Skill Gaps in Occupations and Sectors with High and Risky Returns *

Astrid Oline Ervik†

September 2007

Abstract

In a stochastic two-period overlapping generations (OLG) model, individuals can make a discrete choice between two types of education that qualify for occupations in either a 'safe' sector or a 'risky' sector. The educational choice between 'safe' and 'risky' skills can interpreted as a choice between different subject fields and occupations and does not necessarily involve differences in levels of education. The risky sector is directly affected by shocks, whereas the safe sector is only affected by shocks if they are propagated over from the risky sector. The paper examines the effects of aggregate sector-specific shocks and idiosyncratic sector-specific shocks with and without counter-cyclical variance.

The paper shows that risky sectors may suffer from persistent skill gaps when individuals are risk averse, markets for labour income risk are incomplete, and the goods produced in the risky and safe sectors are not strong substitutes. Skills gaps are defined as the difference between supply of skills when there is complete insurance for labour income risk and supply of skills when there are no markets for labour income risk.

The paper contributes to the understanding of the process of educational and occupational choice in the context of volatile, often innovative, high-skill sectors. Skills gaps in risky, but highly productive sectors may curb economic growth, and the welfare consequences of skills gaps may be severe.

The paper highlights the need for educational policy to give careful consideration to sector- and occupation-specific risk and individuals' preferences for risk when addressing skill gaps in volatile sectors.

JEL Classifications:

Keywords: Educational choice, occupational choice, idiosyncratic shocks, cyclical variance, aggregate shocks, sectoral risk, business cycles.

---

*I am indebted to Steven Durlauf, Petra Geraats, Tom Krebs, Dirk Krueger, Jim Mirrlees, Hamish Low, Paul Ryan, Peter Tinsley, Susanna Sällström-Matthews and seminar participants at the European Association of Labour Economists (EALE) Annual Conference in Seville in 2003, at the CEBR conference on Entrepreneurial Choice in Copenhagen in 2006, the European Workshop in Macroeconomics in Florence 2007, and at seminars at the Norwegian School of Economics and Business Administration (NHH), University of Stavanger, and University of Cambridge for valuable encouragement, discussions and comments. All errors are my own.

†Faculty of Economics and Churchill College, University of Cambridge and Norwegian School of Economics and Business Administration. E-mail: aoe22@cam.ac.uk

1 Introduction

Different sectors of the economy have varying sensitivity to national business cycles, and some sectors are more exposed to technology, demand or stockmarket-related shocks than other sectors. Both sector-specific shocks and national business cycles are likely to translate into sectoral co-movements in employment, output, and value added.[1] Sectoral fluctuations can also affect workers through sector-specific aggregate or idiosyncratic labour income risk.[2] Recent contributions in the literature on labour income risk have documented a considerable idiosyncratic component, together with conditional variance of labour income that is strongly negatively related to the national business cycles. Sensitivity to business cycles and the persistence of shocks on labour income have also been shown to differ over educational and occupational categories, and the majority of labour income shocks are highly persistent.[3]

The aim of this paper is to show how sector-specific aggregate and idiosyncratic productivity shocks can translate into labour income shocks, thereby affecting discrete educational and occupational choices and leading to skills gaps in sectors associated with higher risk. Skills gaps are defined as the difference between supply of skills when markets for labour income risk are complete and supply of skills when markets are incomplete. If markets for risk were complete, skills gaps would be an efficient reflection of preferences for risk. But with incomplete markets, skills gaps capture the social deficiencies in availability of sector- or occupation-specific skills and emphasize the scope for efficiency improvement through policy. In addition to the social inefficiencies of skills gaps, sector-specific shocks have distributional consequences and equity could also be improved through policy. Further consequences of skills gaps in risky sectors, which may well be highly innovative and offer high expected returns, include lower sector productivity and competitiveness than in social optimum. This can force risky sectors towards less skill-intensive development strategies. Moreover, sectors that are exposed to high cyclical variations, maybe even due to cycles of innovation in the Schumpeterian sense, can have external effects on the rest

[1] See Long and Plosser (1983, 1987), Cooper and Haltiwanger (1990), Shea (2002), and Carlino and DeFina (2004) for documentation of co-movement of employment, output, and value-added over national business cycles. Co-movements across cycles are well-documented both within and across sectors. Hornstein (2000) document considerable comovement in activity in manufacturing, services and construction industries over the business cycle, but also highlight that industries are affected to different degrees and respond differently to shocks.

[2] Firms could insure their workers, but this often proves difficult as firms' capital base is also affected by shocks. It is likely that larger firms are better able to insure workers than smaller firms, through better access to capital markets and a broader range of economic activities. This again implies that sectors consisting of many small firms are more likely to expose their workers to labour income risk than sectors with larger firms.

[3] See e.g. Storesletten, Telmer, and Yaron (2004), Meghir and Pistaferri (2004), and Saks and Shore (2005) for empirical studies of labour income risk. Meghir and Pistaferri (2004) and Saks and Shore (2005) also document siginificant differences in the labour income risk structure across occupational and educational categories.

of the economy, and hence skills gaps in such sectors can affect growth across the whole economy.

The paper presents a dynamic general equilibrium model with a 'risky' sector which is directly affected by shocks, of aggregate or idiosyncratic nature and with and without counter-cyclical variance, and a 'safe' sector, which is not directly affected by shocks. However, shocks can be propagated across from the risky to the safe sector through complementarity between 'risky' and 'safe' sector intermediate goods in production of the final consumption good.

The paper shows that skill gaps arise in the risky sector due to the time lag involved in educational investments, individual risk aversion, and lack of insurance markets for labour income risk. Skill gaps are always positive when the risky and safe sector goods are complements, but when they are substitutes and certain other conditions are also met skills gaps can be negative.[4] Moreover, skill gaps are shown to depend on the risk aversion parameter and the personal intertemporal discount rate, where the effect of the latter depends on the relationship between costs of the different types of education. In the presence of aggregate shocks, the size and persistence of the shocks are shown to also matter for the skills gaps, whereas for the simple case of sector-specific independently and identically distributed idiosyncratic shocks skills gaps depend on the mean and variance of the shocks. The strength of comparative statics effects are dependent on the degree of complementarity between the safe and risky sector goods.

Insurance could reduce skills gaps, but it is not possible to insure for aggregate income shocks unless the effect of the shocks go in opposing directions for different sectors. For the case when sector-specific shocks are idiosyncratic, as opposed to aggregate, labour income risk could in principle be insured for if financial markets are complete and individuals pool their risk[5] and thus skills gaps could be reduced. However, even for the case of idiosyncratic labour income risk, informational asymmetries make insurance difficult and explain the limited availability of labour income insurance and other risk-sharing facilities in private markets. Furthermore, in the current model framework, where each generation only lives for two periods, shocks have permanent effects on labour income and are thus difficult to insure for both with explicit and implicit insurance mechanisms, see e.g. Cochrane (1991) and Attanasio and Davis (1996). Other public policy alternatives include redistributive taxes, as suggested by the entrepreneurial choice literature, and scholarship provision, and these alternatives are currently being explored in a

[4]Gould, Moav, and Weinberg (2001) find that labour income risk will increase human capital investments. However, they consider differences in earnings risk for the low-skilled vs. the high-skilled, as opposed to earnings risk that varies with sectoral or occupational choices for similar levels of education.

[5]See Cochrane (1991), Mace (1991), and Attanasio and Davis (1996).

separate paper.

The remainder of the paper is organised as follows. Section 2 discusses related literature. Section 3 analyses a benchmark model with sector-specific idiosyncratic productivity shocks in the risky sector. Section 4 sets out the basic model with aggregate production shocks in the risky sector, solves and discusses the assumptions and results. Section 5 considers a third version of the model, with sector-specific idiosyncratic shocks and counter-cyclical variance. Section **??** considers various options for private markets and public policy to address occupation- or sector-specific income risk. Finally, section **??** concludes with a discussion of the results, their implications and possible extensions of the analysis.

2 Related approaches

A largely recent literature has focused on human capital investments and occupational choice in the presence of labour income risk, but the models are primarily concerned with sources of and consequences of the increasing differences between the high- and low-skilled and to some extent also the increasing labour income risk within skills groups, see e.g. Galor and Moav (2000), Gould, Moav and Weinberg (2001), and Garcia-Penalosa and Wen (2005). In order to replicate and analyse the recent findings in the empirical labour income risk literature, Krebs (2003a, 2003b) considers human capital risk more generally and solves for optimal level of homogeneous human capital investments in the presence of risk. Krebs (2003a) shows how capital markets can be used to manage some of human capital risk. The risk structure specification in Krebs (2003b) is, however, more closely related to the current paper, as it includes both correlated aggregate and idiosyncratic shocks. In order to address differences in skill composition between the US and central European economies, Krueger and Kumar (2004a, 2004b) analyse the choice between general and skill-specific education, and their approach is maybe the most closely related to the current paper, even if the focus is different.

Sector-specific labour income risk is addressed by Dixit and Rob (1994), but they formulate occupational choice as a number of switches between sectors over an infinite lifetime, for which the individuals pay a switching cost at each switch. This framework is suitable for analysing the reallocation of labour between sectors, over time. However, it differs distinctly from the topic of the current paper where occupational choice is formulated as an irreversible decision to invest in discrete types of education when labour income risk varies across the resulting occupations.

The entrepreneurial choice literature considers entrepreneurial risk as a form of income uncertainty and typically prescribe income redistribution through taxes as the means to increasing shares of entrepreneurs in the labour market, see e.g. Kanbur (1979a, 1979b, 1981), Boadway, Marchand and Pestieau (1991), and more recently Poutvaara (2002), Garcia-Penalosa and Wen (2005) and Clemens (2006). This literature features discrete educational or occupational choices where the risky choice, entrepreneurship, is potentially associated with higher returns. Both because of the focus on discrete educational choices and the coupling of risk and higher expected returns, the current paper is more related to this literature. In particular, Poutvaara (2002) who analyses the case where returns are higher for the most risky occupational choice is relevant for the topic of the current paper, and his results are building on, but are not specific to, the entrepreneurial choice literature. His model is, however, very different from that presented in the current paper, with its static supply-side analysis of the effect of linear income taxation on occupational choices.

Wildasin (2000) highlights a possible source of higher labour income risk for highly specialized educational choices. He argues that for highly educated workers, increases in human capital investments lead to more specialization, and even though workers have high expected labour income, specialization implies lower probability of good employment matches and thus considerable regional labour income risk.

Skills gaps caused by market failures are given limited explicit attention, although the collective volume edited by Booth and Snower (1996) has gone some way towards remedying this. In this volume, Acemoglu (1996) analyses market imperfections in education financing and imperfectly competitive wage setting as a source of skills gaps when human and physical capital are complements. In the same volume, Ulph (1996) shows that when highly trained workers are in short supply, firms do not innovate sufficiently and subsequently workers do not train sufficiently, because there is insufficient demand for them from innovating firms. Arrow and Capron (1959) explain how wages adjust only slowly to demand changes due to lags in firms' decision making processes, internal labour markets, prevalence of long-term contracts, and slow diffusion of information of wage changes, thus causing a slow equilibrating process. They show that as long as demand is steadily increasing, the skills gaps will be persistent and not approach zero. Siow (1984) takes future demand conditions into account when analysing occupational choice and labour income uncertainty, but his analysis is restricted to risk neutral individuals. Freeman (1975a, 1975b) also presents models of educational supply responses to earnings uncertainty for risk averse individuals. These models include a demand side and have been labelled cobweb models, because they show that skills supply, due to the gestation period of education, is a lagged response to demand changes, such that skills gaps and subsequent over-supply may arise. But in these models, individuals are myopic and do not have expectations of future demand conditions.

3 Sector-specific idiosyncratic production shocks

The idiosyncratic component of labour income risk has been given the most attention in the labour income risk literature. According to Storesletten, Telmer and Yaron (2001) the aggregate risk associated with labour income is smaller than the aggregate risk associated with financial equity. However, Storesletten, Telmer and Yaron (2004) find that idiosyncratic labour income risk is strongly correlated with aggregate risk. They estimate that the conditional variance of permanent idiosyncratic shocks increase by 75% as the aggregate economy moves from expansion to contraction.

The following section analyses the case where the productivity shocks in the risky sector are of an idiosyncratic nature.

3.1 Production

The production side consists of two intermediate goods sector, the safe sector and the risky sector, and a final good sector, which combines the outputs from the two intermediate goods sectors in production of a final consumption good. The risky sector intermediate good production is affected by idiosyncratic shocks. These are multiplicative productivity shocks and are independently and identically distributed over individuals and time in the risky sector. In a later section of the paper, the effect of lifting this assumption to allow for counter-cyclical variance of the shocks is being explored. The safe sector is not directly affected by shocks.

The production firms within each of the three sectors are assumed to be owned by a large number of entrepreneurs, and the firms are all so small that they do not have any market power. Any profits earned in each period are spent consuming the final consumption good, as there is no intertemporal storage or capital markets that allow for intertemporal smoothing of consumption.

