Risk and Wealth in a Model of Self-Fulfilling Currency Attacks*

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Abstract

Market participants' risk attitudes, wealth and portfolio composition influence their positions in a pegged foreign currency and, therefore, may have important effects on the sustainability of currency pegs. We analyze such effects in a global game model of currency crises with continuous action choices. The model, solved in closed form, generates a rich set of theoretical predictions consistent with many popular and academic (unmodelled) speculations about the onset and timing of currency crises. The results extend linearly to a heterogeneous agent population.

Keywords: Currency crisis, global games, risk aversion, wealth, portfolio.

Jel Classification: F3, D8

1 Introduction

Market participants choose their positions in a pegged foreign currency in the light of their beliefs about the sustainability of the peg, their overall portfolio of assets and their risk aversion. So the risk attitudes and portfolio composition of speculators in foreign exchange markets might be expected to have a significant impact on the sustainability of currency pegs. Many popular and academic speculations about the onset and timing of currency crises build on this view. Did hedge funds' ability and willingness to take large short positions play a role in the currency crises of the 1990s?[1] Was the apparent contagion from the Russian crisis of 1998 to Brazil caused by emerging market investors who lost wealth in Russia withdrawing capital from Brazil?[2] Was the apparent contagion from unstable Asian markets like Indonesia to more

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[1] Krugman (1996) writes that "if we accept the idea that many currency crises are unjustified by fundamentals, there is a strong case for reconsidering the traditional economist's benign attitude toward financial markets; instead of regarding speculators as essentially blameless, mere messengers bringing the bad news, the new models suggest that the George Soroses of the world may be the true villains, tearing down structures that might otherwise have stood indefinitely." (page 246)

[2] See Blustein (2001, chapter 12) for a popular account of this contagion and Baig and Goldfajn (2001) for an academic assessment.

stable markets like Australia caused by cross-hedging, where investors with illiquid exposure to Indonesia hedged with short positions in the Australian dollar?[3] Did increased (illiquid) foreign direct investment in emerging markets create hedging demand for short positions in the currency?[4] Are contingent repo facilities[5] and standstills[6] negotiated by emerging market central banks with New York banks to defend against liquidity crises self-defeating because of the hedging demand for short positions that they created for counterparties?

Standard models of currency crises do not address these issues. In "first generation" models of currency crises (e.g., Krugman (1979)), the timing of the collapse of an unsustainable peg is determined by forward looking arbitrage conditions. In "second generation" models (e.g., Obstfeld (1996)), government actions make currency attacks self-fulfilling giving rise to multiple equilibria. In both cases, each market participant is certain (in equilibrium) if and when an attack will occur. Someone who is certain that an attack will occur will short the currency independent of attitudes to risk and underlying portfolio position.

Thus a necessary ingredient of a model of the role of risk and wealth considerations in a currency attack is *strategic uncertainty*: market participants must face uncertainty about whether an attack will be successful or not at the time they take positions in the market.[7] This is surely a realistic feature of these markets. It is also the driving force behind so-called "global games" models of currency crises (Morris and Shin (1998)). In these models, a small amount of uncertainty about fundamentals leads to a large amount of uncertainty about others' actions for the participant on the margin of attacking the currency. Unfortunately, existing global games models have focussed exclusively on the binary actions case, i.e., each participant faces a binary choice between attacking or not attacking; this makes the model inappropriate to understand risk and wealth considerations.[8]

[3]Concerns about such channels of financial contagion lead the Financial Stability Forum to establish a market dynamics study group, which found some evidence of this phenomenon (Financial Stability Forum (2000) page 117).

[4]The International Monetary Fund (1998) reports that "in the event a (unhedged) foreign direct investor decides to hedge, there will be an incipient capital outflow. If a counterparty with an exactly offsetting need does not emerge at the same time, such a transaction undertaken through a financial intermediary will, when it offsets its position, result in an actual capital outflow. Hedging by multinational corporations was ascribed a significant role by market participants in generating the pressures on the Brazilian real in late 1997" (page 16).

[5]Prior to the collapse of the Argentine currency board, the Central Bank of Argentina negotiated contingent repurchase agreements with international banks for government bonds and other Argentine domestic financial assets. The policy was intended to provide insurance against systemic liquidity shocks, and at one point amounted to around $6 billion in assets. One concern in the design of the policy was the possibility that the international banks would limit their (contingent) country exposure by hedging in other markets. In the end, the facility was triggered in August 2001, with the then $1.8 billion commitments honored by the international banks and later fully repaid by the central bank. See Powell (2002, page 283) on the Argentinian experience and Tirole (2002, page 21) and Giannini (2002, page 534) on theoretical difficulties with such repo agreements.

[6]In March 1999, there was a voluntary agreement by Brazil's creditor banks to maintain interbank and trade lines at existing levels. The banks stuck to their agreement but partially hedged by shorting the liquid Brazilian C-bond (IMF (2000) page 135).

[7]Possibly one could derive related comparative statics in a complete information model with symmetric uncertainty about fundamentals. However, to make risk aversion matter in such a model, a large amount of uncertainty about fundamentals would be required. Our results continue to hold even when uncertainty about fundamentals is arbitrarily small.

[8]An important paper by Goldstein and Pauzner (2001) does study the role of risk aversion and wealth in currency crises using a binary action global game model. We will discuss this paper and the limitations of binary action models for

This paper introduces a benchmark static model for analyzing risk, wealth and portfolio composition effects on currency crises. Agents are characterized by the degree of relative risk aversion, the composition of their portfolio of dollar and peso-denominated assets and their propensity to consume in dollar and peso denominated goods. They earn an interest rate premium from holding pesos. However, there is a possibility that the exchange rate will be devalued by a known amount. Each investor will choose an optimal portfolio given his beliefs about the likelihood of devaluation. Devaluation occurs if the aggregate net sales of pesos exceeds some stochastic threshold (the "fundamentals"). Each agent has a different noisy signal of fundamentals.

This is an example of a global game with continuous actions. Existing theoretical arguments (Frankel, Morris and Pauzner (2003)) establish that there is a unique equilibrium characterized by the critical threshold where the currency is devalued. But no global game with continuous actions has been solved in closed form. We are able to solve the model of this paper in closed form and — despite the many variables introduced into the model — the solution has a simple economic logic and generates a rich set of theoretical predictions.

The problem can be conveniently split into two components. First, consider an agent who attaches a given probability to devaluation. Since the uncertainty the agent faces is binary (devaluation or no devaluation), his portfolio choice problem is very elementary and can be represented in an introductory economics diagram. Thus the comparative static response of that agent to changes in risk aversion, wealth and portfolio composition are especially transparent and easily analyzed in terms of income and substitution effects. Second, consider an equilibrium which is characterized by a threshold at which devaluation takes place. Suppose that the true state is at the threshold. Then, there will be a distribution of signals in the population and thus a distribution of probabilities that agents attach to devaluation. Consider the signal which is at the $(1-\pi)$th percentile in the population (i.e., proportion π of the population have observed higher signals). An agent observing that signal will assign probability π to the state being greater than the threshold. So since percentiles are uniformly distributed, the probability of devaluation will be uniformly distributed in the population. Combining these two steps, there is a simple way of calculating the threshold. Find the optimal portfolio choice of an agent who attaches probability π to the peg being maintained and integrate as π varies uniformly from 0 to 1. This gives the net aggregate position of agents at the critical threshold, and thus pins down the equilibrium.

The critical threshold will always be in the range where, if there was complete information, there would be both an equilibrium with devaluation and an equilibrium without devaluation. The threshold implicitly determines what we call the "sunspot" probability: the probability of devaluation conditional on fundamentals being in the multiple equilibrium range of the complete information model. The sunspot probability is a measure of the likelihood of a self-fulfilling attack when fundamentals do not require it. By showing how the sunspot probability depends on the parameters of the model, we develop comparative static predictions that would

this purpose below.

not arise in a complete information model. We focus on comparative statics under the "one way bet" assumption: the interest rate differential received from holding pesos is much smaller than the capital loss from holding pesos in the event of a devaluation. Some key findings are:

1. **Risk Aversion**. If agents are risk neutral, the sunspot probability is close to 1: since the potential payoff to a successful attack is high, agents will attack even when they assign a low probability to success, and, since they are risk neutral, they will take large short positions. But if agents' constant relative risk aversion is greater than 1, the sunspot probability is less than $\frac{1}{2}$ and, in the limiting case of infinite risk aversion, is close to 0. Here income effects outweigh substitution effects. For any devaluation probability not close to 0 and 1, agents will hold a zero net position.[9]

2. **Wealth**. Without any short-selling constraints, the probability of a crisis is *increasing* in wealth (the opposite of the conventional wisdom underlying some contagion stories). Here it is key that agents can both go long and short pesos. Increased wealth allows agents to take larger short positions, increasing the likelihood of a successful attack. If there are incomplete markets and agents cannot short the currency, we regain the conventional result that increased wealth reduces the likelihood of crises.

3. **Portfolio Effects**. If an agent's illiquid exposure to devaluation is increased, he will reduce or hedge his exposure in liquid assets. Thus many capital commitments such as foreign direct investment or private sector commitment to rollovers may be self-reversing in currency markets. We identify conditions under which an exogenous shift of an agent's wealth out of dollar assets into peso assets is exactly offset by his re-optimized portfolio choice in the currency market.

Other comparative static findings, for reasonable levels of risk aversion, are that attacks are more likely when market participants are foreigners rather than locals; and that increasing the size of the possible devaluation may increase or decrease the likelihood of crisis. The effect of an interest rate defense is theoretically ambiguous but, for reasonable parameter values, an interest rate defense will reduce the likelihood of crisis. For the sake of expositional clarity, most of the paper focusses on the case of a homogeneous population (representative agent economy), but our results and insights extend to a more general framework with many different heterogeneous agents. The devaluation threshold is *linear* in the distribution of characteristics in the population.

Our analysis makes a rich set of empirical predictions from a global games model of currency crises that would probably be hard to replicate with other models that do not build on agents' strategic uncertainty in equilibrium. The task of confronting our model to data is challenging, since the predictions concern micro information about market participants' portfolios that are

[9] In fact, under risk neutrality, our model behaves just like the binary action model of Morris and Shin (1998). Chamley (2003) and others have observed that existing global game models are thus models where attacks occur in the unique equilibrium most of the time when they are possible in the complete information multiple equilibrium model. Our results show that this conclusion is dramatically reversed under reasonable levels of risk aversion.

hard to observe. However, our analysis goes to the heart of much unmodelled discussion of currency crises of the type reviewed in the opening paragraph. Our analysis assumes standard expected utility maximizing agents. To model the particular stories outlined above, and develop empirical applications, we would want to model in more detail the richer behavior of hedge funds, banks, governments and other participants in these markets. But this richer behavior could be easily incorporated in the analysis because of the aggregation properties of this model.

