Risks For The Long Run And The Real Exchange Rate*

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Abstract

Brandt, Cochrane, and Santa-Clara (2004) pointed out that the implicit stochastic discount factors computed using prices, on the one hand, and consumption growth, on the other hand, have very different implications for their cross country correlation. They leave this as an unresolved puzzle. We explain it by combining Epstein and Zin (1989) preferences with a model of predictable returns and by positing a very correlated long run component. We also assume that the intertemporal elasticity of substitution is larger than one. This setup brings the stochastic discount factors computed using prices and quantities close together, by keeping the volatility of the depreciation rate in the order of 12% and the cross country correlation of consumption growth around 30%.

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1 Introduction

Ever since the seminal paper of Mehra and Prescott (1985), researchers have tried to reconcile the implications that prices and quantities have for asset returns. The heart of the problem is characterized by Hansen and Jagannathan (1991) that show that in a no arbitrage framework, stochastic discount factors should be at least as volatile as the highest Sharpe ratio of the return of any asset that they are intended to price. Financial data point in the direction of highly volatile stochastic discount factors, while in the context of an economy with a representative consumer with constant relative risk aversion (CRRA) preferences, quantities would imply lowly volatile stochastic discount factors, unless an unreasonably high coefficient of risk aversion is assumed. Brandt, Cochrane, and Santa-Clara (2004) extend the puzzle to a two country economy[1]. They follow Backus, Foresi, and Telmer (1996) in showing that for any stochastic process for the depreciation rate and returns on domestic and foreign currency denominated assets, there exist stochastic discount factors, whose ratio is equal to the depreciation rate, provided that there are no arbitrage opportunities. This 'accounting' relationship allows them to retrieve the correlation of the stochastic discount factors from their volatilities and the standard deviation of the rate of growth of the exchange rate. Using the Hansen and Jagannathan (1991) bound the variance of the pricing kernels in major industrialized countries is in the order of 20% in annualized terms, while the volatility of the depreciation rate between the US and members of the same set of countries is typically between 11% and 15% also in annualized terms[2]. The implied correlation of the stochastic discount factors is not less than 0.96, when computed in this way. However data on consumption display a very low cross country correlation, that is generally not higher than 0.3 when the US is one of the two countries; the assumption that agents have CRRA preferences then imply a correlation of the stochastic discount factors of the same magnitude. The implication is that exchange rates will have to move around a lot to prevent arbitrage opportunities across countries: the volatility of

[1] Brandt, Cochrane, and Santa-Clara (2004) construct an index that they regard as a measure of international risk sharing, but we prefer to reformulate their puzzle without invoking the concept of risk sharing.

[2] Source: IMF's 'International Financial Statistics'.

the depreciation rate would have to take on values as high as 55%, that is about 5 times what we observe in the actual data. This can be thought as an extension of Mehra and Prescott (1985) equity premium puzzle: in a one country economy, consumption growth does not vary enough to keep track of the equity premium, while in a two country economy, consumption growth does not co-vary enough to explain the relative smoothness of the depreciation rate. We want to explain this puzzle.

This paper is a contribution to a growing literature that goes under the name of 'risks for the long run'. This class of models has been successful in explaining some long standing puzzles of financial economics, including the afore mentioned high excess return of equities over the risk free rate. Examples of these economies can be found in Bansal and Yaron (2004), Bansal, Gallant, and Tauchen (2002) and Hansen, Heaton, and Li (2004). This paper extends this framework to a two country economy, each of them populated by a representative consumer with Epstein and Zin (1989) recursive preferences. We specify consumption growths as the sum of a slowly moving predictable component and an *i.i.d.* shock. We assume the former to be relatively smaller than the latter to respect the empirical finding that consumption growth is almost an *i.i.d.* process. We further assume that the predictable components are highly correlated across countries. This specification makes the puzzle disappear: we manage to obtain highly correlated stochastic discount factors along with a lowly volatile depreciation rate. This result is obtained in combination with consumption growth processes that are as volatile, as correlated and as persistent as observed in the actual data. We also show that the model can describe the international correlation of financial markets. The success of the model reflects the fact that when agents care about the timing of the resolution of uncertainty, long horizon consumption growth prospects are crucial determinants of stochastic discount factor processes. Having modelled these prospects as being highly correlated we obtain highly correlated stochastic discount factors. Because stochastic discount factors do most of the job of adjusting to avoid international arbitrage opportunities, exchange rates can take a lesser role and be less volatile.

What comes with the territory in these models is the difficulty of identifying the small predictable component of consumption growths in a formal econometric

model. Bansal and Yaron (2004) show that the presence of a small predictable component in consumption growth cannot be rejected in the past century of US data and endowing the agents with sure knowledge of its presence helps explaining the behavior of asset returns. Bansal, Gallant, and Tauchen (2002) push the analysis one step forward by using price information to properly identify the properties of the consumption process. Our analysis follows in their footsteps as we demonstrate that consumption data alone contain insufficient evidence for the presence of low frequency components of consumption growths even when a large cross section of countries is employed. However when the econometric system is enriched with restrictions imposed on the model by asset returns and exchange rates, we manage sharply to identify the departure of consumption growths from a purely *i.i.d.* process.

The paper is organized as follows. In section 2 we describe the determinants of the puzzle that we want to explain. In regards to the literature of which this is an extension, we denote it as the 'international equity premium puzzle'. In section 3, we write down a simple model that we can solve and calibrate. This provides a useful instrument to show the internal transmission mechanism of the economy. In section 4, we show how we can match a large number of statistics in the data. In section 5, we extend the model to include in each country a redundant asset that pays dividends, whose process looks like the aggregate dividend paid by the stock market and we explore what our model has to say about the international correlation of financial markets. We also introduce time varying economic uncertainty. The focus of section 6 is on the estimation of the model. We study the afore mentioned difficulties of providing conclusive evidence of predictable components of consumption growths from quantity data only and show the benefits of introducing prices in the picture. Section 7 concludes the paper, hinting to potential extensions of the model and summarizing the main findings.

2 The 'international equity premium puzzle'

We analyze two economies that we denote as home and foreign. Accordingly, we will index variables in the two countries with an h and with an f. Assuming that

the two countries are characterized by the absence of arbitrage opportunities, the following pricing condition has to be satisfied:

$$E_t\left[\exp\left\{m^f_{t+1}\right\}R^f_{t+1}\right] = \quad 1 \quad = E_t\left[\exp\left\{m^h_{t+1}\right\}R^h_{t+1}\right]$$

where m^h_{t+1} and m^f_{t+1} are the logarithms of the stochastic discount factors for returns denominated in home and foreign currencies respectively and R^h_{t+1} and R^f_{t+1} are gross returns. If the asset that in the foreign country delivers the return R^f_{t+1} in expectation is also traded in the home country, then a feasible investment strategy consists in converting home prices into foreign prices at the spot exchange rate e_t:

$$E_t\left[\exp\left\{m^f_{t+1}\right\}R^f_{t+1}\right] = \quad 1 \quad = E_t\left[\exp\left\{m^h_{t+1}\right\}\frac{e_{t+1}}{e_t}R^f_{t+1}\right] \tag{1}$$

Equation (1) gives us a connection between the rate of depreciation of the home currency and the two stochastic variables m^h_{t+1} and m^f_{t+1}. Following Backus, Foresi, and Telmer (1996) and Brandt, Cochrane, and Santa-Clara (2004), we further assume that there is a complete set of markets for currencies and state contingent claims. This uniquely identifies the refinement of condition (1) as

$$\pi_{t+1} \quad = \quad m^f_{t+1} - m^h_{t+1}$$

where $\pi_{t+1} = \log\frac{e_{t+1}}{e_t}$. By taking the variance operator on both sides and by denoting $\sigma_{m^i}, \forall i \in \{h, f\}$ as the standard deviation of the stochastic discount factor in the two countries, ρ_{m^h,m^f} as the correlation of the stochastic discount factors and σ_π as the volatility of the depreciation rate, we obtain:

$$\rho_{m^h,m^f} \quad = \quad \frac{\sigma^2_{m^h} \ + \ \sigma^2_{m^f} - \sigma^2_\pi}{2\sigma_{m^h}\sigma_{m^f}} \tag{2}$$

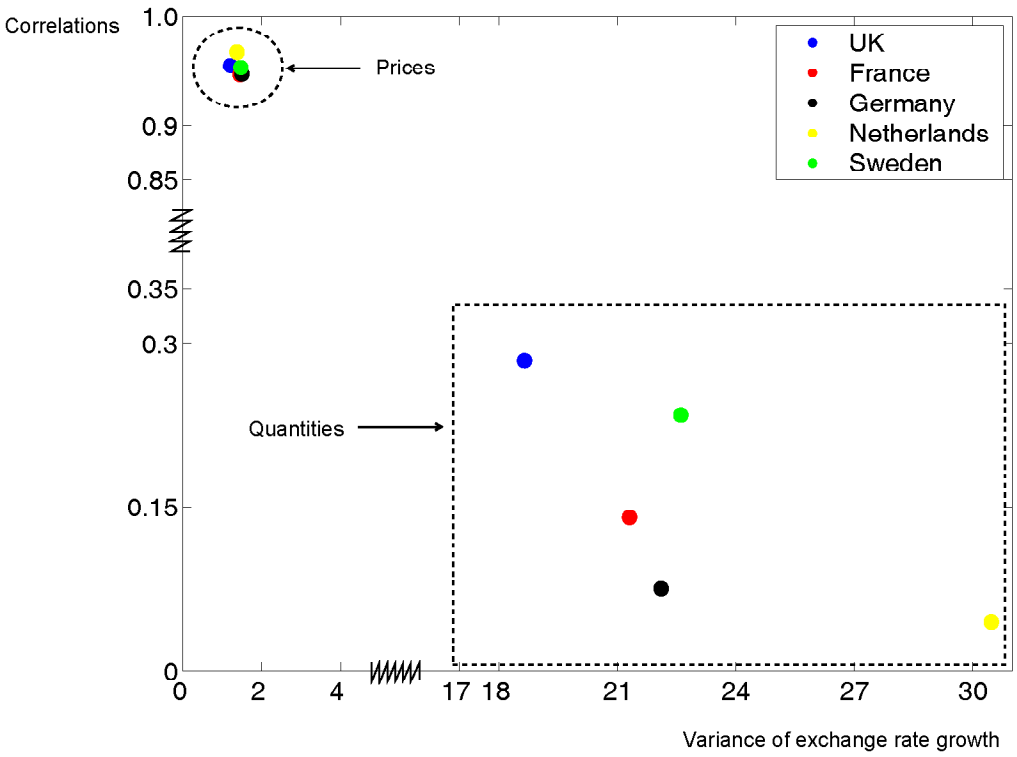


FIG. 1 - The puzzle in a cross section of countries. The horizontal axis reports the variance of the log depreciation rate and the vertical axis reports correlations. The home country is always the US. In the top left corner the variance-correlation of SDF pairs are obtained from prices. In the bottom right part of the graph the variance-correlation pairs are obtained from consumption data.

It is useful to restate the puzzle we are after in terms of equation (2). The Hansen and Jagannathan (1991) bound on the volatility of the logarithm of the stochastic discount factor[3] is in the order of 39% in the US and 37% in the United Kingdom. The standard deviation of the log-depreciation rate between the same countries is in the order of 11%. These numbers and equation (2) imply a correlation of the

[3]Since the Hansen and Jagannathan (1991) bound is defined on the levels of the stochastic discount factors, we assume log-normality of the stochastic discount factors to derive the corresponding bound on σ_{m^h} and σ_{m^f}.

stochastic discount factors of approximatively 0.96. This is what the data on prices have to say about the correlation of the stochastic discount factors in the US and in the UK. The puzzle arises when we look at the restrictions imposed by equation (2) when quantity data are used. In particular, assuming that the two countries have identical constant relative risk aversion (CRRA) preferences wtih coefficient of risk aversion γ, the log-stochastic discount factors are $m_t^i = -\gamma \Delta c_t^i$, $\forall i \in \{h, f\}$ and the correlation ρ_{m^h, m^f} is simply the correlation of consumption growth. This number is about 0.3 that is far below the 0.96 calculated from financial data. Furthermore, consumption growth is notoriously lowly volatile, its standard deviation being 1.37% in the US and 2.83% in the UK. As a consequence, even if we were to set the coefficient of risk aversion to a number as high as 30 to reconcile lowly volatile consumption growths with highly volatile stochastic discount factors, equation (2) would point in the direction of an extremely high variance of the depreciation of the home currency: 18.6%. Since this variance is about 15 times what we observe in the data it is natural to wonder whether there is something wrong with the observed exchange rate's fluctuations of US Dollars versus British Pounds or whether this particular version of consumption based asset pricing model is not suited to describe the data at hand[4].We can also see this as a restatement of Mehra and Prescott (1985) equity premium puzzle. In a one country model, consumption growth does not vary enough to explain the excess return over the risk free rate.In a two country model, consumption growth does not co-vary enough to keep track of movements in the exchange rate. This opens up to the rules of the game we want to play. We want to be able to reconcile the implications that both prices and quantities have for equation (2), by controlling at the same time for risk aversion, cross country correlation of consumption growth and volatility of the depreciation rate. In Figure 1 we show how the dichotomy prices-quantities extends also to other countries when paired with the US.

[4]Brandt, Cochrane, and Santa-Clara (2004) examine market imperfections as the source of this discrepancy, showing that an unreasonable amount of extra volatility should be added to the stochastic discount factors to reconcile the two measure of correlations.

