

# The Risk-Adjusted Cost of Financial Distress

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## ABSTRACT

Financial distress is more likely to happen in bad times. The present value of distress costs therefore depends on risk premia. We estimate this value using risk-adjusted default probabilities derived from corporate bond spreads. For a BBB-rated firm, our benchmark calculations show that the risk-adjusted NPV of distress is 4.5% of pre-distress firm value. In contrast, a valuation that ignores risk premia produces an NPV of 1.4%. We show that risk-adjusted, marginal distress costs can be as large as the marginal tax benefits of debt derived by Graham (2000). Thus, distress risk premia can help explain why firms appear to use debt conservatively.

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Financial distress has both direct and indirect costs (Warner, 1977, Altman, 1984, Franks and Touros, 1989, Weiss, 1990, Asquith, Gertner and Scharfstein, 1994, Opler and Titman, 1994, Sharpe, 1994, Denis and Denis, 1995, Gilson, 1997, Andrade and Kaplan, 1998, Maksimovic and Phillips, 1998). However, whether such costs are high enough to matter for corporate valuation practices and capital structure decisions is hotly debated. Direct costs of distress, such as litigation fees, are relatively small.<sup>1</sup> Indirect costs, such as loss of market share (Opler and Titman, 1994) and inefficient asset sales (Shleifer and Vishny, 1992), are believed to be more important, but they are also much harder to quantify. In a sample of highly leveraged firms, Andrade and Kaplan (1998) estimate losses in value given distress on the order of 10% to 23% of pre-distress firm value.<sup>2</sup>

Irrespective of their exact magnitudes, ex-post losses due to distress must be capitalized to assess their importance for ex-ante capital structure decisions. The existing literature argues that even if ex-post losses amount to 10% to 20% of firm value, ex-ante distress costs are modest because the probability of financial distress is very small for most public firms (Andrade and Kaplan, 1998, Graham, 2000). In this paper, we propose a new way of calculating the NPV of financial distress costs. Our results show that the existing literature has substantially underestimated the magnitude of ex-ante distress costs.

A standard method of calculating ex-ante distress costs is to multiply Andrade and Kaplan's (1998) estimates of ex-post costs by historical probabilities of default (Graham, 2000, Molina, 2005), but this calculation ignores capitalization and discounting. Other researchers have assumed risk neutrality and have discounted the product of historical probabilities and losses in value given default by a risk-free rate (e.g., Altman (1984)).<sup>3</sup> This calculation, however, ignores the fact that distress is more likely to occur in bad times.<sup>4</sup> Thus, risk-averse investors should care more about financial distress than is suggested by risk-free valuations. Our goal in this paper is to quantify the impact of distress risk premia on the NPV of distress costs.

Our approach is based on the following insight: to the extent that financial distress costs occur in states of nature in which bonds default, one can use corporate bond prices to estimate the distress risk adjustment. The asset pricing literature has provided substantial evidence for a systematic component in corporate default risk. It is well known that the spread between corporate and government bonds is too high to be explained only by expected default and that it reflects in part a large risk premium (Elton, Gruber, Agrawal and Mann, 2001, Huang and Huang, 2003, Longstaff,

Mittal, and Neis, 2005, Driessen, 2005, Chen, Collin-Dufresne, and Goldstein, 2005, Cremers, Driessen, Maenhout and Weinbaum 2005, Berndt, Douglas, Duffie, Ferguson, and Schranz, 2005).<sup>5</sup>

As in standard calculations, our new methodology assumes the estimates of ex-post distress costs provided by Andrade and Kaplan (1998) and Altman (1984). Unlike those calculations, however, this method uses observed credit spreads to back out the market-implied, risk-adjusted (or risk-neutral) probabilities of default. Such an approach is common in the credit risk literature (i.e., Duffie and Singleton, 1999, and Lando, 2004). Our calculations also consider tax and liquidity effects (Elton et al., 2001, Chen, Lesmond, and Wei, 2004) and use only the fraction of the spread that is likely to be due to default risk.

Our estimates suggest that risk-adjusted probabilities of default and, consequently, the risk-adjusted NPV of distress costs, are considerably larger than historical default probabilities and the non risk-adjusted NPV of distress, respectively. Consider for instance a firm whose bonds are rated BBB. In our data, the historical 10-year cumulative probability of default for BBB bonds is 5.22%. However, in our benchmark calculations the 10-year cumulative risk-adjusted default probability implied by BBB spreads is 20.88%. This large difference between historical and risk-adjusted probabilities translates into a substantial difference in NPVs of distress costs. Using the average loss in value given distress from Andrade and Kaplan (1998), our NPV formula implies a risk-adjusted distress cost of 4.5%. For the same ex-post loss, the non risk-adjusted NPV of distress is only 1.4% for BBB bonds.

Our results have implications for capital structure. In particular, they suggest that the marginal, risk-adjusted distress costs can be of the same magnitude as the marginal tax benefits of debt computed by Graham (2000). For example, using our benchmark assumptions the increase in risk-adjusted distress costs associated with a change in ratings from AA to BBB is 2.7% of pre-distress firm value.<sup>6</sup> To compare this number with marginal tax benefits of debt, we derive the marginal tax benefit of leverage that is implicit in Graham's (2000) calculations and use the relationship between leverage ratios and bond ratings recently estimated by Molina (2005). The implied gain in tax benefits as the firm moves from an AA to a BBB rating is 2.67% of firm value. Thus, it is not clear that the firm gains much by increasing leverage from AA to BBB levels.<sup>7</sup> These large estimated distress costs may help explain why many US firms appear to be conservative in their use of debt, as suggested by Graham (2000).

This paper proceeds as follows. We first present a simple example of how our valuation approach works. The general methodology is presented in Section II, followed by our empirical estimates of the NPV of distress costs in Section III, and various robustness checks in Section IV. Section V discusses the capital structure implications of our results, and we summarize our findings in Section VI.

## I. Using Credit Spreads to Value Distress Costs: A Simple Example

In this section, we explain our procedure with a simple example. The purpose of the example is both to illustrate the intuition behind our general procedure (Section II) and to provide simple back-of-the-envelope formulas that can be used to value financial distress costs. The formulas are easy to implement and provide a reasonable approximation of the more precise formulas derived later. We start with a one-period example and then present an infinite horizon example.

### A. One-period Example

Suppose that we want to value distress costs for a firm that has issued an annual-coupon bond maturing in exactly one year. The bond's yield is equal to  $y$ , and the bond is priced at par. The bond's recovery rate, which is known with certainty today, is equal to  $\rho$ . Thus, if the bond defaults, creditors recover  $\rho(1 + y)$ . The bond's valuation tree is depicted in Figure 1. The value of the bond equals the present value of expected future cash flows, adjusted for systematic default risk. If we let  $q$  be the risk-adjusted (or risk-neutral), one-year probability of default, we can express the bond's value as:

$$1 = \frac{(1 - q)(1 + y) + q\rho(1 + y)}{1 + r^F}, \quad (1)$$

where  $r^F$  is the one-year risk-free rate.

[Insert Figure 1 here]

In the valuation formula (1), the probability  $q$  incorporates the default risk premium that is implicit in the yield spread  $y - r^F$ . If investors were risk neutral, or if there was no systematic

default risk,  $q$  would be equal to the expected probability of default which we denote by  $q$ . If default risk is priced, then the implied  $q$  is higher than Equation (1) can be solved for  $q$ :

$$q = \frac{y - r^F}{(1 + y)(1 - \rho)}. \quad (2)$$

The basic idea in this paper is that we can use the risk-neutral probability of default,  $q$ , to perform a risk-adjusted valuation of financial distress costs. Consider again Figure 1, which also depicts the valuation tree for distress costs. Let the loss in value given default be equal to  $\phi$  and the present value of distress costs be equal to  $\Phi$ . For simplicity, suppose that  $\phi$  is known with certainty today. If we assume that financial distress can only happen at the end of one year, but never again in future years, then we can express the present value of financial distress costs as:

$$\Phi = \frac{q\phi + (1 - q)0}{1 + r^F}. \quad (3)$$

Formula (3) is similar to that used by Graham (2000) and Molina (2005) to value distress costs. The key difference is that while Graham (2000) and Molina (2005) used *historical* default probabilities, equation (3) uses a risk-adjusted probability of financial distress that is calculated from yield spreads and recovery rates using equation (2).

### B. *Infinite Horizon Example*

To provide a more precise estimate of the present value of financial distress costs, we must recognize that if financial distress does not occur at the end of the first year, it can still happen in future years. If we assume that the marginal risk-adjusted default probability  $q$  and the risk-free rate  $r^F$  do not change after year one,<sup>8</sup> then the valuation tree becomes a sequence of one-year trees that are identical to that depicted in Figure 1. This implies that if financial distress does not happen in year one (an event that happens with probability  $1 - q$ ) the present value of future distress costs at the end of year one is again equal to  $\Phi$ . Replacing 0 with  $\Phi$  in the valuation equation (3) and solving for  $\Phi$ , we obtain:

$$\Phi = \frac{q}{q + r^F} \phi. \quad (4)$$

Equation (4) provides a better approximation of the present value of financial distress costs than does equation (3). Notice also that for a given  $q$  (that is, irrespective of the risk adjustment), equation (3) substantially underestimates the present value of distress costs.

The assumptions that  $q$  and  $r^F$  do not vary with the time horizon are counterfactual. The general procedure that we describe later allows for a term structure of  $q$  and  $r^F$ . For the purpose of illustration, however, suppose that  $q$  and  $r^F$  are indeed constant. In the appendix, we spell out the conditions under which equation (2) can be used to obtain the (constant) risk-adjusted probability of default  $q$ .

To illustrate the impact of the risk adjustment, take the example of BBB-rated bonds. In our data, the historical average 10-year spread on those bonds is approximately 1.9%, and the historical average recovery rate is equal to 0.41.<sup>9</sup> As we discuss in the next section, the credit risk literature suggests that this spread cannot be attributed entirely to default losses because it is also affected by tax and liquidity considerations. Essentially, our benchmark calculations remove 0.51% from this raw spread.<sup>10</sup> The difference (1.39%) is what is usually called the default component of yield spreads. Using this default component, a recovery of 0.41, and a long-term interest rate of 6.7% (the average 10-year treasury rate in our data), equation (2) gives an estimate for  $q$  equal to 2.2%. Using historical data to estimate the marginal default probability yields much lower values. For example, the average marginal default probability over time horizons from 1 to 10 years for bonds of an initial BBB rating is equal to 0.53% (Moody's, 2002). The large difference between risk-neutral and historical probabilities suggests the existence of a substantial default risk premium.

As discussed in the introduction, the literature has estimated ex-post losses in value given default (the term  $\phi$ ) of 10% to 23% of pre-distress firm value. If we use, for example, the midpoint between these estimates ( $\phi = 16.5\%$ ), the NPV of distress for the BBB rating goes from 1.2% (using historical probabilities) to 4.1% (using risk-adjusted probabilities). Clearly, incorporating the risk adjustment makes a large difference to the valuation of financial distress costs. We now turn to the more general model to see if this conclusion is robust.

