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# Trade, Growth, and Technology Equalization

by

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## Abstract

Trade is shown to increase economic growth purely through comparative advantage without recourse to scale effects, technology transfer, research and development, or even international investment. The resulting growth rates are those that would result from technology transfer, even though no technology transfer actually occurs. A balanced growth rate exists, is identical for all countries and therefore the world, and is asymptotically stable if and only if each country has an *absolute* (not just comparative) advantage in something. When balanced growth does not exist, trade reduces but does not eliminate differences between countries' growth rates. Trade therefore does not necessarily guarantee a stable world income distribution. The magnitude of trade's effect on growth depends on the goods imported, not those exported, with interesting implications for the interplay of trade policy and development policy to further a country's economic development.



**Keywords:** trade, growth, technology equalization, comparative advantage, absolute advantage, world income distribution

**JEL classification codes:** O4, F15

## I. Introduction

A long-standing issue in the endogenous growth literature is the effect of international trade on growth rates. Does trade promote economic growth? How? One might ask if trade raises growth rates in a dynamic model in the same way that it raises the level of income in a static model - by allowing countries to exploit their comparative advantages in production. To date, that question has not been answered theoretically. This paper examines that question; the answer turns out to have several surprising elements.

In the existing literature, trade affects growth in three ways - through increased global efficiency in research and development, scale effects, and technology transfer. All are interesting and suggest possibly important channels through which trade may affect growth. Most, however, do not depend on comparative advantage at all, and even those that do also require other driving mechanisms to obtain growth effects of trade.

The present paper show that trade, in and of itself, raises growth through the same comparative advantage mechanism that raises the income level in static models without growth. This result is derived in a model that is deliberately minimal, constructed to have no scale effects, no research and development, no technology transfer, and no international investment so that any effects of trade on growth cannot reflect those influences. The purpose here is to isolate the phenomenon of interest in accord with good scientific method, not to construct of a "realistic" model ready for estimation. The model has two countries producing two goods, with the goods produced in separate sectors. In the main analysis, each good can be used as capital, and both goods are essential for production in both sectors. The trade aspects of the model are a blend of Hecksher-Ohlin and Ricardian approaches, appropriate for a dynamic setting. As in

Hecksher-Ohlin, there are two factors of production, but they are not fixed; as in Ricardo, technology differs across countries. Comparative advantage guarantees trade, but it turns out that important aspects of trade's effect on growth depend on the *absolute* advantages of the trading countries. In particular, the international market equilibrium with trade can be an interior solution or a corner solution. An interior solution occurs if and only if each country has an absolute advantage in the production of a good. When that condition is met, trade raises both countries' growth rates and replicates the result that would emerge if countries exchanged technology, even though no technology transfer actually takes place. A corner solution occurs if one country (the "disadvantaged country") has no absolute advantage in any good. In that case, trade may but need not raise the disadvantaged country's growth rate but always leaves it below the growth rate of the other country, whose growth rate is unaffected by trade. The model thus generalizes Acemoglu and Ventura's (2002) results on trade, growth, and the world income distribution. In the interior case, the model reproduces their result that trade equalizes growth rates and so produces a stable world income distribution; in the corner case, and in contrast to Acemoglu and Ventura, the model shows that trade does not equalize growth rates, so that world income distribution does not stabilize. The effect of trade on growth also depends on the kind of good imported but not the kind exported. The crucial thing is that imports must be factors of production. Importing pure consumption goods has no effect on growth.

We thus are led to a technology equalization theorem, related in its structure to the Hecksher-Ohlin-Samuelson theorem on factor price equalization. The theorem roughly says that trade in goods that are factors of production substitutes for technology transfer - the growth rates of countries with different technologies are the same in the presence of trade as they would have been had those countries agreed to share technological knowledge, except for the corner solution case, where trade leads to no more than partial technology equalization. The theorem also provides conditions for existence and asymptotic stability of the balanced growth rate.

The theorem offers a possible explanation of why some countries persistently have lower growth rates than others. It also has implications for trade policy and its effects. Government policies that create an absolute advantage in something can increase a country's growth rate. However, such policies must be aimed at goods that can be exchanged in the world market for factors of production, not just for consumption goods.

**II. Existing Models and Evidence**

In the existing theoretical literature, trade affects growth through three channels: enhanced research and development, scale effects, and technology transfer. Consider these in turn. (1) In models where growth arises from research and development, comparative advantage plays a role in fostering growth, making this class of models the most closely related to the model we will study below. In Grossman and Helpman's (1990, 1991, 1995) work, for example, some countries have a comparative advantage in research and development. Trade allows those countries to specialize in what they are relatively good at doing - namely, R&D - and so making all countries grow faster. In these models, though, the role of comparative advantage is tangential; the growth effects of trade are driven by research and development, not comparative advantage *per se*. If there were no increase in research and development, trade in itself would lead to no increase in any country's growth rate. A similar conclusion holds for the interesting models of Young (1991), in which trade alters growth by changing countries' patterns of learning-by-doing, and Galor and Mountford (2003), in which trade allows countries to specialize in skilled or unskilled production, which in turn affects the kind of technical progress the countries choose to develop. What we shall study in the present paper are the growth effects of trade and comparative advantage by themselves, apart from the influences of research and development. (2) In models that have scale effects, opening a country to trade effectively increases that economy's size, which then leads to a higher growth rate through the scale effect. For example, Kremer (1990) argues that trade increases the relevant population size and thereby raises the rate of technical progress; Barro and Sala-I-Martin (1997) and Connolly (2000) present models in which trade expands markets and so raises profitability of producing intermediate goods, leading to more production of those goods and consequently higher growth. In these models, it is not trade that matters, only the size of the economy. Anything that increases that size raises growth; there is nothing special about trade. (3) In models of technology transfer, trade is a vehicle for the exchange of technological know-how. Trade opens countries to each other's knowledge, allowing each country to learn and adopt the production techniques of its trading partners and so increasing its stock of knowledge. Because the rate of knowledge accumulation is treated as proportional to the stock of existing knowledge, the technology transfer enabled by trade raises the growth rates of all trading partners. These models often invoke a type of scale effect. Rivera-Batiz and Romer (1991), for example, discuss two such models in which the growth effects of trade explicitly arise from scale effects and not from comparative advantage. More recently, Howitt (2000) has used a technology-transfer model of Schumpeterian growth to study cross-country income differences; in the model, countries that are relatively good at research and development share technology with each other and grow fast compared to countries that are not good at R&D and that end up not being included in the technology sharing.  Acemoglu and Ventura (2002) also study the world income distribution, using a model in which countries are endowed with the ability to produce different sets of intermediate goods.  Opening the countries to trade makes the different goods available to everyone and so leads to higher growth through an increased variety of intermediates.  All these models have two characteristics that are important for our discussion here: the increase in growth ultimately reflects a kind of scale effect (the instantaneous jump in the stock of knowledge), and comparative advantage plays no role.[1]

The existing evidence on whether trade causes growth is not decisive, but it tends to support a statistically and economically significant effect.  Models in which trade works through the scale effect receive mixed support; see, for example, Backus, Kehoe, and Kehoe (1992).  The evidence is more supportive of the possibility that trade affects growth through the research and development and the technology transfer channels.  The classic citation is perhaps Coe and Helpman (1995), who provide evidence that R&D in one country improves growth in a trading partner.  The interesting thing for our purposes here, however, is that there also is evidence that trade may affect growth directly rather than through R&D or technology transfer.  Alcala and Ciccone  (2001), for example, find strong and robust effects of trade on growth.  Their measure of openness to trade does not depend on in any way on technology transfer or R&D and so

[1]One contribution that stands apart from all others is Ventura's (1997) discussion of how trade, through comparative advantage, distributes growth among countries.  Although Ventura's discussion concerns trade and growth, it is unrelated to the issues we examine below.  Ventura addresses the issue of how to use cross-country data to tests implications of growth theory; the analysis below examines the effect of trade on the balanced growth rates of both individual countries and the world as a whole.  Indeed, in Ventura's analysis, trade has no effect on the world's balanced growth rate (see his equation 11), whereas in the endogenous growth model used below, trade generally raises the growth rates of all trading countries and increases the world's rate of growth along the balanced growth path when such a path exists.

suggests that trade in and of itself boosts growth. This conclusion is buttressed by Wacziarg and Welch's (2003) finding that increased trade liberalization is positively associated with increased growth. What is missing is a theoretical argument showing why trade should have such an effect. We now proceed to develop such an argument.