3.1.1 The final good sector

The final good sector produces the aggregate consumption good y_t by combining the outputs from the safe sector $x_{s,t}$ and the risky sector $x_{r,t}$:

$$y_t = F_y\left(x_{s,t}, x_{r,t}\right) \tag{1}$$

$F_y\left(x_{s,t}, x_{r,t}\right) \geq 0$ is the aggregated production technology for the whole of the risky sector. The sector is assumed to consist of a large number of small firms owned by entrepreneurs who maximise profits. Any profits of firms in each period are spent on final good consumption in the current period cannot be stored or reinvested. The marginal products of the safe sector intermediate good $\frac{\partial y_t}{\partial x_{s,t}} > 0$ and the risky sector intermediate good $\frac{\partial y_t}{\partial x_{r,t}} > 0$ are assumed to be positive, and the production function is assumed to be twice differentiable and defined only for non-negative values of the intermediate goods inputs $x_{s,t}, x_{r,t} \geq 0$. The production function is concave, which implies constant or diminishing returns $\frac{\partial y_t^2}{\partial x_{s,t}^2} \leq 0$, $\frac{\partial y_t^2}{\partial x_{r,t}^2} \leq 0$ and $\frac{\partial y_t^2}{\partial x_{s,t}^2} * \frac{\partial y_t^2}{\partial x_{r,t}^2} - \left(\frac{\partial y_t^2}{\partial x_{r,t}^2 \partial x_{s,t}^2}\right)^2 \geq 0$ for all possible combinations of $x_{s,t}$ and $x_{r,t}$.[6]

The final good is assumed to be a numeraire $p_{y,t} = 1$, such that the price of each good or labour unit is equal to the number of units of final good that can be exchanged for that unit. The final good sector maximizes profits $\pi_{y,t}$ by setting the demand for the safe and the risky sector good optimally

$$\pi_{y,t} = p_{y,t} F_y\left(x_{s,t}, x_{r,t}\right) - p_{s,t} x_{s,t} - p_{r,t} x_{r,t} \tag{2}$$

$$\max_{x_{s,t}, x_{r,t}} \pi_{y,t} \Rightarrow \begin{cases} p_{s,t} = \frac{\partial y_t}{\partial x_{s,t}} \\ p_{r,t} = \frac{\partial y_t}{\partial x_{r,t}} \end{cases} \tag{3}$$

where $p_{s,t}$ is the relative price per unit of the safe sector output and $p_{r,t}$ is the relative price per unit of the risky sector output. Supply of the final sector good is thus given by

$$y_t^S = y^S\left(p_{r,t}, p_{s,t}\right) \tag{4}$$

[6] Instead of a final good sector that combines the intermediate outputs of the risky and safe sectors, consumers could combine the two intermediate goods in their consumption bundle. However, the final consumption good sector allows for a tractable analysis of how different ways of combining the two risky and safe sector goods affect the equilibrium outcomes. Furthermore, the final goods sector makes it relatively easy to add growth dynamics to the analysis.

and demand of the intermediate risky sector and safe sector goods are given by the following functions

$$x_{r,t}^{D} = x^{D}\left(p_{r,t}, p_{s,t}\right) \tag{5}$$

$$x_{s,t}^{D} = x^{D}\left(p_{r,t}, p_{s,t}\right) \tag{6}$$

Final good supply and input demand are thus dependent on the prices of the intermediate inputs.

3.1.2 The risky sector

The risky sector also consists of large number of small firms owned by entrepreneurs. Workers in the risky sector are affected by idiosyncratic multiplicative productivity shocks. The aggregate production function in the risky sector is given by the following function

$$x_{r,t} = \int_{i=\underline{a}_{t-1}}^{\overline{a}_{t-1}} z_t^i a_t^i di \tag{7}$$

where the idiosyncratic shock z_t^i enters within the integral over the interval of risky sector workers $r_t = \left[\underline{a}_{t-1}, \overline{a}_{t-1}\right]$ and r_t is a fraction of the working population L_t who are born at time $t-1$.

The idiosyncratic shocks $z_t^i \sim \Lambda\left(\mu, \sigma^2\right)$ are assumed to be lognormally distributed with density function $g(z)$.[7] I also assume that the shocks are independently and identically distributed (i.i.d.) across workers in the risky sector and across time. However, Meghir and Pistaferri (2004) find that the i.i.d. assumption is not supported by data on labour income risk, and this assumption will be altered in the next section of the paper. The lognormal distribution is chosen because it has the advantage that it replicates the income distribution relatively well, and it also has the property that the expectation of z_t^i is related to the variance of the logarithm of the shock $\ln z_t^i$. The expectation of risky sector output at time t is thus given by

$$E\left(x_{r,t}\right) = \int_{i=\underline{a}_{t-1}}^{\overline{a}_{t-1}} \int z_t^i a_t^i g\left(z\right) dz di = \exp\left(\mu + \frac{1}{2}\sigma^2\right) \int_{i=\underline{a}_{t-1}}^{\overline{a}_{t-1}} a_t^i di$$

The labour income is now affected by the idiosyncratic shock, but is otherwise identical to

[7] This implies that the logarithm of the shock is normally distributed, i.e., $\ln z_t^i \sim N\left(\mu, \sigma^2\right)$. For lognormal distribution, the mean and variance of the shock is given by $E\left(z_t^i\right) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$ and $Var\left(z_t^i\right) = \exp\left(2\mu + \sigma^2\right)\left(\exp \sigma^2 - 1\right)$.

equation (38)

$$w_{r,t}\left(z_t^i, a_{t-1}^i\right) = \arg\max_{a_{t-1}^i} \pi\left(r_t\right) = p_{r,t} z_t^i a_t^i \tag{8}$$

Demand of risky sector workers r_t^D is thus a function of the price of the risky sector product and the labour income

$$r_t^D = r^D\left(w_{r,t}^i\left(z_t^i, a_{t-1}^i\right), p_{r,t}\right) \tag{9}$$

and correspondingly supply of the risky sector intermediate good $x_{r,t}^S$ is given by

$$x_{r,t}^S = x_r^S\left(w_{r,t}^i\left(z_t^i, a_{t-1}^i\right), p_{r,t}\right) \tag{10}$$

3.1.3 The safe sector

The safe sector also consists of a large number of small firms owned by silent entrepreneurs, and the aggregate sectoral production technology $F_s() \geq 0$ is given by

$$x_{s,t} = F_s\left(s_t\right) \tag{11}$$

which is defined for non-negative input of workers with the safe sector education $s_t \geq 0$. The production technology has positive and constant or diminishing returns $F_s''\left(s_t\right) \leq 0$. The safe sector is profit-maximizing and sets labour income to equal the value of productivity at the margin:

$$\pi_{s,t} = p_{s,t} x_{s,t} - w_{s,t} s_t = p_{s,t} F_s\left(s_t\right) - w_{s,t} s_t \tag{12}$$

$$w_{s,t} = \arg\max_{s_t} \pi_{s,t} = p_{s,t} F_s^{'}\left(s_t\right) \tag{13}$$

where the labour income of a worker in the safe sector is given by $w_{s,t}$. Demand for safe sector workers s_t^D is given by

$$s_t^D = s^D\left(p_{s,t}, w_{s,t}\right) \tag{14}$$

which is a function of the safe sector labour income $w_{s,t}$ and price of the safe sector product $p_{s,t}$. Correspondingly, supply of the intermediate safe sector good $x_{s,t}$ is given by

$$x_{s,t}^S = x^S\left(p_{s,t}, w_{s,t}\right) \tag{15}$$

3.2 Educational Choice

In each period t, a generation with a continuum of individuals is born. Each generation lives for two periods only. In the first period of their lives individuals invest in education, and in the second period of their lives they work in their chosen occupation. This implies that in this simple version of the model, there is only one generation in the labour market at each time t. We also assume that there is no population growth, which means that each generation can be normalized to 1. Ability a is assumed to be uniformly distributed over the continuum of individuals in each generation, such that $a_t^i \in [0,1]$.

The educational choice can be interpreted as a discrete choice between two different degree subjects that qualify for different occupations, but degree level may or may not be the same. In this basic model set-up, the educational choice, occupational choice and sectoral choice is one and the same. This assumption is modified in the next chapter. But for now, one type of education only qualify workers for jobs in the 'safe' sector $j = s$, and the other type of education only qualify workers for jobs in the 'risky' sector $j = r$.

Individual i in the generation born at time t chooses education and incurs a utility cost $e_j\left(a_t^i\right)$ at time t, where the utility cost is a function of ability a_t^i. The education cost $e_j\left(a_t^i\right)$ is decreasing in ability $e_j'\left(a_t^i\right) < 0$ with $e_j''\left(a_t^i\right) \geq 0$, and it is allowed to differ across the two discrete educational choices. Ability a_t^i is assumed to be uniformly distributed over the unit interval. After individuals have completed their education, they enter the second and last period of their life $t+1$, where they work in the sector that demands the type of skill that they have acquired and consume c_{t+1}.

Individuals maximize expected lifetime utility, where utility of individual i born at time t is given by the additively separable intertemporal utility function specification U_t^i

$$U_t^i = -e_j\left(a_t^i\right) + \frac{1}{1+\rho}\frac{\left(c_{t+1}^i\right)^{1-\gamma} - 1}{1-\gamma} \tag{16}$$

We assume that individuals have constant relative risk aversion (CRRA) such that utility of consumption is given by $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ if $\gamma \neq 1$, and γ is the risk aversion coefficient.[8] The parameter γ is here constant and assumed to be identical for all individuals. Individuals are risk averse when $\gamma > 0$, individuals are risk neutral when $\gamma = 0$, and individuals are risk-loving when $\gamma < 0$. However, the latter case where $\gamma < 0$ is largely being ignored in the discussion of results, due to the very limited support for risk-loving behaviour in the behavioural literature. The parameter ρ is the individual's intertemporal rate of time preference over consumption c_{t+1} and is assumed to lie in the interval $\rho \in (0, 1)$.

The budget constraint for consumption at time $t+1$ is dependent on the choice of education $e_j = e_j\left(a_t^i\right)$ at time t:

$$c_{t+1}^i = w_{j,t+1}^i \text{ if } e_j = e_j\left(a_t^i\right) \quad \forall \quad j = r, s \tag{17}$$

where $j = r$ when the individual chooses education that qualifies for work in the risky sector and $j = s$ when the individual chooses education that qualifies for work in the safe sector. Aggregate consumption demand is given by the sum of labour incomes in the safe and the risky sector and the sum of the profits earned by entrepreneurs in the final good sector and the risky and safe intermediate goods sectors

$$C_{t+1}^D = s_{t+1} w_{s,t+1} + \int_{\underline{a}_{t-1}}^{\overline{a}_{t-1}} w_{r,t+1}\left(a_t^i\right) da_t^i + \pi_{r,t+1} + \pi_{s,t+1} + \pi_{y,t+1} \tag{18}$$

Skills supply is determined by an ability cut-off level, such that the individual with ability a_t^S at the cut-off level is indifferent between the two educational choices. For all ability levels above a_t^S, one educational choice dominates and for all with ability below a_t^S the other educational choice dominates. For the person with cut-off ability, however, the following arbitrage equation $E\left(U\left(c_{t+1}; a_t^S, e_r\right)\right) = E\left(U\left(c_{t+1}; a_t^S, e_s\right)\right)$ must hold with equality. Expected lifetime utility of the safe sector educational choice is given by the following function

$$E\left(U\left(c_{t+1}; a_t^i, e_s\right)\right) = -e_s\left(a_t^i\right) + \frac{1}{1+\rho} \frac{w_{s,t+1}^{1-\gamma} - 1}{1-\gamma}$$

[8]When $\gamma = 1$, marginal utility is given by $u'(c) = c^{-\gamma} = \frac{1}{c}$ and $u(c) = \log c$. If $\gamma \to 1$, the constant -1 ensures that utility is well-defined, and the limit $\lim_{\gamma \to 1} \frac{-\log(c) c^{1-\gamma}}{-1} = \log c$ can be computed using L'Hôpital's rule.

Individuals assign probability to the different possible realisations of the productivity shock in the risky sector such that expected utility of risky sector education is given by

$$E\left(U\left(c_{t+1};a_t^i,e_r\right)\right)=-e_r\left(a_t^i\right)+\frac{\int_z w_r\left(z_{t+1}^i,a_t^i\right)^{1-\gamma}g\left(z\right)dz-1}{\left(1+\rho\right)\left(1-\gamma\right)} \tag{19}$$

The ability cut-off a_t^S is thus determined using the arbitrage equation $E\left(U\left(c_{t+1};a_t^S,e_r\right)\right)=E\left(U\left(c_{t+1};a_t^S,e_s\right)\right)$, which can be rewritten on the following form

$$\int_z w_r\left(z_{t+1}^i,a_t^S\right)^{1-\gamma}g\left(z\right)dz=w_{s,t+1}^{1-\gamma}+\left(1-\gamma\right)\left(1+\rho\right)\left[e_r\left(a_t^S\right)-e_s\left(a_t^S\right)\right] \tag{20}$$

From the equation above, we see that optimal skills supply to the two occupations is implicitly defined as a function of the risk aversion parameter γ, intertemporal discount rate ρ, and the labour incomes in the two occupations

$$a_t^S=a^S\left(\gamma,\rho,w_{r,t+1},w_{s,t+1}\right) \tag{21}$$

This is a unique crossing point for the expected lifetime utility functions if

$$\frac{\partial E\left(U\left(c_{t+1};a_t^i,e_r\right)\right)}{\partial a_t^i}>\frac{\partial E\left(U\left(c_{t+1};a_t^i,e_s\right)\right)}{\partial a_t^i}$$

Because the model assumes that individual productivity and hence also labour income in the risky sector depends on the heterogeneous innate abilities of workers, whereas productivity in the safe sector is only dependent on number of workers, expected lifetime utility increases more strongly with ability in the risky than in the safe sector such that the risky sector employs the highest ability workers. The assumption that individual productivity in the risky sector depends on ability whereas in the safe sector it does not, allows clear separation of workers also in the case where educational costs are independent of ability. The assumption is not essential to the results related to the skills gap, even if workers in the safe sector would also be paid according to their individual productivity. However, analysis of welfare effects will depend on this assumption.s_{t+1}^S and r_{t+1}^S can be interpreted as shares of workers in the safe sector and share of workers in the risky sector in the labour force.

As long as $\frac{\partial E\left(U\left(c_{t+1};a_t^i,e_r\right)\right)}{\partial a_t^i}>\frac{\partial E\left(U\left(c_{t+1};a_t^i,e_s\right)\right)}{\partial a_t^i}$ holds for all values of a_t^i, those with ability in the subset $\left[a_t^S, 1\right]$ will choose the risky education type

$$r_{t+1}^S = 1 - a_t^S$$

as their expected lifetime utility will be higher than the expected utility from the education that qualifies for the safe sector occupation $E\left(U_{i,t}\left(c_{t+1}; e_{r,t}, a_t^i\right)\right) \geq E\left(U_{i,t}\left(c_{t+1}; e_{s,t}, a_t^i\right)\right)$. For those with ability in the subset $\left[0, a_t^S\right]$, the reverse will be true and they will choose the safe education.

