The Rise of the Service Economy*

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Abstract

We present four facts and a model explaining the rise of the service economy. First, the rising share of services in output is a recent phenomenon, starting around the mid-20th century. Second, it reflects increases in both the relative price *and* relative quantity of services to commodities. Third, this rising share is entirely explained by the surge of skill-intensive services, and is contemporaneous with the increases in the relative quantity of skilled labor and the skill premium. Finally, individual services follow a distinct product cycle as an economy grows. They start being provided as market services, but are later produced at home with the purchase of manufactured intermediate inputs and durable goods. In our model, agents make decisions between the market and home provision over a continuum of wants that are satiated sequentially. The disutility of public consumption and economies of scale (in the use of specialized capital and skills) are the key elements explaining the rich dynamics of the service economy. If skilled labor has a comparative advantage in the production of newer services, the theory explains the late rise in the service economy characterized by rising relative prices and quantities of services, and growth in the relative quantity of skilled labor and the skill premium.

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1 Introduction

Four important facts characterize the rise of the service economy. First, it is a recent phenomenon. As measured by its share in GDP, services were roughly constant from 1870 till the mid-20th century, but since then have risen from less than sixty to eighty percent of GDP. Second, the rising nominal share of services in output is driven by increases in both the price and quantity of services relative to commodities. Third, this increase in the share of services is entirely explained by the growth of skill-intensive services, and is contemporaneous with an increasing relative quantity of skilled labor and a rising skill premium. The output of high-skill industries increases by more than twenty-five percent, while the share of low-skill industries actually declines. Over the same period, the wages of college-educated workers rose from 125 percent to more than twice the wage of high school-educated workers, and the fraction of workers who were college-educated rose from just fifteen percent to over sixty percent. The fourth relevant fact, which we establish, is that many of the declining individual services have actually transitioned from being market-purchased to home-produced. Examples of this product cycle include: laundry; dairy, ice and other refrigeration services; street cars and buses; taxis; and motion pictures. These services utilize market-purchased goods as specialized intermediate inputs or durables into home production.[1]

The theory we develop endogenizes this transition from market to home production of services, and is consistent with the relevant facts. We model the home vs. market decision for a particular want as depending on: the disutility of public consumption; the costs of intermediate goods and the benefits of economies of scale/higher utilization of these intermediates available on the market; the opportunity cost of transaction time (market purchases) vs. labor time (home production); and the importance of skilled labor in production. We focus on the importance of the disutility of public consumption, economies of scale, and skill-intensity in explaining the secular time trends.

Beyond this theory of services, the growth model includes two ingredients

[1] Of these facts, only the first two were recognized by a subset of the literature. Lee and Wolpin (2006) make passing mention of the fact that the relative price and relative quantity of services has grown. Kuznets (1957) discussed the lack of a clear relationship between development and the share of service in output before the 50's, while he, as well as Stigler (1956), documented the secular rise in the share of services in employment. With a few exceptions (e.g., Kuznets (1973)) most of the literature overlooked the late nature of the rise in the service economy (e.g., Maddison (1987)).

that are key in reproducing the stylized facts. First, agents order their wants, and then satiate these desires sequentially. Labor productivity grows at an exogenous constant rate, and as income rises, the consumption set expands to ever more luxury services. Wants are initially satisfied through market services because market production has the advantage of economies of scale in the utilization of intermediate goods, and goods costs are relatively more important at lower incomes. Given the disutility of public consumption, at higher incomes service production moves into the home. The simple model yields (temporarily) balanced growth in which the service sector produces a constant share of output, and services follow the market-to-home product cycle.

Second, we assume that agents have skill-biased Engel curves. That is, luxury services are ever more complex services in which specialized skilled workers hold a comparative advantage. Since the acquisition of any particular skill entails paying a fixed cost, agents will choose to attain specialized skills in the production of only one service. Agents will work as highly productive skilled workers on the market, which increases utilization of the skill, but will be less skilled in the broad range of services potentially produced at home.[2] Hence, skill-intensive services have a bias toward market provision, and consequently have slower product cycles. As incomes rise, the service sector expands, the demand for specialized skill increases, investment and returns to skill increase, and the composition of the service sector shifts toward skill-intensive services. Since the service sector is more skill-intensive, the rising returns to skill cause the relative price of services to rise over time. The takeoff of this rising service economy occurs, when incomes are high enough for agents to demand services in which skilled workers have an absolute advantage in production.[3]

Our work touches on a number of literatures. There is a vast literature related to the subject of structural change, for which we provide a (very) incomplete summary. Earlier discussions of the facts and explanations for the changes in the structure of production include Clark (1941), Stigler (1956), Kuznets (1957), Baumol (1967), Chenery and Syrquin (1975), Fuchs (1980), Kravis et. al. (1984), and Maddison (1987). They posited that a combi-

[2]More formally, if specialized, they will only be producing a measure zero set of home services with a skilled technology.

[3]We view college education as producing specialized skills, whereas elementary and high school education produce general skills. We do not explicitly model the investment in general skills.

nation of Engel's law and biased productivity rates were important in explaining labor shifts across sectors. Recent important contributions focusing on these labor market shifts include Rogerson (2005) and Lee and Wolpin (2006). Another recent literature that attempts to explain long run structural change within models that are consistent with Kaldor facts includes: Kongsamut, Rebelo and Xie (2001), Acemoglu and Guerrieri (2005), Foellmi and Zweimuller (2005), Ngai and Pissarides (2005), and Buera and Kaboski (2006). In linking structural change to the rising demand for schooling, our model relates to Kaboski (2006), who focuses on the growth effects of schooling within a model of skill-biased technical change and structural transformation. Our model posits a potential explanation for the important relationship between the relative price of manufactured goods (capital, tradables) and income which has been observed and studied in both the time series and cross-section (e.g., Alessandria and Kaboski, 2006, Greenwood, Hercowitz, and Krusell, 1997, Hsieh and Klenow, 2005). Finally, our modeling of home production draws mainly from the earlier analysis by Becker (1965), while our non-homothetic preferences follow Matsuyama (2000, 2002). Hall and Jones (forthcoming) provide an important contribution in explaining the underlying non-homotheticity for one important area of consumption: healthcare.[4] We instead take the non-homotheticic preferences as given, focusing on their implications rather than their origins.

The paper is organized as follows. Section 2 motivates the need for a theory of services, and presents the evidence for product cycles. We develop a model of the home production vs. market production margin in Section 3, and highlight some of the driving forces at work. Section 4 embeds a simplified version of this model into a growth economy. In Section 5, we characterize this model for the balanced growth case in which skilled labor holds an absolute advantage in all services, and show the driving forces behind the product cycle. The assumption of comparative advantage in Section 6 yields the rise of the service economy and the observed trend in relative prices. Section 7 provides a few interesting extensions, and Section 8 concludes.

[4]For other important growing service industries (finance and education), demand could plausibly be increasing in life expectancy.

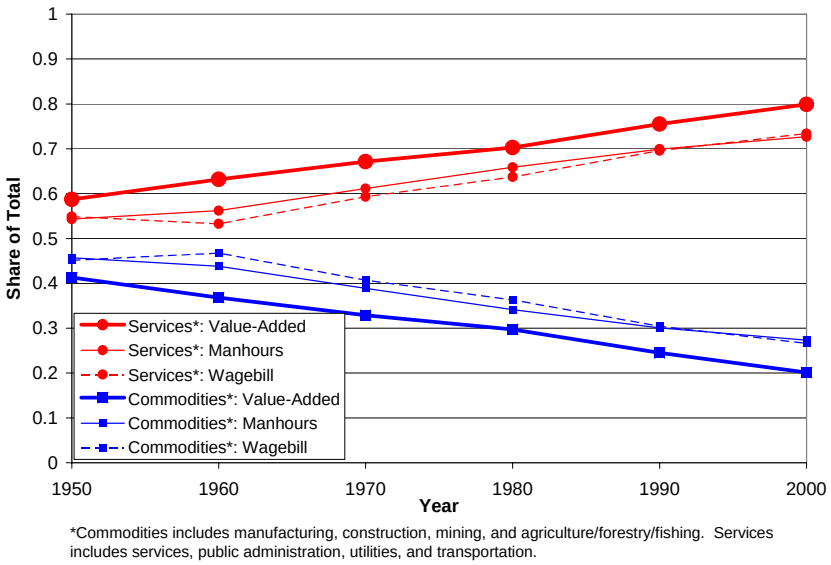


Figure 1: Post-1950 Rise in Services Relative to Commodities

2 Empirical Evidence

In this section we explore the facts associated with the rise of the service economy and briefly discuss the standard explanations for the service-related structural change observed in modern economies, pointing out their failure to account for the observed facts and the need for a theory of services.

Figure 1 shows what we refer to as rise of the service economy after 1950 in the United States. In 1950, the service and commodity sectors are comparable, but by 2000 the service sector is about three times the size of the commodity sector. This post-1950 transformation occurs whether measured as fractions of output, labor employed, or total labor payments in the economy.[5] We have used a broad measure of services (including government, public utilities and transportation along with services), but the same substantial trend exists in the more narrow concept of services as well, and the

[5]Buera and Kaboski (2006) present both time series and cross-sectional evidence of a strong relationship between income and output per worker in different sectors at lower income levels. Since effective labor may differ substantially from raw workers (both in theory and empirically), we argue that output is the variable of most interest.

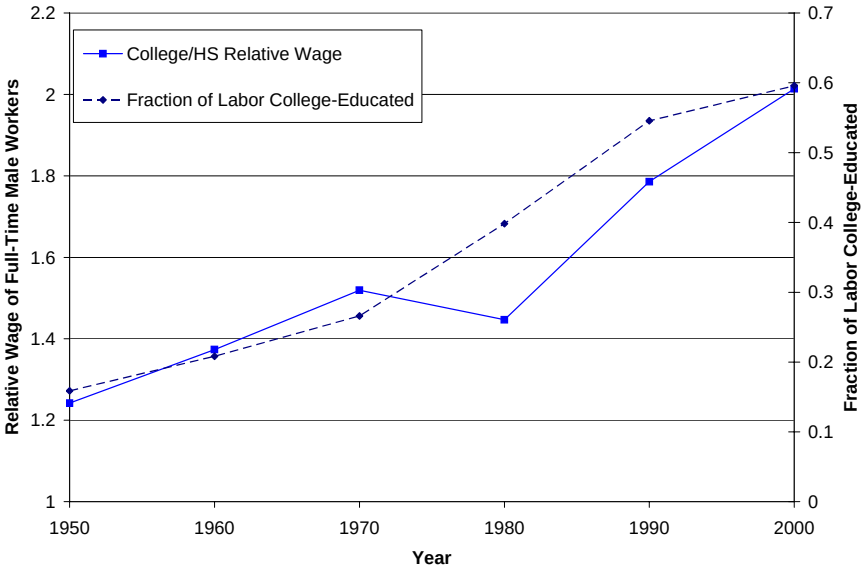


Figure 2: Rise in Relative Wage and Supply of College-Educated Labor

decline in commodities exists for each of the commodity sectors (agriculture, manufacturing, mining, and construction).

While the focus of the paper is to explain the rise in services, economists and policymakers often draw a link between the rise in the service economy, and the growing disparity between high and low-skilled workers over the same period. Figure 2 shows an example of this disparity using high school- and college-educated workers as measures of low- and high-skill, respectively. The wage of college-educated workers rose from 125 percent of the high school-educated wage in 1950 to over 200 percent by 2000. At the same time, the ratio of college- to high school-educated labor in the workforce rose from about fifteen percent to sixty percent. Existing theories do not establish a link between the trends in Figure 1 and Figure 2.

2.1 New Set of Facts

The rising importance of the service economy and the trends in returns to skill are well-known. What is interesting is that they are only recent phenomena, which is our first stylized fact. Figure 3 illustrates 130 years of the

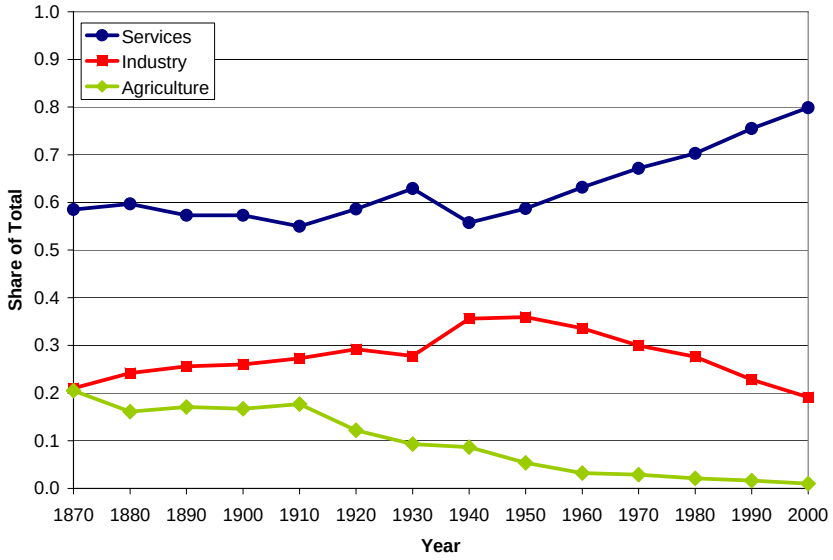


Figure 3: Late Rise of the Service Economy

structural transformation of the US economy between the primary sectors of agriculture, industry, and services. While the decline in agriculture is present even at the beginning of this period, the share of output in services is nearly constant between 1870 and 1950, and rises only during the last fifty years. If government output is omitted from the data for services in Figure 3, the flat share of the service sector before 1950 is even more striking.[6] Similarly, the evidence suggests that, if anything, the skill premium fell over the first half of the twentieth century.[7]

The second stylized fact is that the rise in services reflects increases in both the relative price *and* relative quantity of services. These trends are illustrated in Figure 4, which plots the growth in the *nominal* output of services relative to commodities, and decomposes it into the growth in the relative price of services (as measured), and the growth in the relative *real* quan-

[6]The raw labor numbers differ substantially from output numbers, with agriculture showing a much steeper decline and services showing a much more gradual increase in labor. See Kuznets (1957) for an earlier discussion of these distinct trends.

