# Optimal Monetary Policy and the Sources of Local-Currency Price Stability[1]

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**Abstract**

In this paper we reconsider the policy trade-offs created by stable import prices in local currency, which realistically result from both nominal rigidities and endogenous destination-specific markup adjustment by exporters. The main novelty of our model consists of placing price stickiness at the heart of strategic interactions between upstream producers and local firms. We show that price dispersion at consumer level due to staggered pricing decisions by local firms, affects the desired markup by upstream producers, magnifying their price response to shocks. We solve for the optimal policy under cooperation, unveiling a reason specific to the international dimension of the economy in support of price stability as the main prescription for monetary policy. As stable consumer prices feed back into a low volatility of markups among upstream producers, this contains inefficient deviations from the law of one price at the border.

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# 1 Introduction

The high degree of stability of import prices in local currency, documented both at the border and at consumer level,[1] vis-à-vis large movements in exchange rates, raises issues at the core of the design of optimal monetary policy in open economies. On the one hand, a low elasticity of import prices with respect to the exchange rate can result from factors independent of nominal rigidities. An instance is the presence of costs incurred locally before the imported goods reach the consumers, such as distribution costs or local assembly costs; another instance is optimal markup adjustment by monopolistic firms, which maximize profits through price discrimination across national markets ("pricing to market"). While these real sources of local-currency price stability of imports may result in inefficiencies — like deviations from the law of one price due to pricing to market — they shield price and wage dynamics from currency volatility, thus helping central banks to maintain a low and stable headline inflation. On the other hand, stable import prices and low exchange rate pass-through can also stem from nominal frictions impeding desired markup adjustment, thus interfering with equilibrium movements in relative prices. When local-currency-price stability of imports is due to price stickiness, it creates policy trade-offs between competing objectives, e.g. between stabilizing the prices of domestically produced goods as opposed to the (relative) price of imported goods.

In this paper we reconsider these policy trade-offs in open economies where stable import prices in local currency result from both nominal rigidities and endogenous destination-specific markup adjustment. The main question is how these different mechanisms influence the optimal design of monetary policy. As in standard monetary models, we assume that firms set prices in local currency, adjusting them infrequently according to the Calvo mechanism.[2] Different from most models, we model local downstream firms who make intermediate traded goods available to final users possibly using local inputs. Specifically, we specify that a two country model where each economy produces an array of country-specific, differentiated traded goods. Each good is produced by a monopolist and sold to a continuum of monopolistic downstream firms, active in each market, from which local consumers can directly buy further differentiated final

[1] See, e.g. Engel and Rogers, 1996; Goldberg and Knetter, 1997; Campa and Goldberg, 2005 and Frankel et al. 2004.

[2] See the literature review in section 2 below.

varieties. Because both upstream and downstream firms have monopoly power, final prices reflect double marginalization. Realistically, we assume that intermediate producers are allowed to price-discriminate between domestic and foreign local downstream producers as a group, but not among individual local producers (i.e. they do charge different prices within the same country). Upstream firms exercise their monopoly power by taking into account country-specific differences in the properties of the aggregate demand for their products.

Relative to the literature already modelling vertical interactions in monetary open economies (e.g. Bacchetta and Van Wincoop 2005, Corsetti and Dedola 2005, Devereux and Engel 2004, Monacelli 2005), the main novelty of our contribution consists of placing nominal rigidities at the heart of the strategic interdependence between upstream and downstream firms. Namely, we show that, since only a fraction of downstream firms update their price every period, the optimal price by upstream producers (thus their 'desired markup') is a function of price dispersion at consumer level — due to staggered price decisions by local firms.

A first important set of results establishes three key characteristics of the perceived demand elasticity by upstream producers. First, this elasticity is decreasing in downstream price dispersion: the higher the price dispersion among final producers selling an industry product, the higher the intermediate producer's equilibrium markup on average. Second, the perceived demand elasticity is market-specific, depending on differences in industry price dispersion across domestic and export markets. Sticky prices at consumer level create an incentive for upstream firms to price discriminate across borders, independently of the degree of nominal rigidities in their own prices. Third, the demand elasticity is increasing in the price charged by upstream producers. In other words, downstream nominal rigidities magnify the price response to shocks by those upstream monopolists who adjusts their price during each period.

The latter result is particularly meaningful in light of recent empirical debates on pass-through. Against the backdrop of a large body of literature documenting that, relative to consumer prices, border prices have on average a higher exchange rate elasticity, recent empirical evidence (see Gopinath and Rigobon [2006]) suggests that border prices are revised less frequently than final, consumer prices. Now, the average exchange-rate pass-through depends on both the frequency of import price adjustment, and the size of the adjustment by the firms who revise their prices in local currency. According to our model, downstream price rigidities magnify the incentives for upstream producers to undertake large price revisions. Even if the

average frequency of price revision is the same for intermediate and final producers, in equilibrium our model predicts a higher variability of producer prices "at the dock" in response to exchange rate movements.

Another corollary to our results is that, in line with the observations by Taylor [2000], lower CPI inflation volatility and price dispersion tend to reduce exchange-rate pass-through: while stable inflation reduces at the margin the producers' incentives to price discriminate across countries — which in itself decreases pass-through —, it also reduces the sensitivity of their 'desired markup' to price changes.

These results have immediate implications for policy design: price stability as the main prescription of monetary policy, carries over in an environment with strategic interdependence in vertically integrated industries. We show this result by solving for the (first-best) optimal monetary policy under cooperation. In our findings, benevolent central bankers jointly implement a high degree of consumer price stabilization, as this feeds back into a low volatility of markups at producers' level. When such policy is implemented, holding the productivity of downstream firms constant, the variability of the deviations from the law of one price are much lower at the dock than at the final consumer.

Consistent with the literature (e.g. Corsetti and Pesenti [2005]), the optimal monetary policy is not inward-looking, in the sense that it stabilizes a weighted average of prices of domestically produced and imported goods, where the optimal weights depend, among other things, on the composition of final demand. Clearly, because of home bias in final demand and local currency price stability, the isomorphism between optimal monetary policy in closed and open economies characterized by Clarida, Gertler and Galí [2003] breaks down. Nonetheless, we show that, while efficient monetary policy comes close to stabilizing CPI inflation, a strict inflation targeting would generate too much volatility in consumption and sectoral markups.

These 'international dimensions' of optimal monetary policy, however, do not mechanically entail specific prescriptions regarding the volatility of the real exchange rate. In the literature, optimal monetary policies in models with nominal rigidities in local currency are sometimes associated with a reduced degree of real exchange rate volatility relative to terms of trade volatility — see e.g. Devereux and Engel 2004. Our framework provides an example of an economy in which, under the optimal monetary policy, the real exchange rate can be more or less volatile than the terms of trade, depending on the relative degree of nominal rigidities of

upstream and downstream firms.

This paper is organized as follows. In the next section, we will briefly survey the literature on pricing to market and monetary policy, with the goal of clarifying our contributions to it. Section 3 specifies the model, and provides analytical results on price stickiness and price discrimination. Section 4 describes our calibration of the model. Section 5 sheds light on equilibrium properties of prices, and discusses the allocation under the optimal policy, relative to alternative policy rules and the case of flexible prices. Section 6 concludes. The appendix provides analytical details on the model and the derivation of the main results.

# 2 Local currency price stability and the debate on efficient monetary stabilization

In this section we briefly reconstruct the main development of recent debates on monetary transmission and monetary policy design in open economy, with the goal of clarifying our contribution to the literature. A relevant issue in these debates is whether monetary policy should react to international variables, such as the exchange rate or the terms of trade, beyond the influence that these variables have on the domestic output gap (for example, via external demand) and on the domestic good prices — so that it would have a specific international dimension. As argued below, an answer to this question has been provided by models stressing the stability of import prices in local currency — and their implications for monetary transmission and stabilization policy.

## 2.1 The 'LCP' versus 'PCP' controversy

Our starting point is the controversy in the so-called New Open Economy Macroeconomics (henceforth NOEM) literature, on the role of the exchange rate in the international transmission mechanism. While the main passages in this controversy are well known (see for instance Corsetti and Pesenti 2005b for a synthesis), we find it instructive to go over it in some detail.

Consistent with traditional open macroeconomic models, the seminal NOEM contribution by Obstfeld and Rogoff (1995) encompasses the view that exchange rate movements play the stabilizing role of adjusting international relative prices in response to shocks, when frictions prevent or slow down price adjustment in the local currency. At the heart of this vision is

the idea that nominal depreciation transpires into real depreciation, making domestic goods cheaper in the world markets, hence redirecting world demand towards them: exchange rate movements therefore have 'expenditure switching effects'. Accordingly, NOEM contributions after Obstfeld and Rogoff (1995) draw on the Mundell–Fleming and Keynesian tradition, and posit that firms preset the price for their products in domestic currency, implying that export prices are sticky in the currency of the producers — thus this hypothesis is commonly dubbed 'producer currency pricing' (henceforth PCP). Under this hypothesis, nominal import prices in local currency move one-to-one with the exchange rate and pass-through is perfect. In the baseline model with preset prices, identical demand elasticity across countries can ensure that there is no incentive for producers to charge different prices across borders: in equilibrium there would be no deviations from the law of one price even if national markets were segmented.

In model economies with such PCP, optimal monetary policy rules tend to be 'inward-looking' (and 'iso-morphic' to the rules derived in closed-economy models): welfare-maximizing central banks pursue the stabilization of domestic producers marginal costs and markups — hence of the GDP deflator —, while letting the consumer price index (CPI) fluctuate with movements in the relative price of imports. There is no need for monetary policies to react to international variables — a result that in the baseline NOEM model after Corsetti and Pesenti (2001) goes through under different assumptions regarding nominal rigidities, including staggered price setting and partial adjustment (see, for example, Clarida, Galí and Gertler , 2002, or Benigno and Benigno, 2003 for a generalization of the baseline model).

The high elasticity of import prices to the exchange rate underlying the stylized contributions after Obstfeld and Rogoff (1995), however, is clearly at odds with a large body of empirical studies showing that the exchange rate pass-through on import prices is far from complete in the short run, and deviations from the law of one price are large and persistent (see, for example, Engel and Rogers, 1996; Goldberg and Knetter, 1997; Campa and Goldberg, 2005). Based on this evidence, several contributions have engaged in a thorough critique of the received wisdom on the expenditure switching effects of the exchange rate. Specifically, Betts and Devereux (2000) and Devereux and Engel (2003), among others, posit that firms preset export prices in the currency of the market where they sell their goods. This assumption, commonly dubbed 'local currency pricing' (henceforth LCP), attributes local currency price stability of imports entirely to nominal frictions. The far-reaching implications of LCP for the role of the exchange

rate in the international transmission mechanism have been widely discussed by the literature (see e.g. Engel, 2003).

To the extent that import prices are sticky in the local currency, a Home depreciation does not affect the price of Home goods in the world markets; hence, it has no expenditure switching effects. Instead, it raises ex post markups on Home exports: at given marginal costs, revenues in domestic currency from selling goods abroad rise. In contrast with the received wisdom, nominal depreciation strengthens a country's terms of trade: if export prices are preset during the period, the Home terms of trade improves when the Home currency weakens.