From proposition **??** above, it follows that all workers r_{t+1}^S with ability above the ability cut-off level a_t^S will supply labour to the risky sector. Thus, the supplied shares of safe sector workers s_{t+1}^S equals the cut-off ability level and the remaining workers are those with ability above the cut-off level

$$s_{t+1}^S = a_t^S \tag{22}$$

$$r_{t+1}^S = 1 - s_{t+1}^S \tag{23}$$

3.3 Equilibrium

There is no population growth or economic growth caused by any other factor. Moreover, there is only one generation in the labour market at a time, such that the analysis consists of an eternally repeated two-period problem. Thus, we can focus on the equilibrium solutions for each period t.

Thus, equilibrium is defined by the equations (14), (6), (9), (10), (15), (40) (21), (22), and (23), such that a_{t-1}^*, s_t^*, r_t^*, $x_{r,t}^*$, $x_{s,t}^*$, $p_{r,t}^*$, $p_{s,t}^*$, $w_{r,t}^*\left(a_{t-1}^i, z_t^i\right)$, and $w_{s,t}^*$ are determined endogenously for given values of the exogenous variables γ, ρ, μ, σ and $p_{y,t} = 1$.

The labour income in the risky occupation in equilibrium is a function of individual ability, the idiosyncratic shock and price of the safe sector good, whereas the labour income in the safe occupation is the same for all workers and is a function of marginal productivity and the price of the safe sector intermediate good

$$w_r^*\left(z_t^i, a_{t-1}^i\right) = \frac{\partial y_t}{\partial x_{r,t}} z_t^i a_{t-1}^i \tag{24}$$

$$w_{s,t}^* = \frac{\partial y_t}{\partial x_{s,t}} F_s'\left(a_{t-1}^*\right) \tag{25}$$

Equilibrium prices of intermediate goods are equal to the value of their marginal product

$$p_{r,t}^{*} = \frac{\partial y_t}{\partial x_{r,t}}$$
$$p_{s,t}^{*} = \frac{\partial y_t}{\partial x_{s,t}}$$

If we insert this equation into equation (20) from the supply side and use the properties of the lognormal distribution of z, we can determine ability cut-off level in equilibrium

$$\begin{aligned} & \left(p_{r,t}^{*} a_{t-1}^{*}\right)^{1-\gamma} \exp\left[(1-\gamma)\left(\mu + \frac{1}{2}(1-\gamma)\sigma^2\right)\right] \\ = & \left(p_{s,t}^{*} F_s^{'}\left(a_{t-1}^{*}\right)\right)^{1-\gamma} + (1+\rho)(1-\gamma)\left[e_r\left(a_t^{*}\right) - e_s\left(a_t^{*}\right)\right] \end{aligned} \quad (26)$$

Condition 1 *In order to ensure that the cut-off ability level a_{t-1}^{*} is a unique solution for the optimal skill allocation in equilibrium, expected lifetime utility of the risky educational choice must be a steeper function of ability, and this condition must hold for all levels of ability:*

$$\frac{\partial E\left(U\left(c_{t+1}; a_t^i, e_r\right)\right)}{\partial a_t^i} > \frac{\partial E\left(U\left(c_{t+1}; a_t^i, e_s\right)\right)}{\partial a_t^i} \quad \forall \quad a_t^i \in [0,1]$$

Condition 1 can be presented on the following form

$$-e_r^{\prime}\left(a_t^i\right) + \left(\frac{1}{1+\rho}\right)\frac{p_{r,t}^{*}}{\left(p_{r,t}^{*} a_{t-1}^i\right)^{\gamma}} \exp\left[(1-\gamma)\left(\mu + \frac{1}{2}(1-\gamma)\sigma^2\right)\right] > -e_s^{\prime}\left(a_t^i\right)$$

$\forall \quad a_t^i \in [0,1]$. From this expression, we see that the condition implies that a person with a marginally higher ability level than a^i must have a proportionally higher difference in expected lifetime utility for the risky sector educational choice than for the safe educational choice relative to someone with ability level a^i. That is, the combination of lower educational cost and higher utility arising from higher expected wage for the risky educational choice must be larger than the gain from lower educational cost in the safe sector. This condition is only violated when the educational cost for the safe education drops so much more with increasing ability relative to educational cost for the risky education that this is not sufficiently compensated for through the higher expected labour income in the risky sector.

Proposition 1 *If condition 1 is fulfilled, the allocation of workers between types of education and sectors has a unique equilibrium solution and is constant over time. The equilibrium allocation is implicitly defined as a function of the time-invariant risk aversion coefficient γ, intertemporal* *discount rate ρ, the mean of the log of the shock $\mu = E(\ln z)$ and the variance of the log of the shock $\sigma^2 = Var(\ln z)$.*

$$a_{t-1}^* = s_t^* = 1 - r_t^* = a\begin{pmatrix} \gamma, & \rho, & \mu, & \sigma^2 \\ + & +/- & - & - \end{pmatrix} \tag{27}$$

The directions of the comparative statics effects rely on complementarity or not too strong substitutability between the intermediate goods in final good production.

Proof. The proposition follows directly from equation (26) and condition 1. The directions of comparative statics effects are discussed in more detail below. ■

The directions of the effects on the equilibrium allocation of skills from marginal shifts in the exogenous parameters γ, ρ, μ, and σ^2 are all quite intuitive. If γ goes up such that individuals become more risk averse, the utility derived from labour income in the risky sector goes down and hence fewer wish to work in the risky sector. If the mean μ or variance σ^2 of the log of the idiosyncratic shock goes up, expected labour income increases and a higher share of workers choose the risky sector. The effect on the equilibrium level of a change in the personal intertemporal discount rate also depends on the relationship between the educational costs

$$\frac{\partial a^*}{\partial \rho} < 0 \text{ if } e_r(a^*) > e_s(a^*)$$
$$\frac{\partial a^*}{\partial \rho} > 0 \text{ if } e_r a^* < e_s(a^*)$$

This effect is quite intuitive. If the discount rate ρ increases, workers value present utility higher and are less inclined to forego utility in the present in order to achieve future rewards. Thus, the most expensive type of education becomes less attractive, such that a^* goes up if the risky education is most expensive $e_r(a^*) > e_s(a^*)$ and if the safe education is most expensive the reverse is true.

These effects rely on some conditions which will be discussed below. The directions of effects on a^* of marginal changes in the exogenous variables require that the risky and safe sector goods are not too strongly substitutable. With strong substitutability, a reallocation of skill due to shifts in the risk aversion parameter or income effect due to shifts in μ or σ^2 can lead to a weaker diminishing returns effect than substitutability effect on prices. Such strong substitutability could reverse the direction of effects on labour income and utility levels. This does not happen if the conditions $\frac{\frac{\partial p_{s,t}^*}{\partial a^*}}{p_{s,t}^*} < -\frac{F_s''(a_{t-1}^*)}{F_s'(a_{t-1}^*)}$ and $\frac{\partial p_{r,t}^*}{\partial a^*} > -\frac{p_{r,t}^*}{a_{t-1}^*}$ are fulfilled. A further, but not essential, condition which ensures that direction of the comparative statics effects relates to the relationship between the steepness of the two cost functions for the safe and risky educational choices $e'_s(a_t^*) \geq e'_r(a_t^*)$. The directions of the comparative statics effects could change if the educational cost of safe education would drop much more than the educational cost of the risky education when ability level increases. If the condition is violated, it is possible that exogenous parameter shifts which imply positive income or utility effects for the risky occupation would be offset by a proportionally larger drops in safe sector educational costs. This could mean that the number of safe sector workers could remain constant or even increase.

If the educational costs are the same for both types of education $e_r\left(a_t^i\right) = e_s\left(a_t^i\right)$, only expected utility of second period consumption will differ between the two educational choices. In the safe sector, labour income and consumption is independent of ability whereas in the risky sector both idiosyncratic shocks and ability determine labour income. Because workers in this basic model set-up only work for one period, the personal intertemporal discount rate does not affect the educational choice. It then becomes a trade-off between utility of a fixed labour income or expected utility of labour income that depends both on idiosyncratic shocks and ability. Hence, when $e_r\left(a_t^i\right) = e_s\left(a_t^i\right)$ the skill allocation is independent of the personal intertemporal discount rate ρ

$$a_{t-1}^* = s_t^* = 1 - r_t^* = a\left(\gamma, \mu, \sigma\right)$$

3.4 Skills Gap

The skill gap G capture the under- or in some cases over-supply of skills in the risky occupation due to incomplete markets for labour income insurance. In the presence of idiosyncratic labour income risk, this inefficiency arises both because human capital and productivity are difficult to measure and because there are numerous asymmetrical information problems tied to any such insurance. The inefficient skills supply is contrasted with the efficient skills supply if there would be complete insurance for labour income risk.

The skill gap G is defined as the difference between share of risky sector workers $r_t^I = 1 - a_{t-1}^I$ if labour income risk could be completely insured for and share of risky sector workers $r_t^* = 1 - a_{t-1}^*$ when there is no labour income insurance

$$G_t = r_t^I - r_t^* = a_{t-1}^* - a_{t-1}^I \tag{28}$$

The skills gap in the risky sector is defined in equation (64). If labour income risk could be completely insured, skills supply would be efficient. Complete insurance implies that each risky sector worker receives a income $w^I_{r,t+1}\left(a^i_t\right)$ that is independent of the realisation of the idiosyncratic shock and leaves the expected labour income $E\left(w_r\left(z^i_{t+1},a^i_t\right)\right) = \int_z w_r\left(z^i_{t+1},a^i_t\right) g\left(z\right) dz$ unchanged. This implies that under complete insurance, the labour income in the risky sector at time $t+1$ of a worker with ability a^i_t can be found by equating with the expected income without insurance

$$w^I_{r,t+1}\left(a^i_t\right) = \int_z w_r\left(z^i_{t+1},a^i_t\right) g\left(z\right) dz = \frac{\partial y_{t+1}}{\partial x_{r,t+1}} a^i_t \exp\left(\mu + \frac{1}{2}\sigma^2\right) \tag{29}$$

Expected utility of the risky occupation if complete insurance were available is given by

$$E\left(U\left(c_{t+1},e_{r,t};a^i_t,I\right)\right) = -e_r\left(a^i_t\right) + \frac{\left[\frac{\partial y_{t+1}}{\partial x^I_{r,t+1}} a^i_t \exp\left(\mu + \frac{1}{2}\sigma^2\right)\right]^{1-\gamma} - 1}{\left(1+\rho\right)\left(1-\gamma\right)} \tag{30}$$

Similarly to in the previous section, ability cut-off a^I_t when complete insurance is available is determined such that the person with ability a^I_t is indifferent between the risky and the safe occupation $E\left(U\left(c_{t+1},e_{r,t};a^I_t,I\right)\right) = E\left(U\left(c_{t+1},e_{s,t};a^I_t\right)\right)$. Thus, the following equation implicitly defines the ability cut-off level a^I_t

$$w^I_{r,t+1}\left(a^I_t\right) = \left[w_{s,t+1}\left(a^I_t\right)^{1-\gamma} - \left(1+\rho\right)\left(1-\gamma\right)\left[e_r\left(a^I_t\right) - e_s\left(a^I_t\right)\right]\right]^{\frac{1}{1-\gamma}} \tag{31}$$

such that labour income in the risky sector with complete insurance $w^I_{r,t+1}\left(a^I_t\right)$ is equal to the sum of the utility-weighted labour income in the safe sector and the utility-weighted difference in educational costs. This can be rewritten on the form

$$\left[\frac{\partial y_{t+1}}{\partial x^I_{r,t+1}} a^I_t \exp\left(\mu + \frac{1}{2}\sigma^2\right)\right]^{1-\gamma} - \left[\frac{\partial y_{t+1}}{\partial x_{s,t+1}} F'_s\left(a^I_t\right)\right]^{1-\gamma} - \left(1+\rho\right)\left(1-\gamma\right)\left[e_r\left(a^I_t\right) - e_s\left(a^I_t\right)\right] = 0 \tag{32}$$

If educational costs are the same for both occupations $e_r\left(a^I_t\right) = e_s\left(a^I_t\right)$, the worker with cut-off ability a^I_t would receive the same labour income with complete insurance in the risky sector as he would in the safe sector $w^I_{r,t+1}\left(a^I_t\right) = w_{s,t+1}\left(a^I_t\right)$.

From the equations above, we see that cut-off ability level a^I_t if complete insurance would be available, is an implicitly defined function of the following variables

$$a_t^I = a\left(\gamma, \quad \rho, \quad \mu, \quad \sigma \right) \tag{33}$$

This discussion is not yet complete.

4 Sector-specific aggregate production shocks

Except for the specification pertaining to the sector-specific idiosyncratic production shock in the risky sector and the probability structure of the shocks, the model is kept unchanged. Hence, both the safe sector and the final goods sector are identical to in the analysis above.

4.1 Production

4.1.1 The risky sector

The risky sector produces a good $x_{r,t}$ using workers r_t in the ability subset $\left[\underline{a}_{t-1}, \overline{a}_{t-1}\right]$:

$$x_{r,t} = Z_t \int\limits_{a_{t-1}^i = \underline{a}_{t-1}}^{\overline{a}_{t-1}} a_{t-1}^i di \tag{34}$$

A marginal increase in the lower boundary $\underline{a}_{t-1}$ of the ability interval of risky sector workers decreases output $x_{r,t}^{'}\left(\underline{a}_{t-1}\right) = -Z_t\underline{a}_{t-1} < 0$ as this makes the ability subset of risky sector workers smaller. Similarly, an increase of the upper boundary $x_{r,t}^{'}\left(\overline{a}_{t-1}\right) = Z_t\overline{a}_{t-1} > 0$ has the reverse effect. Individual productivity $x_{r,t}^{'}\left(a_{t-1}^i\right) = Z_t a_{t-1}^i > 0$ of worker i born at time $t-1$ depends on his ability a_{t-1}^i, and this contributes to the spread of labour income in the risky sector. If ability a_{t-1}^i of individual i could increase, the increase in productivity $x_{r,t}^{''}\left(a_{t-1}^i\right) = Z_t > 0$ of individual i would be positive.

