Asset Prices and Exchange Rates

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June 2003

Abstract

This paper develops a simple two-country, two-good model, in which the real exchange rate, stock and bond prices are jointly determined. The model predicts that stock market prices are correlated internationally even though their dividend processes are independent, providing a theoretical argument in favor of financial contagion. The foreign exchange market serves as a propagation channel from one stock market to the other. The model identifies interconnections between stock, bond and foreign exchange markets and characterizes their joint dynamics as a three-factor model. Contemporaneous responses of each market to changes in the factors are shown to have unambiguous signs. These implications enjoy strong empirical support. Estimation of various versions of the model reveals that most of the signs predicted by the model indeed obtain in the data, and the point estimates are in line with the implications of our theory. Furthermore, the uncovered interest rate parity relationship has a risk premium in our model, shown to be volatile. We also derive agents' portfolio holdings and identify economic environments under which they exhibit a home bias, and demonstrate that an international CAPM obtaining in our model has two additional factors.

JEL Classifications: F31, F36, G12, G15
Keywords: Asset pricing, exchange rate, contagion, international finance, international macroeconomics.

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1. Introduction

Financial press has long asserted that stock prices and exchange rates are closely intertwined. In recent times, the depreciation of the dollar against the euro has been argued to put pressure on the investor sentiment and the US stock markets. Similarly, the previous decade associated with high productivity gains and a stock market boom in the US was accompanied by an exchange rate appreciation. Surprisingly, these connections have rarely been highlighted in workhorse models of exchange rate determination. This paper develops a framework in which the same forces that drive exchange rates, also influence countries' stock markets, and argues that a great deal can be learned about foreign exchange markets by examining stock markets, and vice versa. Identifying interrelations between these markets would also shed light on some widely-debated spillovers, such as, for example, international financial contagion.

The innovation of our modeling approach is to draw upon three separate strands of literature: international asset pricing, open economy macroeconomics and international trade, while differing from each one of them in some important dimensions. On the one hand, while encompassing a rich financial markets structure, the overwhelming majority of international asset pricing models assumes that there is a single commodity in the world, implying that the real exchange rate has to be equal to unity. Nontrivial implications on exchange rates in such a framework have been obtained by either introducing barriers to trade into a real model, or by exogenously specifying a monetary policy and focusing on the nominal exchange rate. On the other hand, the international economics literature typically concentrates either on how different patterns of international trade in goods affect the real exchange rate, or on how the nominal exchange rate is linked to bond markets, typically overlooking the implications on equity markets. Ours is a two-country, two-good model where the countries trade in goods as well as in stocks and bonds. To our knowledge, it is the first asset pricing model in which the terms of trade, exchange rate, and asset prices are jointly determined in equilibrium, thus marrying dynamic asset pricing with Ricardian trade theory.[1]

The paper consists of two parts: theoretical and empirical. The first part presents our model of a dynamic world economy under uncertainty. The two countries comprising the economy specialize in producing their own good. The stock market in each country is a claim to the country's output.

[1] Zapatero (1995) employs a similar two-country, two-good economy and discusses the new insights the model provides for empirical studies. Although his model offers valuable perspective on the exchange rate, it contains some counterfactual implications regarding financial markets, which we discuss in Section 2.

Bonds, whose interest rates are uncovered endogenously within the model, provide further opportunities for international borrowing and lending. A representative agent in each country consumes both goods, albeit with a preference bias toward the home good, and invests in the stock and bond markets. Uncertainty in the economy is due to output (supply) shocks in each country and the consumers' demand shocks. While the former are very common in models of international macroeconomics, and especially international business cycles, the latter have received considerably less attention. For the bulk of our analysis, we adopt a very general specification of the demand shocks, imposing more structure later to gain additional insight. We distinguish between the special cases where (i) there are no demand shocks, (ii) demand shocks are due to pure consumer sentiment, (iii) demand shocks are due to preferences encompassing a "catching up with the Joneses" feature or consumer confidence and (iv) preferences are state-independent and demand shocks are driven by differences of opinion. It is ultimately an empirical question − addressed in the second part of the paper − as to which of these specifications is the most plausible.

Our model is extremely tractable, allowing us to characterize, in closed-form, the patterns of responses of stock and bond markets in each country, as well as those of the foreign exchange market, to supply and demand shocks. Since, by design, our model nests a number of fundamental implications of various strands of the international economics literature, directions of some of these responses are familiar. For example, all else equal, a positive shock to a country's output leads to a deterioration of the terms of trade it enjoys and an exchange rate depreciation (consistent with the comparative advantages theory of the international trade literature). At the same time, the national stock market sees a positive return (in line with the asset pricing literature). On the other hand, a positive demand shock in a country leads to an exchange rate appreciation (as in the open economy macroeconomics literature). We unify all these implications in one model, and focus on the interconnections between the stock, bond and foreign exchange markets and on the spillovers.

One important spillover obtaining in our economy provides a natural basis for a theory of international financial contagion. The empirical literature on contagion has concluded that some factors, such as trade and financial linkages, are important in explaining the propagation of shocks. Nevertheless, these channels account for a small proportion of the observed co-movement. The argument is that the correlation in equity and bond prices is an order of magnitude larger than that implied by the correlation in real variables.[2] In our model, the real variables − the countries' output pro-

[2] See, e.g., Eichengreen, Rose, and Wyplosz (1996), Fischer (1998), Goldstein, Kaminsky, and Reinhart (2000), Kaminsky and Reinhart (2000), Claessens, Dornbusch, and Park (2001), Kaminsky, Reinhart, and Végh (2003).

cesses – are unrelated and yet stock returns on the national markets become positively correlated. Contagion is a natural response to a supply shock in one of the countries. As we discussed earlier, a positive output shock leads to a positive return on the domestic stock market; however, it has to be accompanied by an exchange rate depreciation. The latter implies a strengthening of the foreign currency, leading to a rise in the value of the foreign country's output, thereby boosting its stock market. The foreign exchange market thus acts as a channel through which shocks are propagated across countries' stock markets. Another spillover we uncover is what we labeled "divergence": a response of world asset markets to a demand shock in one of the countries. As mentioned earlier, a positive shift in domestic demand causes an exchange rate appreciation. A strengthening of the domestic currency relative to the foreign leads to divergence in world financial markets: it provides a boost to the domestic stock and bond markets, while asset prices abroad fall. This pattern is very close to macroeconomic dynamics observed in the US in the 90's when large capital inflows were pushing the interest rates down, increasing stock prices, and strengthening the dollar. These implications, together with the remaining patterns of responses to innovations in supply and demand, provide a testable theory of the interconnections between stock, bond and foreign exchange markets in different countries.

In our model, uncovered interest parity in its classical form need not be satisfied. The presence of demand shocks implies that the interest rate parity relationship necessarily contains an additional term capturing a pertinent risk premium. We demonstrate that this term is time-varying, driven in part by demand shocks, and possibly quite volatile. We also derive portfolio strategies, shown to consist of holding a mean-variance portfolio and an additional portfolio hedging against future demand shocks. Within our model, we can identify economic environments under which countries' portfolios exhibit a home bias, which is demonstrated to be due to the nature of the demand shocks. In particular, a home bias is induced when the demand shocks are positively correlated with the supply shocks in each country. Agents, who get enthusiastic and demand more consumption (biased toward the home good) when their country is experiencing an output increase, optimally hold a larger fraction of the domestic stock – the claim to the domestic output. This is consistent with our consumer confidence or differences in opinion interpretations of the demand shocks. Finally, due to the presence of the hedging portfolios in the countries' investment strategies, we obtain a multi-factor CAPM in our model. In addition to the standard market factor, our model identifies two further factors: demand shocks of each country – hence pointing to a potential misspecification of the traditional international CAPM.

The second part of the paper tests the implications of our theory on daily data for the US vis-à-vis Germany and the UK. Our model implies that the stock market prices, bond prices and exchange rates are described by a three latent-factor model with time-varying coefficients. The model, as it is, has too many degrees of freedom to fit the data, so we estimate two simplified versions of it in order to increase chances of finding a rejection. First, we force the coefficients to be constant. The two most important findings emerging from this estimation are: (i) demand shocks are twice as important as supply shocks in describing the behavior of asset prices and the exchange rate, (ii) the data reject the hypotheses that our demand shocks are generated by either pure sentiment or "catching up with the Joneses"-type behavior. Rather, our results support the view that the demand shocks are likely to represent differences in opinion or consumer confidence, which is positively correlated with the current performance of the domestic economy.

In our second pass at the data, we directly estimate the pattern of responses of stock, bond and foreign exchange markets to the underlying shocks within a latent-factor model with constant coefficients where only some sign restrictions are imposed. To solve the identification problem, we rely on the natural heteroskedasticity present in the data.[3] Our estimation procedure can uniquely identify twelve coefficients in a system of simultaneous equations corresponding to each bilateral estimation. We find that, for example, in the case of the US vis-à-vis the UK all twelve point estimates have the signs predicted by the model, and eleven are statistically significant. Moreover, the theory entails implications as to how the coefficients are related to each other. For most of the point estimates predicted to be equal by the model, we cannot reject the hypothesis that they are the same, however we do find some rejections, especially in bond markets responses. Finally, we examine the robustness of our implications on the low frequency data, which contains observations of the countries' output and thus allows us to relate the latent factors from our estimation to innovations in output in the data. Although in this exercise the size of our sample is significantly reduced, we are still able to argue that the responses of asset and currency markets entailed by the model hold in the data, and that the output shocks identified by our estimation are indeed highly correlated with the actual output innovations.

In terms of the modeling approach, our work is closest to the international asset pricing literature. As we mentioned earlier, however, most of the models in this literature are casted in a

[3] Our approach follows the identification through heteroskedasticity literature, which is based on Philip Wright's (1928) book. It has been recently extended by Sentana and Fiorentini (2001) for the conditional heteroskedasticity case (see also Rigobon (2002)), and Rigobon (2003) for the case in which heteroskedasticity can be described by different regimes.

single-good framework, in which forces of arbitrage equate the real exchange rate to unity. Nontrivial implications on exchange rates have been obtained either by exogenously specifying a monetary policy and focusing on the nominal exchange rate instead of real (see, for example, Bakshi and Chen (1997), Basak and Gallmeyer (1999) for monetary general equilibrium models, and Solnik (1974), Adler and Dumas (1983) for the international CAPM), or by introducing barriers to trade into a real model (Dumas (1992), Sercu and Uppal (2000)), and thus impeding goods markets arbitrage. To our knowledge, the only other multi-good asset pricing model in which the exchange rate is determined through the terms of trade is Zapatero (1995).[4] In terms of the overall objective of constructing a structural model tying together exchange rates and asset prices and taking it to the data, we know of two papers that are closest to our work. In a mean-variance setting, absent international trade in goods, Hau and Rey (2002) study the relationship between stock market returns and the exchange rate. Brandt, Cochrane, and Santa-Clara (2001) also examine this relationship so as to argue that exchange rates fluctuate less than the implied marginal rates of substitution obtained from stock market returns.

The rest of the paper is organized as follows. Section 2 describes the economy and derives the testable implications of our model. Section 3 presents the empirical analysis. Section 4 discusses caveats and avenues for future research. Section 5 concludes and the Appendix provides all proofs.

2. The Model

2.1. The Economic Setting

We consider a continuous-time pure-exchange world economy along the lines of Lucas (1982). The economy has a finite horizon, $[0, T]$, with uncertainty represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a standard three-dimensional Brownian motion $\vec{w}(t) = (w(t), w^*(t), w^\theta(t))^\top$, $t \in [0, T]$. All stochastic processes are assumed adapted to $\{\mathcal{F}_t; t \in [0, T]\}$, the augmented filtration generated by $\vec{w}$. All stated (in)equalities involving random variables hold P-almost surely. In what follows, given our focus, we assume all processes introduced to be well-defined, without explicitly stating regularity conditions ensuring this.

There are two countries in the world economy: Home and Foreign. Each country produces its

[4]Another analysis that inspired our work is Cass and Pavlova (2003), who employ a similar model to argue that benchmark models of financial equilibrium theory in economics and finance, commonly believed to entail similar implications, can be in fundamental disagreement.

own perishable good via a strictly positive output process modeled as a Lucas' tree:

$$dY(t) = \mu_Y(t)\,Y(t)\,dt + \sigma_Y(t)\,Y(t)\,dw(t) \qquad \text{(Home)}, \tag{1}$$

$$dY^*(t) = \mu_Y^*(t)\,Y^*(t)\,dt + \sigma_Y^*(t)\,Y^*(t)\,dw^*(t) \qquad \text{(Foreign)}, \tag{2}$$

where μ_Y, μ_Y^*, $\sigma_Y > 0$ and $\sigma_Y^* > 0$ are arbitrary adapted processes. Note that the country-specific Brownian motions w and w^* are independent.[5] The prices of the Home and Foreign goods are denoted by p and p^*, respectively. We fix the world numeraire basket to contain α, $\alpha \in [0, 1]$, units of the Home good and (1-α) units of the Foreign good, and normalize the price of this basket to be equal to unity.