An important merit of models assuming LCP is to unveil a clear-cut argument in favour of policies with an 'international dimension.' One way to present the argument is as follows. To the extent that exporters' revenues and markups are exposed to exchange rate uncertainty, firms' optimal pricing strategies internalize the monetary policy of the importing country. In the benchmark model by Corsetti and Pesenti 2005, for instance, foreign firms optimally preset the price of their goods in the Home market one period ahead, by charging the equilibrium markup over expected marginal costs evaluated in Home currency. The (preset) price of Home imports then depends on the joint distribution of Home monetary policy and Foreign productivity shocks: in the model, it is increasing in the variance of nominal marginal costs.

The reason why the isomorphism between closed-economy and open-economy monetary rules breaks down is apparent. Suppose that Home monetary authorities ignore the influence of their decisions on the price of Home imports. Incomplete stabilization of Foreign firms marginal costs and markup in local currency will translate into inefficiently high local prices of their product. On the other hand, if Home monetary authorities want to stabilize Foreign firms' marginal costs, they can only do so at the cost of raising costs and markup uncertainty for Home producers, resulting in inefficient Home good prices. It follows that, to maximize Home welfare, Home policymakers should optimally trade-off the stabilization of marginal costs of all producers (domestic and foreign) selling in the Home markets. The same principle goes through in model with LCP but staggered price adjustment. In this case, what matters is the loss of welfare due not only to the misalignment of import prices, but also the relative price dispersion among domestic and foreign goods.

The general lesson is that, when foreign firms' profits are exposed to exchange rate uncertainty, optimal monetary rules are no longer inward-looking. The optimal response to Foreign

shocks by domestic policymakers depends on the degree of openness of the economy, as indexed by the overall share of imports in the CPI (see Corsetti and Pesenti, 2005a, and Sutherland, 2005, for a discussion of intermediate degrees of pass-through, and Smets and Wouters, 2002, and Monacelli, 2005, for models with staggered price setting).

### 2.2 Real determinants of local currency price stability of imports

While most of the discussion in the NOEM literature has focused on incomplete pass-through as an implication of nominal rigidities for pass through, a low pass-through, in itself, is not necessarily incompatible with expenditure switching effects — a point stressed by Obstfeld (2000) among others. In this respect, Obstfeld and Rogoff (2000) points out that, in the data (and consistent with the received wisdom), nominal depreciation does tend to be associated with deteriorating terms of trade. This piece of evidence clearly sets an empirical hurdle for LCP models, against specifications which assume a very high degree of price stickiness in local currency (see Corsetti, Dedola and Leduc, 2005, for a quantitative assessment). Interestingly, estimates of LCP models attributing incomplete pass-through exclusively to nominal rigidities in local currency, tend to predict that the degree of price stickiness is implausibly higher for imports than for domestic goods — a result suggesting model misspecification (see, for example, Lubik and Schorfheide, 2006).

The key issue is the extent to which the evidence of local currency price stability of imports can be explained by nominal rigidities. In the open economy literature, it is well understood that the low elasticity of import prices with respect to the exchange rate is in large part due to the incidence of distribution (see Burstein, Eichenbaum and Rebelo, 2006 for a recent reconsideration of this point). In addition, several macro and micro contributions have emphasized the role of optimal destination-specific markup adjustment by monopolistic firms depending on market structure (e.g. Dornbusch, 1987), vertical interactions between producers and retailers (e.g. Corsetti and Dedola, 2005), or habits (e.g. Ravn, Schmidt-Grohé and Uribe 2006). Analyses of the relative importance of these different sources of import price stability are provided by several market-specific studies — such as Goldberg and Verboven, 2001, Goldberg and Hellerstein 2007, or Hellerstein 2006. The main result emerging from these contributions is that real factors can explain a large extent of local currency price stability of imports — see for instance the numerical results in previous work by two of us specifying models with sticky wages and flexible prices

(Corsetti and Dedola 2005).

Yet quantitative studies incorporating these factors corroborate the idea that a realistic degree of nominal rigidities can improve substantially the quantitative performance of the model (see for instance Corsetti Dedola and Leduc 2005). The interaction of real and monetary determinants of low exchange rate pass-through and deviations from the law of one price thus defines promising research avenues which are currently been explored by the literature.

As a first instance of these avenues, an important strand of the literature recently emphasizes the need to treat the currency denomination of exports as an endogenous choice by profit maximizing firms. Bacchetta and Van Wincoop, 2005, Devereux, Engel and Storgaard, 2004, Friberg 1998 have developed models where firms can choose whether to price export in domestic or in foreign currency, knowing that price updates will be subject to frictions. A number of factors — from the market share of exporters to the incidence of distribution and the availability of hedging instruments — potentially play a crucial role in this choice —see Engel 2006 for a synthesis.

A specific contribution by the New Open Economy Macroeconomics on this issue, sheds light on the role of monetary policy as a determinant of the LCP versus PCP choice (see Corsetti and Pesenti 2002). The main idea is as follows. Expansionary monetary shocks unrelated to fundamental shocks (e.g. productivity) raise nominal wages and marginal costs while depreciating the currency. Consider a firm located in a country with noisy monetary policy, i.e. hit by frequent monetary shocks unrelated to fundamentals. For such firm, pricing its exports in foreign currency (that is, choosing LCP) is attractive in the following sense: it ensures that revenues from exports in domestic currency will move in parallel with nominal marginal costs, with stabilizing effects on the markup. This is because any expansionary monetary shock depreciating the Home currency would simultaneously raise wages and the domestic currency revenue from unit sales abroad (at an unchanged local price). This observation may help explain why exporters from countries with relatively unstable domestic monetary policies (e.g. some developing countries) prefer to price their exports to developed countries in the importers' currency. The same argument, however, suggests that LCP is not necessarily optimal for exporters producing in countries where monetary policy systematically stabilizes marginal costs (see Goldberg and Tille, 2005, for empirical evidence). For firms located in these country, real factors arguably become more relevant in the choice.

A second research avenue consists of taking the LCP choice as given, and focus on likely interactions between nominal rigidities and price discrimination by profit maximizing firms. This is the avenue we explore in this paper. Different from previous contributions, where price stickiness and price discrimination coexist without feeding into each other, we build a model starting with the observation that the frequency of price changes by local downstream firms selling products to consumers, is bound to affect the elasticity of demand perceived by upstream producers of tradables goods and intermediate inputs.

# 3 Modelling the sources of local currency price stability: price discrimination and nominal rigidities

The world economy consists of two countries of equal size, $H$ and $F$. Each country specializes in one type of tradable good, produced in a number of differentiated industries defined over a continuum of unit mass. Tradable goods are indexed by $h \in [0,1]$ in the Home country and $f \in [0,1]$ in the Foreign country. In each industry, the firm producing the tradable good $h$ (or $f$) is a monopolistic supplier of one good, using labor as the only input to production. These firms set prices in local currency units and in a staggered fashion as in Calvo (1983).

A distinctive feature of our setting is that we model a downstream sector in each country. Specifically, we assume that each producer's good $h$ is sold to consumers in many varieties by a continuum of local firms indexed by $r_h \in [0,1]$. These firms buy the $h$ tradable goods, and turn them into consumer goods — which are not traded across borders — with random productivity. We will distinguish between two cases: one in which local firms use domestic labor as an input; the other in which they do not. Similar to upstream producers, also downstream operate under monopolistic competition and set prices in a staggered fashion as in Calvo (1983).

By the logic of the Calvo adjustment, local downstream firms buying goods from upstream producers charge different prices to final users, with a constant fraction of them reoptimizing prices in each period. In principle, one could assume that upstream firms exercise their monopoly power by charging individual prices which are specific to each downstream firm. However, we find it more realistic and convincing to assume that upstream producers are not able to price-discriminate across individual local firms, but only across groups of them — namely, across domestic and foreign local firms. So, we assume that upstream producers exercise their monopoly

power and set prices by taking into account the total demand for their product in each market, at Home and in the Foreign country.

In what follows, we describe our set up focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy — variables referred to Foreign firms and households are marked with an asterisk.

## 3.1 The Household's Problem

### 3.1.1 Preferences

The representative Home agent maximizes the expected value of her lifetime utility, given by the following standard functional form:

$$V_0 = E\sum_{t=0}^{\infty}\beta^t U\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\chi\frac{\left(\frac{M_{t+1}}{P_t}\right)^{1-\sigma}}{1-\sigma}+\kappa\frac{(1-L_t)^{1-\upsilon}}{1-\upsilon},\right], \tag{1}$$

where instantaneous utility $U$ is a function of a consumption index, $C_t$, leisure, $(1-L_t)$, and real money balances $\frac{M_{t+1}}{P_t}$. Households consume both domestically produced and imported goods. We define $C_t(h, r_h)$ as the Home agent's consumption as of time $t$ of the variety $r_h$ of the Home good $h$, produced and distributed by the firm $r_h$; similarly, $C_t(f, r_f)$ is the Home agent's consumption of the variety $r_f$ of the good $f$, produced and distributed by firm $r_f$. For each good $h$ (or $f$), we assume that one final good variety $r_h$ ($r_f$) is an imperfect substitute for all other final good varieties, with constant elasticity of substitution $\eta > 1$:

$$C_t(h) \equiv \left[\int_0^1 C_t(h,r_h)^{\frac{\eta-1}{\eta}}dr\right]^{\frac{\eta}{\eta-1}}, \qquad C_t(f) \equiv \left[\int_0^1 C_t(f,r_f)^{\frac{\eta-1}{\eta}}dr\right]^{\frac{\eta}{\eta-1}},$$

$C_t(h)$ is the consumption of (all varieties of) the Home good $h$, by the Home agent, at time $t$; similarly, $C_t(f)$ is the same agent's consumption of the Foreign good $f$. We then assume that the good produced by the $h$ industry is an imperfect substitute for all other goods produced by the Home industries, with the same constant elasticity of substitution $\eta > 1$ as between final good varieties. Aggregate consumption of Home and Foreign goods by the Home agent is thus defined as:

$$C_{\mathrm{H},t} \equiv \left[\int_0^1 C_t(h)^{\frac{\eta-1}{\eta}}dh\right]^{\frac{\eta}{\eta-1}}, \qquad C_{\mathrm{F},t} \equiv \left[\int_0^1 C_t(f)^{\frac{\eta-1}{\eta}}df\right]^{\frac{\eta}{\eta-1}},$$

The full consumption basket, $C_t$, in each country, aggregates Home and Foreign goods according to the following standard CES function:

$$C_t \equiv \left[ a_{\mathrm{H}}^{1-\phi} {C_{\mathrm{H},t}}^{\phi} + a_{\mathrm{F}}^{1-\phi} {C_{\mathrm{F},t}}^{\phi} \right]^{\frac{1}{\phi}}, \qquad \phi < 1, \tag{2}$$

where $a_{\mathrm{H}}$ and $a_{\mathrm{F}}$ are the weights on the consumption of Home and Foreign traded goods, respectively and $\frac{1}{1-\phi}$ is the constant elasticity of substitution between $C_{\mathrm{H},t}$ and $C_{\mathrm{F},t}$.

