A Search-Theoretic Model of the Retail Market for Illicit Drugs*

Manolis Galenianos
Pennsylvania State University

Rosalie Liccardo Pacula
RAND Corporation

Nicola Persico
New York University

August 2007
PRELIMINARY AND INCOMPLETE

Abstract

A search-theoretic model of the retail market for illegal drugs is developed. The model produces testable implications regarding the effect of interdiction and enforcement on: (a) the distribution of purity offered in equilibrium; and (b) the duration of the relationships between buyers and sellers. The model is consistent with evidence from the STRIDE and ADAM datasets.

1 Introduction

In this paper we build a search-theoretic model of the retail heroin market that better represents the frictions that are known to exist in illicit markets.

Existing theoretical models of the retail trade in illicit drugs are tied to traditional economic assumptions of an ideal centralized market (the one where demand and supply curves cross), including assumptions of perfect information (Becker Murphy & Grossman, 2006; Poret & Tejedo, 2006; Poret 2002).[1] Of course, retail transactions of illicit drugs do not take place in an ideal centralized market and information asymmetries abound. In the retail trade, the quality of the drugs is effectively chosen by the seller (through cutting) and is largely unobservable to buyers until after they consume. This moral hazard appears to be a big problem: a large fraction of the transactions, for example, are found to be pure "rip-offs" in which the buyer is sold zero-purity heroin. If this opportunistic behavior were too prevalent, buyers would presumably shy away and the

*We thank, without implicating, Peter Reuter who gave the impulse for the writing of this paper.

[1] A somewhat more sophisticated literature exists focusing on consumer demand for illicit drugs (Grossman and Chaloupka, 1998; Becker and Murphy, 1988; Schelling 1984; Stigler and Becker, 1977).

market would collapse. In fact, it is remarkable that these markets exist at all. Because the drugs trade is illegal, this moral hazard is not remedied by a legally enforceable obligation, nor can sellers seek certification by a reputable intermediary. Under these circumstances, long-term relationships between buyers and sellers are key to market viability (Reuter and Caulkins 2004). These long-term relationships, which we shall document empirically, alleviate the moral hazard problem that would otherwise foreclose the possibility of trade.

Another key friction that surely affects the quality of the drugs being traded is the presence of search costs due to police enforcement. Trade in illicit substances is secretive and bi-lateral, and buyers know that searching for new trading partners is risky. Increasing these search costs induces a degree of "buyer lock-in" and so reduces the competitiveness of the market. The effects on the quality of drugs traded are complex, however, because while sellers are able to take advantage of their market power by decreasing the quality of the drugs they sell, at the same time they are able to forge a more durable relationships with their buyers.

In this paper we attempt to provide a coherent account of the effect of these two frictions—moral hazard and search costs—on the functioning of the retail market for illicit drugs. We start with a standard search model a' la Burdett-Mortensen (1998). Searching for a new seller is costly, and consumers are sometimes forced to sample new sellers (for example, because their regular seller is unavailable). A seller always offers the same quality to all buyers. Over time, buyers who start off paired with low-quality sellers search until they find a suitably high-level quality seller, and then they form a stable match. They stick to the match until either (a) the match is exogenously broken (for example, seller goes to jail); or (b) they get to sample another seller (maybe they can't locate their regular seller that day) who happens to sell better quality, in which case they switch. As in Burdett and Mortensen (1998), we get dispersion in the price/quality ratio in equilibrium, which is consistent with the evidence. We modify this model in a couple of ways, in order to capture some key features of the market. First, prices are presumed to be nominally fixed at amounts ($20, or $40), making variability in *pure amount* exchanged the key dimension of these transactions. More crucially, we assume that buyers can only determine the quality of drugs *after* the trade is consummated. Due to this severe moral hazard problem, in our model the market can only be sustained through long term relationships between buyers and sellers. These modification take us a long way towards accounting for some key stylized facts in the market of interest.

A point of keen interest to everyone is, of course, the suppression of the market for illicit substances. According to our model, the retail market for illegal drugs is inherently at risk of collapse from overcutting; its viability is predicated on the presence of long-term buyer-seller relationships. This view suggests the possibility of undermining the retail drugs trade by leveraging the moral hazard problem inherent in the trade. Can we use police enforcement to undermine the viability of long-term buyer-seller relationships? While empirical evidence sufficient to definitively answer this question is not available, our model helps us work through certain effects. We find that police enforcement affects buyer-seller

relationships in complex ways. Increasing the enforcement pressure on buyers, for example, may lead to more stable relationships, as buyers are led to avoid the risk entailed in searching for new trading partners. Perversely, this may reduce the amount of cutting, and support the viability of the market. However, police enforcement that is effective in breaking up long-term relationships helps undermine the viability of the market. Adding another layer of complexity, the quality of drugs traded is also related to the degree of competition among sellers. So if enforcement increases the competitiveness of the market, for example by forcing buyers occasionally to sample a new seller, the result might well be to increase the quality of the drugs traded. Findings from this model suggest that increased enforcement on buyers and sellers at the retail level may have contributed in part to the long term decline in the average price per expected pure gram in cocaine and heroin in the US (Caulkins et al.,2004).

1.1 Related Literature

TO BE ADDED

2 Certain Stylized Facts About the Heroin Market

The model developed in the next section makes certain assumptions about behavior and how the market operates. Key assumptions necessary for the construction of this model are that

- buyers get the urge to consume heroin at some probabilistic rate
- buyers cannot distinguish the quality of the drugs they buy, and hence moral hazard exists in the market,
- moral hazard is tempered by the presence of long-run relationships (loyalty) between buyers and sellers as experience with the quality provided by one seller is established, and
- enforcement can interfere with these long-term relationships.

In this section we attempt to ground these key assumptions of our model into empirical evidence based on what is known about the market for heroin in the United States.

2.1 The data

Much of the information we have regarding drug markets and how buyers and sellers transact comes from two primary data sources: The Arrestee Drug Abuse Monitoring (ADAM) Program and the System to Retrieve Information from Drug Evidence (STRIDE) database. The ADAM data, administered by the U.S.

Department of justice, are quarterly data collected from arrestees in 35 counties across the country. Individuals involved in non-drug and drug-related crimes are selected for inclusion in the study, with the goal of obtaining information about the use, importance and role of drugs among those committing crimes. In addition to interviewing arrestees, urine samples are requested and analyzed for validation of self-reported drug use. Since 1996, the ADAM has obtained a representative sample of all arrestees detained at selected booking facilities within each of the 35 counties. Since 2000, a drug market procurement module has been included as part of the quarterly survey and collects information on the arrestee's most recent heroin purchase for all arrestees who report having used heroin in the previous 30 days. Information collected includes method for acquiring heroin (cash or noncash), place of purchase (inside or street buy), time of exchange, number of dealers they regularly purchase from, whether they last purchased from their regular dealer, difficulties experienced in locating a dealer or buying the drug, the units in which heroin was purchased (e.g. grams, bags), and the price paid for the specific quantity purchased. Although the data is collected from a non-representative sample of all heroin users, information is collected from a particularly relevant group that are likely to frequently engage in the heroin market. Information on the purity of heroin, however, is not available from this survey. Only information on price paid and amount is given.

To understand the relationship between the price and purity of heroin, we use information collected as part of the STRIDE database. STRIDE was created by the Drug Enforcement Agency (DEA) in an effort to collect detailed information on drug transactions by federal, state and local agencies engaged in law enforcement activities targeting drug markets. The primary purposes of the database are to control the inventory of drug acquisitions and to keep current with the types of drugs being distributed and the characteristics of those drugs and to use as part of prosecutions of federal drug offenders. Only acquisitions that are sent to a DEA laboratory for analysis are included in the database, although the acquisition could be made by federal agents, local law enforcement, or informants. Thus the database is not a random, probabilistic, or representative sample of all drug transactions or even those targeted by law enforcement. Instead they represent a convenience sample of drug transactions based on the focus of law enforcement in a given place at a given time. The timing and location of encounters are often erratic and the number of observations from a given location can vary dramatically from year to year, particularly for smaller cities. This has important implications for how these data can be used (Caulkins et al., 2004), yet they remain the only source of information on drug prices and purity over time.

There are two things about the STRIDE data that make them particularly useful for our purposes. First, since the early 1990s, the DEA has implemented a program called the Domestic Monitoring Program (DMP), the purpose of which is to go into selected cities each quarter and see the amount and quality of heroin that can be purchased on the streets for $100. Unlike other transactions represented in STRIDE, the sole purpose of these transactions is to see how much heroin you can get on the streets and determine the average purity. Thus,

unlike other observations in STRIDE, these observations are done for the sole purposes of monitoring and occur as random encounters in large cities. It is these observations that we draw most heavily on for our analyses in this paper. The second useful thing about the STRIDE database is the type of information collected in the database. Unlike ADAM and other existing databases, the STRIDE data contains accurate information on the potency of the drug involved in each transaction, determined through chemical analysis of the drug. Thus, in addition to important information on the amount, type, transaction price, date, location and source of the drug, there is also information on the potency of the drug involved in each trade. Unfortunately, the STRIDE data do not contain information on the relationship between the buyer and seller.

