Optimal Taxation with Endogenous Insurance Markets[*]

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Abstract

In this paper, we study optimal tax policy in a dynamic private information economy. We describe efficient allocations and competitive equilibria. The standard assumption in the literature is that trades are observable by all agents. We show that in such an environment the competitive equilibrium is efficient and that government consumption can be financed by lump-sum taxation. We go on to consider an environment with unobservable trades in competitive markets. We show that efficient allocations have the property that the marginal product of capital is different from the market interest rate associated with unobservable trades. In any competitive equilibrium without taxation, the marginal product of capital and the market interest rate are equated, so that competitive equilibria are not efficient. Taxation of capital income can be welfare-improving because such taxation introduces a wedge between market interest rates and the marginal product of capital and allows agents to obtain better insurance in private markets. We use plausibly calibrated numerical examples to compute optimal taxes and welfare gains and compare results to an economy with a restricted set of tax instruments, and to an economy with observable trades.

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1 Introduction

The main question this paper addresses is how should optimal tax and social insurance policy be designed when private insurance can be provided by competitive markets. The presence of private insurance markets can significantly change policy prescriptions common to the literature on optimal taxation. In particular, we show that under assumptions common in the literature, the only effect of government policy is a crowding out of private insurance without improvement in welfare. We then move on to identify circumstances under which government policy can lead to better allocations than those that can be achieved by competitive markets alone and the specific tax instruments that can bring such a welfare improvement.

We answer this question about the design of optimal policy in a dynamic economy where workers receive unobservable skill shocks to labor supply and can trade assets privately. We study two polar extremes. In one extreme, individual asset trades are publicly observable. In this case, we show that the amount of insurance provided by private insurance companies is efficient and gives agents the correct incentives to supply labor. Any attempt on the part of the government to provide insurance only crowds out insurance provided by private insurers and leaves allocations unchanged. At the other extreme, asset trades are private information. We show that in this case, private insurance schemes do not lead to efficiency. Although each firm takes into account the effects of the hidden asset trades of its policy holders, it does not internalize the effect of these trades on the incentives to supply labor by agents insured by other firms. Because of this externality, we show that competitive equilibrium allocations can be improved upon by the government. This improvement can be done by introducing a wedge, much like a capital income tax, between the private rate of return on savings and the marginal product of capital.

Our paper builds on the literature of government policy in private information economies stemming from the seminal paper of Mirrlees (1971). Mirrlees showed that it is optimal to use distorting taxes to redistribute income across agents with unobservable skills. More closely related to our work are Green (1987) and Atkeson and Lucas (1992) who studied efficient allocations in dynamic, private information economies. However, rather than assuming that the market structure is given and characterizing the optimal provision of insurance by the government (as is done in much of the literature following Atkeson and Lucas (1992)[1]), we instead show that with

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[1] See, for example, Hopenhayn and Nicolini (1997), Werning (2002), Albanesi and Sleet

no exogenous restrictions on private markets there is no role for government policy. Indeed, if all trades are observable and agents can sign binding contracts with private insurers ex ante, the only effect of public provision of social insurance is the crowding out of private insurance. Thus, the welfare gain from introducing public insurance is zero in this setting. This result is reminiscent of that in Prescott and Townsend (1984) who show that the First Welfare Theorem holds in a large class of economies with private information. Private insurers have the same incentives as the government to insure agents, and hence, the equilibrium allocation is efficient.

When asset trades are not observable, however, private insurance is not sufficient for attaining efficiency. Efficiency requires that the interest rate at which agents can trade assets differs from the marginal rate of transformation. This improves the labor supply incentives of agents. We show that in a competitive equilibrium, competition among different insurers implies that the interest rates at which agents trade are equated to the marginal rates of transformation. Thus, privately provided insurance does not lead to efficient allocations in this setting.

The government can introduce a wedge between the interest rate and the marginal rate of transformation by using distorting taxes, an avenue not available to private insurers. We show that such policies are welfare improving. Indeed, combining linear taxes on capital income with non-linear labor income taxes allows the government to achieve the optimum.

It is worth noting that even in the environment with unobservable trades, government insurance can crowd out private insurance by changing nature of private insurance contracts. We show that the estimates of the size of welfare gains from changes in public policy that do not take into account private market responses can give very misleading results. Typically such welfare effects are smaller when private markets are endogenous. We study an application of our theory to a quantitative model of disability insurance to get some understanding of the magnitude of the crowding out effect. Our benchmark is efficient allocations with hidden trades. We consider the effect of complete elimination of the optimally-provided public insurance in two environments. In one environment private markets are exogenously restricted and the only form of insurance available to agents is trades of a risk-free bond. In the other environment we impose no restrictions on private markets. We find that the welfare losses from elimination of public insurance are 97% smaller in the economy where private markets are endogenous.

The papers most closely related to our analysis are those of Arnott and

(2003), Golosov and Tsyvinski (2003), Kocherlakota (2003b).

Stiglitz (1986, 1990) and Greenwald and Stiglitz (1986). They showed that in hidden action or moral hazard economies unobservable insurance purchases cause the competitive equilibria to be constrained inefficient. Bisin and Guaitoli (2004) and Bisin and Rampini (2003) further develop this idea. None of these papers consider hidden information environments, such as those with unobservable skills. More importantly, we concentrate on the effect of asset trading on the dynamic incentives, an aspect that was not considered in these earlier papers. Our paper is also related to the literature on mechanism design with unobservable savings (see, for example, Diamond and Mirrlees (1995), Cole and Kocherlakota (2001), Werning (2002), Kocherlakota (2003a)). In these papers, the authors assume that private insurance markets do not exist and the rate of return on savings is given. If private markets are unrestricted competitive equilibria are efficient in those environments. In contrast, we study an economy in which market interest rates are endogenously determined.

Our results are also related to Diamond and Dybvig's (1983) analysis of bank deposits as means of risk sharing. Jacklin (1987) pointed out that in their model risk sharing breaks down if agents are able to trade among themselves unobservably. The government can improve upon equilibrium allocations by restricting the deposit contracts that banks are allowed to offer and forbidding private investments. Similarly to our model, competitive equilibrium is not efficient in that set up because a contract offered by any bank affects incentives for the agents insured with other banks.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 considers the environment with observable trades and establishes the efficiency result. Section 4 analyzes the economy with unobservable trades. We propose a concept of efficiency in such an environment and provide a partial characterization of the efficient allocations. We show that competitive equilibrium is not efficient and that positive taxes on capital income are welfare improving. Finally, Section 5 presents numerical results.

2   Environment

We consider an economy that lasts for $T \leq \infty$ periods, denoted by $t = 1, ..., T$. In period 1 the economy is endowed with K_1 units of capital. The economy has a continuum of agents with a unit measure. Each agent's preferences are described by a time separable utility function over consumption

of a private good c_t, labor l_t, and consumption of a public good G_t as follows

$$\sum_{t=1}^{T} \beta^t (u(c_t, l_t) + u^g(G_t)).$$

In the above specification, β is a discount factor and $\beta \in (0, 1)$. The utility function is continuous, strictly increasing and strictly concave in consumption of both private and public goods, and decreasing in labor.

Agents are heterogenous and in each period they have idiosyncratic skills θ that belong to a finite distribution $\Theta = \{\theta(1), ..., \theta(N)\}$ where $\theta(1) < \theta(2) < ... < \theta(N)$. These skills evolve stochastically over time. Formally, in period one each agent gets an iid draw of a vector of skills for T periods from the distribution Θ^T with a common probability $\pi(\theta^T)$. The t-th component of θ^T is agent's skill in period t. The probability $\pi(\theta^T)$ is known but the specific realization of it is not. Each agent learns about his realization of θ^T over time. In period t, he knows only his skill realization for the first t periods $\theta^t = (\theta_1, ..., \theta_t)$. No other agent in the economy can observe it. We assume that the law of large numbers holds and in each period there are exactly $\pi(\theta^t)$ agents with the history of shocks θ^t. Our structure implies that there is no aggregate uncertainty.

An agent who supplies l units of labor and has a skill level θ produces $y = \theta l$ units of effective labor. The supply of labor is not observable. In the paper we will use an interpretation that although it is possible to observe how many hours an agent spends at his workplace, it is impossible to determine if he works or consumes leisure there. This interpretation implies that in checking incentive constraints we only need to consider the possibility that agents underreport their skill level.

Effective labor is observable and is a factor of production. Production in this economy is described by a function $F(K, Y)$, where K is the stock of capital, and Y is the aggregate level of effective labor $Y = \sum_{\theta} \pi(\theta) y(\theta)$. We assume that F is continuous, increasing in K and Y and exhibits constant returns to scale. The output can be divided into consumption of private and public goods, and investment.

An *allocation* is a vector $\{c_t, y_t, G_t, K_t\}_{t=1}^{T}$ where $c_t : \Theta^t \to \mathbb{R}_+$, $y_t : \Theta^t \to \mathbb{R}_+$, $K_t, G_t \in \mathbb{R}_+$. Here, $c(\theta^t)$ is private consumption of an agent with history θ^t; $y(\theta^t)$ is the amount of effective units that such a person supplies, and G_t and K_t are the level of public good and capital in period t. An allocation is *feasible* if in every period t it satisfies the feasibility constraint

given by

$$\sum_{\theta^t} \pi(\theta^t)c(\theta^t) + K_{t+1} + G_t \leq F(K_t, \sum_{\theta^t} \pi(\theta^t)y(\theta^t)). \qquad (1)$$

We say that a history θ^j *contains* θ^i for $j \geq i$ if the first i realizations of θ^j are θ^i and we denote it $\theta^i \in \theta^j$. We will also use a notation $c_t(\theta^T)$ which is equivalent to $c(\theta^t)$ for $\theta^t \in \theta^T$. The probability of history θ^{t+1} conditional of the realization of the history θ^t is denoted by $\pi(\theta^{t+1}|\theta^t)$.

3 Observable consumption

We first consider an environment with dynamically evolving private information where the consumption of each agent is publicly observable. This makes our setup similar to those considered in a large literature on efficiency and dynamic optimal contracting with private information including Green (1987), Atkeson and Lucas (1992), Phelan (1994), Golosov, Kocherlakota and Tsyvinski (2003). We first define an efficient allocation and show that in this environment private markets can provide optimal insurance. We go on to analyze the nature of taxes when, in addition to insurance, the government needs to finance its purchases.

Under our notion of efficient allocations, a social planner offers each agent a contract $\{c(\theta^t), y(\theta^t)\}_{t=1}^T$, where $c(\theta^t)$ and $y(\theta^t)$ are the functions of the agent's reported type. Each agent chooses a reporting strategy σ, which is a mapping $\sigma : \Theta^T \to \Theta^T$. We denote the set of all such reporting strategies by Σ. An agent who chooses to report $\sigma(\theta^t)$ after history θ^t provides $y(\sigma(\theta^t))$ units of effective labor and receives $c(\sigma(\theta^t))$ units of consumption from the planner.

