Practical Volatility and Correlation Modeling for Financial Market Risk Management

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Question

What (if anything...) do academics have to offer market risk management practitioners in financial institutions?

Themes

Conditional volatility modeling and conditional density simulation
Flexible models for high-dimensional covariance matrix modeling
Using intraday information in daily risk management

Goal

To encourage further dialog between the academic and practitioner communities, stimulating the development of improved market risk management techniques

Key Points (I)

- Standard "model-free" methods, such as historical simulation, rely on false assumptions of independent returns. Reliable risk measurement requires a *conditional* density model that allows for time-varying volatility.

- For the purpose of risk *measurement*, specifying a univariate density model directly on the portfolio return is likely to be most accurate.

- The GARCH volatility models offer a convenient and parsimonious framework for modeling key dynamic features of returns, including volatility mean-reversion, long-memory, and asymmetric response.

- Although risk measurement can be done from a univariate model for a given set of portfolio weights, risk *management* requires a fully specified multivariate density model. Unfortunately, standard multivariate GARCH models are too heavily parameterized to be useful in realistic large-scale problems.

Key Points (II)

- Recent advances in multivariate GARCH modeling are likely to be useful for medium-scale models, but large scale modeling requires decoupling variance and correlation dynamics.

- High-frequency volatility measures present a promising venue for risk management. Realized volatility and correlation measures give more accurate forecasts of future realizations.

- Risk management requires fully-specified conditional density models, not just conditional covariance models. Resampling returns standardized by the conditional covariance matrix presents an attractive strategy.

- The near log-normality of realized volatility, together with the surprising near-normality of returns standardized by realized volatility, holds promise for log-normal / normal mixture models in financial risk management.

Overview

1) Historical Simulation and Problems
2) RiskMetrics and Problems
3) GARCH Portfolio Variance Models

4) Conditional Covariance and Correlation Models

5) Using Intraday data in Risk Management

6) Conditional Distribution Models via Simulation

7) Issues Going Forward

Risk Measurement
Using VaR from Historical Simulation

- Pseudo portfolio returns calculated from T historical returns

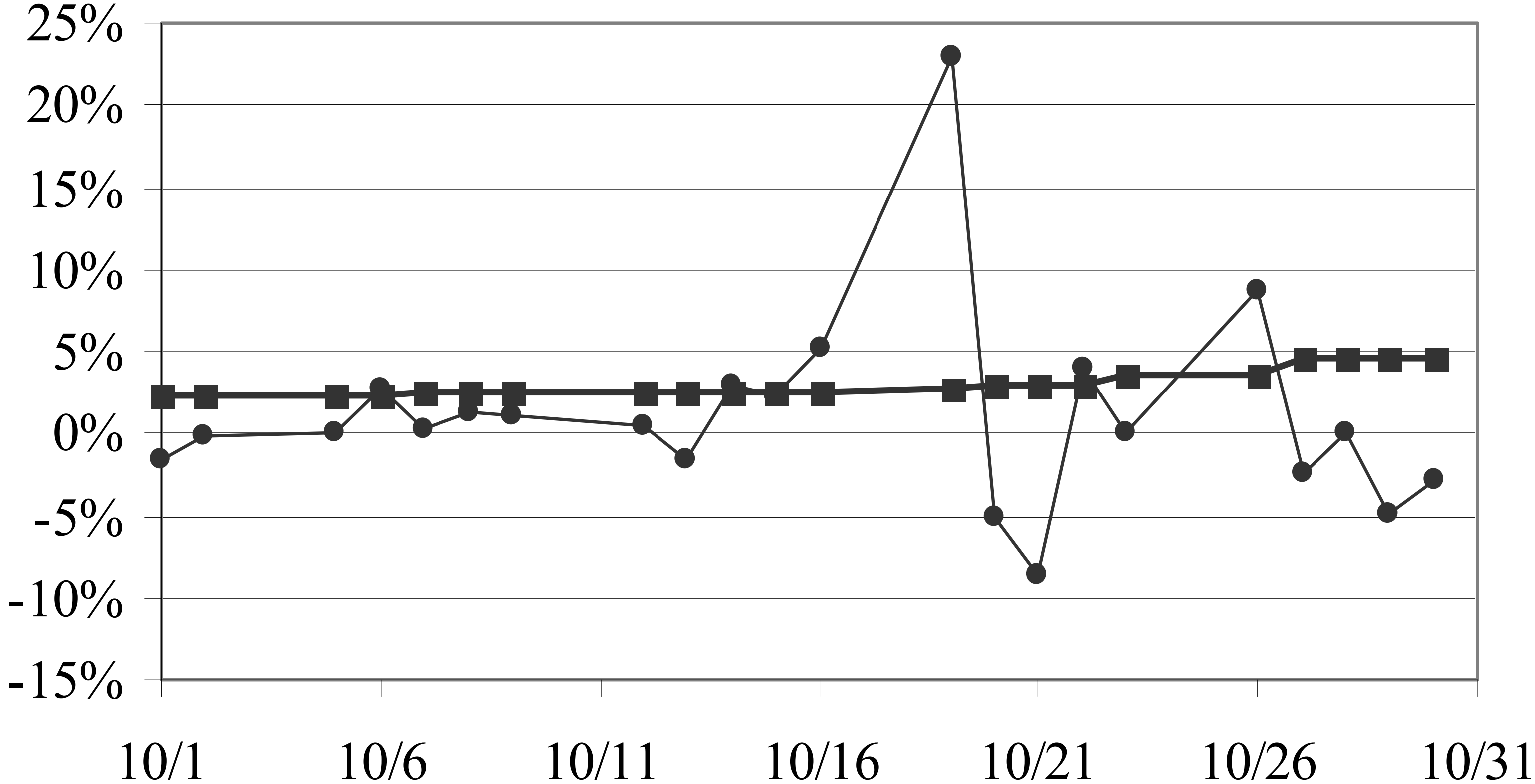
$$r_{w,t} = \sum_{i=1}^{N} w_{i,T} r_{i,t} \equiv W_T' R_t, \quad t = 1, 2, \ldots, T$$