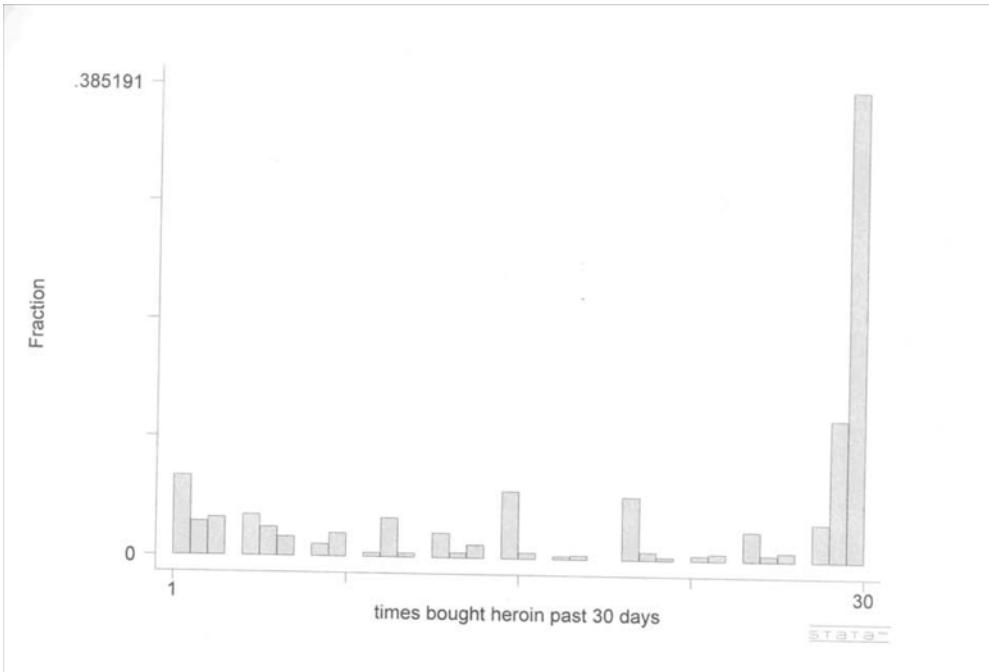
Given that each of these data sets has its strengths and weaknesses, we use both to elicit relevant information to inform our model. To examine issues related to search time and relationships with buyers, we examine data from the 2001-2003 ADAM but limit our analysis to the following cities for which we have a large number of observations from STRIDE consistently reported over time: New York City, San Diego, Washington DC, Chicago, Boston, Detroit, and Philadelphia. By limiting our analysis to these cities, we can empirically adjust transactions for trends in average purity over time, as indicated by the data in STRIDE for these cities.

To examine the relationship between price and purity and the dispersion of purity within cities, we use information available in the 1981 – 2003 STRIDE. Several initial data cleaning steps are taken following Arkes et al (2004). For example, observations measured in quantities that are not easily translated into grams are deleted, as are observations acquired outside the United States borders. Heroin base and heroin hydrochloride observations are retained, as these forms of heroin cannot be physically distinguished without chemical analysis so would be impossible for a buyer to differentiate (Arkes et al., 2004). Because of theoretical model focuses on transactions at the retail level, most of the analyses we conduct examining the retail market behavior focus on observations involving amounts less than less than 1 gram or having a transaction cost less than $200. The vast majority of these retail observations come from a few main cities, including New York City, Washington DC, Philadelphia, Chicago, Boston, and Detroit.

2.2 Frequency of consumption

The ADAM data provide some information on how frequently buyers buy drugs. As can be seen in the Figure below, the great majority of arrestees reporting heroin use in the past thirty days report buying heroin at least 30 times. In addition, over a third of these arrestees reporting use of heroin in the past 30 days report buying it multiple times in a single day. Table 1 shows the breakdown in responses to the question, "How many times did you buy heroin on the day of last purchase?" These two pieces of evidence together support the notion that frequent buyers of heroin get the "urge" to obtain the drug at a fairly regularly rate. In our model, we assume a Poisson rate α, which for

simplicity is assumed to be the same for everyone.



Number of times heroin was bought in past 30 days. (Source: ADAM).

Many buyers report buying heroin multiple times in a day. ADAM data give the following breakdown in reponse to the question "how many times did you buy heroin on the day of last purchase?"

Times bought heroin on same day	Number	Frequency
1	721	61.6%
2	277	23.7%
3	121	10.3%
4	51	4.3%
Total	1170	100%

2.3 Buyer loyalty

In our model some buyers engage in long-run relationships with sellers. These long-run relationships mitigate the moral hazard implicit in the trade. The ADAM data offer some empirical measure of how much loyalty there is in the

retail market. Among heroin users,[2] the fraction who in their last transaction report dealing with a regular, occasional, or new seller are as follows.

Source from whom you bought heroin	Number	Frequency
Regular dealer	787	67.4%
Occasional dealer	299	25.6%
New dealer	82	7%
Total	1168	100%

Thus, a lot of long-run relationships are reported by these consumers, consistent with the equilibrium of our model. Indeed, we may surmise that some of the buyers who report buying from an occasional or new dealer might have tried (and failed to) locate their regular dealer, so the table may underestimate the fraction of buyers who attempt to buy from their regular dealer.

2.4 Temporary interruptions of matches

In our model, matches are interrupted once in a while (with probability γ) and buyers are forced to search for a non-regular seller. This is an important assumption for the equilibrium of the model because it rules out "Diamond paradox" equilibria in which all sellers extract the maximum amount of surplus from buyers and buyers do not search in equilibrium. The ADAM data provide a glimpse of the causes that may lead a match to be interrupted. Among those who report not being able to buy heroin in the past 30 days, the breakdown of the causes is as follows.

Why buyer reports not bought heroin in past 30 days	Number	Frequency
No dealer available	92	33.3%
Dealer did not have any	52	18.8%%
Did not have quality	17	6.2%
Police activity	77	28%
Other	38	13.8%
Total	276	100%

We will take the view that police enforcement is an important contributor to the rate of interruption of matches. The data reported above seem to support this conclusion, especially if we believe that the reason "no dealer available" reflects to some extent police enforcement.

2.5 Moral Hazard

A defining feature of our model is the presence of moral hazard. In equilibrium, moral hazard results in a fraction of all trades being "scams," that is, zero-purity

[2] Those who reported obtaining heroin in the past 12 months.

buys. Zero-purity trades are very frequent in the STRIDE data. For example the percentage of zero-purity trades in all years and all cities is as follows.

	trades for less than			
	\$21	\$51	\$101	\$ ∞
number of trades	1,038	3,901	12,280	30,960
percentage zero purity	31.6%	21.8%	11.9%	6%

The prevalence of zero-purity trades is consistent with the predictions of our model, and strongly suggests the presence of the moral hazard in the retail market. The additional observation that zero-purity trades are more common for small value trades is consistent with (though is not required by) the equilibrium of our model.

The STRIDE data reveal other evidence of moral hazard at the retail level. Moral hazard, in our model, is caused by the buyer's inability to assay the drugs before purchasing. STRIDE provides evidence that, for relatively small transactions, buyers do not really know that they are buying. This evidence comes from regressing the purity of the drugs traded on the price per gram. If buyers knew what they were buying, then we should expect a positive coefficient (the price per gram should reflect purity). And indeed this is what we see for large transactions. But for relatively smaller transactions (less than 1 gram) the coefficient is essentially zero, indicating that buyers who pay more per gram do not receive heroin that is more pure. The coefficients of the regression are reported below.

Amount	N	coeff of purity	t-stat
$< 1g$	5776	0.0028	(1.24)
$(1, 10)g$	2318	0.153	(11.75)
$> 10g$	2261	0.229	(10.03)

Coefficients in regression of purity of the drugs traded on the price per gram.

The fact that small (<1g) trades have such a small coefficient is indicative of the presence of moral hazard in small trades.

2.6 Identification of the retail market: boundaries of moral hazard

In our model, buyers are unable to verify the quality of the drugs they are purchasing. Of course this assumption is plausible only for relatively small transactions. For large transactions we expect the buyers to assay the quality of the drugs before buying, and so there is no moral hazard. What are the boundaries of applicability of our model? That is, can we identify a value threshold above which we expect our model not to hold?

In the equilibrium of our model, all sellers cut to some extent, and sellers engaged in larger monetary transactions tend to cut less (see section 9.5). Thus, as the size of the transaction increases, the quality of the drugs traded also increases. In a model with no moral hazard, in contrast, the purity of the drugs is always assayed irrespective of the value of the transaction (provided of course the value is sufficient to justify the cost of assaying), and so there is no reason why it should correlate with the value of the transaction. We can use the STRIDE data to look into the relationship between the value of the transaction and the purity that is traded. Thus we split the data in small and large value transactions, and within each sample regress the purity of the drugs traded on the value of the transaction.[3] We expect the coefficient on value to be positive across small value transactions, and to be zero for large value transactions. After experimenting with a few reasonable thresholds, the threshold that maximizes the difference in coefficients is $200. The coefficients in the regression are as follows.

```
64 .
65 . regress potency tcost realyear if tcost <201
```

Source	SS	df	MS	Number of obs =	17015
				F(2, 17012) =	1072.83
Model	1372004.82	2	686002.411	Prob > F =	0.0000
Residual	10878062.9	17012	639.434689	R-squared =	0.1120
				Adj R-squared =	0.1119
Total	12250067.8	17014	719.99928	Root MSE =	25.287

potency	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
tcost	.0853881	.0039572	21.58	0.000	.0776316	.0931446
realyear	1.206987	.0309064	39.05	0.000	1.146407	1.267567
_cons	5.238727	.5797263	9.04	0.000	4.102404	6.375051

Relationship between purity and value of the trade in trades below $200.

[3] We control for the year to account for the time trend in wholesale purity (strongly increasing over time).

```
regress potency tcost realyear if tcost >201
```

Source	SS	df	MS		
Model	1888296.41	2	944148.203	Number of obs	= 13944
Residual	9184627.52	13941	658.821284	F(2, 13941)	= 1433.09
				Prob > F	= 0.0000
Total	11072923.9	13943	794.156489	R-squared	= 0.1705
				Adj R-squared	= 0.1704
				Root MSE	= 25.668

potency	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
tcost	.000106	8.61e-06	12.31	0.000	.0000891	.0001228
realyear	1.668539	.0318074	52.46	0.000	1.606192	1.730885
_cons	26.14131	.4327852	60.40	0.000	25.29299	26.98962

Relationship between purity and value of the trade in trades above $200.