Investment opportunities are represented by four securities. Located in the Home country are a bond B, in zero net supply, and a risky stock S, in unit supply. Analogously, Foreign issues a bond B^* and a stock S^*. The bonds B and B^* are money market accounts instantaneously riskless in the local good, and the stocks S and S^* are claims to the local output. The terms of trade, q, are defined as the price of the Home good relative to that of the Foreign good: $q \equiv p/p^*$. The terms of trade are positively related to the real and nominal exchange rates; however, we delay making the exact identification till Section 3, where we map nominal quantities available in the data into the variables employed in the model.

A representative consumer-investor of each country is endowed at time 0 with a total supply of the stock market of his country. Thus, the initial wealth of the Home resident is $W_H(0) = S(0)$ and that of the Foreign resident is $W_F(0) = S^*(0)$. Each consumer i chooses nonnegative consumption of each good $(C_i(t),\, C_i^*(t))$, $i \in \{H,\, F\}$, and a portfolio of the available securities $(x_i^S(t),\, x_i^{S^*}(t),\, x_i^B(t),\, x_i^{B^*}(t))$, where x^j denotes a fraction of wealth W_i invested in security j. The dynamic budget constraint of each consumer takes the standard form

$$\frac{dW_i(t)}{W_i(t)} = x_i^B(t)\,\frac{dB(t)}{B(t)} + x_i^{B^*}(t)\,\frac{dB^*(t)}{B^*(t)} + x_i^S(t)\,\frac{dS(t) + p(t)Y(t)dt}{S(t)} + x_i^{S^*}(t)\,\frac{dS^*(t) + p^*(t)Y^*(t)dt}{S^*(t)}$$
$$-\frac{1}{W_i(t)}(p(t)C_i(t) + p^*(t)C_i^*(t))\,dt\,, \qquad i \in \{H,\, F\}, \tag{3}$$

[5] It is straightforward to extend the model to the case where shocks to the countries output are multi-dimensional and correlated. Our equilibrium characterization of the stock prices and the exchange rate would be the same. Since part of our objective in this paper is to demonstrate that Home and Foreign stock returns become correlated even when the countries' output processes are unrelated, we forgo inclusion of additional (possibly common) components in the shocks structure.

with $W_i(T) \geq 0$. Both representative consumers derive utility from the Home and Foreign goods

$$E\left[\int_0^T \theta_H(t)\left[a_H \log(C_H(t)) + (1-a_H)\log(C_H^*(t))\right]dt\right] \qquad \text{(Home country)}, \tag{4}$$

$$E\left[\int_0^T \theta_F(t)\left[a_F \log(C_F(t)) + (1-a_F)\log(C_F^*(t))\right]dt\right] \qquad \text{(Foreign country)}, \tag{5}$$

where a_H and a_F are the weights on Home goods in the utility function of each country. The objective of making a_H and a_F country-specific is to capture the possible home bias in the countries' consumption baskets. This home bias may in part be due to the presence of non-tradable goods, and by imposing an assumption that $a_H > a_F$, we would model it in a reduced form. (We elaborate on this in Section 4.) Heterogeneity in consumer tastes is required for most of our implications; otherwise demand shocks will have no effect on the real exchange rate or the terms of trade. The "demand shocks," θ_H and θ_F, are arbitrary non-negative adapted stochastic processes driven by $\vec{w}$, with $\theta_H(0) = 1$ and $\theta_F(0) = 1$. The only requirement we impose on these processes is that they be martingales. That is, $E_t[\theta_i(s)] = \theta_i(t),\ s > t,\ i \in \{H,\, F\}$. This specification is very general. In the special cases we consider later in this section, we put more structure on these processes depending on the interpretation we adopt. Note that the presence of the stochastic components θ_H and θ_F in (4)–(5) does not necessarily imply that the countries' preferences are state-dependent. As we discuss in one of the interpretations below, the countries may disagree on probability measures they use to compute expectations. Then, state-independent utilities under their own probability measures give rise to an equivalent representation (4)–(5) of the countries' expected utilities under the true measure.

2.2. Characterization of World Equilibrium

Financial markets in the economy are potentially dynamically complete since there are three independent sources of uncertainty and four securities available for investment. Since endowments are specified in terms of share portfolios, however, this is not sufficient to guarantee that markets are indeed complete in equilibrium, as demonstrated by Cass and Pavlova (2003) for a special case of our economy where θ_H and θ_F are deterministic. Nevertheless, one can still obtain a competitive equilibrium allocation by solving the world social planner's problem because Pareto optimality is preserved even under market incompleteness.[6] The planner chooses countries' consumptions so as

[6] For a special case of our economy where θ_H and θ_F are deterministic, Cass and Pavlova prove that any equilibrium in the economy must be Pareto optimal, and thus the allocation is a solution to the planner's problem. When θ_H and θ_F are stochastic, we verify that the allocation is Pareto optimal.

to maximize the weighted sum of countries' utilities, with weights λ_H and λ_F (see the Appendix), subject to the resource constraints:

$$\max_{\{C_H, C_H^*, C_F, C_F^*\}} \quad E\Bigg[\int_0^T \lambda_H\,\theta_H(t)\,[a_H\log(C_H(t)) + (1-a_H)\log(C_H^*(t))]$$

$$+\lambda_F\,\theta_F(t)\,[a_F\log(C_F(t)) + (1-a_F)\log(C_F^*(t))]dt\Bigg]$$

with multipliers

$$\text{s. t.} \qquad C_H(t) + C_F(t) = Y(t)\,, \qquad \eta(t), \tag{6}$$

$$C_H^*(t) + C_F^*(t) = Y^*(t)\,, \qquad \eta^*(t). \tag{7}$$

Solving the planner's optimization problem, we obtain the sharing rules

$$C_H(t) = \frac{\lambda_H\theta_H(t)a_H}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}\,Y(t), \qquad C_F(t) = \frac{\lambda_F\theta_F(t)a_F}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}\,Y(t)\,,$$

$$C_H^*(t) = \frac{\lambda_H\theta_H(t)(1-a_H)}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}\,Y^*(t), \quad C_F^*(t) = \frac{\lambda_F\theta_F(t)(1-a_F)}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}\,Y^*(t)\,.$$

The competitive equilibrium prices are identified with the Lagrange multipliers associated with the resource constraints. The multiplier on (6), $\eta(t, \omega)$, is the price of one unit of the Home good to be delivered at time t in state ω. Similarly, $\eta^*(t, \omega)$, the multiplier on (7), is the price of one unit of the Foreign good to be delivered at time t in state ω. We find it useful to represent these quantities as products of two components: the state price and the spot good price. The former is the Arrow-Debreu state price, denoted by π, of one unit of the numeraire to be delivered at (t, ω) and the latter is either p (for the Home good) or p^* (for the Foreign good). The equilibrium terms of trade are then simply the ratio of $\eta(t, \omega)$ and $\eta^*(t, \omega)$, which is, of course, the same as the ratio of either country's marginal utilities of the Home and Foreign goods:

$$q(t) = \frac{\eta(t)}{\eta^*(t)} = \frac{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}\,\frac{Y^*(t)}{Y(t)}\,. \tag{8}$$

The terms of trade increase in the Foreign and decrease in the Home output. When the Home output increases, all else equal, the terms of trade deteriorate as the Home good becomes relatively less scarce. Analogously, the terms of trade improve when Foreign's output increases. This is the standard terms of trade effect that takes place in Ricardian models of trade (see Ricardo (1817) and Dornbusch, Fischer, and Samuelson (1977) for seminal contributions). So far, the asset pricing literature has ignored it by assuming a single good.

The terms of trade also depend on the relative weight of the two countries in the planner's problem and the relative demand shock θ_H/θ_F. If we make an additional assumption that each country has a preference bias for the local good, then a positive relative demand shock improves Home's terms of trade: $sign(\partial q/\partial(\theta_H/\theta_F)) = sign(a_H - a_F) > 0$. The presence of this effect relates our model to the open economy macroeconomics literature. In the "dependent economy" model (see Salter (1959), Swan (1960), and Dornbusch (1980), Chapter 6), a demand shift biased toward the domestic good raises the price of the Home good relative to that of the Foreign, thus appreciating the exchange rate. In our model, if a_H is larger than a_F then the relative demand shock does indeed represent a demand shift biased toward the domestic good.

Finally, we determine stock market prices in the economy. Using the no-arbitrage valuation principle, we obtain

$$S(t) = \int_t^T \frac{\pi(s)}{\pi(t)} p(s) Y(s) ds \quad \text{and} \quad S^*(t) = \int_t^T \frac{\pi(s)}{\pi(t)} p^*(s) Y^*(s) ds. \tag{9}$$

Explicit evaluation of these integrals, the details of which are relegated to the Appendix, yields

$$S(t) = \frac{q(t)}{\alpha q(t) + 1 - \alpha} Y(t)(T - t), \tag{10}$$

$$S^*(t) = \frac{1}{\alpha q(t) + 1 - \alpha} Y^*(t)(T - t). \tag{11}$$

Consistent with insights from the asset pricing literature, each country's stock price is positively related to the national output the stock is a claim to. Recall that the innovations to the processes driving the Home and Foreign output are independent. This, however, does not imply that international stock markets are contemporaneously uncorrelated. The correlation is induced through the terms of trade present in (10)–(11). One can combine (10) and (11) to establish a simple relationship tying together the stock prices in the two countries and the prevailing terms of trade:

$$S^*(t) = \frac{1}{q(t)} \frac{Y^*(t)}{Y(t)} S(t). \tag{12}$$

The dynamics of the terms of trade and the international financial markets are jointly determined within our model. Proposition 1 characterizes these dynamics as a function of three sources of uncertainty: two country-specific shocks and the relative demand shock.

Proposition 1. *The dynamics of the Home and Foreign stock and bond markets, and the terms* *of trade are given by*

$$\frac{dS(t)}{S(t)} = I_1(t)dt + \frac{1-\alpha}{\alpha q(t)+1-\alpha}A(t)d\theta(t) + \frac{\alpha q(t)}{\alpha q(t)+1-\alpha}\sigma_Y(t)dw(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha}\sigma_Y^*(t)dw^*(t)\,, \quad (13)$$

$$\frac{dS^*(t)}{S^*(t)} = I_2(t)dt - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha}A(t)d\theta(t) + \frac{\alpha q(t)}{\alpha q(t)+1-\alpha}\sigma_Y(t)dw(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha}\sigma_Y^*(t)dw^*(t)\,, \quad (14)$$

$$\frac{dB(t)}{B(t)} = I_3(t)dt + \frac{1-\alpha}{\alpha q(t)+1-\alpha}A(t)d\theta(t) - \frac{1-\alpha}{\alpha q(t)+1-\alpha}\sigma_Y(t)dw(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha}\sigma_Y^*(t)dw^*(t)\,, \quad (15)$$

$$\frac{dB^*(t)}{B^*(t)} = I_4(t)dt - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha}A(t)d\theta(t) + \frac{\alpha q(t)}{\alpha q(t)+1-\alpha}\sigma_Y(t)dw(t) - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha}\sigma_Y^*(t)dw^*(t)\,, \quad (16)$$

$$\frac{dq(t)}{q(t)} = I_5(t)dt + A(t)d\theta(t) - \sigma_Y(t)dw(t) + \sigma_Y^*(t)dw^*(t)\,, \quad (17)$$

where $\theta(t) \equiv \theta_H(t)/\theta_F(t)$*,* $A(t) \equiv \lambda_H\lambda_F(a_H - a_F)/[(\lambda_H\theta(t)a_H + \lambda_F a_F)(\lambda_H\theta(t)(1-a_H) + \lambda_F(1-a_F))]$*, and* $I_j(t)$*,* $j = 1, \ldots, 5$ *are reported in the Appendix. Furthermore, if* $a_H > a_F$*, the diffusion coefficients of the dynamics of the Home and Foreign stock markets and the terms of trade have the following signs:*

Variable/ Effects of	$d\theta(t)$	$dw(t)$	$dw^*(t)$
$\frac{dS(t)}{S(t)}$	+	+	+
$\frac{dS^*(t)}{S^*(t)}$	−	+	+
$\frac{dB(t)}{B(t)}$	+	−	+
$\frac{dB^*(t)}{B^*(t)}$	−	+	−
$\frac{dq(t)}{q(t)}$	+	−	+

(18)

Proposition 1 identifies some important interconnections between the financial and real markets in the world economy. Under the home bias assumption, it entails unambiguous directions of contemporaneous responses of all markets to innovations in supply and demand, summarized in (18). (18) nests some fundamental implications from various strands of international economics, which we highlighted in our earlier discussion. Our goal is to unify them within a simple model, and focus on the interactions.