### 3.1.2 Budget constraints and asset markets

For simplicity, we posit that domestic and international asset markets are complete and that only domestic residents hold the Home currency, $M_{t+1}$. Households derive income from working, $W_t L_t$, from domestic firms ' profits and from previously accumulated units of currency, as well as from the proceeds from holding state-contingent assets, $B_t$. They pay non-distortionary (lump-sum) net taxes $\mathbb{T}$, denominated in Home currency. Households use their disposable income to consume and invest in state-contingent assets. The individual flow budget constraint for the representative agent $j$ in the Home country is therefore:

$$\begin{aligned} \mathbb{P}_{\mathrm{H},t} C_{\mathrm{H},t} + \mathbb{P}_{\mathrm{F},t} C_{\mathrm{F},t} + \int_s p_{bt,t+1} B_{t+1} + M_{t+1} \leq W_t L_t + M_t + B_t + \int_0^1 \Pi(h) dh \\ + \int_0^1 \int_0^1 \Pi(h, r_h) dh dr_h + \int_0^1 \int_0^1 \Pi(f, r_f) df dr_f + \mathbb{T}_t \end{aligned} \tag{3}$$

where $\Pi(.)$ denotes the agent's share of profits from all firms $h$ and $r$ in the economy. The price indexes are as follows: $\mathbb{P}_{\mathrm{H},t}$ denote the consumer price of the aggregate Home traded good, $\mathbb{P}_{\mathrm{F},t}$ is the consumer price of aggregate Home imports. We will use $P_t$ to denote the consumer price index.

The household's problem then consists of maximizing lifetime utility, defined by (1), subject to the constraints (3).

## 3.2 Production structure and technology

International price discrimination is a key feature of the international economy captured by our model. In what follows we show that, even if Home and Foreign consumers have identical constant-elasticity preferences for consumption, there are different ways in which vertical interactions between upstream and downstream firms causes differences in the elasticity of demand

for the $h$ ($f$) product at wholesale level across national markets. Upstream firms will thus want to charge different prices at Home and in the Foreign country. We will focus our analysis on Home firms — optimal pricing by Foreign firms can be easily derived from it. To distinguish between upstream and downstream firms, we will denote variables referred to the former with an upper bar.

We begin by specifying the technology used by upstream firms producing Home tradables. These firms employ domestic labor to produce a differentiated product $h$ according to the following linear production function:

$$\overline{Y}(h) = \overline{ZL}(h),$$

where $\overline{L}(h)$ is the demand for labor by the producer of the good $h$ and $\overline{Z}$ is a technology shock common to all upstream producers in the Home country, which follows a statistical process to be determined below. The letter $h$ will be indifferently referred to a upstream producer selling to downstream firms $r_h$, or the corresponding 'industry.'

In previous work of ours (Corsetti and Dedola (2005) and in Corsetti, Dedola and Leduc (2005)), we have modelled downstream firms in an industry $h$ which combine the tradable good $h$ with labor according to the fixed proportion production function, such as

$$Y(h, r_h) = X(h, r_h) + ZL(h, r_h) \tag{4}$$

where $X(h, r_h)$ is the demand for tradable good $h$ by firm $r_h$ and $L(h, r_h)$ is the demand for labor by the same firm; $Z$ is a random technology component that affects the amount of nontraded input required to produce the variety $r_h$ and distribute it to consumers. This random shock is country-specific, and hits symmetrically all national downstream firms. As shown by Corsetti and Dedola (2005), the above specification can generate endogenous movements in upstream firms' markups and cross-border price discrimination independently of nominal price rigidities.

In this paper, we make a related, but different point. Namely, we show that vertical interactions among upstream and downstream firms can lead to price discrimination exclusively as a consequence of nominal rigidities. To focus sharply on this case, we specify the production function of downstream firms removing the local input (labor) altogether. Namely, we assume that, in each industry $h$, the production function of firms $r_h$ is linear in the traded input only

$$Y(h, r_h) = ZX(h, r_h). \tag{5}$$

Although we will carry out quantitative analysis of the latter specification only, for the sake of comparison we will also study analytical properties of a model embedding nominal rigidities in our previous framework (with a Leontief production function).

### 3.3 The problem of downstream firms

Since both upstream and downstream firms are subject to nominal rigidities à la Calvo, at any time $t$ downstream firms will buy either from a producer $h$ which updates its price in the same period, or from a producer still charging an old price. Conversely, in each period, upstream producers updating their price will need to consider that only a fraction of downstream firms buying their products will also re-optimize in the period. In characterizing optimal pricing decisions, it is instructive to go over these cases separately. Let $\theta$ be the probability that a downstream firm within the industry $h$ keeps its price fixed — in each period a firm $r_h$ sets a new price with probability $(1-\theta)$. The corresponding probabilities for the upstream producers will be denoted by $\overline{\theta}$ and $\left(1-\overline{\theta}\right)$.

We start with the optimization problem of downstream firms $r_h$ which reset their prices in an industry $h$ where the producer also updates its price in the same period. In this group, the representative firm $r_h$ chooses $P_t(h,r_h)$ to maximize the expected discounted sum of profits:

$$\pi(h) = E_t\left\{\sum_{k=0}^{\infty} p_{bt,t+k}\theta^k \left(P_t(h,r_h)C_{t+k}(h,r_h) - MC_{t+k}(h)C_{t+k}(h,r_h)\right)\right\},$$

where $p_{bt,t+k}$ is the firm's stochastic nominal discount factor between $t$ and $t+k$. This firm faces the following final demand:

$$C_t(h,r_h) = \left(\frac{P_t(h,r_h)}{\mathbb{P}_t(h)}\right)^{-\eta}\left(\frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}}\right)^{-\eta} C_{\mathrm{H},t},$$

where $\mathbb{P}_t(h)$ is the price index of the good (or industry) $h$, and $\mathbb{P}_{\mathrm{H},t}$ is the price index of all Home goods — to be defined below in section 3.2.4. Depending on whether we consider (4) or (5), the firm's marginal cost, $MC_t(h)$, is given by the following expressions:

$$\begin{aligned} MC_t(h) &= \overline{P}_t(h) + \alpha Z_t W_t \\ MC_t(h) &= \frac{\overline{P}_t(h)}{Z_t} \end{aligned}$$

where $\overline{P}_t(h)$ is the price of good $h$ charged by the producer. The optimal price charged to

consumers, $P_t(h)$, can be written in the following standard form:

$$P_t(h) = \frac{\eta}{\eta - 1} \frac{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k MC_{t+k}(h) C_{t+k}(h)}{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k C_{t+k}(h)}. \tag{6}$$

Referring to the appendix for details, the infinite sum above can be written recursively. This allows us to solve for the optimal policy using the non-linear first-order condition and standard numerical methods.

Consider now the problem of firms updating their prices in industries in which the upstream producer keeps its price unchanged during the period. Clearly these firms face a different marginal cost, and thus charge a different price than $P_t(h, r_h)$ above. Denoting these firms' new prices and marginal costs with $\widetilde{P}_t(h)$ and $\widetilde{MC}_t(h)$, we can write the pricing equation as follows:

$$\widetilde{P}_t(h) = \frac{\eta}{\eta - 1} \frac{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k \widetilde{MC}_{t+k}(h) C_{t+k}(h)}{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k C_{t+k}(h)}. \tag{7}$$

where as before the marginal cost will be either one of the following:

$$\begin{aligned} \widetilde{MC}_t(h) &= \overline{P}_{t-1}(h) + \alpha Z_t W_t \\ \widetilde{MC}_t(h) &= \frac{\overline{P}_{t-1}(h)}{Z_t} \end{aligned}$$

depending on which technology they use. Observe that, since the producer doesn't update its price in the period, the downstream firms' marginal cost is a function of last period's price $\overline{P}_{t-1}(h)$.

### 3.4 The problem of upstream firms

We are now coming to the core of our contribution. Hereafter we show that, as downstream firms adjust their prices infrequently, firms producing tradables will endogenously choose to "price to market": price discrimination follows from the solution to these firms' problem . Consistent with the logic of the Calvo model, we posit that, when upstream producers update their prices, they do so simultaneously in the Home and in the Foreign market, in the respective currencies.

The maximization problem is then as follows:

$$Max_{\bar{p}(h),\bar{p}^*(h)} \;\; E_t \left\{ \sum_{k=0}^{\infty} p_{bt,t+k} \theta^k \left( \left[ \overline{P}_t(h) D_{t+k}(h) + \mathcal{E}_t \overline{P}_t^*(h) D_{t+k}^*(h) \right] - \overline{MC}_{\mathrm{H},t+k} \left[ D_{t+k}(h) + D_{t+k}^*(h) \right] \right) \right\} \tag{8}$$

where the marginal cost of the producer is given by:

$$\overline{MC}(h) = \frac{W_t}{\overline{Z}_t},$$

and, depending on the production function downstream, the Home and Foreign demands for the firm's variety are given by:

$$\begin{aligned} D_t(h) &= \int \left( \frac{P_t(h, r_h)}{\mathbb{P}_t(h)} \right)^{-\eta} \left( \frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}} \right)^{-\eta} C_{\mathrm{H},t} dr_h \\ D_t^*(h) &= \int \left( \frac{P_t^*(h, r_h^*)}{\mathbb{P}_t^*(h)} \right)^{-\eta} \left( \frac{\mathbb{P}_t^*(h)}{\mathbb{P}_{\mathrm{H},t}^*} \right)^{-\eta} C_{\mathrm{H},t}^* dr_h^* \end{aligned}$$

for the case of Leontief production function, or

$$\begin{aligned} D_t(h) &= \frac{1}{Z_t} \int \left( \frac{P_t(h, r_h)}{\mathbb{P}_t(h)} \right)^{-\eta} \left( \frac{\mathbb{P}_t(h)}{\mathbb{P}_{\mathrm{H},t}} \right)^{-\eta} C_{\mathrm{H},t} dr_h \\ D_t^*(h) &= \frac{1}{Z_t^*} \int \left( \frac{P_t^*(h, r_h^*)}{\mathbb{P}_t^*(h)} \right)^{-\eta} \left( \frac{\mathbb{P}_t^*(h)}{\mathbb{P}_{\mathrm{H},t}^*} \right)^{-\eta} C_{\mathrm{H},t}^* dr_h^* \end{aligned}$$

in the linear production case. In these expressions, $\mathbb{P}_t^*(h)$ and $\mathbb{P}_{\mathrm{H},t}^*$ denote the price index of industry $h$ and of Home goods, respectively, in the Foreign country, expressed in Foreign currency — also to be defined below. In comparing the two sets of demands above, recall that in the linear production case (5) the firm's productivity affect the quantity of tradable good $h$ needed to satisfy a given final demand for each variety $r_h$: hence the demand for the monopolist's product is scaled by this productivity.

