

Skill Acquisition, Credit Constraints, and Trade

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Abstract

This paper looks at the effect of credit constraints on skill acquisition when agents have heterogeneous abilities and wealth. We use a general equilibrium model and assume credit markets are absent.

Two payment systems for training are explored. Under the first, payment is made up front and as a result, credit constraints are severe. In the second, a form of work study is allowed, and this helps mitigate credit constraints. We consider the system's behavior both in and out of steady state and argue that there can be multiple equilibria as supply need not always be monotonic. Moreover, that stricter credit constraints need not shift supply inward.

Also, opening the economy to trade could reduce welfare in steady state. An increase in the relative price of the skill intensive good raises the cost of education. As education becomes more expensive, credit constraints become more binding and the stock of skilled labor falls as does the supply of the skill intensive good and this permits welfare to fall.

1 Introduction

What is the role of credit constraints in transferring skills across generations and what is the role of trade in this? What is the effect of tighter credit constraints? Will tighter credit constraints reduce the extent of training and raise its price? Will welfare rise or fall and why? These are the questions addressed in this paper. They are clearly of immense relevance for policy since human capital is not good collateral for loans and the ability to acquire skills can be severely limited by wealth. In this paper we explicitly model the acquisition of skills by agents. We assume credit markets are absent, but consider two settings with different degrees of credit market imperfections. In the first one, credit markets are completely absent and the only way an agent can acquire skills is by paying for training in full up front. This is clearly an extreme case and in the second setting we allow credit constraints to be weakened through a pay as you go system. These can be interpreted as an apprenticeship contract where training is provided in exchange for services. Alternatively, they can be interpreted as a form of work study. For example, most Ph.D. students in the U.S. have their education paid for and obtain a stipend in return for teaching or research services. Many undergraduates finance at least part of their education through work study programs.¹

An alternative way of acquiring skills is through firms paying for it. There is a fairly large literature that models such contracts. It deals with issues such as which labor market imperfections would make firms pay for general training that is transferable across firms, the features of such contracts and their rationale, the inefficiency of training levels provided by firms, as well as the success of such apprenticeship programs in providing a skilled labor force. For example, Chang and Wang (1996), Acemoglu and Pischke (1998), and Malcomson et. al. (2003) analyze the effects of asymmetric information between training firms and other potential employers. Acemoglu (1997), Booth and Chatterji (1998) consider the implications of imperfect competition in the skilled labor market. Acemoglu and Pischke (1999a) look at how wage compression can induce general training by firms. All these

¹This system, in our simple model, is equivalent to payment at the end of the period.

papers have a worker's marginal productivity increasing with training by more than the wage the firm pays. This enables the firm to capture some returns to general training and, as a result, the firm finds it profitable to invest in worker training. For a review of this research, see Acemoglu and Pischke (1999b) and Smits and Stromback (2001).

Such issues are not the subject of this paper. Rather, we focus on another, hitherto unstudied aspect of apprenticeships, namely their ability to help circumvent credit constraints. Those with the skills to impart (masters) enter into a contract with the unskilled (apprentices) to "teach as best they know" their technical skills. In return, the apprentice undertakes the tasks assigned to him by the master for the (specified) period of his apprenticeship. He is paid below market wages during this period, receiving payment in the form of training instead.² Contrast this with the alternative where the training fees have to be paid up front. In the absence of credit markets, only those with the wealth to pay the up front fee could afford training. Note however, that even if part of the fee is paid up front, as occurs when the apprentice's wage is negative, the less well off may be able to afford the apprenticeship (work study) route.

Are credit constraints important in the U.S. today and is there evidence that such ways of overcoming credit constraints are important in the U.S. today? It is well understood that inter-generational income correlation is reasonably high.³ Empirical work on college attendance has consistently shown that parental income does predict college attendance and that the effect on college attendance of tuition is greater for lower income families. Should this be taken as evidence of credit constraints? Recent work by Cameron and Heckman (1998) questions such an interpretation of these facts. They estimate decision rules for

²Lane (1996) shows that in late 18th century, apprentices earned 41% of the journeyman (skilled) rate while unskilled workers earned 77%. In some cases, apprentices have even paid for the privilege of learning the trade. In fact, by the 18th century, an up front fee had become the norm. While there was considerable variation in the terms specified between the country and the city as well as across occupations, there were instances of large sums, hundreds of pounds, being paid up front when the trade was particularly well rewarded.

³Solon (1992) finds a ballpark figure of .4. Charles and Hurst (2003) find the pre-bequest correlation in log wealth to be .37.

college attendance that control for family background measures like parents education, family income at 16, and a measure of the child's skill endowment as proxied for by the Armed Forces Qualifying Test (AFQT). They find that family income is not significant which they interpret as evidence that credit constraints are not binding. They argue that such correlations could arise in the absence of credit constraints at the college level: for example, parental wealth could be correlated to the skill endowment of the children⁴ or school could have a consumption value so that higher parental incomes result in more education directly.

Keane and Wolpin (2001) argue that credit constraints do matter. They estimate a structural model and argue that though borrowing constraints are tight, they are mitigated by agents adjusting labor supply. This is why changes in tuition affect college attendance by relatively little, and have a greater effect on poorer families who are likely to be more constrained by their labor supply. See Keane (2002) for a simple and clear summary of the issues here. Since most colleges today, other than the very elite (who go the straight aid route) offer work study to needy applicants, apprenticeships, broadly speaking, seem to have a significant role in overcoming credit constraints in education.

There is a growing literature on education and the role of government in providing it in a dynamic setting. Most of this work works with one final good and focuses on the impact on income distribution. See, for example, the classic work of Becker and Tomes (1979). Glomm and Ravikumar (1992) and Hanushek, Leung and Yilmaz (2004) are more recent examples that explore issues such as the effect of public versus private education and the effects of alternative ways of subsidizing college education.

Our work is more closely related to the literature on endogenous skill formation in international trade. In an influential contribution, Findlay and Kierzkowski (1983) extend the standard Heckscher-Ohlin model by endogenizing the formation of human capital. They show that trade amplifies initial differences in factor endowments through the Stolper-Samuelson effect: trade raises the reward of the abundant factor in each country. Therefore,

⁴This may well be a reflection of credit constraints or differences in the importance given to schooling by parents long before college!

trade leads to a decrease in the accumulation of human capital in skill-scarce countries and does the opposite in skill-abundant countries. However, there are no credit constraints.⁵

Cartiglia (1997) incorporates credit constraints into a Findlay-Kierzkowski type model, but uses a static setting. He shows that, trade leads to convergence in human capital endowments rather than amplification of initial differences. A key element in his paper is that skilled labor is used as an input in the formation of skilled labor. Trade liberalization in a skill-scarce country reduces the cost of education which weakens credit constraints, resulting in a higher investment in human capital. This effect in fact outweighs the Stolper-Samuelson effect of Findlay-Kierzkowski, reversing their results.⁶

Ranjan (2001) also allows for borrowing constraints to affect human capital accumulation in a model with two traded final goods, but only considers small open economies. Lenders can lend at the world rate of interest, while borrowers can only borrow up till an incentive compatible repayment level. This level is higher for more able agents, i.e., for agents who obtain more effective units of skilled labor upon becoming educated. He points out a third effect that operates through changes in the distribution of income which influences the accumulation of human capital though the aggregate effect of trade is indeterminate in his model. Ranjan (2003) looks at the effect of trade liberalization on skill acquisition, the skilled-unskilled wage differential, and the distribution of wealth. Multiple steady state equilibria exist in his model and he shows that trade may induce convergence to the good equilibria. However, he assumes a single non traded final good and traded intermediate goods.

None of these analyze the static and dynamic effects of skill formation. Moreover, they do not model the training sector explicitly and do not compare alternative training arrangements as we do. We develop a simple general equilibrium model where apprenticeships help overcome credit constraints that limit the ability of agents with heterogeneous abilities and wealth to acquire skills. There are two tradable final goods, and two factors, unskilled labor

⁵There is a large literature on trade and human capital creation that builds on this paper which we shall neglect as our focus is on the role of credit constraints.

⁶A similar result obtains in Eicher (1999) via a domestic credit market.

and skilled labor which is produced using skilled labor and unskilled labor. The unskilled labor intensive good is the numeraire and we allow the availability of skilled and unskilled labor in production to be endogenously determined.

In the static version of our model⁷, under either system of training, the response of supply to price depends on the number of skilled agents in the economy. If there are relatively few skilled agents, the normal supply response obtains. However, with many skilled agents, supply can be decreasing in price so that multiple equilibria may exist. The intuition is that in addition to the normal supply response, there is an induced Rybczynski effect which could work in either direction. When there are relatively few skilled agents, an increase in the price of the skill intensive good raises the return to skilled labor and the cost of education. This reduces the number of agents who want to acquire training and thus raises the availability of unskilled labor. The availability of skilled labor for production also rises as less skilled labor is needed in training. Whether relative availability of skilled labor rises or falls depends on the endowment of skilled workers to begin with: if this endowment is large, the relative availability of skilled labor falls, while if it is small, it rises. In the former case, relative supply of the skill intensive good can fall with price, while in the latter case it must rise.

Moreover, the relative supply of the skill intensive good can be higher or lower when credit constraints are relaxed! Again, relaxing credit constraints increases the number of agents who want to be educated at any given price. However, this reduces both the skilled and unskilled labor available for production. If the stock of skilled agents is large, then weaker credit constraints will raise the relative availability of skilled labor in production and hence shift out the relative supply of the skill intensive good. The *opposite* occurs when the stock of skilled agents is small. As a result, weaker credit constraints can result in a higher price of the skill intensive good contrary to what simple intuition might suggest.⁸

⁷In a static setting the stock of skilled labor is exogenously given as are expectations about future prices.

⁸Simple general equilibrium intuition would suggest that weaker credit constraints would raise the availability of skilled labor shifting the relative supply of the skill intensive good a la Rybczynski, and lowering its relative price in autarky.

In steady state, however, non monotonicity of supply and multiplicity of equilibria occur only in the presence of credit constraints. An increase in the price in steady state raises the return to skilled labor today, and hence, the cost of education, but it also raises the return tomorrow. While the increase in the cost of education reduces the demand for training, the increase in the return tomorrow increases it. Since each agent must be able to train more than one worker for there to be skilled workers present in steady state, the latter effect dominates. Thus, an increase in price raises the number of trainees, reducing the availability of unskilled workers for production. In steady state, and in the absence of credit constraints, as the number of trainees rises, so does the number of skilled workers available for production. As a result, an increase in price raises the relative supply of the skill intensive good. In the presence of credit constraints, relative supply need not be monotonic in price. For relative supply to be backward bending, the increase in education cost resulting from an increase in price must constrain enough of the potential pool of trainees.

In steady state, weaker credit constraints always raise the relative supply of the skill intensive good and lower its autarky relative price. Weaker credit constraints raise the demand for training, thus reducing the availability of unskilled labor in production. However, as the number of trainees rise, so does the availability of skilled labor for production and hence relative supply of the skill intensive good.

There may or may not be multiple equilibria in steady state with credit constraints: a key determinant is the distribution of wealth. We show that if there is limited substitutability in consumption, then a fall in the value of output of final goods under trade evaluated at trade prices must reduce welfare.

The rest of the paper proceeds as follows. Section 2 lays out the model. Section 3 analyzes the equilibrium in the single period under each system in isolation and together. Section 4 looks at steady state equilibria and how they differ from the static equilibria in a closed economy. Section 5 studies how trade affects outcomes. Section 6 provides concluding remarks and directions for future research.