[7]For a review of the evidence and for the most comprehensive time series see Goldin and Katz (1999a).

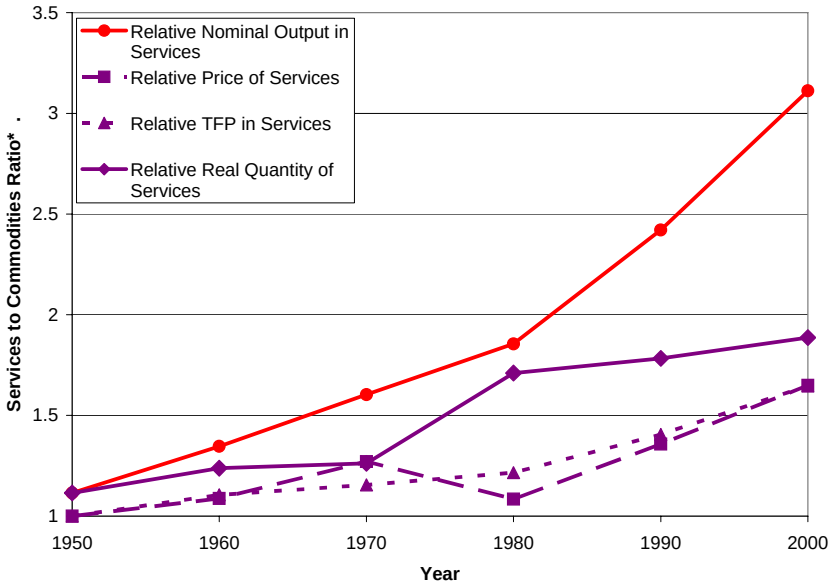


Figure 4: Increase in Relative Price and Quantity of Services to Commodities

tity of services (after deflating).[8] All three trends show substantial growth. Finally, we also present the time series of relative TFP (manufacturing to services) which is strongly related to the relative price of services in many models (e.g. Hsieh and Klenow, 2005) and also in the data.

A third stylized fact is the compositional change in the service sector toward skill-intensive industries. Figure 5 provides evidence of this claim by plotting the absolute change in the labor income share of different service industries between 1950 and 2000 against the skill-intensity of the industry (measured as the fraction of workers with college-education in 2000).[9] The absolute importance of each industry to the total growth in services is its vertical distance from the zero growth line, and the growth in services is driven by more than just a few industries. The skill-intensive service industries

[8]The price data used to construct the relative price and quantity series fluctuates considerably at high frequency. We have smoothed using an eleven-year moving average to show the long run trends of interest.

[9]The detailed industry data comes from IPUMS census. Therefore, output shares are not available at this detailed level. Using manhours instead of labor income yields a very similar picture.

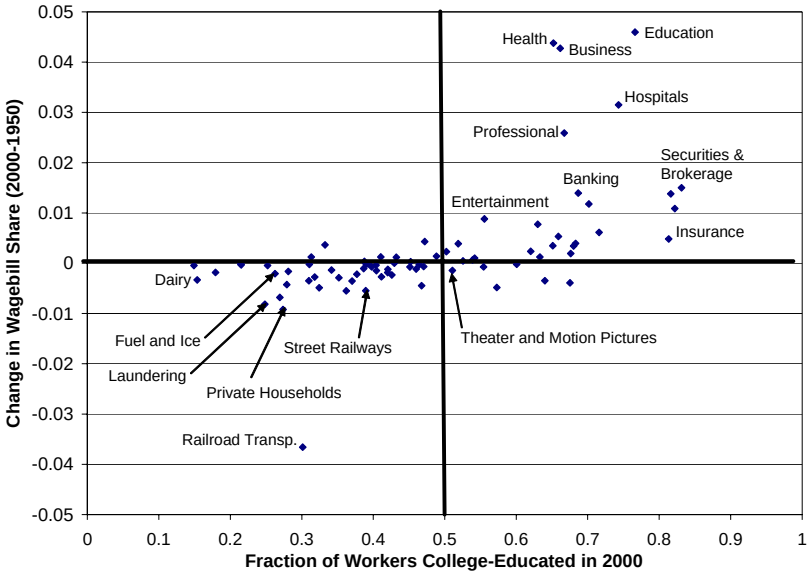


Figure 5: Growth vs. Skill Intensity of Individual Service Industries

tended to grow, while the less-skilled service industries actually contracted. The aggregate importance of this industry-level relationship between growth and skill intensity relationship is illustrated in Figure 6. Figure 6 separates the growth in services in Figure 1 into the contributions of high- and low-skill industries. We categorize broad industries in which at least half of workers are college-educated as high-skill industries.[10] Clearly, the rise of the service economy has been completely driven by high-skill industries, while the importance of low-skill service industries in the overall economy has actually declined.

The fourth stylized fact involves the particular industries which have tended to grow and contract. The growing services are industries with a

[10] The fifty percent cutoff in 2000 is not completely arbitrary. The relative skill intensities of these broad industries are fairly stable over time. As Figure 5 shows, it divided the output of the service sector almost equally in 1950. There is also a large break at this point. The cutoff industries were wholesale trade (low-skill, 47% college-educated) and entertainment and recreational services (high-skill, 56% college-educated). Because this classification relies on classifying broader, more aggregate industries than those in Figure 5, the sum of the vertical distances is not exactly equal to the difference presented in 6.

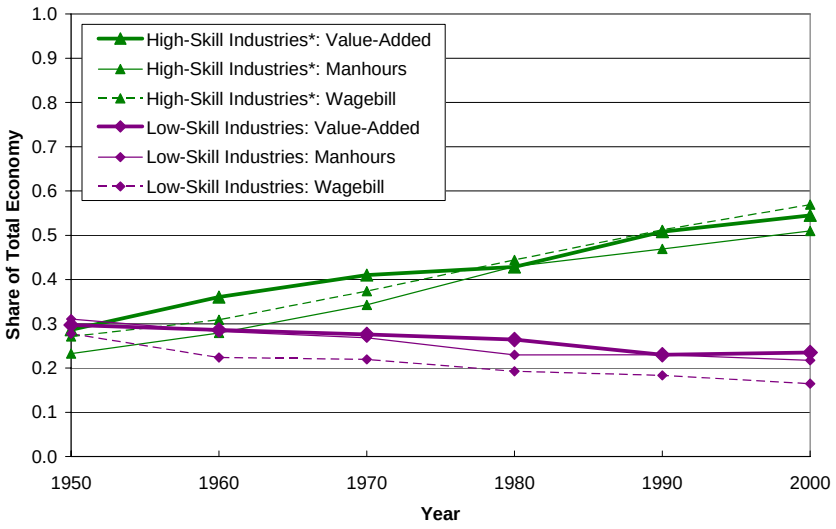


Figure 6: Growth of Low- and High-Skill Services

high-degree of specialized skilled-labor, including education (especially higher education), health care, legal services, banking, real estate and accounting, broadcasting and television, and air transportation. In contrast, the declining industries are less-skilled industries with home production substitutes, such as railroads, private household workers, laundering, railways and bus lines, food stores, dairy retailing, fuel and ice retailing, auto repair, postal service, taxicab service, and theaters and motion pictures. Together, these industries constitute nearly three-quarter of the total decline in all low-skill industries (See Appendix A).[11] Moreover, their decline is contemporaneous with the spread of durable goods that are used as inputs in the home production of these services**.** We return to this evidence in Section 2.3, to establish the importance of the market-to-home product cycle.

[11] We have omitted postal service (56 percent college graduates) and theaters and motion pictures (51 percent college graduates) from this number, since these are technically high skilled in 2000 according to our fifty-percent definition.

2.2 Traditional Explanations

Three traditional explanations have been recently formalized in attempt to explain the growth of the service economy: sector-biased technological progress with low elasticity of substitution across sectors (e.g. Ngai and Pissarides, 2005, Acemoglu and Guerreri, 2005);[12] non-homothetic preferences (e.g., Rebelo, Kongsamut and Xie, 2000); a secular marketization of previously home-produced services (e.g., Rogerson (2005)).[13] We compare each of these explanations to the relevant empirical facts.

At first glance, the first explanation may appear consistent with Fact 2. As Figure 4 showed, measured productivity growth has been slower in services than manufacturing, and the price of services relative to commodities has grown markedly since the postwar period. A sufficiently low elasticity of substitution might explain the increasing share of labor in services. However, a substitution story alone cannot account for the increase in the relative quantities of services in response to its rising relative price.[14] Such a response requires an income effect coming from non-homothetic preferences.

The late onset of the service economy (Fact 1) is particularly damaging for existing explanations involving non-homothetic preferences and Engel's Law, however. In particular, the late rise in services cannot be driven by an initial endowment of services in Stone-Geary type preferences (e.g., Rebelo, Kongsamut and Xie, 2000), since these endowments are most important to structural transformation at lower incomes. Instead it leads us to consider non-homothetic preferences a la Matsuyama (2000, 2002), in which demands are satisfied sequentially (e.g., first food, clothing, shelter, and later education, health care, and banking).

The compositional change in services (Fact 3) is important in understand-

[12] Acemoglu and Guerrieri (2005) features neutral technological progress, but capital accumulation and differences in capital shares across sectors imply a rise in the relative price of services.

[13] Clark (1941), Kuznets (1957), and Baumol (1967) are early citations for these traditional explanations applied to the structural transformation of *labor* across sectors. See Matsuyama (2005) for a review of these traditional explanations of structural change. Buera and Kaboski (2006) explore the quantitative performace of these theories.

[14] Clark (1944) proposed the earliest hypothesis of structural transformation caused by slower productivity growth in the service sector. Since the output share of services had not yet risen, he was interested in only explaining the rise in the service sector's share of the workforce. Biased productivity growth models can potentially explain an ever rising fraction of the workforce in services, but not the growth in relative real quantities observed after 1950.

ing the importance of the market purchase vs. home production margin for the rise in services. In particular, it indicates that the rise in services is not likely the result of a shift from home production of services to market production, a phenomenon stressed by Rogerson (2005). We follow Rogerson (2005), however, in emphasizing the home vs. market production margin.

2.3 Product Cycles and the Need for a Theory of Services

Indeed, the evidence suggests that the product cycle of services, relevant for understanding the recent period at least, involves wants initially being satisfied through market services and later transitioning to home production. Important examples in the data are illustrated in Figure 7, which shows the rise and fall of market services over time and the corresponding diffusion of the relevant commodities.[15]

The top panel graphs the growth and decline of important transportation services (i.e., railways, trolleys, and buses) on the left axis, and the spread of the stock of registered automobiles (i.e., all vehicles and just cars) on the right axis. Because the three services are substitutes for one another, the dynamics of individual service industries are complicated, but clearly the spread of the automobile has played an important role in the decline of all three.

The middle panel similarly displays the dynamics of market services for "domestic work" (i.e., laundering and dry cleaning, domestic servants) together with the diffusion of vacuums, laundry washers, dryers, and dishwashers. Laundering and dry cleaning services show a rise and then fall that corresponds to the spread of the washer and dryer. Domestic servants shows a continuous decline, but seems to also be related to the spread of these household items. Indeed Greenwood et. al. (2005), from whom we take the data on household items, also draw a link between these trends.

Finally, the third panel plots the rise and fall of retail food, drinking, ice and fuel service establishments, and the corresponding expansion of important household amenities: private refrigerators, freezers, electricity and

[15] The model we present gives complicated dynamics for the value of output because of price effects, so instead we count real services and establishments. Related, in the graphs the commodities that diffuse are durables, so we choose measures related to the stocks of these items instead of the flows.

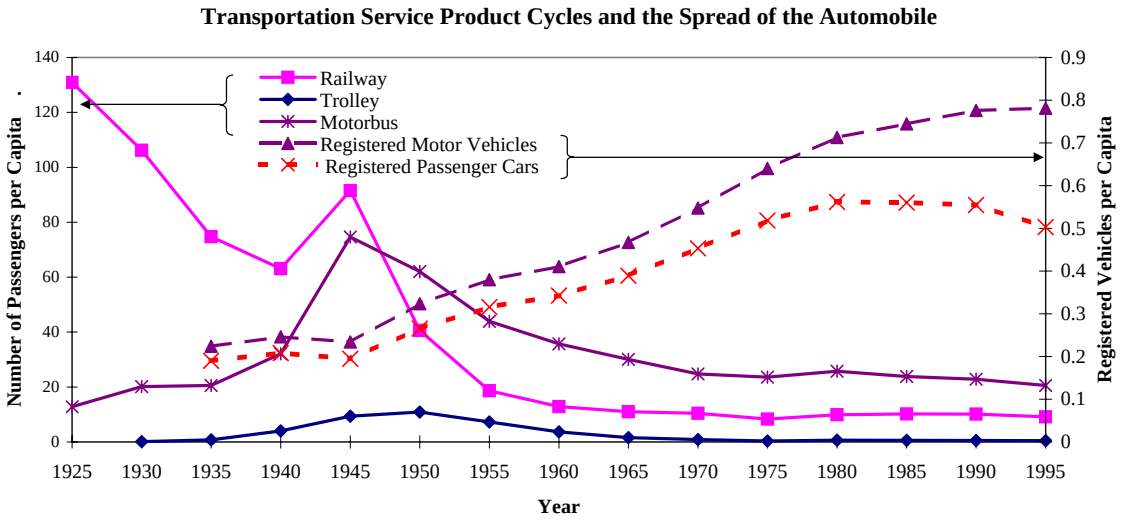
central heating. We argue that the expansion of refrigerators and freezers has moved these services from market production to home production, eliminating the need for frequent shopping, neighborhood grocery stores, and milk, ice and fuel delivery services.