Our analysis follows the approach to modelling currency crises in Morris and Shin (1998), building on the global games analysis of Carlsson and van Damme (1993). Morris and Shin (1998) and other applied papers using these techniques (surveyed in Morris and Shin (2003)) make heavy use of the assumption that each player faces a binary choice (to attack the currency or not). While Frankel, Morris and Pauzner (2003) established existence and uniqueness results in a class of global games with many actions, we have identified a new important class of global games where there is "noise independent selection"; that is, the unique equilibrium is independent of the shape of the noise. In doing so, we demonstrate the robustness of the global game approach to the binary action assumption. Because this selection has an easy and intuitive characterization, it could be used to develop economic insights in other settings. We identify in the body of the paper the features of our model that deliver the noise independence and clean characterization of the equilibrium.

This paper is related to work on contagion by Goldstein and Pauzner (2001). In their model, catastrophic losses in Russia, say, reduce the wealth of investors. If those same investors are also investing in Brazil, and those investors have decreasing absolute risk aversion, then they will reduce their risky exposure to Brazil, thus generating a wealth contagion mechanism. Goldstein and Pauzner emphasize that risk aversion has a large impact on the unique equilibrium even though there may be an arbitrarily small amount of uncertainty about fundamentals, and the same mechanism underlies our results. But by working with a binary action model, they are forced as a modelling assumption to decide if attacking or defending the currency is the riskier action. By allowing for a continuum of actions, we are able to endogenize the amount of "hot money" available in currency attacks and endogenize whether attacking or defending the currency is the riskier action.[10] Our results show how the Goldstein-Pauzner model — and the underlying intuition about contagion — rely on a (perhaps empirically plausible) incomplete markets assumption that people who lost money in Russia were unable to short the Brazilian real. In a complete markets model, their loss of wealth in Russia should reduce their ability to short the real, and under the one way bet assumption and plausible risk aversion, this would actually decrease the likelihood of a Brazilian crisis.

We model a currency attack as a static coordination game played among a continuum of ex ante identical agents. Maintaining these assumptions, we are able to substantially generalize the analysis to allow for risk aversion and portfolio effects. A number of recent papers have

[10] Calvo and Mendoza (2000) modelled this type of contagion using an informational story. Kyle and Xiong (2001), like Goldstein-Pauzner, modelled a wealth effect version of the contagion story, but the mechanism is different, relying on a significant amount of uncertainty in equilibrium. These papers also rely on explicit or implicit assumptions that "attacking" (selling pesos) rather than "defending" (buying pesos) is the safe action.

examined if, when and how the global game approach is robust to a wide variety of features, including public information (exogenous and endogenous), asymmetric players and dynamic effects (with or without signalling),[11] and the continuous action analysis of this paper could significantly add to existing insights in the literature. The key features of the model are the strategic complementarities in actions and the friction (incomplete information) that prevents agents from perfectly coordinating their actions. Other frictions ensuring strategic uncertainty — such as the timing frictions from Calvo (1983) — will deliver similar economic insights.[12]

We describe and solve the model in section 2. Comparative statics with respect to risk aversion, wealth, portfolios, ownership, devaluation size and interest rate differentials are analyzed in section 3. We briefly examine the robustness to various assumptions — the form of asymmetric information about fundamentals, the known size of the potential devaluation, the constant relative risk aversion assumption and the size of agents — in section 4. We conclude in section 5.

2 Basic Model

2.1 Setup

A continuum of agents (measure 1) will realize wealth w_D denominated in dollars and wealth w_P denominated in pesos next period. Each agent must decide his net demand for dollars today, y, with $-y$ being the dollar value of the agent's net demand for pesos.[13] Dollar investments earn an interest rate normalized to zero. Peso investments earn an interest rate of r. The initial peso/dollar exchange rate is fixed at e_0, but there is a possibility that the exchange rate will be devalued next period. Thus the exchange rate next period (e_1) will be either e_0 or $E > e_0$.[14] The agent's final period wealth (denominated in dollars) is given by

$$\begin{aligned} \widetilde{w}\left(y, e_1\right) &= w_D + y + \left(\frac{w_P}{e_1} - y\frac{e_0}{e_1}(1+r)\right) \\ &= w_D + \frac{w_P}{e_1} + y\left(1 - \frac{e_0}{e_1}(1+r)\right). \end{aligned}$$

The agent may consume both foreign goods (x_D, denominated in dollars) and domestic goods (x_P, denominated in pesos). The agent's von-Neumann Morgenstern utility function over foreign and domestic goods is Cobb-Douglas,

$$u\left(x_D, x_P\right) = x_D^{\alpha} x_P^{1-\alpha},$$

[11] A few examples are Atkeson (2000), Dasgupta (2003), Corsetti, Dasgupta, Morris and Shin (2004), Tarashev (2003), Angeletos, Hellwig and Pavan (2003) and Angeletos and Werning (2004).

[12] See Frankel and Pauzner (2000) for an elegant model of timing frictions generating strategic uncertainty in a game theoretic setting, and Guimarães (2004) for a model using such timing frictions in a model of currency crises.

[13] For simplicity, we assume that the agent has zero liquid assets in the current period. If the agent had positive liquid assets, we could simply add their current dollar value to w_D, and our analysis would be unchanged.

[14] The assumption that the size of a potential devaluation is known is discussed in section 4.2.

with $\alpha \in [0,1]$. Letting q_D and q_P be the constant prices of dollar and peso denominated goods, respectively, indirect vNM utility is

$$\begin{aligned} & \left(\frac{\alpha \widetilde{w}\left(y, e_1\right)}{q_D}\right)^{\alpha}\left(\frac{\left(1-\alpha\right) e_1 \widetilde{w}\left(y, e_1\right)}{q_P}\right)^{1-\alpha} \\ = & \left(\frac{\alpha}{q_D}\right)^{\alpha}\left(\frac{1-\alpha}{q_P}\right)^{1-\alpha} e_1^{1-\alpha} \widetilde{w}\left(y, e_1\right). \end{aligned}$$

Dividing through by the constant

$$\left(\frac{\alpha}{q_D}\right)^{\alpha}\left(\frac{1-\alpha}{q_P}\right)^{1-\alpha} e_0^{1-\alpha},$$

we have normalized indirect vNM utility

$$v\left(y, e_1\right)=\left(\frac{e_1}{e_0}\right)^{1-\alpha} \widetilde{w}\left(y, e_1\right).$$

We will assume that the net return to attacking the currency by buying a dollar (and going short in pesos to do so) if there is a devaluation is positive, so

$$v_A=\frac{dv\left(y, E\right)}{dy}=\left(1-\left(1+r\right) \frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)}>0;$$

and the net return to defending the currency by selling a dollar (and purchasing pesos) if there is no devaluation is

$$v_D=-\frac{dv\left(y, e_0\right)}{dy}=r>0.$$

We will often want to make the "one way bet" assumption[15] that

$$v_A>v_D$$

or

$$\left(1-\left(1+r\right) \frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)}>r.$$

Writing

$$t=\frac{v_D}{v_A+v_D}=\frac{r}{\left(1-\left(1+r\right) \frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)}+r},$$

the one way bet assumption is equivalent to the requirement that

$$t<\frac{1}{2}.$$

The agent has constant relative risk aversion ρ over his vNM index. So he chooses y to maximize the expected value of

$$\frac{1}{1-\rho}\left(\left(\frac{e_1}{e_0}\right)^{1-\alpha} \widetilde{w}\left(y, e_1\right)\right)^{1-\rho}$$

[15]Betting in favor of a devaluation is often seen as a one-way bet because the opportunity cost of taking a temporary short position in the currency is small relative to the potential gains from devaluation. See, e.g., Krugman (1979).

The agent's optimal portfolio choice will thus depend of the probability he attaches to devaluation. We assume that devaluation occurs if the aggregate net demand for dollars exceeds a stochastic threshold θ (that is, if $\int y_i > \theta$). This assumption could be understood as a reduced form description of an optimizing decision by the government whether to abandon the peg. Morris and Shin (1998) had a slightly more detailed modelling of government behavior — the government pays an exogenous reputational cost of abandoning the peg — that would give the same results in this setting. A natural interpretation is that θ is the amount of dollars available to the country and the peg can be sustained only if this is enough to cover the demand for dollars ($\int y_i$).

Initially agents have a common prior about θ. This prior is the uniform distribution over the real line. Then each agent i observes a signal x_i, $x_i = \theta + \varepsilon_i$, where the ε_i are distributed in the population according to probability density function f.[16]

The Inada condition implies non-negative wealth next period; $\widetilde{w}(y, E) \geq 0$ implies that

$$y > \underline{y} = -\frac{w_D + \frac{w_P}{E}}{1 - (1+r)\frac{e_0}{E}}; \tag{1}$$

and $\widetilde{w}(y, e_0) \geq 0$ implies

$$y < \overline{y} = \frac{w_D + \frac{w_P}{e_0}}{r}. \tag{2}$$

We will assume that the agent must choose $y \in [\underline{\theta}, \overline{\theta}]$. This implies that if there was complete information, there would be a tripartite division of fundamentals. If $\theta < \underline{\theta}$, a devaluation would occur. If $\underline{\theta} \leq \theta \leq \overline{\theta}$, there would be multiple equilibria. If $\theta > \overline{\theta}$, the peg would be maintained.

Most of our analysis will concern the "complete markets" model, with no limits on an agent's ability to go long or short in dollars and pesos, so that $[\underline{\theta}, \overline{\theta}] = [\underline{y}, \overline{y}]$. Note that $\underline{y}$ and $\overline{y}$ depend on some parameters of the model. In other cases, we will look at various "incomplete markets" scenarios, where there are exogenous limits on the position the agent can take. In those cases, we will have

$$\underline{y} < \underline{\theta} < \overline{\theta} < \overline{y}.$$

2.2 Solution

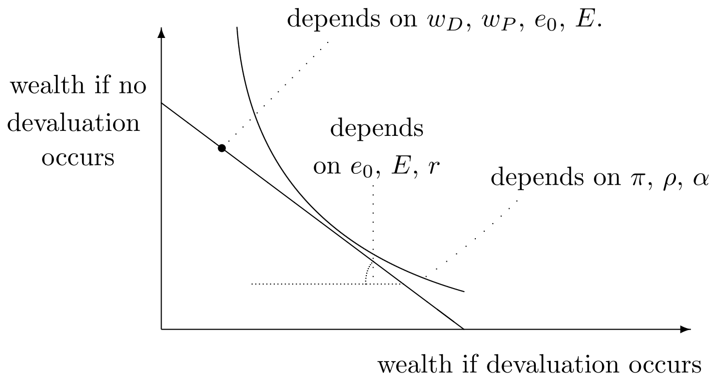
2.2.1 The Single Agent's Portfolio Problem

Suppose that an agent believe that the peg will be maintained with probability π. His problem can be represented by figure 1.

