Monetary policy in Europe vs the US: what explains the difference?*

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COMMENTS WELCOME

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Abstract

This paper compares monetary policy in the US and EMU during the last decade, employing an estimated hybrid New Keynesian cash-in-advance model, driven by five shocks. It appears that the difference between the two monetary policies between 1998 and 2006 is due to both surprises in productivity as well as surprises in wage demands, moving interest rates in opposite directions in Europe and the US, but not due to a more sluggish response in Europe to the same shocks or to different monetary policy surprises.

Keywords:

JEL codes:

1 Introduction

Interest rate paths during the last decade or so have been remarkably different in the US and in Europe, see figure 1. What explains the difference? Three different interpretations for this observation have been put forth.

One interpretation is to argue that monetary policy is simply different. A number of observers have argued that the difference in policy shows the difference between an established central bank in the US, which knows what it is doing and acts decisively, if need be, versus a new central bank in Europe, run by a committee which is too timid and too inertial to anything in time, following the US example with too much caution and delay. A more benign interpretation - recently put forth e.g. by ECB president Trichet in a speech on "Activism and alertness in monetary policy" in Madrid 2006 - argues that instead it is the ECB holding the steering wheel steady, while the monetary policy pursued by the Fed is just erratic.

The second interpretation is that the shocks simply have been different. For example, growth in the US was considerably higher in the second half of the 90s in the US, giving rise to fear of "overheating" there and thereby possibly necessitating policy interventions, which then needed to be reversed, as the US economy spun into a recession. While the decline in growth rates in EMU may have been similarly large between 2000 and 2002, the growth rate only briefly achieved US levels in 2000, though, see figure 2.

The third interpretation is that the structure of the economies are simply different. There are three striking differences in particular:

1. Labor markets are more rigid in Europe than in the US. While one can point to some measures, the evidence here comes more from a variety of sources and qualitative measures, starting with labor market regulations and government interference in the labor market to union memberships and the role of unions in economic policy and the gover-

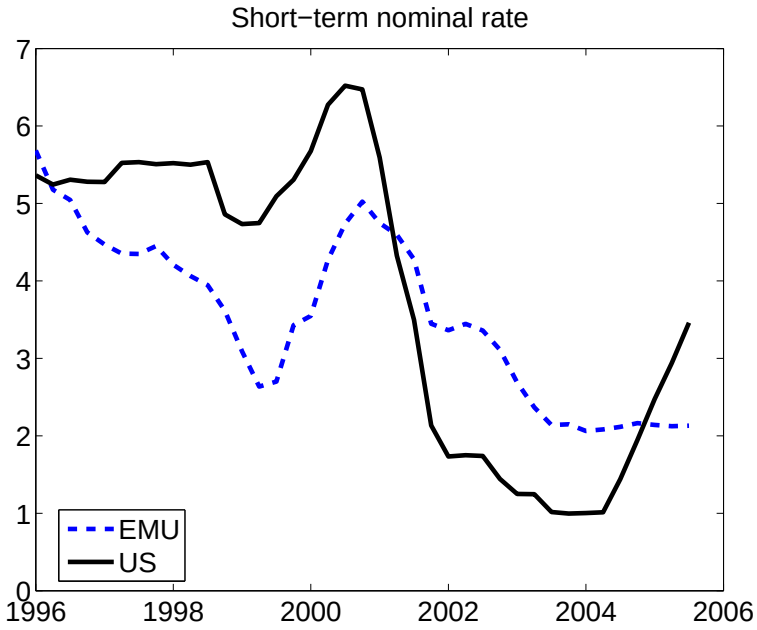


Figure 1: Central bank rates in the EMU and in the US.

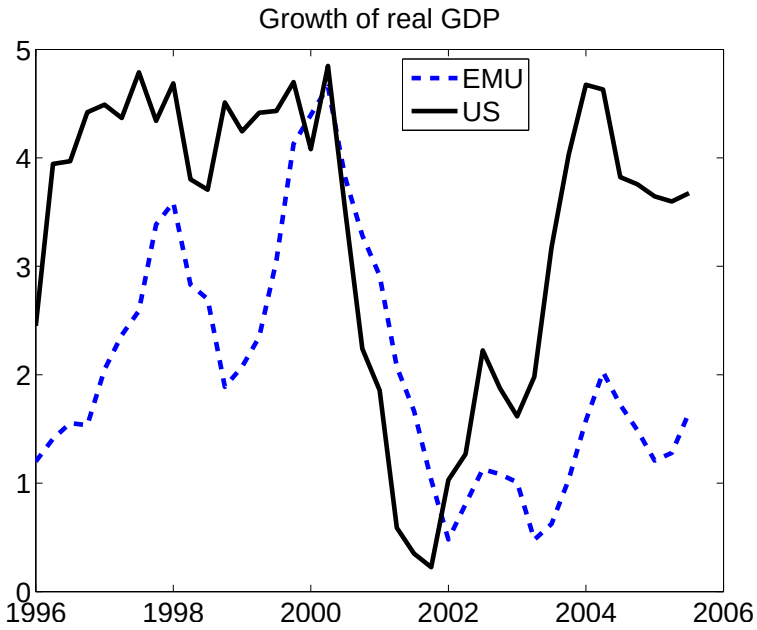


Figure 2: Real GDP growth in the US and EMU.

nance of firms.

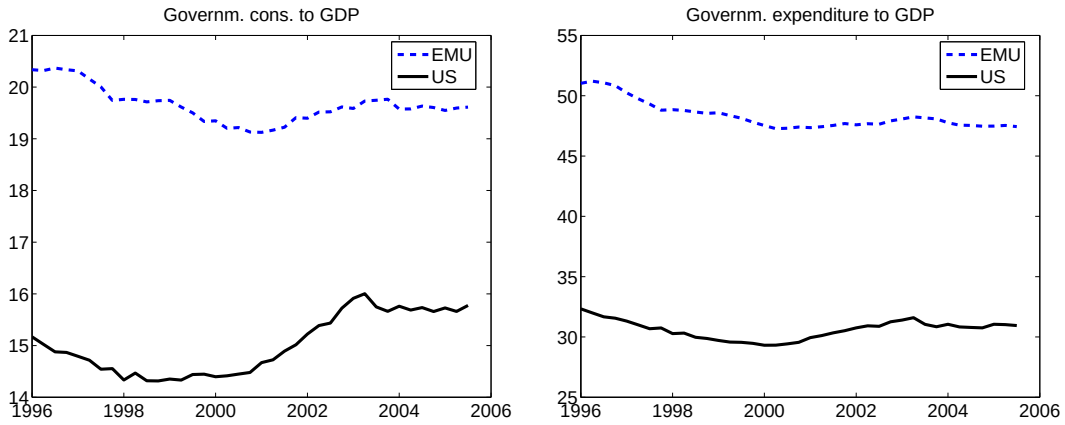
2. The share of government is larger in Europe than it is in the US. For the period from 1985 to 2005, mean government consumption to GDP was 16 percent in the US and 20 percent in the US. For government expenditure, the contrast was even more striking, with 32 percent in the US versus 50 percent in Europe, see also figure **??**. Fiscal policy is arguably also more decentralized in Europe, with Brussels playing a minor role vis-a-vis the nation states in Europe compared to the federal government vis-a-vis state and local governments in the US.

3. A much larger share of business is bank-financed rather than market financed in EMU, compared to the US. For example, de Fiore and Uhlig (2006) document that the ratio of debt-to-equity is .41 in the US and .61 in Europe. Furthermore, the ratio of bank-to-bond finance is 7.3 in the EMU and thus ten times as high as 0.74, the value for the US.

It seems a priori plausible that these differences play a significant factor in the explanations for monetary policy. For example, government spending tends to be rather smooth and acyclical: a larger share of government spending might therefore lessen the role of price rigidities for the private economy.

I seek to provide a tentative answer the lead question of this paper. Recent advances in the modelling of dynamic stochastic general equilibrium models - e.g. Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005) and related work - in particular have made it possible in principle to impose the key structural differences of the economy, estimate monetary policy reaction functions and quantitatively accounting for the movements in key variables by a decomposition into the model-specific structural shocks. This avenue is therefore well-suited for answering the question at hand.

These models are built on recent advances in investigating the role of sticky prices for the economy and the New Keynesian paradigm, see in par-



Government consumption as a share of GDP

Government expenditure as a share of GDP

Figure 3: Government consumption and government expenditure

ticular Clarida, Gali and Gertler (1999) and Woodford (2003). Applying them directly to the task at hand poses three challenges, however.

1. These models emphasize the role of sticky prices and the output gap in driving inflation rates. Frictions from the interaction between financial intermediation and monetary policy typically play no role or a role only insofar as they influence the output gap. This makes it challenging to address the third of the three key differences above.

2. The distortionary role of non-monetary economic policy typically plays a minor role. This makes it hard to address the first the second of the key differences listed above.

3. In quantitative applications and estimations, many observable time series are used. An equivalent number of shocks is then used in order to generate a regular one-step ahead variance-covariance matrix of the prediction errors. This makes it challenging to avoid pushing key dynamic

features of the economy into "measurement errors" instead, which then receive a structural interpretation.

There is an earlier literature, emphasizing financial frictions and the real-locational role of monetary injections. For example, Lucas and Stokey (1987) emphasize the role of cash for some of the transactions, while Bernanke-Gertler and Gilchrist (1999) emphasize credit contracts arising in the presence of asymmetric information. While the New Keynesian approach in focussing on sticky prices may be appealing for a number of reasons, it is useful for the task at hand to bring lessons of that earlier literature into this framework.

There are important contributions in the literature on which I can draw for this task. In their seminal paper, Christiano, Eichenbaum and Evans (2005) or CEE impose a cash-in-advance constraint for firms to pay their wage bill. Firms borrow these funds from financial intermediaries who in turn obtain funds from household deposits as well as central bank cash injection. While this feature of their paper seems there mostly to create some sort of money demand, it opens the possibility of studying financial frictions further. Schmitt-Grohe and Uribe (2004) and Altig, Christiano, Eichenbaum and Linde (2004) assume an additional cost for purchasing consumption goods, which depends on the velocity of the household's cash balances. Christiano, Motto and Rostagno (2003) introduce a fairly rich banking sector, allowing for various monetary aggregates such as bank reserves and demand deposits, to study the role of money in the Great Depression.

To keep the model tractable, yet allow for some potentially important avenues, I largely follow the lead of CEE. I additionally allow for a cash-in-advance constraint on consumption good purchases in order to judge the relative importance of private transactions to firm borrowing. I allow for the possibility that not all cash-injections are permanent, but instead are taken out of the system again at the end of the period (which one might think

of a one-off reverse transaction). Finally, I explicitely account for the cost of borrowing in the profit maximization problem and price setting problem of the firm, giving rise to an interest-rate cost channel, see also Barth and Ramey (2001), Gaiotti and Secchi (2006). This is a contribution to solving the first of the three challenges listed above.

I will explicitely allow for distortionary taxation of labor income, used to finance a stock of government as well as a certain level of government expenditure. I view this as a beginning to make progress on the second challenge above. Certainly, several - although not all - monetary policy models of recent vintage have allowed for such influence of nonmonetary policy: this model is in the same tradition. In particular, Schmitt-Grohe and Uribe (2006) add distortionary income taxation to CEE.

For the third challenge, I use DYNARE and thus off-the-shelve estimation techniques, and discuss some issues arising within the "ABCD"-framework of Fernandez-Villaverde et al(2007). In particular, I will focus on a small set of observable variable, judiciously chosen, and allow for as many shocks as there are variables. It will turn out that one needs to be careful. It is not just enough to insure an invertible mapping from the shocks to the innovations of the variables, but furthermore, it is important to check invertibility of the VAR representation itself. We do this by "visually" inspecting the VAR coefficients in the derived representation, see section 4.

In sum, the model can perhaps best be described as a variant of the CEE model, with the following deviations:

1. The costs of adjusting the capital stock arise from the investment-to-capital ratio, not the investment-to-previous-investment ratio.

2. There is a cash-in-advance constraint for household consumption purchases.

3. Only a fraction of the cash injections, which "liquify" the loan market

for firms, may permanently increase the money supply.

4. The interest rate costs for borrowing part of the input bill explicitly arises in the objective function of the intermediate good firms.

5. Capital utilization is constant.

6. There is a distortionary tax on wage income and firm profits, as well as government debt.

7. There is real wage sluggishness, following Blanchard and Gali (2005)

8. Monetary policy is assumed to follow a Taylor rule.

9. There are six shocks: a productivity shock, an investment-specific shock, a wage setting shock, a monetary policy shock and two fiscal policy shocks.

10. Estimation is in terms of five variables, inverting for the shocks per the recursive law of motion.

Section 2 explains the model. A technical appendix provides the details for the analysis of the model. Section 3 explains the estimation strategy and lists the parameters used for the comparison. Section 4 is devoted to the invertibility issue. Section 5 provides results. Section 6 discusses these results and offers some tentative conclusions.

2 The model

The model is a combination of a cash-in-advance model and a Calvo sticky-price model, amended with a role for a government.

Time is discrete. There are identical households, who supply labor and enjoy final consumption. They own all firms. They use cash for parts of their

transactions. There is a competitive sector of final-goods producing firms. There is a unit interval of monopolistic intermediate-good firms, using labor to produce output and setting sticky prices. They need to borrow a fraction of their input bill from commercial banks. Commercial banks take deposits from households and receive cash injections from the central bank. They lend to intermediate-goods firms. The central bank injects cash and thereby sets the nominal interest rate. The government taxes wage income and uses it to finance government purchases as well as debt repayments. Nominal wages are sluggish on the aggregate level.

A period has four parts:

1. Shocks are realized. The new nominal wage for the period is set. The central bank injects cash Ψ_t to banks.

2. A fraction of intermediate good firms is chosen to reset its price. Intermediate good firms "guess" demand and produce accordingly, hiring labor at the market wage. They are assumed to be required to borrow a fixed fraction of the input bill from banks.

3. Households shop, using cash at hand as well. Government shops, using tax receipts as well as a short-term credit line from the central bank.

4. Financial markets open. Firms pay capital rental payments and wages to households. Firms pay interest to banks. They pay profits to households. Households pay taxes to the government. The government issues new bonds and repays old bonds. The household splits the remaining cash into deposits with banks and cash-at-hand for the next period.

2.1 Households

Household enjoy final consumption c_t and dislike labor n_t according to

$$U = E\left[\sum_{t=0}^{\infty} \beta^t \left(\log(c_t - \chi c_{t-1}) - A n_t^{1+\sigma}\right)\right] \tag{1}$$

where $0 \leq \chi < 1$ is a habit parameter and $1/\sigma > 0$ is the Frisch elasticity of labor supply. Households enter period t, holding deposits D_{t-1} at financial intermediaries and cash-at-hand H_{t-1}. In the second part of the period, they supply labor n_t according to demand at the market wage W_t. In the third part, they use cash-at-hand to shop for a fraction η of consumption,

$$H_{t,res} + \eta P_t c_t = H_{t-1} \tag{2}$$

holding residual cash $H_{t,res} \geq 0$. I essentially assume that there are cash goods and credit goods as in Lucas-Stokey (1987), but that these cash and credit goods are purchased in fixed proportion for consumption, and that investment goods are always credit goods[1]. The latter would be implied by a Leontieff specification for the preferences in cash and credit goods. In principle, the household may spend less cash than available. However, I shall assume that shocks and parameters are such, that the constraint on residual cash is binding, $H_{t,res} = 0$.

