DEBT AND MANAGERIAL RENTS IN A REAL-OPTIONS MODEL OF THE FIRM

Abstract

We present a theory of capital investment and debt and equity financing in a real-options model of a public corporation. The model assumes that managers maximize the present value of their future compensation (managerial rents), subject to constraints imposed by outside shareholders' property rights to the firm's assets. We show that managers adopt an optimal debt policy that generates efficient investment and disinvestment decisions. Optimal debt equals the liquidation value of the firm's assets and is therefore default-risk free. But managers' personal wealth constraints can justify additional risky debt to fund positive-NPV investments. Changes in cash flow can cause changes in investment by tightening or loosening the wealth constraints.

Keywords: investment policy, financing policy, debt, agency (JEL: G31, G32)

1 Introduction

The separation of ownership and control in public corporations creates serious potential agency problems. Given these problems, it is perhaps surprising that most public firms are able to raise large amounts of outside equity and deliver significant value to shareholders. It is also surprising how few corporate meltdowns happen in developed economies, even in occasional episodes of "irrational exuberance."

Of course these agency problems are mitigated by several practical mechanisms, including management compensation schemes, boards of directors, takeovers and the market for corporate control, auditing and disclosure requirements and legal sanctions for managers who lie or steal. Evidently these mechanisms work to some extent.

We argue that the efficiency and staying power of public corporations can also reflect a natural alignment of the interests of outside shareholders and inside managers. We show how alignment works in a dynamic, open-horizon model of a public firm. The model gives managers (and possibly other stakeholders) the ability to capture cash flow. Investors hold property rights to the assets of the firm, however, and managers pay out part of cash flow after debt service in order to prevent intervention by investors. The difference between cash flow and payout goes to insiders as managerial rents. Overall value is nevertheless maximized, providing that the firm adheres to an optimal debt policy in which debt equals the liquidation value of the firm's assets, that is, the assets' value in their best alternative use. Given the liquidation value, optimal debt is fully collateralized, default-risk free and linked to the "book" rather than the market value of the firm. Risky debt is not needed to discipline management. Managers implement the optimal debt policy voluntarily, except when limited personal wealth forces them to resort to additional risky debt in order to finance investment.

Managers' wealth is relevant because investors cannot prevent managers from capturing part of the future cash flows generated by new investment. Therefore the managers have to coinvest. (We show that the amount of available equity financing always falls short of the investment outlay, even when NPV is positive.) Coinvestment can be financed by

corporate borrowing. When the firm borrows, the managers are in effect pledging part of their future rents to service the debt. The commitment is credible because the debt is senior to the managers' claim on future cash flows. A similar pledge to shareholders does not work, because a promise by managers to capture less and pay out more in the future is not contractible.

If managers' wealth constraints are binding at the optimal debt level, then they delay investment *and* use risky debt when the investment is finally made. Risky debt is inefficient because it forces managers to disinvest too early, that is, at a demand level higher than the level for first-best disinvestment.

If the firm does end up with risky debt, it can be stuck with it. It is not in managers' or stockholders' interests to use cash flow to pay down risky debt. Doing so would transfer value to the remaining bondholders. This free-rider problem is a version of the debt overhang problem noted by Myers (1977). Capital released by disinvestment is used to pay down debt, however, and not used to increase immediate dividend payout. Thus our model generates a form of dividend smoothing when the firm disinvests.

When the firm is expanding and managers' wealth constraints are binding, changes in operating cash flow can cause changes in investment by tightening or loosening the managers' wealth constraints. This link between cash flow and investment is not created by information problems or other imperfections that prevent the firm from accessing financial markets. The link reflects managers' inability to use their future rents as collateral, except through corporate borrowing. Thus our theory may help explain the observation by Kaplan and Zingales (1997) and others that changes in investment are sensitive to changes in cash flow (controlling for changes in investment opportunities), even for firms that appear to have easy access to financing.

Our modeling setup follows Myers (2000), Jin and Myers (2005) and Lambrecht and Myers (2005), but none of these papers undertakes a detailed analysis of both investment and financing. This paper is closest to Lambrecht and Myers (2005), which presents a theory of takeovers in industries where firms sooner or later have to shrink and release

capital to investors. Lambrecht and Myers focus on contraction, not expansion, and consider debt financing only in passing. We do adapt Lambrecht and Myers's continuous-time, infinite-horizon model, however.

This paper contributes to the literature on agency problems and financing, payout and investment decisions by public corporations. Prior contributions include Jensen (1986), Stulz (1990), Zwiebel (1996) and Morellec (2004). Jensen (1986) argues that managers prefer to capture and invest free cash flow rather than paying it out. High levels of debt solve this free cash flow problem by forcing payout of cash. Managers do not like debt, but may borrow in order to avoid control challenges. Stulz (1990) assumes that managers always want to invest, but that debt financing can be prearranged to limit the cash available for investment. The optimal amount of cash, which determines optimal financing, trades off the costs of over- or underinvestment when the firm ends up with too much or too little cash relative to investment opportunities. Zwiebel (1996) derives a dynamically consistent capital structure as the optimal response of partially entrenched managers who always want to invest but also fear takeovers. Managers voluntarily issue risky debt, accepting bankruptcy risk as a way of committing to forgo bad investments. Morellec (2004) develops a continuous-time, real-options model in which managers want to invest and also to retain control. Managers avoid control challenges by limiting negative-NPV investment.

Our modeling strategy is different from these papers. We do not invoke private benefits or assume that managers always want to expand. The managers' benefits are inside our model and are valued endogenously. The threat of bankruptcy is not needed to discipline managers. Also, unlike Stulz (1990), we do not assume that outside shareholders can influence financing policy directly. We agree that the threat of takeovers can discipline managers, but takeovers are covered in Lambrecht and Myers (2005) and not considered here.

This paper continues a line of research that uses real-options models to analyze financing and investment decisions. Other papers include Fisher, Heinkel, and Zechner (1989), Mello and Parsons (1992), Leland (1994), Anderson and Sundaresan (1996), Mella-Barral

and Perraudin (1997), Leland (1998), Parrino and Weisbach (1999), Morellec (2004), Childs, Mauer, and Ott (2005) and Mauer and Sarkar (2005). However, with the exception of Morellec (2004), these papers all focus exclusively on conflicts of interests between shareholders and lenders, which are material only when the probability of default is high. We focus on the agency problem faced by all public corporations: the separation of ownership by outside stockholders and control by inside managers.

The outline of the paper is as follows. In section 2 we set out our basic model and assumptions for an all-equity firm. Then we investigate how debt affects managers' closure decisions. We start with closure rather than expansion, because the NPV of a new investment project depends on how the project will be managed, in particular on the managers' decision rule for shutting down the project when its performance degrades. We show that debt financing always speeds up closure and we derive the optimal debt level that forces efficient closure.

Section 3 examines financing for new investment and expansion. If managers are not wealth constrained, the optimal debt policy derived in section 2 still applies. We explore the effects of managers' wealth constraints on the timing of investment and managers' willingness to issue risky debt. We confirm our results when the firm has assets in place and one or more options to invest in additional projects.

Section 4 discusses the implications of our theory and its relationships to standard theories of corporate financing. We recap our conclusions and note some areas for further research.

2 Financing and disinvestment

Consider a firm that generates an operating profit of $\pi\, x_t \;-\; f$ per period, where f is the fixed cost of operations, π is a constant revenue parameter and x_t is a geometric Brownian motion representing exogenous demand shocks:

$$dx_t \;=\; \mu x_t dt \;+\; \sigma x_t dB_t \tag{1}$$

Here μ is a drift term and σ measures the volatility of demand. All investors are risk neutral and can borrow and lend at the risk-free rate of interest, r (with $r > \mu$). As demand falls, the firm will at some point close down. We assume that closure is irreversible and that it releases the firm's stock of capital, which has a constant liquidation value K. With no debt, shareholders have exclusive property rights to the assets in liquidation and receive K at closure. If the all-equity financed firm were managed by the stockholders, then the firm would be closed when demand falls to a value-maximizing threshold $\underline{x}^o$. This first-best closure policy leads to a first-best firm value $V^o(x)$.

The firm is, however, run by managers, who set its payout and closure policies.[1] Managers act in their own interest to maximize the present value of the expected cash flows (managerial rents) that they will take from the firm's operations. But the managers are subject to the threat of collective action by the shareholders, who can either close the firm or manage it privately if they decide to take over. Collective action is costly. If shareholders have to mobilize to take control, they realize only $\alpha V^o(x) = \alpha \max[V^0(x), K]$, where $0 < \alpha < 1$. Thus the threat of collective action constrains the managers, but the cost of collective action creates the space for managerial rents. The size of the space is determined by $1 - \alpha$.[2]

The following proposition is proved and discussed in detail in Lambrecht and Myers (2005):

Proposition 1 *Assume that firm U is all-equity financed. Shareholders face a cost of collective action, but if they absorb that cost and take control of the firm, they can run the firm efficiently or shut it down. If the managers shut down the firm, its capital stock*

[1] We follow Myers (2000) and Lambrecht and Myers (2005), who assume that managers act as a risk-neutral, long-lived, value-maximizing coalition. We could also assume that managers have access to complete financial markets and thus interpret payoffs as certainty equivalents.

[2] The space allows managers to get a return on personal investments in firm-specific human capital. If such investments are important, then the value of the firm can depend on α (see Myers (2000), especially section III). We do not model human-capital investment in this paper, although we discuss it below.

is automatically returned to investors. Then the values of the firm and shareholders' and managers' claims are:

$$\begin{aligned}
V_U(x) &= \tfrac{\pi x}{r-\mu} - \tfrac{f}{r} + \left[K + \tfrac{f}{r} - \tfrac{\pi \underline{x}_U}{r-\mu}\right]\left(\tfrac{x}{\underline{x}_U}\right)^{\lambda} && \text{for} \quad x > \underline{x}_U \\
&= K && \text{for} \quad x \leq \underline{x}_U \\
E_U(x) &= \alpha V^o(x) + (1-\alpha)K\left(\tfrac{x}{\underline{x}_U}\right)^{\lambda} && \text{for} \quad x > \underline{x}_U \\
&= K && \text{for} \quad x \leq \underline{x}_U \\
R_U(x) &= V_U(x) - E_U(x)
\end{aligned}$$

The first-best firm value $V^o(x)$ and closure policy $\underline{x}^o$ are:

$$\begin{aligned}
V^o(x) &= \frac{\pi x}{r-\mu} - \frac{f}{r} + \left[K + \frac{f}{r} - \frac{\pi \underline{x}^o}{r-\mu}\right]\left(\frac{x}{\underline{x}^o}\right)^{\lambda} && \text{for } x > \underline{x}^o \\
&= K && \text{for } x \leq \underline{x}^o \\
\underline{x}^o &= \frac{-\lambda\left(K + \frac{f}{r}\right)(r-\mu)}{(1-\lambda)\pi} && (2)
\end{aligned}$$

where λ is the negative root of the characteristic equation $\frac{1}{2}\sigma^2 p(p-1) + \mu p = r$. The managers' closure threshold $\underline{x}_U$ is:

$$\underline{x}_U = \frac{-\lambda\left[\alpha K + \frac{f}{r}\right](r-\mu)}{(1-\lambda)\pi} \qquad (3)$$

The payout policy $p(x)$ is:

$$\begin{aligned}
p(x) &= \alpha(\pi x - f) && \text{for} \quad x > \underline{x}^o \\
&= r\alpha K && \text{for} \quad \underline{x}_U \leq x \leq \underline{x}^o
\end{aligned}$$

When there are no costs of collective action ($\alpha = 1$), managers close the firm at $\underline{x}_U = \underline{x}^o$ and shareholders realize the first-best firm value $E(x) = V^o(x; \underline{x}^o)$. When the cost of collective action is strictly positive ($\alpha < 1$), managers close the firm at $\underline{x}_U < \underline{x}^o$ and $V_U(x; \underline{x}_U) < V^o(x; \underline{x}^o)$.

With all-equity financing, managers always close the firm inefficiently late. They do not bear the full opportunity cost of the capital stock, because the cash they have to pay out to shareholders depends on αK, not K. That is why αK appears in the numerator of the managers' closure threshold.

Our results hold for a firm with two or many investment projects, as we will show below. But for simplicity we first introduce debt financing for a single-asset firm.

Assume that the firm issues perpetual debt with principal D and a required debt service rD. The market value of the debt is $B(x)$. When $D \leq K$ the debt is default-risk free and $B(x) = D$. The value of the levered firm, the levered equity and the managers' claim are denoted by $V(x)$, $E(x)$ and $R(x)$, respectively. We assume that shareholders' cost of collective action is proportional to the equity value $E(x)$.[3]

Assumption 1 *The net payoff to shareholders when they take over the levered firm is* $\alpha \max_{\underline{x}_e} (V(x; \underline{x}_e) - B(x; \underline{x}_e))$*, where* $\underline{x}_e$ *is the shareholders' optimal closure threshold once they have taken over.*

The cost of collective action allows managers to extract cash flows from shareholders, but we assume that managers cannot extract concessions from the banks or bondholders. In other words, we assume that lenders have a zero cost of collective action. But lenders cannot intervene if interest is paid on schedule:

Assumption 2 *Lenders have a zero cost of collective action but cannot act unless the firm defaults. If the firm defaults lenders get the firm's capital* K*.*

This assumption guarantees that managers pay lenders the full debt service rD as long as the firm continues as a going concern. Failure to service the debt would allow lenders to eject the managers and take the firm's stock of capital K.

[3]We have explored the alternative case where the net payoff from collective action depends on overall firm value, that is, on $\alpha V(x) - B(x)$. In that case the debt liability falls entirely on shareholders. Managers' rents do not depend on the amount of debt and managers' claim is senior to both debt and equity. We reach a Modigliani-Miller result where debt cannot discipline managers and is irrelevant to overall firm value. We believe that our Assumption 1, which gives both managers and stockholders junior claims on the firm, is more realistic and interesting.

Lenders are willing to advance financing only if they have property rights to the firm's assets in liquidation. Their property rights can be protected by laws against fraudulent conveyance and by covenants. We assume these laws and covenants are effective and that lenders are protected against purely dilutive debt issues.

Assumption 3 *Debt covenants prohibit distribution of the capital stock or the proceeds of additional debt issues to shareholders and managers, unless existing lenders are compensated for any decrease in the value of their claims.*

This assumption does not prevent risky debt issues if the proceeds are invested or if existing lenders are made whole. It does rule out liquidating dividends that would distribute the value of the firm's capital stock to managers or shareholders. Assumption 3 also prevents the firm from piling on additional debt if the only motive is to harm existing lenders. We will show that managers and shareholders have an incentive to do just that, even though additional risky debt reduces overall value. We will also show that existing lenders are not harmed by additional borrowing if the proceeds are kept in the firm, either as cash or as an increase in the capital stock K.