**III. The Model Under Autarky**

The analysis is based on an extension of the autarkic model discussed by Barro and Sala-I-Martin (1995, chapter 5). We start with a quick summary of the original model. The model is one of a standard set of endogenous growth models, so we can restrict ourselves here to stating the problem and the solution, consigning all derivation and mathematical detail to the Appendix.[2]

The economy produces two goods, Y and H. Good Y can be used for consumption C or can be used as gross investment I in one kind of capital K. Good H is another type of capital and is produced in a different sector from Y by a different technology. Note in particular that H *need not* be human capital; it is just another type of (physical) capital different from K. Each sector's technology is Cobb-Douglas. Each type of capital depreciates at the rate δ. We have

(1)
$$Y_t = C_t + I_t$$
$$= A(v_t K_t)^{\alpha}(u_t H_t)^{1-\alpha}$$

(2)
$$\dot{K}_t = A(v_t K_t)^{\alpha}(u_t H_t)^{1-\alpha} - C_t - \delta K_t$$

(3)
$$\dot{H}_t + \delta H_t = B[(1-v_t)K_t]^{\eta}[(1-u_t)H_t]^{1-\eta}$$

Where v and u are the fractions of total K and H that are used in the Y-producing sector and A,

[2]Barro and Sala-I-Martin (1995) sketch the derivation of the solution. The Appendix to the present paper contains much detail omitted by Barro and Sala-I-Martin.

B, $\alpha$, and $\eta$ are constants. We can define a gross domestic product:

(4) $$Q = Y + p(\dot{H} + \delta H)$$

Where p is the price of H in terms of Y. The production functions in (1) and (3) omit labor and are extended versions of AK functions, but that is a mere convenience. We could replace H by HL, where is L is labor and H is labor-augmenting technical progress. The model then ceases to be AK in nature. We consider just such a version later and show that the main results still obtain. For now, we suppose that labor is a constant normalized to one.[3]

Utility is CRRA, so lifetime utility is:

(5) $$U = \int_0^{\infty} \frac{C_t^{1-\theta} - 1}{1-\theta} e^{\rho t} dt$$

The balanced growth path solution is obtained in the usual way. We begin by writing the Hamiltonian and necessary conditions. We then differentiate the first-order condition for consumption with respect to time to obtain the growth rate $\gamma_C$ for consumption:

(6) $$\gamma_C = \frac{1}{\theta}\left[\alpha A\left(\frac{vK}{uH}\right)^{-(1-\alpha)} - \delta - \rho\right]$$

A large amount of analysis is then performed on the remaining necessary conditions to show that the growth rates of K, H, Y, and Q all equal the growth rate of C.[4] The common value $\gamma$ for

[3]When we associate H with L in the form HL, joint degree-one homogeneity in K and H arises naturally because the factor share for the composite factor HL is the unitary additive complement of the factor share for K. In contrast, when labor in effect is excluded, joint degree-one homogeneity in K and H is less obvious. We can justify it by appealing to Romer's (1986) learning-by-doing model. Romer considered only one type of capital, but the argument applies just as well to a vector of types of capital, as in the present model.

[4]See the Appendix.

these growth rates is

$$(7) \qquad \gamma = \frac{1}{\theta}\left[A^{\frac{\eta}{1-\alpha+\eta}} B^{\frac{1-\alpha}{1-\alpha+\eta}} \alpha^{\frac{\alpha\eta}{1-\alpha+\eta}} (1-\alpha)^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}} \eta^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}} (1-\eta)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha+\eta}} - \delta - \rho\right]$$

The value of p is

$$(8) \qquad p = \left(\frac{A}{B}\right)\left(\frac{\alpha}{\eta}\right)^{\eta}\left(\frac{1-\alpha}{1-\eta}\right)^{1-\eta}\left[\left(\frac{A}{B}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{1-\alpha}{1-\eta}\right)^{\frac{1}{1-\alpha+\eta}}\right]^{\alpha-\eta}$$

**IV. Fundamentals of Trade and Growth**

We now want to introduce foreign trade. We begin with a simple case that makes the intuition clear and examine generalizations later. We restrict attention to two countries, 1 and 2, which have different technology. No factors are fixed.[5]

In our framework of a two-factor Cobb-Douglas production function, cross-country technology differences are captured in different values of the total factor productivities A and B and of the factor share parameters $\alpha$ and $\eta$. We suppose for now that the factor share for K is the same in the two sectors and in the two countries:

$$\alpha = \eta$$

Obviously, this restriction means the factor share for H also is the same in the two sectors and the two economies, equaling 1-$\alpha$ everywhere. All cross-country technology differences reflect differences in the values of A and B, which generally differ in the two countries. We relax these restrictions later to see that the main results do not depend on them.

The results of the previous section now simplify in several ways. Most important, we

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[5]In reality, it is not clear that any factor(except perhaps land) is fixed in the long run. In a growth model, whose whole point is long-run dynamics, the convenient short run static assumption of fixed factors seems to make little sense.

now have:[6]

$$u = v$$

This in turn implies that the production functions for Y and H are

$$Y = AvK^{\alpha}H^{1-\alpha}$$

$$\dot{H} + \delta H = B(1-v)K^{\alpha}H^{1-\alpha}$$

The autarkic growth rate is

$$\gamma = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B\left(\frac{A}{B}\right)^{\alpha} - \delta - \rho\right] \tag{9}$$

A. Two Countries Not Small Relative to Each Other.

It is traditional to begin (and often end) discussion of trade with the case of a "small" country, in which the country studied is an atomistic agent in the world economy. Here, however, it is preferable to begin with two countries of arbitrary size. The small country is a special case of this more general framework and will be discussed briefly later.[7] To avoid complications of bilateral monopoly, we suppose that each country's economy consists of a large number of competitive firms with identical production functions, preventing either country from acting as a monopolist and guaranteeing a competitive solution.

Let X denote exports of Y, so that $p^{-1}X$ is imports of H. Then the accumulation

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[6]This result follows from (46) in the Appendix.

[7]The general case, in which countries are not small relative to each other, is not only more useful for the present analysis but seems more realistic as well. In mathematical terms, the small country assumption means that the country in question has measure zero. In reality, all countries have positive measure because there are only finitely many of them and yet the world's measure is positive. Perhaps more important, many countries are "large" players in the world economy, whose behavior is determined mostly by them.

constraints for country *i* are

(10) $$Y_i = C_i + \dot{K}_i + \delta K_i + X_i$$

$$= A_i v_i K_i^{\alpha} H_i^{1-\alpha}$$

(11) $$\dot{H}_i + \delta H_i = B_i (1 - v_i) K_i^{\alpha} H_i^{1-\alpha} + \frac{1}{p} X_i$$

When economies are closed, X ≡0, and the autarkic model of section II applies. Each country's growth rate is determined in isolation and is given by (9), which we denote here as $\gamma_{i,\,Au}$ for convenience of subsequent discussion:

(12) $$\gamma_{i,Au} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha} B_i \left(\frac{A_i}{B_i}\right)^{\alpha} - \delta - \rho\right]$$

We will compare this growth rate with that prevailing when there is trade.

