relies on its estimate of private sector signals and sets $\lambda' = \delta'$; the private sector is aware of this strategy but it imagines $\widetilde{\lambda} = \widetilde{\delta}$. This suggests that the systematic bias is larger the less well-informed the central bank is.

## 4.2 The central bank is partially transparent

We consider now the case where the central bank reveals the interest rate and its estimates of private sector variances $V^{CB}_{\eta,k}$ - i.e. $z'$ and $\delta'$ - but not its signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$. The central bank choice of $\lambda'$ rests on its mistaken estimates $z'$ and $\delta'$ but this is now known to the private sector. Accordingly $\widetilde{\lambda} = \lambda'$ and the systematic bias in (18) is eliminated:

$$\begin{aligned} p^i - A\theta &= \frac{\tilde{\varphi}_1 + \tilde{\varphi}_2\widetilde{\lambda}}{\widetilde{\lambda}}\left[\widetilde{\lambda}\left(\tilde{\theta}_1 - \theta_1\right) + \left(\tilde{\theta}_2 - \theta_2\right)\right] \\ &\quad + \left[\left(1 - \tilde{\varphi}_1\right) - \tilde{\varphi}_2\widetilde{\lambda}\right]\left(x_1^i - \theta_1\right) + \left[\left(1 - \tilde{\varphi}_2\right)\widetilde{\lambda} - \tilde{\varphi}_1\right]\frac{x_2^i - \theta_2}{\widetilde{\lambda}} \end{aligned} \tag{19}$$

where $\tilde{\varphi}_k$ is as in (5) and (7) with $\lambda'$ instead of $\lambda$, and the loss is $L^{PT}\left(\delta, \lambda'\right)$. We evaluate this case in Section 4.4 below.

## 4.3 The central bank is fully transparent

The central bank now reveals its estimates $z'$ and $\delta'$ as well as its own signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$. This situation has already been treated in Section 3.2. Since $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are known, the interest rate has no information value anymore and the discrepancy of information precision between the central bank and the previous sector does not create any systematic bias anymore. The optimal forecast of producer $i$ is given by (10) and (11), and the individual loss remains as in (13).

When the central bank is fully transparent, the private sector knows everything that the central bank knows, even though the central bank is mistaken about the precision of private forecasts. Since the interest rate is not informative anymore, the private sector ignores the flawed signal that it conveys.

**Proposition 3** *When the central bank and the private sectors have different information about the private sector precision, by revealing all its information, the central bank allows the private sector not to misinterpret the interest rate signal.*

[16] The bias disappers when $\widetilde{\theta}_1 = \widetilde{\theta}_2$. This woud imply that there is a single fundamental. In that case, revealing $R$ would fully reveal the central bank choice up to the signal errors that appear in the second and third terms.

## 4.4 Welfare comparisons

We now compare the welfare properties of the three transparency regimes:

1) Opacity: the central bank only publishes the interest rate. It chooses $\lambda'$ given by (20), but the private sector believes that it is $\tilde{\lambda}$ as in (15). The corresponding loss $L^{op}\left(\delta, \tilde{\lambda}, \lambda'\right)$, is the unconditional expectation $E(p^i - A\theta)^2$ where $p^i$ is given by (18).

2) Partial transparency: in addition to the interest rate, the central bank reveals its (mistaken) estimates of private sector variances $V^{CB}_{\eta,k}$, hence $\beta'$ and $\delta'$. It chooses $\lambda'$ and the private sector is aware of this. The loss, which is computed using the correct (and private estimates of) variances $V_{\eta,k}$, hence using $z$ and $\delta$, is $L^{PT}\left(\delta, \lambda'\right) = E(p^i - A\theta)^2$ where $p^i$ is set according to (19).

3) Full transparency: the central bank additionally reveals its signals $\tilde{\theta}_1$ and $\tilde{\theta}_2$, which renders the choice of $R$, and therefore of $\lambda$, uninformative. The corresponding loss $L^{FT}(\delta)$ is given by (13).

### 4.4.1 Opacity vs. transparency

From Section 3.1 we know that the optimal choice of $\lambda$ by the central bank depends on the second order condition $1 + z - 2r \gtrless 0$. When $1 + z - 2r \geq 0$ is satisfied, the optimal $\lambda$ is given by the following condition, suitably adjusted from equation (9):

$$\lambda' - 1 \simeq \frac{(1 - 3r + 2r^2) + z'}{(1 - 3r + 2r^2) + (2 - 3r)\, z' + z'^2}\left(\delta' - 1\right) \tag{20}$$

when $\delta$ is in the neighborhood of unity. When $1 + z - 2r < 0$ and this second-order condition is not satisfied, $\lambda \simeq 1$ is a maximum, not a minimum for the loss function; the optimal choice instead is $\lambda = -\frac{1}{\delta}$, see Section 3.1.

When the central bank estimates of private sector variances are not known and the central bank sets $\lambda'$ while the private sector believes that the central bank has chosen $\tilde{\lambda}$, four possibilities may arise: 1) the central bank chooses $\lambda' \simeq 1$ and the private sector correctly believes that $\tilde{\lambda} \simeq 1$; 2) the central bank chooses $\lambda' = -\frac{1}{\delta'}$ and the private sector believes $\tilde{\lambda} = -\frac{1}{\tilde{\delta}}$; 3) the central bank chooses $\lambda' = -\frac{1}{\delta'}$ and the private sector believes that $\tilde{\lambda} \simeq 1$; and 4) the central bank chooses $\lambda' = 1$ and the private sector believes that $\tilde{\lambda} \simeq -\frac{1}{\tilde{\delta}}$.

We know from (18) that $\lambda' \neq \tilde{\lambda}$ produces a systematic bias in the opacity case, which is eliminated under partial and full transparency. This bias therefore tends to make opacity welfare-inferior. The welfare loss due to the systematic bias is to be compared with the two other terms in (18), which correspond to signal errors and are therefore assumed to negligible on average. Intuitively, we expect that the "off-diagonal" cases (3) and (4) above lead to large losses.