The multiplicative shock Z in the risky sector is a proportional technology shock and follows a first-order Markov process. Although the underlying causes of the shock are not modelled explicitly, it can be interpreted as business cycles of recessions and booms or sector-specific technology shocks. This could be due to changes in macroeconomic fundamentals. Another alternative interpretation of an underlying cause of the shock can be the cycles of innovation processes in the Schumpeterian sense.

For simplicity, the shock Z_t can only have two values, reflecting a good state $\overline{Z}$ and a bad state $\underline{Z}$. The size of the shock at time t is given by:

$$\begin{aligned} Z_t &= \underline{Z}, \overline{Z} \quad \forall \quad t \\ 0 &< \underline{Z} < 1 \quad \text{and} \quad \overline{Z} > 1 \end{aligned} \tag{35}$$

Expectation and variance of the shock Z_t are given by

$$\begin{aligned} E\left(Z_t\right) &= \mu_Z \\ Var\left(Z_t\right) &= \sigma_Z^2 \end{aligned} \tag{36}$$

and the shock Z_t is Markov with the following 2×2 transition Matrix:

$$\Gamma_{Z,Z'} = \Pr\left(Z_{t+1} = Z' \mid Z_t = Z\right) = \begin{pmatrix} q_1 & 1 - q_1 \\ q_2 & 1 - q_2 \end{pmatrix}$$

where the transition probabilities are given by $q_k = q\left(Z_{t+1} = \overline{Z} | Z_t = Z_k\right)$ and $1 - q_k = q\left(Z_{t+1} = \underline{Z} | Z_t = Z_k\right)$ where $Z_1 = \overline{Z}$ and $Z_2 = \underline{Z}$. This means the stochastic shock Z follows a homogeneous, first order Markov process. This period's shock only depends on the value of last period's shock, such that the model can be solved recursively. The transition probabilities lie in the range $0 < q_k < 1 \quad \forall \quad k = 1, 2$. History of shocks up to time t are given by $Z^t = \{Z_0, Z_1, \ldots, Z_t\}$, and probability of history Z^t, conditional on initial state Z_0 is given by $p\left(Z^t \mid Z_0\right)$.

The sector is hit by the Markov shock Z_t at time t and sets demand of workers in the risky sector r_t optimally to maximize profits $\pi_{r,t}$:

$$\pi_{r,t} = p_{r,t} Z_t \int_{\underline{a}_{t-1}}^{\overline{a}_{t-1}} a_{t-1}^i da_{t-1}^i - \int_{\underline{a}_{t-1}}^{\overline{a}_{t-1}} w_r\left(a_{t-1}^i\right) da_{t-1}^i \tag{37}$$

$$w_r\left(a_{t-1}^i\right) = \arg\max_{r_t} \pi_{r,t} = p_{r,t} Z_t a_{t-1}^i \tag{38}$$

Labour income w_r in the risky sector is thus directly affected by the multiplicative shock, while also depending on own productivity which varies with innate ability a_{t-1}^i. Because the shock is multiplicative, the absolute effect on labour income will depend on ability.

In this basic model set-up individuals work only for one period, thus making any labour income shock permanent. Hence, the shock can be interpreted as a permanent income shock associated with fluctuations in incidents of job-mobility, unemployment spells, promotions, and demotions caused by sector-specific fluctuations. If the lifetimes of each generation were extended to many periods, it would be possible to also allow for transitory income shocks with low or no persistence, caused by for example cyclical fluctuations in overtime labour supply, bonuses and premia, and possibly also very short-term unemployment in cases where income would quickly be mean-reverting.

When relating this specification to the empirical literature on labour income risk, it is useful to notice that the aggregate shock can be captured by a sector-, education- or occupation-specific time component, see Card and Lemieux (2001). As ability is inherently difficult to control for, the ability-component of labour income would typically be captured by an individual-specific fixed effect in the error term.

From equation 38, demand for risky sector workers r_t^D is given by

$$r_t^D = r^D \left(p_{r,t}, w_{r,t}^i, Z_t\right) \tag{39}$$

which is a function of the risky sector labour income $w_{r,t}\left(a_{t-1}^i\right)$, the shock Z_t, and price of the risky sector product $p_{r,t}$. Supply of the risky sector good is similarly given by

$$x_{r,t}^S = x^S \left(p_{r,t}, w_{r,t}^i, Z_t\right) \tag{40}$$

4.2 Educational Choice

The individual observes the state Z_t and own ability a_t^i at time t and decides whether to educate himself for work in the safe or in the risky sector by comparing the expected intertemporal utility of the two educational choices. The expected intertemporal utilities for the two education choices are expressed in the following equations:

$$E\left(U\left(c_{2t}; a_t^i, e_r\right)\right) = -e_r\left(a_t^i\right) + \frac{q_k\left(\overline{w}_{r,t+1}\left(a_t^i\right)\right)^{1-\gamma} + \left(1-q_k\right)\left(\underline{w}_{r,t+1}\left(a_t^i\right)\right)^{1-\gamma} - 1}{\left(1+\rho\right)\left(1-\gamma\right)} \tag{41}$$

$$E\left(U\left(c_{2t}; a_t^i, e_s\right)\right) = -e_s\left(a_t^i\right) + \frac{1}{1+\rho}\frac{w_{s,t+1}^{1-\gamma} - 1}{1-\gamma} \tag{42}$$

where footscript k indicates the current state Z_t such that $k = 1$ when there is a good productivity shock and $k = 2$ when there is a bad productivity shock. q_k is the Markov transition probability, given the current shock, $\overline{w}_{r,t+1}\left(a_t^i\right)$ denotes the labour income in the risky occupation at time $t+1$ if there is a good productivity shock $Z_{t+1}=\overline{Z}$ and $\underline{w}_{r,t+1}$ denotes the labour income in the risky occupation at time $t+1$ if there is a bad shock $Z_{t+1}=\underline{Z}$. The choice of education is determined by the individuals ability a_t^i because his or her productivity and labour income is dependent on ability and also the utility cost of education, which is payed up front. The case where education cost is independent of ability is also discussed.

As before, supply of skills is determined by ability cut-off level a_t^S, such that everybody with ability $a_t^i > a_t^S$ will have higher expected intertemporal utility $E\left(U_t\right)$ from becoming a worker in the risky sector r than in the safe sector s, and everybody with $a_t^i < a_t^S$ will have higher expected intertemporal utility from becoming a worker in the safe sector s than in the risky sector r:

$$\begin{aligned} E\left(U_{i,t}\left(c_{t+1};e_{r,t},a_t^i\right)\right) &\geq E\left(U_{i,t}\left(c_{t+1};e_{s,t},a_t^i\right)\right) \quad \text{if } a_t^i \geq a_t^S \\ E\left(U_{i,t}\left(c_{t+1};e_{r,t},a_t^i\right)\right) &\leq E\left(U_{i,t}\left(c_{t+1};e_{s,t},a_t^i\right)\right) \quad \text{if } a_t^i \leq a_t^S \end{aligned}$$

Skill supply is dependent on the transition probabilities between states, the degree of risk aversion, the private intertemporal discount rate, the labour income in the risk sector in good and bad states, and the labour income in the safe sector.

This supply side arbitrage equation for the case of states $k=1,2$ at time t determines the ability cut-off level a_t^S implicitly

$$q_k = \frac{w_{s,t+1}^{1-\gamma} - \left(\underline{w}_{r,t+1}\left(a_t^S\right)\right)^{1-\gamma} + (1-\gamma)(1+\rho)\left[e_r\left(a_t^S\right) - e_s\left(a_t^S\right)\right]}{\left(\overline{w}_{r,t+1}\left(a_t^*\right)\right)^{1-\gamma} - \left(\underline{w}_{r,t+1}\left(a_t^*\right)\right)^{1-\gamma}} \quad \forall \quad k=1,2 \tag{43}$$

See condition **??** below for proof that as long as there is an interior solution, all workers with ability above the cut-off level $a_{i,t} \geq a_t^S$ will choose the risky occupation.

From equation (43) above, we see that ability cut-off or skill supply can be written as a function of the transition probabilities between states, the degree of risk aversion, the private intertemporal discount rate, the labour income in the risk sector in good and bad states, and the labour income in the safe sector

$$a_t^S = a^S\left(q_k, \gamma, \rho, \overline{w}_{r,t+1}, \underline{w}_{r,t+1}, w_{s,t+1}\right) \quad \forall \quad k=1,2 \tag{44}$$

In order for the ability cut-off level a_t^S to be strictly bigger than zero and less than one, for the worker with cut-off ability a_t^S the risky sector labour income in the case of a good shock at time $t+1$ must be strictly larger than the utility-weighted sum of the safe sector labour income at time $t+1$ and the discounted difference in utility-weighted education cost for the risky sector occupation and the safe sector occupation, which again must be strictly larger than the risky sector labour income in the case of a bad shock at time $t+1$.

$$\overline{w}_{r,t+1}\left(a_t^S\right) > \left[w_{s,t+1}^{1-\gamma} + (1+\rho)(1-\gamma)\left[e_r\left(a_t^S\right) - e_s\left(a_t^S\right)\right]\right]^{\frac{1}{1-\gamma}} > \underline{w}_{r,t+1}\left(a_t^S\right) \tag{45}$$

We derive the necessary condition for an interior solution $0 < a_t^S < 1$ by manipulating equation (43) and exploiting that $0 < q_k < 1$ where $k = 1, 2$. When fulfilled, the condition implies that for those with ability in the subset

$$[\underline{a}_t, \overline{a}_t] = \left[a_t^S, 1\right]$$

the expected utility of choosing the education that qualifies for the risky sector occupation will be higher than the expected utility from the education that qualifies for the safe sector occupation $E\left(U_{i,t}\left(c_{t+1}; e_{r,t}, a_t^i\right)\right) \geq E\left(U_{i,t}\left(c_{t+1}; e_{s,t}, a_t^i\right)\right)$, whereas for those with ability in the subset $\left[0, a_t^S\right\rangle$ the reverse will be true $E\left(U_{i,t}\left(c_{t+1}; e_{r,t}, a_t^i\right)\right) < E\left(U_{i,t}\left(c_{t+1}; e_{s,t}, a_t^i\right)\right)$.

Because the model assumes that productivity in the risky sector depends on the heterogeneous innate abilities of workers, whereas productivity in the safe sector is only dependent on number of workers, the risky sector employs the highest ability workers as long as the condition above is fulfilled. This assumption allows clear separation of workers and is not essential to the results related to the skills gap, even if workers in the safe sector would also be paid according to their individual productivity. However, analysis of welfare effects will depend on this assumption.

When education costs are the same for the two types of education, such that $e_r\left(a_t^i\right) = e_s\left(a_t^i\right)$, educational choice is independent of the personal intertemporal discount rate. Also, an interior solution requires that labour income in case of a good shock in the risky sector for the person with ability at the cut-off level must be strictly larger than the labour income in the safe sector, which again must be strictly larger than labour income in the case of a bad shock in the risky sector.

The ability cut-off a_t^S is determined implicitly by

$$q_k = \frac{\left(w_{s,t+1}\right)^{1-\gamma} - \left(\underline{w}_{r,t+1}\left(a_t^S\right)\right)^{1-\gamma}}{\left(\overline{w}_{r,t+1}\left(a_t^S\right)\right)^{1-\gamma} - \left(\underline{w}_{r,t+1}\left(a_t^S\right)\right)^{1-\gamma}} \tag{46}$$

and ability cut-off can be written as a function of the transition probabilities between states, the degree of risk aversion, the labour income in the risky sector in good and bad states, and the labour income in the safe sector

$$a_t^S = a^S\left(q_k, \gamma, \overline{w}_{r,t+1}, \underline{w}_{r,t+1}, w_{s,t+1}\right) \quad \forall \quad k = 1,2$$

Again exploiting the fact that $0 < q_k < 1$, the necessary condition for an interior solution is given by

$$\overline{w}_{r,t+1}\left(a_t^S\right) > w_{s,t+1} > \underline{w}_{r,t+1}\left(a_t^S\right) \quad \text{if} \quad e_r\left(a_t^i\right) = e_s\left(a_t^i\right)$$

4.3 Equilibrium

Because there is no population growth or economic growth caused by any other factor in this version of the model and only one generation in the labour market at a time, the analysis consists of an eternally repeated two-period problem where only the realisation of and the persistence of the shock and the equilibrium responses to the shocks are differing over time. Thus, we can focus on the equilibrium solutions for each period t.

Substituting equations (44), (22), (23) and intermediate goods prices (3) from the final good sector into the labour income equations (13) and (38) for the safe and the risky sector respectively, we see that in equilibrium labour incomes in both sectors are dependent on the marginal productivities in the final goods sectors and the marginal productivities in the intermediate goods sectors

$$w_{s,t}^* = \frac{\partial y_t}{\partial x_{s,t}} F_s^{'}\left(a_{t-1}^*\right) \tag{47}$$

$$w_r^*\left(a_{t-1}^i\right) = \frac{\partial y_t}{\partial x_{r,t}} Z_t a_{t-1}^i \tag{48}$$

Labour income in equilibrium will depend on marginal productivity of the worker in his or her chosen sector, the multiplicative shock in the risky sector, and the marginal productivity of the intermediate good in the final good sector. Thus, labour income will depend on the complementarity between the intermediate risky sector good and the intermediate safe sector good in final sector production.