When country *i* is open, it must choose X along with everything else. Countries 1 and 2 are linked through trade, so the solutions for their growth rates must be determined simultaneously. The key variable that guarantees world general equilibrium is the price p, which now is determined to guarantee international trade balance. Because neither country is small, equilibrium p depends on what both countries do, so that p, too, must be determined as part of the simultaneous solution for the two countries. We find the world general equilibrium in two steps. First, we solve each country's quasi-central planning problem, taking p as given; then we impose the trade balance condition $X_1 = - X_2$ to find the equilibrium value of p.[8]

It is important to note here that scale effects, research and development, and technology

[8]Equivalently, we could solve for world general equilibrium as a world central planning problem, obtaining $v_{1t}$, $v_{2t}$, $C_{1t}$, $C_{2t}$, $X_t$, and $p_t$ in a single step. The two-step approach gives a more intuitive view of what each country is doing.

transfer all are absent from this model. Consequently, any effects that trade has on growth will not be due to those influences but rather to comparative advantage alone.

B. Individual Country Solutions.

With trade, country *i*'s Hamiltonian is

$$V_i = \frac{C_i^{1-\theta}-1}{1-\theta}e^{-\rho t} + \phi_i\left[A_i v_i K_i^{\alpha} H_i^{1-\alpha} - C_i - X_i - \delta K_i\right] \tag{13}$$

$$+ \psi_i\left[B_i(1-v_i)K_i^{\alpha}H_i^{1-\alpha} + \frac{1}{p}X_i - \delta H_i\right]$$

where $\phi$ and $\psi$ are the costate variables. The necessary conditions are

$$\dot{\phi}_i = -\frac{\partial V_i}{\partial K_i} = -\phi_i[\alpha A_i v_i\left(\frac{K_i}{H_i}\right)^{\alpha-1} - \delta] - \psi_i \alpha(1-v_i)B_i\left(\frac{K_i}{H_i}\right)^{\alpha-1} \tag{14}$$

$$\dot{\psi}_i = -\frac{\partial V_i}{\partial H_i} = -\phi_i(1-\alpha)v_i A_i\left(\frac{K_i}{H_i}\right)^{\alpha} - \psi_i\left[(1-\alpha)(1-v_i)B_i\left(\frac{K_i}{H_i}\right)^{\alpha} - \delta\right] \tag{15}$$

$$\frac{\partial V_i}{\partial C_i} = 0 = C_i^{-\theta}e^{-\rho t} - \phi_i \tag{16}$$

$$\frac{\partial V_i}{\partial v_i} = 0 = \phi_i A_i K_i^{\alpha} H_i^{1-\alpha} - \psi_i B_i K_i^{\alpha} H_i^{1-\alpha} \tag{17}$$

$$\frac{\partial V_i}{\partial X_i} = 0 = -\phi_i + \frac{1}{p}\psi_i \tag{18}$$

plus initial and transversality conditions, which are unimportant in what follows and so are suppressed for simplicity.

Equation (18) does not depend on any control variable, so we have bang-bang control for $X_i$. When $-\phi_i + (1/p)\psi_i$ is positive, equation (18) cannot be satisfied; the marginal value of X equals $-\phi_i + (1/p)\psi_i$ and always is positive, irrespective of the value X. Consequently, country *i* sets $X_i$ as high as possible, which it does by producing only Y and exporting some of it to obtain H. The opposite holds when $-\phi_i + (1/p)\psi_i$ is negative. Country *i* then sets $X_i$ as low as possible, producing only H and obtaining Y solely through imports (so that $X_i$ is negative). When $-\phi_i + (1/p)\psi_i$ equals zero, country *i* does not engage in trade, and X is zero. To see this, note that $-\phi_i + (1/p)\psi_i = 0$ is equivalent to $p = \psi_i / \phi_i$. The price p is the international price for H in terms of Y, that is, the ratio of marginal utilities of H and Y. The costate variables $\phi_i$ and $\psi_i$ are, respectively, country *i*'s marginal utilities of Y-goods and H-goods from internal production. Their ratio is the marginal value of H-goods in terms of Y-goods if country *i* produces both goods; that is, the ratio is country *i*'s internal price for H in terms of Y. If this internal price equals the external (world) price p, country *i* can obtain the same number of units of Y in exchange for H from its own internal operations as it can by trading on the world market. Country gains nothing from trade, so it sets $X_i$ to zero and behaves autarkically. We thus have three possible values for $X_i$: as high as possible, zero, and as low as possible. The borderline case of $X_i = 0$ generally will prevail no more than momentarily; as $\phi_i$ and $\psi_i$ vary according to their laws of motion (14) and (15), they generally will not satisfy the equality $p = \psi_i / \phi_i$. Furthermore, we are interested here in situations where trade does occur. Consequently, we ignore the knife-edge no-trade case henceforth.

Comparisons of the world price p with the internal prices $\psi_i / \phi_i$ are central to all that follows. The Appendix shows that $\psi_i / \phi_i = A_i / B_i$ when $\alpha=\eta$ as we are assuming here. This fact together with the results of the previous paragraph means that country *i* specializes in producing H when $p > A_i / B_i$ and in producing Y when $p < A_i / B_i$.

C. Balanced Growth Rates in World Equilibrium.

The equilibrium world price p must fall between $A_1 /B_1$ and $A_2 /B_2$; otherwise, both countries would try to specialize in and export the same good, violating international balance of trade. Which country has the higher value of $A_i /B_i$ is arbitrary, so we assume without loss of generality that $A_1 /B_1 > A_2 /B_2$. We then have

$$\frac{A_1}{B_1} \geq p \geq \frac{A_2}{B_2}$$

For p strictly in the interior of the closed interval $[A_2/B_2, A_1/B_1]$, country 1 sets v = u = 1, specializes in production of Y, and trades to obtain H; country 2 does the opposite, producing only H and trading for Y. When p is on the boundary of the interval, equaling either $A_1/B_1$ or $A_2/B_2$, we have extra complications. For now, we restrict attention to the interior, discussing the corner cases afterward. We now calculate the growth rates for the two countries.

Knowing that country 1 specializes in Y allows us to write a simplified maximization problem for it, conditional on the fact that it produces no H. Equations (13) - (18) still characterize the problem, but now we set u = v =1. The Hamiltonian reduces to

(19) $$V_1 = \frac{C_1^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_1 [A_1 K_1^{\alpha} H_1^{1-\alpha} - C_1 - \delta K_1 - X_1] + \psi_1 \left[ (1/p) X_1 - \delta H_1 \right]$$

and the necessary conditions become

(20) $$\dot{\phi}_1 = -\frac{\partial V_1}{\partial K_1} = -\phi_1 [\alpha A_1 \left( \frac{K_1}{H_1} \right)^{\alpha - 1} - \delta]$$

(21) $$\dot{\psi}_1 = -\frac{\partial V_1}{\partial H_1} = -\phi_1 (1-\alpha) A_1 \left( \frac{K_1}{H_1} \right)^{\alpha} + \psi_1 \delta$$

(22) $$\frac{\partial V_1}{\partial C_1} = 0 = C_1^{-\theta} e^{-\rho t} - \phi_1$$

(23) $$\frac{\partial V_1}{\partial X_1} = 0 = -\phi_1 + \frac{1}{p}\psi_1$$

There no longer is a first-order condition for v because it already has been set to one. The first-order conditions for C and X are unchanged from the unconditional problem, but the FOC for X now holds with equality. Having accepted the world price p and agreed to specialize in producing Y, country 1 now chooses $\phi$ and $\psi$ to satisfy (23) exactly.