For large trades, we see that while the purity (potency) of the trade is high (cons = 26% purity), the coefficient on value (tcost) is essentially zero, indicating that the value of the trade does not affect the purity traded. Conversely, for small trades we see that the purity (potency) of the trade is low (cons = 5% purity), the coefficient on value (tcost) is positive and relatively large, indicating that the purity traded increases with the value of the trade.

In sum, trades below $200 are consistent with our model as regards the effect of moral hazard, while trades above $200 show no evidence of moral hazard.[4]

3 Summary: Model Desiderata

We view the stylized facts described above as important features that a good model of the retail market for drugs should capture. Informed by the evidence reported above, we want a model with the following characteristics.

(i) Agents trade an experience good (ie, you have to consume it before knowing its quality, like used cars and restaurant meals),

(ii) Consumption is frequent which allows the formation of long-term relationships (more like restaurant meals than used cars)

(iii) The good is addictive, which motivates our assumption of inelastic demand.

(iv) The market is illegal which means that the usual economic institutions that solve the information/experience-good problem are not present (eg, restaurant guides, consumer reports etc) leaving long-term relationships as the principal way of solving the information problem.

(v) More importantly, although the good is storable, buyers seem to consume their purchases instantaneously probably because credit constraints do not allow

[4] Incidentally, the small coefficient for trades larger than $200 suggests that there is not much cutting going on as the drugs are sold in smaller and smaller quantities down the distribution chain.

them to buy large quantities and/or legally they face much larger penalties if caught with higher quantities.

(vi) Illegality implies some of the market parameters (frictions) are a policy variable.

In the next section we build a model with these characteristics.

4 The Model

Time runs continuously, the horizon is infinite, and the future is discounted at rate r.

4.1 Agents

There is a continuum of buyers (or customers) of measure b and a continuum of sellers (or suppliers) of measure 1. Buyers want to trade with sellers. Each buyer gets the urge to consume at random times which arrive at Poisson rate α and when this occurs he has to purchase the commodity.[5]

4.2 Trading Process & Informational Frictions

The price of the commodity is exogenously fixed at p but the quantity and/or purity that is sold may vary. To keep the analysis simple, we restrict attention to pure quantity=quantity*purity as a measure of the utility that the buyer receives from a transaction and ignore what it consists of (e.g. the same pure quantity can be the result of a large quantity of low purity or a small quantity of high purity). From now on we refer to the *quality* of the good transacted instead of its "pure quantity" and we denote it by q.

Buyers cannot evaluate the quality of what is on sale at the time that they meets with their seller. After buying and consuming the good, however, the quality of the purchase is perfectly revealed. This is important because it affects the buyer's decision of whether to return to the same seller the next time that he wants to consume.

4.3 Search Frictions

The market is characterized by search frictions in the sense that there is no central marketplace where all agents can meet to trade. Rather, buyers and sellers have to trade bilaterally.

A buyer can be in either of two states: *matched*, which means that he has a regular supplier, or *unmatched*. An unmatched buyer has to search in the market at random, incurring utility cost of search, s. A matched buyer also

[5] Implicitly the buyer gets infinite disutility if he does not consume when he gets the urge to do so. Alternatively, he could draw the cost of not consuming from some distribution. Then the cutoff cost (below which he would choose to forgo consumption) would be an equilibrium object depending on market conditions. Furthermore, we could have two types of users: regular and infrequent users, characterized by their α.

has the option of visiting his regular supplier, which does not entail any cost. However, there is a probability γ that the regular supplier is unavailable, in which case the buyer has to search at random and pay s.

The transition between these two states takes place after the trading is done. This transition is partly beyond the control of the agents—all matches are assumed to be exogenously destroyed at rate δ. But buyers are also able to effect the transition from one state to the other, as detailed below.

4.4 Choices

The buyer's decisions concern whether he comes back to some seller after sampling his good. In particular, after an unmatched buyer samples a seller he has to decide whether to make him his regular supplier, that is, whether to *match* with that supplier. If a matched buyer could not purchase from his regular seller and sampled a new seller at random, then he has to decide whether to go back to his old supplier or to *match with the new seller* instead. The substantive implication of this assumption is that a buyer can only be matched with one seller at a time and there is no recall: once a match has been broken, the buyer cannot find that seller again.

The seller chooses the quality level that he supplies to the buyers that visit him. Providing with higher quality is more costly but it may make buyers more likely to come back. The main assumption on sellers' behavior is that, once they decide on the quality level that they offer, they commit to their decision forever. That is, each seller supplies the same quality to all customers at all times and, as a result, a buyer knows the quality that he will receive from a particular seller once he has sampled from him.

We will look for steady state equilibria.

5 The Buyers' Decision Problem

We first consider the buyers' problem, taking the sellers' actions as exogenous.

Let F denote an arbitrary distribution of qualities in the market with support in $[0, \overline{q}]$. The state variables for a buyer is whether he is matched and, if so, what is the quality that he receives from his regular supplier. Let $V(q)$ denote the value of being matched with a seller who offers quality q. Let $\bar{V}$ denote the value function of a buyer who does not have a regular seller.

The value functions in flow terms are given by the following asset pricing equations:

$$r\ \bar{V} = \alpha\ [-s + \int_0^{\overline{q}} [\tilde{q} + \max\{V(\tilde{q}), \bar{V}\} - \bar{V}] dF(\tilde{q})] \tag{1}$$

$$r\ V(q) = \alpha\ [(1-\gamma)\ q + \gamma\ \int_0^{\overline{q}} [-s + \tilde{q} + \max\{V(\tilde{q}), V(q)\} - V(q)]\ dF(\tilde{q})] + \delta\ (\bar{V} - V(q)) \tag{2}$$

The interpretation is as follows. Consider equation (2) first. At rate α the buyer gets the urge to consume. When this happens, he samples a seller at random and incurs the cost of search s. The instantaneous utility that he receives from consuming is a random draw from the distribution of qualities, F. After consuming, the buyer decides whether to keep this seller as his regular supplier, which yields a "capital gain" of $V(\tilde{q}) - \bar{V}$, or to remain unmatched, in which case there is no change in his value. Equation (1) is similar. Again, at rate α the buyer wants to consume. With probability $1 - \gamma$ his supplier is available and the buyer receives quality q. With probability γ the regular seller is unavailable and the buyer has to search in the market. As a result, he incurs cost s and he makes a random draw from F. The only difference from the previous case is that he compares the new seller with his regular supplier when deciding whether to stay with the new draw. Therefore the capital gain of switching to the new seller is $V(\tilde{q}) - V(q)$. Last, at rate δ the match is destroyed and the buyer becomes unmatched leading to a capital loss of $\bar{V} - V(q)$.

Note that $V(q)$ is strictly increasing in its argument. Thus there is at most one quality level R such that $\bar{V} = V(R)$. An unmatched buyer who samples a seller offering quality $q \geq R$ will choose to match with the current seller, while if $q < R$ he will remain unmatched.[6] A matched buyer will switch suppliers if and only if the new seller offers a higher quality.

The rest of the analysis of the buyers' behavior centers on characterizing R as a function of the (still) exogenous distribution F. Using the reservation properties that we derived, we can now simplify the two equations above in the following way (defining $\mathbf{E}(\tilde{q}) \equiv \int_0^{\overline{q}} \tilde{q}\, dF(\tilde{q})$)

$$r\ \bar{V} = \alpha\ [-s + \mathbf{E}(\tilde{q}) + \int_R^{\overline{q}} (V(\tilde{q}) - \bar{V})\ dF(\tilde{q})] \tag{3}$$

$$r\ V(q) = \alpha\ [(1-\gamma)q + \gamma[-s + \mathbf{E}(\tilde{q}) + \int_q^{\overline{q}} (V(\tilde{q}) - V(q)) dF(\tilde{q})] + \delta\ (\bar{V} - V(q)). \tag{4}$$

These two equations can be manipulated to eliminate the value function V, thus expressing R as a function of F alone.

Lemma 1 *We have*

$$R = -s + \mathbf{E}(\tilde{q}) + \alpha\ (1-\gamma)\ \int_R^{\overline{q}} \frac{1 - F(\tilde{q})}{r + \delta + \alpha\ \gamma\ (1 - F(\tilde{q}))}\ d\tilde{q}. \tag{5}$$

Proof. Recalling that $V(R) = \bar{V}$ we can equate the two expression above to get (after a bit of algebra)

$$R + s = \mathbf{E}(\tilde{q}) + \int_R^{\overline{q}} (V(\tilde{q}) - \bar{V})\ dF(\tilde{q}) \tag{6}$$

[6] By convention, he matches when indifferent.

