Productivity, Efficiency, Scale Economies and Technical Change: a New Decomposition Analysis of TFP Applied to the Japanese Prefectures

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November 2004

1. Introduction

The productivity index is generally defined as the ratio of an aggregate output index to an aggregate input index. Depending on an aggregator function, there are so many productivity indexes. The functional approach of the index theory suggests that an aggregator function should be chosen on the basis that the resulting productivity index has a meaningful interpretation and provides an insightful factor decomposition. This paper aims to propose a new decomposition analysis of the productivity change based on the Hicks-Moorsteen-Bjurek (HMB) index, and to show its usefulness using data on 47 Japanese prefectures during the year 1980-2000 .

In the recent literature, two of the most widely used productivity index are the Törnqvist index and the Malmquist index. Nevertheless, the decomposition analyses provided by the Törnqvist and Malmquist productivity indexes are incomplete. The Törnqvist productivity index is decomposable so that the contributions of technical change, efficiency change and scale change are identified. However, the Törnqvist index does not allow any inefficiencies because it presumes the optimizing behavior of a producer. In contrast, the Malmquist productivity index factorizes in technical change and efficiency change, but it is not indicative of the scale effects.

Then, the first purpose of this paper is to synthesize the two productivity indexes so as to develop a framework where the components of efficiency change and the scale effects are simultaneously identifiable. We show that the HMB productivity index serves as a basis of such an integrated framework that the productivity change is decomposed into four factors: technical change, efficiency change, scale change and the input and output mix effects. An appealing feature of this approach is that it can be seen as a synthesis of the Törnqvist and Malmquist indexes. In fact, a combination of the terms of technical change and efficiency change is identical to the Malmquist productivity index. Thus, the HMB index is represented as an augmented Malmquist index with the scale effects. On the other hand, each component of technical, efficiency and scale changes of the HMB index is all reduced to the corresponding part of the Törnqvist index if inefficiencies are removed and the output-oriented distance function is the translog. In this sense, the HMB productivity index is also a generalization of the Törnqvist productivity index.

Recently, Lovell (2003) proposes a similar decomposition of the HMB productivity index. Although Lovell's decomposition proves that the HMB index is decomposable, its implications for the conventional productivity indexes are unclear. Our decomposition of the HMB index is more naturally interpreted as an extension of the conventional approaches.

The second purpose of this paper is to apply the proposed analysis to examining the productivity of the Japanese economy using data pertaining to the 47 prefectures during the year 1980-2000. Implementation of the analysis is illustrated along this application.

Furthermore, it is of interest to clarify the sources of productivity growth in this period of Japan. While the Japanese economy outperformed other developed countries in the 80s, it experienced serious stagnation in the 90s. Interesting enough, the economic growth rate of Japan during the year 1981-1990 was 4.0 percent on average which considerably fell into the average rate of 1.4 percent during the year 1991-2000. Such a sharp contrast provides a good material to clarify the relationship between productivity and business cycle.

In the macroeconomic literature, the persistent empirical fact of procyclical productivity has once received much attention[1]. The procyclicality of productivity is alternatively explained by technology shocks, the increasing returns to scale, or procyclical variations in input utilization. Notice that in the decomposition analysis of the HMB index, efficiency measures the effects on productivity of production deviating from the frontier. Since labor hoarding or excess capacity implies behavior of "off production frontier", the variation in input utilization are expected to be captured by the efficiency change component. Also, the effects of technology shocks and the increasing returns to scale are identified by the technical change component and the scale change component, respectively. As a result, the decomposition analysis of the HMB productivity index can be used to assess relative importance of those factors as sources of fluctuations in productivity.

2. Conventional approach

2.1 Malmquist productivity index

The Malmquist productivity index uses the distance function as an aggregator function. The distance function is defined on the production possibility set at the time t

$$\Omega^t = \{(x,y) \mid x \ can \ produce \ y\} \tag{1}$$

where $x = (x_1, x_2, \cdots, x_n)'$ is an input vector and $y = (y_1, y_2, \cdots, y_m)'$ an output vector. The distance function is defined by rescaling the length of an output or an input vector with the production frontier as a reference. The output-oriented distance function is formerly defined by

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[1] Recent studies include Baily, Bartelsman and Haltiwanger (2001), Basu (1996), Sbordone (1996,1997), and Chirinko (1995).

$$D_o^t(x,y) \equiv \min\{\ \delta \mid (x, y/\delta\ ) \in \Omega^t\ \}. \tag{2}$$

By definition, $D_o^t(x,y) > 1$ implies that y is not producible from x. When $D_o^t(x,y) \le 1$, the output-oriented distance function measures technical efficiency, and $D_o^t(x,y) = 1$ indicates full efficiency in the sense that outputs cannot be obtained more without increasing inputs. Given the production possibility set satisfying the regularity conditions, the output-oriented distance function is monotonic increasing, convex, and linearly homogeneous in outputs, and is monotonic decreasing in inputs.

The input-oriented distance function is defined by

$$D_i^t(x,y) \equiv \max\{\ \delta \mid (x/\delta, y) \in \Omega^t\ \}, \tag{3}$$

where $D_i^t(x,y) < 1$ implies that x cannot produce y. When $D_i^t(x,y) \ge 1$, the input-oriented distance function measures technical efficiency, and $D_i^t(x,y) = 1$ indicates full efficiency in the sense that inputs cannot be reduced more without decreasing outputs. Given the production possibility set satisfying the regularity conditions, the input-oriented distance function is monotonic increasing, concave, and linearly homogeneous in inputs, and is monotonic decreasing in outputs.

The aggregate output index is constructed by comparing the outputs at the period t+1 to the outputs at the period t. There are two alternatives depending on choice of the reference period between t and t+1. In this paper, we in principle employ the geometric average of the two indexes. Formerly, the Malmquist output index over the period t and t+1 is defined by

$$M_y(x^{t+1}, x^t, y^{t+1}, y^t) = \left\{ \frac{D_o^t(x^t, y^{t+1})\ D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)\ D_o^{t+1}(x^{t+1}, y^t)} \right\}^{\frac{1}{2}}. \tag{4}$$

Similarly,

$$M_x(x^{t+1}, x^t, y^{t+1}, y^t) = \left\{ \frac{D_i^t(x^{t+1}, y^t)\ D_i^{t+1}(x^{t+1}, y^{t+1})}{D_i^t(x^t, y^t)\ D_i^{t+1}(x^t, y^{t+1})} \right\}^{\frac{1}{2}} \tag{5}$$

defines the Malmquist input index over the period t and t+1. Notice that for any scalar $\lambda > 0$, $M_y(x^{t+1}, x^t, \lambda y^t, y^t) = M_x(\lambda x^t, x^t, y^{t+1}, y^t) = \lambda$.

Although the ratio of the Malmquist output to input indexes is a natural form of the productivity index, it is not the Malmquist productivity index but the HMB productivity index discussed in the next section. The output-oriented Malmquist productivity index is defined as[2]

$$M_o(x^{t+1}, x^t, y^{t+1}, y^t) = \left\{ \frac{D_o^t(x^{t+1}, y^{t+1}) \; D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t) \; D_o^{t+1}(x^t, y^t)} \right\}^{\frac{1}{2}} \qquad (6)$$

A decomposition of this index to the components of technical change and efficiency change is proposed by Färe, Grosskopf, Norris and Zhang (1992). In the logarithmic form,

$$\ln M_o(x^{t+1}, x^t, y^{t+1}, y^t) = \ln TC^{t+1,t} + \ln EC^{t+1,t} \qquad (7)$$

where the technical change component is

$$\ln TC^{t+1,t} = \ln \left\{ \frac{D_o^{t+1}(x^t, y^t) \, D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t) \, D_o^t(x^{t+1}, y^{t+1})} \right\}^{\frac{1}{2}}, \qquad (8)$$

and the efficiency change component

$$\ln EC^{t+1,t} = \ln \left\{ \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \right\}. \qquad (9)$$

It is, however, difficult to isolate the effects of scale on productivity change from $\ln M_o(x^{t+1}, x^t, y^{t+1}, y^t)$. In fact, unless the technology exhibits the constant returns to scale, the Malmquist productivity index does not meet the proportionality condition required for a proper productivity index. For properness, the productivity of $(\lambda x, \mu y)$ in comparison with (x, y) is expected to be μ/λ for any $\lambda>0$ and $\mu>0$. Unfortunately, $M_o(\lambda x^t, x^t, \mu y^t, y^t) = \mu/\lambda$ holds only under the constant returns to scale which ensures that the output-oriented distance function is

[2] The input-oriented Malmquist productivity index is analogously defined. Replacing the output-oriented distance functions in (5) with the corresponding input-oriented distance functions gives the input-oriented Malmquist productivity index.

homogeneous of degree -1 in inputs.[3] Probably, the constant returns to scale is necessary to make the Malmquist productivity index valid, and this is why the Malmquist index is not indicative of the scale effects.

To circumvent this difficulty, Ray and Desli (1997) and Balk (2001) proposed the generalized Malmquist productivity index. They show that this index satisfies the proportionality condition under a more general technology, and that it is decomposable to the factors including scale change and efficiency change as well as technical change. Their approach uses the Banker's (1984) most productive scale size (MPSS) as a reference to evaluate the effects of scale change. It is thus inapplicable when the MPSS does not exist. Since the MPSS is given by the minimum average cost in the single output case, a U-shaped average cost curve is presumed for the generalized Malmquist index to work well.[4]

In this paper, we will show that the HMB index approach takes a different principle based on the scale elasticity to measure the effects of scale change. Unlike the MPSS, the scale elasticity is estimable from local information of costs. Thus, the HMB approach can be used for such a technology that the generalized Malmquist approach encounters a difficulty.