Differentiating (22) with respect to time, dividing both sides by $\phi$, and manipulating yields the growth equation for consumption

(24) $$\gamma_{C_1} = -\frac{1}{\theta}\left(\frac{\dot{\phi}_1}{\phi_1} + \rho\right)$$

The same kind of manipulations as for the autarkic model show that the growth rates of Y, K, and Q all equal the growth rate of consumption; as before, we denote this common growth rate $\gamma$. We obtain the growth rate of $\phi_1$ from the remaining necessary conditions and the requirements of balanced growth. Because equation (23) now always holds, we have $p = \psi_1/\phi_1$. Trade balance constrains p to lie in the closed interval $[A_2/B_2, A_1/B_1]$, and balanced growth requires that everything that grows must do so at a constant rate. The only growth rate for p consistent with both these requirements is zero, so p must be constant along the balanced growth path. The ratio $\psi_1/\phi_1$ therefore also is constant, implying that the growth rates of $\phi_1$ and $\psi_1$ are equal. We can use this fact together with (20) and (21) to solve for $K_1/H_1$, obtaining

(25) $$\frac{K_1}{H_1} = p\frac{\alpha}{1-\alpha}$$

We then substitute this expression into (20) and divide by $\phi_1$ to obtain the growth rate for $\phi_1$:

(26) $$\frac{\dot{\phi}_1}{\phi_1} = \delta - p^{\alpha}\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_1$$

Substituting this solution into (24) and collecting terms gives us what we are after, the growth rate of country 1 in the presence of trade:

(27) $$\gamma_{1,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_1\left(\frac{1}{p}\right)^{1-\alpha} - \delta - \rho\right]$$

where the subscript T indicates that this growth rate pertains when country 1trades.

Country 2's growth rate is found the same way. Country 2 produces no Y, only H, so its Hamiltonian is

(28) $$V_2 = \frac{C_2^{1-\theta}-1}{1-\theta}e^{-\rho t} + \phi_2[-\delta K_2 - C_2 - X_2] + \psi_2\left[B_2\,K_2^{\eta}H_2^{1-\eta} + (1/p)X_2 - \delta H_2\right]$$

with corresponding necessary conditions. Going through the same steps as for country 1 yields the growth rate

(29) $$\gamma_{2,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_2 p^{\alpha} - \delta - \rho\right]$$

These growth rates both are greater than the growth rates under autarky for any p in the interval $[A_2 /B_2, A_1 /B_1]$, where p must lie:

$$\gamma_{1,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_1\left(\frac{1}{p}\right)^{1-\alpha} - \delta - \rho\right]$$

$$> \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_1\left(\frac{B_1}{A_1}\right)^{1-\alpha} - \delta - \rho\right]$$

$$= \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_1\left(\frac{A_1}{B_1}\right)^{\alpha} - \delta - \rho\right]$$

$$= \gamma_{1,Au}$$

$$\gamma_{2,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_2 p^{\alpha} - \delta - \rho\right]$$

$$> \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_2\left(\frac{A_2}{B_2}\right)^{\alpha} - \delta - \rho\right]$$

$$= \gamma_{2,Au}$$

We thus have the important conclusion that *trade raises the balanced growth rates of both countries* (except for an important qualification explained momentarily).

The growth rates in (27) and (29) are not equal for arbitrary p. A necessary and sufficient condition for equality of growth rates is that p equal the quantity $A_1 / B_2$. When p equals that value, we obtain by substituting into (27) and (29) the common solution

(30) $$\gamma \equiv \gamma_{1,T} = \gamma_{2,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_1^{\alpha}B_2^{1-\alpha} - \delta - \rho\right]$$

Otherwise, if $p > A_1 / B_2$, then $\gamma_{1,T} < \gamma_{2,T}$ and conversely if $p < A_1 / B_2$.[9] In the present discussion, we are confining attention to balanced growth paths for each of the trading countries. For each country to have balanced growth individually, however, it is necessary that the two countries

[9]Recall that we are maintaining the assumption that $A_1 / B_1 > A_2 / B_2$. If we assume the converse, then the foregoing results are reversed.

have the same growth rate, that is, that we have balanced growth for the world. To see this, note that the equations for the growth rates of $K_1$ and $K_2$ become, in the presence of trade and specialization

(31) $$\frac{\dot{K}_1}{K_1} = A_1\left(\frac{K_1}{H_1}\right)^{\alpha-1} - \left(\frac{C_1}{K_1}\right) - \left(\frac{X_1}{K_1}\right) - \delta$$

(32) $$\frac{\dot{K}_2}{K_2} = \left(\frac{X_1}{K_2}\right) - \left(\frac{C_2}{K_2}\right) - \delta$$

where we have used the trade balance condition $X_1 = -X_2$. Balanced growth in each country requires that these two growth rates for K be constant. It is straightforward to show that constancy of these growth rates requires that all the ratios on the right sides of the two equations also must be constant. That in turn requires that $K_1$ and $K_2$ both grow at the same rate as $X_1$, which then implies that $H_i$, $X_i$, $C_i$, and $Y_i$ all must grow at the same rate as well. Balanced growth for the individual countries therefore requires that $p = A_1 / B_2$. When that condition is met, countries have the same growth rate, implying that the world has that growth rate, too.

D. <u>The Importance of Absolute Advantage</u>.

An important issue arises here. Balanced growth requires that p equals $A_1 / B_2$, and, as we have seen above, trade balance requires that p fall between $A_1 / B_1$ and $A_2 / B_2$ (otherwise both countries would try to specialize in and export the same good). There is no guarantee, however, that $A_1 / B_2$ falls between $A_1 / B_1$ and $A_2 / B_2$. The only restriction we have imposed so far is that the ratio $A_1 / B_1$ exceed the ratio $A_2 / B_2$ (in order to guarantee that trade occurs). That restriction puts no limits on the ratio $A_1 / B_2$. If $A_1 / B_2$ falls outside the closed interval $[A_2 / B_2 , A_1 / B_1]$, then p cannot equal it, and balanced growth for the world and for the two countries individually is not possible when the countries trade. Balanced growth thus requires that $A_1/B_2$ falls inside the interval $[A_2/B_2, A_1/B_1]$.

What conditions on the values of $A_1$, $B_1$, $A_2$, and $B_2$ are required to guarantee that $A_1/B_2$ falls inside $[A_2/B_2, A_1/B_1]$? Recall that we are assuming for concreteness that $A_1/B_1 > A_2/B_2$. A little algebra shows that to satisfy this latter condition and also have $A_1/B_2$ inside the interval $[A_2/B_2, A_1/B_1]$ requires that

$$A_1 > A_2 \qquad (33)$$
$$B_1 < B_2$$

that is, country 1 must have an absolute advantage in producing Y, and country 2 must have an absolute advantage in producing H.[10] When that condition is met, balanced growth is possible. Furthermore, the balanced growth solution is globally asymptotically stable because, from equations (27) and (29), $d\gamma_{1,T}/dp < 0$ and $d\gamma_{2,T}/dp > 0$ for all values of p. At $p = A_1/B_2$, we have balanced growth, which means K and H are growing at the same rate, leaving p constant. In contrast, if $p > A_1/B_2$, then $\gamma_{1,T}$ falls and $\gamma_{2,T}$ rises compared to their balanced growth values. As a result, K grows more slowly than H, and p must fall. The same logic shows that p must rise if it is below $A_1/B_2$. The conclusion is that the balanced growth value of $p = A_1/B_2$ is globally asymptotically stable. The balanced growth path therefore is a global attractor for the economy whenever it exists.

When (33) is not satisfied, then $A_1/B_2$ either is greater than $A_1/B_1$ or is less than $A_2/B_2$, and no balanced growth rate will exist for either country or for the world. The world price p will equal whichever of $A_1/B_1$ or $A_2/B_2$ is closer to $A_1/B_2$ (because, as remarked earlier, trade

[10]These inequalities are reversed if $A_1/B_1 < A_2/B_2$.

balance requires that p cannot be outside the interval [$A_2 /B_2$ , $A_1 /B_1$]). The world economy will be at a corner. The growth rate of one country is the same under trade as under autarky; the growth rate of the other country is higher under trade than under autarky but is lower than that of the first country.