- Historical Simulation (HS) 100p % VaR calculated as the order statistic

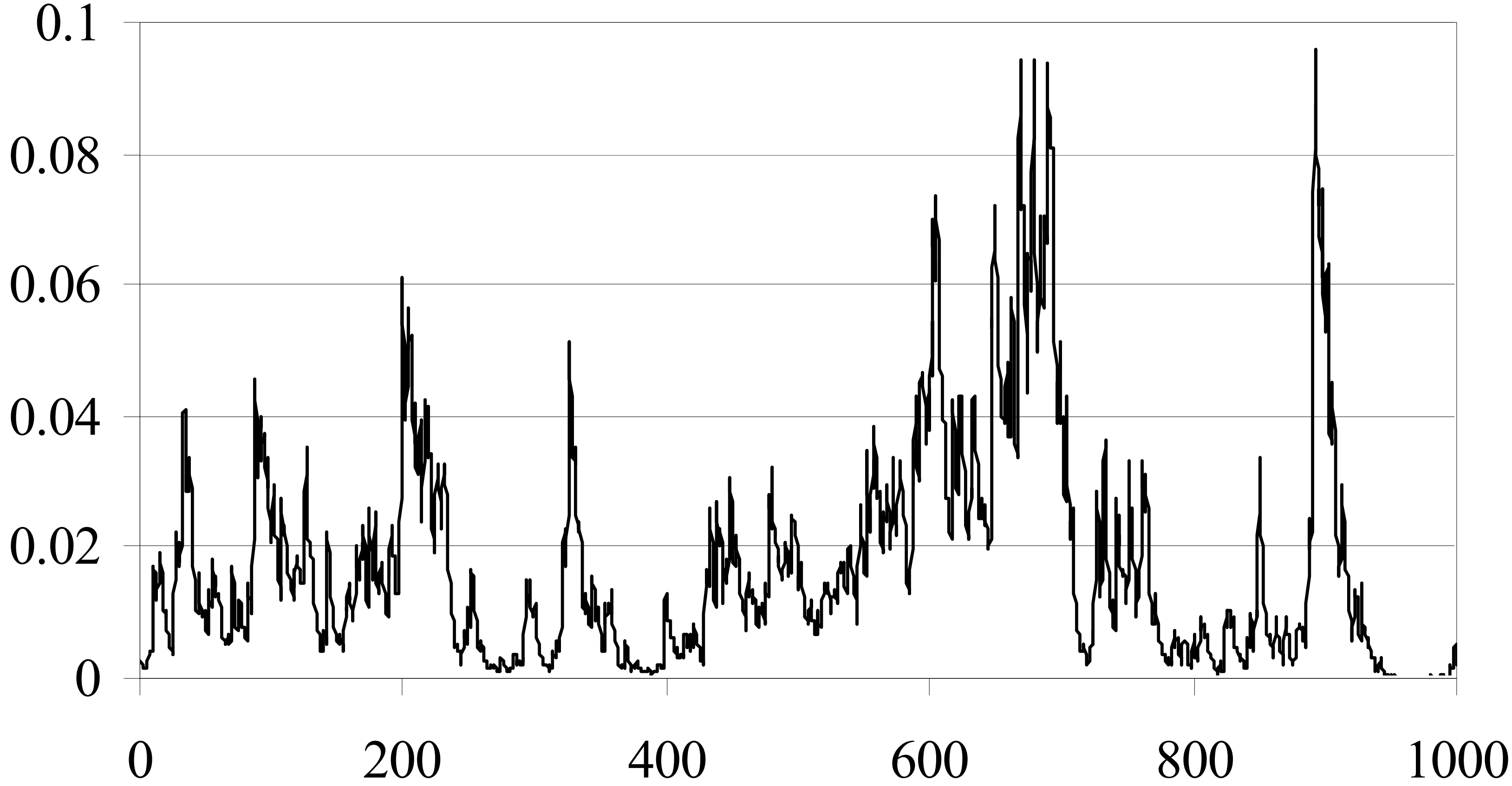
$$HS-VaR^{p}_{T+1|T} \equiv r_w((T+1)p)$$

