Investment Planning Costs and the Effects of Fiscal and Monetary Policy

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Abstract We show that simple "New Keynesian" models with capital but without investment frictions have counterfactual predictions regarding the short-run effects of fiscal policy shocks: fiscal expansions *lower* output, employment, and the real interest rate in these models. We modify the model by assuming that investment projects are costly to start or stop, which is consistent with micro evidence. Adding investment planning costs restores the aggregate expenditure logic of the Keynesian Cross, and eliminates the counterfactual predictions regarding fiscal shocks. The modified model is also better able to match stylized facts on (i) the delayed effects of monetary policy shocks on output, (ii) the size of the liquidity effect, and (iii) the fact that monetary shocks change real interest rates for a significantly shorter time than they change real output. We show that convex *capital* adjustment costs, as in the neoclassical interpretation of Tobin's Q, cannot substitute for investment planning costs in all of these respects.

First draft: November 2002
This version: March 2004

This paper focuses on understanding the dynamics of economy-wide variables following shocks to fiscal and monetary policy. In keeping with a large literature, with notable contributions both early and recent, we do so in dynamic, general-equilibrium models where nominal prices are slow to adjust.[1] In our view, models with nominal rigidities are the most promising vehicles for understanding the size and persistence of the effects of monetary policy shocks. However, we focus on fiscal shocks to stress three points. First, simple "New Keynesian" models often predict strikingly non-Keynesian results in response to changes in fiscal policy. In particular, fiscal *expansions* may *reduce* output and real interest rates on impact in these models! Second, we believe that one should use a single model to explain the short-run behavior of the economy in response to shocks of all kinds, real as well as monetary.[2] Thus, we wish to confront these models, which can be successful in predicting the response of the economy in response to monetary shocks, with fiscal shocks in order to judge whether sticky-price models have the potential to provide a unified explanation of short-run aggregate behavior.[3] Third, we show that a single propagation mechanism, investment planning adjustment costs, can dramatically improve the model's ability to match the estimated responses to both fiscal and monetary shocks.

Our first result is probably the most startling. The standard New Keynesian framework does not model investment explicitly (see, e.g., Woodford, 2002, ch. 3), which is neither realistic nor useful for studying whether expansionary fiscal policy "crowds out" capital formation. We examine the model with capital and investment.[4] We initially analyze this model without investment adjustment costs of any form, so our model is similar in spirit to Tobin's (1955) "Dynamic Aggregative Model," although with sticky prices rather than wages. We then study the usual policy experiment of an increase in government purchases financed by lump-sum taxes. From the point of view of the consumer, the results of the

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[1] A very important early contribution is Tobin (1955), whose insights and results are being rediscovered piecemeal a half-century later. There is a large number of recent related papers: a few of the most relevant ones are Kimball (1995), Christiano, Eichenbaum and Evans (2001), Dotsey and King (2001), and Altig et al. (2002).

[2] In previous work (Basu, Fernald and Kimball, 1998) we studied another important category of real shocks, namely technology shocks. Galì (1999) presents a stylized model of the effects of technology shocks with sticky prices.

[3] Whether these models can explain the effects of monetary shocks is itself a subject of controversy. For differing views, see Chari, Kehoe and McGrattan (2000), Christiano, Eichenbaum and Evans (2001), Dotsey and King (2001), and Woodford (2002, ch. 3).

[4] Kimball (1995) and Dupor (2001) emphasize the importance of investment in sticky-price models.

intervention are strictly neoclassical (see, e.g., Barro, 1981). The extra government expenditure reduces lifetime wealth, reducing consumption and increasing the consumer's supply of labor at any given wage (reducing her consumption of leisure). In a flexible-price model with either perfect competition or with a fixed markup of price over marginal cost, equilibrium labor supply and output would increase, while the real (product) wage would decline.[5] In both flexible- and sticky-price models, the equilibrium real wage will be lower at any given level of output. But in the sticky-price model, the lower wage combined with a price level that is a state variable implies that the equilibrium markup jumps up. This higher inefficiency wedge reduces labor demand and labor input so much that output and employment actually fall.

This paper is part of a research program that asks whether a single model can adequately explain the short-run effects of all the major types of aggregate shocks on economy-wide variables. We have argued before that sticky-price models can explain why improvements in production technology are found to reduce employment and investment in the short run—although they have the stimulative effects predicted by real business cycle models two to three years after the shock.[6] In this paper, we examine the third major category of aggregate shocks—fiscal shocks. The effects of a subset of these shocks, the large expansions in government purchases associated with wars and defense build-ups since World War II, have been documented by Ramey and Shapiro (1998), while Blanchard and Perotti (2002) study the effects of "normal" fiscal shocks. Both sets of authors find that increases in government purchases increase output immediately, and Ramey and Shapiro (1998) find that they increase employment, and real interest rates as well.[7] These findings are consistent with the predictions of both the traditional Keynes-Hicks IS-LM model and the standard neoclassical real business cycle model. But as discussed above, they are not consistent with the predictions of the extended sticky- price model. To the extent that one

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[5] See, for example, Rotemberg and Woodford (1995).

[6] Basu, Fernald and Kimball (1998). See also Galì (1999), Marchetti and Nucci (2004), and Francis and Ramey (2001). Altig et al. (2002) argue that a model with both nominal price and wage rigidity can explain the short-run effects of both monetary shocks and technology shocks. However, they claim that technology improvements are not contractionary. They argue that Galì (1999) and Francis and Ramey (2001) over-difference the hours-per-worker variable that is included in their VARs. This argument does not explain why Basu, Fernald and Kimball (1998), Marchetti and Nucci (2004), and Shea (1998) also find the contractionary technology improvement result, but using very different identification schemes.

[7] These authors also examine the effects of fiscal shocks on a variety of other variables, which we discuss below.

finds sticky-price models attractive (for example, for explaining the effects of money and technology shocks), this is a serious problem.

It is unsurprising therefore that recent modeling on the effects of fiscal shocks have used flexible-price models. Ramey and Shapiro (1998) use a perfectly competitive, two-sector model. In a (1999) paper, Burnside, Eichenbaum, and Fisher [henceforth BEF] use a one-sector model with efficiency wages. BEF (2002) use a competitive, one-sector model with habit formation in consumption and adjustment costs of changing the flow of *investment*—not the stock of capital, as in the neoclassical interpretation of Tobin's *Q* by Abel (1981) and Hayashi (1982). But in these purely real models, of course, monetary shocks have no real effect, which is inconsistent with the vast literature on the real effects of monetary policy summarized by Christiano, Eichenbaum, and Evans (1999).

Our paper attempts to fill this gap. We start with a model that is consistent with the real effects of money on output, and then ask whether sensible modifications will also make it consistent with the short-run effects of fiscal shocks documented by Ramey and Shapiro (1998) and Blanchard and Perotti (2002). We stress the importance of investment precommitments, which we model as investment planning adjustment costs. The idea is that an investment project has a large degree of inertia—it takes time to start, and is not easy to abandon—which strikes us as a reasonable assumption, especially for business fixed investment.[8]

Planning adjustment costs make traditional aggregate demand logic work during the planning delay when investment is close to being predetermined. This means that an unanticipated government purchase shock will raise output during the planning delay, even if the shock does not have a significant effect on lifetime wealth. The standard neoclassical analysis of fiscal shocks, by contrast, relies on the negative wealth effect of a fiscal expansion, which is predicted to increase labor supply and raise output. But a small and relatively short-lived shock has almost no effect on lifetime wealth. In the standard

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[8] This idea is an extension of Christiano and Todd's (1996) "time to plan" model, which has been used by Edge (2000). The investment adjustment cost used by Christiano, Eichenbaum and Evans (2001) and by BEF (2001) has the similar effect of making investment inertial. Similar devices have been used in models without investment by

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neoclassical analysis, such a shock should basically crowd out investment one for one, and leave output and consumption virtually unchanged. Making investment inertial, however, changes this prediction: If investment cannot fall, then output rises instead. We show that for the standard fiscal policy experiment, the sticky-price model augmented with planning adjustment costs predicts that government purchase shocks will have their observed procyclical effects on impact.

A benefit of using the same model to analyze fiscal and monetary shocks is that we can see whether changing the model to explain the effects of one shock improves or worsens its ability to fit responses to other shocks. In this case, the addition of planning adjustment costs greatly improves the model's ability to match the effects of monetary shocks. There is general consensus on the stylized fact that output responds to monetary policy with a 6-12 month lag. Investment planning costs induce this type of hump-shaped response of investment, output, and employment after a monetary shock. Not surprisingly, by generating a lagged response of output to monetary shocks, investment planning adjustment costs also give rise to a liquidity effect: a monetary expansion causes an initial fall in the *real interest rate*, despite the positive effect of output on the *rental rate* of capital. Without some sort of investment friction, sticky-price models with capital cannot generate a liquidity effect—a discovery first made by Tobin (1955).

We then ask whether conventional *Q*-theory adjustment costs can substitute for planning costs along all these different margins. A discussion of terminology sheds light on the difference between the two. We term conventional *Q*-theory *capital adjustment costs*, since they are costs to changing the capital *stock* rapidly. Our proposed model we refer to as *investment adjustment costs*; they are costs to changing the *flow* of investment rapidly. Thus, *Q*-theory adjustment costs induce investment smoothing, but not investment inertia. In some cases, the two induce similar investment behavior. Thus, if there is a shock that creates short-lived variations in the frictionless desired capital stock, then both investment smoothing and investment inertia will produce a small change in actual investment. But for long-lived shocks, the

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Rotemberg and Woodford (1997) and Fuhrer (2001). Edge (2000) has an excellent discussion of micro survey evidence supporting the "time to plan" assumption.

two have very different predictions. Under Q-theory, investment is a completely forward-looking jump variable, so a long-lived shock will cause investment to jump immediately. Which way investment jumps may depend on the duration as well as the type of the shock. For example, a permanent increase in government spending will increase investment, and an instantaneous shock will leave investment more or less unchanged, but a shock of medium length might reduce investment. Thus, an increase in government spending of medium persistence, for example, is predicted to reduce employment and output in the short run in a sticky-price model with capital adjustment costs (although the reductions will be smaller than in a model with no adjustment costs). But with investment adjustment costs, the short-run effects will always be positive, because investment is fixed at its preshock level at the instant the shock occurs.

The paper is structured as follows. In Section I, we lay out the basic optimization problems facing consumers and monopolistically competitive final goods producers, which we hold fixed for the remainder of the paper. Final goods producers are assumed to hire capital and labor in competitive markets. We assume that capital is supplied by perfectly competitive firms, which purchase investment goods and rent them to final goods producers. In Section II, we complete the model by assuming that the capital rental firms do not face adjustment costs—which makes their optimization problem a trivial one— and show the implications of this model for real and monetary shocks. In the next section, we introduce planning adjustment costs, and study how this modification changes the model's predictions. In Section IV, we discuss the pros and cons of introducing an investment adjustment cost via the assumption of planning costs versus the standard Q-theory model with capital adjustment costs. We show that both models can fix some of the problems of a sticky-price model with infinitely interest-elastic investment demand, but the planning cost model appears to match the data better on several dimensions.[9] The final section offers concluding thoughts, and suggests directions for future research.

I. Foundations—Consumer and Firm Optimization

The basic building-blocks of the model are standard. We assume that there is a representative consumer who owns all the assets and supplies all the labor in the economy. The consumer's preferences are assumed to be of the form derived by King, Plosser, and Rebelo (1988), which allows for steady-state growth with variable labor supply. Output is produced by competitive firms as an aggregate of differentiated goods, and each good is produced by a monopolistically competitive firm. The monopolistic competitors rent capital and hire labor in competitive markets. Their nominal output price is adjusted only infrequently, in the stochastic manner analyzed by Calvo (1983).

We present the model in continuous time to take advantage of the sharp distinction between stocks and flows that is possible in a continuous-time setting and simplify several derivations. The discrete-time log-linearization of the model used to produce numerical solutions is presented in the Appendix.[10]

The main innovation in preferences, inspired by Basu and Kimball (2002) and Kimball and Shapiro (2003), is allowing for a continuum of different kinds of labor in a way that yields separable Frisch labor supply functions for each type of labor, even though labor is not additively separable from consumption. We wish to allow for a number of different types of labor in order to model "labor attachment" as parsimoniously as possible. Labor attachment implies that each worker works at one (and only one) firm, so that firms face an upward-sloping labor supply curve, instead of taking the wage as given, as they would if there were an economy-wide labor market. Kimball (1995) and Rotemberg (1996) emphasize the importance of attached factors in generating "real rigidity," in the sense of Ball and Romer (1990).

The representative consumer maximizes

$$\max_{C,N_\ell} E_0 \int_0^\infty e^{-\rho t} \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \left[1 + (1-\sigma)\Omega \int_0^1 \frac{N_\ell^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} d\ell \right]^{\frac{1}{\sigma}} dt \qquad (1)$$

s.t.

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[9] Of course, planning costs and capital adjustment costs are not mutually exclusive. More detailed empirical tests may argue that both are needed to match all the facts, but in the interests of parsimony we study each friction in isolation.

[10] We implement the discrete-time calculations with a time period equal to 1/100 of a year, so numerically the difference between the discrete-time and continuous-time models is negligible.

$$\dot{A} = \Re A + (\overline{R} - \delta)K + \overline{\Theta} + T - C + \int_0^1 \overline{W}_\ell N_\ell \, d\ell. \tag{2}$$

C is consumption, N is labor supply, A is the consumer's stock of assets (equal to the capital stock K in equilibrium), $\Re$ is the (after-tax) real interest rate on consumption bonds, $\overline{R}$ is the after-tax rental rate of capital, δ is the depreciation rate, $\overline{\Theta}$ is after-tax economic profit, $\overline{W}$ is the after-tax real wage, and T is lump-sum transfers from the government. (T can be negative—for example, when we consider changes in government expenditures financed by lump-sum taxes.) The parameters ρ and σ are the subjective discount rate and the elasticity of intertemporal substitution in consumption, while η is the Frisch elasticity of labor supply (the same value for each type of labor).

The current-value Hamiltonian is

$$H = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \left[1 + (1-\sigma)\Omega \int_0^1 \frac{N_\ell^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} d\ell \right]^{\frac{1}{\sigma}} + \lambda \left\{ \Re A + (\overline{R} - \delta)K + \overline{\Theta} + T - C + \int_0^1 \overline{W}_\ell N_\ell \, d\ell \right\}. \tag{3}$$

The first-order condition for optimal consumption can be solved to yield

$$C = \lambda^{-\sigma} \left[1 + (1-\sigma)\Omega \int_0^1 \frac{N_\ell^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} d\ell \right]. \tag{4}$$

Substituting into the current-value Hamiltonian yields the Hamiltonian maximized over C, which we label $\overline{H}$:

$$\overline{H} = \lambda \left\{ \Re A + (\overline{R} - \delta)K + \overline{\Theta} + T - \frac{\lambda^{-\sigma}}{1-\sigma} + \int_0^1 \overline{W}_\ell N_\ell \, d\ell - \lambda^{-\sigma}\Omega \int_0^1 \frac{N_\ell^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} d\ell \right\}. \tag{5}$$

Maximizing H over consumption C and labor $\{N_\ell\}_{\ell \in [0,1]}$ is equivalent to maximizing $\overline{H}$ over $\{N_\ell\}_{\ell \in [0,1]}$. Notice that $\overline{H}$ is additively separable in the different types of labor, even though H is not. The first order condition for the optimal supply of each type of labor, given the optimization of consumption, is

$$\overline{W}_\ell = \Omega \lambda^{-\sigma} N_\ell^{\frac{1}{\eta}} \tag{6}$$

or

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$$N_\ell = \left(\frac{\lambda^\sigma \bar{W}_\ell}{\Omega} \right)^\eta. \tag{7}$$

The relationships between the after-tax factor prices as perceived by the household and the pre-tax prices faced by the firm are:

$$\bar{R} = (1 - \tau_K)(R - \delta_{tax}) + \delta_{tax} = \Re + \delta, \tag{8}$$

and

$$\bar{W} = (1 - \tau_L)W. \tag{9}$$

R and W are *pre-tax* capital rental rates and real wages. τ_K and τ_L are the rates of capital and labor income taxation. Note that we assume that only capital income in excess of depreciation is taxed. We take into account the fact that depreciation for tax purposes (δ_{tax}) is lower than depreciation in the capital accumulation equation (δ), since the latter is adjusted for steady-state growth but the former is not. Apart from this issue, we abstract from steady-state growth in the exposition of the paper.

The government is assumed to finance lump-sum transfers and purchases of real goods and services using its tax revenue. For now we assume that lump-sum transfers adjust to keep the government budget in balance at every instant; in current work we are extending the model to allow for financing with distortionary taxation, but with tax smoothing. The government budget constraint is:

$$G + T = \tau_K \left[(R - \delta_{tax})K + \Theta \right] + \tau_L WN . \tag{10}$$

There is a single composite good in the economy, which is an aggregate of individual varieties of goods using a constant-returns technology. One can think of the aggregation being done by individual agents (consumers, investors, the government), or by competitive final-goods firms. Under either assumption, the assemblers minimize:

$$\int_0^1 P_i Y_i di \tag{11}$$

subject to

$$\int_0^1 \Upsilon (Y_i / Y) di = 1 . \tag{12}$$

The function Υ is increasing and strictly concave, and satisfies $\Upsilon(1) = 1$. Equation (12) implies a demand curve facing the monopolistically competitive firms. Equation (12) implies a demand curve facing the monopolistically competitive firms. We assume that this demand curve is of the "smoothed-off kinked" type discussed by Kimball (1995). The implications of this assumption are discussed below.

Each variety of output Y_i is produced by a monopolistic competitor. These firms use an increasing-returns technology to produce each individual good, subject to the demand functions implied by (12). Writing $x \equiv Y_i/Y$, Kimball (1995) shows that the elasticity of demand of demand facing an individual producer is

$$\varepsilon(x) = -\frac{\Upsilon'(x)}{x\Upsilon''(x)}$$

Hence, the optimal markup of each monopolist is

$$\mu(x) = \frac{\varepsilon(x)}{\varepsilon(x) - 1}, \tag{13}$$

where ε is declining in the firm's market share, x. (In the well-known Spence-Dixit-Stiglitz case, ε is constant.) The markup at the symmetric steady state corresponds to $\varepsilon = \varepsilon(1) \equiv \varepsilon^*$. (We denote a steady-state value with a *.) Kimball (1995) shows that this specification can deliver arbitrarily large "real rigidity" in the sense of Ball and Romer (1990), depending on the curvature of $\varepsilon(x)$. We assume that $\varepsilon(x)$ is much less concave than does Kimball (1995), because factor attachment and the presence of intermediate goods also contribute to generating the degree of real rigidity needed to justify reasonable persistence of output fluctuations driven by shocks to nominal money.

A continuum of monopolistic firms maximize profit, taking the demand curves implied by (12) as given. They produce gross output subject to a Cobb-Douglas production functions, with increasing returns to scale from a fixed cost:

$$Q_i = ZK_i^{\alpha^G} N_i^{\beta^G} M_i^{1-\alpha^G-\beta^G} - F, \tag{14}$$

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where Z is the level of Hicks-neutral technology and F is the fixed cost. Around the steady state, the

degree of returns to scale in the production of *gross output*, Γ^G, is

$$\Gamma^G = \frac{Q^* + F}{Q^*}. \tag{15}$$

Materials input for every firm is assumed to be the same composite commodity as the final good used for

consumption, investment, or government purchases. Thus, each firm uses some of the output of all other

firms as intermediate inputs to production.[11]

As noted before, we assume that labor is "attached" to firms, so one can think of labor input at each

firm as coming from a single worker. However, in the baseline model we do not assume that capital is

attached; obviously, q-theory style adjustment costs, introduced in Section IV, creates some attachment.

We assume that there are zero profits in the steady state. One can show that this implies

$$\Gamma^G = \mu^G, \tag{16}$$

where μ^G is the optimal steady-state markup of all firms facing the demand functions for their products

discussed above (see, e.g., Basu and Fernald, 1997).

The value-added produced by each representative firm is computed as a Divisia index (i.e., in

growth rates) as

$$\frac{\dot{Y}}{Y} \equiv \frac{\dot{Q}/Q - s_M \dot{M}/M}{1 - s_M}, \tag{17}$$

where s_M is the share of expenditure on materials in gross revenue (equal to $1 - \alpha^G - \beta^G$ because of the

zero-profit assumption). The returns to scale in the production of real value added exceeds the returns to

scale in gross output, and equals the markup on real value added:

$$\Gamma = \frac{(1 - s_M)\Gamma^G}{1 - s_M \Gamma^G} = \mu, \tag{18}$$

where the second equality follows from the zero-profit assumption, (16).

The monopolistic firms (but not the final-goods assemblers) are assumed to face the friction that they can adjust their nominal prices only occasionally, with a constant Poisson probability of adjustment of ζ, as in Calvo (1983). Since the Calvo pricing mechanism is well known, we do not exposit the details here. See Kimball (1995) for a treatment of Calvo pricing in continuous time; we present the log-linearized equations in discrete time in the Appendix. In recent years, models with nominal rigidities of the standard Calvo or Taylor specifications have been criticized because they do not imply sufficient persistence of inflation to match the data.[12] Since it is not clear from the literature which sticky-inflation specification of aggregate supply should be preferred, and since we are primarily interested in aggregate-demand issues in this paper, we adopt the conventional Calvo-Taylor specification. However, at various points we discuss how our results might change if inflation were sticky.