This setup has two interpretations. One interpretation is that the planner controls the consumption of an agent directly, and an agent consumes goods that the planner allocates to him. Under the other interpretation, an agent is able to enter observable contractual agreements with other agents and trade various assets with them. The consumption allocations that the social planner allocates can be conditioned on these trades. Since we impose no restrictions on the allocations, the social planner can make any additional contractual agreements unappealing to the agent such that he consumes $c(\sigma(\theta^t))$ after each history θ^t.

The expected utility of the agent who is offered a contract $\{c_t, y_t\}_{t=1}^T$

and chooses a strategy σ is denoted by $W(c, y)(\sigma)$ and given by

$$W(c, y)(\sigma) = \sum_{t=1}^{T} \beta^t \sum_{\theta^t} \pi(\theta^t) u(c_t(\sigma(\theta^t)), y_t(\sigma(\theta^t))/\theta_t).$$

The strategy σ^* is *truth-telling* if an agent reveals his type truthfully after any history: $\sigma^*(\theta^t) = \theta^t$ for all t. The allocation is called *incentive compatible* if the truth telling strategy yields a higher utility than any other strategy

$$W(c, y)(\sigma^*) \geq W(c, y)(\sigma) \text{ for any } \sigma \in \Sigma.$$

An allocation $\{c_t, y_t, K_t, G_t\}_{t=1}^{T}$ is *efficient* if it solves the following planner's problem

$$\max_{c,y,K,G} \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) \beta^t \{u(c(\theta^t), y(\theta^t)/\theta_t) + u^g(G_t)\}$$

s.t

$$W(c, y)(\sigma^*) \geq W(c, y)(\sigma) \text{ for any } \sigma \in \Sigma, \tag{2}$$

$$\sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} + G_t \leq F(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)) \text{ for all } t.$$

The above program states that the planner maximizes the expected utility of an agent subject to the incentive compatibility constraint and to the feasibility constraint.

3.1 Competitive equilibrium

In this subsection, we define a competitive equilibrium for the economy with observable consumption described above when $G_t = 0$ for all t. Consider an economy populated by ex-ante identical agents each of whom is endowed with the same initial capital k_1, so that the aggregate capital stock is K_1. There is a large number of firms with the identical production technology $F(K, Y)$. We assume throughout the paper that all activities at a firm level are observable. All firms are owned equally by all agents. In the beginning of period 1, before any realization of uncertainty, each firm signs a contract $\{c_t, y_t\}_{t=1}^{T}$ with a large number of workers and purchases the initial capital stock k_1 from them. We interpret c_t as the actual consumption of the agent. Such a contract is feasible since consumption and all transactions of agents

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are observable. The price paid for the initial capital is included in the contract[2]. The contracts are offered competitively and workers sign a contract with the firm that promises the highest ex-ante expected utility. We denote the equilibrium utility by $\underline{U}$.

After the contract is signed the worker chooses a reporting strategy σ, supplies $y(\sigma(\theta^t))$ effective labor and receives $c(\sigma(\theta^t))$ units of consumption when his history is θ^t. The agents do not participate in any markets.

Each firm makes new investments k_t for $t > 1$, pays dividends d_t to its owners, and trade bonds with other firms. The price of a bond b_t in period t that pays 1 unit of consumption good in period $t+1$ is q_t. All firms take these prices as given. We consider equilibria where all firms are identical, and we study a problem of a representative firm.

The maximization problem of the firm that faces intertemporal prices q_t and the reservation utility $\underline{U}$ for workers is

$$\max_{c,k,d,y} d_1 + q_1 d_2 + ... + \prod_{i=1}^{T-1} q_i d_T$$

s.t.

$$\sum_{\theta^t} \pi(\theta^t) c(\theta^t) + k_{t+1} + d_t + q_t b_{t+1} \leq F\left(k_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)\right) + b_t,$$

$$W(c,y)(\sigma^*) \geq W(c,y)(\sigma) \text{ for any } \sigma, \tag{3}$$

$$W(c,y)(\sigma^*) \geq \underline{U}.$$

In equilibrium, competition among firms forces them to have zero profits. We now define a competitive equilibrium.

Definition 1 *A competitive equilibrium is a set of allocations $\{c_t, y_t, k_t\}$, prices q_t, dividends d_t, bond trades b_t and utility $\underline{U}$ such that*
(i) Firms choose $\{c_t, y_t, d_t, k_t, b_t\}_{t=1}^T$ to solve firm's problem taking q_t, $\underline{U}$ as given;
(ii) Consumers choose the contract that offers them the highest ex-ante utility;
(iii) Firms make zero profits;
(iv) The aggregate feasibility constraint (1) holds.

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[2] Alternatively, we could assume that firms rent capital from workers. Then the contract would also specify the amount of savings of each agent. Our results are the same in this case.

It is easy to show that in equilibrium the prices are $1/q_t = F_k(k_{t+1}, Y_{t+1})$ and $d_t = 0$ for all t.

This construction closely follows Atkeson and Lucas (1992). One can easily see that in equilibrium the representative firm's problem is dual to the social planner's problem and hence gives the same allocations. This result allows us to prove that competitive equilibrium is efficient in the absence of public goods.

Theorem 1 *Suppose in the efficient allocations $G_t = 0$ for all t. Then the competitive equilibrium is efficient.*

Proof. Suppose the competitive equilibrium is not efficient. Consider an optimal allocation $\{c_t^{sp}, y_t^{sp}, K_t^{sp}\}_{t=1}^T$ with utility level U^*. Such an allocation is feasible for the firm, satisfies incentive compatibility and delivers workers a utility U^* which is strictly higher than the equilibrium utility $\underline{U}$. This allocation also delivers zero profit for the firm, as in the candidate competitive equilibrium allocations. By standard arguments it is possible for the firm to offer another contract $\tilde{U}$, $U^* > \tilde{U} > \underline{U}$ with strictly less resources. It is possible to do so by reducing the consumption of the agent with the lowest skill realization in the first period by ε. This deviation preserves the incentive compatibility, delivers the utility $\tilde{U}$ and firms have strictly positive profits ε. We have a contradiction. ■

This theorem shows that ex-ante insurance can be provided privately in competitive settings. There is no need for public intervention. This result is in contrast with substantial literature[3] that analyses how the government can provide insurance optimally in these environments. Our analysis suggests that the only result of such provision is crowding out of the private insurance. Suppose, for example, that the government introduces a lump sum redistribution between agents $T(\theta^t)$ where in each period $\sum_{\theta^t} \pi(\theta^t)T(\theta^t) = 0$. Such taxes leave the after tax allocations unchanged - firms adjust their contracts optimally so that the new payments to the workers $\{\hat{c}_t\}_{t=1}^T$ reflect the taxes: $\hat{c}(\theta^t) = c(\theta^t) + T(\theta^t)$. The higher level of insurance provided by the government is exactly offset by less insurance available through private markets.

Since agents utility functions are strictly concave, efficient allocations involve redistribution of resources from agents with high productivity to agents with low productivity. This redistribution introduces wedges, or inequalities, between the marginal utilities of consumption and leisure, and between

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[3] See, for example, Green (1987), Werning (2001), Golosov and Tsyvinski (2003), Albanesi and Sleet (2003), and Kocherlakota (2003b).

the marginal utility of consumption and the marginal product of labor. Mirrlees (1971, 1976) used the result that, in the optimum, it is typically true that $u_c(c,l) \neq u_l(c,l)/w$ to provide a justification for the government's use of distorting labor taxes to achieve efficient allocations in the competitive equilibrium settings.[4] Golosov, Kocherlakota and Tsyvinski (2003) showed that, in dynamic settings, there is also an intertemporal wedge since the optimal allocations satisfy (assuming separable utility) for all t, θ^t

$$\frac{u_c(c(\theta^t))}{\beta E_{\theta^t} u_c(c(\theta^{t+1}))} < F_k(K_{t+1}, Y_{t+1}).$$

Golosov and Tsyvinski (2003), Albanesi and Sleet (2003), and Kocherlakota (2003b) used this result to study optimal implementation via taxes. The need for such taxes or wedges arises since one of the objectives of the planner is to provide insurance to the agents against idiosyncratic uncertainty. The implicit assumption in these papers is that the government is the only provider of such insurance available to the agents.

In the absence of governmental policy, firms and agents can write contracts that provide agents with insurance. The availability of such contracts implies that the arrangement in which firms pay the worker the marginal product of his labor in each period may be suboptimal. Since the agents are risk averse, they have incentives to enter into contracts with firms before the realization of uncertainty that provides them with some insurance against low skill shocks. In our economy markets for insurance arise endogenously to supplement or substitute for government insurance.

3.1.1   Provision of public goods

Consider next an economy with public goods. The competitive equilibrium is no longer efficient if there is a need to finance government expenditures such as positive consumption of a public good G. However, these public expenditures can be financed in non-distorting ways. Suppose $\{G_t^{sp}\}_{t=1}^T$ is the optimal amount of public good. One can easily see that competitive equilibrium with lump sum taxes $T_t = G_t^{sp}$ is efficient. Indeed suppose each agent is required to pay some lump sum tax T_t. The incentive constraint in the firm's problem (3) becomes

$$W(c - T, y)(\sigma^*) \geq W(c - T, y)(\sigma) \text{ for any } \sigma.$$

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[4] See also Diamond (2003) for an application of these ideas to the analysis of the design of the social security system.

The firm's allocation of consumption $\{c^f(\theta^t)\}_{t=1}^T$ reflects the lump sum taxes that consumers pay. One can see that the allocation $\{c^f(\theta^t), y^f(\theta^t)\}_{t=1}^T$, where $c^f(\theta^t) = c^{sp}(\theta^t) + T(\theta^t)$, $y^f(\theta^t) = y^{sp}(\theta^t)$ is feasible and allows to finance the public good G_t. Thus the competitive equilibrium with the lump sum taxes T_t is efficient.

Proposition 1 *In an economy with no trades among agents (observable consumption) it is optimal to finance a public good G_t with lump sum taxes $T_t = G_t$ in all t.*

The presence of private information is often cited as an explanation why non-distorting taxes are not feasible. In this economy they are not only feasible but also optimal. The equilibrium insurance offered by the firms takes into account the taxes that agents have to pay, and this insurance is provided optimally.

This analysis suggests that optimal allocations can be achieved without distorting government interventions. It abstracts from many possible sources of inefficiencies. For example, by allowing agents to sign insurance contracts before any realization of uncertainty we abstract from issues arising because of adverse selection. We also assume that the contracts are binding and neither the employer nor the agent can renege on them. However, the analysis above suggests that the existence of private information about individual's skills per se does not justify distorting taxes by the government.