For example, suppose $A_1 /B_2 > A_1 /B_1$, so that $p = A_1 /B_1$. Note that $A_1 /B_2 > A_1 /B_1$ also implies $B_1 > B_2$. Country 1's autarkic growth rate exceeds that of country 2:

$$\begin{aligned}\gamma_{1,Au} &= \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_1\left(\frac{A_1}{B_1}\right)^{\alpha}-\delta-\rho\right]\\ &> \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_2\left(\frac{A_2}{B_2}\right)^{\alpha}-\delta-\rho\right]\\ &= \gamma_{2,Au}\end{aligned}$$

Trade has no effect on country 1's growth rate:

$$\begin{aligned}\gamma_{1,T} &= \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_1\left(\frac{1}{p}\right)^{1-\alpha}-\delta-\rho\right]\\ &= \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_1\left(\frac{B_1}{A_1}\right)^{1-\alpha}-\delta-\rho\right]\\ &= \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_1\left(\frac{A_1}{B_1}\right)^{\alpha}-\delta-\rho\right]\\ &= \gamma_{1,Au}\end{aligned}$$

but raises country 2's growth rate:

$$
\begin{aligned}
\gamma_{2,T} &= \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_2 p^{\alpha} - \delta - \rho\right] \\
&= \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_2\left(\frac{A_1}{B_1}\right)^{\alpha} - \delta - \rho\right] \\
&> \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}B_2\left(\frac{A_2}{B_2}\right)^{\alpha} - \delta - \rho\right] \\
&= \gamma_{2,Au}
\end{aligned}
$$

Also, country 2's growth rate remains below that of country 1 even with trade because $B_2 < B_1$.

This absence of change in one country's growth rate in the corner solutions is the aforementioned qualification to the conclusion that trade increases the growth rates of the trading partners. For a country in a corner, it need not do so.[11] Trade therefore does not in itself guarantee balanced growth.

To see the intuition for these results, let us continue with our example in which $A_1 > A_2$, $B_1 > B_2$, and $A_1 / B_1 > A_2 / B_2$. Country 1 has an absolute advantage in both goods, and country 2 has a comparative advantage in producing the second good, which is H. Country 2 specializes in producing H and trades with the country 1 to obtain Y. Country 1, however, does not specialize in producing Y but rather continues to produce both goods.[12] In effect, country 2 is not "big enough" in a technological sense to fulfill country 1's demand for H. Because country 1 continues to produce H, it continues to use its own technology, and its growth rate continues to

[11]Trade always increases the income *levels* of both trading countries and therefore raises their growth rates temporarily through transition effects, even in the case where one country's growth rate ultimately is left unchanged.

[12]Formally, this conclusion follows immediately by noting that, because $p = A_1 / B_1$ in the corner solution, trade does not alter the necessary conditions in country 1's optimization problem and so does not alter country 1's production decisions.

depend only on its own total factor productivities $A_1$ and $B_1$. Country 2 must continue to produce something (in this example, H), and it continues to use its inferior technology for that good (that is, $B_2$). Its growth rate therefore depends on both $A_1$ and $B_2$. The result is a partial equalization of technology in the corner case; country 2 abandons $A_2$ for $A_1$ but continues to use the inferior $B_2$ technology to produce H.[13] Total world output is higher than under autarky, and both countries have more total output available to them. Average total product in both countries is higher. Marginal total product, however, is higher only in country 2, which abandons its inefficient production of Y. In contrast, trade offers country 1 no technological advantage on the margin, which is why its growth rate does not change even though it does enjoy higher national income.

These results have an important implication for the dynamics of the world income distribution. If a corner solution prevails, countries grow at different rates and their incomes diverge. This outcome contrasts with Acemoglu and Ventura's (2002) conclusion that trade leads to a stable world income distribution. The reason for the difference is that Acemoglu and Ventura restrict their model in such a way that it necessarily yields the equivalent of our interior solution, in which all growth rates are equal. In particular, they specify that each country is endowed with a monopoly in the production of a subset of intermediate goods, which no other country ever is permitted to produce. Each country therefore is specialized from the outset simply by assumption. The specialization is imposed exogenously; comparative advantage plays no role in determining it. Given this exogenous fixed pattern of specialization, trade improves

[13] Asymptotically, of course, balanced growth emerges because the disadvantaged country's lower growth rate means that its share of world output vanishes in the limit, leaving the world growth rate equal to that of the advantaged country.

and equalizes all countries' growth rates by allowing each country to use all intermediate goods that it cannot produce itself. The resulting equilibrium is mathematically equivalent to the interior solution of the model developed here, and corner solutions are excluded *a priori*. In the less constrained analysis of the present model, however, the pattern of production is determined endogenously by comparative advantage, and corner solutions are possible outcomes. In the corners, balanced growth does not occur, and the world income distribution is not stable.

Comparative advantage matters for growth; it determines which good will be produced and traded by each country, and it leads to increased growth for at least some countries through trade. However, absolute advantage matters, too; the pattern of absolute advantages determines whether balanced growth is possible for either individual countries or the world as a whole and whether a particular country will experience a higher growth rate as a result of trade. This importance of absolute advantage is unusual for trade theory.[14]

**V. Two Special Cases**

We now apply the foregoing general analysis to two special cases.

A. <u>A Small Country</u>.

It is usual in trade theory to study a "small" country, so we should see what our theory implies for that case. In the foregoing analysis, we proceeded in two steps, first solving each country's optimization problem taking the world price p as given and then solving for equilibrium p by imposing world trade balance. The analysis of the small country requires only the first step because the world price p is unaffected by the small country's choices. If the small country's TFP ratio $A_S/B_S$ happens to equal p (which equals the world's TFP ratio $A_W/B_W$), the

[14] For two rare exceptions, see Jones (2000) and MacDonald and Markusen (1985).

small country does not bother to trade and has the same growth rate as the rest of the world. Otherwise, trade occurs, and the resulting equilibrium is a corner solution of one of the types analyzed above. If $A_S/B_S > p$, the small country produces only Y and trades to obtain H. If $A_S/B_S < p$, the small country produces only H and trades for Y. The rest of the world is unaffected and behaves like a country in a corner; in particular, its growth rate is unaffected by trade. How trade affects the small country's growth rate depends on the pattern of absolute advantage. If the world has an absolute advantage in one good and the small country has an absolute advantage in the other good, then trade raises small country's growth rate to the world rate. If the small country has an absolute advantage in both goods, its growth rate is higher than the world's but is unaffected by trade.[15] If the small country has an absolute disadvantage in both goods, its growth rate is increased by trade but starts and remains below the world rate.

B. <u>A Pure Consumption Good</u>.

So far, all goods considered have been factors of production. We can examine goods that are not factors of production by considering a special case of the foregoing model in which $\alpha$ equals zero. In that case, K is no longer useful, and all output in the Y sector is devoted to consumption C.[16] The autarkic growth rate for country *i* becomes

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[15]In this case, the small country eventually ceases to be small.

[16]Bond and Trask (1997), in the only other study of pure trade and growth that I know of, use a three-sector model with separate sectors for C, K, and H. They study a single small open economy and assume that H is human capital and non-tradeable. The existence on only one tradeable factor of production makes Bond and Trask's growth implications of trade similar but not identical to those discussed here. The presence of a non-tradeable factor guarantees that growth rates will not be equalized across countries unless the technology for producing that factor is the same for all countries.