The final important component of the model is our treatment of nominal interest rate determination. We can assume that short-run monetary policy keeps the nominal money stock growing at a constant rate, but does not respond to short-run shocks, or assume that it follows a nominal interest rate setting rule of the Taylor (1993) type.[13] In either case, the deviation of the real interest rate from its steady-state value can be written as

$$\Re - \Re^* = a + b_y y + b_\pi \pi + b_p p . \tag{19}$$

y is the log deviation of output from its steady-state value and p the log deviation of the price level from its steady-state trend. (We use the notation that lower-case letters represent log deviations from steady state of the levels denoted by their upper-case counterparts.) π is the *level* of inflation; we keep the real interest rate and the inflation rate in levels rather than in logs, since both can be negative. In the case that (19) is interpreted as a Taylor rule, we are assuming that the central bank puts at least some weight on a price level target. (Monetary rules with a 'nominal anchor' of this sort often have superior properties to

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[11] Basu (1995) suggests that intermediate inputs priced with sticky prices can be an important source of real rigidity. A number of authors, including Bergin and Feenstra (2000) and Dotsey and King (2001), confirm that modeling the use of intermediate goods helps dynamic models match the persistent real effects of monetary shocks.
[12] See, for example, Fuhrer and Moore (1995), Mankiw and Reis (2001), and Christiano, Eichenbaum, and Evans (2001).

those that target only inflation; Gorodnichenko and Shapiro (2004) find that the Fed's policy does target the price level.) The variable is the source of monetary policy shocks; it is either the level of the nominal money supply, or the innovation to the interest rate target.

One major difference between regarding (19) as a money market equilibrium condition with fixed real money balances (an "LM curve") and as a monetary policy rule is the interpretation and magnitude of b_π.[14] If (19) represents a money market equilibrium condition, then $b_\pi = -1,$ implying that the real interest rate falls when inflation rises. If (19) is interpreted as a Taylor rule, and the monetary authority follows the Taylor principle, then b_π should be positive, implying that the monetary authority raises the real rate to slow the economy when inflation rises. (In Taylor's original formulation, $b_\pi = 0.5.$) In the numerical simulations we follow the policy of treating (19) as an LM curve because we find it easier to understand the model responses intuitively when monetary policy is not endogenous. But changing to a simple form of the Taylor rule (one without interest rate smoothing) is as easy as changing a parameter.

The second major difference between treating (19) as a Taylor rule and as an LM curve is the interpretation of the output gap, y. In Taylor's original formulation, and in most descriptions of optimal policy, the output gap is interepreted as the difference between current output and current flexible-price output, where the latter is conditional on the current levels of all state variables other than the price level. Our assumption that y is instead the deviation between current and *steady-state* output reflects our assumption that the Fed observes shocks to the flexible-price output level only with a long lag (by which time the price level has already adjusted to the shock).

Section II. The Model Without Adjustment Costs

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[13] Taylor's original rule was written in terms of one-period lagged targets; in continuous time, it is easiest to write the rule in terms of current values.

[14] The value of b_p also differs in the two specifications—in the LM curve, $b_p = -1$, while with the Taylor rule we know only that $b_p > 0$—but these differences are less important.

We first discuss a diagrammatic approach to understanding short-run equilibrium in the model without adjustment costs at a point in time. We then present impulse responses of the model, and see to what extent the intuition is confirmed by the numerical solution to the full model.

A. An Intuitive Approach: The KE-MP Diagram

We first model an economy without any investment frictions—essentially a standard RBC model with nominal price rigidity. We do so for two reasons. First, we find it useful for building intuition that carries over to the more realistic model. Second, it is useful to understand the case without adjustment costs, because the model with investment planning frictions deviates from this baseline model for less than a year, a shorter period than the length of time over which price stickiness is important.

We model the determination of the rental rate by introducing capital rental firms. These firms are competitive, and simply buy capital and rent it out to the monopolistically competitive producers. Without adjustment costs, they face a trivial problem:

$$Max \int_{t=0}^{\infty} e^{-\int_0^t \Re(s)ds} RKdt$$

$$s.t. \tag{20}$$

$$\dot{K} = I - \delta K$$

In order to analyze the model without adjustment costs, we present log-linear versions of the key optimality conditions from the consumer and firm optimization problems in Section I. Other than determining the dynamics of pricing decisions, the monopolistic firms hire factors to satisfy demand and substitute between capital and labor to minimize costs. Using equations (14) and (17), the log deviation of value-added output from its steady-state level is:

$$y = \Gamma\left(\alpha k + (1-\alpha)(n+z)\right), \tag{21}$$

where $\alpha = \alpha^G/(1-s_M)$. With price above marginal cost so that the firm wants to satisfy demand at a given price, the production function can be inverted to find the amount of labor needed to produce a given level of output:

$$n = \frac{y - \alpha k}{\Gamma(1-\alpha)} - z. \tag{22}$$

This labor requirements equation can be thought of as giving the location of the short-run labor demand curve, which is vertical since the real wage does not appear in the equation.

The constant cost shares for capital and labor that follow from the generalized Cobb-Douglas assumption imply the following relationship between detrended log labor, capital and the pre-tax detrended log wage and rental rate:

$$r + k = w + n. \tag{23}$$

As for the household, the logarithm λ of the marginal value of wealth Λ evolves according to the usual household Euler equation

$$\dot{\lambda} = \rho - \Re.$$

Given the King-Plosser-Rebelo form of the utility function, consumption and labor supply are governed by the equations

$$c = -\sigma\lambda + (1-\sigma)hn \tag{24}$$

$$\bar{w} = -\sigma\lambda + \frac{1}{\eta}n \tag{25}$$

where $\bar{w}$ is the log real after-tax wage (that is, the effective wage as viewed by the household), η is the *Frisch* labor supply elasticity (which is not the same as the consumption-constant labor supply elasticity when consumption and labor are not additively separable) and h is the steady-state ratio of after-tax labor income to consumption:

$$h = \left(\frac{WN}{C}\right)^* \approx 1.$$

Combining equation (23) with the labor supply equation (25), short-run labor demand equation (22), and the linearized version of the definition of the after-tax real wage (9), we get an equation for the log real rental rate as perceived by the firm:

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$$r = w + n - k = \overline{w} + \frac{1}{1-\tau_L^*}\left(\tau_L - \tau_L^*\right) + n - k$$

$$= -\sigma\lambda + \left(1+\eta^{-1}\right)n + \frac{1}{1-\tau_L^*}\left(\tau_L - \tau_L^*\right) - k \qquad (26)$$

$$= \frac{\left(1+\eta^{-1}\right)}{\Gamma(1-\alpha)}y - \left(1+\eta^{-1}\right)z - \sigma\lambda + \frac{1}{1-\tau_L^*}\left(\tau_L - \tau_L^*\right) - \left[1 + \frac{\alpha\left(1+\eta^{-1}\right)}{\Gamma(1-\alpha)}\right]k.$$

(Note that the tax rate is not log-linearized. We represent tax rates, like the real interest rate and inflation, in terms of level deviations from the steady state.)

Combining the real rental rate equation with the log-linearization of equation (8) gives us the deviation of the real interest rate from its steady-state value in terms of the variables on the right-hand side of (26):

$$\mathfrak{R} - \mathfrak{R}^* = \left(1-\tau_K^*\right)R^*\left\{\frac{\left(1+\eta^{-1}\right)}{\Gamma(1-\alpha)}y - \left(1+\eta^{-1}\right)z - \sigma\lambda + \frac{1}{1-\tau_L^*}\left(\tau_L - \tau_L^*\right) - \left[1 + \frac{\alpha\left(1+\eta^{-1}\right)}{\Gamma(1-\alpha)}\right]k\right\}$$
$$- \left(R^* - \delta\right)\left(\tau_K - \tau_K^*\right). \qquad (27)$$

Following Sargent's (1979) terminology for the parallel equation in Tobin's (1955) model, we call this equation the capital market equilibrium (KE) curve.[15] Comparing (26) and (27), both the real interest rate and the rental rate go up with output y, but for a given value of y, they go down with technology, down with the marginal value of wealth λ, down with the capital stock k, but up with the rate of labor taxation, τ_L. Furthermore, for a given value of the rental rate, the real interest rate goes down with the rate of capital taxation.

Several properties of (27) give important intuition about the workings of the New Keynesian model with capital accumulation. First, as noted by Tobin (1955), the real interest rate increases with output, regardless of the reason for the output expansion. For example, a monetary expansion increases the real interest rate.

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[15] However, it is important to keep in mind that our assumption of price rigidity makes the KE curve here differ in some important respects from Tobin's KE curve, which is based on nominal wage rigidity. To take just one

Second, the rental rate of capital—the best measure of the incentive to invest in this model without adjustment costs—often goes *down* with "positive" shocks, such as an improvement in technology, but *up* with "negative" shocks, such as an increase in the rate of labor taxation. The intuition is easiest to see from equation (23). That equation shows that the rental rate of capital goes up with employment and the real wage. The reason is that the marginal product of capital, in a cost-minimization sense, is the savings on the cost of the other input to production, labor. Shocks that lower the amount of labor required for any given level of output, such as an improvement in technology, or that lower the real wage, such as an increase in government purchases (which raises λ), reduce the benefits of having additional capital, and hence lower the rental rate. From this logic, it is clear why an increase in the labor income tax rate raises the rental rate on capital: it raises the pre-tax real wage at any given level of output. It is also clear why the capital income tax rate has a different effect. The capital tax does not enter equation (23) directly, although it can affect that equation by changing the marginal utility of wealth, λ. The direct effect of an increase in the capital tax rate is to create a wedge between the pre-tax rental rate and the real interest rate.

We can study informally the short-run determination of output and the real interest rate in the sticky-price model by graphing equations (19) and (27) in $\Re$-y space in Figure 1a. (The exact numerical solutions of the log-linearized model are presented in the next sub-section.) Stability is most easily guaranteed if the LM/MP curve cuts the KE curve from below, and we shall assume this condition in what follows.

From the discussion above, it is clear that shocks that are normally thought to increase output, like improvements in technology or an increase in government purchases financed by lump-sum taxes, will shift the KE curve down. For a given position of the LM curve, a downward shift of the KE curve reduces output and the real interest rate, as in Figure 1b. Hence, technology improvements will tend to be

example, fiscal policy has no real effects in Tobin's model. Kimball (2002) calls this capital market equilibrium condition based on price stickiness the Net Rental Rate (NRR) curve.

contractionary on impact, as will increases in government purchases—both results very different from what obtains in the same model with flexible prices.

One way of getting intuition for why these shocks lead to an overall reduction in output is to consider the effect on the markup, which is a good summary statistic for the overall distortion in sticky-price models (Goodfriend and King, 1997). "Positive" real shocks in a flexible-price model are normally those that lower marginal cost at each level of output. But in a model where the price level cannot jump, a downward jump in marginal cost causes the markup to rise, which raises the distortions in the economy and lowers output.

The discussion so far has held the inflation rate fixed, but inflation is a shift variable for the LM/MP curve. Most shocks that will expand output in the long run have an immediate negative impact on the inflation rate, since price changes are a strongly forward-looking variable. If the money stock does not respond to the shock (the LM curve case), then the LM curve shifts back, since the lower nominal interest rate raises money demand for all given real rates. This backward shift reinforces the contractionary effect of "positive" real shocks, as shown in Figure 1b . But if the LM/MP curve represents a Taylor-type rule, then the monetary authority would typically respond to lower inflation by lowering the real interest rate, which would shift the MP curve out, ameliorating the contractionary effects of the downward-shifting KE curve.

This is the place where changing the aggregate supply behavior to have sticky inflation is likely to make the largest difference. If inflation is sticky, then the LM/MP curve will not shift in the short run, since in the two scenarios we have just explored the curve shifts because shocks cause the inflation rate to jump. An qualitative version of the effect of a positive real shock with a Taylor rule and sluggish inflation is shown in Figure 1c. It is clear that the contractionary effect will be smaller than in the LM-rule case, but if inflation is sufficiently sluggish then the shock will still be contractionary.

B. Numerical Results

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We solve for the impulse responses of the sticky-price model whose elements and intuition we have sketched above. The log-linearized equations of the full model in discrete time are presented in Appendix A, together with the parameter values assumed. Several key equations and assumptions deserve some mention. First, we assume that the intertemporal elasticity of substitution in consumption is 0.5.[16] The King-Plosser-Rebelo utility function then requires that consumption and labor be complements. This is important, because it implies that changes in labor hours will induce changes in consumption in the same direction. We set the Frisch labor supply elasticity to 1, a low value relative to most DGE business-cycle models, but consistent with the composition-corrected labor supply estimates of Solon, Barsky, and Parker (1994). Following the updated version of the data set described by Jorgenson, Gollop, and Fraumeni (1987), we set $s_M = 0.5$. Following the recent estimates of the markup in, e.g., Basu and Kimball (1997), we set the *value-added* markup to a low value of 1.1 in the steady state, implying that the gross-output markup is 1.048; this choice also calibrates the two returns-to-scale parameters. Finally, we add some curvature to the elasticity of demand: we assume that $\mu'(1)/\mu(1) = 1.2334$, significantly lower than the value of 4.28 assumed by Kimball (1995).

Based on the values in Jones (2002), we set the steady-state values of the capital and labor taxes to 0.39 and 0.23, respectively. We take the LM curve interpretation of the KE-MP diagram, and choose the income elasticity of money demand to be 1, while setting the interest semi-elasticity to –0.5.

Since the theory is laid out in continuous time, we want the discrete-time implementation to come close to a continuous-time model. We thus make periods in the model correspond to a small length of time; we set the period length such that 100 periods equal one year. Thus, the persistence parameters on all AR(1) shocks need to be quite close to 1 to get reasonable persistence. We set all AR(1) shocks to fiscal policy to have a half-life of 2.5 years. The money shock is permanent.

The impulse responses for AR(1) shocks to government purchases, the labor tax, the capital tax, the and the money stock are shown in Figures 2-5. The fiscal shock is defined as a Y^*/G^* percent deviation

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[16] See Basu and Kimball (2002) for estimates of consumption Euler equations based on the King-Plosser-Rebelo

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from the steady state of government purchases (i.e., a shock that would cause a one percent increase in output if private expenditure were unchanged). The tax rate shocks are one *percentage point* increases (e.g., an increase in the capital tax from 0.39 to 0.40). The money shock is defined as a one percent increase in the nominal money stock. In all cases, the thin (green) solid line represents the impulse response of the basic sticky-price model we have laid out so far, while the dotted (dark blue) line shows the impulse response of the "RBC" version of the model—one with the same parameter values for most of the basic real parameters (including the utility function, tax distortions), but with constant returns to scale, and without sticky prices. (For now, ignore the other two lines, which will be discussed later.)

In order to isolate the effects of government purchases, we assume that the marginal government purchases represented by the shock are financed by lump-sum taxes. Similarly, in the case of tax rate shocks, we assume that the extra revenue is rebated to consumers in a lump-sum fashion. However, to facilitate comparison, the steady-state tax rates are the same in all cases, implying identical distortions.

The effects of increases in government purchases and labor taxes generally accord well with the intuition of the simple LM-KE diagram. An increase in government purchases has a slight contractionary effect on output on impact in the sticky-price model, but raises output in the RBC model, as one would expect. The labor tax shows the contrast more dramatically—output in the RBC model falls, while it rises sharply in the sticky-price case. As we explained, however, the capital tax rate has a different effects—an increase in capital taxation makes investment less attractive, so investment and output fall.

The counter-intuitive effects of fiscal shocks in the sticky-price model are not limited to quantities. The intuition of the RBC model says that the rental rate of capital, and hence the real interest rate, should rise with an increase in *G*, but in the sticky-price case both fall—just as the KE-LM framework predicts. (The real wage, however, falls in both models.)

The effects of money shocks are shown in Figure 5. Note that our high real rigidity allows us to meet Chari, Kehoe and McGrattan's (2000) standard that the effects of monetary shocks on output should have a half-life of 2.5 years. This result is due to the combination of real rigidities in the model,

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(1988) utility function. They find estimates around 0.5 are plausible for the recent period (since about 1980).

discussed in Section I. Thus, if the challenge is to present a completely-specified, optimizing DGE model with capital accumulation and sticky prices in which shocks to nominal money can have very persistent real effects, we show that this challenge can easily be met.

However, there are several counterfactual implications of this simple sticky-price model. First, as noted in the intuitive discussion, an expansion in the money supply raises the real interest rate. Second, the model does not predict a delay in the real effects of money on aggregate variables: output, investment, consumption, and hours worked all jump in the first instant, when the money shock is announced. (This prediction will change depending on the time-series properties of the monetary shock. A more complex ARMA process—such as the ARMA(2,2) suggested by Christiano, Eichenbaum, and Evans (1998)—can make the basic model display some hump-shaped dynamics, but the result will be very sensitive to parameter values, including the speed of price change.)

Section III. The Model with Planning Adjustment Costs

The cleanest way to understand how the equations for planning adjustment costs work is to focus again on a competitive capital-owning-and-lending firm that invests in capital and then rents it out to production firms. The firm faces both Q-theory type "capital adjustment costs," controlled by a parameter that can be made zero, and planning adjustment costs. We show that a model with only planning costs has significant implications for the short-run effects of fiscal shocks, predictions that differ from those of the sticky-price model discussed in Section II, and come closer to matching the data. To make the derivations easier to understand we omit tax terms, although the full log-linearized model allows for distortionary taxes and their variation.

A. Capital-Rental Firms Redux

The key to understanding planning adjustment costs is the idea that investment takes time to execute so that an investment *project*, once started, extends over a significant length of time.   The planning

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adjustment costs induce smoothing of investment project starts. To emphasize the distinction between planning adjustment costs (a way of implementing the time-to-plan idea) and time-to-build, we will assume that investment expenditure, when it takes place in the schedule of the project, instantly yields useable capital. Capital rental firms now face a non-trivial optimization problem. We replace (20) with

$$\underset{S}{Max} \int_{t=0}^{\infty} e^{-\int_{\tau=0}^{t} \Re_{\tau} d\tau} \left[RK - KJ\left(I\!\big/_{K} \right) - K\Phi\left(S\!\big/_{K} \right) \right] dt$$

subject to

$$\dot{K} = I - \delta K \tag{28}$$

$$\dot{I} = S - \gamma I$$

$$K_0, I_0 \text{ given}$$

S is investment project starts, KJ(I/K) is the constant-returns Q-theory-type "capital adjustment cost" function, KΦ(S/K) is the constant-returns planning adjustment cost function, and γ is the rate at which investment projects are completed. Both J and Φ are increasing and convex.

Note that planning adjustment costs make I as well as K a state variable. If we write Q for the costate variable of K and ξ for the costate variable of I, the current-value Hamiltonian is

$$H = RK - KJ\left(\frac{I}{K} \right) - K\Phi\left(\frac{S}{K} \right) + Q\left(I - \delta K \right) + \xi\left(S - \gamma I \right) \tag{29}$$

The first-order condition for optimal project starts S is

$$\Phi'\left(S/K \right) = \xi. \tag{30}$$

The Euler equations for Q and ξ are

$$\dot{Q} = \Re Q - \frac{\partial H}{\partial K}$$
$$= \left(\Re + \delta \right) Q - R + \left[J\left(\frac{I}{K} \right) - \frac{I}{K} J'\left(\frac{I}{K} \right) \right] + \left[\Phi\left(\frac{S}{K} \right) - \frac{S}{K} \Phi'\left(\frac{S}{K} \right) \right] \tag{31}$$

and

$$\dot{\xi} = \Re\xi - \frac{\partial H}{\partial I}$$

$$= (\Re + \gamma)\xi - R + J'\left(\frac{I}{K}\right) - Q$$

(32)

These equations are implemented in our simulations in log-linearized form. The standard certainty-equivalence approximation allows us to apply the impulse response functions from the log-linearized equations in a perfect-foresight model to stochastic disturbances.

In log-linearizing, we normalize the steady-state values as follows:

$$J\left(\frac{I^*}{K^*}\right) = 0$$

$$J'\left(\frac{I^*}{K^*}\right) = 1$$

$$\Phi\left(\frac{S^*}{K^*}\right) = \Phi'\left(\frac{S^*}{K^*}\right) = 0$$

It follows that in steady state

$$Q^* = 1$$
$$\xi^* = 0$$
$$S^* = \gamma I^*$$
$$I^* = \delta K^*$$
$$\bar{R}^* = \Re^* + \delta.$$

Then, using small letters to represent detrended logarithms of the corresponding variables, the first-order condition becomes

$$\varphi(s - k) = \xi,$$

(33)

where ξ is not logged, since $\xi^* = 0$, and $\varphi \equiv \dfrac{S^*}{K^*}\Phi''\left(\dfrac{S^*}{K^*}\right) = \delta\gamma\Phi''(\delta\gamma)$.

In log deviations, the accumulation equations become

$$\dot{k} = \delta(i - k)$$

(34)

$$\dot{i} = \gamma(s - i)$$

(35)

and the two Euler equations become

$$\dot{q} = \Re - \Re^* + R^*(q - r) - j\delta(i - k) - \varphi\delta\gamma(s - k) \tag{36}$$

$$\dot{\xi} = (\Re^* + \gamma)\xi - q + j(i - k) \tag{37}$$

where j is the steady-state elasticity $j = \dfrac{\delta J''(\delta)}{J'(\delta)}$.