2.2 Törnqvist productivity index

The Törnqvist indexes aggregate outputs and inputs by weighted geometric average with shares of the cost and revenue as weights. Formally, the Törnqvist output and input indexes, respectively, take the form of

$$T_y(y^{t+1}, y^t ; p^{t+1}, p^t,) = \prod_{i=1}^{m} \left(\frac{y_i^{t+1}}{y_i^t} \right)^{\frac{1}{2}\left(\frac{p_i^t y_i^t}{p^{t\prime} y^t} + \frac{p_i^{t+1} y_i^{t+1}}{p^{t+1\prime} y^{t+1}} \right)} \tag{10}$$

and

$$T_x(x^{t+1}, x^t ; w^{t+1}, w^t) = \prod_{i=1}^{n} \left(\frac{x_i^{t+1}}{x_i^t} \right)^{\frac{1}{2}\left(\frac{w_i^t x_i^t}{w^{t\prime} x^t} + \frac{w_i^{t+1} x_i^{t+1}}{w^{t+1\prime} x^{t+1}} \right)}, \tag{11}$$

where $p^t = (p_1, p_2, \cdots, p_m)'$ is the output price vector and $w^t = (w_1, w_2, \cdots, w_n)'$ is the input price

[3] See Shephard (1970, p.77) or Färe and Primont (1995, p.51).

[4] Orea (2002) argued the technology to which the generalized Malmquist approach becomes inapplicable.

vector. The Törnqvist productivity index is defined by the ratio of the output to input index as

$$T(x^{t+1},x^{t},y^{t+1},y^{t}\,;\,w^{t+1},w^{t},p^{t+1},p^{t}) = \frac{T_y(y^{t+1},y^{t}\,;\,p^{t+1},p^{t})}{T_x(x^{t+1},x^{t}\,;\,w^{t+1},w^{t})}. \qquad (12)$$

A remarkable feature of the Törnqvist productivity index is the "translog exactness". Caves, Christensen and Diewert (1982, theorem 3) states that if no inefficiency exists and the output-oriented-distance function is the translog form as (A.5) in the appendix 1, then the Törnqvist productivity index is exactly equal to the output-oriented Malmquist productivity index adjusted for the effects of scale change. Namely, in the logarithmic form,

$$\ln T(x^{t+1},x^{t},y^{t+1},y^{t}\,;\,w^{t+1},w^{t},p^{t+1},p^{t}) = \ln M_o^{*}(x^{t+1},x^{t},y^{t+1},y^{t}) + \ln SC_T^{t+1,t}. \qquad (13)$$

The first term of the right-hand-side $\ln M_o^{*}$ is the output-oriented Malmquist productivity index in the presence of no inefficiency such that

$$\ln M_o^{*}(x^{t+1},x^{t},y^{t+1},y^{t}) = \frac{1}{2}\ln\left\{D_o^{t}(x^{t+1},y^{t+1})\,/\,D_o^{t+1}(x^{t},y^{t})\right\}. \qquad (14)$$

because of $D_o^{t+1}(x^{t+1},y^{t+1}) = D_o^{t}(x^{t},y^{t}) = 1$. As is easily verified, $\ln M_o^{*}$ is also equal to technical change $\ln TC^{t+1,t}$ without inefficiency. In the Törnqvist index, efficiency change is identically zero because inputs and outputs are assumed to be always on the production frontier.

The second term of the right-hand-side of (13), $\ln SC_T^{t+1,t}$, is the scale change factor such that

$$\ln SC_T^{t+1,t} = \frac{1}{2}\sum_{i=1}^{n}\left\{\frac{w_i^{t}x_i^{t}}{w^{t\prime}x^{t}}(\varepsilon_o^{t}-1) + \frac{w_i^{t+1}x_i^{t+1}}{w^{t+1\prime}x^{t+1}}(\varepsilon_o^{t+1}-1)\right\}\left(\ln x_i^{t+1} - \ln x_i^{t}\right), \qquad (15)$$

and $\varepsilon_o^{\tau} = -\sum_{i=1}^{n}\partial\ln D_o^{\tau}(x,y)/\partial\ln x_i$, $\tau = t, t+1$ is the scale elasticity represented by the output-oriented distance function. The appendix 1 discusses measurement of the scale elasticity in detail.

Although $SC_T^{t+1,t}$ captures the effects of the returns to scale, it includes not only the scale effects but the effects of a change in input mix. For convenience in comparing the Törnqvist approach to the HMB approach, we further decompose $\ln SC_T^{t+1,t}$ into the pure scale effect $\ln \widetilde{SC}_T^{t+1,t}$ and the input mix effect $\ln ME_T^{t+1,t}$ as

$$\ln \widetilde{SC}_T^{t+1,t} = \left(\frac{\varepsilon_o^t + \varepsilon_o^{t+1}}{2} - 1 \right) \ln s^{t+1,t} \tag{16}$$

and

$$\ln ME_T^{t+1,t} = \frac{1}{2}\sum_{i=1}^{n} \frac{w_i^t x_i^t}{w^{t'} x^t}(\varepsilon_o^t - 1)\left(\ln x_i^{t+1} - \ln s^{t+1,t} x_i^t\right) + \frac{1}{2}\sum_{i=1}^{n} \frac{w_i^{t+1} x_i^{t+1}}{w^{t+1'} x^{t+1}}(\varepsilon_o^{t+1} - 1)\left(\frac{\ln x_i^{t+1}}{\ln s^{t+1,t}} - \ln x_i^t \right) \tag{17}$$

where $s^{t+1,t} = M_x(x^{t+1}, x^t, y^{t+1}, y^t)$, i.e., the Malmquist input index is used to measure the change in scale of inputs from the period t to t+1.

Eqs. (13)-(17) provide a decomposition of the Törnqvist productivity index. Since eq. (13) is no longer valid if inefficiency exists, the Törnqvist decomposition does not allow us to draw the effects of efficiency on productivity change. We now turn to the HMB approach that defines the productivity index as the ratio of the Malmquist output to input index.

3. Hicks-Moorsteen-Bjurek (HMB) productivity index

A formal definition of the HMB productivity index was given by Bjurek (1996) as

$$HMB(x^{t+1}, x^t, y^{t+1}, y^t) = \frac{M_y(x^{t+1}, x^t, y^{t+1}, y^t)}{M_x(x^{t+1}, x^t, y^{t+1}, y^t)} . \tag{18}$$

This productivity index is obtained by replacing the Törnqvist indexes in eq. (12) with the Malmquist counterparts. Apply the same replacement to the left-hand-side of eq. (13), and modify the second term of the right-hand-side while keeping the first term, the Malmquist productivity index, unaltered. Then we have

$$\ln HMB(x^{t+1}, x^t, y^{t+1}, y^t) = \ln M_o(x^{t+1}, x^t, y^{t+1}, y^t) + \ln \widetilde{SC}_{HMB}^{t+1,t} + \ln ME_{HMB}^{t+1,t} \tag{19}$$

where

$$\ln\widetilde{SC}_{HMB}^{t+1,t} = \frac{1}{2}\ln\left\{\frac{D_o^t(x^t,y^t)}{D_o^t(s^{t+1,t}x^t,y^t)}\frac{D_i^t(x^t,y^t)}{D_i^t(s^{t+1,t}x^t,y^t)}\right\} + \frac{1}{2}\ln\left\{\frac{D_o^{t+1}(\frac{x^{t+1}}{s^{t+1,t}},y^{t+1})}{D_o^{t+1}(x^{t+1},y^{t+1})}\frac{D_i^{t+1}(\frac{x^{t+1}}{s^{t+1,t}},y^{t+1})}{D_i^{t+1}(x^{t+1},y^{t+1})}\right\},$$

$$\ln ME_{HMB}^{t+1,t} = \frac{1}{2}\ln\left\{\frac{D_o^t(s^{t+1,t}x^t,r^{t+1,t}y^t)}{D_o^t(x^{t+1},y^{t+1})}\right\} + \frac{1}{2}\ln\left\{\frac{D_o^{t+1}(x^t,y^t)}{D_o^{t+1}(\frac{x^{t+1}}{s^{t+1,t}},\frac{y^{t+1}}{r^{t+1,t}})}\right\},$$

and

$$r^{t+1,t} = M_y(x^{t+1},x^t,y^{t+1},y^t).$$

In eq. (19), $\ln\widetilde{SC}_{HMB}^{t+1,t}$ is the components of scale change and $\ln ME_{HMB}^{t+1,t}$ represents the input and output mix effects. If $s^{t+1,t} \neq 1$, the effects of scale change can be rewritten as

$$\ln\widetilde{SC}_{HMB}^{t+1,t} = \left(\frac{\hat{\varepsilon}_o^t + \hat{\varepsilon}_o^{t+1}}{2} - 1\right)\ln s^{t+1,t} \tag{20}$$

where

$$\hat{\varepsilon}_o^t = -\frac{\ln\left\{\frac{D_o^t(s^{t+1,t}x^t,y^t)}{D_o^t(x^t,y^t)}\right\}}{\ln\left\{\frac{D_i^t(s^{t+1,t}x^t,y^t)}{D_i^t(x^t,y^t)}\right\}} \quad \text{and} \quad \hat{\varepsilon}_o^{t+1} = -\frac{\ln\left\{\frac{D_o^{t+1}(x^{t+1},y^{t+1})}{D_o^{t+1}(\frac{x^{t+1}}{s^{t+1,t}},y^{t+1})}\right\}}{\ln\left\{\frac{D_i^{t+1}(x^{t+1},y^{t+1})}{D_i^{t+1}(\frac{x^{t+1}}{s^{t+1,t}},y^{t+1})}\right\}}.$$

We show in the appendix 1 that $\hat{\varepsilon}_o^t$ and $\hat{\varepsilon}_o^{t+1}$ are obtained by taking discrete approximations to the scale elasticity. Recalling eq. (7) decomposes $\ln M_o$ into technical change and efficiency change, we eventually have a decomposition of the HMB productivity index as

$$
\begin{aligned}
&\ln HMB(x^{t+1}, x^{t}, y^{t+1}, y^{t}) \\
&= \ln\left\{\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})}\frac{D_o^t(x^{t}, y^{t})}{D_o^{t+1}(x^{t}, y^{t})}\right\}^{\frac{1}{2}} && (\textit{technical change}) \\
&+ \ln\left\{\frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^{t}, y^{t})}\right\} && (\textit{efficiency change}) \\
&+ \left(\frac{\hat{\varepsilon}_o^t + \hat{\varepsilon}_o^{t+1}}{2} - 1\right)\ln s^{t+1,t} && (\textit{scale change}) \\
&+ \ln\left\{\frac{D_o^t(s^{t+1,t}x^{t}, r^{t+1,t}y^{t})}{D_o^t(x^{t+1}, y^{t+1})}\frac{D_o^{t+1}(x^{t}, y^{t})}{D_o^{t+1}(\frac{x^{t+1}}{s^{t+1,t}}, \frac{y^{t+1}}{r^{t+1,t}})}\right\}^{\frac{1}{2}}. && (\textit{input and output mix effects})
\end{aligned}
\tag{21}
$$

We show in the appendix 2 that if no inefficiency exists and the output-oriented distance function is the translog form, then the first three components appeared in the right-hand-side of eq. (21) each are equivalent to the corresponding terms of the Törnqvist index given by eqs. (13)-(17). Thus, it can be seen that the HMB productivity index naturally extends the Törnqvist productivity index so that the effects of efficiency on productivity change is correctly measurable.