## II. General Valuation Formula

Figure 2 illustrates the timing of the general model that we use to value financial distress costs. Our goal is to calculate  $\Phi_0$ , the NPV of distress costs at an initial date (date 0). In Figure 2,  $\phi_{0,t}$  is the deadweight loss that the firm incurs if distress happens at time  $t$ , where  $t = 1, 2, \dots$

[Insert Figure 2 here]

In all of our analysis, we assume that distress states and default states are the same. Thus, our calculations apply to the distress costs that are incurred upon or after default. This assumption is consistent with the results in Andrade and Kaplan (1998), who report that 26 out of the 31 distressed firms in their sample either default or restructure their debt in the year that the authors classify as the onset of financial distress. Nonetheless, we acknowledge that our approach might not capture some of the indirect costs of distress that are incurred before default (i.e., Titman (1984)). To be consistent with Andrade and Kaplan (1998), who measure the value lost at the onset of distress, we define  $\phi_{0,t}$  as the expectation at time 0 of the capitalized distress costs that occur after default at time  $t$ . After default, the firm might reorganize or it might be liquidated. If the firm does not default at time  $t$ , it moves to period  $t + 1$ , and so on.

We let  $q_{0,t}$  be the risk-adjusted, marginal probability of distress (default) in year  $t$ , conditional on no default until year  $t - 1$ , and evaluated as of date 0. In contrast with Section I, we now allow  $q_{0,t}$  to vary with the time horizon. We also define  $(1 - Q_{0,t}) = \prod_{s=1}^t (1 - q_{0,s})$  as the risk-adjusted probability of surviving beyond year  $t$ , evaluated as of date 0. Conversely,  $Q_{0,t}$  is the cumulative risk-adjusted probability of default before or during year  $t$ . The probability that default occurs exactly at year  $t$  is thus equal to  $(1 - Q_{0,t-1})q_{0,t}$ . Throughout the paper, we will maintain the following assumption:

ASSUMPTION 1: *The deadweight loss  $\phi_{0,t}$  in case of default is constant,  $\phi_{0,t} = \phi$ .*

In particular, this assumption implies that there is no systematic risk associated with  $\phi$ . Assumption 1 could lead us to underestimate the distress risk adjustment if the deadweight losses conditional on distress are higher in bad times, as suggested by Shleifer and Vishny (1992). However, it is also possible that deadweight losses are higher in good times because financial distress might cause the firm to lose profitable growth options (Myers, 1977).

Under assumption 1, we can write the NPV of financial distress as:

$$\Phi_0 = \phi \sum_{t \geq 1} B_{0,t} (1 - Q_{0,t-1}) q_{0,t} , \quad (5)$$

where  $B_{0,t}$  is the price at time zero of a riskless zero-coupon bond paying one dollar at date  $t$ . Equation (5) gives the ex-ante value of financial distress as a function of the term structure of distress probabilities and risk-free rates. In Section III.D, we estimate the average value of  $\Phi_0$

using the historical average term structures of  $B_{0,t}$  and  $Q_{0,t}$ , and in Section IV.F we discuss the impact of time variation in the price of credit risk.

#### A. From Credit Spreads to Probabilities of Distress

As in Section I, we use observed corporate bond yields to estimate the risk-adjusted default probabilities used in equation (5). Specifically, suppose that we observe at date 0 an entire term structure of yields for the firm whose distress costs we want to value; that is, we know the sequence  $\{y_0^t\}_{t=1,2,\dots}$ , where  $y_0^t$  is the date-0 yield on a corporate bond of maturity  $t$ . In addition, suppose we know the coupons  $\{c^t\}_{t=1,2,\dots}$  associated with each bond maturity.<sup>11</sup> For now, we assume that the entire spread between  $y_0^t$  and the reference risk-free rate is due to default losses and relegate the discussion of tax and liquidity effects to Section III. By the definition of the yield, the date-0 value of the bond of maturity  $t$ ,  $V_0^t$ , is

$$V_0^t = \frac{c^t}{(1 + y_0^t)} + \frac{c^t}{(1 + y_0^t)^2} + \dots + \frac{1 + c^t}{(1 + y_0^t)^t}. \quad (6)$$

##### A.1. Bond Recovery

We let  $\rho_\tau^t$  be the dollar amount recovered by creditors if default occurs at date  $\tau \leq t$  for a bond of maturity  $t$ . As discussed by Duffie and Singleton (1999), to obtain risk-neutral probabilities from the term structure of bond yields, we need to make specific assumptions about bond recoveries. Our benchmark valuation uses the following assumption, which was originally employed by Jarrow and Turnbull (1995):

ASSUMPTION 2: *Constant recovery of treasury (RT). In case of default, the creditors recover  $\rho_\tau^t = \rho P_\tau^t$ , where  $P_\tau^t$  is the price at date  $\tau$  of a risk-free bond with the same maturity and coupons as the defaulted bond and  $\rho$  is a constant.*

The idea behind assumption 2 is that default does not change the *timing* of the *promised* cash flows. When default occurs, the risky bond is effectively replaced by a risk-free bond whose cash flows are a fraction  $\rho$  of the cash flows promised initially. In Section IV.B, we discuss other assumptions commonly used in the credit risk literature and we show that our results are robust. The assumption that  $\rho$  is constant is similar to our previous assumption that  $\phi$  is constant. However, there is some evidence in the literature that recovery rates tend to be lower in bad times (Altman et

al., 2003, Allen and Saunders, 2004, Acharya et al., 2005). We verify in Section IV.A the robustness of our results to the introduction of recovery risk.

### A.2. Risk-Neutral Probabilities

Our next task is to derive the term structure of risk-neutral probabilities from observed bond prices. We do so recursively. Under assumption 2, the price  $V_0^1$  of a one-year bond must satisfy

$$V_0^1 = [(1 - Q_{0,1}) + Q_{0,1}\rho] (1 + c^1) B_{0,1}. \quad (7)$$

This equation gives  $Q_{0,1}$  as a function of known quantities. Given  $\{Q_{0,\tau}\}_{\tau=1..t}$ , we show in the appendix that the value of a bond with maturity  $t + 1$  is

$$V_0^{t+1} = \sum_{\tau=1}^t [(1 - Q_{0,\tau}) + Q_{0,\tau}\rho] c^{t+1} B_{0,\tau} + [(1 - Q_{0,t+1}) + Q_{0,t+1}\rho] (1 + c^{t+1}) B_{0,t+1}. \quad (8)$$

This equation can be inverted to obtain  $Q_{0,t+1}$ . Therefore, we can recursively derive the sequence of risk-adjusted probabilities  $\{Q_{0,t}\}_{t=1,2..}$  from  $\{V^t\}_{t=1,2..}$ ,  $\{c^t\}_{t=1,2..}$ ,  $\{B_{0,t}\}_{t=1,2..}$ , and  $\rho$ . This procedure allows us to generalize equation (2). The risk-adjusted probabilities can then be used to value distress costs using equation (5).

## III. Empirical Estimates

We begin by describing the data used in the implementation of equations (5) and (8).

### A. Data on Yield Spreads, Recovery Rates, and Default Rates

We obtained data on corporate yield spreads over treasury bonds from Citigroup's yield book, which covers the period 1985-2004. These data are available for bonds rated A and BBB, separately for maturities 1-3, 3-7, 7-10, and 10+ years. For bonds rated BB and below, these data are available only as an average across all maturities. Because the yield book records AAA and AA as a single category, we rely on Huang and Huang (2003) to obtain separate spreads for the AAA and AA ratings. Table I in Huang and Huang reports average 4-year spreads for 1985-1995 from Duffee (1998) and average 10-year spreads for the period 1973-1993 from the Lehman's bond index. For consistency, we calculate our own averages from the yield book over the period 1985-1995, but we note that average spreads are similar over periods 1985-1995 and 1985-2004.<sup>12</sup> For all ratings, we

linearly interpolate the spreads to estimate the maturities that are not available in the raw data. We assume constant spreads across maturities for BB and B bonds. The spread data used in this study are reported in Table I.

[Insert Table I here]

Our benchmark valuation is based on the average historical spreads in Table I. Thus, the resulting NPVs of distress should be seen as *unconditional* estimates of ex-ante distress costs for each bond rating. We discuss the implications of time variation in yield spreads in Section IV.F.

We also obtained data on average treasury yields and zero coupon yields on government bonds of different maturities from FRED and from JP Morgan. Because high expected inflation in the 1980s had a large effect on government yields, we use a broad time period (1985-2004) to calculate these yields.<sup>13</sup> Treasury data are available for maturities 1, 2, 3, 5, 7, 10, and 20 years, and zero yields are available for all maturities between 1 and 10 years. Again, we used a simple linear interpolation for missing maturities between 1 and 10 years.

Finally, we obtained historical cumulative default probabilities from Moody's (2002). These data are available for 1-year to 17-year horizons for bonds of initial ratings ranging from AAA to B and refer to averages over the period 1970-2001. These default data are similar to those used by Huang and Huang (2003).<sup>14</sup> While these data are not used directly for the risk-adjusted valuations, they are useful for comparison purposes. Moody's (2002) also contains a time series of bond recovery rates for the period 1982-2001.<sup>15</sup> In most of our calculations we assume a constant recovery rate, which we set to its historical average of 0.413.

### *B. Default Component of Yield Spreads*

There is an ongoing debate in the literature about the role of default risk in explaining yield spreads such as those reported in Table I. Because treasuries are more liquid than corporate bonds, part of the spread should reflect a liquidity premium (see Chen et al., 2004). Also, treasuries have a tax-advantage over corporate bonds because they are not subject to state and local taxes (Elton et al., 2001). These arguments suggest that we cannot attribute the entire spreads reported in Table I to default risk.

Researchers have attempted to estimate the default component of corporate bond spreads using a number of different strategies. Huang and Huang (2003) use a calibration approach and found that the default component predicted by many structural models is relatively small.<sup>16</sup> In contrast, Longstaff et al. (2005) argue that credit default swap (CDS) premia are a good approximation of the default components, and suggest that the default component of spreads is much larger than that suggested by Huang and Huang. Chen et al. (2005) use structural credit risk models with a counter-cyclical default boundary and show that such models can explain the entire spread between BBB and AAA bonds when calibrated to match the equity risk premium. Cremers et al. (2005) add jump risk to a structural credit risk model that is calibrated using option data and generate credit spreads that are much closer to CDS premiums than those generated by the models in Huang and Huang. We summarize these recent findings in Table II. With the exception of Huang and Huang, the findings in these papers appear to be reasonably consistent with each other.

