More Holidays For Central Bankers?

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[Preliminary, COMMENTS WELCOME]

Abstract

In their quest for simple, easy to implement and commitment-resembling policies, researchers may have overlooked an obvious candidate that to some extent is already in place - the practice of infrequent monetary policy meetings. We show that under such policy central banks operate as if under short-term sequential commitments and face improved inflation-output variability tradeoff. Improved tradeoff also spreads to periods of central bank's inaction where lower inflationary expectations mute the effects of exogenous shocks. We find that even in the absence of any adjustment costs, holding monetary policy meetings infrequently is preferred to period by period adjustment under discretion. In addition, we solve for the optimal frequency of policy meetings and characterize its determinants. Under reasonable calibration we find that the Federal Open Markets Committee should meet no more than (and possibly less than) twice a year.

1 Introduction

As Clarida et. al. (1999) point out, no major central bank has announced a life-time commitment to a specific monetary policy rule. Thus, theoretical literature has devoted a great deal of attention to designing policies that could in one way or another mimic commitment. In this paper we consider the benefits of a simple policy that to some extent is already in place: the practice of holding infrequent and periodic monetary policy meetings.

Our motivation comes from two observations. First, central banks around the world make monetary policy decisions at discrete times and with differing frequency. Bank of England's recent survey of over ninety central banks found that seven central banks held policy making meetings less than monthly, about thirty six had monthly meetings, while

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the rest made policy decisions more frequently, some even on a daily basis[2]. A natural question is whether there are benefits of such infrequent policy adjustment and what makes some central bankers meet more often than others [3].

Second, in the absence of major shocks to the economy most major central banks hold policy meetings with regular frequency. For example, Bank of Japan's monetary policy meetings take place twice a month, the Governing Council of the European Central Bank meets monthly, while the Federal Open Markets Committee in the U.S. revises the target federal funds rate eight times a year. Moreover, most monetary authorities in developed countries announce the schedule of policy meetings in advance.

Below we argue these practices should be viewed as important characteristics of monetary policy. We show that infrequent but periodic (with pre-announced schedule) policy meetings can serve the commitment purpose when no explicit rule is announced. This is because following each meeting central bankers not only announce a new policy target, but also *de facto* promise to keep it fixed until the next meeting. Such implicit promises can be viewed as sequential short-term commitments. As such they allow the monetary authority to affect short-term private sector forecasts and therefore improve the inflation-output tradeoff at the time of the target adjustment. Since improved tradeoff results in an aggressive response to inflation, inflationary expectations are low even in periods when the central bank rests. This additional effect is of great importance: when comparing a policy of infrequent interventions with pure discretion, we find that the former produces lower sensitivity of inflation to supply shocks in all periods, including periods of central banks' inaction. This line of analysis led us to several findings. First, we show that in the standard New Keynesian model and for most plausible parameter values the policy of infrequent and periodic interventions is preferable to pure discretion, even without any costs of convening policy meetings. Second, we identify key factors contributing to the desirability of infrequent interventions. Thirdly, we solve for the optimal frequency of

[2]See Mahadeva and Sterne (2000), chart 7.5.

[3]At the anecdotal level, many justify observed frequency of policy meetings by administrative difficulties, Board members' opportunity costs, etc. However, this reasoning is not useful in explaining the fact that some major central banks (e.g. Bank of Japan, the ECB and the Bank of England) have policy meetings more often than some smaller ones (e.g. Bank of Canada or the Riksbank).

monetary policy meetings. The optimal frequency is decreasing in the length of price stickiness, the persistence of supply shocks and labor supply elasticity, and is increasing in the volatility of demand shocks and the firms' elasticity of demand. Finally, when the model is calibrated in relation to the U.S. economy, it prescribes a much lower frequency of policy meetings than what is observed: FOMC should meet twice or even once a year.

The analysis below proceeds as follows. We consider a central bank in the Clarida et. al. (1999) world that is not able (or not willing) to make a life-time commitment to a policy rule (i.e. operates under pure discretion). As they show, even in the absence of the Barro and Gordon(1983) natural rate problem, commitment to a simple linear rule is welfare improving because the impact of policy decisions on private sector forecasts improves the inflation-output variability tradeoff. To this we add that a welfare improving commitment need not be life-time. We show that a policy of sequential announcements of short-term commitments (although inferior to life-time commitment) is also preferred to the discretionary outcome and for the same reason. This is discussed in section 2.

Next, in section 3 we characterize a simple policy of infrequent interventions where the central bank vows to adjust the interest rate only every other period. Section 4 discusses model calibration. Then in section 5 we compare the simple policy of discrete interventions with the standard case of discretion. Intuitively, a discretionary policy allows a timely response to exogenous disturbances, but features higher output costs of reducing inflation. On the other hand, under infrequent adjustments the central bank acts as if under commitment every two periods, but leaves some shocks unanswered in non-meeting months. We discuss in detail how benefits of commitment spread over to periods of central bank's inaction making inflation less volatile in all periods, not only when the interest rate is adjusted. We also examine how welfare gains of this simple infrequent adjustment policy depend on parameter values.

Finally, section 6 discusses optimal frequency of policy meetings. The choice of optimal duration between meetings boils down to comparing gains from the ability to affect longer term forecasts with the losses arising from longer periods of inaction. Optimal frequency is decreasing in the length of price stickiness, the persistence of supply shocks and labor supply elasticity, and is increasing in the volatility of demand shocks and the firms' elasticity of demand. In the benchmark specification, chosen to relate to the U.S. economy, it is optimal to revise the target rate twice a year. An alternative calibration suggests even longer periods of FOMC inaction. Section 7 concludes.

2 Short-term Sequential Commitments

2.1 General Framework

To better relate to the existing New Keynesian literature, we consider a central bank in the celebrated world of Clarida et. al. (1999),hereafter CGG99. It seeks to minimize expected quadratic losses of the form:

$$E_0\left(L\right)=\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\alpha x_t^2+\pi_t^2\right) \tag{1}$$

The economy is described by the usual IS and Phillips curve equations:

$$x_t=-\varphi(i_t-E_t\pi_{t+1})+E_tx_{t+1}+g_t \tag{2}$$

$$\pi_t=\lambda x_t+\beta E_t\pi_{t+1}+u_t \tag{3}$$

where the two exogenous state variables u_t (cost-push shifts) and g_t (demand shifts) evolve according to[4]:

$$g_t=\mu g_{t-1}+\hat{g}_t \tag{4}$$

$$u_t=\rho u_{t-1}+\hat{u}_t \tag{5}$$

[4]Exact interpretation of the cost-push and demand shifts depends on the level of generality in the underlying nonlinear model. With government spending, variable markups and technology shocks each shift is a combination of two types of disturbances: g_t incorporates government spending (as in Clarida et. al. (1999) and shocks to the natural rate of interest (as in Woodford (1999)). Similarly, in the general setup u_t is a combination of exogenous variations in firms' markups and shocks to productivity (as in Ireland (2002)).

2.2 Benefits from Life-Time Commitment

CGG99, compare optimal policies with and without commitment. In the absence of commitment, which we will refer to as pure discretion, the central bank does not make any future promises as to how it will adjust the interest rates in the future. Thus every period it solves a static version of (1), taking all forecasts as given. Equilibrium then implies an interest rate policy that always neutralizes demand shocks and makes inflation and output react to supply shocks only:

$$x_t^{nc} = -\tfrac{\lambda}{\lambda^2+\alpha(1-\beta\rho)} u_t; \quad \text{and} \quad \pi_t^{nc} = \tfrac{\alpha}{\lambda^2+\alpha(1-\beta\rho)} u_t; \text{ , or} \\ \pi_t^{nc} = -\tfrac{\alpha}{\lambda} x_t^{nc} \tag{6}$$

The outcome could be improved if the central bank were to announce and commit to a particular policy in advance. Commitment enables the central bank to influence private sector expectations. Such ability effectively reduces output costs of reducing inflation. As CGG99 show, if the central bank announces a rule of the form $\pi = const \cdot u_t$, it can count on private sector forecasts of future inflation to be $E_t\pi_{t+1} = \rho\pi_t$. The Phillips curve equation then becomes:

$$\pi_t = \frac{\lambda}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t \tag{7}$$

The multiplicative constant $\frac{1}{1-\beta\rho}$ captures the effects of current policy actions on expected inflation in the future. To see the importance of this term, recall from CGG99, that under policies that neutralize demand shocks the social loss function can be represented as a multiple of current period losses:

$$-\frac{1}{2}\left(\alpha x_t^2 + \pi_t\right) E_t \sum_{i=0}^{T} \beta^i \left(\frac{u_{t+i}}{u_t}\right)^2$$

where T is the policy horizon (∞ in their case). The modified loss function can be maximized w.r.t. x_t subject to the Phillips curve relation. The F.O.C. is:

$$\alpha x_t + \frac{\partial \pi_t}{\partial x_t} \pi_t = 0$$

The term $\frac{\partial \pi_t}{\partial x_t}$ is therefore crucial in determining inflation-output tradeoff. In the standard case under discretion the Phillips curve is simply (3), hence $\frac{\partial \pi_t}{\partial x_t} = \lambda$ and the solution is (6). On the other hand, under life-time commitment the current period effect of λ is multiplied by the effects on all future discounted forecasts of inflation. The modified Phillips curve (7) implies: $\frac{\partial \pi_t}{\partial x_t} = \frac{\lambda}{1-\beta\rho} > \lambda$. Hence, commitment reduces ceteris paribus per unit output cost of reducing inflation by a factor of $(1 - \beta\rho)$. In equilibrium, we have[5]:

$$\begin{aligned} \pi_t^c &= -\tfrac{\alpha k_1}{\lambda} x_t^c \\ x_t^{rc} &= -\tfrac{\lambda}{\lambda^2+\alpha k_1(1-\beta\rho)} u_t \\ \pi_t^{rc} &= \tfrac{\alpha k_1}{\lambda^2+\alpha k_1(1-\beta\rho)} u_t \end{aligned} \tag{8}$$

where $k_1 = 1 - \beta\rho$.

2.3 Benefits from Short-term Sequential Commitments

The commitment described in the previous section was long-term (in fact, life-time). It may be both unrealistic (e.g. because chairmen of central banks have limited terms in office) and undesirable (e.g. in the presence of model uncertainty or because extraordinary circumstances in the economy may require deviations from the announced rule). As an alternative, central banks could use short-term commitments, i.e. announce policy rules that are valid for a pre-determined period of time. Such short-term promises are also capable of affecting private sector forecasts in periods when the commitment is valid, although not in the long-run. Thus, we should expect it to be suboptimal relative to life-time commitment, but preferred to a case of pure discretion.

As an example, consider a CB that announces a new commitment every other period. Because the promise is only valid for two periods, the CB's powers are restricted to affecting only one period ahead forecasts. We will consider a rule that also neutralizes demand shocks[6]:

$$i_t = \gamma u_t + \frac{1}{\varphi} g_t \tag{9}$$

[5]Note, that this is not a globally optimal policy under commitment, but only within a family of linear interest rate rules that neutralize demand shocks.

[6]This rule is also not globally optimal. We use it to ease exposition and comparison across models.

Thus, the IS equation at the time of announcement becomes:

$$x_t = -\varphi(\gamma u_t - E_t\pi_{t+1}) + E_t x_{t+1} \tag{10}$$

Since the rule is valid in the next period, we have:

$$x_{t+1} = -\varphi(\gamma u_{t+1} - E_{t+1}\pi_{t+2}) + E_{t+1}x_{t+2}$$

Hence, the CB expects people to form the forecasts of next period's output as follows:

$$E_t x_{t+1} = -\varphi\gamma\rho u_t + \varphi E_t\pi_{t+2} + E_t x_{t+2} \equiv -\varphi\gamma\rho u_t + f_{2t}$$

where two period ahead forecasts (summarized in f_{2t}) are taken by the CB as given. Similarly, next period's inflation forecast is:

$$E_t\pi_{t+1} = -\lambda\varphi\gamma\rho u_t + \beta E_t\pi_{t+2} + \lambda f_{2t} \equiv -\lambda\varphi\gamma\rho u_t + f_{3t}$$

Using the expressions for output forecast in the current period IS equation, we obtain:

$$x_t = -\varphi\gamma u_t + \varphi\left(-\lambda\varphi\gamma\rho u_t + f_{3t}\right) - \varphi\gamma\rho u_t + f_{2t}$$

or $\varphi\gamma u_t = -\frac{1}{1+\varphi\lambda\rho+\rho}x_t + f_{4t}$, where f_{4t} captures two period ahead forecasts. The Phillips curve becomes:

$$\begin{aligned}\pi_t &= \lambda x_t + \beta E_t\pi_{t+1} + u_t = \lambda x_t + \beta\left(-\lambda\varphi\gamma\rho u_t + f_{3t}\right) + u_t = \\ &= \lambda\left(1 + \tfrac{\beta\rho}{1+\varphi\lambda\rho+\rho}\right)x_t + f_{5t} + u_t\end{aligned} \tag{11}$$

The extra output term appearing in the Phillips curve represents the effect of the current period announcement on the next period's inflation forecast (compare to equation

7). As discussed above, the sensitivity of inflation with respect to output growth rises by a factor of $\left(1+\frac{\beta\rho}{1+\varphi\lambda\rho+\rho}\right)>1$, thus improving the Central Bank's tradeoff relative to pure discretion. Full solution can be expressed as:

$$\begin{aligned}\pi_t^{1c} &= -\tfrac{\alpha k_2}{\lambda}x_t^{1c}\\ x_t^{1c} &= -\tfrac{\lambda}{\alpha k_2(1-\beta\rho)+\lambda^2}u_t\\ \pi_t^{1c} &= \tfrac{\alpha k_2}{\alpha k_2(1-\beta\rho)+\lambda^2}u_t\end{aligned} \tag{12}$$

where $k_2=\left(1+\frac{\beta\rho}{1+\varphi\lambda\rho+\rho}\right)^{-1}<1$. To summarize, even although $k_1<k_2$ (i.e. lifetime commitment is still preferred), Central Banks that are unwilling to commit forever can achieve a better outcome than pure discretion by offering short-term commitments to policy rules.