4 Unobservable trades

In the previous section we have shown that competitive equilibria are efficient if all trades are observable. In such an environment each firm has the power to be the sole insurer for any agent and can preclude agents from trading with other firms and agents. This power to preclude requires that the firm possesses full information about agents' consumption and assets. In this section we relax the assumption of full observability of trades. We still maintain the assumption that an agent's effective labor y is publicly observable. The agent can, however, trade assets and consume unobservably. We assume that T is finite and describe the extension to the infinite period economy later.

4.1 Retrading market

Consider an environment where all agents have access to a market in which they can trade assets unobservably. We call this market a *retrading market.*

In this market agents trade a security called a risk free bond. A purchases of this security entitles the holder to one unit of consumption in the following period. In the appendix we show that in a large class of environments it is the only security traded in equilibrium.

All trades at period t occur at prices Q_t. The prices are such that the market for bonds clears each period. We assume that all trades are enforceable so that agents cannot default on their liabilities. This assumption precludes agents from borrowing more than they can ever repay in the future.

A social planner offers a contact $\{c(\theta^t), y(\theta^t)\}_{t=1}^T$ to all agents, where y_t is the amount of effective labor that the agent has to provide, and c_t is the endowment of consumption goods that agent receives. Unlike the environment described in the previous section, the amount of consumption goods allocated by the planner is not necessarily equal to the actual consumption of an agent, since the planner has no possibility to preclude an agent from borrowing and lending on the retrading market.

An agent takes the contract offered by the planner and the equilibrium prices $\{Q_t\}_{t=1}^T$ as given and chooses his optimal reporting strategy σ together with holdings (possibly negative) of a risk free security $s_t : \Theta^t \to \mathbb{R}_+$. Total resources available to the agent are the endowment of consumption good $c(\sigma(\theta^t))$ he receives from the planner and his asset holding from the previous period. The actual consumption after retrading is $x_t : \Theta^t \to \mathbb{R}_+$.

The agent maximizes his ex-ante utility:

Agent's Problem

$$\max_{\sigma,x,s} \sum_{t=1}^T \beta^{t-1} \sum_{\theta^t} \pi(\theta^t) u(x(\sigma(\theta^t)), y(\sigma(\theta^t))/\theta_t)$$

s.t. for all θ^t, t

$$x(\sigma(\theta^t)) + Q_t s(\sigma(\theta^t)) = c(\sigma(\theta^t)) + s(\sigma(\theta^{t-1})),$$

$$s(\theta^0) = 0,$$

where $s(\theta^0)$ are the initial asset holdings of the agent before realizations of the shocks.

We denote the value of this problem at the optimum by $V(\{c, y\}, \{Q\})$. Sometimes we will need to compute a value for an arbitrary reporting strategy σ and we denote ex ante utility from following this strategy by $V(\{c, y\}, \{Q\})(\sigma)$.

Equilibrium in the retrading market requires that in each period the total endowment of consumption goods should be equal to the total after

trade consumption:

$$\sum_{\theta^t} \pi(\theta^t) x(\sigma(\theta^t)) = \sum_{\theta^t} \pi(\theta^t) c(\sigma(\theta^t)). \tag{4}$$

We can define equilibrium in the retrading market.

Definition 2 *An equilibrium in the retrading market given the contract* $\{c(\theta^t), y(\theta^t)\}_{t=1}^T$ *consists of prices* Q*, strategies* σ *and allocations* $\{x(\theta^t), s(\theta^t)\}_{t=1}^T$ *such that*
(i) Consumers solve Agent's Problem taking $\{c(\theta^t), y(\theta^t), Q_t\}_{t=1}^T$ *as given;*
(ii) Feasibility constraint on the retrading market (4) is satisfied.

We assume that for any contract $\{c_t, y_t\}_{t=1}^T$ that the social planner offers there exists a unique equilibrium. The ex-ante utility of an agent in the equilibrium in the retrading market is denoted $\hat{V}(\{c, y\})$.

4.2 Efficiency with unobservable trades

The social planner chooses the allocations $\{c_t, y_t, K_t, G_t\}_{t=1}^T$ that maximize the ex ante utility of agents. Using the revelation principle it is easy to show that the social planner offers a contract $\{c_t, y_t\}_{t=1}^T$ so that all agents choose to report their type truthfully to the planner and do not trade on the retrading market.[5]

Definition 3 *An efficient allocation* $\{c_t^{sp}, y_t^{sp}, G_t^{sp}, K_t^{sp}\}_{t=1}^T$ *is the solution to the following Social Planner problem*

$$\max_{c,y,G,K} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t \{u(c(\theta^t), y(\theta^t)/\theta_t) + u^g(G_t)\}$$

s.t. for all t, θ^t

$$\sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} + G_t \leq F(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)),$$

$$\sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t u(c(\theta^t), y(\theta^t)/\theta_t) \geq \hat{V}(\{c, y\}). \tag{5}$$

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[5] The retrading market here is a constraint on the social planner's problem. The idea is similar to that in Hammond (1987) who studied a static environment with multiple goods and side markets where agents can trade unobservably. He showed that for any incentive compatible allocation, side markets must be in a Walrasian equilibrium. Guesnerie (1998) used that set up to study optimal taxation.

The market for hidden trades imposes a stricter constraint (5) than the incentive constraint with observable asset trades (2). We show this by first showing that any allocation that satisfies (5) also satisfies (2). Consider an allocation $\{c(\theta^t), y(\theta^t)\}_{t=1}^T$ that satisfies (5). Suppose there exists some reporting strategy σ for which the incentive constraint (2) is violated. Consider the same strategy σ on the market with hidden trades. The allocation $\{c(\sigma(\theta^t)), y(\sigma(\theta^t))\}_{t=1}^T$ is feasible for the agent, and he can further improve upon it by trading bonds. Therefore the strategy σ also violates the constraint imposed by the market for hidden trades (5).

The reverse relationship does not hold in general - it is typically not true that an allocation that satisfies (2) also satisfies (5). First, it is known that agents' marginal rates of substitution (MRS) defined by

$$\frac{u_c(c(\theta^t), y(\theta^t)/\theta_t)}{\beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u_c(c(\theta^{t+1}), y(\theta^{t+1})/\theta_{t+1})}$$

differ for different histories θ^t under efficient allocations with observable trades[6]. On the opposite, in the environment with hidden trades agents' MRS are necessarily equated. For any reporting strategy σ the allocations $\{x(\sigma(\theta^t)), s(\sigma(\theta^t))\}_{t=1}^T$ that an agent chooses on the retrading market must satisfy the following conditions for all θ^t

$$x(\sigma(\theta^t)) + Q_t s(\sigma(\theta^t)) = c(\sigma(\theta^t)) + s(\sigma(\theta^{t-1})), \tag{6}$$

$$Q_t u'(x(\sigma(\theta^t))) = \beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(x(\sigma(\theta^{t+1}))), \tag{7}$$

$$s(\theta^T) = 0. \tag{8}$$

Condition (7) implies that agents equalize their MRS in each period for all histories θ^t.

Another difference between the two environments is a possibility for agents to use a *double deviation* - agents choose not only a deviating reporting strategy but also hidden asset trades that maximize the utility of the deviation. The possibility of such deviations implies that even if agents' MRS were equalized for an allocation that satisfies (2), such an allocation would not necessarily satisfy (5).

To illustrate this point we rewrite social planner's problem.

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[6]See for example Golosov, Kocherlakota and Tsyvinski (2003).

Lemma 1 *An efficient allocation $\{c_t^{sp}, y_t^{sp}, G_t^{sp}, K_t^{sp}\}_{t=1}^T$ together with the corresponding equilibrium prices on the retrading market $\{Q_t\}_{t=1}^T$ is a solution to the problem*

$$\max_{c,y,G,K,Q} \sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t \{u(c(\theta^t), y(\theta^t)/\theta_t) + u^g(G_t)\}$$

s.t. for all t, θ^t

$$\sum_{\theta^t} \pi(\theta^t) c(\theta^t) + K_{t+1} + G_t \leq F(K_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)),$$

$$\sum_{t=1}^T \sum_{\theta^t} \pi(\theta^t) \beta^t u(c(\theta^t), y(\theta^t)/\theta_t) \geq V(\{c,y\}, \{Q\})(\sigma) \text{ for any } \sigma, \quad (9)$$

$$Q_t u_c(c(\theta^t), y(\theta^t)/\theta_t) = \beta \sum \pi(\theta^{t+1}|\theta^t) u_c(c(\theta^{t+1}), y(\theta^{t+1})/\theta_{t+1}). \quad (10)$$

Proof. We show that any allocation satisfying (5) also satisfies (9) and (10), and vice versa.

Suppose $\{c_t, y_t\}_{t=1}^T$ satisfies (5) and the equilibrium prices on the retrading market are $\{Q_t\}_{t=1}^T$. Then the Euler equation (10) is satisfied otherwise the truth telling agent can improve his utility along some history and (5) would not hold. Similarly (9) is also satisfied. Otherwise, if it did not hold for some strategy $\sigma' \neq \sigma^*$, this strategy would also be optimal on the retrading market and the original allocation would not be incentive compatible.

Suppose $\{c_t, y_t, Q_t\}_{t=1}^T$ satisfies (9) and (10). We need to show that on the retrading market in equilibrium agents choose to reveal their types truthfully, do not trade and consume their consumption allocations $c(\sigma^*)$ and the equilibrium interest rates are equal to Q. An agent who faces prices Q chooses the truthful revelation because of (9). The Euler equation (10) guarantees that the agent optimally chooses to buy no bonds along this truth telling path. That implies that the feasibility condition on the retrading market (4) is satisfied and Q are indeed the equilibrium prices. ∎

In this problem the social planner chooses the prices Q on the retrading market directly. Although the planner has no control over transactions on that market, he has enough power to determine these prices. By the revelation principle, a social planner chooses allocations such that each agent reveals his type truthfully and never re-trades from the allocations he receives. The truth telling agent does not retrade if his marginal rate of substitution

for consumption between the periods t and $t+1$ is exactly equal to the interest rate. In other words, these intertemporal rates of substitution determine the prices of risk free bonds. The incentive constraint should ensure that a deviating agent cannot achieve a higher utility by retrading at those prices.

The possibility of trading assets and using double deviations implies that constraint (9) is stricter than the incentive constraint (2). For any strategy σ the allocation $\{c(\sigma(\theta^t)), y(\sigma(\theta^t))\}_{t=1}^T$ is feasible, but the agent can further improve upon it using hidden trades.