The bias is proportional to $\frac{\lambda' - \tilde{\lambda}}{\tilde{\lambda}\left(1 + \lambda'\right)}$. When the central bank chooses $\lambda' = -\frac{1}{\delta'}$ because $1 + z' - 2r < 0$ while the private sector expects $\tilde{\lambda} \simeq 1$ because $1 + \tilde{z} - 2r < 0$, the bias is proportional to $-\frac{1 + \delta'}{\delta' - 1}$, which is close in absolute value to $\infty$. With $\lambda' \simeq 1$ and $\tilde{\lambda} = -\frac{1}{\tilde{\delta}}$ the bias is proportional to $\frac{1 + \tilde{\delta}}{2} \simeq 1$. Thus, indeed, these biases are infinitely larger than the two other sources of welfare losses and opacity is definitely welfare-dominated by one or the other transparency regimes. In case (2), when $\lambda' = -\frac{1}{\delta'}$ and $\tilde{\lambda} = -\frac{1}{\tilde{\delta}}$, the

systematic bias is proportional to $\dfrac{\tilde{\delta}-\delta'}{\delta'-1}$, which is indeterminate but likely finite. In this case too, which occurs when $1+z'-2r<0$ and $1+\tilde{z}-2r<0$, opacity is welfare inferior.

This leaves us with case (1), when $\lambda' \simeq 1$ and $\tilde{\lambda} \simeq 1$; the bias is then proportional to $\dfrac{\tilde{\delta}-\delta'}{2}$ which is of the same order of magnitude as the other terms in (18). Under reasonable assumptions, the Appendix shows that, in this case as well, opacity is welfare-inferior to either transparency regime. This is the case independently of the relative precision $z$ of central bank information.

**Proposition 4** *When the precision of private information is unknown to the central bank, it is never desirable for the central bank to be opaque, i.e. to only publish the interest rate.*

#### 4.4.2 Partial vs. full transparency

It remains, therefore, to determine which of the transparency regimes, partial or full, is more desirable. To do so, we distinguish the same two cases as in Section 3.

**Relatively high central bank precision and weak beauty contest effect ($1+z-2r \geq 0$)** The optimal policy choice $\lambda'$ of the central bank is given by (20). We take as benchmark the optimal policy choice $\lambda$ when the variances are known and $\delta$ measures the relative private sector information precision (see Section 3). With $\delta$ and $\delta'$ in the neighborhood of unity, we know that $\lambda$ and $\lambda'$ are also in the neighborhood of 1. We assume that the central bank measurement error $\delta'-\delta$ is an exogenous random variable with zero mean. Under partial transparency, this measurement error leads to a policy error, which creates a sort of fog that the private sector must see through. A convenient measure of the fog is:

$$F=\sqrt{\frac{E\left((\delta'-\delta)^2\right)}{(\delta-1)^2}}$$

The optimal policy error is:

$$\lambda'-\lambda \simeq \frac{(1-3r+2r^2)+z}{(1-3r+2r^2)+(2-3r)z+z^2}\left(\delta'-\delta\right)$$

This policy error is itself random and its variance is proportional to the variance of $\left(\delta'-\delta\right)$. With $\delta$ and $\lambda$ close to 1, a Taylor expansion gives:

$$L^{PT}\left(\delta,\lambda'\right)-L^{FT}\left(\delta\right) \simeq \frac{\left[(1-2r)(1-r)+z\right]^2F^2-r^2z^2}{2\beta\left[1-r+z\right]^4\left[1-2r+z\right]}z\left(\delta-1\right)^2 \tag{21}$$

whose sign is ambiguous. The Appendix derives the various possible outcomes summarized in Figure 1, which displays the welfare-dominating regime. The intuition is easy to grasp. The larger is the fog, i.e. the more mistaken is the central bank regarding private sector information precision, the less it can manipulate private sector forecasts to attenuate the common knowledge effect. A thick fog, therefore, calls for the central bank to reveal all its information,which makes the interest rate uninformative. On the other hand, the more reactive is the private sector to each other's expectations, the more pronounced is the common knowledge effect and, therefore, the more useful it is for the central bank to use the interest rate to manipulate private sector forecasts. A large $F$ favors full transparency while a large $r$ favors partial transparency. As a consequence, the combination of a thick fog and weak private sector reactivity calls for full transparency

while partial transparency dominates in the opposite case of a thin fog and strong private sector reactivity.

The situation becomes ambiguous for intermediate combinations of $F$ and $r$. Then, the relative precision $z$ of central bank information matters because the gain from manipulation diminishes as the bank becomes better informed (at the limit, when $z \to \infty$, there is no difference between full and less-than-full transparency). When the fog is relatively thick and the beauty contest effect is strong ($F$ and $r$ are large) but the earlier is relatively more important than the latter ($F > r$), full transparency dominates when the central bank information is relatively precise ($z > F(2r-1)\frac{1-r}{F-r}$) because the gain from manipulation decreases since the beauty effect itself is welfare increasing; it is more important to eliminate the fog. When the central bank is less precisely informed, its need to manipulate private sector expectations rises and partial transparency dominates.

In the opposite case of a relatively thin fog and a relatively low beauty contest effect ($F$ and $r$ are low), but with the former relatively lower than the latter ($F < r$), partial transparency dominates full transparency when the central bank information is precise because it becomes relatively more important to eliminate the interest signal signal than to eliminate the fog.