One such interaction sheds light on the determinants of financial contagion − a puzzling tendency of stock markets across the world to exhibit excess co-movement (for a recent detailed account of this phenomenon, see Kaminsky, Reinhart, and Végh (2003)). Contagion in our model occurs in response to an output shock. *Ceteris paribus,* a positive output shock say in Home causes a positive return on the domestic stock market. At the same time, however, it initiates a Ricardian response of the terms of trade: the terms of trade move against the country experiencing a productivity increase. A flip side of the deterioration of the terms of trade in Home is an improvement of those in Foreign. Hence, the value of Foreign's output has to rise, thereby providing a boost to Foreign's stock market. Note that nothing in this argument relies on the correlation between the countries' output processes. In fact, in a special case of our model where there are no demand shocks (discussed below), we obtain a perfect co-movement of the stock markets despite independence of the countries' output innovations. Bond markets certainly also react to changes in productivity: a positive output shock in Home lowers bond prices in Home and lifts those in Foreign.

While supply shocks move the countries' stock prices in the same direction, demand shocks act in the opposite way. We call this phenomenon "divergence." Thus, a demand shock say in Home causes a relative demand shift biased toward the Home good, thereby boosting Home's terms of trade. Improved terms of trade increase the value of Home's output, and hence lift Home's stock market, while decreasing that of Foreign's output and lowering Foreign's stock market. Similarly, since our bonds are real bonds, a strengthening of Home's terms of trade increases the value of the Home bond and decreases that of the Foreign. A world economy without supply shocks is thus an example of perfect divergence: asset markets of different countries always move in opposite directions.

Finally, Proposition 1 provides analytical characterization of the sensitivities of each market's responses to the supply and demand shocks, and also establishes cross-equation restrictions on how these sensitivities measure up to each other. The system of simultaneous equations describing the joint dynamics of the five markets (13)–(17) establishes a basis for our empirical analysis, carried out in Section 3.

2.3. Special Cases and Interpretations of $\theta_{\mathbf{H}}$ and $\theta_{\mathbf{F}}$

In Proposition 1 we did not commit to specifying the dynamics of the demand shocks θ_H and θ_F. For the empirical analysis to follow, this is advantageous because the relative demand shock $\theta = \theta_H/\theta_F$, in general, depends on the underlying Brownian motions w and w^* inducing a correlation in the error structure that we do not have to impose ex-ante. In this section, we put more structure on the martingales θ_H and θ_F as we consider various economic interpretations of these processes:

$$d\theta_H(t) = \vec{\kappa}_H(t)^\top \theta_H(t) d\vec{w}(t)\,, \qquad d\theta_H(t) = \vec{\kappa}_F(t)^\top \theta_H(t) d\vec{w}(t)\,. \tag{19}$$

A. Deterministic Preference Parameters

It is instructive to consider a special case where θ_H and θ_F are deterministic processes, i.e., $\vec{\kappa}_H$ and

$\vec{\kappa}_F$ are zeros. By substituting equation (8) into equation (12), we find that

$$S^*(t) = \frac{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}{\lambda_H \theta_H(t)(1 - a_H) + \lambda_F \theta_F(t)(1 - a_F)} S(t).$$

As the multiplier term is time deterministic, the two stock markets must be perfectly correlated. This implication achieves our goal of generating international financial contagion without assuming correlation between individual countries' output processes, but it is certainly rather extreme and is easily rejected empirically. (Zapatero (1995) and Cass and Pavlova (2003) obtain this implication in their models as well, dubbing the resulting equilibrium a "peculiar" financial equilibrium.) In what follows, we force θ_H and θ_F to be stochastic processes, which would provoke the divergence effect and hence guarantee imperfect correlation between Home and Foreign stock markets, but would still produce financial contagion.

B. Preference Shocks or Pure Sentiment

The next special case we consider is where θ_H and θ_F are driven by the Brownian motion w^θ, independent of w and w^*. In this case, $\vec{\kappa}_H = (0, 0, \kappa_H)^\top$ and $\vec{\kappa}_F = (0, 0, \kappa_F)^\top$, where κ_H and κ_F are arbitrary adapted processes. Then θ has dynamics

$$d\theta(t) = (\kappa_F(t)^2 - \kappa_H(t)\kappa_F(t))\, \theta(t)\, dt + (\kappa_H(t) - \kappa_F(t))\, \theta(t)\, dw^\theta(t).$$

The process θ can then be interpreted as a relative preference shock or a shock to consumer sentiment.

C. "Catching up with the Joneses"/ Consumer Confidence

Consider the case where the processes θ_H and θ_F depend on the country-specific Brownian motions: $\vec{\kappa}_H = (\kappa_H, 0, 0)^\top$ and $\vec{\kappa}_F = (0, \kappa_F, 0)^\top$, where $\kappa_H, \kappa_F < 0$ may depend on w and w^*.[7] Home and Foreign consumers' preferences then display "catching up with the Joneses" behavior (Abel (1990)). The "benchmark" levels of Home and Foreign consumers, θ_H and θ_F, are negatively correlated with aggregate consumption of their country, Y and Y^*, respectively. Positive news to local aggregate consumption reduces satisfaction of the local consumer, as his consumption bundle becomes less attractive in an improved domestic economic environment. The implication of this interpretation

[7] The labels κ_H and κ_F are used just to save on notation, they need not be the same as in interpretation **B**. Note that under this interpretation $\vec{\kappa}_H$ and $\vec{\kappa}_F$ do not depend on the Brownian motion w^θ. This specification makes w^θ a sunspot in the sense of Cass and Shell (1983): it affects neither preferences nor aggregate endowments. There is then a potential problem with employing the planner's problem in solving for equilibrium allocations because it only identifies nonsunspot equilibria. Since markets are dynamically complete in our economy, however, the sunspot immunity argument of Cass and Shell goes through.

for the relative demand shock, θ, is that

$$d\theta(t) = \kappa_F(t)^2\,\theta(t)\,dt + \kappa_H(t)\,\theta(t)\,dw(t) - \kappa_F(t)\,\theta(t)\,dw^*(t), \qquad \kappa_H(t), \kappa_F(t) < 0.$$

That is, an innovation to θ is negatively correlated with the Home output shock and positively correlated with the Foreign shock.

An alternative kind of dependence of agents' preferences on the country-specific output shock is a form of consumer confidence. The idea is that agents get more enthusiastic when their local economy is doing well and hence demand more consumption. Formally, preferences exhibiting consumer confidence are the same as those under catching up with the Joneses, except that $\kappa_H(t)$, $\kappa_F(t) > 0$.

D. Radon-Nikodym Derivatives Reflecting Heterogeneous Beliefs

The previous two special cases we considered assume that consumer preferences are state-dependent. State-dependence of preferences, however, is not necessarily implied by our specification (4)–(5). Under the current interpretation, the countries have state-independent preferences, but they differ in their assessment of uncertainty underlying the world economy. For example, Home residents may believe that the uncertainty is driven by the (vector) Brownian motion $\vec{w}_H \equiv (w_H,\, w_H^*,\, w_H^\theta)^\top$ and the Foreign residents believe that it is driven by $\vec{w}_F \equiv (w_F,\, w_F^*,\, w_F^\theta)^\top$. All we require is that the "true" probability measure P and the country-specific measures H (Home) and F (Foreign) are equivalent; that is, they all agree on the zero probability events. Under their respective measures, the agents' expected utilities are given by

$$E^i\left[\int_0^T [a_i \log(C_i(t)) + (1-a_i)\log(C_i^*(t))]dt\right], \qquad i \in \{H,\, F\},$$

where E^i denotes the expectation taken under the information set of agent i. Thanks to Girsanov's theorem, the above expectations can be equivalently restated under the true probability measure in the form of (4)–(5). The multiples θ_H and θ_F, appearing in the expressions as a result of the change of measure, are the so-called Radon-Nikodym derivatives of H with respect to P, $\left(\frac{dH}{dP}\right)$, and F with respect to P, $\left(\frac{dF}{dP}\right)$, respectively.

The Radon-Nikodym derivatives θ_H and θ_F may reflect various economic scenarios. One is the case of incomplete information: the consumers do not observe the parameters of the output processes, and need to estimate them. While the diffusion coefficients $\sigma_Y(t)$ and $\sigma_Y^*(t)$ may be estimated by computing quadratic variations of $Y(t)$ and $Y^*(t)$, estimation of mean growth rates of output is rather nontrivial. First, the consumers may be assumed to be Bayesian, endowed with some priors of the growth rates. Then they will be updating their priors each instant as new information arrives, through solving a filtering problem.[8] Second, the consumers may use some updating method other than Bayesian. For example, they may be systematically overly optimistic or pessimistic about the mean growth rates of output. Finally, they may explicitly account for model uncertainty in their decision-making. All these special cases result in consumers employing a probability measure different from the true one. These differences of opinion can be succinctly represented by some Radon-Nikodym derivatives θ_H and θ_F. Under any of these interpretations, we can no longer assume that θ in Proposition 1 is uncorrelated with either w or w^*, which we have to take into account in our econometric tests.

2.4. Interest Rate Parity and Trading Strategies

Uncovered interest rate parity in its classical form is a relationship between local interest rates at Home and Foreign and the expected exchange rate (terms of trade) appreciation. In this section we show that in our setting, such a relationship must have an additional term capturing a pertinent risk premium. This term is, in general, time-varying. We also fully characterize equilibrium interest rates prevailing at Home and Foreign, under different interpretations of the demand shocks. These results are reported in the following Proposition.

Proposition 2. *The uncovered interest rate parity relationship has the form*

$$r^F(t) - \mu_q(t) = r^H(t) + \sigma_q(t)^\top (m^H(t) + \sigma_q(t)), \tag{20}$$

where μ_q and σ_q are the mean growth and volatility parameters in the dynamic representation of q, $dq(t)/q(t) = \mu_q(t)dt + \sigma_q(t)^\top d\vec{w}(t)$, m^H is the Home market price of risk reported in the Appendix, and r^H and r^F are the Home and Foreign real interest rates, i.e., instantaneously riskless returns on local money market accounts specified in terms of the local goods, given by:

(i) under interpretations **A** *and* **B**,

$$r^H(t) = \mu_Y(t) - \sigma_Y(t)^2, \quad r^F(t) = \mu_Y^*(t) - \sigma_Y^*(t)^2,$$

[8] We provide details of the ensuing filtering problems in the Appendix, in the context of the proof of Proposition 2. We consider the case where the agents believe there are only two independent innovation processes, w_i and w_i^*, and the case where, additionally, there is the third one, w_i^θ. The first two innovations drive the output processes Y and Y^*, respectively. Following Detemple and Murthy (1994), we impose additional regularity conditions on $\sigma_Y(t)$ and $\sigma_Y^*(t)$ to make the filtering problem tractable: both processes are bounded and are of the form $\sigma_Y(Y(t), t)$ and $\sigma_Y^*(Y^*(t), t)$, respectively. When the agents perceive that the parameters they are estimating also depend on the third innovation process, which can represent either intrinsic of extrinsic uncertainty (a sunspot), they make use of a public signal carrying information about w_i^θ.

(ii) under interpretation **C***,*

$$r^H(t) = \mu_Y(t) - \sigma_Y(t)^2 + \underbrace{\frac{\lambda_H\theta_H(t)a_H}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}}_{C_H(t)/Y_H(t)}\sigma_Y(t)\kappa_H(t),$$

$$r^F(t) = \mu_Y^*(t) - \sigma_Y^*(t)^2 + \underbrace{\frac{\lambda_F\theta_F(t)(1-a_F)}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}}_{C_F^*(t)/Y_F^*(t)}\sigma_Y^*(t)\kappa_F(t),$$

(iii) under interpretation **D** *(see the Appendix for the detailed description of the economic setting),*

$$r^H(t) = \mu_Y(t) - \sigma_Y(t)^2 + \underbrace{\frac{\lambda_H\theta_H(t)a_H(\mu_H(t)-\mu_Y(t)) + \lambda_F\theta_F(t)a_F(\mu_F(t)-\mu_Y(t))}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}}_{\frac{C_H(t)}{Y(t)}(\mu_H(t)-\mu_Y(t))+\frac{C_F(t)}{Y(t)}(\mu_F(t)-\mu_Y(t))},$$

$$r^F(t) = \mu_Y^*(t) - \sigma_Y^*(t)^2 + \underbrace{\frac{\lambda_H\theta_H(t)(1-a_H)(\mu_H^*(t)-\mu_Y^*(t)) + \lambda_F\theta_F(t)(1-a_F)(\mu_F(t)-\mu_Y^*(t))}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}}_{\frac{C_H^*(t)}{Y^*(t)}(\mu_H^*(t)-\mu_Y^*(t))+\frac{C_F^*(t)}{Y^*(t)}(\mu_F^*(t)-\mu_Y^*(t))},$$

where $\mu_H(t)$ and $\mu_F(t)$ denote perceived or estimated mean growth rates of the Home output by Home and Foreign consumers, respectively, and $\mu_H^*(t)$ and $\mu_F^*(t)$ denote perceived or estimated mean growth rates of the Foreign output by Home and Foreign consumers, respectively.