3 A Policy of Infrequent Interventions

In this section we take more seriously the empirical reality that central banks seem to be unwilling to commit to explicit policy rules. However, they make policy decisions at specific (pre-announced) dates. We therefore consider a central bank that publicly announces the schedule of its policy meetings but makes no other promises. Clearly, such announcements *de facto* represent a promise to leave the policy unchanged between the meetings and therefore can be viewed as a form of short-term commitment. To the extent that such implicit promises are believable[7], it then follows that the central bank should be able to affect short-term private sector forecasts and (in light of the discussion in the previous section) enjoy a better inflation-output variability tradeoff. A formal analysis of such policy must weigh the benefits of short-term commitment with the costs arising from central bank's inaction. We do this next.

[7] In other words, we abstract from the possibility of frequent unscheduled meetings. This is a reasonable assumption for major economies. For example, since 1982, only in 2001 has the FOMC had more than eight meetings a year. As long as the probability of unscheduled meetings is small, their existence should not affect our main results.

3.1 Effects on Short-term Forecasts

To keep things tractable, we begin with a central bank that commits to holding policy meetings every other period. In every meeting period (denote t) it sets a new interest rate and promises to keep it fixed for two periods[8]. Thus it can only affect the forecasts of next period's variables. In particular, the next period IS equation is:

$$x_{t+1} = -\varphi(i_t - E_{t+1}\pi_{t+2}) + E_{t+1}x_{t+2} + g_{t+1}$$

The private sector then forms expectations accordingly:

$$E_t x_{t+1} = -\varphi i_t + \mu g_t + \underbrace{\varphi E_t \pi_{t+2} + E_t x_{t+2}}_{f_{1t}^e} = -\varphi i_t + \mu g_t + f_{1t}^e \tag{13}$$

where the monetary authority takes f_{1t}^e as given due to the short-term nature of its commitment. Similarly, expected next period inflation can be written as:

$$E_t \pi_{t+1} = -\lambda\varphi i_t + \rho u_t + \lambda\mu g_t + f_{2t}^e \tag{14}$$

where $f_{2t}^e = \lambda f_{1t}^e + \beta E_t \pi_{t+2} = (\lambda\varphi + \beta) E_t \pi_{t+2} + \lambda E_t x_{t+2}$.

To account for the influence of the new policy on people's forecasts, use (13) and (14) to substitute for the expectations in (2) and (3). The modified structural equations become:

$$\begin{aligned} x_t &= -\varphi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t = \qquad (15) \\ &= -\varphi i_t + \varphi(-\lambda\varphi i_t) + \varphi f_{2t}^e - \varphi i_t + f_{1t}^e + \varphi(\rho u_t + \lambda\mu g_t) + \mu g_t + g_t = \\ &= -i_t\left(2\varphi + \varphi^2\lambda\right) + (\varphi\lambda\mu + \mu + 1) g_t + \varphi\rho u_t + \varphi f_{2t}^e + f_{1t}^e \end{aligned}$$

Inflation can be re-written as:

[8] In a sense, it behaves like a firm in a two-generational model of Taylor price setting. A fundamental difference lies in the external effects of central bank's actions.

$$
\begin{aligned}
\pi_t &= \lambda x_t + \beta\left(-\lambda\varphi i_t + \rho u_t + \lambda\mu g_t + f_{2t}^e\right) + u_t = \\
&= -i_t(2\lambda\varphi + \varphi^2\lambda^2 + \lambda\beta\varphi) + \left(\varphi\lambda^2\mu + \lambda\mu + \beta\lambda\mu + \lambda\right) g_t + \\
&\quad + \left(\lambda\varphi\rho + \beta\rho + 1\right) u_t + \left(\varphi\lambda + \beta\right) f_{2t}^e + \lambda f_{1t}^e
\end{aligned} \tag{16}
$$

3.2 Central Bank's Problem

In contrast to the case of pure discretion, the policy of using short-term commitments expands the policy horizon to two periods. At the time of interest rate adjustment the central bank's problem is:

$$
\max_{i_t} -\tfrac{1}{2}\left[\left(\alpha x_t^2 + \pi_t^2\right) + \beta E_t\left(\alpha x_{t+1}^2 + \pi_{t+1}^2\right) + F_t\right] \tag{17}
$$

where F_t represents expected losses beyond $t+1$, which are taken as given. The constraints to the problem include the modified IS and Phillips curve at dates t and $t+1$:

$$
x_t = -i_t\left(2\varphi + \varphi^2\lambda\right) + \left(\varphi\lambda\mu + \mu + 1\right) g_t + \varphi\rho u_t + \varphi f_{2t}^e + f_{1t}^e \tag{18}
$$

$$
\begin{aligned}
\pi_t &= -i_t(2\lambda\varphi + \varphi^2\lambda^2 + \lambda\beta\varphi) + \left(\varphi\lambda^2\mu + \lambda\mu + \beta\lambda\mu + \lambda\right) g_t + \\
&\quad + \left(\lambda\varphi\rho + \beta\rho + 1\right) u_t + \left(\varphi\lambda + \beta\right) f_{2t}^e + \lambda f_{1t}^e
\end{aligned} \tag{19}
$$

$$
x_{t+1} = -\varphi(i_t - E_{t+1}\pi_{t+2}) + E_{t+1}x_{t+2} + g_{t+1} \tag{20}
$$

$$
\pi_{t+1} = \lambda x_{t+1} + \beta E_{t+1}\pi_{t+2} + u_{t+1} \tag{21}
$$

where $E_{t+1}\pi_{t+2}$ and $E_{t+1}x_{t+2}$ can not be manipulated by the Central Bank in the absence of commitment beyond period $t+1$. Inserting the constraints into the objective function and maximizing w.r.t. i_t yields the following F.O.C.:

$$
\alpha x_t \tfrac{\partial x_t}{\partial i_t} + \tfrac{\partial \pi_t}{\partial i_t} + \beta E_t\left(\alpha x_{t+1}\tfrac{\partial x_{t+1}}{\partial i_{t+1}} + \pi_{t+1}\tfrac{\partial \pi_{t+1}}{\partial x_{t+1}}\tfrac{\partial x_{t+1}}{\partial i_{t+1}}\right) = 0.
$$

Or:

$$
\alpha\left(2 + \varphi\lambda\right) x_t + \lambda(2 + \varphi\lambda + \beta)\pi_t = -\beta\left(\alpha E_t x_{t+1} + \lambda E_t \pi_{t+1}\right) \tag{22}
$$

Thus, the optimal policy under infrequent interventions tries to achieve a balance between both current and next period inflation and output[9]. A complete solution to the problem requires describing the equilibrium outcomes in periods of interest rate adjustment and when the interest rate is not adjusted. Here we'll describe the solution in general terms[10].

In periods of intervention, the equilibrium is described by (18), (19), and (22). We solve for an optimal policy of the form:

$$y_t = De_t \tag{23}$$

where $y_t = [x_t, \pi_t]'$, and $e_t = [g_t, u_t]'$. To obtain the solution, we can first substitute the interest rate out using one of the equations. Next, note that since at $t+2$ the central bank faces exactly the same problem as in (17)-(21), rational expectations equilibrium requires $E_t y_{t+2} = DP^2 e_t$, where P is a diagonal matrix with persistence parameters (μ and ρ) on the diagonal. Using this forecasting rule and (23) leaves us with a deterministic system of 4 linear equations in unknown coefficients of D which is straightforward to solve[11].

At time $t+1$, when the CB rests, equilibrium is described by:

$$x_{t+1} = -\varphi(i_t - E_{t+1}\pi_{t+2}) + E_{t+1}x_{t+2} + g_{t+1} \tag{24}$$

$$\pi_{t+1} = \lambda x_{t+1} + \beta E_{t+1}\pi_{t+2} + u_{t+1} \tag{25}$$

where the expectations $E_{t+1}\pi_{t+2}$ and $E_{t+1}x_{t+2}$ must be consistent with the way the CB is expected to adjust interest rates, i.e. $E_{t+1}y_{t+2} = DPe_{t+1}$.

We use the unconditional expectation of the loss function (1) to measure social

[9]Note, that in the absence of next period considerations ($\beta = 0$) , the optimality condition reduces to the standard solution without commitment as in (6).

[10]A complete solution is provided in Appendix A.

[11]Note, that unlike the case of pure discretion, matrix D will generally not have zeros in the first column. This is because with interest rate being fixed for two periods, it is no longer optimal to neutralize demand shocks at the time of adjustment. Instead, a policy of minimizing the average effects of demand and supply shocks over two periods is preferred.

loss. For the case of infrequent interventions welfare costs can be expressed as:

$$E(L^d) = \frac{0.5}{1-\beta}\left(0.5E\left(L^{da}\right) + 0.5E\left(L^{dn}\right)\right)$$

where $E\left(L^{da}\right)$ and $E\left(L^{dn}\right)$ represent unconditional expectations of losses in times of adjustments and inaction, respectively (see Appendix A).

Under pure discretion, expected losses are straightforward to compute from (6)[12]:

$$\begin{aligned} Ex^2 &= \left[\tfrac{\lambda}{\lambda^2+\alpha(1-\beta\rho)}\right]^2 \tfrac{\sigma_{\hat{u}}^2}{1-\rho^2} \\ E\pi_t^2 &= \left[\tfrac{\alpha}{\lambda^2+\alpha(1-\beta\rho)}\right]^2 \tfrac{\sigma_{\hat{u}}^2}{1-\rho^2} \\ E\left(L^{alt}\right) &= \tfrac{0.5}{1-\beta}\left(\alpha Ex^2 + E\pi^2\right) \end{aligned}$$

4 Choice of Parameter Values

Our choice of parameter values is guided by micro-foundations behind the model. The IS equation is derived from the consumption Euler equation, where φ is the inter-temporal elasticity of substitution ($\frac{1}{\sigma}$). The plausible range of σ suggested in the literature, lies between 1 (log utility) and 2. In the benchmark model we use an average: $\varphi = 0.67$ ($\sigma = 1.5$). We choose $\beta = 0.997$ since the focus is on monthly frequency. The Phillips curve is commonly derived from Calvo pricing equation (see Appendix C), which restricts λ as follows:

$$\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\sigma + \frac{1-\eta+\vartheta}{\eta}\right) \tag{26}$$

where θ is the probability that a firm will not change its price in any given period, η is the weight of labor in the Cobb-Douglas production function, ϑ is the inverse of the Frisch elasticity of labor supply. Common assumptions in the literature, suggest $\theta = \frac{1}{12}$ and $\eta = \frac{2}{3}$. There is little agreement regarding the appropriate value of labor supply elasticity. We use a high value of elasticity ($\vartheta = 0.2$), as prescribed by Prescott (2003) and used in Rotemberg and Woodford (1992, 1997) and Gali et. al. (2003). The weight

[12] In some of our experiments we report welfare results under the simple life-time commitment policy in eq. (8). Values of the loss function there were computed similarly as for the case of pure discretion.

of output in the loss function can be expressed as follows [13]:

$$\alpha = \frac{\lambda}{q} \tag{27}$$

where q is the demand elasticity. We set the latter at 6, which implies steady state markup over marginal costs of 20%.