Prices (E, e_0 and r) determine the slope of the budget constraint. The indifference curve depends on π, ρ and α. For close-to-risk-neutral-agents, the indifference curve is close to linear and its slope is determined by π. At the limit, the agent will be basically comparing prices and

[16] The assumption of a uniform prior is standard in the global games and used for convenience. As discussed in section 4.1, the results continue to hold for any prior if the noise is small.

Figure 1: Agent's maximization problem, given π



probability and choosing (a point close to) one of the corners. As ρ increases, π gets less and less important in shaping the indifference curve, that gets less and less flat. So, the optimal choice is closer to the 'middle of the graph'. For low risk aversion, comparative statics are dominated by substitution effects. For high risk aversion, comparative statics will be dominated by income effects. Changes in w_D and w_P change the endowment point, but not the slope. Formally, let $y^*(\pi)$ be the dollar position of an agent who believes that the peg will be maintained with probability π,

$$y^*(\pi) = \underset{y \in [\underline{\theta}, \overline{\theta}]}{\arg\max} \left[\pi \left(\widetilde{w}(y, e_0) \right)^{1-\rho} + (1-\pi) \left(\left(\frac{E}{e_0} \right)^{1-\alpha} \widetilde{w}(y, E) \right)^{1-\rho} \right]. \tag{3}$$

2.2.2 Unique Threshold Equilibrium

There will be a unique equilibrium characterized by a critical θ^* such that above that θ^*, the peg will survive and below that θ^*, there will be a devaluation. In such an equilibrium, an agent observing signal x must attach probability $F(x - \theta^*)$ to the peg being maintained. Thus the aggregate demand for dollars at state θ with be

$$\int_{\varepsilon=-\infty}^{\infty} y^*\left(F\left(\theta + \varepsilon - \theta^*\right)\right) f(\varepsilon)\, d\varepsilon.$$

This is a decreasing function of θ (as an agent's demand for dollars (y^*) is decreasing in π). An equilibrium condition is that this expression must be greater than or equal to θ when $\theta \leq \theta^*$, and less than or equal to θ when $\theta \geq \theta^*$. This is true if and only if

$$\theta^* = \int_{\varepsilon=-\infty}^{\infty} y^*\left(F\left(\theta^* + \varepsilon - \theta^*\right)\right) f\left(\varepsilon\right) d\varepsilon \tag{4}$$
$$= \int_{\varepsilon=-\infty}^{\infty} y^*\left(F\left(\varepsilon\right)\right) f\left(\varepsilon\right) d\varepsilon.$$

This argument establishes that there is exactly one such equilibrium characterized by a threshold θ^*.

There are no other equilibria. A general result in Frankel, Morris and Pauzner (2003) can be used to show this. Because the game is one of strategic complementarities, there will be a largest and smallest strategy surviving iterated deletion of strictly dominated strategies. The strategic complementarities also ensure that the largest and smallest strategies are both monotonic - i.e., an agent's dollar position is decreasing in his signal. Thus both are threshold equilibria characterized by a critical value θ^*. But the argument above established that there is at most one such equilibrium, so the largest and smallest strategies surviving iterated deletion must be the same.

2.2.3 Noise Independence and the Distribution of Devaluation Probabilities.

We analyzed an agent's optimal portfolio for given beliefs about the probability of the peg being maintained. But the distribution of beliefs is endogenous to the equilibrium. A key and surprising insight of our analysis is that when $\theta = \theta^*$, this distribution will always be uniform between 0 and 1. Thus the critical threshold must be

$$\theta^* = \int_{\pi=0}^{1} y^*\left(\pi\right) d\pi. \tag{5}$$

We first give some intuition for why this is the case, before giving the simple algebraic derivation of this result.

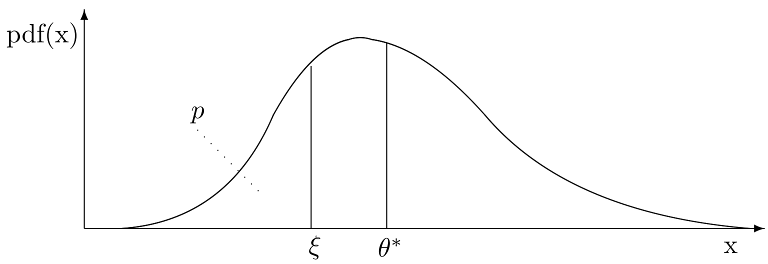
First consider an agent who observes a signal ξ. For him, $\theta = \xi - \varepsilon$ and the probability that the peg will be maintained (i.e., $\theta \geq \theta^*$) is $p = \Pr\left(\varepsilon \leq \xi - \theta^*\right) = F\left(\xi - \theta^*\right)$. Now suppose that the true state is θ^*. What proportion of agents will observe a signal x below ξ? Since $x = \theta^* + \varepsilon$, this is equivalent to asking what proportion of agents will have noise term $\varepsilon \leq \xi - \theta^*$. This proportion is $F\left(\xi - \theta^*\right) = p$. Figure 2 illustrates this intuition.

Thus when the true state is θ^*, the proportion of agents who attach probability less than π to the peg being maintained is π. This is true for all π, so $\text{cdf}(\pi) = \pi$. Therefore π is distributed uniformly in the population.

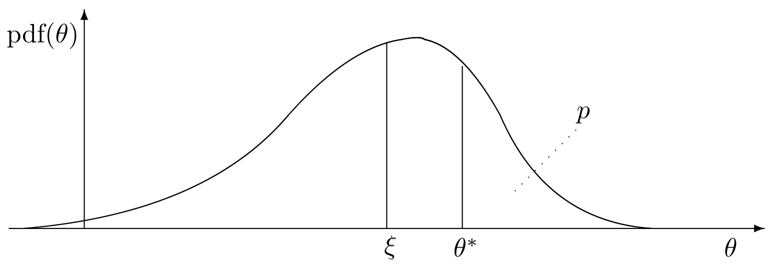
Due to the uniform prior distribution of θ, the agent has no information about how his signal compares to the others. The probability distribution he attaches to the proportion of signals smaller than his is uniform over $[0, 1]$. This is key to the result.

Figure 2: Noise independence

Suppose that $\theta = \theta^*$ and a fraction p of agents get a signal below ξ...



... for an agent with signal ξ, the probability that $\theta > \theta^*$ is exactly p.



In fact, this intuition can be summed up in a change of variables that transforms (4) into (5):

$$\begin{aligned}\theta^* &= \int_{\varepsilon=-\infty}^{\infty} y^*\left(F\left(\varepsilon\right)\right) f\left(\varepsilon\right) d\varepsilon \\ &= \int_{\pi=0}^{1} y^*\left(\pi\right) d\pi, \text{ by change of variables } \pi = F\left(\varepsilon\right). \end{aligned} \tag{6}$$

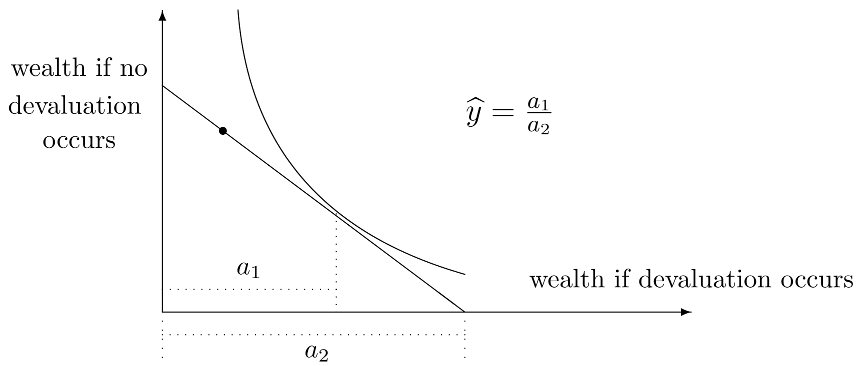
2.3 The "Sunspot Probability" with complete markets

Expression (5) is the key equation in the paper. We now describe an insightful change of variables that allows us to report an even more explicit expression for the threshold in terms of the underlying parameters. Recall that $\underline{y}$ and $\overline{y}$ are the smallest and largest position an agent will take given the Inada conditions and let

$$\widehat{y}\left(\pi\right) = \frac{y^*\left(\pi\right) - \underline{y}}{\overline{y} - \underline{y}};$$

thus $\widehat{y}(\pi) \in [0,1]$ can be interpreted as the proportion of potentially usable resources (given Inada conditions) that the agent puts into dollars. In the graphical representation of the portfolio choice problem, $\widehat{y}$ is measured by the proportion of the distance down the budget constraint to the optimal choice, as shown at figure 3.

Figure 3: $\widehat{y}$



We can similarly define

$$\widehat{\theta} = \frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \qquad \left(= \frac{\theta^* - \underline{y}}{\overline{y} - \underline{y}} \quad \text{with complete markets} \right);$$

this has a couple of interpretations. First, thanks to our normalization of the mass of agents, $\widehat{\theta}$ represents that the average dollar position that each agent must take for the currency to collapse. Second, since $[\underline{\theta}, \overline{\theta}]$ represents the range of fundamentals where multiple equilibria are possible, $\widehat{\theta}$ represents the the probability that devaluation will occur, conditional on the realized θ being in the region where there are multiple equilibria. In multiple equilibrium models, the probability of the bad outcome when multiple equilibria are possible is sometimes called the probability of a "bad sunspot". Thus if the outcomes of the model were interpreted using a sunspot, $\widehat{\theta}$ would be the probability of a bad sunspot, and we dub $\widehat{\theta}$ the sunspot probability.

Simple optimization and algebraic manipulation (see the appendix) then give the following very simple closed form characterizations of $\widehat{y}(\pi)$ and $\widehat{\theta}$ which we will use extensively in our analysis:

$$\widehat{y}(\pi) = \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} \tag{7}$$

$$\widehat{\theta} = \int_{\pi=0}^{1} \widehat{y}(\pi)\, d\pi$$
$$= \int_{\pi=0}^{1} \frac{1}{1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} d\pi \tag{8}$$

A very convenient feature of these expressions is that they depend only on the determinants of t (r, $\frac{E}{e_0}$ and α) and risk aversion ρ, not on portfolio variables (w_D and w_P). As shown at figure 1, changes in w_D and w_P alter the agent's budget constraint but not its slope, so they do not influence $\widehat{y}$, because preferences are homothetic.