B. The Ultra-Short-Run Adjustment of Investment

In our full general equilibrium model, aside from those that govern the policy dynamics and the dynamics of exogenous variables, there are three state variables: capital, the price level, and investment. (It is planning adjustment costs that make investment a state variable here.) With what we consider reasonable parameter values, the adjustment of investment is faster than the adjustment of prices, which in turn is faster than the adjustment of the capital stock. Although it is easy to compute the full equilibrium as we do below, for the sake of building intuition it is useful to look at an approximation that builds on this hierarchy of adjustment speeds. Kimball (1995) makes heuristic use of a "fast-price-adjustment approximation," which treats the overall movement in the capital stock that takes place while prices adjust as negligible. In this section, we make heuristic use of a "fast-investment-adjustment approximation" which treats the overall movement in the price level while investment adjusts as negligible. To put things a different way, we use approximation theory to expand on the hoary pedagogical dichotomy of short-run versus long-run. Instead of just a short-run and a long-run, we have (1) an *ultra short run* in which investment adjusts toward the short-run equilibrium, but the price level and the capital stock move very little, (2) a *short-run* in which the economy is in short-run equilibrium with prices adjusting toward the medium-run (full-employment) equilibrium and (3) a *medium-run* in which the economy is in the medium-run (full-employment) equilibrium familiar from Real Business Cycle Theory with the capital stock adjusting toward long-run equilibrium.

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To complete ultra-short-run equilibrium, we use the equations for household behavior, production firm behavior, and monetary policy that were discussed above. As far as monetary policy goes, in the ultra-short-run analysis we will treat inflation π as known and its ultra-short-run movements *after* any initial jump as negligible. Then the dynamics of any monetary policy specified as in equation (19), with a being a monetary state variable that evolves slowly, will effectively depend only on log output, y, in the ultra-short run. This is what we will assume.

Other than the dynamics of pricing decisions, which affect the economy relatively little in the ultra-short run, the production firm hires factors to satisfy demand and trades off capital and labor to minimize costs. Neglecting taxes, equation (26) for the KE curve simplifies to:

$$r = \frac{\left(1+\eta^{-1}\right)}{\Gamma\left(1-\alpha\right)}y - \left(1+\eta^{-1}\right)z - \sigma\lambda - \left[1 + \frac{\alpha\left(1+\eta^{-1}\right)}{\Gamma\left(1-\alpha\right)}\right]k. \tag{38}$$

In the ultra-short run, and even in the short run, the movements in λ *after* any initial jump in λ can be treated as negligible, especially since λ gets multiplied by the presumptively small (meaning significantly less than 1) elasticity of intertemporal substitution σ in both the consumption and labor supply equations. Thus, overall, the ultra-short-run approximation is defined by treating movements in k, p, π and λ as negligible. "Treated as negligible" means that these variables themselves are treated as if they were constant, *but* the real interest rate $\Re$ and log investment i that appear in the derivatives of λ and k are *not* treated as constant. The stock-flow distinction—whether in prices or in quantities—allows a flow rate of change to move around without the stock being able to move much if the time interval is short.

C. The Return of the Keynesian Cross: Aggregate Expenditure when Investment is a State Variable

A key characteristic of the Keynesian Cross as still taught in many introductory and intermediate macroeconomics courses is that (as the Keynesian Cross is first taught) the level of investment is treated as a known constant. Suppose one begins with a background of the standard sticky price machinery, including the initial steady-state cushion of price over marginal cost that allows the actual markup to vary

some distance before firms would have any hesitation at meeting demand. In that context, planning adjustment costs, by making investment a state variable, allow one to resurrect the simple Keynesian Cross on firm microeconomic foundations. Because firms satisfy demand, output must equal aggregate expenditure: $Y \equiv C + I + G$. In log-linear form,

$$y = \left(\frac{C^*}{Y^*}\right)c + \left(\frac{I^*}{Y^*}\right)i + \left(\frac{G^*}{Y^*}\right)g \tag{39}$$
$$\equiv \upsilon_C c + \upsilon_I i + \upsilon_G g,$$

where $\upsilon_C,$ $\upsilon_I,$ and υ_G are the aggregate demand shares of consumption, investment, and government purchases. With government purchases exogenous and investment a state variable, only consumption needs to be expanded. In conjunction with the consumption equation and the labor requirements equation, this yields

$$y = \upsilon_C\left[-\sigma\lambda + (1-\sigma)hn\right] + \upsilon_I i + \upsilon_G g$$
$$= \upsilon_I i + \upsilon_G g - \upsilon_C\sigma\lambda + \upsilon_C(1-\sigma)h\left[\frac{y-\alpha k}{\Gamma(1-\alpha)} - z\right].$$

The right-hand-side is an aggregate expenditure curve which—because of the permanent income hypothesis—slopes up with output only because of the complementarity between consumption and labor implied by King-Plosser-Rebelo preferences when σ < 1. The New Keynesian multiplier is

$$\frac{1}{1 - \dfrac{\upsilon_C(1-\sigma)h}{\Gamma(1-\alpha)}} \tag{40}$$

and ultra-short-run equilibrium output is

$$y = \frac{\upsilon_I i + \upsilon_G g - \upsilon_C\sigma\lambda - \upsilon_C(1-\sigma)h\left[\dfrac{\alpha k}{\Gamma(1-\alpha)} + z\right]}{1 - \dfrac{\upsilon_C(1-\sigma)h}{\Gamma(1-\alpha)}}. \tag{41}$$

The big advantage of the New Keynesian Cross over the old Keynesian Cross is that because it is microeconomically based, one is actually able to ask questions like "What happens to ultra-short-run aggregate demand when technology improves?" or "What happens when expected future government

purchases rise with current government purchases staying the same?" Future technology and government purchases operate through the term involving λ, while current government purchases and technology appear explicitly in the equation. We graph the New Keynesian Cross diagram in Figure 6.

One drawback of the New Keynesian Cross is that even in the ultra-short run, shocks to g and z will cause λ to jump. Thus, unlike the old Keynesian Cross, one cannot always study the effects of a change in a single exogenous variable without knowing something about intertemporal linkages.

Note that monetary policy does not directly appear in this equation. The effects of monetary policy on the equation operate mainly through the level of investment i. Thus, monetary policy has very little immediate effect on output. This means that an outward shift in the monetary policy curve will automatically generate a substantial liquidity effect.

D. Short-Run Equilibrium

"Short-run equilibrium" is an approximate equilibrium defined as the level of output that would prevail if there were no planning adjustment costs. (Thus, the discussion of Section II implicitly treated short-run equilibrium.) In this section, since we treat both output in the ultra-short run and the short run, we denote the short-run values with a "SR" superscript. In Sections II and IV, where we analyze models without planning adjustment costs, we are always considering short-run equilibrium, and thus drop the superscript.

Short-run equilibrium takes on a different form depending on whether the "capital adjustment cost" parameter j is zero or not. In this section, we will assume $j = 0$, treating planning adjustment costs as a pure *alternative* to capital adjustment costs. With no capital adjustment costs, the short-run equilibrium has the real interest rate equal to the net rental rate of capital, investment evolving only slowly (at about the rate at which price adjusts) and $Q = 1$. (Note that even without capital adjustment costs, planning adjustment costs allow Q to depart from 1 in the ultra-short-run.) In log-linear form,

$$\Re^{SR} - \Re^* = R^* r^{SR}$$
$$s^{SR} = i^{SR}.$$

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Combining the condition $\Re^{SR} - \Re^* = R^* r^{SR}$ with the monetary policy rule (19) and capital market equilibrium condition (38), yields the solution

$$y^{SR} = \frac{\dfrac{-a - b_\pi \pi}{R^*} - \left(1 + \eta^{-1}\right) z - \sigma \lambda - \left[1 + \dfrac{\alpha\left(1 + \eta^{-1}\right)}{\Gamma(1 - \alpha)}\right] k}{\dfrac{b_y}{R^*} - \dfrac{\left(1 + \eta^{-1}\right)}{\Gamma(1 - \alpha)}}. \tag{42}$$

Again, stability is easiest to guarantee if the monetary policy rule is steeper than the capital equilibrium curve, i.e., $b_y > R^*\left(1 + \eta^{-1}\right)\left[\Gamma(1 - \alpha)\right]^{-1}$, which we will assume here. Note that movements in z and λ that raise full-employment output reduce short-run equilibrium output y^{SR} unless the monetary authority adjusts the intercept in the monetary policy rule enough, *or* b_π is a large enough positive number that an associated fall in inflation π can make monetary policy sufficiently more expansionary to raise y^{SR}.

The ultra-short-run equilibrium equation (41) implies that the ultra-short-run dynamics of output are governed by movements in investment. Treating *g, z, λ* and *k* as slow-moving relative to the ultra-short-run,

$$y - y^{SR} = \frac{\upsilon_I\left(i - i^{SR}\right)}{1 - \dfrac{\upsilon_C(1 - \sigma)h}{\Gamma(1 - \alpha)}} \tag{43}$$

Below, we will need the corresponding gap $R - \delta - \Re$ between the net rental rate and the real interest rate. In log-linearized form, this is

$$R^*\left(r - r^{SR}\right) - \left(\Re - \Re^{SR}\right) = \chi\left(i - i^{SR}\right) \tag{44}$$

where

$$\chi = \frac{\left[b_y - R^*\left(1 + \eta^{-1}\right)\Gamma(1 - \alpha)\right]}{1 - \dfrac{\upsilon_C(1 - \sigma)h}{\Gamma(1 - \alpha)}}$$

E. The Dynamics of the Stripped-Down Ultra-Short-Run Model

The full model has four state or historical variables—capital, wealth, the price level, and investment—and four corresponding costate or jump variables—q, λ, the optimal reset price or inflation, and ξ. Of the eight dynamic equations, the ones for investment i and its costate variable ξ are especially fast. The dynamic equation for q is so closely entwined with the dynamics of ξ that it also must be included in even a stripped down model focusing on the high-speed movements of the model. Ignoring the quantities declared ≈ 0 above (and setting the Q-theory adjustment costs to zero, i.e., $j = 0$), one can write

$$\xi - \xi^{SR} = \varphi\left(s - s^{SR}\right)$$

$$\dot{i} = \gamma\left[\left(s - s^{SR}\right) - \left(i - i^{SR}\right)\right]$$

$$\dot{\xi} \approx \left(\mathfrak{R} + \gamma\right)\left(\xi - \xi^{SR}\right) - \left(q - q^{SR}\right)$$

$$\dot{q} \approx \mathfrak{R} - \mathfrak{R}^{SR} + R^*\left[\left(q - q^{SR}\right) - \left(r - r^{SR}\right)\right] - \varphi\delta\gamma\left(s - s^{SR}\right)$$

Using the first-order condition for s to eliminate s from these equations and equation (44) to eliminate the gap between the net rental rate and the real interest rate yields an approximate three-dimensional dynamic system:

$$\begin{bmatrix} \dot{i} \\ \dot{\xi} \\ \dot{q} \end{bmatrix} \approx \begin{bmatrix} -\gamma & \gamma\varphi^{-1} & 0 \\ 0 & \mathfrak{R}^* + \gamma & -1 \\ \chi & \delta\gamma & R^* \end{bmatrix} \begin{bmatrix} (i - i^{SR}) \\ (\xi - \xi^{SR}) \\ (q - q^{SR}) \end{bmatrix} \tag{45}$$

The characteristic equation for an eigenvalue ω of the dynamic matrix is

$$\omega^3 - \left(2\mathfrak{R}^* + \delta\right)\omega^2 + \left[\mathfrak{R}^*\left(\mathfrak{R}^* + \delta\right) - \gamma\left(\mathfrak{R}^* + \delta + \gamma\right)\right]\omega + \gamma\left[\mathfrak{R}^*\left(\mathfrak{R}^* + \delta + \gamma\right) + \chi\varphi^{-1}\right] = 0 \tag{46}$$

Since the characteristic function has a positive intercept and ω^3 as the leading term, it has at least one negative root. Since the second derivative indicates it is strictly concave for negative ω, it must have exactly one negative root. The positive trace $(2\mathfrak{R}^* + \delta)$ of the dynamic matrix must equal the sum of the three roots, so the other two roots must either both be positive or be conjugate complex roots with a

positive real part. We are interested in the negative root, which we will represent as $\omega = -\kappa$, where κ stands for the convergence rate of the ultra-short-run dynamics.

Rather than solving for the convergence rate κ, what we want to do is solve for how big φ would have to be to obtain a given convergence rate. Substituting $\omega = -\kappa$ into the characteristic equation (46) and solving for φ,

$$\frac{1}{\varphi} = \frac{\kappa^3 + \left(2\mathfrak{R}^* + \delta\right)\kappa^2 + \left[\mathfrak{R}^*\left(\mathfrak{R}^* + \delta\right) - \gamma\left(\mathfrak{R}^* + \delta + \gamma\right)\right]\kappa - \gamma\left[\mathfrak{R}^*\left(\mathfrak{R}^* + \delta + \gamma\right)\right]}{\chi\gamma}.$$

It is easy to calculate that when $\kappa = \gamma$ the denominator is 0. In other words, $\kappa = \gamma$ requires $\varphi = \infty$ and anything less than infinite φ will result in $\kappa > \gamma$. Intuitively, a finite planning adjustment cost always allows some adjustment in investment plans so that things adjust faster than the rate at which investment projects mature. But the planning adjustment cost parameter φ falls very quickly with κ. For large κ, φ goes down with the cube of κ. Consider the following numerical example to see that κ need not be that large before the required φ becomes very small:

$$\gamma = 1/\text{year}$$
$$\kappa = 2/\text{year}$$
$$\mathfrak{R}^* = 0.02/\text{year}$$
$$\delta = 0.08/\text{year}$$
$$b_y = 0.5/\text{year}$$
$$\eta = 0.667$$
$$\Gamma = 1.1$$
$$\alpha = 0.3$$
$$\upsilon_I = 0.24$$
$$h = 1$$
$$\chi = \frac{b_y - R^*\left(1 + \eta^{-1}\right)\Gamma(1-\alpha)}{1 - \left[\upsilon_C(1-\sigma)h\right]\left[\Gamma(1-\alpha)\right]^{-1}} = 0.12/\text{year}$$

These values imply $\varphi = 0.019$ year.

On its face this number looks small. To get a clearer picture of the meaning of this value for φ, note that the planning adjustment cost in relation to the cost of maintaining that capital stock is

$$\frac{K\Phi(S/K)}{\delta K} = \frac{\Phi(S/K)}{\delta}.$$

Using a Taylor expansion for Φ around the steady value $S^*/K^* = \delta\gamma$,

$$\Phi(S/K) \approx \frac{1}{2}\Phi''(\delta\gamma)[(S/K) - \delta\gamma]^2 = \frac{\delta\gamma\varphi}{2}[S/(\delta\gamma K) - 1]^2.$$

Thus, in this numerical example,

$$\frac{K\Phi(S/K)}{\delta K} = \frac{\gamma\varphi}{2}[S/(\delta\gamma K) - 1]^2 = 0.0096[S/(\delta\gamma K) - 1]^2.$$

In words, this says that even a doubling of the rate of investment project starts only has to raise overall investment costs by 0.96 % or a 10% increase in the rate of investment project starts only needs to raise overall investment costs by 0.0096% in order to generate approximately a 6 month aggregate delay when projects last about a year. (Since pressure on the planning process is likely to generate errors that affect the entire investment project, this way of gauging the size of the planning adjustment cost seems appropriate.)

For comparison, if $j > 0$, the Q-theory type adjustment cost relative to normal replacement investment costs is

$$K\delta^2 J''(\delta)\frac{[I/(\delta K) - 1]^2}{2\delta K} = \frac{j}{2}[I/(\delta K) - 1]^2.$$

Assuming Q-theory to be true, a value of $j = 0.2$ would be considered low (this is the value chosen by Kimball, 1995). That makes $j/2 = 0.1$, which is an order of magnitude higher than the corresponding coefficient needed on the planning adjustment cost to cause a six-month delay.

F. Numerical Results for the Costly-Planning Model

The results of the exact numerical solutions of the model confirm most of the intuition from our analytical approximations above. The impulse responses of the planning adjustment cost model to the AR(1) shocks discussed in Section IIB are the thick solid (red) lines in Figures 2-5. These uniformly show, as we argued, that planning costs create approximately a one-year delay before the impulse

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responses become essentially identical to those of the basic sticky-price model of Section II. (In the case of a monetary shock, this implies that the peak output effect is delayed 6-9 months, which is the minimum consistent with the data.) However, over the first year, the behavior of the model is quite different, even for modest planning adjustment costs. We set the length of a project, γ, to 1/year, implying that the half-life of an investment project, starting from initial conception to being half-completed (and half-productive), is about 8 months.[17] We set φ to 0.0022, almost an order of magnitude smaller than the already-small investment adjustment cost we calculated in the previous sub-section, in order to demonstrate that tiny planning costs can have a large effect on ultra-short-run behavior.

We find, as we had surmised using the logic of the New Keynesian Cross, that fiscal expansions now raise output and the real interest rate on impact (Figure 2). Output now rises by almost 1 percent following a shock to G that would augment output by 1 percent if private expenditure is unchanged. The real interest rate now rises almost 2 percentage points! However, the increase in output is smaller than one would predict based on the New Keynesian multiplier alone. Since the change in G is a 1 percent increase in aggregate expenditure, and the multiplier for our parameter values is about 2, the change in output predicted by the simple multiplier is about 2 percent. As we noted above, the reason for the difference is that the shock to G induces a change in λ, and the increase in λ is a downward shock to the "consumption function" assumed by the New Keynesian Cross. This intuition is confirmed by the rise in λ and the fall in C shown in the figure; according to a simple interpretation of the New Keynesian Cross, of course, C should rise, not fall.

Although the effect of government purchases is positive on impact, since the planning adjustment costs are unimportant after about a year, while price stickiness matters for 2-3 years, the impulse responses for both the costly-planning and the basic sticky-price model are both noticeably different from those for the basic RBC model. However, the qualitative differences between these two sticky-price models are now generally gone.

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[17] See Edge (2000) for a variety of evidence regarding planning and building times of different types of capital.

Figure 3 shows the results for the labor income tax cut. Here, since the differences relative to the basic RBC model were more pronounced, the addition of investment planning costs makes relatively little difference. The planning period eliminates most of the counterintuitive rise in output following a tax increase, but even with investment planning costs, there is a prolonged investment boom. The behavior of the capital stock, for example, is quite different in both sticky-price models relative to the RBC model.

The results for the capital tax increase in Figure 4 shows a qualitative change once we introduce planning costs: the tax increase is now expansionary. The reason is that the main effect of increased capital taxation is to reduce investment; with that channel blocked for some time, the rise in consumption following the tax increase raises output.

The planning adjustment costs have a striking effect on the model's ability to match the effects of monetary shocks (see Figure 5), as estimated by, e.g., Christiano, Eichenbaum, and Evans (1999). In most of their specifications, they find that (1) monetary policy shocks have a delayed effect on output, with the peak effect coming 3-4 quarters after the shock, (2) a strong negative effect of monetary expansions on the real interest rate, and (3) a real interest rate effect that is much shorter than the output or employment effect (see also Evans and Marshall, 1998). Note that the basic sticky-price model matches none of these three facts (at least for an AR(1) shock to money). The costly-planning version of the model, by contrast, matches all three. The third point is the most interesting for comparison with the Q theory-based model in the next section. There we shall find that the Q-theory model implies the liquidity effect must last as long as the real effects of money on the economy, which is counterfactual. The costly-planning mechanism seems necessary to match the short-lived real interest rate effect.[18]

Section IV. The Q-Theory Alternative: A Short-Run Analysis

A. Discussion of the Theory

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[18] Of course, this argument does not imply that Q-theory is not an useful addition to the basic sticky-price model, merely that it is not sufficient to make that model match all the facts.

We still analyze the model given by equations (28)-(37), but now instead of setting the capital adjustment cost parameter, j, to zero, we set $\varphi = 0$ instead. Without any planning adjustment costs, the three state variables are $k, p,$ and wealth. The associated costate variables are q, π (or the optimal reset price), and λ. With $\varphi = 0$, the ultra-short-run adjustment is effectively instantaneous, so there is no ultra-short-run story to speak of. The medium-run or full-employment equilibrium is the equilibrium that would prevail if all prices were perfectly flexible. Its characteristics are familiar from Real Business Cycle Theory. To look at the short-run dynamics that converge to the medium-run or full-employment equilibrium the capital stock and λ can be treated as moving very slowly, and the model can be analyzed using approximations similar to those employed above, and in Kimball (1995). However, since the treatment is lengthy, and the q-theory model is better-understood than the model with investment planning costs, we proceed directly to the numerical results.

B. Numerical Results for the Q-theory Model

Results for the model with capital adjustment costs are given in Figures 2-5. The model with Q-theory adjustment costs is represented by a heavy dashed (blue) line. We calibrate the Q-theory model by setting $j = 0.5$, which corresponds to a half-life of capital adjustment of one year. Recent empirical work by Hall (2002) using US industry data suggests a median estimate of $j \approx 1$ (albeit with large dispersion across industries).

As one would anticipate from the theory, an increase in government spending that is persistent does raise output persistently in the Q-theory model, about the same amount as in the RBC model. The reason is clear to see: due to the capital adjustment costs, investment jumps down by less in the Q-theory model than in the basic sticky-price model. This effect is offset to a large extent by a larger fall in consumption. Note, however, that the magnitudes of the Q-theory and RBC responses are much smaller than those found by Blanchard-Perotti (2002), who find that a one-dollar increase in government purchases raises GDP by about a dollar on impact. The real interest rate also rises modestly in the Q-theory model, and real wages fall about 0.50 percent.

The *Q*-theory model also produces results fairly similar to those of the basic sticky-price model and investment-planning-cost model in the case of tax shocks. In most cases, the impulse responses resemble those of the basic model, but are damped by the presence of the adjustment costs.