In the forth part of the period, households receive after-tax nominal wages and trade all contingent claims as well as firm shares and government bonds, and pay for the remaining $(1 - \eta)$ share of their purchases ("credit goods"). Netting out all household-to-household trades, the financial market budget constraint is

$$H_t + D_t + q_t B_t + (1 - \eta) P_t c_t + P_t x_t \tag{3}$$

[1]A key reason for introducing the cash-in-advance constraint on only a fraction of the goods is that otherwise the money stock becomes quantitatively large in this model, implying that seignorage is a substantial fraction of the government budget constraint.

$$
\begin{aligned}
= \quad & (1+\tau_t)W_t n_t + (1+i_t)D_{t-1} \\
& + P_t r_t K_{t-1} + (1-\tau_V)V_t + B_{t-1} + H_{t,res}
\end{aligned}
$$

where H_t is cash-at-hand for the next period, D_t is deposited with banks, q_t is the discount price for government bonds B_t, $1+i_t$ is the return paid by banks on deposits D_{t-1}, $P_t r_t$ is the nominal rental rate for capital, V_t is the value added of intermediate good firms, and B_{t-1} are the debt repayments by the government.

One can extend this budget constraint with between-household trades. In particular, let $\Lambda_{t,t+k}$ be the discount price on the financial market at t for an extra unit of cash on the financial market at date $t+k$.

Also, households produce new capital subject according to

$$
k_t = \left(1 - \delta + \varphi\left((1+u_{x,t})\frac{x_t}{k_{t-1}}\right)\right) k_{t-1} \tag{4}
$$

where the adjustment cost function $\varphi(\cdot)$ satisfies

$$
\varphi(\delta) = \delta,\ \varphi'(\delta) = 1,\ \delta\varphi''(\delta) = -\frac{1}{\varpi}
$$

for some $\varpi > 0$, see Jermann (1998), and where $u_{x,t}$ is a possibly persistent investment-specific disturbance,

$$
u_{x,t} = \rho_x u_{x,t-1} + \epsilon_{x,t}. \tag{5}
$$

following Fisher (2006).

2.2 Final good firms

Final good firms take inputs $y_{t,j}$ to produce a final good y_t according to the production function

$$
y_t = \left(\int_0^1 y_{t,j}^{1/(1+\mu)} dj\right)^{1+\mu} \tag{6}
$$

They purchase intermediate goods at price $P_{t,i}$ per unit and sell the final good at price P_t.

2.3 Intermediate good firms

Given a current intermediate goods price $P_{t,j}$, intermediate good firms "guess" their demand $y_{t,j}$ resulting from the demand of final good firms, see equation (62). They thus hire labor $n_{t,j}$ at nominal wages W_t and rent capital $k_{t,j}$ at nominal rental rates $P_t r_t$ to produce output according to

$$y_{t,j} = \begin{cases} \gamma_t k_{t,j}^{\theta} n_{t,j}^{1-\theta} - \Phi & \text{if } \gamma_t n_{t,j} > \Phi \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where γ_t is an exogenous process for the change in technology and Φ is a parameter of the production function and might be thought of as a fixed cost of production. Let $\hat{\gamma}_t = \log(\gamma_t) - \log(\bar{\gamma})$ for some appropriate $\bar{\gamma}$, and assume

$$\begin{aligned} \hat{\gamma}_t &= \rho_{\gamma,L} \hat{\gamma}_{t-1} + u_{\gamma,t} \\ u_{\gamma,t} &= \rho_{\gamma,u} u_{\gamma,t-1} + \epsilon_{\gamma,t} \end{aligned} \tag{8}$$

I assume that the firm needs to obtain a loan $L_{t,j}$ for a fraction ξ_t of the input bill, on which a nominal market interest rate i_t needs to be paid. The rest of the input bill is paid for per trade credit (or more efficient market instruments) to be settled at the end of the period, on which no interest needs is paid. I.e., let MC_t be the nominal marginal costs of producing an extra unit of output, excluding the additional costs of borrowing, see equation (54). Then,

$$L_{t,j} = \xi \mathrm{MC}_t y_{t,j}$$

and the value-added of this firm (or, equivalently, end-of-period profits) are

$$V_{t,j} = (P_{t,j} - (1 + \xi i_t)\mathrm{MC}_t) y_{t,j} \tag{9}$$

Firms get to re-optimize prices with probability $1 - \alpha$, independently of their past. If they cannot re-optimize prices, they will be adjusted at the average inflation rate, i.e.

$$P_{t,j} = \bar{\pi} P_{t-1,j} \tag{10}$$

When given a chance to re-optimize prices, they will choose it so as to maximize discounted value added along the no-optimization-of-prices path[2]

$$\text{NPV}_{t,j} = E\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} V_{t,j}\right] \tag{11}$$

where $\Lambda_{t,t+k}$ is the market price at date t for an extra unit of cash at date $t+k$ on the financial markets in part 4 of the period.

2.4 Commercial banks

Banks compete for deposits from households and can borrow from the central bank. They then compete for giving loans to firms. Banks collect the returns on their loans in the forth part of the period, and then repay households as well as the central bank. In equilibrium, banks make zero profits. Thus, there will be a market nominal rate of return i_t on loans, deposits and central bank money.

2.5 The central bank

The central bank provides cash Ψ_t into the economy via providing loans to the commercial banks at the nominal interest rate i_t. It may be best to think of this as open market operations. The interest earnings on this open market operation constitute seignorage. Additionally, the central bank declares a fraction ν of the cash injection to be seignorage, not to be taken out of the system after repayment by the commercial banks. Thus, the government receives a central bank profit transfer of $(\nu + i_t)\Psi_t$ in part 4 of the period.

[2]Note that I assume that value added or profits are taxed at rate τ_V. Since I hold this rate constant, maximizing the net present value of before-tax value-added is equivalent to maximizing the net present value of after-tax value added, which would be the more appropriate objective.

Note that only $\nu\Psi_t$, but not the interest earnings on the cash injection (or even the entire cash injection) constitute an increase in the money supply,

$$M_t = M_{t-1} + \nu\Psi_t \tag{12}$$

The parameter ν allows the distinction between a short-run liquidity injection and a long-run increase in money supply. If $\nu = 0$, liquidity is provided only temporary, and taken out of the economy after the injection. Seignorage is then given only by the interest earned on the short-term injection. By contrast, $\nu = 1$ means that any short-term injection also increases money supply in the long run.

Recall, that the output gap is defined as the difference between actual output and the output which would emerge in the absence of sticky prices and absence of stickiness in wages, i.e. for $\alpha = 0$ and $\omega = 1$, but keeping the friction of borrowing from banks. In an economy without sticky prices and sticky wages and without the need to borrow from banks, real marginal costs will be constant. The percent deviation of actual real marginal costs

$$\text{mc}_t = \frac{\text{MC}_t}{P_t} \tag{13}$$

from its steady state level can therefore serve as a proxy for the output gap.

I therefore assume that the central bank follows a Taylor rule in setting interest rates, using this ratio, i.e. that

$$i_t = \bar{i} + \rho_{i,L} i_{t-1} + (1 - \rho_{i,L}) \left(\zeta_\pi (\frac{\pi_t}{\bar{\pi}} - 1) + \zeta_x (\frac{\text{mc}_t}{\overline{\text{mc}}} - 1) + u_{i,t} \right) \tag{14}$$

where

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{15}$$

is inflation, where $\bar{\pi}$ is the inflation target, $\bar{i}$ is the steady state nominal rate, ζ_π and ζ_x are coefficients of the policy rule, and where

$$u_{i,t} = \rho_{i,u} u_{i,t-1} + \epsilon_{i,t} \tag{16}$$

is a possibly persistent distortion to the Taylor rule, driven by the monetary policy shock $\epsilon_{i,t}$.

2.6 The government

The budget constraint of the government at the end of the period is given by

$$q_t B_t = B_{t-1} + P_t g_t - \tau_t W_t n_t - \tau_V V_t - (\nu + i_t)\Psi_t \tag{17}$$

The government does not carry cash from one period to the next. However, the government is assumed to finance its purchases within the period via a short-term credit from the central bank. Thus, government spending $P_t g_t$ is akin to a short-term cash injection on the demand side. This is consistent with the view that the central bank acts as the "checking account" bank to the government. Note that I do not allow the government to borrow from the central bank in the long term.

Define real debt

$$b_t = \frac{B_t}{P_t} \tag{18}$$

real seignorage

$$\psi_t = \frac{\Psi_t}{P_t} \tag{19}$$

real value-added

$$v_t = \frac{V_t}{P_t} \tag{20}$$

as well as real wages

$$w_t = \frac{W_t}{P_t} \tag{21}$$

I assume that the government aims at some steady-state debt-to-GDP ratio $\bar{b}/\bar{y}$ as well at some steady state level government-spending-to-GDP ratio $\bar{g}/\bar{y}$. Given all other parameters, let $\bar{\tau}$ be the steady state tax rate on wage income consistent with these targets.

I assume that the government follows the policy rule of adjusting future tax and spending plans, if the current debt level b_t deviates from its target level $\bar{b}$,

$$\tau_t - \bar{\tau} \;=\; \zeta_\tau\left(\frac{b_{t-1} - \bar{b}}{\bar{y}} - 1\right) + u_{\tau,t} \tag{22}$$

$$\frac{g_t}{\bar{y}} = \zeta_g(\frac{b_{t-1}-\bar{b}}{\bar{y}} - 1) + u_{g,t} \tag{23}$$

$$\tag{24}$$

where $\zeta_\tau \geq 0$ and $\zeta_g \leq 0$ such that the dynamics of government debt remains stable, and where both equations are driven by possibly persistent distortions

$$u_{\tau,t} = \rho_\tau u_{\tau,t-1} + \epsilon_{\tau,t} \tag{25}$$

$$u_{g,t} = \rho_g u_{g,t-1} + \epsilon_{g,t} \tag{26}$$

driven by the fiscal tax shock $\epsilon_{\tau,t}$ and the fiscal spending shock $\epsilon_{g,t}$.

2.7 Labor markets and wage setting

I assume that wages move sluggishly on the aggregate level. A common form to generate nominal wage sluggishness is to assume Calvo wage stickiness for wage setters, see Erceg, Henderson and Levin (2000). A different literature has emphasized frictions or sluggishness stemming from bargaining as the route cause, see Shimer (2005) or Hall (2005), giving directly rise to real wage sluggishness. The form I use here has been adapted from Blanchard and Gali (2005) and has been used e.g. in Uhlig (2007).

More specifically, let $W_{t,f}$ be the wage emerging from the first-order condition of the households maximization problem. I assume that

$$W_t = \left((1-\omega)\pi_t W_{t-1} + \omega \Upsilon W_{t,f}\right)(1+u_{w,t}) \tag{27}$$

for some $\Upsilon > 1$ and a possibly persistent stochastic distortion

$$u_{w,t} = \rho_w u_{w,t-1} + \epsilon_{w,t} \tag{28}$$

An alternative interpretation of the distortion $u_{w,t}$ is to view it as being driven by fluctuations in the preference parameter A, manifested in stochastic fluctuations of the market clearing wage $W_{t,f}$. This perspective may be a

reasonable shortcut in order to account for e.g. the fluctuations in female labor supply.

Assuming moderate-size fluctuations, actual wages will exceed the wage stemming from the first-order condition, $W_t > W_{t,f}$, and thus, labor markets will be demand constrained. I.e., I assume that households always supply labor at the going wage. Note that (27) can be rewritten in terms of real wages as

$$w_t = ((1-\omega)w_{t-1} + \omega\Upsilon w_{t,f})(1+u_{w,t}) \tag{29}$$

where $w_{t,f} = W_{t,f}/P_t$.

2.8 Aggregation and market clearing

1. Money market: Post-injection money supply equals end-of-period money demand, is given by

$$M_t \;=\; D_t + H_t \tag{30}$$

2. Final goods market:

$$g_t + c_t + x_t = y_t \tag{31}$$

3. Labor market:

$$n_t = \int_0^1 n_{t,j}dj \tag{32}$$

4. Capital market:

$$k_{t-1} = \int_0^1 k_{t-1,j}dj \tag{33}$$

5. Loan market:

$$D_{t-1} + \Psi_t = L_t = \int_0^1 L_{t,j}dj \tag{34}$$

2.9 Equilibrium and Solution

An equilibrium is an allocation, policy parameters and prices (including returns and profits) such that

1. the allocation solves the problem of the representative household, given prices and policy parameters,
2. the allocation solves the firms maximization problems, given prices, and policy parameters
3. the constraints for the government and the central bank hold,
4. markets clear

To solve for the equilibrium, I characterize the first-order conditions, explicitely solve for the steady state and characterize the dynamics per log-linearization around the steady state. I then compute the recursive law of motion solving these log-linearized equations. Details are available in a technical appendix.

3 Data and estimation.

I assume that $u_{g,t} \equiv 0$, i.e., I assume that there are no fiscal spending shocks. This is reasonable in light of the smoothness[3] in figures 3.

There are five shocks in the model: I therefore need observations on five time series to solve for these shocks:

1. π_t, inflation. I calculate it using the GDP deflator, since I am using real GDP in some other measures. A popular alternative is to use the consumer price index.

[3]It also appeared to be initially sensible for the invertibility issue discussed in the next section, when doing an exploration of the model properties with freely chosen parameters: that issue seemed to disappear with the estimated parameters, though.

2. i_t, the central bank interest rate or short rate.

3. y_t/n_t, i.e. labor productivity. For y_t, I use real GDP. For n_t, I use employment rather than hours worked. In a boom, more part-time labor will be hired, but also, more "uncounted" hours are worked by employees: it may thus be that employment rather than hours is a more reasonable variable to measure fluctuations in labor input. It was also the series that was more easily available.

4. c_t/y_t, the consumption-to-GDP ratio. Cochrane (COMPLETE REFERENCE) in particular has shown that this ratio has predictive power for GDP growth and a number of other variables. Theory implies that this statistic indeed provides key information: so it is included here.

5. b_t/y_t, the debt-to-GDP ratio.

For the EMU, the data has been obtained from the ECB, and is in use for the area-wide model. For the US, the data has been obtained from the Federal Reserve Bank of St. Louis. For debt, I have used the series GFDEBTN, i.e. debt on the federal level.

I have used quarterly data from 1985 to 2005, striking a compromise between getting a reasonably long time span for data and relying on a reasonably stable monetary policy environment. While EMU only exists since 1999, one might argue that the Bundesbank has effectively played the role of a European central bank in the time before.

I am comparing the model in its log-linearized version, i.e. in terms of log-deviations from the steady state, to the data. I therefore take logs of all variables, and removed the means. The resulting five time series used in estimation can therefore be seen in figure 4. In particular, I have linearly detrended labor productivity. If there is a constant time trend in γ_t, it is fairly straightforward to correct all equations for it: essentially, this amounts

to a slight correction in the discount rate. If the time trend is stochastic, the correction would imply a different set of equations, comparing everything to the current level of productivity. Since I log-linearized the model around a steady state with constant productivity, the linear detrending method is therefore more compatible with the theory.