By substituting (22), (23), (47) and (48) into equation (45), which follows from condition **??**, we see that condition **??** requires that for the individual born at time $t-1$ with ability cut-off level $a_{t-1}^* = \underline{a}_{t-1} = s_t^D = s_t^S$ the value of its marginal product in the risky sector at time t if there was a good shock $\overline{Z}$ must be strictly larger than the sum of the utility weighted value of the marginal product in the safe sector and the discounted utility weighted difference in education costs for the risky and the safe sector education at time $t-1$, which again has to be strictly larger than the value of the marginal product of the worker with cut-off ability a_{t-1}^* in the risky sector in the case of a bad shock

$$\frac{\partial y_t}{\partial x_r}\overline{Z}a_{t-1}^* > \left[\left(\frac{\partial y_t}{\partial x_S}F_s^{'}\left(a_{t-1}^*\right)\right)^{1-\gamma} + (1+\rho)(1-\gamma)\left[e_r\left(a_{t-1}^*\right) - e_s\left(a_{t-1}^*\right)\right]\right]^{\frac{1}{1-\gamma}} > \frac{\partial y_t}{\partial x_r}\underline{Z}a_{t-1}^* \tag{49}$$

Because $\frac{\partial y_t}{\partial x_r}\overline{Z}f_r\left(a_{t-1}^*\right) > \frac{\partial y_t}{\partial x_r}\underline{Z}f_r\left(a_{t-1}^*\right)$ must hold, we know that a marginal increase in the value of the shock Z_t for a given level of skills supply a_{t-1}^* must have a positive effect on the labour income in the risky sector $w_{r,t}$. Hence, by differentiating equation (48) with respect to a change in the size of the shock for a given level of skill supply

$$\frac{\partial w_r^*\left(a_{t-1}^i\right)}{\partial Z_t} = a_{t-1}^i\left(\frac{\partial^2 y_t}{\partial x_{r,t}^2}x_{r,t} + \frac{\partial y_t}{\partial x_{r,t}}\right) > 0 \tag{50}$$

and exploiting that $\frac{\partial w_r^*\left(a_{t-1}^i\right)}{\partial Z_t} > 0$, this condition follows:

When the input of risky sector intermediate good increases by one percentage point, the percentage change in final sector marginal productivity $\frac{\partial y_t}{\partial x_{r,t}}$ of the risky sector intermediate good must be larger than -1 and smaller than or equal to 0 to ensure that condition **??** is fulfilled. This ensures that the model has an interior solution for the skills allocation

$$0 \geq \left(x_{r,t}\frac{\partial^2 y_t}{\partial x_{r,t}^2}\right) / \left(\frac{\partial y_t}{\partial x_{r,t}}\right) > -1 \tag{51}$$

In equilibrium, labour incomes $w_{r,t}$ and $w_{s,t}$ and intermediate goods prices $p_{r,t}$ and $p_{s,t}$ adjust for skills and goods markets to clear. The skills market is in equilibrium when $s_t^D = s_t^S = s_t^* = a_{t-1}^*$ and $r_t^D = r_t^S = r_t^*$. The intermediate goods market in equilibrium clears such that $x_{r,t}^D = x_{r,t}^S = x_{r,t}^*$ and $x_{s,t}^D = x_{s,t}^S = x_{s,t}^*$. According to Walras' law for general equilibrium models, the final consumption goods market will clear when the skills market and intermediate goods markets clear such that $C_t^D = y_t^S = y_t^*$. Thus, equilibrium is defined by the equations (5), (6), (14), (15), (39), (40), (44), (22), and (23), such that a^*_{t-1}, s^*_t, r^*_t, $x^*_{r,t}$, $x^*_{s,t}$, $p^*_{r,t}$, $p^*_{s,t}$, $w^*_{r,t}$, and $w^*_{s,t}$ are determined endogenously for given values of the exogenous variables q_{11}, q_{21}, $\overline{Z}$, $\underline{Z}$, γ, ρ and $p_{y,t} = 1$. In addition to the above equations, the necessary conditions **??** and 4.3 have to hold in order to ensure an interior solution for the optimal skills allocation across the safe and risky sectors $0 < s^*_t = a^*_{t-1} = 1 - r^*_t < 1$. Then, the level of safe sector workers at time t is strictly positive and less than one and is a function of the following exogenous variables, where k is the state of the economy in period $t-1$:

$$s^*_t = 1 - r^*_t = a^*_{t-1} = a\begin{pmatrix} q_k, & \gamma, & \rho, & \underline{Z}, & \overline{Z} \\ - & + & & - & - \end{pmatrix} \quad \forall \quad k = 1, 2 \tag{52}$$

Proposition 2 *When the risky and safe sector intermediate inputs are complementary or independent in final good production and the conditions below are fulfilled, optimal share of safe sector s^* workers is decreasing as a result of a marginal increase in the probability of transition to a good state q_k in the next period, is increasing in a marginal increase in the degree of risk aversion γ, and decreasing in increases in the marginal increases in the values of the multiplicative shocks $\underline{Z}$ and $\overline{Z}$. The effect of a marginal change in the personal intertemporal discount rate ρ is dependent on the relative relationship of educational costs $e_r\left(a^i\right)$ and $e_s\left(a^i\right)$, as is discussed below.*

Proof. We use implicit differentiation to determine the direction of effects on optimal share of workers with the safe occupation s^*_t from marginal shifts in the exogenous variables. For an explanation of the effects, see below. ■

The signs of these partial derivatives hold in the case where the risky and safe sector intermediate goods are complementary or independent in final sector production $\frac{\partial^2 y_t}{\partial x_{r,t} \partial x_{s,t}} \geq 0$. If the intermediate goods are substitutes, the direction of the effects on a^* of marginal shifts in the exogenous variables still hold if $a^*_{t-1}\left(\partial p^*_{r,t}/\partial a^*_{t-1}\right)/p^*_{r,t} > -1$. This means that the relative change in the price of the risky sector output at time t around optimum when there is a marginal change in cut-off ability at time $t-1$ must be greater than the relative change in productivity of the worker with the cut-off ability. Similarly, for the safe sector $\left(\partial p^*_{s,t}/\partial a^*_{t-1}\right)/p^*_{s,t} < -F''_s\left(a^*_{t-1}\right)/F'_s\left(a^*_{t-1}\right)$. Furthermore, the restriction on the relative relationship between educational costs $\left(e_s\left(a^*_{t-1}\right)/e_r\left(a^*_{t-1}\right)\right)^\gamma \geq \left(e'_s\left(a^*_{t-1}\right)/e'_r\left(a^*_{t-1}\right)\right)$ for the individual with cut-off ability a^*_{t-1} must hold.[9] Notice that the latter assumption reduces to $e'_s\left(a^*_{t-1}\right) \geq e'_r\left(a^*_{t-1}\right)$ when $\gamma = 0$, which means that for the directions of the effects above to hold, education costs of the safe sector

[9] A less restrictive, but sufficient assumption instead of $\left(e_s\left(a^*_{t-1}\right)/e_r\left(a^*_{t-1}\right)\right)^\gamma \geq \left(e'_s\left(a^*_{t-1}\right)/e'_r\left(a^*_{t-1}\right)\right)$

occupation must decrease less or equally much as education costs for the risky sector occupation for the individual with the new ability cut-off level, relative to the costs for the individual with the old ability cut-off level a_{t-1}^* as a result of a shift in one of the exogenous variables.

Under these assumptions, optimal share of workers in the safe sector $s_t^* = a_{t-1}^*$ decreases with increases in the transition probabilities q_1 and q_2. The decrease is due the increased likelihood of a good shock and consequently higher expected utility in the risky occupation relative to the safe occupation. Optimal share of workers in the safe occupation s_t^* also decreases when the multiplicative shocks $\underline{Z}$ or $\overline{Z}$ increase in value, such that the bad shock has a smaller negative impact or the good shock has a larger positive impact on expected utility in the risky sector relative to the safe sector.

An increase in risk aversion γ increases share of workers in the safe occupation s_t, as reduced taste for risk makes the returns to the risky education choice less attractive. Also, an increase in γ increases the weight individuals give to present utility loss of cost of education relative to future utility gain through consumption, which again works to make the risky occupation less attractive.

The effect on optimal share of safe sector workers s_t^* of a marginal change in ρ is slightly less straightforward

$$\frac{\partial s_t^*}{\partial \rho} \gtreqqless 0 \text{ if } e_r\left(a_{t-1}^*\right) \gtreqqless e_s\left(a_{t-1}^*\right)$$

An increase in the discount rate ρ will lead to an increased optimal share of workers in the safe occupation s_t^* when the education costs are higher for the risky than the safe occupation. This is true because the difference in utility cost of education is assigned higher weight when the value of present utility increases in importance relative to the value of future utility. If education costs are the same for the cut-off ability level, optimal skills supply will be independent of education costs and thus there is no intertemporal weighting of utility and optimal skills supply also becomes independent of the personal intertemporal discount rate. If education costs are higher for the safe sector occupation than the risky occupation, an increase in the discount rate will lead to reduced optimal level of safe sector workers, consistent with the argument described above for the reverse case.

Following from example 4.2, when education costs $e_r\left(a_t^i\right) = e_s\left(a_t^i\right)$ don't differ between

is given by $q_{k1}\left(\overline{w}_{r,t}^*\right)^{-\gamma}\left(d\overline{w}_{r,t}^*/da_{t-1}^*\right) + (1-q_{k1})\left(\underline{w}_{r,t}^*\right)^{-\gamma}\left(d\underline{w}_{r,t}^*/da_{t-1}^*\right) > \left(w_{s,t}^*\right)^{-\gamma}\left(dw_{s,t}^*/da_{t-1}^*\right) + (1+\rho)\left(e_r^{-\gamma}e_r'\left(a_{t-1}^*\right) - e_s^{-\gamma}e_s'\left(a_{t-1}^*\right)\right)$.

educational choices, the equilibrium solution for the skills market is defined by (22), (23) and the following function

$$s_t^* = a\begin{pmatrix} q_k & \gamma & \underline{Z} & \overline{Z} \\ - & + & - & - \end{pmatrix} \quad \forall \quad k = 1, 2 \tag{53}$$

Skills supply is now independent of the personal intertemporal discount rate, as the skills supply is independent of education costs and thus there is no intertemporal trade-off between current and future utility. The condition which ensures that we have an interior solution for optimal skills supply $0 < a_{t-1}^* = s_t = 1 - r_t < 1$ reduces to

$$\frac{\partial y_t}{\partial x_{r,t}} \overline{Z} f_r\left(a_t^*\right) > \frac{\partial y_t}{\partial x_{s,t}} F_s^{'}\left(a_{t-1}^*\right) > \frac{\partial y_t}{\partial x_r} \underline{Z} f_r\left(a_{t-1}^*\right) \tag{54}$$

such that for the worker with cut-off ability a_{t-1}^* the value of the marginal product in the risky sector when there is a good shock must be strictly larger than the value of the marginal product in the safe sector, which again must be strictly larger than the value of the marginal product in the risky sector when there is a bad shock at time t.

Example 1 ***Propagation of shocks to the safe sector.***

In period t when a shock is realised, product price $p_{s,t}$ and labour income $w_{s,t}$ in the safe sector are dependent on the realisation of the shock. The direction and size of these effects depend on the degree of complementarity or substitutability of the safe sector and risky sector intermediate goods in final good production. If the intermediate goods are independent, there will be no effects on price of the safe sector good and safe sector labour income.

The skill allocation in the labour market will not be affected in period t, but the educational choices of the generation born at time t will affected as realisation of the shocks determines the transition probability q_k, which is used to calculate expected utility of risky occupation choice. If the shock at time t is good $Z_t = \overline{Z}$, then the probability of a good shock at time $t+1$ is given by probability $\Pr\left(Z_{t+1} = \overline{Z} \mid Z_t = \overline{Z}\right) = q_1$ and the probability of a bad shock in the subsequent period is given by $\Pr\left(Z_{t+1} = \underline{Z} \mid Z_t = \overline{Z}\right) = 1 - q_1$. If there is a bad shock at time t, then the corresponding transition probabilities are $\Pr\left(Z_{t+1} = \overline{Z} \mid Z_t = \underline{Z}\right) = q_2$ and $\Pr\left(Z_{t+1} = \underline{Z} \mid Z_t = \underline{Z}\right) = 1 - q_2$.

The product price $p_{s,t}$ and labour income $w_{s,t}$ in the safe sector at time t are affected by different realisations of the shock. The differences in prices at time t depending on the realisation of a good shock $\overline{Z}$ and a bad shock $\underline{Z}$ at time t, can be captured by $\frac{\partial p^*_{s,t}}{\partial Z_t}$ and $\frac{\partial w_{s,t}}{\partial Z_t}$

$$\frac{\partial p^*_{s,t}}{\partial Z_t} = \frac{\partial^2 y_t}{\partial x_{r,t} \partial x_{s,t}} \frac{\partial x_{r,t}}{\partial Z_t} \quad (55)$$

$$\frac{\partial w_{s,t}}{\partial Z_t} = \frac{\partial p_{s,t}}{\partial x_{r,t}} F'_s(a^*_t) \quad (56)$$

The realisation of the shock at time t affects the skills allocation in the subsequent period only through the transition probability q_k. If the shock at time t was good, then the probability of a good shock in the next period is given by q_1 whereas if the shock at time t was bad then the probability of a good shock in the next period is q_2. If shocks are persistent, then $q_1 > \frac{1}{2} > q_2$. The implication of a good shock in the current period is captured by the change to the cut-off ability level $\frac{\partial a^*_t}{\partial q}$, which equals the change in number of safe sector workers at time $t+1$

$$\frac{\partial a^*_t}{\partial q_k} = -\frac{\overline{w}^{1-\gamma}_{r,t+1} - \underline{w}^{1-\gamma}_{r,t+1}}{(1-\gamma)\left[q_k \overline{w}^{-\gamma}_{r,t+1} \frac{\partial \overline{w}_{r,t+1}}{\partial a^*_t} + (1-q_k)\underline{w}^{-\gamma}_{r,t+1} \frac{\partial \underline{w}_{r,t+1}}{\partial a^*_t} - w^{-\gamma}_{s,t+1} \frac{\partial w_{s,t}}{\partial a^*_t} - (1+\rho)\left[e_r^{-\gamma} e'_r - e_s^{-\gamma} e'_s\right]\right]} \quad (57)$$

When the two intermediate goods are **complementary** in production of the final sector good $\frac{\partial^2 y_t}{\partial x_{r,t} \partial x_{s,t}} > 0$, any realisation of a shock in the risky sector will be propagated over to the safe sector. A positive production shock in the risky sector $Z_t = \overline{Z} > 1$ implies higher supply of the risky sector intermediate good than in the case of a bad shock $\overline{x}_{r,t} > \underline{x}_{r,t}$, and results in a higher price $p^*_{s,t}$ for the safe sector intermediate good then in the case of a bad shock, as we can see from the differentiation $\frac{\partial p^*_{s,t}}{\partial Z} = \frac{\partial^2 y_t}{\partial x_{r,t} \partial x_{s,t}} \frac{\partial x_{r,t}}{\partial Z_t} > 0$ and hence also a higher labour income for safe sector workers $\frac{\partial w_{s,t}}{\partial Z} = \frac{\partial p_{s,t}}{\partial x_{r,t}} F'_s(a^*_t) > 0$. The degree to which the safe sector is directly affected by the shock depends on the degree of complementarity $\frac{\partial^2 y_t}{\partial x_{r,t} \partial x_{s,t}}$ in final good production. The supply of workers at time $t+1$ will also depend on the realisation of the type of the shock at time t, but only because it determines which transition probabilities $q_1 = q\left(Z_{t+1} = \overline{Z} | Z_t = \overline{Z}\right)$ and $(1-q_1)$ or $q_2 = q\left(Z_{t+1} = \overline{Z} | Z_t = \underline{Z}\right)$ and $(1-q_2)$ which the generation born at time t uses to compute expected utility for the risky and safe sector occupations. The effect of the transition probability q_k on the cut-off ability level is captured by $\frac{\partial a^*_t}{\partial q_k} < 0$.