Although the economy with unobservable retrading typically has lower welfare than the economy with observable trades, we can identify one situation in which the allocations and welfare in both economies are the same. It is the economy analyzed extensively in Werning (2001) where all the uncertainty about skill shocks is realized after the first period.

Proposition 2 *Suppose that all uncertainty is realized after the first period, so that in each period t for each history θ^t there exists some history θ^{t+1} such that $\pi(\theta^{t+1}|\theta^t) = 1$. Suppose that the utility function is separable between consumption and leisure. Then the efficient allocations in the economy with and without observable trades are the same.*

Proof. Let θ be the skill shock in the first period. Since all uncertainty is realized after the first period, it determines the future path of skills. It is a well known result from Werning (2001) and Golosov, Kocherlakota and Tsyvinski (2003) that when trades are observable, the optimal allocations satisfy for all θ, t

$$u'(c_t(\theta)) = F_k(K_{t+1}, Y_{t+1})\beta u'(c_{t+1}(\theta)).$$

We show now that these allocations are also feasible in the economy with unobservable retrading. Suppose prices on the retrading market are $Q_t = 1/F_k(K_{t+1}, Y_{t+1})$. Consider an agent who sends an arbitrary report $\sigma(\theta)$ about his first period skill. Since all uncertainty is realized after the first period, in all the following periods the agent receives the allocations $\{c_t(\sigma(\theta)), y_t(\sigma(\theta))\}_{t=1}^T$ that depend only in his report in the first period. Since allocations received from planner satisfy the Euler equation, it is optimal for the agent to consume these allocations without any additional trades: $x_t(\sigma(\theta)) = c_t(\sigma(\theta))$ for all t. Hence efficient allocations in the economy with observable trades are still incentive compatible if there are hidden retrading markets. It remains to verify that the constructed Q_t's are indeed the equilibrium prices. Since with such prices for all t, θ the following equality holds

$$x_t(\theta) = c_t(\theta),$$

the feasibility constraint (4) is satisfied. ∎

When all uncertainty is realized in the first period, there is no longer any gain from double deviations. Any asset trading occurs after agents have revealed their type to the planner. The possibility of hidden trade does not improve the value of any deviation and the incentive constraints in the two economies become identical.

Our economy differs from standard problems with unobservable savings such as Diamond and Mirrlees (1995), Werning (2002), Doepke and Townsend (2003), and Abraham and Pavoni (2003) where the rate of return on hidden trades is assumed to be exogenous. In our environment the social planner can choose the rate of return by choosing allocations $\{c_t, y_t\}_{t=1}^T$. This additional instrument is important for the planner because it allows to affect the return from double deviations. We show below that since competitive environments typically lack this instrument, competitive equilibria are not efficient. This result is different from the environments with the exogenous rate of return in which competitive equilibria are efficient.

4.3 Competitive equilibrium

In this subsection, we consider a decentralized version of this private information economy with unobservable trades. As in the section on the economy with observable trades, we assume that before any uncertainty is realized an agent signs a long term contract with a firm which is binding for both parties. The environment is identical to the one described in Section 3, but now firms need to take into account that agents are able to retrade their allocations on the retrading market.

The retrading market is identical to the one in the social planner's problem. Every agent who has a contract $\{c_t, y_t\}_{t=1}^T$ with a firm chooses his reporting strategy and asset trades optimally taking prices Q_t for the risk free bond on the retrading market as given. Prices Q_t are the equilibrium prices on the retrading market. For the retrading market to be identical to that in the social planner's problem, we do not allow agents to trade with other firms directly. However, we will show below that allowing such trades does not affect competitive equilibria.

The contracts offered by firms take into account the possibility that agents may retrade. Firms may choose to provide such allocations that agents retrade from them along the truth telling path. The incentive constraint for the firm has the form $V(\{c, y\}, \{Q\})(\sigma^*) \geq V(\{c, y\}, \{Q\})(\sigma)$ for any $\sigma \in \Sigma$.

The problem of the representative firm is similar to the problem de-

scribed in Section 3.1. Each firm is a price taker, it chooses a contract offered to workers $\{c_t, y_t\}_{t=1}^T$, investments k_t, dividends d_t and bond trades b_t to maximize profits.

Firm's Problem 1

$$\max_{c,k,d,y} d_1 + q_1 d_2 + \ldots + \prod_{i=1}^T q_i d_T$$

s.t. for all t

$$\sum_{\theta^t} \pi(\theta^t) c(\theta^t) + k_{t+1} + d_t + q_t b_{t+1} \leq F(k_t, \sum_{\theta^t} \pi(\theta^t) y(\theta^t)) + b_t,$$

$$V(\{c, y\}, \{Q\})(\sigma^*) \geq V(\{c, y\}, \{Q\})(\sigma) \text{ for any } \sigma, \qquad (11)$$

$$V(\{c, y\}, \{Q\})(\sigma^*) \geq \underline{U}.$$

The first constraint in the firm's problem is feasibility. The second is the incentive compatibility. The last constraint says that the firm cannot offer a contract which delivers a lower expected utility than the equilibrium utility $\underline{U}$ from contracts offered by other firms. In equilibrium all firms act identically and make zero profits.

The firm's problem in this economy is very similar to the firm's problem in the economy with observable trades. The only difference comes from the fact that the incentive constraint (11) now has to take into account side trades that are not observable. The definition of the competitive equilibrium is parallel to that in the economy with observable trades.

Definition 4 *A competitive equilibrium is a set of allocations $\{c_t, y_t, k_t\}$, prices q_t, dividends d_t, bond trades b_t, utility $\underline{U}$ and prices Q_t such that*

(i) Firms choose $\{c_t, y_t, d_t, k_t\}_{t=1}^T$ to solve Firm's problem taking q_t, $\underline{U}$ as given;

(ii) Consumers choose the contract that offers them the highest ex-ante utility;

(iii) For any $\{c_t, y_t, Q_t\}_{t=1}^T$ agents choose their reporting strategy and asset trades optimally as described in Agent's Problem;

(iv) Firms make zero profits;

(v) The aggregate feasibility constraint (1) holds;

(vi) The retrading market for the contract $\{c_t, y_t\}_{t=1}^T$ is in equilibrium and Q_t are the equilibrium prices.

It is easy to see that the interest rates in the economy must be equal to the marginal product of capital, so that $1/q_{t-1} = F_k(K_t, Y_t)$ for all t. The prices that firms and agents face are also equalized, $q_t = Q_t$ for all t. Suppose it were not true, so that for example $1/Q_1 < F_k(K_2, Y_2)$. It is optimal for all firms to postpone any payments of the first period wages until the second period. Workers are able to borrow at the interest rates Q_1 and repay from the wages they make in the second. But since all the firms are identical, they all choose to pay no wages in the first period, and then Q_1 can not be the equilibrium interest rate. In other words, if $q_t \neq Q_t$ firms can use agent's ability to borrow and lend at rate Q_t to create arbitrage opportunities. We can summarize this result in the following proposition.

Proposition 3 *In the competitive equilibrium $1/Q_t = F_k(K_{t+1}, Y_{t+1})$ for all t.*

This result suggests that competitive equilibria typically are *not* efficient when asset trades are unobservable. From the maximization problem described in lemma 1, the social planner has the power to choose the interest rates on the retrading market $1/Q$ and usually these interest rates are different from $F_k(K, Y)$. It is difficult to characterize the efficient allocations in this general set up, and we will consider them for two special cases in the next sections.

Although the competitive equilibrium is not efficient it is generally not true that no insurance is provided by firms. In the numerical section below we show that this privately provided insurance can be very significant. This finding stands in contrast with the environments where agent's endowment is not observable such as environments studied in Allen (1985) and Cole and Kocherlakota (2001). There, no insurance is possible when agents can borrow and lend at the rate equal to F_k. When agent's effective labor y is observed, insurance can be provided despite the possibilities of hidden trades.

4.4 Efficient allocations with iid shocks and separable utility

To simplify the analysis we assume that the utility function is separable between consumption and leisure.

Assumption. $u(c, l) = u(c) + v(l)$.

In addition we assume that the skill shocks follow an iid process.

Assumption. $\pi(\theta^t) = \pi(\theta_t) = \pi(\theta)$ for $\theta = \theta_t$ for all θ^t.

We showed that any equilibrium allocation in the retrading market satisfies conditions (6), (7) and (8). When θ is iid, the Euler equation (10)

becomes

$$Q_t u'(c(\sigma(\theta^t))) = \beta \sum_\theta \pi(\theta) u'(c(\sigma(\theta^t), \theta)), \qquad (12)$$

where $c(\sigma(\theta^t), \theta)$ denotes the allocation to the agent who sent report $\sigma(\theta^t)$ in period t and revealed his realization of the shock in period $t+1$ truthfully.

We also assume that consumption allocations are monotonic so that agents who report higher types receive weakly higher consumption. This assumption appears to be very weak and it holds in all the numerical experiments we conducted.

Assumption (monotonicity). For any θ^t, and any θ', θ'' such that $\theta'' > \theta'$ it is optimal for the planner to choose consumption allocations such that $c(\theta^t, \theta'') \geq c(\theta^t, \theta')$.

We prove that in the efficient allocations the interest rates on the retrading market are lower than F_k, which formally stated in the following proposition.

Proposition 4 *In the efficient allocations $F_k(k_t, Y_t) > 1/Q_{t-1}$ for at least one t.*

To prove this result we first present a sequence of lemmas. We show that *any* deviating strategy $\sigma \neq \sigma^*$ involves positive saving after some history, and never borrowing. This result implies that the planner would want to decrease the return on deviations by lowering the interest rates on the retrading market.

Consider the optimal asset trades and consumption on the retrading market $\{x(\sigma(\theta^t)), s(\sigma(\theta^t))\}_{t=1}^T$ for a given strategy σ. They must satisfy (6), (7) and (8).

Lemma 2 *For any strategy σ consider the allocation $\{x_t, s_t\}_{t=1}^T$ that satisfies (6), (7) and (8). This allocation must satisfy*

$$\frac{\sum_\theta \pi(\theta) u'(x(\sigma(\theta^t, \theta)))}{u'(x(\sigma(\theta^t)))} \leq \frac{\sum_\theta \pi(\theta) u'(c(\sigma(\theta^t, \theta)))}{u'(c(\sigma(\theta^t)))} \qquad (13)$$

for all θ^t.

Proof. By the monotonicity assumption and the assumption that the only possible deviations are those in which an agent reports a lower type, it must be true that $c(\sigma(\theta^t), \theta) \geq c(\sigma(\theta^t), \theta)$ for all θ^t, θ, σ. Here we use a notation $\sigma(\theta^t, \theta)$ to denote a report of the agent after history (θ^t, θ) who uses strategy σ.