In its classical form, uncovered interest rate parity is a relationship which assumes that arbitrage will enforce equality of returns on the following two investment strategies. The first one is investing a unit of wealth in the Home money market account resulting in receiving the riskless return at rate r^H, in units of the Home good, over the next instant. The second is investing the same amount in the Foreign money market, at rate r^F, over the next instant and then converting the proceeds paid out in the Foreign good into the Home good using the prevailing terms of trade. In our model, the two strategies are not equivalent because in terms of the Home good, the first strategy is riskless, while the second one is risky. The left-hand side of (20) represents the mean return on the second strategy, given by the Foreign money market rate less the mean terms of trade appreciation. The right-hand side contains the return on the riskless first strategy, plus an additional term capturing the risk premium the risky strategy commands. This risk premium is, in general, time-varying. The only case in which it can be shown to be constant in the context of our model is within interpretation **A** (no demand shocks) under an additional assumption that the mean growths and volatilities of the output processes Y and Y^* are constant. Recall that the role of interpretation **A** is to establish a theoretical benchmark; it is easily rejected empirically.

Under the remaining interpretations, the risk premium term depends on the demand shocks. Hence, uncovered interest rate parity in its classical form − (20) with no risk premium term − does not obtain in our model.

In the empirical section, which comes next, we estimate the variance-covariance matrix of the shocks, and find that the variance of demand shocks is two times larger than that of supply shocks. One implication of this finding is that the risk premium would be very volatile, too. Deviations from the classical uncovered interest rate parity relationship have been found to be very large and volatile, and the need for a very volatile risk premium has been emphasized repeatedly in the empirical literature since Fama (1984). Our model may then shed some light on these issues.

Equilibrium riskless rates of return on money market accounts in each country have to be such that agents are willing to save. In the benchmark case of no demand shocks (interpretation **A**), local interest rates must then be positively related to the growth of aggregate consumption of the local good and negatively related to the aggregate consumption volatility, reflecting the precautionary savings motive. When the demand shocks are interpreted as pure sentiment shocks (**B**), driven by an independent source of uncertainty, the local money market rates are the same as in the benchmark. This is because the sentiment shocks are martingales; otherwise if they had nonzero mean growth terms, these terms would have appeared in the expression for the interest rate as additional inducements for the agents to save, acting analogously to impatience parameters (which we do not model). Since the demand shocks do not co-vary with aggregate consumption uncertainty, they also do not give rise to an additional precautionary savings component. Such a component appears under interpretation **C**, where aggregate consumption of the local good imposes a negative (catching up with the Joneses) or positive (consumer confidence) externality on the agents. In the former case, this gives rise to an additional precautionary savings effect, driving down the interest rates, while in the latter case the interest rates rise. The strength of this additional effect induced by consumption externalities of Home (Foreign) agents is determined by the relative importance of Home (Foreign) consumers in the world economy. This relative importance is captured by the shares of the agents in the aggregate consumption of the local good, which can also be linked to wealth distribution in the economy. An additional component over the benchmark also appears in the interest rate expressions under the differences of opinion interpretation **D**, as highlighted in Proposition 2. Perhaps a better way to present say the Home rate under this interpretation is as a weighted average of the Home rates prevailing if only Home consumers were present in the economy

$(\mu_H(t) - \sigma_Y(t)^2)$ and if only Foreign consumers were present $(\mu_F(t) - \sigma_Y(t)^2)$, with weights again given by the agents' consumption shares.

We now turn to computing the agents' trading strategies. To facilitate comparison with the literature, we consider an additional security: a "world" bond B^W, locally riskless in the numeraire. This bond does not, of course, introduce any new investment opportunities in the economy; it is just a portfolio of α shares of the Home bond and (1-α) shares of the Foreign bond. Let r denote the interest rate on this bond.

The number of non-redundant securities differs across interpretations **A**–**D**, which complicates our exposition. Thus, as we discussed earlier, Home and Foreign stocks are perfectly correlated under interpretation **A**, and hence investments in individual stock markets cannot be uniquely determined. Under the remaining interpretations, we can uniquely identify agents' holdings of national stock markets. One of the bond markets, however, may or may not be redundant depending on the interpretation. In what follows, we replace the Foreign bond by the world bond in the investment opportunity set of the agents. (Investment in the Foreign bond can be recovered from the portfolio allocations to the Home and world bonds, where applicable.)

Before we proceed to reporting our results, we need to define the notion of a *home bias*. We measure a bias in portfolios relative to the (common to the agents) mean-variance portfolio. Thus, a Home (Foreign) resident's portfolio is said to exhibit a home bias if the portfolio weight he assigns to the Home (Foreign) stock is higher than that in the mean-variance portfolio.

Proposition 3. *(i) Countries' portfolios are given by*

$$x_i(t) = \underbrace{(\sigma(t)^\top)^{-1}\mathcal{I}\, m(t)}_{\textit{mean-variance portfolio}} + \underbrace{(\sigma(t)^\top)^{-1}\mathcal{I}\,\vec{\kappa}_i(t)}_{\textit{hedging portfolio}}, \qquad i \in \{H, F\}, \tag{21}$$

where the compositions of the vector of the fractions of wealth invested in risky assets, x_i, of the volatility matrix of the investment opportunity set σ, of the market price of risk m and of the auxiliary matrix $\mathcal{I}$, different across interpretations **A**–**D**, *are provided in the Appendix. The remaining fraction of wealth, $1 - x_i^\top \vec{1}$, is invested in the world bond B^W, where $\vec{1} = (1, \ldots, 1)^\top$.*

(ii) Under interpretation **B**, *countries' portfolios do not exhibit a home bias. Assume further that $a_H > a_F$. Then, under interpretation* **C**, *in the case of $\kappa_H(t), \kappa_F(t) > 0$ (consumer confidence) portfolios always exhibit a home bias, while in the case of $\kappa_H(t), \kappa_F(t) < 0$ (catching up with the Joneses) the portfolio of Home exhibits a home bias if and only if $(1-\alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t) > 0$ and that of Foreign if and only if $\alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1-\alpha)\kappa_H(t) > 0$. Under interpretation* **D**, *the sign of the bias is ambiguous.*

(iii) The international CAPM has the form

$$\frac{E_t(dS^j(t))}{S^j(t)} - r(t) = Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, dW(t)\right) - Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{\lambda_H}{\lambda_H\theta_H(t)+\lambda_F\theta_F(t)}d\theta_H(t)\right)$$
$$-Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{\lambda_F}{\lambda_F\theta_F(t)+\lambda_F\theta_F(t)}d\theta_F(t)\right), \qquad S^j \in \{S,\, S^*\}\,, \quad (22)$$

where $W \equiv W_H + W_F$ is the aggregate wealth and r is the interest rate on the world bond, reported in the Appendix.

In our economy, the fact that the countries have logarithmic preferences does not imply that the their investment behavior is myopic. Their trading strategies involve holding the standard mean-variance portfolio along with a hedging one. Although the countries do not hedge against changes in the investment opportunity set (which is standard for logarithmic preferences), they do hedge against their respective demand shocks.[9] The way they do so depends on the correlation between their demand shocks and output shocks, and hence differs across our interpretations of demand shocks. Thus, if the demand shocks are independent of output shocks − pure sentiment interpretation **B** − portfolio weights the countries assign to the stocks coincide with those in the mean-variance portfolio. If a country's demand shocks are positively related to its output shocks (consumer confidence interpretation **C**)− that is, if a country demands more consumption (biased toward the domestic good) when it is experiencing a positive output shock − consumers increase their portfolio holdings of the domestic stock, a claim to domestic output. On the other hand, if the demand shocks are negatively correlated with the country's output innovations − catching up with the Joneses interpretation **C** − a home bias in portfolios may or may not arise. Proposition 3 provides a necessary and sufficient condition for the bias to occur. Finally, our differences of opinion interpretation **D** is too general to entail unambiguous implications on the sign of the bias. However, if specialized further, it is likely to yield sharper predictions. For example, albeit in a different setting, Uppal and Wang (2003) argue that disparities in agents' ambiguity about home and foreign stock returns can lead to a home bias in portfolios.

[9]Our references to consumers' hedging behavior made in the context of the differences of opinion interpretation of the demand shocks may appear confusing. How can consumers hedge deviations of their beliefs from the true probability measure, which they do not know? In fact, they do not. They hold a portfolio which is mean-variance efficient under *their own* probability measure, given by the product of the inverted volatility matrix $(\sigma^\top)^{-1}$ and the country-specific market prices of risk $m_i = m + \vec{\kappa}_i$, constructed from their own investment opportunity sets. The representation in equation (21) is thus formally equivalent to a mean-variance portfolio under a country's individual beliefs, except it further decomposes a country-specific market price of risk into two components: the "true" market price of risk m and the measure of deviation of agent's beliefs from the true probability measure $\vec{\kappa}_i$, both unobservable from the point of view of an agent.

The presence of the hedging portfolios optimally held by the agents clearly rules out the traditional one-factor CAPM. In addition to the standard market (aggregate wealth) factor, our model identifies two further factors, the Home and Foreign demand shocks, that affect the risk premia on the stocks. The expression in (22) also differs from the international CAPM (see Solnik (1974), Adler and Dumas (1983)). As is standard for logarithmic preferences, the exchange rate does not appear explicitly in (22); however, its determinants – the demand shocks – do. (One would expect the exchange rate to appear as an additional factor if we relaxed the assumption of logarithmic preferences.) The presence of the demand shocks points to a possible misspecification of the CAPM widely tested empirically. Our alternative formulation might then provide an improvement over the standard specification used in the international finance literature.

3. Empirical analysis

In this section we examine the empirical implications of the model. We use daily data for the US vis-à-vis the UK and Germany that has been collected from DataStream. As proxies for our riskless bonds, we use data on the three-month zero-coupon government bonds for each country. The US, UK and German stock market indexes are taken to represent the countries' stock prices. The dollar-pound and dollar-mark exchange rates are used for identifying the real exchange rates and the terms of trade via a procedure described below. The data for the US and the UK are from 1988 until the end of 2002. The data for Germany is shorter, until December 1998, because of the transition to the euro. We did not consider it prudent to complete the data from 1998 to the end of 2002 by extrapolating the euro exchange rates and interest rates to the mark. Moreover, we do not want our exercise to be contaminated by the change in an exchange rate regime that took place with the creation of the euro. All empirical analysis involving Germany is thus performed on a shorter data set.

Figure 1 depicts the evolution of the exchange rate along with stock and bond markets values for the US vis-à-vis Germany and the UK. Even a casual observation of the figure suggests that the bond prices do not bear too much relationship to the exchange rates. This observation is strongly supported by a regression analysis, not reported here, whose results may be interpreted as a failure of uncovered interest rate parity, well-documented in the literature and also found in our sample. On the other hand, the amount of co-movement between the stock indexes and the exchange rates is quite striking. We computed the pertinent correlations and found them to be highly statistically significant. Our theory that stock prices and exchange rates are driven by the same set of factors, then, does not appear to be empirically unfounded.

3.1. Real and Nominal Quantities

Before we proceed with our formal empirical analysis, we need to establish a mapping between the quantities employed in our model and those in the data available to us − the data are in nominal terms, while our model is real. The main issue is to impute the terms of trade from the available data on nominal exchange rates. To do so, we first compute the real exchange rate implied by our model. The real exchange rate, e, is defined as $e(t) = P_H(t)/P_F(t)$, where P_H and P_F are Home and Foreign price indexes, respectively. In practice, a variety of methods have been used for computing a country's price level, or cost of living (see Schultze (2003)). We have chosen to adopt geometric average price indexes, which address the criticism pertaining to the substitution between goods bias, and are consistent with the countries' homothetic preferences:

$$P_H(t) = \left(\frac{p(t)}{a_H}\right)^{a_H} \left(\frac{p^*(t)}{1-a_H}\right)^{1-a_H}, \qquad P_F(t) = \left(\frac{p(t)}{a_F}\right)^{a_F} \left(\frac{p^*(t)}{1-a_F}\right)^{1-a_F}.$$

The real exchange rate, expressed as a function of the terms of trade, is then

$$e(t) = q(t)^{a_H - a_F} \frac{(1-a_F)^{1-a_F} a_F^{a_F}}{(1-a_H)^{1-a_H} a_H^{a_H}}.$$

Finally, to obtain the nominal exchange rate, we need to adjust the real exchange rate for inflation in Home versus Foreign. Unfortunately, daily data on inflation, required for our estimation, are not available. However, our data span a relatively short period of time when international inflation rates were very low. Consequently, as argued by Mussa (1979), our real rates are closely related to the nominal ones. We thus assume that inflation is negligibly small and simply make a level adjustment to the real rate to back out the nominal exchange rate, ε: $\varepsilon(t) = \bar{\varepsilon} e(t)$, where $\bar{\varepsilon}$ is the average nominal exchange rate in the sample.