(34) $$\gamma_{i,Au} = \frac{1}{\theta}\left[B_i - \delta - \rho\right]$$

Because Y no longer is used as a factor of production, the TFP parameter $A_i$ from the Y sector no longer affects the growth rate. With trade, the growth rate for the interior solution is simply

(35) $$\gamma = \gamma_{1,T} = \gamma_{2,T} = \frac{1}{\theta}\left[B_2 - \delta - \rho\right]$$

The growth rate of both countries is determined only by TFP in the H sector of country 2, which is the country that specializes in H because $A_1 / B_1 > A_2 / B_2$. Country 1's TFP parameter $A_1$ plays no role in the growth rate because now Y is not useful as a factor of production.

As before, an interior solution requires that (33) be satisfied; otherwise the world price *p* will be at one of the boundary values $A_1 / B_1$ or $A_2 / B_2$. When *p* equals $A_2 / B_2$, country 2 has an absolute advantage in both goods. Country 1 produces only Y and imports H, and country 2 produces both Y and H and imports Y. The growth rates for the two countries are obtained from (27) and (29) by substituting $A_2 / B_2$ for *p* and noting that $\alpha = 0$:

(36) $$\gamma_{1,T} = \frac{1}{\theta}\left[A_1\left(\frac{B_2}{A_2}\right) - \delta - \rho\right]$$

(37) $$\gamma_{2,T} = \frac{1}{\theta}\left[B_2 - \delta - \rho\right]$$

A little algebra shows that country 1's growth rate $\gamma_{1T}$ is larger than under autarky but smaller than country 2's growth rate $\gamma_{2T}$. Thus, as when $\alpha$ is not zero, trade has no effect on the growth rate of the absolutely advantaged country, raises the growth rate of the absolutely disadvantaged country, but leaves the disadvantaged country's growth rate below that of the advantaged country. Results are slightly different at the other corner. When *p* equals $A_1 / B_1$, country 1 produces both Y and H and imports H, and country 2 produces only H and imports Y. The growth rates are the same as under autarky:

(38) $$\gamma_{1,T} = \frac{1}{\theta}\left[B_1 - \delta - \rho\right]$$

(39) $$\gamma_{2,T} = \frac{1}{\theta}\left[B_2 - \delta - \rho\right]$$

In this case, trade does not change either country's growth rate.[17]

C. An Important Implication for Economic Development.

This last pair of corner results makes clear a strong policy implication for economic development that emerges from our analysis: what is important for economic growth is not the good that is exported but rather the good that is imported. Growth rates depend on TFP in sectors that produce factors of production. Trade can raise a country's growth rate by allowing that country to substitute another country's higher TFP for its own. It is the importation of capital that can raise a country's growth rate; it does not matter what good is exported in payment. It therefore may be very good policy for underdeveloped countries to export consumption goods and natural resources in exchange for capital (human as well as physical); likewise, it may be very bad policy for those countries to try to create their own industries for producing capital. In particular, a country that has an absolute disadvantage in everything but a comparative advantage in consumption goods can increase its growth rate by concentrating on producing consumption goods and exporting them in exchange for capital. More problematic would be the optimal policy for a country with universal absolute disadvantage but comparative advantage in some type of capital. Such a country cannot raise its growth rate by exporting

[17]The mathematical constraints placed on the problem (i.e., $A_1/B_2 \geq A_1/B_1 = p > A_2/B_2$) imply that $B_1 \geq B_2$, so that $\gamma_{1T} \geq \gamma_{2T}$.

capital.[18]

These results suggest a possible reason why some countries' growth rates are persistently below others'. Any country that is absolutely disadvantaged at everything has nothing to offer the world technologically. It will, of course, have comparative advantage in something, and it should produce and export that good. If it obtains a factor of production in return, then its growth rate will be increased by trade. Nonetheless, the growth rate will remain below that of the technological leaders. The country must move to a position of absolute advantage in something to obtain a growth rate equal to that of most advanced countries. It is interesting that it only needs an absolute advantage in producing one good to attain the highest possible growth rate; that is sufficient to result in effective technology equalization with other advanced countries.

An apparent implication is that a country should drop all barriers to trade if it is to be on its growth rate frontier, but things may not be so straightforward. A country with no absolute advantage cannot attain the highest growth rates prevailing in the rest of the world. To do so, it must raise its absolute advantage in something. Optimal policy may be to protect a domestic industry and perhaps also subsidize it until domestic firms have developed sufficient technical knowledge to achieve absolute advantage in the world. Such a policy will impose short-run costs through inefficient trade barriers and through taxation to finance the subsidy, but if the policy raises the growth rate, then eventually output will be higher than it would have been in the

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[18]This case may be no more than an intellectual curiosity. Capital production, by its nature, seems to be the province of technologically advanced countries. It seems likely that a country with absolute disadvantage in everything will have a comparative advantage in natural resources or certain consumption goods, such as fruit, than in any type of capital.

absence of the policy. This is the old infant industry argument in slightly different garb. Whether the benefits of such a policy outweigh the costs requires analysis beyond the scope of the present study. An important aspect of any such policy, however, is that it must favor goods that can be exchanged for factors of production on the world market if it is to increase the economy's growth rate. The framework presented here permits a proper dynamic analysis of the relevant trade-offs.

The feasibility of eliminating an absolute disadvantage undoubtedly depends on the reason the disadvantage exists. Exogenously low TFP in a particular industry, perhaps because a country lacks appropriate climate, is unlikely to be amenable to cost-effective change, but endogenously low TFP is another matter. Industry composition affects TFP (Chanda and Dalgaard, 2002), so policy that encourages structural change may be able to raise TFP and thus create an absolute advantage.[19] Some kinds of choices also can confer absolute disadvantage. Under dynastic management, for example, owners of firms bequeath management control on their heirs rather than search the market for the best managerial talent. Dynastic management is the predominant pattern of firm management in underdeveloped countries, and there may be good reasons for choosing it (such as solving a principal-agent problem). Whatever the reason for its existence, Caselli and Gennaioli (2002) show that dynastic management tends to create persistent differences in total factor productivity, possibly causing persistent absolute disadvantage. Government policies that discourage such practices could raise TFP and eliminate the disadvantage.

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[19]Chanda and Dalgaard's discussion concerns TFP at the national level, but the same argument apparently applies at the industry level.

**VI. Two Generalizations**

We can relax some of the restrictions used in the foregoing analysis to obtain more general results.

A. Inequality Between $\alpha$ and $\eta$.

When the share parameters $\alpha$ and $\eta$ differ from each other, derivations are considerably messier, but the final results are clear generalizations of what we have obtained so far. For example, in the case of two countries that are not small, the growth rate under autarky is

$$\gamma_{i,Au} = \frac{1}{\theta}\left[\alpha^{\frac{\alpha\eta}{1-\alpha+\eta}}(1-\alpha)^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}}\eta^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}}(1-\eta)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha+\eta}}A_i^{\frac{\eta}{1-\alpha+\eta}}B_i^{\frac{1-\alpha}{1-\alpha+\eta}} - \delta - \rho\right]$$

Notice in particular that $A_i$ and $B_i$ are exponentially weighted as before, just with more complicated exponents. The growth rate under trade that is consistent with balanced growth is

$$\gamma = \frac{1}{\theta}\left[\alpha^{\frac{\alpha\eta}{1-\alpha+\eta}}(1-\alpha)^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}}\eta^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}}(1-\eta)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha+\eta}}A_1^{\frac{\eta}{1-\alpha+\eta}}B_2^{\frac{1-\alpha}{1-\alpha+\eta}} - \delta - \rho\right]$$

which is greater than $\gamma_{i,Au}$.