We want to integrate by parts the integral on the right hand side. To do that we first calculate $V'(q)$. Differentiating equation (4) with respect to q yields

$$rV'(q) = \alpha\ [1 - \gamma + \gamma[-V(q)\ F'(q) - V'(q)\ (1 - F(q)) + V(q)\ F'(q)]] - \delta\ V'(q)$$

$$V'(q) = \frac{\alpha\ (1-\gamma)}{r + \delta + \alpha\ \gamma\ (1 - F(q))}$$

Now, integrating by parts

$$\begin{aligned}\int_R^{\overline{q}} (V(\tilde{q}) - \bar{V})\ dF(\tilde{q}) &= \int_R^{\overline{q}} V(\tilde{q})\ dF(\tilde{q}) - (1 - F(R))\bar{V} \\ &= V(\overline{q}) - V(R)\ F(R) - \int_R^{\overline{q}} F(\tilde{q})\ V'(\tilde{q})\ d\tilde{q} - (1 - F(R))V(R) \\ &= \int_R^{\overline{q}} (1 - F(\tilde{q}))\ V'(\tilde{q})\ d\tilde{q}\end{aligned}$$

since $V(\overline{q}) - V(R) = \int_R^{\overline{q}} V'(\tilde{q})\ d\tilde{q}$. Substituting the results of this integration into equation (6) yields equation (5). ■

6 The Sellers' Decision Problem

We assume that sellers maximize their steady state level of profits and concentrate on the case where $r \to 0$ for simplicity. The steady state profits of a seller who chooses to offer quality level q when the distribution of qualities is F, the workers' reservation strategy is R and the price is p are given by

$$\pi(q) = (p - c\ q)\ t(q)$$

The first terms is the seller's margin per sale (with a linear cost c of quality) and $t(q)$ is his steady state number of transactions.

7 Immediate Equilibrium Implications

F denotes the distribution of qualities offered by sellers in equilibrium. The sellers' equilibrium condition is that, for any q in the support of F,

$$\pi(q) = (p - c\ q)\ t(q) = \text{const.}$$

Note that multiplying $t\,(q)$ by a constant does not upset this equilibrium condition. This implies that, in response to an increase the number of buyers to $b' > b$, it remains an equilibrium for sellers to choose their quality according to F provided that consumers do not react to the change. Note furthermore that b does not appear in the consumer's equilibrium equations (1, 2), which means that the consumers' decision problem is not directly affected by b. These obervations imply that increasing the number of buyers b does not affect F or R

(though it increases the number of customers served by each seller by a factor b'/b). If, moreover, we wanted to capture the effect of an increase in the number of sellers (which in our model are normalized to 1), then we would decrease b. So we can state our findings as follows.

Proposition 2 (Total industry profits invariant to number of sellers) *Increasing the number of sellers does not affect the agents' equilibrium behavior. When the number of sellers increases by a factor τ each seller's equilibrium profit decreases by a factor $1/\tau$.*

This proposition highlights an important feature of the equilibrium of our model. Price, quality traded, and total profits in the sector, are unaffected by the ration of sellers to buyers. Deterring entry by sellers is therefore not a useful policy tool in the sense that it has no impact on the supply of drugs in this model. This is because the sellers who are in the market will pick up the slack. Deterring entry by buyers is useful per se, of course, insofar as we want to minimize the number of drugs users, but it has no effect on the quality traded in equilibrium. Within this model, then, the entry behavior of buyers and seller into the market has no bearing on the quality distribution.

Let us now characterize F, using the property that any q in the support of F must yield the same level of profits.

First we characterize the support of F. First, offering $q \in (0, R)$ is dominated by offering $q = 0$ (because customers will switch anyway). So, if any quality is offered below R, it must be zero quality. Next, suppose no quality level is offered between q_1 and q_2, where $R < q_1 < q_2$. Then $\pi(q_1) > \pi(q_2)$, which cannot be part of an equilibrium. Therefore, the support of F above R is connected. We have proved the following lemma.

Lemma 3 (Gap in quality offered) *The set of quality levels offered by sellers in equilibrium is $\{0\} \cup [R, \overline{q}]$ for some $\overline{q} \geq R$.*

We label sellers who offer $q = 0$ "opportunistic sellers."

Suppose there was a discrete mass of sellers offering quality $q^* \geq R$. A seller who offered $q^* + \epsilon$ would poach customers from the whole mass of suppliers offering exactly q^*, leading to a discrete increase in the inflow of buyers. Such a seller would thus get discretely more steady state revenues than any seller offering q^*, with only negligible additional cost. Since this cannot be part of an equilibrium with positive profits, we have shown the following result.

Lemma 4 (Quality dispersion) *There can be no positive mass point in the distribution of qualities offered by the sellers.*

This result also rules out the possibility of a single-quality equilibrium. The main underlying reason is that while sellers have some monopoly power with respect to their customers, once in a while (when they are unavailable) they have to indirectly compete with each other since their customers sample from a new potential supplier. This additional source of competition breaks the

Diamond paradox. How realistic is the quality dispersion predicted by the model? Evidence from STRIDE suggests that considerable quality dispersion is observed in the retail market. We return to this point in Section 9.

8 Steady-State Flow of Trades

In this section we characterize the equilibrium level of sales $t(q)$ that a seller makes who offers quality q. The only equilibrium assumption that is used in this section is that the equilibrium is a steady state, and so the aggregate population statistics (number of matched and unmatched sellers, etc.) are constant over time.

Sales come from two sources: the steady state number of 'loyal' (or regular) customers and the random customers who sample once and may or may not become regular after consuming (if they do become regulars, they are counted in the first category the next time they consume). Denote the flow of regular and occasional buyers by $\alpha_R(q)$ and α_O, respectively. Then total sales are given by

$$t(q) = \alpha_O + \alpha_R(q).$$

We henceforth characterize $\alpha_R(q)$ and α_O.

8.1 Number of matched and unmatched buyers

Let n denote the number of buyers that are unmatched in steady state. In steady state, the flows of buyers from the matched state to the unmatched state and vice versa equal each other. The rate at which buyers become matched is $\alpha\ (1 - F(R))$ while they become unmatched at rate δ. As a result, the steady state condition is

$$n\ \alpha\ (1 - F(R)) = (b - n)\ \delta$$

Isolating n yields

$$n = \frac{b}{1 + (\alpha/\delta)\ (1 - F(R))}.$$

The stock of matched buyers is $b - n$.

8.2 Flow of occasional trades

Since the occasional buyers are either matched buyers whose regular supplier is unavailable or they are unmatched we have that

$$\alpha_O = \alpha[\gamma\ (b - n) + n] = \alpha\ b\ \frac{\delta + \alpha\gamma\ \ L}{\delta + \alpha\ L},$$

where $L = 1 - F(R)$ denotes the proportion of sellers who offer quality above R.

8.3 Number of buyers matched with a seller providing quality q

In order to characterize the number of regular customers a seller has, we need to start by characterizing G, the steady state distribution of qualities received by the $b-n$ buyers who are matched.[7]

At every instant, the inflow into the ranks of customers matched to sellers offering quality levels up to q comes from the unmatched buyers who drew a quality level that they chose to keep (i.e. above R) but which is below q. This inflow is

$$n\ \alpha\ [F(q)-F(R)].$$

(Note that there are also movements within this group (i.e. from some q_1 to q_2 with $R \leq q_1 < q_2 \leq q$) but these do not affect the transition rate in question.) The outflow of customers from sellers offering quality levels up to q is given by the $(b-n)\ G(q)$ buyers who were matched with quality up to q, and who left their matches either because they were exogenously destroyed or because they sampled a higher quality level while their regular supplier was unavailable. This outflow is

$$(b-n)\ G(q)\ [\delta + \alpha\ \gamma\ (1-F(q))].$$

Equating the two flows and solving for the steady state distribution of quality levels received by matched buyers we get

$$\begin{aligned} G(q) &= \frac{n\ \alpha\ (F(q)-F(R))}{(b-n)\ (\delta + \alpha\ \gamma\ (1-F(q)))} \\ &= \frac{F(q)-F(R)}{L\ (1+(\alpha/\delta)\ \gamma\ (1-F(q)))} \end{aligned}$$

for $q \geq R$, and $G(q)=0$ otherwise.

We are now able to characterize the number of regular customers. The average number of buyers matched to a seller offering quality level $q \geq R$ is given by

$$l(q) = \lim_{\epsilon \to 0}\ (b-n)\ \frac{G(q)-G(q-\epsilon)}{F(q)-F(q-\epsilon)}$$

Divide numerator and denominator by ε to get

$$l(q) = (b-n)\ \frac{\partial G(q)/\partial q}{\partial F(q)/\partial q}$$

(we assume here, and later verify, that F is differentiable for $q > R$). Observe now that q only enters into $G(\cdot)$ through $F(\cdot)$, so if we define

$$H(x) = \frac{x-F(R)}{L\ (1+(\alpha/\delta)\ \gamma\ (1-x))},$$

[7] The matched buyers who are not hit by exogenous separations move toward higher quality suppliers over time so the cross-sectional distribution G of the quality that they receive is not the same as the offered distribution F (in fact it first order stochastically dominates it).

then

$$G(q) = H(F(q)).$$

Substituting $H(\cdot)$ into $l(q)$ yields

$$\begin{aligned} l(q) &= (b-n)\ \frac{\frac{\partial H(F)}{\partial F}\frac{\partial F(q)}{\partial q}}{\partial F(q)/\partial q} \\ &= (b-n)\ \frac{\partial H(F)}{\partial F} \\ &= \frac{b\ \alpha}{[\delta+\alpha\gamma\ \ (1-F(q))]^2}\frac{\delta+\alpha\gamma L}{\delta+\alpha L} \end{aligned}$$

where $k_0 = \alpha/\delta$ and $k_1 = \gamma\ \alpha/\delta$.

8.4 Flow of regular trades at quality q

The flow of regular trades for a seller offering quality $q \geq R$ is given by

$$\begin{aligned} \alpha_R(q) &= \alpha\ (1-\gamma)\ l(q) \\ &= \alpha\ (1-\gamma)\frac{b\ \alpha}{[\delta+\alpha\gamma\ \ (1-F(q))]^2}\frac{\delta+\alpha\gamma L}{\delta+\alpha L}. \end{aligned}$$

When $q < R$ then $\alpha_R(q) = 0$.