2.4 Heterogeneous Agents

Our analysis thus far has considered ex ante identical agents who differ only in their private signals and we will focus on this case in our comparative static analyses. However, the model can deal with heterogeneous agents in a very simple way: in fact, there is linear aggregation across the population. This feature ensures that it is trivial to extend the comparative statics to heterogeneous population and offers hope for empirical work and policy analyses that carry out a detailed treatment of market participants.

We will briefly describe the extension for the complete markets model with a finite set of possible types; however, the incomplete markets models would generalize in a similar way and we could deal with a continuum of types essentially by replacing summations with integrals.

An agent of type i is characterized by a coefficient of constant relative risk aversion ρ_i, dollar-denominated wealth w_{Di}, peso-denominated wealth w_{Pi}, and preference parameter α_i. If this agent assigned probability π to the peg being maintained, this agent demand for dollars would be

$$y_i^*(\pi) = \underset{y\in[\underline{\theta},\overline{\theta}]}{\arg\max} \left[\begin{array}{l} \pi \left(w_{Di} + \frac{w_{Pi}}{e_0} - yr \right)^{1-\rho_i} \\ +(1-\pi) \left(\left(\frac{E}{e_0}\right)^{1-\alpha_i} \left(w_{Di} + \frac{w_{Pi}}{E} + y\left(1 - \frac{e_0}{E}(1+r)\right)\right)\right)^{1-\rho_i} \end{array} \right]. \tag{9}$$

If there was a homogenous continuum of agents of type i, we know that the critical threshold would be

$$\theta_i^* = \int_{\pi=0}^{1} y_i^*(\pi)\, d\pi.$$

But if there was a heterogeneous population, with proportion λ_i of type i, then there would be a uniform distribution of probabilities of devaluation at the threshold within each population. Applying the reasoning described to obtain equations (4) and (5), we get the resulting threshold:[17]

$$\theta^* = \sum_{i=1}^{I} \lambda_i \int_{\pi=0}^{1} y_i^* (\pi)\, d\pi$$
$$= \sum_{i=1}^{I} \lambda_i \theta_i^* .$$

Interestingly, the threshold θ^* is just an average of θ_i^* weighted by λ_i. In some models in which risk aversion plays an important role, the existence of a few close-to-risk-neutral agents is enough to make the system behaves as if there were no risk averse agents because the "close-to-risk-neutral" agents provide the hedge to all others. Here, the agents with low ρ will be very aggressive in their bets but they know that the others will not and that is crucial in determining the threshold θ^*.

3 Comparative Statics

For the complete markets model, we obtained a very convenient characterization of the unique equilibrium in the previous section. For equation (8), we know how the sunspot probability $\widehat{\theta}$ depends on risk aversion ρ and the payoff parameter, t, which in turn depends on the interest rate differential r, the devalued exchange rate E and the preference parameter α. The actual threshold is then given by

$$\theta^* = \left(1 - \widehat{\theta}\right) \underline{y} + \widehat{\theta}\overline{y},$$

where $\underline{y}$ and $\overline{y}$ are given by (1) and (2); $\underline{y}$ and $\overline{y}$ depend on wealth and portfolio composition (w_D and w_P), r and E.

Thus we analyze the effect of risk aversion (ρ) and ownership (α) exclusively by looking at their effect on $\widehat{\theta}$ ($\underline{y}$ and $\overline{y}$ are independent of ρ and α); we then analyze the effect of wealth and portfolio composition (w_D and w_P) exclusively by looking at the their effect on $\underline{y}$ and $\overline{y}$ ($\widehat{\theta}$ is independent of w_D and w_P); finally, when analyzing the effect of r and E, we must take both kinds of effects into account.

As well as analyzing our benchmark complete markets scenario, we also look at a number of incomplete markets scenarios, to see how very different comparative statics conclusions may result under reasonable market restrictions.

3.1 Risk Aversion

Most of global games models in the applied literature assume that agents have a binary action set and are risk neutral. Here we perform comparative statics with respect to ρ in order to analyze what happens to θ^* when the agents are risk averse.

[17] An earlier version of this paper, Guimarães and Morris (2003), provides a more detailed derivation of this result and also describes a class of continuum action global games where such an aggregation result holds.

In the "complete markets" case, with $t < \frac{1}{2}$, risk aversion decreases the likelihood of a devaluation. With incomplete markets, the effect of risk aversion in θ^* depends on whether the safe action is holding dollars or pesos, and that depends on specific constraints on positions investors can hold and on their characteristics.

3.1.1 Complete Markets

Here $\widehat{\theta}$ depends on ρ, but $\underline{y}$ and $\overline{y}$ are independent of ρ. When $\rho \to 0$,

$$\widehat{y}(\pi) \to \begin{cases} 0 & \text{if} \quad \pi > 1-t \\ 1 & \text{if} \quad \pi < 1-t \end{cases}$$

An almost risk-neutral agent will bet virtually all his future consumption unless his signal is arbitrarily close to $1-t$. We get:

$$\hat{\theta} \to 1-t$$

$$\theta^* \to t\underline{y} + (1-t)\,\overline{y}$$

In applications, t is often considered to be close to 0. That implies $\hat{\theta}$ close to 1 in the risk neutral case — conditional on θ being in the multiple equilibrium region, the probability of a "bad sunspot" is very high. But with risk aversion, $\hat{\theta}$ drops dramatically.

When $\rho = 1$ (log utility), we get that:

$$\widehat{y}(\pi) = 1-\pi$$

With logarithmic utility, the proportion invested in dollars is equal to the probability of a devaluation and does not depend on any other thing — prices are irrelevant. Then,

$$\hat{\theta} = \frac{1}{2}$$

$$\theta^* = \frac{1}{2}\underline{y} + \frac{1}{2}\overline{y}$$

Note the dramatic impact of risk aversion on $\hat{\theta}$. For example: if $t = 0.05$, the probability of a "bad sunspot" is 95% in the risk neutral case but equals only 50% if agents have logarithmic utility function.

When $\rho \to \infty$:

$$\hat{y}(\pi) \to t$$

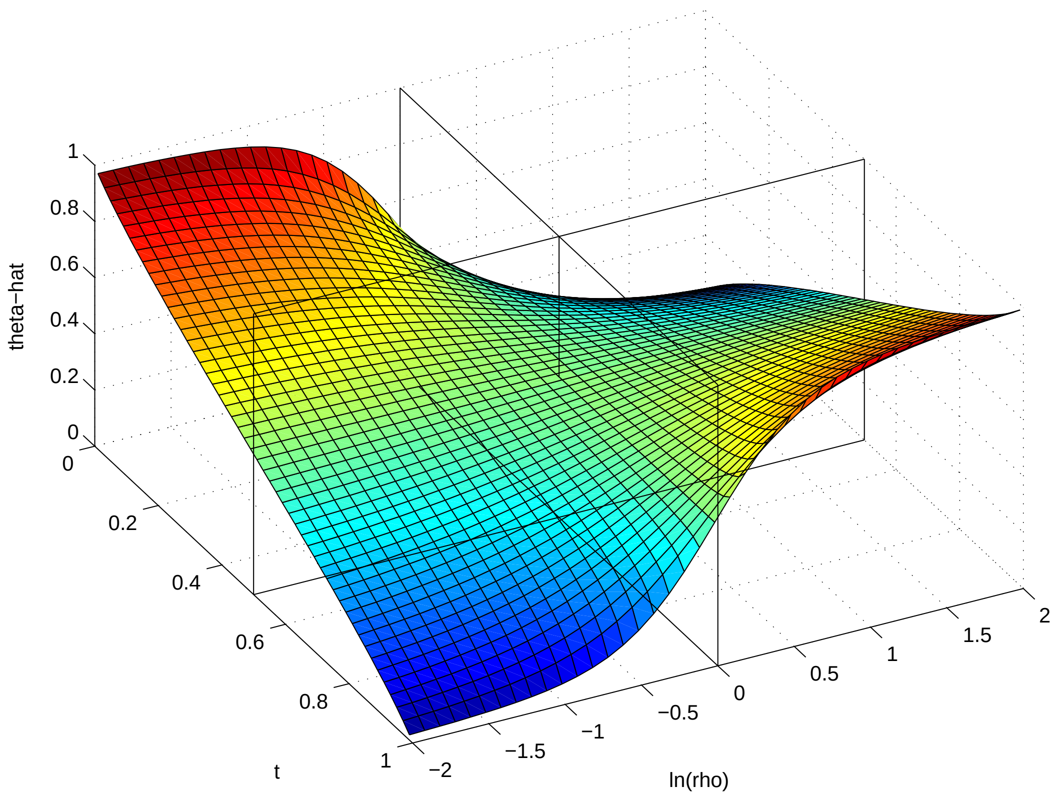
$$\hat{\theta} \to t$$

$$\theta^* \to (1-t)\,\underline{y} + t\overline{y} = \frac{w_P\left(\frac{1}{e_0} - \frac{1}{E}\right)}{\left(1-(1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} + r}.$$

When $w_P = 0$ and ρ tends to ∞, θ^* tends to 0: with no hedging demand because of peso exposure, risk averse agents take zero positions. When the signal x is very accurate and ρ is large, agents will be almost always choosing either $\underline{y}$ or $\overline{y}$, but $\hat{\theta}$ will be close to t anyway.

Figure 4 shows $\hat{\theta}$ as a function of $\log(\rho)$ and t. Under the one way bet assumption ($t < \frac{1}{2}$), risk aversion reduces $\hat{\theta}$ and makes investors less willing to attack the currency. The opposite holds when $t > \frac{1}{2}$. The sunspot probability ($\hat{\theta}$) equals $\frac{1}{2}$ whenever $t = \frac{1}{2}$ or $\rho = 1$.[18]