4. Empirical model

4.1 Output-oriented distance function

The decomposition analysis of the HMB productivity index is implemented using data on 47 prefectures in Japan. Suppose that two inputs, labor and capital, produce a single output, real GDP. Let x_{Li}^t, x_{Ki}^t and y_i^t denote labor, private capital and real GDP of the prefecture i at the year t, respectively. The share of the manufacturing industry in real GDP, h_i^t, is introduced as a control variable to adjust differences in the industrial structure over time and prefectures. Public capital, x_{Gi}^t, is also a control variable which is supposed to create "atmosphere" through influencing the marginal productivity of private capital. The two control variables are denoted by $z_i^t = (h_i^t, x_{Gi}^t)$. The state of technology is represented by time-specific dummies, $A^t = (A_1^t, A_2^t, \cdots, A_T^t)$ where A_τ^t, $\tau = 1, 2, \cdots, T$ takes unity when $t = \tau$ and zero otherwise.

The deterministic part of the output-oriented distance function is specified by the translog form as

$$
\begin{aligned}
\ln D_o^t(x_{Li}^t, x_{Ki}^t, y_i^t ; z_i^t, A^t) &= TL(x_{Li}^t, x_{Ki}^t, y_i^t ; z_i^t, A^t) \\
&\equiv \alpha_0 + \ln y_i^t + \alpha_L \ln x_{Li}^t + \alpha_K \ln x_{Ki}^t \\
&+ \frac{1}{2}\alpha_{LL}\left(\ln x_{Li}^t\right)^2 + \frac{1}{2}\alpha_{KK}\left(\ln x_{Ki}^t\right)^2 \\
&+ \alpha_{LK}\left(\ln x_{Li}^t\right)\left(\ln x_{Ki}^t\right) \\
&+ \gamma_{KG}\left(\ln x_{Ki}^t\right)\left(\ln x_{Gi}^t\right) + \gamma_h h_i^t + \sum_{\tau=1}^{T} \beta_\tau A_\tau^t \\
i = 1, 2, \cdots, N, &\quad t = 1, 2, \cdots, T.
\end{aligned} \tag{22}
$$

In the above specification, homogeneity in an output is imposed on parameters, which also makes monotonicity and convexity in an output hold automatically.

By definition, the output-oriented distance function is not greater than unity whenever it is evaluated at an observation. This condition is introduced to specifying the stochastic part of the output-oriented distance function as

$$
\ln D_o^t(x_{Li}^t, x_{Ki}^t, y_i^t ; z_i^t, A^t) = -u_i^t + v_i^t, \qquad t = 1, 2, \cdots, T \tag{23}
$$

where

$$
\begin{aligned}
u_i^t &= u_i \sum_{\tau=1}^{T} (1 + \varphi_\tau A_\tau^t), \quad (1 + \varphi_\tau) > 0, \\
u_i &\sim |N(0, \sigma_u^2)|, \\
v_i^t &\sim N(0, \sigma_v^2),
\end{aligned}
$$

and u_i and v_i^t are assumed to be independent. Technical efficiency of the ith prefecture at the year t is given by $\exp\{-u_i(1 + \varphi_t)\}$.

Combining eqs. (22) and (23), we have a stochastic frontier production model. The log-likelihood function of this model can be formed by following Battese and Coelli (1992). Specifically,

$$\ln L = const. + NT\ln\theta + N\ln\lambda - \frac{1}{2}\theta^2\sum_{i=1}^{N}\left\{\sum_{t=1}^{T}\xi_{it}^2 - \frac{1-\lambda^2}{\sum_{t=1}^{T}\eta_t^2}\left(\sum_{t=1}^{T}\eta_t\xi_{it}\right)^2\right\}$$
$$+ \sum_{i=1}^{I}\ln\Phi\left(-\theta\sum_{t=1}^{T}\eta_t\xi_{it}\sqrt{\frac{1-\lambda^2}{\sum_{t=1}^{T}\eta_t^2}}\right) \tag{24}$$

where $\Phi(\bullet)$ is the cumulative distribution function of the standard normal distribution,

$$\xi_{it} = -u_i\eta_t + v_{it} = TL(x_{Li}^t, x_{Ki}^t, y_i^t\ ; z_i^t, A^t),$$

$$\theta = 1/\sigma_v,$$

and

$$\lambda = \sigma_v / \sqrt{\sigma_v^2 + \sigma_u^2\sum_{t=1}^{T}\eta_t^2}.$$

4.2 Input-oriented distance function

Once the output-oriented distance function is parametrically specified, it is directly estimable with econometric techniques. However, the HMB approach requires both of input and output-oriented distance functions. This subsection describes a way to obtain the input-oriented distance function values from the estimated output-oriented distance function.[5]

Since $D_o^t(x,y) \le 1$ implies $(x,y) \in \Omega^t$, the input-oriented distance function is alternatively defined as

$$D_i^t(x,y) = \max\left\{\delta \mid D_o^t(x/\delta, y) \le 1\right\}. \tag{25}$$

Letting $D_i^t(x,y) = \delta^*$, $D_o^t(x/\delta^*, y) = 1$ follows under the regular production possibility set. Thus, δ^* is obtained by solving $\ln D_o^t(x/\delta^*, y) = 0$. Given the translog output-oriented distance function

[5] The same procedure is applicable for obtaining the output-oriented distance function values from the input-distance function.

(22), we have

$$\frac{1}{2}(\ln\delta^*)^2 \sum_{i=K,L}\sum_{j=K,L} \alpha_{ij} + \varepsilon_o^t \ln\delta^* + \ln D_o^t(x,y) = 0. \tag{26}$$

If $\sum_i\sum_j \alpha_{ij} = 0$, then

$$\ln D_i^t(x,y) = -\frac{\ln D_o^t(x,y)}{\varepsilon_o^t(x,y)}. \tag{27}$$

If $\sum_i\sum_j \alpha_{ij} \neq 0$, eq. (26) is a quadratic equation. Suppose that there exist real roots δ_1^* and δ_2^*. Then, nonsmaller one of them is relevant by definition of the input distance function. Therefore,

$$\ln D_i^t(x,y) = \max\left\{\delta_1^*, \delta_2^*\right\}. \tag{28}$$

Eq. (26) has real roots as long as the output distance function is increasing in inputs over a sufficiently large domain around (x,y).

4.3 Four components of productivity change

This subsection explains measurement of the four components of the decomposition analysis based on eq. (21). First of all, according to Battese and Coelli (1992), technical efficiency, $\hat{u}_i^t$, is estimated as the conditional expectation of $u_i^t = u_i\varphi_t$ on ξ_{it}.[6] Then, efficiency change is measured as

$$\ln \hat{EC}_i^{t+1,t} = -\hat{u}_i^{t+1} + \hat{u}_i^t.$$

Technical change is computed as

$$\ln \hat{TC}_i^{t+1,t} = \frac{1}{2}\left\{ TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1}; z_i^t, A^t) - TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1}; z_i^{t+1}, A^{t+1})\right\}$$
$$+ \frac{1}{2}\left\{ TL(x_{Li}^t, x_{Ki}^t, y_i^t; z_i^t, A^t) - TL(x_{Li}^t, x_{Ki}^t, y_i^t; z_i^{t+1}, A^{t+1})\right\}.$$

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[6] Note that $\hat{u}_i^t$ is a measure of inefficiency. It decreases as efficiency becomes higher. The lowest value, zero, is attained when no inefficiency exists.

Although $\ln \hat{TC}_i^{t+1,t}$ represents a shift in the production frontier, some amount of the shift is due to changes in the control variables z_i^t. Recalling that the effects of technology shocks are supposed to be captured by the time-specific dummies, we define pure technical change which is solely due to A^t as

$$\ln \hat{\tilde{TC}}_i^{t+1,t} = \frac{1}{2}\left\{ TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1} \; ; z_i^{t+1}, A^t) - TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1} \; ; z_i^{t+1}, A^{t+1}) \right\} + \frac{1}{2}\left\{ TL(x_{Li}^t, x_{Ki}^t, y_i^t \; ; z_i^t, A^t) - TL(x_{Li}^t, x_{Ki}^t, y_i^t \; ; z_i^t, A^{t+1}) \right\}.$$

Scale change component is computed as

$$\ln \hat{\tilde{SC}}_{HMBi}^{t+1,t} = \frac{1}{2}\left\{ TL(x_{Li}^t, x_{Ki}^t, y_i^t \; ; z_i^t, A^t) - TL(\hat{s}_i^{t+1,t} x_{Li}^t, \hat{s}_i^{t+1,t} x_{Ki}^t, y_i^t \; ; z_i^t, A^t) \right\} + \frac{1}{2}\left\{ TL(\frac{x_{Li}^{t+1}}{\hat{s}_i^{t+1,t}}, \frac{x_{Ki}^{t+1}}{\hat{s}_i^{t+1,t}}, y_i^{t+1} \; ; z_i^{t+1}, A^{t+1}) - TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1} \; ; z_i^{t+1}, A^{t+1}) \right\}.$$