2.1 Managers' Disinvestment Policy with Safe Debt

We start with the case of safe debt. If $D \leq K$, then the debt is fully collateralized and default-risk free, because lenders can intervene and recover the stock of capital at no cost if debt service is not paid. With risk-free debt, shareholders would realize $V^o(x; \underline{x}^o) - D$ after absorbing the cost of collective action. Since D is fixed, shareholders in control of the firm would disinvest at the first-best closure point $\underline{x}^o$.

Managers maximize $R(x)$, the present value of the future cash flows they will extract from the firm, subject to the constraints of collective action by shareholders ($E(x) \geq \alpha\left(V^o(x) - D\right)$) and shareholders' property rights at closure ($E(\underline{x}) = K - D$). We show in the appendix that the following payout policy replicates the payoff that shareholders

would get from taking collective action:

$$\begin{aligned} p(x) &= \alpha\left(\pi x - f - rD\right) \quad \text{for } x \geq \underline{x}^o \\ &= \alpha\left(rK - rD\right) \qquad \text{for } x \leq \underline{x}^o \end{aligned}$$

The flow of rents to the managers is:

$$\begin{aligned} r(x) &= (1-\alpha)\left(\pi x - f - rD\right) \qquad \text{for } x \geq \underline{x}^o \\ &= \pi x - f - \alpha rK - (1-\alpha)rD \quad \text{for } x \leq \underline{x}^o \end{aligned}$$

Thus debt squeezes both inside managers and outside shareholders. Both hold residual claims on the firm's assets. When the firm borrows, its managers in effect pledge future rents to cover part of the debt service.

The following proposition sets out the values of the firm and managers' and shareholders' claims when debt is default-risk free. (Proofs to all propositions are given in the appendix.)

Proposition 2 *If debt is risk-free ($D \leq K$) then the values of the firm and of managers' and shareholders' claims are:*

$$\begin{aligned} V(x) &= \tfrac{\pi x}{r-\mu} - \tfrac{f}{r} + \left[K + \tfrac{f}{r} - \tfrac{\pi \underline{x}}{r-\mu}\right]\left(\tfrac{x}{\underline{x}}\right)^{\lambda} && \text{for} \quad x > \underline{x} \\ &= K && \text{for} \quad x \leq \underline{x} \\ R(x) &= (1-\alpha)\left(\tfrac{\pi x}{r-\mu} - \tfrac{f}{r} - D\right) + \left(C^o + C\right)x^{\lambda} && \text{for} \quad x > \underline{x}^o \\ R(x) &= \tfrac{\pi x}{r-\mu} - \tfrac{f}{r} - \alpha K - (1-\alpha)D + Cx^{\lambda} && \text{for} \quad \underline{x}^o \geq x > \underline{x} \\ R(x) &= 0 && \text{for} \quad \underline{x} \geq x \\ E(x) &= V(x) - R(x) - D \\ &= \alpha\left(V^o(x) - D\right) + \left(K - D\right)\left(1 - \alpha\right)\left(\tfrac{x}{\underline{x}}\right)^{\lambda} && \text{for} \quad x > \underline{x} \\ &= K - D && \text{for} \quad x \leq \underline{x} \end{aligned}$$

Expressions for the constants C and C^o are in the appendix. For $0 \leq D \leq K$ the closure threshold $\underline{x}$ is given by:

$$\underline{x}_U \leq \underline{x} = \frac{-\lambda\left[\alpha K + (1-\alpha)D + \tfrac{f}{r}\right](r-\mu)}{(1-\lambda)\pi} \leq \underline{x}^o \tag{4}$$

The payout policy is:

$$\begin{aligned} p(x) &= \alpha\left(\pi x - f - rD\right) \quad \text{for} \quad x > \underline{x}^o \\ &= \alpha\left(rK - rD\right) \qquad \text{for} \quad \underline{x} < x \leq \underline{x}^o \end{aligned}$$

The equity value $E(x)$ has two components. The first $(\alpha\left(V^o(x) - D\right))$ is the value derived from the investors' threat of collective action. The second $((K-D)\left(1-\alpha\right)\left(\frac{x}{\underline{x}}\right)^{\lambda})$ is the extra value from the investors' property rights to the the residual stock of capital after debt is repaid $(K - D)$. Of course when $D = K$ the residual value is zero. In that case the investors' and managers' claims are perfectly aligned as $\alpha\left(V^o(x) - D\right)$ and $(1-\alpha)\left(V^o(x) - D\right)$. Each owns a fraction of the value of the going concern.

The expression for the closure threshold shows why debt speeds up closure. First, debt reduces the conflict of interest between managers and investors. Recall that investors' cash flow rights are less protected than their property rights to the stock of capital K. Since management receives cash flows from operations but nothing in liquidation, they have an incentive to disinvest late. But debt reduces the stock of capital that investors get in liquidation and therefore also the difference between shareholders' cash flow and property rights. When D equals K, shareholders get nothing at closure, and their incentives are exactly aligned with the managers'.

Second, debt raises managers' cost of keeping the firm going. When the firm ought to be closed but still is operating $(\underline{x} < x \leq \underline{x}^o)$, the managers' rents are $\pi x - f - \alpha r K - (1-\alpha) r D$. When the firm is all-equity financed, the cost of capital charged against the operating profits is $\alpha r K$, less than the opportunity cost rK. Introducing debt raises this cost to $\alpha r K + (1-\alpha) r D$. When D equals K the managers bear the full opportunity cost. Thus we have the following corollary.

Corollary 1 *The debt level D^o that maximizes the value of the firm and leads to efficient closure is given by $D^o = K$. If debt is below (above) K then the firm is closed inefficiently late (early).*

The proposition and corollary have several interesting implications. First, since $D^o = K$, the optimal debt is fully backed by collateral and therefore risk-free. Second, there is an optimal "book" leverage ratio $\left(\frac{D}{K}\right)^o = 1$. Contrary to other models, for example in Leland (1994) and related papers, the optimal debt level is not linked to the firm's market value but to the firm's liquidation value K. If K is fixed, then debt is fixed but

still dynamically optimal. Debt does not have to be rebalanced as profitability and market value change.

Third, managers of underlevered firms will voluntarily rebalance and increase debt to the optimal debt level $D^o = K$. This may come as a surprise, because debt reduces the present value of the managers' claim ($\frac{\partial R(x;D)}{\partial D} < 0$) and forces first-best disinvestment. How then can additional debt make managers better off? The reason is that as long as $D < K$, increasing debt creates value that managers can appropriate.

A debt issue that increases principal by a small amount ΔD and raises debt service by $r\Delta D$ changes the total debt value by $\Delta B = \frac{\partial B(x;D)}{\partial D} \Delta D$. Since $\frac{\partial B(x;D)}{\partial D} = 1$ for as long as $D < K$, the proceeds of the debt issue are $\Delta B = \Delta D$.

What is the payoff to managers? Managers have to respect shareholders' property rights. Since a debt issue reducers shareholders' claims ($\frac{\partial E(x;D)}{\partial D} < 0$) managers have to pay $-\frac{\partial E(x;D)}{\partial D} \Delta D$ to keep shareholders happy and to prevent their intervention. The managers' net gain from the debt issue is:

$$\frac{\partial R}{\partial D} \Delta D - \left(-\frac{\partial E}{\partial D}\right) \Delta D + \Delta D = \left(\frac{\partial R}{\partial D} + \frac{\partial E}{\partial D} + 1\right) \Delta D = \frac{\partial V}{\partial D} \Delta D > 0 \qquad (5)$$

Thus the managers capture the entire value gain for the firm as a whole and have an incentive to increase debt for as long as $D < K$. Existing lenders are not harmed because their claims remain default-risk free.

The gain to managers from issuing debt up to the first-best level $D^o = K$ follows from simple present-value arithmetic. By issuing debt and compensating shareholders, the managers make two zero-NPV transactions, one with lenders and one with shareholders. They retain the overall value added for themselves.

2.2 Managers' Disinvestment Policy with Risky Debt

Although optimal debt is risk-free, firms may still end up with risky debt. For example, we show later that wealth-constrained managers may choose to issue risky debt to finance

expansion. We show now that managers would always want to overshoot and issue risky debt, if lenders were not protected by Assumption 3.

To derive the managers' disinvestment policy with risky debt, we first derive the shareholders' payoffs. Assumption 1 states that the net payoff to shareholders from taking collective action is $\alpha\left(V(x;\underline{x}_e) - B(x;\underline{x}_e)\right)$, where $\underline{x}_e$ is the shareholders' optimal closure threshold once they have taken control. We therefore need to determine $\underline{x}_e$ and $B(x;\underline{x}_e)$. We show in the appendix that the debt value for a given closure threshold $\underline{x}_e$ is given by:

$$B(x;\underline{x}_e) \;=\; D \;-\; [D \;-\; K]\left(\frac{x}{\underline{x}_e}\right)^{\lambda} \tag{6}$$

The shareholders' optimal closure threshold, $\underline{x}_e$, is chosen to maximize $V(x;\underline{x}_e) - B(x;\underline{x}_e)$, the equity value after collective action. This optimal threshold is:

$$\underline{x}_e \;=\; \frac{-\lambda\left[D \;+\; \frac{f}{r}\right](r-\mu)}{(1-\lambda)\pi} \;>\; \underline{x}^o \tag{7}$$

Therefore shareholders would close the firm inefficiently early when debt principal exceeds the capital value K.[4] This is a consequence of the Myers (1977) debt overhang problem.

We can now derive the managers' payout and closure policy. To prevent outsiders from taking collective action, managers have to pay:

$$\begin{aligned} p(x) \;&=\; \alpha\left(\pi x \;-\; f \;-\; rD\right) \quad \text{for} \quad x > \underline{x}_e \\ &=\; 0 \qquad\qquad\qquad\qquad\quad \text{for} \quad x \le \underline{x}_e \end{aligned}$$

Rents after payout and debt service are:

$$\begin{aligned} r(x) \;&=\; (1-\alpha)\left(\pi x \;-\; f \;-\; rD\right) \quad \text{for} \quad x > \underline{x}_e \\ &=\; 0 \qquad\qquad\qquad\qquad\quad \text{for} \quad x \le \underline{x}_e \end{aligned}$$

[4] "Closure" here means that the shareholders and managers default and walk away from the firm, leaving it to creditors, who get the capital stock worth K. We do not allow the creditors to run the firm as a going concern, as in Leland (1994) and Mella-Barral and Perraudin (1997). Giving creditors this alternative would not change managers' or stockholders' decisions about closure, but would make risky debt more valuable. Our assumption that creditors can get only K simplifies the formulas for debt and firm value, but does not change our model's basic implications.

At the managers' optimal closure point $\underline{x}$, the debt value and the value of the managers' and shareholders' claims are $B(\underline{x}) = K$, $R(\underline{x}) = 0$ and $E(\underline{x}) = 0$. Therefore the managers' and shareholders' incentives are aligned: prior to closure both receive a fraction of the firm's earnings after interest, and at closure both get nothing. We arrive at the following proposition:

Proposition 3 *Assume that the firm has issued risky debt with principal $D > K$. The values of the firm and the debt are:*

$$\begin{aligned} V(x) &= \tfrac{\pi x}{r-\mu} - \tfrac{f}{r} + \left[K + \tfrac{f}{r} - \tfrac{\pi \underline{x}}{r-\mu}\right]\left(\tfrac{x}{\underline{x}}\right)^{\lambda} && \text{for} \quad x > \underline{x} \\ &= K && \text{for} \quad x \leq \underline{x} \\ B(x) &= D - [D-K]\left(\tfrac{x}{\underline{x}}\right)^{\lambda} && \text{for} \quad x > \underline{x} \\ &= K && \text{for} \quad x \leq \underline{x} \end{aligned}$$

Managers' closure threshold $\underline{x}$ is:

$$\underline{x} = \frac{-\lambda\left[D + \tfrac{f}{r}\right](r-\mu)}{(1-\lambda)\pi} > \underline{x}^{o} \tag{8}$$

Managers close the firm at the demand level that is optimal for shareholders ($\underline{x} = \underline{x}_e$), but closure is inefficiently early ($\underline{x} > \underline{x}^o$). The value of the managers' and shareholders' claims prior to closure are $E(x;\underline{x}) = \alpha\left(V(x;\underline{x}) - B(x;\underline{x})\right)$ and $R(x;\underline{x}) = (1-\alpha)\left(V(x;\underline{x}) - B(x;\underline{x})\right)$. The payout policy is:

$$\begin{aligned} p(x) &= \alpha(\pi x - f - rD) && \text{for} \quad x > \underline{x} \\ &= 0 && \text{for} \quad x \leq \underline{x} \end{aligned}$$

With safe debt, there is a conflict of interest between managers and shareholders, because managers wait too long to close the firm. With risky debt, managers' and shareholders' interests are aligned but adverse to lenders, which leads to an underinvestment problem, manifested here as early *dis*investment.

We have shown that managers of underlevered firms will raise debt to the optimal level $D^o = K$. Will managers of overlevered firms *reduce* debt by repurchasing debt at its market price? The answer is no! Managers' (and shareholders') reluctance to pay off risky debt is a consequence of the debt overhang problem.

Suppose that the principal D is decreased by a small amount ΔD (< 0) by repurchasing debt in the open market, reducing debt service from $r\,D$ to $r\,(D - \Delta D)$. The cost of repurchasing the debt equals the value of the debt repurchased, that is, $-\frac{\Delta D}{D}\,B(x)$. The managers finance as much as possible of the repurchase from equity. The most they can raise without triggering collective action is $\frac{\partial E(x;D)}{\partial D}\,\Delta D$. The net value to managers is therefore:

$$\begin{aligned} & \frac{\partial R(x;D)}{\partial D}\,\Delta D \,+\, \frac{\partial E(x;D)}{\partial D}\,\Delta D \,-\, \Big(-B(x;D)\frac{\Delta D}{D}\Big) \\ = \;& \left[\frac{\partial R(x;D)}{\partial D} \,+\, \frac{\partial E(x;D)}{\partial D} \,+\, \frac{B(x;D)}{D}\right]\Delta D \\ = \;& \left[\Big(\frac{x}{\underline{x}}\Big)^{\lambda} \,-\, 1 \,+\, 1 \,-\, \Big(1 \,-\, \frac{K}{D}\Big)\Big(\frac{x}{\underline{x}}\Big)^{\lambda}\right]\Delta D \\ = \;& \frac{K}{D}\Big(\frac{x}{\underline{x}}\Big)^{\lambda}\,\Delta D \,<\, 0 \;\;\text{since}\;\; \Delta D < 0 \end{aligned} \tag{9}$$

Therefore managers will never raise or retain equity to retire risky debt, even if the debt can be repurchased at market. Paying down debt transfers value to the remaining debt, because the remaining lenders free-ride and capture the value of reduced default risk. (The only possible tactic is to repurchase *all* debt at the market price.[5]) This result holds a fortiori if risky debt cannot be bought back at market price but has to be repaid at par.