For each industry $h$, it is useful to define the relative price dispersion at the consumer level as:

$$S_t(h) \equiv \int \left( \frac{P_t(h, r_h)}{\mathbb{P}_t(h)} \right)^{-\eta} dr_{h.}$$

In the appendix, we show that such price dispersion can be written as:

$$S_t(h) \equiv (1 - \theta) \left( \frac{P_t(h)}{\mathbb{P}_t(h)} \right)^{-\eta} + \theta \pi_t^{\eta}(h) S_{t-1},$$

where $\pi_t^{\eta}(h) = \frac{\mathbb{P}_t(h)}{\mathbb{P}_{t-1}(h)}$. Using these results, we can rewrite the demand faced by each upstream producer as a function of price dispersion. In other words, the producer's demand curve depend on the price dispersion at the consumer level, induced by infrequent price adjustment by downstream firms.[3]

By the first order condition of the producer's problem, the optimal price $\overline{P}_t(h)$ charged to domestic downstream firms is:

$$\overline{P}_t(h) = \frac{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \overline{\theta}^k \left[ \overline{MC}_{t+k}(h) \frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} - D_{t+k}(h) \right]}{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \overline{\theta}^k \frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)}}.$$

In the above expression, the partial derivative $\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)}$ takes into account how the price set by the producer as of $t$, will affect the choice of downstream firms that will have a chance to change their prices in the current period and in the future. This derivative clearly depends on the specification of these firms' production function: to further characterize the optimal producer price $\overline{P}_t(h)$ we now look at the case of the linear and the Leontief production function separately.

### 3.4.1 Strategic substitutability in the linear technology case

**Optimal pricing and price dispersion at consumer level** When the downstream firms' production function is linear as in (5), the derivative of the producer's demand with respect to its own price is:

$$\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} = -\eta (1-\theta) P_{t+k}(h)^{-\eta-1} \mathbb{P}^{\eta}_{\mathrm{H},t+k} C_{\mathrm{H},t+k} \frac{1}{Z_t} \frac{\partial P_{t+k}(h)}{\partial \overline{P}_t(h)}.$$

where the partial derivative $\frac{\partial P_{t+k}(h)}{\partial \overline{P}_t(h)}$ captures the extent to which current and future pricing decision by firms $r_h$ are affected by the current producer pricing decision — recall that $P_{t+k}(h)$ denotes the optimal price set by the (fraction $1-\theta$ of) downstream firms which will reoptimize in each period $t+k$, while facing the traded input price $\overline{P}_t(h)$.

[3] In the Leontief case, for instance, the demand curve can be written as:

$$D_{t+k}(h) = \left[ (1-\theta) P_{t+k}^{-\eta}(h) + \theta \mathbb{P}^{-\eta}_{t+k-1}(h) S_{t+k-1}(h) \right] \mathbb{P}^{\eta}_{\mathrm{H},t+k} C_{\mathrm{H},t+k}.$$

As shown in appendix, it turns out that this derivative is simply equal to the ratio of the two prices, e.g. at time $t$ we have $\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} = \frac{P_t(h)}{\overline{P}_t(h)}$. Using this fact, the impact on current and future demand of a price change by the producer can be simplified as follows:[4]

$$\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} = -\eta\left(1-\theta\right)\frac{P_{t+k}(h)^{-\eta}}{\overline{P}_t(h)}\frac{1}{Z_{t+k}}\mathbb{P}_{\mathrm{H},t+k}^{\eta}C_{\mathrm{H},t+k}.$$

The optimal producer price is as follows:

$$\overline{P}_t(h) = \frac{\eta}{\eta-1}\frac{E_t\sum_{k=0}^{\infty}p_{bt,t+k}\overline{\theta}^k\overline{MC}_{t+k}(h)\left(\frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta}\frac{C_{\mathrm{H},t+k}}{Z_{t+k}}}{E_t\sum_{k=0}^{\infty}p_{bt,t+k}\overline{\theta}^k\left(\frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta}\frac{C_{\mathrm{H},t+k}}{Z_{t+k}}\left[1-\frac{\theta}{(\eta-1)(1-\theta)}\left(\frac{P_{t+k}(h)}{\mathbb{P}_{t+k-1}(h)}\right)^{\eta}S_{t+k-1}(h)\right]}. \quad (9)$$

We emphasize one important and novel result suggested by this expression. From the vantage point of a producer of tradables $h$, the marginal revenue from a price change reflects the fact that only some downstream firms update their prices in any given period. The upstream monopolist does not know which individual firm $r_h$ will be updating their price in the period, but knows that a fraction $1-\theta$ of them will do so, while a fraction $\theta$ will keep their price unchanged. Because of the latter, the producer will optimally charge a price which is higher than she/he would ideally charge if all downstream producers set new prices. The higher the price dispersion, the higher the optimal price.

The implications for optimal pricing of nominal rigidities at the consumer level are apparent when comparing the above expression, with the standard expression obtained by positing that only producers adjust their prices infrequently:

$$\overline{P}_t(h) = \frac{\eta}{\eta-1}\frac{E_t\sum_{k=0}^{\infty}p_{bt,t+k}\overline{\theta}^k\overline{MC}_{t+k}(h)\left(\frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta}\frac{C_{\mathrm{H},t+k}}{Z_{t+k}}}{E_t\sum_{k=0}^{\infty}p_{bt,t+k}\overline{\theta}^k\left(\frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}}\right)^{-\eta}\frac{C_{\mathrm{H},t+k}}{Z_{t+k}}}. \quad (10)$$

[4]We note here that this result is due to the fact that for prices $P_{t+k}$ reset optimally as of $t+k$, $\frac{\partial MC_{t+k+s}(h)}{\partial \overline{P}_t(h)} = \frac{1}{Z_{t+k+s}} = \frac{MC_{t+k+s}(h)}{\overline{P}_t(h)}$, $\forall s > 0$, and

$$\frac{\partial C_{t+k+s}(h,r_h)}{\partial \overline{P}_t(h)} = -\eta\frac{C_{t+k+s}(h,r_h)}{P_{t+k}(h)}\frac{\partial P_{t+k}(h)}{\partial \overline{P}_t(h)}.$$

The difference between the two consists of the term in squared brackets in the denominator of (9), which include price dispersion, and the probability of price updates by downstream firms.

The price (in foreign currency) charged to downstream firms in the foreign country has an analogous form:

$$\overline{P}_t^*(h) = \frac{\eta}{\eta-1}\frac{E_t\sum_{k=0}^{\infty} p_{bt,t+k}\overline{\theta}^k \mathbb{P}_{\mathrm{H},t+k}^{*\eta}\frac{C_{\mathrm{H},t+k}^*}{Z_{t+k}^*}P_{t+k}^*(h)^{-\eta}\frac{\overline{MC}_{t+k}(h)}{\mathcal{E}_{t+k}}}{E_t\sum_{k=0}^{\infty} p_{bt,t+k}\overline{\theta}^k \mathbb{P}_{\mathrm{H},t+k}^{*\eta}\frac{C_{\mathrm{H},t+k}^*}{Z_{t+k}^*}P_{t+k}^*(h)^{-\eta}\left[1-\frac{\theta}{(\eta-1)(1-\theta)}\left(\frac{P_{t+k}^*(h)}{\mathbb{P}_{t+k-1}^*(h)}\right)^{\eta}S_{t+k-1}^*(h)\right]}, \quad (11)$$

Denote with $\mathcal{E}_t$ the Home nominal exchange rate, expressed in units of Home currency per unit of Foreign currency, so that the above price in Home currency is $\mathcal{E}_t\overline{P}_t^*(h)$. Comparing this price with the one charged to domestic downstream firms, (9), one can clearly see that the two are generally different: the law of one price is systematically violated. In this respect, it is worth emphasizing that in our economy deviations from the law of one price across markets are not a mechanical implication of nominal rigidities in local currency. They also depend on the way vertical interactions among upstream and downstream monopolists affect optimal pricing by producers.

**Demand price elasticities and strategic substitutability** To appreciate this point fully, we write the price elasticity of demand at each point in time as perceived by the producer:

$$\begin{aligned}
-\frac{\partial D_{t+k}(h)}{\partial\overline{P}_t(h)}\frac{\overline{P}_t(h)}{D_{t+k}(h)} &= \eta(1-\theta)\frac{P_{t+k}(h)^{-\eta}}{\overline{P}_t(h)}\mathbb{P}_{\mathrm{H},t+k}^{\eta}\frac{C_{\mathrm{H},t+k}}{Z_{t+k}}\frac{\overline{P}_t(h)}{D_{t+k}(h)} \\
&= \frac{\eta(1-\theta)\left(\frac{P_{t+k}(h)}{\mathbb{P}_{t+k}(h)}\right)^{-\eta}}{S_{t+k}(h)} \\
&= \frac{\eta}{1+\frac{\theta}{1-\theta}\left(\frac{P_{t+k}(h)}{\mathbb{P}_{t+k-1}(h)}\right)^{\eta}S_{t+k-1}(h)}.
\end{aligned}$$

This demand elasticity is a function of the producer price $\overline{P}_t(h)$ only indirectly, through the consumer price $P_{t+k}(h)$: absent nominal rigidities at consumer price level ($\theta = 0$) the price elasticity of the producer would be constant and equal to that perceived by the downstream firm, $\eta$ — the price charged to consumers would simply be $\frac{\eta}{\eta-1}\frac{\overline{P}_t(h)}{Z_t}$. The effect arising from nominal rigidities at the consumer level is captured by the term in the denominator: as discussed above, the upstream monopolist does not know whether the individual firm $r_h$ is updating their

price, but takes into account that a fraction $\theta$ of them keep their price unchanged during the period.

Because of nominal rigidities, the demand elasticity is clearly falling in the final price and thus in the upstream producer price. This generates *strategic substitutability* among producers: namely, a rise in marginal costs will lead to an increase in the desired markups by producers. This result is important because it implies that producers prices will be more reactive to shocks to their demand conditions and to marginal costs: when vertical interactions among firms with sticky prices are considered, it may not be necessarily the case that several layers of nominal rigidities bring about more inertia in the dynamic of prices at consumer level.

It is worth stressing that, if monetary policy could remove the within-industry price dispersion, it would make the producer's demand elasticity and thus the desired markup constant. By eliminating within-industry price dispersion, monetary authorities would therefore eliminate the incentive to price discriminate. In this case, the producer's demand elasticity would be given by $\eta(1-\theta)$. Note that the producer's markup in a steady state with no consumer price dispersion is affected by the presence of the Calvo parameter.

### 3.4.2 Price dispersion and local costs as competing determinants of the producers' demand price elasticity: the Leontief technology case

**Optimal pricing and local nontraded inputs** In previous work (see Corsetti Dedola and Leduc 2005) we have shown that, when the production function of the downstream firm is Leontief and include a local input, the effects of shocks on real markups are also country-specific, but are quite different from the ones induced by price dispersion at consumer level described above — namely, they tend to lead to strategic complementarities among producers. In the rest of this subsection we reconsider these different results in the same framework. Our main conclusion is that local inputs in downstream production leads to an attenuation of the main effects of price stickiness on optimal markups by upstream producers, but does not necessarily overturn them.