Innovations to cost and demand shifts are assumed uncorrelated with standard deviations of: $\sigma_{\hat{u}} = \sigma_{\hat{g}} = 0.001$. Finally, the persistence parameters are: $\rho = \mu = 0.8$. In section 6 we consider an alternative calibration of the stochastic processes.

5 Discreteness vs. Discretion

In general, the desirability of a policy of infrequent interventions depends on the size of the gains from short-term commitment relative to losses arising from the inability to respond to exogenous shocks in a timely fashion. In the benchmark model welfare loss under the discrete adjustment policy (0.0045) is smaller than under period by period adjustment with discretion (0.0053) [14]. In this section we explain the reasons for the welfare gain as well as its determinants.

5.1 Why less is more

We begin our explanation by examining in detail the responses of output and inflation under the two policies. In the benchmark model the solution for equilibrium in times of adjustments (eq. (23)) is:

$$\begin{pmatrix} x_t^d \\ \pi_t^d \end{pmatrix} = \begin{pmatrix} 0.0445 & -23.0357 \\ -0.0009 & 2.8620 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{28}$$

Under pure discretion the variables in all periods evolve according to:

[13] Wodford (2003), chapter 6, proposition 6.2.

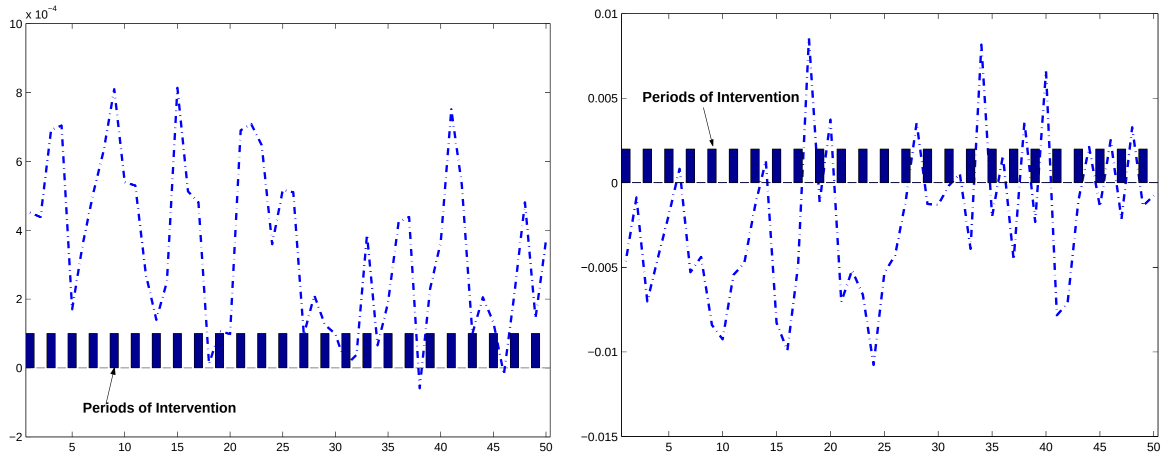
[14] As a reference point, the loss under a simple life-time commitment (eq. 8) is 0.0031.

$$\begin{pmatrix} x_t^{nc} \\ \pi_t^{nc} \end{pmatrix} = \begin{pmatrix} 0.0000 & -19.3301 \\ 0.0000 & 3.2217 \end{pmatrix} \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{29}$$

First, consider the response to cost-push shocks in times of adjustments (second column of each coefficient matrix). Note that under the discrete adjustments policy inflation responds by less and output responds by more to supply shocks. This is a result of two effects. First is the 'substitution effect' of lower output costs of reducing inflation: the central bank tries to "buy" more of inflation reduction at the expense of output[15]. As a result, under infrequent interventions a 1% positive cost-push shock generates a 'saving' of 0.35% in inflation. Second is the presence of demand shocks. Since the policy has to react to demand shocks, it does so at the expense of output, resulting in a larger response of output.

Next we examine periods of central bank's inaction. On the surface level, an easy way to examine volatility of inflation and output is to simulate the model and examine period-by-period differences in absolute deviations of inflation and output under the two policies. Figure 1 displays the resulting series. The left panel indicates that with

Figure 1: Model Simulation: Absolute Deviations of Inflation and Output(left panel: $(|\ \pi^{discretion}\ | - |\ \pi^{discrete}\ |)$, right panel: $(|\ x^{discretion}\ | - |\ x^{discrete}\ |)$)



infrequent interventions volatility of inflation is smaller in all periods, including periods of inaction. The explanation of this comes from the forward looking nature of the model.

[15]CGG99 also make this point.

From equation (28) it follows that rational agents expect aggressive stabilization in periods of intervention. Hence, in contrast to pure discretion, inflationary effects of supply shocks arriving in periods of inaction are not expected to persist for too long. Lower inflationary expectations, in turn, mute the effects of cost-push shocks and contain inflation itself.

A more illuminating illustration of this extra kick from commitment can be constructed directly from equations (28) and (29). Suppose the economy is initially in the steady state (all variables equal zero) when a 1% cost-push shock arrives. Suppose also that the central bank happened to rest at the time of shock arrival. Under pure discretion (see (29)) inflation and output change by 3.22% and −19.33% respectively. The response of inflation under the infrequent adjustment policy can be obtained by combining (24) and (25) with rational forecasts:

$$E_t \begin{pmatrix} x^d_{t+1} \\ \pi^d_{t+1} \end{pmatrix} = \begin{pmatrix} 0.0445 & -23.0357 \\ -0.0009 & 2.8620 \end{pmatrix} \begin{pmatrix} 0.8 & 0 \\ 0 & 0.8 \end{pmatrix} \begin{pmatrix} 0.00 \\ 0.01 \end{pmatrix}$$

Expected output is $E_t x_{t+1} = -0.1843$. Similarly, expected inflation is: $E_t \pi_{t+1} = 0.0229$. From (24) and since $i_t = i_{t-1} = 0$, we can obtain output:

$$x_t = \varphi \cdot 0.0229 - 0.1843 = -0.1690 \quad \text{or} \quad -16.9\%.$$

Inflation's response follows from (25)[16]:

$$\pi_t = -0.1690 \cdot \lambda + \beta \cdot 0.0229 + 0.01 = 0.0298 \quad \text{or} \quad 2.98\%.$$

Thus, while in periods of adjustments inflation is made less volatile *at the expense of output*, in periods of inaction *both* inflation and output can be made less sensitive to supply shocks. To this end, we can conclude that in a world without demand shocks, the policy of infrequent interventions would result in: i) lower volatility of inflation in all periods; ii) higher and lower volatility of output in, respectively, periods of intervention and inaction.

[16]The benchmark calibration implies that λ and α are respectively 0.018 and 0.003.

Figure 2: Comparison of Policies: Role of Shock Volatility

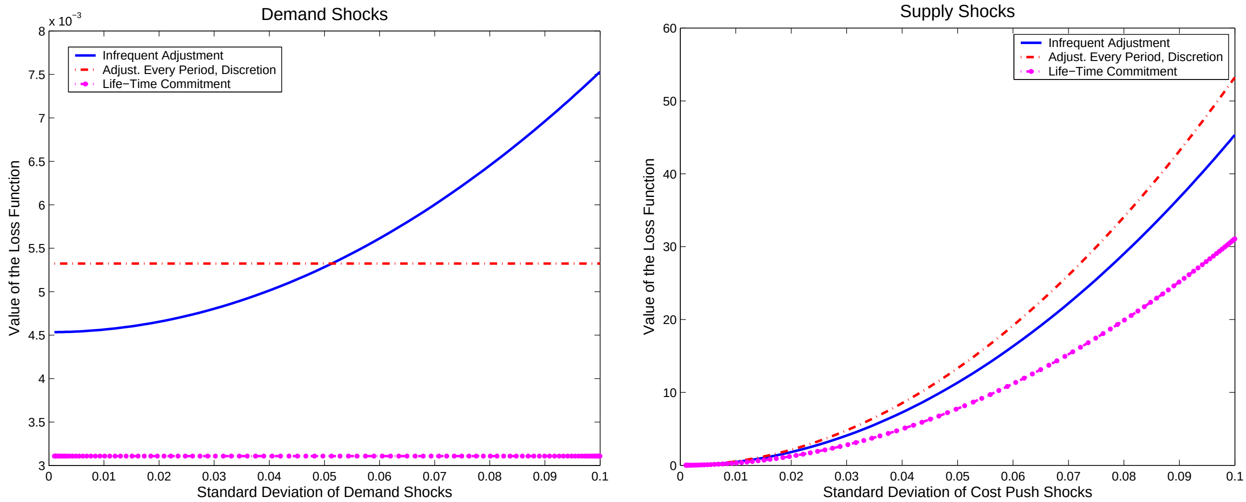
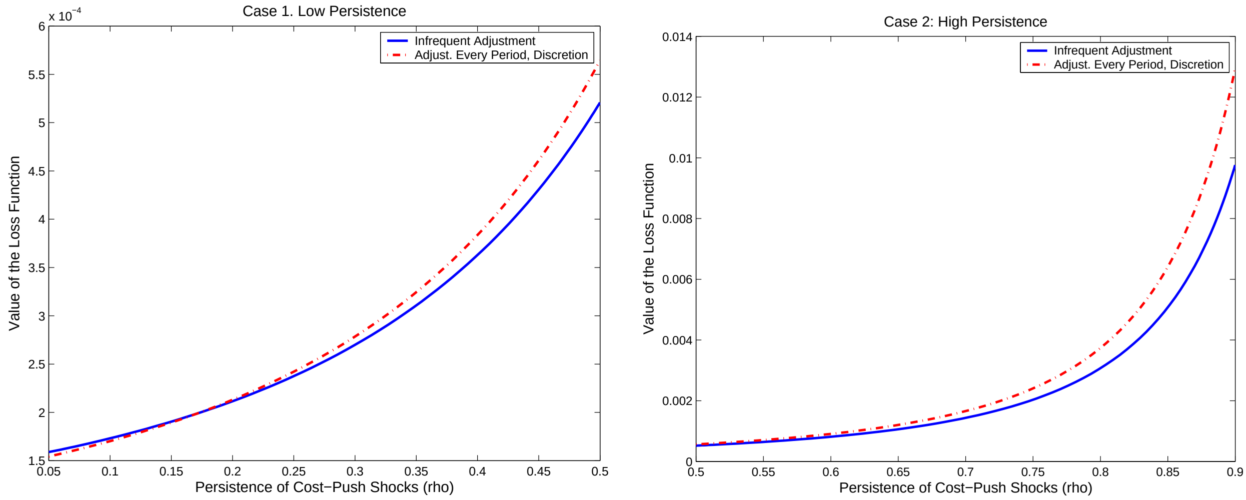


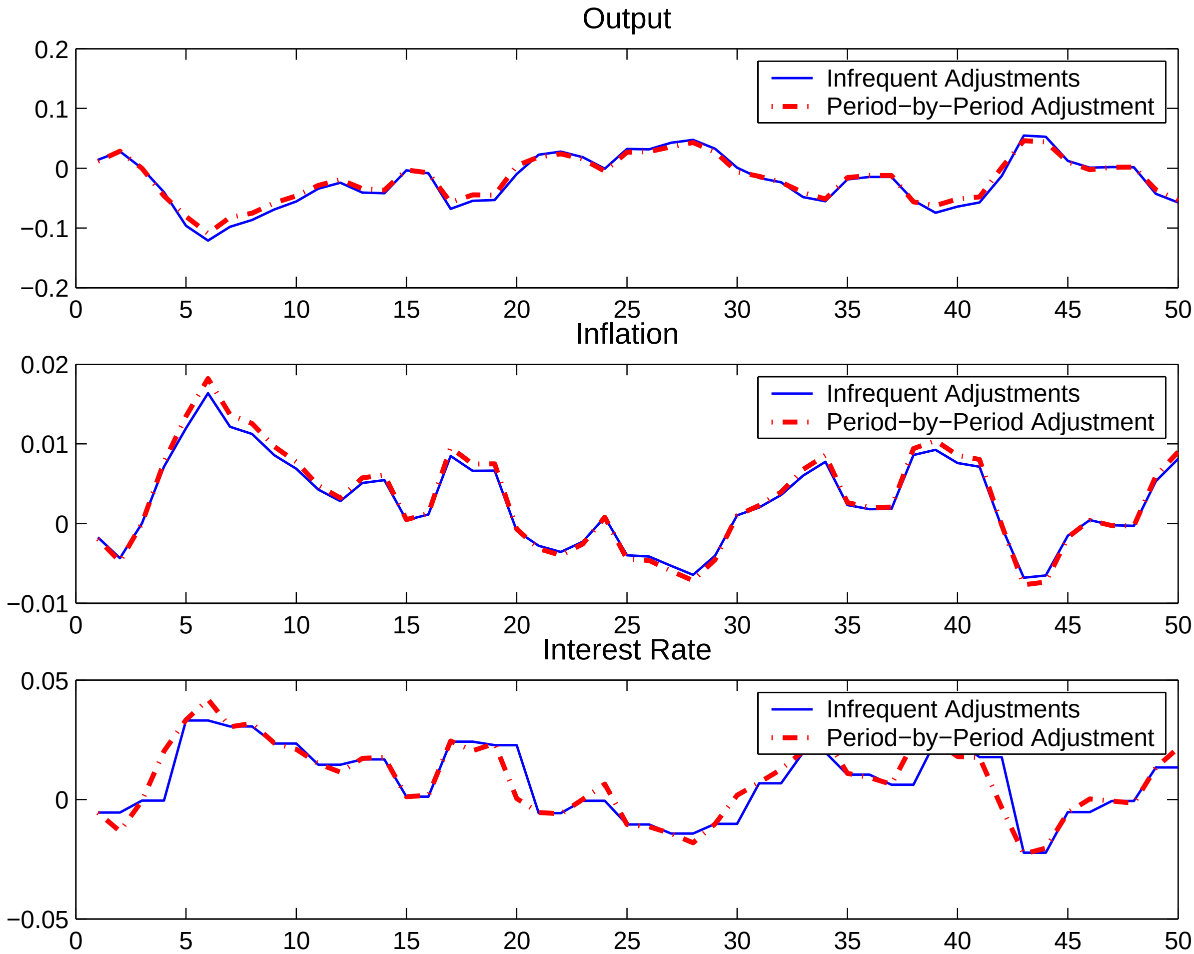
Figure 3: Comparison of Policies: Role of Persistence in Supply Shocks