Conversely, managers would always gain from issuing still more risky debt, if assumption 3 allowed them to do so. Of course existing lenders would resist such an issue. A debt issue that increases principal by a small amount ΔD changes total debt value by $\frac{\partial B(x;D)}{\partial D}\,\Delta D$. New lenders hold a fraction $\frac{\Delta D}{D}$ of the total debt value, leaving existing lenders with $(B \,+\, \Delta B)\left(1 \,-\, \frac{\Delta D}{D}\right)$. Ignoring second order terms and using our expressions for the claim values, the net change in value to the existing bondholders is:

$$\begin{aligned} & (B \,+\, \Delta B)\left(1 \,-\, \frac{\Delta D}{D}\right) \,-\, B \\ = \;& \left(\frac{\partial B}{\partial D} \,-\, \frac{B}{D}\right)\Delta D \end{aligned} \tag{10}$$

[5]This result is not new – see Leland (1994), for example. It is a cornerstone for a growing literature on debt renegotiation.

$$
\begin{aligned}
&= \left(\frac{x}{\underline{x}}\right)^{\lambda}\left[\frac{\lambda\left(D-K\right)}{\underline{x}}\frac{\partial \underline{x}}{\partial D}-\frac{K}{D}\right] \\
&= \frac{\partial V}{\partial D}\Delta D-\frac{K}{D}\left(\frac{x}{\underline{x}}\right)^{\lambda}\Delta D<0 \text{ since } \Delta D>0 \qquad (11)
\end{aligned}
$$

Bondholders' loss equals the destruction in firm value ($\frac{\partial V}{\partial D}\Delta D$) *and* the gain to managers ($\frac{K}{D}\left(\frac{x}{\underline{x}}\right)^{\lambda}\Delta D$).

Even at the optimal debt level $D^o = K$, a debt issue is harmful to existing lenders. Firm value is unchanged at the margin ($\frac{\partial V(x;D=K)}{\partial D}\Delta D = 0$), but there is still a net transfer of value from lenders to managers equal to $\left(\frac{x}{\underline{x}}\right)^{\lambda}\Delta D$. Existing lenders have to hold almost all of the new, unsecured claim ΔD, which is lost in default. The default probability is $\left(\frac{x}{\underline{x}}\right)^{\lambda}$, and therefore the expected loss is approximately $-\Delta D\ \left(\frac{x}{\underline{x}}\right)^{\lambda}$.[6]

We can summarize these results as follows:

Theorem 1 *When the firm is underlevered, managers will raise debt to the optimal level $D^o = K$. The additional debt creates value that managers can appropriate. Existing lenders are not harmed because optimal debt is default-risk free. When the firm is overlevered with risky debt, managers will not pay debt down to the optimal level. They would issue still more risky debt if covenants allowed. The resulting loss to existing lenders would equal the managers' gain plus the destruction in firm value. Shareholders would not gain or lose, because the threat of collective action would force managers to pay out enough of the proceeds of the debt issue to keep shareholders whole.*

Since too much and too little debt are both costly, it is worth asking which of the two costs is more severe. Consider the value loss from marginally over- or undershooting optimal debt. Debt affects firm value only through its effect on the closure threshold, so the difference in the left and right limits of $\frac{\partial V(x;\underline{x})}{\partial D}\Big|_{D=K}$ is determined by $\frac{\partial \underline{x}}{\partial D}$, the sensitivity of the threshold to debt changes. This sensitivity is less when $D < K$ than when $D > K$,

[6]Equation (10) also shows that the loss to existing bondholders equals the difference between the value of a marginal dollar of debt ($\frac{\partial B}{\partial D}\Delta D$) and the value of the average dollar of debt ($\frac{B}{D}\Delta D$). Dilution allows the new debt to be issued at the price of the average dollar of outstanding debt.

since $\frac{\partial \underline{x}}{\partial D} = \frac{-\lambda(1-\alpha)(r-\mu)}{(1-\lambda)\pi}$ for $D < K$ and $\frac{\partial \underline{x}}{\partial D} = \frac{-\lambda(r-\mu)}{(1-\lambda)\pi}$ for $D > K$. Overshooting therefore has the more adverse effect. It also has a long-lasting effect, because managers will not act to reduce debt if it is risky.

2.3 Staged Disinvestment

So far we have assumed that the firm has a single asset, which is liquidated all at once if the firm defaults on risky debt. What if the firm has two or more assets and disinvestment proceeds in stages? Will risky debt distort the timing of staged disinvestment? Will partial disinvestment be accompanied by default? We now consider these and related questions.

Consider a firm that generates operating profits of $\pi_2 x_t - f_2$. The stock of capital is worth K_2 in liquidation. However, when demand falls the firm has the option to scale down and release the amount of capital $K_2 - K_1$. Operating profits then fall to $\pi_1 x_t - f_1$. The remaining stock of capital K_1 is released if and when demand falls enough to force complete closure. We will assume that the first-best policy is to scale down before final closure. As we will show, this assumption restricts the model's parameters as follows.

Assumption 4 *It is optimal for the firm to contract before closure. This is the case if:*

$$\frac{K_2 - K_1 + \frac{f_2 - f_1}{r}}{K_1 + \frac{f_1}{r}} > \frac{\pi_2 - \pi_1}{\pi_1} \tag{12}$$

We first derive the first-best disinvestment policy:

Proposition 4 *First-best value is:*

$$\begin{aligned} V_2{}^o(x) &= \tfrac{\pi_2 x}{r-\mu} - \tfrac{f_2}{r} + \textstyle\sum_{i=1}^{2} \left[K_i - K_{i-1} + \tfrac{f_i - f_{i-1}}{r} - \tfrac{(\pi_i - \pi_{i-1})\underline{x}_i{}^o}{r-\mu}\right] \left(\tfrac{x}{\underline{x}_i{}^o}\right)^{\lambda} \quad \text{for} \quad x > \underline{x}_2{}^o \\ &= K_2 - K_1 + V_1{}^o(x) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{for} \quad x \leq \underline{x}_2{}^o \end{aligned}$$

where $V_1{}^o(x)$ is the (one-stage) first-best firm value as defined in Proposition 1 and $K_0 = \pi_0 = f_0 = D_0 \equiv 0$. The first-best contraction ($i = 2$) and closure ($i = 1$) thresholds are:

$$\underline{x}_i{}^o = \frac{-\lambda \left[K_i - K_{i-1} + \frac{f_i - f_{i-1}}{r}\right](r-\mu)}{(1-\lambda)(\pi_i - \pi_{i-1})} \qquad i = 1, 2 \tag{13}$$

Contraction comes before closure if and only if:

$$\underline{x}_2{}^o > \underline{x}_1{}^o \iff \frac{K_2 - K_1 + \frac{f_2 - f_1}{r}}{K_1 + \frac{f_1}{r}} > \frac{\pi_2 - \pi_1}{\pi_1}, \tag{14}$$

which is satisfied by assumption.

The proposition is a straightforward generalization of the single-stage disinvestment case. The value of the firm $V_2(x)$ is now given by the present value of expected operating cash flow plus options to contract ($i = 2$) and to close ($i = 1$) the firm completely.

Next we consider staged disinvestment by managers when the firm has safe debt outstanding ($D_2 \leq K_2$). The payoff to shareholders from taking collective action is $\alpha\left(V_2{}^o(x) - D_2\right)$. The debt level after contraction is $D_1 \leq K_1$.

Proposition 5 *Assume that managers can disinvest in two stages and that the firm has safe debt before and after disinvestment ($D_2 \leq K_2$ and $D_1 \leq K_1$). Then the values of the firm and managers' and shareholders' claims are:*

$$\begin{aligned}
V_2(x) &= \tfrac{\pi_2 x}{r-\mu} - \tfrac{f_2}{r} + \textstyle\sum_{j=1}^{2}\left[K_j - K_{j-1} + \tfrac{f_j - f_{j-1}}{r} - \tfrac{(\pi_j - \pi_{j-1})\underline{x}_j}{r-\mu}\right]\left(\tfrac{x}{\underline{x}_j}\right)^{\lambda} && \text{for } x > \underline{x}_2 \\
&= K_2 - K_1 + V_1(x) && \text{for } x \leq \underline{x}_2 \\
R_2(x) &= (1-\alpha)\left(\tfrac{\pi_2 x}{r-\mu} - \tfrac{f_2}{r} - D_2\right) + \textstyle\sum_{j=1}^{2}\left(C_j + C_j{}^o\right)x^{\lambda} && \text{for } x > \underline{x}_2{}^o \\
&= \tfrac{\pi_2 x}{r-\mu} - \tfrac{f_2}{r} - \alpha\left(\tfrac{\pi_1 x}{r-\mu} - \tfrac{f_1}{r}\right) - \alpha\left(K_2 - K_1\right) - (1-\alpha)D_2 \\
&\quad + C_1{}^o\, x^{\lambda} + \textstyle\sum_{j=1}^{2} C_j\, x^{\lambda} && \text{for } \underline{x}_2{}^0 > x \geq \underline{x}_2 \\
&= R_1(x) && \text{for } x \leq \underline{x}_2 \\
E_2(x) &= V_2(x) - R_2(x) - D_2 \\
&= \alpha\left(V_2{}^o(x) - D_2\right) + \\
&\quad (1-\alpha)\textstyle\sum_{j=1}^{2}\left[\left(K_j - K_{j-1}\right) - \left(D_j - D_{j-1}\right)\right]\left(\tfrac{x}{\underline{x}_j}\right)^{\lambda} && \text{for } x > \underline{x} \\
&= \left[\left(K_j - K_{j-1}\right) - \left(D_j - D_{j-1}\right)\right] + E_1(x) && \text{for } x \leq \underline{x}
\end{aligned}$$

Expressions for the constants C_j and $C_j{}^o$are given in the appendix. The contraction ($j = 2$) and closure ($j = 1$) thresholds are:

$$\underline{x}_j = \frac{-\lambda\left[\alpha(K_j - K_{j-1}) + (1-\alpha)(D_j - D_{j-1}) + \frac{f_j - f_{j-1}}{r}\right](r-\mu)}{(1-\lambda)(\pi_j - \pi_{j-1})}, \quad j = 1, 2 \tag{15}$$

Payout policy is:

$$\begin{aligned} p(x) &= \alpha(\pi_2 x - f_2 - rD_2) && \text{for} \quad x > \underline{x}_2{}^o \\ &= \alpha(\pi_1 x - f_1 - rD_2) + \alpha(rK_2 - rK_1) && \text{for} \quad \underline{x}_2 < x \leq \underline{x}_2{}^o \end{aligned}$$

The expressions for $V_1(x)$, $E_1(x)$ and $R_1(x)$ are given in proposition 2.

The equity value $E_2(x)$ again consists of two components: the value derived from the threat of collective action and the incremental value from stockholders' property rights when capital is released. A comparison of the managers' closure and contraction thresholds ($\underline{x}_1$, $\underline{x}_2$) with the respective first-best thresholds ($\underline{x}_1{}^o$, $\underline{x}_2{}^o$) immediately reveals that optimal debt still equals the liquidation value of the capital stock, both before and after contraction.

Corollary 2 *The debt levels that maximize the value of the firm and lead to efficient contraction and closure are $D_2{}^o = K_2$ and $D_1{}^o = K_1$. Contraction occurs at the optimal demand threshold, provided that debt is reduced by the amount of capital released, that is, $\underline{x}_2 = \underline{x}_2{}^o \iff D_2 - D_1 = K_2 - K_1$.*

The financing policy $D_2{}^o = K_2$ and $D_1{}^o = K_1$ is optimal because it leads to the first-best firm value at all times. If the debt level were reduced by a smaller (larger) amount, then managers would contract inefficiently late (early).

If the firm is at optimal leverage before contraction, managers will maintain optimal leverage after contraction. Assumption 3 requires the managers to pay the proceeds from contraction out to debt investors or retain the proceeds as cash. If they pay the proceeds out, the remaining debt is optimal. Also, managers will not pay out more than $K_2 - K_1$. We have already shown that managers are better off at the optimal debt level. If they retain the proceeds, debt investors are just as well off as with payout, because *net* debt still equals $D_1{}^o = K_1$.[7] (For simplicity, we assume henceforth that the proceeds from

[7] If the firm retains the cash from contraction, its cash flow is $\pi_1 x - f_1 - (rD_2 - r(K_2 - K_1))$, which is identical to the cash flow when debt is retired, that is, $\pi_1 x - f_1 - rD_1$, where $D_1 = D_2 - (K_2 - K_1)$). Payout to debt investors and retention are equivalent in our model and lead to the same disinvestment policy.

contraction or closure are paid out to debt investors. There is no loss of generality, because we could also interpret D_1 and D_2 as debt net of cash held by the firm.)

Optimal financing therefore leads to a form of dividend smoothing, because the dividend flow $p(x)$ continues when the firm contracts and capital is released. The proceeds from contraction are used to pay off debt, not to pay a one-time extra dividend.

Now suppose the firm is overlevered, with outstanding risky debt before contraction. Consider the shareholders' contraction strategy if they take collective action and run the firm. Shareholders would prefer to pay out the proceeds from contraction as a special dividend, but assumption 3 gives lenders first call on the firm's capital. The proceeds are therefore used to pay off debt ($D_1 = D_2 - (K_2 - K_1)$). The debt value prior to contraction is:

$$\begin{aligned} B_2(x; \underline{x}_{2e}) &= D_2 + (B_1(\underline{x}_{2e}) + K_2 - K_1 - D_2) \left(\frac{x}{\underline{x}_{2e}}\right)^{\lambda} \\ &= D_2 - (D_2 - K_2) \left(\frac{x}{\underline{x}_{1e}}\right)^{\lambda} \end{aligned} \tag{16}$$

where $B_1(x)$ was previously defined as $B(x)$. Stockholders would optimize the demand threshold $\underline{x}_{2e}$, giving:

$$\underline{x}_{2e} = \frac{-\lambda \left[D_2 - D_1 + \frac{f_2 - f_1}{r}\right](r - \mu)}{(1 - \lambda)(\pi_2 - \pi_1)} \tag{17}$$

This result is a straightforward generalization of the one-stage disinvestment threshold. Since $D_1 = D_2 - (K_2 - K_1)$, contraction happens at the efficient demand level ($\underline{x}_{2e} = \underline{x}_2{}^o$), even though the underlying debt is risky. The intuition for this result is as follows. Since equityholders reduce the debt principal by exactly the capital released, contraction does not alter the absolute difference between debt principal and liquidation value ($D_1 - K_1 = D_2 - K_2$) nor does it affect the default probability. Therefore, the risk premium $(D_2 - K_2)\left(\frac{x}{\underline{x}_1}\right)^{\lambda}$ is not changed (in absolute value) and a dollar of capital released reduces the bond value by a full dollar. The debt overhang problem therefore does not influence the shareholders' contraction decision, although it influences the final closure decision.