2 The Model

There are two goods, X and Z , and one basic factor, unskilled labor, U , in the economy. Unskilled labor can be transformed into its skilled counterpart, S . However, if K unskilled workers are taken on by a skilled worker then only $G(K)$ units of the skilled worker's time remains available to him, where $G(K)$ is a decreasing function of K . We assume that

$$G(K) = 1 - AK$$

so that A is the time required per trainee.

While good X , (the agricultural good and the numeraire), uses only unskilled labor in our calculated example, good Z uses both skilled and unskilled labor. All that is needed is that production of X and Z have different relative intensities and Z is relatively more skill-intensive than good X at all factor prices. We normalize units so that it takes one unit of unskilled labor to make a unit of the agricultural good and take it to be the numeraire. Hence, p denotes the relative price of good Z . The production function for the industrial good is

$$Z = S^\alpha U^{1-\alpha}.$$

We use an overlapping generations framework. There are L agents born in each period. Each agent lives for two periods and is endowed with one unit of time in each period. An agent is characterized by two parameters: the probability of becoming skilled, or a master, upon undertaking the needed education, γ , and his initial wealth, y . It is assumed that γ is distributed uniformly in the unit interval ($\gamma \sim U[0, 1]$) and y is distributed according to distribution function $F(\cdot)$ in $[0, y_{\max}]$, where y_{\max} is the maximal wealth level.

In the first period of life an agent makes career choices. He could remain an unskilled worker, work in both periods at the unskilled wage, w . Alternatively, he could spend part of his time acquiring the skills that give him a chance at becoming a master and allowing him, if he so chooses and is successful in his training, to earn the skilled wage in the second period. Agents who try to become skilled but fail can work only as unskilled workers in the second period. Skilled workers could also choose to work as unskilled workers were it

in their interests to do so.

We study two training systems. In the first, which we interpret as an apprenticeship system, where payment for training is not made up front, each skilled worker hires K apprentices. An apprentice supplies β^A hours of his time to the master at a wage w^A and spends $1 - \beta^A$ of his time studying.⁹ If a master takes on K apprentices he obtains $\beta^A K$ units of unskilled labor at cost $w^A \beta^A K$ but has to spend AK hours of his own time in training them. In the second system, called the pay up front system, unskilled workers pay the master a fee, w^C , up-front. The training takes $1 - \beta^C$ units of their time and they work for the remaining time as unskilled workers.¹⁰

We assume that agents consume only at the end of their lifetimes and have identical Cobb-Douglas preferences given by

$$U = (c_X^\delta c_z^{1-\delta})^{1-\theta} b^\theta$$

where c_X is the consumption of the agricultural good, c_Z is the consumption of the industrial good, and b is bequests which are modelled as a “warm glow” from giving. In the early part of the paper we will neglect b , in effect setting $\theta = 0$ as we will keep the distribution of wealth fixed. Hence, optimal consumption of each good is a linear function of lifetime income, Y :

$$c_X = \delta Y, \quad c_Z = (1 - \delta) Y,$$

There are no credit markets, so agents cannot borrow. Hence, each agent has to finance any up-front costs only from his wealth, which comes from bequests. When fees must be paid up front, agents with high ability but low initial wealth are barred from becoming skilled. In the apprenticeship system, credit constraints are less binding. In this manner we explore the implications of apprenticeship as a way of relaxing credit constraints in short-run and long-run settings. Next we set up the problems under the two systems.

⁹The master may use all this time himself or sell it on the open market for w . We allow w^A to be negative so as to allow for the possibility that being a master is so lucrative that workers are willing to pay for the privilege of being an apprentice.

¹⁰If the training technologies in the two systems are the same then $\beta^A = \beta^C = \beta$, which will serve as our base case.

2.1 The Apprenticeship System

Since there are constant returns to scale, we can always think of the masters (skilled labor) as running everything and interpret their returns as the earnings of skilled labor.¹¹ Each master chooses to hire unskilled labor and/or train unskilled workers, who, in return, work part of their time at below market wages and/or pay to be trained. Each apprentice spends $(1 - \beta^A)$ of his time studying and works the rest of the time as an unskilled worker. Unskilled workers are paid wage w_t and apprentices are paid w_t^A , which may be positive or negative. If it is positive then we say that credit constraints do not operate as anyone who wishes to become an apprentice can do so. If w_t^A is negative then unskilled workers must pay masters. Only those who have sufficient initial wealth to do so have the option of becoming apprentices and we say that credit constraints operate.

In each period of time there are M_t masters who are the successful trainees from the last period. Each master chooses how many apprentices to take on, K_t , and how many unskilled workers to hire, u_t , taking into account w_t^A , w_t , and p_t . Profits are

$$\pi_t^A = p_t (G(K_t))^\alpha (\beta^A K_t + u_t)^{1-\alpha} - w_t u_t - \beta^A w_t^A K_t. \quad (1)$$

It is convenient to transform the variables from u_t and K_t to U_t and S_t where $u_t + \beta^A K_t = U_t$ and $S_t = 1 - AK_t$. Doing so and substituting in (1) yields the following profit-maximization problem

$$\max_{U_t, S_t} \pi_t^A = p_t S_t^\alpha U_t^{1-\alpha} - w_t U_t - \frac{\beta^A}{A} (w_t - w_t^A) S_t + \frac{\beta^A}{A} (w_t - w_t^A) \quad (2)$$

This expression has an intuitive interpretation. Think of a master as selling all his time on the market. A unit of master's time allows him to claim $\frac{\beta^A}{A} (w_t - w_t^A)$ from training apprentices. Then he buys back the time, S_t , he needs to produce the good. Hence, the earnings of a master equal the value of output less the cost of hiring all the unskilled labor used, less the opportunity cost of his labor used in production, plus the value of his stock of skilled labor.

¹¹Increasing returns, when they occur, are external in their nature.

The first order conditions with respect to U_t and S_t are:

$$(1 - \alpha) p_t \left(\frac{S_t}{U_t} \right)^\alpha = w_t, \quad (3)$$

$$\alpha p_t \left(\frac{S_t}{U_t} \right)^{\alpha-1} = \frac{\beta^A}{A} (w_t - w_t^A). \quad (4)$$

The marginal value product of an unskilled worker is equated to his wage.¹² Similarly, the value of an additional unit of skilled labor is equated to its opportunity cost.

Using (3) and (4) we get the demand for unskilled relative to skilled labor for each master to be equal to

$$\frac{U_t}{S_t} = \frac{(1 - \alpha) \beta^A (w_t - w_t^A)}{\alpha A w_t} \quad (5)$$

Since product is exhausted due to constant returns to scale and perfect competition, the first three terms in (2) cancel so that the maximized value of profits equals

$$\pi_t^{A*} = \frac{\beta^A}{A} (w_t - w_t^A). \quad (6)$$

Note that this is exactly the opportunity cost of the time a master is endowed with. Had he chosen to just train workers, which he could do without facing diminishing returns, and sell the value of the time they offered as payment at market prices this is exactly what he would have obtained.

2.2 The Pay Up Front System (PUF)

Trainees pay the master tuition, w_t^c , and spend $(1 - \beta^C)$ hours of their time learning skills. In addition, they can work as unskilled workers β^C hours of their time and earn $\beta^C w_t$. A master hires unskilled labor directly and undertakes training of unskilled workers. His profits are

$$\pi_t^C = p_t (G(K_t))^\alpha U_t^{1-\alpha} - w_t U_t + w_t^c K_t \quad (7)$$

¹²Note that we do not have to worry about corner solutions since all inputs are essential in production. In addition, note that the unskilled labor hired from the market can be negative: this just means that a master does not use all the apprentice labor he is entitled to, but sells it for w_t per unit, while paying w_t^A .

Again, it is convenient to substitute for $G(K_t) = 1 - AK_t = S_t$ in (7). Each master solves the following profit-maximization problem:

$$\max_{S_t, U_t} \pi_t^C = p_t S_t^\alpha U_t^{1-\alpha} - w_t U_t - \frac{w_t^c}{A} S_t + \frac{w_t^c}{A} \quad (8)$$

The first order conditions are:

$$(1 - \alpha) p_t \left(\frac{S_t}{U_t} \right)^\alpha = w_t \quad (9)$$

$$\alpha A p_t \left(\frac{S_t}{U_t} \right)^{\alpha-1} = w_t^c. \quad (10)$$

Using (9) and (10) we get the demand for unskilled relative to skilled labor for each master to be

$$\frac{U_t}{S_t} = \frac{(1 - \alpha) w_t^c}{\alpha A w_t} \quad (11)$$

The maximized value of each master's profits is his earnings. Think of the master selling all his skills as a trainer on the market, and buying back his use of S_t and U_t . Since there are constant returns to scale, product is exhausted, and the first three terms in (8) cancel so that the maximized value of profits equals the opportunity cost of the time the master is endowed with

$$\pi_t^{C*} = \frac{w_t^c}{A}. \quad (12)$$

3 Autarky Equilibrium

3.1 Static Autarky Equilibrium

In this section we analyze the autarky equilibrium in each period t . First, we describe equilibrium under the apprenticeship system and then under the pay up front system. Then we argue that if the two coexist, it is equivalent to having the apprentice system as it dominates.

3.1.1 The Apprenticeship System

An equilibrium in period t is characterized by a vector of prices (p_t, w_t^A, w_t) , where p_t is the price of the industrial good, w_t^A is the wage of the apprentice, and w_t is the wage of unskilled worker. The proportion of agents who become apprentices is denoted by $(1 - \tilde{\gamma}_t^A)$.

Since both goods are essential in the consumption, both goods must be produced in autarky. Therefore, the wage of unskilled workers is equal to the price of the agricultural good, i.e., $w_t = 1$. The equilibrium price, p_t , is determined from the condition

$$c\left(\frac{\beta^A}{A}(1 - w_t^A), 1\right) = p_t$$

where $c(\cdot)$ is the unit cost function. The opportunity cost of a unit of skilled labor is $\pi_t^{A*} = \frac{\beta^A}{A}(1 - w_t^A)$, while the unskilled wage is unity. This pins down the price for a given w_t^A . Using the fact that the production function has the Cobb-Douglas form, we see

$$p_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \left(\frac{\beta^A (1 - w_t^A)}{A}\right)^\alpha = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} (\pi_t^{A*})^\alpha \quad (13)$$

Note that as price p_t rises, so does π_t^{A*} which, in turn, implies that w_t^A falls. For a high enough price, w_t^A even turns negative so that workers must pay up front to become apprentices. Even under the apprenticeship system credit constraints become binding when p_t is high enough, i.e., when

$$p_t > p_2 = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \left(\frac{\beta^A}{A}\right)^\alpha.$$

Occupational Choice A young agent in period t , with probability of being talented γ and inherited wealth y , has two options. The first is to work both periods of his life as unskilled worker. This gives a lifetime income of 2. The second option is to invest in skills hoping to become a master. In this case his first period income equals the apprentice's wage $\beta^A w_t^A$. In the second period, with probability γ he earns the master's profit, π_{t+1}^{A*} , and with probability $(1 - \gamma)$ he receives the wage of unskilled worker, $w_t = 1$. The expected lifetime income in this case equals $\beta^A w_t^A + \gamma E_t \pi_{t+1}^{A*} + (1 - \gamma)$. Let $\tilde{\gamma}_t^A$ denote the agent who

is indifferent between these two options. Then $\tilde{\gamma}_t^A$ is determined from

$$2 = \beta^A w_t^A + \gamma E_t \pi_{t+1}^{A*} + (1 - \gamma),$$

or

$$\begin{aligned} \tilde{\gamma}_t^A &= \min \left\{ \frac{1 - \beta^A w_t^A}{E_t \pi_{t+1}^{A*} - 1}, 1 \right\} \\ &= \min \left\{ \frac{1 - \beta^A + A \pi_t^{A*}}{E_t \pi_{t+1}^{A*} - 1}, 1 \right\} \end{aligned} \quad (14)$$

where the second equality follows from (6).