Figure 4: $\hat{\theta}$ as a function of t and $\log(\rho)$

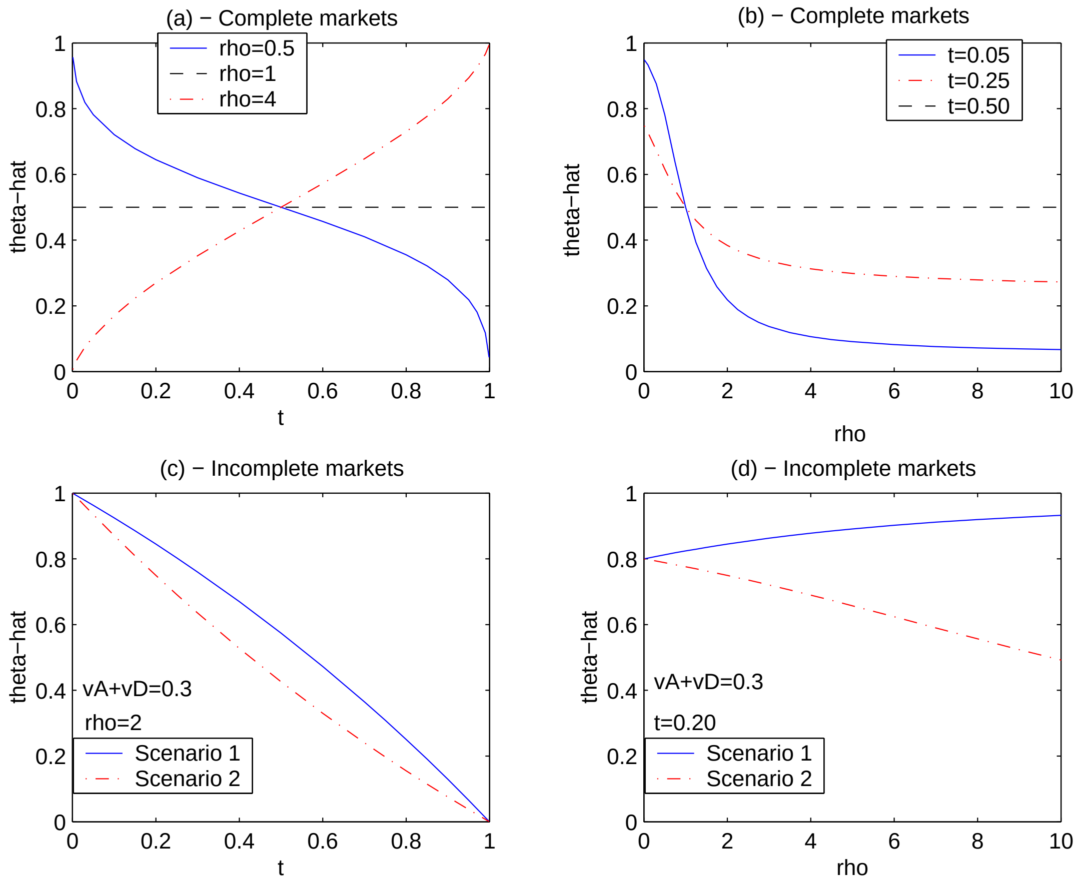


Agents' expectations on others' actions play a crucial role in determining the outcome of the game. Risk aversion influences other agents' positions, which determines θ^*. What drives the results is the impact of risk aversion on what *all others* will do — not on what *a single individual* will do. The impact of a tiny fraction of agents with different levels of risk aversion on θ^* is negligible, as shown in section 2.4.

Figure 5-a cuts figure 4 at some given values of ρ. We can see that the impact of t on the sunspot probability depends crucially on risk aversion. Interestingly, for $\rho > 1$, $\hat{\theta}$ is *increasing* in t: with complete markets, for empirically plausible levels of risk aversion, a higher cost

[18] Analytically, we are able to show that for $t < \frac{1}{2}$, $\hat{\theta}$ is decreasing in ρ for $\rho \geq 1$ — we couldn't prove it for $\rho < 1$ although we believe it also holds in this case. Analogously, for $t > \frac{1}{2}$, we can show that $\hat{\theta}$ is increasing in ρ only for $\rho \leq 1$.

Figure 5: Effects of t and ρ in $\hat{\theta}$



of attacking the currency leads to a *higher* probability of a "bad sunspot". The intuition is that an increase in t generates a substitution effect — attacking is less profitable, an agent has *less* incentives to attack — but also an income effect — an agent gets relatively more consumption if the peg is kept, so there are *more* incentives to attack. Thus, although the gains from a successful currency attack are decreasing in t, the incentives for attacking may not be increasing in t because hedging motivations may dominate the prospects of higher gains (income effects may dominate substitution effects). Note that factors that influence t may also affect $\underline{y}$ and $\overline{y}$, so the overall effects of prices (E and r) on θ^* are not totally captured by figures 4 and 5-a.

Figure 5-b cuts figure 4 at some given values of t and shows that $\hat{\theta}$ reacts strongly to changes in risk aversion for low values of ρ. If t is small, the sunspot probability for empirically plausible degrees of risk aversion is completely different from the risk neutral case.

In sum, when agents are free to short any amount of dollars and pesos, risk aversion has huge impacts on $\hat{\theta}$ (and thus on θ^*). Next, we check what happens when agents positions are restricted and compare results.

3.1.2 Incomplete Markets

With complete markets, it is endogenous whether attacking the currency (high y) or defending the currency (low y) is the riskier action and we can sign the effect of risk aversion on both $\widehat{\theta}$ and θ^*. However, some of our intuition about the effect of risk aversion on currency crises comes from situations where we know that either defending or attacking is riskier for the investor. This intuition depends on some incompleteness of markets. Next, we present scenarios where risk aversion will unambiguously increase the probability of attacks or unambiguously reduce it, independent of the one-way bet assumption (i.e., the size of t). Investing in pesos is the risky action in the first scenario and the safe action in the second scenario.

Incomplete Markets Scenario 1. Foreign investors with all their wealth in dollars cannot go short in either currency. Thus, $w_D > 0$, $w_P = 0$, $\alpha = 1$, $\underline{\theta} = -w_D$ and $\overline{\theta} = 0$ (if $y^* = -w_D$, all his wealth is invested in pesos and if $y^* = 0$, all his wealth is invested in dollars). In this case, $\hat{\theta}$ is given by:

$$\hat{\theta} = \frac{\theta^* + w_D}{w_D}$$

For such an investor, the safe action is to hold his wealth in dollars (i.e., to attack the currency) and the risky action is to hold pesos (i.e., to defend the currency). In particular, this investor's problem is:

$$y^*(\pi) = \underset{y \in [-w_D, 0]}{\arg\max} \left[\pi (w_D - ry)^{1-\rho} + (1-\pi) \left(w_D + y \left(1 - \frac{e_0}{E} (1+r) \right) \right)^{1-\rho} \right]. \tag{10}$$

For any $\rho \in (0, \infty)$, if $\pi < 1-t$, the investor would like to short pesos and go long in dollars, but cannot, so $y^*(\pi) = 0$; if $\pi > 1-t$, the investor will hold less than his whole portfolio in dollars, so $y^*(\pi) < 0$.

Now if $\rho \to 0$, we will have

$$y^*(\pi) \to \begin{cases} -w_D & \text{if} \quad \pi > 1-t \\ 0 & \text{if} \quad \pi < 1-t \end{cases}, \ \hat{\theta} \to 1-t \text{ and } \theta^* \to -tw_D$$

As $\rho \to \infty$, we will have

$$y^*(\pi) \to 0, \ \hat{\theta} = 1 \text{ and } \theta^* = 0.$$

Thus risk aversion increases the probability of attacks (independent of t), because attacking is the safe action by assumption.

Incomplete Markets Scenario 2. Domestic investors with all their wealth in pesos cannot go short in either currency. Thus $w_D = 0$, $w_P > 0$, $\alpha = 0$, $\underline{\theta} = 0$ and $\overline{\theta} = \frac{w_P}{e_0}$ (if $y^* = \frac{w_P}{e_0}$ all his wealth is invested in dollars). In this case, $\hat{\theta}$ is given by:

$$\hat{\theta} = \frac{\theta^* e_0}{w_P}$$

For this investor, the safe action is to hold his wealth in pesos (i.e., to defend the currency) and the risky action is to hold dollars. The investor's problem is

$$y^*(\pi) = \underset{y \in \left[0, \frac{w_P}{e_0}\right]}{\arg\max} \left[\pi \left(\frac{w_P}{e_0} - ry \right)^{1-\rho} + (1-\pi) \left(\frac{w_P}{e_0} + y \left(\frac{E}{e_0} - (1+r) \right) \right)^{1-\rho} \right]. \quad (11)$$

For any $\rho \in (0, \infty)$, if $\pi > 1-t$, the investor would like to short dollars and go long in pesos, but cannot, so $y^*(\pi) = 0$; if $\pi < 1-t$, the investor will hold a positive amount of dollars, $y^*(\pi) > 0$.

Now if $\rho \to 0$, we will have

$$y^*(\pi) \to \begin{cases} 0 & \text{if} \quad \pi > 1-t \\ \frac{w_P}{e_0} & \text{if} \quad \pi < 1-t \end{cases}, \hat{\theta} \to 1-t \text{ and } \theta^* \to (1-t)\frac{w_P}{e_0}$$

As $\rho \to \infty$, we will have

$$y^*(\pi) \to 0, \hat{\theta} = 0 \text{ and } \theta^* = 0.$$

Thus risk aversion reduces the probability of attacks (independent of t), because defending is the safe action by assumption.

If we included both the investors of scenario 1 and the investors of scenario 2, then the heterogeneous agent argument of section 2.4 shows that the threshold would move linearly between the two results as a function of the proportion of investors of both types.

Figures 5-c and 5-d show numerical results for $\hat{\theta}$ in both scenarios. As shown above, when $\rho \to 0$, $\hat{\theta}$ approaches $(1-t)$ — which is the result in a model with risk neutral agents. The effect of risk aversion in the sunspot probability depends on which is the risky action and its sign is independent of t.

The impacts of risk aversion with short-selling constraints, although not at all negligible, are not as huge as in the complete markets case. When agents are free to take any position, they may end up shorting large amount of dollars or pesos and, therefore, facing the risk of getting almost no consumption if their bet goes wrong. Therefore, impacts of risk aversion on their decisions are potentially big. On the other hand, if investors' positions are limited, so are the effects of risk aversion.

With incomplete markets, when $v_A + v_D$ is small, there is little risk and ρ has not much impact on agent's decision. With complete markets, $\hat{\theta}$ does not depend on $v_A + v_D$ because investors choose the amount of risk they will face.

3.2 Ownership

Here we do comparative statics with respect to the parameter α. We focus on the case of complete markets. Our interpretation is that a high α corresponds to foreign investors (who will use terminal wealth to purchase dollar denominated goods) and a low α corresponds to domestic investors. Here $\widehat{\theta}$ in independent depends on α, but $\underline{y}$ and $\overline{y}$ are independent of α.

All the impact of α on the threshold goes through t. As:

$$\frac{d\widehat{\theta}}{d\left(\frac{1-t}{t}\right)} = -\int_0^1 \frac{\left(\frac{\rho-1}{\rho}\right)\left(\frac{1-t}{t}\right)^{-\frac{1}{\rho}}}{\left(1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{\rho-1}{\rho}}\right)^2} d\pi$$

$\frac{d\hat{\theta}}{d\left(\frac{1-t}{t}\right)}$ is positive for $\rho < 1$ and negative for $\rho > 1$. Also, we have that:

$$\frac{d\left(\frac{1-t}{t}\right)}{d\alpha} = -\frac{1}{r}\left(1-(1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{1-\alpha}\ln\left(\frac{E}{e_0}\right) < 0$$

So, the effect of α on θ^* depends on ρ, as shown at table 1.