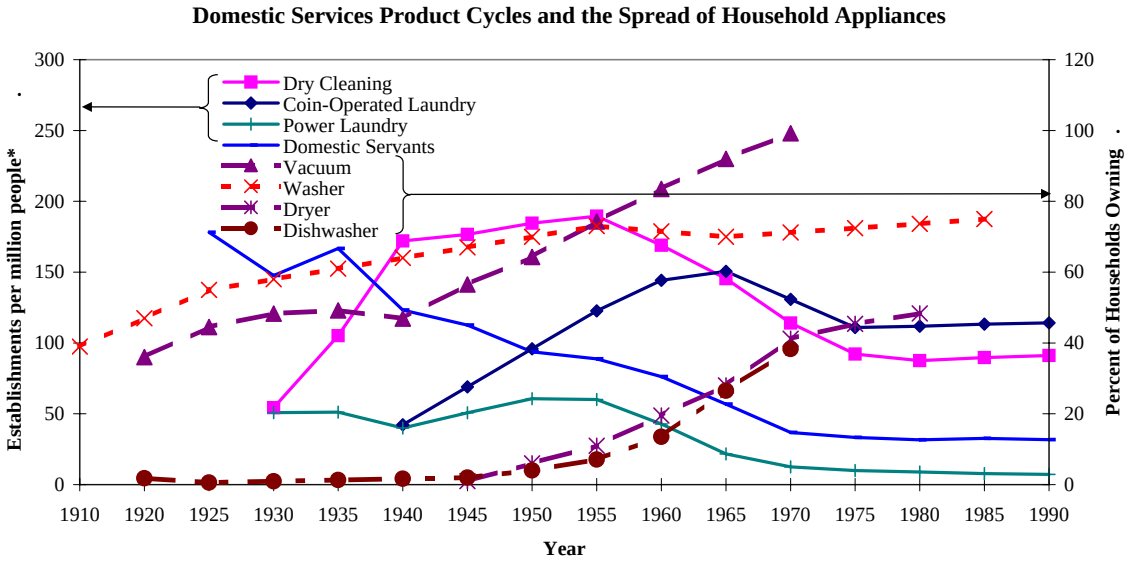
While there are obviously also examples of transitions from home production to market services, we argue that the tendency has been from market to home (again, such industries account for three-quarters of the total decline of declining service industries). The movements certainly underscore the need for a theory of services. As an example, Figure 8 provides a list of wants that can be met either through market services, or through home production and the purchase of an important manufactured good. The first item, casual attire is never provided as a market service, but the second item, formal attire, is often provided as a market service. Items 3-9 are increasingly provided through home production. Indeed, all of these items are associated with low-skill industries in that are declining in the data. In contrast, item 10, air transportation is rarely provided as home production and is associated with a growing industry. Item 11, long distance communication, is the counter-item to casual attire. Although one could conceive of it being home produced, it is exclusively provided as a market service. Item 12, internet surfing, is an item that is consumed on the market by relatively low income people, while item 13, oil changes, are purchased on the market by relatively high income people. Items 14-15 are all produced in small amounts at home, but, in contrast to items 3-9, do not seem to be experiencing a transition toward home. Like long distance communication, surgery (item 16) is another item that is not home produced, but for obviously very different reasons.

There are many important questions requiring a theory of services. Why are some services done on the market, while others are done at home? Why do we see many of the above low-skill services transitioning from market production to home production? How do these transitions affect the market for manufactured goods? The examples in Figure 8 give immediate insight into the important driving factors. The next section develops a model of the market service vs. home production decision to formalize these issues.

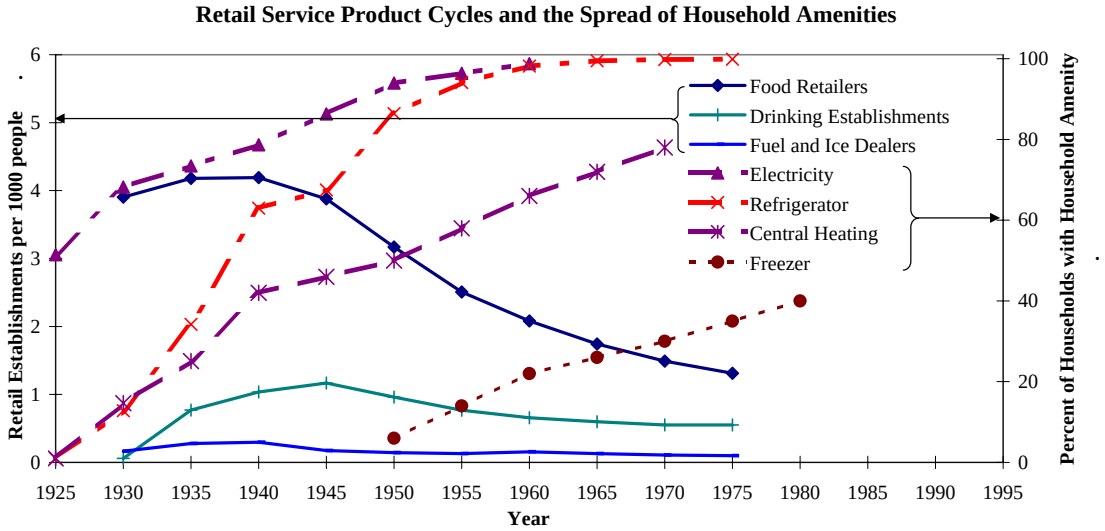
3 Theory of Services: Simple Example

In this section, we model the home vs. market production decision for a single want of an agent with a demand for one unit. Following the examples





*Domestic servants establishments are arbitrarily normalized to ten workers to fit the scale.



Sources: Historical Statistics of the U.S., (2006) [establishments, passengers, and registered vehicles]
Greenwood, Seshadri, and Yorukoglu (2001) [household appliances and amenities]

Figure 7: Important Examples of Product Cycles

	Item	Market Service	Alternative Home Production
1	Casual attire	Blue jeans rental	Blue jeans purchase
2	Formal attire	Tuxedo rental	Tuxedo purchase
3	Keeping food/milk fresh	Local store refrigeration/ Milk delivery	Refrigerator purchase
4	Keeping foods frozen	Ice delivery	Freezer/icemaker purchase
5	Laundry	Laundromat/laundry service	Laundry machine purchase
6	Housework	Cleaning service	Vacuum & dishwasher purchase
7	Ground transportation	Bus, train, or cab	Car purchase
8	Sending messages	Postal service	Fax/e-mail computer purchase
9	Movie watching	Movie theater	DVD player/projection TV purchase
10	Air transportation	Commercial airline	Private jet purchase
11	Long distance communication	Long distance service	Fiber optic network purchase
12	Internet surfing	Internet café	Computer/modem purchase
13	Changing oil	Oil change shop	Wrench, filter & oil purchase
14	Hair cut	Barber shop	Scissors/shears purchase
15	Home sale	Realty agency	"For Sale" sign purchase
16	Surgery	Hospital	Scalpel purchase

Figure 8: Market Service vs. Home Production

in Figure 8, we assume that the production of services (y^s) requires the use of q units of manufactured goods (k^s) in combination with either low (l^s) and/or high-skill labor (h^s) to produce a maximum of $n > 1$ units of a service at a constant marginal cost:

$$y^s = \begin{cases} 0 & \text{if } k^s < q \\ \min\left\{n, T\left[Al^s + Bh^s\right]\right\} & \text{if } k^s \geq q \end{cases} \tag{1}$$

A simple example would be a washing machine that can do a maximum number of loads of laundry a day, with a certain amount of labor required for each load. Since the amount consumed by the agent is one unit, $1/n$ captures the utilization rate at home. T is a skill-neutral productivity parameter, while A and B are low- and high-skilled labor specific productivities, respectively.

We assume for simplicity that services can be produced either at home or on the market, while manufactured goods are only produced on the market. We also simplify by assuming that labor is the only input into manufactured

goods production:[16]

$$y^m = T\left(Al^m + Bh^m\right) \tag{2}$$

In equilibrium, $y^m = k^s$.

Given these production functions we focus on the cost differences in home and market production.

We assume that high-skilled labor is very specialized, so that high-skill workers are only high-skilled in the production of one type of services. Even high-skill workers have productivity B in only one of many potential services. (In Section 4, we will assume a continuum of services on which any specialty has measure zero.) For simplicity here, we assume that the agent has low-skilled productivity at the particular service in question. Whether a market service is produced using low-skilled workers or specialized high-skill workers will depend on comparative advantage as defined by the ratio of their wages w relative to their productivities B/A.

As long as the population is large relative to the efficient scale of production n, free entry will drive prices of market services to their minimum average cost. Using low-skilled labor as the numeraire:

$$p_s = \underbrace{\frac{1}{T}\min\left\{\frac{1}{A}, \frac{w}{B}\right\}}_{\text{labor requirements}} + \underbrace{\frac{q}{Tn}\min\left\{\frac{1}{A}, \frac{w}{B}\right\}}_{\text{goods requirements}}$$

Similarly, for manufactured goods:

$$p_m = \frac{q}{T}\min\left\{\frac{1}{A}, \frac{w}{B}\right\}$$

In addition, agents face transaction costs of purchasing on the market that are proportionate to home labor time, $\frac{c}{TA}$.[17]

An agent with education e, $e \in \{l, h\}$, has one unit of potential labor, a wage w_e, and unearned income y. The agent faces a discrete decision problem

[16] This could be derived from a parallel treatment of manufacturing goods production:

$$y^m = \begin{cases} 0 & \text{if } k^m < q \\ \min\left\{n, \left[Al^m + Bh^m\right]\right\} & \text{if } k^m \geq q \end{cases}$$

Thus, (2) would arise as the limiting expression as $q, n \to \infty$ and $q/n \to 0$. We have implicitly assumed that the minimum scale of manufacturing is quite large, but the returns to scale in production are even larger.

[17] We assume that transaction costs for manufactured goods are negligible.

of whether to consume service, $\chi = 1$, and, if so, whether to home produce, $H = 1$, or to hire a service on the market, $H = 0$. In addition, agents have a constant marginal utility of unspent income x. Formally,

$$\max_{\chi, H \in \{0,1\},\, x} \chi \left[H + (1 - H)\, \delta \right] + \mu x$$

s.t.

$$\chi \left[H \left(p_m + \frac{w_e}{TA} \right) + (1 - H) \left(p_s + w_e \frac{c}{TA} \right) \right] + x \leq w_e + y$$

where μ gives the marginal utility of income. A home produced service has two benefits over market produced. First, since $\delta \in (0, 1)$ home production yields more utility, perhaps because it avoids the disutility of public consumption (e.g., sitting next to others on the bus instead of driving your own car). Second, it avoids the transaction cost of going to the market. A service will be provided by the market iff:

$$\overbrace{(\delta - 1)}^{\text{disutility of joint consumption}} \quad \overbrace{+ \mu \frac{q}{T} \left(1 - \frac{1}{n} \right) \min \left\{ \frac{1}{A}, \frac{w}{B} \right\}}^{\text{scale economies}}$$
$$\underbrace{+ \mu \frac{w_e}{TA} - \mu \frac{1}{T} \min \left\{ \frac{1}{A}, \frac{w}{B} \right\} - \mu c \frac{w_e}{TA}}_{\text{labor cost differential}} \geq 0 \qquad (3)$$

Equation (3) shows the relevant trade-offs. The first term is the difference in direct utility between home and market consumption. This term is negative given the preference for home production. The second term is the utility value of the difference in the costs of manufactured goods used in the production. This term will typically be positive because market production has the benefit of higher scale/utilization of specialized manufactured goods. The third term is the utility value of the difference in labor time costs devoted toward production of the service. The labor time cost of market consumption includes not only the service worker's time, but also the transaction time of the agent. The sign of this term depends on combinations of parameter values: the skill advantage of market production (if $w < \frac{B}{A}$), the opportunity cost w_e, and the transaction cost c.

One can also see the effect of T. At higher T, any potential cost savings in the market is smaller, while the utility cost is independent of T. This benefits is relatively more important for high T and therefore making home production relatively more attractive.

We can define the critical level of productivity T^*, such that the agent is indifferent between market purchase and home production:

$$T^* = \frac{\mu \left[w_e \left(\frac{1}{A} - \frac{c}{A} \right) + \left[\left(1 - \frac{1}{n} \right) q - 1 \right] \min \left\{ \frac{1}{A}, \frac{w}{B} \right\} \right]}{1 - \delta} \tag{4}$$

Simple comparative statics leads to the following proposition:

Proposition 1 *The minimum technology level for home production T^* is:*

1. *decreasing in the transaction costs c and increasing in the utility of market service consumption δ, the manufactured goods requirement q, and the efficient scale of production, n;*

2. *increasing (decreasing) in the agent's wage w_e if the home production time is greater (less) than the market transaction time;*

3. *increasing in the wage of high-skilled workers (w) and decreasing in their relative productivity (i.e., B/A) if high-skill workers have the comparative advantage in production of the service (i.e., $w < \frac{B}{A}$).*

This proposition is useful in organizing and understanding the items discussed earlier in Figure 8 . We have highlighted the role of T^* in the home vs. market decision, since it explains the market to home product cycle as productivity rises, but one can see that the decisions for particular items depend on different combinations of relevant factors (i.e., disutility of market service consumption, the size/cost of required manufactured goods, returns to scale and utilization, the opportunity cost of time of the consumer, and the skill-intensity). Thus, the many factors in the model above allow it to explain a potentially wide range of observations on particular services. In the remaining sections, however, our focus is not on the cross-sectional heterogeneity in consumption wants, but on the secular trends presented in the previous section. We therefore imbed a simplified version of the above model into a dynamic setting to explain the relevant facts. In particular, we focus on the roles of n, δ, and B/A in generating product cycles and the rising importance and changing composition of services.