Agents with a low probability of being talented, $\gamma \in [0, \tilde{\gamma}_t^A]$, work both periods of their life as unskilled workers, while agents with sufficiently high probability, $\gamma \in [\tilde{\gamma}_t^A, 1]$, choose to become apprentices. As expected, higher profits for masters today, i.e., lower wages for apprentices today, raise $\tilde{\gamma}_t^A$ – fewer agents become apprentices. If the expected profits of masters tomorrow rise, i.e., the expected apprentice’s wage tomorrow falls, then $\tilde{\gamma}_t^A$ falls and more agents become apprentices today. Thus

$$\frac{\partial \tilde{\gamma}_t^A}{\partial \pi_t^{A*}} > 0, \quad \frac{\partial \tilde{\gamma}_t^A}{\partial E_t \pi_{t+1}^{A*}} < 0.$$

Equilibrium Since each agent spends a fixed share of his income on the consumption of each good, the relative demand for the industrial good is equal to

$$RD_t = \frac{Z_t^D}{X_t^D} = \frac{(1 - \delta)}{\delta p_t}, \quad (15)$$

where X_t^D and Z_t^D are the aggregate demands for the agricultural and the industrial good respectively.

The derivation of supply of X and Z is slightly more complicated. Take expected profits in the next period, $E_t \pi_{t+1}^{A*}$, and the cutoff level in the previous period, $\tilde{\gamma}_{t-1}^A$, as given. For a particular price, p_t , how do we get supply of the industrial and agricultural goods? Note that for each price, we get the return to skilled labor, π_t^{A*} , and hence the wage for apprentices, from the condition (13) that price equals cost for the industrial good. The level of π_t^{A*} , in turn, determines $\tilde{\gamma}_t^A$ as is apparent from (14). The cutoff level, $\tilde{\gamma}_t^A$, and in

the credit constrained case, the level of the apprentice's wage, w_t^A , together determine how many agents are willing and able to become apprentices (the remainder becomes unskilled workers) and hence the time needed to train them! Removing the skilled labor needed for training from the stock of masters gives the supply of skilled labor available for production, L_t^S . Adding those who choose to become unskilled workers today to the inherited stock of unskilled workers and unskilled labor supplied by apprentices, gives the supply of unskilled labor available for production, L_t^U .

This, in effect, gives the size of the Rybczynski box depicted in Figure 1, where the supply of skilled labor is on horizontal axis and the supply of unskilled labor is on vertical axis. Note that in Figure 1, L_i^S and L_i^U denote the skilled and unskilled labor used in the production of good $i \in \{Z, X\}$. Now we can use the Rybczynski box to get the supply of Z and X .

From (5) we know the relative demand for skilled and unskilled labor in Z , i.e., we know the input mix used in production of Z :

$$\frac{U_t}{S_t} = \frac{1 - \alpha}{\alpha} \pi_t^{A*}$$

Therefore, the ratio of unskilled and skilled labor used in the production of the industrial good must lie on the ray O_zP , which has slope equal to $\frac{1 - \alpha}{\alpha} \pi_t^{A*}$, and whose length is proportional to the supply of Z . Since only unskilled labor is needed in the production of agricultural good, the division of labor between the two goods is given by the intersection of O_zP and O_xA , which is the point P . Hence, the supply of good X is given by O_xP .

More formally, using (3) we get the supply of good Z to be equal to

$$\begin{aligned} Z_t^S &= M_t (S_t)^\alpha (U_t)^{1-\alpha} \\ &= M_t S_t \left(\frac{U_t}{S_t} \right)^{1-\alpha} \\ &= L_Z^S \left(\frac{1 - \alpha}{\alpha} \pi_t^{A*} \right)^{1-\alpha}, \end{aligned} \tag{16}$$

where M_t is the number of masters in period t and $L_Z^S = M_t S_t$. Using this we get that the supply of good Z equals O_zA multiplied by $\left(\frac{1 - \alpha}{\alpha} \pi_t^{A*} \right)^{1-\alpha}$. Then, the relative supply

equals

$$\frac{O_z A}{O_x P} \left(\frac{1 - \alpha}{\alpha} \pi_t^{A*} \right)^{1-\alpha}.$$

Hence, every price p_t corresponds to a point on the relative supply curve as depicted in Figure 2 – moving p_t traces out the relative supply curve.

In Figure 2, price p_1 corresponds to $\pi_t^{A*} = 1$. When price is below p_1 , the return to skilled labor is less than 1 (the wage of unskilled worker), as a result the option of working as unskilled worker is more profitable than the option of being a master. For all prices below p_1 the supply of the industrial good, and hence the relative supply, is zero. At price p_1 skilled workers are indifferent between two options so that relative supply is horizontal. Price p_2 corresponds to $\pi_t^{A*} = \frac{\beta^A}{A}$ and, as we can see from (6), to $w_t^A = 0$. For all prices below p_2 the apprentice's wage is positive, so credit constraints do not operate and the relative supply is denoted by RS_t^{ncc} . For prices above p_2 the apprentice's wage is negative, i.e., workers pay to become apprentices. In this event, agents are subject to credit constraints. Relative supply in this region is denoted by RS_t^{cc} . When price exceeds p_3 the apprentice's wage is so low that the option of investing in skills is dominated and there are no apprentices ($\tilde{\gamma}_t^A = 1$). Finally, at p_4 all unskilled labor is used in the production of the industrial good, the supply of the agricultural good is zero and the relative supply goes to infinity.

Next, we turn to the shape of the relative supply curve and the nature of static equilibrium. An equilibrium in which the apprentice's wage is positive is a non-credit-constrained (NCC) equilibrium. An equilibrium in which the apprentice's wage is negative is a credit-constrained (CC) equilibrium.

Proposition 1 *Under the apprenticeship system, if the number of masters in period t is small enough, i.e., $M_t \leq \tilde{M}^A = \frac{2A}{(1 + A - \beta^A)}$, then relative supply is increasing in price. The static equilibrium could be credit-constrained or not. If there are enough masters, i.e., $M_t > \tilde{M}^A$, then relative supply need not be increasing in price. Multiple equilibria are possible, but there is at most one non-credit-constrained equilibrium.*

We relegate the formal proof to the Appendix and focus on the intuition behind this result here. Suppose that the price increases. This results in a higher return to skilled labor and in a lower apprentice's wage.¹³ As a result, the input mix in production moves away from skilled labor. In a static setting, the fall in the apprentice's wage makes investing in skills less profitable and $\tilde{\gamma}_t^A$ increases. Hence, the supply of unskilled labor in period t rises. As fewer agents wish to be apprentices, masters spend less time training them and the supply of skilled labor available for production increases. Note that both skilled and unskilled labor available for production rise so that their relative availability may rise or fall.

The effects on relative supply can be decomposed into two parts. First, the part due to change in the availability of skilled relative to unskilled labor for production purposes. An increase in the relative skilled labor availability raises Z/X , the relative supply of the skilled labor intensive good, a la Rybczynski. A decrease in the relative skilled labor availability does the opposite. Second, the part due to factor price changes and hence input mix changes. An increase in p moves the input mix towards unskilled labor, the ray in Figure 3 moves from P'' to P' . For given factor supplies, this raises the relative supply of the skill-intensive good. This is the basis of the usual positive supply response in general equilibrium.

In Figure 3(a) both effects raise relative supply of Z . When there are few masters, $M_t > \tilde{M}^A$, then the supply of skilled labor is relatively small to begin with. As a result, any given increase in $\tilde{\gamma}_t^A$ release what amounts to a large percentage increase in the supply of skilled labor so that skilled labor becomes relatively more abundant, and relative supply of Z rises.

If there are many masters, $M_t > \tilde{M}^A$, as in Figure 3(b), then an increase in price results in an increase in the relative availability of unskilled labor. With many masters, the supply of skilled labor is relatively large. Any change in $\tilde{\gamma}_t^A$ translates into a small percentage increase in the supply of skilled labor and, as a result, skilled labor becomes relatively less

¹³If there are credit constraints, this translates into the fee paid by an apprentice going up.

abundant and the relative supply of Z may fall! In this case the two effects work in opposite directions and relative supply may be downward sloping.

It is shown in the Appendix that when $M_t \leq \tilde{M}^A$, relative demand curve can intersect the relative supply curve at most once: either in its non-credit-constrained part, or in its credit-constrained part. If $M_t > \tilde{M}^A$ there may be multiple equilibria with at most one non-credit-constrained one.¹⁴ Multiple equilibria in this static set-up arise from the interaction of credit constraints and prices. When price is low, so is the return to skilled labor. In this case, the apprentice's wage is high, there are no credit constraints and a large fraction of the population becomes apprentices. Since M_t is large, despite this, there is a lot of skilled labor available for production and output is high. Since price is low, demand is high and this can be an equilibrium. On the other hand, if price is high, so is the return to skilled labor and for this, apprentice's wages are negative. Credit constraints operate and many agents cannot become apprentices. While this does free up some skilled labor for production, masters are abundant and there is an ample supply of unskilled workers. Hence the relative supply of skilled workers is low, as is the relative supply of Z .

Note that not all static equilibria are consistent with steady state: for example, if the intersection occurred at prices above p_3 , $\tilde{\gamma}_t^A = 1$ and nobody would invest in skills. If there are no masters in period $t + 1$, then the return to skilled labor would be infinite which is not consistent with expectations or possible in steady state.

Comparative Statics What is the effect of a decrease in expected profits in period $t + 1$ on period t 's equilibrium price? From (14) we can see that a decrease in the expected profit from becoming a master in the next period increases $\tilde{\gamma}_t^A$ – fewer agents choose to become apprentices. This fall in the number of agents wishing to be apprentices results, as argued above, in an increase in the supply of skilled and unskilled labor. However, in contrast to the effects of a price change, there is no change in the returns to skilled or unskilled labor and hence no effect on the input mix. The effect on relative supply depends only on

¹⁴The reason is that independent of the number of masters, relative supply is monotonic in the absence of credit constraints. Hence, it may intersect relative demand at most once.

whether the supply of unskilled labor increases relatively more or less than the supply of skilled labor.

In Figure 3(a) the supply of skilled labor increases relatively more than the supply of unskilled labor since $M_t < \tilde{M}^A$. As we can see at this given price the relative supply goes up:

$$\frac{A'P'}{Ox'P''} > \frac{AP}{OxP}.$$

So, the relative supply curve shifts out, resulting in a lower equilibrium price. From (13) we can see that as a result the return to skilled labor falls and apprentice's wage rises.

In Figure 3(b) the opposite happens – a decrease in master's profit expected in next period results in a higher percentage increase in the supply of unskilled labor relative to that of skilled labor. As a result the relative supply decreases:

$$\frac{A'P''}{Ox'P'} < \frac{AP}{OxP}$$

As the relative supply curve shifts in, the equilibrium price in period t rises, resulting in higher return to skilled labor and lower apprentice's wage.

Lemma 1 summarizes these results.