An estimate of $s_i^{t+1,t}$ is obtained by the technique explained in the last subsection. In the four input-oriented distance functions constituting $s_i^{t+1,t}$, $D_i^t(x_{Li}^t, x_{Ki}^t, y_i^t)$ and $D_i^{t+1}(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1})$ are estimated by substituting $-\hat{u}_i^t$ and $-\hat{u}_i^{t+1}$ into the term of output-oriented distance function in eq. (26), respectively. For the other part of $s_i^{t+1,t}$, estimates of $D_i^t(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^t)$ and $D_i^{t+1}(x_{Li}^t, x_{Ki}^t, y_i^{t+1})$ may be similarly obtainable from eq. (26) using $TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^t \; ; z_i^t, A^t)$ and $TL(x_{Li}^t, x_{Ki}^t, y_i^{t+1} \; ; z_i^{t+1}, A^{t+1})$ as the corresponding output-oriented distance function values. However, $-\hat{u}_i^t$ and $-\hat{u}_i^{t+1}$ differ from $TL(x_{Li}^t, x_{Ki}^t, y_i^t \; ; z_i^t, A^t)$ and $TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1} \; ; z_i^{t+1}, A^{t+1})$ by estimation residuals, i.e., letting $\hat{v}_i^t$ be the residuals, $-\hat{u}_i^t = TL(x_{Li}^t, x_{Ki}^t, y_i^t \; ; z_i^t, A^t) - \hat{v}_i^t$ and $-\hat{u}_i^{t+1} = TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1} \; ; z_i^{t+1}, A^{t+1}) - \hat{v}_i^{t+1}$. Taking account of this, $TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^t \; ; z_i^t, A^t) - \hat{v}_i^t$ and $TL(x_{Li}^t, x_{Ki}^t, y_i^{t+1} \; ; z_i^{t+1}, A^{t+1}) - \hat{v}_i^{t+1}$ are used with eq. (26) in estimation of $D_i^t(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^t)$ and $D_i^{t+1}(x_{Li}^t, x_{Ki}^t, y_i^{t+1})$. By this procedure, the residuals are canceled out in calculation of $\hat{s}_i^{t+1,t}$ and do not affect the analytical results.

Finally, the input and output mix effects are computed by

$$\ln \hat{ME}_{HM}^{t+1,t} = \frac{1}{2}\left\{ TL(\hat{s}_i^{t+1,t} x_{Li}^t, \hat{s}_i^{t+1,t} x_{Ki}^t, \hat{r}_i^{t+1,t} y_i^t \; ; z_i^t, A^t) - TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1} \; ; z_i^t, A^t) \right\} + \frac{1}{2}\left\{ TL(x_{Li}^t, x_{Ki}^t, y_i^t \; ; z_i^{t+1}, A^{t+1}) - TL(\frac{x_{Li}^{t+1}}{\hat{s}_i^{t+1,t}}, \frac{x_{Ki}^{t+1^{t+1}}}{\hat{s}_i^{t+1,t}}, \frac{y_i^{t+1}}{\hat{r}_i^{t+1,t}} \; ; z_i^{t+1}, A^{t+1}) \right\}.$$

where

$$\hat{r}_i^{t+1,t} = \left[\exp\left\{ TL(x_{Li}^t, x_{Ki}^t, y_i^t ; z_i^t, A^t) - TL(x_{Li}^t, x_{Ki}^t, y_i^{t+1} ; z_i^t, A^t) \right\} + \left\{ TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^{t+1} ; z_i^{t+1}) - TL(x_{Li}^{t+1}, x_{Ki}^{t+1}, y_i^t ; z_i^{t+1}, A^{t+1}) \right\} \right]^{\frac{1}{2}}.$$

4.4 Data

The translog output-oriented distance function is estimated with data on 47 prefectures in Japan during the year 1980-2000. The data set is compiled from *CRIEPI Regional Economic Database* constructed by Socio-Economic Research Center, Central Research Institute of Electric Power Industry, Tokyo. From this database, real GDP, labor, private capital, public capital and share of the manufacturing industry are taken.

5. Results

Estimated parameters of the output-oriented distance function is displayed in Table 1. The coefficients of some adjacent time-specific dummies are restricted to be equal, implying that the production frontier dose not shift in these years. Without the restrictions, the output-distance function is estimated with technical regress for many years. Although technical regress is empirically implausible, the concerned coefficients mostly suggest that it is insignificant. We then assume that neither technical progress nor regress occurred in the years when the significant shifts in the production frontier is not detected[7].

The resulting parameter estimates ensure that the output-oriented distance function is monotonic decreasing in labor and private capital within sample. The coefficient of manufacturing industry share is significantly negative, indicating that the manufacturing industry is more productive than other sectors and thereby an increase in its weight advances the production frontier. The interaction term of public and private capital is positively related to the output-oriented distance. This suggests that the marginal productivity of private capital falls with an increase in public capital. In this sense, public and private capital are substitutes. Turning to the stochastic part, we find that the variance of efficiency overwhelms the variance of statistical noise. If efficiency terms are neglected in the empirical production model, it easily leads to misspecification.

Table 2 reports the HMB productivity index and its factorized components based on eq. (21). All figures are measured in percentage change over adjacent years, i.e., measurements of each term

[7] As a result, the production frontier is supposed to halt during the year 1981-1984, 1986, 1991-1995, 1997-1998.

of eq. (21) multiplied by a hundred. To summarize the results, we show the upper and lower quartiles among 47 prefectures for each year. The sum of components does not equal to productivity change because these quartiles represent different prefectures. An overview of Table 2 suggests that the effects on productivity of technical and efficiency changes dominate over those of the other two components.

Figure 1 depicts the development of upper and lower quartiles of the HMB productivity index. The growth rate of nationwide GDP are also designated by the dotted line. Clearly, procyclicality of the productivity change can be seen. Synchronizing with fluctuations in nationwide GDP, the productivity growth sharply fell around 1986, 1992 and 1998, and it rose during the year 1994, 1987-1990, 1996, and 1999-2000. A question is that the procyclical movement is due more to technology shocks or demand shocks.

Figure 2 shows the development of the technical change component. Technical change contributed to raising productivity and GDP growth in the mid and late 80s except 1986, and the last two years in the 90s. Conversely, technical change was responsible for downturns of productivity and GDP in the early and mid 90s. It should be noted that the technical change component did not rise during the year 1994-1996 when a surge of productivity and GDP growth is observed. As we will see below, the efficiency change component played a more important role than the technical change in this period.

In Figure 2, measurements of technical change shown are $\ln\hat{TC}_i^{t+1,t}$ that contains the effects of the control variables. In the present model, the effects of pure technical factor is isolated by $\ln\hat{\tilde{TC}}_i^{t+1,t}$. To give an overview, Figure 3 shows its accumulated index normalized at one in 1980. Technical advances are found in the late 80s and in the last two years of the 90s. In the rest of the period, the technical level is generally unaltered, which is mainly due to the parameter restrictions imposed on the output-oriented distance function. The accumulated index reached 1.16 in 2000, implying that the technology was enhanced to produce sixteen percent more outputs from the same amount of inputs in 1980.

Figure 4 shows the development of the efficiency change component. We see that upturns of productivity and GDP during the year 1994-1996, especially in 1996, were supported by improvement in efficiency. Since demand shocks are captured by the efficiency factor and, as noted above, technology shocks were not prominent, the economic recovery in this period was likely to be driven by fiscal expansion. The efficiency change component also has peaks in 1984 and 1990. In these years, we find a similar pattern to 1996 that improvement in efficiency supported productivity and GDP growth that were not accompanied by technical change.

Conversely, efficiency did not a main contributor to recovery in productivity and GDP in 1999 and 2000, compared with the technical change factor.

Figure 5 shows the scale change component. The effects of scale change are much smaller than those of technical change and efficiency change. It is thus difficult to explain procyclical productivity by the short run increasing returns. As seen from Figure 5, development of upper and lower quartiles of the scale change component exhibit a mirror image. This is because increasing and decreasing returns technologies coexist in 47 prefectures while movement of the scale is rather common among them. By construction, the scale change component is the product of scale change and the scale elasticity minus one, $\hat{\varepsilon}_o^t - 1$. If the scale change is common, the increasing and decreasing returns technologies, $\hat{\varepsilon}_o^t > 1$ and $\hat{\varepsilon}_o^t < 1$, yield symmetric movement of the scale change components. Table 3 shows measurements of the scale elasticity for prefectures having the five largest and the five smallest values. Reflecting the aggromelation economies, prefectures including the metropolitan areas tend to exhibit the scale economies.

Figure 6 shows the input and output mix effects. Since there is a single output in the present specification, the mix effects represent changes in the ratio of two inputs, capital and labor. Thus, Figure 6 suggests that rising the capital to labor ratio over the two decades continuously enhanced productivity. However, this effects are considerably small and do not seem to be an very important factor in productivity change.

In summary, technical change and efficiency change are the two of most important components driving procyclical productivity. We find that relative importance of them varies over periods. Supply shocks captured by technical change drove upturns of productivity in the mid and late 80s and in the last two years in the 90s. Supply shocks also caused downturns in the early and mid 90s. On the other hand, demand shocks captured by efficiency change drove upturns of productivity in 1984, 1990 and 1996 when supply shocks were not detected.

6. Conclusion

This paper provides a new decomposition analysis of productivity employing the HMB approach. We show that the HMB approach can be seen as a synthesis of the conventional approaches widely used in the literature. The proposed analysis is applied to examine productivity of the Japanese economy using data on 47 prefectures during the year 1980-2000 in order to prove its usefulness. The results evidence that supply and demand shocks both drove procyclical productivity, and that their relative importance was mixed.

Before concluding the paper, we point out that the HMB index approach does not provide a fully integrated framework of the productivity and efficiency analyses yet. While the efficiency component in the decomposition analysis represents technical efficiency, it is not indicative of allocative efficiency at all. This is because efficiency is measured along the fixed ray determined by observed combination of inputs and outputs. The next step should be directed toward a further extended HMB approach including allocative efficiency as a component of the decomposition analysis of productivity.