When the technology of the downstream firms is as in (4), the derivative of the producer demand with respect to its price becomes:

$$\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} = -\eta(1-\theta) P_{t+k}(h)^{-\eta-1} \mathbb{P}^{\eta}_{\mathrm{H},t+k} C_{\mathrm{H},t+k} \frac{\partial P_{t+k}(h)}{\partial \overline{P}_t(h)},$$

which is similar to the one derived for the linear case, except that the right-hand side is not scaled by downstream firms' productivity. Moreover, after some simplifications (detailed in the appendix), we find that the derivative of the final price to the producer price is a constant depending on $\eta$. Evaluating this derivative at time $t$ we can write[5]

$$\frac{\partial P_t(h)}{\partial \overline{P}_t(h)} = \frac{\eta}{\eta - 1}.$$

Intuitively, the effect of an increase in the upstream producer price (and thus of the marginal cost) on the price optimally charged by downstream firms in the same period will be proportional to the markup charged by the the latter, $\frac{\eta}{\eta-1}$ — a clear instance of double marginalization.

The derivative of current and future demands with respect to the wholesale price becomes:

$$\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)} = -\frac{\eta^2}{\eta - 1}(1-\theta) P_{t+k}(h)^{-\eta-1} \mathbb{P}^{\eta}_{\mathrm{H},t+k} C_{\mathrm{H},t+k};$$

this is due to the fact that for retail prices $P_{t+k}$ reset as of $t+k$, it will still be true that $\frac{\partial MC_{t+k+s}(h)}{\partial \overline{P}_t(h)} = 1$ and

$$\frac{\partial C_{t+k+s}(h, r_h)}{\partial \overline{P}_t(h)} = -\eta \frac{C_{t+k+s}(h, r_h)}{P_{t+k}(h)} \frac{\partial P_{t+k}(h)}{\partial \overline{P}_t(h)};$$

The optimal domestic producer price is then given by

$$\overline{P}_t(h) = \frac{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \overline{\theta}^k \left( \frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}} \right)^{-\eta} C_{\mathrm{H},t+k} \left[ 1 + \frac{\eta^2}{\eta - 1} \frac{\overline{MC}_{t+k}(h)}{P_{t+k}(h)} + \frac{\theta}{(1-\theta)} \left( \frac{P_{t+k}(h)}{\mathbb{P}_{t+k-1}(h)} \right)^{\eta} S_{t+k-1}(h) \right]}{\frac{\eta^2}{\eta - 1} E_t \sum_{k=0}^{\infty} p_{bt,t+k} \overline{\theta}^{kk} \left( \frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}} \right)^{-\eta} \frac{C_{\mathrm{H},t+k}}{P_{t+k}(h)}};$$

For comparison, we also write the corresponding price absent nominal rigidities among downstream firms ($\theta = 0$):

$$\overline{P}_t(h) = \frac{E_t \sum_{k=0}^{\infty} p_{bt,t+k} \overline{\theta}^k \left( \frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}} \right)^{-\eta} C_{\mathrm{H},t+k} \left[ 1 + \frac{\eta^2}{\eta - 1} \frac{\overline{MC}_{t+k}(h)}{P_{t+k}(h)} \right]}{\frac{\eta^2}{\eta - 1} E_t \sum_{k=0}^{\infty} p_{bt,t+k} \overline{\theta}^{kk} \left( \frac{P_{t+k}(h)}{\mathbb{P}_{\mathrm{H},t+k}} \right)^{-\eta} \frac{C_{\mathrm{H},t+k}}{P_{t+k}(h)}};$$

[5] Since $\frac{\partial MC_{t+k}(h)}{\partial \overline{P}_t(h)} = 1$ and

$$\begin{aligned} \frac{\partial C_{t+k}(h, r_h)}{\partial \overline{P}_t(h)} &= -\eta \left( \frac{P_t(h)}{\mathbb{P}_{\mathrm{H},t+k}} \right)^{-\eta} C_{\mathrm{H},t+k} \frac{1}{P_t(h)} \frac{\partial P_t(h)}{\partial \overline{P}_t(h)} \\ &= -\eta \frac{C_{t+k}(h, r_h)}{P_t(h)} \frac{\partial P_t(h)}{\partial \overline{P}_t(h)}. \end{aligned}$$

as well as the price elasticity of demand at each point in time as perceived by the upstream producer:

$$\begin{aligned} -\frac{\partial D_{t+k}(h)}{\partial \overline{P}_t(h)}\frac{\overline{P}_t(h)}{D_{t+k}(h)} &= \frac{\eta^2}{\eta-1}(1-\theta)\,P_{t+k}(h)^{-\eta-1}\mathbb{P}^{\eta}_{\mathrm{H},t+k}C_{\mathrm{H},t+k}\frac{\overline{P}_t(h)}{D_{t+k}(h)} \\ &= \frac{\frac{\eta^2}{\eta-1}\frac{\overline{P}_t(h)}{P_{t+k}(h)}}{1+\frac{\theta}{1-\theta}\left(\frac{P_{t+k}(h)}{\mathbb{P}_{t+k-1}(h)}\right)^{\eta}S_{t+k-1}(h)}. \end{aligned}$$

Different from the linear production case, the demand elasticity is a now function of the wholesale price $\overline{P}_t(h)$ not only indirectly, through the retail price $P_{t+k}(h)$, but also directly. Specifically, it reflects three effects. The first, arising from the double marginalization due to the presence of two vertically integrated monopolists, is captured by the term $\frac{\eta^2}{\eta-1}$: absent nominal rigidities ($\theta = 0$) and the nontraded input ($\alpha = 0$) among downstream firms, the price elasticity of the producer would be constant and equal to that perceived by these firms, $\eta$ — the price charged to consumers would simply be $\frac{\eta}{\eta-1}\overline{P}_t(h)$.

The second effects, arising from the assumption that the traded input is combined in fixed proportion with labor by downstream firms, is captured by the term $\frac{\overline{P}_t(h)}{P_t(h)}$ : absent nominal rigidities this ratio would be equal to $\frac{\overline{P}_t(h)}{\frac{\eta}{\eta-1}\left(\overline{P}_t(h)+\alpha\frac{W_t}{Z_t}\right)}$ as in Corsetti and Dedola (2005), implying that the demand elasticity is smaller than $\eta$ but increasing in $\overline{P}_t(h)$.

The third and last effect, arising from nominal rigidities, is captured by the term in the denominator of the elasticity expression above. It has already and amply discussed in the linear case in the previous subsection.

**Is the producers' demand elasticity increasing in the tradable price?** As discussed above, the effect of local input (complement in final good production) and price dispersion on producers' markups run against each other: the former makes the producers' demand elasticity increasing, the other decreasing, in the producer price. Which one is more likely to prevail, and under what conditions? It can be shown that the producer's elasticity will be increasing in their

price $\overline{P}_t(h)$ if and only if

$$\begin{aligned}
\frac{\eta^2}{\eta-1}\frac{\overline{P}_t(h)}{P_t(h)}\frac{\dfrac{1}{\overline{P}_t(h)}-\dfrac{1}{P_t(h)}-\dfrac{\dfrac{\eta^2}{\eta-1}\dfrac{\theta}{1-\theta}\left(\dfrac{P_t(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta}S_{t-1}(h)}{P_t(h)\left[1+\dfrac{\theta}{1-\theta}\left(\dfrac{P_t(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta}S_{t-1}(h)\right]}}{\left[1+\dfrac{\theta}{1-\theta}\left(\dfrac{P_t(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta}S_{t-1}(h)\right]} &\geq 0\\
\left(\frac{P_t(h)}{\overline{P}_t(h)}-1\right)+\frac{\theta}{1-\theta}\left(\frac{P_t(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta}S_{t-1}(h)\left[\left(\frac{P_t(h)}{\overline{P}_t(h)}-1\right)-\frac{\eta^2}{\eta-1}\right] &\geq 0.
\end{aligned}$$

Since $P_t(h)-\overline{P}_t(h)>0$ because of the labor input employed by downstream firms, a sufficient condition for this elasticity to be increasing in $\overline{P}_t(h)$ is that the margin $\dfrac{P_t(h)-\overline{P}_t(h)}{\overline{P}_t(h)}$ be larger than $\dfrac{\eta^2}{\eta-1}$ — which is the product between the steady state markup $\dfrac{\eta}{\eta-1}$ and the long-run final demand elasticity $\eta$. Note that $\dfrac{\eta^2}{\eta-1}$, is always much larger than 1; the margin $\dfrac{P_t(h)-\overline{P}_t(h)}{\overline{P}_t(h)}$ cannot be (realistically) expected to exceed 1 by a large amount. It is then likely that the overall price elasticity be falling in $\overline{P}_t(h)$. Observe that, in the case of no price distortions ($S_t(h)=1$) the producer's elasticity will be increasing in $\overline{P}_t(h)$ only if $\dfrac{P_t(h)}{\overline{P}_t(h)}-1>\theta\dfrac{\eta^2}{\eta-1}$.

To appreciate fully the role of price rigidities in affecting the incentive to price discriminate, it is instructive to consider the case in which upstream producer prices are fully flexible — that is, $\overline{\theta}=0$; the derivative $\dfrac{\partial P_t(h)}{\partial\overline{P}_t(h)}$ then becomes:

$$\frac{\partial P_t(h)}{\partial\overline{P}_t(h)}=\frac{\eta}{\eta-1}\left[\frac{E_t\sum_{k=0}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)}{C_t(h,r_h)}+\eta\frac{MC_t(h)}{P_t(h)}\left(\frac{\eta}{\eta-1}-\frac{P_t(h)}{MC_t(h)}\right)\right]^{-1},$$

so that the price elasticity is now equal to:

$$-\frac{\partial D_t(h)}{\partial\overline{P}_t(h)}\frac{\overline{P}_t(h)}{D_t(h)}=$$

$$=\frac{\dfrac{\eta^2}{\eta-1}\dfrac{\overline{P}_t(h)}{P_t(h)}}{\left[1+\dfrac{\theta}{1-\theta}\left(\dfrac{P_t(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta}S_{t-1}(h)\right]\left[1+\dfrac{E_t\sum_{k=1}^{\infty}p_{bt,t+k}\theta^k C_{t+k}(h,r_h)}{C_t(h,r_h)}+\eta\left(\dfrac{\eta}{\eta-1}\dfrac{\overline{P}_t(h)+\alpha\dfrac{W_t}{Z_t}}{P_t(h)}-1\right)\right]}.$$

This demand elasticity is quite different from the one derived before, for the case of multiple layers of nominal rigidities. For once, it is now more likely to be increasing in $\overline{P}_t(h)$: intuitively, the larger the actual markup relative to the desired markup by downstream firms, $\frac{\eta}{\eta-1}$, the higher the price elasticity as perceived by the upstream producer.