B. Labor in the Production Function.

A more important generalization is to introduce non-trivial labor into the production functions:

$$Y_{it} = A_i(v_{it}K_{it})^{\alpha}(u_{it}H_{it}L_{it})^{1-\alpha}$$

$$\dot{H}_{it} + \delta H_{it} = B_i[(1-v_{it})K_{it}]^{\eta}[(1-u_{it})H_{it}L_{it}]^{1-\eta}$$

This alteration is important because it allows completely different interpretations of both the production function and the factor $H$. With labor present, the production function no longer is merely a glorified version of the AK model; production now depends on both reproducible and non-reproducible factors. Previously, we interpreted *H* as some arbitrary type of capital; now, we can interpret it in several other ways. One interpretation is human capital, as in Uzawa (1965), Lucas (1988), and Barro and Sala-i-Martin (1995, chapter 5); the *H*-sector then is the education and training sector.[20] In a similar but not identical vein, *H* can be regarded as the stock of knowledge, as in Jones (2003); the *H*-sector then would be the somewhat amorphous "knowledge industry." Yet another type of knowledge interpretation is to regard *H* as accumulated "learning-by-doing," as in Romer (1986). Romer holds K fixed for simplicity, whereas here it varies, so that we have two state variables and accumulation equations whereas Romer has only one. The *H*-sector in the present model describes the accumulation of learning.[21] Finally, *H* can be treated as general labor-augmenting technical progress, of the type identified by Phelps (1966) as necessary for balanced growth. The *H*-sector then would be the R&D sector that delivers that progress, as in the models of Aghion and Howitt (1998).

Endogenous growth requires non-vanishing marginal returns to the reproducible factors (Jones and Manuelli, 1990). A weakness of the AK model is that it obtains this property by simply assuming it. In contrast, with any of the foregoing new interpretations, the exponent of *H* is naturally the additive unitary complement of capital's share, making both production functions homogeneous of degree one in the reproducible factors and thus guaranteeing perpetual endogenous growth. We can rearrange the production functions as

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[20]Note that human capital is exportable. For example, all foreign students who study in the US and then return home to work take their newly-acquired human capital with them.

[21]Romer distinguishes between firm-level learning and aggregate learning, but we can use Barro and Sala-i-Martin's (1995, chapter 4) method to aggregate over identical firms and obtain an aggregate production function containing only aggregate learning.

$$Y_{it} = (A_i L_{it}^{1-\alpha})(v_{it} K_{it})^{\alpha}(u_{it} H_{it})^{1-\alpha}$$

$$\dot{H}_{it} + \delta H_{it} = (B_i L_{it}^{1-\eta})[(1-v_{it})K_{it}]^{\eta}[(1-u_{it})H_{it}]^{1-\eta}$$

We can consider the terms ($A_i L_{it}^{1-\alpha}$) and ($B_i L_{it}^{1-\eta}$) to be the total factor productivity parameters. Because these terms depend on population size, we now have a scale effect. If population is constant over time but different across countries, then all the previous analysis applies with the only difference being that we have another reason for the TFP parameters to differ across countries. None of the foregoing analysis is changed, and all our earlier conclusions obtain.

We see, then, that the AK nature of the production function in the previous analysis was not essential; the previous results on trade's growth effects are robust to introduction of scale effects and research and development.

If population is growing, the foregoing analysis is invalid; it assumed the existence of constant growth rates along a balanced growth path, whereas growing population will cause time-varying growth rates through the ever-changing TFP parameters. There are ways of eliminating these population-scale effects that preserve endogenous growth; see Peretto (1998, 1999), Peretto and Smulders (2002), and Young (1998). Given the nature of these methods, I see no reason why the results developed above would not be retained in extensions that include trade. Developing such extensions would be useful but analytically complicated. They are well beyond the scope of the present paper.[22]

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[22]Another generalization beyond the scope of the present discussion is a model with more than two countries trading only the two goods K and H. In Ricardian static models, this situation produces an ordering of comparative advantages with countries' choice of specialization depending on how their comparative advantage compares to some critical value. Presumably, such a ranking also would emerge in the extended growth model. The implications for the existence of world balanced growth are unclear.

## V. The Equalization Theorem

The main results we have obtained can be summarized in the following theorem.

*Technology Equalization Theorem.* Suppose the following:

(I) the world comprises two countries, denoted 1 and 2, each having the same constant relative risk aversion utility function
(ii) two goods are produced in separate sectors with Cobb-Douglas production functions
(iii) total factor productivities differ across the two countries for each sector.

Then:

(I) If each country has an absolute advantage in one good, a balanced growth rate exists, is common to all countries and the world as a whole, and is globally asymptotically stable. The balanced growth rate with trade is higher than either country's autarkic growth rate and is the same as the growth rate that would emerge under technology transfer.

(II) If one country has an absolute advantage in both goods, a balanced growth rate does not exist for any country or for the world. The growth rate for the country with the absolute advantages is the same as its autarkic growth rate. The growth rate for the other country depends on what kind of good is imported. If a factor of production is imported, the growth rate with trade is more than the autarkic growth rate but less than the growth rate for the country with the absolute advantages. If a consumption good is imported, the growth rate is the same as the autarkic growth rate.

## VI. Conclusion

We have seen that trade in goods can raise the growth rates of all trading partners through a combination of comparative advantage and absolute advantage, without there being any scale effects, technology transfer, research and development, or international investment. Comparative advantage determines the pattern of trade; absolute advantage determines how trade affects growth rates and whether balanced growth paths exist. If every country has an absolute advantage in at least one good, then a balanced growth path for the world exists and is globally asymptotically stable. Along this balanced growth path, country growth rates are identical and are the same as if countries had exchanged technology; trade leads to a growth outcome equivalent to that which would have emerged from technology transfer. If any country has no absolute in any good, then no balanced growth path for the world is feasible. The technologically backward countries' growth rates generally are increased by trade, as if there were a partial technology transfer, but the resulting growth rate remains lower than that of technologically advanced countries. We thus have the major implications that (1) trade generally increases growth rates, (2) trade need not increase a given country's growth rate and need not lead to growth convergence, and (3) trade can lead to growth outcomes that are equivalent to what would emerge from technology transfer. These effects of trade on growth mean that use of closed-economy models to analyze cross-country data is likely to be misleading.

The analysis suggests that a possible reason why some countries' growth rates persistently lag behind others' is that those countries have an absolute disadvantage in producing everything. It therefore also suggests that those same countries may be able to improve their growth rates if they can design government policies that lead to greater TFP in some industry and thus create an absolute advantage. However, it is important in such a case to create the advantage in a good that can be exchanged for factors of production in the world market because trade raises the growth rate only when it results in importation of factors of production.

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## Appendix

**A. Solution under autarky**. The representative agent maximizes his lifetime utility subject to (1), (2), (3), and the usual initial and terminal conditions. The Hamiltonian is

(40)
$$V = \frac{C_t^{1-\theta}-1}{1-\theta} e^{-\rho t} + \phi_t [A(v_t K_t)^{\alpha} (u_t H_t)^{1-\alpha} - \delta K_t - C_t]$$
$$+ \psi_t \{B[(1-v_t)K_t]^{\eta} [(1-u_t)H_t]^{1-\eta} - \delta H_t\}$$

and the necessary conditions include (1), (2), (3), and[23]

(41)
$$\dot{\phi}_t = -\frac{\partial V}{\partial K} = -\phi_t [\alpha v_t A(v_t K_t)^{\alpha-1} (u_t H_t)^{1-\alpha} - \delta]$$
$$- \psi_t \{\eta (1-v_t) B[(1-v_t)K_t]^{\eta-1} [(1-u_t)H_t]^{1-\eta}\}$$

(42)
$$\dot{\psi}_t = -\frac{\partial V}{\partial H} = -\phi_t [(1-\alpha) u_t A(v_t K_t)^{\alpha} (u_t H_t)^{-\alpha}]$$
$$- \psi_t \{(1-\eta)(1-u_t) B[(1-v_t)K_t]^{\eta} [(1-u_t)H_t]^{-\eta} - \delta\}$$

(43)
$$\frac{\partial V}{\partial C} = 0 = C_t^{-\theta} e^{-\rho t} - \phi_t$$

(44)
$$\frac{\partial V}{\partial v} = 0 = \phi_t \alpha K_t A(v_t K_t)^{\alpha-1} (u_t H_t)^{1-\alpha}$$
$$- \psi_t \eta K_t B[(1-v_t)K_t]^{\eta-1} [(1-u_t)H_t]^{1-\eta}$$

(45)
$$\frac{\partial V}{\partial u} = 0 = \phi_t (1-\alpha) H_t A(v_t K_t)^{\alpha} (u_t H_t)^{-\alpha}$$
$$- \psi_t (1-\eta) H_t B[(1-v_t)K_t]^{\eta} [(1-u_t)H_t]^{-\eta}$$

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[23]The necessary conditions also include the initial and transversality conditions, which are suppressed here and in the sequel for brevity.