3 Setup of the economy

3.1 Structure of the markets

We study an endowment economy with two countries that we shall denote as home (h) and foreign (f). We assume that there are only two goods in the whole economy and that these goods are country specific. To further simplify the setup, we impose that preferences are such that there is complete home bias, meaning that each country is willing to consume only the good that it is endowed with. Markets are complete, implying that returns are equalized across countries after accounting for the exchange rate. An equilibrium of this economy exists, in which each country behaves as in autarky both for consumption and asset holdings.

We motivate this extreme structure of the markets in at least three ways. First Brandt, Cochrane, and Santa-Clara (2004) assume this specification, too. As a matter of making our results comparable to theirs, it appears to be justified to follow in their footsteps. Second, it is a well documented fact that there is a marked home bias in consumption movements. Backus, Kehoe, and Kydland (1992) were the first to notice it and Lewis (1999) shows that there is almost a one for one response of consumption growth to domestic output growth after controlling for a world effect and heteroskedasticity. This appears to be true for the US, the UK, Germany, Japan and other G7 countries. Third, we want to use preferences of the Epstein and Zin (1989) type, that are recursive but not time separable and allowing for trades in the consumption goods market would severely complicate our analysis. There exists a literature that examines the dynamics of allocations when international goods markets are open to trade and agents have non time separable preferences. Anderson (2005) and Kan (1995) provide analytical and theoretical tools to study these economies. In our case, the need to also introduce a slowly moving predictable component in the endowment process would make these frameworks difficult to extend. For the sake of explanation, we prefer to study the no-trade limiting case and leave a more realistic structure of the goods market for future extensions of this paper.

3.2 Preferences and long run risks

We model the two economies as each having a representative consumer with Epstein and Zin (1989) preferences:

$$U_t^i = \left\{(1-\delta)(C_t^i)^{\frac{1-\gamma}{\theta}} + \delta\left[E_t\left[(U_{t+1}^i)^{1-\gamma}\right]\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \forall i \in \{h, f\}$$

where γ is the coefficient of risk aversion and $\theta = \frac{1-\gamma}{1-1/\psi}$ implicitly defines the intertemporal elasticity of substitution ψ. The two economies are assumed to be symmetric, having the same preference and transition laws parameters. The implied pricing equation for the j^{th} asset is

$$E_t\left[M_{t+1}^i R_{j,t+1}^i\right] = 1, \forall i \in \{h, f\}$$

where the pricing kernel M_{t+1}^i is a stochastic process that depends on consumption growth, $\frac{C_{t+1}^i}{C_t^i}$, on the return on the asset that pays the consumption bundle, $R_{c,t+1}^i$ and on the preference parameters:

$$\log M_{t+1}^i = \theta \log \delta - \frac{\theta}{\psi} \log\left(\frac{C_{t+1}^i}{C_t^i}\right) + (\theta - 1) \log R_{c,t+1}^i, \forall i \in \{h, f\} \tag{3}$$

In what follows, we will adopt the convention of denoting log-variables in small letters (hence $m_{t+1}^i = \log M_{t+1}^i$). The price of an assets that entitles to a stream of country i consumption bundle costs $P_{c,t}^i$ and has the following gross return:

$$R_{c,t+1}^i = \frac{v_{c,t+1}^i + 1}{v_{c,t}^i} \exp \Delta c_{t+1} \tag{4}$$

with $v_{c,t+1}^i$ being the price-consumption ratio in each country $i = \{h, f\}$. We complete the system by specifying exogenous laws of motion for consumption as:

$$\Delta c_t^i = x_{t-1}^i + \varepsilon_{c,t}^i \tag{5}$$

$$x_t^i = \rho_x x_{t-1}^i + \varepsilon_{x,t}^i \tag{6}$$

$\forall i = \{h, f\}$. All shocks are *iid* normally distributed within each country, but they are allowed to be cross-country correlated according to the covariance matrix Σ:

$$\begin{aligned} \left[\begin{array}{cccc} \varepsilon^h_{c,t} & \varepsilon^f_{c,t} & \varepsilon^h_{x,t} & \varepsilon^f_{x,t} \end{array} \right]' &\sim N(0, \Sigma) \\ \Sigma &= \sigma^2 \left[\begin{array}{cc} H_c & 0 \\ 0 & \varphi_e^2 H_x \end{array} \right] \end{aligned} \tag{7}$$

where

$$H_c = \left[\begin{array}{cc} 1 & \rho_c^{hf} \\ \rho_c^{hf} & 1 \end{array} \right] \quad H_x = \left[\begin{array}{cc} 1 & \rho_x^{hf} \\ \rho_x^{hf} & 1 \end{array} \right]$$

In Appendix A we show that a first order Taylor expansion of the price-consumption schedule around its steady state v_c^i is:

$$v^i_{c,t} = \overline{v}^i_c \left(1 + \frac{(\psi - 1)}{\psi(1 - \rho_x \delta)} x^i_t \right), \forall i \in \{h, f\} \tag{8}$$

When the intertemporal elasticity of substitution ψ is larger than one, the coefficient on x_t is positive. Bansal and Yaron (2004) show that this one of the crucial ingredients that allows high equity premia to be justified by a reasonable coefficient of risk aversion: intertemporal substitution dominates the wealth effect with the consequence of a higher volatility of the return on the consumption asset.

In the Appendix we show how to derive a first order linear approximation of the model. We obtain the following analytical expressions for stochastic discount factors, exchange rates and returns:

$$\begin{aligned} m^i_{t\,|\,1} &= \log \delta - \frac{1}{\psi} x^i_t + \delta \frac{1 - \frac{1}{\psi}}{1 - \rho_x^{hf}} \varepsilon^i_{x,t\,|\,1} + \varepsilon^i_{c,t\,|\,1} \\ \frac{e_{t+1}}{e_t} &= m^f_{t+1} - m^h_{t+1} \\ r^i_{c,t+1} &= \overline{r}_c + \frac{1}{\psi} x^i_t + \delta \frac{1 - \frac{1}{\psi}}{1 - \rho_x \delta} \varepsilon^i_{x,t+1} + \varepsilon^i_{c,t+1} \\ r^i_{f,t+1} &= \overline{r}_f + \frac{1}{\psi} x^i_t, \qquad \forall i \in \{h, f\} \end{aligned} \tag{9}$$

where $r^i_{f,t+1}$ is the log-risk free rate and r_j is the average return on asset j, $\forall j \in$

$\{c, f\}$. Given the system (9), the following two propositions can be stated.

Proposition 1. *For a given choice of parameters and provided that* $\rho_x^{h,f} \geq \rho_c^{h,f}$*, the lowest cross country correlation of the stochastic discount factors is achieved for*

$$\psi = \tfrac{1}{\gamma}\widetilde{\delta} \quad , \quad \rho_x = 0 \quad , \quad \rho_x^{hf} = \rho_c^{hf}$$

where $\widetilde{\delta} = \frac{1-2\rho_x\delta+\delta^2}{\delta^2(1-\rho_x^2)}$ *. Furthermore, if* $\rho_x^{h,f} > \rho_c^{h,f}$*, then* $(\psi, \rho_x) = \left(\tfrac{1}{\gamma}\widetilde{\delta}, 0\right)$ *is the unique minimizer.*

Proof. See Appendix B. □

Proposition 2. *For a given choice of parameters, the lowest volatility of the depreciation rate is achieved for* $\rho_x^{hf} = 1$*.*

Proof. See Appendix B. □

Proposition 1 and 2 are intended to stress that three crucial ingredients are needed in order to obtain highly correlated stochastic discount factors together with lowly volatile exchange rates. First of all we need to break the tight relationship between risk aversion and intertemporal elasticity of substitution, that constant relative risk aversion preferences would otherwise impose. Indeed, as the subjective discount factor δ approaches unity, as it is the case when the model is calibrated to a monthly or quarterly decision problem, the minimum correlation of stochastic discount factors is obtained for a value of intertemporal elasticity of substitution that is arbitrarily close to the reciprocal of the coefficient of risk aversion. This is independent of the calibration of the rest of the model. Hence it does not seem implausible to think of the contribution of Brandt, Cochrane, and Santa-Clara (2004) as positing a lower bound on what the correlation of stochastic discount factors in a consumption based asset pricing model would look like. Proposition 1 is silent about agents preferring early or late resolution of uncertainty[5]. In the next section we will show that any departure from constant relative risk aversion preferences will do the job of obtaining highly correlated stochastic discount factors. However we

[5]Epstein and Zin (1989) argue that an agent prefers early resolution of uncertainty if $\psi > 1/\gamma$, while late resolution is preferred if $\psi < 1/\gamma$.

can anticipate that we will not want to make the intertemporal elasticity of substitution too small, as this would result in the counterfactual outcome of infinitely volatile returns, as suggested by the system of equations (9).

Proposition 1 stresses the role of the predictable component of consumption growth. The intuition is straightforward. Epstein and Zin (1989) preferences bought us an extra term in the formula for the stochastic discount factors in the two countries, in the form of the return that pays one unit of the consumption bundle at each period. The price of this asset reflects the total expected flow of dividends, in our case consumption, that it entitles to. Since we model consumption as having a predictable component, it is natural to conclude that this term will have an impact on the return of the asset that will be proportional to its persistence. By appropriately raising the correlation of the x_t's we will manage to increase the correlation of the returns on the consumption assets and ultimately the correlation of stochastic discount factors. This is also the way Proposition 2 should be read. As the predictable components of consumption get more and more correlated, the stochastic discount factors behave in the same way, decreasing the role of the exchange rate in eliminating international arbitrage opportunities. In the next section we show how it is only the combination of all the three ingredients put forward by Propositions 1 and 2 that delivers our result.

4 Results from a calibrated economy

4.1 Choice of parameters

In this section we report the results of a calibrated economy of the type discussed earlier. We assume that the countries share the same calibration, as reported in Table 1. As the structure of our two parallel economies mimics those discussed by Bansal and Yaron (2004) and Bansal, Gallant, and Tauchen (2002) most of the coefficients used in our analysis are either estimated or calibrated in those papers. We choose our model to described a monthly decision problem and as a consequence we set the subjective discount factor to 0.998. In terms of Proposition 1, this means that the correlation of stochastic discount factors is minimized for values of $\psi \approx 1/\gamma$.

TABLE 1
BASELINE CALIBRATION.

Ψ	Intertemporal elasticity of substitution	2
γ	Risk aversion	4.25
δ	Subjective discount factor	0.998
ρ	Autoregressive coefficient of the long run component x_t	0.987
ϕ_e	Ratio of long run shock and short run shock volatilities	0.048
σ	Standard error of the short run shock to consumption (in %)	68×10^{-4}
ρ_x^{hf}	Cross country correlation of the long run shock	1.0
ρ_c^{hf}	Cross country correlation of the short run shock to consumption	0.3

Notes - The two countries share the same calibration.

We set the coefficient of risk aversion γ equal to 4.25, that is a number relatively low compared to what is commonly found in the equity premium puzzle literature and with the number proposed by Brandt, Cochrane, and Santa-Clara (2004) in their extension to an international context. The intertemporal elasticity of substitution, ψ is equal to 2 and is the one estimated by Bansal, Gallant, and Tauchen (2002). As far as the calibration of the parameters of the laws of motion of consumption growth is concerned, our goal is to reproduce the average behavior of consumption growth is the United States and in the United Kingdom. We set $\rho_x = .987$, that is the value estimated by Bansal, Gallant, and Tauchen (2002) and that is slightly higher than the 0.979 calibrated by Bansal and Yaron (2004). The standard error of the idiosyncratic shock to the predictable components is extremely small compared to the one of the idiosyncratic shock to consumption growth, allowing the latter to be the main determinant of the volatility of consumption growth. This is in the spirit of a large part of the literature that models consumption growth as an almost i.i.d. process (see among others Tallarini (2000)). The standard deviation of consumption growth implied by our choice of parameters is approximatively 2.4% in annualized terms, that is in between the average growth of per capita consumption of nondurables and services from 1970 to 1998 for the US and the UK.

The choice of the correlations coefficients is driven by the need of matching key features of the data. We set ρ_x^{hf} to 1 as suggested by Proposition 2 to keep the volatility of the depreciation rate to about 11%. The cross country correlation of the

shocks to consumption is chosen so to obtain a correlation of consumption growth in the order of 30%. We will turn to the question of providing evidence for such an extreme correlation coefficient between the shocks to the predictable components of consumption growths in the last section of the paper.