ρ	< 1	$= 1$	> 1
$\frac{d\hat{\theta}}{d\alpha}$	< 0	$= 0$	> 0

Table 1: $\frac{d\hat{\theta}}{d\alpha}$

A higher α implies a higher t — i.e., a higher cost of attacking. A higher t turns investors less inclined to attack the currency if they are not very risk averse but also turn them more interested in holding dollars if $\rho > 1$ by increasing demand for hedging. When we have log utility, α does not impact the threshold: hedging motivations are just enough to offset the prospects of higher gains, as pointed by figures 4 and 5-a and the discussion at section 3.1. For $\rho > 1$, attacks are more likely when market participants are foreigners rather than locals (α is big).

3.3 Wealth

It is often said that a negative wealth shock may threaten a currency peg because investors are forced to withdraw their money. For example, when Russia defaulted its debt in 1998, Brazil experienced a large capital outflow. Here we show that, in the presence of short-selling constraints, a decrease in wealth indeed increases the likelihood of a devaluation. The "contagion" from Russia to Brazil is the focus of Baig and Goldfajn (2001). Interestingly, Baig and Goldfajn argue that the "compensatory liquidation of assets story", according to which institutional investors withdraw their money from Brazil to compensate losses in Russia, does not find empirical support from the data because those who had lost more money in Russia did not present higher rates of withdrawals from Brazil. However, our analysis here, combined with the results of section 2.4, show that a decrease in some agents' wealth would decrease θ^*, increase the likelihood of a crisis and, therefore, lead *all* agents to decrease their exposure in pesos.

Sometimes, wealthy investors are said to have contributed to trigger a crisis. Large short positions taken by hedge funds might have played an important role in the British Pound devaluation in 1992 and, more generally, in the ERM crisis in 1992-3. In the complete markets case, wealth is indeed positively related to the likelihood of a devaluation (if $t < \frac{1}{2}$). Thus, a message from this section is that wealth effects exist regardless of any short selling constraint, but the direction of such effects on the probability of a crisis depends on institutional constraints on agents' position.

3.3.1 Complete Markets

We first examine how a wealth shock could effect the coordination of agents and thus change θ^* even with complete markets. It is important to note that with complete markets, a decrease in wealth will decrease the size of the position that both attacking and defending agents can take (consistent with the Inada condition). We want to find out which effect is more important. For simplicity, we focus on the case where $w_P = 0$, $w_D > 0$ and $\alpha = 1$, so that all wealth belongs to foreigners.[19] We are interested in comparative statics with respect to wealth, w_D. Here the sunspot probability $\widehat{\theta}$ is independent of wealth, but $\underline{y}$ and $\overline{y}$ depend on wealth. We have:

$$
\begin{aligned}
\theta^* &= \hat{\theta}\overline{y} + \left(1 - \hat{\theta}\right)\underline{y} \\
&= w_D \left[\frac{\hat{\theta}}{r} - \frac{1-\hat{\theta}}{1 - \frac{e_0}{E}(1+r)}\right] \\
\frac{d\theta^*}{dw_D} &= \frac{\hat{\theta}}{r} - \frac{1-\hat{\theta}}{1 - \frac{e_0}{E}(1+r)} \\
&= \left(\frac{1-\hat{\theta}}{r}\right)\left(\frac{\hat{\theta}}{1-\hat{\theta}} - \frac{t}{1-t}\right)
\end{aligned}
$$

If $t < \frac{1}{2}$, $\hat{\theta} > t$ for any ρ. So

$$
\frac{d\theta^*}{dw_D} > 0.
$$

Thus increased wealth reduces the probability of a successful currency attack (while the sunspot probability remains unchanged). Note that θ^* is linear in w_D. Our result, with complete markets, is totally different from the usual intuition. If t is small, agents will tend to short pesos and θ^* will be negative (see equation 5). As the utility function is concave, the amount of risk one agent chooses to face depends positively on his wealth. So, a negative wealth shock reduces the munition an agent is willing to use to attack the currency.

As we show now, with incomplete markets, this result may not hold.

[19] The result also holds if all money belong to domestic residents.

3.3.2 Incomplete Markets

Consider again scenario 1 described at section 3.1.2. Foreign investors with all their wealth in dollars cannot go short in either currency. Thus, $w_D > 0$, $w_P = 0$, $\alpha = 1$, $\underline{\theta} = -w_D$ and $\overline{\theta} = 0$. The investor will choose:

$$y^* = \begin{cases} -w_D & \text{if } y^{foc} \leq -w_D \\ y^{foc} & \text{if } y^{foc} \in (-w_D, 0) \\ 0 & \text{if } y^{foc} \geq 0 \end{cases}$$

where

$$y^{foc} = w_D \frac{1 - \left(\frac{\pi\ r}{(1-\pi)v_A}\right)^{-\rho}}{r + v_A \left(\frac{\pi\ r}{(1-\pi)v_A}\right)^{-\rho}}.$$

As in the complete markets case, y^* is given by a linear function of w_D. However, in the present example, an increase in wealth will always reduce y^* (increase y^* in absolute value). A higher wealth increases agents' appetite for risk and make them more willing to hold deposits in pesos.

This result is similar to one obtained in the contagion model by Goldstein and Pauzner (2001).[20] Short selling constraints are implicitly assumed in their paper and agents decide to invest or not one unit of money in the country. We show here that their conclusion still holds when agents have a continuum of actions but depends on positive investments in the country — agents need to be investing, not attacking.

3.4 Portfolio

Foreign direct investment is sometimes said to be more resilient in the face of financial crises. World Bank (1999) states that "FDI is less subject to capital reversals (...) since the presence of large, fixed, illiquid assets makes rapid disinvestment more difficult than the withdrawal of short-term bank lending or the sale of stock holdings". However, agents with illiquid investments in a country will have incentives to hedge. Indeed, IMF (1998) points that such hedging motivations were in effect in Asia and Brazil in 1997.

As discussed in the introduction, the effects of portfolio composition are not related only to FDI. A report from the Financial Stability Forum (2000) describes a perhaps curious setting in which portfolio effects played a role: during the Asian crisis, agents with investment in Indonesia that became illiquid were selling Australian dollars short to (imperfectly) hedge their positions. Another interesting example, brought by IMF (2000), highlights the importance of hedging motivations in some recent experiences with private sector involvement.

[20] Their result is more general in some dimensions: in particular, they allow for any decreasing absolute risk aversion utility function (not just constant relative risk aversion); and they allow for general strategic complementarities in investment in each country.

What happens in our model when we shift wealth between w_D and w_P? Again, $\widehat{y}(\pi)$ and $\widehat{\theta}$ are independent of such reallocations, but $\underline{y}$ and $\overline{y}$ depend on the portfolio composition. Suppose we decrease w_D by 1 unit. In order to obtain a portfolio change with no wealth effect, how many pesos should the agent get? Increasing w_P by e_0 unit does not change an agent's wealth in the no-devaluation case but makes him poorer if there is a devaluation. Increasing w_P by E units makes the agent richer if there is no devaluation and no difference otherwise. If we increase w_P by $e_0.(1+r)$ units, the $1+r$ factor compensates the agent for the risk of a devaluation. In this case, we have, for any given π:

$$-\frac{dy^*(\pi)}{dw_D} + e_0(1+r)\frac{dy^*(\pi)}{dw_P} = 1$$

The net effect of a transfer from dollars to pesos with no wealth effect is 0, because for each dollar deposit shifted to pesos, an agent chooses to go long in 1 extra dollar — $y^*(\pi)$ and θ^* increase by 1 unit. All agents, regardless of their risk aversion, choose to do so.

Now, suppose that w_P was increased by e_0 units (instead of $e_0.(1+r)$). Clearly, this is similar to the above portfolio shift plus a negative wealth effect. In this experiment, agents would still increase their y^*, but such increase would be smaller — especially for low-ρ agents. Conversely, if w_P was increased by E units, the positive wealth effect combined with the portfolio effect would lead agents to go long in more than 1 dollar for each dollar shifted to pesos.

Our benchmark model shows that atomistic agents fully undo the portfolio shift in the currency market when the amount received in pesos compensates them for the risk of a devaluation (the $(1+r)$ factor) and there are no restrictions or costs to hedge their positions. In the examples mentioned in the introduction, hedges are imperfect and/or costly. A model addressing one of those specific questions would need to take its particularities into account.

3.5 The Size of Devaluation

An increase in E represents an increase in the size of the devaluation, if it occurs. In global games models with risk-neutral agents, incentives to attack the currency are increasing in E — which seems intuitive. However, with risk aversion, that may not be the case.

Observe first that an increase in E increases $\underline{y}$ by reducing the maximum amount of pesos an agent can short; $\overline{y}$ does not depend on E. Thus an increase in E shifts the complete information multiple equilibria range unambiguously in the direction of more attacks.

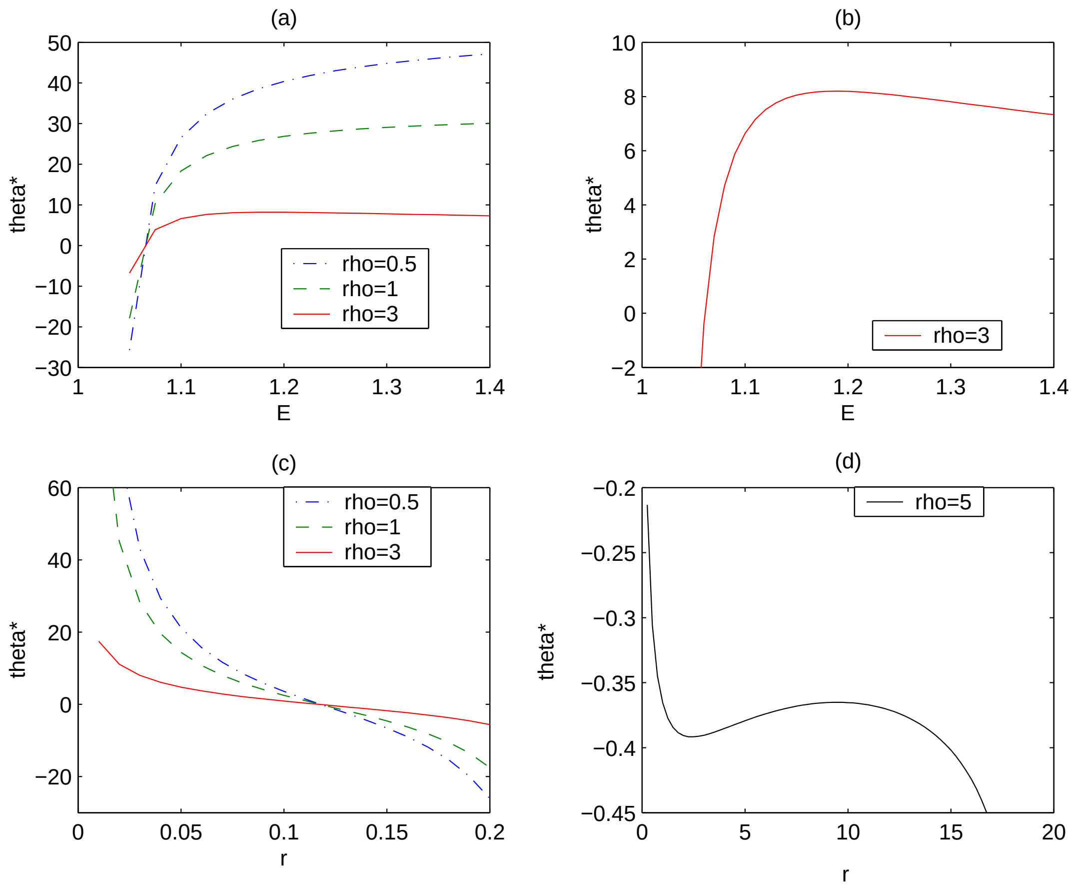
An increase in E decreases t. As we have already observed at section 3.1, $\widehat{\theta}$ is decreasing in t if $\rho < 1$ but increasing in t if $\rho > 1$. Thus for empirically plausible levels of risk aversion, a higher value of E leads to a *higher* probability of a bad sunspot.