Unfortunately, these papers report default components only for a subset of ratings and maturities.<sup>17</sup> Thus, to implement formulas (5) and (8), we must first estimate the default component across all ratings and maturities. We now present two ways to do so.

### *B.1. Method 1: Using the 1-year AAA Spread*

Following Chen et al. (2005), we assume that the component of the spread that is *not* given by default can be inferred from the spreads between AAA bonds and treasuries. Chen et al. use a 4-year maturity in their calculations, but our data on historical default probabilities suggest that, while there has never been any default for AAA bonds up to a 3-year horizon, there is already a small probability of default at a 4-year horizon (0.04%). Thus, it seems appropriate to use a shorter spread to adjust for taxes and liquidity.<sup>18</sup> The 1-year spread in Table I is 0.51%, and we calculate the default components for rating  $i$  and maturity  $t$  as

$$(\text{Default component})_i^t = (\text{spread})_i^t - 0.51\%. \quad (9)$$

Notice that formula (9) allows us to construct spread default components for all ratings and maturities. Table II reports some of the fractions implied by this procedure for select maturities. By construction, the 4-year BBB fraction is virtually identical to that estimated by Chen et al.. Most of the other fractions are very close to those estimated by Longstaff et al. (2005) and Cremers et

al. (2005), suggesting that method 1 produces default components that closely approximate CDS premia. The only real discrepancy is with respect to Huang and Huang (2003), who estimate lower fractions for investment-grade bonds.

[Insert Table II here]

### B.2. Method 2: Using Spreads Over Swaps

As discussed above, Longstaff et al. (2005) argue that CDS premia are a good approximation for the default component of yield spreads. In addition, Blanco et al. (2005) show that the spread over swaps tracks CDS premia very closely. These results suggest that one can use spreads over swaps to estimate the default component. Unfortunately, data on swap rates start only in 2000. Therefore, we cannot use Huang and Huang's spread data (which refers to 1985-1995) and consequently can only provide fraction estimates for A, BBB, BB, and B-rated bonds. Using swap data for 2000-2004, we calculate the average default component for rating  $i$  and maturity  $t$  as

$$(\text{Fraction due to default})_i^t = \frac{(\text{spread})_i^t - (\text{swap}_t - \text{treasury}_t)}{(\text{spread})_i^t}. \quad (10)$$

Table II shows that this alternative approach gives fractions due to default that are very close to those obtained using the AAA spread of method 1.<sup>19</sup> Given these results, it seems safe to choose method 1 as our benchmark approach to calculate default components. An important advantage of method 1 is that it allows us to present valuations for all bond ratings, from AAA to B.

### C. Risk-Neutral Probabilities and Excess Returns

Starting from the spreads reported in Table I, we use equation (9) to estimate the default components, and then we use the default components to derive a term structure of risk-adjusted default probabilities. Each bond yield  $y_0^t$  is computed as the sum of the default component and the corresponding treasury rate. We must make an assumption about coupon rates in order to use equation (6). Our baseline calculations assume that corporate bonds trade at par, so that  $c^t = y_0^t$  and  $V_0^t = 1$  for all  $t$ . We then use equation (8) to generate a sequence of cumulative probabilities of default  $\{Q_{0,t}\}_{t=1,2,\dots,10}$ .

Table III reports the risk-adjusted cumulative default probabilities for select maturities. For comparison purposes, we also report the historical cumulative probabilities of default from Moody's (2002). The risk-adjusted, market-implied probabilities are larger than the historical ones for all ratings and maturities and are substantially so for investment-grade bonds. For instance, the 5-year historical default probability of BBB bonds is 1.95%, while the risk-neutral one is 11.39%. The ratio between risk-neutral and historical probabilities (averaged over maturities) ranges from 3.57 for AAA-rated bonds to 1.21 for B-rated bonds. These ratios indicate the presence of a large credit risk premium. Interestingly, the ratios are highest for investment-grade bonds, especially for the AA, A, and BBB ratings. Cremers et al. (2005) suggest one possible interpretation of this pattern: if the default risk premium is associated with a jump risk premium, it is perhaps not surprising that the risk premium is lower for bonds that are quite likely to default (i.e., BB and B ratings).

[Insert Table III here]

The evidence on holding period excess returns of corporate bonds is also consistent with the existence of the risk premium that we emphasize. Keim and Stambaugh (1986), for example, find that excess returns of BBB bonds over long term government bonds are on average 8 basis points a month in the period of 1928-1978. This excess return is equivalent to approximately 1% a year. Fama and French (1989, 1993) report similar summary statistics for average excess returns.<sup>20</sup> These numbers are largely consistent with the risk-neutral and historical probabilities in Table III. Consider for example the excess return on a zero-coupon security that promises one dollar in 5 years, and defaults like a BBB bond. The risk-adjusted and historical probabilities in Table III imply an annual expected excess return of 1.24% for this security,<sup>21</sup> which is close to the average historical excess returns that the literature reports.

#### *D. Valuation*

We can now use the term structure of risk-neutral probabilities computed in Section III.C in the valuation equation (5). Because we only have cumulative default probabilities up to year 10, we compute a terminal value of financial distress costs at year 10 (details in the appendix). The terminal value is computed by assuming constant marginal risk-adjusted default probabilities and

yearly risk-free rates after year 10. Thus, the formula is very similar to that derived in the infinite horizon example of Section I. As in Section I, we use  $\phi = 16.5\%$  in our benchmark calculations. Graham (2000) and Molina (2005) use numbers in this range to compare tax benefits of debt and costs of financial distress.

[Insert Table IV here]

The second column of Table IV presents our estimates of the risk-adjusted cost of financial distress for different bond ratings. For comparison, we report in the first column the same valuations using the historical default rates.<sup>22</sup> We find that risk is a first-order issue in the valuation of distress costs, which confirms the results of Section I. For instance, distress costs for the BBB rating increase from 1.40% to 4.53% once we adjust for risk. To provide some evidence on the marginal increase in distress costs as the firm moves across ratings, we also report the difference in distress costs between the BBB and the AA rating. An increase in leverage that moves a firm from AA to BBB increases the cost of distress by 2.7%. In contrast, the increase is only 1.11% if we use historical probabilities. Thus, risk adjustment also matters for marginal distress costs.

## IV. Robustness Checks

The estimates in Table IV rely on a set of assumptions about bond recoveries, coupon rates, and deadweight losses given distress. We now check how sensitive our results are to these assumptions.

### A. Recovery Risk

Following assumption 2, the benchmark valuation in the second column of Table IV uses  $\rho = 0.413$  in equation (8). The use of an average historical recovery is common in the credit risk literature. Huang and Huang (2003), Chen et al. (2005), and Cremers et al. (2005), for example, use average historical recoveries of 0.51 in their calibrations. However, there is some evidence in the literature of a systematic component of recovery risk (Altman et al., 2003, and Allen and Saunders, 2004). As discussed by Berndt et al. (2005) and Pan and Singleton (2005), a standard way to incorporate recovery risk into credit risk models is to use a constant *risk-neutral* (as opposed to average historical) recovery rate. Berndt et al. (2005) use a risk-neutral recovery rate of 0.25, which

is the *lowest* cross-sectional sample mean of recovery reported by Altman et al (2003). According to Pan and Singleton (2005), this is a common industry standard for the risk-neutral recovery rate.<sup>23</sup>

We note that the lower the recovery rate plugged into equation (8), the lower the implied risk-neutral probabilities. Low recoveries increase a creditor's loss given default, and thus for a given spread the implied probability of default is higher (see, for example, equation 2). The third column of Table IV reports the results of decreasing the recovery rate to 0.25 without changing the estimate for  $\phi$ . As expected, the risk-adjusted costs of financial distress decrease.<sup>24</sup> For example, the point estimate for the BBB rating goes from 4.53% to 3.70% if bond recovery goes from 0.41 to 0.25. Nonetheless, the risk adjustment is still large, and assuming a lower recovery does not affect the estimated *marginal* costs of distress much. For example, if bond recovery is 0.25, the increase in distress costs for a firm moving from AA to BBB is 2.2%, which is only slightly lower than the corresponding margin when recovery is 0.41 (2.7%). We conclude that our results are robust to the introduction of recovery risk.

### B. *Recovery of Face Value*

Equation 8 is derived under the assumption that recovery is a fraction of a similar risk-free bond assumption 2, or RT assumption). Another commonly used assumption is that recovery is a fraction of the face value of the bond, with zero recovery of coupons (assumption RFV). In the appendix, we show how to derive the term structure of risk-neutral probabilities from the default component of the spreads under assumption RFV. The fourth column of Table IV shows the valuation results with this alternative assumption. The implied risk-neutral probabilities of default are lower, and thus the valuation results are slightly lower than those obtained under RT. However, it is clear from the fourth column that the two assumptions generate very similar costs of financial distress. The AA minus BBB margin, for example, goes from 2.69% (under RT) to 2.47% (RFV). We conclude that the valuation is robust to alternative recovery assumptions.

### C. *Coupon Rates*

The risk-neutral probabilities in Table III are derived under the assumption that the bond coupons are equal to the adjusted bond yields (the default component of the yield plus the corresponding treasury rate). To show the robustness of our results, Table IV contains the valuations

assuming that coupons are equal to 0.5 times the adjusted yields (in the fifth column) or 1.5 times the adjusted yields (in the sixth column). Risk-adjusted probabilities, and thus risk-adjusted distress costs, are higher with higher coupons. However, it is clear from Table IV that the results are relatively robust to variations in coupon rates. The BBB minus AA margin, for example, goes from 2.64% (when coupons are 0.5 times the yield) to 2.77% (when coupons are 1.5 times the yield). Thus, changes in assumed coupon rates have small effects on the marginal costs of financial distress.

#### *D. Using Huang and Huang's (2003) Fractions*

As discussed in Section III.B, Huang and Huang (2003) estimate smaller default components of spreads than those we have used to construct Tables 3 and 4. Not surprisingly, using Huang and Huang's fractions leads to lower costs of financial distress, as shown in the seventh column. The difference is more pronounced for ratings between AAA and BBB. The BBB minus AA margin, for example, decreases to 1.65%. This margin is close to that calculated using historical probabilities. These results highlight the importance of more recent papers, such as those by Longstaff et al. (2005), Chen et al. (2005), and Cremers et al. (2005), which suggest that credit risk can explain a larger fraction of spreads.