Figure 1

**Proposition 5** *When the precision of the private sector is unknown to the central bank and when the precision of central bank information is high relative to the beauty contest effect, full transparency is welfare-superior when the policy errors generated by the central bank's imperfect knowledge of the private sector information precision are large - the fog around the interest rate is thick. Partial transparency is welfare-superior when the fog is not too thick and when the private sector is highly reactive - the beauty contest effect is strong. The precision of central bank information only matters in intermediate situations; it affects the ability of the central bank to optimally manipulate private sector expectations via its choice of the interest rate.*

Pulling together the results so far, we can contrast the cases when information precision is known or not. When information precision is known, opacity welfare dominates because it allows the central bank to set the interest rate in a way that offsets the common knowledge effect of its publication; transparency removes the information content of the interest rate, leaving the distortionary common knowledge to fully operate. When the central bank is mistaken about private sector precision, and the private sector knows it, a systematic bias arises. The central bank incorrectly sets the interest rate and the private sector cannot unscramble this mistake from the interest rate signal. This raises price volatility and reduces welfare. When the central bank reveals its estimate of private sector information, the systematic bias disappears.

When the central bank only reveals its estimate of private sector information, the interest rate remains a source of common knowledge effect. Given that the central bank holds mistaken beliefs about private sector information, its choice of the interest rate - optimally set to offset the common knowledge effect of its publication - is inaccurate and a source of price volatility, which we call fog. By additionally revealing its signals about the fundamentals, as before, the interest rate becomes uninformative. Thus we face a trade-off: not revealing its estimate of private sector information allows the central bank to optimally manipulate the interest rate but it creates some fog; when it does not reveal its estimate of private sector information, the central bank cannot manipulate the interest rate anymore, but it does not creates any fog either.

In contrast to M&S, the relative precision of central bank information plays a secondary and ambiguous role. It matters mostly when the trade-off between fog and interest rate manipulation is finely balanced. Then, more central bank precision reduces the need to manipulate private sector expectations since the beauty effect through the interest rate, but it also reduces the fog and thus tends to favor using the interest rate.

**The case of relatively low central bank precision and strong beauty contest effect** $(1 + z - 2r < 0)$ When $1 + z - 2r < 0$, the optimal policy choice is $\lambda = -\frac{1}{\delta}$. The Appendix shows that $L^{PT}\left(\delta, \lambda'\right) - L^{FT}\left(\delta\right)$ is of the same as the sign of $(V - 4)$, assuming that $V = E(\lambda' - \lambda)^2$ is exogenous. Since $V$ is unconditional, it is reasonable to consider that it is small, which implies that $L^{PT}\left(\delta, \lambda'\right) < L^{FT}\left(\delta\right)$. Partial transparency now dominates full transparency because it allows a relatively poorly informed central bank to conceal the signal value of the interest rate by increasing its variance.

**Proposition 6** *When the precision of private information is unknown to the central bank and when the precision of central bank information is low relative to the beauty contest effect, partial transparency welfare-dominates full transparency. Partial transparency allows the central bank to reduce the common knowledge effect of its interest rate without facing the knowledge effect of its full information set.*

# 5 The private sector is mistaken about its own variances

In Sections 4 and 4.4.2 we assume that the private sector precision is unknown to the central bank but known to the private sector. This section briefly looks at a more symmetric setup where the private sector does not know either its own variance. Its estimates of $\delta$ and $z$ are denoted $\hat{\delta}$ and $\hat{z}$. Unaware of its erroneous forecasts, the private sector therefore uses $\hat{\delta}$ and $\hat{\beta}$ to estimate $\lambda$ in the central bank interest rate rule. Note that the losses are still computed using the true values $\delta$ and $z$.

We use the following notation for the welfare losses:

- The central bank is opaque, i.e. it only publishes the interest rate: $L^{op}(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta)$

- The central bank is partially transparent, i.e. it additionally reveals its variance estimates: $L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta)$

- The central bank is fully transparent, i.e. it additionally reveals its signals: $L^{FT}(\beta', \hat{\delta}, \delta)$.

We cannot, in general compare the losses. Instead we look at the limit cases when the central bank precision of information is either very large ($z \to \infty$) or very low ($z \to 0$) relatively to private sector information. The following results are established in the Appendix:

When central bank information precision is relatively high and the beauty contest effect is relatively weak $(1 + z - 2r \geq 0)$ so that optimal monetary policy is $\lambda$ close to 1:

When $z \to \infty$

$$L^{op}(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) > L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) > L^{FT}(\hat{\beta}, \hat{\delta}, \delta)$$

When $z \to 0$

$$L^{op}(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) > L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) \gtrless L^{FT}(\hat{\beta}, \hat{\delta}, \delta)$$

Thus we reach the same conclusion as in Section 4: it is generally preferable for the central bank to be transparent. When $z$ is large, it is even better for the central bank to be fully transparent. When $z$ is low, the relative desirability of partial and full transparency is ambiguous. In this case, because a partially transparent central bank, which uses

the interest rate to affect private sector expectations, makes a large mistake as it chooses $\lambda'$which affects $L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta)$ while the loss under full transparency $L^{FT}(\hat{\beta}, \hat{\delta}, \delta)$ is mostly affected by the private sector error $\hat{\beta}$ (the central bank's error on $\hat{\delta}$ affects both losses).

When central bank information precision is relatively low and the beauty contest effect is relatively strong $(1 + z - 2r < 0)$ so that optimal monetary policy is $\lambda = -\frac{1}{\delta}$, we only consider the polar case $z \to 0$. Here again, the conclusions from Section 4 generally hold.

# 6 Conclusions

In a world of imperfectly-informed agents who face a strong incentive to converge to the mean (the beauty contest effect) signals that are known to be observed by all (common knowledge) have an impact on expectations that is not justified by their precision. Everyone follows the flute-payer, in other terms. M&S have observed that when this observation is applied to monetary policy, it may justify central bank opacity if the central bank is poorly informed. The flute-player becomes the head-sheep that jumps off the cliff.

This warning against central bank transparency has been criticized by Svensson (2005), who doubts that central banks are relatively less well informed than private agents. Others have observed that when the welfare criterion includes the beauty contest effect, central bank transparency raises welfare. Yet, the flute-player effect stands and reduces welfare.