Managers will follow the same contraction policy. Managers' and shareholders' interests are aligned, because debt is risky both before and after contraction, and residual

claims are worthless when the firm is finally shut down completely. Assumption 3 requires that proceeds from contraction are used to pay down debt, so $D_2 - D_1 = K_2 - K_1$ and the firm is shrunk at the efficient time ($\underline{x}_2 = \underline{x}_{2e} = \underline{x}_2{}^o$).

$$\begin{aligned} E_2(x;\underline{x}_2) &= \alpha\left(V_2(x;\underline{x}_2) - B_2(x;\underline{x}_2)\right) \\ R_2(x;\underline{x}_2) &= (1-\alpha)\left(V_2(x;\underline{x}_2) - B_2(x;\underline{x}_2)\right) \end{aligned} \tag{18}$$

Thus our model predicts efficient contraction decisions, accompanied by reductions in debt, even when the firm is overlevered. The model also predicts that the remaining debt gets riskier and riskier – not in absolute terms but relative to the debt's remaining market value – as the firm approaches terminal shutdown.

With these several results in hand, it's worth repeating the economic role that debt plays in our model. Debt at the optimal level forces managers to bear (their share of) the full opportunity cost of the capital that the firm is using. With all-equity financing, the managers bear less than their share of the opportunity cost and therefore wait to long to disinvest. Shareholders ultimately have full property rights to the firm's capital, but cannot force efficient disinvestment, because of costs of collective action, and more generally because of other costs of monitoring and control. The higher the costs of intervention, monitoring and control – in our model, the lower the parameter α – the greater the gain in overall value by moving from equity to optimal debt financing.

Managers will voluntarily increase debt to the optimal level. Unfortunately, managers are also tempted to issue risky debt, and once debt is risky, managers will not issue or retain equity to pay it down. Thus covenants are necessary to prevent runaway debt issues. Assumption 3 plays this role in our model.

In the next section we show another economic role for debt: corporate borrowing allows managers to use their future rents as collateral to finance their share of new investment. Corporate borrowing also relaxes constraints on new investment caused by managers' limited wealth.

3 Managers' investment and debt policies

In this section we analyze how debt financing affects managers' investment and expansion decisions, starting with the single-asset case. We derive the first-best investment policy and also managers' investment policy. We assume first that managers are not wealth constrained and then consider the effects of such constraints. We generalize to cover a firm with assets in place that can expand by investing in another project.

3.1 First-best Investment

Assume that a firm has no debt and no assets, except an option to invest I in a project that will be managed according to Proposition 1. To rule out money machines we require that $K \leq I$, so that in liquidation the capital stock is worth no more than the amount invested. The option to invest (a perpetual call) is exercised according to a trigger strategy as soon as demand exceeds a threshold level $\overline{x}$. The value of the option at the first-best demand level $\overline{x}^o$ is[8]

$$\begin{aligned} \tilde{V}(x;\overline{x}^o) &= (V^o(\overline{x}^o;\underline{x}^o) - I)\left(\tfrac{x}{\overline{x}^o}\right)^\beta \quad \text{for} \quad x < \overline{x}^o \\ &= V^o(x;\underline{x}^o) - I \qquad\qquad\quad \text{for} \quad x \geq \overline{x}^o \end{aligned}$$

where $\overline{x}^o$ maximizes $\tilde{V}(x;\overline{x}^o)$. (We use the symbol $\tilde{}$ to indicate value before the investment is made.) This threshold is the solution to the first-order condition

$$\overline{x}^o\, V^{o\prime}(\overline{x}^o) - \beta\,(V^o(\overline{x}^o) - I) = 0 \tag{19}$$

which is equivalent to the smoothpasting condition $\tilde{V}'(\overline{x}^o) = V^{o'}(\overline{x}^o)$. Standard results from real options theory tell us that the first-best threshold $\overline{x}^o$ increases with the volatility of demand σ and fixed costs of operation f and decreases with the revenue parameter π.

[8]The proof is standard. For example, see Dixit and Pindyck (1994), chapter 6.

3.2 Investment when Managers are not Wealth Constrained

When the managers decide, the value of the option to invest is again $\tilde{V}(x;\overline{x})$ but the investment threshold $\overline{x}$ may not be first-best. Following assumption 1, we assume shareholders' payoff from collective action *before* investment is the first-best option value after absorbing the cost of collective action (i.e. $\alpha\,\tilde{V}(x;\overline{x}^o)$). In other words, shareholders can invest optimally if they take over and run the firm.[9]

Suppose that upon investment at $\overline{x}$ managers issue debt with principal D for a total value of $B(x;D)$. This investment also allows managers to raise more equity, and they raise as much as possible – they leave no money on the table for shareholders. The net proceeds of the equity issue are $E(\overline{x};D) - \alpha\tilde{V}(\overline{x};\overline{x}^o)$. The managers' payoff from investing is given by:

$$\begin{aligned} & R(\overline{x};D) \;-\; \Big[I \;-\; \Big(E(\overline{x};D) \;-\; \alpha\tilde{V}(\overline{x};\overline{x}^o)\Big) \;-\; B(\overline{x};D)\Big] \\ =\;& V(\overline{x};D) \;-\; I \;-\; \alpha\tilde{V}(\overline{x};\overline{x}^o) \end{aligned} \tag{20}$$

For a given threshold $\overline{x}$, managers choose the debt principal D that maximizes their payoff. Since only the first term $V(\overline{x};D)$ depends on D, managers choose the optimal debt level $D^o = K$. Their investment policy $\overline{x}$ is the solution to the smoothpasting condition:

$$\begin{aligned} & (1-\alpha)\tilde{V}'(\overline{x}) \;=\; V'(\overline{x};D=K) \;-\; \alpha\tilde{V}'(\overline{x};\overline{x}^o) \\ \Longleftrightarrow\quad & \tilde{V}'(\overline{x}) \;=\; V^{o\prime}(\overline{x}) \end{aligned}$$

This is the same optimality condition as for the first-best investment case, and therefore managers adopt the first-best investment policy $\overline{x} = \overline{x}^o$.

Although investment value is maximized, managers still have to co-invest:

$$Coinvestment \;=\; I \;-\; \Big(E(\overline{x}^o;D) \;-\; \alpha\tilde{V}(\overline{x}^o;\overline{x}^o)\Big) \;-\; B(\overline{x}^o;D)$$

[9] We have explored the alternative case in which only managers can exercise the investment option and shareholders get nothing if they take over before the investment is made. It turns out that managers follow the same investment and debt-financing policies derived below. The managers extract a larger fraction of value, however, and are less likely to be wealth-constrained. But the firm would have no value to investors until the investment option is exercised. In other words, the present value of future growth opportunities would not show up at all in stock price.

$$
\begin{aligned}
&= I - \alpha\,(V^o(\overline{x}^o) - K) + \alpha\,(V^o(\overline{x}^o) - I) - K \\
&= (1-\alpha)\,(I - K) > 0 \qquad (21)
\end{aligned}
$$

The amount of additional equity financing does not depend on the NPV of the new investment, because the value of the option to invest is already impounded in the equity price prior to expansion.[10] Only the investment cost I has not yet been sunk. Under the optimal capital structure, shareholders and managers own residual claims to the fractions α and $1-\alpha$ of the firm's cash flows. They also have to contribute these fractions of $I - K$, the investment cost that is not debt financed.

We summarize our results in the following proposition.

Proposition 6 *When managers are not wealth constrained, they invest at the first-best demand threshold* $\overline{x}^o$ *and issue the optimal amount of debt* $D^o = K$ *. With optimal debt, they also disinvest at the first-best demand threshold* $\underline{x}^o$*. Shareholders contribute* $\alpha\,(I - K)$ *and managers coinvest* $(1 - \alpha)\,(I - K)$*.*

Debt financing at $D^o = K$ gives management a credible commitment device that allows efficient investment and closure. We obtain a simple financing policy resembling the pecking order. The first source of financing is debt, which covers the collateralizable part K of the investment I. The second source is investment by shareholders, who put in the fraction α of the remaining investment $I - K$. Finally, managers have to put up $(1 - \alpha)\,(I - K)$, the difference between the investment required and the amount of debt and equity financing.

If for some reason managers can *not* adopt the optimal debt policy, then investment value is reduced and investment is deferred.

[10] If managers hold the key to expansion option and shareholders can not take over and invest on their own, then investment unlocks the value to shareholders and allows managers to issue equity against the investment's NPV. The amount of coinvestment would be reduced by αNPV.

Proposition 7 *If new investment is financed with more or less than optimal debt ($D \neq K$), then the value of the investment option is reduced and investment takes place inefficiently late, at a demand threshold $\overline{x}$ higher than the first-best threshold $\overline{x}^o$.*

The intuition is straightforward: too much or too little debt causes a shortfall from first-best value $V^o(x)$, which results in underinvestment. The value loss happens because managers cannot commit to closure at the efficient threshold $\underline{x}^o$.

3.3 Investment with wealth-constrained managers

Assume that managers have wealth W at their disposal. The optimal financing policy set out in Proposition 6 is no longer feasible if $(1 - \alpha)(I - K) > W$. If this wealth constraint is binding at the first-best threshold $\overline{x}^o$, managers have no recourse to additional equity, because they cannot credibly pledge to pay out more future rents as additional dividends. But managers can, through the firm, issue debt and pre-commit their share of future cash flows as collateral.

If the wealth constraint is binding at $\overline{x}^o$, investment is feasible at $x \geq \overline{x}^o$ only if the following inequality is satisfied.

$$\begin{aligned} & I - [\alpha(V(x;\underline{x}) - B(x;D)) - \alpha(V^o(x;\underline{x}^o) - I)] - B(x;D) \leq W \\ \iff \quad & (1-\alpha)\, B(x;D) - \alpha(V^o(x;\underline{x}^o) - V(x;\underline{x})) \geq (1-\alpha)I - W \end{aligned} \tag{22}$$

The term in brackets on the top line of the equation is the additional equity value generated when the investment is made. (Equity is always worth at least $\alpha(V^o(x;\underline{x}^o) - I)$, because shareholders could take over and invest on their own.) The term $V^o(x;\underline{x}^o) - V(x;\underline{x})$ on the second line of the equation captures the debt-overhang cost of risky debt and too-early disinvestment.

Wealth-constrained managers have only two choices: they can defer investment or borrow more than K in order to invest. Suppose they borrow more. At $D = K$ the debt is risk-free and there is no debt overhang $(V^o(x;\underline{x}^o) - V(x;\underline{x}) = 0)$. Increasing the

debt principal to $K+1$ dollars raises almost a dollar of extra debt and relaxes the wealth constraint by about $1-\alpha$ dollars. Further increases in principal (holding x constant) relax the wealth constraint by smaller amounts, however, because the debt becomes riskier and the debt overhang more severe. The alternative is to delay investment until demand reaches a higher level where debt is safer and the debt overhang is mitigated. But delaying the investment beyond $\overline{x}^o$ is itself costly. The managers balance the cost of delay against the costs of additional risky debt.

We now derive the managers' policy. Define $D^*(x)$ as the lowest value for D that allows the investment to be financed for a given level of demand x. In other words, D^* is the lowest value for D that satisfies equation (22). A higher level of of demand loosens the wealth constraint, reducing the need for debt financing (that is, $\frac{\partial D^*(x)}{\partial x} \leq 0$, where $\frac{\partial D^*(x)}{\partial x} = 0$ when the financing constraint is not binding). Using equation (20) and the fact that $\overline{x} \geq \overline{x}^o$, the managers' payoff upon investment at $\overline{x}$ can be written as:

$$V(\overline{x}; D) \; - \; I \; - \; \alpha \left(V^o(\overline{x}; \underline{x}^o) \; - \; I \right) \tag{23}$$

The value of the managers' option to invest is:

$$\begin{aligned} \tilde{R}(x; \overline{x}) &= \left[V(\overline{x}; \underline{x}(D^*(\overline{x}))) \; - \; I \; - \; \alpha \left(V^o(\overline{x}; \underline{x}^o) \; - \; I \right) \right] \left(\frac{x}{\overline{x}} \right)^{\beta} \\ &= \left[(1-\alpha) \left(V^o(\overline{x}; \underline{x}^o) \; - \; I \right) \; - \; \left[V^o(\overline{x}; \underline{x}^o) \; - \; V(\overline{x}; \underline{x}(D^*(\overline{x}))) \right] \right] \left(\frac{x}{\overline{x}} \right)^{\beta} \quad \text{for} \quad x < \overline{x} \end{aligned}$$

where $\overline{x}$ is chosen to maximize $\tilde{R}(x; \overline{x})$ and the wealth constraint is incorporated in the optimization through the function $D^*(x)$.

Wealth constraints force managers to take on risky debt. The optimal investment threshold $\overline{x}$ trades off the cost of debt overhang against the cost of delayed investment. Increasing the investment threshold decreases the former cost but increases the latter. Thus managerial wealth constraints lead to delayed investment *and* issues of risky debt.

Proposition 8 *If the managers' wealth constraint is binding , with* $(1-\alpha)(I-K) > W$*, then the investment takes place inefficiently late at* $\overline{x} > \overline{x}^o$ *and uses risky debt* $(D > K)$ *as partial financing for the investment.*

3.4 Example

With these results we can investigate the sensitivity of investment to wealth constraints. This depends on the signs of $\frac{\partial \overline{x}}{\partial W}$ and $\frac{\partial^2 \overline{x}}{\partial W^2}$, which require total differentiation of the first-order condition and the budget constraint with respect to W. We have calculated those derivatives, but the solutions are very complicated and signs cannot be determined analytically.[11] We can, however, analyze managers' investment and debt-financing decisions numerically.

Figure 1a shows the managers' investment threshold $\overline{x}$ as a function of their wealth W.[12] For $W \geq (1-\alpha)(I-K) = 20$, the wealth constraint is not binding and investment occurs at the first-best threshold $\overline{x}^o = 0.216$. Optimal debt is $D = K = 100$. As W falls below 20, the wealth constraint binds, the investment threshold rises and investment is financed by risky debt ($D > 100$). The plot shows that $\frac{\partial \overline{x}}{\partial W} \leq 0$ and $\frac{\partial^2 \overline{x}}{\partial W^2} \geq W$. In other words, as the financing constraint becomes tighter the investment threshold rises at an increasing rate, because the debt overhang problem becomes more serious.