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Equation (12) implies then that for any θ^t

$$Q_t u'(c(\sigma(\theta^t))) \le \beta \sum_\theta \pi(\theta) u'(c(\sigma(\theta^t, \theta))).$$

Combining this inequality with (7) we obtain the lemma. ∎

The intuition for this result is simple. The marginal rate of substitution of a truth telling agent is equal to the price of a risk free bond. When an agent reports a lower type, he gets lower consumption allocations. When shocks are iid that implies that consuming these allocations without any additional asset trading increases agent's MRS above the bond price Q, since fewer resources are available in the next period. However it is optimal for the agent to retrade his consumption allocations to equalize his MRS with bond prices. This implies inequality (13).

Since future deviations imply fewer resources, it is optimal for the agent to save in the anticipation of those deviations, and borrowing is always suboptimal. The following lemmas formalize this intuition.

Lemma 3 *For any strategy σ consider the allocation $\{x_t, s_t\}_{t=1}^T$ that satisfies (6), (7) and (8). Suppose $s(\sigma(\theta^t)) < 0$ for some θ^t. Then $x(\sigma(\theta^t)) < c(\sigma(\theta^t))$ and $s(\sigma(\theta^{t-1})) < 0$ for $\theta^{t-1} \in \theta^t$.*

Proof. Suppose that $x(\sigma(\theta^t)) \ge c(\sigma(\theta^t))$. This implies that

$$\frac{\sum_\theta \pi(\theta) u'(x(\sigma(\theta^t, \theta)))}{u'(c(\sigma(\theta^t)))} \le \frac{\sum_\theta \pi(\theta) u'(x(\sigma(\theta^t, \theta)))}{u'(x(\sigma(\theta^t)))}.$$

Combining this with (13) we obtain

$$\sum_\theta \pi(\theta) u'(x(\sigma(\theta^t, \theta))) \le \sum_\theta \pi(\theta) u'(c(\sigma(\theta^t, \theta))).$$

This inequality implies that there must be at least one θ such that $x(\sigma(\theta^t, \theta)) \ge c(\sigma(\theta^t, \theta))$. Then from (6) it follows that $s(\sigma(\theta^t, \theta)) < 0$. Using the previous argument since $x(\sigma(\theta^t, \theta)) \ge c(\sigma(\theta^t, \theta))$ it must be true that there exists some node θ' such that $x(\sigma(\theta^t, \theta, \theta')) \ge c(\sigma(\theta^t, \theta, \theta'))$ and $s(\sigma(\theta^t, \theta, \theta')) < 0$. Continuing this induction there exists a node θ^T such that $x(\sigma(\theta^T)) \ge c(\sigma(\theta^T))$ and $s(\sigma(\theta^T)) < 0$. But this is impossible since in the last period it must be true that $s(\sigma(\theta^T)) = 0$ for all θ^T. A contradiction.

Negative assets in the previous period $s(\sigma(\theta^{t-1})) < 0$ for $\theta^{t-1} \in \theta^t$ follow from the budget constraint (6) and $x(\sigma(\theta^t)) - c(\sigma(\theta^t)) < 0$. ∎

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It is easiest to understand the intuition for this result in the case when consumption allocations that an agent receives along his deviation strategy σ satisfy the Euler equation, i.e. (13) holds with equality, since inequality further strengthen this intuition. Agent's actual consumption x also satisfies the Euler equation. This implies that an agent chooses to have a higher consumption $x(\theta^t)$ than his endowment $c(\theta^t)$ only if his consumption is also higher in the future. This is possible only if an agent starts with a positive amount of assets and saves some resources for the next period.

The previous results imply that it is optimal for an agent to borrow only if he borrowed in the previous period. But then borrowing can never be optimal since each agent has a zero initial asset position. The next proposition formalizes this intuition.

Proposition 5 *Consider any strategy σ together with trades and after-trade consumption on the retrading market $\{x_t, s_t\}_{t=1}^{T}$. If $s(\sigma(\theta^t)) < 0$ then there exists another pair $\{\hat{x}_t, \hat{s}_t\}_{t=1}^{T}$ that is feasible and gives a higher utility.*

Proof. Consider any reporting strategy σ. The optimal consumption/saving pair $\{x_t, s_t\}_{t=1}^{T}$ should satisfy (6), (7) and (8). The previous lemma showed that if $s(\sigma(\theta^t)) < 0$ for some θ^t than $s(\sigma(\theta^{t-1})) < 0$. Continuing this backward induction we obtain that it must be true that $s(\theta^0) < 0$ which violates the initial condition $s(\theta^0) = 0$. Therefore there is no node in which it is optimal to borrow. ∎

In the solution to the social planner's problem in lemma 1, the incentive constraint (9) binds for some strategies σ. The next proposition shows that such strategies imply savings in some states and never borrowing.

Proposition 6 *The only binding incentive constraints in the Social planner problem are those where $s(\sigma(\theta^t)) > 0$ for some θ^t.*

Proof. Consider any deviating strategy σ together with consumption/saving pair $\{x_t, s_t\}_{t=1}^{T}$ that binds in the social planners problem. We established before that for such allocations it must be true that $s(\sigma(\theta^t)) \geq 0$ for all θ^t. We will show that the inequality is strict for some θ^t. The allocations along the truth telling strategy σ^* are such that the optimal saving behavior is $s(\sigma^*(\theta^t)) = 0$ for all θ^t. For any other strategy there exists some $\hat{\theta}^t$ so that $c(\sigma(\hat{\theta}^t)) \neq c(\sigma^*(\hat{\theta}^t))$. But then (6) and (7) can not hold simultaneously with zero savings in each node, therefore there must be some θ^t such that $s(\sigma(\theta^t)) > 0$. ∎

The previous propositions showed that if an agent decides to deviate, he always optimally chooses to have positive savings. A decrease in the interest

rates reduces returns on savings and lowers the utility from deviations. The next proposition shows that the social planner chooses interest rates to be lower than the return on capital.

Proposition 7 *Let T be finite. In the optimum $F_k(k_t, Y_t) > 1/Q_{t-1}$ for at least one t.*

Proof. Suppose $F_k(k_t, Y_t) \leq 1/Q_{t-1}$ for all t. Consider the following perturbation. Choose prices $\hat{Q}_t > Q_t$ for all t close to the original prices Q_t and consumption allocations sufficiently close to the original allocations $\hat{c}(\theta^t) = c(\theta^t) + \varepsilon(\theta^t)$ such that new bond prices are consistent with the perturbed allocations for all θ^t:

$$\hat{Q}_t u_c(\hat{c}(\theta^t)) = \beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u_c(\hat{c}(\theta^{t+1})), \tag{14}$$

and the present value of consumption for any history θ^T remains the same under Q_t :

$$\hat{c}_1(\theta^T) + Q_1 \hat{c}_2(\theta^T) + \dots + \prod_{i=1}^{T} Q_i \hat{c}_T(\theta^T) \tag{15}$$
$$= c_1(\theta^T) + Q_1 c_2(\theta^T) + \dots + \prod_{i=1}^{T} Q_i c_T(\theta^T).$$

where $c_t(\theta^T)$ denotes consumption in period t of the person who had a history $\theta^t \in \theta^T$. In the appendix we show that such a perturbation is feasible if $F_k(k_t, Y_t) \leq 1/Q_{t-1}$ for all t. Substituting expression for $\hat{c}(\theta^t)$ in (15) we get that for all θ^T

$$\varepsilon_1(\theta^T) + Q_1 \varepsilon_2(\theta^T) + \dots + \prod_{i=1}^{T} Q_i \varepsilon_T(\theta^T) = 0. \tag{16}$$

The expected utility of consumption under the new allocations is

$$\sum_{t=1}^{T} \beta^{t-1} \sum_{\theta^t} \pi(\theta^t) u(\hat{c}(\theta^t))$$
$$= \sum_{t=1}^{T} \beta^{t-1} \sum_{\theta^t} \pi(\theta^t) u(c(\theta^t)) + \sum_{t=1}^{T} \beta^{t-1} \sum_{\theta^t} \pi(\theta^t) u'(c(\theta^t)) \varepsilon(\theta^t).$$

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Applying (10) to the last term we get

$$\sum_{t=1}^{T} \beta^{t-1} \sum_{\theta^t} \pi(\theta^t) u'(c(\theta^t)) \varepsilon(\theta^t)$$

$$= \beta^{T-1} \sum_{\theta^T} \{ \pi(\theta^T)(\varepsilon_T(\theta^T) + Q_T^{-1} \varepsilon_{T-1}(\theta^T) + ... + \prod_{i=1}^{T} Q_i^{-1} \varepsilon_1(\theta^T)) \}$$

$$= 0,$$

where the last equality follows from (16). From this we can see that the utility from consumption under the perturbed allocation is the same as under the original allocation.

We want to show that the perturbed allocations relax the incentive constraint for the planner. Then there exists a way for the planner to further improve upon them.

First, suppose that truth telling is an equilibrium. From (14) the price of a risk free bond in such equilibrium is $\hat{Q}$ and no agent wants to retrade from the allocations $\{\hat{c}_t\}_{t=1}^{T}$.

We can show that no agent chooses to deviate from the new allocations. Suppose there exists such a deviation σ and the associated asset holdings $s(\sigma)$. According to proposition 6, $s(\sigma(\theta^t)) \geq 0$ for all θ^t with a strict inequality for at least one history.

Because of (15) under original allocations $\{c_t\}_{t=1}^{T}$ any agent could also choose allocations $\{\hat{c}_t\}_{t=1}^{T}$. Moreover, that agent would be able to carry out the same deviation σ. Since under the original allocations the interest rates were higher, $Q_t^{-1} > \hat{Q}_t^{-1}$ for all t, the return on savings would be strictly higher. Therefore utility from the deviation σ would be strictly higher under the original allocation $\{c_t\}_{t=1}^{T}$ than under the new perturbed allocation $\{\hat{c}_t\}_{t=1}^{T}$. Since the original allocation was incentive compatible, this deviation gives no higher utility than $\sum_{t=1}^{T} \beta^{t-1} \sum_{\theta^t} \pi(\theta^t) u(c(\theta^t))$. Therefore the utility from any deviation under the perturbed allocation will be strictly less than $\sum_{t=1}^{T} \beta^{t-1} \sum_{\theta^t} \pi(\theta^t) u(\hat{c}(\theta^t))$ and the incentive constraint is relaxed. ∎

Although the proof is lengthy, its intuition is quite straightforward. If $F_k(k_t, Y_t) \leq 1/Q_{t-1}$ in all periods it is possible to transfer resources from the last periods back to the first periods in such a way that it gives the same utility to the agent who chooses the truthful reporting strategy. Such a transfer lowers the interest rates in the economy since transfers become more frontloaded. Lower interest rates decrease the return on savings that agents do when they deviate, and this relaxes the incentive constraints.