#### *E. Changes in $\phi$*

Panel A of Table IV assumes that  $\phi=16.5\%$ , which is the midpoint of the 10-23% range reported in Andrade and Kaplan (1998). In Panel B of Table IV we report valuation results for the endpoints of this range.<sup>25</sup> Not surprisingly, direct changes in  $\phi$  have a large impact on the valuations, both for historical and risk-adjusted probabilities. For example, the risk-adjusted BBB valuation increases from 1.95% (if  $\phi = 10\%$ ) to 6.32% (if  $\phi = 23\%$ ). Because the impact of changes in  $\phi$  is higher if default probabilities are high, the effect on the margins is also large, especially when compared with the other assumptions in Table IV. The AA-BBB margin increases from 1.63% to 3.75% as  $\phi$  goes from 10% to 23%. Thus, it is important to consider a range of values for  $\phi$  in the capital structure exercises in the next section. On the other hand, the difference between historical and risk-adjusted valuations remains substantial, irrespective of  $\phi$ . For example, if  $\phi = 10\%$ , the increase in the BBB valuation that can be attributed to the risk adjustment is still equal to 1.90%. Thus, ignoring the risk adjustment substantially undervalues the costs of distress, for all  $\phi$  values in this range.

F. *Time Variation in Spreads*<sup>26</sup>

We have conducted our analysis using average historical spreads to calculate risk-adjusted probabilities. Conceptually, we have answered the following question: what are the costs of financial distress for an average firm about to be created, assuming that aggregate business conditions are and will remain at historical averages?

In reality, however, the market price of credit risk (as captured by credit spreads) varies over time (see Berndt et al. (2005), and Pan and Singleton (2005)). This insight has two important implications for our paper. First, we might underestimate the size of the risk adjustment because a risk-adjusted ex-ante valuation should put more probability weight on episodes of high spreads than on those of low spreads. Second, the (conditional) NPV of financial distress costs will change over time as credit spreads vary, and this may change the optimal leverage.

To understand these points more clearly, consider Figure 3. Figure 3 depicts a simple example that we use to gain some intuition; it is not a full-fledged model. Suppose that there are two periods, and three dates. We assume that the firm makes its leverage decision at time 0. We must then compute the NPV of distress costs at that date. At time 1, an aggregate shock is realized, which affects the market price of risk and the future risk neutral default rates: agents learn that  $q$  is either high,  $q^h$ , or low,  $q^l$ . At time 2, financial distress occurs with probability  $q = q^h$  or  $q^l$ .

[Insert Figure 3 here]

The first point we want to discuss is the bias from using the historical average instead of the correct risk neutral average. Let  $x^Q$  be the *risk neutral* probability that  $q$  jumps to  $q^h$  at time 1. The correct ex-ante NPV of financial distress would then be

$$\Phi^Q = \frac{x^Q q_H + (1 - x^Q) q_L}{(1 + r_F)^2} \phi \quad (11)$$

However, the risk-adjusted valuation that we performed in Section II used *historical average* spreads to compute risk-adjusted probabilities of distress. In the example in Figure 3, the naive NPV of distress using that methodology would be

$$\Phi^P = \frac{x^P q_H + (1 - x^P) q_L}{(1 + r_F)^2} \phi \quad (12)$$

where  $x^P$  is the true (historical) probability that  $q$  jumps to  $q^h$  at time 1. In reality, investors are likely to assign a risk premium to the uncertainty about spreads<sup>27</sup>. In other words, it is likely that  $x^Q > x^P$ . In this case, equations (11) and (12) show that our previous calculations underestimate the true average NPV of financial distress.

The second point that Figure 3 helps clarify is that distress costs depend on the state realized at time 1. If the firm could adjust its capital structure at time 1, it would make different choices in the high and low states because the NPV of distress costs is larger in the high state.

Developing a full-fledged model of the time variation in  $q$  is beyond the scope of this paper. However, we feel that it might be useful to have some sense of the potential impact of time variation in spreads on conditional distress costs. We therefore present some back-of-the-envelope calculations. As described in Section III.A, we have monthly time-series data from 1985 to 2004 for all ratings between A and B. We use these data to compute the standard deviation in spreads separately for each rating and maturity, as a fraction of average 1985-2004 spreads for that rating/maturity. These ratios range from 50% to 80% for A bonds (depending on maturity), 36% to 70% for BBB bonds, 38% for BB bonds, and 33% for B bonds. We then scale our benchmark average spreads, which are calculated using 1985-1995 data, uniformly up and down using these ratios. Using these scaled spreads, we repeat the valuation exercises performed in Sections III.C and III.D.<sup>28</sup> We emphasize that these calculations are only meant as an illustration. In particular, these valuations assume that the spreads remain indefinitely at the low and high levels.

With these caveats in mind, we find that the NPV of financial distress costs varies substantially between the high and low scenarios. For example, for BB bonds the NPV of distress goes from 4.73% (low spreads) to 8.38% (high spreads). The impact of time variation on margins, however, is less clear. For example, the difference in distress costs between A and BBB bonds is highest when spreads are low. It is equal to 0.89% if spreads are low, and 0.53% when spreads are high. On the other hand, the difference between A and BB bonds shows the opposite pattern. It is equal to 1.78% when spreads are low, and 3.73% when spreads are high. While these results are suggestive of the potential effect of time variation in spreads, more research is required to establish their exact impact on marginal distress costs and capital structure choices.

## V. Implications for Capital Structure

The existing literature suggests that distress costs are too small to overcome the tax benefits of increased leverage, and thus that corporations may be using debt too conservatively (Graham, 2000). This quote from Andrade and Kaplan (1998) captures well the consensus view:

“[...] from an ex-ante perspective that trades off expected costs of financial distress against the tax and incentive benefits of debt, the costs of financial distress seem low [...]. If the costs are 10 percent, then the expected costs of distress [...] are modest because the probability of financial distress is very small for most public companies.” (Andrade and Kaplan, page 1488-1489).

In other words, using estimates for  $\phi$  that are in the same range as those used in Table IV should produce relatively small NPVs of distress costs *because the probability of financial distress is too low*. In this section, we attempt to verify whether this conclusion continues to hold if we compare marginal, risk-adjusted costs of financial distress to marginal tax benefits of debt.

Naturally, the calculations that we perform in this section are subject to the limitations of the static trade-off model of capital structure. Our point is not to argue that this model is the correct one or to provide a full characterization of firms' optimal financial policies. We simply want to verify whether the magnitude of the distress costs that we calculate is comparable to that of tax benefits of debt. To compare the distress costs displayed in Table IV with the tax benefits of debt, we need to estimate the tax benefits that the average firm can expect at each bond rating. To do this, we follow closely the analysis in Graham (2000), who estimates the marginal tax benefits of debt, and Molina (2005), who relates leverage ratios to bond ratings.

#### A. *The Marginal Tax Benefit of Debt*

Graham (2000) estimates the marginal tax benefit of debt as a function of the amount of interest deducted, and calculates total tax benefits of debt by integrating under this function. The marginal tax benefit is constant up to a certain amount of leverage, and then it starts declining because firms do not pay taxes in all states of nature and because higher leverage decreases additional marginal benefits (as there is less income to shield). Essentially, we can think of the tax benefits of debt in Graham (2000) as being equal to  $\tau^*D$  (where  $\tau^*$  takes into account both personal and corporate taxes) for leverage ratios that are low enough such that the firm has not reached the point at which

marginal benefits start decreasing (see footnote 13 in Graham's paper). Graham calls this point the *kink* in the firm's tax benefit function. A firm with a kink of 2 can double its interest deductions and still keep a constant marginal benefit of debt.

In Graham's sample, the average firm in COMPUSTAT (in the time period 1980-1994) has a kink of 2.356 and a leverage ratio of approximately 0.34. He estimates that the average firm could have gained 7.3% of their market value if it had levered up to its kink. In addition, because the firm remains in the flat portion of the marginal benefit curve until its kink reaches one, these numbers allow us to compute the implied marginal benefit of debt in the flat portion of the curve ( $\tau^*$ ). If we assume that the typical firm needs to increase leverage by 2.356 times to move to a kink equal to one, we can back out the value of  $\tau^*$  as 0.157. Tax benefits of debt can then be calculated as 0.157 times the leverage ratio, assuming leverage is low enough that we remain in the flat portion. To the extent that the approximation is not true for high leverage ratios, we are probably overestimating tax benefits of debt for these leverage values.<sup>29</sup>

#### *B. The Relation Between Leverage and Bond Ratings*

To compute the tax benefits of debt at each bond rating, we need to assign a typical leverage ratio to each bond rating. As discussed by Molina (2005), the endogeneity of the leverage decision affects the relationship between leverage and ratings. In particular, because less risky and more profitable firms can have higher leverage without greatly increasing the probability of financial distress, the impact of leverage on bond ratings might appear to be too small.

[Insert Table V here]

The leverage data used in this exercise are reported in Table V. The first column reports Molina's predicted leverage values for each bond rating from his Table VI (Molina (2005), p. 1445). According to Molina, these values give an estimate of the impact of leverage on ratings for the average firm in Graham's sample. To verify the robustness of our results, we also use the simple descriptive statistics in Molina's (2005) Table IV (Molina (2005), p. 1442). Molina's data, which corresponds to the ratio of long-term debt to book assets for each rating in the period 1998-2002, are reported in the second column of Table V. As discussed by Molina, despite the aforementioned endogeneity

problem, the rating changes in these summary statistics actually resemble those predicted by the model. In addition, we report in the third column of Table V the relation between leverage and ratings that is used by Huang and Huang (2003). These leverage data come from Standard and Poor's (1999) and have been used by several authors to calibrate credit risk models (i.e., Cremers et al., 2005).

### C. Tax Benefits versus Distress Costs

Table VI depicts our estimates of the tax benefits of debt for each bond rating. If we use the leverage ratios from Molina's (2005) regression model (Panel A), the increase in tax benefits as the firm moves from the AA to the BBB rating is 2.67%. Under the benchmark valuation of distress costs (see Table IV), this marginal gain is of a similar magnitude as marginal *risk-adjusted* distress costs (2.69% according to Table IV). Analysis of Table IV also shows that the similarity between marginal tax benefits of debt and marginal financial distress costs holds, irrespective of our specific assumptions about coupons and recoveries as long as we use the benchmark assumption of  $\phi = 16.5\%$ .

[Insert Table VI here]

To further compare marginal tax benefits and distress costs, Table VI also reports the difference between the present value of tax benefits and the cost of distress for each bond rating. Under the static trade-off model of capital structure, the firm is assumed to maximize this difference. Because the specific assumption about  $\phi$  substantially affects marginal distress costs (see Panel B of Table IV), we report results obtained for  $\phi = 10\%$  and  $\phi = 23\%$ , as well as for the benchmark case of  $\phi = 16.5\%$ .

Table VI illustrates our conclusion that the distress risk adjustment substantially reduces the net gains that the average firm can expect from leveraging up. For example, if  $\phi = 16.5\%$  and we ignore the risk adjustment (second column), the firm can increase value by 3% to 4% if it levers up from zero to somewhere around a BBB bond rating. However, once we incorporate the distress risk adjustment, the net gain from leveraging up never goes above 1%. The gains from leveraging up are higher if  $\phi$  becomes closer to 10%, as shown in the third and fourth columns. However, the distress

risk adjustment substantially reduces the gains from leveraging up, even for these lower values of  $\phi$ . For values of  $\phi$  closer to 23% (fifth and sixth columns), marginal distress costs are uniformly higher than marginal tax benefits.