Lemma 1 *If there are few masters in period t , i.e., $M_t \leq \tilde{M}^A$, then a decrease in expected earnings of a master in the subsequent period results in a lower equilibrium price in the current period, as well as a lower return to skilled labor and a higher apprentice's wage. If $M_t > \tilde{M}^A$, then the opposite occurs.*¹⁵

3.1.2 Pay Up Front System

An equilibrium in period t is characterized by a vector of prices (p_t, w_t^C, w_t) , where p_t is the price of the industrial good, w_t^C is the tuition trainees pay to masters, and w_t is the wage of unskilled worker. As shown below, agents with abilities above the cutoff level, $\tilde{\gamma}^C$, choose to get trained, so that $(1 - \tilde{\gamma}_t^C)$ is the proportion of agents who become trainees.

As in the previous section, the wage of unskilled workers is equal to the price of the agricultural good, i.e., $w_t = 1$.

¹⁵It will become obvious that this lemma also holds for the PUF system.

Occupational Choice Let $\tilde{\gamma}_t^C$ denote the ability of the agent who is indifferent between the two options: to work both periods as unskilled worker or to invest in skills hoping to become a master in the second period. The first option gives a total lifetime income of 2. If agent chooses the second option, then his first period income equals the wage of unskilled worker multiplied by β^C units of time less the tuition. In the second period, with probability γ he earns the master's profit, π_{t+1}^{C*} , and with probability $(1 - \gamma)$ he receives the wage of unskilled worker, $w_t = 1$. The expected lifetime income in this case equals $\beta^C - w_t^C + \gamma E_t \pi_{t+1}^{C*} + (1 - \gamma)$. Then $\tilde{\gamma}_t^C$ is determined from

$$2 = \beta^C - w_t^C + \gamma E_t \pi_{t+1}^{C*} + (1 - \gamma),$$

or

$$\begin{aligned} \tilde{\gamma}_t^C &= \min \left\{ \frac{1 - \beta^C + w_t^C}{E_t \pi_{t+1}^{C*} - 1}, 1 \right\} \\ &= \min \left\{ \frac{1 - \beta^C + A \pi_t^{C*}}{E_t \pi_{t+1}^{C*} - 1}, 1 \right\} \end{aligned} \quad (17)$$

where the second equality follows from (12).

Comparing (14) and (17) we can see that in both apprenticeship and pay up front systems the proportion of agents wanting to invest in skills is given by the same function. As in the apprenticeship system, higher profit for masters today is associated with a lower payoff for trainees, whether through a reduction in the apprentice's wage or an increase in the cost of tuition, which raises $\tilde{\gamma}_t^C$ – fewer agents become trainees. Similarly, if the expected profits of masters tomorrow rise, then $\tilde{\gamma}_t^C$ falls and more agents want to become trainees today. Thus,

$$\frac{\partial \tilde{\gamma}_t^C}{\partial \pi_t^{C*}} > 0, \quad \frac{\partial \tilde{\gamma}_t^C}{\partial E_t \pi_{t+1}^{C*}} < 0.$$

Equilibrium As in the previous section, the relative demand is given by (15). We can use the approach used in the analysis of the apprenticeship system to derive the relative supply here as well. It is easy to check that when there are few masters, the relative supply under the pay up front system is upward-sloping, but if the number of masters is large then

the relative supply can be downward-sloping. Moreover, note that the comparative statics results of Lemma 1 have their analogue for the PUF system.

Proposition 2 *Under the PUF system, if $M_t \leq \tilde{M}^C = \frac{2A}{(1+A-\beta^C)}$ then relative supply is increasing in price and there is a unique equilibrium. If $M_t > \tilde{M}^C$, then relative supply need not be increasing in price and there may be multiple equilibria.*

Suppose that the training technologies are the same in the two systems: $\beta^A = \beta^C = \beta$.¹⁶ What can we say about the relative position of relative supply curves under the pay up front system and under the apprenticeship system? As before, from the condition that the price of the good Z equals cost, we get the master' profit corresponding to any price:

$$p_t = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} (\pi_t^{C*})^\alpha$$

Then, from (17) we get $\tilde{\gamma}_t^C$. These $\tilde{\gamma}_t^C$ and π_t^{C*} determine the available supply of skilled and unskilled labor in a way that is very similar to the apprenticeship system when agents are subject to credit constraints. For any price there is a common level of earnings for skilled labor, π_t^* , under both systems, and corresponding to this, an education cost – the tuition fee, w_t^C , in the pay up front system or the implicit price of $\beta(1 - w_t^A)$ under the apprenticeship system. All of the education cost is paid up front in the PUF system, while only $\max\{0, -\beta w_t^A\}$ is paid up front in the apprenticeship system, so that more is always paid up front under the PUF system. Hence, at any given p_t (or, at any π_t^* corresponding to this price) a larger proportion of agents are credit-constrained under the PUF system.

At any given price p_t , both systems have the same cutoff level for ability. However, since credit constraints are stricter in the PUF system, fewer agents actually become trainees, the rest of those who want to do so cannot and remain unskilled so that the supply of unskilled workers is larger at any given p under the PUF system. The smaller number of trainees under the PUF system requires less skilled labor to train them. As a result, the skilled labor available for production is also larger under the PUF system. What can we say

¹⁶Note that in this case $\tilde{M}^A = \tilde{M}^C = \tilde{M}$.

about relative availability? As before, when there are few masters the relative availability of skilled labor is higher under the PUF system, while when there are many masters the relative availability of skilled labor is lower under the PUF system. Hence, the effect on relative supply follows from the relative factor availability under the two systems.

If there are few masters, $M \leq \tilde{M}$, then at any given p , the relative supply of the skill-intensive good is more than that under the apprenticeship system. If there are many masters, $M > \tilde{M}$, it is less than that in the apprenticeship system. This is depicted in Figures 4(a) and 4(b). In Figure 4(a) the number of masters is small, and, as a result, the relative supply curves for both systems are upward-sloping. At any price below p_3 (so that the proportion of agents who invest in skills is positive) the relative supply curve under the PUF system is to the right of the relative supply curve under the apprenticeship system. When price is above p_3 there are no agents who invest in skills and the two curves coincide. Therefore, there is a unique equilibrium in each system, and the equilibrium price under the PUF system is lower than the equilibrium price under the apprenticeship system. Since the equilibrium price is lower under the PUF system, we cannot conclude that the tuition in the PUF system is higher than the full fee under the apprenticeship system.

In Figure 4(b) the number of masters is large. The relative supply curve under the PUF system is to the left of the relative supply curve under the apprenticeship system. And, as a result, the equilibrium price in the PUF system is higher. Moreover, as the price is higher and so is the full fee and hence the up-front education fee under the PUF system.

Proposition 3 summarizes these results.

Proposition 3 *If the training technologies are the same in the two systems then at any given p , relative supply of the skill-intensive good is higher under the PUF system than that under the apprenticeship system as long as $M \leq \tilde{M}$. As a result, the equilibrium price is lower than the equilibrium price under the apprenticeship system. If $M > \tilde{M}$, relative supply of the skill-intensive good is lower under the PUF system than that under the apprenticeship system. Consequently, the equilibrium price under the PUF system is higher than the equilibrium price under the apprenticeship system.*

3.1.3 Coexistence of Both Systems

Next we argue that when both systems co-exist, the outcome is exactly what it would have been under the apprenticeship system so that we are not losing anything by studying the two in isolation. How can this be? Suppose both systems coexisted and the cutoff levels (the $\tilde{\gamma}$'s) for the two differed.¹⁷ Then as both marginal agents have the same payoff from not being educated and the same expected payoff from being skilled tomorrow

$$\begin{aligned} 2 &= \beta w_t^A + \tilde{\gamma}^A E_t \pi_{t+1}^* + (1 - \tilde{\gamma}^A) \\ &= \beta - w_t^C + \tilde{\gamma}^C E_t \pi_{t+1}^* + (1 - \tilde{\gamma}^C). \end{aligned}$$

Since the high wealth agents have all the options open to the low wealth ones, the only possible difference is that the wealthy have options that are not open to low wealth agents. In other words, the full cost of education for the wealthy is less than that for the poor or $w_t^C < \beta(1 - w_t^A)$ so that

$$(\tilde{\gamma}^A - \tilde{\gamma}^C) E_t \pi_{t+1}^* > (\tilde{\gamma}^A - \tilde{\gamma}^C)$$

or $\tilde{\gamma}^A > \tilde{\gamma}^C$: some wealthy agents can become trainees while less wealthy ones of the same ability cannot.

However, a skilled worker can teach under the PUF system or take apprentices. Since his maximized income under the former is $\frac{w_t^C}{A}$ and under the latter is $\frac{\beta(1 - w_t^A)}{A}$, all skilled agents will take apprentices and only the the apprentice system will exist.

3.2 Steady State Autarky Equilibrium

In this section we solve for autarky steady state equilibrium. First, we describe steady state equilibrium under the apprenticeship system, and then under the PUF system.

¹⁷If they are the same, it is equivalent to the apprentice system.

3.2.1 Apprenticeship System

Steady state equilibrium is characterized by a vector of prices (p, w^A, w) , where p is the price of the industrial good, w^A is the wage of the apprentice, and w is the wage of unskilled worker. The proportion of agents who become apprentices is denoted by $(1 - \tilde{\gamma}^A)$.

As in the previous section the wage of unskilled workers is equal to the price of the agricultural good, i.e., $w = 1$.

Occupational Choice Let $\tilde{\gamma}^A$ denote the ability of the agent who is indifferent between the two options: to work both periods as unskilled worker or to invest in skills hoping to become a master in the second period. The first option gives a total lifetime income of 2. If agent chooses the second option, then his first period income equals the apprentice's wage rate $\beta^A w^A$. In the second period, with probability γ he earns the master's profit, π^{A*} , with probability $(1 - \gamma)$ he receives the wage of unskilled worker, $w = 1$. The lifetime income in this case equals $\beta^A w^A + \gamma \pi^{A*} + (1 - \gamma)$. Then $\tilde{\gamma}^A$ is determined from

$$2 = \beta^A w^A + \gamma \pi^{A*} + (1 - \gamma)$$

Using $\pi^{A*} = \frac{\beta^A}{A}(1 - w_t^A)$ we have

$$\begin{aligned} \tilde{\gamma}^A &= \min \left\{ \frac{(1 - \beta^A) + A\pi^{A*}}{\pi^{A*} - 1}, 1 \right\} \\ &= \begin{cases} 1, & \text{if } \pi^{A*} \leq \frac{2 - \beta^A}{1 - A} \\ \frac{(1 - \beta^A) + A\pi^{A*}}{\pi^{A*} - 1}, & \text{if } \pi^{A*} > \frac{2 - \beta^A}{1 - A} \end{cases} \end{aligned} \quad (18)$$

Note that a higher return to skilled labor means a lower apprentice's wage w^A , hence there is a trade-off between a higher earnings tomorrow and a lower wage today. But since a fall in w^A raises the earnings of a master in the next period by more than it reduces the wage of an apprentice in the current period, a higher π^{A*} leads to a lower $\tilde{\gamma}^A$: more agents become apprentices. Thus,

$$\frac{\partial \tilde{\gamma}^A}{\partial \pi^{A*}} < 0.$$

Note that the relationship between $\tilde{\gamma}^A$ and the return to skilled labor has the opposite sign from that in the static set-up. In steady state the return to the option of investing in skills is increasing in π^{A*} as explained above. In the static set-up, the master's earnings expected in the next period are fixed and agents take into account only the apprentice's wage in the current period, and as a higher π_t^{A*} means a lower apprentice's wage, the return to the option of investing in skills is decreasing in π^{A*} .