The linearized model has been estimated, using DYNARE. In so doing, I have fixed a number of parameters, and estimated others. A list is given in tables 3 and 3.

For the parameters fixed a priori, I have set $\bar{n} = 1$, backing out the preference parameter A, rather than vice versa. In order to capture the different importance of banking in Europe versus the US, I have fixed $\xi = 1$ for Europe, and $\xi = 0.2$ for the US. A good calibration for these numbers would be sensible: the results here instead should be taken as indicative for what would happen for reasonable, although perhaps not sufficiently carefully calibrated values for these parameters. I have used 1 for the inverse Frisch elasticity σ of labor supply. All the other parameters are fairly standard.

For the estimated parameters, I have chosen rather uninformative priors. For parameters which should sensibly be in the unit interval, I used a uniform distribution, or, equivalently, a beta distribution with mean 0.5 and a standard deviation of $1/\sqrt{12} = 0.29$. For parameters which ought to be positive, I have used an inverted gamma distribution with infinite variance. I have used a normal distribution centered at zero and a standard deviation of 1 for ζ_g, which is certainly wide.

4 A, B, C and D's of VAR's

The title of this section refers to Fernandez-Villaverde et al (2007), of course. I use their framework to estimate and investigate the model at hand.

Let x_t be the list of log-deviations from steady state for all variables in

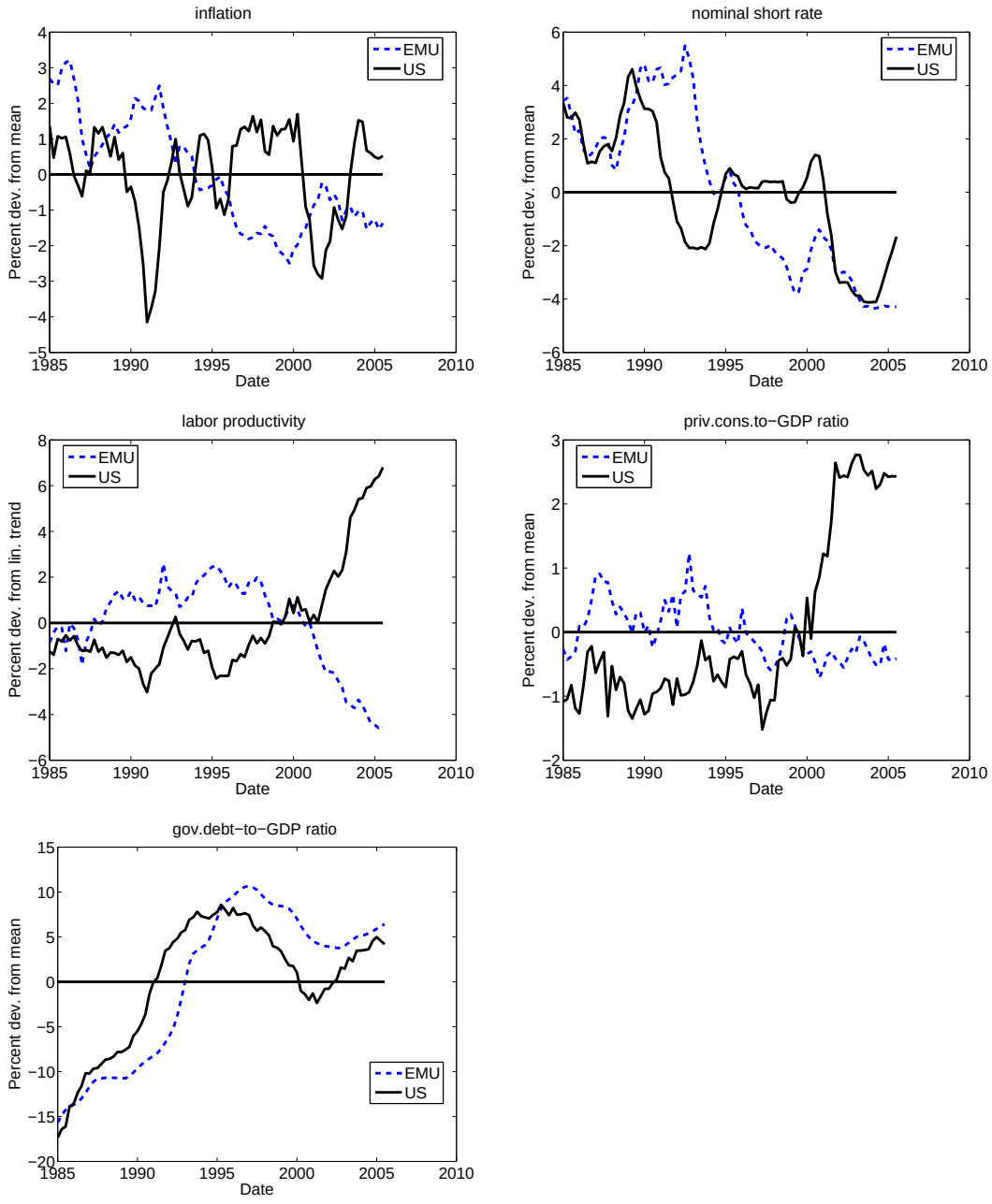


Figure 4: Data used for estimation

Parameter	US	EMU, if different	interpretation
$\bar{\gamma}$	1		Productivity
β	0.99		discount factor
θ	0.36		capital share
δ	0.02		depreciation rate
σ	1		inverse Frisch elast.
Φ	0.8 * markup		fixed cost
ϖ	2		cost of adjustment of capital
Υ	1.1		wage markup
ξ	0.2	1	bank financing share
$\bar{g}/\bar{y}$	0.15	0.2	gov. spending to GDP
$\bar{b}/\bar{y}$	0.62		debt-to-GDP ratio
$\bar{\pi}$	1.033		inflation
τ_V	0.2		profit or value added tax

Table 1: List of parameters fixed a priori.

Parameter	Distribution	mean	std.dev.	interpretation
η	beta	0.5	0.29	cash-in-advance share
ν	beta	0.5	0.29	permanent liquidity
α	beta	0.5	0.29	Calvo prob. of stickiness
μ	inv.gamma	1	∞	markup
χ	beta	0.5	0.29	habit share
ω	beta	0.5	0.29	wage sluggishness
ρ_w	beta	0.5	0.29	autocorr. wage disturb.
$\rho_{\gamma,L}$	beta	0.5	0.29	autoregr. techn.
$\rho_{\gamma,u}$	beta	0.5	0.29	autocorr. techn. disturb.
ρ_x	beta	0.5	0.29	autocorr. inv disturb.
$\rho_{i,L}$	beta	0.5	0.29	autoregr. int. rate
$\rho_{i,u}$	beta	0.5	0.29	autocorr. int.rate disturb.
ρ_τ	beta	0.5	0.29	autocorr. tax disturb.
ζ_τ	inv.gamma	1	∞	tax rule
ζ_g	normal	0	1	spending rule
ζ_π	inv.gamma	1.5	∞	Taylor rule: on inflation
ζ_x	inv.gamma	0.5	∞	Taylor rule: on markup
stderr(ϵ_γ)	inv.gamma	0.2	∞	std.err techn.
stderr(ϵ_i)	inv.gamma	0.2	∞	std.err int.rate
stderr(ϵ_τ)	inv.gamma	0.2	∞	std.err tax rate
stderr(ϵ_x)	inv.gamma	0.2	∞	std.err inv. shock
stderr(ϵ_w)	inv.gamma	0.2	∞	std.err wage shock

Table 2: List of estimated parameters

the economy, including the exogenous disturbances $u_{i,t}$ etc.. Let y_t be a list of observable variables, and let ϵ_t be the vector of iid shocks, driving the system. Solving the linearized model with e.g. the methods exposited in Uhlig (1999) provides a recursive law of motion

$$x_t = Ax_{t-1} + B\epsilon_t \tag{35}$$

$$y_t = Cx_{t-1} + D\epsilon_t \tag{36}$$

Assume that D is square and invertible, and that the eigenvalues of $(A - BD^{-1}C)$ are strictly less than one in modulus. Fernandez-Villaverde et al (2007) show that

$$y_t = C\sum_{j=0}^{\infty}(A - BD^{-1}C)^{-j}y_{t-j-1} + D\epsilon_t \tag{37}$$

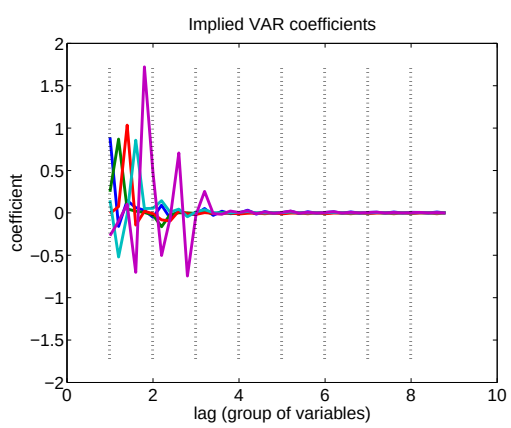
provides a vector autoregression, and that $D\epsilon_t$ are the one-step ahead forecasts for y_t.

Let

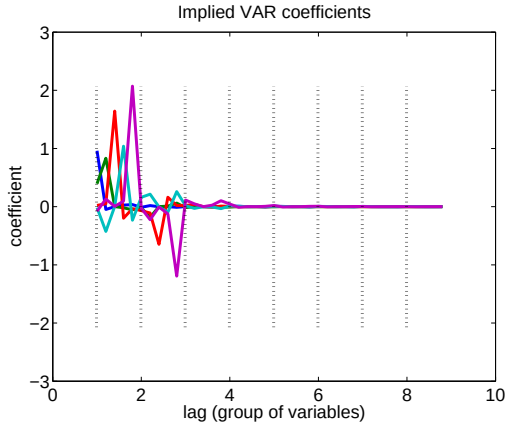
$$y_t = C\sum_{j=0}^{k}(A - BD^{-1}C)^{-1}y_{t-j-1} + D\epsilon_t + \vartheta_{k,t} \tag{38}$$

be a finite-order approximation to the infinite-order VAR in (37), defining the approximation error $\vartheta_{k,t}$. Given a recursive law of motion as in (35,36), and assuming D to be square and invertible, it is always possible to calculate the finite-order approximation (38). In practice, one would drop $\vartheta_{k,t}$ from this equation, hoping that it is small, and then use (38) to calculate the residuals ϵ_t or to estimate the model.

But $\vartheta_{k,t}$ may not be small, either because the eigenvalues of $(A-BD^{-1}C)$ are not strictly less than one in modulus, or because they are only just below one, with the coefficients in (**??**) only gradually dying out with increasing lag length. This is the problem emphasized by Chari, Kehoe and McGrattan (2005).



VAR coefficients for y_t, US parameterization.



VAR coefficients for y_t, EMU parameterization.

Figure 5: Coefficients in the derived VAR representation for y_t.

Thus, even before attempting estimation as well as a "verification" ex post, it is useful to examine how fast the coefficients in (38) die out at a specific parameterization of the model. Grouping the coefficients together according to lag length, I do this in figure5 for the coefficient specifications below for the US and EMU. Note that the VAR coefficients die out quite fast.

5 Results

5.1 Estimates

The results of the estimation are provided in table 5.1: the results are taken directly from Dynare, using standard settings. While some of the confidence intervals are perhaps too tight - most likely pointing to yet insufficient sampling - the estimates all appear to be reasonable.

Taking these estimates at face value, there are some interesting differences

as well similarities in the comparison of the US to EMU. The markup is about 50% in both regions, and the habit share is close to 60%. Wages appear to be slightly more flexible in the US, with $\omega = 0.24$ there as opposed to $\omega = 0.2$ in the EMU, but the difference is small. Wage disturbances are very persistent, and slightly more so in EMU. Productivity, tax disturbances and investment-specific disturbances are all essentially random walks, but the disturbance for productivity is considerably more persistent in EMU, $\rho_{\gamma,u} = 0.65$.

The fraction of cash required for consumption transactions is about one half in EMU and about two thirds in the US. Monetary injections seem to be temporary in the US, $\nu = 0$, but permanent in Europe, $\nu = 1$. Interest rates show a persistence of $\rho_{i,L}$ = one-half in EMU and about two thirds in the US. The Taylor rule coefficients are about 1.2 on inflation and 0.27 on markup, which is reasonable. The standard errors of interest rate shocks are 1.6% in EMU to 2% in the US, but recall that these coefficients are multiplied with $(1 - \rho_{i,L})$, thus reducing their standard deviations to about 0.8% in both regions.

The feedback coefficient for taxes range from 0.4 in EMU to 0.5 in the US, but note that the feedback coefficient for government spending is negative in the US, thus contributing to balancing the budget in the long run, while it is positive in EMU: if debt is already high there, there seems to be more temptation to also increase government spending (and not letting taxes rise quite as much as in the US). I.e., disturbances to the government budgets take longer in EMU to be undone.

5.2 Impulse responses

To understand the properties of the model, I have calculated the impulse responses, with the estimated parameters set at the posterior means rounded to two digits, except for setting $\rho_x = \rho_\tau = 0.97$ in order to render the

Parameter	mean	conf. interval	mean	conf. interval
		US		EMU
η	0.7503	[0.5587, 0.9883]	0.5432	[0.3917, 0.6133]
ν	0.0041	[0.0000, 0.0013]	0.9882	[0.9777, 0.9991]
α	0.5786	[0.5128, 0.6177]	0.7736	[0.7615, 0.7811]
μ	0.4647	[0.3477, 0.5724]	0.5238	[0.4441, 0.5867]
χ	0.6081	[0.5040, 0.6981]	0.5387	[0.4590, 0.6088]
ω	0.2378	[0.1363, 0.2663]	0.1963	[0.1915, 0.2106]
ρ_w	0.9260	[0.8953, 0.9563]	0.9754	[0.9752, 0.9754]
$\rho_{\gamma,L}$	0.9231	[0.8631, 0.9458]	0.9851	[0.9816, 0.9907]
$\rho_{\gamma,u}$	0.0913	[0.0013, 0.1457]	0.6475	[0.6208 0.6634]
ρ_x	0.9975	[0.9956 , 0.9999]	0.9772	[0.9768, 0.9780]
$\rho_{i,L}$	0.6702	[0.6582, 0.7015]	0.5146	[0.4268,0.5343]
$\rho_{i,u}$	0.3059	[0.2629, 0.3145]	0.4053	[0.3837, 0.4488]
ρ_τ	0.9827	[0.9755, 0.9982]	0.9996	[0.9995, 0.9997]
ζ_τ	0.4939	[0.4028, 0.6027]	0.3910	[0.3907, 0.3936]
ζ_g	-0.0641	[-0.0674, -0.0415]	0.1353	[0.1352, 0.1354]
ζ_π	1.1826	[1.1690, 1.1821]	1.2225	[1.2221, 1.2223]
ζ_x	0.2632	[0.1526, 0.3874]	0.2821	[0.2837, 0.2864]
stderr(ϵ_γ)	0.2645	[0.2103, 0.2967]	0.3383	[0.3146, 0.3691]
stderr(ϵ_i)	2.0644	[1.8291, 2.2849]	1.5764	[1.3440, 1.6256]
stderr(ϵ_τ)	1.5574	[1.4501, 1.7362]	0.2230	[0.2019, 0.2423]
stderr(ϵ_x)	3.4922	[3.2199, 3.8078]	3.2013	[2.9195, 3.4144]
stderr(ϵ_w)	0.8569	[0.5834, 0.8733]	0.7796	[0.7149, 0.7972]

Table 3: Estimation results

solution slightly more stable. Figure 6 shows the impulse response of the nominal interest rate. Figure 7 shows the response of output and figure 8 shows the response of inflation. A technical appendix also shows the impulse responses of the remaining variables used for estimating the model, i.e. labor productivity, the consumption-to-output ratio and the debt-to-output ratio.