4 A Model of Services and Economic Growth

In this section we extend the model by introducing a continuum of discrete wants or services indexed by their complexity z, $z \in \mathbb{R}$, in a continuous time overlapping generations model. For simplicity, we assume that generations live for an arbitrarily small interval of time. The continuum of wants and the preferences are similar to Matsuyama (2000, 2002).[18]

4.1 Preferences

Let the function $\chi(z) : \mathbb{R} \to \{0,1\}$ indicates whether a particular want is being satisfied. Wants can either be satisfied by hiring a service from a market provider or by purchasing the required manufactured goods and providing the service at home. Define the function $H(z) : \mathbb{R} \to \{0,1\}$, which takes the value 1 if want z is satisfied by home production and 0 otherwise. Together the set of indicator functions mapping $\mathbb{R}$ into $\{0,1\}^2$ define the consumption set. Preferences over wants and the method of satisfying those wants (i.e., over indicator functions $(\chi(z), H(z))$) are represented by the following utility function:

$$\tilde{u}(\chi, H) = \int_{z_0}^{+\infty} [H(z) + \delta(1 - H(z))] \chi(z) \, dz + z_0$$

where wants $z \leq z_0$ correspond to basic needs that must always be satisfied and z_0 is an arbitrarily small constant.

For expositional purposes and without loss of generality (as will become clear when discussing the production technology), the consumer's problem can be restricted to the choice over the restricted consumption set defined by step functions of the type:

$$\chi(z) = \begin{cases} 1 \text{ if } z \leq \bar{z} \\ 0 \text{ if } z > \bar{z} \end{cases}$$

and

$$H(z) = \begin{cases} 1 \text{ if } z \leq \underline{z} \\ 0 \text{ if } z > \underline{z} \end{cases}$$

[18] As discussed in Matsuyama (2002), an alternative interpretation of z is the place that a particular want occupies in the ordering of wants. Lower z corresponding to more basic wants that are satisfied first.

where $\bar{z}$ denotes the most complex want that is satisfied, and $\underline{z}$ denotes the most complex want that is home-produced.

Preferences over the restricted consumption set can then be represented as a utility function over two thresholds $\underline{z}$ and $\bar{z}$:

$$u\left(\underline{z},\bar{z}\right)=\begin{cases} -\infty & \text{if } \bar{z}<z_0 \\ \underline{z}\left(1-\delta\right)+\delta\bar{z} & \text{if } \bar{z}\geq z_0 \end{cases}$$

4.2 Technologies

We extend the technologies introduced in Section 3 by allowing for constant total factor productivity growth at rate γ, and by assuming that low (high) skilled labor requirements for producing more complex services and goods increase at a constant rate λ^l (λ^h). Formally:

$$y^s\left(z,t\right)=\begin{cases} 0 & \text{if } k^s<1 \\ \min\left\{n,e^{\gamma t}\left[Ae^{-\lambda^l z}l^s\left(z,t\right)+Be^{-\lambda^h z}h^s\left(z,t\right)\right]\right\} & \text{if } k^s\geq 1 \end{cases}$$

We consider the case $\lambda^l\geq\lambda^h$. If $\lambda^l>\lambda^h$, high-skill labor has a comparative advantage in the production of more complex services. Figure 9 illustrates the average cost function associated with this technology when skilled labor is employed.

Again, for simplicity, we assume that intermediate goods specific to the production of z services are manufactured on the market with a linear technology:

$$y^m\left(z,t\right)=e^{\gamma t}\left[Ae^{-\lambda^l z}l^m\left(z,t\right)+Be^{-\lambda^h z}h^m\left(z,t\right)\right]$$

4.3 Schooling

Individuals can become specialized workers by spending a fraction θ of their time endowment acquiring skills. The population is heterogeneous with respect to the time required to acquire specialized skills. In particular, $\theta\in[0,1]$ is distributed according to the c.d.f. $F\left(\theta\right)$.

As long as $F\left(0\right)=0$ individuals can specialize in at most a finite number of services. Thus, given that individuals consume a continuum of them, only a measure zero of home-produced services will be done using high-skilled labor and, therefore, individuals will choose to specialized in at most one service.

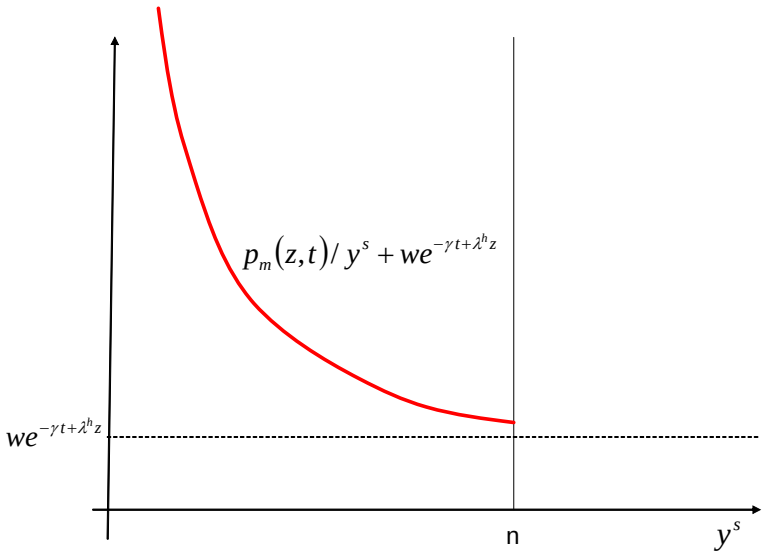


Figure 9: Average Cost of High-skill Services

4.4 Firms' Problem

Given wages, free entry of manufacturing firms implies that they will price at average cost:

$$p_m(z,t) = \min\left\{\frac{e^{-\gamma t+\lambda^l z}}{A}, w\frac{e^{-\gamma t+\lambda^h z}}{B}\right\} \tag{5}$$

Similarly, free entry of market service providers implies:

$$p_s(z,t) = \left(1+\frac{1}{n}\right)\min\left\{\frac{e^{-\gamma t+\lambda^l z}}{A}, w\frac{e^{-\gamma t+\lambda^h z}}{B}\right\} \tag{6}$$

Provided high-skilled workers have a comparative advantage in the production of more complex goods and services, i.e., $\lambda^l > \lambda^h$, there exists a cut-off service $\hat{z}$ above which (i.e., $z \geq \hat{z}$) all output is produced by high-skilled, specialized labor, and below which all output is produced by low-skilled labor. This marginal service $\hat{z}$ is given by:

$$\hat{z} = \frac{1}{\lambda^l - \lambda^h}\log\left(w\frac{A}{B}\right).$$

As the skill premium increases, more of the less complex services and manufactured goods are going to be produced with low-skilled labor.

4.5 Consumer's Problem

The demand for market services and manufactured goods of an individual with skill e, $e \in \{l, h\}$ solves:

$$\begin{aligned} & V^e(\theta) = \max_{0 \leq \underline{z}_e \leq \bar{z}_e} (1-\delta)\,\underline{z}_e + \delta \bar{z}_e \\ & \text{s.t.} \\ & \int_{-\infty}^{\underline{z}_e} p_m(z,t)\,dz + \int_{\underline{z}_e}^{\bar{z}_e} p_s(z,t)\,dz \\ = \;& w_e \left(1 - \int_{-\infty}^{\underline{z}_e} \frac{e^{-\gamma t + \lambda^l z}}{A} dz - \theta \mathcal{I}(e)\right) \end{aligned} \tag{7}$$

where $\mathcal{I}(e)$ is an indicator function that equals one if $e = h$ and zero otherwise. Given prices satisfying (5) and (6), it can be shown the budget set is strictly convex, implying that this is a strictly convex problem and, therefore, that first order conditions are also sufficient.[19]

At an interior optimum the marginal rate of substitution between home producing a service that is currently market purchased (decreasing $\underline{z}_e$) and consuming a new, not yet consumed market service (increasing $\bar{z}_e$) equals the marginal rate of transformation:

$$\frac{1-\delta}{\delta} \geq \frac{p_m(\underline{z}_e, t) + w_e \frac{e^{-\gamma t + \lambda^l \underline{z}_e}}{A} - p_s(\underline{z}_e, t)}{p_s(\bar{z}_e, t)} \; (= \text{ if } \underline{z}_e < \bar{z}_e) \tag{8}$$

Figure 10 illustrates this optimality condition for both a low-skilled individual and a high-skilled individual who is indifferent to becoming skilled, i.e., $\theta = \hat{\theta}$.

Given the value function associated with the agent's problem in (7), $V^e(\theta)$, $e = l, h$, we can solve for the decision rule to become a specialized, skilled worker. Individuals choose to specialize iff:

$$V^h(\theta) \geq V^l. \tag{9}$$

[19] In Appendix B, we show that the budget set is strictly convex provided that $\lambda_z^l \geq \lambda_z^h$. This assumption is clearly the economically more interesting case and is maintained throughout the paper.

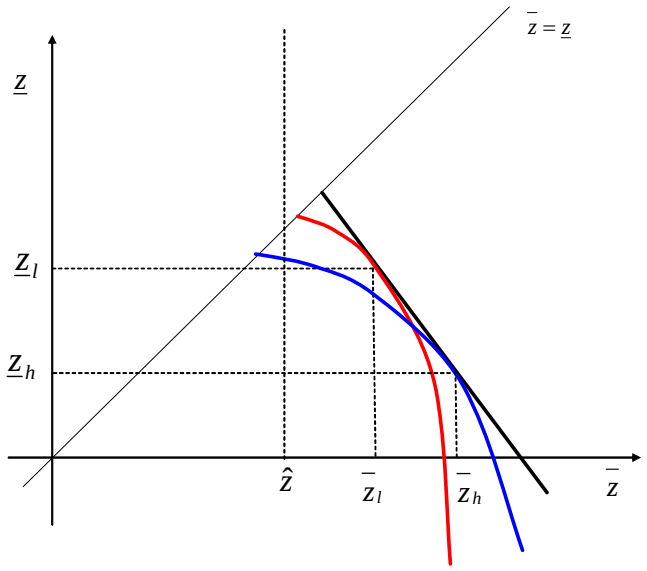


Figure 10: Optimality for Low and Marginal High Skilled ($\theta = \hat{\theta}$) Individuals

As is implicit in condition (9), the value of remaining low-skilled is independent of θ, while the value of becoming high-skilled is a strictly increasing function of θ. Moreover, for $w > 1$, $V^h(0) > V^l$ and $V^l > V^h(1)$, implying that there exist a unique cost $\hat{\theta}$ such that individuals with schooling cost lower than $\hat{\theta}$ strictly prefer to become skilled.

From the analysis of the consumer's problem, we obtain strong cross-sectional predictions for the consumption of market services by low- and high-skilled individuals, and for individuals of different ability.

High-skilled individuals have a high opportunity cost of their time, and therefore, face larger costs of home-producing services. This substitution effect implies that high-skilled workers consume more market services than low-skilled workers. At the same time, low θ high-skilled individuals have a higher net labor endowment, and therefore, they consume more of both market- and home-produced services. For the marginal high-skilled individual, $\theta = \hat{\theta}$, there is only a substitution effect, implying that $\overline{z}_h(\theta, t) > \overline{z}_l$ and $\underline{z}_h(\theta, t) < \underline{z}_l$ (see Figure 10). Among high-skilled workers with identical opportunity costs of time, high income workers consume more manufactured goods and a wider range of services overall, i.e., $\overline{z}_h(\theta', t) > \overline{z}_h(\theta, t)$ and $\underline{z}_h(\theta', t) > \underline{z}_h(\theta, t)$ for $\theta' < \theta$. For low θ high-skilled agents the substitution

effect dominates the income effect, i.e., $\underline{z}_l > \underline{z}_h(0,t)$.

These observations are summarized in the following proposition:

Proposition 2 *If the skill premium is positive,$w > 1$, there exist $0 < \hat{\theta} < 1$ such that $V^h(\theta) \gtreqless V^l$ iff $\theta \lesseqgtr \hat{\theta}$. Moreover, high-skill workers ($\theta < \hat{\theta}$) consume some market services that low-skill workers do not consume, $\overline{z}_h(\theta,t) > \bar{z}_l(t)$, and home produce a (weakly) smaller range of services than low-skill workers, $\underline{z}_h(\theta,t) \leq \underline{z}_l(t)$. Also, the thresholds $\overline{z}_h(\theta,t)$ and $\underline{z}_h(\theta,t)$ are decreasing in θ.*

Proof. See appendix B. ■

4.6 Competitive equilibrium

A competitive equilibrium is given by price functions $p_m(z,t)$, $p_s(z,t)$, $w(t)$, schooling threshold $\hat{\theta}$, demand for manufactured goods and market services $\underline{z}_l(t)$, $\bar{z}_l(t)$, $\underline{z}_h(\theta,t)$, $\bar{z}_h(\theta,t)$, and the threshold of comparative advantage $\hat{z}(t)$ such that consumers demands and schooling decisions solve (7) and (9); prices solve zero profits conditions (5) and (6); and labor markets clear (see Appendix C).

Next, we characterize the dynamics of the model for the case where high-skilled labor has only an absolute advantage, i.e., $\lambda^l = \lambda^h$ (Section 5), and the case where high-skilled labor has a comparative advantage in more complex goods, i.e., $\lambda^l > \lambda^h$ (Section 6).