Note also that for low levels of profit ($\pi^{A*} \leq \frac{2 - \beta^A}{1 - A}$) the option of investing in skills is dominated and all agents choose to work as unskilled labor. What happens if the return to skilled labor becomes large? Using $\pi^{A*} = \frac{\beta^A}{A}(1 - w_t^A)$ we can rewrite the return to the option of investing in skills as $\beta^A + (1 - \gamma) + \pi^{A*}(\gamma - A)$. Then, for $\gamma < A$ this return is clearly less than the lifetime income of unskilled worker. Hence, all agents with $\gamma < A$ choose not to invest in skills. As profit goes to infinity ($\pi^{A*} \rightarrow \infty$) the proportion A of agents work as unskilled labor and $(1 - A)$ become apprentices, i.e., $\tilde{\gamma}^A \rightarrow A$. Therefore, in steady state the proportion of agents who become apprentices never exceeds $1 - A$.

Equilibrium As in the previous section, the relative demand equals

$$RD = \frac{Z^d}{X^d} = \frac{(1 - \delta)}{\delta p}$$

We can use the approach used in the static set-up to derive the relative supply here as well. Take any price p . From the condition that price equals cost for the industrial good we get the return to skilled labor, π^{A*} :

$$p = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} (\pi^{A*})^\alpha$$

Then, the level π^{A*} uniquely determines $\tilde{\gamma}^A$ as is apparent from (18). The cutoff level $\tilde{\gamma}^A$ and, in the credit constrained case, the level of the apprentice's wage, together determine the available supply of skilled and unskilled labor. Note the following difference between steady state and static supply of skilled and unskilled labor. In static set-up the number of masters as well as the amount of unskilled labor from the previous period is exogenous, while in steady state these are not fixed but endogenous and determined by $\tilde{\gamma}^A$.

Next, we can use the Rybczynski box to derive the supply of good X and good Z . As in the previous section every p gives π^{A*} , γ^A , X^S and Z^S . In other words, every p corresponds to a point on the relative supply curve as depicted in Figure 5.

In Figure 5 π_1 denotes the return to skilled labor such that $\gamma^A = 1$. Price p_1 corresponds to this π_1 . For all prices below p_1 nobody invests in skills, so production of the industrial good is zero. Hence, relative supply is zero as well. Price p_2 corresponds to $\pi^{A*} = \frac{\beta^A}{A}$ and, as in the static set-up, to $w^A = 0$. For all prices below p_2 the apprentice's wage is positive, so that credit constraints are not binding and relative supply is denoted by RS^{ncc} . For prices above p_2 the apprentice's wage is negative, and agents are subject to credit constraints. Relative supply in this region is denoted by RS^{cc} . Finally, at p_3 all unskilled labor is used in the production of the industrial good, the supply of the agricultural good is zero and relative supply goes to infinity.

Next, we turn to the shape of the relative supply curve and the nature of steady state equilibrium. First, we prove that when credit constraints are not operating the relative supply curve is upward-sloping. Suppose that price rises. Then the return to skilled labor rises as well, and this results in lower $\tilde{\gamma}^A$: more agents choose to invest in skills at a higher price. Since more agents become apprentices, the supply of unskilled labor falls. What can we say about the supply of skilled labor? Subtracting the skilled labor needed for training from the stock of masters gives the supply of skilled labor available for production:

$$\begin{aligned} L^S &= M - AK \\ &= \frac{1}{2} \left(1 - (\tilde{\gamma}^A)^2 \right) - A (1 - \tilde{\gamma}^A), \end{aligned}$$

where $M = \frac{1}{2} \left(1 - (\tilde{\gamma}^A)^2 \right)$ is the number of masters and $K = 1 - \tilde{\gamma}^A$ is the number of apprentices. As price rises, the number of apprentices increases, as does the number of masters. The total effect on the supply of skilled labor is positive – L^S increases with the price since¹⁸:

$$\frac{dL^S}{dp} = \frac{d\tilde{\gamma}^A}{dp} (A - \tilde{\gamma}^A) > 0$$

¹⁸Agents with $\gamma < A$ will not invest in skills so that $\tilde{\gamma}^A > A$.

Hence, a higher price leads to a higher supply of skilled labor available for production of good Z and a lower aggregate supply of unskilled labor.

What about the output of the skill-intensive good? The supply of the skill-intensive good is

$$\begin{aligned} Z^S &= (L^S)^\alpha (L_z^U)^{1-\alpha} \\ &= L^S \left(\frac{1-\alpha}{\alpha} \pi^{A*} \right)^\alpha \end{aligned}$$

from (16).

Then, as L^S and π^{A*} both increase, we can conclude that the supply of good Z increases as well. Since the amount of unskilled labor used in the production of skill-intensive good equals $L_z^U = \left(\frac{1-\alpha}{\alpha} \pi^{A*} \right) L^S$, it is clear that L_z^U goes up. As the total available unskilled labor decreases and the amount of unskilled labor used in the production of good Z increases at this higher price, the amount of unskilled labor available for production of good X decreases, so that supply of the agricultural good falls. Thus, we can conclude that when credit constraints are not operating, relative supply is increasing in price.

When agents are subject to credit constraints, the effects of an increase in price on the supply of unskilled and skilled labor are ambiguous and depend on the distribution of wealth. If a higher price results in unskilled labor becoming relatively more abundant, then the relative supply of the skill-intensive good may fall. If, for example, there are many agents who become credit-constrained at this higher price, then a large proportion of agents who invested in skills at lower price cannot afford to do so. Hence, the supply of unskilled labor rises, and the supply of skilled labor available for production of good Z falls. As a result, the relative supply of good Z may be lower at this higher price. Thus, when credit constraints operate, the shape of the relative supply curve depends on the distribution of wealth and can be either upward-sloping or downward-sloping.

As relative supply need not be monotonic multiple steady state equilibria may arise. When price is low, so is the return to skilled labor. In this case, the apprentice's wage is positive, credit constraints are not binding and a large fraction of the population become apprentices. There is a lot of skilled labor available for production and output is high. At

this low price, demand is high and this can be an equilibrium. On the other hand, if price is high, so is the return to skilled labor and, as a result, apprentice's wage is negative. Credit constraints operate and many agents cannot become apprentices. This results in a relatively small supply of skilled labor and an ample supply of unskilled workers. Hence the relative supply of skilled workers is low, as is the relative supply of the skill-intensive good.

Proposition 4 summarizes these results.

Proposition 4 *Under the apprenticeship system, if $p \leq p_2$ then credit constraints are not binding and steady state relative supply is increasing in price. If $p > p_2$ then credit constraints are binding, steady state relative supply need not be increasing in price, and multiple steady state equilibria may exist.*

3.2.2 PUF System

Steady state equilibrium is characterized by a vector of prices (p, w^C, w) , where p is the price of the industrial good, w^C is the tuition trainees pay to masters, and w is the wage of unskilled worker. The proportion of agents who become trainees is denoted by $(1 - \tilde{\gamma}^C)$. As before, the wage of unskilled workers is equal to the price of the agricultural good.

Occupational Choice Let $\tilde{\gamma}^C$ denote the ability of the agent who is indifferent between the two options: to work both periods of his life as unskilled worker or get training. The first option gives a total lifetime income of 2. If the agent chooses the second option, then his lifetime income equals $-w^C + \beta^C + \gamma\pi^{C*} + (1 - \gamma)$. Thus $\tilde{\gamma}^C$ is determined from

$$2 = -w^C + \beta^C + \gamma\pi^{C*} + (1 - \gamma)$$

Using $\pi^{C*} = \frac{w^C}{A}$ we have

$$\tilde{\gamma}^C = \min \left\{ \frac{1 - \beta^C + w^C}{\pi^{C*} - 1}, 1 \right\} = \begin{cases} 1, & \text{if } \pi^{C*} \leq \frac{2 - \beta^C}{1 - A} \\ \frac{(1 - \beta^C) + A\pi^{C*}}{\pi^{C*} - 1}, & \text{if } \pi^{C*} > \frac{2 - \beta^C}{1 - A} \end{cases} \quad (19)$$

Comparing (18) and (19) we can see that in both apprenticeship and PUF systems the proportion of agents wanting to invest in skills in steady state is given by the same function. As in the apprenticeship system, a higher return to skilled labor decreases $\tilde{\gamma}^C$ – more agents want to become trainees.

Equilibrium We can use the approach used in the analysis of steady state equilibrium under the apprenticeship system to derive the relative supply here as well. Since under the PUF system, credit constraints operate at all prices, we can conclude that the shape of the relative supply curve depends on the distribution of wealth and can be either upward-sloping or downward-sloping. There may or may not be multiple equilibria in steady state: a key determinant again is the distribution of wealth.

Proposition 5 *Under the PUF system credit constraints are always binding. Hence, steady state relative supply need not be increasing in price. Multiple steady state equilibria may exist.*

Suppose that the two training technologies are the same: $\beta^A = \beta^C = \beta$. What can we say about the relative position of steady state relative supply curves under the PUF system and under the apprenticeship system? At any given price, both systems have the same cutoff level of ability. However, since credit constraints are stricter in the PUF system, fewer agents can become trainees, the rest of those who want to do so cannot, and remain unskilled so that the supply of unskilled workers is larger at any given price under the PUF system. The smaller number of agents acquiring skills under the PUF system results in a lower supply of skilled labor available for production in the steady state. As a result, at any price the supply of good Z relative to good X is lower under the PUF system. Figure 6 depicts relative supply curves in both system, where relative supply under the PUF system is denoted by RS^C , relative supply under the apprenticeship system when credit constraints are not binding is denoted by RS^{Ancc} and when credit constraints are binding by RS^{Acc} . As depicted, relative supply curve under the PUF system is to the left of the relative supply curve under the apprenticeship system. And, as a result, the equilibrium price under the

PUF system is higher. Moreover, as the price is higher, so is the full fee and hence the up-front education fee under the PUF system. This shift out in relative supply of the skill intensive good when a country has better credit markets, in combination with better credit markets raising the mean wealth, drives the result in Ranjan (2001) that at a given price, the country with better credit markets will export the skill intensive good.

Proposition 6 summarizes these results.

Proposition 6 *If the two training technologies are the same, then at any given p , steady state relative supply is lower under the PUF system than that under the apprenticeship system. As a result, the steady state autarky equilibrium price is higher under the PUF system.*

Note that although there can be multiple equilibria, the autarky steady state prices can be compared. Also, if technology is identical, the country with the weaker credit constraints (the one with the apprenticeship system) will export the skill intensive good. The price under trade must rise for the country with the apprenticeship system but could rise or fall for the country with the PUF system as the equilibrium could jump from a low (high) price one to a high (low) price one.

4 Effects of Trade

Having described the closed economy, we turn to the analysis of the effects of opening the economy up to trade. First consider the welfare effects of trade when the country is small and cannot affect the world price of the industrial good, denoted by p^T .

Suppose that the country opens up to trade in period t . From the condition that the price equals unit cost for the industrial good, the return to skilled labor is determined by the world price p^T . Let π_{t+i}^{T*} denote the return to skilled labor in period $t+i$ under trade. Then for all periods after opening up to trade, profits are constant at level π^{T*} , where

$$\pi^{T*} = \alpha (1 - \alpha)^{\frac{(1-\alpha)}{\alpha}} (p^T)^{\frac{1}{\alpha}}.$$

Similarly, the cut-off level, which depends on master's earnings in the current period and in the next period, is also fixed after first period at:

$$\tilde{\gamma}^T = \frac{(1 - \beta) + A\pi^{T*}}{\pi^{T*} - 1}.$$

In period t the number of masters is inherited from the previous period. Hence, the output levels are determined by the number of masters as well as by the world price. In period $t + 1$, the number of masters equals $\frac{1}{2} \left(1 - (\tilde{\gamma}^T)^2 \right)$ and depends only on p^T , so that the output levels in period $t + 1$ are determined only by the world price. Thus, we can conclude that starting from period $t + 1$ the relative price of the industrial good, the return to skilled labor, the education fee, and the cut-off level are fixed, so that the economy is in steady state equilibrium. Then, for any given world price the relative supply is given by the corresponding point on the steady state relative supply curve constructed in Section 3. Similarly, the point on the relative demand curve corresponding to the world price gives the relative demand for the industrial good in trade steady state equilibrium.