- Dynamics only from updating of historical sample

Historical Simulation VaR is Bad. Part 1
Oct. 1987, Long S&P500 Position: 1%VaR and Loss



Historical Simulation VaR is Bad. Part 2
True Conditional Coverage of Nominal 1% HS-VaR



Volatility Updating is Needed

- Simplest Approach: Continue to work with univariate pseudo-portfolio returns but assume conditional variance of the RiskMetrics / Exponential Smoother form

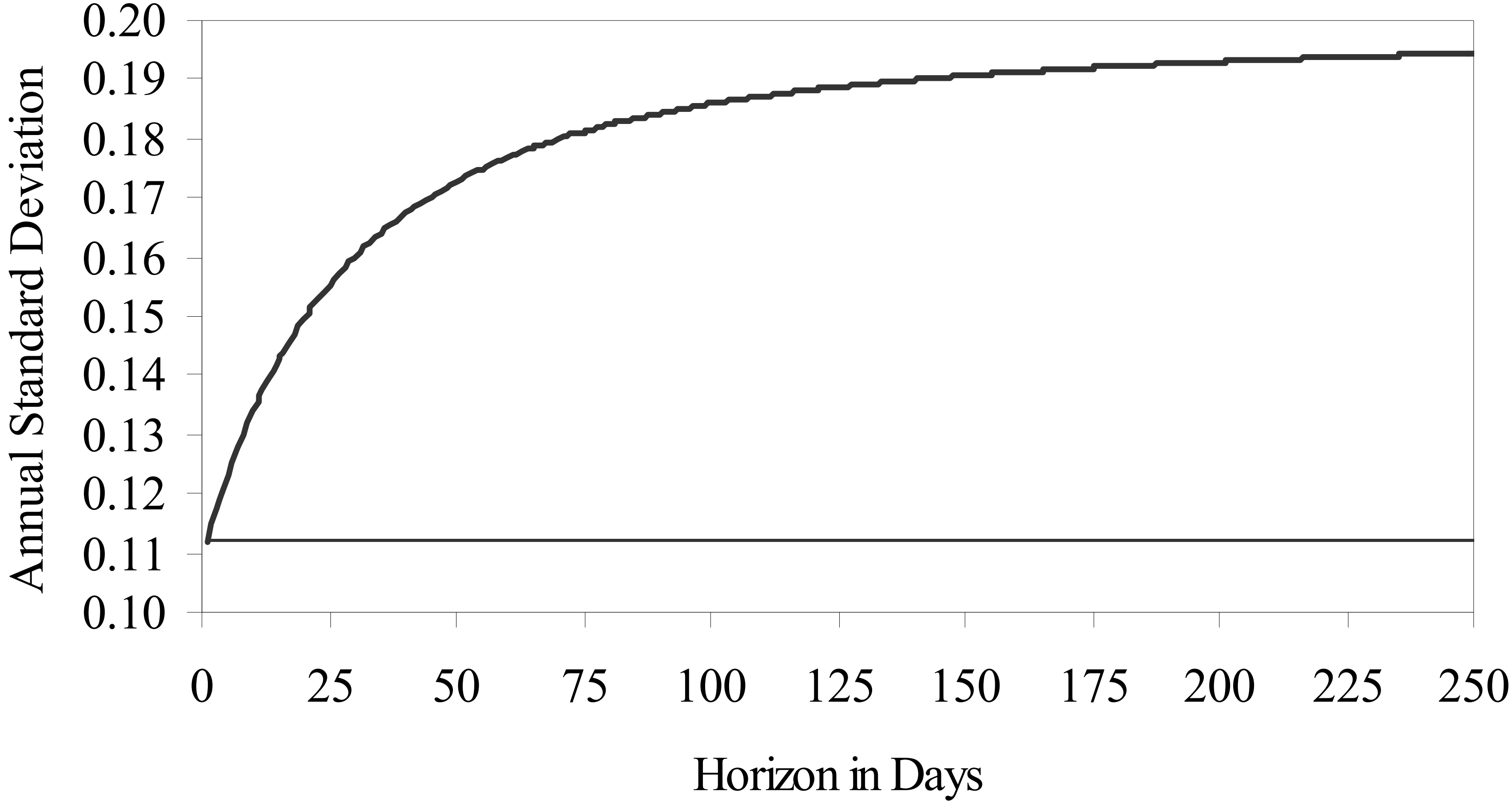
$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{w,t-1}^2$$

