Comparative Advantage and Heterogeneous Firms *

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Abstract

This paper presents a model that incorporates endowment-based inter-industry trade, variety-driven intra-industry trade, firm heterogeneity and industry entry and exit. The model simultaneously explains why some countries export more in certain industries than in others and why, within industries, some firms export and others do not. We show how the theorems of the Heckscher-Ohlin model carry over to this general framework and derive novel implications for *firm-level responses* to a decline in trade costs. Opening to trade increases both the number of firms and the amount of firm entry and exit in comparative advantage industries relative to comparative disadvantage industries. We also show that industry productivity gains due to reallocation across firms depend on country and industry size and are strongest in comparative advantage industries.

Keywords: Heckscher-Ohlin, international trade, inter-industry trade, intra-industry trade, trade costs, entry and exit

JEL classification: F11, F12, L11

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1. Introduction

This paper introduces heterogeneous firms to a world where trade is driven by both comparative advantage and horizontal product differentiation. The number of firms, as well as their size and export behavior, depends upon the interaction of country endowments, industry factor usage and firm productivity. Movement from autarky to free trade differentially affects industries across countries and firms across countries and industries. Some of the implications generated by the model reinforce the intuition of existing frameworks. Others, focusing on heterogeneous firms and comparative advantage, are new.

As in Helpman and Krugman (1985), we integrate new trade theory assumptions about imperfect competition and scale economies into the factor proportions framework of the Heckscher-Ohlin model. However, we extend that analysis of inter- and intra-industry trade to a framework with heterogeneous firms. This introduction of firm diversity into the factor proportions framework permits us to study how economic activity is reallocated across and within firms, industries, and countries as trade costs fall. Following the literature on heterogeneous firms and international trade (Bernard et al. 2003; Melitz 2003), we model firms as varying in terms of productivity. Unlike these models, however, we allow for multiple industries, multiple factors of production, and asymmetric countries. We find that the export behavior of firms and the aggregate productivity response of industries varies with comparative advantage in response to trade liberalization.

Our framework simultaneously explains why some countries export more in certain industries than others (Heckscher-Ohlin based comparative advantage); why nonetheless we observe two-way trade within industries (firm-level product differentiation combined with increasing returns to scale); and why, within industries engaged in these two forms of trade, some firms export and others do not (heterogeneous firms and export costs). Our analysis is consistent with a host of existing stylized facts about industries, including heterogeneous firm productivity, ongoing entry and exit, the positive covariation in entry and exit rates across industries, and higher productivity of exporting firms.[1] It also suggests the need for new empirical research examining the relationship between firm heterogeneity and comparative advantage.[2]

[1] For empirical evidence on these points, see Bartelsman and Doms (2000), Bernard and Jensen (1995, 1999), Clerides, Lach and Tybout (1998), Davis and Haltiwanger (1991), Dunne, Roberts and Samuelson (1989), and Roberts and Tybout (1997a) among others.

[2] For recent empirical evidence of Heckscher-Ohlin forces operating at the level of individual firms and products within industries, see Bernard, Jensen and Schott (2004) and Schott (2004).

Adopting Samuelson's (1949) and Dixit and Norman's (1980) concept of the integrated equilibrium, we characterize the conditions under which factor price equalization occurs and show how the theorems of the Heckscher-Ohlin model carry over, with slight modifications, to our more general setting. Countries that open to international trade enjoy gains from three sources: specialization according to comparative advantage; the potential availability of wider product variety; and the reallocation of economic activity from less to more productive firms.

Adding heterogeneous firms to a model of comparative advantage and intra-industry trade also provides a rich set of new implications for within-industry responses to trade liberalization and factor accumulation. These responses include changes in the number of firms, firm size, the degree of firm entry and exit, the productivity range of producing firms, and the proportion of exporting firms. We explore these responses in open economies with costless trade and then in trading economies with fixed and variable trade costs.

We find important interactions between comparative advantage and economic outcomes when countries move from autarky to free trade. Beside the traditional reallocation of activity across sectors, each country experiences increases in the number of firms, the amount of firm entry and exit, and average firm size in the comparative advantage industry relative to the comparative disadvantage industry. Since firm entry is associated with job creation and firm exit with job destruction, a corollary of relatively more entry and exit in the comparative advantage industry is greater job creation and destruction.

As in previous single industry models of heterogeneous firms, we find that declining trade costs increase overall industry productivity as activity is reallocated from less to more productive firms. Declining trade costs also increase the probability of exporting. In addition, we obtain a variety of novel results on how within-industry reallocations are systematically shaped by the interaction of countries' relative factor abundance and industries' relative factor intensity.

With trade costs, higher-productivity firms export, while some lower-productivity firms may choose only to serve the domestic market. The export opportunities offered to high productivity firms raise the *ex ante* expected value of entry into an industry, increasing entry, reducing *ex post* profitability for low productivity firms, and raising the threshold productivity below which firms earn zero profit and exit the industry.

Comparative advantage means that attractiveness of export opportunities varies systematically across countries and industries. In a country's comparative advantage industry, the greater relative attractiveness of export opportunities means a higher probability of exporting, a larger increase in the *ex ante* expected value of entry, and a greater increase in the zero-profit productivity threshold. Thus, comparative advantage industries see the largest changes in industry composition and hence the greatest improvements in aggregate productivity, as high productivity exporting firms expand in size and low productivity firms exit.

The analysis also identifies a separate and more subtle relationship between firm heterogeneity, factor abundance and factor intensity. If the different stages of economic activity, in particular entry and production, have different factor intensities, movements in factor prices following declines in trade costs have uneven effects on firm entry/exit decisions across countries and industries. For example, if entry is skill intensive (e.g. due to a highly skilled product development component of entry costs), a fall in the relative wage of skilled workers in a labor-abundant country following a decline in trade costs will reduce entry costs relative to revenue from production, thereby inducing increased entry, lower *ex post* profitability, and an increase in the zero-profit productivity threshold.

In each case, whether we are concerned with the uneven effects of trade on firms according to their export status or with differences in factor intensity across the various stages of economic activity, firm heterogeneity interacts with factor abundance and factor intensity to shape the ways in which firms, industries and countries respond to trade.

The remainder of the paper is structured as follows. Section 2 develops the model and solves for general equilibrium with costless trade. Section 3 explores the properties of the free trade equilibrium, highlighting new results on firm and industry changes in response to trade liberalization and showing how the theorems of the standard Heckscher-Ohlin model carry over to our heterogeneous-firm approach. In the interests of realism, Section 4 introduces fixed and variable trade costs to the model. In Section 5, we present the properties of the costly trade model focusing on the effects of changes in trade costs on productivity, firm size, job churning and exporting across industries and countries. In Section 6, we parameterize the model and examine, in turn, the effects of changing trade costs and of changing relative country endowments. Section 7 concludes.

2. Model with Costless Trade

We consider a world of two countries, two industries, two factors and a continuum of heterogeneous firms. We make the standard Heckscher-Ohlin assumption that the countries are identical in terms of preferences and technologies, but differ in terms of factor endowments. Factors of production can move between industries within countries but cannot move across countries. We use H to index the skill-abundant home country and F to index the skill-scarce foreign country, so that $\overline{S}^H/\overline{L}^H > \overline{S}^F/\overline{L}^F$ where the bars indicate country endowments.

2.1. Consumption

The representative consumer's utility depends on consumption of the output of two industries (i), each of which contains a large number of differentiated varieties (ω) produced by heterogeneous firms.[3] For simplicity, we assume that the upper tier of utility determining consumption of the two goods is Cobb-Douglas and that the lower tier of utility determining consumption of varieties takes the CES form[4],

$$U^H = C_1^{\alpha_1} C_2^{\alpha_2}, \qquad \alpha_1 + \alpha_2 = 1, \ \alpha_1 = \alpha. \tag{1}$$

where, to simplify notation, we omit the country superscript except where important.

C_i is a consumption index defined over consumption of individual varieties, $q_i(\omega)$, with dual price index, P_i, defined over prices of varieties, $p_i(\omega)$,

$$C_i = \left[\int_{\omega\in\Omega_i} q_i(\omega)^\rho d\omega\right]^{\frac{1}{\rho}}, \qquad P_i = \left[\int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \quad \text{where } \sigma > 1. \tag{2}$$

2.2. Production

The production side of the model follows Melitz (2003) in that production involves a fixed and variable cost every period, and only variable costs move systematically with firm productivity. We differ from Melitz (2003) in that both entry and production require multiple factors of production (skilled and unskilled labor) whose intensity of use varies across industries. To avoid undue complexity, we assume that the cost function takes the

[3] Allowing one industry to produce a homogenous good under conditions of perfect competition and constant returns to scale (eg Agriculture) is a special case of this framework where, in one industry, the elasticity of substitution between varieties is infinite and the fixed production cost is zero.

[4] We use the terms "good", "sector", and "industry" synonymously while variety is reserved for a horizontally differentied version within an industry. All we require is a utility function with an upper tier where goods are substitutes and a lower tier where consumer preferences exhibit a love of variety for each good. See Melitz and Ottaviano (2003) for a single industry model where love of variety takes the quasi-linear form. We concentrate on the CES case to make our results comparable with the existing inter and intra-industry trade literature (Helpman and Krugman 1985) and to focus on the effects of relative factor abundance with homothetic preferences.

Cobb-Douglas form,[5]

$$\Gamma_i = \left[f_i + \frac{q_i}{\varphi}\right] (w_S)^{\beta_i} (w_L)^{1-\beta_i}, \qquad 1 > \beta_1 > \beta_2 > 0 \tag{3}$$

where w_S is the skilled wage and w_L the unskilled wage. All firms share the same fixed overhead cost, $f_i > 0$, but have different productivity levels indexed by $\varphi \subseteq (0, \infty)$. The fixed production cost implies that, in equilibrium, each firm will choose to produce a unique variety. Within an industry, fixed and variable costs use skilled and unskilled labor in the same proportions. Looking across industries, production in industry 1 is skill intensive relative to industry 2.

Consumer love of variety and costless trade implies that all firms that produce also export. Since firms face the same elasticity of demand in both the domestic market, d, and the export market, x, and trade is costless, profit maximization implies the same equilibrium price in the two markets, equal to a constant mark-up over marginal cost:

$$p_i(\varphi) = p_{id}(\varphi) = p_{ix}(\varphi) = \frac{(w_S)^{\beta_i}(w_L)^{1-\beta_i}}{\rho\varphi}. \tag{4}$$

With this pricing rule, firms' equilibrium domestic revenue, $r_{id}(\varphi)$, and domestic profits, $\pi_{id}(\varphi)$, will be proportional to productivity:

$$\begin{aligned} r_{id}(\varphi) &= \alpha_i R \left(\frac{\rho P_i \varphi}{(w_S)^{\beta_i}(w_L)^{1-\beta_i}}\right)^{\sigma-1} \\ \pi_{id}(\varphi) &= \frac{r_{id}(\varphi)}{\sigma} - f_i (w_S)^{\beta_i}(w_L)^{1-\beta_i}. \end{aligned} \tag{5}$$

For given firm productivity φ, domestic revenue is increasing in the share of expenditure allocated to a good, α_i, increasing in aggregate domestic expenditure (equals aggregate domestic revenue, R), increasing in the industry price index, P_i, which corresponds to an inverse measure of the degree of competition in a market, and increasing in ρ which is an inverse measure of the size of the mark-up of price over marginal cost. Firm revenue is decreasing in own price and hence in own production costs.

The equilibrium pricing rule implies that the relative revenue of two firms with different productivity levels within the same industry and market depends solely on their relative productivity, as is clear from equation (5): $r_{id}(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_{id}(\varphi')$.

[5] As will become evident in the discussion of the FPE equilibrium below, the analysis generalises in a relatively straightforward way to any homothetic cost function, for which the ratio of marginal cost to average cost will be a function of output alone. See Yeaple (2003) for a single-industry model where there is a continuum of workers with different skill levels, giving rise to heterogeneous firm productivity.

Revenue in the export market is determined analogously to the domestic market and, with firms charging the same equilibrium price, relative revenue in the two markets for a firm of given productivity, φ, will depend on relative country size, R^F/R^H, and the relative price index, P_i^F/P_i^H. With the prices of individual varieties equalized and all firms exporting under costless trade, the price indices will be the same in the two countries, $P_i^F = P_i^H$, and relative revenue will depend solely on relative country size:

$$\begin{aligned} r_i(\varphi) &= r_{id}(\varphi) + r_{ix}(\varphi) = \left[1 + \left(\frac{R^F}{R}\right)\right] r_{id}(\varphi) \\ \pi_i(\varphi) &= \frac{r_i(\varphi)}{\sigma} - f_i(w_S)^{\beta_i}(w_L)^{1-\beta_i}. \end{aligned} \tag{6}$$

To produce in an industry, firms must pay a fixed entry cost, which is thereafter sunk. The entry cost also uses skilled and unskilled labor, and we begin by assuming that the factor intensity of entry and production are the same, so that the industry sunk entry cost takes the form:

$$f_{ei}(w_S)^{\beta_i}(w_L)^{1-\beta_i}, \qquad f_{ei} > 0. \tag{7}$$

We relax the assumption of common factor intensities below and show how factor intensity differences between entry and production lead to interactions between country comparative advantage and the behavior of heterogeneous firms.