When the two intermediate goods are **independent** in production of the final sector good such that $\frac{\partial^2 y_t(x_{r,t}, x_{s,t})}{\partial x_{r,t} \partial x_{s,t}} = 0$, e.g. with additively separable production technology $y_t = F_{yr}(x_{r,t}) + F_{ys}(x_{s,t})$, shocks in the risky sector are not propagated across to the safe sector through shifts in intermediate goods prices. The safe sector is then only affected indirectly through the second order effect of reallocation of labour due to shifts in the labour market expectations of the subsequent generation, $\frac{\partial a_t^*}{\partial q_k} < 0$.

With **substitutability** in production of the final good $\frac{\partial^2 y_t}{\partial x_{r,t} \partial x_{s,t}} < 0$, a shock in the risky sector will have the reverse effect on the safe sector. Thus, an increase in risky sector output due to a good shock will have a negative effect on the price of the safe sector good $p_{s,t}$ and the labour income of safe sector workers $w_{s,t}$. Again, labour market expectations of the next generation will also depend on the realisation of the shock at time t.

Example 2 ***Propagation of a permanent increase in the size of a shock.***

In addition to the immediate price effects on the safe sector good and safe sector labour income described in the section above, a permanent increase in the size of a shock will affect the expected utility of the risky sector occupation and hence affect the cut-off ability level a_t^* at time t. The cut-off ability level will determine the skills allocation in the labour market in the subsequent period. The size and direction of the effect will again depend on the complementarity, substitutability or independence of intermediate sector goods in final sector production.

The new equilibrium outcome for the skill allocation will also affect safe sector labour income and price of the safe sector intermediate good.

A permanent increase in the size of the multiplicative shock Z at time t, will affect educational choices at time t, leading to a change in the skill allocation $a_t^* = s_{t+1}^* = 1 - r_{t+1}^*$ at time $t+1$, where $\frac{\partial a_t^*}{\partial \overline{Z}}$ captures the shift in skill allocation at time $t+1$ as a result of a marginal permanent increase in the size of the good shock

$$\frac{\partial a_t^*}{\partial \overline{Z}} = -\frac{q_k \overline{w}_{r,t+1}^{-\gamma} \frac{\partial \overline{w}_{r,t+1}}{\partial \overline{Z}} - w_{s,t+1}^{-\gamma} \frac{\partial w_{s,t+1}}{\partial \overline{Z}}}{q_k \overline{w}_{r,t+1}^{-\gamma} \frac{\partial \overline{w}_{r,t+1}}{\partial a_t^*} + (1-q_k)\, \underline{w}_{r,t+1}^{-\gamma} \frac{\partial \underline{w}_{r,t+1}}{\partial a_t^*} - w_{s,t+1}^{-\gamma} \frac{\partial w_{s,t}}{\partial a_t^*} - (1+\rho)\left[e_r^{-\gamma} e_r' - e_s^{-\gamma} e_s'\right]} \tag{58}$$

Similarly, $\frac{\partial a_t^*}{\partial \underline{Z}}$ captures the shift in the skill allocation as a result of a permanent marginal increase in the size of the bad shock

$$\frac{\partial a_t^*}{\partial \underline{Z}} = -\frac{(1-q_k)\, \underline{w}_{r,t+1}^{-\gamma} \frac{\partial \underline{w}_{r,t+1}}{\partial \underline{Z}} - w_{s,t+1}^{-\gamma} \frac{\partial w_{s,t+1}}{\partial \underline{Z}}}{q_k \overline{w}_{r,t+1}^{-\gamma} \frac{\partial \overline{w}_{r,t+1}}{\partial a_t^*} + (1-q_k)\, \underline{w}_{r,t+1}^{-\gamma} \frac{\partial \underline{w}_{r,t+1}}{\partial a_t^*} - w_{s,t+1}^{-\gamma} \frac{\partial w_{s,t}}{\partial a_t^*} - (1+\rho)\left[e_r^{-\gamma} e_r' - e_s^{-\gamma} e_s'\right]} \tag{59}$$

The change in a_t^* affects both intermediate good product prices $p_{r,t+1}^*$ and $p_{,t+1}^*$

$$\frac{\partial p_{s,t+1}}{\partial a_t^*} = -\frac{\partial^2 y_{t+1}}{\partial x_{r,t+1}\partial x_{s,t+1}} Z f_r\left(a_t^*\right) + \frac{\partial^2 y_{t+1}}{\partial x_{r,t+1}^2} F_s'\left(a_t^*\right) \tag{60}$$

$$\frac{\partial p_{r,t+1}}{\partial a_t^*} = \frac{\partial^2 y_{t+1}}{\partial x_{r,t+1}\partial x_{s,t+1}} F_s'\left(a_t^*\right) - \frac{\partial^2 y_{t+1}}{\partial x_{r,t+1}^2} Z f_r\left(a_t^*\right) \tag{61}$$

and labour incomes $w_{r,t+1}^*$ and $w_{s,t+1}^*$

$$\frac{\partial w_{s,t+1}^*}{\partial Z} = \left(\frac{\partial p_{s,t+1}}{\partial a_t^*} F_s^{'}\left(a_t^*\right) + p_{s,t+1} F_s^{''}\left(a_t^*\right)\right)\frac{\partial a_t^*}{\partial Z} \tag{62}$$

$$\frac{\partial w_{r,t+1}^*\left(a_{t-1}^i\right)}{\partial Z} = \left(\frac{\partial p_{r,t+1}}{\partial a_t^*} Z f_r\left(a_{t-1}^i\right) + p_{r,t+1} Z f_r^{'}\left(a_{t-1}^i\right)\right)\frac{\partial a_t^*}{\partial Z} \tag{63}$$

We can see that the effects on prices and hence also on labour incomes in equilibrium at time $t+1$ are dependent on the sign and size of the cross-derivative $\frac{\partial^2 y_{t+1}}{\partial x_{r,t+1}\partial x_{s,t+1}}$ in final sector production $y_{t+1} = F_y\left(x_{s,t+1}, x_{r,t+1}\right)$.

When the risky sector and safe sector intermediate goods are **complementary** in final good production, a permanent exogenous increase in the size of the current shock at time t, $\triangle Z_t = \triangle\overline{Z} > 0$ or $\triangle Z_t = \underline{Z} > 0$ will have the following effects. At time t, the output $x_{r,t}$ in the risky sector increases and consequently the price of the safe sector good $p_{s,t}$ increases due to increased demand for the safe sector good and consequently the labour income $w_{s,t}$ in the safe sector also increases. Furthermore, the generation born at time t internalizes the permanent change in the size of the shock when the skills allocation decision is made. Thus, there will be fewer safe sector workers and more risky sector workers at time $t+1$ as we can see from $\frac{\partial a_t^*}{\partial Z} < 0$. This means a higher price $p_{s,t+1}$ for safe sector goods and increased labour income for safe sector workers $w_{s,t+1}$, whereas the price $p_{r,t+1}$ of risky sector goods and the risky sector labour income $w_{r,t+1}$ will decrease.

When the risky and safe sector intermediate goods are **independent** in final good production, the price $p_{s,t}$ and labour income $w_{s,t}$ in the risky sector at time t will be unaffected by an increase in Z_t. However, the change in the size of the shock affects expected utility of the risky sector educational choice through the transition probability q_k and thus the allocation of workers between the safe and the risky in the preceding periods. At time $t+1$, there will be fewer workers in the safe sector $\frac{\partial a_{t+1}^*}{\partial Z} < 0$ and the price of the safe sector good $p_{s,t}$ and the safe sector labour income $w_{s,t}$ will both increase.

When the risky and safe sector intermediate goods are **substitutable** in final good production, price $p_{s,t}$ and labour income $w_{s,t}$ in the risky sector at time t will decrease if the size of the shock increases Z_t as more efficient production in the risky sector leads to lower demand for the safe sector intermediate good. The change in the shock also affects skills supply in subsequent periods, as captured by $\frac{\partial a^*_{t+1}}{\partial Z}$ which will be negative if the substitutability effect is not too strong. However, it may be zero or positive for high degrees of substitutability such that price changes and labour income changes are dominated by the substitutability. This is true when the effect of diminishing returns in the final goods sector is dominated by the substitutability effects such that the product price of the safe sector good decreases $\frac{\partial p_{s,t+1}}{\partial Z} < 0$ and the product price of the risky sector good increases $\frac{\partial p_{r,t+1}}{\partial Z} > 0$. If ability cut-off increases as a result of $\frac{\partial a^*_{t+1}}{\partial Z} \geq 0$, these price effects due to substitutability are dominating diminishing returns in the safe sector and the positive marginal individual productivity effect in the risky sector.

4.4 Skill Gap

Skills gaps capture the under- or in some cases over-supply of skills in the risky occupation due to incomplete markets for labour income insurance. This inefficiency arises because the labour income risk arises due to aggregate shocks are uninsurable and because labour income risk is inheritably difficult to insure for due to moral hazard problems. The inefficient skills supply is contrasted with the efficient skills supply if labour income risky could be completely insured.

The skills gap G is defined as the difference between share of risky sector workers $r^I_{t+1} = 1 - a^I_t$ if labour income risk could be completely insured for and share of risky sector workers $r^*_{t+1} = 1 - a^*_t$ when there is no labour income insurance

$$G_{t+1} = r^I_{t+1} - r^*_{t+1} = a^*_t - a^I_t \tag{64}$$

Due to the time lag of educational investments, share of risky sector workers at time $t+1$ is determined by ability cut-off level at time t.

Complete insurance implies that expected income $E\left(w_{r,t+1}\left(a^i_t\right); I\right)$ in the risky sector with and without insurance I stays the same

$$E\left(w_{r,t+1}\left(a^i_t\right); I\right) = E\left(w_{r,t+1}\left(a^i_t\right)\right) = q_k \overline{w}_{r,t+1}\left(a^i_t\right) + (1 - q_k)\, \underline{w}_{r,t+1}\left(a^i_t\right) \tag{65}$$

for all individuals i.[10] The expected utility of the risky occupation with insurance is hence given

[10] Expected income of a risky sector worker with full insurance cover I at insurance premium p_I and ability level

by

$$E\left(U\left(c_{t+1},e_{r,t};a_t^i,I\right)\right)=-\frac{e_r\left(a_t^i\right)^{1-\gamma}-1}{1-\gamma}+\frac{\left(q_k\overline{w}_{r,t+1}\left(a_t^i\right)+\left(1-q_k\right)\underline{w}_{r,t+1}\left(a_t^i\right)\right)^{1-\gamma}-1}{\left(1+\rho\right)\left(1-\gamma\right)} \tag{68}$$

Proposition 3 *The cut-off ability level a_t^I if complete insurance would be available, is again an implicitly defined function of the following variables*

$$a_t^I=a\left(\begin{array}{ccccc} q_k, & \gamma, & \rho, & \underline{Z}, & \overline{Z}\end{array}\right)$$

given the state $k=1,2$. The variables that determine the cut-off ability level and hence the skills allocation across sectors are all exogenous and do not vary over time, with the exception of the transition probability of the shock q_k, which can take on two values depending on the realisation of the shock in the previous period. Thus, the cut-off ability fluctuates between two values over time.