4.2 Correlation of stochastic discount factors

In the previous section we have discussed how it is the combination of long run risks, Epstein-Zin preferences and cross country highly correlated x_t to drive the results. Figure 2 details this finding with regard to the correlation of stochastic discount factors. Using equation (9), we study how this correlation varies with the coefficient of intertemporal substitution. In each of the two panels, the dark line is drawn according to our baseline calibration. In light of Proposition 1, we are not surprised that the minimum of this graph happens in the vicinity of $\psi \approx 1/\gamma$. By moving away from $1/\gamma$ in either direction, there is a sharp increase in the correlation of stochastic discount factors. However, as stressed in the discussion of the two propositions, increasing the correlation of stochastic discount factors by lowering the intertemporal elasticity of substitution would have the unappealing side effect of pushing the volatilities of returns to infinity. It is also for this reason that in our baseline calibration, we set $\psi = 2$, that leaves us with a correlation that is above 0.9, as price data seem to suggest. Interestingly enough, we do not need to take a stand on the intertemporal elasticity of substitution being greater or smaller than 1, as far as our goal of obtaining highly correlated stochastic discount factors is concerned. Figure 2 shows that any ψ larger than 0.5 would do the job of making the stochastic discount factors of the two countries at least 80% correlated, provided that the remaining coefficients are calibrated as in Table 1. This comment deserves particular attention, given that the intertemporal elasticity of substitution being greater or smaller than 1 has given rise to a long lasting debate in the literature. Hall (1988) and Lustig and Nieuwerburgh (2005) estimate this number to be below unity. Guvenen (2005) points out that limited participation in the stock market is able to produce findings consistent with both capital and consumption fluctuations as long as most of the wealth is held by a small fraction of population with a high EIS. Attanasio and Weber (1989) use the *Family Expenditure Survey* to document

an intertemporal elasticity of substitution in excess of one in the UK. This paper follows the tradition of the risks for the long run literature in calibrating $\psi = 2$ with the objective of appropriately describing the first two moments of returns, as it will be discussed in the next two sections. In discussing the extensions of this setup we will also argue that a generalization of this model that includes more than one predictable component in the law of motion of consumption growth has the potential of being able to match key moments of returns with a relatively lower intertemporal elasticity of substitution, provided that this coefficient is still higher than the reciprocal of risk aversion.

What happens if we perturb our baseline calibration? The left panel of Figure 2 shows that decreasing the correlation of the long run shocks would have the consequence of noticeably decreasing also the correlation of stochastic discount factors. This effect comes through a reduced correlation of the returns on the consumption asset, as we document in the last subsection of our calibration exercise. The right panel of Figure 2 shows that reducing the persistence of the predictable components of consumption growths in the two countries would also largely affect the success of our analysis. Once again, the reduced correlation of stochastic discount factors would come from a reduced correlation in returns. The intuition is that if we reduce the persistence of the predictable part of consumption growth, then the stream of dividends paid by the consumption asset is mainly affect by the idiosyncratic shock to consumption, whose international correlation is quite low. So why do not increase the persistence of the predictable components? Because, by doing this we would increase (at an increasing rate) the share of consumption growth variance that is explained by its predictable component, eventually contradicting the empirical evidence on consumption growth being almost an *i.i.d.* process. We do not report what happens if we calibrate the coefficient of risk aversion differently, because the outcome is made obvious by Proposition 1: given that the subjective discount factor is close to 1, lowering (increasing) the coefficient of risk aversion would move the minimum of the correlation of stochastic discount factors to the right (left) in Figure 2.

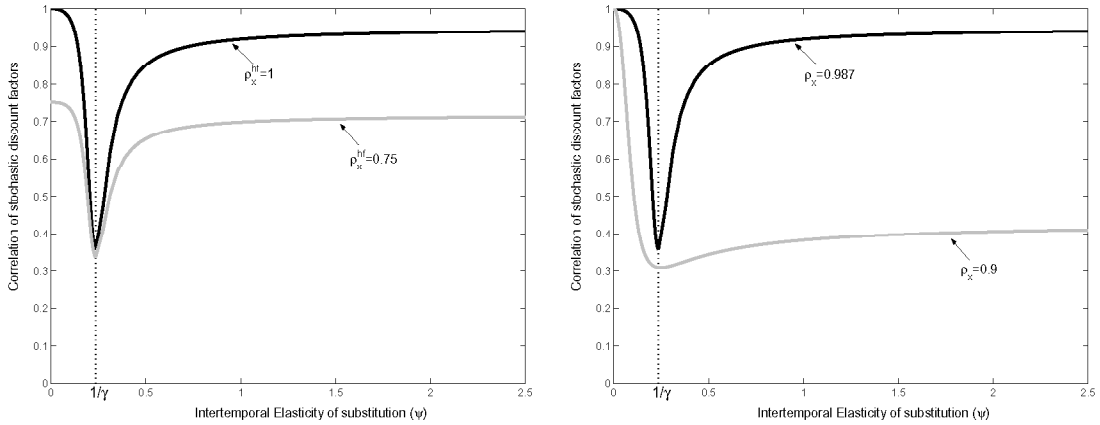


FIG. 2 - The role of intertemporal elasticity of substitution. In both panels, the dark line reports the correlation of stochastic discount factors when ψ changes. The grey line on the left panel is drawn for a smaller value of ρ_x^{hf} everything else equal and the grey line on the right panel is drawn for a lower ρ_x.

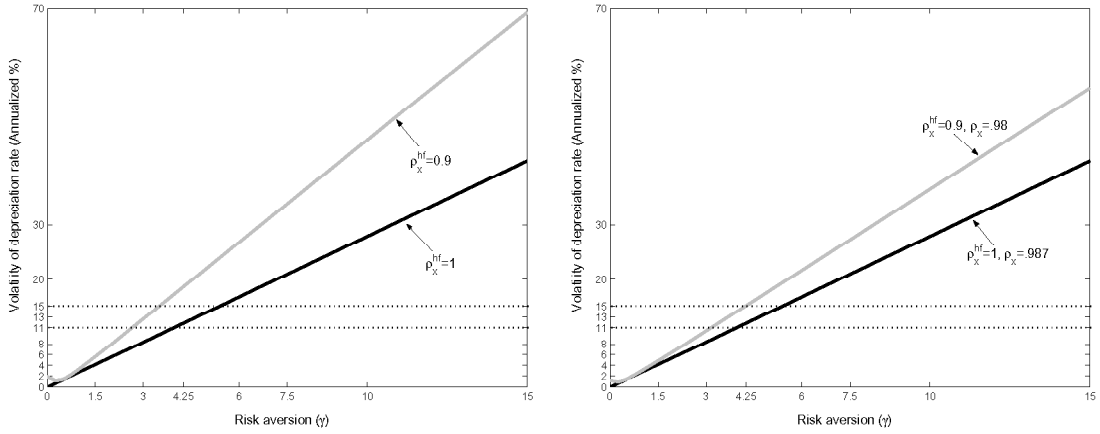


FIG. 3 - The role of risk aversion. In both panels, the dark line reports the volatility of the depreciation rate when γ changes. The grey line in the left panel is drawn for a smaller value of ρ_x^{hf} everything else equal and the grey line in the right panel is drawn for lower ρ_x and ρ_x^{hf}.

4.3 Volatility of the depreciation rate

As formulated in section 2, the puzzle has two parts: one is the correlation of stochastic discount factors that we discussed in the previous subsection and the other one is the volatility of the depreciation rate that we analyze here. Figure 3 plots the volatility of the depreciation rate as obtained from the linear approximation of the model against the coefficient of risk aversion, γ. The two horizontal dashed lines represent the region in which the volatility of the depreciation rate typically falls for major industrialized countries. In our baseline calibration we set $\gamma = 4.25$ that leaves us with a volatility that is well within the region of interest, as documented by the dark lines in the two panels of Figure 3. Increasing the risk aversion of the representative consumers of the two countries to the high levels impelled by the equity premium puzzle literature would push the volatility to counterfactual levels similar to those that motivated Brandt, Cochrane, and Santa-Clara (2004) to postulate the existence of a puzzle. Also in this case, our calibration proves itself crucial. Reducing the correlation of the long run shocks is going to reduce the correlation of stochastic discount factors as we documented before and this will open up to a greater role for the exchange rate in avoiding international arbitrage opportunities. The left panel of Figure 3 shows that unless agents are made less risk averse, we would be left with too much volatility of the depreciation rate at the calibrated level of risk aversion. This effect could be offset if were also to decrease the persistence of the predictable component, because this would result in the volatility of the stochastic discount factors to fall more than their international correlation. However, as we have shown in Figure 2 this is something that we rather avoid and even more so considering the implications that a lowly volatile stochastic discount factor would have for asset returns.

4.4 The role of returns

Table 2 reports the behavior of other relevant variables as we increase the persistence of the long run component. As already observed, the correlation of the stochastic discount factors is increasing and even more so the closer we get to $\rho_x = 1$. The standard deviation of consumption growth does not change too much, except for the

TABLE 2
THE ROLE OF RETURNS.

ρ_x	ρ_{m^h,m^f}	σ_π	$\sigma_{\Delta c}$	$\sigma_{\varepsilon_c}/\sigma_{\Delta c}$	σ_{r_c}	$\rho_{r_c^h,r_c^f}$	$\rho_{\Delta c^h,\Delta c^f}$	$\rho_{\Delta c^f,r_c^h}$
0.00	0.30	11.85	2.36	1.00	2.36	0.30	0.30	0.30
0.70	0.31	11.85	2.36	1.00	2.36	0.31	0.30	0.30
0.90	0.41	11.85	2.37	0.99	2.43	0.34	0.31	0.30
0.987	0.94	11.85	2.46	0.92	4.88	0.84	0.35	0.16
0.999	1.00	11.85	3.46	0.46	44.75	1.00	0.67	0.03

Notes - All figures are annualized. All coefficients are set to the numbers reported in Table 1, except for ρ_x that takes the values reported in the first column.

case in which $\rho_x = 0.999$. This is the result of our choice of setting the volatility of the long run term to a tiny number compared to the volatility of ε_c. Indeed the ratio $\sigma_{\varepsilon_c}/\sigma_{\Delta c}$ indicates that the contribution of long run risks to the volatility of consumption growth is always small unless $\rho_x \approx 1$. The two covariance terms that seem to be driving the results are the correlation of the consumption assets and the correlation of consumption growth and consumption asset returns across countries. As the persistence of the x_t component rises, the returns on the consumption asset will mainly reflect the long run perspectives of consumption growth, implying a low correlation with Δc_t in any country, that isdriven for a large part by the short run shock ε_c, as we have already discussed before. By the same token, returns are going to be increasingly correlated across country as a result of our choice of setting ρ_x^{hf} to one. Since these returns enter the stochastic discount factors of the two countries, a considerably higher correlation of the two can be achieved under our benchmark calibration. Last notice how our choice of setting the correlation of the long run components to one, as suggested by Proposition 2, manages to keep the volatility of the depreciation rate to a constant 11.85%.

5 Other moments of international financial markets

In this section we study two generalizations of the model: the introduction of an asset that pays a stream of dividends that mimics the process of US dividends and

the introduction of conditional heteroskedasticity. In both cases the focus is on how well our model can match key features of financial markets, given the choice of coefficients that we have described in the previous section and the new ones that we add in this section.

5.1 Introducing dividends

We introduce a redundant asset that pays dividends as following a process that is similar to the one that we have used for consumption growth:

$$\Delta d_t^i = \mu_d + \lambda x_{t-1} + \varepsilon_{d,t}^i, \forall i \in \{h, f\} \tag{10}$$

with $\varepsilon_{d,t}^i$ *i.i.d.* normal with mean zero and variance $(\sigma\varphi_d)^2$, $\forall i \in \{h, f\}$. The coefficient λ is referred to as the leverage and is usually set to a number larger than 1. The six shocks of the economy follow a multivariate normal process with covariance matrix $\widetilde{\Sigma}$:

$$\begin{aligned} \left[\begin{array}{cccccc} \varepsilon_{c,t}^h & \varepsilon_{c,t}^f & \varepsilon_{x,t}^h & \varepsilon_{x,t}^f & \varepsilon_{d,t}^h & \varepsilon_{d,t}^f \end{array} \right]' &\sim N(0, \widetilde{\Sigma}) \\ \widetilde{\Sigma} &= \left[\begin{array}{cc} \Sigma & 0 \\ 0 & \sigma^2 \varphi_d^2 H_d \end{array} \right] \end{aligned} \tag{11}$$

where

$$H_d = \left[\begin{array}{cc} 1 & \rho_d^{hf} \\ \rho_d^{hf} & 1 \end{array} \right]$$

and Σ is defined as in the previous section. In the Appendix we derive the price-dividend ratio, $v_{d,t}$ using a first order linear approximation around its steady state. To better describe the nonlinearities in returns we also derive numerical solutions of price-dividend and price-consumption ratios. The Appendix describe in details the numerical algorithm that we employ to minimize the Euler equation errors.

The introduction of the dividends process requires the calibration of four additional parameters. Our guidelines in choosing these coefficients will be to appropriately describe the first two moments of dividend growth, the cross country correlation of dividend growth and the leverage in the US and the UK.

Table 3
Calibration Of Additional Parameters.

λ	Leverage	3.0
μ_d	Average dividend growth	.0007
φ_d	Volatility ratio of short run shocks to dividend and consumption growth	5.0
ρ_d^{hf}	Cross country correlation of the short run shock to dividends	-0.1

Notes - The coefficients refer to a monthly calibration. The remaining coefficients are calibrated according to Table 1.

Table 3 reports the baseline calibration of λ^i, μ_d^i, φ_d^i and ρ_d^{hf}, $\forall i \in \{h, f\}$. By construction the variance of dividend growth explained by its predictable component is very small and in the order of 3%. The coefficient λ is set in such a way that the ratio $\sigma_{\Delta d}/\sigma_{\Delta c}$ is in the range $(4, 8)$ estimated by Ludvigson, Lettau, and Wachter (2004). In our baseline calibration we set $\lambda = 3.0$ that implies $\sigma_{\Delta d}/\sigma_{\Delta c} = 4.86$. The cross country correlation of the short run shocks to dividends, ρ_d^{hf}, is set to target the almost null correlation of dividend growths between the US and the UK.