9 Equilibrium Quality Dispersion and Its Comparative Statics

Depending on the parameter values, in equilibrium there may be a fraction of sellers who offer zero quality. In the appendix we show that in equilibrium a positive measure of sellers offer zero quality if and only if

$$\frac{p/c}{s} > \frac{1}{k_1^2} + \frac{1}{k_0 - k_1}\ (1+\frac{1}{k_1})^2,$$

In this section we characterize such equilibria.The quality distribution is denoted by F. The maximal quality offered in equilibrium is denoted by $\overline{q}$. The lowest *positive* quality offered in equilibrium is denoted by R.

9.1 Maximal quality offered

The seller who offers zero makes profits of

$$\begin{aligned} \pi(0) &= p\ \alpha_O \\ &= p\ \alpha\ b\ \frac{\delta+\alpha\gamma L}{\delta+\alpha L} \end{aligned}$$

The seller who offers $\overline{q}$ makes profits of

$$\begin{aligned}\pi(\overline{q}) &= (p - c\ \overline{q})\ (\alpha_O + \alpha_R(\overline{q})) \\ &= (p - c\ \overline{q})\alpha b\ (1 + (1-\gamma)\ \ (\alpha/\delta))\frac{\delta + \alpha\gamma L}{\delta + \alpha L}.\end{aligned}$$

Equating the two profits and solving for $\overline{q}$ yields

$$\overline{q}\ \ = \frac{p}{c} \cdot \frac{\alpha(1-\gamma)}{\delta + \alpha(1-\gamma)}$$

Since $\overline{q}$ is increasing in $\alpha(1-\gamma)$, we have the following lemma.

Lemma 5 *The maximal quality offered is increasing in α, p and is decreasing in γ, δ, c.*

9.2 Solving for the distribution of quality

The profits of sellers offering 0 and q are given by

$$\begin{aligned}\pi(0) &= \alpha_0\ p \\ \pi(q) &= (\alpha_0 + \alpha_R(q))\ (p - c\ q),\ \text{for } q \geq R.\end{aligned}$$

Substitute for α_0 and $\alpha_R(q)$ and equate $\pi(0)$ and $\pi(q)$ in order to solve for the distribution of qualities offered in equilibrium.[8] After some algebra, we get the following function (which, for convenience, is defined on the entire $\mathbf{R}_+$):

$$\mathbb{F}(q) = 1 + \frac{\delta}{\alpha\gamma}\left(1 - \sqrt{\frac{\alpha\,(1-\gamma)}{\delta}}\ \sqrt{\frac{p/c - q}{q}}\right). \tag{7}$$

On the interval $[R, \overline{q}]$, the c.d.f. of qualities offered in equilibrium coincides with the function $\mathbb{F}(q)$. Outside of that interval, only zero quality is offered. Formally,

$$F(q) = \begin{cases} \mathbb{F}(q) & \text{for } q \in [R, \overline{q}]. \\ \mathbb{F}(R) & \text{for } q \in [0, R]. \end{cases}$$

[8] Here is the algebra.

$$\begin{aligned}\pi(0) &= \pi(q) \\ p &= (1 + \frac{k_0 - k_1}{[1 + k_1\ (1 - F(q))]^2})\ (p - c\ q) \\ \frac{p}{p - c\ q} - 1 &= \frac{k_0 - k_1}{[1 + k_1\ (1 - F(q))]^2} \\ (1 + k_1\ (1 - F(q)))^2 &= (k_0 - k_1)\ \frac{p/c - q}{q} \\ 1 + k_1\ (1 - F(q)) &= \sqrt{k_0 - k_1}\ \sqrt{\frac{p/c - q}{q}}\end{aligned}$$

Differentiating this expression we get the density of the qualities offered in equilibrium,

$$f(q) = \frac{1}{2}\sqrt{\frac{\delta(1-\gamma)}{\alpha\gamma^2}} \cdot \sqrt{\frac{\left(\frac{p}{c}\right)^2}{\left(\frac{p}{c}-q\right)q^3}} \text{ for } q \in [R, \overline{q}]. \tag{8}$$

Proposition 6 *The density distribution of sellers offering quality q is decreasing in q if $\alpha(1-\gamma) \leq 3\delta$; if the opposite inequality holds, then the density is U-shaped in q.*

Proof. The function $f(q)$ defined in expression (8) is U-shaped with a minimum at $q = \frac{3}{4}\frac{p}{c}$. The support of f, of course, is bounded above by $\overline{q}$. So if $\overline{q} > \frac{3}{4}\frac{p}{c}$, that is, if $\alpha(1-\gamma) > 3\delta$, then f is u-shaped; otherwise, f is monotonically decreasing on its support. ■

9.3 Uniqueness of R

If R is part of an equilibrium it solves equation.(5). Rewriting $E(\tilde{q})$ and rearranging yields

$$R\mathbb{F}(R) + s = \int_R^{\overline{q}} (1 - \mathbb{F}(\tilde{q}))\, d\tilde{q} + \alpha(1-\gamma)\int_R^{\overline{q}} \frac{1-\mathbb{F}(\tilde{q})}{\delta + \alpha\gamma\,(1-\mathbb{F}(\tilde{q}))}\, d\tilde{q} \tag{9}$$

As characterized in (7), the expression for $\mathbb{F}$ does not depend on R. Thus the LHS of (9) is increasing in R, and the RHS is decreasing in R. So there is at most a one R that is compatible with equilibrium.

Proposition 7 *The equilibrium is unique.*

9.4 Comparative statics with respect to s

As characterized in (7), the expression for $\mathbb{F}$ does not depend on s. Thus in equation (9) s enters the LHS only, and then only as an additive constant. So, as s increases to s' the equilibrium R decreases to R'. Moreover, since $\mathbb{F}(q)$ does not depend on s or R, as s increases the shape of $F(q)$ is unchanged for $q > R$. Thus, an increase in s results in a stochastically dominant shift of the distribution F. These observations can be rephrased as follows.

Proposition 8 *As the consumer's search cost increases, the average quality of drugs offered by sellers increases. After the change, some sellers who were previously offering zero quality now offer a positive quality. These sellers will offer a previously unavailable range of quality, which is just inferior to the lowest quality previously offered. The distribution of positive qualities previously offered by sellers is unchanged.*

It should be noted that although the quality of drugs offered improves when search costs increase, consumers are not necessarily better off, due to the direct effect of the increase in s.

9.5 Comparative statics with respect to c, p

The stylized fact (according to Reuter, personal communication) is that quality traded decreases in response to interdiction at the source. We mimick the effect of interdiction at the source by increasing c in our model. What happens to the distribution of qualities traded? To anwer this question, we notice that from equation (7) we have

$$\frac{\partial \mathbb{F}(q)}{\partial c} > 0.$$

So a small increase in c increases F in the interval $(R, \bar{q}]$. This would suggest the possibility of a stochastic dominance result, but actually we are only able to prove a weaker property, namely, that all the quantiles of F which lie above R (including the median, if $F(R) < 1/2$) become smaller in response to a small increase in c.

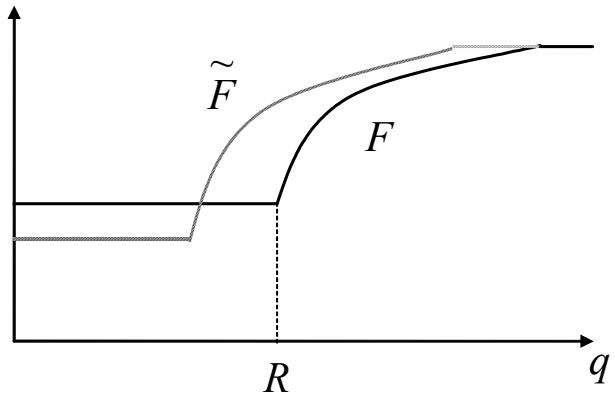
Proposition 9 *Let ϕ denote any quantile in the positive part of the support of the quality distribution. Now increase c (or, equivalently, decrease p), and let $\tilde{\phi}$ denote the same quantile in the new equilibrium distribution. Then $\tilde{\phi} < \phi$.*

Proof. Let a tilde denote equilibrium objects after the increase in c. Fix $y > F(0)$ and denote the corresponding quantiles before and after the increase in c by $\phi = F^{-1}(y)$ and $\tilde{\phi} = \tilde{F}^{-1}(y)$. We can write

$$\tilde{\phi} = \tilde{F}^{-1}(y) \leq \tilde{\mathbb{F}}^{-1}(y) < \mathbb{F}^{-1}(y) = F^{-1}(y) = \phi$$

where the weak inequality reflects the definition of $\tilde{F}(q) = \max\left[\tilde{\mathbb{F}}(q), \tilde{\mathbb{F}}(\tilde{R})\right]$; the strict inequality comes from $\partial \mathbb{F}(q)/\partial c > 0$; and the second-to-last equality follows from $y > F(0)$. ■

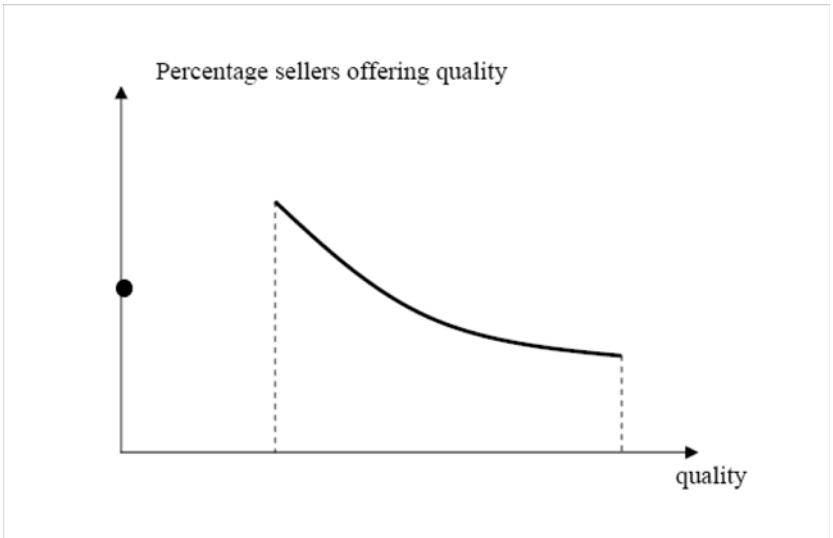
This result does not imply a stochastic dominance shift of F in response to an increase in c, because the mass of sellers offering zero quality might actually decrease. In fact, R decreases in response to the increase in $\mathbb{F}(\cdot)$. To see this, observe that R depends on c only through the increase in $\mathbb{F}(\cdot)$ in equation (9). The RHS of (9) is monotonic in $1-\mathbb{F}$ for fixed R, so as $\mathbb{F}$ increases the RHS goes down. Meanwhile, for fixed R the LHS of (9) goes up. So R needs to decrease to maintain equation (9). A picture illustrates the case in which increasing c does *not* result in a stochastic dominance shift of F.