After entry, firms draw their productivity, φ, from a distribution, $g(\varphi)$, which is assumed to be common to industries and countries.[6] Firms then face an exogenous probability of death each period, δ, which we interpret as due to *force majeure* events beyond managers' control.[7]

A firm drawing productivity φ will produce in an industry if its revenue, $r_i(\varphi)$, at least covers the fixed costs of production, i.e $\pi_i \geq 0$. This defines a **zero-profit productivity cutoff**, φ_i^*, in each industry such that:

$$r_i(\varphi_i^*) = \sigma f_i(w_S)^{\beta_i}(w_L)^{1-\beta_i}. \tag{8}$$

[6] Combining the assumption of identical cost functions within an industry across countries with the assumption of a common productivity distribution yields the standard H-O assumption of common technologies across countries. It is straightforward to allow for differences in productivity distributions across countries and industries.

[7] The assumption that the probability of death is independent of firm characteristics is made for tractability to enable us to focus on the complex general equilibrium implications of international trade for firms, industries and countries. An existing literature examines industry dynamics in closed economies where productivity affects the probability of firm death (see, for example, Hopenhayn 1992 and Jovanovic 1982).

Firms drawing productivity below φ_i^* exit immediately, while those drawing productivity equal to or above φ_i^* engage in profitable production. The value of a firm is, therefore, equal to zero if it draws a productivity below the zero-profit productivity cutoff and exits, or equal to the stream of future profits discounted by the probability of death if it draws a productivity above the cutoff value and produces:

$$\begin{aligned} v_i(\varphi) &= \max\left\{0, \sum_{t=0}^{\infty}(1-\delta)^t \pi_i(\varphi)\right\} \\ &= \max\left\{0, \frac{\pi_i(\varphi)}{\delta}\right\}. \end{aligned} \tag{9}$$

The *ex post* distribution of firm productivity, $\mu_i(\varphi)$, is conditional on successful entry and is truncated at the zero-profit productivity cutoff:

$$\mu_i(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_i^*)} & \text{if} \quad \varphi \geq \varphi_i^* \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $1 - G(\varphi_i^*)$ is the *ex ante* probability of successful entry in an industry.

There is an unbounded competitive fringe of potential entrants and, in an equilibrium with positive production of both goods, we require the expected value of entry, V_i, to equal the sunk entry cost in each industry. The expected value of entry is the *ex ante* probability of successful entry multiplied by the expected profitability of producing the good until death, and the **free entry condition** is thus:

$$V_i = \frac{[1 - G(\varphi_i^*)]\,\overline{\pi}_i}{\delta} = f_{ei}(w_S)^{\beta_i}(w_L)^{1-\beta_i}, \tag{11}$$

where $\overline{\pi}_i$ is expected or average firm profitability from successful entry. Equilibrium revenue and profit in each market are constant elasticity functions of firm productivity (equation (5)) and, therefore, average revenue and profit are equal respectively to the revenue and profit of a firm with weighted average productivity, $\bar{r}_i = r_i(\tilde{\varphi}_i)$ and $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$, where weighted average productivity is determined by the *ex post* productivity distribution and hence the zero-profit productivity cutoff below which firms exit the industry:

$$\widetilde{\varphi}_i(\varphi_i^*) = \left[\frac{1}{1 - G(\varphi_i^*)} \int_{\varphi_i^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}. \tag{12}$$

It proves useful for the ensuing analysis to re-write the free entry condition in a more convenient form. The equation for equilibrium profits above gives us an expression for the profits of a firm with weighted average productivity, $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$. Given the equilibrium pricing rule, the revenue of a firm with weighted average productivity is proportional to the revenue of a firm with the zero-profit productivity, $r_i(\tilde{\varphi}_i) = (\tilde{\varphi}_i/\varphi_i^*)^{\sigma-1}\, r_i(\varphi_i^*)$, where the latter is proportional to the fixed cost of production in equilibrium, $r_i(\varphi_i^*) = \sigma f_i (w_S)^{\beta_i}(w_L)^{1-\beta_i}$. Combining these results with the definition of weighted average productivity above, the free entry condition can be written so that it is a function solely of the zero-profit productivity cutoff and parameters of the model:

$$V_i = \frac{f_i}{\delta}\int_{\varphi_i^*}^{\infty}\left[\left(\frac{\varphi}{\varphi_i^*}\right)^{\sigma-1} - 1\right] g(\varphi)d\varphi = f_{ei} \tag{13}$$

Terms in factor prices have cancelled because firm revenue and the sunk cost of entry are each proportional to factor costs, and entry and production have so far been assumed to have the same factor intensity. Since the left-hand side of equation (13) is monotonically decreasing in φ_i^*, this relationship alone uniquely pins down the zero-profit productivity cutoff independent of factor prices and other endogenous variables of the model. When we allow entry and production to have different factor intensities, this will no longer be the case. The free entry condition will contain terms in factor prices, and movements in relative factor prices have important implications for heterogeneous firms' decisions about whether or not to exit the industry based on their observed productivity.

This way of writing the free entry condition also makes clear how the zero-profit productivity cutoff is increasing in fixed production costs, f_i, and decreasing in the probability of firm death, δ. Higher fixed production costs imply that firms must draw a higher productivity in order to earn sufficient revenue to cover the fixed costs of production. A higher probability of firm death reduces the mass of entrants into an industry, increasing *ex post* profitability, and therefore enabling firms of lower productivity to survive in the market.

2.3. Goods Markets

The steady-state equilibrium is characterized by a constant mass of firms entering an industry each period, M_{ei}, and a constant mass of firms producing within the industry, M_i. Thus, in steady-state equilibrium, the mass of firms who enter and draw a productivity sufficiently high to produce must equal the mass of firms who die:

$$[1 - G(\varphi_i^*)]M_{ei} = \delta M_i. \tag{14}$$

As noted above, under costless trade, firms charge the same price in the domestic and export markets and all firms export. Hence, the industry price indices are equalized across countries: $P_i^F = P_i^H$. Firms' equilibrium pricing rule implies that the price charged for individual varieties is inversely related to firm productivity, while the price indices are weighted averages of the prices charged by firms with different productivities, with the weights determined by the *ex post* productivity distribution. Exploiting this property of the price indices, they may be written as functions of the mass of firms producing and the price charged by a firm with weighted average productivity in each country:

$$P_i = P_i^H = P_i^F = \left[M_i^H \left(p_i^H(\widetilde{\varphi}_i^H) \right)^{1-\sigma} + M_i^F \left(p_i^F(\widetilde{\varphi}_i^F) \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{15}$$

In equilibrium, we also require the goods market to clear at the world level, which requires the share of a good in the value of world production (in world revenue) to equal the share of a good in world expenditure:

$$\frac{R_1 + R_1^F}{R + R^F} = \alpha_1 = \alpha. \tag{16}$$

2.4. Labor Markets

Labor market clearing requires the demand for labor used in production and entry to equal labor supply as determined by countries' endowments:

$$\begin{aligned} S_1 + S_2 &= \overline{S}, \qquad & S_i &= S_i^p + S_i^e \\ L_1 + L_2 &= \overline{L}, \qquad & L_i &= L_i^p + L_i^e \end{aligned} \tag{17}$$

where S denotes skilled labor, L corresponds to unskilled labor, the superscript p refers to a factor used in production, and the superscript e refers to a factor used in entry.

2.5. Integrated Equilibrium and Factor Price Equalization

In this section, we characterize the conditions for a free trade equilibrium characterized by factor price equalization (FPE). We begin by solving for the equilibrium of the integrated world economy, where both goods and factors are mobile, before showing that there exists a set of allocations of world factor endowments to the two countries individually such that the free trade equilibrium, with only goods mobile, replicates the resource allocation of the integrated world economy.

The integrated equilibrium is referenced by a vector of nine variables - the zero-profit cutoff productivities in each sector, the prices for individual varieties within each industry as a function of productivity, the industry price indices, aggregate revenue, and the two factor prices: $\{\varphi_1^*, \varphi_2^*, P_1, P_2, R, p_1(\varphi), p_2(\varphi), w_S, w_L\}$. All other endogenous variables may be written as functions of these quantities. The equilibrium vector is determined by nine equilibrium conditions: firms' pricing rule (equation (4) for each sector), free entry (equation (13) for each sector), labor market clearing (equation (17) for the two factors), the values for the equilibrium price indices implied by consumer and producer optimization (equation (15) for each sector), and goods market clearing (equation (16)). Throughout the following, we choose the skilled wage for the numeraire, and so $w_S = 1$.

These conditions for integrated equilibrium are analogous to those in the standard framework of inter and intra-industry trade with homogenous firms (Helpman and Krugman 1985). A key difference is that firm heterogeneity modifies the free entry condition which, instead of equating price and average cost, now equates the expected value and sunk costs of entry and defines the equilibrium zero-profit productivity cutoff, φ_i^*, below which firms exit the industry. Movements in this productivity cutoff involve within-industry resource reallocations from less to more productive firms and, in a way made precise below, may occur differentially across countries and industries in accordance with patterns of comparative advantage.

Proposition 1 *There exists a unique integrated equilibrium, referenced by the vector* $\{\hat{\varphi}_1^*, \hat{\varphi}_2^*, \hat{P}_1, \hat{P}_2, \hat{R}, \hat{p}_1(\varphi), \hat{p}_2(\varphi), \hat{w}_S, \hat{w}_L\}$. *Under free trade, there exist a set of allocations of world factor endowments to the two countries individually such that the unique free trade equilibrium is characterized by factor price equalization (FPE) and replicates the resource allocation of the integrated world economy.*

Proof. See Appendix ■

Factor price equalization requires that countries' endowments are sufficiently similar in the sense that their relative endowments of skilled and unskilled labor lie in between the integrated equilibrium factor intensities in the two sectors. Integrated equilibrium and the factor price equalization set are illustrated diagrammatically in Figure 1, where the factor employment vector for each sector is a composite of resources used in production and entry. In the remainder of this section and in the next, we concentrate on equilibria characterized by FPE. In section 4. below, we analyze non-FPE equilibria associated with the existence of fixed and variable costs of trade.

One variable of particular interest that may be derived from the integrated equilibrium vector is the mass of firms in each sector, M_i, which depends upon total industry revenue,

R_i, and average firm size, r_i:

$$M_i = \frac{R_i}{\overline{r}_i}. \tag{18}$$

Free entry into each sector implies that total payments to labor used in both entry and production equal total industry revenue, $R_i = w_S S_i + w_L L_i$.[8] Therefore, industry revenue may be obtained from free trade equilibrium factor prices $\{1, w_L\}$ and from the equilibrium allocation of labor to the two sectors $\{L_i, S_i\}$, which is uniquely pinned down by equilibrium factor prices and countries' endowments. From free trade equilibrium factor prices and the zero-profit cutoff productivities $\{\varphi_i^*\}$, we can solve for average firm size, $\overline{r}_i = r_i(\widetilde{\varphi}_i) = (\tilde{\varphi}_i/\varphi_i^*)^{\sigma-1}\sigma f_i(w_S)^{\beta_i}(w_L)^{1-\beta_i}$. The existence of fixed production costs that use skilled and unskilled labor implies that, in equilibrium, average firm size is proportional to factor prices. Combining the expressions for industry revenue and average firm size yields the equilibrium mass of firms.

3. Properties of the Free Trade Equilibrium

We now examine the properties of the free trade equilibrium and the effects of moving from autarky to free trade. The analysis identifies a number of adjustment margins along which individual firms respond following the opening of trade - entry, exit and the overall mass of firms within an industry; average firm size; and the threshold productivity below which firms exit the industry. We begin by establishing that the relationship between production structure and factor abundance in the Heckscher-Ohlin model continues to hold in our heterogeneous firm framework. We then move on to the novel part of our analysis, which is to show how firm-level margins of adjustment vary systematically across firms, industries and countries in accordance with comparative advantage.

3.1. Endowments and Production Structure

As in the Heckscher-Ohlin model, cross-country differences in factor abundance interact with cross-industry differences in factor intensity to determine production structure. Under autarky, home's relative skill abundance leads to a lower relative price of skilled labor and of the skill-intensive good. The opening of trade leads to a convergence in relative goods prices and relative factor prices, so that the relative skilled wage rises in the skill-abundant home country and falls in the labor-abundant foreign country.

[8] See the proof of Proposition 1 for a formal derivation of this result.

The rise in the relative price of the skill-intensive good in home results in a reallocation of resources towards the skill-intensive sector, as each country specializes according to its pattern of comparative advantage. In the move from autarky to free trade, home devotes an increased share of both its skilled and unskilled labor to the skill-intensive industry. In the free trade equilibrium, the share of skilled and unskilled labor that home devotes to the skill-intensive sector is greater than in the labor-abundant foreign country.

Proposition 2 ***(a)*** *Under free trade, countries devote a larger share of both types of labor to their comparative advantage industry;* ***(b)*** *The move from autarky to free trade increases the share of both types of labor allocated to the comparative advantage industry*

Proof. See Appendix ∎

Though, in the interests of brevity, we do not consider this in detail, all four major theorems of the Heckscher-Ohlin model (Rybczynski, Heckscher-Ohlin, Stolper-Samuelson, and Factor Price Equalization) continue to hold, with only minor modifications to take into account monopolistic competition and increasing returns to scale.

3.2. Firm-level Responses

Our heterogeneous-firm framework allows trade liberalization to have differential effects on firms within the same country and industry as well as on firms across countries and industries. Our primary focus is on changes in the distribution of productivity at producing firms. However, we also consider the number and average size of firms as well as the amount of steady-state firm entry and exit.[9]

Proposition 3 *In the free trade equilibrium:* ***(a)*** *the zero-profit productivity cutoff and average firm size are the same within industries across countries, ($\varphi_i^{*H} = \varphi_i^{*F}$ and $\bar{r}_i^H = \bar{r}_i^F$);* ***(b)*** *countries have a larger relative mass of firms in their comparative advantage industry ($M_1^H/M_2^H > M_1^F/M_2^F$);* ***(c)*** *countries experience relatively more entry and exit in their comparative advantage industry ($M_{e1}^H/M_{e2}^H > M_{e1}^F/M_{e2}^F$)*

Proof. See Appendix ∎

[9] Firm entry is associated with job creation and firm exit with job destruction, so a corollary of increased entry and exit is a higher rate of job turnover within an industry.

Proposition 4 *With identical factor intensities in entry and production, a move from autarky to free trade* ***(a)*** *leaves the zero-profit productivity cutoff (φ_i^*) unchanged,* ***(b)*** *increases relative average firm size in a country's comparative advantage industry ($\bar{r}_1^H/\bar{r}_2^H$ and $\bar{r}_2^F/\bar{r}_1^F$),* ***(c)*** *increases the relative mass of firms in a country's comparative advantage industry (M_1^H/M_2^H and M_2^F/M_1^F), and* ***(d)*** *increases the relative amount of entry and exit in a country's comparative advantage industry (M_{e1}^H/M_{e2}^H and M_{e2}^F/M_{e1}^F)*

Proof. See Appendix ■

The opening of free trade results in an expansion of market size as home firms gain access to the foreign market and an increase in the degree of competition within each market as foreign varieties gain access to the home market. With no trade costs, all firms are exporters and are affected in the same way by the opening of trade. Since the revenue of the firm with the lowest level of productivity in the market, $r_i(\varphi_i^*)$, is pinned down by the fixed cost of production and the revenue of all other firms is proportional to the revenue of the firm with the lowest level of productivity, the zero-profit productivity cutoff, φ_i^*, remains unchanged by the opening of trade and is identical across countries within industries in the free trade equilibrium. This can be seen from the free entry condition (13), where φ_i^* is pinned down by model parameters which, under the assumption of identical technologies, are the same across countries. Although factor prices change following the opening of trade, the assumption that entry and production have the same factor intensity means that changes in factor prices affect both the value of entry (firm revenue) and entry costs in the same way and thus do not alter the entry decision.

In equilibrium, average firm size is proportional to the fixed costs of production, since these pin down the revenue of a firm with the lowest level of productivity in the market, $r_i(\varphi_i^*)$, and average revenue is proportional to $r_i(\varphi_i^*)$. Since skilled and unskilled wages are the same across countries in the free trade FPE equilibrium, so are the fixed costs of production within each industry, and hence so is average firm size. In the move from autarky to free trade, the skilled wage rises in the skill-abundant country, which increases fixed costs of production in the skill-intensive industry relative to the labor-intensive industry. The result is a rise in relative average firm size in the skill-intensive industry.

Average firm size is one of the adjustment margins through which firms, industries and countries respond to trade. A second margin is the mass of producing firms. In the move from autarky to free trade, the skill-abundant country increases the share of skilled and unskilled labor in the skill-intensive industry and sees production costs rise relative to those in the labor-intensive industry due to the increase in the relative skilled wage. Both of these considerations increase total industry revenue (which equals total labor payments, $R_i = w_S S_i + w_L L_i$) in the comparative-advantage industry relative to the comparative-disadvantage industry. Although average relative firm size also rises in the skill-intensive industry, it rises by less than relative industry revenue, resulting in an increase in the relative mass of firms, $M_i = R_i/\bar{r}_i$, in the skill-intensive industry.

This change in the relative mass of firms is associated with changes in entry/exit behavior. In steady-state equilibrium, the mass of firms who enter an industry and draw a productivity sufficiently high to produce each period is equal to the mass of firms who die. With identical zero-profit productivity cutoffs in the two countries that are unchanged by the opening of trade, the rise in the relative mass of firms in the skill-intensive industry in the skill-abundant country is associated with a rise in the relative amount of entry and exit in the comparative-advantage industry, since $M_{ei} = \delta M_i/[1 - G(\varphi_i^*)]$.

The opening of free trade is thus associated with changes in average firm size, changes in the mass of firms, and changes in the extent of entry and exit. These changes occur differentially across countries and industries in accordance with comparative advantage, with average firm size, the mass of firms, and the extent of entry and exit rising in comparative advantage industries relative to comparative disadvantage industries.

3.3. Comparative Advantage and Firm Entry and Exit

If we relax the assumption that entry and production have the same factor intensity, we introduce an additional margin of adjustment in response to trade liberalization. Changes in relative factor prices now influence a firm's decision of whether or not to enter (or exit) an industry and thus shift the zero-profit productivity cutoff. Even under free trade, movements in relative factor prices can induce productivity changes that vary across industries and countries according to comparative advantage.

Country factor abundance determines whether skilled wages rise or fall following the opening of trade, and the factor intensity of entry and production determines whether these changes in factor prices have a greater effect on the revenue from producing in an industry or on the sunk costs of entering the industry.

The skill intensity of entry is now indexed by the Cobb-Douglas exponent η_i, while the skill intensity of production continues to be captured by the exponent β_i.[10] The

[10] For simplicity, we continue to restrict fixed and variable costs of production to have the same factor intensity, though this assumption can also be relaxed. See Flam and Helpman (1987) for an analysis of

determination of integrated equilibrium is essentially unchanged, but now relative factor prices enter the free entry condition, which becomes:

$$V_i = \frac{f_i}{\delta} \int_{\varphi_i^*}^{\infty} \left[\left(\frac{\varphi}{\varphi_i^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi = f_{ei} \left(\frac{w_S}{w_L} \right)^{\eta_i - \beta_i}. \tag{19}$$

Besides this change to the free entry condition, labor market clearing and the relationship between industry revenue and labor payments now have to incorporate differences in factor intensity between entry and production. The production sector consists of four activities - entry in the skill-intensive sector, production in the skill-intensive sector, entry in the labor-intensive sector, and production in the labor-intensive sector - and their differing factor intensities influence the factor price equalization set.

We begin by considering the case where entry is more skill intensive than production, which is consistent with the idea that entry may involve skill-intensive product development costs. We also assume that the labor-intensive activity (production) in the skill-intensive industry uses skills more intensely than the most skill-intensive activity (entry) in the labor-intensive industry. Other permutations are possible and it is straightforward to consider them in our framework.

In the skill-abundant home country, the move from autarky to free trade leads to a rise in the relative skilled wage. With entry more skill intensive than production ($\eta_i > \beta_i$), this increases the sunk entry cost relative to the expected value of entry, as can be seen from the right-hand side of equation (19), which is increasing in w_S/w_L.

In equilibrium, the expected value of entry must rise to equal the new higher sunk cost which, in equation (19), requires a fall in the zero-profit productivity cutoff, φ_i^*. Intuitively, the rise in the relative value of the sunk entry cost reduces the mass of firms who enter the industry, increasing *ex-post* profitability, thereby enabling less productive firms to survive in the industry, and reducing the zero-profit productivity cutoff.

The reverse is true in the labor-abundant country, where the relative skilled wage falls following the opening of trade, reducing the value of the sunk entry cost relative to the expected value of entry, and increasing the equilibrium zero-profit productivity cutoff.

If entry is more skill intensive than production in one industry, but less skill intensive than production in the other industry, the effects of changes in relative factor prices following the opening of trade are asymmetric across sectors. Thus, in the skill-abundant country, where the relative skilled wage rises, the zero-profit productivity cutoff will increase in an industry where entry is less skill intensive than production.

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differing factor intensities in fixed and variable production costs when firms are homogeneous.

In the next section, we show how relaxing the assumption of costless trade also introduces interdependence between the mechanisms of HO theory and heterogeneous firms' entry and exit decisions. To keep the analysis tractable, we return to our benchmark case where the factor intensity of entry and production is the same. Again the interaction of country factor abundance and the factor intensity of economic activities leads to movements in the zero-profit productivity cutoff and to changes in industry productivity.

4. Model with Costly Trade

The assumption of perfectly costless trade is unrealistic and at odds with recent empirical work that has documented sizeable trade costs.[11] In this section, we follow Melitz (2003) in considering both fixed and variable costs of trade; the distinctive feature of our analysis is how these interact with comparative advantage. The basic setup remains the same as under free trade. However, in order to export a manufacturing variety to a particular market, we assume that a firm must incur a fixed export cost, which uses both skilled and unskilled labor with the same factor intensities as production.[12] In addition, firms may also face variable trade costs which take the standard iceberg form whereby, in industry i, a fraction $\tau_i > 1$ units of a good must be shipped in order for one unit to arrive.[13]

4.1. Consumption and Production

Profit maximization implies that equilibrium prices are again a constant mark-up over marginal cost, with export prices a constant multiple of domestic prices due to the variable costs of trade:

$$p_{ix}^H(\varphi) = \tau_i p_{id}^H(\varphi) = \frac{\tau_i (w_S^H)^{\beta_i} (w_L^H)^{1-\beta_i}}{\rho\varphi}. \tag{20}$$

Given firms' pricing rules, equilibrium revenue in the export market is proportional to that in the domestic market. However, the price differences between the two markets mean that relative revenue in the export market depends now directly on variable trade costs. Furthermore, price indices now vary across the two countries due to variation in the mass of firms producing in an industry, firms charging different prices in domestic and export

[11] See, in particular, Anderson and van Wincoop (2003) and Hummels (2001).

[12] See Roberts and Tybout (1997b) for survey evidence that firms producing differentiated products face substantial fixed costs in entering export markets.

[13] In the analysis below, we write out expressions for home explicitly; those for foreign are directly analogous.

markets (variable trade costs), and the existence of both exporters and non-exporters (fixed and variable trade costs). Hence, relative price indices enter as a determinant of relative revenue in the export market:

$$r_{ix}^{H}(\varphi) = \tau_i^{1-\sigma}\left(\frac{P_i^F}{P_i^H}\right)^{\sigma-1}\left(\frac{R^F}{R^H}\right) r_{id}^{H}(\varphi). \tag{21}$$

The wedge between revenue in the export and domestic markets in equation (21) will typically vary across countries and industries, and will prove important in determining how trade liberalization increases the expected value of entry into an industry. Total revenue received by home firms is:

$$r_i^H(\varphi) = \begin{cases} r_{id}^H(\varphi) & \text{if does not export} \\ r_{id}^H(\varphi)\left[1+\tau_i^{1-\sigma}\left(\frac{P_i^F}{P_i^H}\right)^{\sigma-1}\left(\frac{R^F}{R^H}\right)\right] & \text{if exports.} \end{cases} \tag{22}$$

Consumer love of variety and fixed production costs imply that no firm will ever export without also producing for the domestic market. Therefore, we may separate each firm's profit into components earned from domestic sales, $\pi_{id}^H(\varphi)$, and foreign sales, $\pi_{ix}^H(\varphi)$, where we apportion the entire fixed production cost to domestic profit and the fixed exporting cost to foreign profit:[14]

$$\begin{aligned} \pi_{id}^H(\varphi) &= \frac{r_{id}^H(\varphi)}{\sigma} - f_i (w_S^H)^{\beta_i}(w_L^H)^{1-\beta_i} \\ \pi_{ix}^H(\varphi) &= \frac{r_{ix}^H(\varphi)}{\sigma} - f_{ix} (w_S^H)^{\beta_i}(w_L^H)^{1-\beta_i} \end{aligned} \tag{23}$$

where both skilled and unskilled labor are used in the fixed cost of exporting, $f_{ix}(w_S^H)^{\beta_i}(w_L^H)^{1-\beta_i}$. A firm which produces for its domestic market also exports if $\pi_{ix}^H(\varphi) > 0$, and total firm profit is given by:

$$\pi_i^H(\varphi) = \pi_{id}^H(\varphi) + \max\left\{0, \pi_{ix}^H(\varphi)\right\}. \tag{24}$$

4.2. Decision to Produce and Export

After firms have paid the sunk cost of entering an industry, they draw their productivity, φ, from the distribution $g(\varphi)$. There are now two cutoff productivities, the **costly-trade zero-profit productivity cutoff**, φ_i^{*H}, above which firms produce for the domestic market

[14] Since no firm ever exports without also serving the domestic market, this is merely an accounting device which simplifies the exposition below.

and the **costly-trade exporting productivity cutoff**, φ_{ix}^{*H}, above which firms produce for both the domestic and export markets:

$$\begin{aligned} r_{id}^{H}(\varphi_i^{*H}) &= \sigma f_i (w_S^H)^{\beta_i} (w_L^H)^{1-\beta_i} \\ r_{ix}^{H}(\varphi_{ix}^{*H}) &= \sigma f_{ix} (w_S^H)^{\beta_i} (w_L^H)^{1-\beta_i}. \end{aligned} \tag{25}$$

Combining these two expressions, we obtain one equation linking the revenues of a firm at the zero-profit productivity cutoff to those of a firm at the exporting productivity cutoff. A second equation is obtained from the relationship between the revenues of two firms with different productivities within the same market, $r_{id}(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_{id}(\varphi')$, and from the relationship between revenues in the export and domestic markets, equation (21). The two equations together yield an equilibrium relationship between the two productivity cutoffs:

$$\varphi_{ix}^{*H} = \Lambda_i^H \ \varphi_i^{*H} \qquad \text{where} \qquad \Lambda_i^H \equiv \tau_i \left(\frac{P_i^H}{P_i^F}\right) \left(\frac{R^H}{R^F} \frac{f_{ix}}{f_i}\right)^{\frac{1}{\sigma-1}} \tag{26}$$

The exporting productivity cutoff will be high relative to the zero-profit productivity cutoff when the fixed cost of exporting, f_{ix}, is large relative to the fixed cost of production, f_i. In this case, the revenue required to cover the fixed export cost is large relative to the revenue required to cover fixed production costs, implying that only firms of high productivity will find it profitable to serve both the domestic and foreign markets. The exporting productivity cutoff will also be high relative to the zero-profit productivity cutoff when the home price index, P_i^H, is high relative to the foreign price index, P_i^F, and the home market, R^H, is large relative to the foreign market, R^F. Here, only high productivity firms receive enough revenue in the relatively small and competitive foreign market to cover the fixed cost of exporting. Finally, higher variable trade costs increase the exporting productivity cutoff relative to the zero-profit productivity cutoff by increasing prices and reducing revenue in the export market.

For values of $\Lambda_i^k > 1$, there is selection into markets, i.e. only the most productive firms export. Since empirical evidence strongly supports selection into export markets and the interior equilibrium is the most interesting one, we focus throughout the following on parameter values where $\Lambda_i^k > 1$ across countries k and industries i.[15]

Firms' decisions concerning production for the domestic and foreign markets are summarized graphically in Figure 2. Of the mass of firms M_{ei}^H who enter the industry each

[15] For empirical evidence on selection into export markets, see Bernard and Jensen (1995, 1999), Clerides, Lach and Tybout (1998), and Roberts and Tybout (1997a).

period, a fraction, φ_i^{*H}, draw a productivity level sufficiently low that they are unable to cover fixed production costs and exit the industry immediately; a fraction, $[\varphi_{ix}^{*H} - \varphi_i^{*H}]$, draw an intermediate productivity level such that they are able to cover fixed production costs and serve the domestic market, but are unable to earn enough total revenue (home and foreign) to cover both fixed production and export costs; and a fraction, φ_{ix}^{*H}, draw a productivity level sufficiently high that it is profitable to serve both the home and foreign markets in equilibrium.

The *ex ante* probability of successful entry is $[1 - G(\varphi_i^{*H})]$ and the *ex ante* probability of exporting conditional on successful entry is:

$$\chi_i^H = \frac{[1 - G(\varphi_{ix}^{*H})]}{[1 - G(\varphi_i^{*H})]}. \tag{27}$$

4.3. Free Entry

In an equilibrium with positive production of both goods, we again require the expected value of entry, V_i^H, to equal the sunk entry cost in each industry. The expected value of entry is now the *ex ante* probability of successful entry times the expected profitability of producing the good for the domestic market until death, plus the *ex ante* probability of successful entry times the probability of exporting times the expected profitability of producing the good for the export market until death:

$$V_i = \frac{[1 - G(\varphi_i^*)]}{\delta} \left[\bar{\pi}_{id}^H + \chi_i^H \bar{\pi}_{ix}^H\right] = f_{ei}(w_S)^{\beta_i}(w_L)^{1-\beta_i} \tag{28}$$

where average profitability in each market is equal to the profit of a firm with weighted average productivity, $\bar{\pi}_{id}^H = \pi_{id}^H(\widetilde{\varphi}_i^H)$ and $\bar{\pi}_{ix}^H = \pi_{ix}^H(\widetilde{\varphi}_{ix}^H)$. Some lower-productivity firms do not export leading to higher weighted average productivity in the export market than in the domestic market. Weighted average productivity is defined as in equation (12), where the relevant cutoff for the domestic market is the zero-profit productivity, φ_i^*, and the relevant cutoff for the export market is the exporting productivity, φ_{ix}^*.

Following the same line of reasoning as under free trade, we can write the free entry condition as a function of the two productivity cutoffs and model parameters:

$$V_i^k = \frac{f_i}{\delta} \int_{\varphi_i^{*k}}^{\infty} \left[\left(\frac{\varphi}{\varphi_i^{*k}}\right)^{\sigma-1} - 1\right] g(\varphi) d\varphi \tag{29}$$

$$+\frac{f_{ix}}{\delta} \int_{\varphi_{ix}^{*k}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{ix}^{*k}}\right)^{\sigma-1} - 1\right] g(\varphi) d\varphi = f_{ei} \qquad i \in \{1,2\}, k \in \{H,F\}.$$

The expression for the expected value of entry now consists of two separate terms that take into account the difference in profitability between firms who only serve the domestic market and those who serve both domestic and export markets (previously, under free trade, all firms exported). In equilibrium, φ_{ix}^{*} and φ_{i}^{*} are related according to equation (26). The distance in productivity between the least productive firm able to survive in the domestic market and the least productive firm able to survive in the export market depends on industry price indices and country size, and will hence vary systematically across countries and industries as considered further below.

4.4. Goods and Labor Markets

Again, in steady-state, the mass of firms who enter an industry and draw a productivity high enough to produce equals the mass of firms who die.

Using the equilibrium pricing rule, the industry price indices may be written as:

$$P_i^H = \left[M_i^H \left(p_{id}^H(\widetilde{\varphi}_i^H) \right)^{1-\sigma} + \chi_i^F M_i^F \left(\tau_i \; p_{id}^F(\widetilde{\varphi}_{ix}^F) \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} . \tag{30}$$

In general, the price indices for an industry will now vary across countries because of differences in the mass of domestic and foreign firms, differences in domestic and export prices (variable trade costs captured by τ_i), and differences in the porportion of exporting firms (fixed and variable trade costs reflected in χ_i^F and $\widetilde{\varphi}_{ix}^F$).

In equilibrium, we also require that the sum of domestic and foreign expenditure on domestic varieties equals the value of domestic production (total industry revenue, R_i) for each industry and country:

$$R_i^H = \alpha_i R^H M_i^H \left(\frac{p_{id}^H(\tilde{\varphi}_i^H)}{P_i^H} \right)^{1-\sigma} + \alpha_i R^F \chi_i^H M_i^H \left(\frac{\tau_i \; p_{id}^H(\tilde{\varphi}_{ix}^H)}{P_i^F} \right)^{1-\sigma} \tag{31}$$

where, with free entry into each industry, total industry revenue again equals total labor payments, $R_i^H = w_S^H S_i^H + w_L^H L_i^H$. Imposing this condition for both countries and industries implies that the goods market clears at the world level.

The first term on the right-hand side of equation (31) captures home expenditure on home varieties, which equals the mass of varieties sold domestically, M_i^H, times expenditure on a variety with weighted average productivity.[16] The second term on the right-hand side of

[16] Expenditure on a variety with weighted average productivity depends negatively on the domestic price of a variety with weighted average productivity, $p_{id}^H(\tilde{\varphi}_i^H)$, positively on the price of competing varieties

equation (31) captures foreign expenditure on home varieties. The key differences between the two terms are that only some of the varieties produced in home are exported to foreign (captured by the probability of exporting χ_i^H), the price charged by home producers in the export market is higher than in the domestic market (variable trade costs τ_i), and that weighted average productivity in the export market is greater than in the domestic market because of selection into export markets by higher productivity firms.

4.5. Costly Trade Equilibrium

The costly trade equilibrium is referenced by a vector of thirteen variables in home and foreign: $\{\varphi_1^{*k},\ \varphi_2^{*k},\ \varphi_{1x}^{*k},\ \varphi_{2x}^{*k},\ P_1^k,\ P_2^k,\ p_1^k(\varphi),\ p_2^k(\varphi),\ p_{1x}^k(\varphi),\ p_{2x}^k(\varphi),\ w_S^k,\ w_L^k,\ R^k\}$ for $k \in \{H, F\}$. All other endogenous variables may be written as functions of these quantities. The equilibrium vector is determined by the following equilibrium conditions for each country: firms' pricing rule (equation (20) for each industry and for the domestic and export market separately), free entry (equation (29) for each sector), the relationship between the two productivity cutoffs (equation (26) for each sector), labor market clearing (equation (17) for the two factors), the values for the equilibrium price indices implied by consumer and producer optimization (equation (30) for each sector), and world expenditure on a country's output of a good equals the value of its production (equation (31) for each sector).

Proposition 5 *There exists a unique costly trade equilibrium referenced by the pair of equilibrium vectors,* $\{\hat{\varphi}_1^{*k},\ \hat{\varphi}_2^{*k},\ \hat{\varphi}_{1x}^{*k},\ \hat{\varphi}_{2x}^{*k},\ \hat{P}_1^k,\ \hat{P}_2^k,\ \hat{p}_1^k(\varphi),\ \hat{p}_2^k(\varphi),\ \hat{p}_{1x}^k(\varphi),\ \hat{p}_{2x}^k(\varphi),\ \hat{w}_S^k,\ \hat{w}_L^k,\ \hat{R}^k\}$ *for* $k \in \{H, F\}$.

Proof. See Appendix. ■

Following the opening of costly trade, the relative price indices for the two goods will lie in between their autarky and free trade values, and will differ across the two countries, with the skill-abundant country characterized by a lower relative price index for the skill-intensive good. These cross-country differences in relative price indices will be reflected in general equilibrium in cross-country differences in relative factor prices, with the skill-abundant country having a lower relative skilled wage. Cross-country differences in both relative

(including those produced in foreign and exported to home) as summarized in the domestic price index, P_i^H, positively on the share of consumer expenditure devoted to a good in equilibrium, α_i, and positively on aggregate home expenditure (equals aggregate revenue, R^H).

price indices and relative factor prices have important implications for how heterogeneous firms adjust to trade, as examined in the next section.

5. Properties of the Costly Trade Equilibrium

As in Bernard et al. (2003) and Melitz (2003), the opening to costly trade is followed by compositional changes within industries, which increase aggregate industry productivity. Unlike those single-industry models, the degree of within-industry reallocation varies systematically with comparative advantage, as driven by Heckscher-Ohlin considerations of relative factor abundance and factor intensity. In this section we examine the effects of trade liberalization on productivity and exporting. In the following section we solve the model numerically and describe other adjustment margins including firm size, the number of firms, and the extent of entry and exit.[17]

Access to foreign markets raises the *ex post* profitability of more productive firms who find it profitable to export relative to less productive firms who choose only to serve the domestic market. This differential effect of costly trade on the profitability of high and low productivity firms is central to the within-industry compositional changes following the opening of trade (previously, under free trade, all firms exported and were affected by trade in the same way).

The increase in *ex post* profitability for more productive exporting firms raises the expected value of entry in an industry because there is a positive *ex ante* probability of drawing a productivity sufficiently high to export. This increases the mass of entrants in the industry, reducing the *ex post* profitability of low productivity firms which only serve the domestic market. As a result, some lower-productivity domestic firms are no longer able to survive in the market and the zero-profit productivity cutoff, φ_i^*, increases. This change did not occur in the transition from autarky to free trade because under free trade these low productivity firms also had access to foreign markets.

The size of the increase in φ_i^* within industries will depend on the extent to which access to foreign markets raises *ex post* profitability for high productivity exporting firms. We saw earlier that this depends systematically on characteristics of countries and industries: market size (as captured by aggregate revenue, R), the degree of competition in a market

[17] Relative factor prices and the shares of labor employed in a sector move in the same direction from autarky to costly trade as from autarky to free trade. Therefore, the free trade propositions on average firm size, the mass of firms, and the mass of entrants, which rely on these changes in factor prices and allocation, continue to apply.

(as captured by the price index P_i), and trade costs. These influence both the zero-profit productivity cutoff, φ_i^*, and the exporting productivity cutoff, φ_{ix}^*, determining the magnitude of within-industry reallocation and the probability of exporting.

We begin by showing how these characteristics of countries and industries matter by comparing costly trade equilibria with different values for countries' revenue, price indices and trade costs. Of course, aggregate revenue and price indices are endogenously determined and, in the next stage of the analysis, we link these endogenous variables to exogenous characteristics of countries and industries - relative factor abundance and factor intensity. We establish that, following the opening of costly trade, there is a higher probability of exporting and the zero-profit productivity cutoff increases by more in comparative advantage industries. Industries with comparative advantage experience the largest changes in within-industry composition and the greatest increases in aggregate industry productivity.

Proposition 6 ***(a)*** *The opening of a closed economy to international trade with positive fixed and variable trade costs will increase the zero-profit productivity cutoff,* φ_i^**, below which firms exit the industry;*
(b) *other things equal, the zero-profit productivity cutoff,* φ_i^**, will rise by more if a country is small relative to its trade partner (*R^H *small relative to* R^F*), if domestic competition is high relative to foreign competition within the industry (*P_i^H *low relative to* P_i^F*), or if fixed and variable trade costs are low (*f_{ix} *and* τ_i *small);*
(c) *the probability of exporting (*$\chi_i = [1 - G(\varphi_{ix}^*)]/[1 - G(\varphi_i^*)]$*) will be greater when home is small relative to foreign (*R^H *small relative to* R^F*), when domestic competition is high relative to foreign competition within the industry (*P_i^H *low relative to* P_i^F*), or when fixed and variable trade costs are low (*f_{ix} *and* τ_i *small).*

Proof. See Appendix ■

The increase in relative profitability for high-productivity exporting firms will be greatest when the home market is small relative to the foreign market, when the degree of competition in the domestic market is high relative to the foreign market, and when trade costs are low. Each of these considerations increases revenue in the export market relative to revenue in the domestic market, raising the probability that a domestic firm will find it profitable to export, and increasing the *ex ante* value of entry in the industry. As noted above, a higher expected value of entry raises the mass of entrants, reducing the profitability of lower-productivity firms who only serve the domestic market, and raising the amount by which the zero-profit productivity cutoff increases following the opening of trade.

Proposition 7 *In the absence of cross-industry differences in variable trade costs or in the ratio of fixed exporting to fixed production costs,*
(a) *the opening of a closed economy to costly international trade will raise the zero-profit productivity cutoff in a country's comparative advantage industry (φ_1^{*H} and φ_2^{*F}) by more than in the country's comparative disadvantage industry (φ_2^{*H} and φ_1^{*F});*
(b) *the probability of exporting will be greater in a country's comparative advantage industry (χ_1^H and χ_2^F) than in the country's comparative disadvantage industry (χ_2^H and χ_1^F)*

Proof. See Appendix ■

The relationship between comparative advantage, the probability of exporting and the extent to which the zero-profit productivity cutoff rises following the opening of costly trade is best seen by examining the relationship between the zero-profit and exporting productivity cutoffs across industries. From the equilibrium relationship between the two productivity cutoffs derived above (equation (26)), we have:

$$\frac{\Lambda_1^H}{\Lambda_2^H} \equiv \frac{\varphi_{1x}^{*H}/\varphi_1^{*H}}{\varphi_{2x}^{*H}/\varphi_2^{*H}} = \frac{\tau_1}{\tau_2}\left(\frac{f_{1x}/f_1}{f_{2x}/f_2}\right)^{\frac{1}{\sigma-1}} \frac{P_1^H/P_2^H}{P_1^F/P_2^F}. \tag{32}$$

Under both autarky and costly trade, the relative price index for the skill-intensive good, P_1^k/P_2^k, will be lower in the skill-abundant home country, with these relative price index differences only eliminated under free trade. Shutting down cross-industry differences in variable trade costs, fixed exporting costs and fixed production costs, we get the intuitive result that the fraction of exporters will be larger in a country's comparative advantage industry. This occurs because the exporting productivity cutoff in the comparative advantage industry is closer to the zero-profit productivity cutoff ($P_1^H/P_2^H < P_1^F/P_2^F$ implies $\Lambda_1^H < \Lambda_2^H$ if $f_x/f = f_{ix}/f_i$ and $\tau = \tau_i$) and thus leads to to a higher probability of exporting $\chi_i^k = [1 - G(\Lambda_i^k \varphi_i^{*k})]/[1 - G(\varphi_i^{*k})]$ in a country's comparative advantage industry.

The lower relative price index for the skill-intensive good in the skill-abundant home country results from home devoting a greater share of its skilled and unskilled labor to the skill-intensive industry, generating a larger relative mass of firms in the skill-intensive industry. With variable trade costs introducing a wedge between domestic and export prices, and fixed and variable trade costs separating firms into exporters and non-exporters, these cross-country differences in the relative mass of firms translate into cross-country differences in relative price indices.

The lower relative price index in a country's comparative advantage industry raises revenue in the export market relative to revenue in the domestic market compared to the other industry. Because of the relatively smaller mass of local producers in the foreign market, home firms face relatively less intense competition in the export market than they do in the domestic market. This increases *ex post* profitability for exporting firms and the *ex ante* expected value of entry in the industry, and thus increases the zero-profit productivity cutoff by more than in an industry of comparative disadvantage. Not only is the probability of exporting higher in a country's comparative advantage industry but, following the opening to costly trade, average productivity will increase by more in a country's comparative advantage industry than in the other industry.

The implications of comparative advantage for the probability of exporting and the zero-profit productivity cutoff are summarized in Figure 3. In comparative advantage industries, the probability of exporting is higher (φ^*_{ix} closer to φ^*_i) and the zero-profit productivity cutoff increases by more following the opening of trade (φ^{*CT}_i further from φ^{*A}_i, where CT indexes costly trade and A autarky) than in comparative disadvantage industries.

Following the opening of trade, there are between-industry reallocations of resources as in Heckscher-Ohlin theory. The skill-abundant country specializes in the skill-intensive sector. At the same time, there are within-industry reallocations of resources between less and more productive firms and the between and within-industry reallocations interact in systematic ways.

In the comparative advantage industry, the relatively small mass of firms in the foreign market reduces the competition faced overseas by domestic firms. Exporting, only undertaken by higher productivity firms, is relatively more profitable in the comparative advantage industry. This results in a higher probability of exporting and a greater increase in the zero-profit productivity cutoff, inducing more low productivity firms to exit and a greater increase in average industry productivity

6. Costly Trade - Numerical Solutions

We now parameterize the costly-trade model and solve it numerically to examine the effects of changes in trade costs and changes in relative factor abundance within a costly trade equilibrium. Across industries and countries, we consider not only responses of productivity and the probability of exporting but also adjustments in average firm size, the mass of firms, and the degree of firm entry and exit.

The analysis so far has allowed for a wide variety of *ex ante* distributions of firm productivity. To numerically solve the model, we assume a Pareto productivity distribution, $g(\varphi) = ak^a\varphi^{-(a+1)}$, where $k > 0$ is the minimum value for productivity $\varphi \geq k$ and $a > 0$ is a shape parameter. The Pareto distribution approximates the observed firm productivity distribution within industries, and we assume $a > \sigma - 1$ which is required in the model for log firm sales to have a finite variance.

Following the empirical results in Bernard, Eaton, Jenson and Kortum (BEJK, 2003) based on US plant and aggregate trade data, we set the elasticity of substitution $\sigma = 3.8$. BEJK (2003) find that the standard deviation of log US plant sales is 1.67, and because the standard deviation of log sales in the model is $1/(a - \sigma + 1)$, we set the Pareto shape parameter $a = 3.4$, which satisfies the requirement $a > \sigma - 1$ above.

We choose symmetric values for country factor endowments $\{\bar{S}^H = 1200, \bar{L}^H = 1000, \bar{S}^F = 1000, \bar{L}^F = 1200\}$, industry factor intensities $\{\beta_1 = 0.6, \beta_2 = 0.4\}$, and the share of the two goods in consumer expenditure ($\alpha_1 = \alpha = 0.5$). Changing the fixed cost of entry, f_{ei}, rescales the mass of firms in an industry and, without loss of generality, we set $f_{ei} = 2$. We set the minimum value for productivity k=0.2.

Fixed production costs are set equal to 5% of entry fixed costs, $f_i = 0.1$, and fixed exporting costs are set equal to fixed production costs, $f_{ix} = f_i$. Given our choice of factor endowments, the two countries have equal revenue under free trade. Therefore, equating fixed production and exporting costs is convenient because it ensures that, when variable costs $\tau = 1$ all firms export, because $\Lambda_i^H \equiv \tau_i \left(\frac{P_i^H}{P_i^F}\right) \left(\frac{R^H}{R^F} \frac{f_{ix}}{f_i}\right)^{\frac{1}{\sigma-1}}$.

Exit in the model includes both the endogenous decision of firms with low productivity draws to leave the industry and exogenous death due to *force majeure* events. Changes in the probability of exogenous firm death, δ, rescale the mass of entrants relative to the mass of firms and, without loss of generality, we set $\delta = 0.025$.

The first exercise we undertake is to reduce the variable costs of trade from 100% to 0% (from $\tau = 2$ to $\tau = 1$). Figure 4 plots the evolution of key outcomes in the skill-abundant home country by industry, including the mass of firms (top left), the mass of entrants (top right), average firm productivity (middle left), average firm size (middle right), the probability of exporting (bottom left), and the relative skilled wage (bottom right). Given our choice of symmetric values for country factor endowments and industry factor intensities, the evolution of outcomes for the labor-abundant foreign country is the exact mirror image of the pattern for home.

As variable trade costs fall, countries specialize according to patterns of comparative advantage. The number of firms falls in both sectors as trade costs fall, but the relative mass of firms decreases faster in the labor-intensive, comparative-disadvantage industry. The mass of entrants rises in the skill-intensive industry and falls in the comparative-disadvantage sector. The falling number of firms and rising mass of entrants lead to higher entry and exit rates and more job churning in the comparative-advantage sector. Starting from high trade costs, we see average firm size increasing in both industries, but the change in relative factor prices means that average firm size increases initially by more in the skill-intensive industry.[18]

As discussed in the previous section, falling trade costs lead to a rise in the probability of exporting and a rise in average industry productivity in both sectors. The rise is initially larger for home's industry of comparative advantage - the skill-intensive industry 1. Finally, as variable trade costs fall, factor prices converge across countries, with the relative skilled wage rising in the skill-abundant home country.

The second exercise we undertake is to hold variable trade costs constant at the value $\tau = 1.3$, which corresponds roughly to the values in Obstfeld and Rogoff (2001) and Ghironi and Melitz (2004), and change the extent of countries' relative factor abundance. In this case, we begin with equal values for factor endowments $\{\bar{S}^H = 1100, \bar{L}^H = 1100, \bar{S}^F = 1100, \bar{L}^F = 1100\}$, and then increase home's skill endowment and reduce the raw labor endowment by equal amounts, while making the reverse changes in the foreign country.

Figure 5 graphs the same set of outcomes for this second simulation. The horizontal axis is now countries' relative factor abundance, as measured by $(\bar{S}^H/\bar{L}^H)/(\bar{S}^F/\bar{L}^F)$. Given the symmetry of the changes in country endowments and of the cross-industry differences in factor intensity, the evolution of outcomes for the labor-abundant foreign country is again the exact mirror image of the pattern for home.

As relative factor abundance increases, countries reallocate resources towards their comparative advantage industry. The increase in output in the comparative advantage industry is achieved through a variety of adjustment margins: the mass of entrants, the mass of firms, average productivity and average firm size all increase in the skill-intensive industry in the skill-abundant country. The probability of exporting rises in the comparative advantage industry and declines in the comparative disadvantage industry, while the relative skilled wage falls as home's relative skill abundance increases.

Thus, in the costly trade equilibrium, all the adjustment margins from our analysis of free trade are present (average firm size, the mass of firms, and entry/exit) and vary systematically across countries and industries in accordance with comparative advantage.

[18] Under autarky average firm size in the two sectors differs due to a factor price ratio different from 1 and factor intensity differences across sectors. This would become apparent in Figure 4 if we considered higher trade costs, larger factor endowment differences across countries and/or larger factor intensity differences across industries.

In addition, even when entry and production have identical factor intensities, changes in trade costs or changes in relative factor abundance have uneven effects on the productivity range of producing firms and the productivity range of exporting firms because of the interaction of country factor abundance and industry factor intensity.

7. Conclusions

This paper combines the new literature on heterogeneous firms in trade with an old tradition of endowment-based comparative advantage. The combination of factor endowment differences across countries, factor intensity differences across industries, and heterogeneous firms within industries is able to simultaneously explain inter-industry trade (countries are net exporters in their industries of comparative advantage), intra-industry trade (even within an industry where a country is a net importer, two-way trade occurs), and selection into export markets (within both net exporting and net importing sectors, some firms export while many others do not).

Our analysis yields a rich set of predictions for the margins along which economies adjust to trade. In general, following the opening of trade, there will be changes in the mass of firms within industries, changes in average firm size, changes in the mass of entering/exiting firms, and changes in industry productivity. The productivity changes will occur whenever the factor intensity of production differs from the factor intensity of entry or where the existence of trade costs induces selection into export markets.

The between-industry reallocations of Heckscher-Ohlin theory interact with the within-industry reallocations of heterogeneous-firm models. When opening to costly trade, productivity increases by more in comparative advantage industries while the range of exporting firms will tend to be smaller in comparative disadvantage industries.

Interesting areas for further research include the empirical testing of these theoretical predictions concerning the role of comparative advantage in industry productivity dynamics and extensions of the theory to introduce additional sources of firm heterogeneity, dynamic firm productivity, and multiple products within industries.

More generally, our analysis provides an example of the rich insights to be gained by combining microeconomic modelling of firms with general equilibrium analyses of trade. It points to fruitful further research placing individual firm behavior at the center of economies' adjustment to trade.