Proof. The person with cut-off ability a_t^I is indifferent between the risky and the safe occupation, $E\left(U\left(c_{t+1},e_{r,t};a_t^I,I\right)\right)=E\left(U\left(c_{t+1},e_{s,t};a_t^I\right)\right)$. Thus, the following equation implicitly defines the ability cut-off level a_t^I if complete insurance would be available

$$q_k=\frac{\left[w_{s,t+1}^{1-\gamma}+\left(1+\rho\right)\left(e_r\left(a_t^I\right)^{1-\gamma}-e_s\left(a_t^I\right)^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}-\underline{w}_{r,t+1}\left(a_t^I\right)}{\overline{w}_{r,t+1}\left(a_t^I\right)-\underline{w}_{r,t+1}\left(a_t^I\right)} \tag{69}$$

such that everybody with ability above the cut-off level chooses the risky occupation and everybody with ability below chooses the safe occupation. By inserting the labour income equations

a_t^i is given by

$$\begin{aligned}E\left(w_{r,t+1}\left(a_t^i\right);I\right) &= q_k\left(\overline{w}_{r,t+1}\left(a_t^i\right)-p_I I\left(a_t^i\right)\right)+\left(1-q_k\right)\left(\underline{w}_{r,t+1}\left(a_t^i\right)+\left(1-p_I\right)I\left(a_t^i\right)\right) \quad (66)\\ &= \underline{w}_{r,t+1}\left(a_t^i\right)+\left(1-p_I-q_k\right)I\left(a_t^i\right)+q_k\left(\overline{w}_{r,t+1}\left(a_t^i\right)-\underline{w}_{r,t+1}\left(a_t^i\right)\right)\end{aligned}$$

Setting the equation above equal to equation (65), we find that complete insurance means that the amount of insurance cover $I\left(a_t^i\right)$ is equal to the income difference between the good and the bad state

$$I\left(a_t^i\right)=\overline{w}_{r,t+1}\left(a_t^i\right)-\underline{w}_{r,t+1}\left(a_t^i\right) \tag{67}$$

The insurance premium $p_I=1-q_k$ must be equal to the probability of the bad shock $1-q_k$, given the current state $k=1,2$. When this is true, the premium is considered to be actuarially fair. In private insurance markets, the insurance premium $p_I>1-q_k$ is normally larger than the probability of the bad state to cover overhead costs etc. In this case, individuals will prefer not to take up complete insurance such that $I\left(a_t^i\right)<\overline{w}_{r,t+1}\left(a_t^i\right)-\underline{w}_{r,t+1}\left(a_t^i\right)$.

determined by the demand side, equation (69) can be rewritten as

$$\left[\left(q_k\overline{Z}+(1-q_k)\underline{Z}\right)\frac{\partial y_{t+1}}{\partial x_{r,t+1}}f_r\left(a_t^I\right)\right]^{1-\gamma}-\left[\frac{\partial y_{t+1}}{\partial x_{s,t+1}}F_s^{'}\left(a_t^I\right)\right]^{1-\gamma}-(1+\rho)\left[e_r\left(a_t^I\right)^{1-\gamma}-e_s\left(a_t^I\right)^{1-\gamma}\right]=0 \tag{70}$$

$\forall\ k=1,2$, which shows that a_t^I is determined by the exogenous variables q_k, γ, ρ, $\underline{Z}$ and $\overline{Z}$. ■

Example 3 *Equation (69) can be rewritten on the form*

$$E\left(w_{r,t+1}\left(a_t^i\right);I\right)=\left[w_{s,t+1}^{1-\gamma}+(1+\rho)\left(e_r\left(a_t^I\right)^{1-\gamma}-e_s\left(a_t^I\right)^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}$$

which makes it easy to spot that when educational costs are the same for both occupations $e_r\left(a_t^I\right)=e_s\left(a_t^I\right)$, *expected labour income for the worker in the risky sector with cut-off ability* a_t^I *is equal to the labour income in the safe sector* $q_k\overline{w}_{r,t+1}\left(a_t^i\right)+(1-q_k)\underline{w}_{r,t+1}\left(a_t^i\right)=w_{s,t+1}$.

The relationship between labour incomes in the risky and safe sector as expressed in condition **??** still holds if complete insurance were available.

Condition 2 *By exploiting that* $0<q_k<1$ *and manipulating equation (69), we find that the following condition secures an interior solution for* a_t^I

$$\overline{w}_{r,t+1}\left(a_t^I\right)>\left[w_{s,t+1}^{1-\gamma}+(1+\rho)\left(e_r\left(a_t^I\right)^{1-\gamma}-e_s\left(a_t^I\right)^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}>\underline{w}_{r,t+1}\left(a_t^I\right)$$

This condition is identical to condition **??**.

Proposition 4 *The skills gap is positive when the risky sector and safe sector intermediate goods are complements in production of the final consumption good.*

The skills gap can be negative when the risky sector and safe sector intermediate goods are substitutes in production of the final consumption good. This is true if shifts in exogenous variables lead to substitutability effects that are so strong that they dominate the diminishing returns effects on prices as skills are reallocated between sectors.

The size of the skills gap at time $t+1$ *depends on the probability* q_k *of the good state in the next period, given state* $k=1,2$ *in the current period, the constant relative risk aversion parameter* γ, *the personal intertemporal discount rate* ρ, *and the size of the shocks* $\overline{Z}$ *and* $\underline{Z}$. *Only the transition probability of the shock* q_k *varies over time and causes fluctuations in the skills gap.*

The other variables are all exogenous.

$$G_{t+1} = g\left(q_k, \quad \gamma, \quad \rho, \quad \underline{Z}, \quad \overline{Z} \right)$$

Notice that the results in the two-period framework would also extend to an OLG model with more periods. The skill gaps will persist when individuals are allowed to save and borrow against a share of future labour income, although they will of course have a smaller magnitude. Even if individuals live forever, they will most likely not be able to perfectly smooth consumption by using capital markets. A reason is the nature of the sector-specific shock, which will lead everybody in the risky sector to demand capital at the same time and cause the interest rate to go up if the sector is large relative to the rest of the economy. However, if the risky sector is small or there are other sectors with offsetting shocks, the effect on the interest rate may be negligeable. For further discussion of consumption smoothing, insurance and policy alternatives, see section **??** below.

If employees in the risky sector could find employment in other sectors and receive labour incomes which would be higher than those in the risky sector during bad shocks, this would be a type of insurance for employees in the risky sector. Hence, to become educated for employment in the risky sector would be less risky, and skill gaps would be smaller or possibly eliminated. This type of occupational mobility as a way of responding to labour income shocks is being analysed in a separate paper

5 Sector-specific idiosyncratic production shocks with countercyclical variance

The following section analyses the case where the productivity shocks in the risky sector are idiosyncratic, but now the variance of the shocks is state-dependent. The variance follows a similar Markov process as the aggregate shock did, with one good and one bad state and with exogenously given transition probabilities between states. Except for the specification pertaining to the production structure in the risky sector and the probability structure of the shocks, the model is kept unchanged.

5.1 Production

5.1.1 The risky sector

Individual productivity is now affected by idiosyncratic productivity shocks to the workers employed in the risky sector such that the production function is now given by

$$x_{r,t} = Z_t \int_{r_t \in L} z_t^i f\left(a_t^i\right) di \tag{71}$$

where the idiosyncratic shock z_t^i enters within the integral over the interval of risky sector workers $r_t = \left[a_{t-1}^*, 1\right]$, where r_t is a fraction of the working population L_t who are born at time $t-1$. The idiosyncratic shocks $z_{t,k}^i \sim \Lambda\left(\mu, \sigma_k^2\right) \; \forall \; k = 1, 2$ are assumed to be lognormally distributed with density function $g_k(z)$.[11] I also assume that the shocks are independently and identically distributed (i.i.d.) across individuals within the risky sector, but the shocks are now not i.i.d. across time. This implies a labour income risk structure that is in line with the empirical findings by Meghir and Pistaferri (2004) and Storesletten, Telmer and Yaron (2004). All other equations on the production side are identical to those in section 3 above.

5.2 Educational Choice

Notice that the shock is now independently and identically distributed across time and across individuals. Hence, the shock is independent of previous realisations and there are no transition probabilities. Instead, individuals assign probability to the different possible realisations in the risky sector such that expected utility of risky sector education is now given by

$$\begin{aligned} & E\left(U\left(c_{t+1}; a_t^i, e_r\right)\right) \qquad (72) \\ = \; & -\frac{e_r\left(a_t^i\right)^{1-\gamma} - 1}{1-\gamma} + \frac{q_k \int_z w_r\left(z_{t+1}^i, a_t^i\right)^{1-\gamma} g_1(z)\, dz + (1-q_k)\int_z w_r\left(z_{t+1}^i, a_t^i\right)^{1-\gamma} g_2(z)\, dz - 1}{(1+\rho)(1-\gamma)} \end{aligned}$$

$\forall \;\; k = 1, 2$. Expected utility of the safe sector educational choice $E\left(U\left(c_{t+1}; a_t^i, e_s\right)\right)$ is given by equation (42) as before, and ability cut-off a_t^S is determined from the supply side such that

[11] This implies that the logarithm of the shock is normally distributed, i.e., $\ln z_t^i \sim N\left(\mu, \sigma^2\right)$. For lognormal distribution, the mean and variance of the shock is given by $E\left(z_t^i\right) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$ and $Var\left(z_t^i\right) = \exp\left(2\mu + \sigma^2\right)\left(\exp \sigma^2 - 1\right)$.

$E\left(U\left(c_{t+1}; a_t^i, e_r\right)\right) = E\left(U\left(c_{t+1}; a_t^i, e_s\right)\right)$ such that

$$\begin{aligned} & q_k \int_z w_r\left(z_{t+1}^i, a_t^i\right)^{1-\gamma} g_1(z)\, dz + (1-q_k) \int_z w_r\left(z_{t+1}^i, a_t^i\right)^{1-\gamma} g_2(z)\, dz \\ = & \\ & w_{s,t+1}^{1-\gamma} + (1+\rho)\left[e_r\left(a_t^S\right)^{1-\gamma} - e_s\left(a_t^S\right)^{1-\gamma}\right] \\ \forall \quad k = & \; 1,2 \end{aligned} \tag{73}$$

From the equation above, we see that optimal skills supply to the two occupations is implicitly defined as a function of the CRRA γ, intertemporal discount rate ρ, and the labour incomes in the two occupations

$$a_t^S = a^S\left(\gamma, \rho, q_k, w_{r,t+1}, w_{s,t+1}\right) \tag{74}$$

5.3 Equilibrium

The equilibrium corresponds to the case with idiosyncratic shocks which are i.i.d. across time, except for the slight changes relating to the variance and probability structure of the states of the economy as specified above. Now, ability cut-off level in equilibrium is determined by the following equation

$$\begin{aligned} & \left[\frac{\partial y_t}{\partial x_{r,t}} f_r\left(a_{t-1}^*\right)\right]^{1-\gamma}\left[q_k Z_1^{1-\gamma} \int_z \left(z_t^i\right)^{1-\gamma} g_1(z)\, dz + (1-q_k) Z_2^{1-\gamma} \int_z \left(z_t^i\right)^{1-\gamma} g_2(z)\, dz\right] \\ = & \\ & \left[\frac{\partial y_t}{\partial x_{s,t}} F_s^{'}\left(a_{t-1}^*\right)\right]^{1-\gamma} + (1+\rho)\left[e_r\left(a_t^*\right)^{1-\gamma} - e_s\left(a_t^*\right)^{1-\gamma}\right] \end{aligned} \tag{75}$$

given the state $k = 1, 2$. By using the properties of the lognormal distribution, the left hand side (LHS) can be rewritten as

$$\begin{aligned} LHS &= \left[\frac{\partial y_t}{\partial x_{r,t}} f_r\left(a_{t-1}^*\right)\right]^{1-\gamma} \\ &\quad * \left[q_k Z_1^{1-\gamma} \exp\left((1-\gamma)\left(\mu + \frac{1}{2}(1-\gamma)\sigma_1^2\right)\right) + (1-q_k) Z_2^{1-\gamma} \exp\left((1-\gamma)\left(\mu + \frac{1}{2}(1-\gamma)\sigma_2^2\right)\right)\right] \\ &= \exp\left((1-\gamma)\mu\right)\left[\frac{\partial y_t}{\partial x_{r,t}} f_r\left(a_{t-1}^*\right)\right]^{1-\gamma} \\ &\quad * \left[q_k Z_1^{1-\gamma} \exp\left(\frac{(1-\gamma)^2\sigma_1^2}{2}\right) + (1-q_k) Z_2^{1-\gamma} \exp\left(\frac{(1-\gamma)^2\sigma_2^2}{2}\right)\right] \end{aligned} \tag{76}$$

where $k = 1$ is the good state and $k = 2$ is the bad state, such that $\sigma_1^2 < \sigma_2^2$. By inserting this manipulated LHS expression into equation (26), we see that the ability cut-off in equilibrium is an implicitly defined function of exogenous variables.

Proposition 5 *The supply of risky sector workers is dependent on the constant relative risk aversion γ, intertemporal discount rate ρ, the mean of the log of the shock $\mu = E(\ln z)$, the variances of the log of the shock $\sigma_1^2 = Var(\ln z | k = 1)$ during a boom and $\sigma_2^2 = Var(\ln z | k = 2)$ during a recession, and the transition probability between states of the economy q_k given the current state $k = 1, 2$.*

$$a_{t-1}^* = s_t^* = 1 - r_t^* = a\left(\gamma, \rho, \mu, \sigma_1, \sigma_2, q_k, Z_1, Z_2\right) \quad \forall \quad k = 1, 2 \tag{77}$$

Proposition 6 *The skill allocation s_t^*, determined by the cut-off ability level a_{t-1}^*, jumps between two values over time, depending on the value of the transition probability between states q_k at time $t-1$. The transition probability jumps between two different values depending on the state k of the economy. All other variables that determine the equilibrium cut-off ability level are constant over time.*

5.4 Skill Gap

The skills gap G capture the under- or in some cases over-supply of skills in the risky occupation due to incomplete markets for labour income insurance. The inefficient skills supply is contrasted with the efficient skills supply if labour income risky could be completely insured.