- The smoothing parameter λ can be estimated via QMLE but is often calibrated to 0.94 for daily data.

- The conditional variance for the *k*-day aggregate return is simply

$$Var(r_{w,t+k} + r_{w,t+k-1} + \ldots + r_{w,t+1} \mid \mathscr{F}_t) \equiv \sigma_{t:t+k|t}^2 = k\,\sigma_{t+1}^2$$

Volatility Term Structures
Risk Metrics versus Mean Reverting Variance



GARCH Variance

- Variance Mean Reversion

$$\sigma_t^2 = \sigma^2 + \alpha(r_{w,t-1}^2 - \sigma^2) + \beta(\sigma_{t-1}^2 - \sigma^2)$$

- Long Memory e.g. Approximately via Components

$$\sigma_t^2 = q_t + \alpha(r_{w,t-1}^2 - q_t) + \beta(\sigma_{t-1}^2 - q_t)$$

$$q_t = \sigma^2 + \rho(q_{t-1} - \sigma^2) + \varphi(r_{w,t-1}^2 - \sigma_{t-1}^2)$$

- Asymmetric Variance Response (“Leverage Effect”)

$$\sigma_t^2 = \omega + \alpha r_{w,t-1}^2 + \gamma r_{w,t-1}^2 \mathbf{1}(r_{w,t-1} < 0) + \beta \sigma_{t-1}^2$$

Market Risk Measurement vs. Management

The aggregate approach above is well-suited for risk measurement, but not for active management, e.g.

- Calculating optimal portfolio weights
- Sensitivity risk measures
- Sub-portfolio analysis

We need multivariate dynamic model. Challenges:

- Positive semidefinite (psd) covariance matrix
- $\frac{1}{2}N(N+1)$ distinct elements

RiskMetrics Covariances

The N by N covariance version of RiskMetrics assumes

$$\Omega_t = \lambda\Omega_{t-1} + (1 - \lambda)R_{t-1}R_{t-1}^{\prime}$$

One parameter!

Notice that this model implies fully persistent variances and covariances with same smoothing parameter across elements.

Multivariate GARCH(1,1)

- In general we have (using "vector half" operator)

$$vech(\Omega_t) = vech(C) + B\,vech(\Omega_{t-1}) + A\,vech(R_{t-1}R_{t-1}^{/})$$

- $O(N^4)$ parameters. $N = 5$ => 465 parameters...

- Scalar Multivariate GARCH(1,1) represents a parsimonious (covariance targeting) extreme

$$\Omega_t = \Omega + \beta(\Omega_{t-1} - \Omega) + \alpha(R_{t-1}R_{t-1} - \Omega)$$

- But we want more flexibility while retaining tractability!

Dimension Reduction I: Flex GARCH

- Ledoit, Santa-Clara and Wolf (2003)

- Decentralizes the estimation procedure by estimating N(N+1)/2 bivariate GARCH models with certain parameter constraints, and then "pasting" them together to form the matrices A, B, and C.