From Section 3 we know that when credit constraints operate, steady state relative supply need not be increasing in price and multiple equilibria may exist under either system. As we show below, non-monotonicity of relative supply may result in trade equilibria where the country ends up importing the industrial good at world prices higher than its autarky price and, as a result, loses from opening up to trade.

Consider the situation depicted in Figure 7. The relative supply is non-monotonic and there are two stable steady state autarky equilibria: E_1^A and E_2^A . At equilibrium E_1^A the price, p_1^A , is low and relative supply of the industrial good is high, while at E_2^A the price, p_2^A , is high and supply is low. Suppose that initially the economy is in autarky equilibrium E_1^A . What happens when the country opens up to trade and faces the world price of the industrial good which is higher than its autarky price? As the price rises, the earnings of a master rise as well, so that the option of investing in skills becomes more attractive. As more agents decide to invest in skills, the up-front education fee goes up. Credit constraints become tighter and more agents are unable to afford education. Consequently, the supply of the skilled labor decreases, resulting in lower output of the industrial good as well as

lower relative supply, denoted by RS^T . As the price goes up, the relative demand for the industrial good, denoted by RD^T , falls. If a significant proportion of agents becomes credit constrained at this higher price, then the decrease in relative supply is considerable and exceeds the decrease in relative demand: opening up to trade allows only few rich agents to invest in skills and, as a result, the supply of the skill-intensive good falls dramatically and cannot satisfy domestic demand at this price. Thus, the country imports the industrial good even though the world price is higher than the autarky price! Since the country loses its comparative advantage in the industrial good and has to import this good at a higher price, the aggregate welfare can be lower with trade.

Proposition 7 *Opening up an economy to trade need not be welfare-improving. Aggregate welfare may fall with trade if a country imports the industrial good at world prices higher than autarky price. If substitution in consumption is small enough and if the value of autarkic output at trade prices exceeds that of the trade output, welfare must fall due to trade.*

Proof. Let $e(P, u)$ be the usual expenditure function. Let p, Q, C, u denote the price, output and consumption vectors while u denotes utility. The superscripts A and T refer to the autarky and trade outcomes. Thus, $Q^T = (X^T, Z^T)$, while $C^T = (c_X^T, c_Z^T)$. We know that

$$\begin{aligned} e(p^A, u^A) &= p^A Q^A = p^A C^A \text{ and } Q^A = C^A \\ e(p^T, u^T) &= p^T Q^T = p^T C^T \\ e(p^A, u^T) &< p^A C^T, \\ e(p^T, u^A) &< p^T C^A = p^T Q^A. \end{aligned}$$

Thus,

$$\begin{aligned} e(p^T, u^T) - e(p^T, u^A) &> p^T C^T - p^T C^A \\ &= p^T Q^T - p^T Q^A. \end{aligned} \tag{20}$$

If there is no substitution in consumption, the first inequality is an equality. Hence, if

$$p^T Q^A > p^T Q^T \tag{21}$$

then $e(p^T, u^T) - e(p^T, u^A) < 0$ so $u^T < u^A$. By continuity, if substitution in consumption is small enough and (21) holds, $u^T < u^A$. ■

The outcome in Figure 7 is depicted in Figure 8 in a way that highlights the condition given above. The production possibility envelope in steady in the presence of credit constraints need not have the usual shape: it can be bowed in as depicted.¹⁹ At p^T , and similarly at p^A , there is a steady state availability of skilled and unskilled labor which defines the PPF at the given price in steady state. These PPFs are tangent to their respective price lines at Q^T and Q^A respectively. The relative demand for good Z exceeds relative supply at p^T but equals it at p^A as shown. If there is no substitution in consumption, demand always lies along the ray OA and if as depicted, (21) holds, then welfare must be lower under trade.

When would we expect trade between countries to reduce welfare in steady state? The intuition is simple: when the price of the skill intensive good rises through trade, credit constraints are tightened and by making the distortion worse, trade reduces welfare. This suggests that if a credit constrained country trade with countries that have a comparative disadvantage in the skill intensive good, whether this is because they are ex ante identical, but ex post different, or whether there are more fundamental differences, opening up to trade could hurt it. This suggests that developed countries, who presumably have a comparative advantage in the skill intensive good, need to ensure access to education to fully reap the gains from trade.

¹⁹It must also lie inside the steady state PPF envelope (which has the usual bowed out shape as in Findlay and Kierzkowski (1983)) in the absence of credit constraints. This makes sense as credit constraints result in inefficiency: the wrong people are trained, which shifts production possibilities inwards.

5 Conclusions

In this paper we develop a model where apprenticeships help overcome credit constraints that limit the ability of agents with heterogeneous abilities and wealth to acquire skills. We show that in the static version of our model, under either system, the response of supply to price depends on the number of skilled agents in the economy. If there are relatively few skilled agents, the normal supply response obtains. However, with many skilled agents, supply can be decreasing in price so that multiple equilibria may exist. In steady state, however, such non monotonicity of supply and multiplicity of equilibrium obtains only in the presence of credit constraints. Since credit constraints are stricter in the PUF system, relative supply of the skill intensive good is always higher, at any given price, under the apprenticeship system. There may or may not be multiple equilibria in steady state: a key determinant is the distribution of wealth. Finally, we show that opening the economy to trade could easily reduce welfare.

What can we say about endogenizing the distribution of wealth in our model? First it is clear that in the absence of credit constraints there will be a non degenerate distribution of wealth in steady state. Intuitively one can see this as follows: suppose that we start out with no bequests. Then there will be only three levels of wealth to begin with corresponding to the bequest of the skilled, the unskilled and the unskilled who tried to become skilled. But as each of this group could end up in each of the three above situations, there will be 3^2 wealth levels in the next iteration. In this manner, all wealth levels get filled in. A better understanding of this distribution, how it changes if the skilled have access to better draws of talent than the unskilled, and its interaction with the general equilibrium of the system are among our plans for extending this paper.

Second, that in the presence of credit constraints, the wealth distribution will become degenerate if even the most talented of the the children of the unskilled cannot afford to become skilled. In this event, being unskilled will be an absorbing state. With heterogeneous ability, and absolute credit constraints as we have, we need to have government provide the up front payments for some fraction of the most able poor to get a non degenerate

distribution of wealth in steady state. A complete understanding of how such scholarships affect equilibrium is the subject of future work.

Appendix

Proof of Proposition 1. For a given price p_t the return to skilled labor is determined from the condition that the price is equal to cost (13). Then, for given expected profits in the next period we get $\tilde{\gamma}_t^A$ corresponding to this π_t^{A*} from (14). For given $\tilde{\gamma}_{t-1}^A$ the available supplies of skilled and unskilled labor when there are no credit constraints equal

$$L_{t,ncc}^S = M_t - A(1 - \tilde{\gamma}_t^A) \quad (22)$$

$$L_{t,ncc}^U = \beta^A(1 - \tilde{\gamma}_t^A) + \tilde{\gamma}_t^A + 1 - M_t \quad (23)$$

When agents are subject to credit constraints ($w_t^A < 0$) the supply of unskilled labor is

$$\begin{aligned} L_{t,cc}^S &= M_t - A(1 - \tilde{\gamma}_t^A)(1 - F(-w_t^A)) \\ &= L_{t,ncc}^S + A(1 - \tilde{\gamma}_t^A)F(-w_t^A) \end{aligned} \quad (24)$$

and the supply of skilled labor is

$$\begin{aligned} L_{t,cc}^U &= \tilde{\gamma}_t^A + F(-w_t^A)(1 - \tilde{\gamma}_t^A) + \beta^A(1 - F(-w_t^A))(1 - \tilde{\gamma}_t^A) + 1 - M_t \\ &= (\beta^A(1 - \tilde{\gamma}_t^A) + \tilde{\gamma}_t^A)(1 - F(-w_t^A)) + F(-w_t^A) + 1 - M_t \\ &= L_{ncc}^U + (1 - \beta^A)(1 - \tilde{\gamma}_t^A)F(-w_t^A) \end{aligned} \quad (25)$$

The supply of the agricultural good is equal to the total unskilled labor available less the total unskilled labor used by the masters. Thus

$$X_t^s = L_t^U - M_t U_t$$

The aggregate supply of the industrial good is equal to

$$Z_t^S = M_t (S_t)^\alpha (U_t)^{1-\alpha}$$

Using (3) it follows that

$$Z_t^S = M_t U_t \left(\frac{S_t}{U_t} \right)^\alpha = L_{Zt}^U \frac{1}{(1-\alpha)p_t}$$

Then, relative supply equals

$$RS_t = \frac{Z_t^S}{X_t^S} = \frac{1}{(1-\alpha)p_t} \frac{L_{Zt}^U}{L_{Xt}^U}$$

Differentiating this expression with respect to p_t we get

$$\frac{dRS_t}{dp_t} = \frac{1}{(1-\alpha)p_t} \left(\left(\frac{L_{Zt}^U}{L_{Xt}^U} \right)' - \left(\frac{L_{Zt}^U}{L_{Xt}^U} \right) \frac{1}{p_t} \right)$$

So, we have that if

$$\frac{d(L_{Zt}^U)/L_{Zt}^U}{dp_t} - \frac{d(L_{Xt}^U)/L_{Xt}^U}{dp_t} > \frac{1}{p_t}$$

then the relative supply curve is upward-sloping. This condition is always satisfied if

$$\frac{d(L_t^S)/L_t^S}{dp_t} > \frac{d(L_t^U)/L_t^U}{dp_t}$$

Using (22) – (25) we get

$$\begin{aligned} \frac{d(L_t^S)}{dp_t} L_t^U - \frac{d(L_t^U)}{dp_t} L_t^S &= \left(\frac{dL_t^S}{d\tilde{\gamma}_t^A} L_t^U - \frac{dL_t^U}{d\tilde{\gamma}_t^A} L_t^S \right) \frac{d\tilde{\gamma}_t^A}{d(\pi_t^{A*})} \frac{d(\pi_t^{A*})}{dp_t} \\ &= (2A - M_t (1 + A - \beta^A)) \frac{d\tilde{\gamma}_t^A}{d(\pi_t^{A*})} \frac{d(\pi_t^{A*})}{dp_t} \end{aligned}$$

Therefore, if the number of masters is less than $\tilde{M}^A = \frac{2A}{(1 + A - \beta^A)}$, then the percentage increases in the supply of skilled labor is more than that in the supply of unskilled labor and, as a result, the relative supply curve is upward-sloping.