5.2 Role of Demand Shocks

Next, we consider the effects if demand shocks. The first column of the coefficient matrix (eq. 28) suggests that they are less important both in periods of adjustment and (in light of the discussion above) inaction. One confirmation of this is obtained by comparing the prediction of the previous paragraph with Figure 1. Clearly, deviations from the prediction are small and rare, and more so in case of inflation. Another illustration is provides in Figure 2, which plots social losses for various standard deviations of exogenous innovations. The left panel, suggests that for the central bank to choose discretion over discreteness in the benchmark model, the standard deviation of demand shocks must be 50 times larger than the standard deviation of the supply shocks. On the other hand, the right panel confirms increasing desirability of the infrequent interventions policy when cost-push shocks are very volatile.

Figure 4: Model Simulation

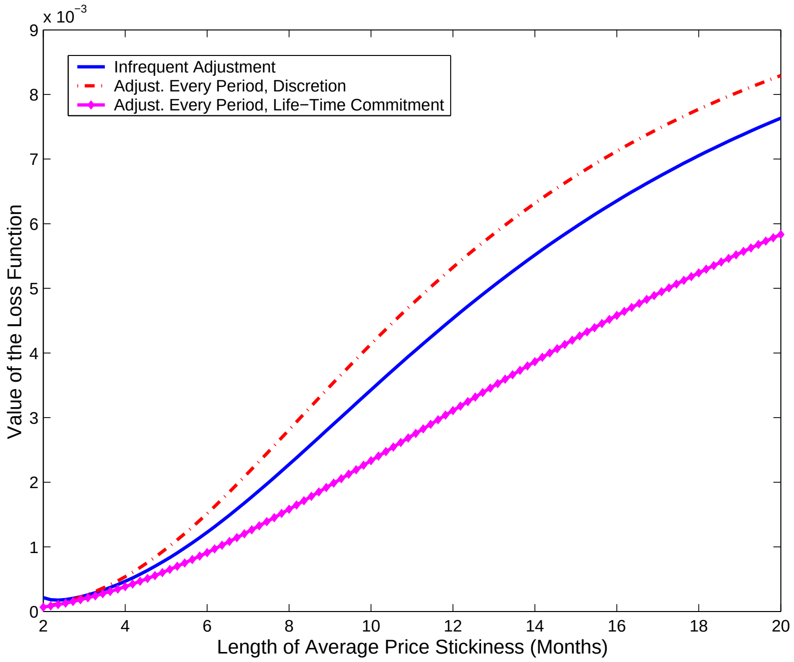


5.3 Persistence of Supply Shocks

As is evident from Figure 3, desirability of infrequent interventions is increasing in the persistence of cost-push shocks[17]. Intuitively, the more persistent they are, the greater is the pre-determined component of conditional short-term forecasts of inflation and output, which, in turn, raises the importance of the central bank's ability to affect those forecasts. A straightforward way to see this is to examine the simple commitment rules of section 1 (equations (8) and (12)). Note that both k_1 and k_2 reduce to unity when $\rho = 0$, and are larger when it is large. Hence, gains from commitment are increasing in $\rho = 0$. In the

[17]Persistence of g_t was not found to affect ranking of the policies.

Figure 5: Comparison of Policies: Role of Price Stickiness



benchmark model with infrequent interventions, when ρ is roughly less than 0.17, benefits from commitment are not sufficient to outweigh the effects of demand shocks. However, this threshold is far below what is empirically plausible.

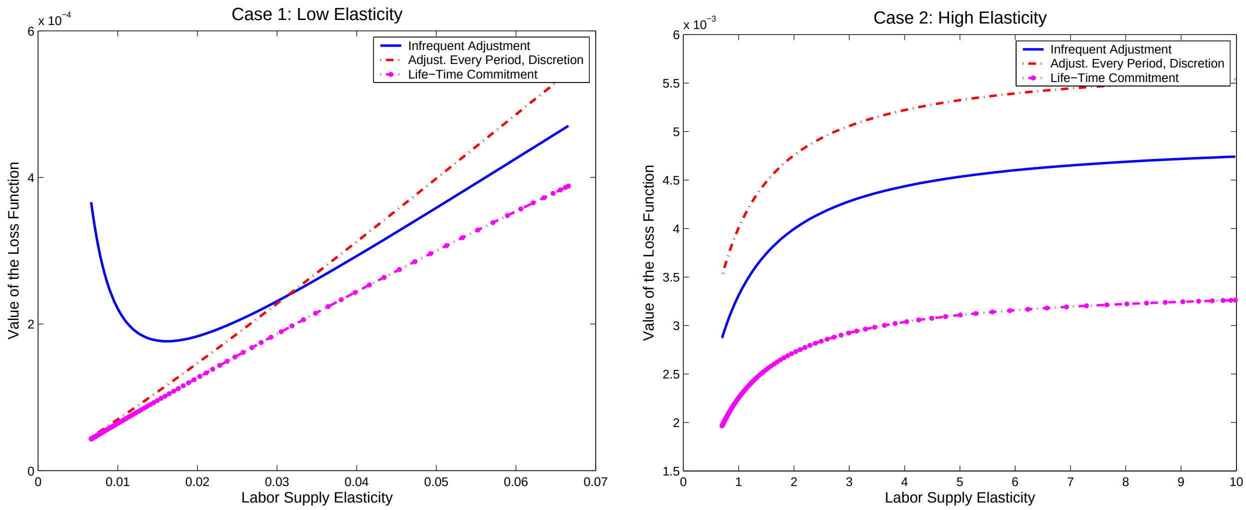
As a summary of the previous discussion, Figure 4 displays simulated inflation and output series under the two policies. We see that, as predicted, the policy of discrete interventions achieves a more stable inflation and a slightly less stable output [18]. Next we examine how the two policies perform with variations in other parameters.

5.4 The Role of Price Stickiness

Duration of average price stickiness in the Calvo model is determined by the firms' probability of not adjusting their prices $\left(duration = \frac{1}{1-\theta}\right)$. Thus, longer price stickiness decreases λ (see eq. 26), resulting in i) smaller sensitivity of inflation to output fluctuations (hence smaller effect of demand shocks on inflation); and ii) lesser weight of output in the social loss function (see eq. 27). In light of the discussion above, both effects work to increase the desirability of a discrete adjustment policy (see Figure 5).

[18] Differences between the two policies will become more evident in section 6 when we solve for the optimal frequency of policy meetings

Figure 6: Comparison of Policies: Role of Labor Supply Elasticity



5.5 Labor Supply Elasticity

Perhaps the most controversial parameter in macroeconomics is the elasticity of labor supply, $\frac{1}{\vartheta}$. Most empirical estimates based on micro level data suggest more realistic values of elasticity in the range of near zero to 0.5 (See, for example, Altonji (1986) or Domeij and Floden (2002)). On the other hand, most of the macro literature suggests quite the opposite (e.g. Woodford and Rotemberg (1992)). Prescott (2003) argues that a highly elastic labor supply is more plausible to account for cross-country variations in labor effort. In the context of our model, when labor is inelastic (high ϑ), firms marginal cost schedules are very steep, which raises the impact of exogenous shocks on inflation and makes output stability a priority. The opposite is true when labor supply is highly elastic. Figure (6) suggests that discrete adjustment policy is preferred when elasticity is greater than roughly 0.03, which is plausible from both camps' perspectives.

6 Optimal Frequency of Interventions

Next, we seek to characterize the optimal frequency of interventions. This is done in two steps. First we develop a solution to a more general model where the central bank revises the interest rate every $T + 1$ periods[19]. Then we choose optimal T based on

[19] The solution is provided in Appendix B.

expected social losses. Figure 7 plots values of the social loss function for various T in the benchmark model. The optimal frequency of policy meetings is twice a year ($T = 5$)[20]. Under optimal frequency of adjustments the value of the loss function reduces to 0.0034 and is much closer to the case of simple commitment than under the previous section's policy. To further illustrate gains from moving to the optimal frequency of adjustments,

Figure 7: Social Losses and Frequency of Interventions

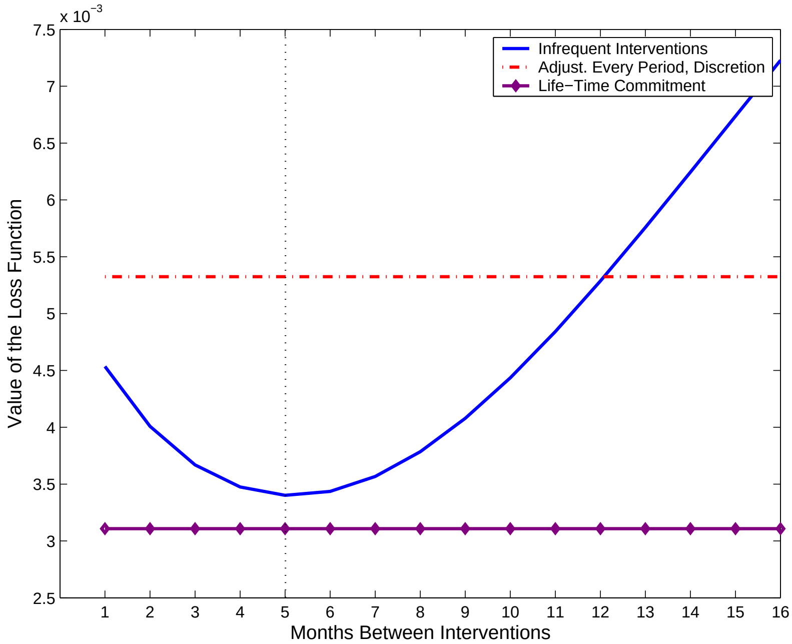
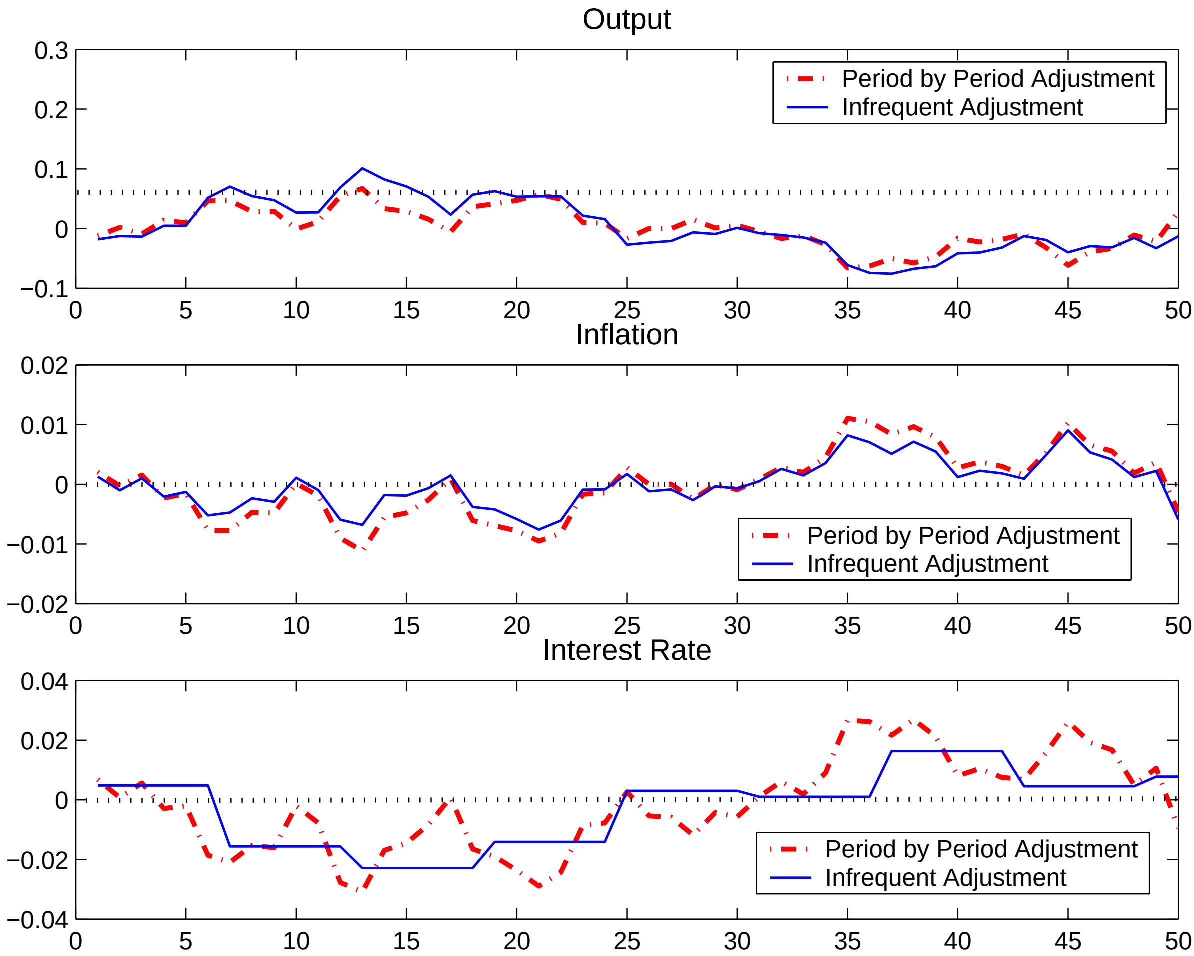