5 Absolute Advantage: Product Cycle with Balanced Growth

In this section we briefly describe the equilibrium of an economy where high-skilled labor holds only an absolute advantage, i.e., $\lambda^l = \lambda^h = \lambda$. Beside gaining insights from the closed-form solutions, the study of this case is interesting in itself because it corresponds closely to the solution of the general case, $\lambda^l \geq \lambda^h$, for low t.

In this economy, the relative wage is pinned down by the relative productivity:

$$w = \frac{B}{A}.$$

The equilibrium allocation when $\lambda^l = \lambda^h = \lambda$ follows a balanced growth path and yields an analytic solution (see Appendix D). We briefly summarize some characteristics of the solution below:

Proposition 3 *The following is true about the balanced growth path allocation when* $\lambda^l = \lambda^h = \lambda$*:*

1. *The fraction of high-skilled workers* $F\left(\widehat{\theta}\right) \in (0,1)$*, is constant over time, increasing in their relative productivity,* B/A*, and increasing in the efficient scale,* n*.*

2. *The thresholds defining the demand of high- and low-skilled individuals for manufactured goods and services* $\bar{z}_l(t)$*,* $\underline{z}_l(t)$*,* $\overline{z}_h(\theta,t)$*, and* $\underline{z}_h(\theta,t)$ *all increase linearly with time with a slope* γ/λ*. Thus, the model yields a product cycle from not consumed to purchased on the market and finally produced at home for all consumers.*

3. *The share of services in output* $\tilde{y}^s$ *and consumption* $\tilde{c}^s$ *are constant, with* $\tilde{c}^s = \frac{n+1}{n}\tilde{y}^s$*.*

Proof. See Appendix D. ■

To illustrate the product cycle it is useful to solve for the value added share of industries providing goods and services associated with a particular set of wants $[z, z+\Delta)$, i.e., $y^s(z,\Delta)$ and $y^m(z,\Delta)$, where z indexes the sector and Δ the level of aggregation. Similarly, we can solve for the share of consumption spent on goods and services of a particular industry. Figure 11 shows these product cycles for an economy with only low-skilled labor. (The general model requires an additional integration over θ, and the particular shape may depend on the distribution of θ). Value-added in both the service and corresponding manufacturing industry rises initially as more of the wants are satisfied (through market services), then falls with rising productivity/falling costs. Eventually costs fall enough to initiate direct consumption of the manufactured good and home production of the service (the dotted line). At this point, mass consumption of the manufactured good begins, and the manufacturing industry again rises, but the service industry declines even more sharply.

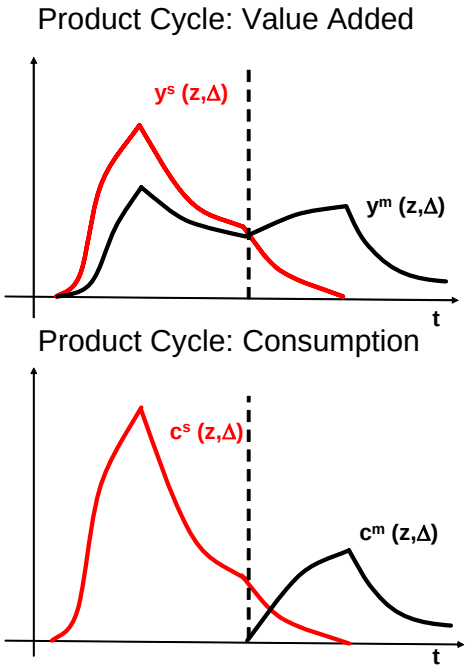


Figure 11: Product Cycle for a Particular Industry $[z, z + \Delta]$. Single skill.

6 Comparative Advantage: The Rise of the Service Economy

In this section, we characterize the dynamics of the economy when skilled labor has a comparative advantage in the production of more complex goods, i.e., $\lambda^l > \lambda^h$. We show that the model generates a transition from an economy with unspecialized labor and a constant share of services to an economy with a rising skill premium, an increasing share of high-skill, specialized labor, and a rising share of services in the economy. Moreover, the model generates both a rise in the relative price of services to manufactured goods and a rise in relative quantity of services to commodities that together contribute to the rising share of services in the economy. Thus, the model is consistent with the four facts of Section 2.

Figure 12.a illustrates the dynamics of services in the model economy. Both measured as a share of value added, consumption, and labor the model generates the transition from an economy with a stable services sector to one with a rising service economy. As illustrated by Figure 12.b and 12.c, the rise in the service economy is associated with a surge in specialized labor and a rising skill premium. Intuitively, it is relatively more costly to home produce services for which high-skilled labor has a comparative advantage, since all home production is perform with low-skilled productivity. For this reason, the product cycle of skill-intensive services is substantially longer, and therefore, a larger fraction of services end up being provided in the market. At the same time, the rising skill premium makes it relatively more costly for high-skilled workers to home-produce services, further increasing the share of market services in the economy.

In the long run, the share of services in output asymptotes to a constant governed by the efficient scale parameter n, $\lim_{t\to\infty} \tilde{y}^s(t) = \frac{n}{n+1}$, and the share of service in consumption converges to one. The fraction of individuals who become high-skilled goes to one as the skill premium grows unbounded.

Formally, this discussion is formalized in the following proposition.

Proposition 4 *The following is true about the economy where high-skilled labor has a comparative advantage in more complex goods, i.e., $\lambda^l > \lambda^h$: i) There exists a threshold time t_0 marking the beginning of the service economy. For $t < t_0$, $w = 1$, $\hat{\theta} = 0$ (i.e., there is no specialized high-skilled labor), and the share of services is constant. ii) As t grows large: $\lim_{t\to+\infty} \tilde{y}^s(t) = \frac{n}{n+1}$, $\lim_{t\to+\infty} \tilde{c}^s(t) = 1$, $\lim_{t\to+\infty} w = +\infty$ and $\lim_{t\to+\infty} \hat{\theta} = 1$.*

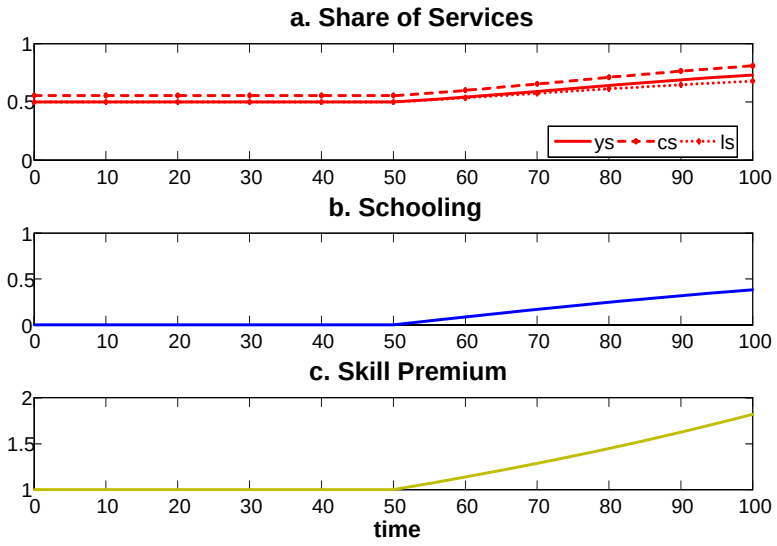


Figure 12: The Rise of the Service Economy and Related Stories

Proof. See Appendix E. ■

Along the transition, the model generates a rising relative price of services to commodities, a fact that is often taken as exogenous by most theories of structural change.[20] In the model, as the skill premium rises, the relative price of services to commodities also increases, as the former are more skill-intensive. This is stated formally in the following proposition.

Proposition 5 $\frac{\partial P_s(t)/\partial t}{P_s(t)} - \frac{\partial P_m(t)/\partial t}{P_m(t)} > 0$ *for* $t \geq t_0$, *where* $\frac{\partial P_s(t)/\partial t}{P_s(t)}$ *and* $\frac{\partial P_m(t)/\partial t}{P_m(t)}$ *are the continuous-time changes in chain-weighted price indices for market services and manufactured goods, respectively.*

Proof. See Appendix E. ■

As discussed before, the share of services in the economy also increases due to the lengthening of product cycles and the substitution away from home produced services by high-skilled workers. Thus, the model generates both a rise in relative price and relative quantities of services to manufactured goods.

[20] A recent exception is given by Acemoglu and Guerrieri (2005).

7 Extensions

We briefly review three extensions that might be considered potential alternative explanations for the rise in the service economy: 1) transaction costs for service purchases, 2) biased productivity across sectors (i.e., service vs. manufacturing), and 3) skill-biased technical change. These extensions also help clarify the intuition for results of the model analyzed in the previous section.[21]

7.1 Transaction Costs

We analyzed three interesting variations in which the inherent benefit of home production over market consumption is in avoiding market transaction costs, rather than the disutility of public consumption. That is, both home and market services provide one unit of utility, but the purchase of any service from the market requires a transaction cost of $Ce^{-\gamma^c t}$ units of time. Here γ^c captures potentially falling transaction costs over time. The simplified expression of the problem is:

$$\begin{aligned} & V^e(\theta) = \max_{0 \leq \underline{z}_e \leq \bar{z}_e} \bar{z}_e \\ & \text{s.t.} \\ & \int_{-\infty}^{\underline{z}_e} p_m(z,t)\,dz + \int_{\underline{z}_e}^{\bar{z}_e} p_s(z,t)\,dz \\ = \;& w_e \left(1 - \int_{-\infty}^{\underline{z}_l} \frac{e^{-\gamma t + \lambda^l z}}{A} dz - \int_{\underline{z}_l}^{\bar{z}_e} Ce^{-\gamma^c t} dz - \theta \mathcal{I}(e)\right) \end{aligned}$$

The first variaton is the analog of the balanced case in which $\lambda^l = \lambda^h = \lambda$ and $\gamma^c = 0$. Thus, instead of a constant utility cost there is a constant time cost C. This model reproduces the balanced growth and product cycles, comparable to the balanced case examined above. The second variation is the case of comparative advantage, in which $\lambda^l > \lambda^h$ and $\gamma^c = 0$. This model produces the result that asymptotically market labor converges to zero, and all labor is used in either transaction costs, or home production. Despite its infinitesimal productivity relative to market production, home production is undertaken in order to avoid transaction costs.

[21] Analysis of the extensions discussed are relatively straightforward and available upon request from the authors.

A third variation, which produces the rise of the service economy, is $\lambda^l = \lambda^h = \lambda$, but $\gamma^c > 0$. In this case, the fall in transaction costs causes market production to be relatively more attractive. The share of the service sector in consumption (and output) increases over time and converges to 1 $\left(\text{and } \frac{n}{n+1}, \text{ respectively}\right)$ asymptotically. Furthermore, although the relative wage $w = \frac{B}{A}$ is constant, $\hat{\theta}$ increases over time because of the reduced time in home production and coverges asymptotically to $1 - \frac{A}{B}$. Thus, this model produces the rise share of services and level of skill in the economy, but has no predictions about the skill-intensity of services (i.e., skills could be employed in manufacturing).

7.2 Biased Productivity Across Sectors

Instead of comparative advantage driving the rise of the service economy, one can produce the rise using differential rates of technological progress across services and manufacturing. To highlight this assume that $\lambda^l = \lambda^h = \lambda$, but productivity growth differs in services (γ^s) and manufacturing (γ^m). The model is therefore identical to the balanced growth case, except the production functions take the following form:

$$\begin{aligned} y^s(z,t) &= \begin{cases} 0 & \text{if } k^s < 1 \\ \min\left\{n, e^{\gamma^s t}\left[Ae^{\lambda z} l^s(z,t) + Be^{\lambda z} h^s(z,t)\right]\right\} & \text{if } k^s \geq 1 \end{cases} \\ y^m(z,t) &= e^{\gamma^m t}\left[Ae^{\lambda z} l^s(z,t) + Be^{\lambda z} h^s(z,t)\right] \end{aligned}$$

Interestingly, in this model, the necessary assumption for the rise of the service economy is $\gamma^s > \gamma^m$, i.e., the service sector must have higher productivity growth. This is precisely the opposite of the required assumption in the standard model with biased sectoral productivity growth (e.g., Ngai and Pissarides, 2005). Intuitively, when manufacturing productivity growth is relatively slow, the service sector expands in order to save on manufactured goods costs through higher utilization on the market.

7.3 Skill-Biased Technical Change

Rather than assuming biased productivity across sectors, assume instead that productivity growth differs across high-skilled $\left(\gamma^h\right)$ and low-skilled $\left(\gamma^l\right)$ workers. Again focusing on the $\lambda^l = \lambda^h = \lambda$ case, the production functions

are therefore:

$$\begin{aligned} y^s(z,t) &= \begin{cases} 0 & \text{if } k^s < 1 \\ \min\left\{n, e^{-\lambda z}\left[Ae^{\gamma^l t} l^s(z,t) + Be^{\gamma^h t} h^s(z,t)\right]\right\} & \text{if } k^s \geq 1 \end{cases} \\ y^m(z,t) &= e^{-\lambda z}\left[Ae^{\gamma^l t} l^s(z,t) + Be^{\gamma^h t} h^s(z,t)\right] \end{aligned}$$

In this model, the relative wage is given by relative productivities:

$$w = \frac{B}{A} e^{(\gamma^h - \gamma^l)t}$$

Since this relative productivity is common to all z, labor costs are equated for low- and high-skilled workers and there is indeterminacy about who works in which z. The model therefore does not predict increasing skill-intensity of services, nor the rising relative price of services, but otherwise looks very similar asymptotically to the comparative advantage model studied in the previous section. Namely, the relative wage goes to infinity, and $\hat{\theta} \to 1$, $\tilde{c}^s \to 1$, and $\tilde{y}^s \to n/(n+1)$. In this model, however, the longer product cycles are not the result of new goods being more skill-intensive. (They may be produced by low-skilled workers.) Instead, longer product cycles for skilled workers are driven by their rising opportunity cost w. As their relative wage rises, high-skilled workers stay with market services longer, transitioning them to home production at a slower rate. In contrast, the product cycles of low-skilled workers remain constant. The rise in the service sector is driven by the lengthening product cycles of high-skilled workers, and the rising fraction of high-skilled workers in the workforce.