For our empirical investigation of the dynamics reported in Proposition 1, we use the variable

$$q(t) = \left(\frac{\varepsilon(t)}{\bar{\varepsilon}} \frac{(1-a_H)^{1-a_H} a_H^{a_H}}{(1-a_F)^{1-a_F} a_F^{a_F}}\right)^{1/(a_H - a_F)} \tag{23}$$

as a proxy for the terms of trade. Taking logs and applying Itô's lemma, we obtain an equation to replace (17) in Proposition 1:

$$\frac{d\varepsilon(t)}{\varepsilon(t)} = I_6(t)dt + (a_H - a_F)A(t)d\theta(t) - (a_H - a_F)\sigma_Y(t)dw(t) + (a_H - a_F)\sigma_Y^*(t)dw^*(t),$$

where $I_6(t) = (a_H - a_F)I_5(t) + \frac{1}{2}(a_H - a_F)(a_H - a_F - 1)|\sigma_q(t)|^2$. Under our assumption of no inflation over the time period we are considering, the remaining equations in Proposition 1, (13)–(16), are unchanged when prices are expressed in nominal terms. Finally, since the nominal exchange rate, ε, is monotonically increasing with the terms of trade q, the signs of the coefficients identified in Proposition 1 are maintained for nominal prices. Moreover, the signs of the effects of the shocks on $\frac{d\varepsilon(t)}{\varepsilon(t)}$ are the same as those for $\frac{dq(t)}{q(t)}$ reported in (18).

For a practical implementation of this conversion of units, we need to calibrate parameters a_H, a_F and α used in the expressions. It is reasonable to assume that about three quarters of a country's consumption comes from the locally produced good. Such home bias is primarily due to the presence of non-tradable goods, which comprise a large fraction of national consumption. While we do not explicitly account for non-tradable goods, their presence is modeled in reduced form, through parameters a_H and a_F (see Section 4 for further elaboration). It is thus reasonable to set a_H equal to 0.75 and a_F equal to 0.25. We also need a sensible value for α, the weight of the US-produced goods in the world numeraire basket, or a price index. By setting $\alpha = 0.75$ we are implicitly assuming that the US economy is about three times the UK economy. Of course, we run robustness checks where we vary these parameters, which we discuss later in this section. The calibrated values of a_H and a_F are used for computing the series for the terms of trade via (23). We then use the numeraire basket (identified by α) and the terms of trade to convert stock and bond prices for each country into a common numeraire.

3.2. Cross equation restrictions

We now turn to testing our model's implications on the dynamic behavior of asset prices and exchange rates. They are summarized in Proposition 1, in the system of equations (13)–(17) and the accompanying table with the predicted signs of the coefficients (18). For convenience, we reproduce this system below, in a matrix form:

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{dB(t)}{B(t)} \\ \frac{dB^*(t)}{B^*(t)} \\ \frac{d\varepsilon(t)}{\varepsilon(t)} \end{bmatrix} = \vec{I} + \underbrace{\begin{bmatrix} b(t) & 1-b(t) & b(t) \\ -1+b(t) & 1-b(t) & b(t) \\ b(t) & -b(t) & b(t) \\ -1+b(t) & 1-b(t) & -1+b(t) \\ (a_H - a_F) & -(a_H - a_F) & (a_H - a_F) \end{bmatrix}}_{\mathcal{B}_1} \begin{bmatrix} A(t)\, d\theta(t) \\ \sigma_Y(t)\, dw(t) \\ \sigma_Y^*(t)\, dw^*(t) \end{bmatrix}, \tag{M1}$$

where $\vec{I}$ is a (vector) intercept term and

$$b(t) \equiv \frac{1-\alpha}{\alpha q\left(t\right)+1-\alpha}\,, \quad \text{with} \quad q(t)=\left(\frac{\varepsilon(t)}{\overline{\varepsilon}}\frac{(1-a_H)^{1-a_H}a_H^{a_H}}{(1-a_F)^{1-a_F}a_F^{a_F}}\right)^{1/(a_H-a_F)}. \tag{24}$$

Unfortunately, while we can construct all the left-hand side variables in (M1), we do not have data on the variables on the right-hand side of the system. Consequently, in our estimation we treat the innovations $A\left(t\right)d\theta\left(t\right)$, $\sigma_Y\left(t\right)dw\left(t\right)$, and $\sigma_Y^*\left(t\right)dw^*\left(t\right)$ as latent factors that we extract from stock and bond prices and exchange rates. The first factor, $A\left(t\right)d\theta\left(t\right)$, captures the relative demand shock, while the remaining two, $\sigma_Y\left(t\right)dw\left(t\right)$ and $\sigma_Y^*\left(t\right)dw^*\left(t\right)$, represent Home and Foreign supply shocks, respectively. (There is a slight abuse of terminology in this section; earlier we referred to $d\theta$, dw and dw^* as the demand and supply shocks.) Note that the factor loadings in matrix $\mathcal{B}_1$ are fully described by the quantity $b(t)$, which depends only on the weight of the US-produced goods in the world numeraire basket α, the nominal exchange rate ε, and the preference for domestic good parameters a_H and a_F.

Perhaps the most direct approach to testing our model would be to take Proposition 1 literally and estimate the structural latent factor model (M1). Note, however, that since the factors are in general heteroskedastic and may be correlated in our model, a priori there is no reason to put any restrictions on their variance-covariance matrix. The only set of restrictions our theory entails is those on the elements of matrix $\mathcal{B}_1$. This estimation strategy, in practice, imposes too few constraints on the model: implicitly, we would be fitting five series with three unrestricted factors. As a result, the fit of the model within sample would be very good and the likelihood of rejecting the model very low.[10] For this reason, we have decided to adopt a different approach. We seek support for our theoretical implications from the following two tests. The first one considers a simplified version of (M1), where factor loadings are assumed constant, and the factors are homoskedastic. We impose all the cross equation constraints, i.e., force the factor loadings in $\mathcal{B}_1$ to be related to each other exactly as specified. In the second test, we drop the cross equations restrictions as well as the assumption of homoskedasticity of the factors, and estimate the model imposing only the sign restrictions from (18). The remainder of this subsection implements the first test, and the next subsection is devoted to the second one.

The key to executing the first test is recognizing that under the assumptions of constant coefficients ($b(t)=b$) and homoskedasticity we can estimate the model using the unconditional covariance

[10] We have indeed estimated the model following this strategy. The fit within sample has been unrealistically good. Results are available from the authors upon request, and from http://web.mit.edu/~rigobon/www/.

	US-UK		US-Germany	
	Point Estimate	Std. Error	Point Estimate	Std. Error
a_H-a_F	0.5219	0.0010	0.5450	0.0021
b	0.2569	0.0016	0.2588	0.0019
σ_θ	0.7702	0.0087	0.7616	0.0107
σ_Y	0.3282	0.0057	0.2390	0.0049
σ_Y^*	0.3824	0.0067	0.4968	0.0095
$\rho_{\theta,Y}$	0.4294	0.0056	0.3108	0.0058
$\rho_{\theta,Y}^*$	-0.6658	0.0132	-0.9902	0.0209

Table 1: Constant coefficients and cross equation restrictions: Estimation of Model (M1) for the US vis-à-vis the UK and Germany. σ_θ denotes the standard deviation of the relative demand shock $A(t)d\theta(t)$, and $\rho_{\theta,Y}$, $\theta_{\theta,Y}^*$ are the correlations of this factor with the supply shocks $\sigma_Y(t)dw(t)$ and $\sigma_Y^*(t)dw^*(t)$, respectively. All standard deviations are in percentage terms.

matrix computed from the aforementioned five time series for each pair of countries. We allow the covariance matrix of the factors to be non-diagonal, so that the test can incorporate all of our interpretations of θ and suggest which ones appear plausible in the data. We still force the two supply shocks to have a covariance of zero, but the other two covariance terms are allowed to differ from zero. We estimate the coefficients b and a_H-a_F, and the covariance matrix by GMM. We have 15 second moments estimated from the data (the covariance matrix of the observed returns), which have to be explained by seven coefficients: two coefficients identifying matrix $\mathcal{B}_1$, b and a_H-a_F, and five coefficients describing the covariance matrix of the latent factors (three variances and two correlations). The results are reported in Table 1.

Consider first the case of the US vis-à-vis the UK. As we discussed earlier, the reasonable calibration of parameters a_H and a_F are 0.75 and 0.25, respectively. The point estimate of a_H-a_F, 0.5219, is then very close to the calibrated value. Furthermore, after our accounting for the level effect of the nominal exchange rate and the calibration of α, the average value of the terms of trade from (23) is, roughly, one. Then, the calibrated value of the coefficient b from (24) is about 0.25. The estimate we obtain in the data is 0.2569 $-$ very close indeed.

The remaining five estimates summarize the covariance matrix of the three latent factors. First, note that the demand shocks are very important both in magnitude and significance, rejecting our interpretation **A**, where we assume that only the supply shocks drive the economy. In fact, the standard deviation of the demand shocks is twice as large as that of the supply shocks. The standard deviations of the two supply shocks have similar orders of magnitude. Second, note that the estimates of the correlations constructed from the covariance matrix, indicate that the relative demand shock is positively correlated with US's supply shock, and negatively correlated with UK's.

The estimates are statistically significant, strongly rejecting the pure sentiment interpretation **B**. Furthermore, the catching up with the Joneses interpretation **C**, under which the the relative demand shock should negatively co-vary with the US supply shock and positively with the UK's, is rejected, too. The story the data is telling is the reverse: instead of becoming relatively unhappy when their country's aggregate consumption increases, agents appear to get enthusiastic when their economy is doing well. This speaks in favor of our consumer confidence interpretation **C**. Alternatively, these results may be viewed as evidence of differences in opinion (interpretation **D**).

The results for the case of US vis-à-vis Germany are very similar. The point estimates of a_H-a_F and b are almost identical to the previous case, as one would expect because the relative sizes of the economies and their preference biases toward domestic goods are close. The variances are also similar, although, in the case of Germany, its supply shocks are larger than UK's. The pattern of the correlations is close to the previous case: again, the relative demand shock is positively correlated with the US supply innovations, and negatively correlated with the German $-$ in support of the consumer confidence interpretation.

In summary, the structural parameters estimated in this exercise are close to those from calibration of $\mathcal{B}_1$. Furthermore, we find that demand shocks are very important in explaining short run variation of asset prices: their variance is twice the size of that of supply shocks. Furthermore, in the data demand shocks are neither independent of supply shocks, nor support the catching up with the Joneses interpretation. Rather, the demand shocks are due to either differences of opinion or variations in consumer confidence, positively related to fluctuations in national output.

3.3. Sign Restrictions

The previous section studied a very restrictive version of the model. In this section we relax the cross equation restrictions on the coefficients of matrix $\mathcal{B}_1$, and also allow the variance of the latent factors to change through time. We continue to maintain the assumption that the coefficients are constant, but now impose only the sign restrictions arising from Proposition 1. Indeed, one of the principal implications of the model is the unambiguous prediction for the signs of the responses of observed prices to innovations in the latent factors, summarized in (18). We thus estimate the following latent-factor model

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{dB(t)}{B(t)} \\ \frac{dB^*(t)}{B^*(t)} \\ \frac{d\varepsilon(t)}{\varepsilon(t)} \end{bmatrix} = \vec{I} + \underbrace{\begin{bmatrix} \alpha_{11} & 1 & \alpha_{13} \\ \alpha_{21} & 1 & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ 1 & \alpha_{52} & \alpha_{53} \end{bmatrix}}_{\mathcal{B}_2} \begin{bmatrix} A(t)d\theta(t) \\ \sigma_Y(t)dw(t) \\ \sigma_Y^*(t)dw^*(t) \end{bmatrix}, \tag{M2}$$

where α_{ij} are constant. The sign restrictions, entailed by our theory, require that α_{11}, α_{13}, α_{22}, α_{31}, α_{33}, α_{42}, α_{53}, are positive and that α_{21}, α_{32}, α_{41}, α_{43}, α_{52}, are negative. In order to estimate the model, we have to normalize some of the coefficients in matrix $\mathcal{B}_2$. The coefficients set equal to unity reflect the following normalization: the productivity shock in the US has a unitary effect on the US and the foreign stock market (responses of the two markets should be identical, according to our theory) and the demand shock has a unitary effect on the exchange rate. We continue to assume that the supply shocks in the US are orthogonal to those in the foreign country, but allow the demand shocks to be correlated with the supply shocks. We also assume that heteroskedasticity can be described by two regimes: a high and a low volatility regime.

In model (M2), there is a problem of identification in the absence of heteroskedasticity. The time series for the left-hand side variables allow us to estimate their variance covariance matrix, yielding a total of 15 moments. On the other hand, once we allow for correlation between demand and supply shocks, the right hand side has 17 moments. Hence, standard identification is not possible. Under the assumptions of constant coefficients and heteroskedasticity, however, the presence of only two heteroskedastic regimes is sufficient to solve the problem of identification − even if the covariance matrices across the two regimes are completely unrestricted. In other words, if there are two heteroskedastic regimes, we can compute two distinct variance-covariance matrices, which provide 30 moments. The right hand side has 12 unknowns in the matrix $\mathcal{B}_2$. In addition, there are two factor covariance matrices, each containing five unknowns. In total, there are 22 variables to be estimated.[11] Obviously, this system of equations satisfies the order condition, meaning that the number of unknowns is smaller than the number of equations. However, a priori it is unclear whether the 30 moments computed from the two regimes necessarily provide us with enough independent

[11]This identification strategy is related to the recent "identification through heteroskedasticity" literature. See Wright (1928) for the original contribution, and Sentana (1992), Sentana and Fiorentini (2001) and Rigobon (2003) for recent developments. For applications see Caporale, Cipollini, and Demetriades (2002), Caporale, Cipollini, and Spagnolo (2002), Rigobon and Sack (2003).

equations. Therefore, in the actual estimation we use more than two regimes; in fact, we use as many as possible.

The estimation procedure is as follows.

(a) We compute two-day returns for each of the five variables we employ.

(b) We run a VAR to clean for serial correlation and recover the residuals. We use five lags in all of our specifications, but the results are unaffected by increasing the number of lags.

(c) We determine the heteroskedastic regimes using the residuals from the VAR. To do so, for each residual we first compute the rolling window variance. We also calculate the variance in the entire sample (similar to the average of the rolling window variance) and the standard deviation of the rolling window variance. We define a cut-off and identify the regime that occurs when the rolling window variance is above the average plus the cut-off times the standard deviation as highly volatile. The results we present assume the cut-off of one and a half standard deviations, but we have examined regimes defined by 0.5, 1.0 and 2 standard deviations cut-offs, and the overall message is the same.

(d) After computing (for each residual) the high and low volatility regimes, we compute the covariance matrix in all of the 32 possible combinations of high and low variances. Obviously not all of the regimes have enough observations to estimate a covariance matrix; in such cases, the observations are dropped.

(e) Finally, we estimate the parameters in matrix $\mathcal{B}_2$ and the covariance matrices by GMM imposing the sign restrictions specified above.

We have made several assumptions in this procedure. First, we have chosen to use two-day returns. When the model is estimated on weekly returns, very similar results are found. Employing daily returns is problematic due to nonsyncronous trading in the US, and the UK and German markets, and some of our results do change. We discuss this issue in detail below. Second, we assume that heteroskedasticity of the latent factors can be characterized by two different regimes. In the literature, however, heteroskedasticity is typically described by an ARCH or GARCH-type model. Indeed, the identification strategy discussed above can be carried out exactly in a GARCH setup.[12] We have chosen to use the regimes instead because the estimates are consistent even if

[12] See Sentana and Fiorentini (2001) and Rigobon (2002).

the underlying process is ARCH or GARCH, but if there were regime shifts in the data, a GARCH model would have a hard time picking them up, while our methodology would not. Since there is no reason to assume a particular structure for the evolution of the covariance matrices, we have decided to take the safe strategy and estimate the model using regimes.

In Table 2, we present the results from estimating the model for the US vis-à-vis the UK case. The first three columns present the estimates of the elements of $\mathcal{B}_2$ for the model where the covariance matrix of the latent factors is forced to be diagonal. The second set shows the results for the model where the covariance matrix allows for correlation between the demand shock and the supply shocks. Remember, however, that in all of the estimations we always force the supply shocks of the US and the UK to have zero correlation.

In our estimation of the first (no-correlation) model, we found 13 variance regimes with enough observations to compute covariance matrices. Of the possible 12 coefficients, six are statistically significant at more than 5 percent confidence, and the other three are statistically significant at (about) one-sided 10 percent confidence. Note that not a single one of the sign restrictions is binding, as evidenced by the coefficients being away from zero. (If a sign restriction is binding, the corresponding coefficient should be exactly zero.)

It is important to discuss the interpretation of these coefficients. Remember that our model predicts that α_{11}, α_{13}, α_{23}, α_{31}, α_{32}, α_{33} are all equal in absolute value. Note that the estimates of the coefficients in the US and the UK stock markets dynamics equations, α_{11}, α_{13}, and α_{23}, are 1.2152, 0.8237, and 1.2642, respectively, $-$ they are close to each other and in fact are not statistically different. The only major deviation occurs in the US bond price dynamics equation. The estimates of the pertinent coefficients, α_{31}, α_{32}, and α_{33} are, in absolute value, 0.4169, 0.3497, and 0.0287. These estimates are not only statistically different from zero, but also statistically different from each other and from one. This observation suggests that even though the model performs well for the stock markets, it does a poorer job explaining the US bond returns. The model also implies that α_{12}, α_{21}, α_{22}, α_{41}, α_{42}, and α_{43} are all the same. In our estimation, α_{12} and α_{22} have been normalized to one; the absolute values of the other four are 1.844, 1.222, 0.924, and 0.094. The first three estimates are statistically different from zero but not from one. In this case, the UK bond prices are well explained by the model and the only rejection occurs in the last coefficient. Finally, given our normalization, the coefficients in the exchange rate dynamics equation, α_{52} and α_{53}, should be close to one in absolute value. Again, one is, but the other one is

	US-UK					
	No Correlation			Correlations $\neq 0$		
	Point Estimate	Std. Error	t Statistic	Point Estimate	Std. Error	t Statistic
α_{11}	1.2152	0.7513	1.62	0.7123	0.0036	19.90
α_{13}	0.8237	0.5531	1.49	0.6030	0.0141	4.27
α_{21}	-1.8443	0.4542	-4.06	-0.6670	0.0053	-12.54
α_{23}	1.2642	0.9541	1.33	0.5453	0.0117	4.66
α_{31}	0.4169	0.0240	17.35	0.2846	0.0059	4.82
α_{32}	-0.3497	0.1590	-2.20	-0.1416	0.0030	-4.71
α_{33}	0.0287	0.0172	1.68	0.5916	0.0048	12.28
α_{41}	-1.2224	0.0849	-14.41	-0.4541	0.0158	-2.86
α_{42}	0.9239	0.4194	2.20	0.3949	0.0059	6.69
α_{43}	-0.0936	0.0557	-1.68	-0.5128	0.0970	-0.52
α_{52}	-1.0724	0.5246	-2.04	-0.3509	0.0052	-6.69
α_{53}	0.1633	0.1182	1.38	0.6574	0.0073	8.99

Table 2: Sign restrictions: Estimation of Model (M2) for the US vis-à-vis the UK.

not. In this estimation, the overall impact of supply shocks coming from the UK is very small.

The second set of columns presents estimates of the same model, except that now we do not restrict the covariance matrix of the factors to be diagonal. Here, all but one coefficient are statistically significant. Furthermore, the results are much closer to those predicted by the model. For instance, the absolute values of α_{21}, α_{41}, α_{42}, α_{43} are 0.67, 0.45, 0.40, and 0.51. These estimates are close to each other and different from zero, consistent with our theoretical implications. Moreover, for the second set of coefficients, predicted to be equal in absolute value, α_{11}, α_{13}, α_{23}, α_{31}, α_{32} and α_{33}, the pertinent estimates are 0.71, 0.60, 0.54, 0.28, 0.14, and 0.59. Again, the estimates are much closer to each other than in the covariance restricted model. Even more importantly, given the normalization we have imposed and the calibrated values of the coefficients, these coefficients should be close to 0.346 (as implied by combining the estimated values of $b(t)$ from the previous section with the current normalization: 0.257/(1-0.257)), which is indeed approximately the estimated values.

Table 3 shows the analogous results for the case of the US vis-à-vis Germany. Notice that for the zero correlation model, there are seven estimates that are statistically significant at more than 5 percent confidence level. As before, the estimates are statistically different from zero indicating that the sign restrictions are not binding. As for the interpretation of the coefficients, we note that the estimates of α_{11}, α_{13}, α_{23}, α_{31}, α_{32}, α_{33} are 2.56, 0.82, 0.96, 0.94, 0.11, 0.12 in absolute value. In this case, only the fifth estimate is statistically different from the other four, and different from 0.346 − which, again, is the coefficient that we would have obtained if we imposed the normalization

	US-Germany					
	No correlation			Correlations $\neq 0$		
	Point Estimate	Std. Error	t Statistic	Point Estimate	Std. Error	t Statistic
α_{11}	2.5564	0.8806	2.90	0.3563	0.1641	2.17
α_{13}	0.8196	1.4812	0.55	0.4684	0.3014	1.55
α_{21}	-2.9888	1.3326	-2.24	-0.0809	0.0489	-1.66
α_{23}	0.9651	1.7437	0.55	0.6630	0.0538	12.33
α_{31}	0.9384	0.3183	2.95	0.3105	0.0607	5.11
α_{32}	-0.1101	0.0419	-2.63	-0.0196	0.0069	-2.85
α_{33}	0.1197	0.2163	0.55	0.6398	0.1470	4.35
α_{41}	-2.6330	0.8842	-2.98	-0.4954	0.1150	-4.31
α_{42}	0.3714	0.1360	2.73	0.0173	0.0183	0.95
α_{43}	-0.3739	0.6755	-0.55	-0.6995	0.4202	-1.66
α_{52}	-0.5130	0.1767	-2.90	-0.0313	0.0378	-0.83
α_{53}	0.7649	1.3876	0.55	0.7488	0.4617	1.62

Table 3: Sign restrictions: Estimation of Model (M2) for the US vis-à-vis Germany

to the estimates obtained from the previous exercise. For the other coefficients, however, we cannot reject the hypothesis that they are different from 0.346. Moreover, the estimates of α_{21}, α_{41}, α_{42}, and α_{43} are 2.98, 2.63, 0.37, and 0.37 in absolute value. For all of them except the third one, we cannot reject the hypothesis that they are equal to one. Finally, the estimates of α_{52} and α_{53} are also sensible: the estimate of α_{53} is not statistically different from the normalized value of one, while that of α_{52} is different but very close (the upper bound of the confidence interval is 0.85).

We run several robustness checks. In the interest of space, we only summarize the results.[13] We have estimated the model allowing for different cut-offs to determine the volatility regimes. The results have been found to be qualitatively the same. However, it is important to mention that if the cut-off is too big (more than 2.5 standard deviations), then very few regimes are found. In principle, this makes the identification harder. On the other hand, if the threshold is too small (0.5), the covariance matrices are very well estimated but their differences across regimes are small. As a result, this implies that the estimates are also noisy. Nevertheless, the message in the end is similar. We have also performed several other robustness checks: we have included more than five lags in the original VAR – this has made no difference. We have also varied parameters a_H and 1-a_F from 0.75 to 0.9, and α from 0.65 to 0.9. This only changes the average value of the coefficients without changing dramatically the relative importance of the variances. In other words, in a variance decomposition exercise the results are (roughly) unaltered. However, remember that both a_H and 1-a_F have to bounded away from 1/2 to be consistent with the home bias in consumption.

[13]The results can be replicated by using the programs available from http://web.mit.edu/~rigobon/www/.

Finally, we varied frequencies with which we compute the returns. In the estimation presented above we used two-day returns, however we have also examined weekly and daily returns. When we have estimated the model using weekly returns we have found that the number of regimes is drastically reduced and the estimates become less precise. However, the point estimates are very close to those using two-day returns. It is important to mention that when we have used daily returns we have found different results. For example, several of the coefficients have been estimated to be zero, indicating that the sign restrictions have been binding. The reason for this is nonsyncronous trading. It is to be expected that some of these coefficients are zero because several of the US innovations occur at times when the UK and German stock markets are closed. Indeed, most GDP and productivity related announcements in the US occur in the afternoon, when European markets are closed. Our model is not designed to account for this nonsynchronous trading: all prices respond instantly to supply and demand innovations, and we do not have intraday data to compute returns over the time intervals when all pairs of markets of interest are open.

In summary, estimation of the unrestricted version of the model lends strong support to our theory. Not only are the signs of the coefficients predominantly as predicted in Section 2, but also the magnitudes of a number of the coefficients are consistent with a reasonable calibration of the model. Nevertheless, some rejections of our implications are found − mostly regarding the bond prices dynamics.