Table 4 shows that our baseline calibration is able to match key features of international financial markets. Stochastic discount factors that have the property of being highly correlated produce excess returns whose first two unconditional moments are in the range that we observe for the US and the UK. This framework also delivers low average and low volatility excess returns. Bansal and Yaron (2004) showed that this class of models had the potential of appropriately describing this moments in the US. We extend their analysis to show that a similar setup can also explain the same moments in the United Kingdom. We push our analysis one step forward, showing that the model can also deliver the average correlation of excess returns in the two countries. This is accomplished in spite of the low calibrated correlation of dividend growth. One dimension along which our model fails to match the data is the correlation of risk free rates. Even though we use a numerical solution of the model to construct the moments reported in Table 4, a linear approximation of the model can help explaining the result. The Appendix shows that the approximate solution for the risk free rates in this economy is a linear function of the predictable component of consumption growth only and as a consequence the correlation of risk

TABLE 4
INTRODUCING DIVIDENDS.

		US	UK	Model
$\rho\left(m^h, m^f\right)$	Correlation of SDF	-	-	0.93
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	Volatility of depreciation rate	11.21		11.83
$E\left(r_d - r_f\right)$	Average excess return	7.02	9.17	7.01
$\sigma\left(r_d - r_f\right)$	Volatility of excess return	17.13	22.83	19.60
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f\right)$	Correlation of excess returns	0.60		0.58
$E\left(r_f\right)$	Average risk free rate	1.47	1.62	1.33
$\sigma\left(r_f\right)$	Volatility of risk free rate	1.53	2.92	1.19
$\rho\left(r_f^h, r_f^f\right)$	Correlation of risk free rates	0.65		1.00
$\sigma\left(r_c\right)$	Volatility of return on cons.	-	-	4.74
$\rho\left(r_c^h, r_c^f\right)$	Correlation of returns on cons.	-	-	0.85
$\sigma\left(\Delta c\right)$	Volatility of consumption growth	1.37	2.86	2.45
$\frac{\sigma^2(x)}{\sigma^2(\Delta c)} \times 100$	Share of predictable cons. variance	-	-	8.18
$\sigma\left(\Delta d\right)$	Volatility of dividend growth	16.85	6.87	11.96
$\rho\left(\Delta c^h, \Delta c^f\right)$	Correlation of consumption growth	0.28		0.35
$\rho\left(\Delta d^h, \Delta d^f\right)$	Correlation of dividend growth	-0.03		-0.07

Notes - All figures are annualized. All coefficients are set to the numbers reported in Table 1 and Table 4.

free rates will entirely reflect the perfect correlation of the x's. If on the one hand this can be regarded as a failure of the model, on the other hand this invites us to reflect on the importance of risk free rates and other yields to extent that the econometrician wants to identify the low frequency component of consumption growth. In a later section, we discuss how a generalization of this framework that includes more than one predictable component of consumption growth in each country has the potential of reconciling the correlation of risk free rates that we observe in the data and that we obtain from the model.

5.2 Introducing stochastic volatility

To allow for time varying economic uncertainty, we let shocks to consumption, dividends and to their predictable components be conditionally heteroskedastic. In

particular we replace the coefficient σ in (7) and (11) with the time varying coefficient σ_t, whose square follows the autoregressive process:

$$\left(\sigma_t^2\right)^i = \overline{\sigma}^2 + \nu_1 \left[\left(\sigma_{t-1}^2\right)^i - \overline{\sigma}^2\right] + \sigma_w \varepsilon_{\sigma,t}^i \tag{12}$$

Our guidelines in calibrating the coefficients of equation (12) are given by the recognition that a linearized approximation of the model implies the following expression for the conditional variance of returns to the dividend asset:

$$Var_t\left(r_{t+1}^d\right) = (1-\nu_1)k_0 + \nu_1 Var_{t-1}\left(r_t^d\right) + k_1 \sigma_w \varepsilon_{\sigma,t}$$

with k_0 and k_1 defined as in the Appendix. The literature on conditional variance (see inter alia Bollerslev, Engle, and Nelson (1994)) recommends to choose a highly persistent autoregressive coefficient with a small contribution by the innovation shock. As a consequence, we choose $\nu_1 = 0.96$ and $\sigma_w = 2.3 \times 10^{-6}$. We set $\overline{\sigma} = 68 \times 10^{-4}$, so that the unconditional variance of shocks remains unchanged compared to the previous sections. We also assume that the shocks to the stochastic volatility process are *i.i.d.* both within and across countries.

Table 5 shows the results when stochastic volatility is introduced in the model. The findings of the table favor the statement that our results seem to be robust to the introduction of time varying economic uncertainty. In fact none of the numbers reported seem be dramatically different from the corresponding row of Table 4. It is probably worth noticing that the introduction of stochastic volatility managed to break the perfect correlation of risk free rates. This is due to the the fact that the shocks to the stochastic volatility process are assumed to be uncorrelated. However increasing the contribution of stochastic volatility as a mean of further reduction of the correlation of risk free rates would have the unappealing consequence of increasing the volatility of the depreciation rate and decreasing the correlation of stochastic discount factors, as the first two lines of Table 5 seem to suggest.

TABLE 5
STOCHASTIC VOLATILITY.

		US	UK	Model
$\rho\left(m^h, m^f\right)$	Correlation of SDF	-	-	0.92
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	Volatility of depreciation rate	11.21		12.67
$E\left(r_d - r_f\right)$	Average excess return	7.02	9.17	7.03
$\sigma\left(r_d - r_f\right)$	Volatility of excess return	17.13	22.83	19.41
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f\right)$	Correlation of excess returns	0.60		0.57
$E\left(r_f\right)$	Average risk free rate	1.47	1.62	1.33
$\sigma\left(r_f\right)$	Volatility of risk free rate	1.53	2.92	1.22
$\rho\left(r_f^h, r_f^f\right)$	Correlation of risk free rates	0.65		0.98
$\sigma\left(r_c\right)$	Volatility of return on cons.	-	-	4.75
$\rho\left(r_c^h, r_c^f\right)$	Correlation of returns on cons.	-	-	0.85
$\sigma\left(\Delta c\right)$	Volatility of consumption growth	1.37	2.86	2.45
$\frac{\sigma^2(x)}{\sigma^2(\Delta c)} \times 100$	Share of predictable cons. variance	-	-	8.18
$\sigma\left(\Delta d\right)$	Volatility of dividend growth	16.85	6.87	11.96
$\rho\left(\Delta c^h, \Delta c^f\right)$	Correlation of consumption growth	0.28		0.35
$\rho\left(\Delta d^h, \Delta d^f\right)$	Correlation of dividend growth	-0.03		-0.07

Notes - All figures are annualized. All coefficients are set to the numbers reported in Table 1 and Table 4.

6 Estimating long run risks

The previous sections have shown that it is the combination of three crucial ingredients that can address the puzzle raised by Brandt, Cochrane, and Santa-Clara (2004). In particular, we need to disentangle the intertemporal elasticity of substitution from the coefficient of risk aversion and we need to model consumption growth has having a highly persistent and cross country correlated predictable component. Can we justify the assumptions that we made about the consumption processes on the grounds of the data that we have? The literature (Bansal and Yaron (2004) and Hansen, Heaton, and Li (2004)) has typically struggled to find conclusive evidence of the consumption process that we adopt in our model, when the focus is on the US only. We study the implications of adding one more country to the analysis, allowing for shocks to consumption and its predictable component to be cross-country

correlated.

6.1 The data

We use the same data set of Campbell (2003). Data are quarterly observations on private consumption of nondurables and services obtained from NIPA for the US and from IMF's 'International Financial Statistics' (henceforth IFS). These quantities are deflated using the Consumer Price Index provided by CRSP for the United States and by IFS for the other countries[6]. The common sample spans from 1970.1 to 1998.4. In the last part of this section we also use price data. In particular, the real US Dollar-British Pound Exchange Rate is derived as the ratio of the price index in UK currency and the MSCI[7] Price Index in dollar terms. The risk free rates are T-bill rates from CRSP for US and the rate at which 91-day T-bills are allotted in the UK. Stock market returns and dividends are for value-weighted portfolios obtained from CRSP for the US and from Morgan Stanley Capital Perspective.

6.2 Spectral analysis

Since we want to identify x_t^h and x_t^f, the low frequency components of consumption growth, we start our empirical investigation by looking at the spectral densities of consumption growth in the US and the UK. Denoting as $Y_t = \left[\begin{array}{cc} \Delta c_t^h & \Delta c_t^f \end{array} \right]$ the vector of consumption growth in the US and the UK at time t, the population spectrum at frequency ω is

$$S_Y(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{k=+\infty} \Gamma_k e^{-ik\omega} \tag{13}$$

[6]Quarterly series extracted from monthly series by selecting the CPI on the last month of the quarter

[7]Tax credit on dividends, applicable to the UK, is available only to local investors. Since the MSCI calculates returns from the perspective of US investors, it excludes from its indices the tax credits which are available only to local investors. Hence to obtain the gross return to local investors, we add back the tax credits of 25% for the UK.

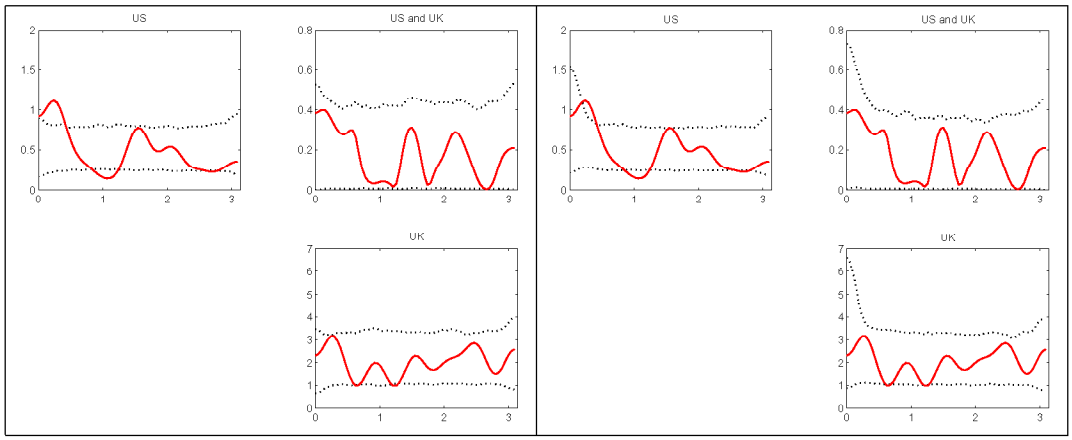


FIG. 4 - Sample periodogram. The grey line is the data sample periodogram, while the black line is the theoretical periodogram along with the 95% confidence interval (dashed lines).

where $\Gamma_j = E\left[Y_t - \overline{Y}\right]\left[Y_{t-j} - \overline{Y}\right]'$ and $\overline{Y} = E[Y_t]$. We follow Hamilton (1994) in estimating $\Gamma_j, \forall j$ with their sample counterparts:

$$\widehat{\Gamma}_j = \frac{\sum_{t=j+1}^{T}\left(Y_t - \overline{Y}\right)\left(Y_t - \overline{Y}\right)'}{T}$$

and smoothing (13) with a 10 period Bartlett (1964) window

$$\widehat{\widehat{S}}_Y(\omega) = \frac{1}{2\pi}\sum_{k=-10}^{k=10}\left(1 - \frac{|k|}{10}\right)\widehat{\Gamma}_k e^{-i\omega k} \tag{14}$$

As a measure of the covariance explained at different frequencies, we consider the coherence, defined as:

$$K^2_{\Delta c^h, \Delta c^f}(\omega) = \frac{\left|\widehat{S}_{\Delta c^h, \Delta c^f}(\omega)\right|^2}{\widehat{S}_{\Delta c^h}(\omega)\,\widehat{S}_{\Delta c^f}(\omega)}$$

The two panels of Figure 4 report the estimated spectra and coherence of US and UK consumption growth (continuous lines) along with the theoretical periodograms and the 95% confidence intervals (dashed lines) under the alternative assumptions

that consumption growth is *i.i.d.* (left panel) and that it has a small predictable component (right panel). The latter are computed simulating 1000 independent samples according to model (5) with the calibration of Table 1 expect for σ_c^h, σ_c^f and ρ_c^{hf}, that are set so that the simulated series have the same sample variance and covariance of the actual series. The main finding of Figure 4 is that the two alternative assumptions look equally likely to say the least, as the estimated spectra and coherence typically lie within the 95% confidence intervals. This anticipates a recurrent result of the next subsections: quantity data are not enough to identify the low frequency component of consumption growth.