## 3.5 Price indexes and market clearing

### 3.5.1 Price indexes

We conclude this section by formally defining the price indexes repeatedly used in the analysis so far, and writing down the market clearing conditions in the goods market. In an industry in which the producer updates its price, the price index of the good $h$ at consumer level is given:

$$\mathbb{P}_t(h) = \left[\int_0^1 P_t(h,r_h)^{1-\eta} dr_h\right]^{\frac{1}{1-\eta}} = \left[(1-\theta)P_t(h)^{1-\eta} + \theta\mathbb{P}_{t-1}(h)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

In an industry in which the producer doesn't update its price, the corresponding price index is:

$$\widetilde{\mathbb{P}}_t(h) = \left[\int_0^1 P_t(h,r_h)^{1-\eta} dr_h\right]^{\frac{1}{1-\eta}} = \left[(1-\theta)\widetilde{P}_t(h)^{1-\eta} + \theta\widetilde{\mathbb{P}}_{t-1}(h)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

The price index of Home tradables consumed at home thus becomes:

$$\begin{aligned}
\mathbb{P}_{\mathrm{H},t} &= \left[\int_0^1 \mathbb{P}_t(h)^{1-\eta} dh\right]^{\frac{1}{1-\eta}}. \\
\mathbb{P}_{\mathrm{H},t}^{1-\eta} &= \left[(1-\overline{\theta})\mathbb{P}_t(h)^{1-\eta} + \overline{\theta}\widetilde{\mathbb{P}}_t(h)^{1-\eta}\right] \\
&= (1-\theta)\left[(1-\overline{\theta})P_t(h)^{1-\eta} + \overline{\theta}\widetilde{P}_t(h)^{1-\eta}\right] + \theta\left[(1-\overline{\theta})\mathbb{P}_{t-1}(h)^{1-\eta} + \overline{\theta}\widetilde{\mathbb{P}}_{t-1}(h)^{1-\eta}\right] \\
&= (1-\theta)\left[(1-\overline{\theta})P_t(h)^{1-\eta} + \overline{\theta}\widetilde{P}_t(h)^{1-\eta}\right] + \theta\mathbb{P}_{\mathrm{H},t-1}^{1-\eta}
\end{aligned}$$

The price index associated with the consumption basket, $C_t$, is:

$$CPI_t = \left[a_{\mathrm{H}}\mathbb{P}_{\mathrm{H},t}^{\frac{\phi}{\phi-1}} + a_{\mathrm{F}}\mathbb{P}_{\mathrm{F},t}^{\frac{\phi}{\phi-1}}\right]^{\frac{\phi-1}{\phi}}.$$

### 3.5.2 Equilibrium in the goods market

To characterize the equilibrium conditions in the goods market, we equate supply to demand at each firm level. Integrating over all downstream firms in a given industry we get:

$$\overline{Y}_t(h) = a_{\mathrm{H}}\left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta}\left(\frac{P_{\mathrm{H},t}}{CPI_t}\right)^{\frac{1}{\phi-1}}\frac{C_{\mathrm{H},t}}{Z_t}S_t(h) + (1-a_{\mathrm{H}})\left(\frac{\mathbb{P}_t^*(h)}{P_{\mathrm{H},t}^*}\right)^{-\eta}\left(\frac{P_{\mathrm{H},t}^*}{CPI_t^*}\right)^{\frac{1}{\phi-1}}\frac{C_{\mathrm{H},t}^*}{Z_t^*}S_t^*(f),$$

Integrating over all industries, aggregate output is:

$$\overline{Y}_t = a_{\mathrm{H}} \left( \frac{P_{\mathrm{H},t}}{CPI_t} \right)^{\frac{1}{\phi-1}} \frac{C_{\mathrm{H},t}}{Z_t} \overline{S}_t + (1 - a_{\mathrm{H}}) \left( \frac{P^*_{\mathrm{H},t}}{CPI^*_t} \right)^{\frac{1}{\phi-1}} \frac{C^*_{\mathrm{H},t}}{Z^*_t} \overline{S}^*_t$$

where the price dispersion term, $\overline{S}_t$, is defined as:

$$\overline{S}_t = \int \left( \frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}} \right)^{-\eta} S_t(h) dh,$$

and captures the relative price dispersion within and across industries. Since $\overline{S}_t$ and $S_t(h)$ are bounded below by 1, price dispersion imply a real resource cost.

# 4 Calibration

This section describes the benchmark calibration for our numerical experiments, which we assume symmetric across countries. We used dynare++ to solve for the optimal monetary policy and to simulate our different economies. In each exercise, we report statistics averaged over 100 simulations of 100 periods each.

## 4.1 Preferences and production

We posit that the period-by-period utility function has the form already shown by (1), that we reproduce here for convenience:

$$U \left[ C_t, \frac{M_{t+1}}{P_t}, L_t \right] = \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{\left( \frac{M_{t+1}}{P_t} \right)^{1-\sigma}}{1-\sigma} + \kappa \frac{(1-L_t)^{1-\upsilon}}{1-\upsilon}, \tag{12}$$

We set $\kappa$ so that in steady state, one third of the time endowment is spent working. In our benchmark calibration, we assume $\upsilon$ equal to $\sigma$ (risk aversion), which we in turn set to 2. Since the utility function is separable in consumption and real money balances, money demand is determined residually and does not play any role in our results. We therefore set $\chi$ arbitrarily to 0.1.

We set the constant elasticity of substitution across brands, $\eta$, so that the markup of downstream firms in steady state is 15 percent. Following Backus, Kehoe, and Kydland [1992], we chose $\phi$ so that the trade elasticity is 1.5. As regards the weights of domestic and foreign tradables in the tradables consumption basket ($C_{\mathrm{T}}$), $a_{\mathrm{H}}$ and $a_{\mathrm{F}}$ (normalized $a_{\mathrm{H}} + a_{\mathrm{F}} = 1$) are set such that imports are 10 percent of aggregate output in steady state, roughly in line with the

average ratio for the U.S. in the last 30 years. We pick the steady state value of $Z$ to ensure that the price of traded goods accounts for 50 percent of the final price in steady state. This value corresponds to the empirical estimates by Burstein, Neves and Rebelo [2003], for the distribution margin only. In our specification, downstream firms can do more than distributing goods, suggesting that the value we select is on the conservative side.

As benchmark, we set the probability that upstream and downstream firms update their prices to 50 percent. This overall frequency of price adjustment is in line with the evidence in Bils and Klenow [2004] and Nakamura and Steinsson [2007], if sales are treated as price changes. However, we also experiment with different values — for instance we study the case in which downstream firms update their prices less frequently than intermediate goods producers, since these authors also find evidence that finished goods prices are less flexible than the prices of intermediate goods (see also Gopinath and Rigobon 2006).

### 4.2 Productivity shocks

Let the vector $\mathbf{Z} \equiv \{Z, \overline{Z}, Z^*, \overline{Z^*}\}$ represent the sectoral technology shocks in the domestic and foreign economies. We assume that sectoral disturbances to technology follow a trend-stationary AR(1) process

$$\mathbf{Z}^{'} = \boldsymbol{\lambda}\mathbf{Z} + \mathbf{u}, \tag{13}$$

whereas has variance-covariance matrix $V(\mathbf{u})$, and $\boldsymbol{\lambda}$ is a $4x4$ matrix of coefficients describing the autocorrelation properties of the shocks, that are the same for both sectoral shocks. Since we assume a symmetric economic structure across countries, we also impose symmetry on the autocorrelation and variance-covariance matrices of the above process. Because of lack of sectoral data on productivity, we posit that sectoral shocks follow a rather conventional process.[6] First, in line with most of the international business cycle literature — e.g., BKK — we assume that these shocks are very persistent, and set their autocorrelation to 0.95. Second, the standard deviation of the innovations is set to 0.007. For simplicity, we set the shock correlation and the spillovers across countries and sectors to zero.

[6] In Corsetti, Dedola and Leduc [2004] we estimated this vector process with annual data, the only frequency for which sectoral productivity is available for several OECD countries. If we use a quarterly version of that process we get broadly similar results to those reported here.

### 4.3 Monetary policy

To characterize the optimal monetary policy, we let the planner choose the growth rates of money in the Home and Foreign economies, to maximizes world welfare subject to the first-order conditions for households and firms and the economy-wide resource constraints. We assume that the planner places equal weights on Home and Foreign welfare, so that world welfare is given by the following expression:

$$\text{welfare} = \frac{V_0 + V_0^*}{2}$$

We follow an approach similar to that in Khan, King, and Wolman (2003) and consider an optimal policy that has been in place for a long enough time that initial conditions do not matter. When solving our economies, we assume the presence of fiscal subsidies, financed via lump-sum taxation, to ensure that all prices would equal marginal costs if prices were fully flexible.

In describing our results, we also compare the optimal policy to other well-known policy rules. We first consider a Taylor-type rule that sets the short-term nominal interest rate as a function of the deviations of CPI inflation and GDP from steady state values:

$$R_t = \rho R_{t-1} + \chi(1-\rho)E(\pi_t - \pi^{ss}) + \gamma(1-\rho)(y_t - y^{ss}). \tag{14}$$

We parameterize the policy rule using the estimates in Lubik and Schorfheide [2004]: $\rho = 0.84$, $\chi = 2.19$, $\gamma = 0.3$. We also consider inflation targeting rules in which the central bank stabilizes either the inflation rate at the consumer or producer level, which we label CPI and GDP inflation targeting, respectively.

## 5 Nominal rigidities, price discrimination and optimal monetary policy

### 5.1 The response of producers and consumer prices to shocks

In light of the analytical results of Section 3, we now discuss key properties of our model regarding the behavior of prices and markups in response to productivity shocks, looking at impulse responses. Figures 1 and 2 show the impulse responses of prices, markups and inflation — all in percentage deviations from their steady state values — to productivity shocks, distinguishing

between the intermediate and final production sectors. For simplicity, throughout these exercises we assume that central banks in the two countries set monetary policy to implement a strict CPI inflation targeting.

### 5.1.1 Shocks to the productivity of upstream firms

Figure 1 focuses on the effects of a productivity increase in the production of Home goods. In response to this shock, upstream producers which update their prices will lower them both in the domestic and the foreign market (see the first chart on the upper left corner). Note that the price falls more abroad than in the domestic market, even when expressed in the same currency, violating the law of one price — in the Figure, a positive value for the deviation from the law of one price means that domestic prices are larger than foreign prices.

This result is mirrored by the response of the desired and actual markups of the upstream Home producers, in both markets, shown by the second and third graph in the Figure 1. As discussed in Section 3, the presence of downstream nominal rigidities leads to pricing substitutability at the level of upstream producers. As a result, the *desired markup* by these producers fall with the prices: this is true in both markets, but relatively more in the Foreign one. However, because of sticky prices, their *actual aggregate markup* rises. Given that both the real and — because of CPI targeting — the nominal exchange rate depreciate together, this means that exchange-rate pass-through for the producers adjusting their prices is larger than one, as shown by our analytical results for the linear technology.[7]

The impact of the same shock on *consumer prices* clearly differs, depending on whether the monopolist in a particular industry $h$ updates its price, or leaves it unchanged. Consider first the case of an industry in which the wholesale price has fallen. Downstream firms face a drop in their marginal costs: those which can reset prices will then react by lowering them, both domestically and abroad. As we have seen above that the intermediate good price falls more in the Foreign country, Foreign downstream firms decrease their price by more than the domestic ones. Hence the deviations from the law of one price at consumer level have the same sign as the ones characterizing producer prices.