3.4. Including Output: Low Frequency Data

Our empirical analysis thus far has relied on the implications of the model in identifying the latent factors − output and demand shocks − from the high frequency data on stock, bond and foreign exchange markets. Indeed, the key to our identification of the contemporaneous coefficients and the relative importance of each of the shocks have been the sign restrictions from the model (in combination with heteroskedasticity clearly present in our data). This approach is in the same spirit as that used in the macroeconomics literature, especially the strand of it that focuses on labor markets, where, similarly, theory-supplied restrictions are used to disentangle supply and demand shocks (see Blanchard and Diamond (1989), Davis and Haltiwanger (1990), Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1996); finally see Fisher (1976) for a comprehensive treatment of the problem of identification using sign restrictions.) In this section we want to make our identification more convincing by introducing a measure of output into the system of equations, and thus relating our factors to innovations in output. This strategy, however, comes at a cost of moving to lower frequencies, at which output is reported. Because we rely on heteroskedasticity in the data, we cannot use a GDP series sampled at quarterly frequency (we would have too few observations in each of the regimes); therefore as a proxy for output we use industrial production data at monthly frequency. Industrial production is obviously a noisy measure of output and productivity, but one that we expect to be highly correlated with both of them.

The procedure used is similar to the one in the previous section in that we estimate the following system of equations:

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{dB(t)}{B(t)} \\ \frac{dB^*(t)}{B^*(t)} \\ \frac{d\varepsilon(t)}{\varepsilon(t)} \\ \frac{dY(t)}{Y(t)} \\ \frac{dY^*(t)}{Y^*(t)} \end{bmatrix} = \vec{I} + \underbrace{\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ 1 & \alpha_{52} & \alpha_{53} \\ \alpha_{61} & 1 & \alpha_{63} \\ \alpha_{71} & \alpha_{72} & 1 \end{bmatrix}}_{\mathcal{B}_3} \begin{bmatrix} A(t)d\theta(t) \\ \sigma_Y(t)dw(t) \\ \sigma_Y^*(t)dw^*(t) \end{bmatrix}, \tag{M3}$$

where all the data are now monthly, and $\frac{dY(t)}{Y(t)}$ and $\frac{dY^*(t)}{Y^*(t)}$ are constructed from the industrial production series collected from Datastream. We adopt a more natural normalization of matrix $\mathcal{B}_3$, in which the productivity shocks now have a unitary effect on the output measures. It is important to mention that production is very simplistic in our model: it is driven solely by changes in productivity. In real life, of course, industrial production is hardly a pure measure of productivity − it is likely to be affected by demand shocks as well − and hence we do not impose a constraint that either α_{61} or α_{71} is zero.

Before estimating (M3), it might be worth pointing out why this model cannot be estimated by OLS. It is well-known from econometric theory that a naïve OLS estimation of equation (12) − the equation linking together stock prices and the exchange rate − would yield incorrect implications because of the omitted variable bias due to the latent factors. It is worth demonstrating how severe this bias actually is. For example, in the case of the US vis-à-vis the UK the naïve estimation of (log-linearized) equation (12) yields

$$\frac{dS^*(t)}{S^*(t)} = \underset{(2.17)}{0.1007}\frac{d\varepsilon(t)}{\varepsilon(t)} - \underset{(0.18)}{0.0089}\frac{dY^*(t)}{Y^*(t)} + \underset{(15.92)}{0.7431}\frac{dS(t)}{S(t)} + \underset{(0.46)}{0.0218}\frac{dY(t)}{Y(t)}.$$

	Low Frequency estimates					
	US-UK			US-Germany		
	Point Estimate	Std. Error	t Statistic	Point Estimate	Std. Error	t Statistic
α_{11}	0.0678	0.0580	1.2	0.0639	0.0484	1.3
α_{12}	1.3535	0.2208	6.1	1.3557	0.2062	6.6
α_{13}	0.7478	0.0331	22.6	0.0620	0.0500	1.2
α_{21}	-0.0804	0.0763	1.1	-0.0371	0.0177	2.1
α_{22}	0.7323	0.0553	13.2	0.7576	0.0224	33.8
α_{23}	1.0253	0.1069	9.6	1.1242	0.1265	8.9
α_{31}	0.7471	0.2812	2.7	0.0122	0.1123	0.1
α_{32}	1.1751	0.4691	2.5	0.8391	0.2432	3.4
α_{33}	-0.4100	0.2417	1.7	-1.1462	0.1986	5.8
α_{41}	-0.7411	0.2749	2.7	0.1120	0.1153	1.0
α_{42}	-1.0800	0.4566	2.4	-0.8701	0.2434	3.6
α_{43}	0.4649	0.2385	1.9	1.1040	0.2005	5.5
α_{52}	-0.6502	0.1758	3.7	-0.2717	0.1494	1.8
α_{53}	0.8584	0.0897	9.6	0.0513	0.0337	1.5
α_{61}	-0.8032	0.1173	6.8	-0.2796	0.1713	1.6
α_{63}	0.0658	0.0506	1.3	0.7660	0.0147	52.1
α_{71}	0.9499	0.1204	7.9	-1.2283	0.0947	13.0
α_{72}	0.0761	0.0713	1.1	0.7219	0.0714	10.1

Table 4: Sign restrictions: Estimation of Model (M3) for the US vis-à-vis the UK and Germany.

Several nonsensical results emerge. First, the signs on the exchange rate and bother the US and the UK outputs agree with neither our theory nor the standard reasoning of policymakers. Second, the coefficients are significant only for the exchange rate and the US stock market, with the latter encompassing a very high t-statistic. In other words, the only conclusion that we derive from this regression is that contagion is extremely important. In fact, the adjusted R^2 of this regression is 54 percent, but if the US stock market is excluded, the R^2 drops to 2 percent. The results for Germany-US are qualitatively even worse. In that case the only significant coefficient is the US stock market, and incorrect signs occur in the same places.

The OLS results should not be interpreted as a rejection to the model. Do we believe that a positive productivity shock has no impact on stock prices? What these OLS regressions demonstrate is that they are simply not fit to capture the effects we are interested in. We thus estimate the coefficients of our system (M3) through the sign restrictions imposed by the model in combination with heteroskedasticity in the data. We have normalized each variable to have a variance of one, and hence the coefficients do not have a structural interpretation. The results are shown in Table 4 for each pair of countries.

In our description of the results, let us first concentrate on the US supply shock (captured by the coefficients α_{i2}, $i = 1, \ldots, 7$). Notice that the shock clearly has a positive effect on the US stock market (1.35, significant), as well as the foreign stock market (0.73 and 0.76, significant). It also increases the US bond prices (1.17 and 0.84, significant), reduces foreign bond prices (-1.08 and -0.87, significant), and depreciates the exchange rate. The last effect is quite pronounced in the case of the UK (-0.65, significant), but weaker in the case of Germany (-0.27, with a t-statistic of 1.8). Furthermore, a US productivity shock has a positive impact on industrial production abroad, with German output being highly responsive (0.72, significant), but UK's a lot less (0.08, not significant). These responses are in much agreement with our earlier analysis. The effects of the US shock on both the US and foreign stock markets are positive and significant. The latter represents contagion, which is detected even by the OLS regressions. The negative exchange rate reaction is exactly as predicted by the model. The bond prices, unfortunately, are moving in the wrong direction. The relationship between output processes across nations is something taken as a primitive in our model, so we are in no position to comment on how industrial production of one country should respond to changes in that of the other. Thus, we merely report our estimates.

The pattern of responses to the foreign supply shock is very similar (captured by α_{i3}, $i = 1, \ldots 7$). For the UK, and increase in productivity induces a rise in the US (0.75) and the UK stock markets (1.03). At the same time, it reduces the US bond prices and increases the UK bond prices (-0.41 and 0.47, not significant), while generating an appreciation of the dollar (0.86, significant) and an increase in the US output (0.07, not significant). Again, all the signs are as implied by our theory, except those for the bond prices. In the case of Germany, all the signs are the same, but the significance of the estimates varies. For example, the effects on the US stock market and on the exchange rate are not significant, however, the effects on bond prices and output are significant.

Finally, the demand shock produces the following responses (captured by α_{i1}, $i = 1, \ldots 7$). For the UK case, a positive demand shift biased toward the US increases the US stock market (0.07, not significant) decreases the UK stock market (-0.08, not significant), increases the US bond prices (0.75, significant) and reduces the UK bond prices (-0.74, significant). It also appreciates the exchange rate (this is the normalization), decreases the US output (-0.80, significant) and increases the UK output (0.95, significant). The coefficients on stock and bond markets' responses, and the exchange rate are all in line with the predictions of the model. The relationship between the demand shock and output is somewhat puzzling, however, as we explained earlier, no definite predictions regarding this relationship can be made within our model. The coefficients for Germany exhibit the same pattern except for the coefficient on the German output, which is now negative

	Correlations (percentage terms)							
	US-UK				US-Germany			
	$A(t)d\theta(t)$	$\sigma_Y(t)dw(t)$	$\sigma_Y^*(t)dw^*(t)$	R^2	$A(t)d\theta(t)$	$\sigma_Y(t)dw(t)$	$\sigma_Y^*(t)dw^*(t)$	R^2
$\frac{dS(t)}{S(t)}$	16.4	64.5	45.7	60.9	21.7	53.4	38.2	56.8
$\frac{dS^*(t)}{S^*(t)}$	-7.9	38.4	68.8	60.5	-1.8	21.1	35.4	53.3
$\frac{dB(t)}{B(t)}$	76.6	52.7	-54.1	94.2	54.8	33.6	-33.9	86.0
$\frac{dB^*(t)}{B^*(t)}$	-75.4	-48.2	59.2	94.0	-52.7	-26.1	29.8	81.2
$\frac{d\varepsilon(t)}{\varepsilon(t)}$	50.2	-27.8	46.5	67.8	24.1	-11.0	41.5	57.3
$\frac{dY(t)}{Y(t)}$	-29.4	49.8	22.6	43.2	-28.9	44.1	18.2	30.1
$\frac{dY^*(t)}{Y^*(t)}$	34.1	23.8	49.4	41.4	-0.6	5.3	1.4	7.2

Table 5: Correlation between the latent factors and the observed variables.

and significant. The only other differences are in the significance of the coefficients.

Our next step is to investigate the nature of the latent factors that are derived from the estimation. We thus compare the supply shocks from our estimation to the actual output innovations in the data. Figure 2 presents both estimated and actual series for each country. As can be seen from the plots, the latent factor follows the output innovations very closely in all cases. To substantiate this assessment, we compute the correlation between the latent factors and the observed variables. Table 5 reports the results.

Notice that US supply shock is highly correlated with the US stock market, bond prices, and output (close to a 50 percent correlation). Likewise, the UK supply shock is very highly correlated with the UK stock market, bond prices, and output. Estimation for the case of Germany produces similar patterns, except that the correlation of output with the corresponding supply shock is rather small. The demand shock for both the US-UK and US-Germany estimations is highly correlated with the exchange rate, as well as bond prices. The latter has an implication for uncovered interest rate parity. Recall the prediction of our model (made in Section 2) that deviations from the classical interest rate parity relationship should to be attributed primarily to demand shocks. The pattern of estimated correlations seems to support that view. Finally, we report the within sample explained variation of the observed variables by the latent factors − this is a quasi R^2. As can be seen, except for the German output, the latent factor has a very high explanatory power.

Overall, the message from this exercise is very similar to the one from our previous estimations. By relying on sign restrictions and heteroskedasticity, we are able to estimate the coefficients of the model, which are shown to conform closely with the implications of our theory.

4. Caveats and Future Research

Our framework is certainly very stylized and several assumptions require further exploration. First, our model is real; both stocks and bonds are claims to a stream of goods. While perhaps this is a reasonable way to model stocks, in real life, bonds are IOU's specified in nominal terms. This is presumably the reason why our model's implications regarding stock prices and exchange rate dynamics find solid empirical support, while those for bond prices do not fare as well in the data. A natural next step is to introduce money formally into the model, which would allow for a distinction between real and nominal assets, and derive similar implications for nominal asset prices and exchange rates. We believe that the main mechanism will survive this extension − due to trade in goods and in asset markets, real exchange rates will continue to be determined by the same factors that drive real stock market returns.

Second, we have assumed that all goods are traded, and the literature on PPP and real exchange rate determination has demonstrated the importance of the non-tradable goods sector. In our model we have oversimplified this aspect by assuming different weights on Home and Foreign goods (a_H and 1-a_H) in the utility function. This is a reduced form for a two-sector economy with a tradable and a non-tradable sectors, where the non-tradables are produced with tradable Home goods.[14] However, extending the model to deal explicitly with transfers across the tradable and non-tradable sectors might find broader applications in international finance.