Figure 8 presents simulated series under the optimal frequency of interventions. It shows that choosing the frequency of meetings optimally can achieve sizable gains in stabilizing inflation (compare with Figure 4). Another observation is that interest rate movements are significantly smoother. This is interesting in light of the recent evidence on interest rate smoothing in developed countries. Smoothness in this model stems in part from lengthier duration of interest fixity and in part from the central bank's ability to affect longer-term forecasts. The latter is related to Woodford (1999)'s arguments for interest rate smoothing: when monetary authority can affect longer term forecasts, it takes a smaller change in interest rates to achieve the desired effect on output and inflation. Finally, Figure 9 explores how the optimal time between policy interventions varies with model parameters. Interpretation there is directly related to the discussion in previous sections.

[20]Note, that our solution restricts T to integers only. One option to approximate a more general solution is to use a polynomial fit based on Figure 7. See e.g. Miranda and Fackler (2002).

Figure 8: Model Simulation Under Optimal Frequency of Adjustments

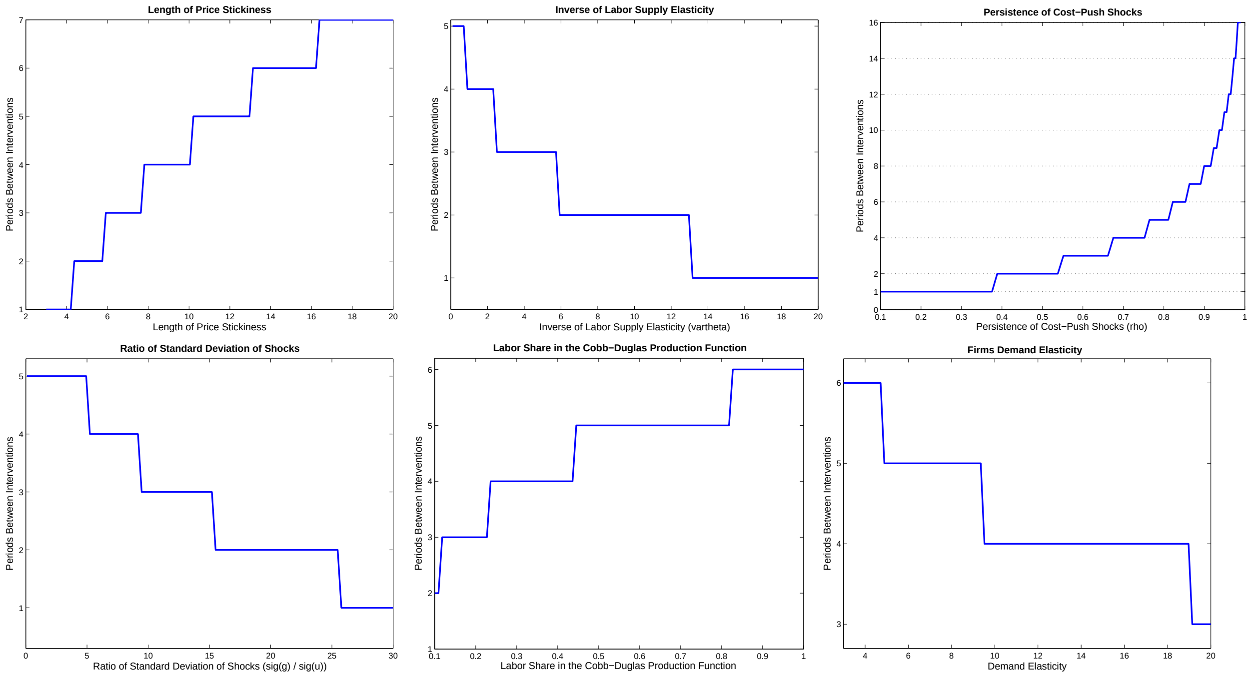


Factors that decrease λ (see eq. 26) tend to raise benefits of infrequent interventions and imply longer durations of inaction (the reasons are the same as in section 5.3). Similarly, more persistent supply shocks raise the importance of longer term forecasts and increase the optimal duration between meetings. Interestingly, with higher persistence, optimal length of inaction rises exponentially. Finally, higher volatility of demand shocks relative to supply shocks raises expected losses from inaction and works to increase the desirable frequency of monetary policy meetings.

6.1 How Often Should the FOMC Meet?

Most parameters used in calibrating the benchmark model are commonly used in relation to the U.S. economy. Since many of them are a source of disagreements, we consider 5 months as a conservative estimate of the optimal duration between FOMC meetings

Figure 9: Optimal Frequency of Adjustment and Parameter Values

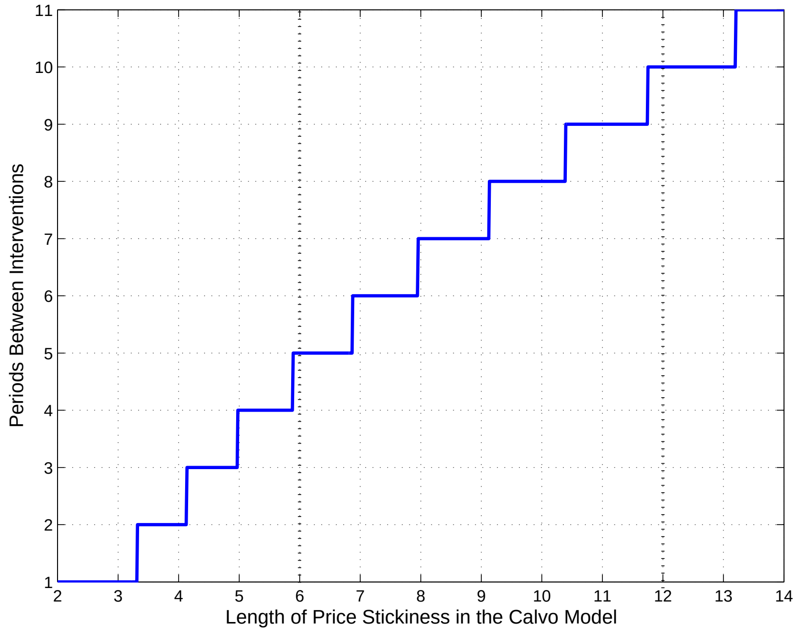


and examine how this policy prescription changes with key parameters. Most important parameters are those, describing the stochastic processes of demand and supply shocks. Ireland (2002) estimates these based on the U.S. post-WWII data. The principal difference between his estimates and our benchmark model is higher persistence of both shocks[21] in the range of 0.95-0.96. High persistence is quite plausible, especially in the short-run [22]. Second we take a middle ground with respect to labor supply elasticity and set $\vartheta = 1$, instead of 0.2. Finally, disagreements exist about the exact interpretation of the average length of price stickiness in the Calvo model about the value of its empirical counterpart. Therefore, we examine optimal duration between policy meetings as a function of the price stickiness in the Calvo model. The result estimate is presented in Figure 10. Assuming the duration of 6 and 12 months in the Calvo model as boundaries of plausibility our alternative calibration suggests that the FOMC should roughly meet between twice and once a year.

[21] Ireland (2002) estimates a system, that matches (2) - (5) combined with a Taylor-type rule describing monetary policy. The analogue of the demand shock follows AR(1) with a persistence parameter of 0.9590 and a standard deviation of innovations of 0.001. The supply shock analogue is a linear combination of productivity and markup variations, both assumed uncorrelated. The translation to our model implies that the process for u_t can be approximately described by $\rho = 0.9586$, and $\sigma_{\hat{u}} = 0.0021$.

[22] Referring to Figure 9, such persistence implies that the FOMC should meet roughly once a year

Figure 10: Optimal Duration Between FOMC Meetings: Alternative Calibration



7 Conclusion

In this paper we have examined the issue of optimal frequency of monetary policy meetings. Viewing infrequent adjustment of monetary policy as simple short-term sequential commitments, we showed that it is preferred to period by period adjustment under discretion. Crucial in our argument is the finding that benefits from commitment spread to periods of central bank's inaction. This happens because expectations of aggressive inflation stabilization at the time of policy adjustment contain inflation and mute effects of exogenous shocks in times when the central bank rests. In addition we have provided a solution for the optimal frequency of policy meetings. For most sensible parameters describing the U.S. economy, the model prescribes holding FOMC meetings twice a year or less.

The analysis in the paper suggests several directions for fruitful research. First, in light of Fuhrer (1997) and Mankiw and Reis (2002), it would be interesting to examine optimal frequency of policy decisions in an environment with the presence of backward-looking agents.

Second, one of the real world determinants of frequency of policy meetings is uncertainty faced by the central bank itself. Parameter and model uncertainty on one hand raises the attractiveness of short-term relative to life-time commitments. On the other hand, it is likely to call for more frequent interventions. A complete analysis of these issues would certainly increase our understanding of the proper policy design.

Thirdly, central banks around world use targets at different horizons. The exact horizon of the target is likely to affect optimal frequency of policy meetings.

Fourth, our analysis could be extended by explicitly introducing emergency/unscheduled meetings. Their presence is likely to lower inflationary expectations, requiring less scheduled meetings.

Fifth, the Federal Reserve uses often uses additional tools in affecting private sector expectations, such as bias announcements[23]. Private sector and the media perceive bias announcements as indications of future policy changes. The latter clearly gives the Fed an extra leverage in influencing private sector forecasts.

Many more extensions are possible.

[23] Conley, Dupor and Mirzoev (2004) discuss the usefulness of bias announcements in the estimation of monetary policy rules.

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A Equilibrium in the Model with Interventions Every Other Period

Here we describe how the optimal policy and equilibrium conditions were solved for under a policy adjusting the interest rate every other period. The solution here corresponds to a slightly more general case, where the loss function in equation (1) also has an interest rate stabilization term ωi_t. When $\omega = 0$ the solution corresponds to the model in section 3.

A.1 Periods of Central Bank Intervention

The equilibrium in periods when the Central Bank adjusts the target rate is described by four equations:

$$\alpha\left(2+\varphi\lambda\right)x_t + \lambda(2+\varphi\lambda+\beta)\pi_t + \tfrac{\omega}{\varphi}\left(1+\beta\right)i_t = -\beta\left(\alpha E_t x_{t+1} + \lambda E_t \pi_{t+1}\right) \tag{30}$$

$$x_t = -\varphi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t \tag{31}$$

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t \tag{32}$$

$$E_t x_{t+1} = -\varphi i_t + \varphi E_t \pi_{t+2} + E_t x_{t+2} + \mu g_t \tag{33}$$

$$E_t \pi_{t+1} = -\lambda\varphi i_t + \left(\lambda\varphi+\beta\right)E_t\pi_{t+2} + \lambda E_t x_{t+2} + \rho u_t + \lambda\mu g_t \tag{34}$$

First we reduce the system by substituting one period ahead forecasts into current period equilibrium equations. This yields:

$$\begin{aligned} &\alpha\left(2+\varphi\lambda\right)x_t + \lambda\left(2+\varphi\lambda+\beta\right)\pi_t - \beta\varphi\left(\alpha+\lambda^2\right)i_t + \tfrac{\omega}{\varphi}\left(1+\beta\right)i_t = \\ &= -\beta\left(\alpha\varphi+\varphi\lambda^2+\lambda\beta\right)E_t\pi_{t+2} - \beta\left(\lambda^2+\alpha\right)E_t x_{t+2} - \rho\lambda\beta u_t - \alpha\beta\mu g_t - \lambda^2\beta\mu g_t \end{aligned} \tag{35}$$

$$\begin{aligned} x_t + i_t\varphi\left(2+\varphi\lambda\right) = \varphi\left(\lambda\varphi+\beta+1\right)E_t\pi_{t+2} + \left(\lambda\varphi+1\right)E_t x_{t+2} + \\ + \left(\varphi\lambda\mu+\mu+1\right)g_t + \varphi\rho u_t \end{aligned} \tag{36}$$

$$\pi_t - \lambda x_t + \beta\lambda\varphi i_t = \left(\lambda\beta\varphi+\beta^2\right)E_t\pi_{t+2} + \beta\lambda E_t x_{t+2} + \left(\beta\rho+1\right)u_t + \beta\lambda\mu g_t \tag{37}$$

Next, since expectations of i_t do not appear in the system, we can substitute the interest rate out. The last equation implies:

$$i_t = -\frac{1}{\beta\varphi\lambda}\pi_t + \frac{1}{\beta\varphi}x_t + E_t\pi_{t+2}\left(1+\frac{\beta}{\varphi\lambda}\right) + \frac{1}{\varphi}E_t x_{t+2} + \frac{\beta\rho+1}{\beta\varphi\lambda}u_t + \frac{\mu}{\varphi}g_t \tag{38}$$

The system becomes two-dimensional:

$$\begin{aligned} &\left(\alpha+\varphi\lambda\alpha-\lambda^2+\tfrac{1+\beta}{\beta\varphi}\tfrac{\omega}{\varphi}\right)x_t + \left(2\lambda+\varphi\lambda^2+\beta\lambda+\tfrac{\alpha+\lambda^2}{\lambda}-\tfrac{1+\beta}{\beta\varphi\lambda}\tfrac{\omega}{\varphi}\right)\pi_t = -\left(1+\beta\right)\tfrac{\omega}{\varphi}\tfrac{\mu}{\varphi}g_t \\ &\left(\beta^2\tfrac{\alpha}{\lambda} - \left(1+\beta\right)\tfrac{\omega}{\varphi}\left(1+\tfrac{\beta}{\varphi\lambda}\right)\right)E_t\pi_{t+2} - \tfrac{1+\beta}{\varphi}\tfrac{\omega}{\varphi}E_t x_{t+2} + \left(\left(\alpha+\lambda^2\right)\tfrac{\beta\rho+1}{\lambda} - \rho\lambda\beta - \left(1+\beta\right)\tfrac{\omega}{\varphi}\tfrac{\beta\rho+1}{\beta\varphi\lambda}\right)u_t \end{aligned} \tag{39}$$

$$\tfrac{\beta+2+\lambda\varphi}{\beta}x_t - \tfrac{2+\lambda\varphi}{\beta\lambda}\pi_t = \left(-\varphi-2\tfrac{\beta}{\lambda}\right)E_t\pi_{t+2} - E_t x_{t+2} + \left(1-\mu\right)g_t + \left(\varphi\rho - \left(2+\varphi\lambda\right)\tfrac{\beta\rho+1}{\beta\lambda}\right)u_t \tag{40}$$

It can be represented as:

$$Ay_t = BE_t\left(y_{t+2}\right) + Ce_t$$

$$e_{t+1} = Pe_t + \hat{e}_{t+1}$$

where $y_t = [x_t, \pi_t]'$ and $e_t = [g_t, u_t]'$. We are looking for the solution of the type: $y_t = De_t$,so that $E_t y_{t+2} = DP^2 e_t$. Insert the solution into the system:

$$ADe_t = BDP^2 e_t + Ce_t$$

Thus the matrix of coefficients D must satisfy:

$$AD - BDP^2 - C = 0 \tag{41}$$

where the matrices are: $A = \begin{pmatrix} \alpha + \varphi\lambda\alpha - \lambda^2 + \frac{1+\beta}{\beta\varphi}\frac{\omega}{\varphi} & \left(2\lambda + \varphi\lambda^2 + \beta\lambda + \frac{\alpha+\lambda^2}{\lambda} - \frac{1+\beta}{\beta\varphi\lambda}\frac{\omega}{\varphi}\right) \\ \frac{\beta+2+\lambda\varphi}{\beta} & -\frac{2+\lambda\varphi}{\beta\lambda} \end{pmatrix}$;
$y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}$; $P^2 = \begin{pmatrix} \mu^2 & 0 \\ 0 & \rho^2 \end{pmatrix}$; $D = \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix}$;
$B = \begin{pmatrix} -\frac{1+\beta}{\varphi}\frac{\omega}{\varphi} & \left(\beta^2\frac{\alpha}{\lambda} - (1+\beta)\frac{\omega}{\varphi}\left(1 + \frac{\beta}{\varphi\lambda}\right)\right) \\ -1 & -\varphi - 2\frac{\beta}{\lambda} \end{pmatrix}$;
$C = \begin{pmatrix} -(1+\beta)\frac{\omega}{\varphi}\frac{\mu}{\varphi} & \left((\alpha+\lambda^2)\frac{\beta\rho+1}{\lambda} - \rho\lambda\beta - \frac{\omega}{\varphi}(1+\beta)\frac{\beta\rho+1}{\beta\varphi\lambda}\right) \\ 1-\mu & \varphi\rho - (2+\varphi\lambda)\frac{\beta\rho+1}{\beta\lambda} \end{pmatrix}$;
The full equation is:

$$\begin{gathered} \begin{pmatrix} \alpha + \varphi\lambda\alpha - \lambda^2 + \frac{1+\beta}{\beta\varphi}\frac{\omega}{\varphi} & \left(2\lambda + \varphi\lambda^2 + \beta\lambda + \frac{\alpha+\lambda^2}{\lambda} - \frac{1+\beta}{\beta\varphi\lambda}\frac{\omega}{\varphi}\right) \\ \frac{\beta+2+\lambda\varphi}{\beta} & -\frac{2+\lambda\varphi}{\beta\lambda} \end{pmatrix}\begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix} - \\ -\begin{pmatrix} -\frac{1+\beta}{\varphi}\frac{\omega}{\varphi} & \left(\beta^2\frac{\alpha}{\lambda} - (1+\beta)\omega\left(1+\frac{\beta}{\varphi\lambda}\right)\right) \\ -1 & -\varphi - 2\frac{\beta}{\lambda} \end{pmatrix}\begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix}\begin{pmatrix} \mu^2 & 0 \\ 0 & \rho^2 \end{pmatrix} = \\ = \begin{pmatrix} -(1+\beta)\frac{\omega}{\varphi}\frac{\mu}{\varphi} & \left((\alpha+\lambda^2)\frac{\beta\rho+1}{\lambda} - \rho\lambda\beta - (1+\beta)\frac{\omega}{\varphi}\frac{\beta\rho+1}{\beta\varphi\lambda}\right) \\ 1-\mu & \varphi\rho - (2+\varphi\lambda)\frac{\beta\rho+1}{\beta\lambda} \end{pmatrix} \end{gathered} \tag{42}$$

To save space, denote:
$a_1 = \alpha + \varphi\lambda\alpha - \lambda^2 + \frac{1+\beta}{\beta\varphi}\frac{\omega}{\varphi}$; $a_2 = \left(2\lambda + \varphi\lambda^2 + \beta\lambda + \frac{\alpha+\lambda^2}{\lambda} - \frac{1+\beta}{\beta\varphi\lambda}\frac{\omega}{\varphi}\right)$;
$a_3 = \frac{\beta+2+\lambda\varphi}{\beta}$; $a_4 = -\frac{2+\lambda\varphi}{\beta\lambda}$;
$b_1 = -\frac{1+\beta}{\varphi}\frac{\omega}{\varphi}$; $b_2 = \left(\beta^2\frac{\alpha}{\lambda} - (1+\beta)\frac{\omega}{\varphi}\left(1+\frac{\beta}{\varphi\lambda}\right)\right)$;
$b_3 = -1$; $b_4 = -\varphi - 2\frac{\beta}{\lambda}$;
$c_1 = -(1+\beta)\frac{\omega}{\varphi}\frac{\mu}{\varphi}$; $c_2 = \left((\alpha+\lambda^2)\frac{\beta\rho+1}{\lambda} - \rho\lambda\beta - (1+\beta)\frac{\omega}{\varphi}\frac{\beta\rho+1}{\beta\varphi\lambda}\right)$;

$c_3 = 1-\mu$; $c_4 = \varphi\rho - (2+\varphi\lambda)\frac{\beta\rho+1}{\beta\lambda}$;
$h_1 = (a_1 - b_1\mu^2)$; $h_2 = (a_2 - b_2\mu^2)$
$h_3 = (a_3 - b_3\mu^2)$; $h_4 = (a_4 - b_4\mu^2)$
$h_5 = (a_1 - b_1\rho^2)$; $h_6 = (a_2 - b_2\rho^2)$
$h_7 = (a_3 - b_3\rho^2)$; $h_8 = (a_4 - b_4\rho^2)$
Then the system becomes:

$$\begin{aligned} h_1 d_1 + h_2 d_3 &= c_1; & h_3 d_1 + h_4 d_3 &= c_3; \\ h_5 d_2 + h_6 d_4 &= c_2; & h_7 d_2 + h_8 d_4 &= c_4. \end{aligned} \tag{43}$$

The solution is:

$$\begin{aligned} d_1 &= \frac{h_4 c_1 - c_3 h_2}{-h_3 h_2 + h_4 h_1}; & d_2 &= \frac{h_8 c_2 - c_4 h_6}{-h_7 h_6 + h_8 h_5}; \\ d_3 &= -\frac{-h_1 c_3 + c_1 h_3}{-h_3 h_2 + h_4 h_1}; & d_4 &= \frac{h_5 c_4 - c_2 h_7}{-h_7 h_6 + h_8 h_5}. \end{aligned} \tag{44}$$

Finally, using the solution we can re-write the interest rate equation as follows:

$$i_t = \left(-\frac{1}{\beta\varphi\lambda}\pi_t + \frac{1}{\beta\varphi}x_t + E_t\pi_{t+2}\left(1+\frac{\beta}{\varphi\lambda}\right) + \frac{1}{\varphi}E_t x_{t+2} + \frac{\beta\rho+1}{\beta\varphi\lambda}u_t + \frac{\mu}{\varphi}g_t\right) = \tag{45}$$

$$= \begin{pmatrix} \frac{1}{\beta\varphi} \\ -\frac{1}{\beta\varphi\lambda} \end{pmatrix}' y_t + \begin{pmatrix} \frac{1}{\varphi} \\ 1+\frac{\beta}{\varphi\lambda} \end{pmatrix}' E_t y_{t+2} + \begin{pmatrix} \frac{\mu}{\varphi} \\ \frac{\beta\rho+1}{\beta\varphi\lambda} \end{pmatrix}' e_t =$$

$$= \begin{pmatrix} \frac{1}{\beta\varphi} \\ -\frac{1}{\beta\varphi\lambda} \end{pmatrix}' De_t + \begin{pmatrix} \frac{1}{\varphi} \\ 1+\frac{\beta}{\varphi\lambda} \end{pmatrix}' DP^2 e_t + \begin{pmatrix} \frac{\mu}{\varphi} \\ \frac{\beta\rho+1}{\beta\varphi\lambda} \end{pmatrix}' e_t$$

A.2 Periods of Non-Adjustment

In periods of non-adjustment the equilibrium is described by:

$$x_t = -\varphi(i_{t-1} - E_t\pi_{t+1}) + E_t x_{t+1} + g_t \tag{46}$$

$$\pi_t - \lambda x_t = \beta E_t \pi_{t+1} + u_t \tag{47}$$

Note, that the expectations $E_t\pi_{t+1}$ and $E_t x_{t+1}$ must be consistent with the way the CB is expected to adjust interest rates, rather than with the interest rate being fixed. Hence, they must satisfy $E_t y_{t+1} = DPe_t$, where D matrix has been solved for in the previous section. The values of x_t and π_t are therefore straightforward to compute.