In these figures, I have also considered two "intermediate" parameterizations to judge the contribution of two features in particular: the higher (assumed) requirement for bank lending in the EMU parameterization, and the parameterization of the labor market with ω and ρ_w. Starting from the US parameterization, I have first only changed the parameter ξ from 0.2 to 1. Next I also have changed the parameters for the labor market to the EMU estimates.

It turns out that the banking requirement ξ matters only for a few key responses and variables. For example, the response of output to monetary policy shocks as well as to wage disturbances moves sizeably, when changing ξ: note e.g. the positive response of output one year after the initial monetary policy shock, when the banking parameter is set to unity. The change in labor market parameters - even though not large - matters considerably for e.g. the response of output to a wage disturbance.

Note also that the difference in the monetary policy reaction function in EMU compared to the US is rather minor and most likely due more to the differences in persistence of e.g. productivity, rather than a more "sluggish response" by a committee of monetary policy makers in Europe.

5.3 Answering the question

Equipped with these tools, I can finally provide an answer to the question with which this paper started out. The answer is provided[4] graphically in

[4]It would be even better to provide standard errors in these graphs, based on the posterior distribution for the parameters given above. This is planned for a future version

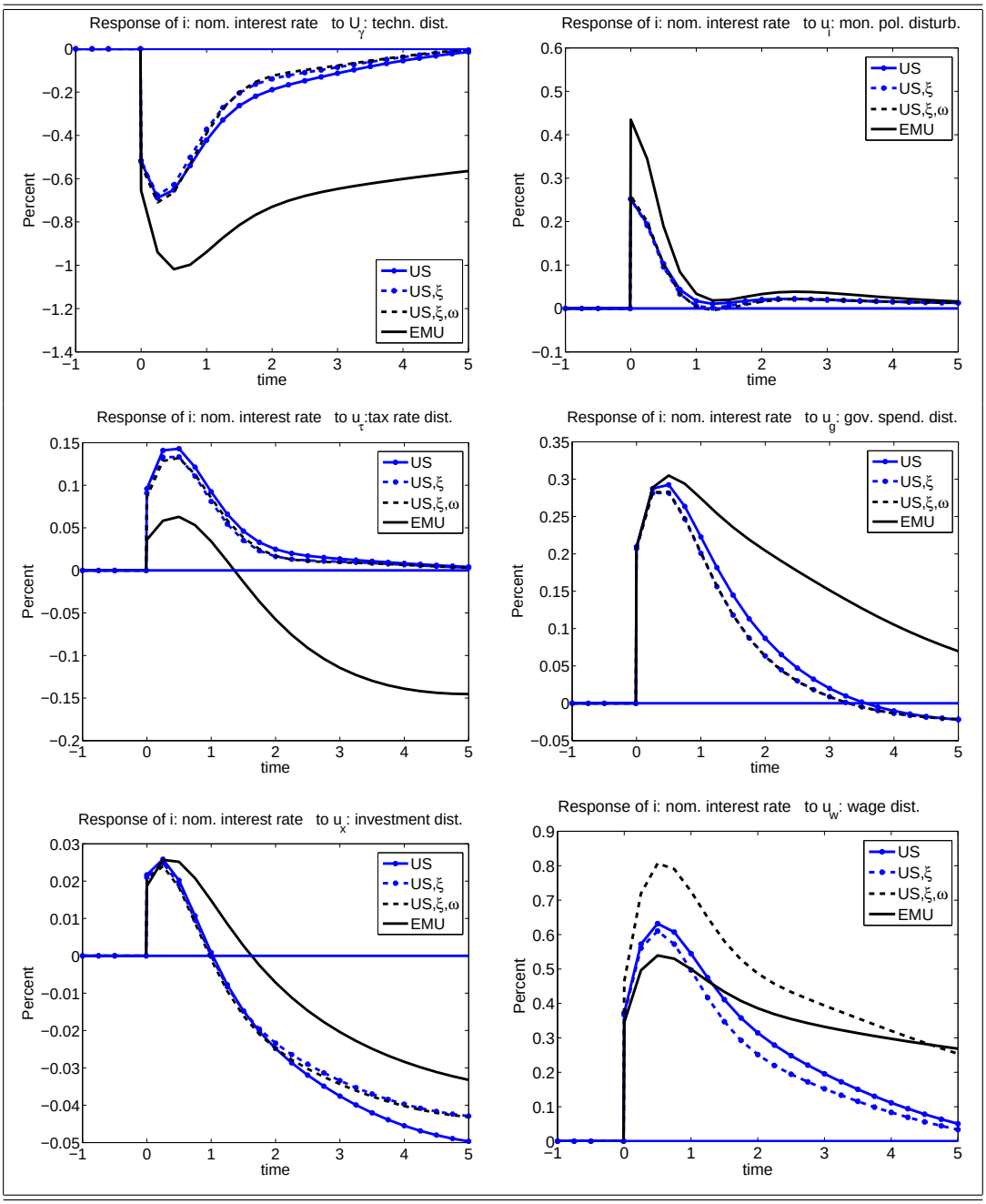


Figure 6: Impulse responses of interest rates.

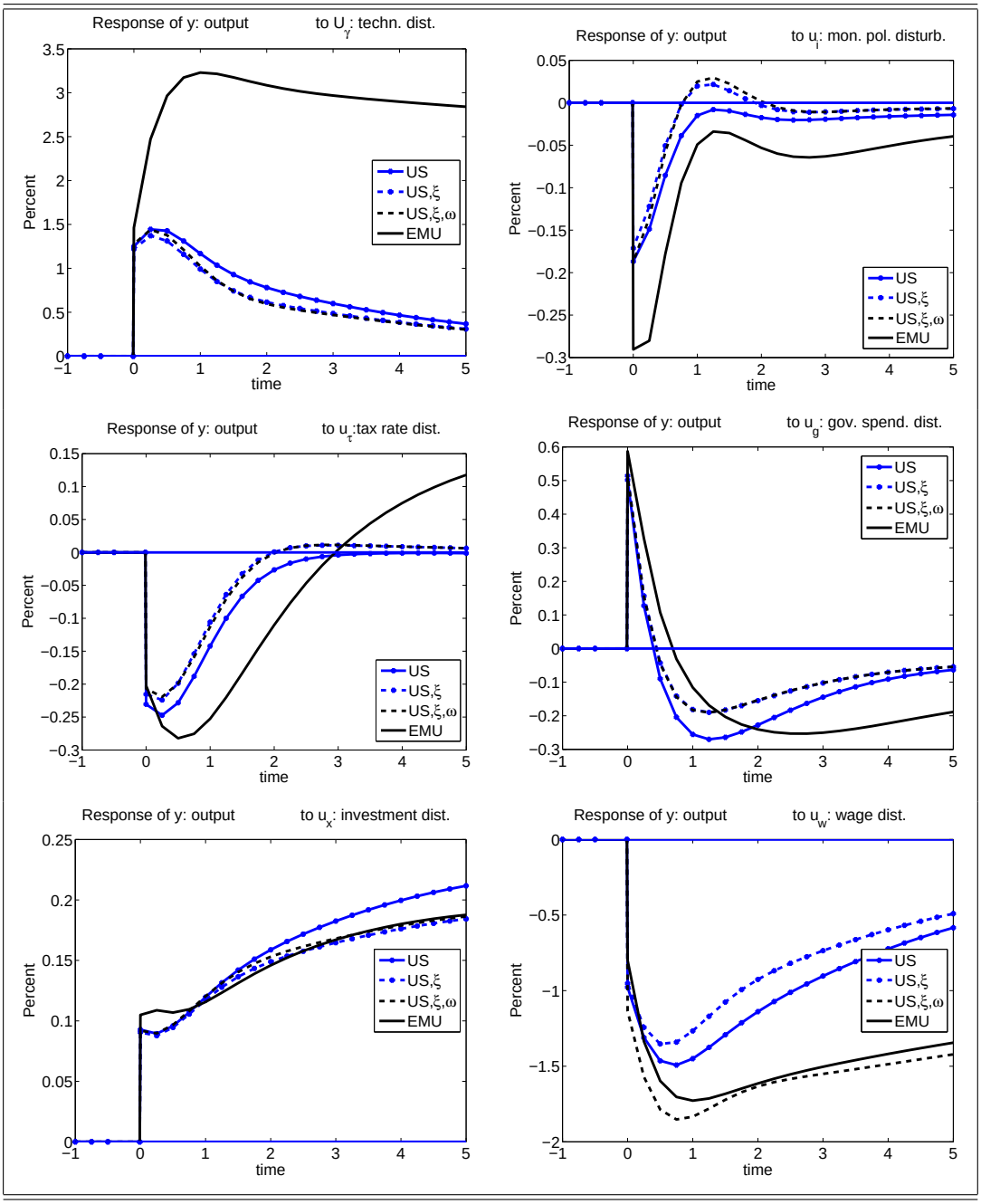


Figure 7: Impulse responses of interest rates.

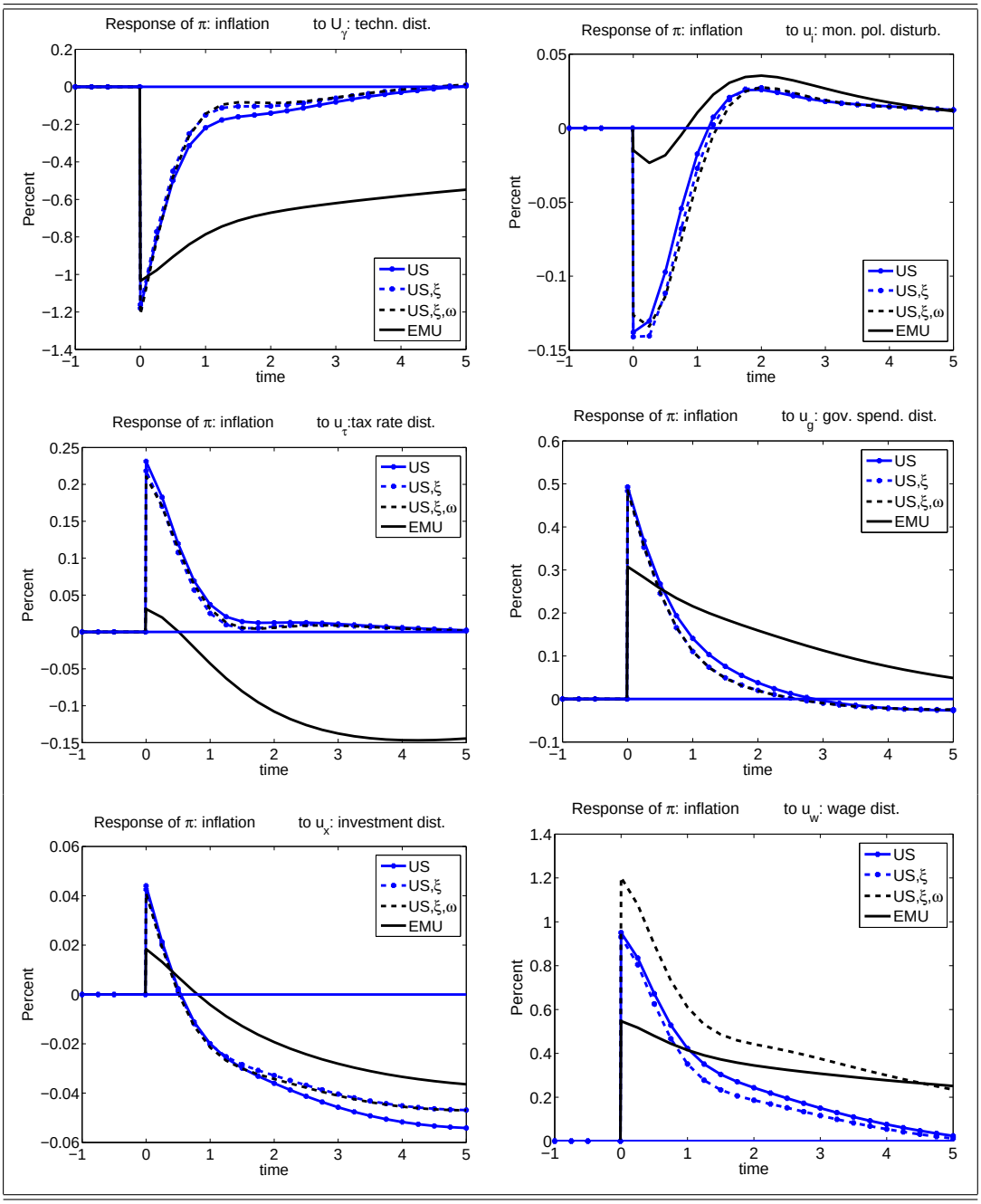


Figure 8: Impulse responses of interest rates.

figure 9. Note that all figures there have been drawn on the same scale for comparison. This figure decomposes the surprise movements in the US and the EMU into the five shocks, and adding up their contributions to the cumulative forecast error, compared to the no-shock prediction in 1998. I.e., the sequence of shocks, shown in figure 10, give rise to impulse responses of the short-term interest rate or central bank interest rate: these impulse responses are cumulated at each point in time, for all present and past shocks back to 1998 shown.

It turns out that three main sources of the movements come from technology shocks, from monetary policy shocks and finally, from wage shocks. Interestingly, the monetary policy shocks provide a fairly similar pattern for both Europe and the US. The top right hand plot in figure 9 shows that monetary policy was tighter in both the US and EMU in 2000, but considerably looser in 2004, than can be explained by all other variables and historical experience. If one views these shocks as policy mistakes, one would conclude that pretty much the same mistakes have been made in both regions, and that, if anything, the Fed seemed to follow the ECB rather than the other way around.

Surprise movements in productivity provide for a key difference between the US and EMU. Note that movements in labor productivity in the new millenium were sharply different in the US and in EMU, as evidenced by the left figure in the second row of figure 4. Figure 6 shows that monetary policy reacts to surprise rises in productivity and thus the surprise fall in marginal costs by lowering interest rates, see the top left panel. The central bank can afford to do so, since inflation is falling anyhow, as a result, see the top left panel in figure 8. Together, it then may no longer surprise, that the productivity movements in this millenium led to a considerable downward drift of interest rates in the US, but upward pressure in the EMU, see the

of this paper.