In this paper, we add a new dimension by recognizing that central banks do not have any more the option of being completely opaque (the flute player must play the flute). At the very least, they reveal the interest rate that they have chosen. Given that they already create a distortionary common knowledge effect, should they reveal additional information? The answer is not unambiguous, it depends on the details of what is known by whom, but a few new intuitions emerge.

To start with, the interest rate is a signal that can be manipulated by the central bank to raise welfare. In our model, this is in fact the only use that the central makes of the interest rate. An optimizing flute player does not jump of the cliff; he may also decide to charm away his followers in the desired direction; he may even hide in the bushes to distill his music when the fog is thick.

The paper also attracts attention to what is known about information precision. When there is no uncertainty about central bank and private sector precision, opacity dominates. The reason is that, under transparency, the central bank can no longer manipulate the interest rate and yet creates a common knowledge effect when it reveals its information. This conclusion echoes the traditional arguments in favor of "central bank mystique". Once information precision is not perfectly known, transparency - partial or full - dominates opacity. The reason is that the central bank makes an unavoidable policy error when its beliefs about private sector information are mistaken. The flute player is now disoriented and must ask the crowd to help him find the way. Maybe central banks have become more transparent once they lost confidence in their superior knowledge.

Alternatively, the trend towards transparency may reflect better central bank information precision. The difference between this paper and M&S is that their central bank, like the Federal Reserve before 1993, does not publish the interest rate while our central bank, like any modern central bank, immediately indicates its interest rate decisions. The old Fed may have found it desirable to be fully opaque, as Morris and Shin suggest is desirable when the central bank information is poor; the new Fed may raise welfare by being transparent.

Obviously, the present paper suffers from a number of limitations that should be kept in mind before drawing firm policy conclusions. To start with, the interest rate plays no direct macroeconomic role in our model. Its only function is to convey some information

about the central bank signals. This explains why we find that, in some instances, the central bank should considerably raise the variance of the interest rate, as if the flute player would be hiding to avoid being followed. While somewhat unrealistic, this assumption allows us to isolate the information content of the interest rate. In addition, we treat as exogenous the variances of the forecast errors.

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## Appendix

### Derivation of (6)

Iterating expectations as in (3), we have:

$$p_i = (1-r)\sum r^k E^i\left(\bar{E}^k(A\theta)\right) = AM\frac{1-r}{1-rM}\begin{bmatrix} R \\ x_1^i \\ x_2^i \end{bmatrix}$$

where:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ \frac{\alpha}{\mu} & \frac{\beta}{\delta} & -\frac{\nu\alpha}{\mu} \\ \frac{\alpha}{\nu} & -\frac{\mu\alpha}{\nu} & \beta \end{bmatrix}$$

Note that:

$$\bar{E}\begin{bmatrix} R \\ \theta_1 \\ \theta_2 \end{bmatrix} = M\begin{bmatrix} R \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

Using:

$$1 - rM = \begin{bmatrix} 1-r & 0 & 0 \\ -r\frac{\alpha}{\mu} & 1-r\frac{\beta}{\delta} & r\frac{\nu\alpha}{\mu} \\ -r\frac{\alpha}{\nu} & r\frac{\mu\alpha}{\nu} & 1-r\beta \end{bmatrix}$$

we obtain:

$$(1-r)(1-rM)^{-1} = \frac{1}{\Delta}\begin{bmatrix} \Delta & \left(r\frac{\alpha}{\mu}\right)(1-r\beta) - r^2\frac{\alpha^2}{\mu} & r\frac{\alpha}{\nu}\left(1-r\frac{\beta}{\delta}\right) - r^2\frac{\alpha^2}{\nu} \\ 0 & (1-r)(1-r\beta) & -(1-r)\,r\frac{\mu\alpha}{\nu} \\ 0 & -(1-r)\,r\frac{\nu\alpha}{\mu} & (1-r)\left(1-r\frac{\beta}{\delta}\right) \end{bmatrix}^t$$

with:

$$\Delta = \left[\left(1-r\frac{\beta}{\delta}\right)(1-r\beta) - r^2\alpha^2\right]$$

As a consequence:

$$\begin{aligned} & M\frac{1-r}{1-rM} \\ = \; & \frac{1}{\Delta}\begin{bmatrix} 1 & 0 & 0 \\ \frac{\alpha}{\mu} & \frac{\beta}{\delta} & -\frac{\nu\alpha}{\mu} \\ \frac{\alpha}{\nu} & -\frac{\mu\alpha}{\nu} & \beta \end{bmatrix} \times \\ & \times \begin{bmatrix} \Delta & 0 & 0 \\ \left(r\frac{\alpha}{\mu}\right)(1-r\beta) - r^2\frac{\alpha^2}{\mu} & (1-r)(1-r\beta) & -(1-r)\,r\frac{\nu\alpha}{\mu} \\ r\frac{\alpha}{\nu}\left(1-r\frac{\beta}{\delta}\right) - r^2\frac{\alpha^2}{\nu} & -(1-r)\,r\frac{\mu\alpha}{\nu} & (1-r)\left(1-r\frac{\beta}{\delta}\right) \end{bmatrix} \\ = \; & \begin{bmatrix} 1 & 0 & 0 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{bmatrix} \end{aligned}$$

with :

$$X_1 = \frac{\alpha}{\mu} + \frac{\beta}{\delta\Delta}\left[\left(r\frac{\alpha}{\mu}\right)(1-r\beta) - r^2\frac{\alpha^2}{\mu}\right] - \frac{\alpha}{\Delta\mu}\left[r\alpha\left(1-r\frac{\beta}{\delta}\right) - r\alpha^2\right]$$

$$X_2 = \frac{\alpha}{\nu} - \frac{\alpha}{\nu\Delta}\left[(r\alpha)(1-r\beta) - r^2\alpha^2\right] + \frac{\beta}{\Delta}\left[r\frac{\alpha}{\nu}\left(1 - r\frac{\beta}{\delta}\right) - r^2\frac{\alpha^2}{\nu}\right]$$