It is important to realize, however, that the predictions of the *Q*-theory model depend sensitively on the exact persistence of the shock. We confirmed this statement by varying the persistence parameters of the real shock processes (the results are not reported in the paper). Indeed, a fiscal shock of medium persistence (a half-life of about a year) led to a small decline in output on impact.

The most noticeable differences between investment and capital adjustment costs are seen in the results for monetary shocks, in Figure 5. Both the *Q*-theory model and the basic model imply that output jumps to its maximum at the time of the shock, and then declines monotonically thereafter. The investment planning model, by contrast, displays hump-shaped impulse responses, in keeping with the estimated effects of monetary policy shocks. (However, output does jump on impact, reflecting the fact that consumption is not inertial. Also, the peak output response occurs a little less than a year after the shock, which is sooner than most studies indicate. Adding consumption inertia to the model, perhaps via habit formation as Fuhrer (2000) suggests, may solve both problems.) Of course, planning costs make investment inertial and lead to a hump in investment. It is worth noting, however, that even absent any consumption frictions, the non-separability between consumption and labor in the King-Plosser-Rebelo utility function leads to a hump-shaped impulse response for consumption due to the hump-shaped time-path of employment (which in turn is due to the delay in investment).

The behavior of the real interest rate is also noteworthy. As we noted before, the model without investment frictions leads expansionary monetary policy to raise the real interest rate, because the variations in the real interest rate occur in lockstep with changes in the marginal product of capital. *Q*-theory-style adjustment costs do lead the model to predict a liquidity effect, but it is small and very persistent. On the other hand, the investment delay model predicts a large, fast-moving liquidity effect, which is much more in keeping with the data.

VI. Conclusion

We have exposited a mechanism—investment planning costs—and a way of using that mechanism to study sticky-price models with investment in an intuitive but still fairly rigorous fashion. We believe that the inertial-investment mechanism is supported by the data, and needs to be an integral part of sticky-price models with capital accumulation. Our graphical approach to studying these models, which is based on a set of useful approximations, may have some pedagogical value of its own, as it allows researchers to move away from the tradition of treating DGE computer models as black boxes.

We show that sticky-price models without investment frictions cannot match key estimates from the data. Most importantly, investment frictions are needed to generate a liquidity effect of expansionary monetary policy, and to match the stylized fact that fiscal expansions increase output. Almost as importantly, output inertia is needed to match the observed delay with which monetary policy changes affect real output, and to match the observation that the liquidity effect is much less persistent than the real output effect. We show that investment planning costs enable the model to match these aspects of the data, but standard *Q*-theory capital adjustment costs do not.

In order to focus on the analysis of planning costs, we have kept the rest of the model as simple as possible. However, in order to make the model more realistic, and match impulse responses from the data in a serious manner, it is clear that at least three additional mechanisms need to be added. First, the specification of monetary policy needs to allow the monetary authority to react to current economic conditions, as in the rule of Taylor (1993), but also needs to add nominal interest rate smoothing, which is an observed feature of monetary rules. Second, to keep the challenge facing monetary policy-makers from being unrealistically simple, one needs to add inflation inertia. As the model now stands, adding a nominal interest rate rule in which the Fed targets inflation would make both disinflation and reacting to real shocks very simple, since the inflation rate would often jump almost to its steady-state value on

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impact. Third, in order to exhibit complete and longer-lasting output inertia, the model probably needs to incorporate inertial consumption as well as investment. (An open-economy model would need to add a net export friction as well—perhaps not unrealistic in light of the common explanations of the *J*-curve.) While we wish to undertake these projects, we believe that including these realistic complications would distract attention from the simple message of this paper, and thus defer the task to future research.

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Set of Equations Common Across Models

$$c_t = (-\sigma)\lambda_t + (1-s)(h)n_t \qquad (A.1)$$

$$\bar{w}_t = (-\sigma)\,\lambda_t + \left(\frac{1}{\eta}\right)n_t \qquad (A.2)$$

$$\lambda_t = \Re_t + E_t\left[\lambda_{t+1}\right] \qquad (A.3)$$

$$n_t = r_t - w_t + k_t \qquad (A.4)$$

$$k_t = (\delta)\,i_{t-1} + (1-\delta)\,k_{t-1} \qquad (A.5)$$

$$y_t = (\Gamma)(\alpha)\,k_t + (\Gamma)(1-\alpha)\,n_t + z_t \qquad (A.6)$$

$$y_t = \left(\frac{c^*}{y^*}\right)c_t + \left(\frac{i^*}{y^*}\right)i_t + \left(\frac{g^*}{y^*}\right)g_t \qquad (A.7)$$

$$\bar{w}_t = w_t - \left(\frac{1}{1-\tau_n^*}\right)\tau_t^n \qquad (A.8)$$

$$\bar{r}_t = \left(\frac{R^*}{\bar{R}^*}\right)(1-\tau_k^*)\,r_t - \left(\frac{R^* - \delta_{tax}}{\bar{R}^*}\right)\tau_t^k \qquad (A.9)$$

$$\Re_t^n = \Re_t + \pi_t \qquad (A.10)$$

$$\pi_t = p_{t+1} - p_t \qquad (A.11)$$

$$pd_t = p_t + \left(\frac{1-(\mu^G s_m)}{\epsilon^*\omega}\right)\left[(\alpha)\,r_t + (1-\alpha)\,w_t - \left(\frac{1}{\Gamma}\right)z_t\right] - \left(\frac{1}{\epsilon^*\omega}\right)[(\chi)\,y_t]$$
$$\qquad (A.12)$$

$$\Re_t^n = (\Upsilon_y)\,y_t - (\Upsilon_m)\,m_t^d \qquad (A.13)$$

$$m_t^d = m_t - p_t \qquad (A.14)$$

1

Benchmark Parameter Values

$year = 100$ number of periods per year

$\sigma \quad = 0.5$ labor-constant elasticity of inter-temporal substitution

$\eta \quad = 1$ Frisch labor supply elasticity

$h \quad = 1$ steady state ratio of labor income to consumption

$\Upsilon_y = \frac{2}{year}$ coefficient on output in LM rule

$\Upsilon_m = \frac{2}{year}$ coefficient on money balances in LM rule

$\alpha \ = 0.3$ cost share of capital

$s_m \ = 0.5$ revenue share of intermediate inputs

$\varpi \ = 1.7789$ elasticity of Marginal Cost / Marginal Revenue with respect to firm output, holding aggregate output fixed

$\Delta = \frac{0.02}{per}$ steady state growth

$\delta \ = \frac{0.08}{year}$ depreciation rate

$\delta_{tax} = \delta - \Delta$ growth adjusted depreciation rate

$\Re^* = \frac{0.02}{per}$ steady state discount rate

$\epsilon^* = 11$ elasticity of demand on the firm's relative output

$\Gamma = 1.1$ degree of returns to scale to a balanced expansion in sticky price models

$\Gamma^{RBC} = 1$ degree of returns to scale for the RBC model

$u^G = 1.0476$ gross output markup

$\frac{c^*}{y^*} = 0.6182$ steady state share of private consumption expenditures

$\frac{g^*}{y^*} = 0.2018$ steady state share of government expenditures

$\frac{i^*}{y^*} = 0.18$ steady state share of investment expenditures

$\tau_n^* \ = 0.231$ steady state labor income tax rate

$\tau_k^* \ = 0.39$ steady state capital income tax rate

$\chi \ = 0$ elasticity of marginal revenue to income

$R^* = 0.001256$ before–tax steady state rental rate

$\bar{R}^* = 0.001$ after–tax steady state rental rate

Closing the Basic Real Business Cycle Model

$$\Re_t = (\Re^* + \delta) E_t [\bar{r}_{t+1}] \qquad (A.15)$$

$$p_t = pd_t \qquad (A.16)$$

Closing the Basic Sticky Price Model

$$\Re_t = (\Re^* + \delta) E_t [\bar{r}_{t+1}] \qquad (A.17)$$

$$p_t = p_{t-1} + (\zeta) \ [pr_{t-1} - p_{t-1}] \qquad (A.18)$$

$$pr_{t+1} = pr_t + (\zeta) \ [pr_t - pd_t] \qquad (A.19)$$

Closing the Investment Adjustment Costs (Time to Plan) Model

$$
\begin{aligned}
q_{t+1} \quad = \quad & q_t + (\Re^* + \delta) q_t - (\Re^* + \delta) E_t [\bar{r}_{t+1}] + \Re_t \\
& - (\gamma \delta \varphi) \ (S_t - k_t)
\end{aligned}
\qquad (A.20)
$$

$$v_{t+1} = v_t + (\Re^* + \gamma) v_t - q_t \qquad (A.21)$$

$$i_t = \gamma S_{t-1} + (1 - \gamma) i_{t-1} \qquad (A.22)$$

$$S_t = k_t + \left(\frac{1}{\varphi}\right) v_t \qquad (A.23)$$

$$p_t = p_{t-1} + (\zeta) \ [pr_{t-1} - p_{t-1}] \qquad (A.24)$$

$$pr_{t+1} = pr_t + (\zeta) \ [pr_t - pd_t] \qquad (A.25)$$

Closing the Capital Adjustment Costs (Q) Model

$$q_{t+1} = (1 + \Re^*) q_t - (\Re^* + \delta) E_t [\bar{r}_{t+1}] + \Re_t \qquad (A.26)$$

$$q_t = (j) [i_t - k_t] \qquad (A.27)$$

$$p_t = p_{t-1} + (\zeta) \ [pr_{t-1} - p_{t-1}] \qquad (A.28)$$

$$pr_{t+1} = pr_t + (\zeta) \ [pr_t - pd_t] \qquad (A.29)$$

Exogenous Shocks

$$z_t = (\rho_a)z_{t-1} + \varepsilon^z{}_t \tag{A.30}$$

$$g_t = (\rho_g)\,g_{t-1} + \left(\frac{1}{\left(\frac{g^*}{y^*}\right)}\right)\varepsilon^g{}_t \tag{A.31}$$

$$\tau^L_t = (\rho_{\tau^L})\,\tau^L_{t-1} + \varepsilon^{tL}{}_t \tag{A.32}$$

$$\tau^K_t = (\rho_{\tau^K})\,\tau^K_{t-1} + \varepsilon^{tK}{}_t \tag{A.33}$$

$$m_t = m_{t-1} + \varepsilon^m{}_t \tag{A.34}$$

Additional Parameter Values

Basic Sticky Price Model

$\zeta = \frac{1.38}{year}$ microeconomic rate of price adjustment

Time to Plan Model

$\gamma = \frac{1}{year}$ rate of completion of projects

$\varphi = \frac{year}{70}$ (inverse of) sensitivity of project starts to marginal value of investment

Q-Theory Model

$j = 0.5$ inverse of the elasticity of the investment-capital ratio to marginal q

Exogenous shocks

$\rho_a = 0.9999$ autocorrelation of technology shock

$\rho_g = 0.9972$ autocorrelation in government purchases

$\rho_{\tau^L} = 0.9972$ autocorrelation in labor income tax

$\rho_{\tau^K} = 0.9972$ autocorrelation in capital income tax

4

Figure 1a
The KE-MP Diagram
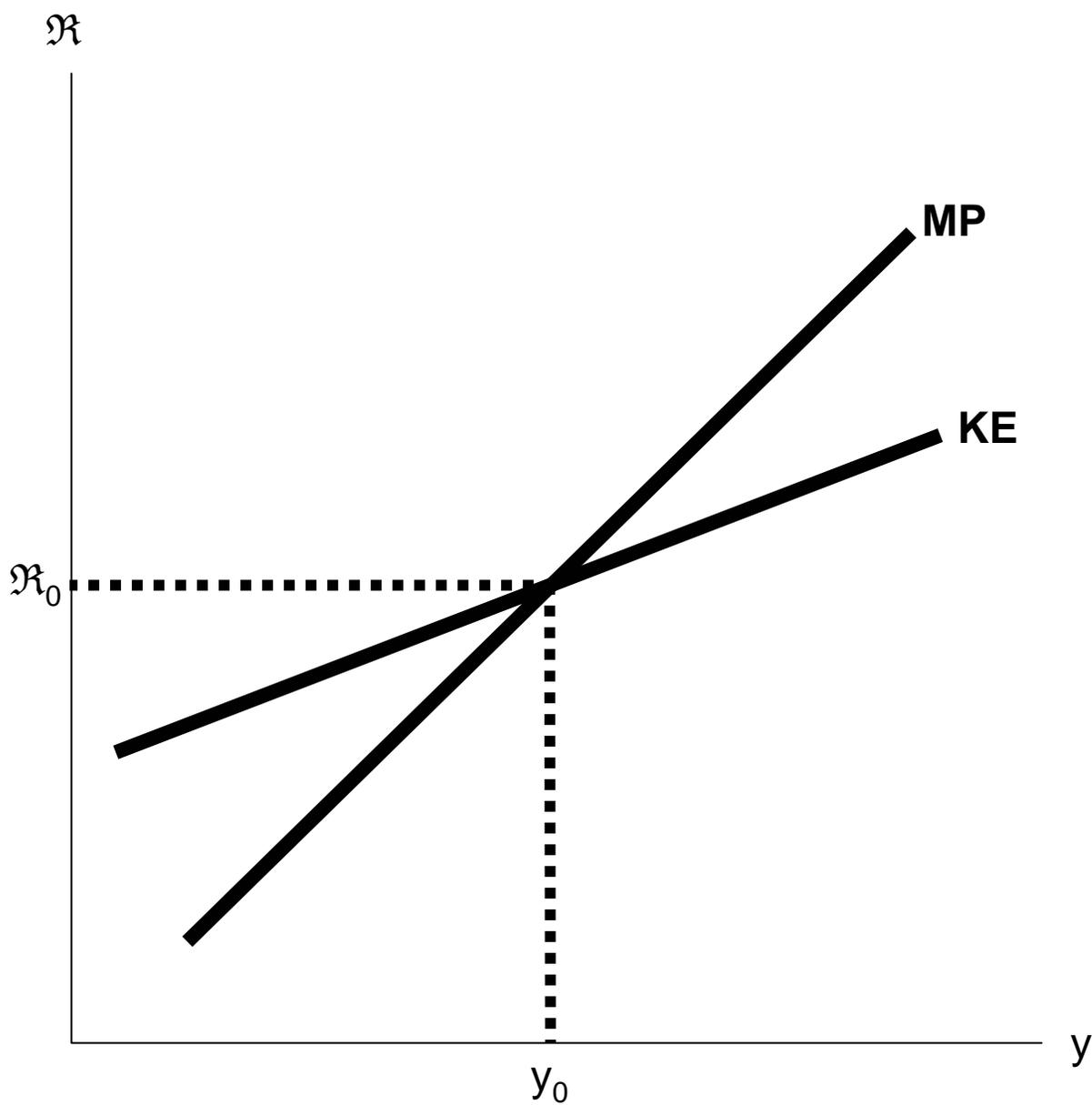
Figure 1b
The Effect of a Positive Real Supply Shock
Given a Constant Money Growth Rule
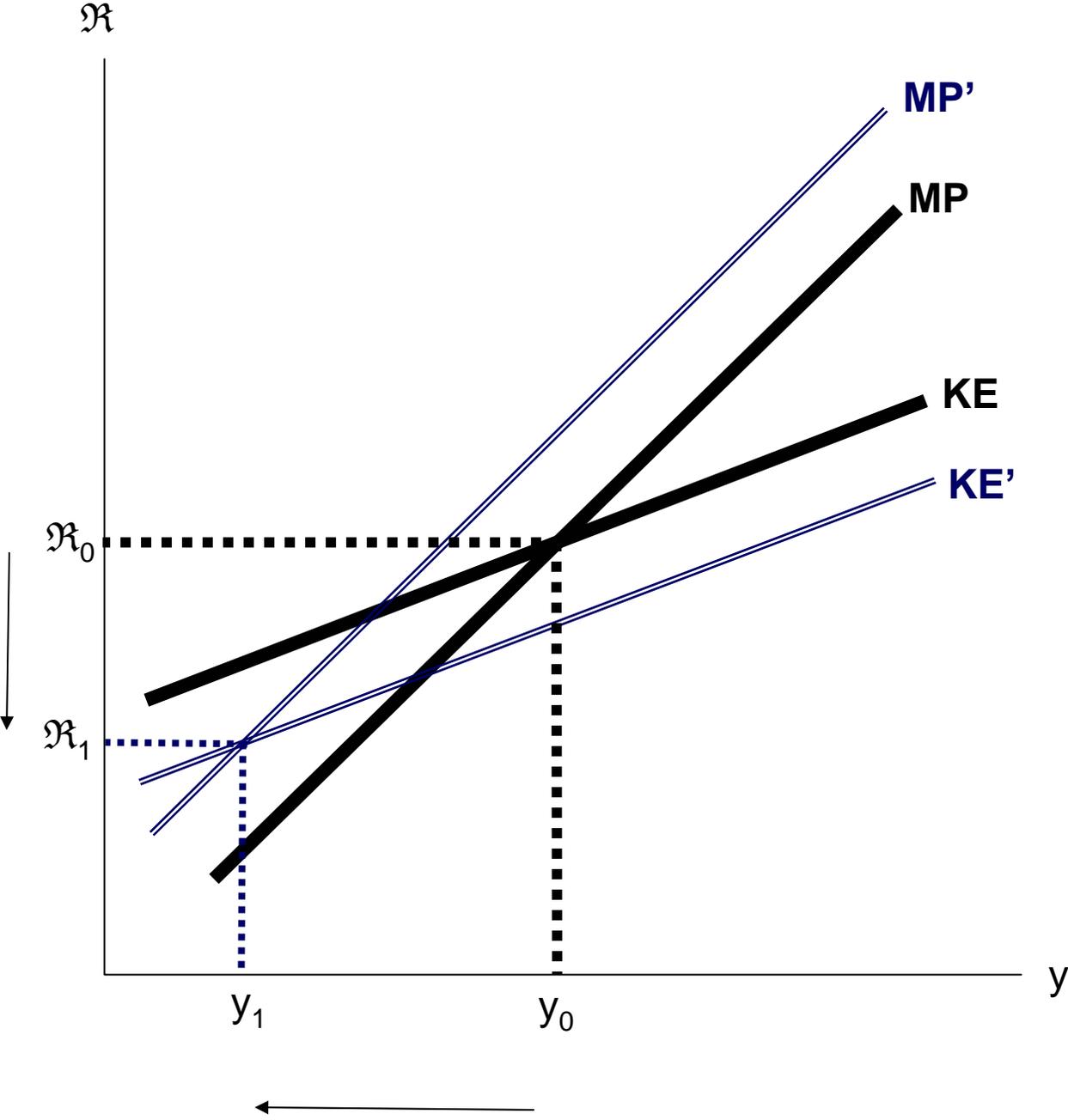
Figure 1c
The Effect of a Positive Real Supply Shock
Given a Taylor Rule and Sluggish Inflation
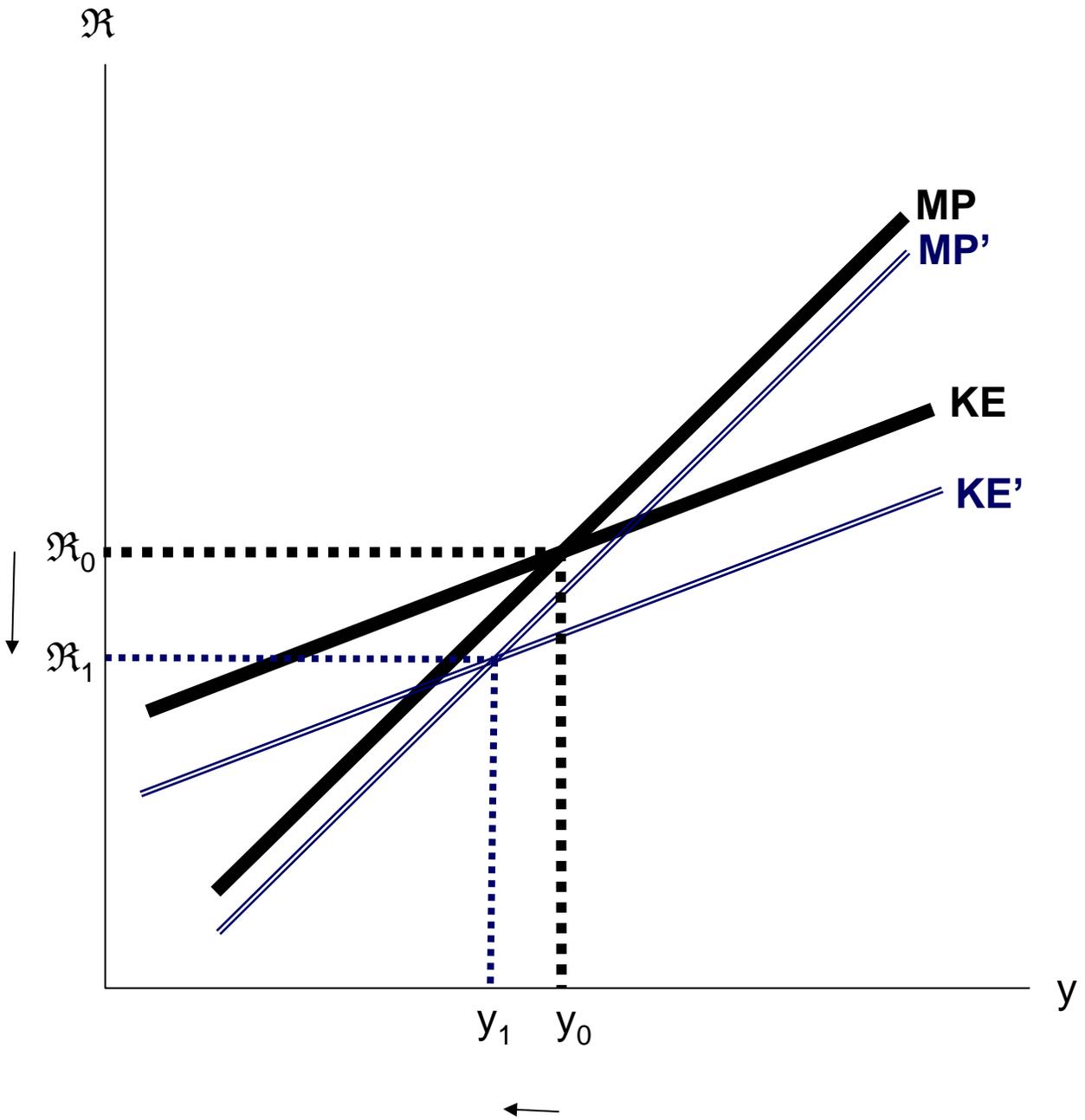
FIGURE 2. AR(1) POSITIVE GOVERNMENT SHOCK