- Clever pasting ensures psd.

- Flex-GARCH appears to be a viable modeling approach when N is less than 30.

Dimension Reduction II: Dynamic Conditional Correlations

- Think of covariance matrix as $\Omega_t \equiv D_t \Gamma_t D_t$

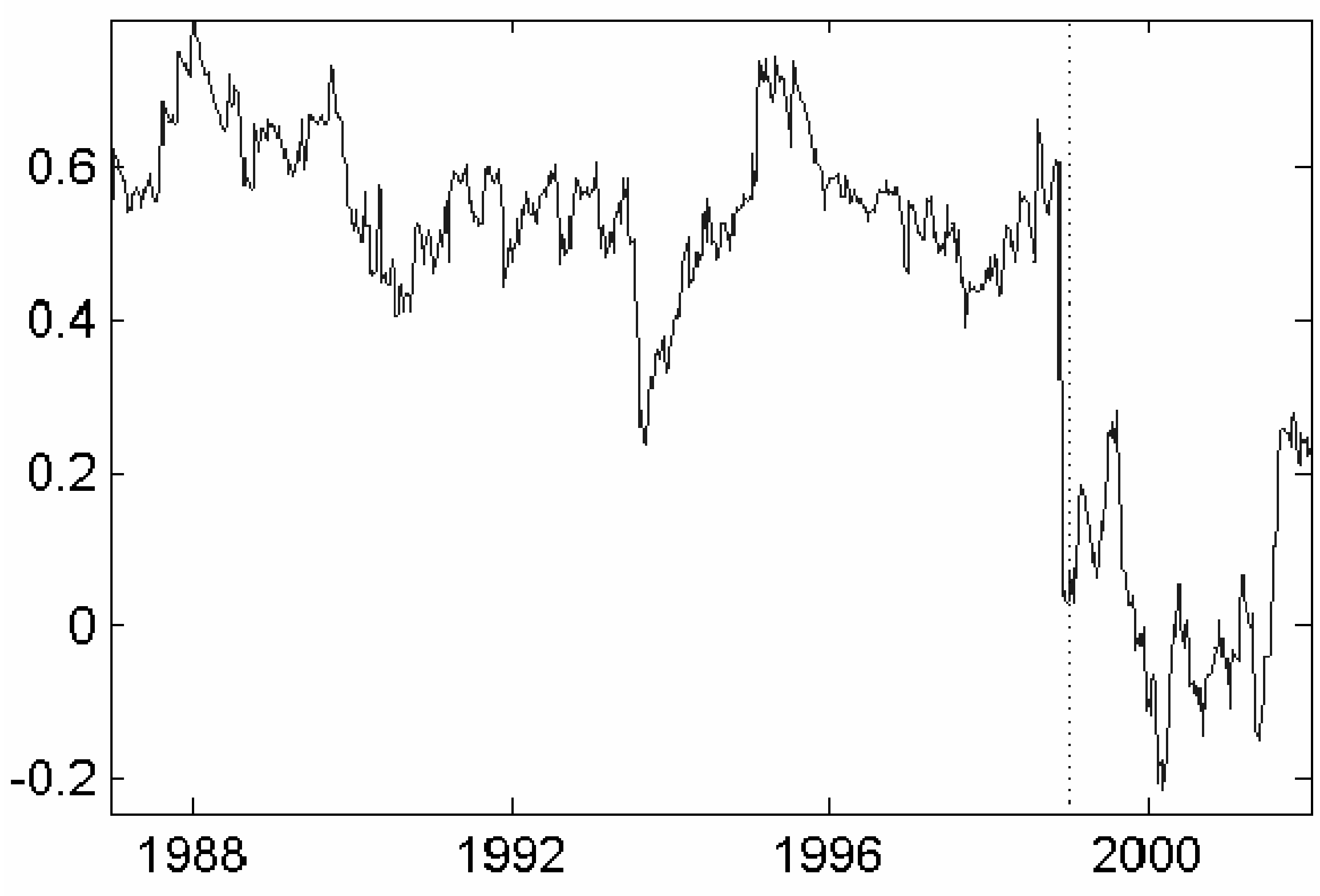
- 1) Estimate variance dynamics asset by asset to get D_t
- 2) Standardize returns to get $z_t = r_t / \sigma_t$
- 3) Estimate joint correlation dynamics from

$$Q_t = C + \beta Q_{t-1} + \alpha (Z_{t-1} Z'_{t-1})$$

with normalization: $\left[\Gamma_t\right]_{i,j} = q_{i,j,t} / (\sqrt{q_{i,i,t}} \sqrt{q_{j,j,t}})$

- Key: N+1 low dimensional optimization problems due to stepwise estimation and correlation targeting.