$$Y_1 = \frac{\beta}{\delta\Delta}(1-r)(1-r\beta) + (1-r)\,r\frac{\alpha^2}{\Delta}$$

$$Y_2 = -\frac{\mu\alpha}{\nu\Delta}(1-r)(1-r\beta) - \frac{\beta}{\Delta}(1-r)\,r\frac{\mu\alpha}{\nu}$$

$$Z_1 = -\frac{\beta}{\delta\Delta}(1-r)\,r\frac{\nu\alpha}{\mu} - \frac{\nu\alpha}{\mu}(1-r)\left(1 - r\frac{\beta}{\delta}\right)$$

$$Z_2 = \frac{\beta}{\Delta}(1-r)\left(1 - r\frac{\beta}{\delta}\right) + (1-r)\,r\frac{\alpha^2}{\Delta}$$

Multiplying on the right hand-side by $\begin{bmatrix} R \\ x_1^i \\ x_2^i \end{bmatrix}$ and on the left hand-side by $A$ leads directly, after some simplifications, to (6).

Proof of Proposition 1

We offer here a proof in the general case with $N$ fundamentals $\theta_k$, $k = 1, ..., N$, assuming that the price level is given by:

$$p^i = (1-r)\sum_{n=0}^{\infty} r^n E^i\left(\bar{E}^n(A\theta)\right)$$

where $A\theta = \sum_{k=1}^{N}\theta_k$. The Central Bank and private agents receive, respectively, the following signals about the fundamentals:

$$\widetilde{\theta}_k = \theta_k + \varepsilon_k \qquad k = 1, ..., N$$

$$x_k^i = \theta_k + \eta_k^i \qquad k = 1, ....N$$

The Central Bank sets the interest rate according the linear rule

$$R = \sum_{k=1}^{N}\mu_k\tilde{\theta}_k$$

constrained by $\sum_{k=1}^{N}\mu_k = 1$.

When the Central Bank releases only its interest rate, the private sector expectations are given by:

$$E^i(\theta_k) = \gamma_k\left(\frac{R - \sum_{j=1, j\neq k}^{N}\mu_j x_j^i}{\mu_k}\right) + (1-\gamma_k)x_k^i \tag{22a}$$

where :

$$\gamma_k = \frac{\frac{1}{\beta_k}\mu_k^2}{\sum_{j=1}^{N}\mu_j^2\left(\frac{1}{\alpha_j} + \frac{1}{\beta_j}\right)}$$

A straightforward generalization of case with two fundamentals leads to the individual price level:

$$p^i = \left(\sum_{j=1}^{N}\frac{1}{\mu_j}\varphi_j\right)R + \sum_{k=1}^{N}\left(1 - \sum_{j=1}^{N}\frac{\mu_k}{\mu_j}\varphi_j\right)x_k^i \tag{23}$$

with

$$\varphi_k = \frac{\gamma_k}{1 - r(1 - \sum_{k=1}^{N} \gamma_k)}$$

Under full transparency, the price level is given by:

$$p_i = \sum_{k=1}^{N} \left( \overline{\varphi}_k \tilde{\theta}_k + (1 - \overline{\varphi}_k) \, x_k^i \right) \tag{24}$$

where:

$$\overline{\varphi}_k = \frac{z_k}{z_k + (1 - r)}$$

with $z_k = \alpha_k / \beta_k$. A straightforward computation shows that choosing

$$\frac{\mu_k}{\mu_j} = \frac{\overline{\varphi}_k}{\overline{\varphi}_j}$$

implies $\sum_{j=1}^{N} \frac{\mu_k}{\mu_j} \varphi_j = \overline{\varphi}_k$. Using $\sum_{k=1}^{N} \mu_k = 1$, this determines a unique set for $\mu_k$ for $k = 1, N$ such that the price level under opacity and under transparency are identical.

This establishes that it is always possible under opacity to reproduce the transparency outcome. It follows that opacity is always at least as good as transparency; it is in general strictly better since there is no reason to assume that the particular choice that reproduces transparency is optimal.

The result is independent from chosen loss function and is therefore completely general.

### Proof of Proposition 4

We first compare $L^{op}\left(\delta, \tilde{\lambda}, \lambda'\right)$ and $L^{PT}\left(\delta, \lambda'\right)$, the losses under opacity and partial transparency. We assume that $\lambda' - \lambda$ is white noise $u_1$ with exogenous variance $V$. We also need to consider the private sector forecast error $\tilde{\lambda} - \lambda' = E^{pr}\left(\lambda - \lambda'\right)$. We assume that this step involves an additive error $u_2$ which is white noise with unconditional variance $V_2$ and $u_1$ and $u_2$ are taken as orthogonal. Intuitively, we consider that expectations of expectations additively increase the variance of forecast errors.

Using (18) and (19) we obtain in the neighborhood of $\delta = 1$:

$$\begin{aligned} L\left(\delta, \tilde{\lambda}, \lambda'\right) - L^{PT}\left(\delta, \lambda'\right) &= L^{PT}\left(\delta, \tilde{\lambda}\right) - L^{PT}\left(\delta, \lambda'\right) \\ &\quad + E\left( \frac{\frac{\left(\tilde{\lambda} - \lambda'\right)^2}{\left(\tilde{\lambda}(1 + \lambda')\right)^2} \left( \frac{2}{\alpha} + \left(\theta_1 - \theta_2\right)^2 \right) \tilde{\lambda}^2 (\delta \tilde{\lambda} + 1)^2 \tilde{z}^2}{\left[ (1 - r)\left(1 + \tilde{\lambda}^2\right) + \left(1 + \tilde{\lambda}^2 \delta\right) \tilde{z} \right]^2} \right) \end{aligned}$$