OUTPUT

EMPLOYMENT

CONSUMPTION

INVESTMENT

RBC Model
BASIC STICKY PRICE Model
TIME TO PLAN Model
ADJUSTMENT COSTS Model

100 periods=1year

FIGURE 2. AR(1) POSITIVE GOVERNMENT SHOCK (cont'd)

BEFORE TAX RENTAL RATE

AFTER TAX RENTAL RATE

BEFORE TAX WAGE RATE

AFTER TAX WAGE RATE

RBC Model
BASIC STICKY PRICE Model
TIME TO PLAN Model
ADJUSTMENT COSTS Model

100 periods=1year
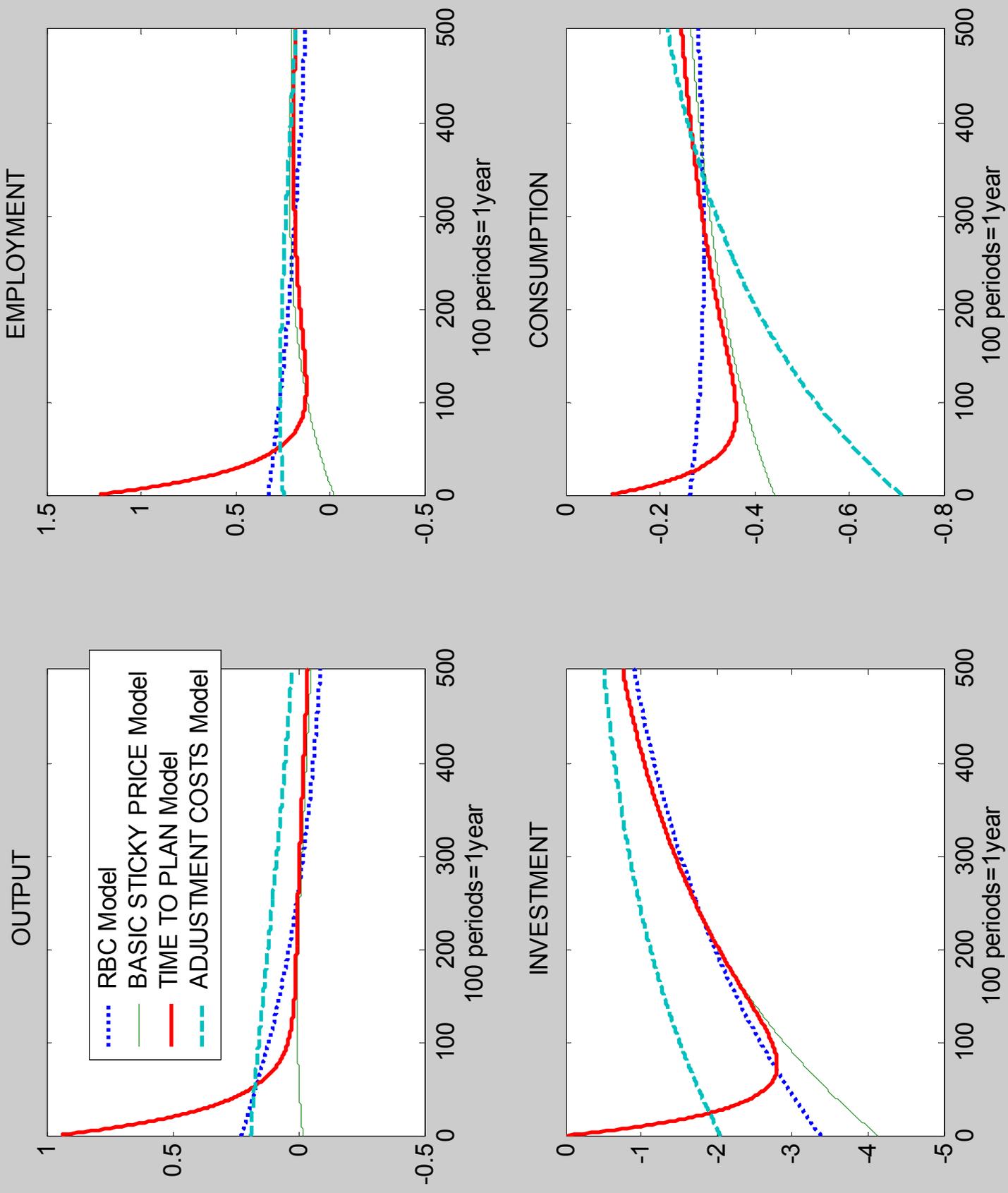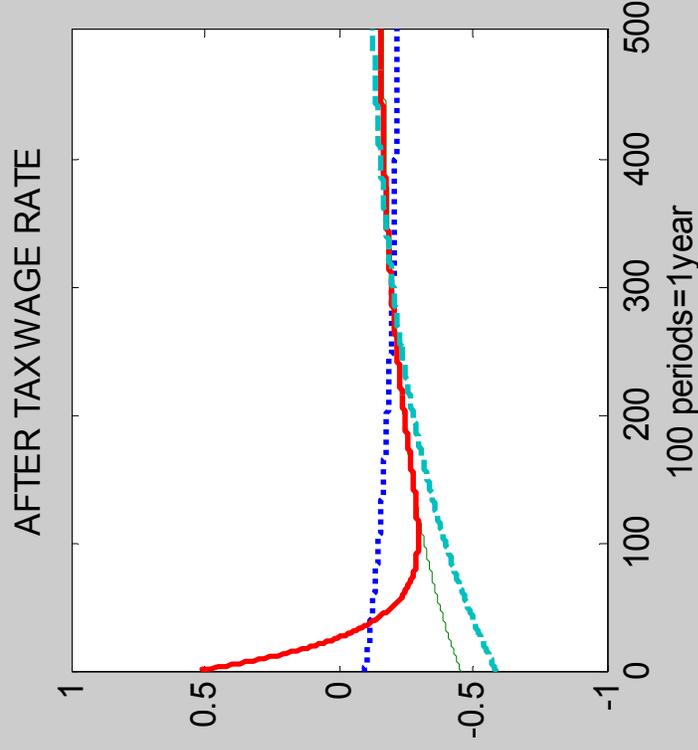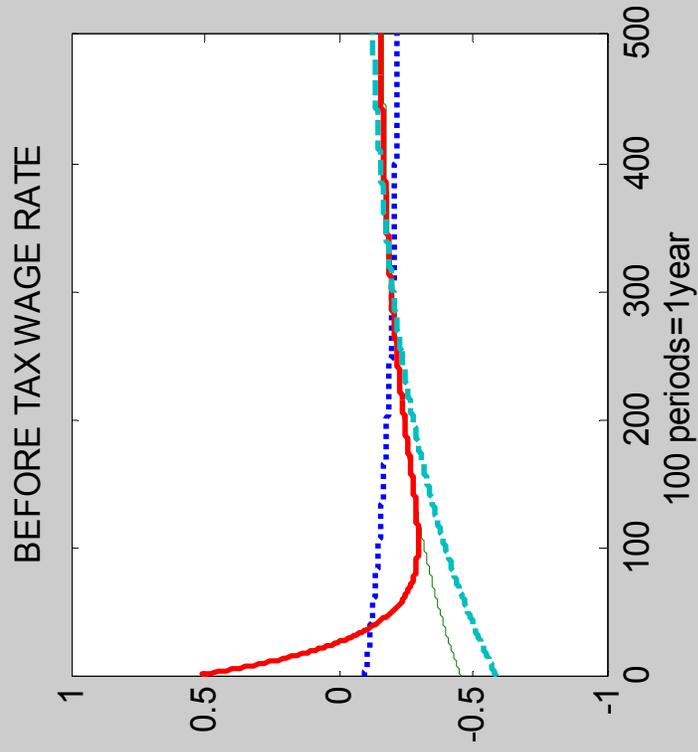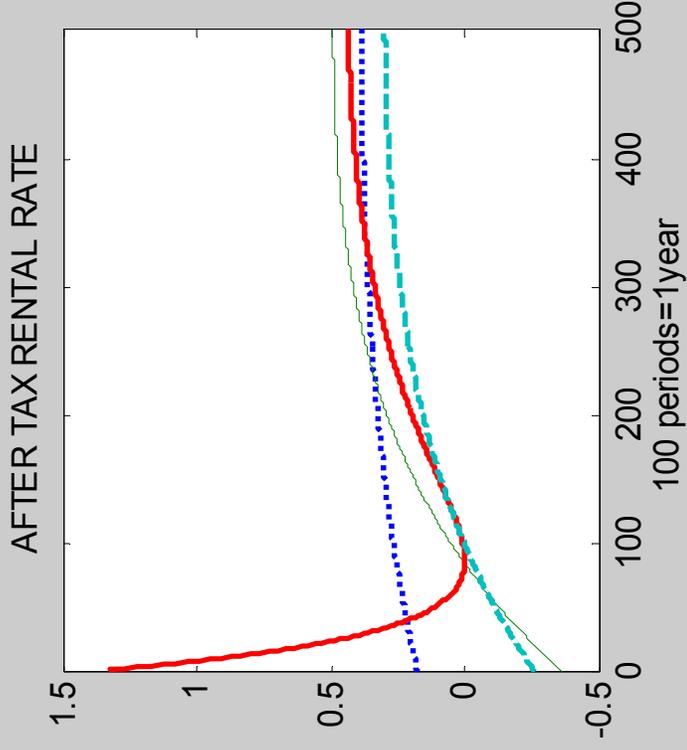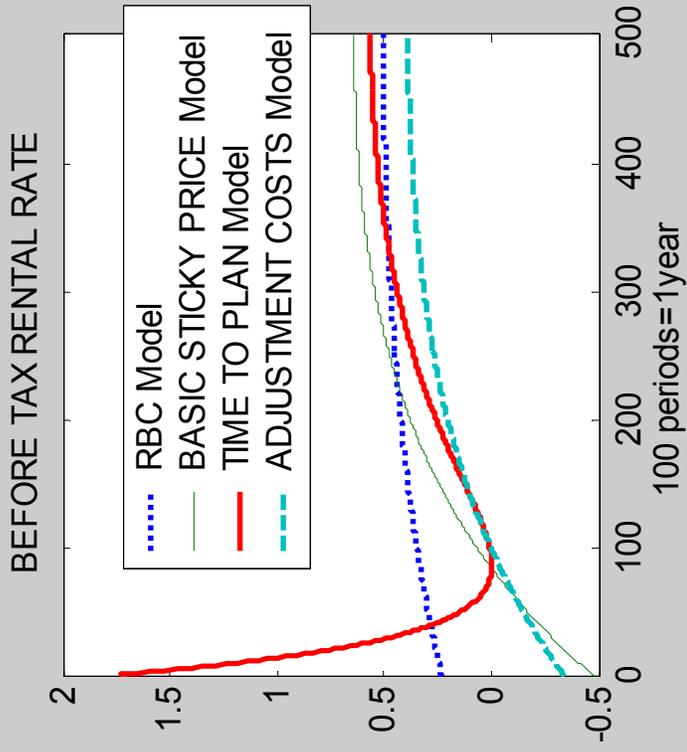
FIGURE 2. AR(1) POSITIVE GOVERNMENT SHOCK (cont'd)
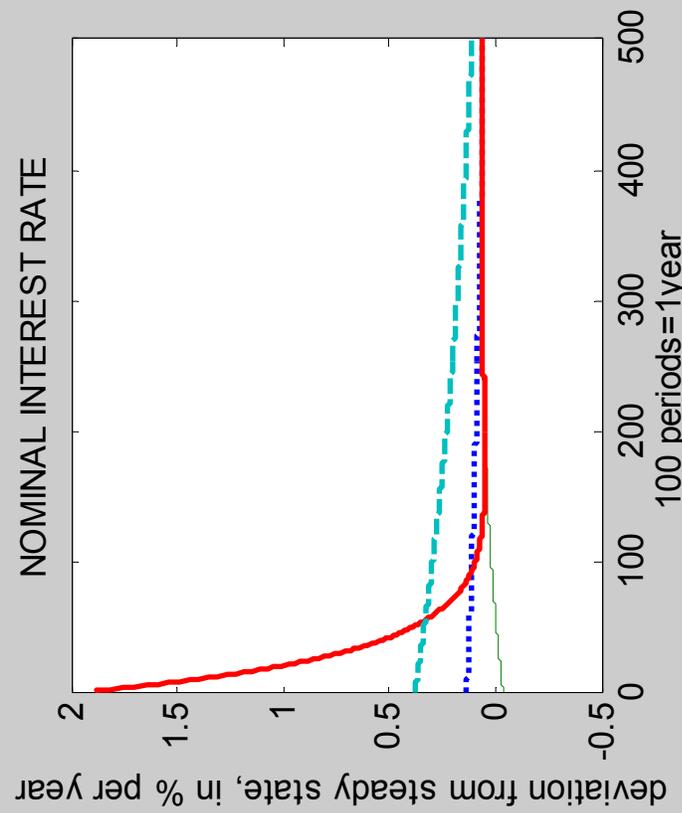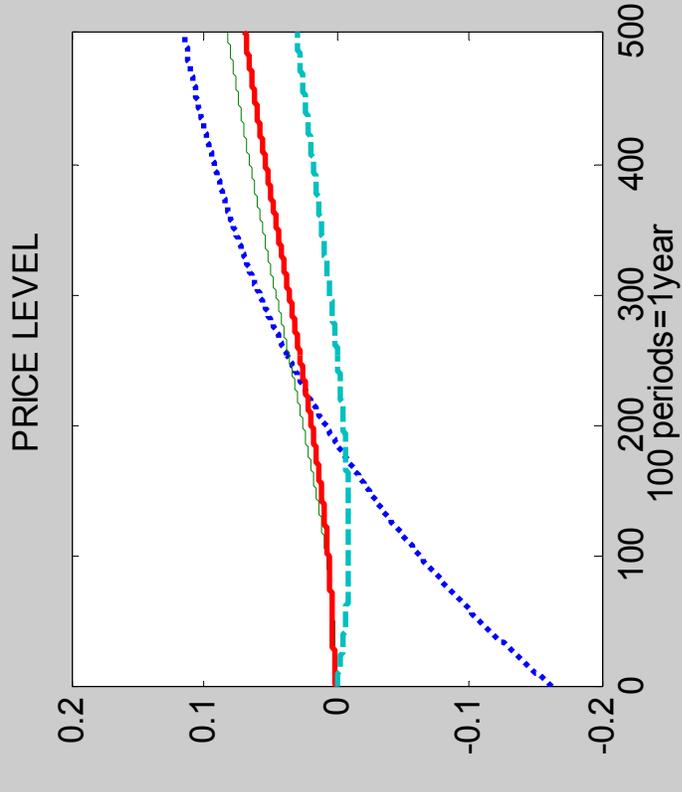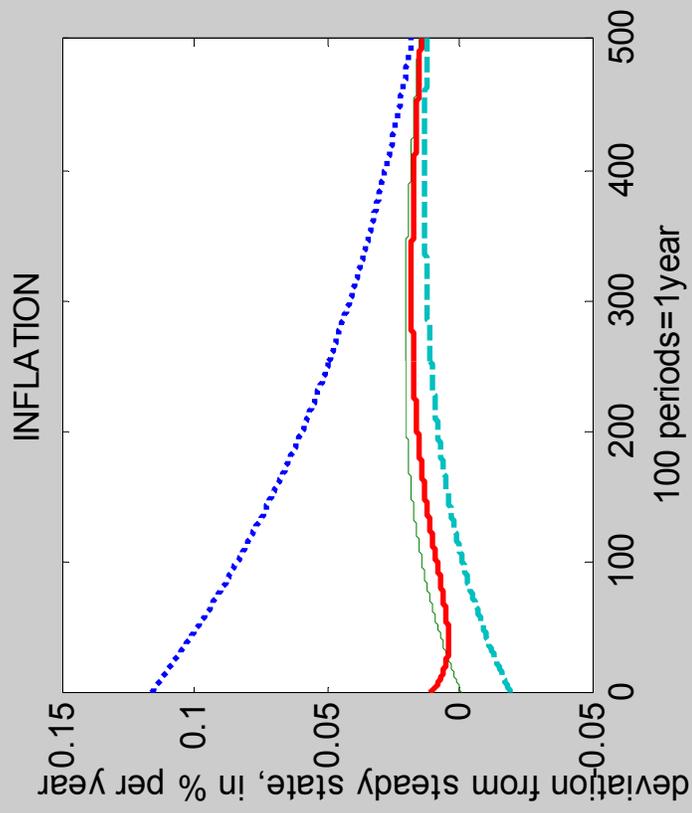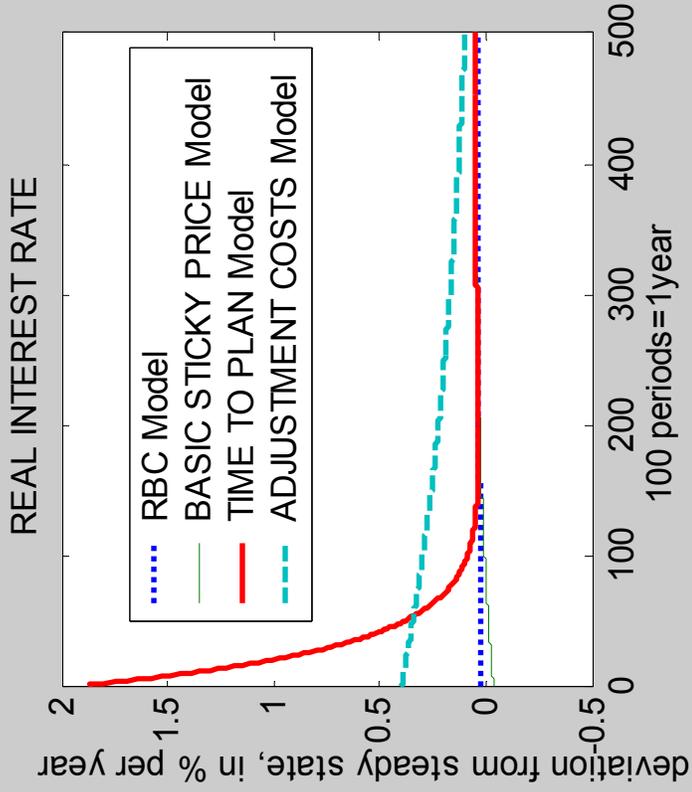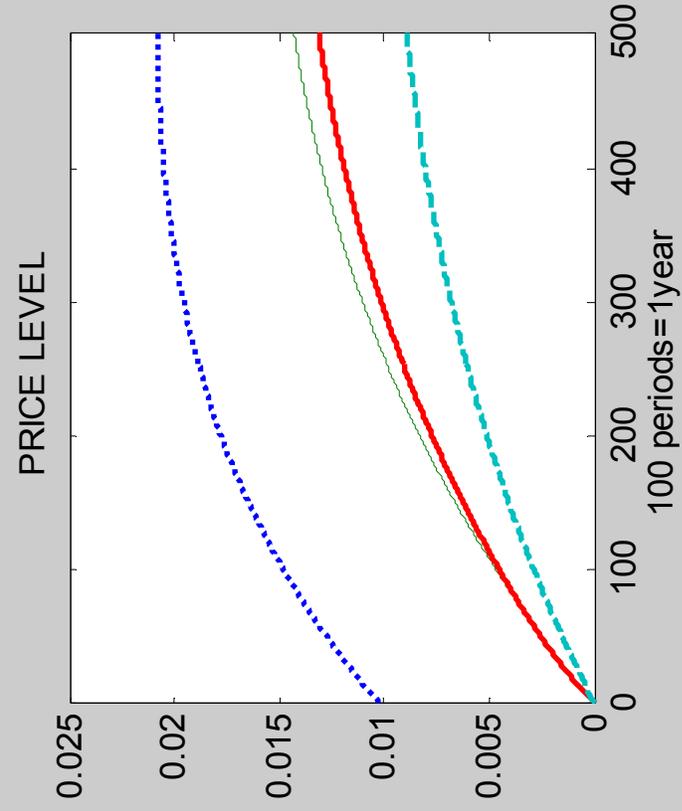
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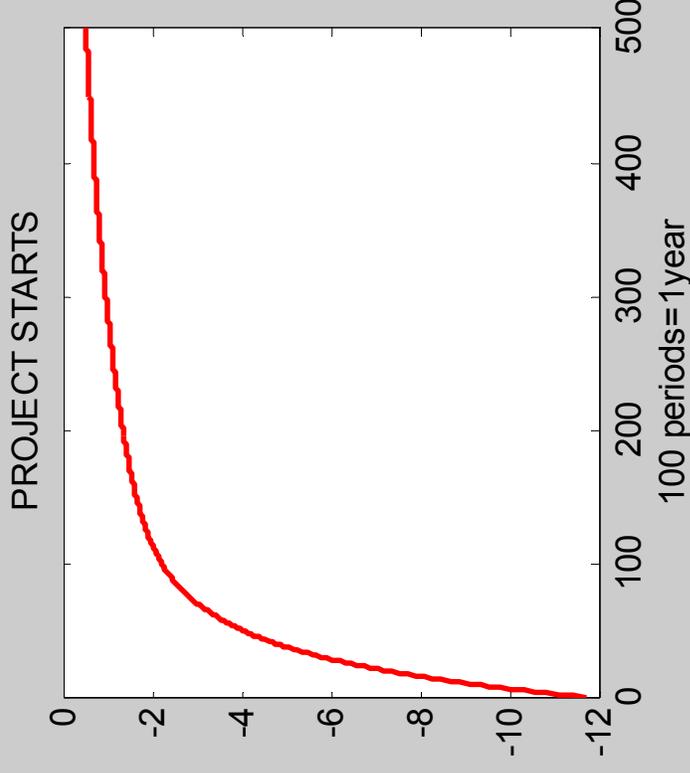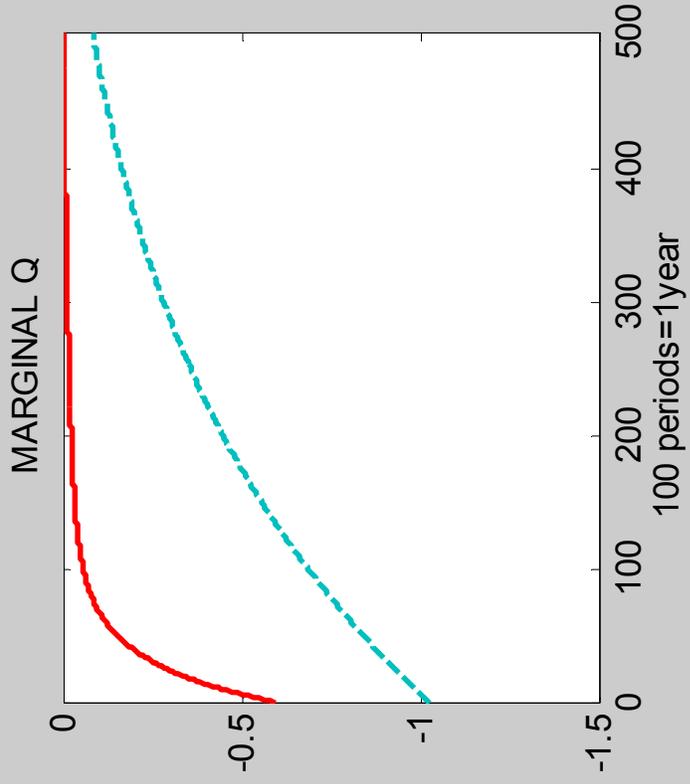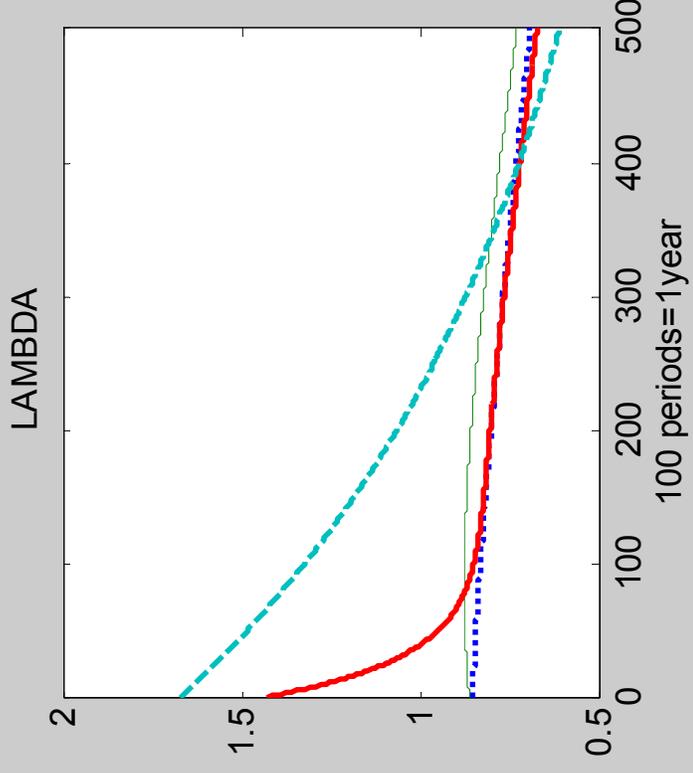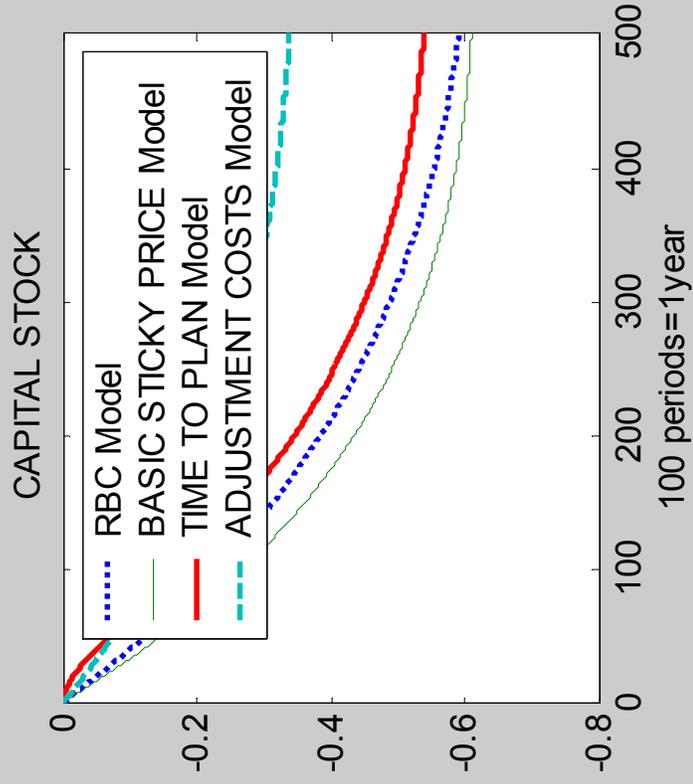
FIGURE 3. AR(1) LABOR TAX INCREASE