Interestingly, we observe that consumer prices fall also in industries in which the upstream

[7] In the analytical section, we also show that this result is attenuated when the production function include local labor input, with a low degree of substitutability with the intermediate tradable goods.

producers *do not* update their prices during the period — albeit by a smaller amount than the other case. This is so for two reasons. First, although marginal costs of downstream firms in these industry do not fall in the period, these firms nonetheless take into account that the productivity shock is persistent: they thus anticipate that their marginal costs are likely to decrease in the future. Second, a lower price helps these firms respond to increasing competition by firms operating in the other industries, where the price of intermediate product have already gone down. It is worth pointing out an interesting result concerning deviations from the law of one price in this case. In the industries where the price of the tradable goods have remained constant, on impact consumer prices decrease by more at Home than abroad: the deviations from the law of one price are larger, but of the opposite sign, relative to the case of industries updating their upstream prices in the period.

### 5.1.2 Shock to the productivity of downstream firms

Figure 2 displays the impact of an increase in the productivity of Home downstream firms. Consider first the case of industries where the upstream producers keep their current price fixed in the period. Since productivity shocks are country specific, they lower the marginal costs of downstream firms at Home, but do not affect the costs of downstream firms in the industry located in the Foreign country. As a result, domestic firms updating their prices will optimally lower them, while foreign local firms will keep their prices virtually unchanged. This causes deviations from the law of one price, which are further magnified by the Home currency depreciation.

More complex is the case of industries in which upstream producers change their prices, as the overall effects of the shock will depend on a number of general equilibrium considerations. Key to understand these effects is that higher productivity by downstream firms causes an increase in their output, and thus in real domestic consumption. The increase in downstream output does not affect real wages directly — under the linear production function specified above, it has no direct impact on the demand for labor, since these firms are assumed not to employ any labor input. However, it does so indirectly: higher domestic consumption is associated with a positive income effect which ultimately translates into a downward shift in hours worked. Facing higher labor costs, upstream firms which can reset their prices raise them, thus increasing the marginal costs of downstream firms. This feedback effect on consumer prices is therefore positive.

Because of currency depreciation, domestic upstream producers slightly lower their wholesale prices in foreign currency. However, in Home currency these prices are higher than the one charged in the domestic market, causing a negative deviation from the law of one price. Consistently, the desired markup of Home producers increases in the Home market, while falls abroad — in line with the price changes. Yet, reflecting again the aggregate effects of nominal rigidities, actual markups fall everywhere in the economy.

### 5.2 Properties of optimal stabilization policy

We now turn to the analysis of the key features of cooperative optimal monetary policy under commitment. The results for our baseline calibration are reported in Table 1, which reports results on the volatility of macro variables under the alternative monetary policies specified in Section 4 — the optimal policy, CPI inflation targeting, GDP inflation targeting, and the Taylor-type rule. Note that, as we assume subsidies which exactly offset steady-state markups, the long-run inflation rates are equal to zero in all these economies.

As a benchmark, the first column of the table shows results for the flexible-price version of our economy. With flexible prices, the demand elasticity facing producers is constant, and the law of one price holds at the dock (the volatility of deviations from the law of one price at the dock is correspondingly zero). However, the law of one price does not hold for final goods: country-specific productivity shocks hitting the downstream firms, drive a wedge between goods' prices across countries (expressed in a common currency). The deviations from the law of one price at final good level are fairly volatile.

The second column of Table 1 displays results for our sticky-price economy with the optimal policy in place. Comparing this column with the first one suggests that benevolent policymakers should attempt to replicate the flexible-price allocation, although their success in this task can only be partial because of the multiple layers of nominal frictions in the economy. All real variables but employment and the terms of trade display more volatility in the sticky-price economy with the optimal monetary policy in place, than in the flexible-price economy. The largest discrepancy, not surprisingly, is in the volatilities of producers' and distributors' markups, which are constant in the flexible-price allocation.

Interestingly, for the Home industries, markups are twice as volatile at the upstream level as at the downstream level. This is remarkable in light of the fact that, in our experiment, we

assume the same degree of nominal rigidities at either level. The volatility differential reflects the real components of markup movements in producers' prices, arising from vertical interactions. In turn, fluctuations in producers' markups translate into inefficient deviations in the law of one price at the border. Observe that deviations from the law of one price in final prices are much closer to those in the flexible price economy, and that, per effect of the exchange rate movements, the markups of Home downstream firms selling imported goods have the highest volatility.

A notable implication of these effects is that the real exchange rate is more volatile in the economy with nominal frictions than under flexible prices, while the terms of trade is less volatile. Moreover, in our model economy and in accord to the stylized facts, the real exchange rate is more volatile than the terms of trade — in our numerical experiments, this is so irrespective of the degree of nominal rigidity in consumer prices. This finding contrasts with a strong conclusion by Devereux and Engel [2007], who argue that the optimal stabilization policy should reduce the variability of the real exchange rates significantly below that of the terms of trade. In these authors' view, the fact that empirically we observe the opposite pattern suggests that policymakers around the world fail to stabilize currency movements efficiently. Our findings caution against these strong policy prescriptions on the exchange rate volatility.[8]

In order to reduce inefficiencies due to price stickiness, monetary policy mitigates fluctuations in the major components of consumer price inflation (relatively more than in the components of other price indexes). However, it falls short of completely stabilizing headline inflation. Key to understand this result is the notion that reducing fluctuations in final prices limits the extent of markup fluctuations and price discrimination across countries. As we have discussed extensively above, the elasticity of the producer's demand curve depends on the industry's dispersion of final goods prices. As is clear from Table 1, markup fluctuations are more pronounced for final varieties of imported goods. This is why they receive a somewhat larger weight in the optimal policy regime, than in the CPI-inflation targeting regime.

[8]In this respect we note that, if we let producer prices to be completely flexible, the real exchange rate becomes less volatile than the terms of trade — in line with the case discussed by Devereux and Engel [2006]. This numerical result shows that the optimal ranking of volatility between the real exchange rate and the terms of trade depend, among other things, on the relative degree of price stickiness among upstream and downstream producers.

The last three columns of Table 1 report results for alternative policy rules, namely, CPI inflation targeting, GDP inflation targeting and a Taylor rule. Compared to the optimal policy rules, these alternative rules bring about noticeably larger volatility in most real variables, particularly in the markups and the deviations of the law of one price, for both consumer prices and prices at the dock. Focus on the pure CPI inflation targeting regime, presented in the third column of Table 1: such monetary policy regime leads to more volatility in the upstream price of both imported and domestically produced goods. Relative to the optimal policy, this translates into higher markup volatility and more price discrimination. This is so because complete stabilization of headline consumer price inflation brings about suboptimally large movements in sectoral inflation rates, especially inflation in the final price of domestic and imported goods, which ultimately affect the desired markups by upstream producers.

Likewise, stabilizing the prices of domestically produced goods — the case dubbed GDP inflation targeting in the fourth column of Table 1 — also leads to too much volatility in sectoral inflation rates, especially in inflation of imported goods at the border and at consumer level. Interestingly, consumption is less volatile than under the optimal policy; but this is achieved by generating more volatility in all other real variables, especially in employment.

# 6 Concluding remarks

A general prescription from the closed economy literature on optimal monetary policy is to keep a low and stable inflation rate. Since price dispersions bring about misallocations of resources, central banks should attempt to minimize them. Although this prescription can be modified depending on the number of frictions in the economy, targeting inflation is generally a robust finding. For policymakers in an open economy, however, minimizing the cost associated with price dispersion is compounded by the fact that they not only need to keep an eye on internal relative prices, but also on international ones, like the real exchange rate and the terms of trade.

In this paper we reconsider the policy trade-offs created by stable import prices in local currency, which realistically result from both nominal rigidities and endogenous destination-specific markup adjustment by exporters. The main novelty of our model consists of placing price stickiness at the heart of strategic interactions between upstream producers and local firms. We show that price dispersion at consumer level due to staggered pricing decisions by

local firms, affects the desired markup by upstream producers, magnifying their price response to shocks. We solve for the optimal policy under cooperation, unveiling a reason specific to the international dimension of the economy in support of price stability as the main prescription for monetary policy. As stable consumer prices feed back into a low volatility of markups among upstream producers, this contains inefficient deviations from the law of one price at the border — reducing the misalignment in international relative prices.

# 7 Appendix (to be completed)

## 7.1 Price dispersions

We can write the within-industry price dispersion of consumer prices as:

$$\begin{aligned}
S_t(h) &\equiv \int \left(\frac{P_t(h,r_h)}{\mathbb{P}_t(h)}\right)^{-\eta} dr_{h.} \\
&= \mathbb{P}_t^{\eta}(h) \sum_{j=0}^{\infty} (1-\theta)\theta^j P_{t-j}(h)^{-\eta} \\
&= (1-\theta)\left(\frac{P_t(h)}{\mathbb{P}_t(h)}\right)^{-\eta} + \theta \mathbb{P}_t^{\eta}(h) \sum_{j=1}^{\infty} (1-\theta)\theta^j P_{t-j}(h)^{-\eta} \\
&= (1-\theta)\left(\frac{P_t(h)}{\mathbb{P}_t(h)}\right)^{-\eta} + \theta \pi_t^{\eta}(h) \sum_{j=1}^{\infty} (1-\theta)\theta^j \left(\frac{P_{t-1}(h)}{\mathbb{P}_{t-1}(h)}\right)^{-\eta} \\
&= (1-\theta)\left(\frac{P_t(h)}{\mathbb{P}_t(h)}\right)^{-\eta} + \theta \pi_t^{\eta}(h) S_{t-1}(h)
\end{aligned}$$