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A Appendix

A1. Proof of Proposition 1

Proof. (a) Begin with existence of a unique integrated equilibrium.
Choose the skilled wage as numeraire, so $w_S = 1$.
From the free entry condition (13), $V_i \to \infty$ as $\varphi_i^* \to 0$; $V_i \to 0$ as $\varphi_i^* \to \infty$; and V_i is monotonically decreasing in φ_i^*. Thus, equation (13) defines a unique equilibrium value of the zero-profit productivity cutoff, $\hat{\varphi}_i^*$, as a function of parameters.
From equation (12), φ_i^* uniquely determines weighted average productivity, $\widetilde{\varphi}_i(\varphi_i^*)$. Combining $\bar{r}_i = r_i(\tilde{\varphi}_i) = (\tilde{\varphi}_i/\varphi_i^*)^{\sigma-1} r_i(\varphi_i^*)$ with the zero-profit cutoff condition (8), average revenue and profitability may be expressed as functions of φ_i^* and factor prices alone:

$$\begin{aligned} \overline{r}_i &= r_i(\widetilde{\varphi}_i) = \left(\frac{\widetilde{\varphi}_i(\varphi_i^*)}{\varphi_i^*}\right)^{\sigma-1} \sigma f_i (w_S)^{\beta_i} (w_L)^{1-\beta_i} \qquad (33) \\ \overline{\pi}_i &= \pi_i(\widetilde{\varphi}_i) = \left[\left(\frac{\widetilde{\varphi}_i(\varphi_i^*)}{\varphi_i^*}\right)^{\sigma-1} - 1\right] f_i (w_S)^{\beta_i} (w_L)^{1-\beta_i}. \end{aligned}$$

In each sector, the total value of payments to labor used in production equals total revenue minus total profits:

$$w_S S_i^p + w_L L_i^p = R_i - \Pi_i \qquad (34)$$

while, combining free entry (11) and steady-state stability (14), the total value of payments to labor used in entry equals total profits:

$$\Pi_i = M_i \overline{\pi}_i = M_{ei} f_{ei} (w_S)^{\beta_1} (w_L)^{1-\beta_1} = w_S S_i^e + w_L L_i^e. \qquad (35)$$

Thus, total payments to labor in each sector equal total revenue:

$$w_S S_i + w_L L_i = R_i. \qquad (36)$$

where $S_i = S_i^p + S_i^e$ and $L_i = L_i^p + L_i^e$. Since this is true for both sectors, aggregate revenue equals aggregate income:

$$w_S \overline{S} + w_L \overline{L} = R. \qquad (37)$$

Since production technologies in each sector are Cobb-Douglas with the same factor intensities for production and entry, payments to skilled and unskilled labor are a constant share of total industry revenue, yielding the following equilibrium labor demands:

$$S_i = \frac{\beta_i R_i}{w_S}, \qquad L_i = \frac{(1-\beta_i) R_i}{w_L} \qquad (38)$$

Combining these expressions with labor market clearing, and using the fact that the representative consumer allocates constant shares of expenditure $\{\alpha, (1-\alpha)\}$ to the two sectors, we obtain the integrated equilibrium labor allocation:

$$S_1 = \left(\frac{\beta_1\alpha}{\beta_1\alpha + \beta_2(1-\alpha)}\right)\overline{S}, \quad S_2 = \left(\frac{\beta_2(1-\alpha)}{\beta_1\alpha + \beta_2(1-\alpha)}\right)\overline{S}. \tag{39}$$

$$\begin{aligned} L_1 &= \left(\frac{(1-\beta_1)\alpha}{(1-\beta_1)\alpha + (1-\beta_2)(1-\alpha)}\right)\overline{L} \\ L_2 &= \left(\frac{(1-\beta_2)(1-\alpha)}{(1-\beta_1)\alpha + (1-\beta_2)(1-\alpha)}\right)\overline{L}. \end{aligned} \tag{40}$$

Substituting equilibrium employment into the expression for unskilled labor demand, and simplifying using equilibrium consumer expenditure shares and our choice of numeraire, yields the integrated equilibrium unskilled wage:

$$w_L = \left(\frac{\overline{S}}{\overline{L}}\right)\left(\frac{1-\beta_2-\alpha\beta_1+\alpha\beta_2}{\beta_2+\alpha\beta_1-\alpha\beta_2}\right) \tag{41}$$

The integrated equilibrium vector is $\{\varphi_1^*, \varphi_2^*, P_1, P_2, R, p_1(\varphi), p_2(\varphi), w_S, w_L\}$. We have solved for $\{\varphi_1^*, \varphi_2^*\}$ and $\{w_S, w_L\}$. $\{p_1(\varphi), p_2(\varphi)\}$ follow immediately from the pricing rule (4) and equilibrium wages. $\{R\}$ follows from equation (37) and equilibrium wages. $\{P_1, P_2\}$ may be determined from the analogue of equation (15) for the integrated world economy: $P_i = M_i^{1/(1-\sigma)} p_i(\tilde{\varphi}_i)$, where $\tilde{\varphi}_i$ is uniquely determined by φ_i^* and $M_i = R_i/\bar{r}_i$. From equation (33), $\bar{r}_i$ is determined by φ_i^* and wages for which we have solved. From equation (36), R_i is determined by wages and labor allocations for which we have also solved. We have thus fully characterized the integrated equilibrium vector.

(b) Now establish the existence of a FPE equilibrium which replicates the integrated equilibrium resource allocation.

FPE and our choice of the skilled wage in one country as numeraire implies:

$$w_S^H = w_S^F = w_S = 1, \qquad w_L^H = w_L^F = w_L. \tag{42}$$

Cost minimization implies the same equilibrium factor intensities in the two countries:

$$\frac{S_i^H}{L_i^H} = \frac{S_i^F}{L_i^F} = \frac{\beta_i}{1-\beta_i}\frac{w_L}{w_S} = \frac{\beta_i}{1-\beta_i}w_L \tag{43}$$

The factor market clearing conditions in each country $k \in \{H, F\}$ may be expressed as follows:

$$\begin{aligned} \lambda_{L1}^{k}\left(\frac{S_1^k}{L_1^k}\right)+\left(1-\lambda_{L1}^{k}\right)\left(\frac{S_2^k}{L_2^k}\right) &= \frac{\overline{S}^k}{\overline{L}^k}, \qquad \lambda_{Li}^{k}\equiv\frac{L_i^k}{\overline{L}^k} \\ \lambda_{S1}^{k}\left(\frac{L_1^k}{S_1^k}\right)+\left(1-\lambda_{S1}^{k}\right)\frac{L_2^k}{S_2^k} &= \frac{\overline{L}^k}{\overline{S}^k}, \qquad \lambda_{\text{Si}}^{k}\equiv\frac{S_i^k}{\overline{S}^k}. \end{aligned} \tag{44}$$

Substituting for equilibrium factor intensities in the above and rearranging yields the free trade equilibrium labor allocations in each country as a function of endowments and the common unskilled relative wage for which we solve below:

$$L_1^k=\frac{\frac{1}{w_L^k}\overline{S}^k-\left(\frac{\beta_2}{1-\beta_2}\right)\overline{L}^k}{\left(\frac{\beta_1}{1-\beta_1}\right)-\left(\frac{\beta_2}{1-\beta_2}\right)}, \quad L_2^k=\frac{\left(\frac{\beta_1}{1-\beta_1}\right)\overline{L}^k-\frac{1}{w_L}\overline{S}^k}{\left(\frac{\beta_1}{1-\beta_1}\right)-\left(\frac{\beta_2}{1-\beta_2}\right)} \tag{45}$$

$$\begin{aligned} S_1^k &= \frac{\left(\frac{\beta_1}{1-\beta_1}\right)\overline{S}^k-\left(\frac{\beta_1}{1-\beta_1}\right)\left(\frac{\beta_2}{1-\beta_2}\right)w_L\overline{L}^k}{\left(\frac{\beta_1}{1-\beta_1}\right)-\left(\frac{\beta_2}{1-\beta_2}\right)} \\ S_2^k &= \frac{\left(\frac{\beta_1}{1-\beta_1}\right)\left(\frac{\beta_2}{1-\beta_2}\right)w_L\overline{L}^k-\left(\frac{\beta_2}{1-\beta_2}\right)\overline{S}^k}{\left(\frac{\beta_1}{1-\beta_1}\right)-\left(\frac{\beta_2}{1-\beta_2}\right)} \end{aligned} \tag{46}$$

Applying the same arguments as in the integrated equilibrium, aggregate income in each country equals aggregate revenue:

$$R^k=\overline{S}^k+w_L\overline{L}^k. \tag{47}$$

In both countries, total industry payments to unskilled labor are a constant share $(1-\beta_i)$ of total industry revenue, while world expenditure on a good equals a constant share of world revenue:

$$w_L\left(L_1^H+L_1^F\right)=(1-\beta_1)\alpha\left[(\overline{S}^H+\overline{S}^F)+w_L(\overline{L}^H+\overline{L}^F)\right] \tag{48}$$

where the superscript W indicates a world value. Substituting for free trade equilibrium employment levels $\{L_1^H, L_1^F\}$ and rearranging yields the equilibrium unskilled wage, which equals the value for the integrated world economy in equation (41).

The FPE set is characterized by the requirement that both countries' relative endowments of skilled and unskilled workers lie inbetween the integrated equilibrium factor intensities of the two sectors:

$$\left(\frac{\beta_1}{1-\beta_1}\right)\widehat{w}_L^{IE} > \frac{\overline{S}^H}{\overline{L}^H} > \frac{\overline{S}^F}{\overline{L}^F} > \left(\frac{\beta_2}{1-\beta_2}\right)\widehat{w}_L^{IE} \tag{49}$$

where the superscript IE indicates an integrated equilibrium value.
The free trade equilibrium is referenced by the vector $\{\varphi_1^{*k}, \varphi_2^{*k}, P_1^k, P_2^k, R^k, p_1^k(\varphi), p_2^k(\varphi), w_S^k, w_L^k\}$ for each country $k \in \{H, F\}$.
We have already solved for $\{w_S^H = w_S^F = 1,\, w_L^H = w_L^F\}$. $\{p_1^H(\varphi) = p_1^F(\varphi),\, p_2^H(\varphi) = p_2^F(\varphi)\}$ follow immediately from the pricing rule (4) and equilibrium wages. $\{R^k\}$ follows from equation (47) and equilibrium wages. $\{\varphi_1^{*H} = \varphi_1^{*F},\, \varphi_2^{*H} = \varphi_2^{*H}\}$ are determined by the free entry condition alone (equation (13)). $\{P_1^H = P_1^F,\, P_2^H = P_2^F\}$ are determined from equation (15), where $\tilde{\varphi}_i$ is uniquely determined by φ_i^* and $M_i^k = R_i^k/\bar{r}_i^k$. From equation (33), $\bar{r}_i^H = \bar{r}_i^F$ is determined by equilibrium φ_i^{*k} and factor prices for which we have solved. From equation (36), R_i^k is determined by wages and labor allocations in each country for which we have also solved. We have thus completed our characterization of the FPE equilibrium vector.
We have already established that, for country factor endowments within the FPE set, free trade equilibrium wages equal their value in the integrated equilibrium. Since the free entry condition is the same in the two countries and a function of parameters alone, the free trade zero-profit cutoff productivities also equal their integrated equilibrium values. Thus, $\{\varphi_1^{*k}$, φ_2^{*k}, P_1^k, P_2^k, $p_1^k(\varphi)$, $p_2^k(\varphi)$, w_S^k, $w_L^k\}$ are the same as in integrated equilibrium. Aggregate revenue, industry revenue, the mass of firms, and labor allocations will vary across countries in the free trade equilibrium. However, their sum across countries equals the values in the integrated equilibrium. ■

A2. Proof of Proposition 2

Proof. (a) In the FPE equilibrium, $S_1^H/L_1^H = S_1^F/L_1^F$ and $S_2^H/L_2^H = S_2^F/L_2^F$. Industry 1 is relatively skill intensive: $S_1^k/L_1^k > S_2^k/L_2^k$ for $k \in \{H, F\}$. Country H is relatively skill abundant: $S^H/L^H > S^F/L^F$. From equation (44), it follows immediately that $\lambda_{L1}^H > \lambda_{L1}^F$ and $\lambda_{S1}^H > \lambda_{S1}^F$.
(b) The FPE equilibrium relative wage equals the value for the integrated world economy. Choose a country's skilled wage for the numeraire, $w_S^H = w_S^F = 1$. Compare the closed economy equilibrium relative unskilled wage in equation (41) for an individual country and the integrated world economy. It follows immediately that $\bar{S}^H/\bar{L}^H > (\bar{S}^H+\bar{S}^F)/(\bar{L}^H+\bar{L}^F)$

implies $w_L^A > w_L^{FT}$, where the superscripts A and FT indicate autarky and free trade respectively. Therefore, in the move from autarky to free trade, the unskilled relative wage, w_L, will fall in a skill-abundant country and rise in a labor-abundant country.
From (38) and (44), cost minimization and factor market clearing imply:

$$\begin{aligned} \lambda_{L1}^k\left(\frac{\beta_1}{1-\beta_1}w_L\right)+\left(1-\lambda_{L1}^k\right)\left(\frac{\beta_2}{1-\beta_2}w_L\right) &= \frac{\overline{S}^k}{\overline{L}^k} \\ \lambda_{S1}^k\left(\frac{1-\beta_1}{\beta_1}\frac{1}{w_L}\right)+\left(1-\lambda_{S1}^k\right)\left(\frac{1-\beta_2}{\beta_2}\frac{1}{w_L}\right) &= \frac{\overline{L}^k}{\overline{S}^k} \end{aligned} \tag{50}$$

where we have used $w_S = 1$. In equation (50), $w_L^{FT} < w_L^A$ implies $\lambda_{L1}^{FT} > \lambda_{L1}^A$ and $\lambda_{S1}^{FT} > \lambda_{S1}^A$, since $\beta_1 > \beta_2$. Thus, following the opening of trade, the skill-abundant country will devote an increased share of its skilled and unskilled labor to the skill-intensive industry, and the labor-abundant country will devote an increased share of its skilled and unskilled labor to the labor-intensive industry, which establishes the Proposition. ■

A3. Proof of Proposition 3

Proof. (a) A common value for the zero-profit productivity cutoff within industries across countries follows immediately from the assumption of identical technologies and equation (13) which uniquely pins down φ_i^* as a function of model parameters alone. A common φ_i^* implies, from equation (12), the same value of $\widetilde{\varphi}_i$ within industries across countries. Combining this with factor price equalization implies, from equation (33), that the two countries have the same average firm size within an industry.
(b) Since $M_i^k = R_i^k/\overline{r}_i^k$ and $R_i^k = w_L^k L_i^k + w_S^k S_i^k$ for country $k \in \{H, F\}$, we have:

$$\frac{M_1^k}{M_2^k} = \left[\frac{w_L L_1^k + S_1^k}{w_L L_2^k + S_2^k}\right]\frac{\overline{r}_2^k}{\overline{r}_1^k} \tag{51}$$

where, with factor price equalization, $w_L^k = w_L$, and $w_S^k = w_S = 1$. Substituting for equilibrium labor allocations from equations (45) and (46), noting that average revenue can be expressed as in equation (33), and simplifying terms we obtain:

$$\frac{M_1^k}{M_2^k} = \left[\frac{(1-\beta_2)\frac{\overline{S}^k}{\overline{L}^k}-\beta_2 w_L}{\beta_1 w_L-(1-\beta_1)\frac{\overline{S}^k}{\overline{L}^k}}\right]\left(\frac{\widetilde{\varphi}_2/\varphi_2^*}{\widetilde{\varphi}_1/\varphi_1^*}\right)^{\sigma-1}\left(\frac{f_2}{f_1}\right)\left(\frac{w_L}{1}\right)^{\beta_1-\beta_2} \tag{52}$$

where, from equations (13) and (12), φ_i^* and $\widetilde{\varphi}_i$ are determined by model parameters and are the same across countries. From equation (52), $S^H/L^H > S^F/L^H$ implies $M_1^H/M_2^H >$

M_1^F/M_2^F.

(c) From the steady-state stability conditions (14):

$$\frac{M_{e1}}{M_{e2}} = \frac{[1-G(\varphi_2^*)]}{[1-G(\varphi_1^*)]}\frac{M_1}{M_2} \tag{53}$$

Since φ_i^* is the same within industries across countries, $M_1^H/M_2^H > M_1^F/M_2^F$ implies $M_{e1}^H/M_{e2}^H > M_{e1}^F/M_{e2}^F$.

■

A4. Proof of Proposition 4

Proof. (a) The zero-profit productivity cutoff remains unchanged in the move from autarky to free trade because the free entry condition (13) uniquely pins down φ_i^* as a function of model parameters alone.

(b) From equation (33), relative firm average size in the two sectors is:

$$\frac{\overline{r}_1}{\overline{r}_2} = \left(\frac{\widetilde{\varphi}_1/\varphi_1^*}{\widetilde{\varphi}_2/\varphi_2^*}\right)^{\sigma-1}\left(\frac{f_1}{f_2}\right)\left(\frac{1}{w_L}\right)^{\beta_1-\beta_2} \tag{54}$$

where φ_i^* remains unchanged in the move from autarky to free trade and hence, from equation (12), so does $\widetilde{\varphi}_i$. In the skill-abundant country, the relative unskilled wage falls following the opening of trade, increasing relative average firm size in the skill-intensive industry, since $\beta_1 > \beta_2$. The converse is true for the labor-abundant country, where average firm size rises in the labor-intensive industry.

(c) The relative mass of firms in the two sectors may be determined from equation (52). Moving from autarky to free trade, the relative unskilled wage falls in the skill-abundant country. Taking logarithms and totally differentiating with respective to the relative unskilled wage in equation (52), and rearranging, it can be shown that:

$$\begin{aligned}\frac{d\log(M_1^k/M_2^k)}{dw_L} \overset{\text{sign}}{=} & -\beta_2(1-\beta_2)\left[w_L+\frac{\bar{S}^H}{\bar{L}^H}\right]\underbrace{\left[\beta_1 w_L-(1-\beta_1)\frac{\bar{S}^H}{\bar{L}^H}\right]}_{+\text{ve}} \\ & -\beta_1(1-\beta_1)\left[w_L+\frac{\bar{S}^H}{\bar{L}^H}\right]\underbrace{\left[(1-\beta_2)\frac{\bar{S}^H}{\bar{L}^H}-\beta_2 w_L\right]}_{+\text{ve}} < 0\end{aligned}$$

Thus, the relative mass of firms in the skill-intensive industry rises in the skill-abundant country. The converse is true in the labor-abundant country, where the relative unskilled

wage increases, and the relative mass of firms in the labor-intensive industry rises.
(d) The relative mass of entrants in the two sectors may be determined from equation (53). Since the zero-profit productivity cutoff is unchanged in the move from autarky to free trade, the relative mass of entrants will move proportionately with the relative mass of firms, which establishes the Proposition. ■

A5. Proof of Proposition 5

Proof. We choose the skilled wage in one country as numeraire, $w_S^H = 1$.
Suppose that the equilibrium wage vector $\{1, w_L^H, w_S^F, w_L^F\}$ is known.
The free trade equilibrium allocations of skilled and unskilled labor in equations (45) and (46) were determined using labor market clearing (equation (17)) and equilibrium industry factor intensities (equation (43)). The expressions for the costly trade equilibrium allocations of skilled and unskilled labor are the same, except that the relative unskilled wage will now generally vary across countries, so that terms in what was previously the common unskilled wage, w_L, need to be replaced with country-specific values for the relative unskilled wage, w_L^k/w_S^k.
Using the costly trade analogues of equations (45) and (46), the wage vector uniquely pins down equilibrium labor allocations in home and foreign: $\{L_1^H, L_2^H, L_1^F, L_2^F, S_1^H, S_2^H, S_1^F, S_2^F\}$. These equilibrium allocations now include labor used in entry, production and exporting: $L_i^k = L_i^{kp} + L_i^{ke} + L_i^{kx}$ and $S_i^k = S_i^{kp} + S_i^{ke} + S_i^{kx}$.
Following the same line of reasoning as in the proof of Proposition 1, it may be shown that total industry payments to labor used in production, entry and exporting equal total industry revenue:

$$R_1^k = w_S^k S_1^k + w_L^k L_1^k, \qquad R_2^k = w_S^k S_2^k + w_L^k L_2^k. \tag{55}$$

Thus, the wage vector and equilibrium labor allocations uniquely pin down total industry revenue $\{R_1^H, R_2^H, R_1^F, R_2^F\}$ and hence each country's aggregate revenue $\{R^H, R^F\}$.
The pricing rule (20) determines equilibrium variety prices in the domestic and export markets for each country $\{p_{1d}^H(\varphi), p_{1x}^H(\varphi), p_{2d}^H(\varphi), p_{2x}^H(\varphi), p_{1d}^F(\varphi), p_{1x}^F(\varphi), p_{2d}^F(\varphi), p_{2x}^F(\varphi)\}$ as a function of the wage vector.
With wages, variety prices, total industry revenue, and aggregate revenue known, the equilibrium zero-profit cutoff productivities $\{\varphi_1^{*k}, \varphi_2^{*k}\}$, the exporting-cutoff productivities $\{\varphi_{1x}^{*k}, \varphi_{2x}^{*k}\}$, and price indices $\{P_1^k, P_2^k\}$ are the solution to the system of six simultaneous equations in each country k defined by (29), (26) and (30) for each industry i. In solving this

system of six simultaneous equations in each country, we substitute out for the equilibrium mass of firms, $M_i^k = R_i^k / \overline{r}_i^k$, probability of exporting, $\chi_i^k = \frac{[1-G(\varphi_{ix}^{*k})]}{[1-G(\varphi_i^{*k})]}$, and average firm revenue, $\overline{r}_i^k = \left(\frac{\widetilde{\varphi}_i^k(\varphi_i^{*k})}{\varphi_i^{*k}}\right)^{\sigma-1} \sigma f_i (w_S^k)^{\beta_i} (w_L^k)^{1-\beta_i}$, using the fact that these are functions of elements of the six unknowns $\{\varphi_1^{*k}, \varphi_2^{*k}, \varphi_{1x}^{*k}, \varphi_{2x}^{*k}, P_1^k, P_2^k\}$ as well as the known wage vector and equilibrium industry revenue for which we have already solved.

Thus, given the wage vector $\{1, w_L^H, w_S^F, w_L^F\}$, we have solved for all other elements of the equilibrium vector $\{\varphi_1^{*k}, \varphi_2^{*k}, \varphi_{1x}^{*k}, \varphi_{2x}^{*k}, P_1^k, P_2^k, p_1^k(\varphi), p_2^k(\varphi), p_{1x}^k(\varphi), p_{2x}^k(\varphi), R^k\}$ for $k \in \{H, F\}$.

The equilibrium wage vector itself is pinned down by the requirement that the value of total industry revenue, $R_i^k = w_S^k S_i^k + w_L^k L_i^k$, equals the sum of domestic and foreign expenditure on domestic varieties (equation (31) for each country and industry). ■

A6. Proof of Proposition 6

Proof. (a) Under autarky, the free entry condition is given by (13), which equals the value for the integrated world economy (a closed economy). Under costly trade, the free entry condition becomes (29), where the relationship between the productivity cutoffs is governed by equation (26), so that $\varphi_{ix}^{*k} = \Lambda_i^k \varphi_i^*$. The expected value of entry, V_i^k, in equation (29) equals its value in the closed economy (equation (13)), plus a positive term reflecting the probability of drawing a productivity high enough to export. Since, using equation (26), V_i^k is monotonically decreasing in φ_i^{*k}, the costly trade equilibrium must be characterized by a higher value of φ_i^{*k} than under autarky, in order for V_i^k to equal the sunk entry cost f_{ei}.

(b) The additional positive term in the costly trade free entry condition (29) will be larger, and hence the increase in φ_i^{*k} will be greater, when Λ_i^k is smaller (when φ_{ix}^{*k} is close to φ_i^{*k}). The second part of the proposition follows immediately from the definition of Λ_i^k in equation (26).

(c) The probability of exporting, $\chi_i^k = [1 - G(\Lambda_i^k \varphi_i^{*k})]/[1 - G(\varphi_i^{*k})]$, is monotonically decreasing in Λ_i^k. Hence, the final part of the proposition also follows immediately from the definition of Λ_i^k in equation (26). ■

A7. Proof of Proposition 7

Proof. At the free trade equilibrium, the relative price indices of the two sectors are the same in the two countries and are determined according to equation (15). Under autarky, the relative price indices generally differ across countries k and are given by:

$$\frac{P_1^k}{P_2^k} = \left(\frac{M_1^k}{M_2^k}\right)^{\frac{1}{1-\sigma}} \frac{p_1^k(\varphi_1^k)}{p_2^k(\varphi_2^k)}, \tag{56}$$

where the closed economy relative mass of firms, M_i^k/M_2^k, may be determined following a similar line of reasoning as under free trade (equation (52)). The mass of firms $M_i^k = R_i^k/\bar{r}_i^k$, equilibrium average revenue is given by equation (33), while, under autarky, $R_i^k = \alpha_i R^k$. Substituting for the relative mass of firms in the above, and simplifying using the pricing rule (20), the autarky relative price index becomes:

$$\frac{P_1^k}{P_2^k} = \left(\frac{\alpha}{1-\alpha}\right) \frac{\varphi_2^{*k}}{\varphi_1^{*k}} \left(\frac{f_2}{f_1}\right)^{\frac{1}{1-\sigma}} \left(\frac{w_L^k}{w_S^k}\right)^{\frac{\sigma(\beta_1-\beta_2)}{1-\sigma}} \tag{57}$$

To make comparisons across the two countries under autarky, we require consistent units of measurement and we choose skilled labor as the numeraire in each country, $w_S^H = 1$ and $w_S^F = 1$. The closed economy relative unskilled wage is given by equation (41), substituting a country's relative endowments for world relative endowments (exploiting the fact that the integrated world economy is closed). From equation (41), the closed economy with a larger relative supply of skilled labor is characterized by a higher relative wage of unskilled workers, w_L^k.

In equation (57), $\beta_1 > \beta_2$ and $\sigma > 1$, while identical technologies implies $\varphi_i^{*H} = \varphi_i^{*F}$. Hence, the higher relative wage of unskilled workers in the skill-abundant closed economy is reflected in a lower relative price index for the skill-intensive good: $P_1^H/P_2^H > P_1^F/P_2^F$.

Under costly trade, from equation (30), the relative price indices may be expressed as:

$$\frac{P_1^k}{P_2^k} = \left[\frac{M_1^k\left(p_{1d}^k(\widetilde{\varphi}_1^k)\right)^{1-\sigma} + \chi_1^j M_1^j \left(\tau_1\, p_{1d}^j(\widetilde{\varphi}_{1x}^j)\right)^{1-\sigma}}{M_2^k\left(p_{2d}^k(\widetilde{\varphi}_2^k)\right)^{1-\sigma} + \chi_2^j M_2^j \left(\tau_2\, p_{2d}^j(\widetilde{\varphi}_{2x}^j)\right)^{1-\sigma}}\right]^{1/(1-\sigma)}, \tag{58}$$

for $k, j \in \{H, F\}$, $j \neq k$. As $\tau_i \to \infty$ and $f_{ix} \to \infty$ for $i \in \{1, 2\}$, the costly trade relative price index converges to its autarkic value. In equation (58), $\mu_i^k \to 0$, while M_i^k and $p_{1d}^k(\widetilde{\varphi}_1^k)$ converge to their autarky values.

As $\tau_i \to 1$ and $f_{ix} \to 0$ for $i \in \{1, 2\}$, the costly trade relative price index converges to its common free trade value. In equation (58), $\mu_i^k \to 1$, while M_i^k, $p_{id}^k(\widetilde{\varphi}_i^k)$ and $p_{id}^k(\widetilde{\varphi}_{ix}^k)$ converge to their free trade values, where $p_{id}^k(\widetilde{\varphi}_i^k) = p_{id}^k(\widetilde{\varphi}_{ix}^k) = p_{id}^j(\widetilde{\varphi}_i^j) = p_{id}^j(\widetilde{\varphi}_{ix}^j)$.

For intermediate fixed and variable trade costs where selection into export markets occurs, the relative price indices will lie inbetween the two countries' autarky values and the common free trade value: $P_1^H/P_2^H < P_1^F/P_2^F$.

In the absence of cross-industry differences in τ_i or f_{ix}/f_i, this difference in relative prices indices implies, from equation (32), that Λ_i^k will be smaller in a country's comparative advantage industry than in the country's comparative disadvantage industry ($\Lambda_1^H < \Lambda_2^H$ and $\Lambda_2^F < \Lambda_1^F$).