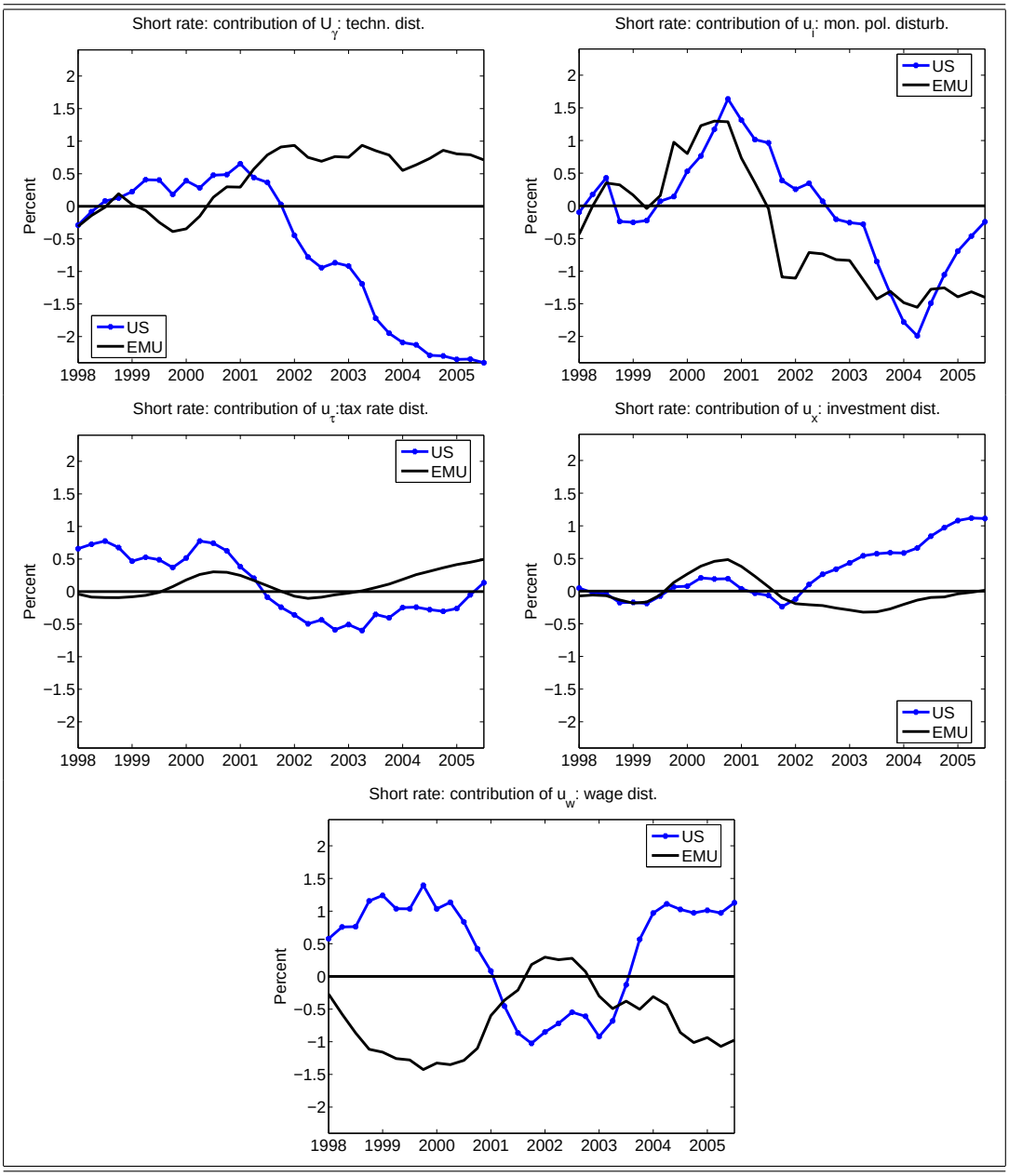


Figure 9: Contribution of each shock to the cummulative forecast error, compared to the no-shock prediction starting in 1996.

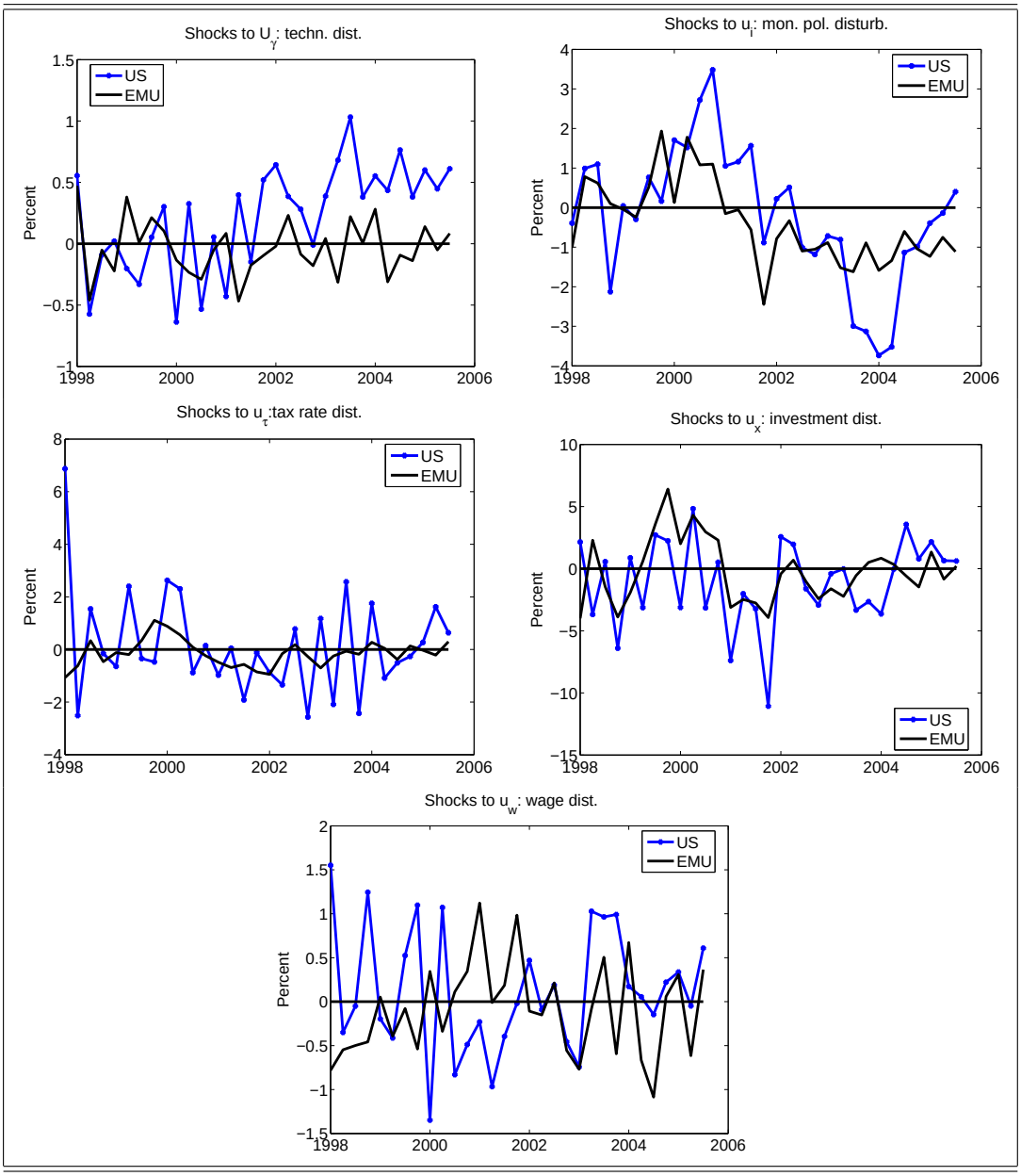


Figure 10: Sequence of shocks.

top left panel in figure 9.

Finally, surprise wage increases in the US have contributed to raising interest rates before 2000 and after 2004, with the opposite movements in the EMU, see the bottom panel in figure 9.

While the reaction function of US and EMU monetary policy to both wage shocks and productivity shocks differ quantitatively, see figure 6, they do not differ qualitatively. The differences in the interest rate movements in figure 9 arises due to different shocks, actually almost moving in oppositive direction for both variables.

In sum, it appears that the difference between the two monetary policies seen in figure 1 is due to both surprises in productivity as well as surprises in wage demands, moving interest rates in opposite directions in Europe and the US. , but not due to a more sluggish response in Europe to the same shocks or to different monetary policy surprises.

6 Discussion and Conclusion

The conclusion from this quantitative exercise appears to be that the difference between the two monetary policies seen in figure 1 is due to both surprises in productivity as well as surprises in wage demands, moving interest rates in opposite directions in Europe and the US. , but not due to a more sluggish response in Europe to the same shocks or to different monetary policy surprises.

But a number of words of caution are in order. First, these conclusions hinge on a particular choice of shocks propagating in the economy. There is a tradeoff between missing an important disturbance as explanation versus adding spurious shocks and thus risking to misinterpret important economic dynamics as movements in these spurious disturbances instead.

Second, the conclusions hinge on the particular model chosen. Is there

any sense that they are correct across a wide range of models or approaches? The model may be faulty in a number of crucial features - or improve on these features compared to other models: how are we to judge this? Acknowledging misspecification of the theory and seeking robust approaches to answer the key question may be a way to proceed further, see e.g. Hansen and Sargent (2001).

Third, while the paper has provided an accounting method for explaining the different interest paths in the US and the EMU, it has not asked whether this difference is, in fact, optimal or what the optimal reaction function should have been. I.e., it may be the case that US monetary policy has behaved badly and EMU monetary policy has done the right thing, or the other way around: the analysis above has not addressed this issue all. The tools for pursuing this question are provided in e.g. Schmitt-Grohe and Uribe (2004, 2005) or Levin et al (2005). One could even combine the perspective of optimality with the acknowledgement of misspecification and a desire for robustness, see e.g. Levin and Williams (2003).

At the end of the day, there appears to be little else than delivering quantitative answers, based on thoughtfully chosen assumptions. This paper hopes to make a contribution to that end. Along its novel features it has provided a possibility for considering traditional lending channels of monetary policy alongside the sticky-price perspective pursued by the more recent new Keynesian literature. To that end a hybrid New Keynesian cash in advance model has been provided, estimated and used to quantitatively answer the question at hand.

Some progress has been made. But much more needs to be done.

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Technical Appendix

A Analysis

A.1 First-order conditions

A.1.1 Households

Households solve

$$\max_{c_t,b_t,d_t,h_t} E\left[\sum_{t=0}^{\infty}\beta^t\left(\log(c_t-\chi c_{t-1})-An_t^{1+\sigma}\right)\right]$$

$$s.t.\quad \eta c_t=\frac{h_{t-1}}{\pi_t} \tag{39}$$

$$h_t+d_t+q_tb_t+(1-\eta)c_t+x_t \tag{40}$$
$$=(1-\tau_t)w_tn_t+\frac{1+i_t}{\pi_t}d_{t-1}+r_tk_{t-1}+(1-\tau_V)v_t+\frac{b_{t-1}}{\pi_t}$$

$$k_t=\left(1-\delta+\varphi\left((1+u_{x,t})\frac{x_t}{k_{t-1}}\right)\right)k_{t-1} \tag{41}$$

Let ϱ_t be the Lagrange multiplier on the first constraint (39), λ_t on the second constraint (40) and ς_t the Lagrange multiplier on the third constraint (41). Note that

$$\Lambda_t=\frac{\lambda_t}{P_t} \tag{42}$$

would therefore be the Lagrange multiplier on the second constraint written in nominal terms. Therefore,

$$\Lambda_{t,t+k}=\beta^k\frac{\Lambda_{t+k}}{\Lambda_t}=\frac{\lambda_{t+k}P_t}{\lambda_tP_{t+k}} \tag{43}$$

The first-order conditions are

$$\frac{\partial L}{\partial c_t}:\quad \eta\varrho_t+(1-\eta)\lambda_t=\frac{1}{c_t-\chi c_{t-1}}-\beta\chi E_t\left[\frac{1}{c_{t+1}-\chi c_t}\right] \tag{44}$$

$$\frac{\partial L}{\partial h_t}:\qquad \lambda_t=\beta E_t[\frac{\varrho_{t+1}}{\pi_{t+1}}] \tag{45}$$

$$\frac{\partial L}{\partial d_t}: \qquad \lambda_t = \qquad \beta E_t\left[\lambda_{t+1}\frac{1+i_{t+1}}{\pi_{t+1}}\right] \tag{46}$$

$$\frac{\partial L}{\partial b_t}: \qquad \lambda_t q_t = \qquad \beta E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \tag{47}$$

$$\frac{\partial L}{\partial x_t}: \qquad \lambda_t = \qquad (1+u_{x,t})\varphi'\left((1+u_{x,t})\frac{x_t}{k_{t-1}}\right)\varsigma_t \tag{48}$$

$$\begin{aligned}\frac{\partial L}{\partial k_t}: \qquad \varsigma_t = \qquad & \beta E_t\left[\lambda_{t+1}r_{t+1}\right] \\ & +\beta E_t\left[\varsigma_{t+1}\left(1-\delta+\varphi\left((1+u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\right)\right] \\ & -\beta E_t\left[\varsigma_{t+1}\left((1+u_{x,t+1})\varphi'\left((1+u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\frac{x_{t+1}}{k_t}\right)\right]\end{aligned} \tag{49}$$

Also note that the first-order condition with respect to labor determines the target real wage $w_{t,f}$,

$$\frac{\partial L}{\partial n_t}: \quad \lambda_t(1-\tau_t)w_{t,f} = \quad (1+\sigma)An_t^{\sigma} \tag{50}$$

A.1.2 Final good firms

Maximizing profits

$$P_t y_t - \int_0^1 P_{t,j}y_{t,j}dj$$

subject to the production function (6) results in the demand function

$$y_{t,j} = \left(\frac{P_t}{P_{t,j}}\right)^{\frac{1+\mu}{\mu}} y_t \tag{51}$$

and the price aggregation

$$P_t = \left(\int_0^1 P_{t,j}^{-1/\mu}dj\right)^{-\mu} \tag{52}$$

$$= \left((1-\alpha)\left(P_t^*\right)^{-1/\mu} + \alpha\left(\bar{\pi}P_{t-1}\right)^{-1/\mu}\right)^{-\mu} \tag{53}$$

A.1.3 Intermediate good firms

Cost minimization leads to the nominal marginal costs of producing an extra unit of output,

$$\mathrm{MC}_t = P_t \theta^{-\theta}(1-\theta)^{\theta-1}\gamma_t^{-1} r_t^{\theta} w_t^{1-\theta} \tag{54}$$

and therefore to the real marginal costs

$$\mathrm{mc}_t = \theta^{-\theta}(1-\theta)^{\theta-1}\gamma_t^{-1} r_t^{\theta} w_t^{1-\theta}, \tag{55}$$

excluding the costs of borrowing from bank.