The skills gap in the risky sector is defined in equation (64). If labour income risk could be completely insured, skills supply would be efficient. Complete insurance implies that each risky sector worker receives a income $w^I_{r,t+1}\left(a^i_t\right)$ that is independent of the realisation of the idiosyncratic shock and leaves the expected labour income $E\left(w_r\left(z^i_{t+1}, a^i_t\right)\right) = \int_z w_r\left(z^i_{t+1}, a^i_t\right) g\left(z\right) dz$ unchanged. This implies that under complete insurance, the labour income in the risky sector at time $t+1$ of a worker with ability a^i_t can be found by equating with the expected income without insurance

$$w^I_{r,t+1}\left(a^i_t\right) = q_k Z_1 \int_z w_r\left(z^i_{t+1}, a^i_t\right) g_1\left(z\right) dz + \left(1-q_k\right) Z_2 \int_z w_r\left(z^i_{t+1}, a^i_t\right) g_2\left(z\right) dz \quad (78)$$

$$= \left(q_k Z_1 \exp\frac{\sigma_1^2}{2} + \left(1-q_k\right) Z_2 \exp\frac{\sigma_2^2}{2}\right) \frac{\partial y_{t+1}}{\partial x_{r,t+1}} f_r\left(a^i_t\right) \exp\mu \quad (79)$$

Expected utility of the risky occupation if complete insurance were available is given by

$$E\left(U\left(c_{t+1}, e_{r,t}; a^i_t, I\right)\right) = -\frac{e_r\left(a^i_t\right)^{1-\gamma} - 1}{1-\gamma} + \frac{\left[\left(q_k Z_1 \exp\frac{\sigma_1^2}{2} + \left(1-q_k\right) Z_2 \exp\frac{\sigma_2^2}{2}\right) \frac{\partial y_{t+1}}{\partial x_{r,t+1}} f_r\left(a^i_t\right) \exp\mu\right]^{1-\gamma} - 1}{\left(1+\rho\right)\left(1-\gamma\right)} \quad (80)$$

Similarly to in the previous section, ability cut-off a^I_t when complete insurance is available is determined such that the person with ability a^I_t is indifferent between the risky and the safe occupation $E\left(U\left(c_{t+1}, e_{r,t}; a^I_t, I\right)\right) = E\left(U\left(c_{t+1}, e_{s,t}; a^I_t\right)\right)$. Thus, the following equation implicitly defines the ability cut-off level a^I_t

$$w^I_{r,t+1}\left(a^I_t\right) = \left[w_{s,t+1}\left(a^I_t\right)^{1-\gamma} - \left(1+\rho\right)\left[e_r\left(a^I_t\right)^{1-\gamma} - e_s\left(a^I_t\right)^{1-\gamma}\right]\right]^{\frac{1}{1-\gamma}} \quad (81)$$

such that labour income in the risky sector with complete insurance $w^I_{r,t+1}\left(a^I_t\right)$ is equal to the sum of the utility-weighted labour income in the safe sector and the utility-weighted difference in educational costs. This can be rewritten on the form

$$\left[\left(q_k Z_1 \exp\frac{\sigma_1^2}{2} + \left(1-q_k\right) Z_2 \exp\frac{\sigma_2^2}{2}\right) \frac{\partial y_{t+1}}{\partial x_{r,t+1}} f_r\left(a^I_t\right) \exp\mu\right]^{1-\gamma} - \left[\frac{\partial y_{t+1}}{\partial x_{s,t+1}} F_s'\left(a^I_t\right)\right]^{1-\gamma} - \left(1+\rho\right)\left[e_r\left(a^I_t\right)^{1-\gamma} - e_s\left(a\right.\right. \quad (82)$$

If educational costs are the same for both occupations $e_r\left(a^I_t\right) = e_s\left(a^I_t\right)$, the worker with cut-off ability a^I_t would receive the same labour income with complete insurance in the risky sector as he would in the safe sector $w^I_{r,t+1}\left(a^I_t\right) = w_{s,t+1}\left(a^I_t\right)$.

From the equations above, we see that

$$a_t^I = a\Big(\ \gamma, \quad \rho, \quad \mu, \quad \sigma_1, \quad \sigma_1, \quad q_k\ , Z_1, Z_2\Big) \tag{83}$$

The skills gap is thus equal to the difference between supply of risky sector workers under complete insurance and supply of risky sector workers when there are no insurance markets for labour income risk.

$$\begin{aligned} G_{t+1} &= r_{t+1}^I - r_{t+1}^* \\ &= a_t^*\left(\gamma, \rho, \mu, \sigma_1, \sigma_2, q_k, Z_1, Z_2\right) - a_t^I\left(\gamma, \rho, \mu, \sigma_1, \sigma_2, q_k, Z_1, Z_2\right) \\ &= G\left(\gamma, \rho, \mu, \sigma_1, \sigma_2, q_k, Z_1, Z_2\right) \quad \forall \quad k = 1, 2 \end{aligned} \tag{84}$$

6 Public policy

To be completed - discussion of progressive taxation and education subsidy.

7 Conclusion

Heterogeneity of earnings risk appear significant in the empirical literature, see Meghir and Pistaferri (2004), Saks and Shore (2005) and Blundell and Preston (1998). This paper has explored one such type of heterogeneity, namely sectoral or occupational heterogeneity. The paper has shown that skills gaps can be explained by individuals' risk aversion, the lack of insurance for income risk, and the uncertainty over future labour market outcomes when individuals make their education choices. The model gives the following results. When sector-specific shocks are aggregate, the size of skills gaps depends on the degree of risk aversion of individuals, the persistence and size of shocks, their discount rate and the differences in cost of types of education. When sector-specific shocks are idiosyncratic, the size of skills gaps depends on risk aversion, personal intertemporal discount rates, the mean and variance of the shocks. Skills gaps reflect market inefficiencies and highlight the scope for efficiency improvements through policy. Policy can also address distributional consequences of uninsurable sector-specific risk.

The results in this paper has implications for education policies, as it shows that simply increasing the capacity of the education system does not necessarily alleviate skills gaps. Also, it shows that increasing the number of study places for skills for which there are skills gaps does not secure increased supply of skills. Instead, policy makers need to focus on the incentives that lead individuals to choose different types of education. Policy prescriptions that may change incentives for high risk education investments is currently being explored in a further extension to this analysis.

The analysis can be extended in the following ways. A simple calibration of the model will give some indication of the actual size of skills gaps and the size of comparative statics effects. For the calibration, we can use data on labour income, allocation of the working population between educational and occupational choices and into sectors, in addition to assumptions on intertemporal discount rates and risk preferences. By subsequently using the calibration and assuming complete insurance, we can back out the size of the skills gap. We can also do sensitivity analysis with respect to choice of risk and time preference parameters and assumptions about production technologies.

In a separate paper, occupational choices in the presence of aggregate and idiosyncratic shocks when there is some mobility of workers between sectors is being analysed. Sectoral mobility functions as a form of insurance for sectoral labour income risk. This paper also studies efficiency-improvement effects and welfare effects of progressive taxation and scholarship provision. Innovative risk management methods as proposed by Shiller (2003), in particular livelihood insurance tied to occupational income profiles and intergenerational pooling of risk are highly relevant and could be explored in future work.

Capital markets can also be added to the models presented in the current paper. This allows for precautionary savings to partly alleviate skills gaps. The model would then benefit from allowing each generation to live for more periods. Furthermore, introduction of capital markets can allow for bequests and thus give interesting welfare and growth dynamics in the economy.

Extending the model to allow each generation to live for more than two periods will enable distinction between permanent and transitory shocks to labour income and thereby allow for insurance in the case of transitory idiosyncratic shocks. The distinction between permanent and transitory shocks is highlighted by both Storesletten et al (2004) and Meghir and Pistaferri (2004). Storesletten, Telmer and Yaron (2004) report that 44.5% of lifetime income uncertainty can be attributed to persistent labour income shocks, whereas 0.7% only can be attributed to transitory shocks. Meghir and Pistaferri (2004) conclude that earnings variance can be best characterised by separately identified permanent income shocks and serially correlated transitory shocks, thus violating the assumption of independently and identically distributed shocks across time. In the current model framework, which considers effects on lifetime income as each generation only works for one period, there are no transitory shocks. By extending lifetimes in the current model to many periods, it would be possible to capture transitory shocks too.

The model can be extended to examine cases where risk aversion and discount rates are heterogeneous. The consequences of other forms of labour income heterogeneity, e.g. age and cohort differences, could also be explored in further work.

References

[1] Acemoglu, D. (1996): Credit constraints, investment externalities and growth, in A. Booth and D. Snower (eds.), *Acquiring skills: market failures, their symptoms and policy responses*, Centre for Economic Policy Research, Cambridge University Press, Cambridge.

[2] Anderson, Steffen, Glenn Harrison, Morten I. Lau and Elisabet Rutström (2006): Eliciting Risk and Time Preferences, mimeo April 2006. Earlier version: CEBR Discussion Paper No. 2005-16, July 2005, Copenhagen, Denmark.

[3] Arcidiacono, P. (2004): Ability sorting and the returns to college major, Journal of Econometrics, 121, 343-375.

[4] Arrow, K. and W. Capron (1959): Dynamic Shortages and Price Rises: The Engineering-Science Case, *Quarterly Journal of Economics*, 73, 2, 292-308.

[5] Attanasio, Orazio P. and Steven J. Davis (1996): Relative Wage Movements and the Distribution of Consumption, *Journal of Political Economy*, pp. 1227-1262.

[6] Boadway, R., M. Marchand and P. Pestieau (1991): Optimal Linear Income Taxation in Models with Occupational Choice, *Journal of Public Economics*, 46, 133-162.

[7] Booth, A. and D. Snower (eds.) (1996): *Acquiring Skills: Market Failures, Their Symptoms and Policy Responses*, CEPR and Cambridge University Press.

[8] Card, David and Lemieux (2001): Can Falling Supply Explain the Rising Returns to College for Younger Men? A Cohort-Based Analysis, *The Quarterly Journal of Economics*, Vol. 116, No. 2. (May), pp. 705-746.

[9] Carlino, Gerald A. and Robert H. DeFina (2004): How strong is co-movement in employment over the business cycle? Evidence from state/sector data, Journal of Urban Economics 55 (2004) 298-315.

[10] Clemens,Christiane (2006): Entrepreneurship and Growth. - An overlapping Generations Approach, *Topics in Economic Analysis & Policy*, www.bepress.com/bejeap. *Forthcoming.*

[11] Cochrane, J. H., (1991), A simple test of consumption insurance, Journal of Political Economy 99, 957-76.

[12] Cooper, R. and J. Haltiwanger (1990): Inventories and the propagation of sectoral shocks, American Economic Review 80 (1990) 170-190.

[13] Dixit, A. and R. Rob (1994): Switching Costs and Sectoral Adjustments in General Equilibrium with Uninsured Risk, *Journal of Economic Theory*, 62, 48-69.

[14] Elston, Julie Ann, Glenn W. Harrison, and E. Elisabet Rutström (2006): Characterizing the Entrepreneur Using Field Experiments, Mimeo. University of Central Florida.

[15] Freeman, R. (1975): Supply and Salary Adjustments to the Changing Science Manpower Market, *American Economic Review*, 65, 27-39.

[16] Freeman, R. (1976): Legal Cobwebs: The Changing Market for Lawyers, *Review of Economics and Statistics*, 57, 171-179.

[17] Galor and Moav (2000): Ability-Biased Technological Transition, Wage Inequality, and Economic Growth, Quarterly Journal of Economics, 469-497.

[18] Garcia-Penalosa, C. and J. Wen (2005) Redistribution and Occupational Choice in a Schumpeterian Growth Model, CESifo Working Paper (2004). Revised Version- March 15, 2005.

[19] Gould, Moav, Weinberg (2001): Precautionary Demand for Education, Inequality and Technological Progress, Journal of Economic Growth, 6, 285-315.

[20] Hornstein, A (2000): The Business Cycle and Industry Comovement, Federal Reserve Bank of Richmond *Economic Quarterly* Volume 86/1 Winter 2000, pp. 27-48.

[21] Kanbur, S.M. (1979): Of Risk Taking and the Personal Distribution of Income, *Journal of Political Economy*, 87, 769-797.

[22] Kanbur, S.M. (1979): Impatience, Information and Risk Taking in a General Equilibrium Model of Occupational Choice, Review of Economic Studies, 46, 4, pp. 707-718.

[23] Kanbur, S.M. (1981): Risk Taking and Taxation: An Alternative Perspective, *Journal of Public Economics*, 15, 163-184.

[24] Krebs, T. (2003a): Human Capital Risk and Economic Growth, Quarterly Journal of Economics, May, 709-744.

[25] Krebs, T (2003b): Growth and welfare effects of business cycles in economies with idiosyncratic human capital risk, Review of Economic Dynamics, 6, 846-868.

[26] Long, J.B. and C.I. Plosser (1983): Real business cycles, Journal of Political Economy 91 (1983) 39-69.

[27] Long, J.B. and C.I. Plosser (1987): Sectoral versus aggregate shocks in the business cycle, American Economic Review 70 (1987) 333-336.

[28] Mace, B. J., (1991), Full insurance in the presence of aggregate uncertainty, Journal of Political Economy 99, 928-56.

[29] Meghir, Costas and Luigi Pistaferri (2004): Income Variance Dynamics and Heterogeneity, Econometrica, 72, 1, pp. 1-32.

[30] Orazem, P. and J. Mattila (1991): Human Capital, Uncertain Wage Distributions, and Occupational and Educational Choices, *International Economic Review*, 32, 1, Feb., 103-122.

[31] Poutvaara, P. (2002): Income Redistribution and Risky Occupational Choices, *Topics in Economic Analysis & Policy*, 2, 1, 8, www.bepress.com/bejeap.

[32] Romer, P. (2000). 'Should the Government Subsidize Supply or Demand in the Market for Scientists and Engineers?,' NBER Working Papers 7723, National Bureau of Economic Research.

[33] Saks, R. and S. Shore (2005): Risk and Career Choice, Advances in Economic Analysis & Policy, 5, 1, article 7, www.bepress.com.

[34] Shea, J. (2002): Complementarities and co-movements, Journal of Money, Credit and Banking 34, 412-433.

[35] Shiller, R. (2003): The New Financial Order: Risk in the 21st Century, Princeton University Press.

[36] Siow, A. (1984): Occupational Choice under Uncertainty, *Econometrica*, 52, 3, May, 631-645.

[37] Storesletten, Telmer and Yaron (2001): Asset....

[38] Storesletten, K, C. Telmer, and A. Yaron (2004): Cyclical Dynamics in Idiosyncratic Labor-Market Risk, *Journal of Political Economy*, vol. 112 (3), pp. 695-717.

[39] Ulph, D. (1996): Dynamic competition for market share and the failure of the market for skilled labor, in A. Booth and D. Snower (eds.), *Acquiring skills: market failures, their symptoms and policy responses*, Centre for Economic Policy Research, Cambridge University Press, Cambridge.

[40] Zarkin, G. (1985): Occupational Choice: An Application to the Market for Public School Teachers, *Quarterly Journal of Economics*, 100, May, 409-446.

[41] Wang, Y. (1994): Stationary Markov Equilibria in An OLG Model with Correlated Production Shocks, *International Economic Review*, 35, 3, Aug., 731-744..

[42] Wildasin, D. (2000): Labor-Market Integration, Investment in Risky Human Capital, and Fiscal Competition, *American Economic Review*, 90, 1, 73-95.

A Appendix: