Job Displacement Risk and the Cost of Business Cycles

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Abstract

This paper analyzes the welfare costs of business cycles when workers face uninsurable job displacement risk that has a cyclical component. Using a simple dynamic general equilibrium model with incomplete markets, this paper shows the following general result: for sufficiently high degree of risk aversion (at least one), the introduction of cyclical variations in the permanent earnings losses of displaced workers can generate arbitrarily large cost of business cycles even if there is no employment risk (displaced workers are immediately re-employed) and the second moments of the distribution of individual income shocks are (almost) constant over the cycle. In other words, the previous literature, which has either focused on cyclical fluctuations in employment risk or assumed that income changes of workers are (log)-normally distributed, might have severely under-estimated the cost of business cycles. In addition to the theoretical analysis, this paper also conducts a quantitative study of the cost of business cycles using empirical evidence about the permanent earnings losses of displaced U.S. workers. The quantitative analysis suggests that cyclical variations in job displacement risk generate sizable cost of business cycles.

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I. Introduction

In a highly influential contribution, Lucas (1987) argues that standard macroeconomic theory implies that the welfare cost of business cycles is negligible. In other words, Lucas (1987) argues that from a welfare point of view, business cycle research and counter-cyclical stabilization policy are irrelevant. His argument is based on a representative-agent model with no production and "standard" preferences. More specifically, Lucas (1987) assumes that i) there is no uninsurable idiosyncratic risk (complete markets), ii) there is no link between business cycles and economic growth, and iii) preferences allow for a time-additive expected utility representation with moderate degree of (relative) risk aversion. In principle, any one of these three assumptions could be questioned, and an extensive literature has subsequently studied how weakening these assumptions could change the surprisingly strong conclusion drawn by Lucas (1987). In his recent survey, Lucas (2003) summarizes the findings of this literature in the following way: *"But I argue in the end that, based what we know now, it is unrealistic to hope for gains larger than a tenth of a percent from better countercyclical policy"*.

This paper removes the complete-markets assumption made by Lucas (1987), and argues that the introduction of uninsurable idiosyncratic labor market risk (incomplete markets) generates substantial welfare costs of business cycles. In accordance with the previous literature, this paper takes as a starting point of the analysis the observation that idiosyncratic labor market risk has a cyclical component (labor income risk is high during recessions)[1], and assumes that the elimination of business cycles leads to the elimination of the cycli-

[1]Clearly, the job displacement rate and the unemployment duration increase during recessions (Hall, 1995). Furthermore, the magnitude of permanent income losses of displaced workers depends on cyclical conditions (Jacobson, LaLonde, and Sullivan, 1993), and the business cycle has a strong effect on the wages of new hires (Beaudry and DiNardo, 1991). Finally, Storesletten, Telmer, and Yaron (2004), and to a certain extent also Meghir and Pistaferri (2004), find that the variance of persistent income shocks is sensitive to business cycle conditions.

cal fluctuations in idiosyncratic labor market risk. In contrast to the previous literature, however, this paper broadens the scope of the analysis. More specifically, the previous literature has either focused on cyclical fluctuations in employment risk or assumed that income changes of workers are (log)-normally distributed.[2] In contrast, this paper considers cyclical variations in the permanent earnings losses of displaced workers (earnings risk as opposed to employment risk) and does not assume that income changes of individual workers are (log)-normally distributed.

The paper presents two results, one theoretical and one empirical, that suggest that the costs of business cycles are substantially larger than reported by the previous literature. First, this paper shows that for sufficiently high degree of risk aversion (at least one), the introduction of cyclical variations in the permanent earnings losses of displaced workers can generate arbitrarily large cost of business cycles even if there is no employment risk (displaced workers are immediately re-employed) and the second moments of the distribution of individual income shocks are (almost) constant over the cycle.[3] In other words, the previous literature, which has either neglected the permanent earnings losses of displaced workers or dealt with them only indirectly using second-moment analysis, may have severely under-estimated the cost of business cycles.

In addition to the theoretical analysis, this paper also provides a first quantitative analysis of the welfare effects of cyclical fluctuations in job displacement risk. More specifically, this paper uses evidence about job displacement rates and permanent earnings losses of displaced U.S. workers obtained by the empirical literature to calibrate the model economy, and then

[2] See section II and Lucas (2003) for a survey of the literature.

[3] The main idea underlying the proof of this result is that extreme income losses (personal disasters) that occur during recessions can make the cost of business cycles very large even if the probability of such extreme events is very small. Krebs (2004) uses a similar idea to show that any arbitrage-free asset return process is an equilibrium outcome for given preferences and given process of first and second moments of individual income changes.

computes the cost of business cycles for the calibrated version of the model. The quantitative analysis shows that cyclical fluctuations in job displacement risk generate sizable welfare cost of business cycles. For example, for high-tenure workers with log-utility preferences (degree of risk aversion of one), the welfare cost of business cycles is around .4 percent of lifetime consumption, and this cost increases to 1.3 percent if we assume a degree of relative risk aversion of two. In contrast, for the same economy with no cyclical fluctuations in the earnings losses of displaced workers, the welfare cost of business cycles is nil. Moreover, the implied cyclical variations in the second moments of the distribution of individual income shocks are so small that a log-normal view of the world would suggest negligible cost of business cycles.

At this stage, it is worth pointing out that the results reported here do not necessarily imply that macroeconomic stabilization policy leads to substantial welfare gains. More specifically, this paper follows the previous literature and uses a "black-box" approach to the elimination of business cycles in the sense that it does not explicitly model the interaction between counter-cyclical stabilization policy and the business cycle. Thus, without further extension, the current approach is not well-suited for studying the design of stabilization policy, and for gaining new insights into the interaction between stabilization policy and idiosyncratic labor market risk. Moreover, in accordance with Lucas (1987), the current paper disregards any link between short-run fluctuations in aggregate output (business cycles) and the mean of aggregate output (economic growth).[4] Despite these limitations, the current analysis seems well-suited for addressing the issue that is the topic of Lucas (2003), namely whether our current knowledge suffices to rule out substantial welfare cost of business cycles.

[4] For example, the elimination of business cycles might affect the aggregate stock of physical capital (Krusell and Smith, 1999) or human capital (Krebs, 2003a) in the economy. Moreover, business cycles affect the re-allocation of factors across heterogenous production units, which in turn affects aggregate economic growth. See, for example, Haltiwanger (2000) for a recent survey of the theoretical and empirical literature on the growth effects of factor re-allocation.

As shown in this paper, sizable cost of business cycles cannot be ruled out once we take into account the permanent earnings losses of displaced workers.

Finally, a comment regarding modeling strategy is in order. The objective of the current paper is to study the welfare effects of business cycles when workers face uninsurable job displacement risk. As it is well-known, equilibria of incomplete-market models with idiosyncratic risk and aggregate shocks are in general difficult to compute and (almost) impossible to analyze theoretically. In order to focus on the main economic issues, the analysis presented here is therefore based on a highly tractable incomplete-market model along the lines of Constantinides and Duffie (1996) and Krebs (2004). The model is simple enough to allow for closed-form solutions for the welfare cost of business cycles, yet rich enough to establish a tight link between the theoretical earnings process and the empirical literature on job displacement risk.[5]

The paper is organized as follows. Section II surveys the previous literature. Section III develops the model that is used to discuss the effect of job displacement risk on the welfare cost of business cycles. Section IV derives a closed-form expression for the welfare cost of business cycles, and uses these expressions to prove the main theoretical result: cyclical fluctuations in job displacement risk may have an arbitrarily large effect on the cost of business cycles even if there is no employment risk and the second moments of the distribution of income shocks have no cyclical component (proposition 2). Section V provides a quantitative analysis of the effect of cyclical job displacement risk on the cost of business cycles, and section VI concludes.

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[5]An alternative approach would be to use preferences and estimates of the process of individual consumption risk to derive the welfare cost of business cycles. Although in general quite attractive, such an approach has the drawback that the empirical literature on labor market risk has mainly focused on cyclical variations in individual income risk (for an important exception, see Brav, Constantinides, and Gescy, 2002). Put differently, we know much more about cyclical variations in individual income risk than we know about cyclical variations in individual consumption risk.

II. Previous Literature

We now briefly survey the literature on the cost of business cycles in models with incomplete markets. Most of the previous literature has focused on quantitative results derived from analyzing calibrated model economies.

Atkeson and Phelan (1994), Imrohoroglu (1989), and Krusell and Smith (1999,2002) all study models of worker unemployment, and assume that cyclical fluctuations in unemployment rates and unemployment durations are the only sources of cyclical variations in idiosyncratic labor market risk. In other words, these papers assume that the earnings of displaced workers fully recover after re-employment at a new job, and therefore rule out by assumption the type of effect studied here. This assumption means that idiosyncratic income shocks have a relatively low persistence, and that their effect on consumption and welfare is therefore relatively small (consumption smoothing through self-insurance or borrowing and lending works well). Gomes, Greenwood, and Rebelo (2001) extent the analysis of unemployment risk and allow for endogenous search. In this case, business cycles may have a positive effect on welfare (option value of search).[6]

The papers by Gomes, Greenwood, and Rebelo (2001) and Krusell and Smith (1999,2002) employ versions of the neoclassical model with aggregate productivity shocks that generate cyclical fluctuations in the aggregate wage, and in this sense these papers take into account cyclical fluctuations in the income losses of displaced workers. However, in the calibrated model economies considered by these authors, the cyclical fluctuations in the aggregate wage have relatively low persistence and small amplitude, and therefore do not generate sizable cost of business cycles.[7] Beaudry and DiNardo (2001) consider a model of unemployment

[6]See also Manuelli, Jones, Siu, and Staccetti (2003) and Jovanovic (2003) for representative-agent models in which aggregate fluctuations have a positive effect on growth.

[7]Similarly, the cyclical fluctuations in unemployment durations considered by these authors introduce

in which the wage of new hires varies over the business cycle, and argue that these cyclical variations are much more persistent and stronger than the cyclical variations in the aggregate wage. In this sense, the analysis conducted in Beaudry and DiNardo (2001) is closely related to the current analysis. However, in contrast to Lucas (2003) and most of the work in the literature, Beaudry and Page (2001) simply assume that the elimination of business cycles eliminates all idiosyncratic risk, and this assumption makes it difficult to judge to what extent the welfare cost reported in Beaudry and Pages (2001) are truly cost of business cycle.

Krebs (2003a) and Storesletten, Telmer, and Yaron (2001) discuss the cost of business cycles when individual income shocks are (log)-normally distributed and the variance of these shocks depend on business cycle conditions, but they do not condition on the job displacement event. According to the model analyzed in this paper, log-income changes are only normally distributed after we condition on the individual job displacement event (and business cycles conditions). In a certain sense, the models considered by Krebs (2003a) and Storesletten et al. (2001) are mis-specified. The subsequent analysis will show that this type of mis-specification can lead to a serious under-estimate of the cost of business cycles (proposition 3).

Finally, Krebs (2003b) and Rogerson and Schindler (2002) study the welfare cost of job displacement risk and also focus on the income losses of displaced workers. However, Krebs (2003b) and Rogerson and Schindler (2002) analyze the welfare gain from eliminating job displacement risk in an economy with constant job displacement risk, whereas the current paper studies the welfare gains from eliminating the *cyclical variations* in job displacement risk (keeping average job displacement risk constant).

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cyclical variations in the income losses of displaced workers.

III. Model

The model is an incomplete-market version of the Lucas asset pricing model (Lucas, 1978) similar to the one considered in Constantinides and Duffie (1996) and Krebs (2004).[8] It provides a formal approach to the intuitive idea that consumption equals permanent income. The model features ex-ante identical, long-lived households (workers) with homothetic preferences that make consumption/saving choices in the face of uninsurable income shocks. Income shocks are permanent, which implies that self-insurance is an ineffective means to smooth out income fluctuations. Indeed, the economy is set up in a way so that in equilibrium households will not self-insure at all. That is, income shocks translate one-to-one into consumption changes (proposition 1). Notice that this result does not depend on the assumption that aggregate saving is zero, even though we will make it to simplify the analysis. For example, Krebs (2003a,2003b) considers a production economy with only permanent income shocks (log-income follows a random walk) and ex-ante identical households, and shows again that self-insurance is highly ineffective.[9] Deaton (1991) and Carroll (1997) provide a partial equilibrium analysis of the effect of permanent income shocks on consumption and saving, and also conclude that the main effect of permanent income shocks is to change consumption.

There is strong empirical evidence that individual labor income risk has a substantial permanent (or highly persistent) component,[10] and the empirical estimates of this perma-

[8]To simplify the analysis, the current paper does not allow for assets in positive net supply. See Constantinides and Duffie (1996) and Krebs (2004) for an extension of the model to this case.

[9]One implication of the random walk assumption is that the cross-sectional distributions of income and consumption diverge. However, Constantinides and Duffie (1996) show how to modify the model by introducing death probabilities so that a stationary distribution of income and consumption always exists. Indeed, they show that by choosing the death probabilities appropriately, the model can match any cross-sectional distribution of income and consumption.

[10]See, for example, Carroll and Samwick (1997), Jacobson, LaLonde, and Sullivan (1993), Meghir and Pistaferri (2004), Ruhm (1991), and Storesletten, Telmer, and Yaron (2004).

nent component will be used in the quantitative section (section IV) to calibrate the model economy. The same empirical literature also provides clear evidence in favor of a substantial transitory component of labor income risk, and in this sense the current model is not consistent with a certain dimension of the data. More specifically, the job displacement event has two effects on the earnings of a displaced worker. First, the worker goes through a period of unemployment with no earnings (the transitory effect). Second, the worker finds a new job, but receives a permanently lower wage (the permanent effect).[11] In the current paper, we disregard the first effect, and only focus on the second effect. Given that the first effect has been extensively studied by the previous literature (Atkeson and Phelan (1994), Imrohoroglu (1989), Krusell and Smith (1999,2002)), this modeling choice seems appropriate.

III.1. Economy

Time is discrete and open ended. Labor income of worker i in period t is denoted by y_{it}. Labor income is random and defined by an initial level y_{i0} and the law of motion

$$y_{i,t+1} = (1+g)(1+\theta_{i,t+1})(1+\eta_{i,t+1})\, y_{it} \ , \tag{1}$$

where g is the (constant) aggregate growth rate of labor income and $\theta_{i,t+1}$ and $\eta_{i,t+1}$ describe shocks to the labor income of worker i.[12] We assume that for each i and t, the two random variable $\theta_{i,t+1}$ and $\eta_{i,t+1}$ are independently distributed. Further, we assume that the sequence of random variables $\{\theta_{it}\}$ is i.i.d. across workers and over time with log-normal distribution

[11] Of course, this description is very simplistic in the sense that the typical displaced worker first goes through a period of multiple job spells before settling into a new long-term employment relationship (Hall, 1995). In this more complicated case, the permanent earnings losses that are the focus of the current paper should be defined as the earnings difference between the old long-term job and the new long-term job.

[12] Notice that we do not allow the growth rate g to vary over the business cycle. Thus, any welfare cost of business cycles reported here are solely due to cyclical variations in *idiosyncratic* labor market risk.

function:[13]

$$log(1+\theta_{i,t+1}) \sim N(-\sigma^2/2, \sigma^2)\ . \tag{2}$$

The sequence of random variables $\{\eta_{it}\}$ is also i.i.d. across workers, but not over time. More specifically, we assume that there is an aggregate state process $\{S_t\}$ that is i.i.d., and that the distribution of η_{it} depends on the aggregate state of the economy in the following way:

$$\eta_{i,t+1} = \begin{cases} -d_S & with\ \ probability\ \ p_S\ \ if\ \ S_{t+1} = S \\ \frac{p_S d_S}{1-p_S} & with\ \ probability\ \ (1-p_S)\ \ if\ \ S_{t+1} = S\ , \end{cases} \tag{3}$$

The random variable η_{it} is the cyclical component of labor income risk, and we interpret this component as describing job displacement risk. The number d_S is the (permanent) income loss of a worker who is displaced when the aggregate state is S, and the number p_S is the corresponding displacement probability. To ensure that the random variable η_{it} has mean zero, we assume that the worker gains income $\frac{p_S}{1-p_S}d_S$ if he is not displaced. Notice that the i.i.d. assumption means that income *changes* associated with the displacement event are unpredictable, which implies that the corresponding income losses are permanent. The income losses of displaced workers, and in particular its permanent component, have been extensively studied by the empirical literature (Beaudry and DiNardo (1991), Faber (1997), Jacobson, LaLonde, and Sullivan (1993), Neal (1995), Ruhm (1991), Topel (1991)). In the quantitative section V we will use the estimates of this empirical literature to calibrate the model economy. Notice also that the specification (3) allows the size of the income loss of displaced workers to have a cyclical component (d_S depends on S), something that finds support in the data (Beaudry and DiNardo (1995) and Jacobson, LaLonde, and Sullivan

[13]The term $-\sigma^2/2$ ensures that the mean of income growth is independent of σ^2, a property that is useful since it allows us to vary income risk without changing the mean growth rate. Notice that this type of specifying the distribution of income shocks is standard in the literature. See, for example, Carroll (1997), Constantinides and Duffie (1996), and Storesletten, Telmer, and Yaron (2001). To understand the economic meaning of this assumption, notice that with this specification we have $E[\theta_{i,t+1}] = 0$ and $var[\theta_{i,t+1}] = e^{\sigma^2} - 1$ using the standard formula for log-normal distributions. Thus, any increase in σ^2 increases $var[\theta_{i,t+1}]$, but leaves $E[\theta_{i,t+1}]$ unchanged.

(1993)), and is particularly important from the point of view of the current paper.[14]

The random variable θ_{it} is the a-cyclical component of labor income risk, and we interpret it as containing any labor income risk beyond job displacement risk. To relate this variable to the empirical literature, let us take logs in equation (1):

$$log y_{i,t+1} = log y_{it} + log(1+g) + log(1+\theta_{i,t+1}) + log(1+\eta_{i,t+1}) \ . \tag{4}$$

Equation (4) says log-labor income approximately follows a random walk with drift and heteroscedastic error term $\epsilon_{i,t+1} = log(1+\theta_{i,t+1}) + log(1+\eta_{i,t+1})$, which is yet another way of saying that income shocks are permanent. An extensive empirical literature has estimated the parameters of the permanent component of income shocks under the log-normal distribution assumption[15] and the estimates obtained by this literature can be used to find a value of $var((log\, y_{i,t+1} - log\, y_{it}))$, and therefore indirectly a value for $var(log(1+\theta_{i,t+1})) = \sigma^2$ in (2). Notice, however, that even though two recent contributions by Meghir and Pistaferri (2004) and Storesletten et al. (2004) have allowed the variance of log-income changes to depend on the aggregate state S, this literature (in contrast to the literature on job displacement mentioned above) has not conditioned their estimates on the displacement event and has not taken into account any deviations from the log-normal distribution assumption.[16] Clearly, if the specification (4) is correct, then the error term $\epsilon_{i,t+1} = log(1+\theta_{i,t+1}) + log(1+\eta_{i,t+1})$ is not normally distributed (indeed, it is the mixture of two normal distributions with different

[14] Most of this literature interprets the income losses as arising from the loss of firm-specific human capital. See, for example, Jovanovic (1979) for a theoretical model of firm-specific human capital. Neal (1995), however, argues that most of the income losses estimated by this empirical literature are due to the loss of industry-specific (or occupation-specific) human capital.

[15] See, for example, Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten, et al. (2004). Notice that even though Storesletten, et al. (2004) specify the permanent component to be AR(1), they estimate an autocorrelation coefficient close to one (the random walk case).

[16] Geweke and Keane (2000) allow for deviations from the log-normal distribution assumption. In contrast to Meghir and Pistaferri (2004) and Storesletten et al. (2004), however, they do not condition their estimates of the labor income parameters on the aggregate state of the economy and they do not decompose labor income risk into a transitory and a permanent component.

means), which implies that one of the identifying assumptions of this literature is violated.

Each worker begins life with no initial financial wealth. Workers have the opportunity to borrow and lend (dissave and save) at the risk-free rate r_t. There are no insurance markets for idiosyncratic labor income risk. In other words, there are no assets with payoffs that, conditional on the aggregate state S, are correlated with either θ_{it} or η_{it}. Thus, the sequential budget constraint of worker i reads:[17]

$$\begin{aligned} a_{i,t+1} &= (1+r_t)a_{it} + y_{it} - c_{it} \\ a_{i,t+1} &\geq -M \ , \ a_{i0} = 0 \ . \end{aligned} \tag{5}$$

Here c_{it} denotes consumption of household i in period t and a_{it} his asset holdings (wealth excluding current interest payments) at the beginning of period t. The real number M represents an explicit debt constraint that rules out Ponzi schemes.

Workers have identical preferences that allow for a time-additive expected utility representation:

$$U(\{c_{it}\}) = E\left[\sum_{t=0}^{\infty} \beta^t u(c_{it})\right] \ . \tag{6}$$

Moreover, we assume that the one-period utility function, u, is given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, for $\gamma \neq 1$ and $u(c) = logc$ for $\gamma = 1$. That is, we assume that preferences exhibit constant degree of relative risk aversion γ.

Finally, we assume that the following condition is satisfied:

$$\begin{aligned} & \beta E\left[\left((1+g)^{1-\gamma}(1+\theta_{i,t+1})(1+\eta_{i,t+1})\right)^{1-\gamma}\right] \\ = \ & \beta\,(1+g)^{1-\gamma}\, e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2} \sum_S \pi_S \left(p_S(1-d_S)^{1-\gamma} + (1-p_S)\left(1+\frac{p_S d_S}{1-p_S}\right)^{1-\gamma}\right) \ , \\ < \ & 1 \end{aligned} \tag{7}$$

[17]Notice that the analysis remains unchanged if we assume that agents have the opportunity to trade assets whose payoffs depend on the aggregate state S (Krebs, 2004). Our assumption that there are no insurance markets means that one should interpret y_{it} as income after transfer payments from the government.

where π_S is the probability that aggregate state S occurs. This inequality will ensure that in equilibrium the expected lifetime utility of workers is finite for any budget-feasible plan, and that any solution to the Euler equation also solves a corresponding transversality condition. Notice that this condition is automatically satisfied if $\gamma = 1$ (log-utility).

III.2. Equilibrium

For a given interest rate process $\{r_t\}$, each household chooses a consumption-saving plan that maximizes expected lifetime utility (6) subject to the budget constraint (5). In equilibrium, the asset (bond) market must clear. In an exchange economy, this means aggregate saving is zero:[18]

$$\sum_i a_{it} = 0 \; . \tag{8}$$

Notice that (5) and (8) imply goods market clearing (Walras' law).

The Euler equations associated with the consumption-saving problem of worker i read

$$c_{it}^{-\gamma} = \beta(1 + r_{t+1})E[c_{i,t+1}^{-\gamma}|F_{it}] \; , \tag{9}$$

where F_{it} represents the information that is available to household i in period t. In the following, we assume that F_{it} contains any variable that has been realized up to time t. In particular, it contains θ_{it}, η_{it}, and S_t. The Euler equation (9) says that the marginal utility cost of saving one more unit of the good is equal to the expected marginal utility gain of doing so.

Suppose the interest rate is constant and given by:

$$1 + r = \left(\beta \, E\left[\left(\frac{y_{i,t+1}}{y_{it}}\right)^{-\gamma}\right]\right)^{-1} \quad . \tag{10}$$

[18]The notation suggests that there are a finite number of households, but all propositions in this paper remain valid for the case of a continuum of households.

Evaluating the expectations in (10) using (2) and (3) yields:

$$r \;=\; \left(\beta\,(1+g)^{-\gamma}\,e^{\frac{1}{2}\gamma(\gamma+1)\sigma^2}\sum_S \pi_S\left[p_S(1-d_S)^{-\gamma}+(1-p_S)\left(1+\frac{p_S d_S}{1-p_S}\right)^{-\gamma}\right]\right)^{-1} \;. \quad (11)$$

Given this interest rate, the Euler equation (9) is satisfied if workers consume all their income: $c_{it} = y_{it}$.[19] If $c_{it} = y_{it}$, then $a_{it} = 0$ (budget constraint). In Krebs (2004) it is shown that the consumption-saving plan $c_{it} = y_{it}$ and $a_{it} = 0$ also satisfies a corresponding transversality condition if the interest rate is given by (11). Thus, it maximizes expected lifetime utility. Clearly, the choice $a_{it} = 0$ also satisfies the market clearing condition (8). Thus, we have found an equilibrium.

The result that equilibrium consumption equals income,$c_{it} = y_{it}$, means that equilibrium welfare (expected lifetime utility), U, is given by:

$$U \;\doteq\; E\left[\sum_{t=0}^{\infty}\beta^t u(c_{it})\right] \;=\; E\left[\sum_{t=0}^{\infty}\beta^t u(y_{it})\right] \quad (12)$$

Using the preference specification (6) and the definition of the income process (1)-(3), direct calculation yields the following formula for equilibrium welfare:

$$\begin{aligned} U &= \frac{y_{i0}^{1-\gamma}}{(1-\gamma)\left(1-\beta E\left[((1+g)^{1-\gamma}(1+\theta_i)(1+\eta_i))^{1-\gamma}\right]\right)} \quad if \;\; \gamma \neq 1 \quad (13)\\ U &= \frac{1}{1-\beta}log\, y_{i0} + \frac{\beta}{(1-\beta)^2}E\left[log\left((1+g)(1+\theta_i)(1+\eta_i)\right)\right] \quad if \;\; \gamma = 1 \;\; . \end{aligned}$$

Using the distributional assumption (2) and (3), we can evaluate the expectations in (13) and find:

$$U \;=\; \frac{y_{i0}^{1-\gamma}}{(1-\gamma)\left(1-\beta\,(1+g)^{1-\gamma}\,e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2/2}\sum_S \pi_S\left[p_S(1-d_S)^{1-\gamma}+(1-p_S)\left(1-\frac{p_S d_S}{1-p_S}\right)^{1-\gamma}\right]\right)}$$

[19]Notice that here we use the fact that θ_{it} and η_{it} do not predict future idiosyncratic shocks to income. That is, we have used the fact that (θ_{it}, η_{it}) and $(\theta_{i,t+1}, \eta_{i,t+1})$ are uncorrelated. Without this assumption, the Euler equation (9) would not hold at $a_{it} = 0$ and $c_{it} = y_{it}$ for the interest rate (10).

$$U = \frac{1}{1-\beta} log\, y_{i0} + \frac{\beta}{(1-\beta)^2}\left(log(1+g) - \sigma^2/2 + \sum_S \pi_S \left[p_S \log(1-d_S) + (1-p_S) log\left(1 + p_S d_S/(1-p_S)\right)\right] \right) . \quad (14)$$

We summarize the preceding discussion in the following proposition:

Proposition 1. The consumption-saving plan $\{a_{it}, c_{it}\}$, where $a_{it} = 0$ and $c_{it} = y_{it}$, in conjunction with the interest rate process (9) constitute an equilibrium. Welfare (expected lifetime utility) of workers in this equilibrium is given by (13), respectively (14).

IV. Cost of Business Cycles: Qualitative Analysis

In this section, we use the model laid out in section II to derive an explicit formula for the welfare cost of business cycles (proposition 2). Using this welfare expression, we then show the main theoretical result, namely that the cost of business cycles might be arbitrarily large even if the second moments of the distribution of individual income changes are constant over the business cycle (proposition 3). Indeed, the result we prove here is somewhat stronger in the sense that our proof assumes that job displacement probabilities are constant over the cycle. In other words, cyclical fluctuations in the earnings losses of displaced workers are sufficient to prove the result.

IV.1. Eliminating Business Cycles

We begin this section with a discussion of how the elimination of business cycles affects the nature of idiosyncratic risk. In our model, the elimination of business cycles amounts to moving from an economy with fluctuations in the aggregate state S (the economy with business cycles) to an economy with constant S (the economy without business cycles). In other words, we are moving from an economy with income risk defined by the S-independent distribution of income shocks θ_i and $S-$ *dependent* distribution of income shocks η_i to an economy with income risk defined by the S-independent distributions of income shocks $\bar{\theta}_i$ and $\bar{\eta}_i$. The question that arises is how to find the distributions of $\bar{\theta}_i$ and $\bar{\eta}_i$ given the distributions of θ_i and η_i. Following Lucas (1987) and the subsequent literature, we will answer this question without an explicit model of the interaction between macroeconomic stabilization policy and the business cycle.

For economies without uninsurable idiosyncratic risk (complete markets), Lucas (1987) postulates that the elimination of business cycles amounts to replacing all S-dependent economic variables by their mean value. That is, we take the expectations over S.[20] Extending this approach to economies with uninsurable idiosyncratic risk (incomplete markets), we postulate that eliminating business cycles means that we replace all S-dependent economic variables by their expected value with respect to S conditional on the idiosyncratic state of an individual worker. This "integration principle" has been used by several previous authors (Krebs (2003a), Krusell and Smith (1999,2002), and Lucas (2003)), and we use this approach in the current paper. For economic variables that have no S-dependence, the integration principle implies that the elimination of business cycles has no effect. Thus, for the a-cyclical component of individual income shocks we have $\bar{\theta}_i = \theta_i$, and therefore

$$log(1+\bar{\theta}_i) \sim N\left(-\sigma^2/2, \sigma^2\right) \ . \tag{15}$$

For the cyclical component of income shocks, η_i, the integration principle reads:

$$\bar{\eta}_i = E\left[\eta_i | s_i\right] \ , \tag{16}$$

where $s_i = 0$ if worker i is not displaced and $s_i = 1$ if worker i is displaced. Taking the

[20]Notice that this approach fundamentally differs from an approach that calculates the welfare cost of one recession (Hall, 1995). Put differently, Lucas (1987,2003) implicitly assumes that we can only reduce the severity of recessions by making the good times somewhat less attractive.

expectations in (16) yields:

$$\bar{\eta}_i = \begin{cases} -\bar{d} \quad with \;\; probability \; \bar{p} \\ \frac{\bar{p}\bar{d}}{1-\bar{p}} \quad with \;\; probability \;\; (1-\bar{p}) \end{cases} , \tag{17}$$

where

$$\begin{aligned} \bar{p} &= \sum_S \pi_S p_S \\ \bar{d} &= \sum_S \frac{\pi_S \, p_S}{\bar{p}} \, d_S \, . \end{aligned}$$

IV.2. Cost of Business Cycles

Equations (16) and (17) show how the elimination of business cycles affect the labor income process. We can use this information in conjunction with our welfare formula (13), respectively (14), to calculate the welfare cost of business cycles. More precisely, we define the welfare cost of business cycles as the number Δ that solves

$$E_0\left[\sum_{t=0}^{\infty} \beta^t u\left(c_{it}(1+\Delta)\right)\right] = E_0\left[\sum_{t=0}^{\infty} \beta^t u(\bar{c}_{it})\right] , \tag{18}$$

where c_{it} is consumption in the economy with business cycles and $\bar{c}_{it}$ is consumption in the economy without business cycles. That is, we define the welfare cost of business cycles as the percentage of consumption in each date-event that workers have to receive in order to be fully compensated for the cyclical variations in labor income risk. Using the definition (18) in conjunction with the welfare formula (14), we find the following formula for the cost of business cycles:

$$\begin{aligned} \Delta &= \left(\frac{1-\beta\,(1+g)^{1-\gamma}\, E\left[(1+\theta_i)^{1-\gamma}(1+\eta_i)^{1-\gamma}\right]}{1-\beta\,(1+g)^{1-\gamma}\, E\left[(1+\theta_i)^{1-\gamma}(1+\bar{\eta}_i)^{1-\gamma}\right]}\right)^{\frac{1}{1-\gamma}} - 1 \quad if \;\; \gamma \neq 1 \\ \Delta &= \frac{\beta}{1-\beta}\left(E\left[log\left((1+\theta_i)(1+\bar{\eta}_i)\right)\right] - E\left[log\left((1+\theta_i)(1+\eta_i)\right)\right]\right) \quad if \;\; \gamma = 1 \, . \end{aligned} \tag{19}$$

Using (15) and (17), we can evaluate the expectations in (19) and find:

$$\Delta = \left(\frac{1-\beta\,(1+g)^{1-\gamma}\, e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2}\sum_S \pi_S\left[p_S(1-d_S)^{1-\gamma} + (1-p_S)\left(1+\frac{p_S d_S}{1-p_S}\right)^{1-\gamma}\right]}{1-\beta\,(1+g)^{1-\gamma}\, e^{\frac{1}{2}\gamma(\gamma-1)\sigma^2}\left[\bar{p}(1-\bar{d})^{1-\gamma} + (1-\bar{p})\left(1+\frac{\bar{p}\bar{d}}{1-\bar{p}}\right)^{1-\gamma}\right]}\right)^{\frac{1}{1-\gamma}} - 1$$

$$U = \frac{\beta}{1-\beta}\left(\bar{p}log(1-\bar{d}) + (1-\bar{p})log\left(1+\bar{p}\bar{d}/(1-\bar{p})\right)\right) \tag{20}$$
$$- \frac{\beta}{1-\beta}\left(\sum_S \pi_S\left[p_S log(1-d_S) + (1-p_S)log\left(1+p_S d_S/(1-p_S)\right)\right]\right) .$$

Several facts about (20) are noteworthy. First, the cost of business cycles is the same for all worker. This is a result of the joint assumption of homothetic preferences and permanent income shocks with a distribution that is independent of workers' characteristics. Second, the welfare cost of business cycles is non-negative: $\Delta \geq 0$. This fact immediately follows from the concavity of the utility function in conjunction with the fact that η_i is a mean-preserving spread of $\bar{\eta}_i$. Thus, cyclical variations in idiosyncratic labor income risk never decrease the welfare cost of business cycles. Finally, and most importantly, the cost of business cycles is nil, $\Delta = 0$, if the income losses of displaced workers are constant: $d_S = \bar{d}$ for all aggregate states S. This last result immediately follows from the welfare expression (20) using $d_S = \bar{d}$ and $\bar{p} = \sum_S \pi_S p_S$. Thus, if the only source of cyclical fluctuations in labor income risk is state-dependent displacement probabilities, then the introduction of uninsurable idiosyncratic labor market risk does not change the cost of business cycles.

We summarize the preceding discussion in the following proposition.

Proposition 2. The welfare cost of business cycles is given by (20). The welfare cost of business cycles is always non-negative, $\Delta \geq 0$. The cost of business cycles is nil, $\Delta = 0$, if (and only if) income losses of displaced workers have no cyclical component: $d_S = \bar{d}$ for all aggregate states S. That is, cyclical fluctuations in job displacement probabilities by themselves do not generate cost of business cycles.

Proposition 2 shows that there is not much hope for generating cost of business cycles through cyclical variations in job displacement probabilities only. Matters are different, however, once we allow for cyclical variations in the income losses of displaced workers: $d_S \neq d_{S'}$ for some pair of aggregate states $S \neq S'$. Inspection of (20) suggests that we have $\Delta \to \infty$ when $d_S \to 1$ for some S if $\gamma \geq 1$. That is, for high enough degree of risk aversion, the cost of business cycles becomes arbitrarily large when the income losses of displaced workers during certain macroeconomic conditions (recessions) become arbitrarily large. In the appendix, we show that this result still holds even if the second moments of the distribution of income changes are (almost) constant. The trick here is to send simultaneously the displacement probability to zero, $p \to 0$, and to do this in a way so that the cost of business cycles is still growing without bounds, but the second moments of the distribution of income shocks is (almost) not affected. In short, we have the following proposition:

Proposition 3. Suppose the degree of relative risk aversion is large enough: $\gamma \geq 1$. Then there is a process of job displacement risk so that i) the cost of business cycles is arbitrarily large and ii) the second moments of the distribution of individual income shocks are (almost) constant over the business cycle. More precisely, denote the second moment of the distribution of individual income shocks by $var\,(y_{i,t+1}/y_{it}|S_{t+1}) = \sigma_y^2(S_{t+1})$ and assume $\gamma \geq 1$.[21] Then for any real numbers $\epsilon > 0$ and any $\bar{\Delta} > 0$, we can find a process of job displacement risk defined by displacement probabilities p_S and earnings losses of displaced workers d_S so that i) the implied cost of business cycles is $\Delta = \bar{\Delta}$ and ii) the implied second moments of the distribution of income shocks satisfy $|\sigma_y^2(S) - \sigma_y^2(S')| < \epsilon$ for all S, S'.

V. Cost of Business Cycles: Quantitative Analysis

In this section, we analyze the quantitative importance of the main theoretical result derived in the previous section (proposition 3). To this end, we first discuss the calibration of the model economy (section V.1), and then report the quantitative results (section V.2).

[21]The proposition still holds if we consider the variance of log-income changes, $\sigma^2_{log\,y} \doteq var\,(logy_{i,t+1} - logy_{it}|S_{t+1})$, but for this version of proposition 3 the conditions $\gamma > 1$ is required.

V.1. Calibration

Following the previous literature (Imrohoroglu, 1989, Krebs, 2003a, Krusell and Smith, 1999, and Storesletten et al., 2001) we assume now that there are two aggregate states, $S = L, H$, corresponding to low economic activity (economic contraction) and high economic activity (economic expansion). We further follow the literature and disregard any asymmetry in the business cycle, that is, we assume that on average both aggregate states have the same likelihood of occurences. In our setting without persistence in the aggregate state process, this means that $\pi_L = \pi_H = .5$. We choose the period length to be one year to be consistent with the empirical work on labor market risk (see below). Thus, the choice of $\pi_L = \pi_H = .5$ implies an average duration of both good and bad times of two years, which is also the value considered in Imrohoroglu (1989), Krebs (2003a), and Krusell and Smith (1999,2002). We choose an average growth rate of labor income of $g = .02$.

The process of job displacement risk is defined by the parameters p_L, p_H, d_L, and d_H. We choose these parameters so that the model economy matches the first and second moments of the job displacement rate and the permanent earnings losses of U.S. displaced workers that have been estimated by the empirical literature. That is, we try to match previous estimates of i) the average probability of job displacement, ii) the cyclical variations of the job displacement probabilities, iii) the average permanent earnings loss of displaced workers, and iv) the cyclical variations in the permanent income loss of displaced workers. We now turn to a discussion of the estimates of these moments by the empirical literature.

There are several studies of the incidence and consequence of job displacement for U.S. workers (Beaudry and Nardi (1991), Farber (1997), Jacobson, LaLonde, and Sullivan (1993), Neal (1995), Topel (1991), and Ruhm (1991)). One of the most detailed studies of the long-term consequences of job loss for high-tenure workers is Jacobsen et al. (1993), and we will base our choice of d_L and d_H for the baseline economy mainly on their findings. Jacobsen et al. (1993) use longitudinal data on the earnings of high-tenure workers (workers with at least six years of tenure) in Pennsylvania from 1974 to 1986 to estimate the earnings losses of displaced workers. In their restricted sample, they confine attention to workers that are separated from distressed firms, where they define a distressed firm as a firm that experienced an employment contraction of at least 30%. For these workers, they find a very large drop in earnings the year following the displacement event (around 50%). Moreover, and more importantly from the point of view of the current paper, these income losses have a sizable component that is highly persistent. More specifically, even 6 years after separation the earnings of the displaced workers is 25% below the earnings of workers with similar characteristics that have not been displaced. We take this number as an estimate of the average permanent income loss of displaced workers and therefore require $.5d_L + .5d_H = .25$.[22]

The result that high-tenure, displaced workers experience a permanent earnings loss of about 25% is also consistent with other estimates in the empirical literature. For example, Topel (1990) analyzes individual earnings data from the Panel Study of Income Dynamics (PSID) and estimates that displaced workers with at least 10 years of seniority prior to displacement suffer a permanent income loss of 25%.[23] Ruhm (1991) also uses PSID income data to estimate the permanent income losses of displaced workers, but he does not split the sample into low- and high-tenure workers. For the entire sample of all displaced workers, he finds earnings losses that are substantial ($10 - 13\%$), but significantly lower than the

[22] Jacobson et al. (1993) also consider a larger sample of workers that consists of all separated workers including those who quit their job and those who get laid off due to "slack work". For workers who quit, we expect smaller earnings losses than before, and for workers who are laid off because of "slack work", we would expect higher earnings losses (Gibbon and Katz, 1991). Jacobson et al (1993) find that the average earnings losses of all separated workers are smaller than the earnings losses of workers that are displaced because of mass lay-offs, which suggest that the "quit-effect" dominates.

[23] Topel (1990) and others in the literature interpret these results as a confirmation of a human capital theory in which workers invest in firm-specific human capital, but Neal (1995) presents results that suggest that most of the income losses of displaced workers are due to the destruction of industry-specific (or occupation-specific) human capital. For a theoretical model of job separation with firm-specific human capital, see Jovanovic (1979).

25% Jacobson et al. (1993) find for high-tenure workers. Since the permanent income losses of low-tenure workers are likely to be small, this difference in findings is clearly not too surprising. Finally, using evidence from the Displaced Workers Survey (DWS), Farber (1997) finds earnings losses similar to Ruhm (1991) for a sample that again includes both low- and high-tenure workers.

Jacobson et al (1993) also find that the permanent earnings losses of high-tenure, displaced workers have a strong cyclical component. More specifically, they define the cyclical labor market condition by the unemployment rate and the deviation from trend employment, and estimate that workers who become displaced during the worst cyclical conditions experience a permanent income loss of 37%, whereas this income loss is only 13% for those workers who experience job displacement during the best cyclical conditions.[24] In short, the spread is $37\% - 13\% = 24\%$. Clearly, focusing only on the most extreme cyclical conditions is overstating the cyclicality of income losses, but these estimates indicate a very strong cyclical component. Beaudry and DiNardo (1991) provide additional evidence in favor of the view that earnings losses of displaced workers display strong cyclical fluctuations. More specifically, they show that wages of new hires decrease on average by approximately $3-4.5\%$ for every percent increase in the unemployment rate.[25] Thus, assuming a spread of the unemployment rate of 5 percent between economic contractions and economic expansions, their finding implies a variation of income losses over the cycle of somewhere between 15% and 22.5%.[26] Taken together, the empirical evidence suggests that a value of 16% is a reasonable

[24]See table 2 and the corresponding discussion in Jacobson et al (1993). Note that they derive this result exploiting differences across local labor markets. In other words, their result is a cross-sectional result, and the time-series conclusions used here are therefore somewhat tentative.

[25]Bils (1985) finds that the real wage of all workers goes up by $1.5 - 2$ percent for each percentage point increase in the unemployment rate.

[26]Notice that a 5 percent spread in the unemployment rate is consistent with the fluctuations in the U.S. unemployment rate over the last 30 years. Note also that Imrohoroglu (1989) and Krusell and Smith (1999,2002) use a spread in the unemployment rate that is even larger.

estimate of the change in the permanent income losses of displaced workers over the cycle, which in the model is given by the simple difference $d_L - d_H$. Combining the requirement $d_L - d_H = .16$ with the restriction $.5d_L = .5d_H = .25$ yields the parameter choice $d_L = .33$ and $d_H = .17$.

To complete the calibration of the job displacement process, we need to assign values for the job displacement probabilities p_L and p_H. Jacobson et al (1993) report that in their sample of high-tenure workers, the fraction of workers that experience at least one job displacement event due to mass layoffs is equal to 28 percent over a time span of 13 years (see table 1 in Jacobson et al (1993)). Assuming that the job displacement event is i.i.d. over time, this means that the probability of experiencing job displacement within a particular year is equal to $1 - (1 - .28)^{1/13} = .025$. An annual job displacement probability of 2.5%, however, is likely to be an underestimate of the actual job displacement probability since the sample of workers displaced by mass-layoffs constructed by Jacobson et al. (1993) excludes many workers that ought to be counted as displaced workers, namely all those workers who lost their jobs for "exogenous reasons" but worked for firms that did not contract by at least 30 percent .[27] Another reason why the job displacement rates reported in Jacobson et al. (1993) are not a sufficient basis for calibrating the current model is that they do not report to what degree these job displacement rates depend on cyclical labor market conditions. In this regard, the results reported in Farber (1997) are more useful since he computes job displacement rates for seven three-year periods. For male workers of age 35-44, these job displacement rates have a mean of .0384 and a standard deviation of .0061 (annualized). Clearly, the fact that Farber considers both low- and high tenure workers somewhat overstates the average job displacement rate. On the other hand, the fact that he considers three-year averages is likely to understate the degree of cyclical variation. In our baseline

[27] Another reason why a job displacement rate of 2.5% is an underestimate is that Jacobson et al (1993) eliminate all displaced workers who subsequently report no earnings.

model, we consider a job displacement process with an average (annual) displacement probability of 3% and a standard deviation of .5%, which seems a conservative estimate. In short, we impose the restrictions $.5p_L + .5p_H = .03$ and $\sqrt{.5(p_L - .03)^2 + .5(p_H - .03)^2} = .005$, which yields $p_L = .035$ and $p_H = .025$.

The parameter choice $d_L = .33$, $d_H = .17$, $p_L = .35$, and $p_H = .25$ matches certain moments of the job displacement process for high-tenure U.S. workers. Clearly, high-tenure U.S. workers are different from the average U.S. worker in the sense that high-tenure workers have lower displacement rates and higher earnings losses when displaced. To see how the cost of business cycles are affected when we match the first and second moments of the job displacement process for all U.S. workers, we also consider a version of the model in which we choose $d_L = .20$, $d_H = .10$, $p_L = .045$, and $p_H = .035$. That is, we consider a model with a job displacement rate that has a mean of 4 percent and a standard deviation of .5 percent as well as earnings losses of displaced workers with a mean of 15 percent and a standard deviation of 5 percent.

To complete the calibration of the labor income process, it remains to find a value for the variance $var\,(log(1+\theta_{i,t+1})) = \sigma^2$. To find this value, notice first that the average variance of log-income changes is

$$\begin{aligned} \sigma^2_{log\,y} &\doteq \sum_S \pi_S\, var\,((log\,y_{i,t+1} - log\,y_{it})|S) \\ &= \sigma^2 + \sum_S \pi_S\, var\,(log(1+\eta_{i,t+1})|S) \ . \end{aligned} \tag{21}$$

The variance term $\sigma^2_{log\,y}$ has been estimated by an extensive empirical literature. More specifically, work by, for example, Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten, et al. (2004) decompose individual income of workers into a transitory and permanent component, and estimate the variance of each component separately. As argued before (see equation 4), the estimates of the variance of the permanent component of labor income changes correspond to $\sigma^2_{log\,y}$. Averaged over business cycle conditions and educational groups (if applicable), the estimates of this literature lie around a value of at least $\sigma^2_{log\, y} = .022$ (a standard deviation of 15 percent). Using the already assigned values for the displacement parameters p_S and d_S, equation (21) pins down σ^2. Using our previous values, we find $\sigma^2 = .02067$ (a standard deviation of $\sigma = .1438$) for the first model (high-tenure workers), and $\sigma^2 = .02154$ (a standard deviation of $\sigma = .1468$) for the second model (all workers).

We choose the preference parameters as follows. We follow the bulk of the business cycle literature and choose an annual discount factor of $\beta = .96$. In comparison, Imrohoroglu (1989) also chooses $\beta = .96$, but Storesletten et al. (2001) pick $\beta = .95$. Krusell and Smith (1999,2002) assume a stochastic discount factor with a mean of .95. For the degree of relative risk aversion, the standard choice in the macroeconomic literature is $\gamma = 1$ (log-utility), and this is also our choice for the baseline model. In comparison, Krusell and Smith (1999,2002) use log-utility, Storesletten et al. (2001) focus on $\gamma = 4$, and Imrohoroglu considers the two cases $\gamma = 1.5$ and $\gamma = 6$.

To sum up, for the baseline economy we choose $g = .02$, $\pi_L = \pi_H = .5$, $\sigma^2 = .02067$, $p_L = .035$, $p_H = .025$, $d_L = .33$, $d_H = .17$, $\beta = .96$, and $\gamma = 1$. Using equation (9), we find that the implied real interest rate is $r = .389\%$. This defines our baseline economy (model 1) with displacement risk of a typical high-tenure U.S. worker. However, we also consider a calibrated model economy (model 2) with displacement risk $p_L = .045$, $p_H = .035$, $d_L = .20$, $d_H = .10$ describing the typical U.S. worker with any years of tenure. Finally, we also consider the effect of changes in preference parameters. More specifically, we also consider increases in risk aversion to $\gamma = 2$ and $\gamma = 3$. When we vary the degree of risk aversion, we consider two cases. In the first case, we keep the discount factor at $\beta = .96$ and let the interest rate adjust. For $\gamma = 2$, this implies $r = 1.34\%$ and for $\gamma = 3$ this means $r = -3.27\%$. In the second case, we adjust the discount factor so that the implied real interest rate remains at $r = 3.89\%$.

V.2. Results

Table 1 present the results of our quantitative study. The first column in table 1 shows the cost of business cycles for the baseline labor income process (high-tenure workers) and different preferences parameters. If $\beta = .96$ and $\gamma = 1$ (log-utility), which is the standard assumption in the macroeconomic literature, we have a welfare cost of business cycles that is equal to $\Delta = .423\%$. In comparison, for the same preference parameters, Lucas (1987) finds $\Delta = .05\%$ using a representative-agent model. Thus, the cost of business cycles found here is roughly one order of magnitude larger than the ones found in Lucas (1987). Notice also that in the current model without fluctuations in aggregate income, the cost of business cycles is nil when the representative-agent assumption is made (column 4).

The next column shows the cost of business cycles when job displacement probabilities are constant $p_L = p_H$. In this case, the cost of business cycles is somewhat smaller than in the previous case, but the difference is very small. Thus, introducing cyclical variations in job displacement probabilities amplifies the effect of cyclical variations in the income losses of displaced workers, but this effect is quantitatively negligible. Notice, however, that cyclical variations by themselves have no effect on the cost of business cycles (proposition 2). Put differently, if we assume $d_L = d_H$, then in the current model we have no cost of business cycles: $\Delta = 0$.

The third column shows the cost of business cycles when, incorrectly according to the current paper, the researcher assumes that log-income changes are normally distributed. More precisely, according to model 1 , the standard deviation of log-income changes, $\sigma^2_{log\,y}$, varies between $\sigma_{log\,y}(L) = .1538$ and $\sigma_{log\,y} = .1460$. In the third column of table 1, we consider a labor income process that has, conditional on the aggregate state S, a symmetric two-state support, and choose the support so that we match the given S-dependent standard deviations. That is, we consider the case $\theta_{it} = 0$ ($\sigma^2 = 0$), $p_L = p_H = .5$, and $log(1-d_L) =$ $.154$ and $log(1-d_H) = .146$, which is the two-state approximation of a labor income process for which log-income changes are normally distributed with given S-dependent variance.[28] The results reported in the third column show that the cost of business cycles becomes negligible once the log-normal approach is taken. Put differently, if we generate the cyclical variation in income risk $\sigma_{log\,y}(L) = .154$ and $\sigma_{log\,y} = .146$ using the log-normal approach, then the resulting cost of business cycles is only a very small fraction of the cost of business cycles that is suggested by a more detailed study of job displacement risk. More generally, if the true distribution is not symmetric, then assuming symmetry as in Krebs (2003a) and Storesletten et al. (2001) can lead to a serious under-estimation of the cost of business cycles.

Table 1 also shows how the cost of business cycles changes when the degree of risk aversion is varied. More specifically, we choose $\gamma = 2$ and $\gamma = 3$ and either keep the discount factor at $\beta = .96$ or readjust β so that the implied interest rate remains constant. Table 1 shows that for fixed β, the cost of business cycles is increasing in γ and strongly convex. For example, if $\gamma = 2$ we have $\Delta = 1.255\%$ and for $\gamma = 3$ we find $\Delta = 12.68\%$. The strong convexity of the cost of business cycles as a function of γ has been emphasized by Storesletten et al. (2001). In contrast, once we readjust the discount factor β to keep the interest rate constant, the cost of business cycles becomes almost linear. For example, if $\gamma = 2$ we have $\Delta = .746\%$ and for $\gamma = 3$ we find $\Delta = 1.142\%$.

Table 2 shows the welfare results if we calibrate the model so that the implied job displacement process matches the estimated first and second moments of job displacement rates and earnings losses for all U.S. workers (model 2). This version of the model generates cost

[28] More precisely, this is the two-state approximation of the normal distribution using the Gauss-Hermite quadrature (Judd, 1998). The welfare results barely change if a four-state approximation is used, which suggests that the two-state approximation is already very good.

of business cycles that are still sizable, but a comparison of table 1 and table 2 shows that now the cost are significantly lower than the cost in model 1. For example, in the case of $\gamma = 1$ and $\beta = .96$, we have $\Delta = .423\%$ for model 1, but only $\Delta = .174\%$ for model 2.

Finally, we provide a welfare calculation similar to the one made in Hall (1995). More specifically, suppose that we could eliminate recessions (the bad times) without affecting economic expansions (the goods times). That is, suppose that we replace the job displacement process defined by $p_L = .035$, $p_H = .025$, $d_L = .33$, and $d_H = .17$ with a job displacement process defined by $\bar{p} = p_H = .025$ and $\bar{d} = d_H = .17$, and then calculate the welfare gain expressed as before in equivalent consumption units. This number corresponds to the welfare cost of recessions.[29] For model 1 with $\gamma = 1$ and $\beta = .96$, we find a cost of recessions of 2.36 percent of lifetime consumption. For model 2 with the same preference parameters, the cost of recessions is still 1.00 percent of lifetime consumption. In short, the cost of recessions are very large indeed.

VI. Conclusion

This paper analyzes the welfare costs of business cycles when workers face uninsurable job displacement risk that has a cyclical component. Using a simple dynamic general equilibrium model with incomplete markets, this paper shows the following: for sufficiently high degree of risk aversion (above one), the introduction of cyclical variations in the permanent earnings losses of displaced workers can generate arbitrarily large cost of business cycles even if there is no employment risk (displaced workers are immediately re-employed) and the second moments of the distribution of individual income shocks are (almost) constant over the cycle. In other words, the previous literature, which has either focused on cyclical fluctuations in employment risk or assumed that income changes of workers are (log)-normally distributed,

[29] Hall (1995) computes the cost of one recession, whereas here we compute the cost of all (future) recessions.

might have severely under-estimated the cost of business cycles. In addition to the theoretical analysis, this paper also conducts a quantitative study of the cost of business cycles using empirical evidence about the permanent earnings losses of displaced U.S. workers. The quantitative analysis suggests that the cost of business cycles due to the cyclical variations in job displacement risk is quite substantial.