In the competitive equilibrium it is true that the interest rates on the retrading market and the marginal product of capital are equated. This implies that the competitive equilibrium is not efficient.

Theorem 2 *The competitive equilibrium is not efficient.*

Proof. Follows from propositions 3 and 7 ∎

Intuitively, the competitive equilibrium is not efficient because a contract offered by one company to its workers affects the return on trades and thus incentives to reveal information truthfully for agents insured by other companies. Individual firms can not internalize this effect. Competition between different insurers implies that interest rates at which agents trade are equated with the marginal rates of transformation. The planner is able to choose the interest rates optimally. Thus, privately provided insurance does not lead to efficient allocations in this setting[7]. In the next section we will explore how distortionary taxes can introduce the wedge between the equilibrium interest rates on the retrading market and the marginal product of capital.

4.4.1 Tax policy with iid shocks

We showed in the previous section that efficiency requires that the interest rates on the retrading market are lower than the marginal product of capital. In the competitive equilibrium without government interventions they are equated and the equilibrium allocations are not efficient. Government interventions in a form of distortionary taxes on capital can reintroduce this wedge in competitive equilibrium. In this section we show that such policy improves welfare.

We proceed as follows. First, we re-write the firm's problem in its dual form. The dual form is convenient to use since it maximizes total utility of agents similar to the social planner's problem. Second, we show that positive linear taxes on capital income improve the welfare when agent's optimal deviations involve oversaving.

Consider a dual version of the Firm's problem. Since all the firms are making zero profit in equilibrium, their problem can be rewritten in the following form.

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[7]See Greenwald and Stiglitz (1986) for a discussion how economies with private information are similar to the economies with externalities. Arnott and Stigliz (1990) discuss how unobservable insurance purchases create externality-like effect in static moral hazard models.

Firm's Problem 2

$$\max_{c,y,k} \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t)\beta^t u(c(\theta^t), y(\theta^t)/\theta_t)$$

s.t. for all t

$$\sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t)\beta^t u(c(\theta^t), y(\theta^t)/\theta_t) \geq V(\{c,y\},\{Q\})(\sigma) \text{ for any } \sigma,$$

$$\sum_{\theta^t} \pi(\theta^t)c(\theta^t) + k_{t+1} \leq F(k_t, \sum_{\theta^t} \pi(\theta^t)y(\theta^t)).$$

Claim 1 *In a competitive equilibrium, the solution to Firm's Problem 1 coincides with the solution to Firm's Problem 2.*

Proof. First we will show that w.l.o.g. we can use utility of the consumer $\sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t)\beta^t u(c(\theta^t), y(\theta^t)/\theta_t)$ instead of the indirect utility function $V(\{c,y\},R)(\sigma^*)$. Consider any solution to Firm's Problem 1 $\{c_t, y_t\}_{t=1}^{T}$ and the resulting equilibrium allocations of consumption $\{x_t\}_{t=1}^{T}$. For each history θ^T the present value of firms' payment and agent's consumption must be the same

$$x_1(\theta^T) + Q_1 x_2(\theta^T) + ... + \prod_{i=1}^{T} Q_i x_T(\theta^T)$$
$$= c_1(\theta^T) + Q_1 c_2(\theta^T) + ... + \prod_{i=1}^{T} Q_i c_T(\theta^T).$$

>From proposition 3 $1/Q_{t-1} = F_k(t)$ for all t, which implies that the cost of providing $\{x_t\}_{t=1}^{T}$ directly to agents must be exactly the same as the cost of providing $\{c_t\}_{t=1}^{T}$. Therefore w.l.o.g. we can assume that firms provide each agent with x directly so that the truth telling agent does not retrade.

Finally, since in equilibrium firm's profits are zero, $d_t = 0$ for all t and firm's problem 1 can be re-written in its dual form as in problem 2. ∎

This result allows to directly compare the Firm's problem and the Social planner's problem. These two problems are very similar. The planner however has an additional choice variable - prices on the retrading market Q. The social planner choosing efficient allocations takes into account how these allocations affect the interest rates in the economy. The competition among firms makes the interest rates on the retrading market equal to the marginal rate of transformation.

Unlike the economy with observable asset trades, distorting taxes are welfare improving in this environment. Consider a simple linear tax τ imposed on capital income Rk, where $R \equiv F_k(K, Y)$. The revenues from this tax are distributed equally among all agents. As argued in proposition 1 such a lump sum distribution has the same effect as returning lump sum rebates directly to firms. In the following proposition we show that such a tax system is welfare improving.

Proposition 8 *There is a positive tax τ on capital income together with the lump sum rebate T that improves the welfare in the competitive equilibrium.*

Proof. >From proposition 6 the only binding incentive constraints in the firm's problem must be those constraints that involve savings only. Let t be a time period for which there exists a binding strategy σ and a history θ^t such that $s(\sigma(\theta^t)) > 0$. We know that for all other $\hat{\sigma}, \hat{\theta}^t$ savings are non-negative: $s(\hat{\sigma}(\hat{\theta}^t)) \geq 0$.

Consider a linear tax τ on the return on capital Rk in period $t + 1$. The tax revenues are rebated in the lump sum amount T to the firms. Let $k(\tau, T)$ denote firm's investment in period t as a function of (τ, T). The feasibility constraint for the government is $\tau Rk(\tau, T) = T$. Using the implicit function theorem we obtain

$$T'(\tau) = \frac{Rk(\tau, T) - \tau Rk_\tau(\tau, T)}{1 - \tau Rk_T(\tau, T)}.$$

Let $W(\tau, T)$ be the value of the objection function in the Firm's problem 2 when the firm faces taxes T and τ. It coincides with the ex-ante utility of agents and represents welfare in the economy. Consider the derivative dW of this function at zero capital taxes

$$dW(\tau, T(\tau))|_0 = W_\tau(0, 0) + W_T(0, 0)T'(0) = W_\tau + W_T Rk.$$

All the variables on the right hand side are evaluated at zero taxes.

Let γ_{ic} be the Lagrangian multiplier on the incentive constraint, and γ_t be the multiplier on the feasibility constraint in period t in Firm's Problem 2. From the envelope theorem

$$W_T = \gamma_{t+1},$$

$$W_\tau = -\gamma_{t+1}Rk - \gamma_{ic}V_Q\frac{\partial Q_t}{\partial \tau}.$$

In equilibrium $1/Q_t = \tau R_{t+1}$, therefore $\partial Q_t / \partial \tau = -R_{t+1}/\tau^2 < 0$. By proposition 6 any deviation involves savings and higher interest rates increase return on savings, therefore $V_Q < 0$. Combining these effects we see that capital taxes are welfare improving:

$$dW(\tau, T(\tau))|_0 = -\gamma_{ic} V_Q \frac{\partial Q_t}{\partial \tau} > 0.$$

■

Similarly to the economy with observable trades, lump sum taxes have no effect on the insurance agents receive. Taxes on capital income have two effects. On the one hand, they distort investment decisions of firms and create a deadweight loss. On the other hand, they also lower the return on savings in the retrading market. This improves the incentives of agents to reveal their private information truthfully and firms are able to provide better insurance – private markets change endogenously in response to government policy. At least for small capital taxes the second effect dominates the first one, and welfare improves.

The capital taxes alone are not sufficient to achieve the efficient outcome in the competitive settings. To see this, suppose taxes were set in such a way that the after-tax return on capital were equal to the interest rates on the retrading market under the efficient allocations, $1/Q^{sp}$. Then the firm would have the same incentive constraint (11) as the social planner. The feasibility constraint would be different however. While the planner's decisions are undistorted, firms' savings are affected by distorting taxes. In general the government has to impose additional non-linear taxes on labor income to achieve the efficient allocations.

4.5   Extension to the infinite period economy

In this section we outline the extension of our results to the infinite period economy that is similar to Aiyagari (1994, 1995).

The problem of the agent who is offered a menu $\{c_t, y_t\}_{t=1}^{\infty}$ is similar to the one in the finite case. However, we need to impose an additional upper bound B on the asset holdings to prevent Ponzi schemes. This bound is arbitrarily large but finite. In the economy with any finite number of agents such a bound can be enforced since agents observe their individual asset trades with any other agent. The sufficient condition in such an economy to rule out a Ponzi scheme is that each agent bounds the amount of asset trades he does with any other agent. This condition is more difficult to ensure when there is a continuum of agents, since each agent has a measure

zero. However, we will assume that there exists mechanisms that rule out Ponzi type asset holdings.

Agent's problem in the infinite period economy

$$\max_{\sigma,x,s} \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\theta} \pi(\theta^t) u(x(\sigma(\theta^t)), y(\sigma(\theta^t))/\theta_t)$$

s.t. for all θ^t, t

$$x(\sigma(\theta^t)) + Q_t s(\sigma(\theta^t)) = c(\sigma(\theta^t)) + s(\sigma(\theta^{t-1})),$$

$$s(\sigma(\theta^t)) \geq -B,$$

$$s(\theta^0) = 0.$$

As before, the prices Q_t are such that asset markets clear in each period. The definition of the efficient allocations and equilibrium in the retrading market remains unchanged.

It is generally not known when efficient allocations converge to a stationary distribution in a limit. Green (1987), Thomas and Worrall (1990), Atkeson and Lucas (1992) showed that in the economies with no asset trades there is typically no stationary distribution - expected utility for each agent converges to zero with probability one. The economy converges to a steady state if an additional lower bound on promised utility is imposed on the planner's problem as in Atkeson and Lucas (1995). There are no similar results for the economies with asset trades in this context. However, we can show that if a stationary distribution exists in which the aggregate interest rates Q_t, aggregate capital K_t and effective labor Y_t converge to constant values, the interest rates are lower than the marginal rate of transformation $F_k(K,Y)$.[8] The proofs of the next two lemmas follow closely Aiyagari (1995) and Ljungqvist and Sargent (2000).

Lemma 4 *In a stationary equilibrium $F_k(K,Y) = \beta^{-1}$.*

Proof. The FOCs for G_t and K_t imply that $u^{g\prime}(G_t) = \beta F_k(K_{t+1}, Y_{t+1}) u^{g\prime}(G_{t+1})$. Since in a stationary equilibrium G_t, K_t, Y_t are all constant, we immediately get that $F_k(K,Y) = \beta^{-1}$ ∎

Lemma 5 *In a stationary equilibrium $Q > \beta$ if $var(c(\theta)) > 0$.*

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[8] Similar assumption are made in the steady state analysis in Ramsey problems. See Chamley (1986) and Aiyagari (1995).