Appendix 1 Scale elasticity in terms of the output-oriented distance function

This appendix reviews the output-oriented scale elasticity and shows that $\hat{\varepsilon}_o^t$ is obtained by a discrete approximation. For any $\lambda > 0$ and $(x, y) \in \Omega^\tau$, μ is supposed to satisfy

$$D_o^t(\lambda x, \mu y) = 1. \tag{A.1}$$

Since the output distance function is linearly homogenous in y,

$$\mu = 1 / D_o^t(\lambda x, y). \tag{A.2}$$

The output-oriented scale elasticity at (x, y) relative to Ω^τ is defined as

$$\varepsilon_o^t(x, y) = \left.\frac{d\ln\mu}{d\ln\lambda}\right|_{\lambda = 1}. \tag{A.3}$$

Logarithmically differentiating both sides of (A.2), we have

$$\varepsilon_o^t(x, y) = -\nabla_{\ln x}\ln D_o^t(x, y)' i \tag{A.4}$$

where $i = (1, 1, \cdots, 1)'$.

Suppose that the output-oriented distance function takes the translog form with a time-varying constant and time-varying first order parameters

$$\begin{aligned}\ln D_o^t(x, y) = \alpha_0^t + \sum_i \alpha_i^t \ln x_i + \sum_i \beta_i^t \ln y_i + \frac{1}{2}\sum_i \sum_j \alpha_{ij} \ln x_i \ln x_j \\ + \frac{1}{2}\sum_i \sum_j \beta_{ij} \ln y_i \ln y_j + \sum_i \sum_j \gamma_{ij} \ln x_i \ln y_j,\end{aligned} \tag{A.5}$$

$$\alpha_{ij} = \alpha_{ji},\ i,j = 1, 2, \cdots, m, \quad \beta_{ij} = \beta_{ji},\ i,j = 1, 2, \cdots, n.$$

Since the constant and first order parameters are time-varying, they are allowed to depend on control variables and the time trend. Thus, eq. (21) specified in the main text is a special case of eq. (A.5). From (A.4), the scale elasticity implied by (A.5) can be written as

$$\varepsilon_o^t = -\left(\sum_i \alpha_i^t + \sum_i \sum_j \alpha_{ij}^t \ln x_j + \sum_i \sum_j \gamma_{ij}^t \ln y_j \right). \tag{A.6}$$

In the main text, we abbreviate $\varepsilon_o^t(x^t, y^t)$ as ε_o^τ, $\tau = t, t+1$.

To measure the scale elasticity with eq. (A.4), the output-oriented distance function parameters are required to evaluate the first order derivatives. Instead of differentiation, a discrete approximation enables us to measure the scale elasticity with nonparametric techniques. Taking a discrete approximation to eq. (A.3) at (x, y), we have

$$\hat{\varepsilon}_o^t(x,y) = \left. \frac{\Delta\mu}{\Delta\lambda} \frac{\lambda}{\mu} \right|_{\lambda=1} = \frac{1}{\Delta\lambda} \left\{ \frac{D_o^t(\lambda x, y)}{D_o^t((1+\Delta\lambda)x, y)} - 1 \right\}. \tag{A.7}$$

Set $x = x^t$, $y = y^t$ and $\Delta\lambda = s^{t+1,t} - 1$. We then have

$$\hat{\varepsilon}_o^t(x^t, y^t) = -\frac{\ln\left\{ \dfrac{D_o^t(s^{t+1,t} x^t, y^t)}{D_o^t(x^t, y^t)} \right\}}{\ln\left\{ \dfrac{D_i^t(s^{t+1,t} x^t, y^t)}{D_i^t(x^t, y^t)} \right\}}. \tag{A.8}$$

An approximation formula $\ln(1+x) \doteq x$ is utilized here. Similarly, setting $x = x^{t+1}$, $y = y^{t+1}$ and $\Delta\lambda = 1/s^{t+1,t} - 1$ yields

$$\hat{\varepsilon}_o^{t+1}(x^{t+1}, y^{t+1}) = -\frac{\ln\left\{ \dfrac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(\frac{x^{t+1}}{s^{t+1,t}}, y^{t+1})} \right\}}{\ln\left\{ \dfrac{D_i^{t+1}(x^{t+1}, y^{t+1})}{D_i^{t+1}(\frac{x^{t+1}}{s^{t+1,t}}, y^{t+1})} \right\}}. \tag{A.9}$$

In the main text, $\hat{\varepsilon}_o^t(x^t, y^t)$ and $\hat{\varepsilon}_o^{t+1}(x^{t+1}, y^{t+1})$ are abbreviated as $\hat{\varepsilon}_o^t$ and $\hat{\varepsilon}_o^{t+1}$, respectively.

Estimation of the scale elasticity by taking a discrete approximation was initially proposed by Löthgren and Tambour (1996) in the context of the data envelopment analysis. Our estimates $\hat{\varepsilon}_o^t$ and $\hat{\varepsilon}_o^{t+1}$ always lie between their upper and lower bound estimates.

Appendix 2 Törnqvist and the HMB productivity indexes in the presence of no inefficiency

This appendix shows that all components of the HMB productivity index except the input and output mix effects are reduced to the corresponding components of the Törnqvist index under the translog output-oriented distance function if no inefficiency exists. This is basically proven by a slightly extended version of the translog identity lemma as the following.

<u>Translog identity lemma</u>: Suppose that $D_o^t(x,y)$, $t = g, h, k, l$ takes the translog form defined by eq. (A.5). If no inefficiency exist and $\ln x_i^k - \ln x_i^h = \ln x_i^g - \ln x_i^l$, for all i, then

$$\ln\left\{\frac{D_o^k(x^k,u^k)\,D_o^l(x^g,y^l)}{D_o^k(x^h,y^k)\,D_o^l(x^l,y^l)}\right\}^{\frac{1}{2}} = \frac{1}{2}\sum_{i=1}^{n}\left\{\frac{\partial \ln D_o^k(x^k,y^k)}{\partial \ln x_i^k} + \frac{\partial \ln D_o^l(x^l,y^l)}{\partial \ln x_i^l}\right\}\left(\ln x_i^k - \ln x_i^h\right).$$

Proof: This lemma can be demonstrated by following the proof of the translog identity lemma in Cave et al. (1982). Q.E.D.

The decomposition of the HMB productivity index is given by

$$\ln HMB(x^{t+1},x^t,y^{t+1},y^t) = \ln M_o(x^{t+1},x^t,y^{t+1},y^t) + \ln \widetilde{SC}_{HMB}^{t+1,t} + \ln ME_{HMB}^{t+1,t} \tag{18}$$

and from eqs. (13)-(17), the Törnqvist productivity index is decomposed as

$$\ln T(x^{t+1},x^t,y^{t+1},y^t\,;\,w^{t+1},w^t,p^{t+1},p^t) = \ln M_o^*(x^{t+1},x^t,y^{t+1},y^t) + \ln \widetilde{SC}_T^{t+1,t} + \ln ME_T^{t+1,t}.$$

<u>Proposition 1 (Scale effects)</u>: If i) the output-oriented distance function takes the translog form (A.5) and ii) no inefficiency exists, then $\ln \widetilde{SC}_{HMB}^{t+1,t} = \ln \widetilde{SC}_T^{t+1,t}$.

Proof: It follows from the definition of $\hat{\varepsilon}_o^t$ and $\hat{\varepsilon}_o^{t+1}$ and homogeneity of the input-oriented distance function in inputs that

$$\ln \widetilde{SC}_{HMB}^{t+1,t} = \ln\left\{\frac{D_o^t(x^t,y^t)}{D_o^t(s^{t+1,t}x^t,y^t)}\,\frac{D_o^{t+1}(\frac{x^{t+1}}{s^{t+1,t}},y^{t+1})}{D_o^{t+1}(x^{t+1},y^{t+1})}\right\}^{\frac{1}{2}} - \ln s^{t+1,t}.$$

Applying the translog identity lemma to the first term of the right-hand-side, we have

$$\ln \widetilde{SC}_{HMB}^{t+1,t} = -\frac{1}{2}\sum_{i=1}^{n}\left\{\frac{\partial \ln D_o^t(x^t, y^t)}{\partial \ln x_i^t} + \frac{\partial \ln D_o^{t+1}(x^{t+1}, y^{t+1})}{\partial \ln x_i^{t+1}}\right\}\ln s^{t+1,t} - \ln s^{t+1,t}.$$

Since no inefficiency exists, the cost minimization is achieved. As shown by Caves et al. (1982), the first order condition for cost minimization is expressed in terms of the output-oriented distance function as

$$\frac{\partial \ln D_o^\tau(x^\tau, y^\tau)}{\partial \ln x_i^\tau} = -\varepsilon_o^\tau \frac{w_i^\tau x_i^\tau}{w^{\tau\prime} x^\tau}, \quad \tau = t, t+1.$$

Substituting this to the above equation, we have $\ln \widetilde{SC}_{HMB}^{t+1,t} = \ln \widetilde{SC}_{T}^{t+1,t}$. Q.E.D.

Without inefficiency, it is trivial that $\ln M_o$ is equal to $\ln M_o^*$ by definition, and is also trivial that $\ln TC^{t+1,t} = \ln M_o^*$ and $\ln EC^{t+1,t} = 0$. Therefore, together with the proposition 1, one can say that the components of technical change, efficiency change and scale change are equivalent to the Törnqvist counterparts under the conditions i) and ii).

Since $\ln HMB = \ln T_y - \ln T_x$ and $\ln T = \ln M_y - \ln M_x$, the Törnqvist and the Malmquist indexes are equivalent if both of their output and input indexes coincide. A straightforward application of the translog lemma to the Malmquist output index yields $\ln T_y = \ln M_y$.

<u>Proposition 2 (Output index)</u>: If i) the output-oriented distance function takes the translog form (A.5) and ii) no inefficiency exists, $\ln T_y = \ln M_y$.

Proof: This proposition is the Theorem 2 by Caves et al. (1982). Q.E.D.

If also $\ln T_x = \ln M_x$ could be shown, the Törnqvist index and the HMB index are fully equivalent. Unfortunately, this is not the case because the translog identity lemma cannot be applied to the input-oriented distance function even though the output-oriented distance function is translog. Two productivity indexes differ by the difference between $\ln T_x$ and $\ln M_x$ under the conditions i) and ii).

References

Baily, M. N., E. J. Bartelsman and J. Haltiwanger (2001) "Labor Productivity: Structural Change and Cyclical Dynamics," Review of Economics and Statistics 83, 420-433.

Balk, B. (2001) "Scale Efficiency and Productivity Change," Journal of Productivity Analysis 15, 159-183.