10 Testable Implications of the model

10.1 Quality dispersion and rip-offs

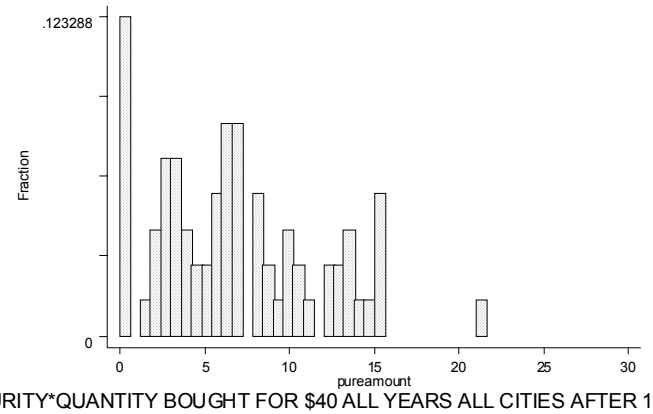
Our model predicts rip-offs (in the language of Section 9, $\mathbb{F}(R) > 0$), a gap in the quality offered above zero quality, and dispersion in the set of positive qualities offered. A graphical rendition of the distribution of qualities traded in the equilibrium of our model is presented in the following figure.



Quality dispersion predicted from the model

Evidence available in the STRIDE data is apparently consistent with our predictions concerning quality dispersion. An example of the quality dispersion seen in the data is reported in the next figure, where we show the quality (purity

x quantity) of heroin involved in transactions costing $40 from 1996 forward.[9]



Pure quality traded at $40, from STRIDE

The data appear broadly consistent with our predictions. There is a large fraction of rip-offs, with over 12% of the transactions involving sellers who sold heroin consisting of zero purity. Even more striking is the gap between observed between zero potency observations (the pure rip offs) and the low quality sellers, as predicted by our model. Finally, the STRIDE data support the notion of purity dispersion in the market, as no single modal level of quality emerges. Instead, we get dispersion in the price/quality ratio consistent with other models of search (Burdett and Mortensen, 1998).[10]

10.2 Loyalty of frequent and occasional buyers

Are frequent buyers more loyal to their supplier? In our model the frequency with which buyers purchase is captured by the parameter α. Although we do not have differences in α across buyers in our model, we can conjecture about the behavior of buyers with different α's. if we introduced in our model a very frequent buyer (α close to ∞) that buyer would quickly become matched with

[9] The presence of price and purity dispersion in the market for illicit drugs has been noted before by Caulkins (1995) and Reuter and Caulkins (2004). Reuter and Caulkins (2004) examine 14 years of purchase data for cocaine and heroin from STRIDE and find extremely high price and purity dispersion. Their paper does not offer a formal theoretical model that generates dispersion, however.

[10] Heterogeneity in people's willingness to search and search costs could also generate a range of transaction prices for even a homogeneous good (Stigler, 1961), and the same is true for drug markets (Reuter and Caulkins, 2004). This effect could partly account for the quality dispersion we observe. In our model, all the dispersion comes from the different length in the relationship between buyers and sellers. Buyers who have been around for a long time tend to be matched with better sellers. Even with the assumption that all sellers make the same level of profit, the dispersion of quality is sustained because some sellers offer low (even zero) quality, making a large profit per transactions but with relatively few repeat customers, and other sellers offer better quality, getting lower profit per transaction but more repeat customers.

a seller, and so would not search in equilibrium. Conversely, if we introduced in the model a very infrequent buyer (α close to zero) that buyer typically would take a long time before getting hooked up with a regular seller, and until he did he would search by repeatedly sampling from opportunistic sellers. Thus the infrequent buyer would be expected to be less loyal to his sellers because the initial sampling phase would last very long. In addition, once sellers and buyers are matched, the frequent buyer would be expected to be matched to a better quality seller, and so to be less likely to switch sellers when matches separate.[11] In sum, our model suggests that frequent buyers should be more likely to be loyal to their sellers.

This prediction is consistent with the evidence from the ADAM data. The next table displays a stochastic dominance relationship between the source of frequent and infrequent buyers.[12]

Source from whom you bought heroin	Infrequent buyers	Frequent buyers
Regular dealer	290 58.3%	468 76%
Occasional dealer	162 32.6%	117 19%
New dealer	45 9%	31 5%
Total	497 100%	616 100%

This evidence is consistent with our model.

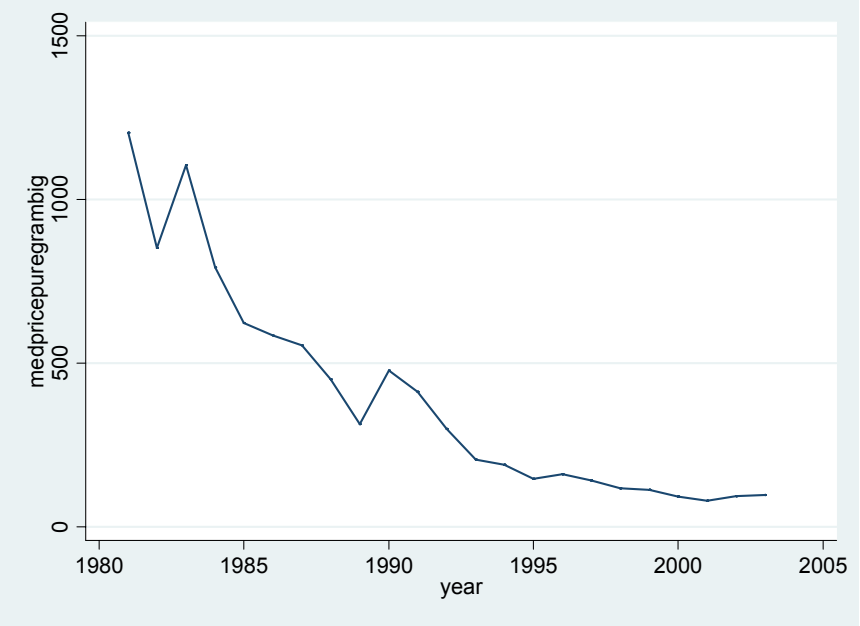
10.3 Trends in wholesale prices and retail quality

In our model the wholesale price is a parameter in the choice of retail sellers. The STRIDE data reveal a strong time trend in the availability of wholesale heroin. The wholesale price per pure gram[13] has decreased dramatically over time.

[11] A more frequent consumer (higher α) has a higher reservation quality, ie is pickier with his suppliers, because he consumes more per unit of time once he settles with some seller. As a result, the average matched frequent consumer enjoys higher quality than the average matched casual consumer. Therefore, when forced to search, the casual consumer is more likely to find a better deal than he currently enjoys and switch.

[12] For the purpose of this table a frequent buyer is defined as a buyer who purchased more than 28 times in the past 30 days. A Pearson chi-square test rejects the hypothesis that the distribution of frequent and infrequent buyers are the same (p=0.000).

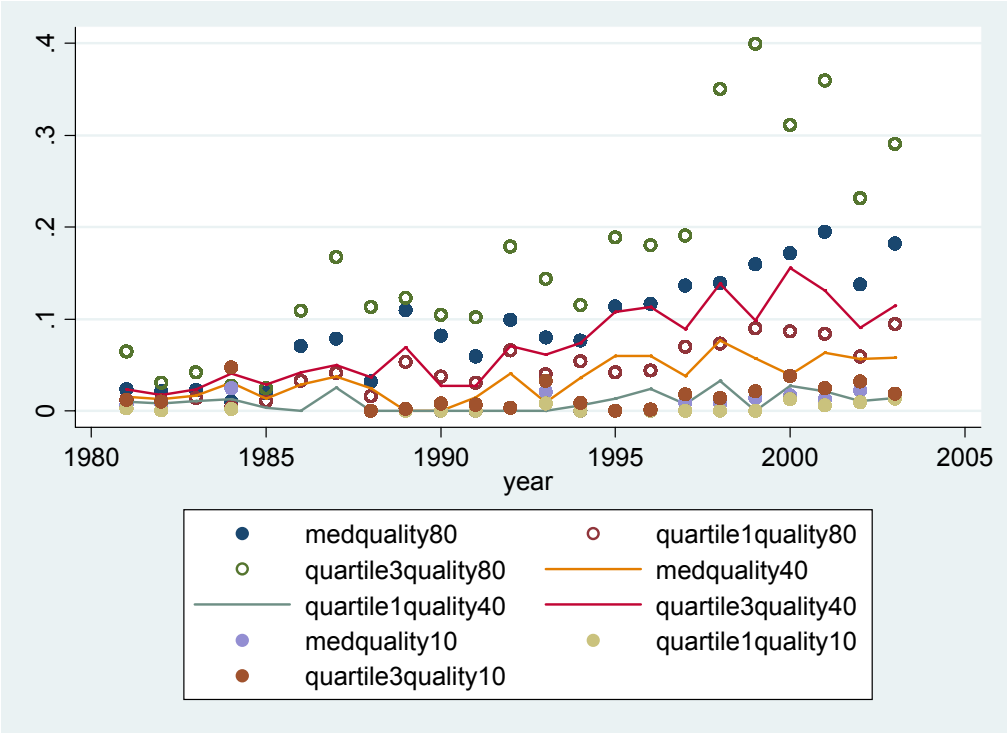
[13] Computed as the median price per pure gram for trades exceeding $1,000 in inflation-adjusted value.