6.3 A state space model

The way in which we model consumption growth falls directly in the class of state-space models. In particular, if we denote $Y_t' = \left[\begin{array}{cc} \Delta c_t^h & \Delta c_t^f \end{array} \right]$ as the vector of variables that are observable to the econometrician and $X_t' = \left[\begin{array}{cc} x_{t-1}^h & x_{t-1}^f \end{array} \right]$ as the vector of variables that are unobservable to the econometrician, we have the following representation:

$$\begin{aligned} Y_t - \left[\begin{array}{c} \mu_c^h \\ \mu_c^f \end{array} \right] &= +X_t + \left(\left[\begin{array}{c} \sigma^h \\ \sigma^f \end{array} \right]' \left[\begin{array}{cc} 1 & \rho_c^{hf} \\ \rho_c^{hf} & 1 \end{array} \right] \left[\begin{array}{c} \sigma^h \\ \sigma^f \end{array} \right] \right)^{\frac{1}{2}} \left[\begin{array}{c} \varepsilon_{c,t}^h \\ \varepsilon_{c,t}^f \end{array} \right] \\ X_{t+1} &= \left[\begin{array}{cc} \rho_x^h & 0 \\ 0 & \rho_x^f \end{array} \right] X_t + \left(\left[\begin{array}{c} \sigma^h \varphi_e^h \\ \sigma^f \varphi_e^f \end{array} \right]' \left[\begin{array}{cc} 1 & \rho_x^{hf} \\ \rho_x^{hf} & 1 \end{array} \right] \left[\begin{array}{c} \sigma^h \varphi_e^h \\ \sigma^f \varphi_e^f \end{array} \right] \right)^{\frac{1}{2}} \left[\begin{array}{c} \varepsilon_{x,t}^h \\ \varepsilon_{x,t}^f \end{array} \right] \end{aligned}$$

or, in more compact notation:

$$\begin{aligned} Y_t - \overline{Y} &= AX_t + C\varepsilon_{c,t} \\ X_{t+1} &= BX_t + D\varepsilon_{x,t} \end{aligned} \tag{15}$$

Notice that we are now allowing for an asymmetric calibration of the two economies. Aside from one technicality regarding time aggregation of the monthly model into quarterly data on observed consumption growth that we discuss in the Appendix, this model can be directly estimated via maximum likelihood, using the Kalman filter

TABLE 6
ESTIMATING LONG RUN RISKS.

	Real Data	Simulated data	
Sample size	T=120	T=120	T=10000
ρ^h	0.909	0.941	0.987
	[0.547, 0.995]	[0.731,1.000]	[0.983,0.990]
ρ^f	0.940	0.934	0.987
	[0.308, 0.995]	[0.630,1.000]	[0.983,0.991]
φ_e^h	0.351	0.142	0.049
	[0.000, 0.424]	[0.000,0.475]	[0.039,0.059]
φ_e^f	0.184	0.119	0.049
	[0.000, 1.092]	[0.000,1.165]	[0.041,0.057]
σ^h	38.103	66.083	67.978
	[28.562, 38.105]	[51.208,71.617]	[67.101,69.054]
σ^f	77.944	66.078	68.034
	[72.280, 78.009]	[37.388,68.164]	[67.282,68.757]
ρ_x	0.897	0.844	0.986
	[0.696, 1.000]	[0.467,1.000]	[0.953,1.000]
ρ_c	0.208	0.312	0.302
	[0.004, 0.406]	[0.066,0.530]	[0.285,0.318]

Notes - Estimation of consumption growth processes in the US and in the UK. The second column reports the results of the estimation on real data, while the last two columns report the results of the estimation over 100 independent samples simulated according to the baseline calibration for sample sizes T=120 and T=10000.

as in Hansen and Sargent (2004). We also impose that the volatility coefficients σ_c^h, σ_c^f, φ_e^h and φ_e^f are nonnegative, that the autoregressive coefficients ρ^h and ρ^f and the correlation coefficients ρ_x^{hf} and ρ_c^{hf} are smaller than 1 in absolute value. Due to these constraint, the standard asymptotic theory to compute standard errors of estimated coefficients does not apply. Therefore we follow Runkle (1987) and we construct standard errors by bootstrapping 1000 independent samples from the estimated smoothed residuals.

The second column of Table 6 reports the estimation results on actual consumption growth data obtained from using Kalman filter and maximum likelihood as detailed in the Appendix. The point estimates of autoregressive coefficients and

TABLE 7
ESTIMATING LONG RUN RISKS WITH MORE COUTRIES.

	US, UK and Germany	US, UK and Japan	US, UK, Germany and Japan
ρ^{US}	0.911	0.906	0.919
	[.530, .990]	[.552, .984]	[.796, .996]
ρ^{UK}	0.928	0.947	0.934
	[.049, .986]	[.214, .978]	[.746, .994]
ρ^{Ger}	0.932	-	0.993
	[.274, .985]	-	[.614, .996]
ρ^{Jpn}	-	0.989	0.99
	-	[.215, .989]	[.504, .998]
φ_e^{US}	0.351	0.040	0.177
	[.000,1.035]	[.000,.348]	[.000,.206]
φ_e^{UK}	0.317	0.182	0.232
	[.000,1.484]	[.000,1.557]	[.000,.322]
φ_e^{Ger}	0.019	-	0.002
	[.000,.774]	-	[.000,.407]
φ_e^{Jpn}	-	0.087	0.090
	-	[.000,1.558]	[.000,1.312]
σ^{US}	38.105	33.622	38.106
	[28.002,38.108]	[27.903,38.104]	[33.409,38.106]
σ^{UK}	69.861	69.864	69.863
	[69.798,69.907]	[69.844,69.898]	[69.693,69.956]
σ^{Ger}	70.436	-	69.861
	[70.106,70.450]	-	[69.847,69.866]
σ^{Jpn}	-	70.452	70.440
	-	[70.435,70.490]	[70.429,70.452]
$\rho_x^{US,UK}$	0.920	0.973	0.913
	[.890, .999]	[.861, .999]	[.700, .999]
$\rho_x^{US,Ger}$	0.899	-	0.891
	[.874, 1.000]	-	[.700, .997]
$\rho_x^{US,Jpn}$	-	0.991	0.905
	-	[.877, 1.000]	[.701, 1.000]
$\rho_x^{UK,Ger}$	0.891	-	0.917
	[.851, 1.000]	-	[.700, .999]
$\rho_x^{UK,Jpn}$	-	0.955	0.898
	-	[.648, 1.000]	[.699, 1.000]
$\rho_x^{Ger,Jpn}$	-	-	0.884
	-	-	[.698, .999]
$\rho_c^{US,UK}$	0.142	0.169	0.216
	[-.048, .341]	[-.015, .423]	[.089, .400]
$\rho_c^{US,Ger}$	0.111	-	0.151
	[-.088, .285]	-	[-.022, .337]
$\rho_c^{US,Jpn}$	-	0.237	0.275
	-	[.028, .483]	[.082, .446]
$\rho_c^{UK,Ger}$	0.272	-	0.309
	[.096, .469]	-	[.222, .464]
$\rho_c^{UK,Jpn}$	-	0.182	0.194
	-	[-.052, .358]	[.017, .332]
$\rho_c^{Ger,Jpn}$	-	-	-0.06
	-	-	[-.219, .202]

Notes - Estimation of consumption growth processes in the US, the UK, Germany and Japan. In squared brackets we report the 95% confidence intervals.

correlations would not be too far from the values that we calibrated them to in Table 1 if it was not for the big 95% confidence intervals associated to them. Furthermore the identification coefficients of the x_t processes, φ_e^h and φ_e^f are not significantly different from zero. We take these findings as confirming the graphical intuition of Figure 4 that we cannot reject neither provide positive evidence of this low frequency component from consumption data only. To check the power of our estimation procedure we also simulated 100 independent samples according to the baseline calibration reported in Table 1 for sample sizes T=120 (the same of the actual data sample) and T=10000 (a much bigger sample). The results for the Monte Carlo experiment of sample size $T = 120$ confirms the difficulty of tightly identifying the coefficients of the model that we experienced on the actual data sample. When we push the sample size to infinity, we find out, not surprisingly, that the estimation procedure is consistent. The punch line seems to be that we simply do not have enough quantity observations in the actual data.

6.4 More quantities, better estimates?

In principle, if the low frequency component of consumption growth is really perfectly correlated across countries, estimating the model including consumption observations from a cross section of countries should help to better identify the coefficients. We explore this idea by including also Germany and Japan in our analysis. The estimation procedure follows the same logic and technique of the previous subsection, while we relegate the construction of the matrices A, B, C and D in (15) to the Appendix. The results as reported in Table 7 show that the inclusion of additional countries does not help in tightening the confidence intervals around the estimated coefficients. This is not entirely a negative results, as it would have been a reduction of uncertainty around the wrong values of the parameter. It simply confirms that quantities are not enough to identify the low frequency component of consumption growth.

6.5 More prices, better estimates?

The spirit of this paper and of the literature that this paper belongs to is to try to understand the properties of the consumption process that would allow us to price assets in a no arbitrage framework. Even though these characteristics of consumption cannot be clearly identified from quantity data only, exchange rates and asset returns should help in this task. We may extend the procedure adopted in the previous subsections to estimate a larger system that also includes exchange rates and asset returns, but we prefer not to do so. The reason is that our model has been shown to provide an excellent description of a number of unconditional moments of international data, but, at least in this basic setup, it is still unsatisfactory along a few dimensions. The unconditional correlation of risk free rates and the unconditional correlation of consumption differentials and exchange rates are certainly hard to get numbers that would require more attention in the research to come. Furthermore the conditional distribution of exchange rates has typically been object of debate as described by Backus and Smith (1993) and more recently by Rey and Gourinchas (2005). Also, our assumptions of complete home bias in consumption and of complete markets are likely to provide an additional source of noise in the exchange rates. For all these reasons, we implement a GMM style estimation, in which we only focus on the set of moments that are the focus of attention of this paper. Alternatively this can be interpreted as a way of providing statistical support to our calibration as described in Tables 1 and 3. In order to increase the speed of the estimation routine, we restrict our attention to the linearized version of the model[8], as summarized by the system of equations (5), (6), (9) and (10).

In particular, we focus on the variances, the first two auto-covariances, the covariance and the first order cross auto-covariance of consumption growths, on the variances and the covariance of dividend growths, on the variance of the depreciation rate, on the variances of risk free rates, on the variances, the first order auto-covariances and the covariance of excess returns and on the average excess returns. There is a total of 22 moments conditions to identify 17 coefficients, as

[8] If we were to use a numerical solution of the model, this would entail re-solving for the equilibrium price consumption and price dividend ratios schedules, every time a new vector of parameters is searched for.

TABLE 8
PRICES AND LONG RUN RISKS.

	Consumption only		Whole Model	
	Point Estimate	95% CI	Point Estimate	95% CI
ρ^h	0.736	[0.349,0.996]	0.997	[0.927,1.000]
ρ^f	0.904	[0.015,0.997]	0.996	[0.912,0.999]
φ_e^h	1.422	[0.190,17.318]	0.024	[0.005,0.213]
φ_e^f	0.182	[0.000,3.502]	0.041	[0.003,0.157]
σ^h	27.629	[4.527,34.799]	32.792	[28.036,36.676]
σ^f	79.916	[44.407,87.421]	79.569	[69.767,88.879]
ρ_x^{hf}	0.999	[0.353,1.000]	0.998	[0.853,1.000]
ρ_c^{hf}	0.222	[-0.988,0.999]	0.349	[0.215,0.492]
ψ^h	-	-	0.530	[0.368,6.613]
ψ^f	-	-	6.363	[1.521,28.438]
γ^h	-	-	4.334	[3.231,8.524]
γ^f	-	-	3.625	[1.915,4.529]
λ^h	-	-	3.008	[2.176,14.133]
λ^f	-	-	3.655	[1.854,33.271]
φ_d^h	-	-	13.677	[12.022,15.227]
φ_d^f	-	-	3.693	[0.321,4.083]
ρ_d^{hf}	-	-	0.072	[-0.094,0.268]

	Unconditional moments (at point estimates)	
$\sigma\left(\Delta c^h\right)$	1.406	1.588
$\sigma\left(\Delta c^f\right)$	2.819	2.812
$\rho\left(\Delta c^h, \Delta c^f\right)$	0.296	0.332
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	-	11.692
$\rho\left(m^h, m^f\right)$	-	0.922
$\rho\left(r_d^h, r_d^f\right)$	-	0.475

Notes - The second column reports the results of the estimation on consumption data only, while the fourth column reports the results of the estimation of the whole model. In parenthesis we report the 95% confidence intervals. The subjective discount factors δ^h and δ^f are calibrated as in Table 1. The bottom panel of the table reports the unconditional moments in annualized terms of the variables of interest using the point estimates.

detailed in Table 8. Due to the analytical difficulty of computing these moment conditions for quarterly data starting from a model that is specified at a monthly frequency, we follow Lee and Ingram (1991) and Duffie and Singleton (1993) in implementing a Simulated Method of Moments. This amounts to computing the afore mentioned moments from the data set that we have and minimizing the distance between them and their counterparts obtained from simulating the model. Since the model includes consumption growth and returns moment conditions, the use of a Newey and West (1987) weighting matrix would suggest to put a much bigger weight on consumption relative to returns moments. To avoid this unappealing consequence, we use an identity weighting matrix. The estimator is going to be inefficient, but consistency is guaranteed. One more caveat is brought into the problem by the fact that we need to constrain the correlation coefficients to be bounded by 1 in absolute value[9]. Although this constraint is not binding in the estimation, Andrews (2002) warns that theoretically when the true parameter is on the boundary, GMM/SMM estimators are not asymptotically normally distributed. As a consequence, the asymptotic results of Hansen (1982) cannot be employed to derive confidence intervals of the estimated coefficients. We derive 95% confidence intervals using a Monte Carlo procedure. After consistently estimating the parameters of the model using a simulated sample of $120,000$ monthly observations, we use these coefficients to generate $1,000$ independent samples of length 120 quarters, the same that we have in the data. We then estimate the coefficients of interest using each of these simulated samples and construct confidence regions based on the parameters' distribution across samples.

Table 8 shows that adding price restrictions helps in tightening the point estimates of the consumption laws of motion parameters to the values that we specified in the theoretical analysis of the model. This procedure is still unable to properly identify the preference parameters and the coefficients that describe the laws of motion of dividends. Our explanation of this difficulty relies on the fact that we are only employing 13 moment conditions that involve these parameters in the form of exchange rate, risk free rates and stock returns. Future extensions should focus on

[9]We also impose nonnegativity constraints on volatilities and on preference parameters, although they are never binding.

generalizing the set of properties of the model, allowing for a wider set of moments restrictions to be used in the empirical analysis.