Comparing the expected value of entry, V_i^k, under autarky (equation (13), which equals the value for the integrated world economy) with the expected value of entry under costly trade (equation (29)), the expression for V_i^k under costly trade contains an additional positive term. From the equilibrium relationship between the two productivity cutoffs in equation (26), this additional positive term will be larger when Λ_i^k is small (when φ_{ix}^{*k} is close to φ_i^{*k}).

Using the equilibrium relationship between the two productivity cutoffs, the expected value of entry, V_i^k, in the costly trade free entry condition (29) is monotonically decreasing in the zero-profit productivity cutoff, φ_i^{*k}. It follows immediately that the industry with the smaller value of Λ_i^k, and hence the larger increase in the expected value of entry following the opening of costly trade, must experience a larger increase in φ_i^{*k} in order for V_i^k to equal the sunk entry cost f_{ei}. This establishes the Proposition.

(c) Since Λ_i^k is smaller in a country's comparative advantage industry than in the comparative disadvantage industry, and the probability of exporting, $\chi_i^k = [1 - G(\Lambda_i^k \varphi_i^{*k})]/[1 - G(\varphi_i^{*k})]$, is monotonically decreasing in Λ_i^k, it follows immediately that the probability of exporting will be greater in a country's comparative advantage industry, which establishes the Proposition. ■

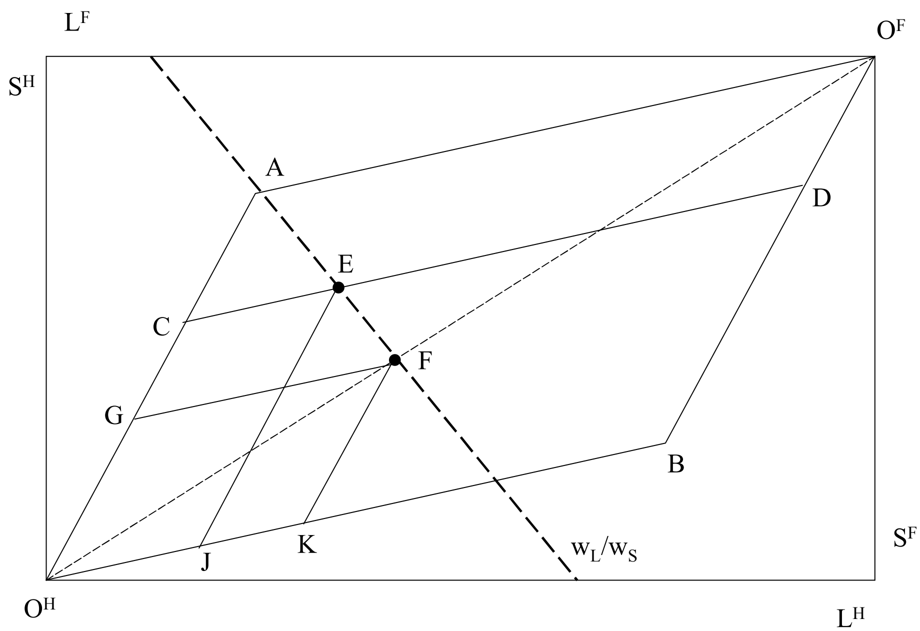

Figure 1: integrated equilibrium and factor price equalization

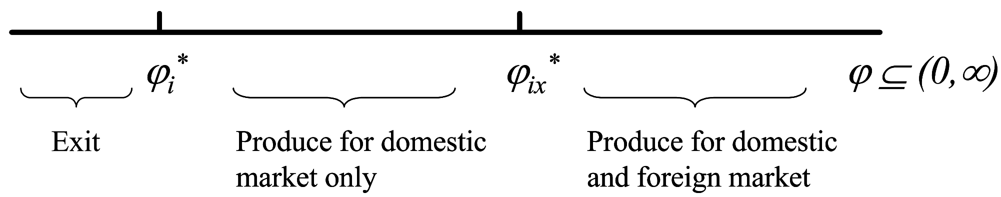

Figure 2: Zero-profit and exporting productivity cutoffs with costly trade

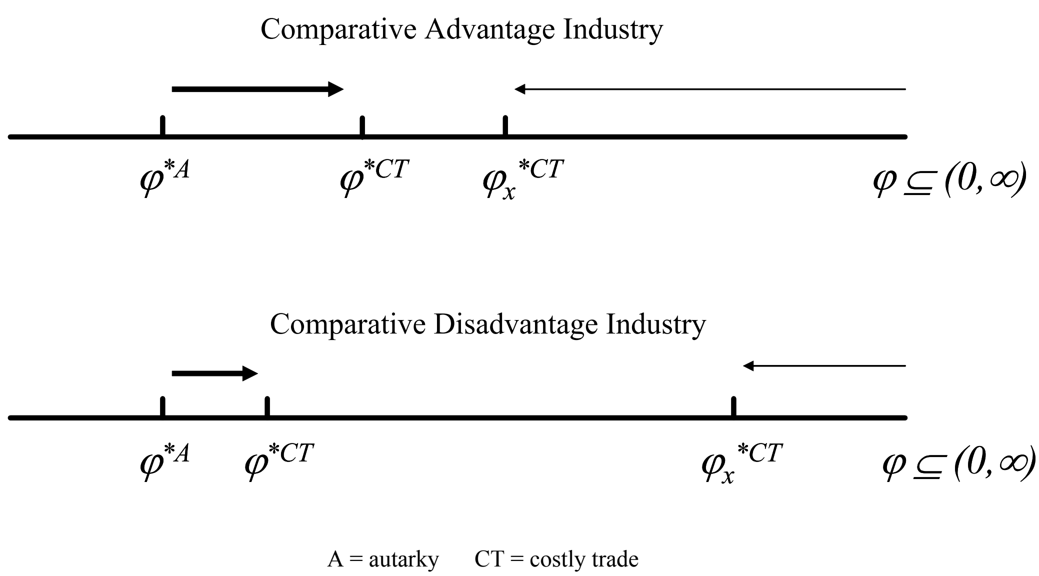

Figure 3: From autarky to costly trade: differential movements of the productivity cutoffs across industries

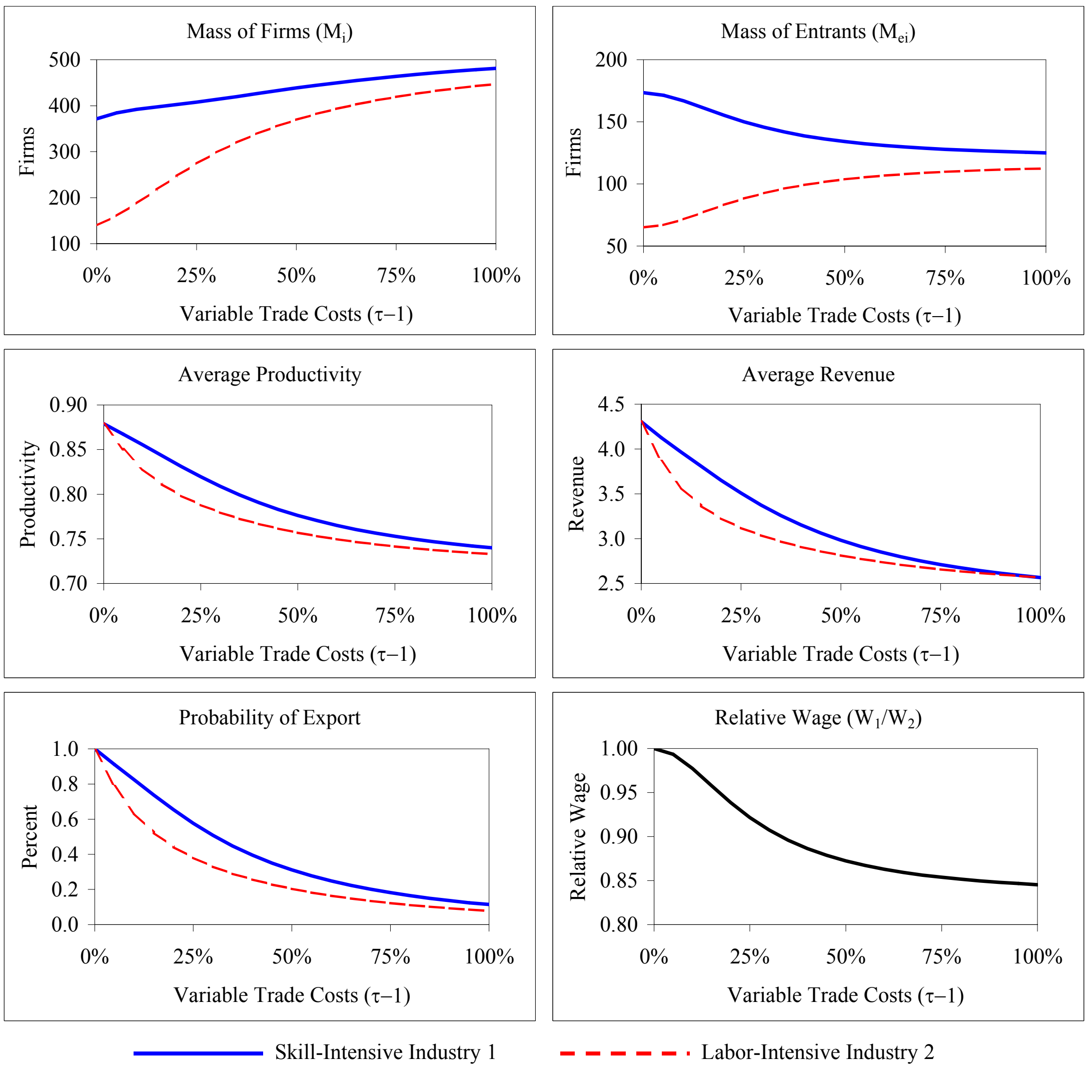

Figure 4: Numerical Solutions with Declining Variable Trade Costs

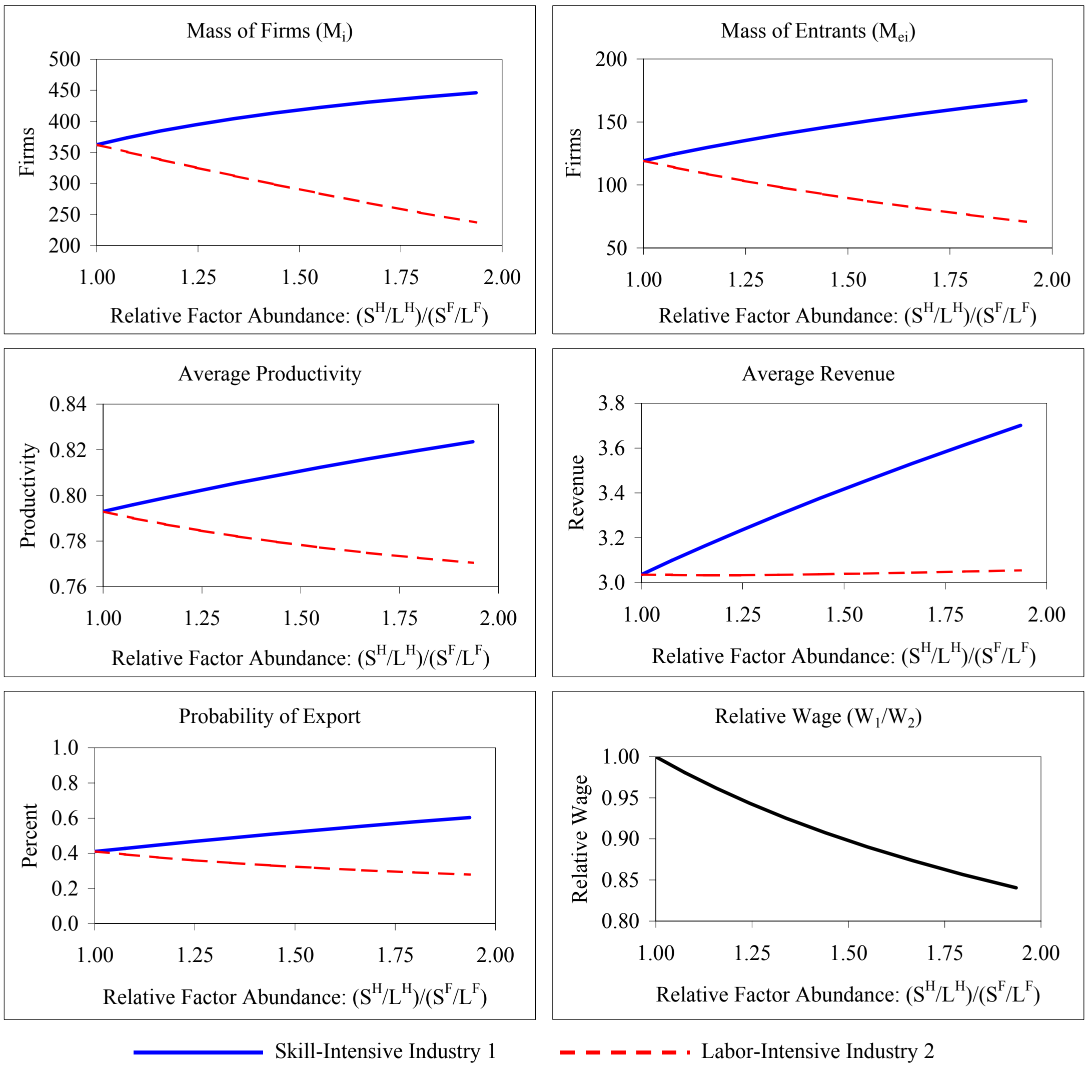

Figure 5: Numerical Solutions with Symmetric Changes in Home and Foreign Skill Abundance