Combining the two effects, we know that for $\rho \leq 1$, we must have θ^* increasing in E. But for $\rho > 1$, the combined effect may go either way for reasonable values of the parameters. Figure 6-a shows θ^* as function of E for different levels of risk aversion.[21] We can see that for $\rho \leq 1$, θ^* is increasing in E: higher rewards for a successful attack make it more likely. For $\rho = 3$,

[21] Parameters of this example: $w_D = 1$, $w_P = 1$, $r = 0.03$, $e_0 = 1$, $\alpha = 0.5$. The non-monotonicity of θ^* as function of E holds for empirically plausible values of parameters regardless of the values of α, w_D and w_P.

however, θ^* is decreasing in E for sufficiently high values of E (i.e., for sufficiently low values of t). Figure 6-b plots the same function θ^* using different scale and shows clearly that, for $\rho = 3$, an increase E may turn a devaluation less likely.

Figure 6: Effects of E and r



3.6 Interest Rate Defense

How does an increase in r affect the likelihood of currency crises? In global games models with risk neutral agents, for a given signal (x_i) an agent gets on fundamentals, incentives to hold *pesos* are increasing in r. However, if agents are risk averse, interest rate defense of a peg may be much less effective.[22]

Again, $\widehat{\theta}$ depends on r (through t), but so do $\underline{y}$ and $\overline{y}$. Now, observe that an increase in r reduces $\underline{y}$, since the reduced return to attacking the currency allows investors to take a larger peso position and thus a larger (negative) dollar position; an increase in r also reduces $\overline{y}$, since

[22] Angeletos, Hellwig and Pavan (2003) propose a global games model in which the interest rate works as a signal from the Central Bank. In their model, as in ours, interest rate defense is not as effective as suggested by a standard global game model of currency crises.

the largest dollar position consistent with being able to pay the interest differential on the short peso position is reduced. Thus an increase in r shifts the complete information multiple equilibria range unambiguously in the direction of less attacks.

An increase in r increases t. As we have already observed, this increases $\widehat{\theta}$ if $\rho < 1$ but decreases $\widehat{\theta}$ if $\rho > 1$. Again, for empirically plausible levels of risk aversion, an interest rate defense paradoxically increases the probability of a bad sunspot — a higher interest rate means that the full hedging demand for dollars is increased.

Which effect wins out overall? For $\rho \leq 1$, we must have θ^* decreasing in r. We find that this comparative static is maintained for $\rho > 1$ for reasonable values of the parameters. Figure 6-c shows θ^* for different levels of risk aversion.[23] Regardless of the value of ρ, an increase in r turns a devaluation less likely.

However, it is possible to construct examples where θ^* is increasing in r. Figure 6-d shows an example of such curious behavior of θ^*. The parameters in such example ($e_0 = 1$, $E = 20$ and $r \in (200\%, 900\%)$) are too unrealistic in the context of currency crisis.[24] However, in other applications in which the difference between agent's utility in the 2 possible states is too big, we may find this perverse effect of an interest rate defense.[25]

4 Discussion of assumptions

At this point, it is useful to review the role of some of the assumptions made in our analysis.

4.1 The Uniform Prior Assumption

We made the convenient assumption that θ was uniformly distributed on the real line. This is a standard simplifying assumption in the global games literature (see Morris and Shin (2003)). If we bounded the support of the noise distribution f, we could have had θ uniform on a bounded interval, with no change in the analysis. In addition, if θ were drawn from a smooth, but non-uniform, prior and we let the variance of the noise shrink to zero (i.e., the support f shrinks to zero), then the limiting equilibrium threshold is equal to threshold identified under the uniform prior assumption. Intuitively, if the noise is small, variation in the density of the prior becomes irrelevant. Thus our results should be understood as applying if either private information is very accurate or uncertainty is large but there is not too much prior or public information about θ.

4.2 The Known Devaluation Assumption

A crucial assumption in our model is that the exchange rate at period 1, conditional on the occurrence of a devaluation, is common knowledge and constant (equal to E). The size of the

[23] Parameters of this example: $w_D = 1$, $w_P = 1$, $E = 1.25$, $e_0 = 1$, $\alpha = 0.5$. We could not find an example of θ^* increasing in r for any reasonable values of E, e_0 and r.

[24] Other parameters are: $w_P = 1$, $w_D = 1$, $\alpha = 0.5$.

[25] One could extend our model to the case of debt crises. If there is risk of total default, could an interest rate defense make matters worse?

devaluation is independent of θ, which represents the ability of the government to defend the peg. A more realistic assumption would be that if a devaluation occurred in state θ, then the new exchange rate would be $e_1(\theta)$, where e_1 is a decreasing function of θ.[26] Both our noise independence property and our ability to get a closed form solution would go away in this model. In particular, we heavily exploited the fact that we were always evaluating two state gambles. In general, there would be a complicated interaction between the binary uncertainty about whether will be a devaluation or not, and the richer uncertainty about the size of the devaluation.

However, there is some hope of extending our results if there was a small amount of uncertainty about θ. In this case, agents' uncertainty about the size of the devaluation would go away even as strategic uncertainty about others' actions remained (this is the key insight of the global games approach). Thus if we restricted the noise to have finite support and let the support shrink to zero, then the existing analysis might apply.

If there is no uncertainty about the size of the devaluation, a bit of algebra on the results of section 2.3 shows that θ^* is the unique value of θ solving:

$$\begin{aligned}\theta &= \frac{w_D + \frac{w_P}{e_0}}{r}\left(\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{E}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1}d\pi\right)\\ &+\frac{w_D+\frac{w_P}{E}}{1-(1+r)\frac{e_0}{E}}\left(1-\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{E}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1}d\pi\right)\end{aligned}$$

If agents' demand for dollars was increasing in E (implying that the right hand side is increasing in E), then with uncertainty about the size of a devaluation, our candidate solution would be the unique value of θ solving:

$$\begin{aligned}\theta &= \frac{w_D + \frac{w_P}{e_0}}{r}\left(\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{e_1(\theta)}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1}d\pi\right) \quad (12)\\ &+\frac{w_D+\frac{w_P}{e_1(\theta)}}{1-(1+r)\frac{e_0}{e_1(\theta)}}\left(1-\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{e_1(\theta)}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1}d\pi\right)\end{aligned}$$

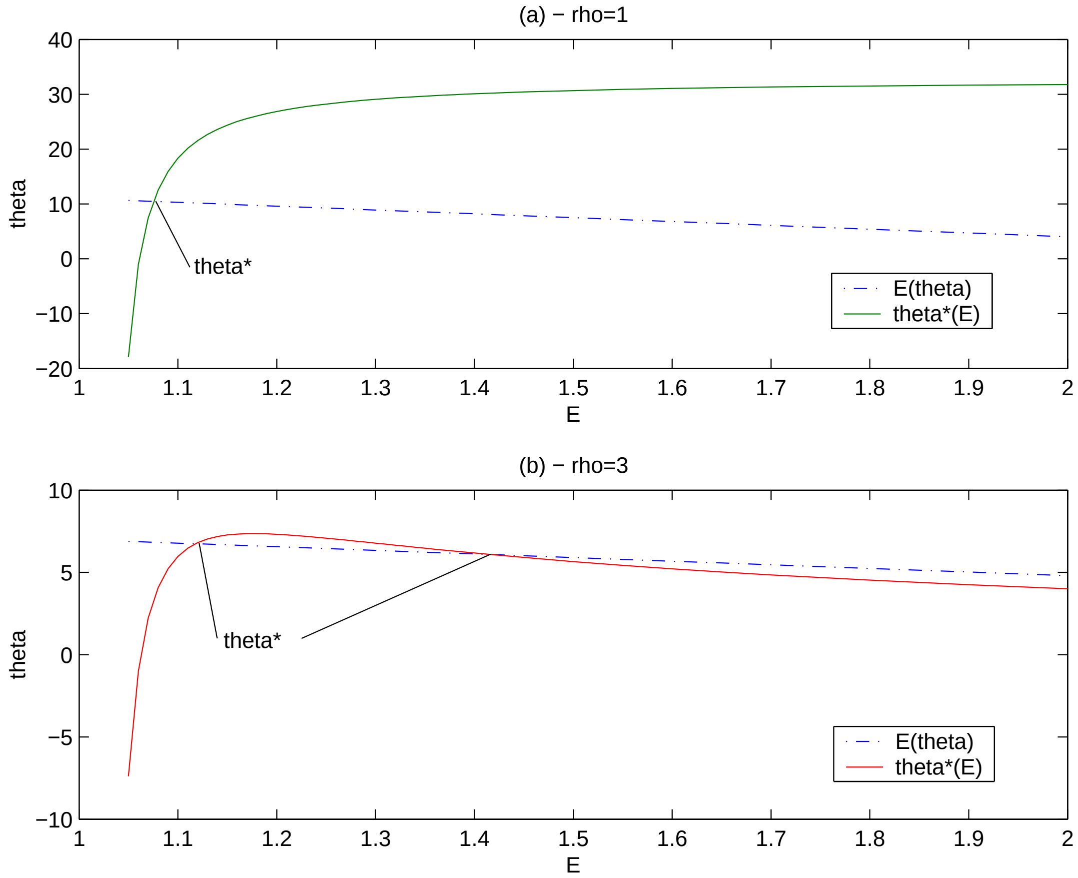
A unique solution will exist since the left hand side is increasing in θ and the right hand side is decreasing in θ.