Time-Varying Bond Return Correlation
Germany and Japan (Capiello, Engle, Sheppard)



Using Intraday Information

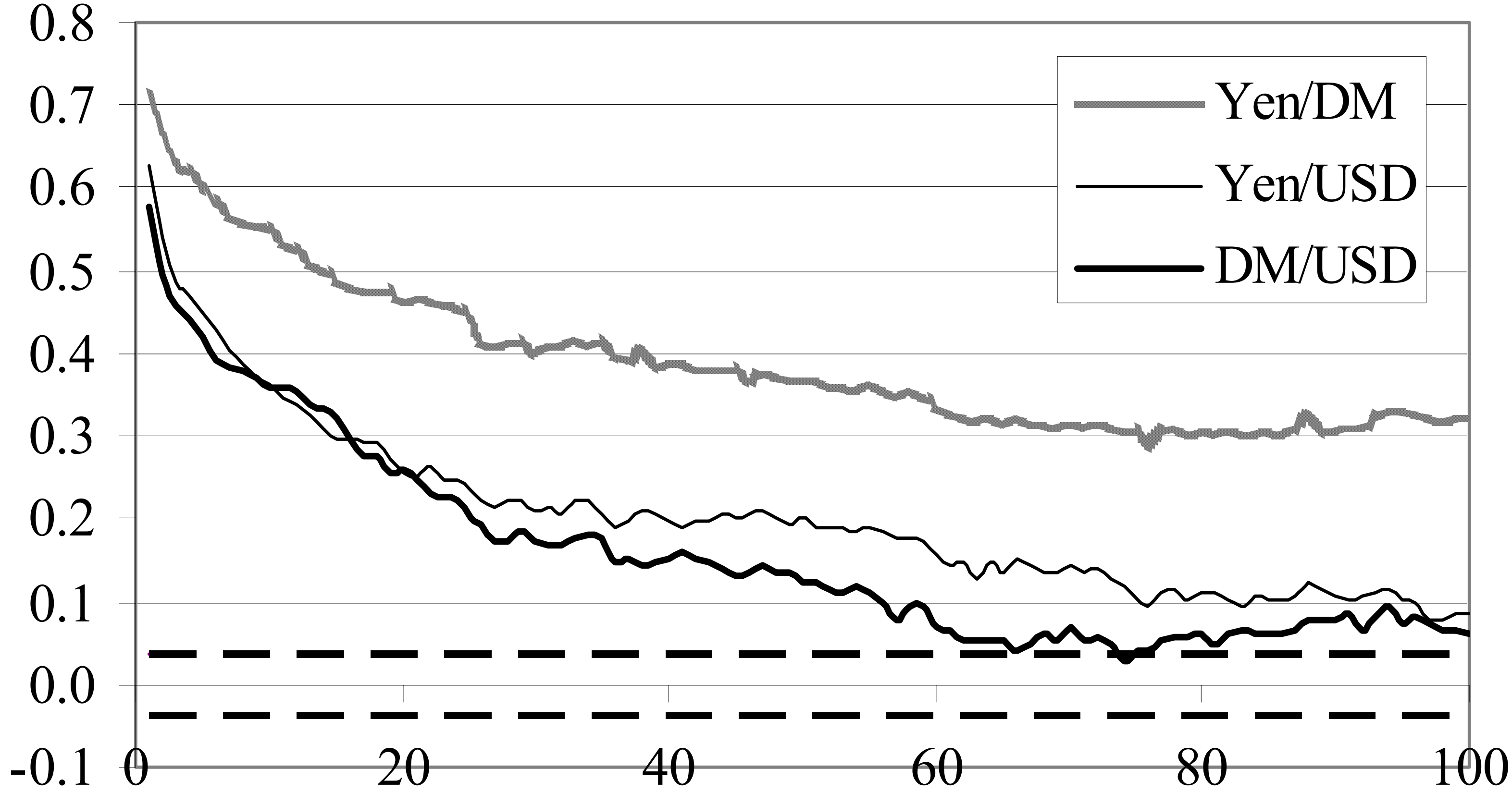
- Following Andersen, Bollerslev, Diebold and Labys (2003), define the realized variance as

$$\sigma^2_{t,\Delta} \equiv \sum_{j=1}^{1/\Delta} r^2_{t-1+j\Delta,\Delta},$$

- Theoretically, sampling infinitely often, we approach the true integrated volatility of the underlying continuous time process on day t. Natural multivariate extension.