The second, and related, conclusion is that the distress risk adjustment generally moves the optimal bond rating generated by these simple calculations towards higher ratings. For example, if  $\phi = 16.5\%$  and we ignore the risk adjustment, a firm should increase leverage until it reaches a rating of A to BBB, because this rating is associated with the largest differences between tax benefits and distress costs. However, after incorporating the distress risk adjustment, the difference becomes essentially flat or decreasing for all ratings lower than AA. Naturally, the result is even stronger for higher values of  $\phi$ .

Both conclusions are driven by the finding that marginal risk-adjusted distress costs are very close to marginal tax benefits of debt. Figure 4 gives a visual picture of these results. In Figure 4 we plot the difference between tax benefits and distress costs for the benchmark case ( $\phi = 16.5\%$ ), both for non risk-adjusted and risk-adjusted distress costs. Clearly, the marginal gains from increasing leverage are very flat for any rating above AA if distress costs are risk adjusted. The visual difference with the inverted U-shape generated by the non risk-adjusted valuation is very clear.

[Insert Figure 4 here]

In Panel B of Table VI, we vary the relationship between leverage and ratings for the benchmark case of  $\phi = 16.5\%$ . The net gains from leveraging up are even lower than those in Panel A if we use Molina's summary statistics to compute marginal tax benefits of debt (first column). However, if we use historical probabilities to value financial distress costs, the firm can still gain around 3% in value by moving from zero leverage to a BBB rating (second column). These gains disappear once distress costs are risk adjusted (third column). Marginal tax benefits are higher if we use the leverage ratios from Huang and Huang (fourth column), resulting in large net gains from leverage if distress costs are not risk adjusted (fifth column). However, the sixth column shows that the difference between tax benefits and risk-adjusted distress costs is relatively flat, even for these leverage ratios. This difference increases from 1.73% (AAA rating) to a maximum of 2.26% for the BBB rating. We conclude that the results are robust to variations in the ratings-leverage relationship.

*D. Interpretation and Comparison with Previous Literature*

Table VI and Figure 4 show that risk-adjusted costs of financial distress can counteract the marginal tax benefits of debt estimated by Graham (2000). These results suggest that financial distress costs can help explain why firms use debt conservatively, as suggested by Graham. We note, however, that Graham’s evidence for debt conservatism is not based solely on the observation that the average firm appears to use too little debt. His data also show that firms that appear to have low costs of financial distress have lower leverage (higher kinks). Our results do not address this cross-sectional aspect of debt conservatism.

Molina (2005) argues that the bigger impact of leverage on bond ratings and probabilities of distress that he finds after correcting for the endogeneity of the leverage decision can also help explain why firms use debt conservatively. However, Molina does not perform a full-fledged valuation of financial distress costs, as we have done in this paper. His calculations are based on the same approximation of marginal costs of financial distress used by Graham (2000), which is to use  $\Phi = p\phi$ , where  $p$  is the 10-year cumulative historical default rate. As we discussed in Section I, this formula underestimates the NPV of financial distress costs, irrespective of the risk-adjustment issue.<sup>30</sup> Thus, we believe the results on Table VI and Figure 4 provide a more precise comparison between the NPV of distress costs and the capitalized tax benefits of debt.

## **VI. Final Remarks**

In this paper, we developed a method to estimate the present value of the costs of financial distress that takes into account the systematic component in the risk of distress. Our formulas are easy to implement (particularly those in Section I) and should be useful for research and teaching purposes. We find that the traditional practice of using historical default rates severely underestimates the average value of distress costs, as well as the effects of changes in leverage on marginal distress costs. The marginal distress costs that we find can help explain the apparent reluctance of firms to increase their leverage, despite the existence of substantial tax benefits of debt.

One caveat is that we risk-adjust distress costs using historical average bond spreads. There is evidence, however, of significant variation in credit risk premia over time (Berndt et al. (2005),

Pan and Singleton (2005)). Time variation in distress costs could lead firms to optimally reduce their leverage in times when credit spreads are high, as emphasized in the market timing literature.

The large risk adjustments found herein are derived from the significant risk premia that investors appear to require if they are to hold corporate bonds. These large risk premia might justify the reluctance of firms to lever up, if their goal is to maximize the wealth of risk-averse investors. Thus, our results suggest that bond spreads and capital structure decisions are mutually consistent. Considering simultaneously the option market and the bond market, Cremers et al. (2005) show that the implied volatilities and jump risks implicit in option prices can explain credit spreads across firms and over time. In other words, they find that corporate bond spreads and option prices are also mutually consistent. Taken together, these results suggest that market participants, from options and bonds traders to corporate managers, seem to respond similarly to the market price of risk.

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## Appendix. Proofs

*Proof of Equation (2) in the perpetuity example of Section I:* Suppose that the promised return on a perpetual bond is constant over time and that given default creditors recover a fraction of the bond's market value just prior to default, including the coupons that are due on the year that default occurs. This "recovery of market value" assumption comes from Duffie and Singleton (1999). In addition, we maintain the assumptions that the recovery rate is non-stochastic and that the risk-free rate is constant (equal to  $r^F$ ).

Let the bond's promised yearly return be equal to  $y$ . Without loss of generality, assume that the bond is priced at par such that the yearly coupon is also equal to  $y$ . Next year, if the bond does not default creditors receive a coupon equal to  $y$ . The value of the remaining promised payments is constant over time and equal to one. Thus, creditors receive  $(1 + y)$  if there is no default. The recovery of market value assumption implies that creditors will receive  $\rho(1 + y)$  if there is default in year one. Thus, the bond valuation tree is identical to that presented in Figure 1, and the bond's date zero value can be expressed by equation (1). Formula (2) then follows. Q.E.D.

*Proof of Equation (8):* To understand the recursive formula, consider a two-year bond. If the bond defaults in year 1, we have:

$$E^Q [\rho_1^2] = \rho [c^2 + (1 + c^2)B_{1,2}], \quad (A1)$$

where  $B_{1,2}$  is the date-1 price of a zero that pays one at date 2, and  $E^Q [\cdot]$  are expectations under the risk-neutral measure. If the bond defaults in year 2, we have  $E^Q [\rho_2^2] = \rho(1 + c^2)$ . We can then write the valuation equation for a two-period bond as:

$$\begin{aligned} V_0^2 = & (1 - Q_{0,1})c^2 B_{0,1} + Q_{0,1}\rho [c^2 + (1 + c^2)B_{1,2}] B_{0,1} + \\ & + [(1 - Q_{0,1})(1 - q_{0,2}) + (1 - Q_{0,1})q_{0,2}\rho] (1 + c^2)B_{0,2} . \end{aligned} \quad (A2)$$

Using the facts that  $B_{0,2} = B_{1,2}B_{0,1}$ ,  $(1 - Q_{0,1})(1 - q_{0,2}) = (1 - Q_{0,2})$ , and  $Q_{0,1} + (1 - Q_{0,1})q_{0,2} = Q_{0,2}$ , we can rewrite equation (A2) as

$$V_0^2 = [(1 - Q_{0,1}) + Q_{0,1}\rho]c^2 B_{0,1} + [(1 - Q_{0,2}) + Q_{0,2}\rho] (1 + c^2) B_{0,2} . \quad (A3)$$

Given  $Q_{0,1}$ , we can solve equation (A3) for  $Q_{0,2}$ . A similar reasoning leads to equation (8). Q.E.D.

*Terminal Value calculation (Section III.D):* We assume that the marginal, risk-adjusted probability of default is constant after year 10, that is:

$$q_{0,t} = q_{0,10} = 1 - \frac{(1 - Q_{0,10})}{(1 - Q_{0,9})}, \text{ for } t > 10. \quad (A4)$$

Similarly, we assume that the yearly zero coupon rate is constant after year 10, that is, the yearly risk-free rate after year 10 is given by:

$$r_{0,10}^F = \frac{B_{0,9}}{B_{0,10}} - 1 \quad (A5)$$

Given these assumptions, we can compute a terminal cost of financial distress at year 10. We can expand equation (5) as:

$$\Phi_0 = \phi \left[ \sum_{t=1}^{10} B_{0,t}(1 - Q_{0,t-1})q_{0,t} + (1 - Q_{0,10})q_{0,11}B_{0,11} + (1 - Q_{0,11})q_{0,12}B_{0,12} + \dots \right]. \quad (A6)$$

Using the assumptions that  $q_{0,t} = q_{0,10}$  and  $r_{0,t}^F = r_{0,10}^F$  for  $t > 10$ , we can write:

$$\Phi_0 = \phi \left[ \sum_{t=1}^{10} B_{0,t}(1 - Q_{0,t-1})q_{0,t} + \frac{B_{0,10}(1 - Q_{0,10})q_{0,10}}{q_{0,10} + r_{0,10}^F} \right]. \quad (A7)$$

*Different Recovery Assumptions (Section IV.B):* In addition to assumption 2, the credit risk literature has used the following assumptions about bond recoveries:

(i) *Recovery of face value (RFV)*:  $E(\rho_\tau^t) = E(\rho)$ . This assumption has been used by Brennan and Schwartz (1980) and Duffee (1998). In words, if default occurs at time  $\tau < t$ , creditors receive a fraction of face value immediately upon default. There is zero recovery of coupons.

(ii) *Recovery of market value (RMV)*:  $E(\rho_\tau^t) = E(\rho V_\tau^t)$ , where  $V_\tau^t$  is the market value prior to default at date  $\tau$  of the corporate bond, contingent on survival up to date  $\tau$ . This assumption comes from Duffee and Singleton (1999).

Duffee and Singleton (1999) compare risk-neutral probabilities that are generated by assumptions RMV and RFV, and find that the two alternative assumptions generate very similar results unless corporate bonds trade at significant premia or discounts or if the term structure of interest rates is steeply increasing or decreasing. For simplicity, we focus only on assumption RFV for the robustness checks. Under assumption RFV,  $E(\rho_\tau^t) = \rho$  for all  $\tau, t$ , and the valuation formula becomes:

$$V_0^{t+1} = c^{t+1} \sum_{\tau=1}^t (1 - Q_{0,\tau}) B_{0,\tau} + \rho \sum_{\tau=1}^{t+1} (1 - Q_{0,\tau-1}) q_{0,\tau} B_{0,\tau} + (1 + c^{t+1}) (1 - Q_{0,t+1}) B_{0,t+1} \quad (\text{A8})$$

Again, this formula can be easily inverted to obtain  $Q_{0,t+1}$  if one has the sequence  $\{Q_{0,\tau}\}_{\tau=1,t}$  and the yield on a coupon paying bond with maturity  $t + 1$ . Notice that  $Q_{0,0} = 1$  and that  $q_{0,t+1} = 1 - \frac{(1 - Q_{0,t+1})}{(1 - Q_{0,t})}$ .