7 Conclusions and Extensions

7.1 More factors and yield curve evidence

Risk free rates and yields in general should bear a lot of information about the predictable component of consumption growth, as pointed out by the linearized system (9). Figure 5 reports the spectra and coherence of real risk free rates in the US and the UK. The sharp increase observed at low frequencies in all the subplots seems to confirm the idea that there is potentially a lot of information to be used to tighten the estimates provide earlier in the paper. Unfortunately the version of the model that we analyze in this paper prevents us from being able to do this, due to the implied perfect correlation of yields on bonds of any maturity. Is there anything that we can do to better exploit yield curve data? In the Appendix we show how to derive risk free rates and stochastic discount factors when consumption growth in the two countries is assumed to be affected by two factors[10]:

$$\begin{aligned}
\Delta c_t^h &= \mu_c^h + \lambda_1 z_{1,t-1} + \lambda_2 z_{2,t-1} + \varepsilon_t^{c,h} \\
\Delta c_t^f &= \mu_c^f + \lambda_2 z_{1,t-1} + \lambda_1 z_{2,t-1} + \varepsilon_t^{c,f} \\
z_{1,t} &= \rho_1 z_{1,t-1} + \varepsilon_t^{z_1} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \varepsilon_t^{z_2}
\end{aligned}$$

Notice that in the particular case in which $\lambda_1 = 1$, $\lambda_2 = 0$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{c,f}\right) = 0.3$, $\rho\left(\varepsilon_t^{z_1}, \varepsilon_t^{z_2}\right) = 1$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{z_1}\right) = \rho\left(\varepsilon_t^{c,f}, \varepsilon_t^{z_2}\right) = 0$, these laws of motion are identical to the ones that we have discussed in section 3. Table 9 shows that the inclusion of an extra factor can break the perfect correlation of risk free rates and at the same

[10]The analytical results in the Appendix are derived under the assumption that $\psi = 1$, that is when Epstein and Zin (1989) preferences collapse to Hansen's risk sensitive preference studied, inter alia, in a related paper by Tallarini (2000).

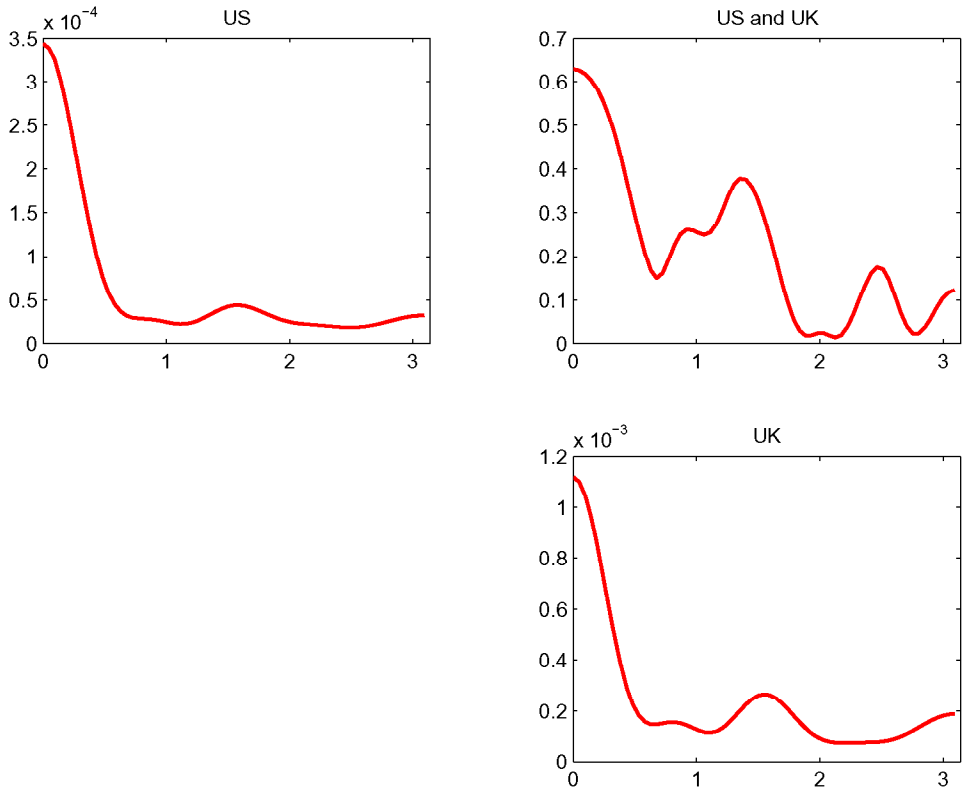


FIG. 5 - Spectra and coherence of real risk free rates in the United States and in the United Kingdom.

time preserve all the appealing results shown in the previous sections. A unified framework to study consumption, yield curves and long run risks is a priority for future research.

7.2 What is x_t?

In the paper we let data tell us what consumption growth should reasonably look like and imposing a subset of the equilibrium conditions impelled by the Euler equations we derived asset prices. Now the question becomes: where is the low

TABLE 9
ONE VS. TWO FACTORS.

	$\frac{Var(\sum \lambda z)}{Var(\Delta c)}$	$\rho\left(m^h, m^f\right)$	$\rho\left(r^h, r^f\right)$	$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	$\sigma(r)$	$\sigma(m)$
One Factor	8.18%	0.91	1.00	11.84	1.54	28.71
Two Factors	9.37%	0.92	0.61	12.99	1.59	35.00

Notes - Results in the row 'One factor' refer to the calibration $\lambda_1 = 1$, $\lambda_2 = 0$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{c,f}\right) = 0.3$, $\rho\left(\varepsilon_t^{z1}, \varepsilon_t^{z2}\right) = 1$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{z1}\right) = \rho\left(\varepsilon_t^{c,f}, \varepsilon_t^{z2}\right) = 0$. Results in the row 'Two factors' refer to the calibration $\lambda_1 = 1$, $\lambda_2 = 5$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{c,f}\right) = 0.3$, $\rho\left(\varepsilon_t^{z1}, \varepsilon_t^{z2}\right) = .3$, $\rho\left(\varepsilon_t^{c,h}, \varepsilon_t^{z1}\right) = \rho\left(\varepsilon_t^{c,f}, \varepsilon_t^{z2}\right) = 1$ and $\varphi_e = 0.0096$. All other parameters are calibrated as in Table 1 with the exception of $\Psi = 1$.

frequency component of consumption growth coming from? It seems reasonable to think of x_t as being embedded in the technology process and affecting consumption through a fully specified real business cycle model. What is the degree of persistence of technological progress that would justify our consumption process? Our model also requires the predictable component to be perfectly correlated across countries, leading us to further investigate the question of what are the causes of this common business cycle component. We believe that future research should focus on the deeper economic reasons that link quantities and prices in a fully specified model.

7.3 Relaxing the assumption of complete home bias

Throughout the paper we maintained the assumption of complete home bias in consumption. This proved a powerful instrument to simplify the solution of the model and we justified it by saying that domestically produced goods typically represent a large share of domestic consumption. This limiting situation should be taken as the starting point to show the ability of the model to solve the puzzle raised by Brandt, Cochrane, and Santa-Clara (2004) and to extend the analysis to the more realistic case of traded and non-traded goods.

7.4 Concluding remarks

We have shown that allowing for an intertemporal elasticity of substitution larger than one and for a persistent and highly cross-correlated forecastable component of consumption growth in the economy described by Brandt, Cochrane, and Santa-

Clara (2004) it is possible to reconcile the measure of cross country correlation of the stochastic discount factors obtained from data on consumption and from data on prices. This result is achieved in combination with a lowly volatile depreciation rate and without requiring a high correlation of the consumption processes and high coefficients of risk aversion. We have also shown how key features of the data can be described by the same parametrization that allows us to meet our primary goal, extending in this way the set of properties of the models that take into account long run risks beyond what pointed out by Bansal and Yaron (2004). We are optimistic about the positive findings of this paper to hold also in more general setups that have the potential of extending the set of properties of this class of models.

Appendix A. Derivation of moments

We express the relevant moments of the variables of the model as a function of the set Υ of deeper parameters defined as $\Upsilon = \left\{\theta, \psi, \delta, \rho_x, \lambda, \sigma_c, \sigma_x, \sigma_d, \rho_c^{hf}, \rho_x^{hf}, \rho_d^{hf}\right\}$. We assume that:

$$\begin{aligned} \Delta c_{t+1}^i &= x_t^i + \varepsilon_{c,t+1}^i,\ \varepsilon_{c,t}^i \sim_{iid} \mathcal{N}\left(0, \sigma_c^2\right) \\ \Delta d_{t+1}^i &= \lambda x_t^i + \varepsilon_{d,t+1}^i,\ \varepsilon_{d,t}^i \sim_{iid} \mathcal{N}\left(0, \sigma_c^2\right) \\ x_{t+1}^i &= \rho_x^i x_t^i + \varepsilon_{x,t+1}^i,\ \varepsilon_{x,t}^i \sim_{iid} \mathcal{N}\left(0, \sigma_x^2\right) \forall i \in \{h, f\} \end{aligned} \tag{A.1}$$

$$\tag{A.2}$$

The Euler equation for the asset that pays one unit of the consumption bundle at each period is:

$$1 = E_t\left[M_{t+1}^i R_{c,t+1}^i\right] \tag{A.3}$$

with $M_{t+1} = ADD$. The return on the consumption asset can be expressed in terms of the price-consumption ratio $v_{c,t}$ as $R_{c,t+1}^i = \frac{v_{c,t+1}^i + 1}{v_{c,t}^i} \exp \Delta c_{t+1}^i, \forall i \in \{h, f\}$. This implies that (A.3) can be written as:

$$(v_{c,t}^i)^\theta = E_t\left[\delta e^{\Delta c_{t+1}^i (1-\gamma)} (1 + v_{c,t+1}^i)^\theta\right] \tag{A.4}$$

Linearizing (A.4) around the steady state of $v_{c,t}$ defined as $v_c^{i,ss} = \frac{\delta^i}{1-\delta^i}$, we obtain:

$$v_{c,t}^i = E_t\left[\delta + \frac{\delta}{1-\delta}\left(1 - \frac{1}{\psi}\right)\Delta c_{t+1}^i + \delta v_{c,t+1}^i\right]$$

that can be solved forward delivering the price-consumption ratio as a function of the state variable x_t^i nd of the deeper parameters of the model:

$$v_{c,t}^i = \alpha_c^i + \beta_c^i x_t^i \tag{A.5}$$

with $\alpha_c^i = \frac{\delta^i}{1-\delta^i}$ and $\beta_c^i = \frac{\frac{\delta^i}{1-\delta^i}\left(1-\frac{1}{\psi^i}\right)}{1-\rho_x^i\delta^i}$, $\forall i \in \{h,f\}$. Similarly, we can solve for the price-dividend ratio $v_{d,t}^i$

$$v_{d,t}^i = \alpha_d^i + \beta_d^i x_t^i \tag{A.6}$$

with $\alpha_d^i = \frac{\delta^i}{1-\delta^i}$ and $\beta_d^i = \frac{\frac{\delta^i}{1-\delta^i}\left(\lambda^i-\frac{1}{\psi^i}\right)}{1-\rho_x^i\delta^i}$, $\forall i \in \{h,f\}$.

Log-returns on the assets that pay streams of consumption and stream of dividends follow immediately from (A.1),(A.5) and (A.6):

$$r_{c,t+1}^i = -log(\delta) + \left[\frac{1}{\psi}\right] x_t^i + \left[\delta\left(1-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]\epsilon_{x,t+1}^i + \epsilon_{c,t+1}^i \tag{A.7}$$

$$r_{d,t\mid 1}^i = -log(\delta) + \left[\frac{1}{\psi}\right] x_t^i + \left[\delta\left(\lambda-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]\epsilon_{x,t\mid 1}^i + \epsilon_{d,t\mid 1}^i \tag{A.8}$$

Also the log-stochastic discount factor is:

$$m_{t+1}^i = log\delta - \frac{1}{\psi}x_t^i - \gamma\epsilon_{c,t+1}^i + \frac{\delta\left(1-\gamma\Psi\right)}{\psi(1-\rho_x\delta)}\epsilon_{x,t+1}^i \tag{A.9}$$

The log-risk free rate r_f^i is obtained as the solution of:

$$\begin{aligned} r_f^i &= -E_t\left[\delta^\theta \exp\left\{-\frac{\theta}{\psi}\Delta c_{t+1}^i + (\theta-1)\, r_{c,t+1}^i\right\}\right] \\ &= -\log(\delta) + \frac{1}{2}\left[\left[\gamma\right]^2\sigma_c^2 + \left[\delta\left(1-\frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]^2\sigma_x^2\right] + \frac{1}{\psi}x_t^i \end{aligned} \tag{A.10}$$

The following moments follow directly from the previous formulas and from the assumption that both countries share the same parametrization:

$$\begin{aligned} Var(\pi) &= 2\left[\left(1-\rho_x^{hf}\right)\Gamma_0\sigma_x^2 + \left(1-\rho_c^{hf}\right)\gamma^2\sigma_c^2\right] \\ corr(m_t^h, m_t^f) &= \frac{\Gamma_0\rho_x^{hf}\sigma_x^2 + \gamma^2\rho_c^{hf}\sigma_c^2}{\Gamma_0\sigma_x^2 + \gamma^2\sigma_c^2} \\ Var(r_{c,t+1}^i) &= \Gamma_0\sigma_x^2 + \sigma_c^2 \\ Cov(r_{c,t+1}^h, r_{c,t+1}^f) &= \Gamma_0\rho_x^{hf}\sigma_x^2 + \rho_c^{hf}\sigma_c^2 \\ Cov(\Delta c_{t+1}^i, r_{c,t+1}^i) &= \Gamma_1 + \sigma_c^2 \\ Cov(\Delta c_{t+1}^h, r_{c,t+1}^f) &= \Gamma_1\rho_x^{hf} + \rho_c^{hf}\sigma_c^2 \\ corr\left(r_{d,t}^h - r_f^h, r_{d,t}^f - r_f^f\right) &= \frac{\Gamma_2\rho_x^{hf}\sigma_x^2 + \rho_d^{hf}\sigma_d^2}{\Gamma_2\sigma_x^2 + \sigma_d^2} \end{aligned} \tag{A.11}$$

where $\Gamma_0 = \left(\frac{1}{\psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\frac{\delta(1-\gamma\psi)}{\psi(1-\rho_x\delta)}\right]^2$, $\Gamma_1 = \frac{\sigma_x^2}{\psi(1-\rho_x^2)}$ and $\Gamma_2 = \left[\delta\left(\lambda - \frac{1}{\psi}\right)\frac{1}{1-\rho_x\delta}\right]^2$.