- In practice, prices sampled at 15-30 minute intervals, depending on the market, are therefore often used.

Sample Autocorrelations of Realized Volatility
Three Currencies



Uses of Intraday Data in Financial Risk Management

- Realized variance and covariance can be treated as observed (with error) which enables the use of standard AR(F)IMA techniques for covariance matrix forecasting.

- DCC structure may be useful for realized covariance matrix forecasting in order to ensure psd.

Dimension Reduction III: Map assets in portfolio to a smaller set of liquid “base” assets which are themselves modeled using realized variances and covariances.

Conditional Distribution Modeling

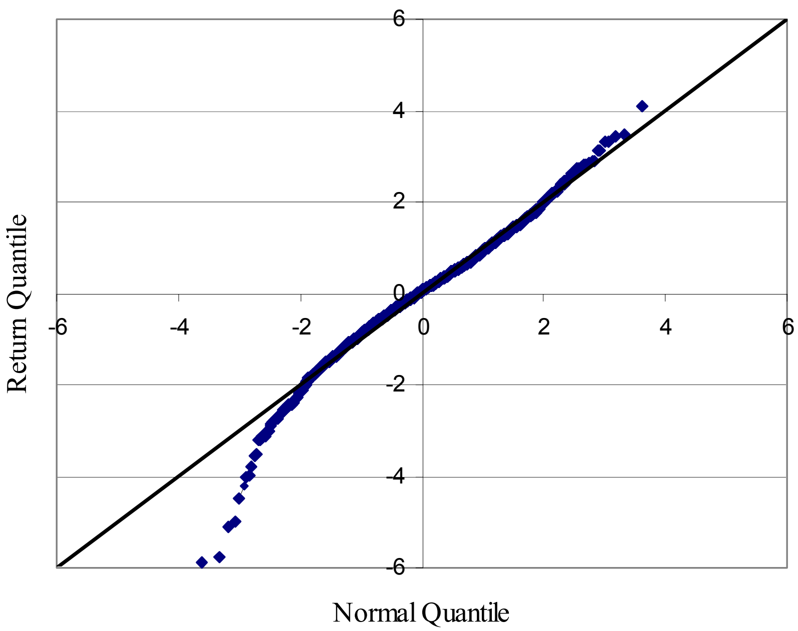
- Risk measurement and management requires knowledge of the entire conditional distribution and not only the second moment.

- Parametric approaches exist: normal and non-normal.

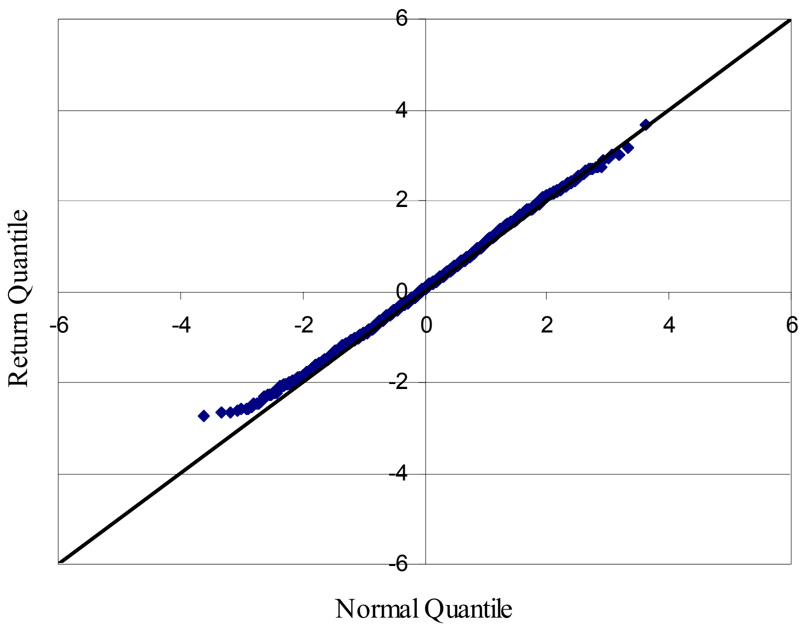
- We favor conditional bootstrapping a.k.a. filtered historical simulation. Can be done in the univariate and multivariate case. Dynamic standardization needed. Standardization by realized volatility is interesting...