A.3 Solution for the Social Loss Under Discrete Interventions

Since half of the time the interest rate is left unadjusted, its unconditional variance is the same in both periods. However, unconditional variances of inflation and output are different in the two periods. We decompose the total social loss under the proposed policy into three parts: losses due to output and inflation in the two types of periods and the interest rate component:

$$E\left(L^d\right) = \frac{1}{2}\left(E\left(L^{da}\right) + E\left(L^{dn}\right)\right) + \frac{0.5}{1-\beta}\omega E\left(r^2\right) \tag{48}$$

where $E\left(L^{da}\right)$ and $E\left(L^{dn}\right)$ are respectively expected losses in periods of adjustment and in periods when the Central Bank rests.

A.3.1 Losses In Periods of Interventions

When the CB adjusts the interest rate equilibrium variables evolve as:

$$\begin{aligned} x_t^{da} &= d_1 u_t + d_2 g_t \\ \pi_t^{da} &= d_3 u_t + d_4 g_t \end{aligned} \tag{49}$$

Since the two shocks are assumed uncorrelated, we have:

$$E\left(L^{da}\right) = \frac{0.5}{1-\beta}\left[\alpha\left(\frac{d_1^2}{1-\mu^2}\sigma_{\hat{g}}^2 + \frac{d_2^2}{1-\rho^2}\sigma_{\hat{u}}^2\right) + \frac{d_3^2}{1-\mu^2}\sigma_{\hat{g}}^2 + \frac{d_4^2}{1-\rho^2}\sigma_{\hat{u}}^2\right] \tag{50}$$

A.3.2 Losses In Periods of Non-Adjustment

To describe social losses in periods of non-adjustment, re-write the interest rate in (29) as:

$$i_t = \Psi e_t$$

where $\Psi = \begin{pmatrix} \frac{1}{\beta\varphi} \\ -\frac{1}{\beta\varphi\lambda} \end{pmatrix}' D + \begin{pmatrix} \frac{1}{\varphi} \\ 1 + \frac{\beta}{\varphi\lambda} \end{pmatrix}' DP^2 + \begin{pmatrix} \frac{\mu}{\varphi} \\ \frac{\beta\rho+1}{\beta\varphi\lambda} \end{pmatrix}'$.

Next, re-write the IS equation as:

$$x_t = -\varphi\Psi e_{t-1} + \begin{pmatrix} 1 \\ \varphi \end{pmatrix}' DPe_t + \begin{pmatrix} 1 \\ 0 \end{pmatrix}' e_t = -\varphi\Psi e_{t-1} + \Phi e_t$$

Hence:

$$Ex_t^2 = (-\varphi\Psi)\,\Omega\,(-\varphi\Psi)' + \Phi\Omega\Phi' - 2\varphi\Psi\Sigma\Phi' \tag{51}$$

where Ω is the matrix of unconditional variances and Σ is the covariance matrix of $[e_{t-1}, e_t]$:

$$\Omega = \begin{pmatrix} \frac{1}{1-\mu^2}\sigma_{\hat{g}}^2 & 0 \\ 0 & \frac{1}{1-\rho^2}\sigma_{\hat{u}}^2 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \frac{\mu}{1-\mu^2}\sigma_{\hat{g}}^2 & 0 \\ 0 & \frac{\rho}{1-\rho^2}\sigma_{\hat{u}}^2 \end{pmatrix}$$

Finally, the Phillips curve can be represented as:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t = -\lambda\varphi\Psi e_{t-1} + \underbrace{\left(\lambda\Phi + \begin{pmatrix} 0 \\ 1 \end{pmatrix}' (\beta DP + I_{2x2})\right)}_{\equiv\Gamma} e_t$$

The variance of inflation is:

$$E\pi_t^2 = (-\lambda\varphi\Psi)\,\Omega\,(-\lambda\varphi\Psi)' + \Gamma\Omega\Gamma' - 2\lambda\varphi\Psi\Sigma\Gamma' \tag{52}$$

Hence, expected losses in periods of non-adjustment can be expressed as:

$$E\left(L^{dn}\right) = \tfrac{0.5}{1-\beta}\left\{ \alpha\left[(-\varphi\Psi)\,\Omega\,(-\varphi\Psi)' + \Phi\Omega\Phi' - 2\varphi\Psi\Sigma\Phi'\right] + \right. \\ \left. + (-\lambda\varphi\Psi)\,\Omega\,(-\lambda\varphi\Psi)' + \Gamma\Omega\Gamma' - 2\lambda\varphi\Psi\Sigma\Gamma' \right\} \tag{53}$$

Finally, the interest rate variability is:

$$Er^2 = \Psi\Omega\Psi' \tag{54}$$

B Solution to a Problem with Arbitrary Frequency of Interventions

Now we assume that the Central Bank fixes the interest rate at t until some future date $t+T$. The logic of the solution is the same as for $T=1$. First we would like to eliminate all endogenous forecasts until date $t+T$ to understand the full impact of such policy.

B.1 Impact of Interest Rate on Future Output and Inflation

Since the forecasts beyond date $t+T$ when the commitment expires are taken by the CB as given, at date $t+T$ we have:

$$\frac{\partial E_t x_{t+T}}{\partial i_t} = -\varphi; \qquad \text{and} \qquad \frac{\partial E_t \pi_{t+T}}{\partial i_t} = \lambda \frac{\partial E_t x_{t+T}}{\partial i_t} = -\lambda\varphi$$

At date $T-1$ the impact is:

$$\begin{aligned} \tfrac{\partial E_t x_{t+T-1}}{\partial i_t} &= -\varphi + \tfrac{\partial E_t x_{t+T}}{\partial i_t} + \varphi \tfrac{\partial E_t \pi_{t+T}}{\partial i_t} = \\ &= -\varphi + \tfrac{\partial E_t x_{t+T}}{\partial i_t}(1+\varphi\lambda); \end{aligned}$$

and:

$$\tfrac{\partial E_t \pi_{t+T-1}}{\partial i_t} = \lambda \tfrac{\partial E_t x_{t+T-1}}{\partial i_t} + \lambda\beta \tfrac{\partial E_t x_{t+T}}{\partial i_t}$$

Continuing in this fashion and noting that $\frac{\partial E_t \pi_{t+j}}{\partial i_t} = \frac{\partial \pi_{t+j}}{\partial i_t} \equiv B_j^\pi$ and $\frac{\partial E_t x_{t+j}}{\partial i_t} = \frac{\partial x_{t+j}}{\partial i_t} \equiv B_j^x$, $T \geq j \geq 0$ we can see that the impact can be expressed in a recursive form:

$$\begin{aligned} B_j^x &= -\varphi + B_{j+1}^x + \lambda\varphi \sum_{k=1}^{T-j} \beta^{k-1} B_{j+k}^x \\ B_j^\pi &= \lambda \sum_{k=0}^{T-j} \beta^k B_{j+k}^x = \lambda B_j^x + \beta B_{j+1}^\pi \end{aligned} \tag{55}$$

for any $j \in [0,T)$ and for $j=T$, the impact is

$$B_{t+T}^x = -\varphi; \qquad \text{and} \qquad B_{t+T}^\pi = -\lambda\varphi \tag{56}$$

B.2 First Order Condition

Then, the Central Bank's first order condition can be expressed as:

$$E_t \sum_{j=0}^{T} \beta^j \left(\alpha x_{t+j} B_{t+j}^x + \pi_{t+j} B_{t+j}^\pi \right) = 0 \tag{57}$$

To solve for the equilibrium we perform repeated substitutions to express each $E_t x_{t+j}$ and $E_t \pi_{t+j}$, $j \in [0,T]$ above as a function of the current period interest rate and the variables which the Central Bank takes as given: exogenous states u_t and g_t and time $t+T+1$ forecasts of inflation and output. Each forecast in the equation above can be represented as:

$$E_t x_{t+j} = B_j^x i_t + A_j^{xx} E_t x_{t+T+1} + A_j^{x\pi} E_t \pi_{t+T+1} + A_j^{xg} g_t + A_j^{xu} u_t \tag{58}$$

$$E_t \pi_j = B_j^\pi i_t + A_j^{\pi x} E_t x_{t+T+1} + A_j^{\pi\pi} E_t \pi_{t+T+1} + A_j^{\pi g} g_t + A_j^{\pi u} u_t \tag{59}$$

where coefficients B and A are constants. In the same way as we derived the coefficients on i_t is possible to show that other coefficients can be computed recursively as follows:

Expected Inflation:

$$A_j^{x\pi} = A_{j+1}^{x\pi} + \lambda\varphi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{x\pi} + \varphi\beta^{T-j}$$
$$A_j^{\pi\pi} = \lambda \sum_{k=0}^{T-j} \beta^k A_{j+k}^{x\pi} + \beta^{T-j+1} = \lambda A_j^{x\pi} + \lambda\beta \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{x\pi} + \beta^{T-j+1} \tag{60}$$

Expected output:

$$A_j^{xx} = A_{j+1}^{xx} + \lambda\varphi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{xx}$$
$$A_j^{\pi x} = \lambda \sum_{k=0}^{T-j} \beta^k A_{j+k}^{xx} = \lambda A_j^{xx} + \beta A_{j+1}^{\pi x} \tag{61}$$

Demand Shocks:

$$A_j^{xg} = \mu^j + A_{j+1}^{xg} + \lambda\varphi \sum_{k=1}^{T-j} \beta^{k-1} A_{j+k}^{xg}$$
$$A_j^{\pi g} = \lambda \sum_{k=0}^{T-j} \beta^k A_{j+k}^{xg} \tag{62}$$

Cost-Push Shocks:

$$A_j^{xu} = A_{j+1}^{xu} + \varphi A_{j+1}^{\pi u}$$
$$A_j^{\pi u} = \lambda A_j^{xu} + \beta A_{j+1}^{\pi u} + \rho^j \tag{63}$$

for any $j \in [0, T)$ and for $j = T$ the initial values are:

$$A_{t+T}^{x\pi} = \varphi; \quad \text{and} \quad A_{t+T}^{\pi\pi} = \beta + \lambda\varphi$$
$$A_{t+T}^{xx} = 1; \quad \text{and} \quad A_{t+T}^{\pi x} = \lambda$$
$$A_{t+T}^{xg} = \mu^T; \quad \text{and} \quad A_{t+T}^{\pi g} = \lambda\mu^T$$
$$A_{t+T}^{xu} = 0; \quad \text{and} \quad A_{t+T}^{\pi u} = \rho^T \tag{64}$$

B.3 Equilibrium in Period of Adjustment

So far we have:

$$\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} \begin{pmatrix} E_t x_{t+j} \\ E_t \pi_{t+j} \end{pmatrix} = 0 \tag{65}$$

As before, we guess that at time t the response of inflation and output takes the form:

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = D \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{66}$$

So that:

$$\begin{pmatrix} E_t x_{t+T+1} \\ E_t \pi_{t+T+1} \end{pmatrix} = DP^{T+1} \begin{pmatrix} g_t \\ u_t \end{pmatrix}$$

It follows that:

$$\begin{pmatrix} E_t x_{t+j} \\ E_t \pi_{t+j} \end{pmatrix} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix} i_t + A_j \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{67}$$

where:

$$A_j = \underbrace{\begin{pmatrix} A_j^{xx} & A_j^{x\pi} \\ A_j^{\pi x} & A_j^{\pi\pi} \end{pmatrix} DP^{T+1}}_{\equiv A_{1j}} + \underbrace{\begin{pmatrix} A_j^{xg} & A_j^{xu} \\ A_j^{\pi g} & A_j^{\pi u} \end{pmatrix}}_{\equiv A_{2j}} \tag{68}$$

The first order condition becomes:

$$\left[\sum_{j=0}^{T} \beta^j \left(\alpha \left(B_j^x\right)^2 + \left(B_j^\pi\right)^2\right)\right] i_t + \left[\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_j\right] \begin{pmatrix} g_t \\ u_t \end{pmatrix} = 0$$

We can express the optimal interest rate as:

$$i_t = \underbrace{-\left[\sum_{j=0}^{T} \beta^j \left(\alpha \left(B_j^x\right)^2 + \left(B_j^\pi\right)^2\right)\right]^{-1}}_{\equiv C_1} \cdot \left[\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_j\right] \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{69}$$

$$i_t = C_1 \cdot \left(A_3 DP^{T+1} + A_4\right) \begin{pmatrix} g_t \\ u_t \end{pmatrix} \tag{70}$$

where

$$A_3 = \left[\sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_{1j}\right]$$

$$A_4 = \sum_{j=0}^{T} \beta^j \begin{pmatrix} \alpha B_j^x & B_j^\pi \end{pmatrix} A_{2j}$$

To solve for equilibrium in periods when the interest rate is adjusted, we combine the previous equation with forecast equation when $j = 0$[24]:

$$D \begin{pmatrix} g_t \\ u_t \end{pmatrix} = \underbrace{\begin{pmatrix} B_t^x \\ B_t^\pi \end{pmatrix}}_{B_0^x} i_t + A_0 \begin{pmatrix} g_t \\ u_t \end{pmatrix}$$

Hence, D must satisfy:

$$D = B_0^x C_1 \cdot \left(A_3 DP^{T+1} + A_4\right) + \left(A_{1_0} DP^{T+1} + A_{2_0}\right)$$

or:

$$D - \underbrace{(B_0^x C_1 A_3 + A_{1_0})}_{\equiv C_2} DP^{T+1} = \underbrace{B_0^x C_1 A_4 + A_{2_0}}_{\equiv C_3}$$

This is a system of 4 linear equations in coefficients of D which are straightforward to solve.