Appendix B. Proof of Propositions

Proof of Proposition 1. For any choice of the parameters that satisfy

$$\rho_x^i \neq 1 \quad \text{and} \quad \rho_x^i \delta^i \neq 1$$

the following three partial derivatives

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \rho_x^{hf}} = \frac{\Gamma_0 \sigma_x^2}{\Gamma_0 \sigma_x^2 + \gamma^2 \sigma_c^2} > 0 \tag{B.1}$$

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \Psi} = -\frac{2\Gamma_0 \gamma^2 \left(\rho_x^{hf} - \rho_c^{hf}\right) \sigma_x^2 \sigma_c^2}{\psi \left(\Gamma_0 \sigma_x^2 + \gamma^2 \sigma_c^2\right)^2} \tag{B.2}$$

$$\frac{\partial corr(m_t^h, m_t^f)}{\partial \rho_x} = \frac{2\left[\frac{\rho_x}{(1-\rho_x^2)^2} + \frac{\delta^3(1-\gamma\psi)^2}{(1-\rho_x\delta)^3}\right]\gamma^2(\rho_x^{hf} - \rho_c^{hf})\sigma_x^2\sigma_c^2}{\psi^2\left[\Gamma_0\sigma_x^2 + \gamma^2\sigma_c^2\right]^2} \tag{B.3}$$

exist and are well defined. (B.1) is positive for all the values of the parameters that respect the two conditions, implying that the correlation of the two stochastic discount factors is strictly increasing with respect to ρ_x^{hf}. When $\rho_x^{hf} = \rho_c^{hf}$ (B.2) is always zero, meaning that changes in Ψ, γ or δ do not affect the correlation of the two stochastic discount factors. If $\rho_x^{hf} \neq \rho_c^{hf}$, this derivative is zero only if $\psi = \frac{1}{\gamma}\widetilde{\delta}$, where $\widetilde{\delta} = \frac{1-2\rho_x\delta+\delta^2}{\delta^2(1-\rho_x^2)}$. In particular, when $\rho_x^{hf} > \rho_c^{hf}$ the sign of the derivative is positive for $\Psi > \frac{1}{\gamma}\widetilde{\delta}$ and negative for $\psi < \frac{1}{\gamma}\widetilde{\delta}$. Notice that

$$\lim_{\delta \to 1^-} \frac{1}{\gamma}\widetilde{\delta} \geq \frac{1}{\gamma} \quad \text{and} \quad \lim_{\rho_x \to 1^-} \lim_{\delta \to 1^-} \frac{1}{\gamma}\widetilde{\delta} = \frac{1}{\gamma}$$

So, for a high persistence of the long run component and an individual discount factor close to one, the minimum of the cross correlation of the two discount factors is achieved for $\psi = \frac{1}{\gamma}$, that is when the Epstein-Zin preferences collapse to the standard CES utility function. (B.3) is always positive when $\rho_x^{hf} > \rho_c^{hf}$, negative when $\rho_x^{hf} < \rho_c^{hf}$ and equal to zero when $\rho_x^{hf} = \rho_c^{hf}$. Therefore if $\rho_x^{hf} > \rho_c^{hf}$, the minimum is achieved for ρ_x=0. When $\rho_x^{hf} > \rho_c^{hf}$, (B.1), (B.2) and (B.3) imply the existence of one unique minimizer at $\left(\rho_x = 0, \rho_x^{hf} = 0, \psi = \frac{1}{\gamma}\tilde{\delta}\right)$. □

Proof of Proposition 2. The partial derivative of (A.11) with respect to ρ_x^{hf} exists and is well defined provided that $\rho_x \neq 1$ and $\rho_x\delta \neq 1$:

$$\frac{\partial Var(\pi)}{\partial \rho_x^{hf}} = -2\left[\left(\frac{1}{\psi}\right)^2 \frac{1}{1-\rho_x^2} + \left[\frac{\delta(1-\gamma\psi)}{\psi(1-\rho_x\delta)}\right]^2\right]\sigma_x^2 \leq 0 \tag{B.4}$$

In particular, this derivative is always negative, implying that the volatility of the log-depreciation rate achieves its minimum when $\rho_x^{hf} = 1$. □

Appendix C. Numerical algorithm for the approximation of the price dividend ratios

We describe the procedure to numerically approximate the price-consumption and price-dividend ratios for the most general case in which stochastic volatility is in the model, too. We discretize the support of x and σ into I_x and I_σ points respectively, to get $I - I_x \cdot I_\sigma$ nodes $(x, \sigma)_i$, $\forall i - 1, ..., I$. In what follows, we will refer to x_i and σ_i as the first and the second entry of $(\alpha, s)_i$ respectively. Specify J known linearly independent basis functions $\phi_j\left((x, \sigma)_i\right)$, $j \in \{1, ..., J\}$. In our solution, we employ a third order polynomial in x combined with a first order polynomial in σ, implying that $J = 6$. The goal is to find basis coefficients c_j, $j = 1, ..., J$ that best approximate the Euler equation

$$v_c^i = V\left((x, \sigma)_i\right) \approx \sum_{j=1}^{J} c_j \phi_j\left((x, \sigma)_i\right) = \sum_{j=1}^{J} c_j \phi_{j,i} \tag{C.1}$$

$\forall i = 1, ..., I$ or, in the equivalent matrix notation:

$$v_c \approx \Phi c$$

where v_c is the $I \times 1$ vector of approximated value functions at each node, Φ is the $I \times J$ collocation matrix and $c = [c_1, ..., c_J]'$ is the vector of approximation coefficients. We also discretize the support of the three shocks in K_1, K_2 and K_3 points and denote $w_{1,k}$, $w_{2,k}$ and $w_{3,k}$ the approximated probability masses associated to each of the nodes. The shocks are assumed to independent. Under these assumptions, we get for each node $i \in \{1, ..., I\}$:

$$\begin{aligned} v_{c,i} = \delta \Bigg[& \sum_{\varepsilon_c \in K_1} \sum_{\varepsilon_x \in K_2} \sum_{\varepsilon_\sigma \in K_3} w_{1,\varepsilon_c} w_{2,\varepsilon_x} w_{3,\varepsilon_\sigma} \exp\left\{\theta\left(1 - \frac{1}{\psi}\right) \Delta c'_{i,\varepsilon_c}\right\} \\ & \left(1 + \sum_{j=1}^{J} c_j \phi_j\left(x'_{i,\varepsilon_x}, \sigma'_{i,\varepsilon_\sigma}\right)\right)^{\theta} \Bigg]^{\frac{1}{\theta}} \end{aligned} \tag{C.2}$$

where

$$\begin{aligned} \sigma'_{i,\varepsilon_\sigma} &= \overline{\sigma} + \nu_1\left(\sigma_i - \overline{\sigma}\right) + \sigma_w \varepsilon_\sigma \\ \Delta c'_{i,\varepsilon_c} &= x_i + \sigma'_{i,\varepsilon_\sigma} \varepsilon_c \\ x'_{i,\varepsilon_x} &= \rho_x x_i + \varphi_e \sigma'_{i,\varepsilon_\sigma} \varepsilon_x \end{aligned}$$

We can now use the following algorithm to solve the Euler equation recursively:

1. guess an initial vector of basis coefficients c^1
2. for each node $(s, \alpha)_i$ compute the right hand side of equation (C.2) using c^1 and call $v(c^1)$ the outcome
3. solve for $c^2 = \left(\Phi'\Phi\right)^{-1} \Phi' v(c^1)$
4. replace c^1 with c^2 and iterate until convergence.

Having solved for the price-consumption ratio v_c, we can solve the Euler equation for the price-dividend ratio in a similar way:

$$\begin{aligned} v_{d,i} &= \delta^{\theta} \sum_{\varepsilon_c \in K_1} \sum_{\varepsilon_x \in K_2} \sum_{\varepsilon_\sigma \in K_3} \sum_{\varepsilon_d \in K_1} w_{1,\varepsilon_c} w_{2,\varepsilon_x} w_{3,\varepsilon_\sigma} w_{4,\varepsilon_d} \exp\left\{ m'_{i,(\varepsilon_c,\varepsilon_x,\varepsilon_\sigma)} \right\} \\ & \left(1 + \sum_{j=1}^{J} d_j \phi_j \left(x'_{i,\varepsilon_x}, \sigma'_{i,\varepsilon_\sigma} \right) \right) \end{aligned} \tag{C.3}$$

where

$$m'_{i,(\varepsilon_c,\varepsilon_x,\varepsilon_\sigma)} = \left(\theta - 1 - \frac{\theta}{\psi} \right) \Delta c'_{i,\varepsilon_c} (\theta - 1) \log \left(\frac{1 + v'_{i,(\varepsilon_c,\varepsilon_x,\varepsilon_\sigma)}}{v_i} \right)$$

Appendix D. Details of estimation of state space models

Since we have quarterly data, but we specified the model to describe a monthly decision problem, the first thing that we need to do is to aggregate consumption data:

$$\begin{aligned} \Delta c^i_t &= \widetilde{x}^i_t + \varepsilon^i_{c,t} \\ \widetilde{x}^i_{t+1} &= \left(1 + \rho^i_x + \rho^{i^2}_x \right) \left(\rho^{i^3}_x x^i_{1,t-1} + y^i_{1,t} + \rho^i_x y^i_{2,t} + \rho^{i^2}_x y^i_{3,t} \right) \\ x^i_{1,t} &- \rho^{i^3}_x x^i_{1,t-1} + \left(y^i_{1,t} + \rho^i_x + y^i_{2,t} + \rho^{i^2}_x y^i_{3,t} \right) \\ y^i_{j,t+1} &= \varepsilon^i_{x_j,t+1}, \forall j \in \{1,2,3\} \end{aligned} \tag{D.1}$$

$$\tag{D.2}$$

where $\varepsilon^i_{c,t} \sim_{iid} \mathcal{N}\left(0, 3\sigma^{i^2}_c\right)$ and $\varepsilon^i_{x_j,t} \sim_{iid} \mathcal{N}\left(0, \varphi^{i^2}_e \sigma^{i^2}_c\right)$. We also assume that $\varepsilon^i_{c,t}$ and $\varepsilon^i_{x,t}$ are uncorrelated within the same country. Denoting $Y^i_t = \Delta c^i_t$ and $X^i_t = \left[\begin{array}{ccccc} \widetilde{x}^i_t & x^i_{1,t-1} & y^i_{1,t} & y^i_{2,t} & y^i_{3,t} \end{array} \right]'$, $\forall i \in \{h, f\}$, we obtain the following state space representation within each country:

$$\begin{aligned} Y^i_t &= A^i X^i_t + C^i v^i_t \\ X^i_{t+1} &= B^i X^i_t + D^i w^i_{t+1} \end{aligned}$$

where A^i, B^i, C^i and D^i can be easily built from (D.1), $\forall i \in \{h, f\}$. The whole system can now be constructed by stacking the observable and unobservable variables into vectors $Y_t = \left[\begin{array}{cc} \Delta c^h_t & \Delta c^f_t \end{array} \right]'$ and $X_t = \left[\begin{array}{cc} X^{h'}_t & X^{f'}_t \end{array} \right]$:

$$\begin{aligned} Y_t &= A X_t + C v_t \\ X_{t+1} &= B X_t + D w_{t+1} \end{aligned}$$

where

$$A = \left[\begin{array}{cc} A^h & 0 \\ 0 & A^f \end{array} \right] \quad C = \left[\begin{array}{cc} \sigma^h_c & 0 \\ \sigma^f_c \rho_c & \sigma^f_c \sqrt{1 - \rho^2_c} \end{array} \right]$$

$$B = \left[\begin{array}{cc} B^h & 0 \\ 0 & B^f \end{array} \right] \quad D = \left[\begin{array}{ccc} \left[\begin{array}{cc} 1 & 0 \end{array} \right] & \otimes & D^h \\ \left[\begin{array}{cc} \rho_x & \sqrt{1 - \rho^2_x} \end{array} \right] & \otimes & D^f \end{array} \right]$$

and v_t and w_{t+1} are bivariate independent vectors distributed as standard normals. Standard methods as those described in Hansen and Sargent (2004) can be used to obtain the likelihood function:

$$L\left(Y_1,...,Y_T\right) = \frac{1}{2}\sum_{t=1}^{T}\left\{p\log(2\pi)+\log\left|\Omega_t\right|+a_t'\Omega_t^{-1}a_t\right\}$$

where $\Omega_t = A\Sigma_t A'+CC'$, Σ_t solves the matrix Riccati difference equation $\Sigma_{t+1} = B\Sigma_t B'+DD'-B\Sigma_t A'\left(A\Sigma_t A'+CC'\right)^{-1}A\Sigma_t B'$, a_t comes from the innovations representation

$$\begin{aligned} a_t &= Y_t - A\widehat{X}_t \\ \widehat{X}_{t+1} &= B\widehat{X}_t + K_t a_t \end{aligned}$$

and $K_t = B\Sigma_t A'\left(A\Sigma_t A'+CC'\right)^{-1}$ is the Kalman gain.