Similarly, we can express the across-industry dispersion in consumer prices as follows:

$$
\begin{aligned}
\overline{S}_t &= \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} S_t(h)dh \\
&= \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} (1-\theta)\left(\frac{P_t(h)}{\mathbb{P}_t(h)}\right)^{-\eta} dh + \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} \theta \pi_{t-1}^{\eta}(h) S_{t-1}(h)dh \\
&= (1-\theta)\int \left(\frac{P_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} dh + \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} \theta \pi_{t}^{\eta}(h) S_{t-1}dh \\
&= (1-\theta)\int \left(\frac{P_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} dh + \int \left(\frac{\mathbb{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} \theta \left(\frac{\mathbb{P}_t(h)}{\mathbb{P}_{t-1}(h)}\right)^{\eta} S_{t-1}(h)dh \\
&= (1-\theta)\left[(1-\overline{\theta})\left(\frac{P_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} + \overline{\theta}\left(\frac{\widetilde{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta}\right] + \theta\int \left(\frac{\mathbb{P}_{t-1}(h)}{P_{\mathrm{H},t}}\right)^{-\eta} S_{t-1}dh \\
&= (1-\theta)\left[(1-\overline{\theta})\left(\frac{P_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} + \overline{\theta}\left(\frac{\widetilde{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta}\right] + \theta\int \left(\frac{P_{\mathrm{H},t}}{P_{\mathrm{H},t-1}}\right)^{\eta}\left(\frac{\mathbb{P}_{t-1}(h)}{P_{\mathrm{H},t-1}}\right)^{-\eta} S_{t-1}dh \\
&= (1-\theta)\left[(1-\overline{\theta})\left(\frac{P_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} + \overline{\theta}\left(\frac{\widetilde{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta}\right] + \theta\pi_{\mathrm{H},t}^{\eta}\int \left(\frac{\mathbb{P}_{t-1}(h)}{P_{\mathrm{H},t-1}}\right)^{-\eta} S_{t-1}(h)dh \\
&= (1-\theta)\left[(1-\overline{\theta})\left(\frac{P_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta} + \overline{\theta}\left(\frac{\widetilde{P}_t(h)}{P_{\mathrm{H},t}}\right)^{-\eta}\right] + \theta\pi_{\mathrm{H},t}^{\eta}\overline{S}_{t-1}
\end{aligned}
$$

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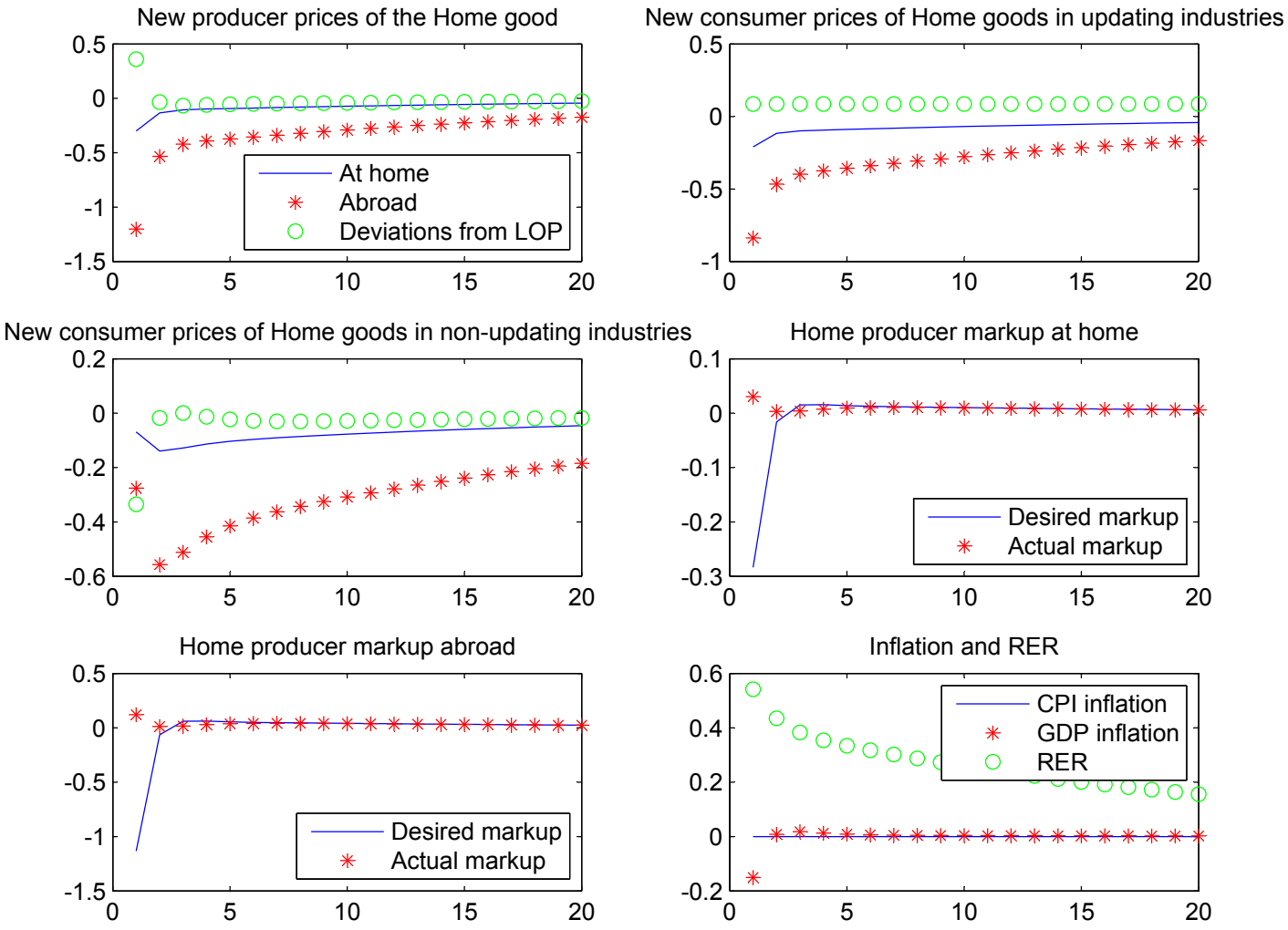
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**Table 1. Volatility Under Alternative Policies: Benchmark calibration (in percent)**

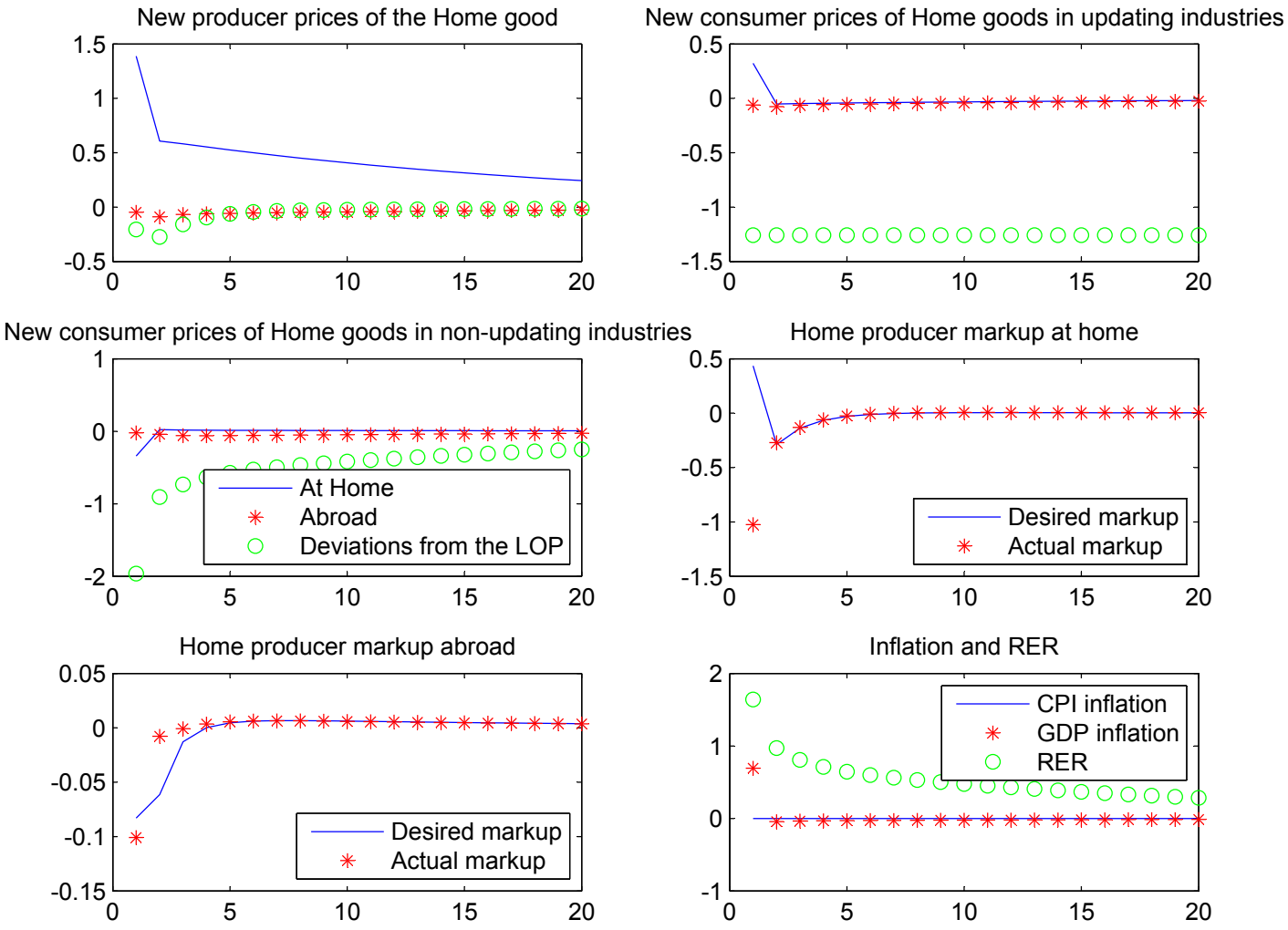
| Standard deviation | *Policies* | | | | |
|---|---|---|---|---|---|
| | Flexible Prices | Optimal Policy | CPI Inflation Targeting | GDP Inflation Targeting | Taylor Rule |
| *Inflation Rates* | | | | | |
| CPI | - | 0.08 | 0 | 0.42 | 0.75 |
| Domestic goods | - | 0.14 | 0.11 | 0.42 | 0.80 |
| Imported goods | - | 0.43 | 0.43 | 0.65 | 0.72 |
| Home producer prices | - | 0.61 | 0.97 | 0 | 0.63 |
| Import prices at the dock | - | 0.95 | 1.12 | 1.04 | 1.08 |
| Export prices at the dock | - | 0.95 | 1.12 | 0.89 | 1.08 |
| *International Prices* | | | | | |
| Real exchange rate (CPI based) | 2.93 | 3.29 | 3.74 | 3.04 | 3.48 |
| Terms of trade | 2.23 | 2.18 | 2.51 | 2.28 | 1.74 |
| *Deviations from the LOP* | | | | | |
| at producer level | 0 | 0.50 | 0.77 | 1.55 | 1.10 |
| at consumer level | 2.47 | 2.70 | 3.15 | 2.88 | 3.19 |
| *Markups of Home producers* | | | | | |
| Home producer selling at Home | 0 | 0.59 | 1.07 | 0.87 | 0.76 |
| Home producer selling abroad | 0 | 0.60 | 0.96 | 1.04 | 1.14 |
| Final producers of domestic goods | 0 | 0.30 | 0.62 | 0.62 | 0.80 |
| Final producers of imported goods | 0 | 0.95 | 1.22 | 1.25 | 1.65 |
| *Quantities* | | | | | |
| Home consumption | 1.43 | 1.50 | 1.61 | 1.46 | 1.58 |
| Home hours | 0.61 | 0.61 | 0.61 | 0.63 | 0.56 |

Figure 1. Productivity Increase in the Production of Home Goods




Responses are in percent deviations from steady state.

Figure 2. Productivity Increase in the Home Distribution Sector




Responses are in percent deviations from steady state.