Cost minimization also implies that

$$\begin{aligned} r_t k_{t,j} &= \theta \mathrm{mc}_t \gamma_t k_{t,j}^{\theta} n_{t,j}^{1-\theta} && (56) \\ w_t n_{t,j} &= (1-\theta)\mathrm{mc}_t \gamma_t k_{t,j}^{\theta} n_{t,j}^{1-\theta} && (57) \end{aligned}$$

Therefore the capital-labor ratio $k_{t,j}/n_{t,j}$ is the same across all firms, and equal to the aggregate ratio k_{t-1}/n_t. Aggregating (56) and (57) across all firms yields

$$\begin{aligned} r_t k_{t-1} &= \theta \mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \\ w_t n_t &= (1-\theta)\mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} && (58) \end{aligned}$$

Note that (58) follows from (58) with (55) or vice versa. I will therefore drop (58) when collecting all equations below. Alternatively, observe that (58) and (55) imply the more intuitive equation

$$\frac{1}{\theta} r_t k_{t-1} = \frac{1}{1-\theta} w_t n_t \tag{59}$$

To calculate the aggregate production function, observe that

$$\begin{aligned} \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} &= \int_0^1 (y_{t,j} + \Phi) dj \\ &= \left(\int_0^1 \left(\frac{P_t}{P_{t,j}} \right)^{\frac{1+\mu}{\mu}} dj \right) y_t + \Phi \end{aligned}$$

so that

$$y_t = \left(\frac{S_t}{P_t}\right)^{\frac{1+\mu}{\mu}} \left(\gamma_t k_{t-1}^{\theta} n_t^{1-\theta} - \Phi\right) \tag{60}$$

where

$$\begin{aligned} S_t &= \left(\int_0^1 P_{t,j}^{-\frac{1+\mu}{\mu}} dj\right)^{-\frac{\mu}{1+\mu}} \\ &= \left((1-\alpha)\left(P_t^*\right)^{-\frac{1+\mu}{\mu}} + \alpha\left(\bar{\pi} S_{t-1}\right)^{-\frac{1+\mu}{\mu}}\right)^{-\frac{\mu}{1+\mu}} \end{aligned} \tag{61}$$

and where $(S_t/P_t)^{\frac{1+\mu}{\mu}}$ can be thought of as a correction of the Solow residual due to sticky prices. This correction is known to disappear in a first-order log-linear approximation, see also (124) below, but it may be relevant in higher-order approximations.

When a firm can re-optimize its price $P_t^* = P_{t,j}$, it seeks to maximize the objective (63), taking into account the dependence of demand on its chosen price in future dates, if prices cannot be re-optimized, and taking into account the costs of borrowing from banks,

$$y_t(P_{t-k}^*) = \left(\frac{P_t}{\bar{\pi}^k P_{t-k}^*}\right)^{\frac{1+\mu}{\mu}} y_t \tag{62}$$

This problem can be rewritten as

$$\max_{P_t^*} E\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} y_{t+k}(P_t^*)\left(P_t^* - P_{t+k}(1+\xi i_{t+k})\mathrm{mc}_{t+k}\right)\right] \tag{63}$$

The first-order condition becomes - as usual (or with some calculation) -

$$\begin{aligned} & P_t^* E_t\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} \bar{\pi}^k y_{t+k}(P_t^*)\right] \\ &= (1+\mu) E_t\left[\sum_{k=0}^{\infty} \alpha^k \Lambda_{t,t+k} y_{t+k}(P_t^*) P_{t+k}(1+\xi i_{t+k})\mathrm{mc}_{t+k}\right] \end{aligned} \tag{64}$$

which essentially says that P^* is a markup of μ over nominal marginal costs inclusive of the costs of borrowing,

$$P_{t+k}(1+\xi i_{t+k})\mathrm{mc}_{t+k},$$

appropriately discounted.

Aggregating (9) across all firms delivers

$$v_t = y_t - (1 + \xi i_t)\mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{65}$$

A.2 Banks

Note that the required loan quantity per intermediate good firms is

$$L_{t,j} = \xi \mathrm{MC}_t (y_{t,j} + \Phi) \tag{66}$$

Aggregating, and equalizing to available funds yields in real terms

$$\frac{d_{t-1}}{\pi_t} + \psi_t = \xi \mathrm{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{67}$$

which I shall use instead of (34).

A.3 Parameters

The fundamental parameters are

$$A, \beta, \chi, \sigma, \theta, \delta, \Phi, \mu, \bar{\gamma}, \rho_{\gamma,L}, \rho_{\gamma,u}, \rho_x, \varpi$$

and the variance of the technology shock and investment-specific shock. The parameters for prices, wages and credit markets are

$$\Upsilon, \alpha, \omega, \xi, \rho_w, \eta$$

and the variance of the wage shock. The policy parameters are

$$\bar{g}, \bar{b}, \bar{\pi}, \nu, \tau_V$$

as well as the feedback coefficients

$$\zeta_\tau, \zeta_g, \zeta_\pi, \zeta_x, \rho_{i,L}, \rho_{i,u}, \rho_\tau, \rho_g$$

and variances of the policy shocks.

A.4 Collecting the equations

The equations characterizing the equilibrium are (HH: "household", "FG": final good firms, "IG": intermediate good firms, "CB": central bank, "GOV": government, "MC": labor market and market clearing):

$$\text{HH:} \quad 0 = -\eta c_t + \frac{h_{t-1}}{\pi_t} \tag{68}$$

$$\begin{aligned}\text{HH:} \quad 0 = & -h_t - d_t - q_t b_t - (1-\eta)c_t - x_t \\ & +(1-\tau_t)w_t n_t + r_t k_{t-1} + \frac{1+i_t}{\pi_t} d_{t-1} + (1-\tau_V)v_t + \frac{b_{t-1}}{\pi_t}\end{aligned} \tag{69}$$

$$\text{HH:} \quad 0 = -k_t + \left(1 - \delta + \varphi\left(\frac{x_t}{k_{t-1}}\right)\right) k_{t-1} \tag{70}$$

$$\text{HH:} \quad 0 = -\eta \varrho_t - (1-\eta)\lambda_t + \frac{1}{c_t - \chi c_{t-1}} - \beta\chi E_t\left[\frac{1}{c_{t+1} - \chi c_t}\right] \tag{71}$$

$$\text{HH:} \quad 0 = -\lambda_t + \beta E_t[\frac{\varrho_{t+1}}{\pi_{t+1}}] \tag{72}$$

$$\text{HH:} \quad 0 = -\lambda_t + \beta E_t\left[\lambda_{t+1}\frac{1+i_{t+1}}{\pi_{t+1}}\right] \tag{73}$$

$$\text{HH:} \quad 0 = -\lambda_t q_t + \beta E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \tag{74}$$

$$\text{HH:} \quad 0 = -\lambda_t + (1+u_{x,t})\varphi'\left((1+u_{x,t})\frac{x_t}{k_{t-1}}\right)\varsigma_t \tag{75}$$

$$\begin{aligned}\text{HH:} \quad 0 = & -\varsigma_t + \beta E_t\left[\lambda_{t+1} r_{t+1}\right] \\ & +\beta E_t\left[\varsigma_{t+1}\left(1 - \delta + \varphi\left((1+u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\right)\right] \\ & -\beta E_t\left[\varsigma_{t+1}\left((1+u_{x,t+1})\varphi'\left((1+u_{x,t+1})\frac{x_{t+1}}{k_t}\right)\frac{x_{t+1}}{k_t}\right)\right]\end{aligned} \tag{76}$$

$$\text{HH:} \quad 0 = -\lambda_t(1-\tau_t)w_{t,f} + (1+\sigma)An_t^{\sigma} \tag{77}$$

$$\text{FG:} \quad 0 = -y_t(P^*_{t-k}) + \left(\frac{P_t}{\bar{\pi}^k P^*_{t-k}}\right)^{\frac{1+\mu}{\mu}} y_t \tag{78}$$

$$\text{FG:} \quad 0 = -P_t + \left((1-\alpha)\left(P_t^*\right)^{-1/\mu} + \alpha\left(\bar{\pi}P_{t-1}\right)^{-1/\mu}\right)^{-\mu} \tag{79}$$

$$\text{IG:} \quad 0 = -S_t + \left((1-\alpha)\left(P_t^*\right)^{-\frac{1+\mu}{\mu}} + \alpha\left(\bar{\pi}S_{t-1}\right)^{-\frac{1+\mu}{\mu}}\right)^{-\frac{\mu}{1+\mu}} \tag{80}$$

$$\text{IG:}\quad 0 = -y_t + \left(\frac{S_t}{P_t}\right)^{\frac{1+\mu}{\mu}} \left(\gamma_t k_{t-1}^{\theta} n_t^{1-\theta} - \Phi\right) \tag{81}$$

$$\text{IG:}\quad 0 = -\text{mc}_t + \theta^{-\theta}(1-\theta)^{\theta-1}\gamma_t^{-1} r_t^{\theta} w_t^{1-\theta} \tag{82}$$

$$\text{IG:}\quad 0 = -r_t k_{t-1} + \theta \text{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{83}$$

$$\text{IG:}\quad 0 = -P_t^* E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k \frac{\lambda_{t+k} P_t}{\lambda_t P_{t+k}} \bar{\pi}^k y_{t+k}(P_t^*)\right] + (1+\mu) E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k \frac{\lambda_{t+k} P_t}{\lambda_t P_{t+k}} y_{t+k}(P_t^*) P_{t+k} (1+\xi i_{t+k}) \text{mc}_{t+k}\right] \tag{84}$$

$$\text{IG:}\quad 0 = v_t - y_t + (1+\xi i_t)\text{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{85}$$

$$\text{CB:}\quad 0 = -i_t + \bar{i} + \rho_{i,L} i_{t-1} + (1-\rho_{i,L})\left(\zeta_{\pi}(\frac{\pi_t}{\bar{\pi}} - 1) + \zeta_x(\frac{\text{mc}_t}{\overline{\text{mc}}} - 1) + u_{i,t}\right) \tag{86}$$

$$\text{CB:}\quad 0 = -m_t + \frac{m_{t-1}}{\pi_t} + \nu\psi_t \tag{87}$$

$$\text{GOV:}\quad 0 = -q_t b_t + \frac{b_{t-1}}{\pi_t} + g_t - \tau_t w_t n_t - \tau_V v_t - (\nu + i_t)\psi_t \tag{88}$$

$$\text{GOV:}\quad 0 = -\tau_t + \bar{\tau} + \zeta_{\tau}(\frac{b_{t-1} - \bar{b}}{\bar{y}} - 1) + u_{\tau,t} \tag{89}$$

$$\text{GOV:}\quad 0 = -\frac{g_t}{\bar{y}} + \zeta_g(\frac{b_{t-1} - \bar{b}}{\bar{b}} - 1) + u_{g,t} \tag{90}$$

$$\text{MC:}\quad 0 = -w_t + ((1-\omega)w_{t-1} + \omega\Upsilon w_{t,f})(1+u_{w,t}) \tag{91}$$

$$\text{MC:}\quad 0 = -m_t + d_t + h_t \tag{92}$$

$$\text{MC:}\quad 0 = -y_t + g_t + c_t + x_t \tag{93}$$

$$\text{MC:}\quad 0 = -\frac{d_{t-1}}{\pi_t} - \psi_t + \xi \text{mc}_t \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{94}$$

$$\text{MC:}\quad 0 = -\pi_t + \frac{P_t}{P_{t-1}} \tag{95}$$

together with the specification for the exogenous processes

$$\text{techn.:}\quad \hat{\gamma}_t = \rho_{\gamma,L}\hat{\gamma}_{t-1} + \epsilon_{\gamma,t} \tag{96}$$

$$\text{mon.pol:}\quad u_{i,t} = \rho_{i,u} u_{i,t-1} + \epsilon_{i,t} \tag{97}$$

$$\text{taxes:}\quad u_{\tau,t} = \rho_{\tau} u_{\tau,t-1} + \epsilon_{\tau,t} \tag{98}$$

$$\text{gov.spend.:} \quad u_{g,t} = \rho_g u_{g,t-1} + \epsilon_{g,t} \tag{99}$$

$$\text{investment:} \quad u_{x,t} = \rho_x u_{x,t-1} + \epsilon_{x,t} \tag{100}$$

$$\text{wages:} \quad u_{w,t} = \rho_w u_{w,t-1} + \epsilon_{w,t} \tag{101}$$

The equations above determine the quantities

$$b_t, c_t, g_t, n_t, v_t, y_t, \mathrm{mc}_t, k_t, x_t, r_t$$

the demand function

$$y_t(P^*_{t-k}),$$

real money balances

$$d_t, h_t, m_t, \psi_t,$$

multipliers

$$\varrho_t, \lambda_t, \varsigma_t$$

prices and tax rate

$$i_t, P_t, P^*_t, q_t, S_t, w_t, w_{t,f}, \pi_t, \tau_t$$

as well as the exogenous processes

$$\gamma_t, u_{i,t}, u_{\tau,t}, u_{g,t}, u_{x,t}, u_{w,t}$$

Note that these are 34 equations for 33 variables. One may drop either the household budget constraint, the government budget constraint or one of the market clearing conditions, due to Walras' law.

A.5 Steady state

A.5.1 Household

To calculate the steady state, and since my focus is not on a steady state comparison across various parameters, I assume a value for $\bar{n}$ and instead

back out the compatible preference parameter A. The capital accumulation equation (70) implies

$$\bar{x} = \delta \bar{k} \tag{102}$$

The first-order conditions (72) and (75) of the households imply

$$\bar{\varrho} = \frac{\bar{\pi}}{\beta} \bar{\lambda} \tag{103}$$

$$\bar{\varsigma} = \bar{\lambda} \tag{104}$$

For the rental rate of capital, the first-order condition (76) implies

$$\bar{r} = \frac{1}{\beta} - 1 + \delta \tag{105}$$

The first-order conditions (73) and (74) imply

$$1 + \bar{i} = \frac{1}{\bar{q}} = \frac{\bar{\pi}}{\beta} \tag{106}$$

A.5.2 Firms

We shall assume that the parameters imply $\bar{v} > 0$. Equations (79), (80) and (95) deliver

$$\bar{P}_t^* = \bar{S}_t = \bar{P}_t = \bar{\pi} \bar{P}_{t-1}$$

With equation (78),

$$y(\bar{P}_{t-k}^*) = \bar{y}$$

The markup equation (84) for the intermediate good implies

$$\overline{\text{mc}} = \frac{1}{(1+\mu)(1+\xi \bar{i})} \tag{107}$$

This and equation (83) imply

$$r\bar{k} = \theta \overline{\text{mc}} \bar{\gamma} \bar{k}^{\theta} \bar{n}^{1-\theta}$$

or

$$\bar{k} = \left(\frac{\theta\overline{\text{mc}}\bar{\gamma}}{\bar{r}}\right)^{\frac{1}{1-\theta}} \bar{n} \tag{108}$$

From this and (82) or, equivalently, (58), obtain

$$\begin{aligned} \bar{w} &= (1-\theta)\overline{\text{mc}}\bar{\gamma}\left(\frac{\bar{k}}{\bar{n}}\right)^{\theta} \\ &= (1-\theta)\theta^{\frac{\theta}{1-\theta}}\left(\overline{\text{mc}}\bar{\gamma}\right)^{\frac{1}{1-\theta}} \bar{r}^{\frac{-\theta}{1-\theta}} \end{aligned} \tag{109}$$

With this as well as equations (81,93,71,91,85)

$$\bar{y} = \bar{\gamma}\bar{k}^{\theta}\bar{n}^{1-\theta} - \Phi \tag{110}$$

$$\bar{c} = \bar{y} - \bar{g} - \delta\bar{k} \tag{111}$$

$$\bar{\lambda} = \left(\eta\frac{\bar{\pi}}{\beta} + 1 - \eta\right)^{-1} \frac{1-\beta\chi}{1-\chi}\,\frac{1}{\bar{c}} \tag{112}$$

$$\bar{w}_f = \bar{w}/\Upsilon \tag{113}$$

$$\bar{v} = \bar{y} - (1+\xi\bar{i})\overline{\text{mc}}\bar{\gamma}\bar{k}^{\theta}\bar{n}^{1-\theta} \tag{114}$$

$$= \frac{\mu}{1+\mu}\bar{\gamma}\bar{k}^{\theta}\bar{n}^{1-\theta} - \Phi \tag{115}$$

$$= \bar{y} - \frac{1}{1+\mu}(\bar{y} + \Phi) \tag{116}$$

which now allows to solve for the steady state values of the Lagrange multipliers in (103).