FIGURE 3. AR(1) LABOR TAX INCREASE (cont'd)

AFTER TAX RENTAL RATE

AFTER TAX WAGE RATE

BEFORE TAX RENTAL RATE

BEFORE TAX WAGE RATE

RBC Model
BASIC STICKY PRICE Model
TIME TO PLAN Model
ADJUSTMENT COSTS Model

100 periods=1year
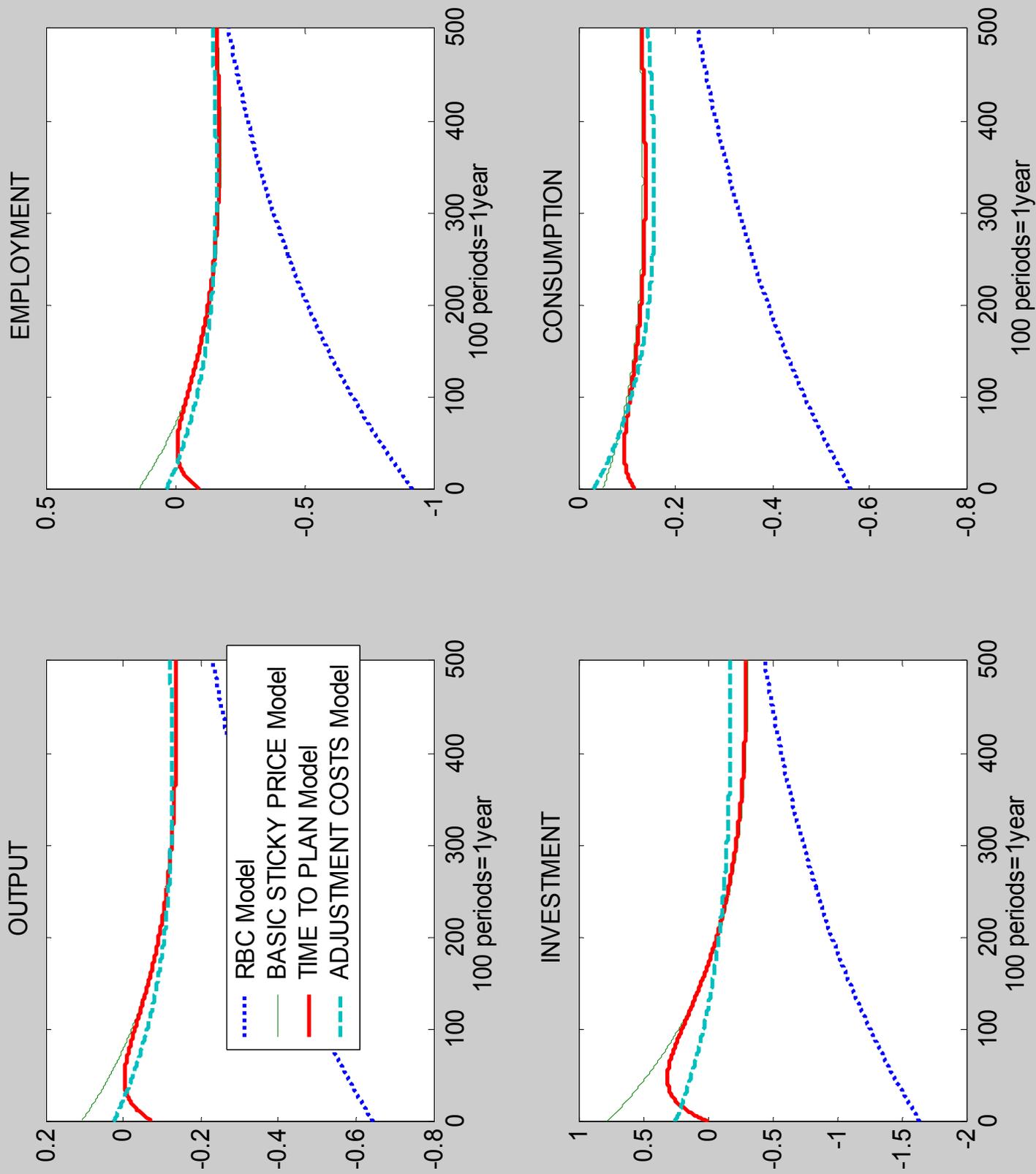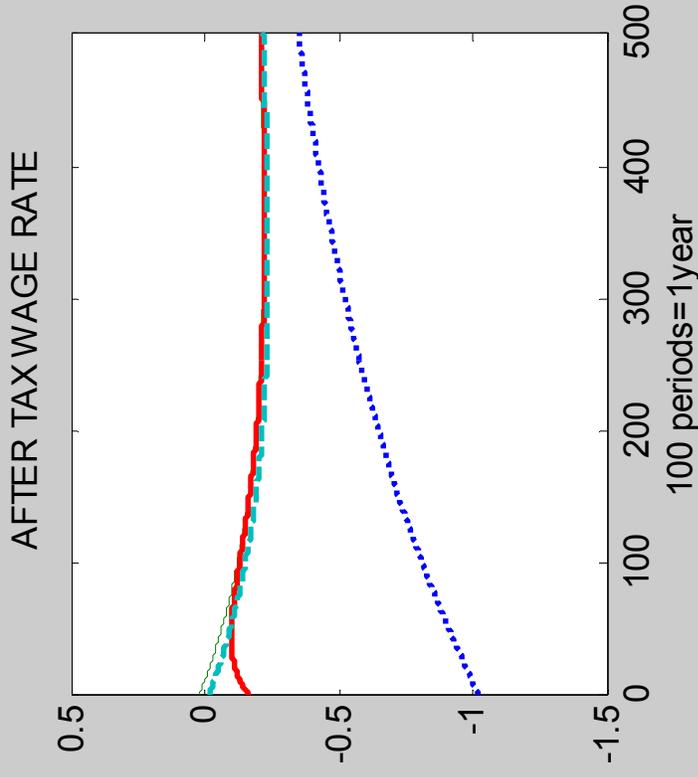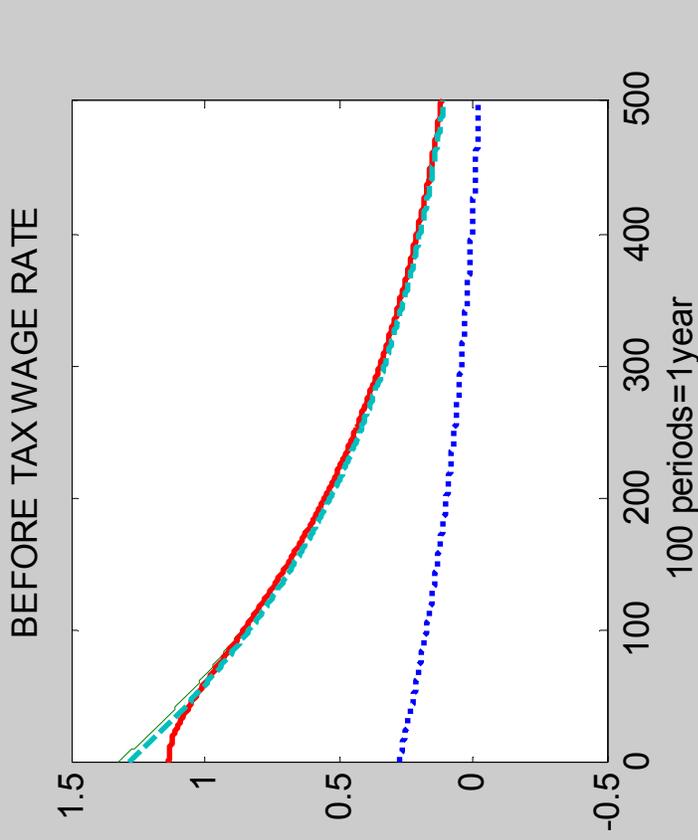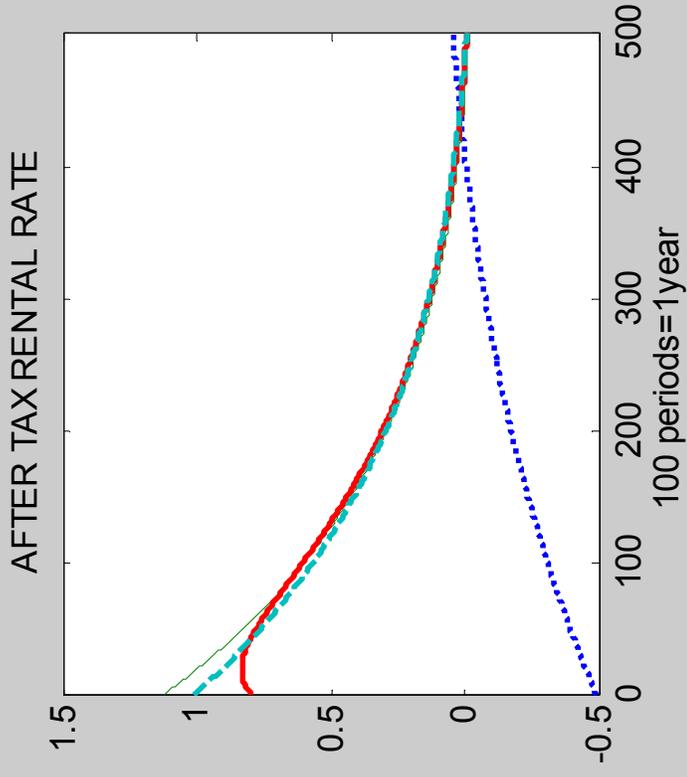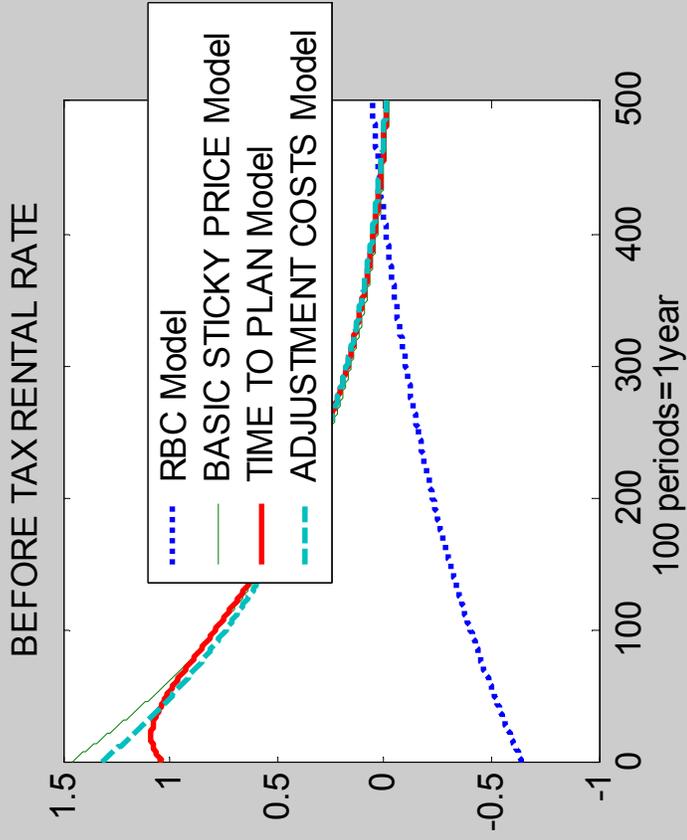
FIGURE 3.   AR(1) LABOR TAX INCREASE (cont'd)
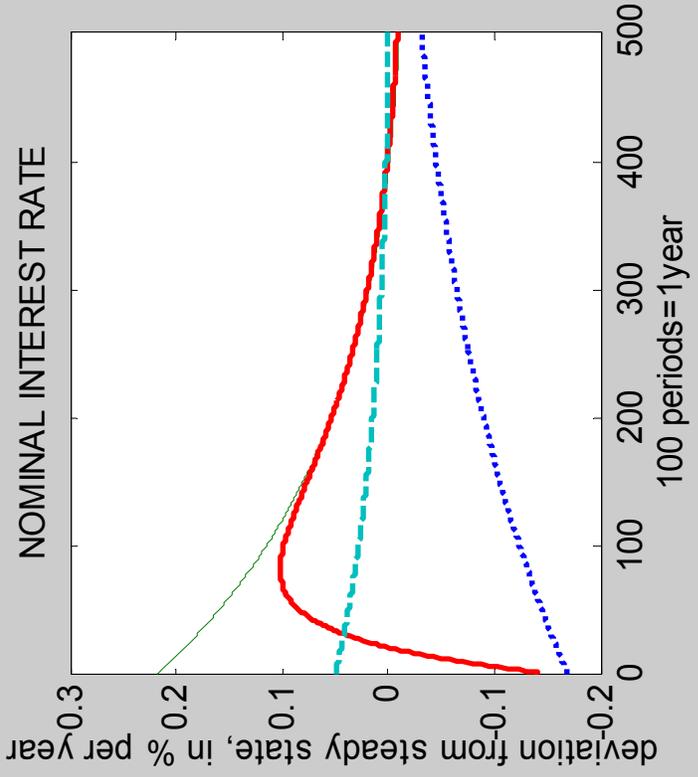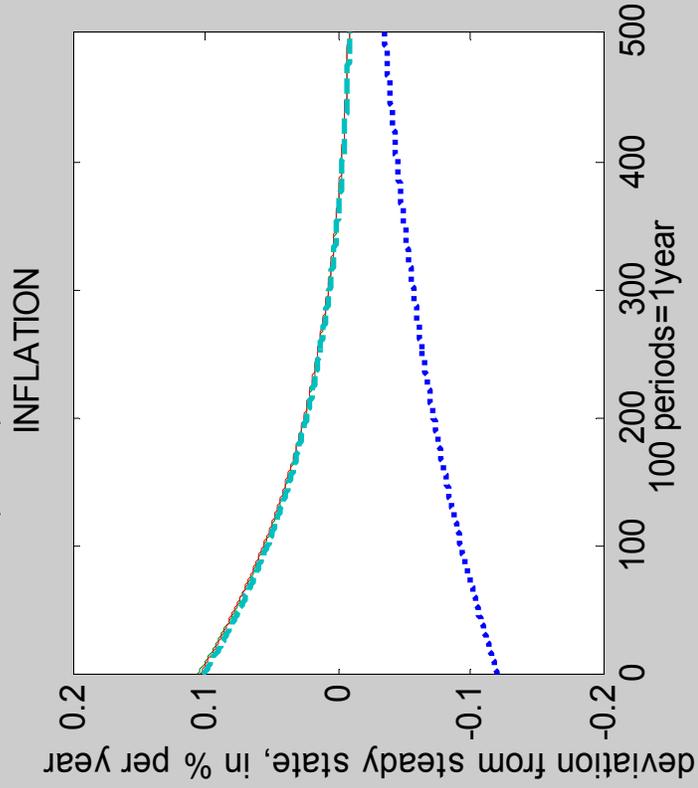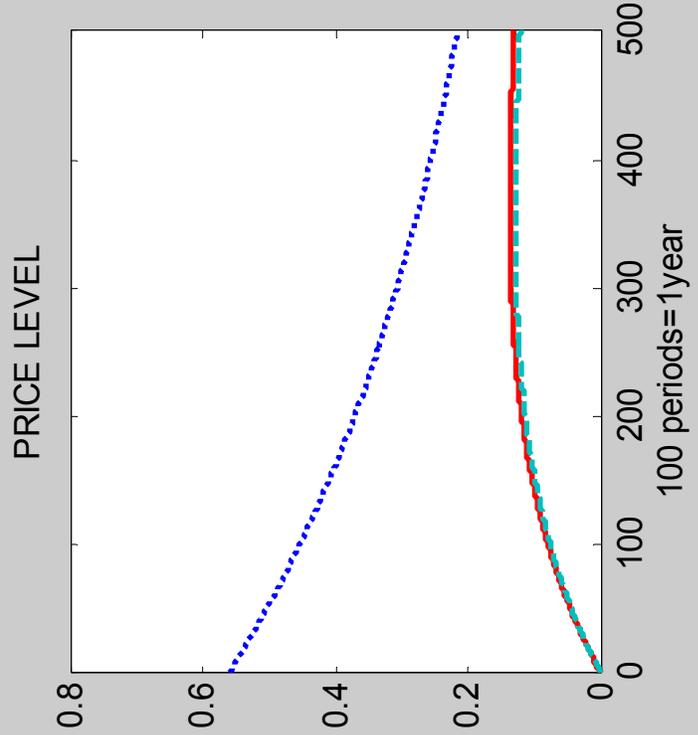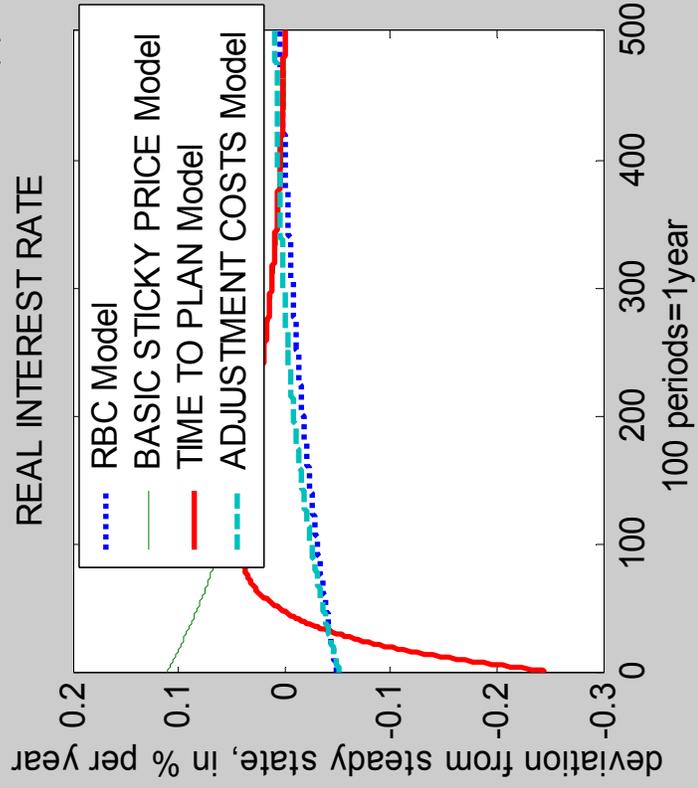
FIGURE 3. AR(1) LABOR TAX INCREASE (cont'd)
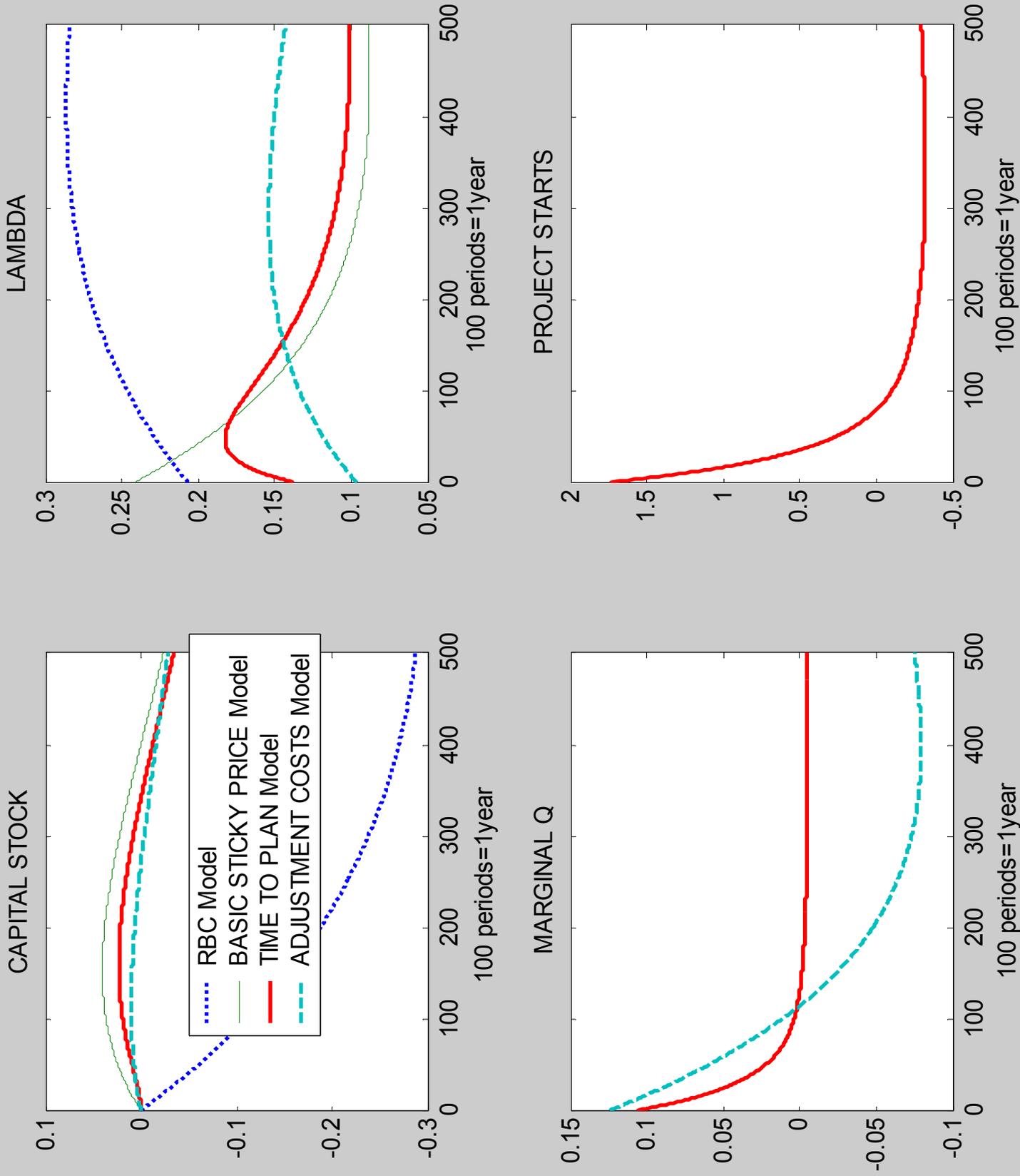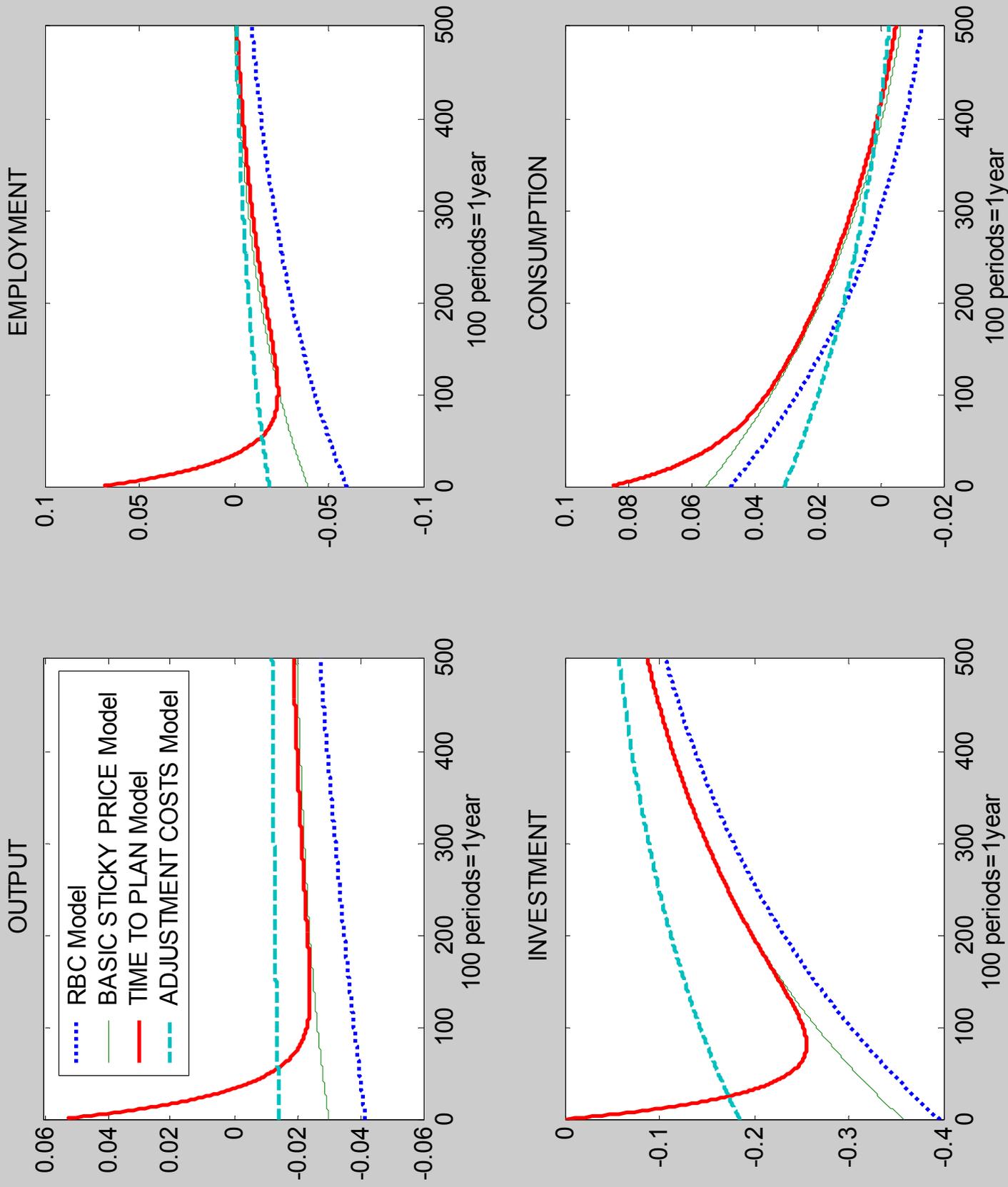
FIGURE 4.   AR(1) CAPITAL TAX INCREASE