This example highlights that there are two reasons for longer product cycles in the comparative advantage model in Section 6. The rising skill intensity of services lengthens product cycles for both low- and high-skilled workers, while the rising opportunity cost is an additional factor in the longer product cycles of high-skilled workers.

8 Conclusions

We have focused on the household's decision between home production and market production in explaining the rise of the service economy. In many ways, however, the problem could be restated as a manufacturing firm's decision between in-house provision of services and out-sourced production.

The disutility of market provision would be interpreted as a lower value of market services because they are less customized perhaps. The issue would be most comparable for small firms for which scale and utilization of specialized skills and goods may be an issue. It is possible that shipping and delivery has undergone a transition from external to in-house provision with the falling cost of trucks, while high-skill services such as advertising, accounting and legal services have not, for example.[22]

Secondly, the model has avoided reference to female labor supply, which has had strong impacts on the U.S. labor market over the same period, is of great importance in considering the home production vs. market purchase decision, and has been linked to the growth in services (Lee and Wolpin, 2006). Trade is yet a third potentially important factor in understanding structural transformation, and the model may have important implications for the link between trade, home production and structural composition. A fourth question is whether the model's concepts of returns to scale of market services and mass consumption of manufactured goods in home production could also generate the observed expansion of manufacturing earlier in development, and the extent to which the theory can qualitatively and quantitatively account for the facts involving long run structural transformation. All of these questions are subjects of ongoing research.

[22] The Council of Economic Advisers (2004) mention the importance of temporary manpower agencies and increased outsourcing in the growth of service employment.

Appendix A: Skill-Intensity and Labor Income Growth of Detailed Service Industries

Declining Services			Growing Services		
Fraction College-Educated in 2000	**Change in Share of Labor Income (2000-1950)**	**Industry**	**Fraction College-Educated in 2000**	**Change in Share of Labor Income (2000-1950)**	**Industry**
0.301	-0.0366	Railroads and railway	0.766	0.0460	Educational services
0.274	-0.0092	Private households	0.652	0.0438	Medical and other health services, except hospitals
0.248	-0.0082	Laundering, cleaning, and dyeing	0.662	0.0428	Misc business services
0.362	-0.0055	Street railways and bus lines	0.743	0.0315	Hospitals
0.389	-0.0055	Food and related products	0.667	0.0259	Misc professional and related
0.324	-0.0049	Food stores, except dairy	0.831	0.0150	Security and commodity brokerage and invest companies
0.573	-0.0048	Postal service	0.687	0.0140	Banking and credit
0.468	-0.0045	Apparel and accessories stores, except shoe	0.816	0.0138	Legal services
0.279	-0.0043	Water transportation	0.822	0.0109	Engineering and architectural services
0.370	-0.0035	Electric-gas utilities	0.555	0.0088	Misc entertainment and recreation services
0.310	-0.0035	Gasoline service stations	0.630	0.0077	Real estate
0.640	-0.0035	Telephone	0.716	0.0061	Insurance
0.154	-0.0033	Dairy prods stores and milk retailing	0.659	0.0053	Household appliance and radio stores
0.352	-0.0029	Hardware and farm implement stores	0.813	0.0048	Accounting, auditing, and bookkeeping services
0.318	-0.0028	Auto repair services and garages	0.472	0.0043	Services incidental to transportation
0.411	-0.0027	Petroleum products	0.683	0.0039	Radio broadcasting and television
0.426	-0.0023	General merchandise	0.519	0.0039	Misc retail stores
0.377	-0.0022	Taxicab service	0.333	0.0036	Eating and drinking places
0.263	-0.0021	Fuel and ice retailing	0.651	0.0035	Air transportation
0.420	-0.0019	Dry goods apparel	0.680	0.0034	Telegraph
0.179	-0.0018	Five and ten cent stores	0.620	0.0024	Welfare and religious services
0.281	-0.0016	Farm prods--raw materials	0.502	0.0023	Misc wholesale trade
0.404	-0.0015	Gas and steam supply systems	0.676	0.0019	Advertising
0.511	-0.0014	Theaters and motion pictures	0.488	0.0014	Machinery, equipment, and supplies
0.341	-0.0014	Sanitary services	0.411	0.0013	Hotels and lodging places
0.420	-0.0012	Lumber and building material retailing	0.313	0.0012	Trucking service
0.460	-0.0012	Furniture and house furnishings stores	0.633	0.0012	Nonprofit membership organizs.
0.387	-0.0011	Shoe stores	0.432	0.0012	Motor vehicles and accessories retailing
0.451	-0.0007	Other and not specified utilities	0.542	0.0010	Not specified wholesale trade
0.554	-0.0007	Electric light and power	0.539	0.0007	Electrical goods, hardware, and plumbing equipment
0.398	-0.0007	Liquor stores	0.526	0.0005	Drug stores
0.470	-0.0007	Jewelry stores	0.388	0.0004	Motor vehicles and equipment
0.252	-0.0005	Bowling alleys, and billiard and pool parlors	0.452	0.0003	Misc personal services
0.149	-0.0005	Shoe repair shops	0.429	0.0000	Water supply
0.404	-0.0004	Misc repair services			
0.215	-0.0003	Real estate-insurance-law offices			
0.464	-0.0003	Not specified retail trade			
0.311	-0.0002	Warehousing and storage			
0.601	-0.0002	Drugs, chemicals, and allied products			
0.215	-0.0002	Dressmaking shops			
0.390	-0.0001	Petroleum and gasoline pipe lines			
0.392	-0.0001	Retail florists			

B Characterization of the Consumer's Problem

In this section we characterize the properties of the budget set for the case skill labor has a (weak) comparative advantage in the production of more complex wants: $\lambda_z^h \leq \lambda_z^l$, and characterize the choice of market and home consumption of services for low-skill and high-skill individuals (Proposition 2).

The slope of the budget set equals:

$$\begin{aligned}\frac{d\underline{z}_l}{d\bar{z}_l} &= -\frac{p_s\left(\bar{z}_l,t\right)}{p_m\left(\underline{z}_l,t\right)+\frac{e^{-\gamma t+\lambda^l\underline{z}_l}}{A}-p_s\left(\underline{z}_l,t\right)} \\ &= -\frac{\min\left\{\frac{e^{-\gamma t+\lambda^l\bar{z}_l}}{A},w\frac{e^{-\gamma t+\lambda^h\bar{z}_l}}{B}\right\}}{\frac{e^{-\gamma t+\lambda^l\underline{z}_l}}{A}-\frac{1}{n}\min\left\{\frac{e^{-\gamma t+\lambda^l\underline{z}_l}}{A},w\frac{e^{-\gamma t+\lambda^h\underline{z}_l}}{B}\right\}}.\end{aligned}$$

The curvature of the budget set:

$$\begin{aligned}\frac{\partial^2\underline{z}_l}{\partial\bar{z}_l^2} &= \frac{d\underline{z}_l}{d\bar{z}_l}\begin{cases}\lambda^l+\lambda^l e^{\lambda^l\bar{z}_l}e^{-\lambda^l\underline{z}_l}\left(1-\frac{1}{n}\right) & \text{if } \underline{z}_l\leq\bar{z}_l\leq\hat{z} \\ \frac{A}{B}w\left[\lambda^h+\lambda^l e^{\lambda^h\bar{z}_l}e^{-\lambda^l\underline{z}_l}\left(1-\frac{1}{n}\right)\right] & \text{if } \underline{z}_l\leq\hat{z}<\bar{z}_l \\ \left[\lambda^h+w\frac{e^{\lambda^h\bar{z}_l}}{B}\frac{\lambda_z^l\left[\frac{e^{\lambda^l\underline{z}_l}}{A}-\frac{\lambda^h}{\lambda^l}\frac{1}{n}w\frac{e^{\lambda^h\underline{z}_l}}{B}\right]}{\left[\frac{e^{\lambda^l\underline{z}_l}}{A}-\frac{1}{n}w\frac{e^{\lambda^h\underline{z}_l}}{B}\right]^2}\right] & \text{if } \hat{z}<\underline{z}_l\leq\bar{z}_l\end{cases} \\ &\leq 0\end{aligned}$$

Similar expressions can be obtained for the budget set of skill individuals.

The strict convexity of the budget set guarantees that the first order condition is a sufficient for a maxima.

Proof of Proposition 2:. $\bar{z}_l$, $\underline{z}_l$, $\bar{z}_h=\bar{z}_h\left(0\right)$, and $\underline{z}_h=\underline{z}_h\left(0\right)$ solve the following system of equations for $e=l,h$:

$$\left(1-\delta\right)p_s\left(\bar{z}_e,t\right)=\delta\left[p_m\left(\underline{z}_e,t\right)+w_e e^{-\gamma t+\lambda^l\underline{z}_e}-p_s\left(\underline{z}_e,t\right)\right]$$

and

$$\int_{-\infty}^{\underline{z}_e}p_m\left(z,t\right)dz+\int_{\underline{z}_e}^{\bar{z}_e}p_s\left(z,t\right)dz=w_e\left(1-\frac{e^{-\gamma t+\lambda^l\underline{z}_e}}{\lambda^l}\right)$$

In the case there is a positive demand for skill labor, i.e., $\hat{z} < \bar{z}_l < \bar{z}_h$,[23] and $\underline{z}_e > \hat{z}$ (a similar argument can be done for $\underline{z}_l < \hat{z}$), the first order condition and budget constraint can be written as:

$$\left(1+\frac{1}{n}\right) w \frac{e^{-\gamma t+\lambda^h \bar{z}_e}}{B} = \frac{\delta}{1-\delta}\left[\frac{e^{-\gamma t+\lambda^l \underline{z}_l}}{A} w_e - \frac{w}{n}\frac{e^{-\gamma t+\lambda^h \underline{z}_e}}{B}\right]$$

and

$$\begin{aligned} & \frac{e^{-\gamma t+\lambda^l \hat{z}}}{A\lambda^l} + \frac{w}{B\lambda^h}\left(e^{-\gamma t+\lambda^h \underline{z}_e} - e^{-\gamma t+\lambda^h \hat{z}}\right) \\ & + \left(1+\frac{1}{n}\right)\frac{w}{B\lambda^h}\left(e^{-\gamma t+\lambda^h \bar{z}_e} - e^{-\gamma t+\lambda^h \underline{z}_e}\right) \\ = \; & w_e\left(1-\frac{e^{-\gamma t+\lambda^l \underline{z}_e}}{A\lambda^l}\right) \end{aligned}$$

Solving for $e^{-\gamma t+\lambda^h \bar{z}_e}$ from the first order condition and substituying in the budget constraint we obtain an equation in $\underline{z}_l$:

$$\begin{aligned} & w_e\left(\frac{e^{-\gamma t+\lambda^l \underline{z}_e}}{A}\left(\frac{1}{\lambda^l}+\frac{\delta}{1-\delta}\frac{1}{\lambda^h}\right)-1\right) \\ = \; & \frac{1}{n}\frac{e^{-\gamma t+\lambda^h \underline{z}_e}}{B\lambda^h(1-\delta)} + \frac{e^{-\gamma t+\lambda^l \hat{z}}}{A}\left(\frac{1}{\lambda^h}-\frac{1}{\lambda^l}\right) \end{aligned}$$

Notice that the left hand side and the right hand side of this equality define two increasing functions of $\underline{z}_e$. Moreover, the function defined by the left hand side intersects the function defined by the right hand side from below and at a point where $\left(\frac{e^{-\gamma t+\lambda^l \underline{z}_e}}{A}\left(\frac{1}{\lambda^l}+\frac{\delta}{1-\delta}\frac{1}{\lambda^h}\right)-1\right) > 0$. Therefore, for $e = h$, $w_h = w > 1 = w_l$ and the intersection occurs to the left, i.e., $\underline{z}_h < \underline{z}_l$. It is straightforward to show that $\bar{z}(\theta) > \bar{z}(\theta)$ and $\underline{z}_h(\theta) > \underline{z}_h(\theta')$ for $\theta > \theta'$, and that $\bar{z}_h(\theta) \geq z_l$ for $\theta > \hat{\theta}$. This completes the proof of Proposition 2. ■

[23] Since skilled individual spend a positive amount on goods produced by unskilled individuals, trade balance between unskilled and skilled agents requires that $\hat{z} < \bar{z}_l$ whenever skilled intensive goods are produced.

C Market Clearing

Market clearing conditions are given by the resource constraints of low-skilled

$$\begin{aligned}\int_{\theta:V^h(\theta)<V^l} dF(\theta) &= \frac{e^{-\gamma t}}{A}\int_{\theta:V^h(\theta)\geq V^l}\left[\int_{z<\min\{\bar{z}_h(\theta),\hat{z}\}} l(z,t)\,dz\right]dF(\theta)\\ &\quad+\int_{\theta:V^h(\theta)<V^l}\left[\int_{\hat{z}\leq z<\bar{z}_h(\theta)} l(z,t)\,dz+\int_{z\leq \underline{z}_l}\frac{e^{\lambda^l z}}{A}dz\right]dF(\theta)\end{aligned}$$

and high-skilled workers

$$\begin{aligned}\int_{\theta:V^h(\theta)\geq V^l} dF(\theta) &= \frac{e^{-\gamma t}}{B}\int_{\theta:V^h(\theta)\geq V^l}\left[\int_{\hat{z}\leq z<\bar{z}_h(\theta)} l(z,t)\,dz+\int_{z\leq \underline{z}_h(\theta)}\frac{e^{\lambda^l z}}{A}dz\right]dF(\theta)\\ &\quad+\int_{\theta:V^h(\theta)<V^l}\left[\int_{\hat{z}\leq z<\bar{z}_h(\theta)} l(z,t)\,dz\right]dF(\theta)\end{aligned}$$

D Derivations Associated with the Balance Case (i.e. No Comparative Advantage)

This section provides the analysis of the balance case discussed in Section 5, and prove Proposition 3.