Proof. Any allocation in stationary equilibrium should satisfy $Qu'(c_t) \geq \beta E_t u'(c_{t+1})$. If we define $M_t = (1/Q)^t \beta^t u'(c_t)$ this equation can be re-written as $E_t(M_{t+1} - M_t) \leq 0$. Supermartingale convergence theorem implies that $M_t \rightarrow M$ a.s. Chamberlain and Wilson (1984) showed that for $\beta/Q \geq 1$ that would imply that $u'(c_t) \rightarrow 0$ or $c_t \rightarrow \infty$. This is possible only if $k_t \rightarrow \infty$, but this implies that $F_k = 0$ in the stationary equilibrium. But it can not be true by the previous lemma. Therefore the only possibility is $Q > \beta$. ∎

In the infinite horizon economy the competitive equilibrium is not efficient since it necessarily has $1/Q = F_k(K, Y)$.

The consumption-saving decisions of agents in our environment are similar to those in Bewley (1986) and Aiyagari (1995). In Bewley's model, labor income of agents follows an exogenous stochastic process. In our economy the endowments of the agents are determined by the planner. This has important consequences for determining the optimal tax policy. Taking informational constraints into account allows us to find the optimal policy without imposing restrictions on the tax instruments as done in Aiyagari (1995). Although the result that savings should be distorted in the steady state is the same in both models, these models produce very different qualitative and quantitative tax policy recommendations along the transition path. We return to this point in the next section where we discuss optimal taxation in numerical examples.

5   Numerical example

In this section we compute optimal allocations and tax policy in economies with observable and unobservable asset trades. As a benchmark, we will use a disability insurance environment analyzed in Diamond and Mirrlees (1978, 1995) and Golosov and Tsyvinski (2003). We consider three types of experiments. First, we compute the efficient allocations in an economy where private trades are observable. In particular, we study the pattern and the size of intertemporal wedges. Second, we compute the optimal allocations for the economy where agents are allowed to trade unobservably. We find that the intertemporal wedge in this economy is smaller than in the economy with observable trades. We then compare the welfare losses from the unobservability of trades. Third, we compute the competitive equilibrium in the economy with unobservable trades. We compare welfare in the competitive equilibrium to welfare of the optimal allocation with unobservable trades and with a version of Bewley's economy where the only form of insurance available to agents is trading of a risk free bond. We find

that, even in the environment with unobservable trades, private markets can achieve allocations that are nearly optimal. This result indicates that the large welfare gains from introducing government insurance found in the literature on optimal dynamic contracting may be misleading as they treat private markets exogenously. To a large extent, public provision of insurance crowds out private insurance[9].

We consider an economy that lasts for ten periods. In each period agent's skill is drawn from a two point distribution $\Theta = \{0, 1\}$. The state $\theta = 0$ is absorbing. Any agent who receives a zero productivity shock in any period, has zero productivity in all the subsequent periods. The production function is $F(K, Y) = rK + wY$. We assume that there is no utility from the consumption of the public good G.

In the numerical exercises described below each period is assumed to be five years. We choose the following parameter values: $\beta = 0.8$, $r = \beta^{-1}$, $w = 1.21$. Each agent is endowed with $k_1 = 0.69$ units of initial capital. The parametrization is described in Golosov and Tsyvinski (2003). We adjust those parameters to represent a five year time period. The stochastic process for skills is shown in figure 1. It matches disability shocks among the US population for 20-65 year old. The utility function is $u(c, l) = \ln(c) + 1.5 \ln(1 - l)$.

5.1 Observable trades

In this subsection, we compute optimal allocations and intertemporal wedges for an economy where trades are observable.

It is well known that in the economy with private information without hidden retrading, savings decisions of each agent are distorted. In particular optimal allocations satisfy for all θ^t the following inequality:

$$u'(c(\theta^t)) \leq \beta r \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(c(\theta^{t+1})).$$

This inequality is strict if $var(c(\theta^{t+1})) > 0$. We define the wedge $\tau(\theta^t)$ that each agent faces as

$$\tau(\theta^t) = 1 - \frac{1}{r-1} \left[\frac{u'(c(\theta^t))}{\beta \sum_{\theta^{t+1}} \pi(\theta^{t+1}|\theta^t) u'(c(\theta^{t+1}))} - 1 \right]. \qquad (17)$$

---

[9]Crowding out of private markets by government policies also occurs in Attanasio and Rios-Rull (2000) and Krueger and Perri (2001) who study economies with limited commitment.

The wedge is defined to be consistent with a wedge from a linear tax imposed on the net capital income $(r - 1)k$. The standard Euler equation with linear taxes on capital income is

$$u'(c_t) = \beta \left[(1 - \tau)(r - 1) + 1\right] E_t u'(c_{t+1}),$$

and we use this expression to define the savings wedge τ.

This wedge is history specific: agents who had a different history of shocks θ^t face different wedges. The wedge is equal to zero for the agent whose current skill is zero (since it is an absorbing state) and is strictly positive for the other agents. In the computed example the wedge of the agent who has positive productivity increases over the lifetime and reaches 8%. See figure 2.

5.2 Unobservable trades

In this subsection, we compute the optimal allocation for the economy where trades are unobservable. We compare the welfare for this economy to that of the economy without private information and to the economy with private information but observable trades. When agents can trade assets unobservably, efficiency requires that equilibrium interest rates on the retrading markets are lower than r. Although the stochastic process for skills is not iid, it is straightforward to modify the proof of proposition 6 to show that for any binding deviating strategy σ, savings are always non negative: $s(\sigma(\theta^t)) \geq 0$ with a strict inequality for some θ^t. It implies that proposition 7 holds in this economy.

We define the wedge in the same way as we defined it in (17) for the economy with observable trades. This wedge is not history specific - for every period t all agents face the same wedge τ_t. Figure 3 shows the computed wedge in this example. Note that it is strictly positive in each period but smaller than the wedge in the economy with observable trades. It never exceeds two percent.

The ex ante utility of agents is lower in the economy with unobservable trades than in the economy with observable trades. When trades are not observable the set of incentive compatible allocations is smaller, and the provision of insurance to agents is more difficult.

We use the following measure to compare welfare in the two economies. Let $\{c_t^{no}, y_t^{no}\}_{t=1}^{T}$ be the allocations that solve the social planner's problem with non-observable consumption. The ax-ante utility of such allocations is $\sum_{t=1}^{T} \beta^t \sum_{\theta^t} \pi(\theta^t) u(c^{no}(\theta^t), y^{no}(\theta^t)/\theta)$. If ex-ante utility in the economy with observable trades is U^o, we find such a number κ, that increasing

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consumption of each agent by $\kappa\%$ would make the ex-ante utility of the agent equal to U^o, i.e.

$$\sum_{t=1}^{T} \beta^t \sum_{\theta^t} \pi(\theta^t) u((1+\kappa)c^{no}(\theta^t), y^{no}(\theta^t)/\theta) = U^o.$$

We find the welfare losses from unobservable retrading to be 0.2%. The small absolute magnitude of this number is not surprising since the probability of zero productivity shock is low. The welfare loss compared to the first best outcomes - the economy with no private information - is 1.1%.

5.2.1   Crowding out

In this subsection we address the question as to what extend private markets are able to provide insurance in such an environment. We find that most optimal provision can be done privately with very small gains from public interventions. This contrasts with a large body of literature that studies social insurance when private markets are absent or exogenously restricted. For example Hansen and Imrohoroglu (1992), Wang and Williamson (1996), Hopenhayan and Nicolini (1997), and Alvarez and Veraciero (1998) and many others found large welfare effects of public policy when markets are exogenously incomplete. In this section, we show that this private provision of insurance, though not efficient, is a significant improvement over the autarkic allocations with self-insurance.

Consider an economy where there is no private provision of insurance. In the absence of taxes each agent is able to borrow and lend at the interest rate r, and, if he has a positive productivity, supplies labor at the wage rate w. This setup is equivalent to that in Aiyagari (1994). The agent's problem is

$$\max_{c,y,k} \sum_{t=1}^{T} \sum_{\theta^t} \pi(\theta^t) \beta^t \{u(c(\theta^t)) + v(y(\theta^t)/\theta_t)\}$$

s.t. for all θ^t

$$c(\theta^t) + s(\theta^t) = wy(\theta^t) + rs(\theta^{t-1}),$$
$$s(\theta^0) = k_1.$$

where we use a convention that if $\theta = 0$ then $v(y(\theta^t)/\theta_t) = v(0)$.

Thus, similarly to Bewley (1986), Huggett (1993) and Ayiagari (1994) the only insurance available is a self-insurance with a risk-free bond.

We find that competitive equilibrium allocations provide welfare which is 1.08% higher than welfare in the economy where a risk-free bond is the only

form of insurance available to agents. Welfare under efficient allocations is 1.11% higher. These findings show that competitive equilibrium without government interventions provides about 97% of the optimal insurance.

This example suggests that it is important to consider responses of private markets to changes in the government policy. Consider the environment we described where the optimal insurance is provided by the government. Since there are no gains from additional insurance, all private insurance markets are absent. To an outside observer such an economy appears to be identical to Aiyagari's economy where the only private asset available is a risk free bond. Taking exogenous such a structure of private markets would suggest that the removal of public insurance decreases welfare by 1.11%. This argument however does not take into account that private markets may emerge and the actual welfare losses would be much smaller.

The analysis above assumes that private markets function perfectly. In such circumstances most of the optimal insurance can be provided with no government interventions. One may argue that legal restrictions or market imperfections decrease the amount of insurance available privately, and public insurance is needed in such circumstances. The size of crowding out depends on the particular form of the assumed imperfections, and additional work would be needed to compute it. In general, unless such imperfections are assumed to be very severe, the welfare effects of the optimal public policy may be small.

6   Conclusion

This paper studies dynamic optimal taxation in an economy with informational frictions and endogenous insurance markets. Existence of private information is commonly used to justify government interventions via taxes to achieve optimal redistribution or insurance. We derive two sets of results. First, we show that, if private trades are observable, as commonly assumed in the existing literature, there is no role for government intervention. Competitive markets provide the optimal amount of insurance. The only effect of government policy is a crowding out of private insurance without improvement in welfare. Second, we relax the assumption of observable trades and study environments where trades are unobservable. We show that competitive equilibria are not optimal. A government, even the one that has the same information as private parties, can improve upon any allocations that can be achieved by markets. A linear tax levied on firm's capital income affects the rate of return in hidden asset markets and improves the insurance

provided to agents by insurance firms.

The main economic message of our paper is twofold. First, understanding the interaction of private insurance markets and public tax policy may have very important implications for the design of optimal taxation and social insurance. Second, we find a specific tax instrument that can affect the insurance provided through private markets and lead to the welfare improvement over any allocation achieved by private markets alone.