When $1 + z - 2r > 0$ so that $\lambda \approx \tilde{\lambda} \approx \lambda' \approx 1$, a Taylor expansion of the above equation gives:

$$L^{op}\left(\delta, \tilde{\lambda}\right) - L^{PT}\left(\delta, \lambda'\right) = E\left[ \left(\lambda' - \lambda\right)^2 - \left(\tilde{\lambda} - \lambda\right)^2 \right] \frac{1}{\beta} \frac{2\left(\lambda^2 - 3\right)\left(z^2 + z\left(1 - 2r\right)\right)\lambda}{\left(1 - r + z\right)^2 \left(1 + \lambda^2\right)^3}$$

Computing the expectations yields:

$$\begin{aligned} & L^{op}\left(\delta,\tilde{\lambda},\lambda'\right)-L^{PT}\left(\delta,\lambda'\right) \\ = \; & E\frac{\left(\tilde{\lambda}-\lambda'\right)^2}{\left(\tilde{\lambda}(1+\lambda')\right)^2}\left(\frac{2}{\alpha}+(\theta_1-\theta_2)^2\right)\frac{\tilde{z}^2\tilde{\lambda}^2(\delta\tilde{\lambda}+1)^2}{\left((1-r)\left(1+\tilde{\lambda}^2\right)+\left(1+\tilde{\lambda}^2\delta\right)\tilde{z}\right)^2} \\ & +E\left[\left(\lambda'-\lambda\right)^2-\left(\tilde{\lambda}-\lambda\right)^2\right]\frac{1}{\beta}\frac{2\left(\lambda^2-3\right)\left(z^2+z\left(1-2r\right)\right)\lambda}{\left((1-r)+z\right)^2\left(1+\lambda^2\right)^3} \end{aligned}$$

At the lowest order in $\lambda-\lambda'$ we have:

$$\begin{aligned} & L^{op}\left(\delta,\tilde{\lambda},\lambda'\right)-L^{PT}\left(\delta,\lambda'\right) \\ = \; & \frac{1}{4}E\left(\tilde{\lambda}-\lambda'\right)^2\left(\frac{2}{\alpha_{\varepsilon}}+(\theta_1-\theta_2)^2\right)\frac{\tilde{z}^2}{\left(1-r+\tilde{z}\right)^2} \\ & -\frac{1}{2}E\left[\left(\lambda'-\lambda\right)^2-\left(\tilde{\lambda}-\lambda\right)^2\right]\frac{1}{\beta}\frac{(1+z-2r)}{(1-r+z)^2} \\ = \; & \frac{1}{4}V_2\left(\frac{2}{\alpha}+(\theta_1-\theta_2)^2\right)\frac{\tilde{z}^2}{\left(1-r+\tilde{z}\right)^2}+\frac{1}{2}V_2\frac{(\alpha+\beta\left(1-2r\right))}{\left(\beta\left(1-r\right)+\alpha\right)^2}>0 \end{aligned}$$

Note that we do not need to further compare opacity and full transparency. If full transparency welfare-dominates partial transparency it also welfare-dominates opacity and is the best regime. If partial transparency welfare-dominates full transparency, since it welfare-dominates opacity, it is the best regime.

### Derivation of results of Section 4.4.2

The study of the sign of (21) leads to the following results:

- When $F>1$, $L^{PT}\left(\delta,\lambda'\right)>L^{FT}\left(\delta\right)$, i.e. full transparency dominates partial transparency when the fog is large. Under partial transparency the policy error creates a fog that is dissipated under full transparency when the private sector does not pay any attention to the interest rate.
- When $1>F>r$ and $r<1/2$, we still have $L^{PT}\left(\delta,\lambda'\right)>L^{FT}\left(\delta\right)$. The fog is less thick but the beauty contest effect is not too strong.
- When $1>F>r$ and $r>1/2$, we need to dsitinguish two cases:
  - if $z>F\left(2r-1\right)\frac{1-r}{F-r}$, we have $L^{PT}\left(\delta,\lambda'\right)>L^{FT}\left(\delta\right)$.
  - if $z<F\left(2r-1\right)\frac{1-r}{F-r}$, we now have $L^{PT}\left(\delta,\lambda'\right)>L^{FT}\left(\delta\right)$.
- When $r>F>0$ and $r<1/2$:
  - if $z>F\left(1-2r\right)\frac{1-r}{r-F}$ : $L^{PT}\left(\delta,\lambda'\right)<L^{FT}\left(\delta\right)$
  - if $z<F\left(1-2r\right)\frac{1-r}{r-F}$ : $L^{PT}\left(\delta,\lambda'\right)>L^{FT}\left(\delta\right)$
- When $r>F>0$ and $r>1/2$, $L^{PT}\left(\delta,\lambda'\right)<L^{FT}\left(\delta\right)$.

**Proof** We first look at two limit cases, keeping in mind the second order condition $1+z-2r>0$. Let $V=E(\lambda-\lambda')^2$.

When $z\to 0$, the second order condition becomes $1-2r>0$, $L^{PT}\left(\delta,\lambda'\right)-L^{FT}\left(\delta\right)\to\frac{V}{2\beta}\frac{1-2r}{(1-r)^2}z$ is of the same sign as $1-2r$.

When $z\to\infty$, $L^{PT}\left(\delta,\lambda'\right)-L^{FT}\left(\delta\right)\to\frac{V}{2\beta}>0$.

Next, in order to deal with the general case, note that the denominator of (21) is positive when $1+z-2r>0$. Denote $N$ the numerator :

$$\begin{aligned} N &= (\delta-1)^2 z\left[F^2\left[(1-2r)(1-r)+z\right]^2-r^2z^2\right] \\ &= (\delta-1)^2\left[\left(1-3r+2r^2+z\right)F+rz\right]\left[\left(1-3r+2r^2+z\right)F-rz\right] \end{aligned}$$

Noting that $\left(1-3r+2r^2+z\right)F+rz=\left((1-2r)(1-r)+z\right)F+rz$ is positive given our assumption when $1+z-2r>0$, the sign of $N$ reduces thus to to sign of $\left(1-3r+2r^2\right)F+(F-r)z$. This last quantity is positive for $z>(1-2r)(1-r)\frac{F}{r-F}$, when $F-r>0$, and negative when $F-r<0$.