## ENDNOTES

1. Warner (1977) and Weiss (1990), for example, estimate costs on the order of 3% to 5% of firm value at the time of distress.

2. Altman (1984) reports similar cost estimates of 11% to 17% of firm value three years prior to bankruptcy. However, Andrade and Kaplan (1998) argue that part of these costs might not be genuine financial distress costs, but rather consequences of the economic shocks that drove firms into distress. An additional difficulty in estimating ex-post distress costs is that firms are more likely to have high leverage and to become distressed if distress costs are expected to be low. Thus, any sample of ex-post distressed firms is likely to have low ex-ante distress costs.

3. Structural models in the tradition of Leland (1994) and Leland and Toft (1996) are typically written directly under the risk-neutral measure. Others (e.g., Titman and Tsyplakov (2004), and Hennessy and Whited (2005)) assume risk neutrality and discount the costs of financial distress by the risk-free rate. In either case, these models do not emphasize the difference between objective and risk-adjusted probabilities of distress.

4. More precisely, we mean to say that distress tends to occur in states in which the pricing kernel is high. As we discuss in the next paragraph and elsewhere in the paper, there is substantial evidence that default risk has a systematic component.

5. See also Pan and Singleton (2005) for related evidence on sovereign bonds.

6. For comparison purposes, the increase in marginal, non risk-adjusted distress costs is only 1.11%.

7. This conclusion generally holds for variations in the assumptions used in the benchmark valuations. The results are most sensitive to the estimate of losses given distress, as we show in Section IV.

8. In a multi-period model, the probability  $q_{0,t}$  should be interpreted as the *marginal*, risk-adjusted default probability in year  $t$ , *conditional* on survival up to year  $t - 1$ , and evaluated at date 0. In this simple example we assume that  $q_{0,t} = q$  for all  $t$ .

9. See Section III.A for a detailed description of the data.

10. This adjustment factor is the historical spread over treasuries on a one-year AAA bond. In Section III.B we discuss alternative ways to adjust for taxes and liquidity, and we argue that most (but not all) of them imply similar default component of spreads.

11. For simplicity, we use a discrete model in which all payments (coupons, face value, and recoveries) that refer to year  $t$  happen exactly at the end of year  $t$ .

12. For example, the average 10+ year spread for BBB bonds in the yield book data is 1.90% for both time periods. Average B-bond spreads are 5.45% if we use 1985-1995 and 5.63% if we use 1985-2004. In addition, the yield book data and the Huang and Huang data are similar for comparable ratings and maturities. For example, the 10 year spread for BBB bonds is 1.94% in Huang and Huang.

13. Some average treasury yields that we use are 5.74% (1-year), 6.32% (5-year), and 6.73% (10-year).

14. The default probabilities are calculated using a cohort method. For example, the 5-year default rate for AA bonds in year  $t$  is calculated using a cohort of bonds that were initially rated AA in year  $t - 5$ .

15. More specifically, these data refer to cross-sectional average recoveries for original issue speculative grade bonds.

16. In particular, Huang and Huang's results imply that the distress probabilities in Leland (1994) and Leland and Toft (1996) incorporate a relatively low risk-adjustment.

17. Chen et al. consider only BBB bonds in their analysis, while Longstaff et al. do not provide estimates for AAA and B bonds. In addition, Huang and Huang (2003) provide estimates for 4- and 10-year maturities only, while Longstaff et al. and Chen et al. consider only one maturity

(5-year and 4-year, respectively). Cremers et al. (2005) report 10-year credit spreads for ratings between AAA and BBB.

18. In any case, the difference between 1-year and 4-year AAA spreads (0.04%) is negligible, so using the 4-year spread would produce virtually identical results.

19. In fact, AAA spreads are very close to the difference between swap and treasury rates (see Feldhutter and Lando (2005) for some additional evidence on this point). Thus, it is not surprising that both methods give similar results.

20. More recently, Saita (2006) also finds high holding period returns and Sharpe ratios for portfolios of corporate bonds.

21. To compute this number, we use the same assumptions about recoveries and risk free rates that were used to compute the probabilities in Table III.

22. Notice that equation (5) only requires default probabilities and risk-free rates to translate  $\phi$  estimates into NPV estimates. We assume that the historical marginal default probability is fixed after year 10 for each rating to compute a terminal value, and we estimate the long-term marginal default probability as the average marginal probability between years 10 and 17.

23. Pan and Singleton (2005) use the term structure of sovereign CDS spreads to separately estimate risk-neutral recoveries and default intensities and estimate recovery rates that are larger than the commonly used value of 0.25.

24. Recall, however, that we are also assuming a constant  $\phi$ . If the reason for a low value of  $\rho$  in bad times is precisely a high value of  $\phi$ , then it is less clear that using historical values for  $\rho$  and  $\phi$  leads us to overestimate distress costs.

25. Notice that unlike the robustness checks above, which only affect risk-adjusted probabilities, these variations also impact the valuation using historical probabilities.

26. We thank our referee for suggesting this discussion to us.

27. See Pan and Singleton (2005) for evidence on the risk premium associated with time variation in default probabilities for sovereign bonds.

28. In these exercises, we keep all parameters fixed at their benchmark values, including recovery rates (0.41), losses given distress (0.165), and risk-free rates.

29. These tax benefit calculations also ignore risk adjustments. We derived a risk adjustment in a previous version of the paper, assuming perpetual debt. If  $D$  is taken to be the market value of debt, the risk adjustment does not have a substantial effect on Graham's formula because it is already incorporated in  $D$ . In fact, with zero recovery rates the interest tax shields are exactly a fraction  $\tau$  of the cash flows to bondholders in all states, and thus by arbitrage the value of tax benefits must be exactly equal to  $\tau D$ . With non-zero recovery, there is a risk adjustment that reduces tax benefits, but it is quantitatively small.

30. In addition, there are two differences between our calculations and those performed by Molina. First, his marginal tax benefits of debt are smaller than those we use because he uses more recent data from Graham that implies a  $\tau^*$  of around 13%. Second, when comparing marginal tax benefits with marginal costs of distress (Table VII), he uses the *minimum* change in leverage that induces a rating downgrade. In contrast, we use *average* leverage values for each rating in Table VII.

**Table I**  
**Term Structure of Yield Spreads**

This Table gives the spread data used in this study. The spread data for A, BBB, BB and B bonds come from Citigroup's yieldbook, and refers to average corporate bond spreads over treasuries, for the period 1985-1995. The original data contains spreads for maturities 1-3 years, 3-7 years, 7-10 years and 10+ years for A and BBB bonds. We assign these spreads, respectively, to maturities 2, 5, 8 and 10, and we linearly interpolate the spreads to estimate the maturities that are not available in the raw data. The spreads for BB and B bonds are reported as an average across all maturities. Data for AAA and AA bonds comes from Huang and Huang (2003). The original data contains maturities 4 (1985-1995 averages, from Duffee, 1998), and 10 (1973-1993 averages, from Lehman's bond index). We linearly interpolate to estimate the maturities that are not available in the raw data.

Maturity	Ratings					
	AAA	AA	A	BBB	BB	B
1	0.51%	0.52%	1.09%	1.57%	3.32%	5.45%
2	0.52%	0.56%	1.16%	1.67%	3.32%	5.45%
3	0.54%	0.61%	1.23%	1.76%	3.32%	5.45%
4	0.55%	0.65%	1.30%	1.85%	3.32%	5.45%
5	0.56%	0.69%	1.38%	1.94%	3.32%	5.45%
6	0.58%	0.74%	1.28%	1.89%	3.32%	5.45%
7	0.59%	0.78%	1.18%	1.84%	3.32%	5.45%
8	0.60%	0.82%	1.08%	1.79%	3.32%	5.45%
9	0.62%	0.87%	1.20%	1.84%	3.32%	5.45%
10	0.63%	0.91%	1.32%	1.90%	3.32%	5.45%

**Table II**  
**Fraction of the Yield Spread Due to Default**

This Table reports the fractions of yield spreads over benchmark treasury bonds that are due to default, for each credit rating and different maturities. The first column uses Huang and Huang (2003)'s Table 7, which reports calibration results from their model under the assumption that market asset risk premia are counter-cyclically time varying. The second column uses Longstaff et al.'s (2005) Table IV, which reports model-based ratios of the default component to total corporate spread. The third column uses results from Chen et al. (2005). The fraction reported for BBB bonds is the ratio of the BBB minus AAA spread over the BBB minus treasury spread. The fourth column uses results from Cremers et al. (2005). The fractions reported are the ratios between the 10-year spreads in Cremers et al.'s Table 4 (model with priced jumps), and the corresponding 10-year spreads in Table I of this paper. The fifth and sixth columns report for each rating and maturity the ratio between the default component of the spread and the total spread, where the default component is calculated as the spread minus the 1-year AAA spread. The seventh and eighth columns report for each rating and maturity the ratio between the default component of the spread and the total spread, where the default component is calculated as the spread minus the difference between swap and treasury rates, for the period 2000-2004. NA = not available.

	Huang and Huang (2003)	Longstaff et al. (2005)	Chen et al. (2005)	Cremers et al. (2005)	Method 1 (AAA spread)		Method 2 (spreads over swaps)	
Credit Rating	10-year spread	5-year spread	4-year spread	10-year spread	4-year spread	10-year spread	5-year spread	10-year spread
AAA	0.208	NA	0.000	0.603	0.073	0.190	NA	NA
AA	0.200	0.510	NA	0.505	0.215	0.440	NA	NA
A	0.234	0.560	NA	0.512	0.609	0.613	0.511	0.570
BBB	0.336	0.710	0.702	0.627	0.724	0.731	0.732	0.729
BB	0.633	0.830	NA	NA	0.846	0.846	0.872	0.872
B	0.833	NA	NA	NA	0.906	0.906	0.916	0.916

**Table III**  
**Risk Neutral and Historical Default Probabilities**

This Table reports cumulative risk-neutral probabilities of default calculated from bond yield spreads, as explained in the text. The Table also reports historical cumulative probabilities of default (data from Moodys, averages 1970-2001), and ratios between the risk-neutral probabilities and the historical ones for 5-year and 10-year maturities. In the last column, we report the average ratio between risk-neutral and historical probabilities across all maturities from 1 to 10.