FIGURE 4. AR(1) CAPITAL TAX INCREASE

BEFORE TAX RENTAL RATE

AFTER TAX RENTAL RATE

BEFORE TAX WAGE RATE

AFTER TAX WAGE RATE

100 periods=1year

RBC Model
BASIC STICKY PRICE Model
TIME TO PLAN Model
ADJUSTMENT COSTS Model
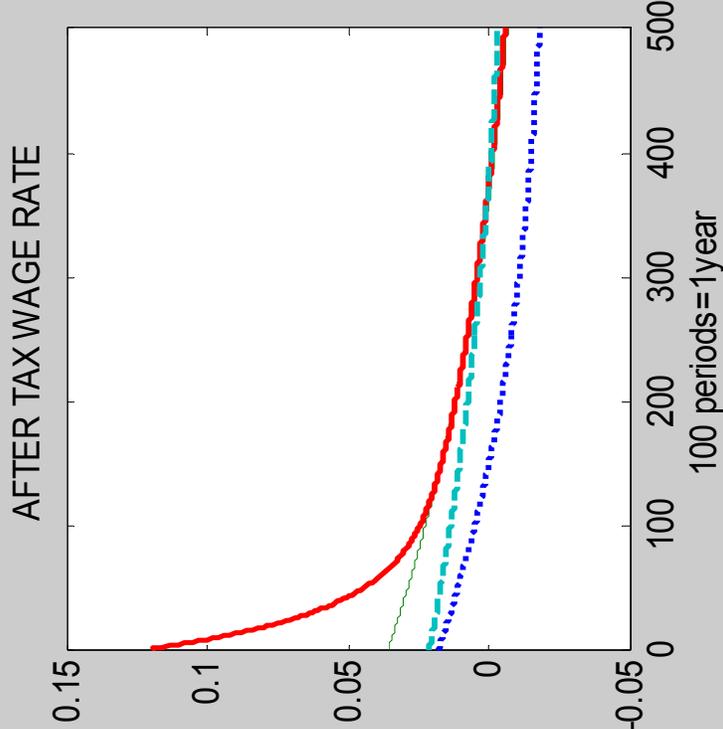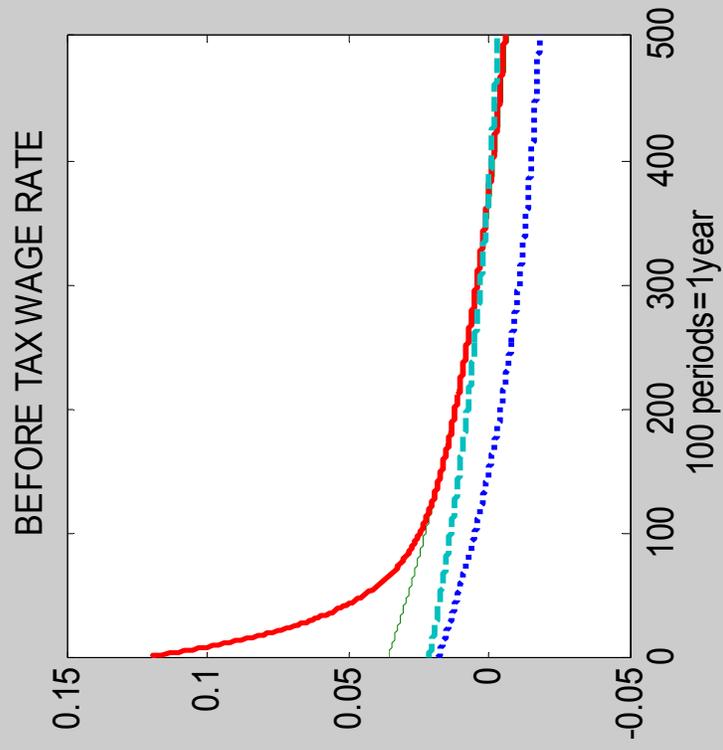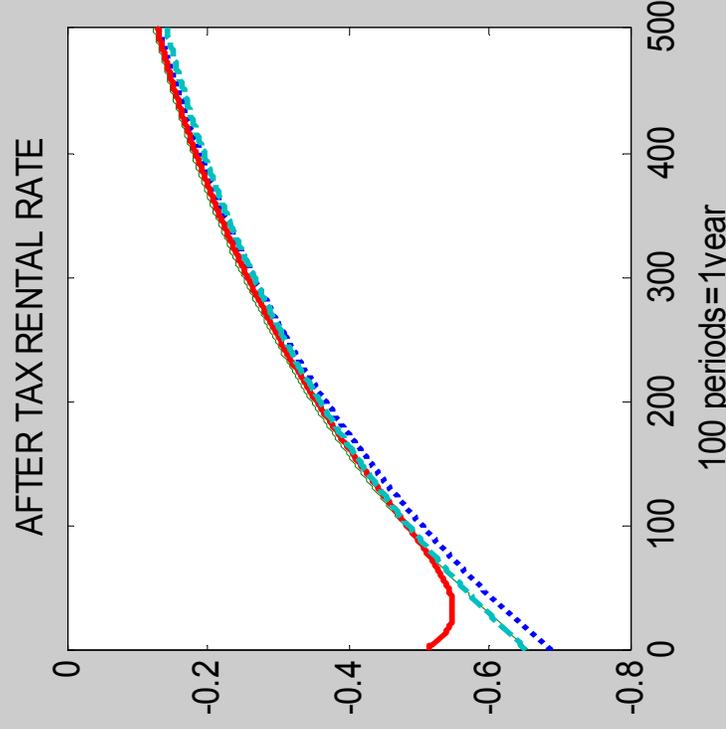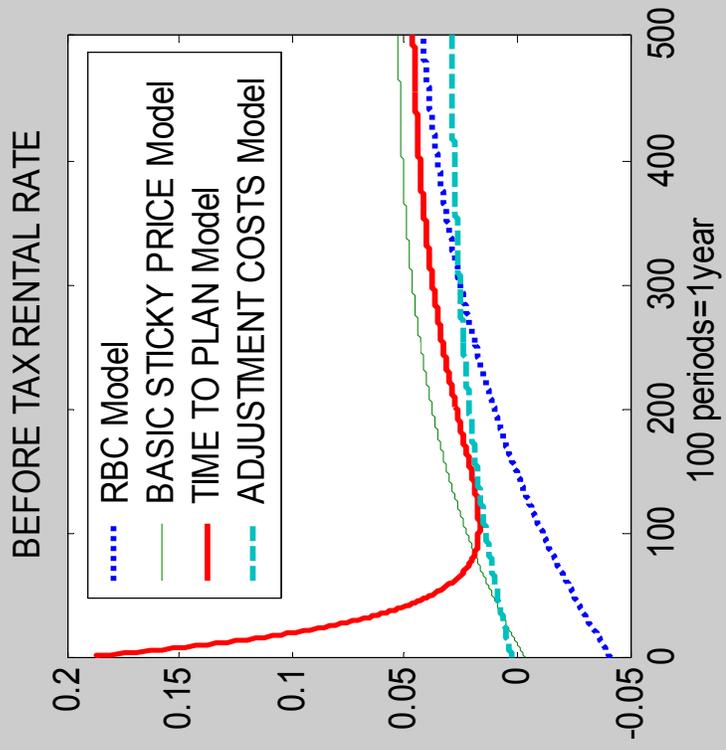
FIGURE 4. AR(1) CAPITAL TAX INCREASE (cont'd)
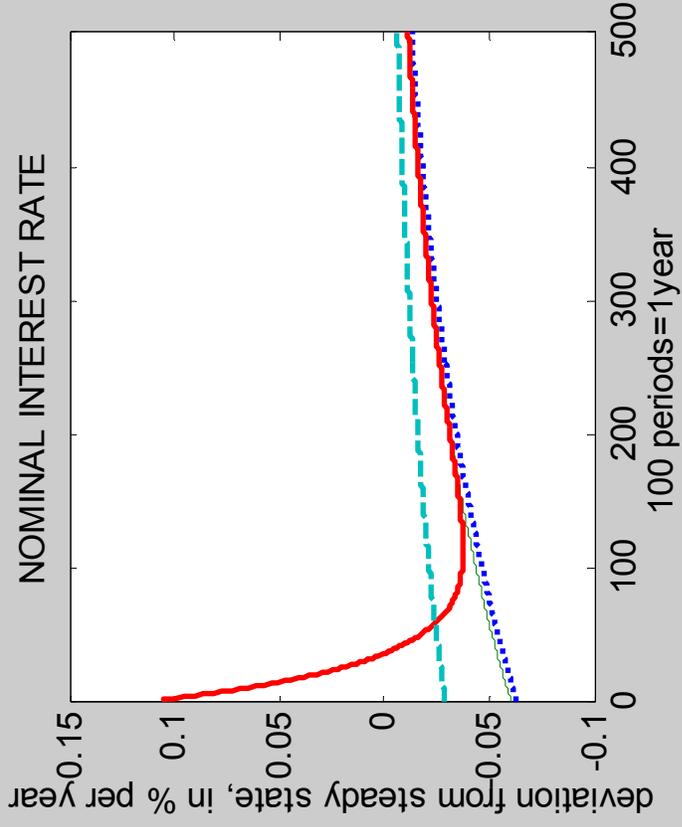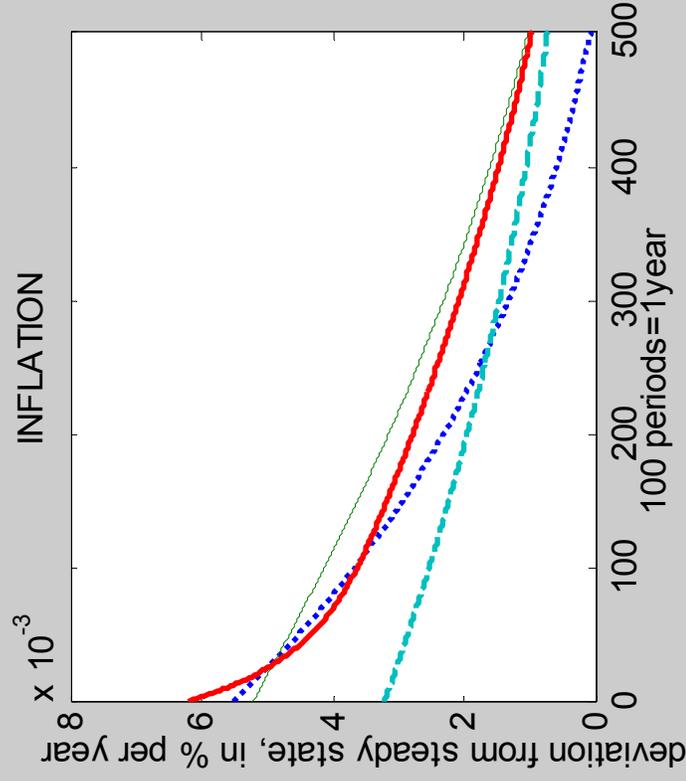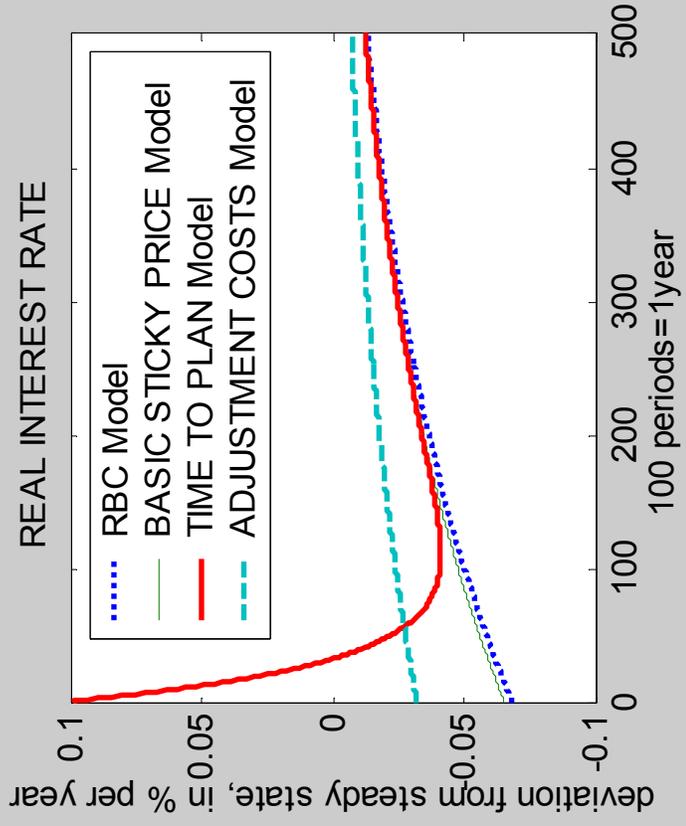
FIGURE 4. AR(1) CAPITAL TAX INCREASE (cont'd)
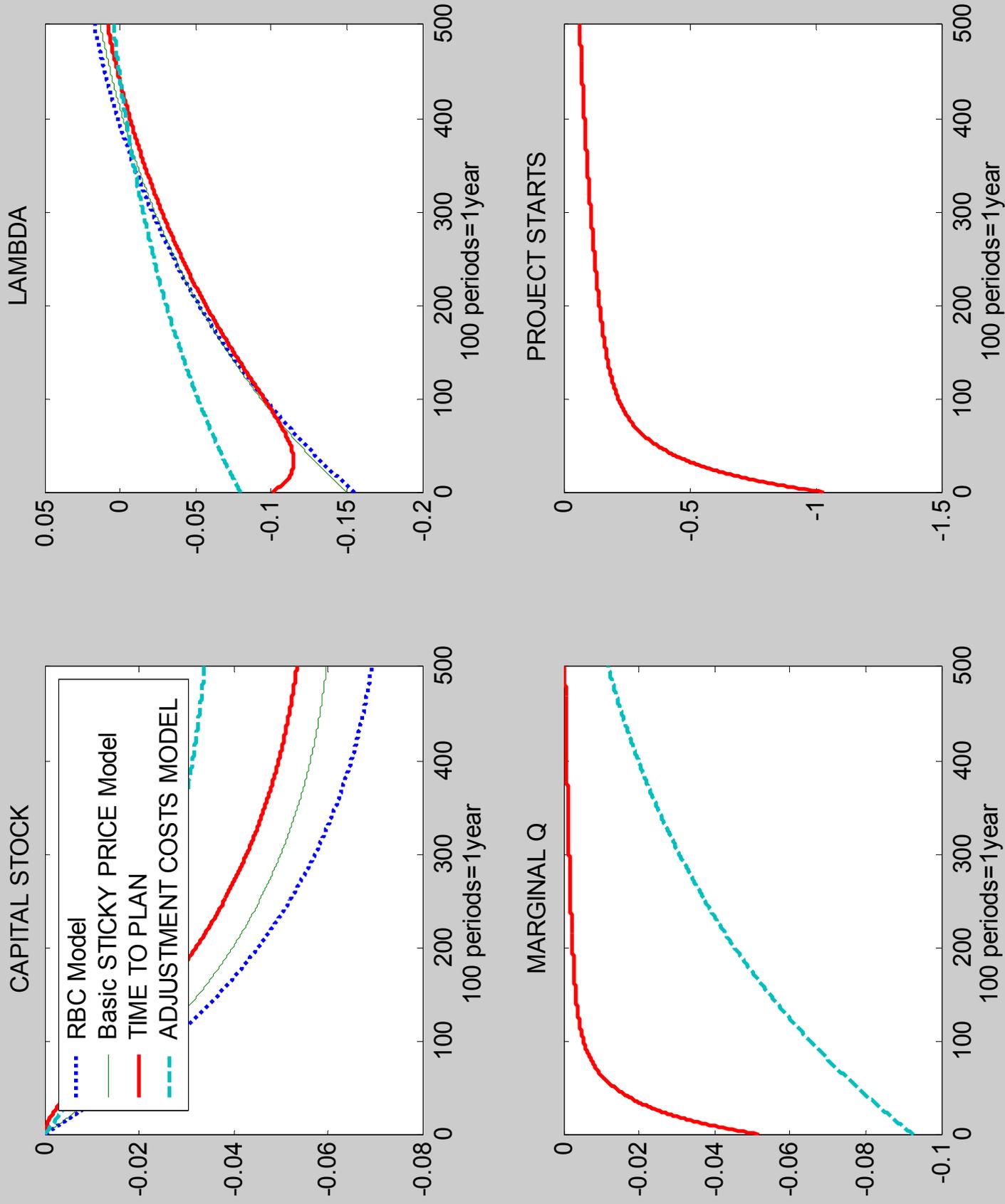
FIGURE 5.   PERMANENT MONEY SHOCK