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7 Appendix

7.1 Absence of shock-specific securities

The assumption that agents can trade only a risk free bond is not restrictive. In many environments risk free bonds emerge as the only asset traded in equilibrium. Consider a market structure described in Section 4.1. Suppose each agent observes the identity of the agent he transact with, but not private characteristics of that agent. In these settings no Arrow-type securities, for which the payment depends on the reports of the agents, are traded in equilibrium. The structure of securities markets is similar to the one studied in Bisin and Gottardi (1999). Let $a^i(\theta)$ be a security that pays one unit of consumption good if an agent i reports θ to the planner in the next period, and zero otherwise. For simplicity we assume that the lowest skill, $\theta(1)$, is strictly positive, so that no agent incurs infinite disutility from reporting any other type.

Claim 2 *There is no equilibrium where securities $a^i(\theta)$ are traded. Only a risk free bond is traded in equilibrium.*

Proof. We will show that, for any price $q^i(\theta)$ for a security $a^i(\theta)$, either an agent i can make an infinite return or has a higher return on a risk free security. Since, in the bilateral trades, agents can see each other's type, the price for each security may be different depending on whether the agent, who controls the outcome of it, buys or sells the security.

Case 1. An agent wants to buy a security that pays one unit of consumption good if he sends report θ in the next period.

We show that a price for such a security will be $q^i(\theta) \geq Q$. Suppose on the opposite that $q^i(\theta) < Q$. Under such prices the agent could buy infinitely many securities that pay in state θ and sell a risk-free debt for this amount. Then, in the next period, he claims the state θ. Since an agent incurs only finite disutility from providing $y(\theta)$ units of labor if his type is $\hat{\theta}$, this strategy yields an infinite utility for the agent. The seller of the security incurs losses, so it cannot be the equilibrium price.

If $q^i(\theta) \geq Q$ an agent prefers not to sell such a security since it pays 1 unit of consumption in only one state θ, while risk-free bond pays one unit of consumption in all states and is cheaper.

Case 2. An agent wants to sell a security that pays one unit of consumption good if he sends a report θ in the next period.

The price of such a security is zero. Suppose not. Then the agent can sell infinitely many of such securities and in the next period claim any state

other than θ. The agent makes infinite profits and utility. Thus, this case is also not possible. ■

The intuition for the proof is simple. An agent can choose which skill to report in the next period. As long as there are gains from reporting any state θ, he will report it with probability one. But that makes such a security $a^i(\theta)$ equivalent to a risk free bond, hence no type-specific securities are traded in equilibrium.

7.2 Feasibility of the perturbation

Here we will prove that the perturbation discussed in proposition 7 is feasible. First, we need to consider several intermediate results.

Lemma 6 *Suppose that for some θ^t $\hat{c}(\theta^t) < c(\theta^t)$. Then there exists a history θ^T with $\theta^t \in \theta^T$ such that $\hat{c}_k(\theta^T) < c_k(\theta^T)$ for $t \leq k \leq T$.*

Proof. $>$From (10) and (14) it follows that if $\hat{c}(\theta^t) < c(\theta^t)$ that there is at least one $\theta^{t+1} > \theta^t$ such that $\hat{c}(\theta^{t+1}) < c(\theta^{t+1})$. The rest follows by induction. ■

Lemma 7 *For any two nodes $\theta^{T\prime}$ and $\theta^{T\prime\prime}$ that follow the same history θ^{T-1} it must be true that $c_T(\theta^{T\prime}) - \hat{c}_T(\theta^{T\prime}) = c_T(\theta^{T\prime\prime}) - \hat{c}_T(\theta^{T\prime\prime}) > 0$.*

Proof. Equation (16) shows that for any two histories $\theta^{T\prime}$ and $\theta^{T\prime\prime}$ that have the same initial history θ^t in common (i.e. $\theta^t \in \theta^{T\prime}$ and $\theta^t \in \theta^{T\prime\prime}$) it must be true that

$$
\prod_{i=1}^{t} Q_i \varepsilon_{t+1}(\theta^{T\prime}) + \prod_{i=1}^{t+1} Q_i \varepsilon_{t+2}(\theta^{T\prime}) + ... + \prod_{i=1}^{T-1} Q_i \varepsilon_T(\theta^{T\prime}) \ (18)
$$
$$
= \prod_{i=1}^{t} Q_i \varepsilon_{t+1}(\theta^{T\prime\prime}) + \prod_{i=1}^{t+1} Q_i \varepsilon_{t+2}(\theta^{T\prime\prime}) + ... + \prod_{i=1}^{T-1} Q_i \varepsilon_T(\theta^{T\prime\prime}).
$$

This immediately implies that if $\theta^{T\prime}$ and $\theta^{T\prime\prime}$ have a common history θ^{T-1} then $\varepsilon_T(\theta^{T\prime\prime}) = \varepsilon(\theta^{T\prime})$ and $c_T(\theta^{T\prime}) - \hat{c}_T(\theta^{T\prime}) = c_T(\theta^{T\prime\prime}) - \hat{c}_T(\theta^{T\prime\prime})$. Suppose $\varepsilon_T(\theta^{T\prime}) > 0$ so that $c_T(\theta^{T\prime}) - \hat{c}_T(\theta^{T\prime}) < 0$. The Euler condition (10) implies that $\varepsilon_{T-1}(\theta^{T\prime}) > 0$. From (16) there must by some i such that $\varepsilon_i(\theta^{T\prime}) < 0$. Pick such i that for any $j > i$ $\varepsilon_j(\theta^{T\prime}) \geq 0$. Consider this node θ^i. From the previous lemma there is a history $\theta^{T\prime\prime\prime}$ that contains θ^i so that $\varepsilon_j(\theta^{T\prime\prime\prime}) < 0$ for all $j > i$. However $\theta^{T\prime}$ also contains θ^i and $\varepsilon_j(\theta^{T\prime}) > 0$ for all $j > i$. This contradicts (18). Therefore it must be true that $\varepsilon_{T-1}(\theta^{T\prime}) = \varepsilon_{T-1}(\theta^{T\prime\prime}) < 0$ ■

Lemma 8 *Consider the sequence $\{\varepsilon(\theta^T)\}$ for any history θ^T. Then there exists a sequence $\delta_t(\theta^T) < 0$ for $t = 2, 3, ...T$ s.t.*

$$\varepsilon_t(\theta^T) = \delta_t(\theta^T) - \delta_{t+1}(\theta^T)Q_t \text{ for } t > 1, \tag{19}$$

$$\varepsilon_1(\theta^T) = -\delta_2(\theta^T)Q_1. \tag{20}$$

Proof. Let $\delta_T(\theta^T) = \varepsilon_T(\theta^T)$. By the previous lemma it is negative. Define $\delta_{T-1}(\theta^T)$ such that (19) holds. Suppose $\delta_{T-1}(\theta^T) \geq 0$, so that $\varepsilon_{T-1}(\theta^T) \geq 0$. Then (16) holds only if there is some t^*, $1 \leq t^* \leq T - 2$, so that $c_{t^*}(\theta^T) > \hat{c}_{t^*}(\theta^T)$, i.e. $\varepsilon_{t^*}(\theta^T) < 0$. Let's pick t^* closest to $T - 1$. That means that for all $i, t^* + 1 \leq i \leq T - 2$ $\varepsilon_i(\theta^T) \geq 0$. By lemma 9 there is some history $\theta^{T\prime}$ that follows θ^t so that for all $t \geq t^*$ $\varepsilon_t(\theta^{T\prime}) < 0$. Define

$$D \equiv \prod_{i=1}^{t^*+1} Q_i \varepsilon_{t^*+1}(\theta^{T\prime}) + ... + \prod_{i=1}^{T} Q_i \varepsilon_T(\theta^{T\prime}).$$

>From (16) it should also be true that

$$
\begin{aligned}
D &= \prod_{i=1}^{t^*+1} Q_i \varepsilon_{t^*+1}(\theta^T) + ... + \prod_{i=1}^{T-1} Q_i \varepsilon_{T-1}(\theta^T) + \prod_{i=1}^{T} Q_i \varepsilon_T(\theta^T) \\
&= \prod_{i=1}^{t^*+1} Q_i \varepsilon_{t^*+1}(\theta^T) + ... + \prod_{i=1}^{T-2} Q_i \varepsilon_{T-2}(\theta^T) + \prod_{i=1}^{T-1} Q_i \delta_{T-1}(\theta^T).
\end{aligned}
$$

Combine these equations for histories θ^T and $\theta^{T\prime}$

$$
\begin{aligned}
0 &> \prod_{i=1}^{t^*+1} Q_i \varepsilon_{t^*+1}(\theta^{T\prime}) + ... + \prod_{i=1}^{T} Q_i \varepsilon_T(\theta^{T\prime}) \\
&= \prod_{i=1}^{t^*+1} Q_i \varepsilon_{t^*+1}(\theta^T) + ... + \prod_{i=1}^{T-2} Q_i \varepsilon_{T-2}(\theta^T) + \prod_{i=1}^{T-1} Q_i \delta_{T-1}(\theta^T) \\
&\geq 0.
\end{aligned}
$$

The left hand side of this equation is negative since all the elements are negative. The right hand side is positive since $\delta_{T-1}(\theta^T) \geq 0$ and all other elements are non-negative. This is a contradiction, therefore it must be true that $\delta_{T-1}(\theta^T) < 0$.

By backward induction we establish the same result for all $\delta_t(\theta^T)$. Finally (20) holds from (16) and construction of δ. ∎

Proposition 9 *The perturbation in proposition 7 is feasible if $1/Q_{t-1} \geq F_k(t)$ for all t.*

Proof. By the previous result we can represent

$$\varepsilon_1(\theta^T) + \frac{1}{F_k(2)}\varepsilon_2(\theta^T) + \ldots + \prod_{i=1}^{T}\frac{1}{F_k(i)}\varepsilon_T(\theta^T)$$

$$= -\delta_2(\theta^T)Q_1 + \frac{1}{F_k(2)}\left(\delta_2(\theta^T) - \delta_3(\theta^T)Q_2\right) + \ldots + \prod_{i=1}^{T}\frac{1}{F_k(i)}\delta_T(\theta^T)$$

$$= \delta_2(\theta^T)\left(\frac{1}{F_k(2)} - Q_1\right) + \frac{\delta_3(\theta^T)}{F_k(2)}\left(\frac{1}{F_k(3)} - Q_2\right) + \ldots + \prod_{i=1}^{T-1}\frac{\delta_T(\theta^T)}{F_k(i)}\left(\frac{1}{F_k(T)} - Q_{T-1}\right)$$

$$\leq 0.$$

The last equality is true since all $\delta \leq 0$ and $\dfrac{1}{F_k(t)} \geq Q_{t-1}$. ∎