Proof of Proposition 3:. In the case $\lambda^l=\lambda^h=\lambda$ the consumer's problem specializes to:

$$\begin{aligned}&\max_{0\leq \underline{z}_e\leq \bar{z}_e}(1-\delta)\,\underline{z}_e+\delta\bar{z}_e\\ &\text{s.t.}\\ &\left(1-\frac{1}{n}\right)\frac{e^{-\gamma t+\lambda\underline{z}_e}}{A\lambda}+\left(1+\frac{1}{n}\right)\frac{e^{-\gamma t+\lambda\bar{z}_e}}{A\lambda}=w_e\left(1-\theta\mathcal{I}(e)\right)\end{aligned}$$

From the first order conditions we obtain:

$$\frac{\left(w_e-\frac{1}{n}\right)}{1-\delta}e^{-\gamma t+\lambda\underline{z}_e}=\frac{\left(1+\frac{1}{n}\right)}{\delta}e^{-\gamma t+\lambda\underline{z}_e} \tag{10}$$

Implying that consumption expansion path correspond to 45^o lines with a negative constant term:

$$\underline{z}_e = \frac{1}{\lambda}\log\left(\frac{1-\delta}{\delta}\frac{1+\frac{1}{n}}{w_e-\frac{1}{n}}\right)+\bar{z}_e$$

The expansion path is illustrated in Figure 13.Substituting (10) into the

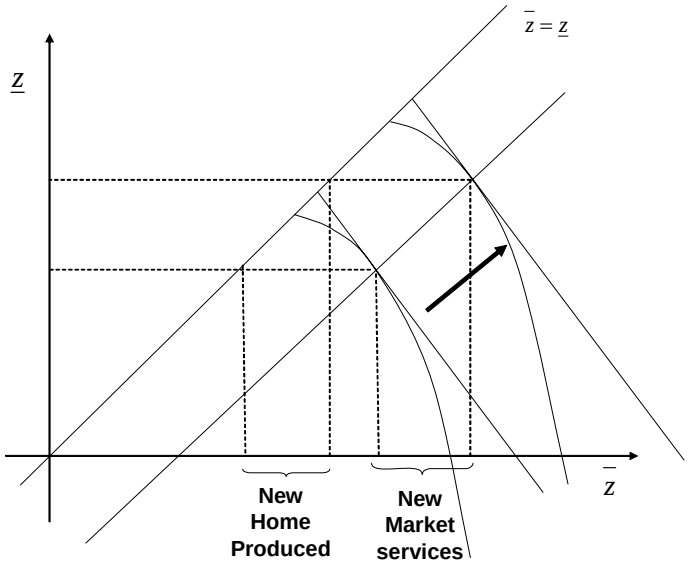


Figure 13: Expansion Path: Balance Case

budget constraint we obtain an expression for the frontier want satisfied by an individual of skill e at time t:

$$\overline{z}_e(t) = \frac{\gamma}{\lambda}t + \frac{1}{\lambda}\log\left(\frac{\delta A\lambda w_e\left(1-\theta\mathcal{I}(e)\right)}{1+\frac{1}{n}}\right) \tag{11}$$

Substituying (11) and (10) into the utility function we obtain an expression for the value function associated with different skill choices by an individual of talent θ:

$$\begin{aligned} V^e(\theta) &= (1-\delta)\ln\left[\frac{(1-\delta)\lambda A w_e}{w_e-\frac{1}{n}}\right]+\delta\ln\left[\frac{\delta\lambda A w_e}{1+\frac{1}{n}}\right] \\ &\quad +\ln\left[(1-\theta\mathcal{I}(e))\right] \end{aligned}$$

The talent of the marginal specialized worker $\hat{\theta}$ then solves:

$$V^l = V^h\left(\hat{\theta}\right)$$

implying

$$\widehat{\theta} = 1 - \frac{A}{B}\left(\frac{\frac{B}{A} - \frac{1}{n}}{1 - \frac{1}{n}}\right)^{1-\delta}.$$

For this economy aggregate output equals:

$$\begin{aligned} Y(t) &= \left(1 - F\left(\hat{\theta}\right)\right)\left[\int_{-\infty}^{\underline{z}_l(t)} p_m(z,t)\,dz + \int_{\underline{z}_l(t)}^{\bar{z}_l(t)} p_s(z,t)\,dz\right] \\ &\quad + \int_0^{\hat{\theta}}\left[\int_{-\infty}^{\underline{z}_h(\theta,t)} p_m(z,t)\,dz + \int_{\underline{z}_h(\theta,t)}^{\bar{z}_h(\theta,t)} p_s(z,t)\,dz\right] dF(\theta) \end{aligned}$$

The aggregate value added of the service sector equals:

$$\begin{aligned} Y^s(t) &= \left(1 - F\left(\hat{\theta}\right)\right)\left[\int_{\underline{z}_l(t)}^{\bar{z}_l(t)}\left(p_s(z,t) - \frac{1}{n}p_m(z,t)\right)dz\right] \\ &\quad + \int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta,t)}^{\bar{z}_h(\theta,t)}\left(p_s(z,t) - \frac{1}{n}p_m(z,t)\right)dz\right] dF(\theta) \end{aligned}$$

The aggregate consumption of services equals:

$$\begin{aligned} C^s(t) &= \left(1 - F\left(\hat{\theta}\right)\right)\left[\int_{\underline{z}_l(t)}^{\bar{z}_l(t)} p_s(z,t)\,dz\right] \\ &\quad + \int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta,t)}^{\bar{z}_h(\theta,t)} p_s(z,t)\,dz\right] dF(\theta) \end{aligned}$$

Using that $p_s(z,t) = \left(1 + \frac{1}{n}\right)p_m(z,t)$ it is clear from these equations that:

$$c^s(t) = C^s(t)/Y(t) = \frac{Y^s(t)/Y}{1 + \frac{1}{n}} = \frac{y^s(t)}{1 + \frac{1}{n}}.$$

■

E Long Run Properties of the Case with Comparative Advantage ($\lambda^l > \lambda^h$)

Proof of Proposition 4. As long as

$$\overline{z}_l = \frac{\gamma}{\lambda^l} t + \frac{1}{\lambda^l} \log\left(\frac{\delta A \lambda^l}{1+\frac{1}{n}}\right) < \frac{1}{\lambda^l - \lambda^h} \log\left(\frac{A}{B}\right)$$

there is no demand for skills and the economy is in a balance growth path with a constant share of services. This condition provides an equation to solve for t_0:

$$t_0 = \frac{\lambda^l}{\gamma} \frac{1}{\lambda^l - \lambda^h} \log\left(\frac{A}{B}\right) - \frac{1}{\gamma} \log\left(\frac{\delta A \lambda^l}{1+\frac{1}{n}}\right)$$

The share of services is given by the following equation:

$$\begin{aligned} y^s &= \frac{\int_{\underline{z}_l}^{\bar{z}_l} \frac{e^{-\gamma t + \lambda^l z}}{A} dz}{\int_{-\infty}^{\underline{z}_l} \frac{e^{-\gamma t + \lambda^l z}}{A} dz + \left(1+\frac{1}{n}\right) \int_{\underline{z}_l}^{\bar{z}_l} \frac{e^{-\gamma t + \lambda^l z}}{A} dz} \\ &= \frac{\frac{\delta}{1+\frac{1}{n}} - \frac{1-\delta}{1-\frac{1}{n}}}{\delta - \frac{1-\delta}{1-\frac{1}{n}}} \\ &= \frac{n}{n+1} \frac{\delta - \frac{n+1}{n-1}(1-\delta)}{\delta - \frac{1-\delta}{n-1}} \end{aligned}$$

where the second equality uses the expression for $\underline{z}_l$ and $\bar{z}_l$ derived for the balanced case, i.e., equations (10) and (11).

To show the second part of Proposition 4 define $\bar{Z}_e(t) = e^{-\gamma t} e^{\lambda^h \bar{z}_e(t)}$, $\underline{Z}_e(t) = e^{-\gamma t} e^{\lambda^l \underline{z}_e(t)}$, $\hat{Z}_e(t) = e^{-\gamma t} e^{\lambda^l \hat{z}_e(t)}$, and $T(t) = \left(e^{-\gamma t}\right)^{1-\lambda^h/\lambda^l}$. After substituting for the equilibrium prices (5) and (6) the budget constrained can be written as

$$\begin{aligned} &\frac{\hat{Z}_e(t)}{A\lambda^l} + w(t) \frac{T(t)\,\underline{Z}_e(t)^{\lambda^h/\lambda^l} - T(t)\,\hat{Z}_e(t)^{\lambda^h/\lambda^l}}{B\lambda^h} \\ &\quad + \left(1+\frac{1}{n}\right) \frac{w(t)}{B\lambda^h} \left[\bar{Z}_e(t) - T(t)\,\underline{Z}_e(t)^{\lambda^h/\lambda^l}\right] \\ &\qquad \leq w_e(t) \left(1 - \frac{\underline{Z}_e(t)}{A\lambda_z^l} - \theta \mathcal{I}(e)\right) \end{aligned}$$

The first order condition is:

$$\frac{1-\delta}{\delta} \geq \frac{\frac{w_e(t)}{w(t)}\frac{B}{A}\frac{\underline{Z}_e(t)}{\bar{Z}_e(t)} - \frac{1}{n}\frac{T(t)\underline{Z}_e(t)^{\lambda^h/\lambda^l}}{\bar{Z}_e(t)}}{\left(1+\frac{1}{n}\right)} (= if \underline{z}_e < \bar{z}_e)$$

Figure 14 illustrate the expansion path implied by the first order condition. Let $w = \lim_{t\to+\infty} w(t)$, using that $\lim_{t\to+\infty} T(t) = 0$, $\lim_{t\to+\infty} \underline{Z}_e(t) = \underline{Z}_e < +\infty$, $\lim_{t\to+\infty} \bar{Z}_e(t) = \bar{Z}_e < +\infty$, and that either $\lim_{t\to+\infty} \hat{Z} = 0$ or $\lim_{t\to+\infty} w(t) = +\infty$:

$$\begin{aligned} &\left(1+\frac{1}{n}\right)\frac{1}{B\lambda^h}\bar{Z}_e \\ &\leq \frac{w_e}{w}\left(1 - \frac{\underline{Z}_e}{A\lambda^l} - \theta I(e=h)\right) \end{aligned}$$

and the first order condition simplifies to:

$$\frac{1-\delta}{\delta} = \frac{\frac{w_e}{w}\frac{B}{A}}{\left(1+\frac{1}{n}\right)}\frac{\underline{Z}_e}{\bar{Z}_e}$$

These last two equations imply:

$$\bar{Z}_e = \frac{w_e}{w}\frac{B(1-\theta\mathcal{I}(e))}{\left(1+\frac{1}{n}\right)\left(\frac{1}{\lambda^h} + \frac{1-\delta}{\delta}\frac{1}{\lambda^l}\right)}$$

and

$$\underline{Z}_e = \frac{1-\delta}{\delta}\frac{A(1-\theta\mathcal{I}(e))}{\left(\frac{1}{\lambda^h} + \frac{1-\delta}{\delta}\frac{1}{\lambda^l}\right)}.$$

Substituting into the objective function:

$$\begin{aligned} V^e(\theta) &= \frac{1-\delta}{\lambda^l}\log \underline{Z}_e(\theta) + \frac{\delta}{\lambda^h}\log \bar{Z}_e(\theta) \\ &= \frac{1-\delta}{\lambda^l}\log\frac{A(1-\delta)}{\delta} + \frac{\delta}{\lambda^h}\log\frac{B}{1+\frac{1}{n}} \\ &\quad + \frac{\delta}{\lambda^h}\log\frac{w_e}{w} + \left[\frac{1-\delta}{\lambda^l} + \frac{\delta}{\lambda^h}\right]\log\frac{(1-\theta\mathcal{I}(e))}{\left(\frac{1}{\lambda^h} + \frac{1-\delta}{\delta}\frac{1}{\lambda^l}\right)} \end{aligned}$$

Thus, $\hat{\theta}$ solve

$$\frac{\delta}{\lambda^h}\log w + \left[\frac{1-\delta}{\lambda^l} + \frac{\delta}{\lambda^h}\right]\log\left(1-\hat{\theta}\right) = 0$$

or

$$\hat{\theta}(w) = 1 - w^{-\frac{\delta\lambda^l}{(1-\delta)\lambda^h + \delta\lambda^l}}.$$

Skilled labor market clearing implies:

$$\begin{aligned} F\left(\hat{\theta}(w)\right) &= \int_0^{\hat{\theta}(w)} \left[\left(1+\frac{1}{n}\right)\frac{\bar{Z}_h(\theta)}{B\lambda^h} + \frac{\underline{Z}_h(\theta)}{A\lambda^l} + \theta\right] dF(\theta) \\ &\quad + \left(1 - F\left(\hat{\theta}(w)\right)\right)\left(1+\frac{1}{n}\right)\frac{\bar{Z}_l}{B\lambda^h} \\ &= \int_0^{\hat{\theta}(w)} dF(\theta) + \\ &\quad + \left(1 - F\left(\hat{\theta}(w)\right)\right)\frac{1}{w}\frac{1-\delta}{\delta\left(\frac{1}{\lambda^h} + \frac{1-\delta}{\delta}\frac{1}{\lambda^l}\right)} \end{aligned}$$

or

$$F\left(\hat{\theta}(w)\right) = F\left(\hat{\theta}(w)\right) + \left(1 - F\left(\hat{\theta}(w)\right)\right)\frac{\delta}{w}$$