A.6 Monetary quantities

Cash demand is given by (68) or

$$\bar{h} = \eta\bar{\pi}\bar{c} \tag{117}$$

To calculate the other monetary quantities, combine the three steady state relationships of (87,92,94),

$$\nu\bar{\psi} = \bar{m}\left(1 - \frac{1}{\bar{\pi}}\right)$$

$$\begin{aligned} \bar{m} &= \bar{d} + \bar{h} \\ \xi\overline{\mathrm{mc}}(\bar{y} - \Phi) &= \frac{\bar{d}}{\bar{\pi}} + \bar{\psi} \end{aligned}$$

to obtain

$$\bar{m} = \frac{\bar{\pi}\nu}{\bar{\pi} + \nu - 1}\left(\xi\overline{\mathrm{mc}}(\bar{y} - \Phi) + \frac{\bar{h}}{\bar{\pi}}\right) \tag{118}$$

$$\bar{\psi} = \frac{\bar{\pi} - 1}{\bar{\pi}\nu}\bar{m} \tag{119}$$

$$\bar{d} = \bar{m} - \bar{h} \tag{120}$$

Note that the fraction appearing in the equation for $\bar{m}$ equals 1, if either $\bar{\pi} = 1$ or $\nu = 1$. For technical reasons, one must set $\nu \neq 0$. Note that $\bar{\pi} = 1$ implies $\bar{\psi} = 0$.

A.6.1 Remaining equations

The steady state government budget constraint (88)

$$\bar{\tau}\bar{w}\bar{n} = \frac{1 - \beta}{\bar{\pi}}\bar{b} + \bar{g} - \tau_V\bar{v} - (\nu + \bar{i})\bar{\psi} \tag{121}$$

can be solved for the steady state level of taxes $\bar{\tau}$. With this and (77), calculate the preference parameter A per

$$A = \frac{\bar{\lambda}(1 - \bar{\tau})}{1 + \sigma}\bar{n}^{-\sigma}\bar{w}_f \tag{122}$$

Note finally that (96) to (99) deliver

$$\bar{z} = \bar{a} = \bar{f}_\tau = \bar{f}_g = 0$$

A.7 Log-linearization

Let hat on variables denote the logarithmic deviation from steady state values, e.g. $\hat{c}_t = \log(c_t) - \log(\bar{c})$. For nominal quantities, in particular prices, I

use likewise e.g. $\hat{P}_t = \log(P_t) - \log(\bar{P}_t)$, where I note that $\bar{P}_t = \bar{\pi}^k \bar{P}_0$, starting from some initial level $\bar{P}_0$. I make the following exceptions for the notation, so as to allow zero values in steady state or to obtain meaningful quantities:

$$\begin{aligned} r_t &= \bar{r} + \hat{r}_t \\ i_t &= \bar{i} + \hat{i}_t \\ \tau_t &= \bar{\tau} + \hat{\tau}_t \\ g_t &= \bar{g} + \hat{g}_t \bar{y} \\ b_t &= \bar{b} + \hat{b}_t \bar{y} \\ \psi_t &= \bar{\psi} + \bar{m}\, \hat{\psi}_t \end{aligned}$$

Hence, $\hat{r}_t$, $\hat{\tau}_t$ and $\hat{i}_t$ are in percent, $\hat{g}_t$ and $\hat{b}_t$ are in percent of steady-state output and $\hat{\psi}_t$ is in percent of the steady state money supply. Most equations can be log-linearized in a straight-forward manner, but some equations require a bit more thought. They are explained now.

A.7.1 Pricing decisions

The following derivation is standard in the literature on New Keynesian models and is replicated here for completeness.

Equations (79) and (80) log-linearize to

$$\begin{aligned} \hat{P}_t &= (1-\alpha)\hat{P}_t^* + \alpha \hat{P}_{t-1} \\ \hat{S}_t &= (1-\alpha)\hat{P}_t^* + \alpha \hat{S}_{t-1} \end{aligned} \tag{123}$$

and thus

$$\hat{S}_t = \hat{P}_t \tag{124}$$

This substantiates the claim that the correction to the Solow residual in (60) vanishes in a first-order approximation.

The first-order condition (84) of the intermediate good firms log-linearizes to

$$\hat{P}_t^* = (1-\alpha\beta)E_t\left[\sum_{k=0}^{\infty}(\alpha\beta)^k\left(\xi\overline{\text{mc}}\hat{i}_{t+k} + (1+\xi\bar{i})\widehat{\text{mc}}_{t+k} + \hat{P}_{t+k}\right)\right] \tag{125}$$

or

$$\hat{P}_t^* = (1-\alpha\beta)\left(\xi\overline{\text{mc}}\,\hat{i}_t + (1+\xi\bar{i})\widehat{\text{mc}}_t + \hat{P}_t\right) + \alpha\beta E_t[\hat{P}_{t+1}^*] \tag{126}$$

From equation (123), substitute $\hat{P}_t^*$ and $\hat{P}_{t+1}^*$ per

$$\hat{P}_t^* = \frac{1}{1-\alpha}\left(\hat{P}_t - \alpha\hat{P}_{t-1}\right).$$

Combine terms to obtain the New Keynesian Phillips curve

$$\hat{\pi}_t = \beta E_t\left[\hat{\pi}_{t+1}\right] + \kappa\left(\xi\overline{\text{mc}}\,\hat{i}_t + (1+\xi\bar{i})\widehat{\text{mc}}_t\right) \tag{127}$$

where

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tag{128}$$

One may view the driving term

$$\xi\overline{\text{mc}}\,\hat{i}_t + (1+\xi\bar{i})\widehat{\text{mc}}_t$$

either as reflecting marginal costs inclusive of the costs of borrowing or as a correction to net marginal cost by an interest rate cost channel, as emphasized by Christiano et al (**complete reference!**).

A.7.2 Collecting log-linearized equations without expectations

We shall drop the budget constraint of the household - appealing to Walras' law - as well as equations from pricing decisions and demand, which are no longer needed. All remaining equations without expectations are, in log-linearized form:

$$\text{HH:}\quad 0 = \quad -\hat{c}_t + \hat{h}_{t-1} - \hat{\pi}_t \tag{129}$$

$$\text{HH:}\quad 0 = -\hat{k}_t + \delta\hat{x}_t + (1-\delta)\hat{k}_{t-1} + \delta u_{x,t} \tag{130}$$

$$\begin{aligned}\text{HH:}\quad 0 = & -\hat{\varsigma}_t + \hat{\lambda}_t \\ & + \frac{1}{\varpi}\left(\hat{x}_t - \hat{k}_{t-1}\right) - \left(1 - \frac{1}{\varpi}\right) u_{x,t}\end{aligned} \tag{131}$$

$$\text{HH:}\quad 0 = -\hat{\lambda}_t + \frac{\hat{\tau}_t}{1-\bar{\tau}} - \hat{w}_{t,f} + \sigma\hat{n}_t \tag{132}$$

$$\text{IG:}\quad 0 = -\frac{\bar{y}}{\bar{y}+\Phi}\hat{y}_t + \hat{\gamma}_t + \theta\hat{k}_{t-1} + (1-\theta)\hat{n}_t \tag{133}$$

$$\text{IG:}\quad 0 = -\widehat{\text{mc}}_t + \theta\frac{\hat{r}_t}{\bar{r}} + (1-\theta)\hat{w}_t - \hat{\gamma}_t \tag{134}$$

$$\text{IG:}\quad 0 = -\frac{\hat{r}_t}{\bar{r}} + \widehat{\text{mc}}_t + \gamma_t + (1-\theta)\left(\hat{n}_t - \hat{k}_{t-1}\right) \tag{135}$$

$$\begin{aligned}\text{IG:}\quad 0 = & \frac{\bar{v}}{\bar{y}-\bar{v}}\hat{v}_t - \frac{\bar{y}}{\bar{y}-\bar{v}}\hat{y}_t + \frac{\xi}{1+\xi\bar{i}}\hat{i}_t \\ & + \widehat{\text{mc}}_t + \hat{\gamma}_t + \theta\hat{k}_{t-1} + (1-\theta)n_t\end{aligned} \tag{136}$$

$$\text{CB:}\quad 0 = -\hat{i}_t + \rho_{i,L}\hat{i}_{t-1} + (1-\rho_{i,L})\left(\zeta_\pi\hat{\pi}_t + \zeta_x\widehat{\text{mc}}_t + u_{i,t}\right) \tag{137}$$

$$\text{CB:}\quad 0 = -\hat{m}_t + \frac{1}{\bar{\pi}}(\hat{m}_{t-1} - \hat{\pi}_t) + \nu\hat{\psi}_t \tag{138}$$

$$\begin{aligned}\text{GOV:}\quad 0 = & -\bar{q}(\bar{b}\hat{q}_t + \bar{y}\hat{b}_t) + \frac{1}{\bar{\pi}}(\bar{y}\hat{b}_{t-1} - \bar{b}\hat{\pi}_t) + \bar{y}\hat{g}_t \\ & - \bar{w}\bar{n}\hat{\tau}_t - \bar{\tau}\bar{w}\bar{n}(\hat{w}_t + \hat{n}_t) - \tau_V\bar{v}\hat{v}_t - (\nu+\bar{i})\bar{m}\hat{\psi}_t - \bar{\psi}\,\hat{i}_t\end{aligned} \tag{139}$$

$$\text{GOV:}\quad 0 = -\hat{\tau}_t + \zeta_\tau\hat{b}_{t-1} + u_{\tau,t} \tag{140}$$

$$\text{GOV:}\quad 0 = -\hat{g}_t + \zeta_g\hat{b}_{t-1} + u_{g,t} \tag{141}$$

$$\text{MC:}\quad 0 = -\hat{w}_t + (1-\omega)\hat{w}_{t-1} + \omega\hat{w}_{t,f} + u_{w,t} \tag{142}$$

$$\text{MC:}\quad 0 = -\bar{m}\hat{m}_t + \bar{d}\hat{d}_t + \bar{h}\hat{h}_t \tag{143}$$

$$\text{MC:}\quad 0 = -\bar{y}\hat{y}_t + \bar{y}\hat{g}_t + \bar{c}\hat{c}_t + \bar{x}\hat{x}_t \tag{144}$$

$$\begin{aligned}\text{MC:}\quad 0 = & \frac{\bar{d}}{\bar{d}+\bar{\pi}\bar{\psi}}(\hat{d}_{t-1} - \hat{\pi}_t) + \frac{\bar{\pi}\bar{m}}{\bar{d}+\bar{\pi}\bar{\psi}}\hat{\psi}_t \\ & + \widehat{\text{mc}}_t + \hat{\gamma}_t + \theta\hat{k}_{t-1} + (1-\theta)\hat{n}_t\end{aligned} \tag{145}$$

together with the specification for the exogenous processes

$$\text{techn.:}\quad \hat{\gamma}_t = \quad \rho_{\gamma,L}\hat{\gamma}_{t-1} + u_{\gamma,t} \tag{146}$$

$$u_{\gamma,t} = \rho_{\gamma,u} u_{\gamma,t-1} + \epsilon_{\gamma,t} \tag{147}$$

$$\text{mon.pol:} \quad u_{i,t} = \rho_{i,u} u_{i,t-1} + \epsilon_{i,t} \tag{148}$$

$$\text{taxes:} \quad u_{\tau,t} = \rho_{\tau} u_{\tau,t-1} + \epsilon_{\tau,t} \tag{149}$$

$$\text{gov.spend.:} \quad u_{g,t} = \rho_{g} u_{g,t-1} + \epsilon_{g,t} \tag{150}$$

$$\text{investment:} \quad u_{x,t} = \rho_{x} u_{x,t-1} + \epsilon_{x,t} \tag{151}$$

$$\text{wages:} \quad u_{w,t} = \rho_{w} u_{w,t-1} + \epsilon_{w,t} \tag{152}$$

A.7.3 Collecting loglinearized equations with expectations

All equations with expectations in log-linearized form are:

$$\text{HH:} \quad 0 = -\frac{(1-\beta\chi)(1-\chi)}{\eta\frac{\bar{\pi}}{\beta}+1-\eta}\left(\eta\frac{\bar{\pi}}{\beta}\hat{\varrho}_t + (1-\eta)\hat{\lambda}_t\right) + \chi\hat{c}_{t-1} - (1+\beta\chi^2)\hat{c}_t + \beta\chi E_t[\hat{c}_{t+1}] \tag{153}$$

$$\text{HH:} \quad 0 = -\hat{\lambda}_t + E_t[\hat{\varrho}_{t+1} - \hat{\pi}_{t+1}] \tag{154}$$

$$\text{HH:} \quad 0 = -\hat{\lambda}_t + E_t\left[\hat{\lambda}_{t+1} + \frac{\hat{i}_{t+1}}{1+\bar{i}} - \hat{\pi}_{t+1}\right] \tag{155}$$

$$\text{HH:} \quad 0 = -\hat{\lambda}_t - \hat{q}_t + E_t\left[\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\right] \tag{156}$$

$$\text{HH:} \quad 0 = -\hat{\varsigma}_t + \beta E_t\left[\bar{r}\hat{\lambda}_{t+1} + \hat{r}_{t+1}\right] + \beta E_t\left[(1-\delta)\hat{\varsigma}_{t+1} + \frac{\delta}{\varpi}\left(\hat{x}_{t+1} - \hat{k}_t + u_{x,t+1}\right)\right] \tag{157}$$

$$\text{FG, IG:} \quad 0 = -\hat{\pi}_t + \beta E_t\left[\hat{\pi}_{t+1}\right] + \kappa\left(\xi\overline{\text{mc}}\,\hat{i}_t + (1+\xi\bar{i})\widehat{\text{mc}}_t\right) \tag{158}$$

These equations and the equations without expectations determine the evolution of the log-deviations for the quantities

$$\hat{b}_t, \hat{c}_t, \hat{g}_t, \hat{n}_t, \hat{v}_t, \hat{y}_t, \widehat{\text{mc}}_t, \hat{k}_t, \hat{x}_t, \hat{r}_t$$

real money balances

$$\hat{d}_t, \hat{h}_t, \hat{m}_t, \hat{\psi}_t,$$

multipliers

$$\hat{\varrho}_t, \hat{\lambda}_t, \hat{\varsigma}_t$$

prices and tax rate

$$\hat{i}_t, \hat{q}_t, \hat{w}_t, \hat{w}_{t,f}, \hat{\pi}_t, \hat{\tau}_t$$

Note that there are 23 equations for 23 variables, plus the equations for the exogenous processes.