Basu, S. (1996) "Procyclical Productivity: Increasing Returns or Cyclical Utilization?," Quarterly Journal of Economics 111, 719-751.

Battese, G. E. and T. J. Coelli (1992) "Frontier Production Functions, Technical Efficiency and Panel Data: With Application to Paddy Farmers in India," Journal of Productivity Analysis 3, 153-169.

Banker, R. D. (1984) "Estimating Most Productive Scale Size Using Data Envelopment Analysis," European Journal of Operational Research, 17, 35-44.

Bjurek, H. (1996) "The Malmquist Total Factor productivity," Scandinavian Journal of Economics 98, 303-313.

Caves, D. W., L. R. Christensen and W. E. Diewert (1982) "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity," Econometrica 50, 1393-1414.

Chirinko, R. (1995) "Nonconvexities, Labor Hoarding, Technology Shocks, and procyclical Productivity a Structural Econometric Analysis," Journal of Econometrics 66, 61-98.

Färe, R., S. Grosskopf, M. Norris and Z. Zhang (1994) "Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries," American Economic Review 84, 66-83.

Färe, R. and D. Primont (1995) Multi-Output Production and Duality: Theory and Applications, Kluwer Academic Publishers.

Lovell, C. A. K. (2003) "The Decomposition of Malmquist Productivity Indexes," Journal of Productivity Analysis 20, 437-458.

Löthgren, M. And M. Tambour (1996) "Alternative Approaches to Estimate Returns to Scale in DEA-Models," Working Paper series in Economics and Finance, Working Paper No. 90, Stockholm School of Economics.

Orea, L. (2002) "Parametric Decomposition of a Generalized Malmquist Productivity Index," Journal of Productivity Analysis 18, 5-22.

Ray, S. and E. Desli, "Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries: Comment," American Economic Review 87, 1033-1039.

Sbordone, A. M. (1997) "Interpreting the Procyclical Productivity of Manufacturing Sectors: External Effect or Labor Hoarding?," Journal of Money, Credit, and Banking 29, 26-45.

Sbordone, A. M. (1996) "Cyclical Productivity in a Model of Labor Hoarding," Journal of Monetary Economics 38, 331-361.

Shephard, R. W. (1970) Theory of Cost and production Functions, Princeton University Press.

Table 1. Parameter estimates of the output-oriented distance function

	Parameter Estimate	Standard Error		Parameter Estimate	Standard Error
α_0	0.1862	(0.0144)**	φ_5	0.0776	(0.0533)
α_L	-0.8353	(0.0194)**	φ_6	0.0196	(0.0539)
α_K	-0.2021	(0.0184)**	φ_7	0.1034	(0.0701)
α_{LL}	-0.0032	(0.0270)	φ_8	0.1665	(0.0698) *
α_{KK}	-0.0738	(0.0174)**	φ_9	0.1568	(0.0707) *
α_{LK}	-0.0389	(0.0227)	φ_{10}	-0.0307	(0.0493)
γ_{KG}	0.0577	(0.0107)**	φ_{11}	0.0142	(0.0502)
γ_h	-0.6080	(0.0385)**	φ_{12}	0.0675	(0.0515)
β_5	-0.0447	(0.0068)**	φ_{13}	0.0952	(0.0542)
β_7	-0.0844	(0.0096)**	φ_{14}	0.0440	(0.0539)
β_8	-0.1202	(0.0102)**	φ_{15}	0.0014	(0.0539)
β_9	-0.1423	(0.0112)**	φ_{16}	-0.2200	(0.0504) **
β_{10}	-0.1325	(0.0109)**	φ_{17}	-0.1557	(0.0513) **
β_{16}	-0.1183	(0.0141)**	φ_{18}	-0.1705	(0.0524) **
β_{19}	-0.1288	(0.0169)**	φ_{19}	-0.2385	(0.0640) **
β_{20}	-0.1507	(0.0176)**	φ_{20}	-0.2525	(0.0621) **
φ_1	0.0376	(0.0340)	σ_u	0.1519	(0.0183) **
φ_2	0.0042	(0.0341)	σ_v	0.0242	(0.0006) **
φ_3	-0.0160	(0.0358)			
φ_4	-0.1556	(0.0356)**			

Note: "*" indicates significance at the 5% level.

"**" indicates significance at the 1% level.

Table 2. Decomposition of the HMB productivity index

unit: percent

year	Productivity change		Technical change		Efficiency change		Scale change		Input and output mix effects	
	lower quartile	upper quartile	lower quartile	upper quartile	lower quartile	upper quartile	lower quartile	upper quartile	lower quartile	upper quartile
1981	-1.06	1.14	0.12	0.98	-0.70	-0.29	-0.20	0.00	-0.01	0.02
1982	0.18	2.12	0.00	0.82	0.26	0.62	-0.06	0.02	0.00	0.04
1983	0.17	2.25	0.36	1.09	0.16	0.38	-0.06	0.02	0.00	0.04
1984	1.51	3.23	-0.04	0.83	1.07	2.60	-0.06	0.05	0.01	0.06
1985	0.61	2.87	4.65	5.57	-4.35	-1.79	-0.10	0.04	0.03	0.16
1986	-0.28	1.37	-0.88	0.13	0.45	1.08	-0.03	0.04	0.01	0.07
1987	2.05	4.49	4.02	4.57	-1.56	-0.64	-0.07	0.04	0.02	0.11
1988	2.01	4.67	3.95	4.57	-1.18	-0.48	-0.09	0.06	0.01	0.05
1989	1.81	3.45	2.18	2.91	0.07	0.18	-0.08	0.06	0.02	0.06
1990	2.08	3.44	-0.94	-0.27	1.44	3.50	-0.09	0.07	0.01	0.06
1991	-1.40	0.54	-0.02	0.55	-0.84	-0.35	-0.13	0.13	0.02	0.08
1992	-2.89	-0.81	-0.76	-0.04	-0.99	-0.41	-0.06	0.08	0.01	0.03
1993	-1.85	0.16	-0.52	0.30	-0.52	-0.21	-0.05	0.07	0.00	0.02
1994	-0.21	1.87	-0.03	0.43	0.39	0.95	-0.04	0.04	0.00	0.01
1995	0.35	1.83	0.16	0.70	0.33	0.80	-0.04	0.07	0.00	0.01
1996	1.64	3.08	-1.61	-1.01	1.70	4.13	-0.03	0.07	0.00	0.01
1997	-1.34	-0.14	-0.15	0.50	-1.20	-0.49	0.00	0.03	0.01	0.03
1998	-1.57	0.50	-0.83	-0.24	0.11	0.28	0.00	0.02	0.01	0.02
1999	1.78	3.11	0.83	1.48	0.52	1.27	-0.02	0.02	0.01	0.03
2000	1.80	4.10	2.34	3.08	0.11	0.26	-0.01	0.02	0.01	0.03

Table 3. Scale elasticity

the five largest		the five smallest	
Tokyo	1.22	Tottori	0.84
Osaka	1.17	Kochi	0.86
Aichi	1.17	Okinawa	0.87
Kanagawa	1.15	Tokushima	0.88
Hyogo	1.10	Shimane	0.88