B.4 Equilibrium in Periods of Non-Adjustment

[24] $A_0 = A_{1_0} DP^{T+1} + A_{2_0}$

In periods when the interest rate is fixed ($0 < j \leq T$), equilibrium is described by:

$$x_{t+j} = -\varphi i_t + E_{t+j} x_{t+j+1} + \varphi E_{t+j} \pi_{t+j+1} + g_{t+j} \tag{71}$$

$$\pi_{t+j} = \lambda x_{t+j} + \beta E_{t+j} \pi_{t+j+1} + u_{t+j} \tag{72}$$

with $T \geq j > 0$. These equations can be also expressed as:

$$x_{t+j} = B_j^x i_t + A_j^{xx} E_{t+j} x_{t+T+1} + A_j^{x\pi} E_{t+j} \pi_{t+T+1} + A_j^{xg} \frac{g_{t+j}}{\mu^j} + A_j^{xu} \frac{u_{t+j}}{\rho^j} \tag{73}$$

$$\pi_{t+j} = B_j^\pi i_t + A_j^{\pi x} E_{t+j} x_{t+T+1} + A_j^{\pi\pi} E_{t+j} \pi_{t+T+1} + A_j^{\pi g} \frac{g_{t+j}}{\mu^j} + A_j^{\pi u} \frac{u_{t+j}}{\rho^j} \tag{74}$$

where the forecasts of $(t + T + 1)$ variables must be consistent with interest rate adjustment policy:

$$\begin{pmatrix} E_{t+j} x_{t+T+1} \\ E_{t+j} \pi_{t+T+1} \end{pmatrix} = DP^{T+1-j} \begin{pmatrix} g_{t+j} \\ u_{t+j} \end{pmatrix}$$

Solution for the interest rate can be expressed as $i_t = \Psi e_t$, where $e_t = \begin{pmatrix} g_{t+j} & u_{t+j} \end{pmatrix}'$. Collecting terms, we obtain:

$$\begin{pmatrix} x_{t+j} \\ \pi_{t+j} \end{pmatrix} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix} \Psi e_t + \left[A_{1j} DP^{T+1-j} + A_{2j} \left(P^j\right)^{-1} \right] e_{t+j}$$

Let:

$$C_{5j} = \begin{pmatrix} B_j^x \\ B_j^\pi \end{pmatrix}$$

$$C_{6j} = A_{1j} DP^{T+1-j} + A_{2j} \left(P^j\right)^{-1}$$

Hence:

$$\begin{pmatrix} x_{t+j} \\ \pi_{t+j} \end{pmatrix} = C_{5j} e_t + C_{6j} e_{t+j}$$

Unconditional variances can be expressed as:

$$Ex_{t+j}^2 = (c_{5j_1} \Psi) \Omega (c_{5j_1} \Psi)' + (c_{6j_1}) \Omega (c_{6j_1})' + 2 (c_{5j_1} \Psi) \Sigma_j (c_{6j_1})' \tag{75}$$

$$E\pi_{t+j}^2 = (c_{5j_2} \Psi) \Omega (c_{5j_2} \Psi)' + (c_{6j_2}) \Omega (c_{6j_2})' + 2 (c_{5j_2} \Psi) \Sigma_j (c_{6j_2})' \tag{76}$$

where c_{5j_1} and c_{5j_2} are first and second elements of C_{5j}, c_{6j_1} and c_{6j_2} are rows of C_{6j}, Ω is unconditional covariance matrix of e_t and Σ_j is the unconditional correlation matrix $E(e_t e_{t+j})$.

C Derivation of the New Keynesian Phillips Curve under Calvo Pricing

The derivation below is available in many places (the most complete source is Woodford (2003)). We reproduce it below for mere convenience of the reader.

Firms under Calvo pricing face a fixed probability θ of not adjusting prices every period. Firms who get to set new prices face the following problem:

$$\max_{P_{it}} \sum_{j=0}^{\infty} \theta^j \underbrace{Q_{t+j}}_{\text{disc. factor}} \frac{P_{it} - MC_{t+j}}{P_{t+j}} \underbrace{\left(\frac{P_{it+j}}{P_{t+j}}\right)^{-q} nC_{t+j}}_{Y^d_{t+j} \text{ (real demand)}} \tag{77}$$

FOC:

$$P_{it} E_t \sum_{j=0}^{\infty} \theta^j Q_{t+j} \frac{1}{P_{t+j}} Y^d_{t+j} = \frac{q}{q-1} E_t \sum_{j=0}^{\infty} \theta^j Q_{t+j} \frac{MC_{t+j}}{P_{t+j}} Y^d_{t+j} \tag{78}$$

The first order condition can be loglinearized using a general principle that $E_t f(x_{t+1}) \approx \bar{x} \cdot f'(\bar{x}) \cdot E_t(\hat{x}_{t+1})$. If fims are owned by the consumers, then the discount factor is: $Q_{t+j} = \beta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma}$. However, $(C_t)^{\sigma} P_t$ can be taken out of the expectations operator and will cancel on both sides of the FOC. Hence, the summation term on the LHS becomes:

$$\sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j})^{-\sigma} Y_{t+j} \frac{1}{P_{t+j}} = \left\{ ... + \theta^j \beta^j (C_{t+j})^{-\sigma} \frac{1}{P_{t+j}} Y_{t+j} + ... \right\} =$$

$$\left\{ ... + \theta^j \beta^j QY(-\sigma c_{t+j} - p_{t+j} + \hat{Y}_{t+j}) + .. \right\} =$$

$$= YC^{-\sigma} P^{-1} \sum_{j=0}^{\infty} \theta^j \beta^j (-\sigma c_{t+j} - p_{t+j} + \hat{Y}_{t+j})$$

The total LHS can be expressed as:

$$\begin{aligned} & P_{it} \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j})^{-\sigma} Y_{t+j} \frac{1}{P_{t+j}} = \\ & \left(P_i Y C^{-\sigma} P^{-1}\right) \left\{ \frac{1}{1-\beta\theta} \hat{p}_{it} + \sum_{j=0}^{\infty} \beta^j \theta^j (-\sigma \hat{c}_{t+j} - \hat{p}_{t+j} + \hat{Y}_{t+j}) \right\}. \end{aligned} \tag{79}$$

The RHS similarly is:

$$\begin{aligned} & \frac{q}{q-1} E_t \sum_{j=0}^{\infty} \theta^j \beta^j (C_{t+j})^{-\sigma} \frac{MC_{t+j}}{P_{t+j}} Y_{t+j} = \left\{ ... + \theta^j \beta^j (C_{t+j})^{-\sigma} \frac{MC_{t+j}}{P_{t+j}} Y_{t+j} + ... \right\} = \\ & = \frac{q}{q-1} \left\{ ... + \theta^j \beta^j Y \cdot MC \cdot C^{-\sigma} P^{-1} (-\sigma \hat{c}_{t+j} - \hat{p}_{t+j} + \widehat{MC}_{t+j} + \hat{Y}_{t+j}) + .. \right\}) \end{aligned} \tag{80}$$

Combining the two sides, and noting that in the steady state $\frac{\omega}{\omega-1} MC = P_i$:

$$\hat{p}_{it} = (1 - \beta\theta) E_t \sum_{j=0}^{\infty} \theta^j \beta^j \left(\widehat{MC}_{t+j}\right) \tag{81}$$

or:

$$\hat{p}_{it} = (1-\beta\theta)\widehat{MC}_t + \beta\theta E_t\left(\hat{p}_{it+1}\right) \tag{82}$$

This is the dynamics of the Home reset price. Recall, that the Home price index follows:

$$\begin{aligned} &\hat{p}_t = \theta\hat{p}_{t-1} + (1-\theta)\hat{p}_{it} \\ &\Rightarrow \\ &\hat{p}_{it} = \tfrac{1}{1-\theta}\hat{p}_t - \tfrac{\theta}{1-\theta}\hat{p}_{t-1} = \tfrac{\theta}{1-\theta}\left(\hat{p}_t - \hat{p}_{t-1}\right) + \hat{p}_t \end{aligned} \tag{83}$$

Plug back into pricing equation:

$$\frac{\theta}{1-\theta}\left(\hat{p}_t - \hat{p}_{t-1}\right) + \hat{p}_t = (1-\beta\theta)\widehat{MC}_t + \beta\theta E_t\left(\frac{\theta}{1-\theta}\left(\hat{p}_{t+1} - \hat{p}_t\right) + \hat{p}_{t+1}\right)$$

Bring $\hat{p}_t$ to the right side and add and subtract $\beta\theta\hat{p}_t$

$$\frac{\theta}{1-\theta}\pi_t = (1-\beta\theta)\left(\widehat{MC}_t - \hat{p}_t\right) + \beta\theta E_t\left(\frac{\theta}{1-\theta}\left(\hat{p}_{t+1} - \hat{p}_t\right) + \hat{p}_{t+1} - \hat{p}_t\right)$$

We get:

$$\pi_t = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\widehat{MC}_t - \hat{p}_t\right) + \beta E_t\left(\pi_{t+1}\right) \tag{84}$$

Finally, define deviations of the real marginal costs from the steady state as: $\widehat{mc}_{t+j} = \widehat{MC}_{t+j} - \hat{p}_{t+j}$. Then:

$$\pi_t = \delta\widehat{mc}_t + \beta E_t\left(\pi_{t+1}\right) \tag{85}$$

where $\delta = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. The forward looking Phillips curve is obtained by introducing technology and consumer preferences. We'll assume Cobb-Dublas technology and that capital is fixed in the short-run (normalized to equal 1). The production function then exhibits decreasing returns to scale:

$$y_t(i) = A_t L_t^{\eta}, \;\; \eta < 1$$

Hence, the cost function can be written as:

$$Cost_t(i) = W_t A_t^{-\frac{1}{\eta}} y_t^{\frac{1}{\eta}}(i)$$

Real marginal costs become:

$$mc_t(i) = \frac{1}{\eta} w_t A_t^{-\frac{1}{\eta}} y_t^{\frac{1-\eta}{\eta}}(i)$$

Real wage is defined from the labor supply condition in the consumer problem:

$$-\frac{u_L}{u_c} = w_t$$

We'll parametrize the utility of a consumer employed in sector i as:

$$u(C_t, L_t(i)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\vartheta}(i)}{1+\vartheta}$$

and impose equilibrium conditions: $Y = C$, and $y(i) = AL^{\eta}(i)$. Marginal costs become:

$$mc_t(i) = \frac{1}{\eta}\frac{y_t^{\frac{\vartheta}{\eta}}(i)}{Y_t^{-\sigma}}A_t^{-\frac{1}{\eta}}y_t^{\frac{1-\eta}{\eta}}(i)$$

Loglinearizing and averaging across firms:

$$\widehat{mc}_t = \left(\sigma + \frac{1-\eta+\vartheta}{\eta}\right)x_t - \frac{1}{\eta}\hat{A}_t$$

Finally, combine this with the inflation equation derived earlier and label the cost push shock as $u_t = -\frac{1}{\eta}\hat{A}_t$ [25]. The result is:

$$\pi_t = \lambda x_t + \beta E_t\left(\pi_{t+1}\right) + u_t \tag{86}$$

where $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\sigma + \frac{1-\eta+\vartheta}{\eta}\right)$.

[25] Note: many also assume that markups are stochastic. In that case u_t would be a linear combination of shocks to markups and productivity (see Ireland (2002))