Credit Rating	5-year			10-year			Average Ratio
	Historical	Risk-Neutral	Ratio	Historical	Risk-Neutral	Ratio	
AAA	0.14%	0.54%	3.83	0.80%	1.65%	2.07	3.57
AA	0.31%	1.65%	5.31	0.96%	6.75%	7.04	6.22
A	0.51%	7.07%	13.86	1.63%	12.72%	7.80	9.95
BBB	1.95%	11.39%	5.84	5.22%	20.88%	4.00	4.84
BB	11.42%	21.07%	1.85	21.48%	39.16%	1.82	1.86
B	31.00%	34.90%	1.13	46.52%	62.48%	1.34	1.21

**Table IV**  
**Risk-Adjusted Costs of Financial Distress**

This Table reports our estimates of the NPV of the costs of financial distress expressed as a percentage of pre-distress firm value, calculated using historical probabilities (first column), and risk-adjusted probabilities (remaining columns). It also reports in the last row the increase in the NPV of distress costs that is associated with a rating change from AA to BBB. In Panel A we use an estimate for the loss in value given distress of 16.5%. The valuation in the second column (benchmark valuation) assumes recovery of treasury, a recovery rate of 0.41, bond coupons equal to the default component of the yields, and uses method 1 (1-year AAA spread) to calculate the default component of spreads. In the third column we change the recovery rate to 0.25. In the fourth column we use a recovery of face value (RFV) assumption. In the fifth column we assume that coupons are one half times the default component of spreads, and in the sixth column we assume that coupons are one and a half times the default component of spreads. In the seventh column we use Huang and Huang's (2003) fractions due to default to calculate the default component of spreads. In Panel B we vary the estimate for the loss in value given distress, and report the NPV of distress costs calculated using historical probabilities (first, third and fifth columns), and risk-adjusted probabilities (remaining columns). The risk-adjusted valuations make the same assumptions as the benchmark valuation in Panel A. In the first and second columns we assume a loss given default of 16.5%. In the third and fourth columns we assume a loss given default of 10% and in the fifth and sixth columns we assume a loss given default of 23%.

Panel A ( $\phi = 0.165$ )

Credit Rating	Historical	Benchmark	Recovery 0.25	RFV	Coupon 0.5*Yield	Coupon 1.5*Yield	Huang and Huang (2003)
AAA	0.25%	0.32%	0.25%	0.31%	0.06%	0.50%	0.49%
AA	0.29%	1.84%	1.47%	1.77%	1.52%	2.07%	0.63%
A	0.51%	3.83%	3.17%	3.66%	3.49%	4.10%	1.14%
BBB	1.40%	4.53%	3.70%	4.24%	4.29%	4.71%	2.28%
BB	4.21%	6.81%	5.59%	6.15%	6.70%	6.88%	5.52%
B	7.25%	9.54%	8.04%	8.44%	9.47%	9.58%	9.15%
BBB minus AA	1.11%	2.69%	2.23%	2.47%	2.77%	2.64%	1.65%

Table IV (cont.)  
Risk-Adjusted Costs of Financial Distress

Panel B (variations in  $\phi$  )

	$\phi = 0.165$		$\phi = 0.10$		$\phi = 0.23$	
Credit Rating	Historical	Risk-adjusted	Historical	Risk-adjusted	Historical	Risk-adjusted
AAA	0.25%	0.32%	0.15%	0.19%	0.35%	0.45%
AA	0.29%	1.84%	0.17%	1.11%	0.40%	2.56%
A	0.51%	3.83%	0.31%	2.32%	0.71%	5.34%
BBB	1.40%	4.53%	0.85%	2.75%	1.95%	6.32%
BB	4.21%	6.81%	2.55%	4.13%	5.87%	9.50%
B	7.25%	9.54%	4.39%	5.78%	10.10%	13.30%
BBB minus AA	1.11%	2.69%	0.67%	1.63%	1.55%	3.75%

## Table V Typical Leverage Ratios for Each Bond Rating

This Table reports typical leverage ratios calculated for different bond ratings. The first two columns are drawn from Molina (2005). The first column shows predicted book leverage ratios from Molina's Table VI. These values are calculated using Molina's regression model (Table V), with values of the control variables set equal to those of the average firm with a kink of approximately two in Graham's (2000) sample. Column II replicates the book leverage ratios in the simple summary statistics of Molina's Table IV. Column III reports average leverage ratios for firms of a given credit rating, from Huang and Huang (2003). The original source of these data is Standard and Poor's (1999).

Molina (2005)			
Credit rating	Summary statistics	Regression model	Huang and Huang (2003)
AAA	9.00%	3.00%	13.08%
AA	17.00%	16.00%	21.18%
A	22.00%	28.00%	31.98%
BBB	28.00%	33.00%	43.28%
BB	34.00%	46.00%	53.53%
B	42.00%	57.00%	65.70%

## Table VI Tax Benefits of Debt against Costs of Financial Distress

This Table reports tax benefits of debt and the difference between tax benefits of debt and costs of financial distress. The risk-adjusted valuations assume recovery of treasury, a recovery rate of 0.41, bond coupons equal to the default component of the yields, and uses method 1 (1-year AAA spread) to calculate the default component of spreads. In Panel A, the relation between ratings and leverage is estimated using Molina's (2005) regression model. This relation is reported in this paper in the first column of Table VI. The first column depicts tax benefits of debt calculated for each leverage ratio as explained in the text. The remaining columns show the difference between tax benefits and distress costs. In the second and third columns we assume that losses given default are equal to 16.5%. In the fourth and fifth columns we assume that losses given default are equal to 10%, and in the sixth and seventh columns we assume a loss given default of 23%. In Panel B, we also report results that obtain when we change the relationship between leverage and bond ratings. The second to fourth columns use Molina's (2005) summary statistics and the fifth to seventh columns use the relation between leverage and ratings reported by Huang and Huang (2003). We assume a loss given default equal to 0.165 in all calculations reported in Panel B.

### Panel A: Predicted Leverage Ratios from Molina (2005)

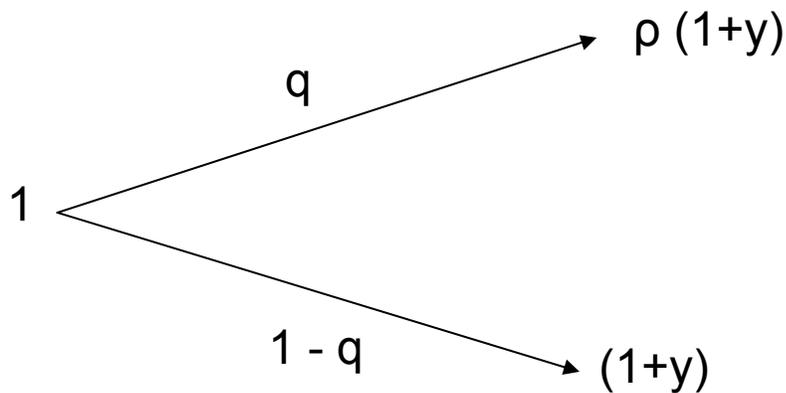
		Tax benefits minus cost of distress					
		$\phi = 0.165$		$\phi = 0.10$		$\phi = 0.23$	
Credit rating	Tax benefits of debt	Historical	Risk-adjusted	Historical	Risk-adjusted	Historical	Risk-adjusted
AAA	0.47%	0.22%	0.15%	0.32%	0.28%	0.12%	0.02%
AA	2.51%	2.22%	0.67%	2.34%	1.40%	2.11%	-0.05%
A	4.40%	3.89%	0.56%	4.09%	2.07%	3.69%	-0.95%
BBB	5.18%	3.78%	0.65%	4.33%	2.43%	3.23%	-1.14%
BB	7.22%	3.01%	0.41%	4.67%	3.09%	1.35%	-2.28%
B	8.95%	1.70%	-0.59%	4.56%	3.17%	-1.15%	-4.35%
BBB minus AA	2.67%						

Table VI (cont.)  
Tax Benefits of Debt against Costs of Financial Distress

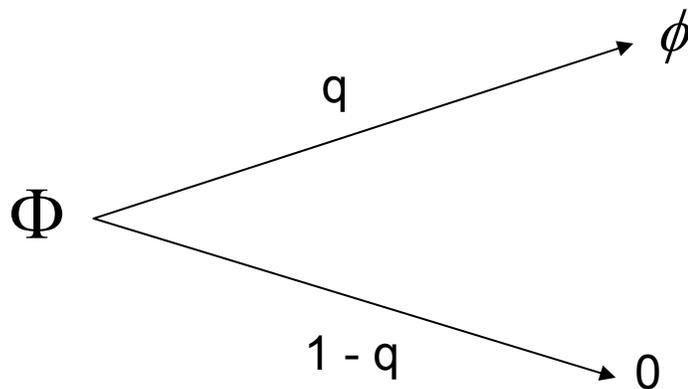
Panel B:  $\phi = 0.165$ , Variations in Leverage Ratios

Molina's (2005) summary statistics				Huang and Huang (2003)		
Credit rating	Tax benefits	Tax benefits minus cost of distress		Tax benefits	Tax benefits minus cost of distress	
		Historical	Risk-adjusted		Historical	Risk-adjusted
AAA	1.41%	1.16%	1.09%	2.05%	1.80%	1.73%
AA	2.67%	2.38%	0.83%	3.33%	3.04%	1.49%
A	3.45%	2.94%	-0.38%	5.02%	4.51%	1.19%
BBB	4.40%	3.00%	-0.14%	6.79%	5.40%	2.26%
BB	5.34%	1.13%	-1.48%	8.40%	4.19%	1.59%
B	6.59%	-0.65%	-2.95%	10.31%	3.07%	0.77%
BBB minus AA	1.73%			3.47%		