Figure 1b plots the closure threshold against managers' wealth. When the wealth constraint is not binding ($W \geq 20$), managers issue optimal debt at $D^o = K$, which induces first-best closure. As wealth drops below $W = 20$, risky debt substitutes for equity and co-investment by managers. The closure point $\underline{x}$ increases with the debt principal D.

Figure 1c plots the debt principal and the market value of the debt upon investment $B(\overline{x})$. As the wealth constraint becomes more severe the firm issues more debt, but at a decreasing rate.

Figure 1d plots two measures of market-value leverage: the solid line is the market-

[11] The main reason is that the closure threshold $\underline{x}$ depends on D and therefore (through the budget constraint) on W. This dependence feeds back into the investment threshold $\overline{x}$.

[12] The parameters used to generate the figure are $\mu = -0.02$, $\sigma = 0.2$, $r = 0.05$, $\alpha = 0.8$, $K = 100$, $I = 200$ and $\pi = 100$.

value debt ratio when investment is made at $\overline{x}$ ($\frac{B(\overline{x})}{V(\overline{x})}$) and the dashed line is the ratio assuming that demand subsequently drops to the first-best investment threshold $\overline{x}^o$ $\left(\frac{B(\overline{x}^o)}{V(\overline{x}^o)}\right)$. The figure illustrates the interaction between managers' use of risky debt and postponed investment. As wealth drops below 20, managers first rely primarily on debt, with little effect on investment timing. This causes leverage at investment to increase. However, once wealth falls below about 14, the rate of increase in debt slows down while the investment threshold rises at an increasing rate. Overall firm value at investment rises faster than the market value of the debt and leverage at investment decreases. Market leverage at investment (the solid line) reaches a peak of about 42% for a wealth level of about 14. This translates into leverage of 47% (on the dashed line) if demand drops from $\overline{x}$ to $\overline{x}^o$. Since the debt principal increases as the wealth constraint becomes more severe, firms with managers that are more wealth constrained at the time of investment will be saddled with higher leverage if the state variable drops after investment.

Figure 1e plots the credit spread. The solid line is the spread when the investment option is exercised at $\overline{x}$, that is, $\frac{rD^*(\overline{x})}{B(\overline{x};D^*(\overline{x}))} - r$. The dashed line is the spread at $\overline{x}^o$ assuming investment took place at $\overline{x}$, that is, $\frac{rD^*(\overline{x})}{B(\overline{x}^o;D^*(\overline{x}))} - r$. When $W \geq 20$, the debt is risk-free and the credit spread is zero. Tighter wealth constraints lead to more risky debt and higher credit spreads. As in Figure 1d, the credit spread at investment first increases and then decreases as managers' wealth is reduced.

Figure 1f calculates the percentage loss in NPV at $\overline{x}^o$ due to the financing constraint, that is, $\left(1 - \frac{\tilde{V}(\overline{x}^o;\overline{x})}{V^o(\overline{x}^o)-I}\right) * 100$. The loss rises from zero for $W \geq 20$ to about 67% for $W = 0$. The loss is determined by two key effects. First, wealth constraints delay investment to a higher threshold $\overline{x} > \overline{x}^o$. Second, the constraints lead to inefficiently late closure, reducing project value when it is finally undertaken.

These experiments confirm that limited managerial wealth could have important impacts on capital investment and financing. But at this point we must ask what a managerial wealth constraint really means. For a small, private firm, it could literally mean an entrepreneur's finite personal wealth. For public corporations, the CEO's personal bank account is probably irrelevant: we interpret the constraint as a limit to coinvestment by

a larger mass of managers (defined broadly to include other inside stakeholders), who receive rents in various forms, including job security, costly perquisites or private benefits and returns on firm-specific human capital.

The first source of cash for coinvestment is the managers' share of current or recent operating cash flow. Imagine a discrete-time version of our model, as in Myers (2000). In each period, the managers choose between taking cash as immediate rents or foregoing the rents and coinvesting. If current rents are easily adjusted[13] and cash flow is ample relative to optimal investment, the wealth constraint does not bind. If it does bind, investment is deferred. Investment (and risky debt financing) occurs later if demand and cash flow are sufficiently high. If wealth-constrained managers have several deferred investments, then a period of increasing cash flows leads to a series of catch-up investments.

In our set-up, there is a natural positive correlation between cash flow and investment, because the state variable, demand, drives both.[14] But wealth constraints should amplify the correlation, especially for firms where coinvestment requirements are likely to be substantial. These would include firms with intangible, firm-specific investments (high

[13]In our theory, managers would cut current rents to zero if necessary to coinvest in positive-NPV projects. In practice, the flow of rents is probably not easily adjusted in the short run. The implications of "sticky rents" are beyond the scope of this paper, but it does appear that coinvestment would be least constrained following increases in cash flow, when managers have a free hand to allocate the incremental cash to rents or coinvestment. Also, sticky rents could prompt borrowing to maintain rents in the short run.

[14]Accountants would treat the costs of rents as expenses, which would be deducted from reported net cash flow. But in our model rents are a constant fraction $1-\alpha$ of gross cash flow, so reported cash flow would be the remaining fraction α. Thus percentage changes in gross and reported cash flow should be the same, unless managers cut rents in order to coinvest. If they do cut back rents for this purpose, then investment will be associated with increased reported cash flow because reported expenses are lower. This connection between coinvestment, lower expenses and higher reported cash flow should amplify the correlation between changes in cash flow and changes in investment.

$\frac{I}{K}$) and a high cost of collective action (low α).

Our results may help clarify empirical research on the effects of cash flow on investment. Changes in cash flow and investment are strongly correlated, even after controls for changes in investment opportunities are included. A strand of papers started by Fazzari, Hubbard, and Petersen (1988) interprets the correlation as evidence that firms are financially constrained. Kaplan and Zingales (1997), however, find similar strong correlations for firms that should have easy access to financial markets. We say that changes in cash flow can affect investment by changing the ability of managers (and inside stakeholders generally) to coinvest with outside shareholders. These managerial wealth constraints could bind and delay investment, even for firms that would not be classified as financially constrained by conventional financial measures.

So far we have considered managers as choosing between immediate rents and coinvestment to generate future rents. But managers who worry about future wealth constraints will try to store up rents for later use. The natural storage mechanism is paying down debt, that is, operating at conservative debt levels below the level of the firm's capital stock. We will discuss this storage strategy shortly.

3.5 Managers' Expansion Policy

We complete our analysis by considering investment by a firm that already has assets in place. We are particularly interested in the timing and financing of new investment when the firm already has debt outstanding. Assume that the firm has invested in a single asset with liquidation value K_1 financed by debt with principal D_1. There is one opportunity to expand by installing extra capital, which results in a total (cumulative) liquidation value K_2 ($> K_1$). The investment required is I_2 and the new debt principal is D_2. After investment, the firm can disinvest by the two-stage process discussed previously. Finally, we assume for this expansion case that existing lenders are compensated if issue of additional risky debt to finance expansion would undercut the value of their claim.[15]

[15]Assumption 3 does not completely protect the outstanding debt in this expansion

There are two possibilities prior to expansion: demand can first drop to the abandonment threshold $\underline{\tilde{x}}_1$, extinguishing the expansion option, or increase to the expansion threshold $\overline{x}_2$. The abandonment threshold $\underline{\tilde{x}}_1$ is, in general, not equal to the abandonment threshold $\underline{x}_1$ previously derived for the case in which the firm has no option to expand. First-best value is determined by the optimal closure thresholds prior to expansion ($\underline{\tilde{x}}_1^o$) and after expansion ($\underline{x}_1{}^o$) and by the optimal expansion threshold $\overline{x}_2{}^o$.[16] The following proposition tells us under what conditions managers achieve the first-best outcome.

Proposition 9 *Assume that the firm has an asset in place and the options to close down if demand falls or to expand if demand improves. Managers achieve first-best expansion and firm value if prior to expansion the firm has the optimal capital structure* $D_1{}^o = K_1$ *and if at the time of expansion the managers' wealth constraint is not binding. Under first-best financing, shareholders and managers invest* $\alpha\left[I_2 - (K_2 - K_1)\right]$ *and* $(1-\alpha)\left[I_2 - (K_2 - K_1)\right]$ *respectively. Debt covers the remaining investment* $K_2 - K_1$.

This proposition is a straightforward generalization of the single-asset investment case. Managers again adopt the first-best investment threshold ($\overline{x}_2 = \overline{x}_2{}^o$) and financing policy ($D_2 = K_2$), provided that they are not wealth constrained at the time of expansion and that the firm is optimally financed in the run-up to expansion. However, the firm will have risky debt outstanding ($D_1 > K_1$) if managers were wealth-constrained when the initial investment was made.

Suppose that outstanding debt is at the first-best level $D_1 = K_1$, but that managers' wealth constraints bind at the optimal trigger for expansion. In this case, the managers

case, since it allows issue of additional risky debt as long as the proceeds are used for the additional investment I_2. If the outstanding debt were not protected (by a senior claim to K_1, for example), then managers would have an incentive to borrow the entire investment I_2 even if they were not wealth-constrained. (The existence of wealth constraints is presumably not contractible: it's hard to imagine a constraint that allows issue of risky debt "only as necessary to relax managers' wealth constraints.")

[16] Strictly speaking the value of the firm also depends on the contraction threshold $\underline{x}_2$. But contraction happens efficiently at $\underline{x}_2 = \underline{x}_2{}^o$ even if the firm has risky debt.

again strike a tradeoff between the cost of debt overhang and the cost of inefficiently late expansion. They delay investment and cover the shortfall by issuing risky debt.

If the firm has risky debt outstanding, then first-best value will not be achieved, even if at the time of expansion managers are no longer wealth-constrained. With too much debt prior to expansion, the firm will be closed inefficiently early and the expansion option will be given up too soon. This feeds back into the value and exercise policy of the expansion option. Furthermore, debt remains risky after expansion. To restore the optimal debt policy ${D^o}_2 = K_2$ insiders should borrow only $K_2 - D_1$ upon expansion. However, this would give existing lenders a free gift of reduced default losses and increased value. Given that the debt level will be reduced by $K_2 - K_1$ upon subsequent contraction, it follows that insiders can set D_2 equal to $D_1 + (K_2 - K_1)$ upon expansion without reducing the existing bondholders' claim. Since the existing debt is risky (i.e. $D_1 > K_1$) it follows that the debt policy after expansion remains suboptimal (i.e. $D_2 > K_2$).

The design of debt covenants to assure efficient investment when the firm already has risky debt is an intriguing topic for further research. Here we simply stress the tradeoffs between safety for lenders vs. financing for expansion. Complete safety for lenders would be feasible in our model, since debt principal could be limited to the value of the capital stock, but it is not optimal, because managers' wealth constraints make real investment contingent on the ability to issue risky debt.

Risky debt and delay of investment are of course both costly. To avoid these costs, managers of firms with valuable growth opportunites could stockpile some of their firm-specific wealth by paying down debt, using a combination of foregone rents and retained or newly issued equity. For example, suppose they arrange to finance the asset in place with $D_1 = K_1 - (1 - \alpha)I_2$, creating unused debt capacity of $(1 - \alpha)I_2$, which could be drawn down later for coinvestment. Operating at $D_1 < K_1$ could be costly if it leads to late closure, but managers would still have an incentive to relever to $D_1 = K_1$ if demand falls close to the optimal closure threshold ${\underline{x}_1}^o$.

This suggests a pecking-order debt policy, where firms pay down debt when cash flow

(and potential rents) are high relative to optimal current investment and use the stockpiled debt capacity when investment opportunities expand and managers' wealth constraints would be binding. The dynamics of this alternative debt policy are another intriguing area for further research.

4 Empirical Implications and conclusions

We begin this final section by summarizing some of the paper's empirical implications.

4.1 Empirical Implications

1. We predict a target debt level that tracks the liquidation value of the firm's stock of capital, defined as the value of its assets in their best alternative use. New debt is issued when the firm expands and the stock of capital is increased. If assets are sold, the proceeds are used to pay down debt and not to pay dividends.[17] There is a constant "book" leverage target $\frac{D}{K} = 1$ for firms whose managers do not run into wealth constraints.

2. Highly profitable firms have lower market leverage. Market leverage is given by $\frac{D}{V(x)}$ (or $\frac{B(x)}{V(x)}$ if the firm has risky debt), which falls as cash flow and profits increase. The volatility of demand and of going-concern profits does not affect debt levels, except when wealth constraints are binding and risky debt is issued (in which case higher volatility reduces debt levels). Of course we are ignoring deadweight costs of financial distress. We could include costs of bankruptcy or liquidation by netting them against the value of the capital stock. Then we predict that debt should be lower when such costs are relatively high.

3. Firms with more tangible assets (defined as assets with high liquidation value relative to going-concern value) should carry more debt.

4. We predict dividend smoothing, in the sense that capital released is not paid out as

[17]The proceeds could also held in cash or marketable securities. The effect would be the same. It is the net debt that matters.

a lump-sum dividend but is used to reduce debt (and therefore future interest payments). In our continuous-time model, investors end up receiving at all times a dividend flow and never a one-time dividend.[18]

5. Firms with financially constrained managers will finance expansion using risky debt. The difference between the debt principal and the liquidation value of the firm's capital ($D-K$) increases whenever wealth constraints bind at the time of expansion. The absolute difference is not reduced if subsequently managers are no longer wealth-constrained. Debt becomes riskier when the firm disinvests, because the absolute shortfall $D-K$ does not decrease as the underlying capital stock declines. Book leverage therefore depends on how severe wealth constraints have been in the past and on whether the firm is still expanding or is nearing the end of its life-cycle.

6. If the firm is doing well and product-market demand is high enough that disinvestment options are deep out of the money, then the price-earnings ratio is independent of α, although the ratio of market to book value is increasing in α. If α is determined by the quality of governance and investor protection, then this hypothesis could be tested using international data. In other words, for firms with similar P/E ratios, we expect the firms based in countries with better investor protection to have higher market-to-book value ratios. Of course it would be necessary to control for differences in accounting or other factors that could affect the level of reported earnings and the difference between reported book values and the net liquidation values identified in our model.

7. We have proposed new measures of financial constraints. Firms that invest in intangible, firm-specific assets will require more coinvestment by managers. Managers' wealth constraints are more likely to bind in such firms. Investment is more likely to be retarded by managers' wealth constraints in firms with a high cost of collective action (low α). High costs of collective action should be associated with dispersed ownership or other

[18] Our model typically generates a dividend flow that is proportional to the firm's cash flow. In reality dividends are smoother than earnings or cash flow, especially on the downside. There is some downward stickiness in dividends. Our model does capture the stickiness if firms are under-levered ($D < K$): during bad times investors receive a constant dividend $r\alpha(K-D)$.

variables that reflect weak governance or poor investor protection. The difference between the amount of outstanding debt and the liquidation value of the stock of capital $(D - K)$ may also be a measure of past and present constraints on coinvestment by managers.