However, remember that the solution $z=(1-2r)(1-r)\frac{F}{r-F}$ to the equation $N=0$ has to belong to the domain $1+z-2r>0$, which imposes the constraint $r(F-1)\frac{1-2r}{F-r}>0$. If this condition is satisfied, $N$ cancels for $z=(1-2r)(1-r)\frac{F}{r-F}$. If it is not, the sign of $N$ will be constant. As a consequence, to study when $r(F-1)\frac{1-2r}{F-r}>0$ we need to consider three cases $F>1$, $1>F>r$, and $r>F$. A straightforward study of each of these cases gives the results above:

When F$>1$, $r(F-1)\frac{1-2r}{F-r}>0$ implies $1-2r>0$, but in such a case, the solution $z$ to $N=0$ is negative, which is impossible. As a consequence $N>0\ \forall z$.

When $F<r$, the same reasonning apply, $1-2r>0$ is imposed and a solution to $N=0$ is $z=(1-2r)(1-r)\frac{F}{r-F}$, leading to the following conditions:

- if $z>(1-2r)(1-r)\frac{F}{r-F}$ then $N<0$
- if $z<(1-2r)(1-r)\frac{F}{r-F}$ then $N>0$.
- if $1-2r<0$, there is no solution to $N=0$ and one has always $N<0$.

When $1>F>r$, $r(F-1)\frac{1-2r}{F-r}>0$ imposes $1-2r<0$. In that case, if $z>F(2r-1)\frac{1-r}{F-r}$, $N>0$. If instead $z<F(2r-1)\frac{1-r}{F-r}$, $N<0$. If $1-2r>0$, one has always $N>0$.

Proof of Propostion 6

When $1+z-2r+\frac{\alpha}{\beta}<0$. with $V=E(\lambda-\lambda')^2$ treated as exogenous, we have:

$$L^{PT}\left(\delta,\lambda'\right)=L^{PT}(\delta,\lambda)+\frac{z(2r-1-z)}{2(z+1-r)^2}E\left(\lambda'+\frac{1}{\delta}\right)^2$$

so that:

$$\begin{aligned} L^{PT}\left(\delta,\lambda'\right)-L^{FT}(\delta) &= L^{PT}(\delta,\lambda)-L^{FT}(\delta)+\frac{z(2r-1-z)}{2(z+1-r)^2}E\left((\lambda-\lambda')^2\right) \\ &\simeq z\frac{(2r-1-z)}{2\beta(z+1-r)^2}(V-4) \end{aligned}$$

as announced in the text.

Derivation of results in Section 5

**Relatively high central bank precision and weak beauty contest effect** $(1+z-2r\geq 0)$ When the central bank is partially transparent, denoting $\hat{z}=a/\hat{\beta}$, the

unconditional expectation of the loss is:

$$\begin{aligned} L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) &= E(\frac{\left(1+\lambda'^2\right)\left(\hat{\delta}\lambda'+1\right)^2\hat{z} + \delta\frac{\tilde{z}}{\hat{z}}\left(\left(1-r\right)\left(1+\lambda'^2\right)+\left(1-\lambda'\right)\hat{z}\right)^2}{\hat{\beta}\left(\left(1-r\right)\left(1+\lambda'^2\right)+\left(1+\lambda'^2\hat{\delta}\right)\hat{z}\right)^2} \\ &+\frac{\frac{\tilde{z}}{\hat{z}}\left(\left(1-r\right)\left(1+\lambda'^2\right)+\left(\lambda'-1\right)\lambda'\hat{\delta}\hat{z}\right)^2}{\hat{\beta}\left(\left(1-r\right)\left(1+\lambda'^2\right)+\left(1+\lambda'^2\hat{\delta}\right)\hat{z}\right)^2}) \end{aligned}$$

So:

$$L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) = L^{PT}(\lambda', \delta) + B$$

with :

$$B = \frac{2}{\beta} z \frac{\left(1-r\right)^2\left(2r+1+z\right)}{\left(z+1-r\right)^4} E\left(\frac{\hat{\beta}}{\beta}-1\right)^2 - \frac{1}{2\beta} z \frac{\left(1-r\right)\left(3zr+r-1-z\right)}{\left(z+1-r\right)^4} E\left(\hat{\delta}-\delta\right)^2$$

When the central bank is opaque, the unconditional expectation of the loss is:

$$\begin{aligned} L^{op}(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) &= L^{op}(\tilde{\lambda}, \delta) + A \\ &= L^{PT}(\tilde{\lambda}, \delta) + \text{Bias term} + A \end{aligned}$$

The loss decomposes into three terms where $A$.comes from the correction due to $\hat{\beta}, \hat{\delta}$. At our order of approximation $A = B$. $L_1(\tilde{\lambda}, \delta)$ and the bias term is similar to the one obtained in Section 4.