Therefore, in the long run the skill premium goes to infinity and the entire population becomes skilled. ■

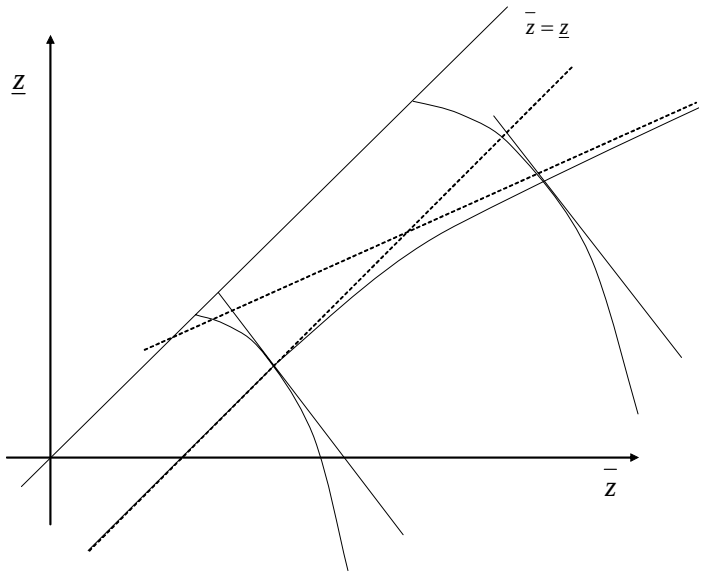


Figure 14: Expansion Path: Non Balance Case

Proof of Proposition 5. The evolution of the relative price of service to manufacturing equals:

$$\frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right) = \frac{1}{P_m(t)^2}\left\{\frac{\partial P_s(t)}{\partial t}P_m(t) - \frac{\partial P_m(t)}{\partial t}P_s(t)\right\}$$

or

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right)\\
=\ &\frac{\partial w/\partial t}{P_m^2(t)}\\
&\left\{\left[\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}\frac{\partial p}{\partial w}dz + \int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}\frac{\partial p}{\partial w}dz\right]dF(\theta)\right]\right.\\
&\quad\left[\left(1-F(\hat{\theta})\right)\left[\int_{-\infty}^{\underline{z}_l}pdz + \frac{1}{n}\int_{\underline{z}_l}^{\bar{z}_l}pdz\right]\right.\\
&\quad\left.+\int_0^{\hat{\theta}}\left[\int_{-\infty}^{\underline{z}_h(\theta)}pdz + \frac{1}{n}\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}pdz\right]dF(\theta)\right]\\
&\quad-\left[\left(1-F(\hat{\theta})\right)\left[\int_{-\infty}^{\underline{z}_l}\frac{\partial p}{\partial w}dz + \frac{1}{n}\int_{\underline{z}_l}^{\bar{z}_l}\frac{\partial p}{\partial w}dz\right]\right.\\
&\quad\left.+\int_0^{\hat{\theta}}\left[\int_{-\infty}^{\underline{z}_h(\theta)}\frac{\partial p}{\partial w}dz + \frac{1}{n}\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}\frac{\partial p}{\partial w}dz\right]dF(\theta)\right]\\
&\quad\left.\left[\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}pdz + \int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}pdz\right]dF(\theta)\right]\right\}
\end{aligned}$$

where $P_s(t)$ and $P_m(t)$ are the continuous time chain-weighted producers price deflators,[24] and $p(z,w)$ is such that:

$$p_m(z,t) = e^{-\lambda_t t}p(z,w),$$

[24] $P_s(t)$ and $P_m(t)$ also equal $Y_s(t)$ and $Y_m(t)$ respectively.

where to save in notation we use $p(z,w)=p$. Simplyfiying:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right)\\
=\;&\frac{\partial w/\partial t}{P_m^2(t)}\\
&\left\{\left[\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}\frac{\partial p}{\partial w}dz+\int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}\frac{\partial p}{\partial w}dz\right]dF(\theta)\right]\right.\\
&\quad\left[\left(1-F(\hat{\theta})\right)\int_{-\infty}^{\underline{z}_l}pdz+\int_0^{\hat{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)}pdzdF(\theta)\right]\\
&-\left[\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}pdz+\int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}pdz\right]dF(\theta)\right]\\
&\left.\quad\left[\left(1-F(\hat{\theta})\right)\int_{-\infty}^{\underline{z}_l}\frac{\partial p}{\partial w}dz+\int_0^{\hat{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)}\frac{\partial p}{\partial w}dzdF(\theta)\right]\right\}
\end{aligned}$$

Notice that when t is such that $\underline{z}_l(t)$, $\underline{z}_h(\theta,t)<\hat{z}$ all θ:

$$\begin{aligned}
&\left(1-F(\hat{\theta})\right)\int_{-\infty}^{\underline{z}_l}\frac{\partial p}{\partial w}dz+\int_0^{\hat{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)}\frac{\partial p}{\partial w}dzdF(\theta)\\
=\;&0
\end{aligned}$$

It is also easy to see that when t is such that $\underline{z}_l(t)$, $\underline{z}_h(\theta,t)>\hat{z}$ all θ:

$$\begin{aligned}
&\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}\frac{\partial p}{\partial w}dz+\int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}\frac{\partial p}{\partial w}dz\right]dF(\theta)\\
=\;&\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}pdz+\int_0^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}pdz\right]dF(\theta)
\end{aligned}$$

In both case we have that:

$$\frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right)>0.$$

To prove the general case it is useful to define $\tilde{\theta}$ such that $\underline{z}_h\left(\tilde{\theta},t\right)=\hat{z}$, i.e., we have $\underline{z}_h\left(\theta,t\right)>\hat{z}$ for $\theta\in[0,\tilde{\theta})$ and $\underline{z}_h\left(\theta,t_0\right)<\hat{z}$ for $\theta\in(\tilde{\theta},\hat{\theta}]$.

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{P_s\left(t\right)}{P_m\left(t\right)}\right)\\
=\;&\frac{\partial w/\partial t}{P_m^2\left(t\right)}\\
&\left\{\left[\begin{array}{c}\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}\frac{\partial p}{\partial w}dz+\\ \int_0^{\tilde{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}\frac{\partial p}{\partial w}dz\right]dF(\theta)+\int_{\tilde{\theta}}^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}\frac{\partial p}{\partial w}dz\right]dF(\theta)\end{array}\right]\right.\\
&\left[\left(1-F(\hat{\theta})\right)\int_{-\infty}^{\underline{z}_l}pdz+\right.\\
&\left.\int_0^{\tilde{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)}pdzdF(\theta)+\int_{\tilde{\theta}}^{\hat{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)}pdzdF(\theta)\right]\\
&-\left[\begin{array}{c}\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l}pdz+\\ \int_0^{\tilde{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}pdz\right]dF(\theta)+\int_{\tilde{\theta}}^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)}pdz\right]dF(\theta)\end{array}\right]\\
&\left[\left(1-F(\hat{\theta})\right)\int_{-\infty}^{\underline{z}_l}\frac{\partial p}{\partial w}dz+\right.\\
&\left.\left.\int_0^{\tilde{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)}\frac{\partial p}{\partial w}dzdF(\theta)+\int_{\tilde{\theta}}^{\hat{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)}\frac{\partial p}{\partial w}dzdF(\theta)\right]\right\}
\end{aligned}
$$

Using that $\frac{\partial p}{\partial w} = \frac{1}{w}p$ for all high-skill goods:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right) \\
= \;& \frac{\partial w/\partial t}{P_m^2(t)}\frac{1}{w} \\
&\left\{\left[\begin{array}{c}\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l} pdz + \\ \int_0^{\tilde{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)} pdz\right]dF(\theta) + \int_{\tilde{\theta}}^{\hat{\theta}}\left[\int_{\hat{z}}^{\bar{z}_h(\theta)} pdz\right]dF(\theta)\end{array}\right]\right. \\
&\left[\left(1-F(\hat{\theta})\right)\int_{-\infty}^{\underline{z}_l} pdz + \right. \\
&\left.\int_0^{\tilde{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)} pdzdF(\theta) + \int_{\tilde{\theta}}^{\hat{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)} pdzdF(\theta)\right] \\
&-\left[\begin{array}{c}\left(1-F(\hat{\theta})\right)\int_{\underline{z}_l}^{\bar{z}_l} pdz + \int_0^{\tilde{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)} pdz\right]dF(\theta) \\ + \int_{\tilde{\theta}}^{\hat{\theta}}\left[\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)} pdz\right]dF(\theta)\end{array}\right] \\
&\left.\left[\left(1-F(\hat{\theta})\right)\int_{\hat{z}}^{\underline{z}_l} pdz + \int_0^{\tilde{\theta}}\int_{\hat{z}}^{\underline{z}_h(\theta)} pdzdF(\theta)\right]\right\}
\end{aligned}$$

Defining $A_1 = \int_0^{\tilde{\theta}}\int_{-\infty}^{\hat{z}} pdzdF(\theta)$, $B_1 = \int_0^{\tilde{\theta}}\int_{\hat{z}}^{\underline{z}_h(\theta)} pdzdF(\theta)$, $C_1 = \int_0^{\tilde{\theta}}\int_{\underline{z}_h(\theta)}^{\bar{z}_h(\theta)} pdzdF(\theta)$, $A_2 = \int_{\tilde{\theta}}^{\hat{\theta}}\int_{-\infty}^{\underline{z}_h(\theta)} pdzdF(\theta)$, $B_2 = \int_{\tilde{\theta}}^{\hat{\theta}}\int_{\underline{z}_h(\theta)}^{\hat{z}} pdzdF(\theta)$, $C_2 = \int_{\tilde{\theta}}^{\hat{\theta}}\int_{\hat{z}}^{\bar{z}_h(\theta)} pdzdF(\theta)$, $A_3 = \left(1-F\left(\hat{\theta}\right)\right)\int_{-\infty}^{\hat{z}} pdz$, $B_3 = \left(1-F\left(\hat{\theta}\right)\right)\int_{\hat{z}}^{\underline{z}_l} pdz$, $C_3 = \left(1-F\left(\hat{\theta}\right)\right)\int_{\underline{z}_l}^{\bar{z}_l} pdz$, we can rewrite the previous expression in more compact form:

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right) \\
= \;& \frac{\partial w/\partial t}{P_m^2(t)}\frac{1}{w} \\
&\left\{\left[C_3 + C_1 + C_2\right]\left[A_3 + B_3 + A_1 + B_1 + A_2\right]\right. \\
&\left.-\left[C_3 + C_1 + B_2 + C_2\right]\left[A_3 + B_3 + B_1\right]\right\}
\end{aligned}$$

$$
\begin{aligned}
& \frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right) \\
= \; & \frac{\partial w/\partial t}{P_m^2(t)}\left(1+\frac{1}{n}\right)\frac{1}{w} \\
& \left\{\left[C_3+C_1+C_2\right]\left[A_1+A_2\right]-B_2\left[A_3+B_3+B_1\right]\right\}
\end{aligned}
$$

Therefore,

$$
\frac{\partial}{\partial t}\left(\frac{P_s(t)}{P_m(t)}\right) \overset{s}{=} \frac{C_1+C_2+C_3}{B_1+B_3}-\frac{B_2}{A_1+A_2+A_3}
$$

Intuitively, the inflation of services is higher than the inflation of manufactured goods if and only if the output share of skilled services in total services is greater than the output share of skilled manufactured sectors in total manufacturing. Defining $\tilde{B}_1=\int_0^{\tilde{\theta}}\int_{\hat{z}}^{\bar{z}_l} pdzdF(\theta)$, $\tilde{B}_2=\int_{\tilde{\theta}}^{\hat{\theta}}\int_{-\infty}^{\hat{z}} pdzdF(\theta)$, $\tilde{C}_2=\int_{\tilde{\theta}}^{\hat{\theta}}\int_{\hat{z}}^{\bar{z}_l} pdzdF(\theta)$ we have:

$$
\begin{aligned}
\frac{C_1+C_2+C_3}{B_1+B_3} &> \frac{\tilde{C}_2}{\tilde{B}_1+B_3} \\
&= \frac{\int_{\tilde{\theta}}^{\hat{\theta}}\int_{\hat{z}}^{\bar{z}_l} pdzdF(\theta)}{\int_0^{\tilde{\theta}}\int_{\hat{z}}^{\bar{z}_l} pdzdF(\theta)+\left(1-F\left(\hat{\theta}\right)\right)\int_{\hat{z}}^{\bar{z}_l} pdz} \\
&= \frac{\int_{\tilde{\theta}}^{\hat{\theta}} dF(\theta)}{\int_0^{\tilde{\theta}} dF(\theta)+\left(1-F\left(\hat{\theta}\right)\right)}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{B_2}{A_1+A_2+A_3} &< \frac{\tilde{B}_2}{A_1+A_2} \\
&= \frac{\int_{\tilde{\theta}}^{\hat{\theta}}\int_{-\infty}^{\bar{z}_l} pdzdF(\theta)}{\int_0^{\tilde{\theta}}\int_{-\infty}^{\bar{z}_l} pdzdF(\theta)+\left(1-F\left(\hat{\theta}\right)\right)\int_{-\infty}^{\bar{z}_l} pdz} \\
&= \frac{\int_{\tilde{\theta}}^{\hat{\theta}} dF(\theta)}{\int_0^{\tilde{\theta}} dF(\theta)+\left(1-F\left(\hat{\theta}\right)\right)}.
\end{aligned}
$$

Implying:

$$\frac{C_1 + C_2 + C_3}{B_1 + B_3} > \frac{B_2}{A_1 + A_2 + A_3}.$$

■

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