QQ Plot of S&P500 Daily Standardized Returns

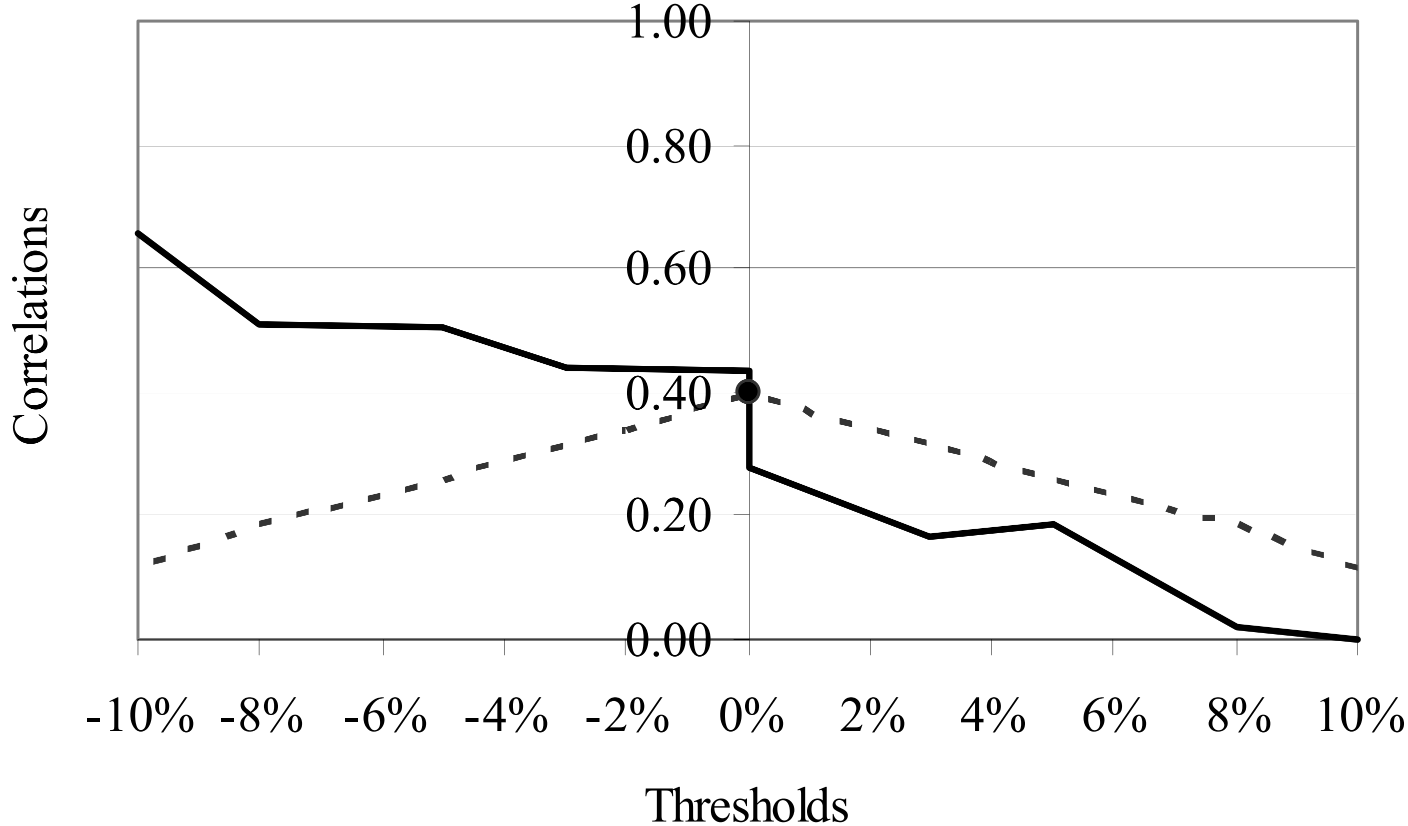
From NGARCH(1,1)



From Realized Volatility



Multivariate Extremes also Require Attention: US vs JP Equity. Longin and Solnik (2001)



Further Issues

- Option Valuation with GARCH and SV

- Market Microstructure noise in RV modeling