Our results at section 3.5 imply that, if $\rho \leq 1$, agents' demand for dollars is increasing in E and, therefore, the right hand side of equation (12) is increasing in E. In this case, we have a unique equilibrium, as shown at figure 7-a. The decreasing function is the (exogenous)

[26] This is essentially the model analyzed in Morris and Shin (1998). In that risk neutral incomplete markets model, the uniqueness result continues to hold.

relation between fundamentals (θ) and the exchange rate (the inverse of $e_1(\theta)$). The increasing function is what θ^* would be if E was a known constant. The intersection of both curves gives the unique equilibrium.[27]

Figure 7: Equilibria



However, for $\rho > 1$, agents' demand for dollars may be decreasing in E, so there might be multiple solutions to equation (12) and thus multiple equilibria in the game. Figure 7-b shows an example for $\rho = 3$ in which the (exogenous) relation between fundamentals and the exchange rate crosses the function $\theta^*(E)$ more than once. In this case, we would require additional restrictions — e.g., upper bounds on the slope of $e_1(.)$ — to restore uniqueness.

4.3 The Infinitesimal Agent Assumption

We assumed a continuum of agents. With a finite set of agents, each agent would anticipate his own impact on whether a crisis would occur. The direction of this effect would vary across the scenarios we considered. If agents are risk neutral, $\alpha = 1$ and $w_P = 0$, then each agent has a private interest in having a successful attack. So attacks would be more likely with large

[27]Parameters used for the graphs at 7-a and 7-b: $w_D = 1$, $w_P = 1$, $e_0 = 1$, $r = 0.03$ and $\alpha = 0$.

traders (this is the case analyzed by Corsetti *et al* (2004)). But if $w_P > 0$ (e.g., the agent has foreign direct investment in the country) then an agent has a lot to lose from devaluation. In a continuum model, this does not influence his best response. But with large traders, this would make attacks less likely.

4.4 Constant Relative Risk Aversion Assumption

The constant relative risk aversion assumption is extensively used in our analysis, as it lies behind the convenient decomposition of comparative statics into the effects on the sunspot probability and the Inada limits $\underline{y}$ and $\overline{y}$. However, many qualitative conclusions go through with more general function forms. This was discussed in a working paper version of this paper (Guimarães and Morris, 2003).

5 Conclusion

We built a 'global-games' model of currency crisis in order to analyze the impact on agents behavior of issues related to risk and wealth. While our analysis concerns currency crises, the modelling may be relevant to a wide array of macroeconomic issues. The analysis of risk and wealth is central to macro. Self-fulfilling beliefs and strategic complementarities play an important role in many macroeconomic settings. In the marriage of these two strands in this paper, risk, wealth and portfolio effects play a central role in determining how strategic complementarities translate into economic outcomes.

Under our naive, static, complete markets model of agents' portfolio choices, we were able to derive a number of striking predictions about the likelihood of currency crises. However, our conclusions were sensitive to the market assumptions: plausible sounding incomplete market restrictions can have a dramatic impact on comparative statics. Real currency markets reflect the transaction, hedging and speculative demands of many private traders, the policy interventions of central banks and the strategies of large institutions such as hedge funds that may be hard to explain and model as the aggregation of individual utility maximizing behavior. One message of this paper is that if currency crises are self-fulfilling, the motives and strategies of market participants may be important in a way they are not in models where an arbitrage condition (and not strategic considerations) pins down the equilibrium.

References

[1] Angeletos, George-Marios; Hellwig, Christian; and Pavan, Alessandro, 2003, "Coordination and policy traps", mimeo.

[2] Angeletos, George; and Werning, Ivan, 2004, "Information Aggretation and Equilibrium Multiplicity", mimeo.

[3] Atkeson, Andrew, 2000, "Discussion of Morris and Shin", *NBER Macroeconomics Annual*.

[4] Baig, Taimur; and Goldfajn, Ilan, 2001, The Russian default and the contagion to Brazil". In: Claessens, Stijn and Forbes, Kristin (eds.), *International Financial Contagion*, Kluwer.

[5] Blustein, Paul, 2001, *The Chastening*, Perseus Books.

[6] Calvo, Guillermo, 1983, "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics* 12, 383-398.

[7] Calvo, Guillermo; and Enrique Mendoza, 2000, "Rational contagion and the globalization of securities markets", *Journal of International Economics* 51, 79-113.

[8] Carlsson, Hans; and van Damme, Eric, 1993, "Global games and equilibrium selection", *Econometrica* 61, 989-1018.

[9] Chamley, Christophe, 2003, "Dynamic speculative attacks", *American Economic Review* 93, 603-621.

[10] Corsetti, Giancarlo; Dasgupta, Amil; Morris, Stephen and Shin, Hyun Song, 2004, "Does one Soros make a difference? A theory of currency crises with large and small traders", *Review of Economic Studies* 71, 87-114.

[11] Dasgupta, Amil, 2003,"Coordination, Learning, and Delay", mimeo LSE.

[12] Frankel, David; and Pauzner, Ady, 2000, "Resolving indeterminacy in dynamic settings: the role of shocks," *Quarterly Journal of Economics* 115, 283-304.

[13] Frankel, David; Morris, Stephen and Pauzner, Ady, 2003, "Equilibrium selection in global games with strategic complementarities", *Journal of Economic Theory* 108, 1-44.

[14] Financial Stability Forum, 2000, "Report of the Working Group on Highly Leveraged Institutions", available at http://www.fsforum.org/publications/Rep_WG_HLI00.pdf.

[15] Giannini, Curzio, 2002, "Pitfalls in International Crisis Lending," in *Financial Crises, Contagion, and the Lender of Last Resort*, (C. Goodhart and G. Illing, Eds). Oxford University Press.

[16] Goldstein, Itay and Pauzner, Ady, 2001, "Contagion of self-fulfilling financial crises due to diversification of investment portfolios", *Journal of Economic Theory*, forthcoming.

[17] Guimarães, Bernardo, 2004, "Dynamics of currency crises", mimeo.

[18] Guimarães, Bernardo and Stephen Morris, 2003, "Risk and Wealth in a Model of Self-Fulfilling Currency Attacks", Cowles Foundation Discussion Paper 1433.

[19] International Monetary Fund, 1998. "International Capital Markets, Chapter II - The Asian crisis: capital markets dynamics and spillover". IMF, Washington, DC.

[20] International Monetary Fund, 2000. "International Capital Markets, Chapter V - Private sector involvement in crisis prevention and resolution: market views and recent experience". IMF, Washington, DC.

[21] Kraay, Aart, 2003, "Do high interest rates defend currencies during speculative attacks?", *Journal of International Economics* 59, 297-321.

[22] Krugman, Paul, 1979, "A Model of Balance-of-Payments Crises", *Journal of Money, Credit and Banking* 11, 311-325.

[23] Krugman, Paul, 1996, "Are currency crises self-fulfilling?", *NBER Macroeconomics Annual.*

[24] Kyle, Albert and Xiong, Wei, 2001, "Contagion as a Wealth Effect", *Journal of Finance* 56, 1401-1440.

[25] Morris, Stephen and Shin, Hyun S., 1998, "Unique equilibrium in a model of self-fulfilling currency attacks", *American Economic Review* 88, 587-597.

[26] Morris, Stephen and Shin, Hyun S., 2003, "Global games: theory and applications", in *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)* (M. Dewatripont, L. Hansen and S. Turnovsky, Eds). Cambridge, England: Cambridge University Press.

[27] Obstfeld, Maurice, 1996, "Models of currency crises with self-fulfilling features", *European Economic Review* 40, 1037-1047.

[28] Powell, Andrew, 2002, "Safety-First Monetary and Financial Policies for Emerging Economies," in *Financial Risks, Stability, and Globalization* (O.E.G.Johnson, Ed.). IMF.

[29] Tarashev, Nikola, 2003, "Currency crises and the informational role of interest rates," mimeo.

[30] Tirole, Jean, 2002, *Financial Crises, Liquidity and the International Monetary System.* Princeton University Press.

[31] World Bank, 1999, "Global Development Finance, Chapter 3, Foreign Investment Resilient in the Face of Financial Crisis."

A Derivation of $\widehat{\theta}$

The first order conditions for (3) imply that

$$\pi \left(\widetilde{w}\left(y, e_0\right)\right)^{-\rho} v_D = (1-\pi)\left(\left(\frac{E}{e_0}\right)^{1-\alpha} \widetilde{w}\left(y, E\right)\right)^{-\rho} v_A.$$

Thus

$$\left(\frac{\left(\frac{E}{e_0}\right)^{1-\alpha}\widetilde{w}\left(y,E\right)}{\widetilde{w}\left(y,e_0\right)}\right)^{-\rho}=\left(\frac{\pi}{1-\pi}\right)\left(\frac{t}{1-t}\right),$$

$$\frac{\widetilde{w}\left(y,E\right)}{\widetilde{w}\left(y,e_0\right)}=\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha},$$

and

$$y^*\left(\pi\right)=\frac{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}\left(w_D+\frac{w_P}{e_0}\right)-\left(w_D+\frac{w_P}{E}\right)}{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}r+1-\frac{e_0}{E}\left(1+r\right)}.$$

But observe that

$$\underline{y}=y^*\left(1\right)\text{ and }\overline{y}=y^*\left(0\right),$$

so

$$\begin{aligned}\widehat{y}\left(\pi\right)&=\frac{y^*\left(\pi\right)-y^*\left(1\right)}{y^*\left(0\right)-y^*\left(1\right)}\\&=\frac{\frac{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}\left(w_D+\frac{w_P}{e_0}\right)-\left(w_D+\frac{w_P}{E}\right)}{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}r+1-\frac{e_0}{E}\left(1+r\right)}+\frac{\left(w_D+\frac{w_P}{E}\right)}{1-\frac{e_0}{E}\left(1+r\right)}}{\frac{w_D+\frac{w_P}{e_0}}{r}+\frac{\left(w_D+\frac{w_P}{E}\right)}{1-\frac{e_0}{E}\left(1+r\right)}}\\&=\frac{1}{1+\frac{1-\frac{e_0}{E}\left(1+r\right)}{r}\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{t}{1-t}\right)^{\frac{1}{\rho}}\left(\frac{E}{e_0}\right)^{\left(1-\alpha\right)}}\\&=\frac{1}{1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}.\end{aligned}$$

Thus

$$\begin{aligned}\widehat{\theta}&=\frac{\int\limits_{\pi=0}^{1}y^*\left(\pi\right)d\pi-\underline{y}}{\overline{y}-\underline{y}}\\&=\int\limits_{\pi=0}^{1}\widehat{y}\left(\pi\right)d\pi\\&=\int\limits_{\pi=0}^{1}\frac{1}{1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}d\pi\end{aligned}$$