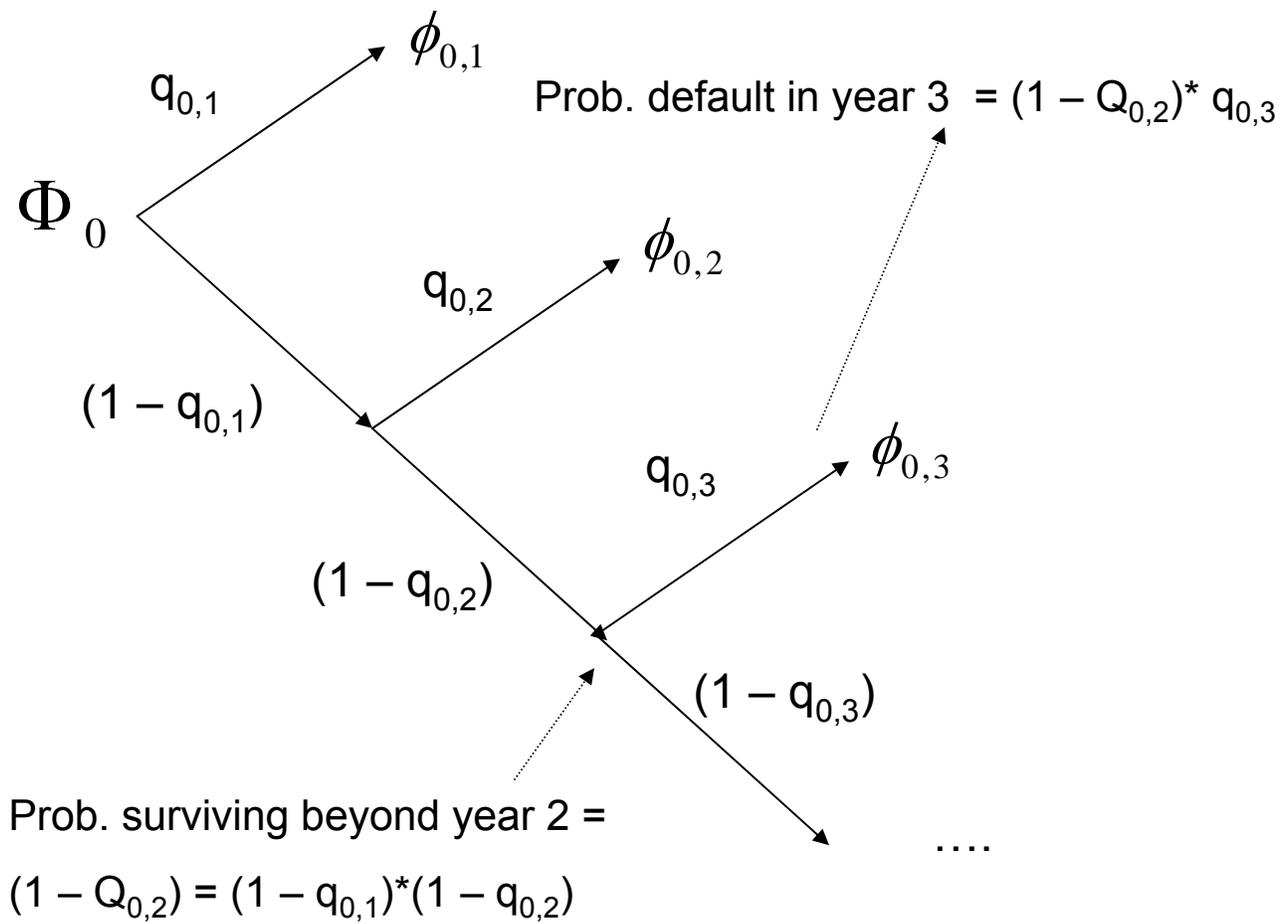
A. One-year par bond valuation tree



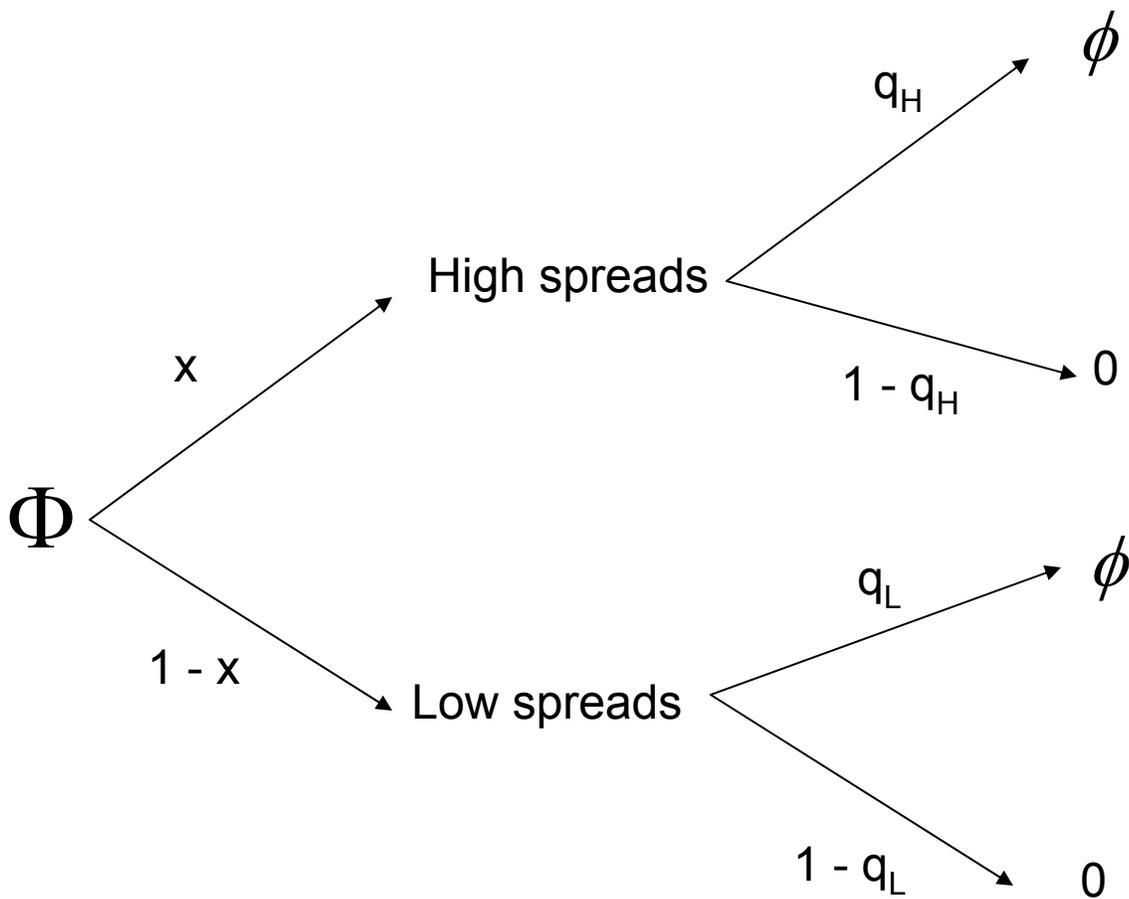
B. One-year valuation tree for distress costs



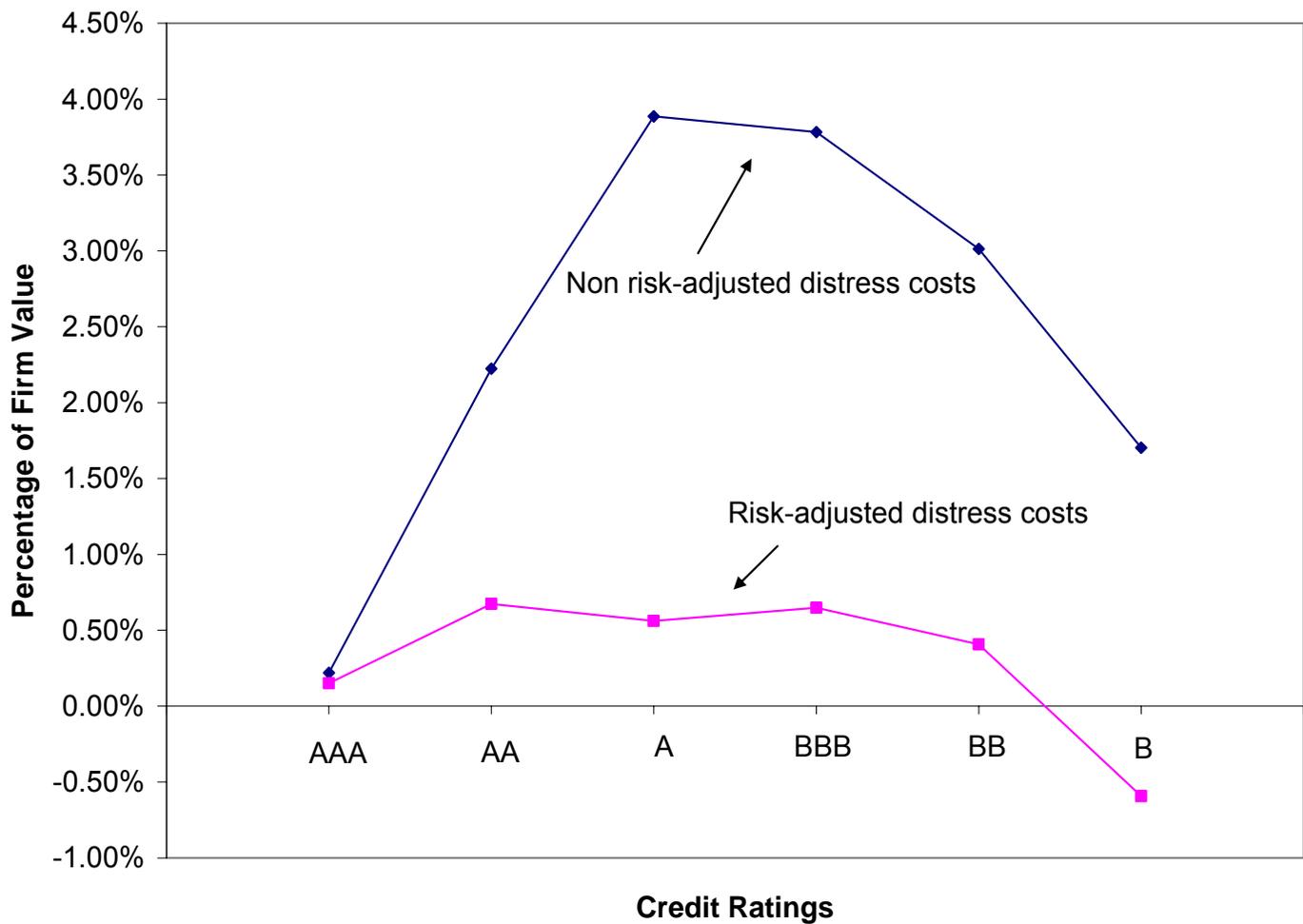
**Figure 1. Valuation trees, one-period example.** This figure shows the trees for the valuations described in Section I.A. Panel A shows the payoff for bond investors, and Panel B shows the deadweight costs of financial distress in default and non-default states. The 1-year risk-adjusted probability of default is equal to  $q$ .



**Figure 2: Valuation tree, general model.** This figure shows the valuation tree for the model in Section II. It shows the time evolution of deadweight costs of financial distress, for a firm that is currently at the initial node (date 0). The subscripts (0,t) refer to the current date and to a future default date (date t). The probability  $q_{0,t}$  is thus the risk-adjusted, marginal probability of default in year t, conditional on no default until year t-1, and evaluated as of date 0.



**Figure 3: Valuation tree, time variation in spreads.** This figure shows the valuation tree for the model in Section IV.F. It shows the time evolution of spreads and risk-adjusted default probabilities, for a firm that is currently at the initial node (time 0). The probability that spreads will be high next period is equal to  $x$ . The probability  $q_H$  is the probability of default in time 2 conditional on high spreads, and  $q_L$  is the probability of default in time 2 conditional on low spreads.



**Figure 4:** This figure shows the difference between the present value of tax benefits of debt and the NPV of distress costs, expressed as a percentage of pre-distress firm value, as a function of the firm's bond rating. The upper curve uses the NPV of distress costs calculated with historical probabilities of default, and the lower curve uses the NPV of distress that is calculated with risk-adjusted default probabilities. The present value of tax benefits assumes the marginal tax benefits estimated by Graham (2000), and uses the relation between leverage and bond ratings estimated by Molina (2005).