Note that q_t is the inverse of the one-period risk free return $R_{t,f}$ from period t to $t+1$. Hence

$$\hat{R}_{t,f} = -\hat{q}_t$$

Note that generally $\hat{i}_t \neq \hat{R}_{t-1,f}$, since $\hat{i}_t$ can react to shocks within period t.

Define

$$\begin{aligned} \hat{r}_t^* &= \hat{r}_t + \frac{1}{\varpi}\left(\hat{x}_t - \hat{k}_{t-1}\right) - \left(1 - \delta - \frac{1}{\varpi}\right) u_{x,t} \\ &\quad - \frac{1}{\beta\varpi}\left(\hat{x}_{t-1} - \hat{k}_{t-2}\right) + \frac{1}{\beta}\left(1 - \frac{1}{\varpi}\right) u_{x,t-1} \end{aligned} \tag{159}$$

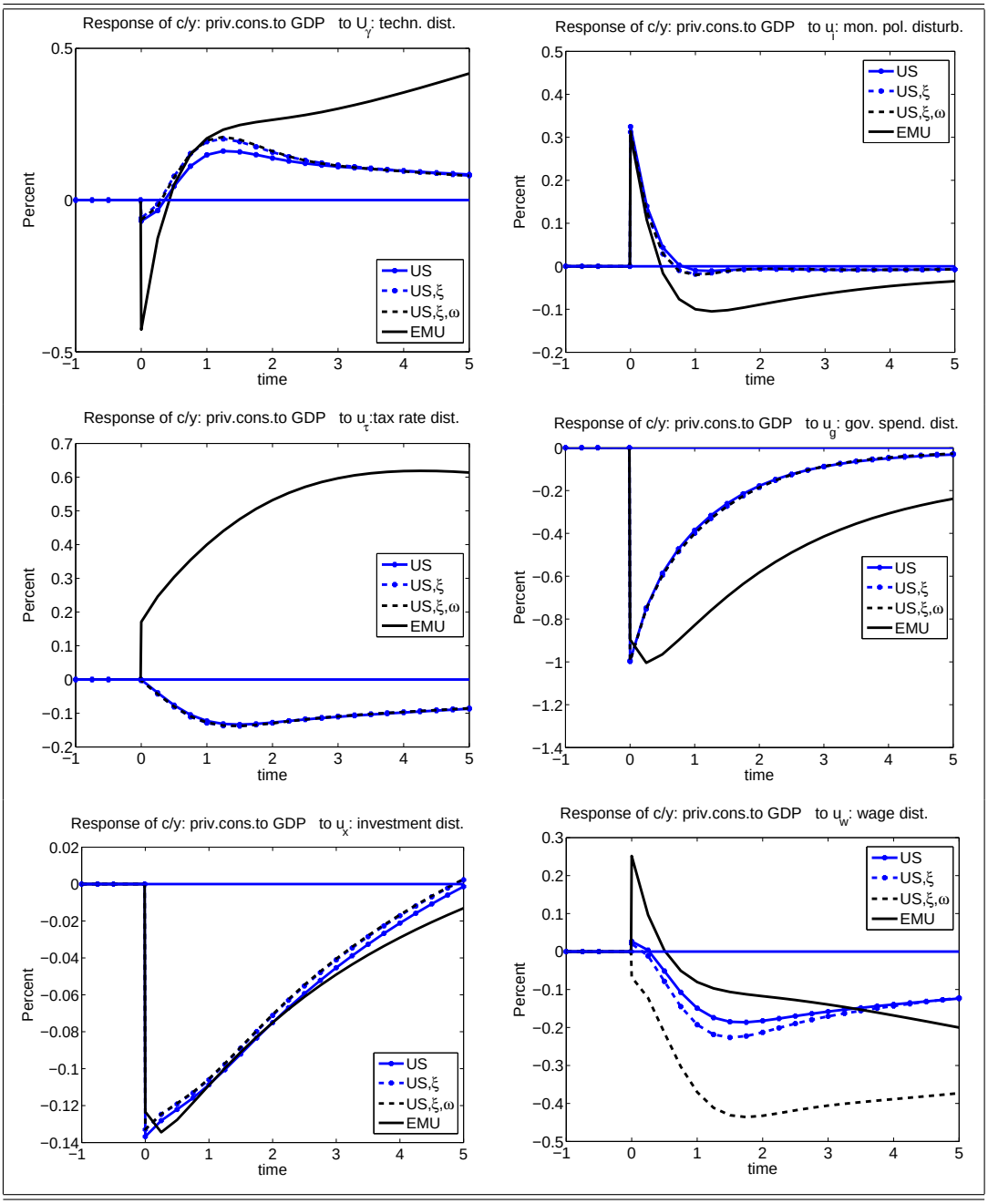
One may interpret this as the log-deviation of the return to capital, taking into account the cost of adjustment and the additional discounting due to the extra period of being able to spend the rental rate on consumption. With this definition and the help of equation (131), one can rewrite (157) as

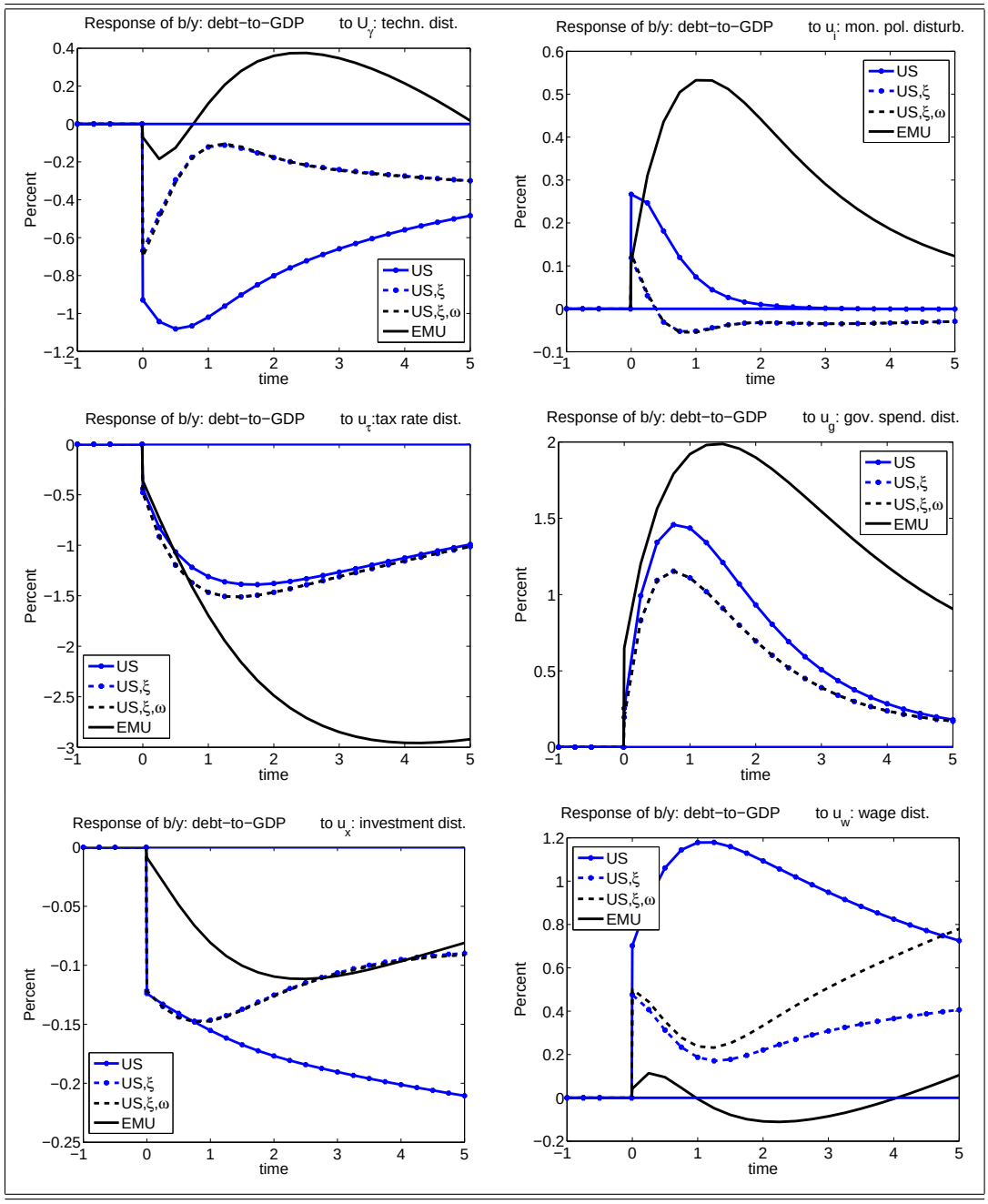
$$\begin{aligned} 0 &= -\hat{\lambda}_t + \eta(\hat{\lambda}_t - \hat{\varrho}_t) \\ &\quad + E_t\left[\lambda_{t+1} + \beta\eta(1-\delta)\left(\hat{\varrho}_{t+1} - \hat{\lambda}_{t+1}\right) + \beta\hat{r}_{t+1}^*\right] \end{aligned} \tag{160}$$

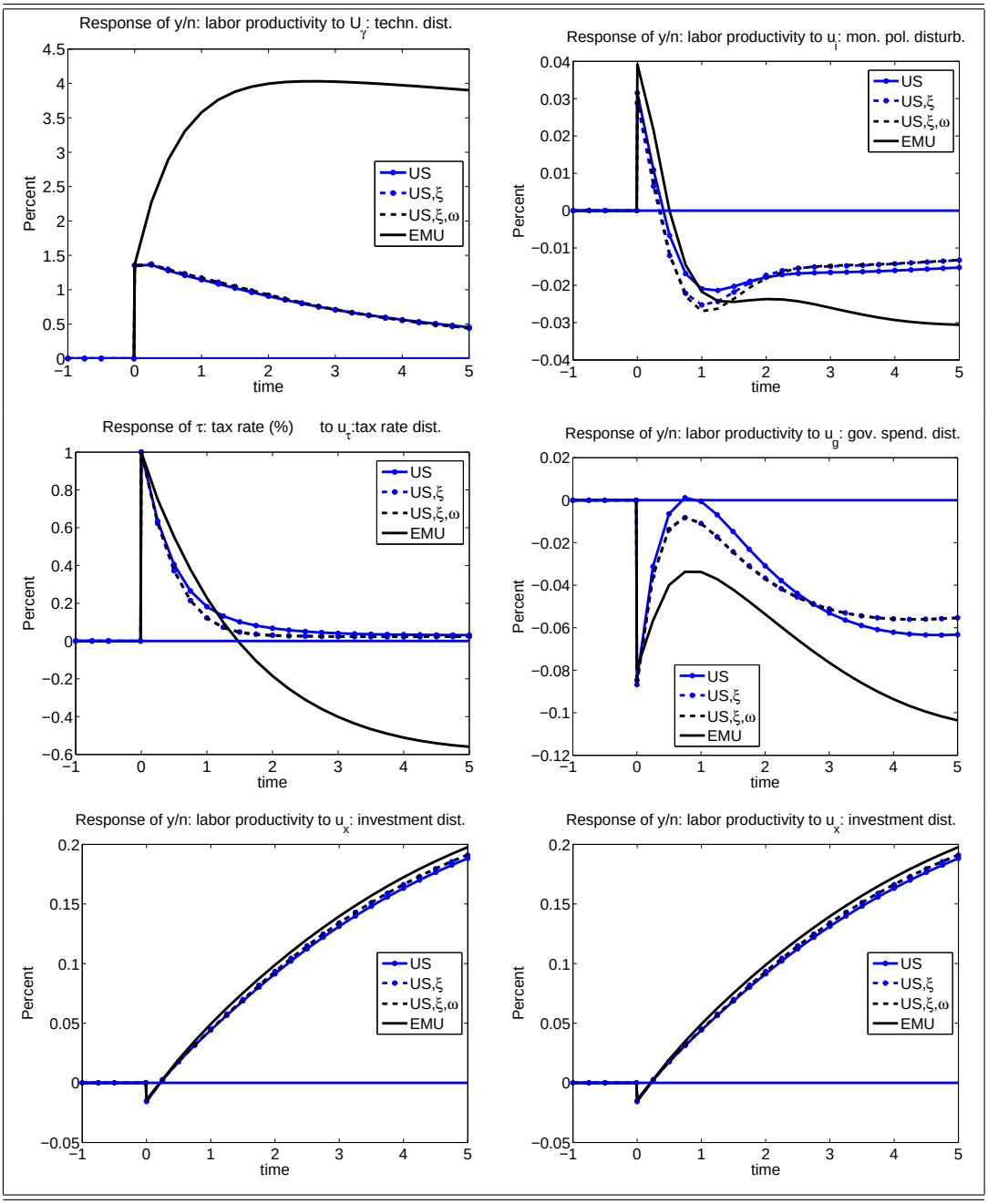
which may be a more intuitive or familiar expression.

B Figures

Shown here are the impulse responses of the three variables used for estimating the model, not shown in the body of the paper.


Response of c/y: priv.cons.to GDP to U_γ: techn. dist.
Response of c/y: priv.cons.to GDP to u_i: mon. pol. disturb.
Response of c/y: priv.cons.to GDP to u_τ:tax rate dist.
Response of c/y: priv.cons.to GDP to u_g: gov. spend. dist.
Response of c/y: priv.cons.to GDP to u_x: investment dist.
Response of c/y: priv.cons.to GDP to u_w: wage dist.
Percent
time
US
US,ξ
US,ξ,ω
EMU


Response of b/y: debt−to−GDP to U_y: techn. dist.
Response of b/y: debt−to−GDP to u_i: mon. pol. disturb.
Response of b/y: debt−to−GDP to u_τ:tax rate dist.
Response of b/y: debt−to−GDP to u_g: gov. spend. dist.
Response of b/y: debt−to−GDP to u_x: investment dist.
Response of b/y: debt−to−GDP to u_w: wage dist.
Percent
time
US
US,ξ
US,ξ,ω
EMU


Response of y/n: labor productivity to U_γ: techn. dist.
Response of y/n: labor productivity to u_l: mon. pol. disturb.
Response of τ: tax rate (%) to u_τ:tax rate dist.
Response of y/n: labor productivity to u_g: gov. spend. dist.
Response of y/n: labor productivity to u_x: investment dist.
Response of y/n: labor productivity to u_x: investment dist.
Percent
time
US
US,ξ
US,ξ,ω
EMU