Note: Elasticities are averaged over the period 1981-2000

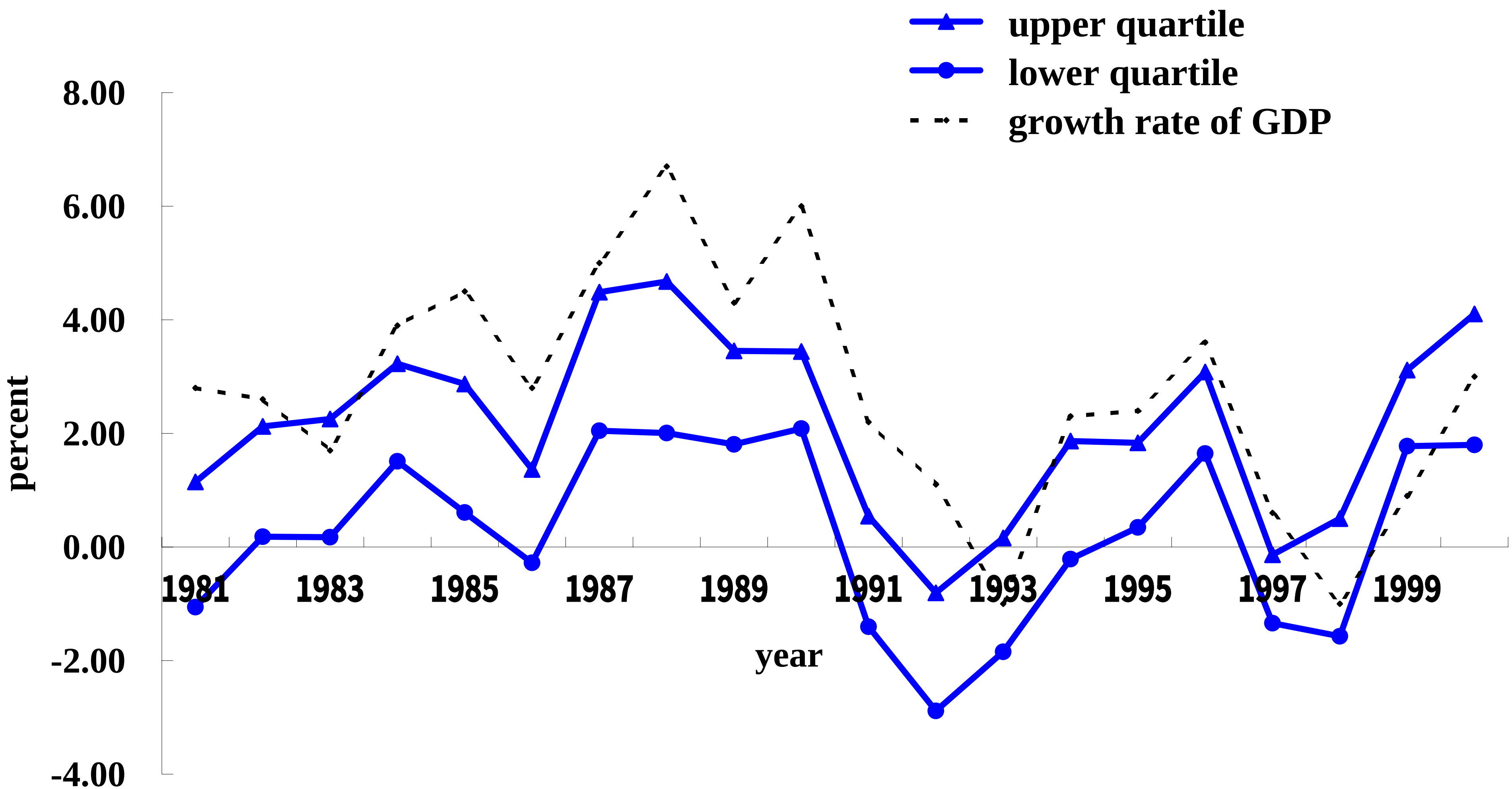

Fig. 1 HMB productivity index

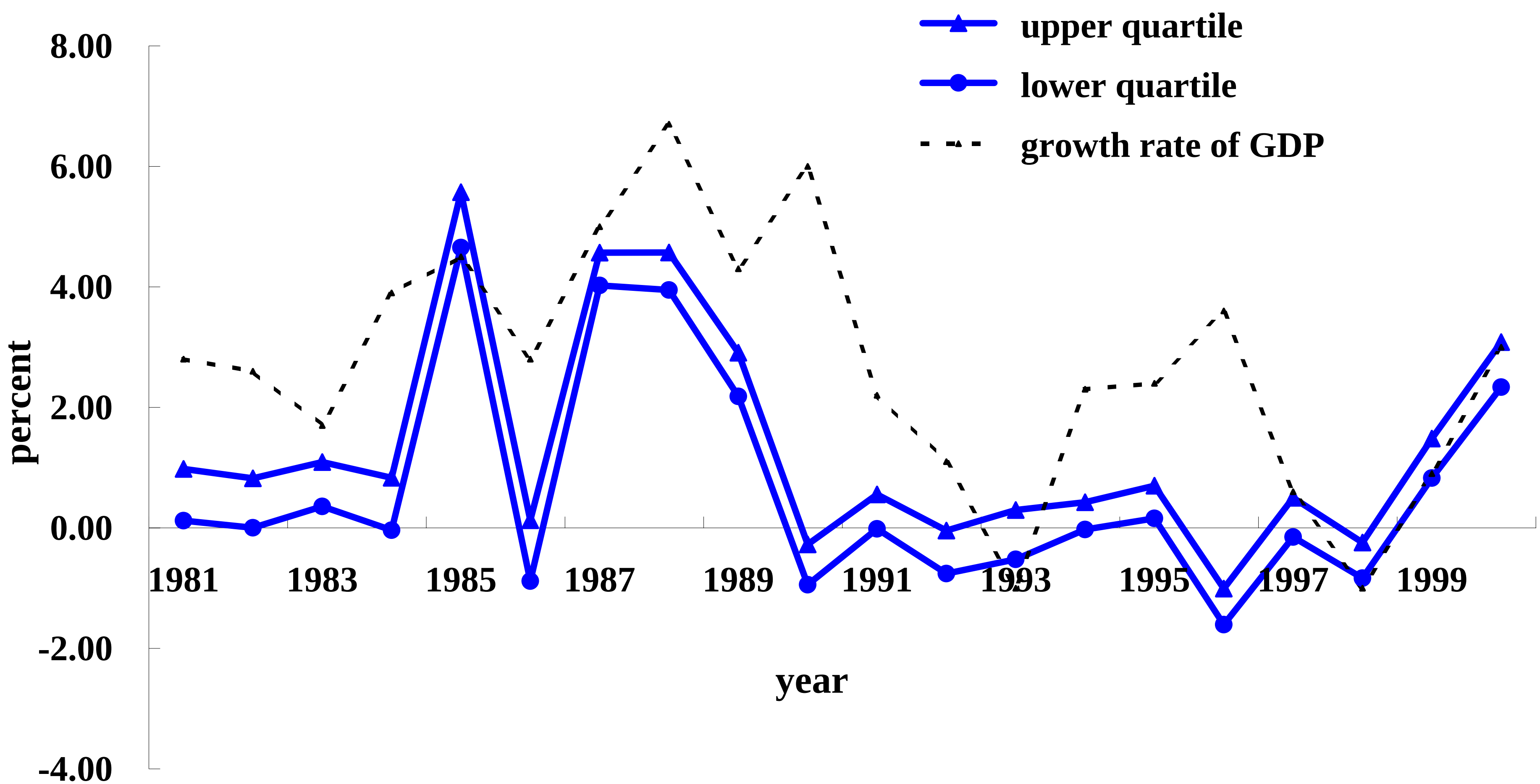

Fig. 2 The effects of technical change on productivity

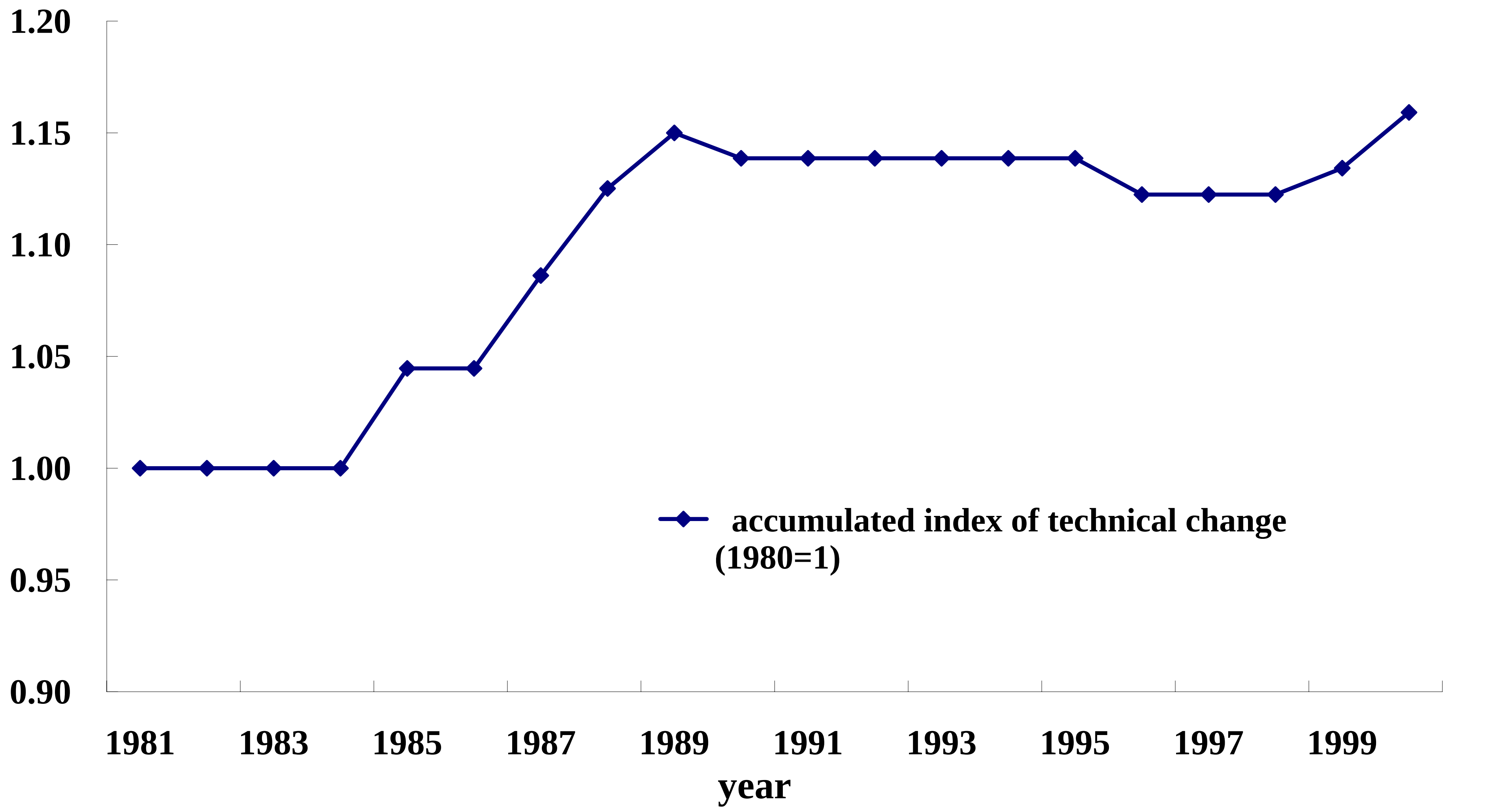

Fig. 3 Accumulated technical change