Wholesale price of a pure gram of heroin. (Source: STRIDE)

What impact does our model predict is the impact of this decline in wholesale price on retail quality?[14] In the model, retail sellers strategically choose how much to cut the drugs that they obtain wholesale, trading off the gain from cutting against the risk of losing the customer. Therefore, the greater availability (lower price per pure gram) of wholesale heroin need not necessarily translate into higher retail quality (see Section 9.5). Nevertheless, for realistic values of the parameters we expect at least the quality quantiles above 50% to be inversely correlated with the wholesale price of pure heroin (c in our model). Next we plot a (somewhat complex) picture of the quartiles of the quality distribution of trades with value \$10, \$40, and \$80.[15] First, looking at the time trends, for each of these three trades we see that the quartiles tend to be increasing over time, consistent with the results of Section 9.5 (comparative static with respect to c).

[14] Again, quality is defined as purity*quantity.
[15] Not adjusted for inflation.



Trends of the quality (purity*quantity) sold at different amounts.

10.4 Larger transaction yield higher quality distributions

The figure presented in the previous section allows us to compare the distribution of qualities traded at different amounts. Comparing across the three trade values, we see that each quartile in a given trade is smaller than the same quartile in a larger trade. This is consistent with the comparative static developed in Section 9.5 (comparative static with respect to p).

11 Conclusions

(PRELIMINARY)

In this paper we offer a theoretical framework that incorporates some key elements of the retail markets for illicit drugs. A prominent role is played by the ability of sellers to cut the drugs without being immediately caught by the customers. This moral hazard, in our model, puts the retail drugs market at risk of collapse for "overcutting." Consistent with this view, the data reveal a large amount of "scam" transactions. The countervailing force that supports

trade in our model is the presence of repeated interactions, which also is found in the data.

If it's true that long-term relationships contribute to the viability of the retail drugs market—and we believe the evidence suggests it is true—then a number of important consequences follow. First, increasing enforcement on buyers (greater risk of being caught, greater penalties) may lead to unintended consequences: buyers will try to avoid these "search costs" by sticking with their regular sellers more, thus boosting the length of the relationship. In equilibrium, this leads sellers to increase the quality of the drugs they sell. Second, only enforcement that permanently breaks up relationships between buyers and sellers has a positive impact on the market (reduced quality). "Temporary" enforcement induces buyers to sample from other sellers, which increases the competition among sellers and may, perversely, lead sellers to offer better quality. Findings from this model suggest that increased enforcement on buyers and sellers at the retail level may have contributed in part to the long term decline in the average price per expected pure gram in cocaine and heroin in the US (Caulkins et al., 2004), though more evidence needs to be marshalled before drawing conclusions concerning the effect of enforcement.

A message from this paper is that we should pay more attention to the length of the relationship between buyers and sellers as a measure of the effectiveness of local enforcement. A particularly interesting measure, in light of our analysis, is the correlation between purity of drugs transacted and length of the buyer-seller relationship—a point on which unfortunately there is scant empirical evidence to date.

References

[1] Akerlof GA (1970). "The Market for Lemons: Quality Uncertainty and the Market Mechanism" Quarterly Journal of Economics 84: 488-500.

[2] Arkes, J., R.L. Pacula, S. Paddock, J. Caulkins, and P. Reuter. (2004). Technical Report for the Price and Purity of Illicit Drugs Through 2003. Office of National Drug Control Policy. November.

[3] Becker GS and KM Murphy. 1988. "A Theory of Rational Addiction" Journal of Political Economy 96: 675-700.

[4] Becker, GS; Murphy KM and Grossman M. 2006. "The Market for Illegal Goods: The Case of Drugs" Journal of Political Economy 114 (1): 38-60.

[5] Burdett, Kenneth, and Kenneth Judd 1983. "Equilibrium Price Dispersion." Econometrica 51: 955-969.

[6] Burdett, Kenneth, and Dale T. Mortensen 1998. "Wage Differentials, Employer Size, and Unemployment." International Economic Review: 257-273.

[7] Caulkins, J., R.L. Pacula, J. Arkes, P. Reuter, S. Paddock, M. Iguchi, and J. Riley. (2004). The Price and Purity of Illicit Drugs: 1981 Through the Second Quarter of 2003. Office of National Drug Control Policy. November.

[8] Grossman, M. and F.J Chaloupka 1998. The demand for cocaine by young adults: A rational addiction approach. Journal of Health Economics17(4): 427-474.

[9] Milgrom P and J Roberts (1986). "Price and Advertising Signals of Product Quality" Journal of Political Economy 94: 796-821.

[10] Miron, J.A. and J. Zwiebel 1995. The economic case against drug prohibition. Journal of Economic Perspectives 9(4): 175-192.

[11] Mortensen, Dale T. *Wage Dispersion: Why Are Similar Workers Paid Differently?* Cambridge: Zeuthen Lecture Book Series (2003).

[12] Poret S 2002. "Paradoxical effects of law enforcement policies: the case of the illicit drug market" International Review of Law and Economics 22: 465-493.

[13] Poret S and C Tejedo. 2006. "Law enforcement and concentration in illicit drug markets" European Journal of Political Economy 22: 99-114.

[14] Reuter P and J Caulkins (2004) "Illegal Lemons: Price Dispersion in the Cocaine and Heroin Markets" UN Bulletin on Narcotics.

[15] Reuter P. and MA Kleiman. 1986. "Risks and prices: An economic analysis of drug enforcement." In M. Tonry & N. Morris (Eds.) Crime and justice: An annual review of research, Vol 7, (128-179). Chicago: University of Chicago Press.

[16] Schelling T 1984. "Self-command in practice, in policy, and in a theory of rational choice." American Economic Review 74: 1-11.

[17] Stigler GJ 1961 "The Economics of Information" Journal of Political Economy 69: 213-225.

[18] Stigler GJ and GS Becker. 1977. "De Gustibus Non Est Disputandum" American Economic Review 67: 76-90.

[19] Wilson CA. 1979. "Equilibrium and Adverse Selection" American Economic Review 69: 313-317.

[20] _____ . 1980. "The Nature of Equilibrium in Markets with Adverse Selection" Bell Journal of Economics 11: 108-130.

A Equilibrium for Arbitrary Number of Opportunistic Sellers

We have previously shown that F consists of a continuous part on $[R, \overline{q}]$ and potentially a mass point at 0, if $R > 0$. We proceed to characterize F for an arbitrary number of opportunistic sellers $F(0) \in [0, 1)$.[16] Subsequently we characterize $F(0)$.

To characterize F, we use the equilibrium condition that all sellers make the same steady state profits. There are two steps to characterizing the distribution of qualities. We first fix an arbitrary R and derive F such that all sellers make the same profits. We then put our solution together with equation (5) which describes the optimal buyer behavior for an arbitrary F. This gives us the unique fixed point in the strategies of the agents.

Proposition 10 (Distribution of Qualities for arbitrary R) *The distribution of qualities offered in the market is given by*

$$F(q) = 1 + \frac{1}{k_1}\left[1 - \sqrt{h(q)}\right] \tag{10}$$

$$\text{where } h(q) = \frac{p/c - q}{(q - R)/(k_0 - k_1) + (p/c - R)/(1 + k_1\ L)^2} \tag{11}$$

Proof. Fix buyers' behavior. In equilibrium $\bar{\pi} = \pi(R) = (p - R\ c)\ t(R)$ and this expression only depends on model parameters and R. Taking $F(0) = 1 - L$ and R as given we want to construct F such that

$$\begin{aligned} \pi(R) &= \pi(q) \Leftrightarrow \\ (p - c\ R)\ [\alpha_R(q) + \alpha_O] &= (p - c\ q)\ [\alpha_R(q) + \alpha_O] \Leftrightarrow \\ \alpha_R(q) &= \frac{p - c\ R}{p - c\ q}\ (\alpha_O + \alpha_R(R)) - \alpha_O \end{aligned} \tag{12}$$

Recalling that $\alpha_O = \alpha\ b\ \frac{1+k_1\ L}{1+k_0\ L}$ we can use equations (**??**) and (**??**) to construct F such that profits are equalized. After some algebra, equation (12) becomes

$$F(q) = 1 + \frac{1}{k_1} - \frac{1}{k_1}\ \sqrt{\frac{p/c - q}{(q - R)/(k_0 - k_1) + (p/c - R)/(1 + k_1\ L)^2}} \tag{13}$$

$$\equiv 1 + \frac{1}{k_1} - \frac{1}{k_1}\ \sqrt{h(q)} \tag{14}$$

■

As a result we now have an expression for the distribution of qualities that are offered in the market given some arbitrary R and L. Furthermore, we can

[16] It is easy to show that if $F(0) = 1$ then $R = 0$ which contradicts the no mass-point lemma above. Therefore, $F(0) \in [0, 1)$.

solve for the highest quality on the market by setting $F(\overline{q}) = 1$ to get

$$\overline{q} = \frac{k_0 - k_1}{k_0 - k_1 + 1} \left[\frac{p}{c} + \frac{R}{k_0 - k_1} - \frac{p/c - R}{(1 + k_1\ L)^2}\right] \tag{15}$$

The next step is to integrate our expression for F into (5) which gave R as a function of F. In this way, we will end with the equilibrium R depending only on model parameters and L. The next proposition states the result.