Henceforth, time subscripts will be omitted except when clarity demands otherwise.

Differentiating (43) with respect to time, rearranging, and using (41) and (44), gives the growth rate $\gamma_C$ for consumption:

(6) $$\gamma_C = \frac{1}{\theta}\left[\alpha A\left(\frac{vK}{uH}\right)^{-(1-\alpha)} - \delta - \rho\right]$$

Using (44) and (45), we can obtain the relation

(46) $$\left(\frac{\eta}{1-\eta}\right)\left(\frac{v}{1-v}\right) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{u}{1-u}\right)$$

Competitive equilibrium requires that the rate of return on K be the same in the two sectors of production, and similarly for the rate of return on H:

(47) $$\frac{\partial Y}{\partial(vK)} = p\frac{\partial(\dot{H}+\delta H)}{\partial[(1-v)K]}$$

$$\Leftrightarrow \quad \alpha A\left(\frac{vK}{uH}\right)^{\alpha-1} = p\eta B\left[\frac{(1-v)K}{(1-u)H}\right]^{\eta-1}$$

(48) $$\frac{\partial Y}{\partial(uH)} = p\frac{\partial(\dot{H}+\delta H)}{\partial[(1-u)H]}$$

$$\Leftrightarrow \quad (1-\alpha)A\left(\frac{vK}{uH}\right)^{\alpha} = p(1-\eta)B\left[\frac{(1-v)K}{(1-u)H}\right]^{\eta}$$

where p is the price of H in terms of Y. We obtain p from (46) and either (47) or (48):

(49) $$p = \left(\frac{A}{B}\right)\left(\frac{\alpha}{\eta}\right)^{\eta}\left(\frac{1-\alpha}{1-\eta}\right)^{1-\eta}\left(\frac{vK}{uH}\right)^{\alpha-\eta}$$

We now can define gross domestic product:

(4) $$Q = Y + p(\dot{H}+\delta H)$$

We want to obtain the balanced growth solution for this economy. Balanced growth requires that all variables grow at constant rates (which may be zero for some variables). We begin with the growth rate of p. We can write p in two ways. Equation (49) is one; the other is obtained by noting that the costate variables $\phi$ and $\psi$ are, respectively, the marginal value of Y-goods and of H-goods in terms of utility. Consequently, their ratio is the price of H in terms of Y, which is p:

$$p = \frac{\psi}{\phi} \tag{50}$$

which implies immediately that

$$\gamma_p \equiv \frac{\dot{p}}{p} = \frac{\dot{\psi}}{\psi} - \frac{\dot{\phi}}{\phi} \tag{51}$$

We then can use (41), (42), and (46) to obtain (after much manipulation) one expression for $\gamma_p$:

$$\gamma_p = (\alpha - \eta)(\gamma_Z + \gamma_K - \gamma_H) \tag{52}$$

where $Z \equiv v/u$.[24] The other equation for $\gamma_p$ is obtained from (49):

$$\gamma_p = A\left\{\alpha\left(Z\frac{K}{H}\right)^{\alpha-1} - (1-\alpha)\left[\left(\frac{A}{B}\right)\left(\frac{\alpha}{\eta}\right)^{\eta}\left(\frac{1-\alpha}{1-\eta}\right)\right]^{-1}\left(Z\frac{K}{H}\right)^{\eta}\right\} \tag{53}$$

Equating these two expression for $\gamma_p$ gives

$$\gamma_Z + \gamma_K - \gamma_H = \frac{A}{\alpha - \eta}\left\{\alpha\left(Z\frac{K}{H}\right)^{\alpha-1} - (1-\alpha)\left[\left(\frac{A}{B}\right)\left(\frac{\alpha}{\eta}\right)^{\eta}\left(\frac{1-\alpha}{1-\eta}\right)\right]^{-1}\left(Z\frac{K}{H}\right)^{\eta}\right\} \tag{54}$$

We obtain the growth rates of K, H, and C from (6), (41), and (42):

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[24]We can write Z as

$$Z \equiv \frac{\alpha(1-\eta)}{(1-\alpha)\eta + (\alpha-\eta)u}$$

by solving (46) for v in terms of u and substituting. This transformation is used in deriving some of the results that follow.

(55) $$\gamma_K = A\left(Z\frac{K}{H}\right)^{\alpha-1} - \frac{C}{K} - \delta$$

(56) $$\gamma_H = B\left(\frac{\eta}{1-\eta}\right)^{\eta}\left(\frac{1-\alpha}{\alpha}\right)^{\eta}\left(Z\frac{K}{H}\right)^{\alpha-1} - \delta$$

(57) $$\gamma_C = \frac{1}{\theta}\left[\alpha A\left(Z\frac{K}{H}\right)^{(\alpha-1)} - \delta - \rho\right]$$

We can use these four equations and the definition of Z to establish the following characteristics of the balanced growth path:

(I) The growth rates of C, K, Y, H, and Q are equal

$$\gamma \equiv \gamma_C = \gamma_K = \gamma_Y = \gamma_H = \gamma_Q$$

(ii) The ratios C/K and K/H therefore are constant.

(iii) The growth rates of p, u, v, and Z are zero

$$\gamma_p = \gamma_u = \gamma_v = \gamma_Z = 0$$

(iv) The value of Z(K/H) is

(58) $$Z\frac{K}{H} = \left(\frac{A}{B}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{1-\alpha}{1-\eta}\right)^{\frac{1}{1-\alpha+\eta}}$$

(v) The value of $\gamma$ is

(7) $$\gamma = \frac{1}{\theta}\left[A^{\frac{\eta}{1-\alpha+\eta}} B^{\frac{1-\alpha}{1-\alpha+\eta}} \alpha^{\frac{\alpha\eta}{1-\alpha+\eta}} (1-\alpha)^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}} \eta^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}} (1-\eta)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha+\eta}} - \delta - \rho\right]$$

(vi) The value of p is

(8) $$p = \left(\frac{A}{B}\right)\left(\frac{\alpha}{\eta}\right)^{\eta}\left(\frac{1-\alpha}{1-\eta}\right)^{1-\eta}\left[\left(\frac{A}{B}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{1-\alpha}{1-\eta}\right)^{\frac{1}{1-\alpha+\eta}}\right]^{\alpha-\eta}$$

**B. The ratio $\psi/\phi$ with trade when $\alpha=\eta$.** Each country takes the world price p as given. We compute an internal price $p_i$ for country *i* to compare with the world price to see whether country *i* should import or export Y. The internal price is the rate at which country *i* can transform Y into H without resorting to international trade, that is, by moving factors of production across sectors. That rate of transformation is given by (49) above. However, the internal price also must equal the ratio of marginal values of H and Y, which is $\psi/\phi$ by (50) above. We thus have:

$$p_i = \frac{\psi_i}{\phi_i} = \frac{A_i}{B_i}$$

where the second equality follows from (8) and the restriction $\alpha=\eta$. If $p > p_i$, H is more valuable on the world market than it is at home. Country *i* can sell a unit of H for more Y on the world market than it can obtain by reducing production of H by one unit and diverting the released factors of production to producing Y. Country *i* specializes in the production of H. The opposite holds when $p < p_i$.