4.2 Conclusions

The theory set out in this paper differs from traditional corporate-finance theory in important ways. Traditional theory takes the objective as maximizing shareholder value and assumes that managers are the shareholders' imperfect agents. We assume that the ultimate objective is to maximize the overall value of the firm, which is shared between managers and outside shareholders. The managers pursue only their own interests, however. They do not care about shareholder value, and simply maximize the value of the rents they can extract from the firm. But to retain their jobs and rents, the managers also have to deliver value to shareholders. Shareholder value becomes a constraint, not the objective. Nevertheless, the managers, while acting in their own interest, end up also maximizing the value of the firm, providing that they can follow their optimal debt policy. With the optimal debt policy, managers' and shareholders' interests are aligned. Managers and shareholders effectively become co-owners of the firm. The managers are also co-investors, who may have to put up part of the financing for new capital investments, for example by reducing their immediate capture of rents.

We assume perfect, frictionless financial markets and thoroughly rational behavior by managers and investors. We do introduce one important problem of incomplete contracts, namely the managers' inability to pledge their future rents in exchange for additional finance from equity investors. Managers cannot commit not to capture future rents. But they can borrow against those rents if the firm does the borrowing and the managers' claims on the firm's cash flow are junior to debt service. Managers will turn to additional, risky debt financing if they are wealth-constrained and their immediate flow of rents is not sufficient to cover their coinvestment. Thus we see why managers may turn to risky debt rather than equity to finance expansion, and also why changes in firms' cash flows affect capital investment. These links between cash flow, debt financing and investment are

caused by incomplete contracting and managers' limited wealth, not by financial-market imperfections.

Of course we have left out many other frictions and imperfections emphasized in traditional theory. We have not considered taxes, bankruptcy costs or asymmetric information. We have also ignored managers' personal risk-aversion (although our optimal debt policy does not force managers to accept the risk of bankruptcy). But we have tackled agency issues head on, and we believe that many of our results, especially about debt policy and the implications of managers' personal wealth constraints, are original.

We have noted several areas for future research, including the possibility of a dynamic debt policy like a pecking order, in which managers forego current rents and use them to pay down debt, thus creating a reservoir of debt capacity for future coinvestment. We have set the simplest debt covenants and not considered whether other, contingent covenants could help support efficient investment. We have not considered debt maturity or alternatives to plain-vanilla debt and equity. For example, convertible debt could be viewed as a way of paying down debt and relaxing managers' wealth constraints when good news arrives about investment opportunities.

Finally, we have taken the parameter α, which determines the division of cash flow between managers and shareholders, as given. Our model takes α as a measure of dispersed shareholders' cost of collective action. More generally α could depend on many things, including the effectiveness of corporate governance, costs of monitoring and the extent of takeover defenses. Suppose we call α a governance parameter. Tighter governance (higher α) is one solution to the costs of managerial wealth constraints. But tight governance can replace the managers' financing constraint with an incentive problem.

The α parameter essentially determines how much of an investment's value is paid out to managers now versus later. Better governance increases α, reduces managers' future rents and decreases the amount of cash $(1-\alpha)(I-K)$ that managers need to co-invest. But, reducing future rents undercuts managers' incentives. The parameter α should be high enough to support financing when managers could be wealth constrained and at the

same time low enough that they have the incentive (in the form of future rents) to commit firm-specific human capital and to run the firm efficiently. In this paper we have ignored incentives and focused on financing. A meaningful analysis of incentives should include managerial compensation mechanisms (such as stock ownersip or contingent bonuses) and analyze how compensation affects managers' effort, decisions and willingness to invest in firm-specific human capital. These issues are also left for further research.

5 Appendix

Proof of proposition 1

See Lambrecht and Myers (2005). Alternatively, the proof can be obtained by setting $D = 0$ in the proof to proposition 2.

Proof of proposition 2

Managers maximize $R(x)$ with respect to the payout policy $p(x)$ and a closure policy $\underline{x}$ at which they stop servicing the payout. Assume for now that at $\underline{x}$ managers act non-cooperatively and have to be forced out, which means that outside investors have to take collective action and receive $\alpha\,(K - D)$ at $\underline{x}$. We return afterwards to the case of cooperation and its implications for the solution.

We first prove that there exists a payout policy such that for any closure policy $\underline{x}\ (\leq \underline{x}^o)$ the cost of collective action constraint is always binding, i.e. $E(x) = \alpha\,(V^o(x) - D)$. This policy is given by:

$$\begin{aligned} p(x) &= \alpha\,(\pi x - f - rD) \quad \text{for} \quad x > \underline{x}^o \\ &= \alpha\,(rK - rD) \qquad\quad \text{for} \quad \underline{x}^0 \geq x \geq \underline{x} \end{aligned}$$

Indeed, define $H(x)$ as the value of a claim on the above payout policy plus a payment

$\alpha\,(K\,-\,D)$ at $\underline{x}$. Then $H(x)$ must satisfy the following differential equations:

$$\begin{aligned} r\,H(x) &= \alpha\,(\pi x\,-\,f\,-\,rD)\,+\,\mu x H'(x)\,+\,\tfrac{1}{2}\sigma^2 x^2 H''(x) \quad \text{for} \quad x\,>\,\underline{x}^o \\ r\,H(x) &= r\alpha K\,-\,r\alpha D\,+\,\mu x H'(x)\,+\,\tfrac{1}{2}\sigma^2 x^2 H''(x) \qquad\quad \text{for} \quad x\,\leq\,\underline{x}^o \end{aligned}$$

Let us define $\overline{H}(x)\;\equiv\;H(x)$ when $x\,>\,\underline{x}^o$ and $\underline{H}(x)\,\equiv\,H(x)$ when $x\,\leq\,\underline{x}^o$. Then the general solution for $\overline{H}(x)$ and $\underline{H}(x)$ is given by:

$$\begin{aligned} \overline{H}(x) &= \alpha\left(\tfrac{\pi x}{r-\mu}\,-\,\tfrac{f}{r}\,-\,D\right)\,+\,\overline{A}_h\,x^\lambda\,+\,\overline{B}_h\,x^\beta \\ \underline{H}(x) &= \alpha\,(K\,-\,D)\,+\,\underline{A}_h\,x^\lambda\,+\,\underline{B}_h\,x^\beta \end{aligned}$$

The constants $\overline{A}_h$, $\overline{B}_h$, $\underline{A}_h$ and $\underline{B}_h$ are the solutions to the following boundary conditions. First, the no-bubble condition requires that $\lim_{x\to\infty}\overline{H}(x)\;=\;\alpha\left(\frac{\pi\,x}{r-\mu}\,-\,\frac{f}{r}\,-\,D\right)$, which implies that $\overline{B}_h\,=\,0$. Second, at $\underline{x}$ the insiders stop paying out dividends and have to be forced out. Outsiders receive $\alpha\,(K\,-\,D)$ and hence $\underline{H}(\underline{x})=\alpha\,(K\,-\,D)$, or equivalently:

$$\underline{H}(\underline{x})\;=\;\alpha\,(K\,-\,D)\;+\;\underline{A}_h\underline{x}^\lambda\;+\;\underline{B}_h\underline{x}^\beta\;=\;\alpha\,(K\,-\,D) \tag{24}$$

Third, in order to rule out arbitrage opportunities $H(x)$ must be continuous and differentiable at the payout switch $\underline{x}^o$, so $\underline{H}(\underline{x}^o)\,=\,\overline{H}(\underline{x}^o)$ and $\underline{H}'(\underline{x}^o)\,=\,\overline{H}'(\underline{x}^o)$. Or equivalently,

$$\begin{aligned} \alpha\,(K\,-\,D)\,+\underline{A}_h\underline{x}^{o\lambda}\,+\,\underline{B}_h\underline{x}^{o\beta} &= \alpha\left(\frac{\pi\underline{x}^o}{r-\mu}\,-\,\frac{f}{r}\,-\,D\right)\,+\,\overline{A}_h\underline{x}^{o\lambda} \\ \lambda\underline{A}_h\underline{x}^{o\lambda}\,+\,\beta\,\underline{B}_h\underline{x}^{o\beta} &= \alpha\left(\frac{\pi\underline{x}^o}{r-\mu}\right)\,+\,\lambda\,\overline{A}_h\underline{x}^{o\lambda} \end{aligned} \tag{25}$$

Combining the above two equations allows us to substitute out $\overline{A}_h$. Simplifying, and substituting for $\underline{x}^o$ gives:

$$\begin{aligned} (\beta-\lambda)\;\underline{B}_h\underline{x}^{o\beta} &= \frac{\alpha\pi\underline{x}^o(1-\lambda)}{r-\mu}\,+\,\alpha\lambda K\,+\,\frac{\lambda f}{r} \quad \text{or equivalently :} \\ (\beta-\lambda)\;\underline{B}_h\underline{x}^{o\beta} &= 0 \end{aligned} \tag{26}$$

Consequently, $\underline{B}_h\;=\;0$; substituting into (24) gives $\underline{A}_h\;=\;0$, and hence $\underline{H}(x)\;=\;\alpha\,(K\,-\,D)$. Substituting this into the value-matching condition at $\underline{x}^o$ allows us to solve for $\overline{A}_h$, which gives us $\overline{H}(x)$. Combining our results for $\underline{H}(x)$ and $\overline{H}(x)$ gives: $H(x)\;=\;\alpha\,(V^o(x)\,-\,D)$, and hence the collective action constraint is always binding, irrespective of the closure threshold $\underline{x}$. Consequently, the payout policy is optimal for the insiders, as any reduction in the payout would cause the constraint to be violated.

Given the payout policy $p(x)$ that is imposed on the insiders, we can now derive the outsiders' claim value $R(x)$ and their optimal closure policy. Under the payout policy $p(x)$ the claim $R(x)$ must satisfy the following differential equations:

$$\begin{aligned} r\,R(x) &= (1-\alpha)\,(\pi x - f - rD) + \mu x R'(x) + \tfrac{1}{2}\sigma^2 x^2 R''(x) && \text{for} \quad x > \underline{x}^o \\ r\,R(x) &= (\pi x - f) - r\alpha K - (1-\alpha) rD + \mu x R'(x) + \tfrac{1}{2}\sigma^2 x^2 R''(x) && \text{for} \quad x \leq \underline{x}^o \end{aligned}$$

Let us define $\overline{R}(x) \equiv R(x)$ when $x > \underline{x}^o$ and $\underline{R}(x) \equiv R(x)$ when $x \leq \underline{x}^o$. Then the general solution for $\overline{R}(x)$ and $\underline{R}(x)$ is given by:

$$\begin{aligned} \overline{R}(x) &= (1-\alpha)\left(\tfrac{\pi x}{r-\mu} - \tfrac{f}{r} - D\right) + \overline{A}_r\, x^{\lambda} + \overline{B}_r\, x^{\beta} \\ \underline{R}(x) &= \left(\tfrac{\pi x}{r-\mu} - \tfrac{f}{r}\right) - \alpha K - (1-\alpha) D + \underline{A}_r\, x^{\lambda} + \underline{B}_r\, x^{\beta} \end{aligned}$$

The constants $\overline{A}_r$, $\overline{B}_r$, $\underline{A}_r$, $\underline{B}_r$ and the managerial abandonment threshold $\underline{x}$ are the solutions to the following boundary conditions. First, the no-bubble condition requires that $\lim_{x\to\infty} \overline{R}(x) = (1-\alpha)\left(\tfrac{\pi x}{r-\mu} - \tfrac{f}{r} - D\right)$, which implies that $\overline{B}_r = 0$. Second, at $\underline{x}$ the insiders stop paying out dividends and are forced out. This means that their claim value is zero at $\underline{x}$, i.e. $\underline{R}(\underline{x}) = 0$. Third, in order to rule out arbitrage opportunities inside equity value must be continuous and differentiable at the payout switch $\underline{x}^o$, so $\underline{R}(\underline{x}^o) = \overline{R}(\underline{x}^o)$ and $\underline{R}'(\underline{x}^o) = \overline{R}'(\underline{x}^o)$. Finally, since the management optimally chooses the closure threshold, $\underline{x}$, it satisfies the following smooth-pasting condition: $\underline{R}'(\underline{x}) = 0$.

In summary, we have five equations (two value-matching and two smooth-pasting conditions, and one no-bubble condition) and five unknowns ($\overline{A}_r$, $\overline{B}_r$, $\underline{A}_r$, $\underline{B}_r$ and $\underline{x}$). The solution method is exactly as before. Combining the two boundary conditions at $\underline{x}^o$ gives $\underline{B}_r = 0$. Substituting into $\underline{R}(\underline{x}) = 0$ we find that:

$$\underline{A}_r = \left[\alpha K + (1-\alpha)\, D + \frac{f}{r} - \frac{\pi \underline{x}}{r-\mu}\right] \underline{x}^{-\lambda} \equiv C \tag{27}$$

Substituting into the condition $\underline{R}(\underline{x}^o) = \overline{R}(\underline{x}^o)$ allows us to solve for $\overline{A}_r$:

$$\overline{A}_r = -\alpha \left[K + \frac{f}{r} - \frac{\pi \underline{x}^o}{r-\mu}\right] \underline{x}^{o-\lambda} + \underline{A}_r \tag{28}$$

Defining $C^o \equiv -\alpha \left[K + \tfrac{f}{r} - \tfrac{\pi \underline{x}^o}{r-\mu}\right] \underline{x}^{o-\lambda}$ and $C \equiv \underline{A}_r$ gives the expression for $R(x)$ as given in the proposition.

Finally, solving $\underline{R}'(\underline{x}) = 0$ for $\underline{x}$ gives the optimal closure threshold $\underline{x}$:

$$\underline{x} = \frac{-\lambda\left(\alpha K + (1-\alpha)D + \frac{f}{r}\right)(r-\mu)}{(1-\lambda)\pi} \tag{29}$$

The second order condition for a maximum is given by:

$$\left(\frac{x}{\underline{x}}\right)^{\lambda}\frac{1}{\underline{x}}\left[\frac{-\pi(1-\lambda)}{r-\mu}\right] < 0 \tag{30}$$

which is always satisfied.