When the central bank is fully transparent, the unconditional expectation of the loss is:

$$L^{FT}(\beta', \hat{\delta}, \delta) = E\left(\frac{\hat{z} + \frac{\tilde{z}}{\hat{z}}\left(1-r\right)^2}{\hat{\beta}\left(\hat{z}+\left(1-r\right)\right)^2} + \frac{\left(\hat{\delta}\right)^2\hat{z} + \delta\frac{\tilde{z}}{\hat{z}}\left(1-r\right)^2}{\hat{\beta}\left(\hat{\delta}\hat{z}+\left(1-r\right)\right)^2}\right)$$

that can be written as:

$$L^{FT}(\hat{\beta}, \hat{\delta}, \delta) = L^{FT}(\delta) + C$$

with:

$$C = -z\frac{\left(1-r\right)\left(3zr+r-z-1\right)}{\beta\left(z+\left(1-r\right)\right)^4} E\left(\hat{\delta}-\delta\right)^2 + 2\left(1-r\right)^2 z \frac{1+2r+z}{\beta\left(z+\left(1-r\right)\right)^4} E\left(\frac{\hat{\beta}}{\beta}-1\right)^2$$

Given the results from Section 4, noting that $A = B$, we have:

$$\begin{aligned} L^{op}(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) - L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) &= L^{op}(\tilde{\lambda}, \delta) + A - L^{PT}(\lambda', \delta) - B \\ &= L^{PT}(\tilde{\lambda}, \delta) - L^{PT}(\lambda', \delta) + \text{Bias term} > 0 \end{aligned}$$

The other comparison is:

$$\begin{aligned} L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) - L^{FT}(\hat{\beta}, \hat{\delta}, \delta) &= L^{PT}(\lambda', \delta) - L^{FT}(\delta) + B - C \\ & L^{PT}(\lambda', \delta) - L^{FT}(\delta) + \frac{1}{2} z \frac{\left(1-r\right)\left(3zr-z+r-1\right)}{\beta\left(z+\left(1-r\right)\right)^4}\left(\hat{\delta}-\delta\right)^2 \end{aligned}$$

Using our previous results about the losses $L_1(\lambda', \delta)$ and $L_2(\delta)$, we can study our limit cases:

When $z \to 0$ $\quad L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) - L^{FT}(\hat{\beta}, \hat{\delta}, \delta) = E\left(\lambda' - \lambda\right)^2 \frac{z(1-2r)}{2\beta\left((1-r)\right)^2} - \frac{z}{2\beta\left((1-r)\right)^2} E\left(\hat{\delta} - \delta\right)^2$

which can be positive or negative, depending on the relative magnitudes of $E\left(\lambda' - \lambda\right)^2$ and $E\left(\hat{\delta} - \delta\right)^2$.

$$\text{When } z \to \infty \qquad L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) - L^{FT}(\hat{\beta}, \hat{\delta}, \delta) \approx E\left(\lambda' - \lambda\right)^2 \frac{1}{2\beta} > 0$$

**Relatively low central bank precision and strong beauty contest effect** $(1 + z - 2r < 0)$ When the optimal policy is $\lambda = -\frac{1}{\delta}$:

$$\begin{aligned} L^{FT}(\hat{\beta}, \hat{\delta}, \delta) &= \frac{1+\delta}{\beta} + \left(\delta^2 + 1\right) \frac{(2r-1)\, z}{\beta\left(-1+r\right)^2} + \frac{1}{\beta} \frac{z}{\left(1-r\right)^2} E\left(\hat{\delta} - \delta\right)^2 \\ &\quad + \frac{1}{\beta} z \left(\left(\delta^2 + 1\right)(1+2r)\right) \frac{E\left(\frac{\hat{\beta}}{\beta} - 1\right)^2}{\left(1-r\right)^2} \\ &\approx L^{FT}(\delta) + \frac{1}{\beta} \frac{z}{\left(1-r\right)^2} E\left(\hat{\delta} - \delta\right)^2 + \frac{1}{\beta} 2z\,(1+2r) \frac{E\left(\frac{\hat{\beta}}{\beta} - 1\right)^2}{\left(1-r\right)^2} > L^{FT}(\delta) \end{aligned}$$

for $\delta$ close to 1.

In Section 4 we have seen that $L^{op}\left(\delta, \tilde{\lambda}, \lambda'\right) - L^{PT}\left(\delta, \lambda'\right) > 0$. By the same arguments as above, we have:

$$L^{op}(\lambda', \tilde{\lambda}, \hat{\beta}, \hat{\delta}, \delta) - L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) > 0$$

Now computing $L^{PT}(\lambda', \delta, \beta', \hat{\delta})$ as previously leads to:

$$L^{PT}(\lambda', \hat{\beta}, \hat{\delta}, \delta) = \frac{1+\delta}{\beta} + \frac{1}{\beta} z\,(\text{Variance terms})$$

where the variance terms are of order $E\left(\hat{\delta} - \delta\right)^2$, $E\left(\frac{\hat{\beta}}{\beta} - 1\right)^2$, $E\left(\lambda' - \lambda\right)^2$. This formula yields the same result as in Section 4.

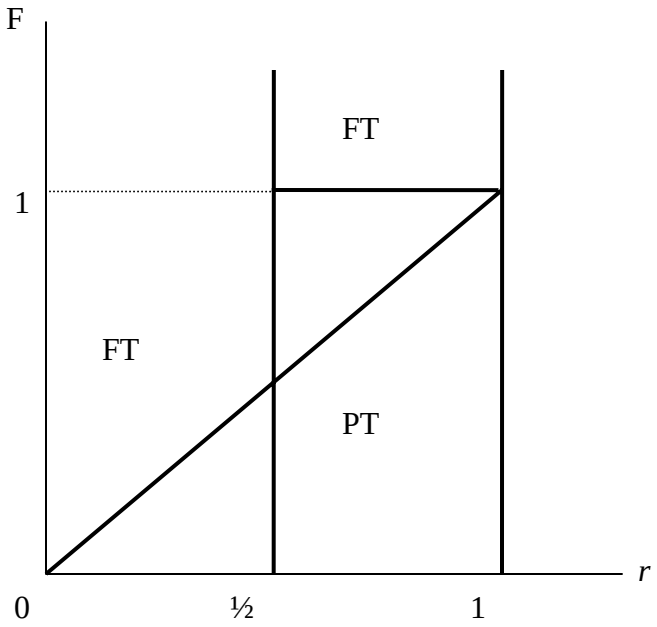



**Figure 1. Optimal Transparency Regimes**
**(Case $1 + z - 2r \geq 0$)**