Appendix

Proof of proposition 3.

We have

$$var\left[y_{i,t+1}/y_{it}|S_{t+1}\right] = var\left[\,(1+g)(1+\theta_{i,t+1})(1+\eta_{i,t+1})|S_{t+1}\right] \qquad (A1)$$

$$= (1+g)^2 var\left[\,\theta_{i,t+1}+\eta_{i,t+1}+\theta_{i,t+1}\eta_{i,t+1}|S_{t+1}\right],$$

$$= (1+g)^2\left(var\left[\,\theta_{i,t+1}|S_{t+1}\right]+var\left[\,\eta_{i,t+1}|S_{1+1}\right]+var\left[\,\theta_{i,t+1}\eta_{i,t+1}|S_{t+1}\right]\right)$$

$$= (1+g)^2\left(var\left[\,\theta_{i,t+1}|S_{t+1}\right]+var\left[\,\eta_{i,t+1}|S_{1+1}\right]+var\left[\,\theta_{i,t+1}|S_{t+1}\right]var\left[\,\eta_{i,t+1}|S_{t+1}\right]\right)\ ,$$

where the third and fourth lines follow from $cov\left[\theta_{i,t+1},\eta_{i,t+1}|S_{t+1}\right]=0$ and $E\left[\theta_{i,t+1}|S_{t+1}\right]=E\left[\eta_{i,t+1}|S_{t+1}\right]=0$.

To simplify the notation, we consider the case of two aggregate states: $S=L,H$. We further assume $d_H=0$ and $p_L=p_H=p$. That is, if $S_{t+1}=H$, then $\eta_{i,t+1}=0$. Thus, for $S_{t+1}=H$ we have:

$$var\left[y_{i,t+1}/y_{it}|H\right]=(1+g)^2e^{\sigma^2}-1\ . \qquad (A2)$$

On the other hand, for $S_{t+1}=L$ we have:

$$var\left[y_{i,t+1}/y_{it}|L\right]=(1+g)^2\left[e^{\sigma^2}-1+\frac{pd^2}{1-p}+\left(e^{\sigma^2}-1\right)\frac{pd^2}{1-p}\right]\ . \qquad (A3)$$

Take any $\epsilon>0$ and d with $0<d<1$, and choose $p=\frac{\epsilon}{\epsilon+d^2(1+g)^2e^{\sigma^2}}$. Clearly, by construction we have $0<p<1$ and

$$var\left[y_{i,t+1}/y_{it}|L\right]-var\left[y_{i,t+1}/y_{it}|H\right] = (1+g)^2\frac{pd^2}{1-p}e^{\sigma^2} \qquad (A4)$$

$$=\epsilon\ .$$

That is, the choice of $p=\frac{\epsilon}{\epsilon+d^2(1+g)^2e^{\sigma^2}}$ ensures that the cyclical variation in the second moments of the distribution of income growth rates is always equal to ϵ, and therefore arbitrarily small. We now show that for any number $\bar{\Delta} > 0$ we can always find a number d with $0 < d < 1$ (and a corresponding number $p = \frac{\epsilon}{\epsilon + d^2(1+g)^2 e^{\sigma^2}}$) so that the implied welfare cost of business cycles is equal to $\bar{\Delta}$.

To simplify notation, introduce $c = (1+g)^2 e^{\sigma^2} > 0$, so that $p = \frac{\epsilon}{\epsilon + c\,d^2}$. Consider the cost of business cycles (19) for $\gamma > 1$ (the argument for the log-utility case is the same). Notice first that the expectations term in (19) can be written as

$$\beta E\left[(1+\theta_{i,t+1})^{1-\gamma}(1+\eta_{i,t+1})^{1-\gamma}\right] = \beta e^{-\frac{1}{2}\gamma(1-\gamma)\sigma^2} f(\epsilon, d) \qquad (A5)$$

and

$$\beta E\left[(1+\theta_{i,t+1})^{1-\gamma}(1+\bar{\eta}_{i,t+1})^{1-\gamma}\right] = \beta e^{-\frac{1}{2}\gamma(1-\gamma)\sigma^2} \bar{f}(\epsilon, d)\ ,$$

where we introduced

$$f(\epsilon, d) = \pi_L \left(\frac{\epsilon}{\epsilon + cd^2}(1-d)^{1-\gamma} + \left(1 - \frac{\epsilon}{\epsilon + cd^2}\right)\left(1 + \frac{\frac{\epsilon}{\epsilon + cd^2} d}{1 - \frac{\epsilon}{\epsilon + cd^2}}\right)^{1-\gamma} \right) + \pi_H \qquad (A6)$$

and

$$\bar{f}(\epsilon, d) = \frac{\epsilon \pi_L}{\epsilon + cd^2}(1 - d\pi_L)^{1-\gamma} + \left(1 - \frac{\epsilon \pi_L}{\epsilon + cd^2}\right)\left(1 + \frac{\frac{\epsilon \pi_L}{\epsilon + cd^2} d\pi_L}{1 - \frac{\epsilon \pi_L}{\epsilon + cd^2}}\right)^{1-\gamma} .$$

From the fact that η_i is a mean-preserving spread of $\bar{\eta}_i$, it follows immediately that

$$\bar{f}(\epsilon, d) < f(\epsilon, d) \qquad (A7)$$

for any $\gamma > 1$. Further, for any $\epsilon > 0$, there is a number d with $0 < d < 1$ so that

$$f(\epsilon, d) = \left(\beta e^{-\frac{1}{2}\gamma(1-\gamma)\sigma^2}\right)^{-1} . \qquad (A8)$$

The last equation (A8) follows from the continuity of f in conjunction with $f(\epsilon, 0) = \pi_L + \pi_H = 1$, $\lim_{d \to -1} f(\epsilon, d) = +\infty$, and the fact that the right-hand-side of (A8) is strictly greater than one by assumption (7). Using (A5)-(A7) in the welfare expression (20) yields the desired result, namely the existence of a d and p so that (A4) is satisfied and the cost of business cycles is equal to $\bar{\Delta}$.

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Table I. Cost of Business Cycles: Model 1[30]

Model	Baseline	$p_L = p_H$	log-normal	complete markets
$\beta = .96\ ,\ \gamma = 1$	.423 %	.420 %	.021 %	0 %
$\beta = .96\ ,\ \gamma = 2$	1.255 %	1.235 %	.046 %	0 %
$\beta = .96\ ,\ \gamma = 3$	12.682 %	10.853 %	.220 %	0 %
$\beta = .937\ ,\ \gamma = 2$	.746 %	.737 %	.028 %	0 %
$\beta = .894\ ,\ \gamma = 3$	1.142 %	1.112 %	.032 %	0 %

Table II. Cost of Business Cycles: Model 2[31]

Model	Baseline	$p_L = p_H$	log-normal	complete markets
$\beta = .96\ ,\ \gamma = 1$	.174 %	.171 %	.005 %	0 %
$\beta = .96\ ,\ \gamma = 2$	.441 %	.434 %	.012 %	0 %
$\beta = .96\ ,\ \gamma = 3$	2.549 %	2.445 %	.055 %	0 %
$\beta = .936\ ,\ \gamma = 2$	.261 %	.257 %	.007 %	0 %
$\beta = .893\ ,\ \gamma = 3$	.340 %	.334 %	.008 %	0 %

[30] Cost of business cycles are expressed as percentage of lifetime consumption. Job displacement process of high-tenure workers. The baseline column assumes displacement probabilities $p_L = .035$ and $p_H = .025$ and corresponding income losses of $d_L = .33$ and $d_H = .17$. The column $p_L = p_H$ has constant displacement probabilities of $p_L = p_H = .03$ using the same income losses.

[31] The first column assumes displacement probabilities $p_L = .045$ and $p_H = .035$ and corresponding income losses of $d_L = .20$ and $d_H = .10$. The column $p_L = p_H$ has constant displacement probabilities of $p_L = p_H = .04$ using the same income losses.