Proposition 11 *Fix the proportion of opportunistic sellers to some $F(0) \in [0, 1)$. The equilibrium reservation quality is given by*

$$R = \frac{p/c\ (k_0 - k_1)\ (k_1\ L)^2 - s\ (1 + k_1\ L)^2}{(1 - L)\ (1 + k_1\ L)^2 + (k_0 - k_1)\ (k_1\ L)^2}, \quad \textit{if} \quad \frac{p/c}{s} \geq \frac{1}{k_0 - k_1}\ (1 + \frac{1}{k_1\ L})^2 \tag{16}$$

$$R = 0, \quad \textit{otherwise.} \tag{17}$$

Proof. We first derive $\hat{R}$ and then check whether it is greater than zero.

Note that $F(q) = 1 - L\ \forall\ q \leq R$. This means that

$$\begin{aligned}
\mathbf{E}[\tilde{q}] &= \int_0^{\overline{q}} \tilde{q}\ dF(\tilde{q}) \\
&= \int_R^{\overline{q}} \tilde{q}\ dF(\tilde{q}) \\
&= (1 - F(R))\ R + \int_R^{\overline{q}} (1 - F(\tilde{q}))\ d\tilde{q}
\end{aligned}$$

where the last equality is a direct result of integration by parts. We can therefore rewrite (5) as (recalling that $k_0 = \alpha/\delta, k_1 = k_0\ \gamma$)

$$\hat{R} = -s + L\ \hat{R} + \int_{\hat{R}}^{\overline{q}} (1 - F(\tilde{q}))\ d\tilde{q} + (k_0 - k_1) \int_{\hat{R}}^{\overline{q}} \frac{1 - F(\tilde{q})}{1 + k_1\ (1 - F(\tilde{q}))}\ d\tilde{q}$$

$$\hat{R}\ (1 - L) = -s + \int_{\hat{R}}^{\overline{q}} \frac{(k_0 - k_1)\ (1 - F(\tilde{q}))}{1 + k_1\ (1 - F(\tilde{q}))}\ d\tilde{q} \tag{18}$$

Using (8), this expression can be simplified to

$$k_1\ [\hat{R}\ (1 - L) + s] - (\overline{q} - \hat{R})\ (k_0 - k_1 - 1) = \int_{\hat{R}}^{\overline{q}} [\sqrt{h(q)} - \frac{k_0 - k_1}{\sqrt{h(q)}}] dq \tag{19}$$

where the key is to evaluate the following integral:

$$I = \int_{\hat{R}}^{\overline{q}} [\sqrt{h(q)} - \frac{k_0 - k_1}{\sqrt{h(q)}}]\ dq \tag{20}$$

This integration is somewhat complicated. In order to solve it note that the expression inside the square root takes the form $h(q) = \frac{a_1 - q}{q\ a_2 + a_3}$ where $a_1, a_2, a_3 > 0$. Then we can change variables successively and we show in the appendix the solution yields equation (16). It is straightforward to show that $\hat{R} < p/c$. The last thing to check is hat $\hat{R} \geq 0$ which yields the condition following equation (16). ■

The last step is to determine the equilibrium number of opportunistic sellers. We pin it down by equalizing profits across opportunistic and non-opportunistic sellers. The following proposition states the result.

Proposition 12 *A positive measure of sellers offer zero quality only if*

$$\frac{p/c}{s} > \frac{1}{k_1^2} + \frac{1}{k_0 - k_1}\ (1 + \frac{1}{k_1})^2, \tag{21}$$

and in that case $F(0)$ is uniquely defined. Otherwise, $F(0) = 0$.

Proof. Let $\pi_O = \pi(0)$ and $\pi_L = \pi(q),\ q \in [R, \overline{q}]$. Then,

$$\pi_O = \alpha_O\ p \tag{22}$$

$$\pi_L = (p - c\ R)\ (\alpha_O + \alpha_R(R)) \tag{23}$$

Let $\Delta\pi(L) \equiv \pi_L(R) - \pi_O$ where the profits are determined for a fixed L. The goal is to characterize L^* such that $\Delta\pi(L^*) = 0$. Algebra shows that

$$\begin{aligned}\Delta\pi(L) &= b\ \alpha\ c\ \frac{1 + k_1\ L}{1 + k_0\ L}\ [(p/c - R)\ \frac{k_0 - k_1}{(1 + k_1\ L)^2} - R] \\ &= \frac{b\ \alpha\ c\ (k_0 - k_1)\ (1 + k_1\ L)\ s}{(1 + k_0\ L)\ [(1 - L)\ (1 + k_1\ L)^2 + (k_0 - k_1)\ (k_1\ L)^2]}\ N(L),\end{aligned} \tag{24}$$

where

$$N(L) = \frac{p/c}{s}\ (1 - L - (k_1\ L)^2) + \frac{(1 + k_1\ L)^2}{k_0 - k_1} + 1 \tag{25}$$

Observe that equation (24) is equal to zero only if $N(L) = 0$. Furthermore, $N(L)$ is a quadratic equation in L and

$$N(0) = \frac{p/c}{s} + \frac{1}{k_0 - k_1} + 1 > 0$$

$$N(1) = -\frac{p/c}{s}\ k_1^2 + \frac{1 + k_1^2}{k_0 - k_1} + 1$$

Therefore, $N(1) < 0$ is necessary and sufficient for a unique solution which is the same as equation (21). In that case the number of opportunistic sellers is characterized by one of the roots of the quadratic.

If $N(1) > 0$ then there it is easy to show that there is no root and the non-opportunistic sellers always make higher steady state profits. As a result, $F(0) = 0$. ■

B Solving for lowest positive quality offered

We want to solve in closed form for the RHS of (9). Let $k_0 = \alpha/\delta$ and $k_1 = \gamma\ \alpha/\delta$.

$$\int_R^{\overline{q}} (1 - F(q))\ dq + \frac{k_0 - k_1}{k_1} \int_R^{\overline{q}} \frac{k_1\ (1 - F(q))}{1 + k_1\ (1 - F(q))}\ dq$$

$$= \frac{1}{k_1} \int_R^{\overline{q}} (\sqrt{k_0 - k_1} \sqrt{\frac{p/c - q}{q}} - 1)\ dq + \frac{k_0 - k_1}{k_1} \int_R^{\overline{q}} \frac{\sqrt{k_0 - k_1} \sqrt{\frac{p/c-q}{q}} - 1}{\sqrt{k_0 - k_1} \sqrt{\frac{p/c-q}{q}}}\ dq$$

$$= \frac{1}{k_1} \int_R^{\overline{q}} (\sqrt{k_0 - k_1} \sqrt{\frac{p/c - q}{q}} - 1)\ dq + \frac{k_0 - k_1}{k_1} \int_R^{\overline{q}} (1 - \frac{1}{\sqrt{k_0 - k_1} \sqrt{\frac{p/c-q}{q}}})\ dq$$

$$= \frac{1}{k_1} \{(\overline{q} - R)\ (-1 + k_0 - k_1) + \sqrt{k_0 - k_1}\ [\int_R^{\overline{q}} \sqrt{\frac{p/c - q}{q}}\ dq - \int_R^{\overline{q}} \frac{1}{\sqrt{\frac{p/c-q}{q}}}\ dq]\}$$

Solving for these integrals is not that simple but here it is. First, do a change of variables: $x = \frac{p/c-q}{q} \Rightarrow q = \frac{p/c-1}{x}$, $dq = -\frac{p/c-1}{x^2}\ dx$, $\overline{x} = \frac{p/c-\overline{q}}{\overline{q}}$, $\underline{x} = \frac{p/c-R}{R}$. Therefore the first integral can be rewritten as

$$\begin{aligned}
\int_R^{\overline{q}} \sqrt{\frac{p/c - q}{q}}\ dq &= \int_{\underline{x}}^{\overline{x}} \sqrt{x}\ (-\frac{p/c - 1}{x^2})\ dx \\
&= -(p/c - 1) \int_{\underline{x}}^{\overline{x}} x^{-3/4}\ dx \\
&= -(p/c - 1)\ (-\frac{2}{\sqrt{x}}|_{\underline{x}}^{\overline{x}}) \\
&= 2\ (p/c - 1)\ (\sqrt{\frac{\overline{q}}{p/c - \overline{q}}} - \sqrt{\frac{R}{p/c - R}})
\end{aligned}$$

Similarly,

$$\begin{aligned}
\int_R^{\overline{q}} \frac{1}{\sqrt{\frac{p/c-q}{q}}}\ dq &= \int_{\underline{x}}^{\overline{x}} \frac{1}{\sqrt{x}}\ (-\frac{p/c - 1}{x^2})\ dx \\
&= -(p/c - 1) \int_{\underline{x}}^{\overline{x}} x^{-5/4}\ dx \\
&= -(p/c - 1)\ (-4\ x^{-1/4}|_{\underline{x}}^{\overline{x}}) \\
&= 4\ (p/c - 1)\ ((\frac{\overline{q}}{p/c - \overline{q}})^{1/4} - (\frac{R}{p/c - R})^{1/4})
\end{aligned}$$