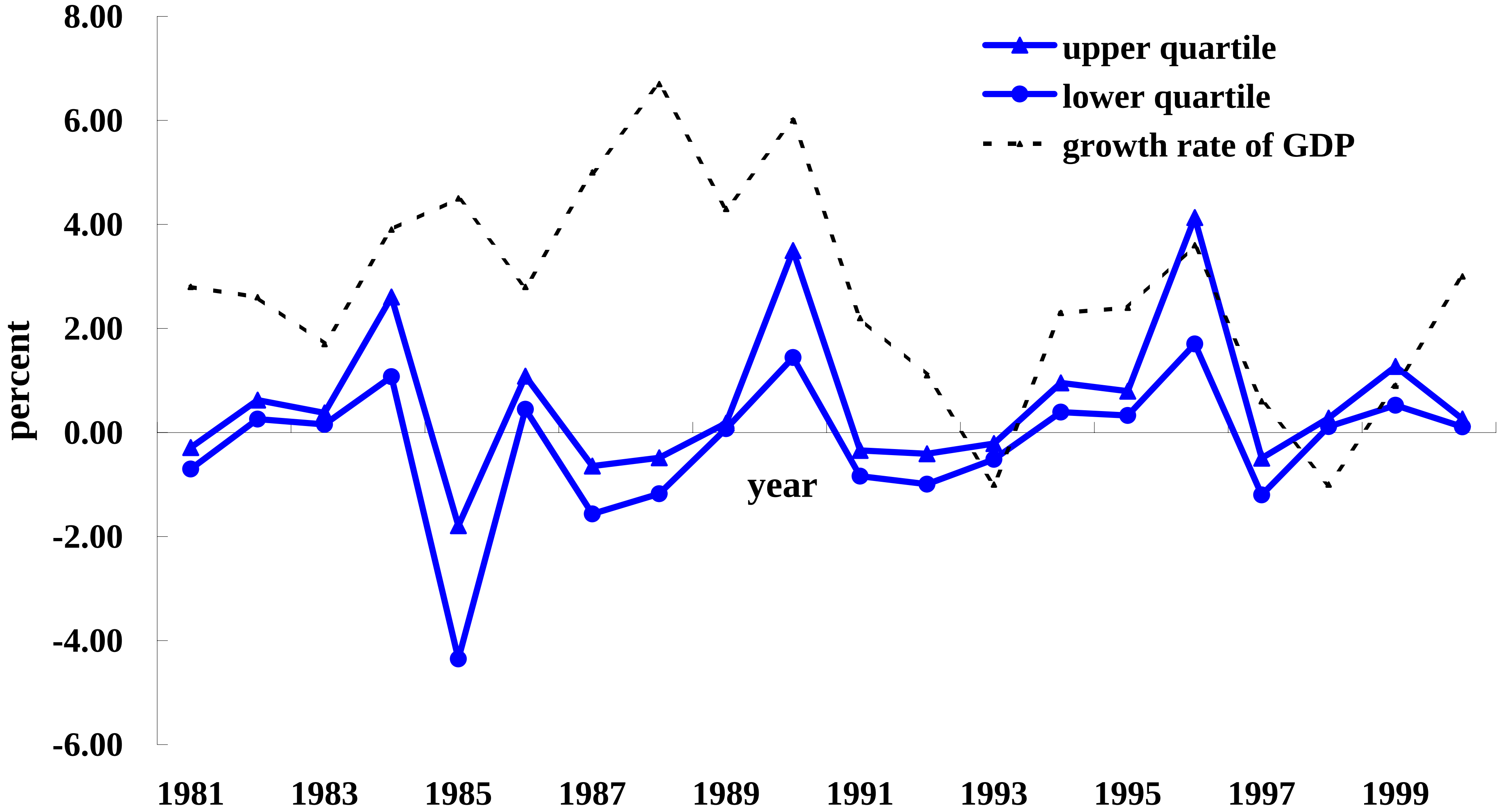

Fig. 4 The effects of Efficiency change on productivity

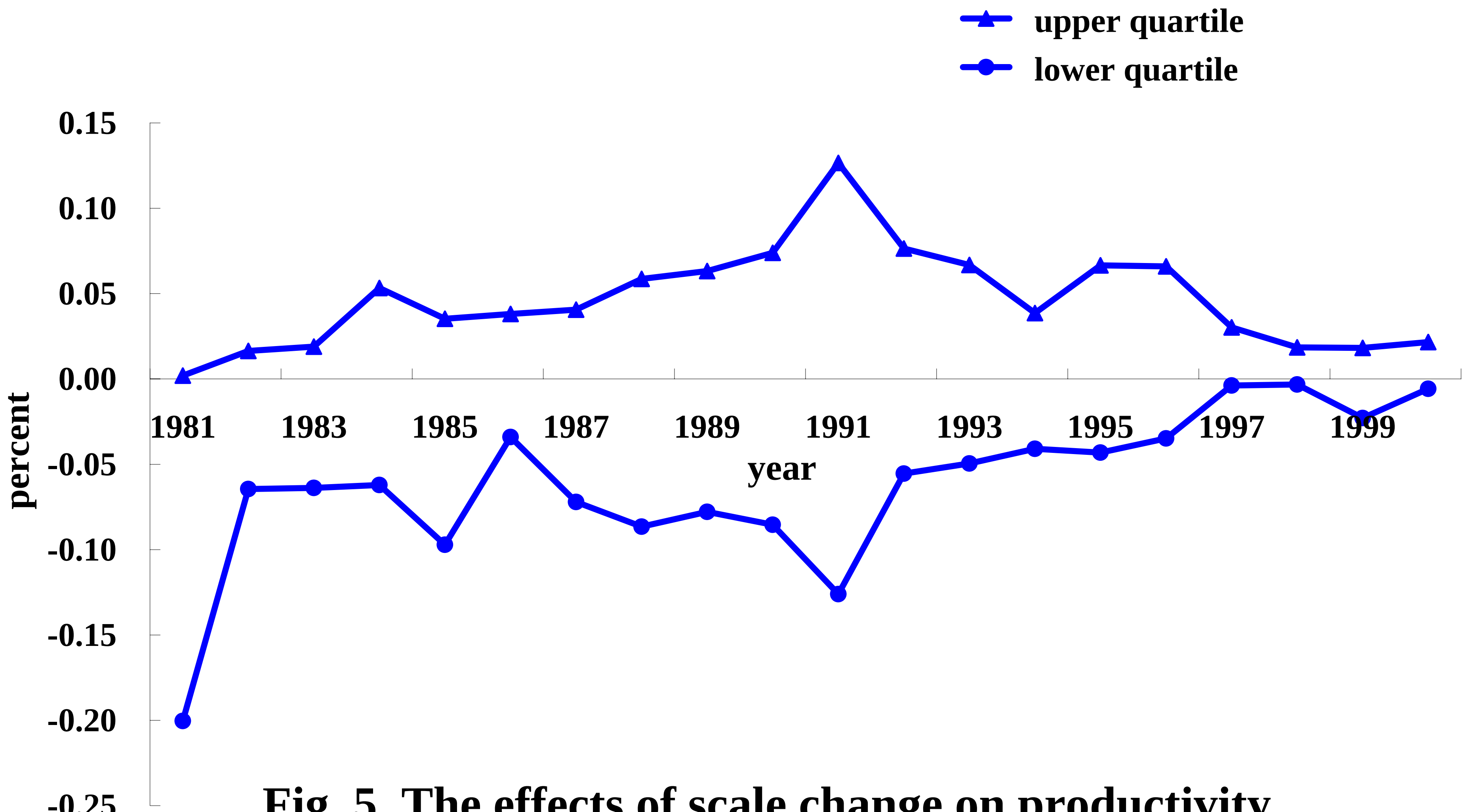

Fig. 5 The effects of scale change on productivity

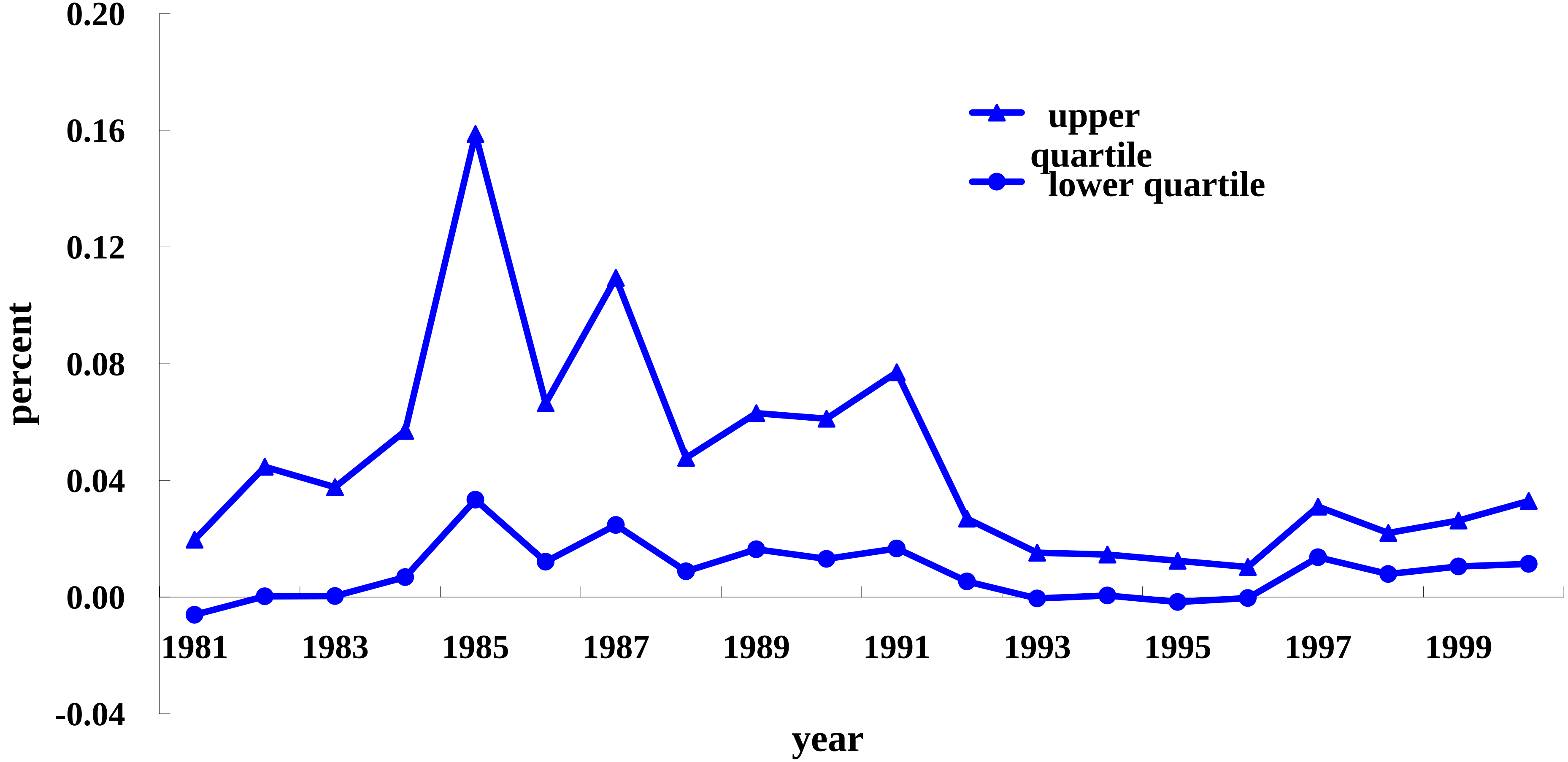

Fig. 6 The input and output mix effects on productivity