When the state space model extends to three and four countries, the afore mentioned matrices A, B, C and D should be modified as follows (the subscript 3 or 4 refers to the two alternative extensions):

$$A_3 = \begin{bmatrix} A & 0 \\ 0 & A^g \end{bmatrix} \quad C_3 = \begin{bmatrix} \sigma_c^h & 0 & 0 \\ \sigma_c^f\rho_c^{hf} & \sigma_c^f\sqrt{1-\rho_c^{hf^2}} & 0 \\ \sigma_c^g\rho_c^{hg} & \sigma_c^g\frac{\rho_c^{gf}-\rho_c^{hf}\rho_c^{hg}}{\sqrt{1-\rho_c^{hf^2}}} & \sigma_c^g\tau_{1,c} \end{bmatrix}$$

$$B_3 = \begin{bmatrix} B & 0 \\ 0 & B^g \end{bmatrix} \quad D_3 = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} & \otimes & D^h \\ \begin{bmatrix} \rho_x^{hf} & \sqrt{1-\rho_x^{hf^2}} & 0 \end{bmatrix} & \otimes & D^f \\ \begin{bmatrix} \rho_x^{hg} & \frac{\left(\rho_x^{fg}-\rho_x^{hf}\rho_x^{hg}\right)}{\sqrt{1-\rho_x^{hf^2}}} & \tau_{1,x} \end{bmatrix} & \otimes & D^g \end{bmatrix}$$

and

$$A_4 - \begin{bmatrix} A_3 & 0 \\ 0 & A^l \end{bmatrix} \quad C_4 - \begin{bmatrix} \sigma_c^h & 0 & 0 & 0 \\ \sigma_c^f\rho_c^{hf} & \sigma_c^f\sqrt{1-\rho_c^{hf^2}} & 0 & 0 \\ \sigma_c^g\rho_c^{hg} & \sigma_c^g\frac{\rho_c^{gf}-\rho_c^{hf}\rho_c^{hg}}{\sqrt{1-\rho_c^{hf^2}}} & \sigma_c^g\tau_{1,c} & 0 \\ \sigma_c^l\rho_c^{hl} & \sigma_c^l\frac{\left(\rho_c^{fl}-\rho_c^{hf}\rho_c^{hl}\right)}{\sqrt{1-\rho_c^{hf^2}}} & \sigma_c^l\tau_{2,c} & \sigma_c^l\tau_{3,c} \end{bmatrix}$$

$$B_4 = \begin{bmatrix} B_3 & 0 \\ 0 & B^l \end{bmatrix} \quad D_4 = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} & \otimes & D^h \\ \begin{bmatrix} \rho_x^{hf} & \sqrt{1-\rho_x^{hf^2}} & 0 & 0 \end{bmatrix} & \otimes & D^f \\ \begin{bmatrix} \rho_x^{hg} & \frac{\left(\rho_x^{fg}-\rho_x^{hf}\rho_x^{hg}\right)}{\sqrt{1-\rho_x^{hf^2}}} & \tau_{1,x} & 0 \end{bmatrix} & \otimes & D^g \\ \begin{bmatrix} \rho_x^{hl} & \frac{\left(\rho_x^{fl}-\rho_x^{hf}\rho_x^{hl}\right)}{\sqrt{1-\rho_x^{hf^2}}} & \tau_{2,x} & \tau_{3,x} \end{bmatrix} & \otimes & D^l \end{bmatrix}$$

where:

$$\begin{aligned}
\tau_{1,j} &= \frac{\sqrt{1-\rho_j^{hf^2}-\rho_j^{hg^2}-\rho_j^{fg^2}+2\rho_j^{hf}\rho_j^{hg}\rho_j^{fg}}}{\sqrt{1-\rho_j^{hf^2}}} \\
\tau_{2,j} &= \frac{\rho_j^{gl}-\rho_j^{fg}\rho_j^{fl}-\rho_j^{fl}\rho_j^{fg}\rho_j^{hf^2}\rho_j^{gl}+\rho_j^{hf}\rho_j^{hg}\rho_j^{fl}+\rho_j^{hf}\rho_j^{hl}\rho_j^{fg}}{\sqrt{\left(1-\rho_j^{hf^2}\right)\left(1-\rho_j^{hf^2}-\rho_j^{hg^2}-\rho_j^{fg^2}+2\rho_j^{hf}\rho_j^{hg}\rho_j^{fg}\right)}} \\
\tau_{3,j} &= \sqrt{\frac{1-\rho_j^{hf^2}-\rho_j^{hl^2}-\rho_j^{fl^2}+2\rho_j^{hf}\rho_j^{hl}\rho_j^{fl}}{1-\rho_j^{hf^2}} - \frac{\left(\rho_j^{gl}-\rho_j^{gl}\rho_j^{hf^2}-\rho_j^{hg}\rho_j^{hl^2}-\rho_j^{fg}\rho_j^{fl}+\rho_j^{hf}\rho_j^{hg}\rho_j^{fl}+\rho_j^{hf}\rho_j^{hl}\rho_j^{fg}\right)^2}{\left(1-\rho_j^{hf^2}\right)\left(1-\rho_j^{hg^2}-\rho_j^{fg^2}-\rho_j^{hf^2}\rho_j^{hg^2}+2\rho_j^{hf}\rho_j^{hg}\rho_j^{fg}\right)}}
\end{aligned}$$

for $j \in \{c, x\}$.

Appendix E. Details of 'two factors' economy

We specialize Epstein and Zin (1989) preferences to the case of $\psi = 1$ (risk sensitive preferences):

$$U_t^i = \log c_t^i \quad \theta_1^i \log E_t \left[\beta^{\frac{U_{t+1}^i}{\theta_1^i}} \right], \forall i \subset \{h, f\} \tag{E.3}$$

where $\theta_1^i = -\frac{\beta}{(1-\gamma^i)(1-\beta)}$. Consumption in the two countries evolves according to the following laws of motion:

$$\begin{aligned}
\Delta c_t^h &= \log c_t^h - \log c_{t-1}^h = \mu_c^h + \lambda_1 z_{1,t-1} + \lambda_2 z_{2,t-1} + \varepsilon_t^{c,h} \\
\Delta c_t^f &= \log c_t^f - \log c_{t-1}^f = \mu_c^f + \lambda_2 z_{1,t-1} + \lambda_1 z_{2,t-1} + \varepsilon_t^{c,f} \\
z_{1,t} &= \rho_1 z_{1,t-1} + \varepsilon_t^{z_1} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \varepsilon_t^{z_2}
\end{aligned}$$

Find the solution of the difference equation (E.3) by guessing that it is a linear function of $\log c_t^i$, $z_{1,t}$ and $z_{2,t}$:

$$\begin{aligned}
U_t^h &= \log c_t^h - \theta_1^h \log E_t \left\{ -\frac{\beta}{\theta_1^h} \left[A + B\left(\mu_c^h + \log c_t^h + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \varepsilon_{t+1}^{c,h}\right) + \right.\right. \\
&\quad \left.\left. D\left(\rho_1 z_{1,t} + \varepsilon_{t+1}^{z_1}\right) + F\left(\rho_2 z_{2,t} + \varepsilon_{t+1}^{z_2}\right) \right] \right\} \\
&= \beta \left[A + B\mu_c^h - \frac{\beta}{2\theta_1^h}\left(B^2\sigma_c^2 + F\sigma_{z_2}^2 + D\sigma_{z_1}^2\right) \right] + (1 + B\beta)\log c_t^h + \\
&\quad \beta\left(B\lambda_1 + D\rho_1\right) z_{1,t} + \beta\left(B\lambda_2 + F\rho_2\right) z_{2,t}
\end{aligned}$$

This implies:

$$\begin{aligned}
A^i &= \frac{\beta}{1-\beta}\left[B^i\mu_c^i - \frac{\beta}{2\theta_1^i}\left(B^{i^2}\sigma_c^2 + D^{i^2}\sigma_{z_1}^2 + F^{i^2}\sigma_{z_2}^2\right) \right], B^i = \frac{1}{1-\beta} \\
D^h &= \frac{\beta\lambda_1}{(1-\beta)(1-\beta\rho_1)}, F^h = \frac{\beta\lambda_2}{(1-\beta)(1-\beta\rho_2)}, D^f = F^h, F^f = D^h
\end{aligned}$$

Pricing kernels must satisfy:

$$M_{t+1}^{i} = \beta \frac{c_t^i}{c_{t+1}^i} \frac{\exp\left\{-\beta \frac{U_{t+1}^i}{\theta_1^i}\right\}}{E_t\left[\exp\left\{-\beta \frac{U_{t+1}^i}{\theta_1^i}\right\}\right]}$$

Using the solution derived above, we can obtain an expression for the logarithm of the stochastic discount factor:

$$\begin{aligned} m_{t+1}^h &= k_m^h - \lambda_1 z_{1,t} - \lambda_2 z_{2,t} - \left(1 + \beta \frac{B^h}{\theta_1^h}\right) \varepsilon_{t+1}^{c^h} - \frac{\beta}{\theta_1^h} D^h \varepsilon_{t+1}^{z_1} - \frac{\beta}{\theta_1^h} F^h \varepsilon_{t+1}^{z_2} \\ m_{t+1}^f &= k_m^f - \lambda_2 z_{1,t} - \lambda_1 z_{2,t} - \left(1 + \beta \frac{B^f}{\theta_1^f}\right) \varepsilon_{t+1}^{c^f} - \frac{\beta}{\theta_1^f} D^f \varepsilon_{t+1}^{z_1} - \frac{\beta}{\theta_1^f} F^f \varepsilon_{t+1}^{z_2} \end{aligned} \quad \text{(E.4)}$$

where $k_m^i = \log\beta - \mu_c^i - \left(\frac{\beta B^i}{\theta_1^i}\right)^2 \frac{\sigma_c^2}{2} - \left(\frac{\beta D^i}{\theta_1^i}\right)^2 \frac{\sigma_{z_1}^2}{2} - \left(\frac{\beta F^i}{\theta_1^i}\right)^2 \frac{\sigma_{z_2}^2}{2}$. Risk free rates are:

$$\begin{aligned} r_f^h &= -logE_t \exp m_{t+1}^h = k_0^h + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} \\ r_f^f &= -logE_t \exp m_{t+1}^f = k_0^f + \lambda_2 z_{1,t} + \lambda_1 z_{2,t} \end{aligned}$$

where $k_0^i = -k_m^i - \left(1 + \beta \frac{B^i}{\theta_1^i}\right)^2 \frac{\sigma_c^2}{2} - \left(\frac{\beta}{\theta_1^i} D^i\right)^2 \frac{\sigma_{z_1}^2}{2} - \left(\frac{\beta}{\theta_1^i} F^i\right)^2 \frac{\sigma_{z_2}^2}{2}$. Bond prices can be derived recursively as:

$$P_t^{n^h} = \exp\left\{k_{0,n}^h + k_{1,n}^h z_{1,t} + k_{2,n}^h z_{2,t}\right\}$$

where

$$\begin{aligned} k_{0,n}^h &= -k_0^h + k_{0,n-1}^h + k_{1,n-1}^{h^2} \frac{\sigma_{z_1}^2}{2} + k_{2,n-1}^{h^2} \frac{\sigma_{z_2}^2}{2} \\ k_{1,n}^h &= -\lambda_1 + \rho_1 k_{1,n-1}^h \\ k_{2,n}^h &= -\lambda_2 + \rho_2 k_{2,n-1}^h \end{aligned}$$

and $k_{0,1}^h = -k_0^h$, $k_{1,1}^h = -\lambda_1$ and $k_{2,1}^h = -\lambda_2$. Similarly, in the foreign country:

$$P_t^{n^f} = \exp\left\{k_{0,n}^f + k_{1,n}^f z_{1,t} + k_{2,n}^f z_{2,t}\right\}$$

where

$$\begin{aligned} k_{0,n}^f &= -k_0^f + k_{0,n-1}^f + k_{1,n-1}^{f^2} \frac{\sigma_{z_1}^2}{2} + k_{2,n-1}^{f^2} \frac{\sigma_{z_2}^2}{2} \\ k_{1,n}^f &= -\lambda_2 + \rho_1 k_{1,n-1}^f \\ k_{2,n}^f &= -\lambda_1 + \rho_2 k_{2,n-1}^f \end{aligned}$$

and $k_{0,1}^f = -k_0^f$, $k_{1,1}^f = -\lambda_2$ and $k_{2,1}^f = -\lambda_1$.

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