OUTPUT

RBC Model
BASIC STICKY PRICE Model
TIME TO PLAN Model
ADJUSTMENT COSTS Model

100 periods=1year

EMPLOYMENT

100 periods=1year

CONSUMPTION

100 periods=1year

INVESTMENT

100 periods=1year
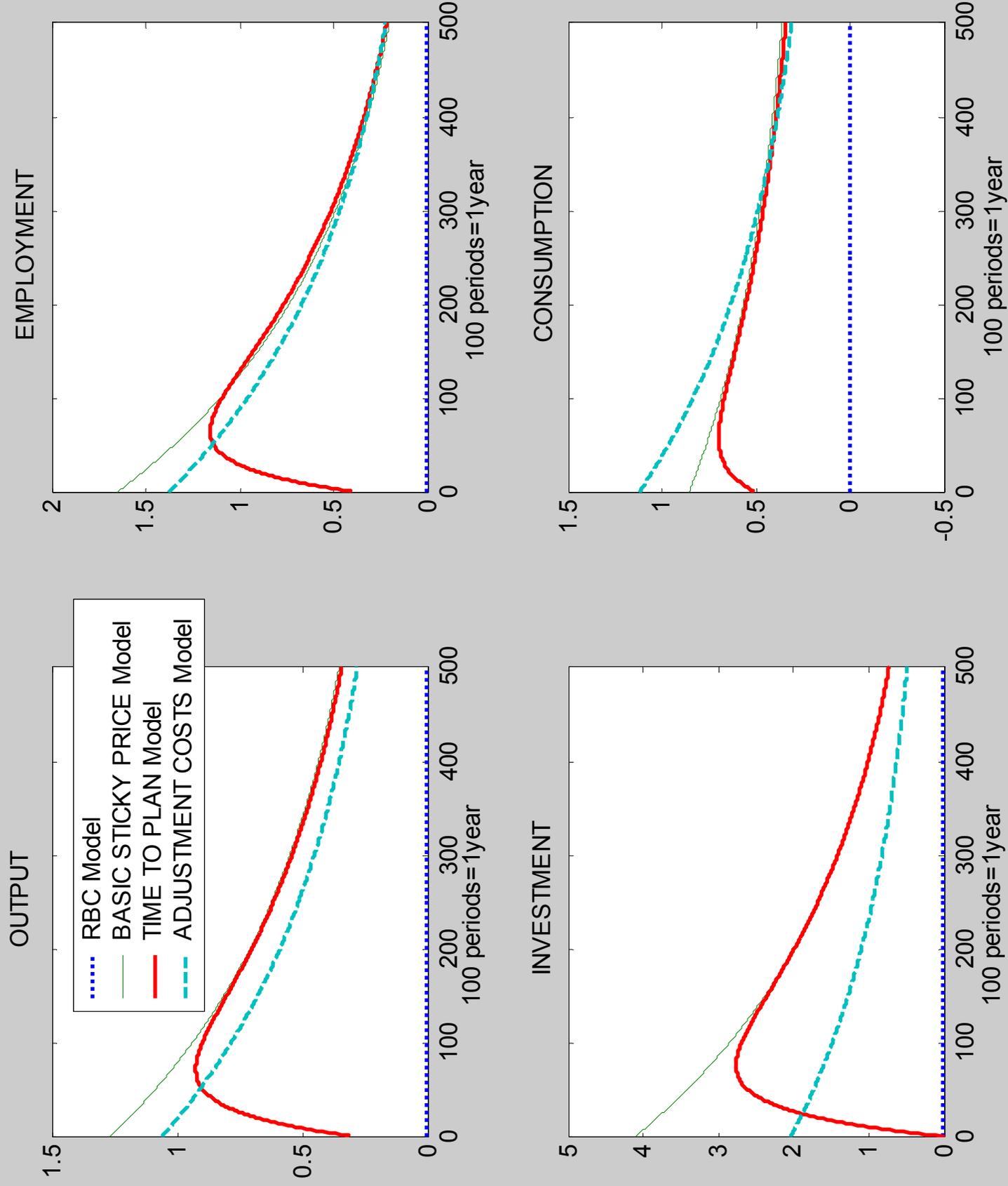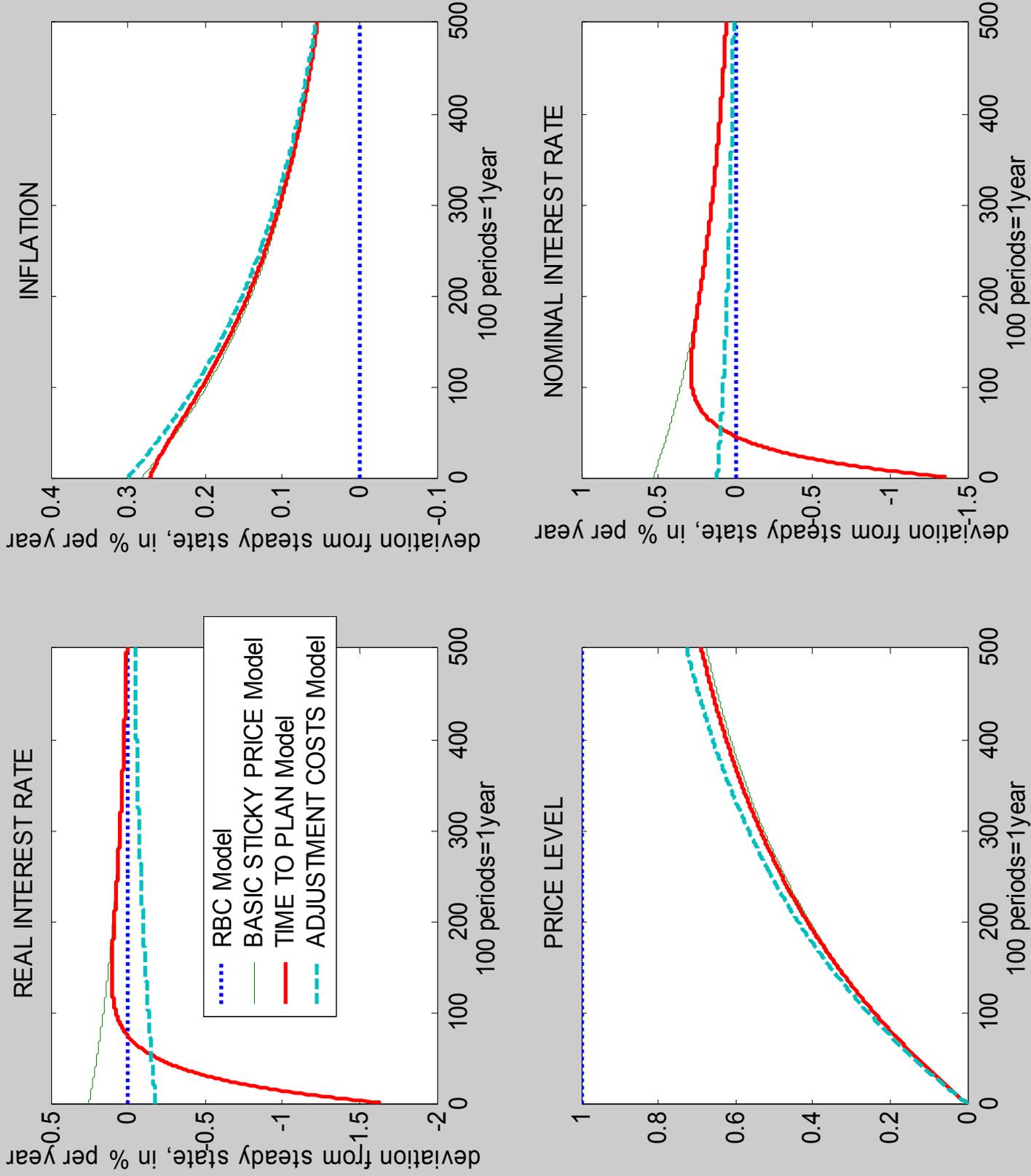
FIGURE 5. POSITIVE PERMANENT MONEY SHOCK (cont'd)
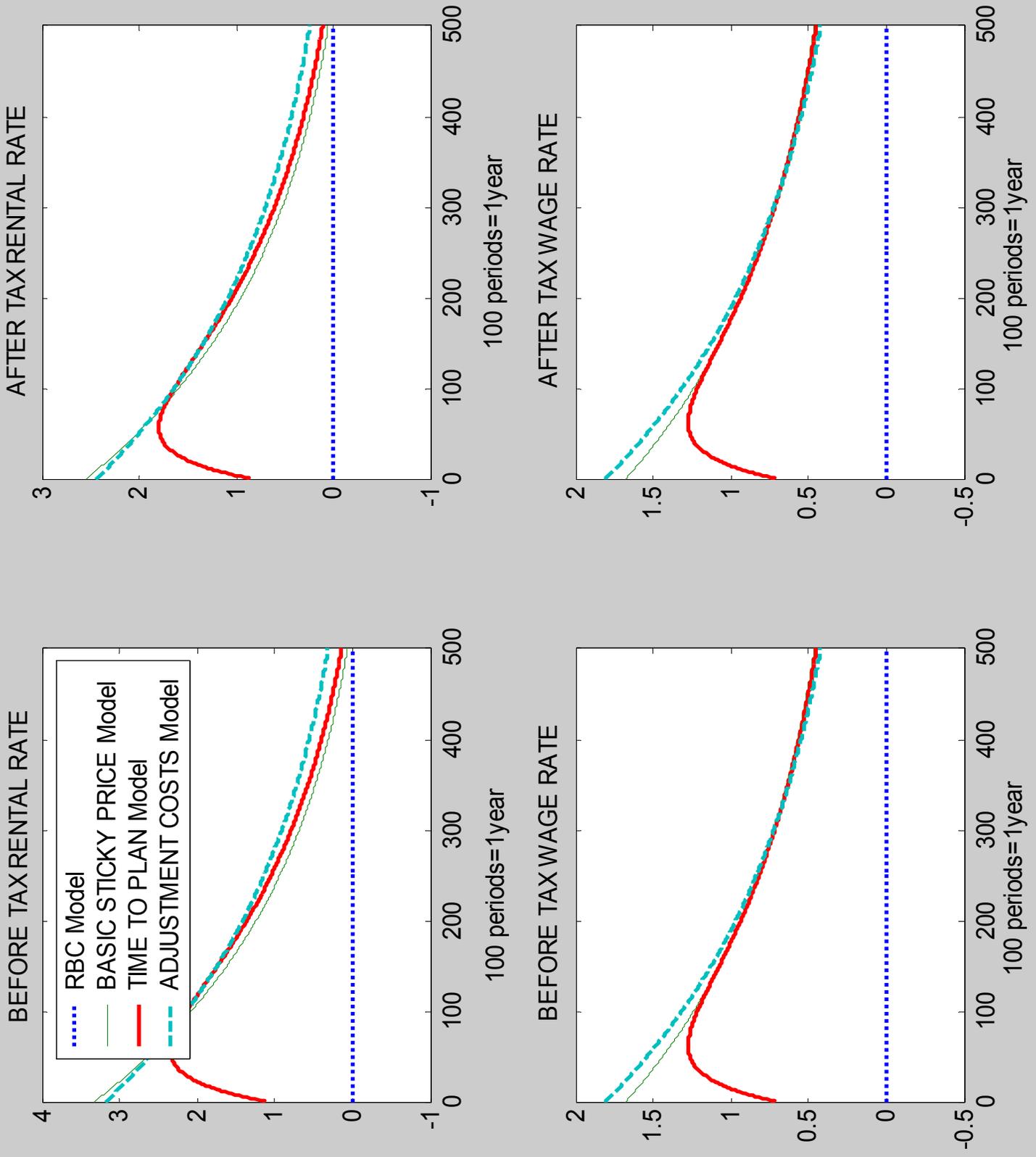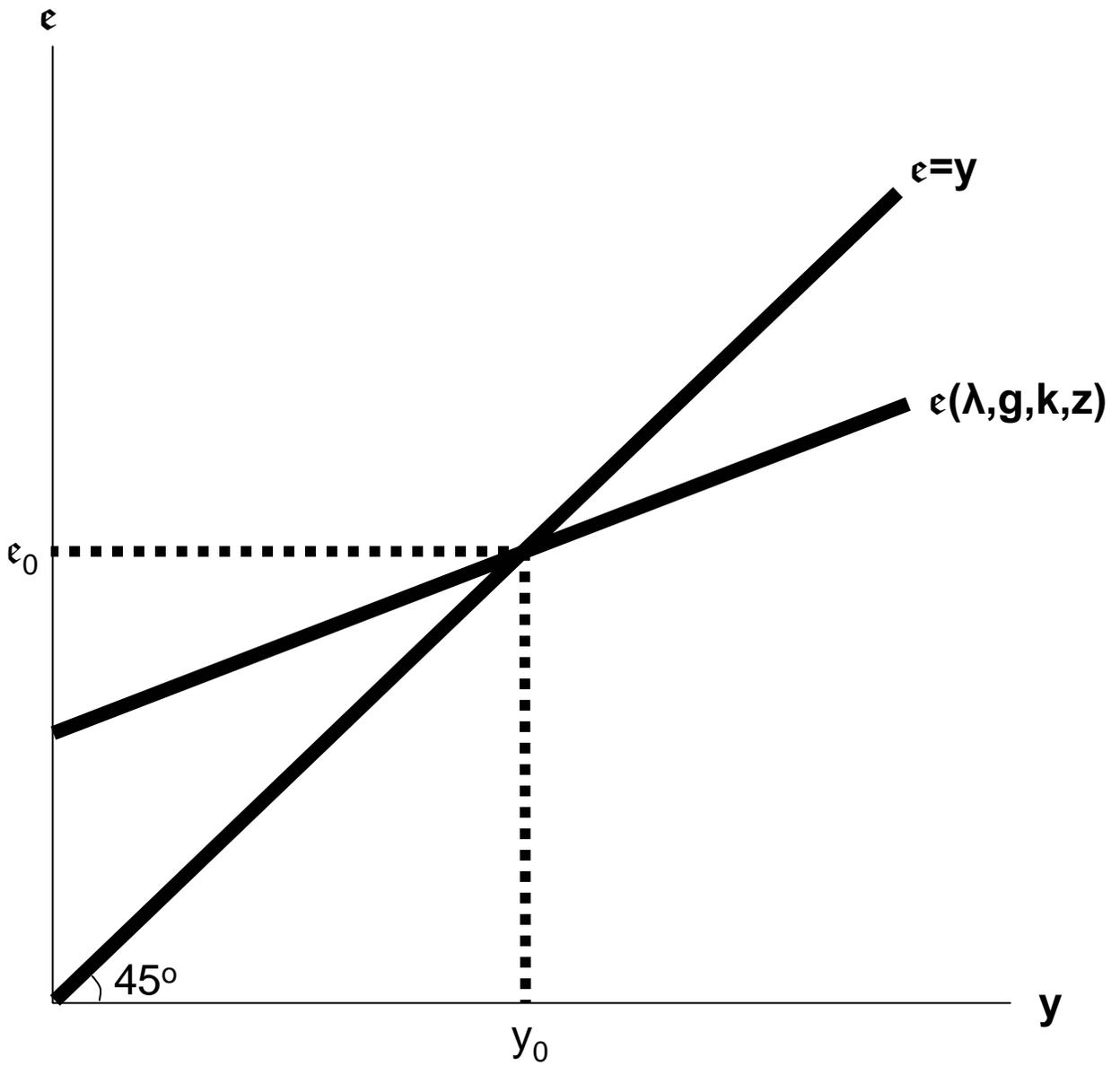
FIGURE 5.   POSITIVE PERMANENT MONEY SHOCK   (cont'd)

Figure 6a
The New Keynesian Cross

Figure 6b
The Ultra-Short Run Effect of
Government Purchases on Output