Finally, we solve for the outside equity value $E(x)$. If insiders do not cooperate then the outside equity value is given by $E(x) = H(x) = \alpha\left(V^o(x) - D\right)$. However, by offering insiders an infinitesimal bribe it should be possible to avoid the deadweight cost of collective action and, as in Lambrecht and Myers (2005), we therefore consider it to be the natural equilibrium. Cooperation would not alter the insiders' closure or payout policy because we assumed that the stock of capital is protected by property rights and that promises to return this capital in the future cannot be used to obtain concessions on payout. However, it would mean that at $\underline{x}$ outsiders receive $(K - D)$ instead of $\alpha(K - D)$. Going through the same derivation as for $H(x)$, but replacing the condition $E(\underline{x}) = \alpha(K - D)$ by $E(\underline{x}) = K - D$, gives:

$$\begin{aligned} E(x) &= \alpha V^o(x) + (K - D)(1-\alpha)\left(\tfrac{x}{\underline{x}}\right)^{\lambda} && \text{for} \quad x > \underline{x} \\ E(x) &= K - D && \text{for} \quad x \leq \underline{x} \end{aligned}$$

Substituting our expressions for $V(x)$, $R(x)$ and $E(x)$ one can easily verify that $V(x) = R(x) + E(x) + D$, i.e. in the absence of any dead weight cost or other frictions the managers', shareholders' and bondholders' claim values sum up to the total firm value.

Finally, we show that there exists a debt level that induces optimal closure and achieves the first-best firm value. We know that the firm value is optimized for $\underline{x} = \underline{x}^o$. Comparing the closure threshold $\underline{x}$ (as defined by equation (29)) with the first-best closure threshold $\underline{x}^o$ it follows immediately that the optimal debt level is given by $D^o = K$.

Proof of proposition 3

$B(x)$ must satisfy the following differential equation for $x > \underline{x}$:

$$rB(x) = rD + \mu x B'(x) + \frac{1}{2}\sigma^2 x^2 B''(x) \tag{31}$$

The general solution is given by: $B(x) = D + A_b\, x^{\lambda} + B_b\, x^{\beta}$. The constants A_b and B_b are determined by the following boundary conditions. First, $\lim_{x\to\infty} B(x) = D$ and hence $B_b = 0$. Second, $B(\underline{x}) = D + A_b \underline{x}^{\lambda} = K$. Hence, $A_b = -(D-K)\underline{x}^{-\lambda}$. Finally, the closure threshold $\underline{x}$ is the solution to the smooth-pasting condition $R'(\underline{x}) = \alpha\,(V'(\underline{x}) - B'(\underline{x})) = 0$.

Proof of proposition 4

The proof can be found in Dixit and Pindyck (1994) (chapter 10, pp. 323-326).

Proof of proposition 5

The proof is analogous to the one for proposition 2. We therefore only give the key steps. Insiders maximize their claim $R_2(x)$ subject to the constraint $E_2(x) \geq \alpha\,(V_2{}^o(x) - D_2)$, where $V_2{}^o(x)$ is the first-best firm value as defined in proposition 4. Similar to the proof for proposition 2 one can show that the following payout policy replicates exactly the payoff outside equityholders would get from taking collective action:

$$\begin{aligned} p(x) &= \alpha\,(\pi_2 x - f_2 - rD_2) && \text{for} \quad x > \underline{x_2}{}^o \\ &= \alpha\,(\pi_1 x - f_1) + r\alpha\,(K_2 - K_1 - D_2) && \text{for} \quad \underline{x_2}{}^0 \geq x \geq \underline{x}_2 \end{aligned}$$

It follows that the managers' rents are given by:

$$\begin{aligned} r(x) &= (1-\alpha)\,(\pi_2 x - f_2 - rD_2) && \text{for} \quad x > \underline{x_2}{}^o \\ &= \pi_2 x - f_2 - \alpha\,(\pi_1 x - f_1) - r\alpha\,(K_2 - K_1 - D_2) - rD_2 && \text{for} \quad \underline{x_2}{}^0 \geq x \geq \underline{x}_2 \end{aligned}$$

The solution for $R_2(x)$ can be written as:

$$\begin{aligned} R_2(x) &= (1-\alpha)\left(\tfrac{\pi_2 x}{r-\mu} - \tfrac{f_2}{r} - D_2\right) + \textstyle\sum_{j=1}^{2}(C_j + C_j{}^o)\, x^{\lambda} + \overline{B}_2\, x^{\beta} \equiv \overline{R}_2(x) && \text{for} \quad x > \underline{x_2}{}^o \\ &= \tfrac{\pi_2 x}{r-\mu} - \tfrac{f_2}{r} - \alpha\left(\tfrac{\pi_1 x}{r-\mu} - \tfrac{f_1}{r}\right) - \alpha\,(K_2 - K_1) - (1-\alpha)D_2 \\ &\quad + C_1{}^o\, x^{\lambda} + \textstyle\sum_{j=1}^{2} C_j\, x^{\lambda} + \underline{B}_2\, x^{\beta} \equiv \underline{R}_2(x) && \text{for} \quad \underline{x_2}{}^0 > x \geq \underline{x} \\ &= R_1(x) && \text{for} \quad x \leq \underline{x}_2 \end{aligned}$$

where the constants C_2, $C_2{}^o$, $\underline{B}_2$ and $\overline{B}_2$ and the contraction threshold $\underline{x}_2$ are determined by five boundary conditions, and where $R_1(x)$, C_1 and $C_1{}^o$ are as previously defined in proposition 2, i.e.:

$$\begin{aligned} \overline{R}_1(x) &= (1-\alpha)\left(\tfrac{\pi_1 x}{r-\mu} - \tfrac{f_1}{r} - D_1\right] + C_1\, x^{\lambda} + C_1{}^o x^{\lambda} && \text{for} \quad x > \underline{x_1}{}^o \\ \underline{R}_1(x) &= \tfrac{\pi_1 x}{r-\mu} - \tfrac{f_1}{r} - \alpha K_1 - (1-\alpha)D_1 + C_1 x^{\lambda} && \text{for} \quad \underline{x_1}{}^0 > x \geq \underline{x}_1 \end{aligned}$$

Define in what follows $\overline{R}_2(x) \equiv R_2(x)$ for $x \geq \underline{x}_2{}^o$ and $\underline{R}_2(x) \equiv R_2(x)$ for $\underline{x}_2{}^0 > x \geq \underline{x}_2$. The no-bubble condition requires that $\lim_{x\to\infty} R_2(x) = (1-\alpha)\left(\frac{\pi_2 x}{r-\mu} - \frac{f_2}{r} - D_2\right)$, which implies that $\overline{B}_2 = 0$. Second, there are the value-matching conditions: $\overline{R}_2(\underline{x}_2{}^o) = \underline{R}_2(\underline{x}_2{}^o)$ and $\underline{R}_2(\underline{x}_2) = \overline{R}_1(\underline{x}_2)$. Third, there are the smooth-pasting conditions $\overline{R}_2'(\underline{x}_2{}^o) = \underline{R}_2'(\underline{x}_2{}^o)$ and $\underline{R}_2'(\underline{x}_2) = \overline{R}_1'(\underline{x}_2)$. This gives a system of 5 equations in 5 unknowns. Solving the system gives the solution in proposition 5

Proof of proposition 6 (the proof is given in the main text)

Proof of proposition 7

For $\overline{x} < \overline{x}^o$ the value of the option to invest can be written as:

$$\begin{aligned}
\tilde{R}(x;\overline{x}) &= \left[V(\overline{x};D) - I - \alpha\tilde{V}(\overline{x};\overline{x}^o)\right]\left(\frac{x}{\overline{x}}\right)^\beta \\
&= \left[V(\overline{x};D) - I\right]\left(\frac{x}{\overline{x}}\right)^\beta - \alpha\tilde{V}(x;\overline{x}^o) \\
&\equiv \left[\frac{\pi\overline{x}}{r-\mu} - \frac{f}{r} - I + A\overline{x}^\lambda\right]\left(\frac{x}{\overline{x}}\right)^\beta - \alpha\tilde{V}(x;\overline{x}^o) \qquad (32)
\end{aligned}$$

where $A\overline{x}^\lambda$ represents the value of the closure option upon investment at $\overline{x}$, i.e.:

$$A = \left(K + \frac{f}{r} - \frac{\pi\underline{x}}{r-\mu}\right)\underline{x}^{-\lambda} \qquad (33)$$

and where $A < A^o$ if $D \neq K$.

For $\overline{x} \geq \overline{x}^o$ the value of the option to invest can be written as:

$$\begin{aligned}
\tilde{R}(x;\overline{x}) &= \left[(1-\alpha)\left(V^o(\overline{x};\underline{x}^o) - I\right) - \left[V^o(\overline{x};\underline{x}^o) - V(\overline{x};D)\right]\right]\left(\frac{x}{\overline{x}}\right)^\beta \\
&\equiv \left[(1-\alpha)\left(\frac{\pi\overline{x}}{r-\mu} - \frac{f}{r} - I + A^o\overline{x}^\lambda\right) - \left(A^o - A\right)\overline{x}^\lambda\right]\left(\frac{x}{\overline{x}}\right)^\beta \\
&= (1-\alpha)\left[\frac{\pi\overline{x}}{r-\mu} - \frac{f}{r} - I + \left(A^o - \frac{A^o - A}{1-\alpha}\right)\overline{x}^\lambda\right]\left(\frac{x}{\overline{x}}\right)^\beta \\
&\equiv (1-\alpha)\left[\frac{\pi\overline{x}}{r-\mu} - \frac{f}{r} - I + B\overline{x}^\lambda\right]\left(\frac{x}{\overline{x}}\right)^\beta - \alpha\tilde{V}(x;\overline{x}^o) \qquad (34)
\end{aligned}$$

where $B < A^o$ if $D \neq K$ and where $B \leq A$.

Define $C \equiv A$ when $\overline{x} < \overline{x}^o$ and $C \equiv B$ when $\overline{x} \geq \overline{x}^o$. Optimizing the investment option with respect to the threshold $\overline{x}$ gives the following first order condition:

$$\frac{\pi\overline{x}}{r-\mu}(1-\beta) + \left(I + \frac{f}{r}\right)\beta + C\overline{x}^\lambda(\lambda - \beta) = 0 \qquad (35)$$

The abandonment option is maximized for the first-best closure threshold $\underline{x}^o$ (in which case $C = A^o$), which in turns leads to first-best investment (i.e. $\overline{x} = \overline{x}^o$).

However, this closure threshold is only obtained if the optimal debt level $D^o = K$ is in place. In all other cases we have $C < A^o$. To find out how debt financing affects the investment threshold $\overline{x}$, it is therefore sufficient to analyse the sign $\frac{\partial \overline{x}}{\partial C}$ since the debt level only enters into the above first order condition through the closure threshold $\underline{x}$, and therefore through C. Therefore, if we can prove that $\frac{\partial \overline{x}}{\partial C} < 0$, then this implies that financing at a suboptimal debt level (whether the firm is over or underlevered) leads to inefficiently late investment (i.e. $\overline{x} > \overline{x}^o$).

Totally differentiating the first order condition with respect to C gives:

$$\frac{\partial \overline{x}}{\partial C} < 0 \iff \lambda C \overline{x}^{\lambda}(\lambda - \beta) - \frac{\pi(\beta - 1)\overline{x}}{r - \mu} < 0 \tag{36}$$

Substituting out C by A and using the first-order condition gives:[19]

$$\pi \overline{x} > \frac{-\lambda\beta\left(I + \frac{f}{r}\right)(r - \mu)}{(1 - \lambda)(\beta - 1)} \Rightarrow \frac{\partial \overline{x}}{\partial C} < 0 \tag{37}$$

λ and β are the negative and positive roots of the characteristic quadratic equation. Using the properties of the sum and product of the roots of a quadratic equation, it follows that the above condition simplifies to:

$$\pi \overline{x} > rI + f \Rightarrow \frac{\partial \overline{x}}{\partial C} < 0 \tag{38}$$

The above condition requires that upon investment the profit flow $\pi\overline{x} - f$ exceeds what is called the "Jorgensonian user cost of capital". This condition must be satisfied upon investment as otherwise it would be better to wait and to invest I at the risk-free rate of interest. Therefore $\frac{\partial \overline{x}}{\partial C} < 0$.

It follows that if the firm adopts a suboptimal debt level (i.e. $D \neq K$) then investment takes place inefficiently late (i.e. $\overline{x} > \overline{x}^o$).

Proof of proposition 8

[19]Since $B < A$, the inequality certainly holds for $C = B$ if we can prove it holds for $C = A$.

The use of risky debt is an immediate consequence of the wealth constraint. It follows then immediately from proposition 7 that this leads to underinvestment (i.e. $\overline{x} > \overline{x}^o$).

Proof of proposition 9

In what follows we first derive the first-best abandonment and expansion thresholds $\underline{\tilde{x}}_1^o$ and $\overline{x}_2{}^o$. We then derive under what conditions managers implement the first-best closure and expansion policy.

First-best Expansion

It is straightforward to show that prior to expansion at $\overline{x}_2{}^o$ and prior to closure at $\underline{\tilde{x}}_1^o$ the value of the firm is given by:

$$\tilde{V}_1^o(x) \;=\; \frac{\pi_1 x}{r-\mu} \;-\; \frac{f_1}{r} \;+\; \tilde{A}\, x^{\lambda} \;+\; \tilde{B}\, x^{\beta} \quad \text{for } \; \underline{\tilde{x}}_1^o \;<\; x \;<\; \overline{x}_2{}^o \tag{39}$$

where $\tilde{A}$ and $\tilde{B}$ are constants to be determined by the boundary conditions ('value matching conditions') at closure ($\underline{\tilde{x}}_1^o$) and at expansion ($\overline{x}_2{}^o$). These value matching conditions are given by:

$$\tilde{V}_1^o(\overline{x}_2{}^o) \;=\; V_2{}^o(\overline{x}_2{}^o; \underline{x}_2{}^o, \underline{x}_1{}^o) \;-\; I_2 \tag{40}$$

$$\tilde{V}_1^o(\underline{\tilde{x}}_1^o) \;=\; K_1 \tag{41}$$

Solving gives the constants $\tilde{A}$ and $\tilde{B}$ as functions of the control variables $\overline{x}_2{}^o$, $\underline{\tilde{x}}_1^o$, $\underline{x}_2{}^o$ and $\underline{x}_1{}^o$ where $\underline{x}_1{}^o$ and $\underline{x}_2{}^o$ denote the closure and contraction threshold after expansion as previously derived.

The optimal closure threshold (prior to expansion) $\underline{\tilde{x}}_1^o$ and the expansion threshold $\overline{x}_2{}^o$ are the solution to the smooth-pasting conditions:

$$\tilde{V}_1^{o\prime}(\overline{x}_2{}^o) \;=\; V_2{}^{o\prime}(\overline{x}_2{}^o) \tag{42}$$

$$\tilde{V}_1^{o\prime}(\underline{\tilde{x}}_1^o) \;=\; 0 \tag{43}$$

Managerial Expansion

Consider now the case where the decision to expand is taken by managers. Prior to expansion the firm will either be optimally financed ($D_1 = K_1$) if the firm was previously

not wealth constrained, or it will have risky debt ($D_1 > K_1$) if the firm was financially constrained when it last invested.