We can rewrite the relative supply in the following way:

$$RS_t = \frac{1}{p_t} R(p_t)$$

where

$$\begin{aligned} R(p_t) &= \frac{1}{(1-\alpha)} \frac{L_{Zt}^U}{L_{Xt}^U} \\ &= \frac{1}{(1-\alpha)} \frac{\frac{1-\alpha}{\alpha} \pi_t^{A*} L_t^S}{L_t^U - \frac{1-\alpha}{\alpha} \pi_t^{A*} L_t^S} \end{aligned}$$

Using (22) and (23) it is straightforward to show that in the region when the apprentice's wage is positive, $R(p_t)$ is increasing in p_t . Since $\frac{RD_t}{p_t} = \frac{(1-\delta)}{\delta}$ is a constant and $\frac{RS_t}{p_t} = R(p_t)$ is an increasing function, there is at most one p_t at which $\frac{RD_t}{p_t}$ and $\frac{RS_t}{p_t}$ intersect. Therefore, there exists at most one equilibrium of NCC-type. ■

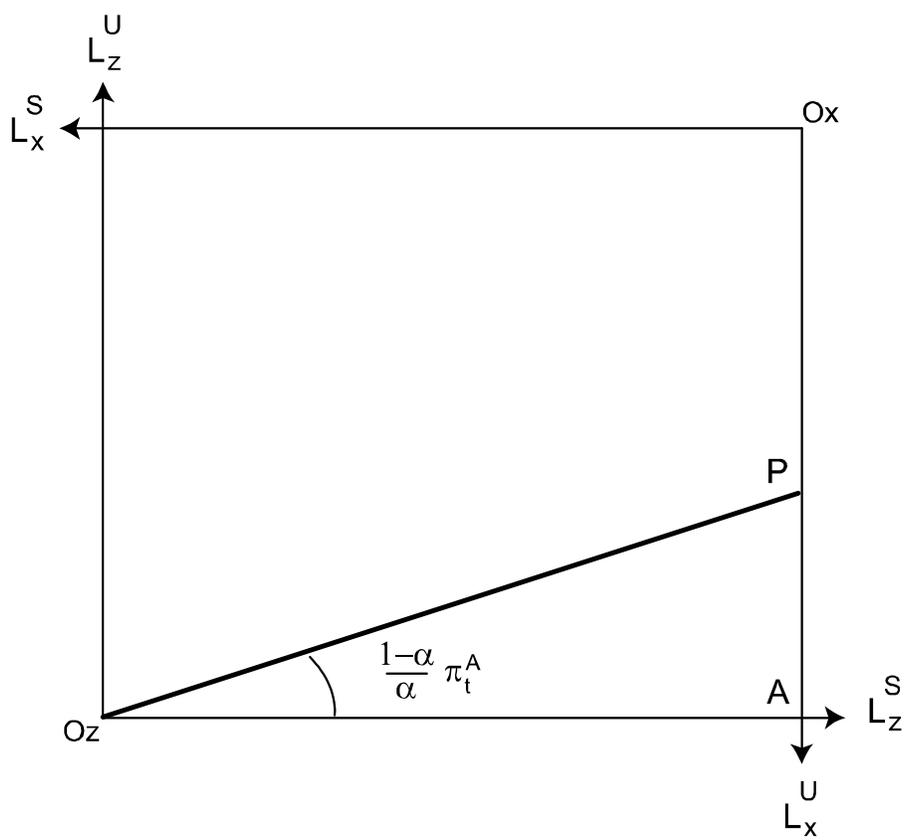


Figure 1. Output and Factor Availability

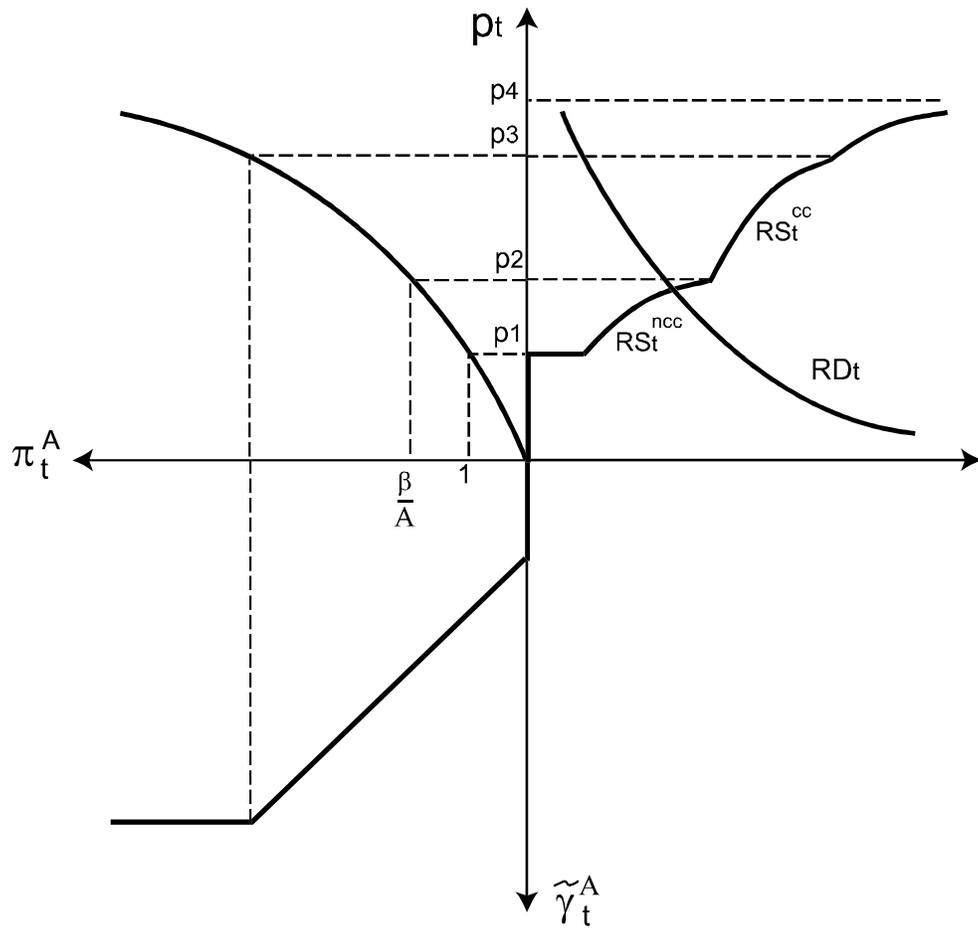


Figure 2. Relative Supply: Apprenticeship System

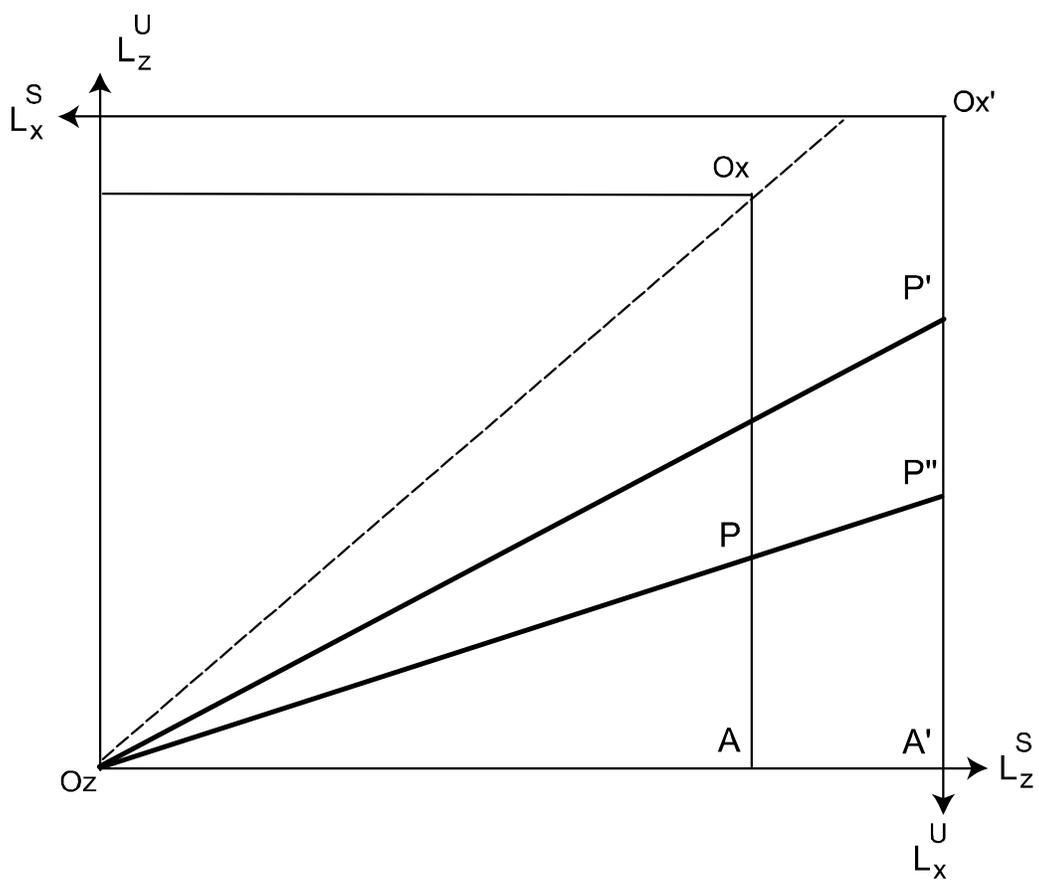


Figure 3(a). Small M_t

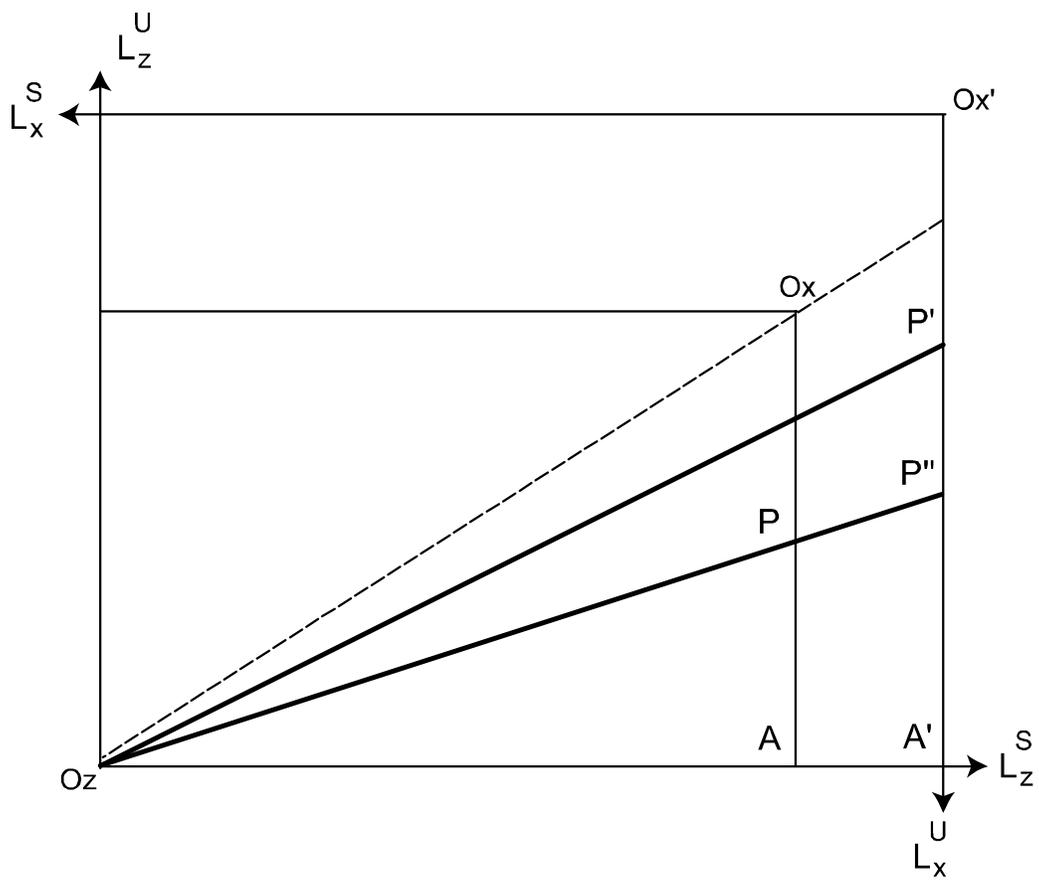
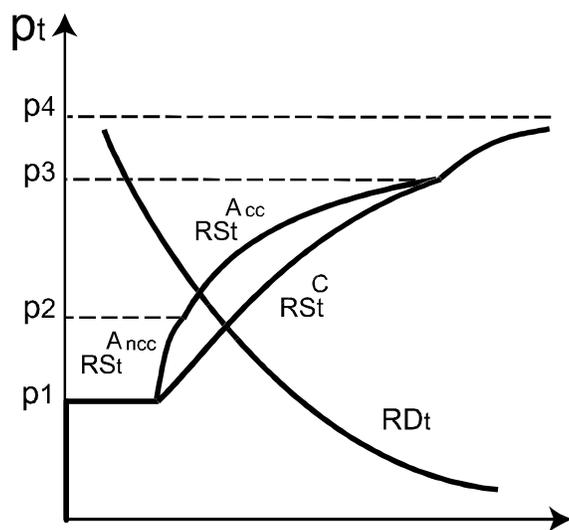
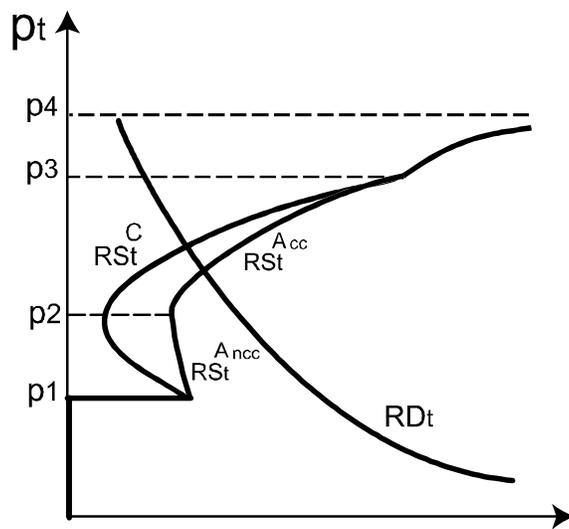


Figure 3(b). Large M_t



(a) Small Mt



(b) Large Mt

Figure 4. Relative Supply: PUF System

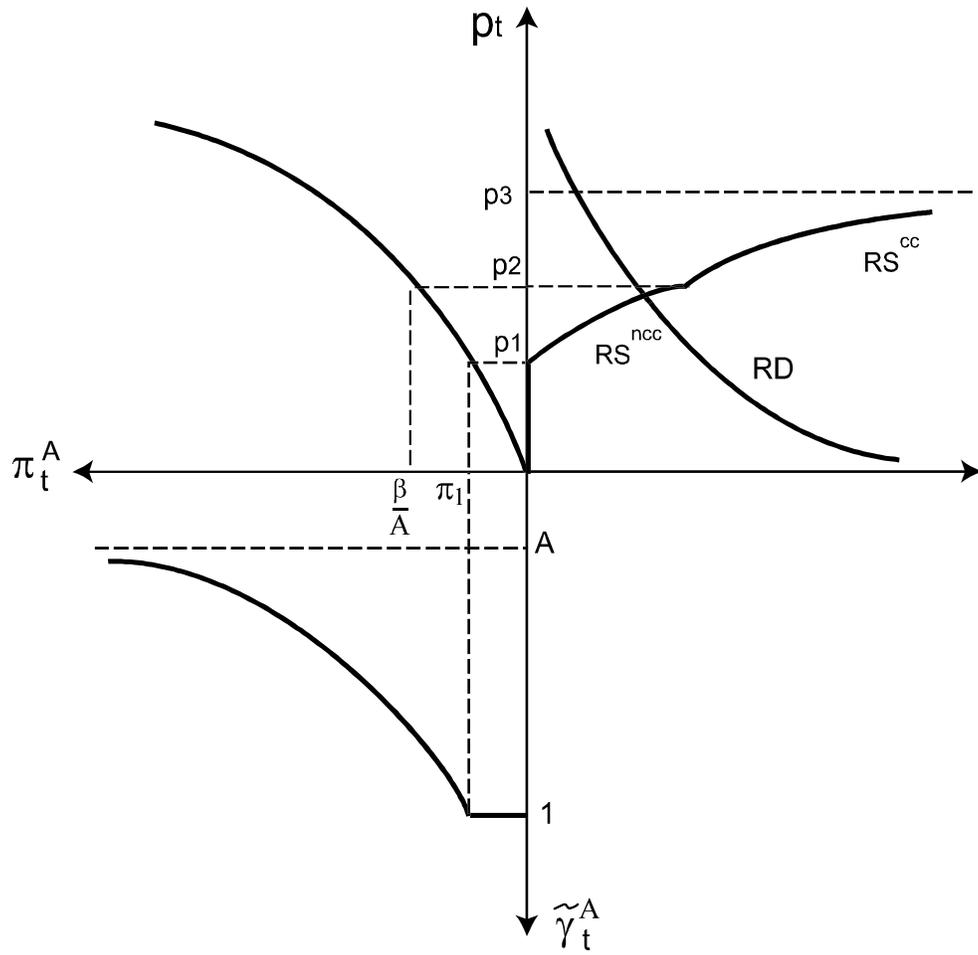


Figure 5. Steady State Relative Supply: Apprenticeship System

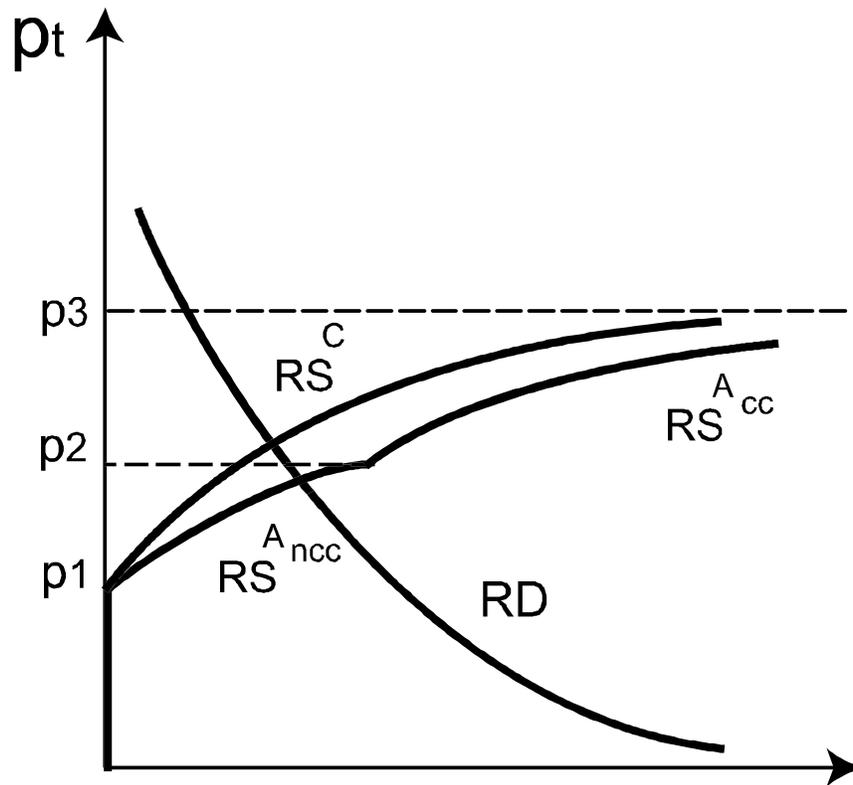


Figure 6. Steady State Relative Supply: PUF System

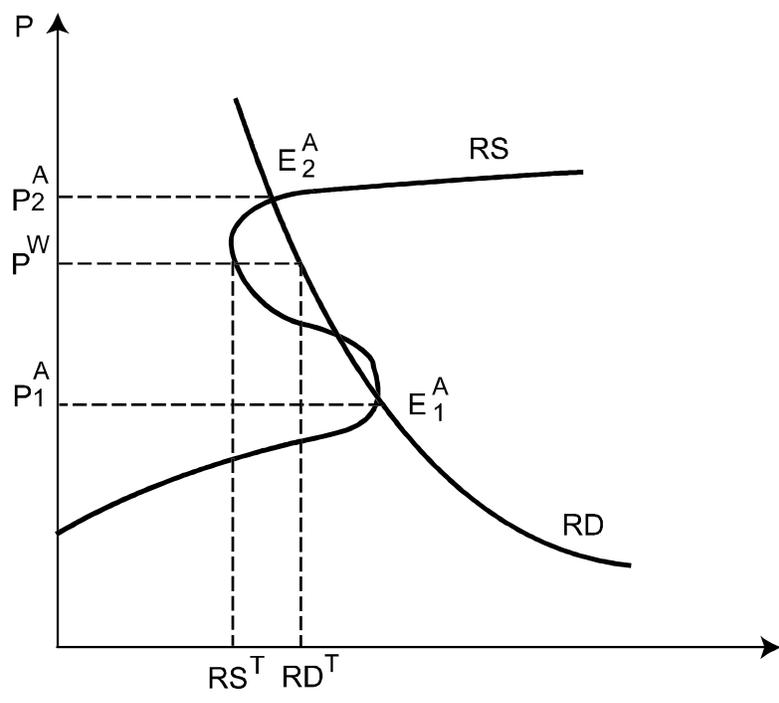


Figure 7. Effects of Trade

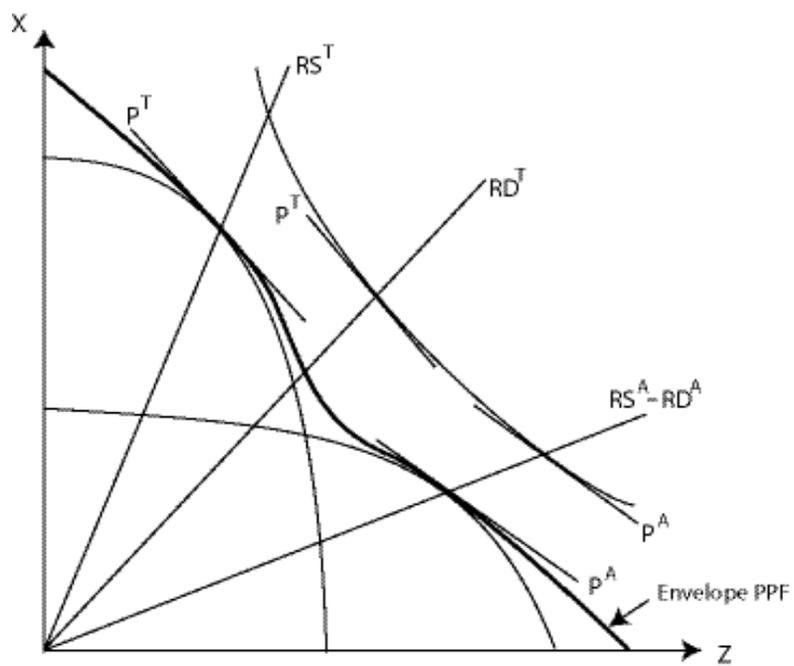


Figure 8. Welfare Reducing Trade

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