Since $D_1 \geq K_1$ it follows that the payoff from collective action to stockholders is given by: $\tilde{E}_1(x; D_1) = \alpha\left(\tilde{V}_1(x; \underline{\tilde{x}}_1, \overline{x}_2) - \tilde{B}_1(x; D_1)\right)$, where $\tilde{V}_1(x; \underline{\tilde{x}}_1, \overline{x}_2)$ is as previously derived, except that the closure and expansion threshold are chosen by managers and may not be first best. $\tilde{B}_1(x; D_1)$ is the value of the debt with principal D_1, prior to expansion.

The payout policy that is required to avoid collective action by stockholders is given by:

$$\begin{aligned} p(x) &= \alpha\left(\pi_1 x - f_1 - rD_1\right) \quad \text{for} \quad \underline{\tilde{x}}_1 < x < \overline{x}_2 \\ &= 0 \qquad\qquad\qquad\qquad\quad \text{for} \quad x \leq \underline{\tilde{x}}_1 \end{aligned}$$

The value of the managers' claim prior to expansion is given by:

$$\tilde{R}_1(x) = \tilde{V}_1(x; D_1) - \tilde{E}_1(x; D_1) - \tilde{B}_1(x; D_1) = (1-\alpha)\left(\tilde{V}_1(x) - \tilde{B}_1(x; D_1)\right)$$

The value matching conditions at $\overline{x}_2$ and $\underline{\tilde{x}}_1$ are given by:

$$\begin{aligned} \tilde{R}_1(\overline{x}_2) &= (1-\alpha)\left(\tilde{V}_1(\overline{x}_2) - \tilde{B}_1(\overline{x}_2)\right) \\ &= R_2(\overline{x}_2) - \left[I_2 - \left(E_2(\overline{x}_2) - \tilde{E}_1(\overline{x}_2)\right) - \left(B_2(\overline{x}_2) - \tilde{B}_1(\overline{x}_2)\right)\right] \\ \tilde{R}_1(\underline{\tilde{x}}_1) &= (1-\alpha)\left(\tilde{V}_1(\underline{\tilde{x}}_1) - \tilde{B}_1(\underline{\tilde{x}}_1)\right) = 0 \end{aligned} \tag{44}$$

One can verify that both valuematching conditions are satisfied by construction.

Given the existing debt with face value D_1 and given a wealth level W, managers have three control variables: D_2, $\underline{\tilde{x}}_1$ and $\overline{x}_2$. We solve for these variables dynamically backwards. Upon expansion managers maximize their claim with respect to D_2, given the level of the state variable $\overline{x}_2$. Once we have determined the optimal debt level D_2 as a function of $\overline{x}_2$, we can then solve backwards for the optimal expansion and closure thresholds $\overline{x}_2$ and $\underline{\tilde{x}}_1$.

Optimal managerial debt policy upon expansion: D_2

The payoff to managers upon expansion at $\overline{x}_2$ is given by:

$$R_2(\overline{x}_2; D_2) - \left[I_2 - \left(E_2(\overline{x}_2; D_2) - \tilde{E}_1(\overline{x}_2; D_1)\right) - \left(B_2(\overline{x}_2; D_2) - \tilde{B}_1(\overline{x}_2; D_1)\right)\right]$$

$$= \; V_2(\overline{x}_2; D_2) \; - \; I_2 \; - \; \tilde{E}_1(\overline{x}_2; D_1) \; - \; \tilde{B}_1(\overline{x}_2; D_1) \tag{45}$$

If the optimal capital structure is in place prior to expansion ($D_1 = K_1$) and if there are no wealth constraints then this payoff is maximized by setting $D_2 = K_2$, in which case $V_2(\overline{x}_2; D_2 = K_2) = V_2{}^o(\overline{x}_2)$. If, however, the wealth constraint is binding at $\overline{x}_2{}^o$ then, as shown before, a higher debt level has to be adopted leaving the firm with risky debt. We know that $V_2(x; D_2) < V_2{}^o(x)$ if $D_2 > K_2$. The absence of wealth constraints is therefore a necessary condition for efficiency.

Optimal managerial expansion and closure policy: $\overline{x}_2$ and $\underline{\tilde{x}}_1$.

The smoothpasting conditions at $\overline{x}_2$ and $\underline{\tilde{x}}_1$ can be written as:

$$\begin{aligned}
& (1-\alpha)\left(\tilde{V}_1'(\overline{x}_2; D_1) \; - \; B_1'(\overline{x}_2; D_1)\right) \; = \; V_2'(\overline{x}_2; D_2) \; - \; \tilde{E}_1'(\overline{x}_2; D_1) \; - \; \tilde{B}_1'(\overline{x}_2; D_1) \\
\iff \quad & \tilde{V}_1'(\overline{x}_2; D_1) \; = \; V_2{}'(\overline{x}_2; D_2) \\
& (1-\alpha)\left(\tilde{V}_1'(\underline{\tilde{x}}_1; D_1) \; - \; \tilde{B}_1'(\underline{\tilde{x}}_1; D_1)\right) \; = \; 0 \iff \tilde{V}_1'(\underline{\tilde{x}}_1; D_1) \; = \; \tilde{B}'(\underline{\tilde{x}}_1; D_1)
\end{aligned}$$

The smoothpasting conditions at $\overline{x}_2$ and $\underline{\tilde{x}}_1$ are equivalent to the one in the first-best scenario (see equations (42) and (43)) if and only if $D_2 = K_2$ and $D_1 = K_1$. This requires that the firm is not wealth constrained at the time of expansion and that the existing debt prior to expansion is risk-free.

References

Anderson, R., and S. Sundaresan (1996): "Design and Valuation of Debt Contracts," *Review of Financial Studies*, 9(1), 37–68.

Childs, P. D., D. C. Mauer, and S. H. Ott (2005): "Interactions of Corporate Financing and Investment Decisions: The Effects of Agency Conflicts," *Journal of Financial Economics*, 76, 667–690.

Dixit, A., and R. Pindyck (1994): *Investment Under Uncertainty*. Princeton University Press, Princeton, New Jersey.

Fazzari, S. M., R. G. Hubbard, and B. C. Petersen (1988): "Financing Constraints and Corporate Investment," *Brookings Papers on Economic Activity*, 1, 141–195.

Fisher, E., R. Heinkel, and J. Zechner (1989): "Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance*, 44, 19–40.

Jensen, M. C. (1986): "Agency Costs of Free Cash Flow, Corporate Finance and Takeovers," *American Economic Review*, 76, 323–329.

Jin, L., and S. C. Myers (2005): "R2 around the World: New Theory and New Tests," *Journal of Financial Economics*, forthcoming.

Kaplan, S. N., and L. Zingales (1997): "Do Investment-Cash Flow Sensitivities Provide Useful Measures of Financing Constraints?," *Quarterly Journal of Economics*, 20, 169–215.

Lambrecht, B. M., and S. C. Myers (2005): "A Theory of Takeovers and Disinvestment," Ssrn working paper 644863, *forthcoming Journal of Finance*.

Leland, H. E. (1994): "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, 49(4), 1213–1252.

——— (1998): "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53(4), 1213–1243.

Mauer, D. C., and S. Sarkar (2005): "Real Options, Agency Conflicts, and Optimal Capital Structure," *Journal of Banking and Finance*, 29, 1405–1428.

Mella-Barral, P., and W. R. Perraudin (1997): "Strategic Debt Service," *Journal of Finance*, 52, 531–556.

Mello, A., and J. Parsons (1992): "Measuring the Agency Cost of Debt," *Journal of Finance*, 47(5), 1887–1904.

Morellec, E. (2004): "Can Managerial Discretion Explain Observed Leverage Ratios?," *Review of Financial Studies*, 17, 257–294.

Myers, S. C. (1977): "Determinants of Corporate Borrowing," *Journal of Financial Economics*, 5, 147–176.

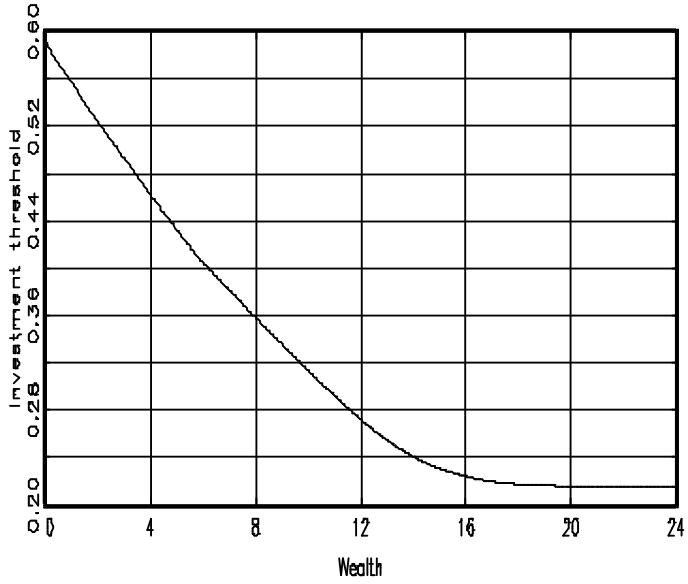
——— (2000): "Outside Equity," *Journal of Finance*, 55(3), 1005–1037.

Parrino, R., and M. S. Weisbach (1999): "Measuring Investment Distortions Arising from Stockholder-bondholder Conflicts," *Journal of Financial Economics*, 53(1), 3–42.

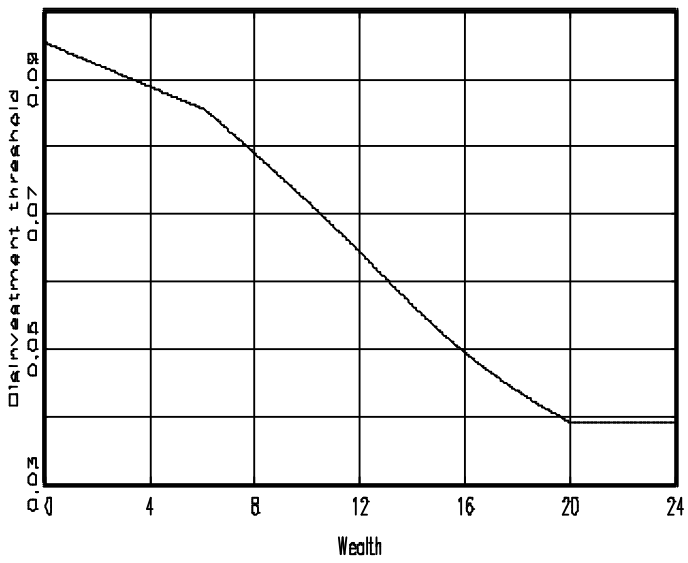
Stulz, R. M. (1990): "Managerial discretion and optimal financing policies," *Journal of Financial Economics*, 26(1), 3–27.

Zwiebel, J. (1996): "Dynamic Capital Structure under Managerial Entrenchment," *American Economic Review*, 86(5), 1197–1215.

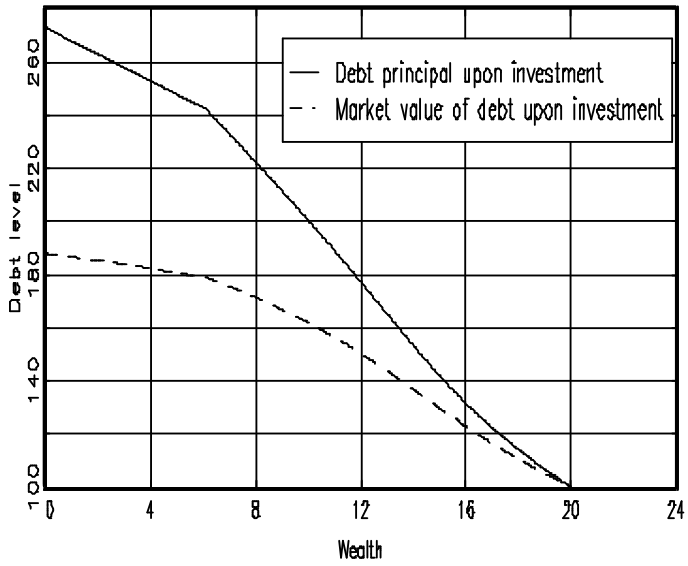
1a Managerial investment threshold



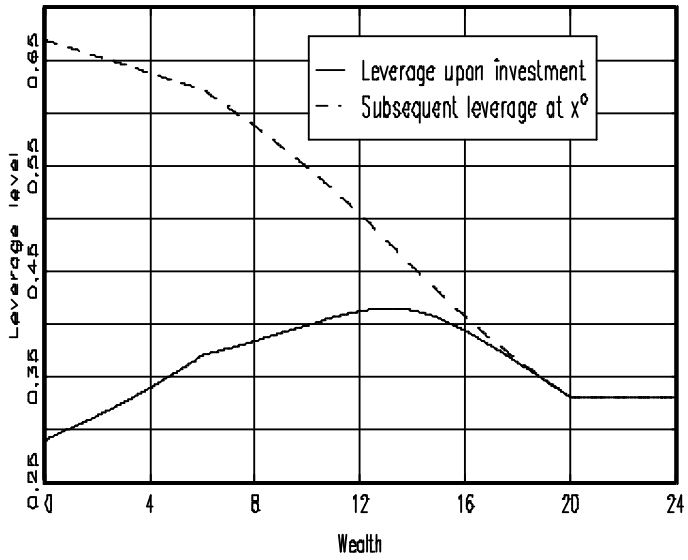
1b Managerial disinvestment threshold



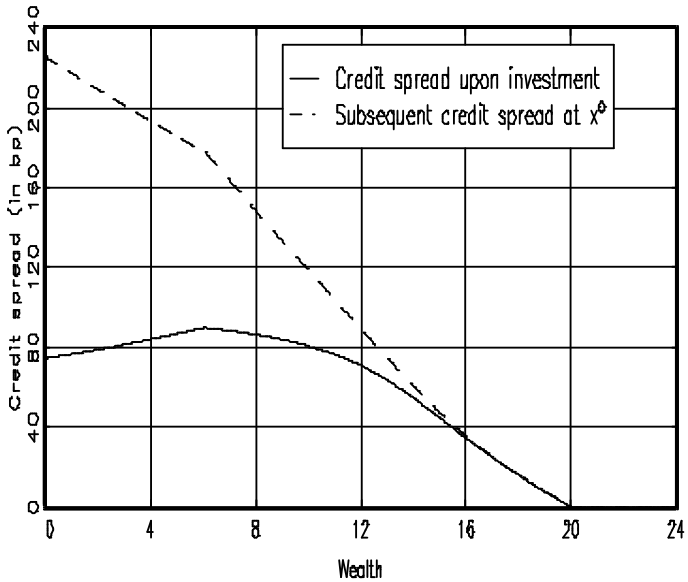
1c Debt Levels



1d Market leverage



1e Credit spread (in bp)



1f Percent loss in NPV (at first-best investment threshold)