Third, we have employed a log-linear utility in our model. This specification has an apparent advantage of allowing us to compute stock prices in closed form. As a result, we obtain a parsimonious structural model with only three latent factors, making the problem of its estimation manageable. We are aware of very few models in the asset pricing literature capable of producing closed-form expressions for stock prices for an economy with multiple risky stocks. An apparent drawback of the logarithmic specification is, of course, the myopic behavior it induces. The agents are not genuinely myopic in our model − they hedge against future demand shocks, however, it would still be desirable to examine different preferences specifications. This extension is likely to

[14] For example, consider an economy where each country additionally consumes a local non-tradable good. That is, each country's representative agent has a log-linear utility defined over the (tradable) domestic good, the non-tradable good, and the foreign good, with weights a_{1i}, a_{2i}, and 1-a_{1i}-a_{2i}, $i \in \{H, F\}$. Furthermore, assume that the non-tradable good is produced using the local tradable good in each country with a constant returns to scale technology, which transforms one unit of the domestic good into a one unit of the non-tradable good. Due to nature of the technology employed, the utility function of each country effectively assigns a weight of $a_{1i} + a_{2i}$ to domestic goods. A reasonable calibration of the share of non-tradables in a country's consumption, would then give rise to a preference bias toward the domestic good, supporting the specification of the utility function adopted in our model.

entail the cost of imposing further assumptions on the parameters of the countries' output processes.

We have attempted to relate the supply and demand shocks, treated as latent factors in our estimation, to actual output innovations in the data by moving to lower frequencies at which output data are available. A different approach would be to use survey data and macroeconomic announcements as proxies for productivity and demand innovations, and study the direct effects of these innovations on asset prices and exchange rates along the lines of Andersen, Bollerslev, Diebold, and Vega (2003), Rigobon and Sack (2002) and Rigobon and Sack (2003). Also left for future research are tests of our model's implications on capital flows and the CAPM.

5. Conclusions

The empirical literature on exchange rate determination has devoted a tremendous amount of effort to explaining its short run fluctuations by interest rate differentials. It is fair to say that this strategy has not been very successful. In this paper we attempt to offer an alternative view on the exchange rate: its movements should be influenced by the same set of factors that govern stock market returns. This new perspective not only renders new insights on exchange rates, but also identifies important interconnections between foreign exchange and financial markets and sheds light on the apparent excessive correlation of stock markets worldwide.

The factor decomposition implied by our model shows that the variance of demand shocks is two times larger than that of output innovations. In our model, the additional term in the uncovered interest rate parity relationship − the risk premium − is driven primarily by demand shocks. This entails large time-varying deviations from interest rate parity. Moreover, we find strong support for the view that demand shocks are driven by differences of opinion or consumer confidence rather than pure sentiment, or catching up with the Joneses behavior. Our strategy of testing the model relies on imposing a number of cross-equation restrictions, as well as overidentifying restrictions obtained from several heteroskedastic regimes detected in the sample. This approach is designed to increase chances of rejecting the model − and yet very few rejections are found.

This paper is a first attempt to unify the international trade, open economy macroeconomics and asset pricing literatures and uncover the impact of demand and supply shocks on the joint dynamics of exchange rates, stock and bond prices. In order to do so, we have been forced to make several simplifying and sometimes unrealistic assumptions that future research must relax. We feel that deepening our stylized framework may lead to fruitful further insights.

Appendix

The state prices densities associated with the Home and Foreign goods are proportional to the marginal utilities of the countries with respect to the corresponding good. They are given by

$$\xi(t) = \frac{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}{Y(t)} \qquad \text{(Home good)}, \tag{A.1}$$

$$\xi^*(t) = \frac{\lambda_H \theta_H(t)(1-a_H) + \lambda_F \theta_F(t)(1-a_F)}{Y^*(t)} \qquad \text{(Foreign good)}. \tag{A.2}$$

These state price densities are, of course, equal to the respective Arrow-Debreu state prices η and η^* per unit probability P. Finally, the state-price density associated with the numeraire basket, henceforth the state price density, is given by $\hat{\xi}(t) \equiv \alpha\xi(t) + (1-\alpha)\xi^*(t)$.

Derivation of Stock Prices. Through the identification of π with the Lagrange multipliers η and η^*, $\pi(t) = \alpha\eta(t) + (1-\alpha)\eta^*(t)$, we can derive the price of the Home stock to be

$$S(t) = \int_0^T \frac{1}{\alpha\eta(t) + (1-\alpha)\eta^*(t)} \eta(s) Y(s) ds = E_t\left[\int_t^T \frac{1}{\hat{\xi}(t)} \xi(s) Y(s) ds\right] = \frac{q(t)}{\alpha q(t) + 1 - \alpha} Y(t)(T-t)\,,$$

where we first employed the definition of a conditional expectation appearing on transitioning from the state prices to the state price densities associated with the respective good and then used the fact that θ_H and θ_F are martingales. The Foreign stock price is determined through an analogous procedure.

Weights in the Planner's Problem. To conform with the competitive equilibrium allocation, the weights λ_H and λ_F in the planner's problem are chosen to reflect the countries' initial endowments. In particular, they are identified with the reciprocals of Lagrange multipliers associated with each country's Arrow-Debreu (static) budget constraint. Since in equilibrium these multipliers, and hence the weights, cannot be individually determined, we adopt a normalization $a_H\theta_H(0)\lambda_H + a_F\theta_F(0)\lambda_F = 1$. To pin down λ_H, note that $W_H(0) = \frac{1}{\hat{\xi}(0)} E\left[\int_0^T [\xi(t)C_H(t) + \xi^*(t)C_H^*(t)]dt\right] = \frac{1}{\hat{\xi}(0)}\lambda_H\theta_H(0)T$, where we used (8) and (A.1)–(A.2). On the other hand, $W_H(0) = S(0)$. This together with (10) yields $\lambda_H = 1/\theta_H(0)$. Consequently, $\lambda_F = (1-a_H)/(a_F\theta_F(0))$.

Proof of Proposition 1. Equation (8) can be equivalently restated as

$$\log q(t) = \log\left(\frac{\lambda_H\theta(t)a_H + \lambda_F a_F}{\lambda_H\theta(t)(1-a_H) + \lambda_F(1-a_F)}\right) + \log Y^*(t) - \log Y(t).$$

Applying Itô's lemma to both sides and simplifying, we obtain

$$\begin{aligned}\frac{dq(t)}{q(t)} &= \text{Itô terms } dt + \frac{\lambda_H a_H}{\lambda_H\theta(t)a_H + \lambda_F a_F} d\theta(t) - \frac{\lambda_H(1-a_H)}{\lambda_H\theta(t)(1-a_H) + \lambda_F(1-a_F)} d\theta(t) + \frac{dY^*(t)}{Y^*(t)} - \frac{dY(t)}{Y(t)} \\ &= \text{Itô terms } dt + A(t)d\theta(t) - \frac{dY(t)}{Y(t)} + \frac{dY^*(t)}{Y^*(t)}\,.\end{aligned}$$

Equations (10)–(11) are equivalent to

$$\log S(t) = \log q(t) - \log(\alpha q(t) + 1 - \alpha) + \log Y(t) + \log(T - t)\,, \tag{A.3}$$

$$\log S^*(t) = -\log(\alpha q(t) + 1 - \alpha) + \log Y^*(t) + \log(T - t)\,, \tag{A.4}$$

respectively. Applying Itô's lemma to both sides of (A.3) and (A.4), we have

$$\begin{aligned}
\frac{dS(t)}{S(t)} &= \text{Itô terms } dt + \left(1 - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha}\right)\frac{dq(t)}{q(t)} + \frac{dY(t)}{Y(t)} \\
&= \text{Itô terms } dt + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha} A(t)\, d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha}\frac{dY(t)}{Y(t)} + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha}\frac{dY^*(t)}{Y^*(t)} \\
\frac{dS^*(t)}{S(t)} &= \text{Itô terms } dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha}\frac{dq(t)}{q(t)} + \frac{dY^*(t)}{Y^*(t)} \\
&= \text{Itô terms } dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} A(t)\, d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha}\frac{dY(t)}{Y(t)} + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha}\frac{dY^*(t)}{Y^*(t)}
\end{aligned}$$

Substituting the dynamics of Y and Y^* from (1)–(2), we arrive at the expressions in Proposition 1. The Itô terms (mean growth rates) appearing in the above equations enter nowhere in our estimation procedure. Computation of these terms is straightforward but tedious, so in the interest of space we report just the end result: $I_5(t) = -\mu_Y(t) + \mu_Y^*(t) + \sigma_Y(t)^2 + A(t)\sigma_Y^*(t)\frac{d[\theta(t),\, w^*(t)]}{dt} - A(t)\sigma_Y(t)\frac{d[\theta(t),\, w(t)]}{dt}$, $I_1(t) = \mu_Y(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha}\left(I_5(t) + \sigma_Y(t)A(t)\frac{d[\theta(t),\, w(t)]}{dt} - \sigma_Y(t)^2\right) - \frac{q(t)}{\alpha q(t)+1-\alpha}\frac{Y(t)}{S(t)}$ and $I_2(t) = \mu_Y^*(t) - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha}\left(I_5(t) + \sigma_Y(t)^* A(t)\frac{d[\theta(t),\, w^*(t)]}{dt} + \sigma_Y^*(t)^2\right) - \frac{1}{\alpha q(t)+1-\alpha}\frac{Y^*(t)}{S^*(t)}$, where $[\theta(t),\, w(t)]$ and $[\theta(t),\, w^*(t)]$ are quadratic covariations of $\theta(t)$ with $w(t)$ and $w^*(t)$, respectively. Further simplification of the quadratic covariation terms requires committing to an interpretation of θ (see Section 2.3).

Home and Foreign bonds are riskless in terms of the local good. That is,

$$dB^i(t) = r^i(t)B^i(t)dt, \qquad i \in \{H,\, F\}\,,$$

where B^i is in units of good i. r^i is the local money market rate, the exact form of of which need not concern us in this proposition (the rates r^H and r^F differ across our interpretations of the demand shocks, and are reported in Proposition 2). Converted into the common numeraire,

$$B(t) = p(t)B^H(t) = \frac{q(t)}{\alpha q(t) + 1 - \alpha}\, B^H(t), \quad B^*(t) = p^*(t)B^F(t) = \frac{1}{\alpha q(t) + 1 - \alpha} B^F(t)\,.$$

Taking logs and then applying Itô's lemma leads to the required expressions. Again, we report the mean growth terms without providing the details of the computations: $I_3(t) = r^H(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha} I_5(t)$ and $I_4(t) = r^F(t) - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} I_5(t)$.

The sign restrictions follow from observing that $sign(A(t)) = sign(a_H - a_F) > 0$, $0 \le \alpha \le 1$ and $q(t)$, $\sigma_Y(t)$, $\sigma_Y^*(t) > 0$. *Q.E.D.*

Proof of Proposition 2. The state price density associated with the Home good, ξ, has a representation

$$d\xi(t) = -r^H(t)\xi(t)dt - m^H(t)\xi(t)d\vec{w}(t). \tag{A.5}$$

The Home money market rate, r^H, and the market price of risk, m^H, are obtained via an application of Itô's lemma to (A.1) taking into account the dynamics of θ_H and θ_F. In particular, under interpretation **A**,

$$d\xi(t) = -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t),$$

whose drift term yields the required money market rate. The Home market price of risk is given by $m^H(t) = (\sigma_Y(t),\, 0,\, 0)^\top$.

Under interpretation **B**,

$$d\xi(t) = -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t) + \frac{\lambda_H a_H \kappa_H(t)\theta_H(t)dw^\theta(t) + \lambda_F a_F \kappa_F \theta_F(t)dw^\theta(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}\xi(t).$$

Since the quadratic covariation between the demand shocks θ_H, θ_F and Y is zero, the interest rate has the same form as under interpretation **A**. The Home market price of risk is given by $m^H(t) = (\sigma_Y(t),\, 0,\, -\frac{\lambda_H a_H \kappa_H(t)\theta_H(t) + \lambda_F a_F \kappa_F \theta_F(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F})^\top$.

Under interpretation **C**,

$$\begin{aligned} d\xi(t) \;&=\; -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t) + \frac{\lambda_H a_H \kappa_H(t)\theta_H(t)dw(t) + \lambda_F a_F \kappa_F(t)\theta_F(t)dw^*(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}\xi(t) \\ &\quad - \frac{\lambda_H a_H \kappa_H(t)\theta_H(t)\sigma_Y(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}\xi(t)dt\,. \end{aligned}$$

The last term reflects the nonzero quadratic covariation between θ_H and Y, lending an additional term to the Home interest rate. Note that the Home state price density does not depend on the Brownian motion w^θ, or in other words, the risk associated with w^θ is not priced (the sunspot equilibrium, in which w^θ matters does not obtain in our model). The Home market price of risk is given by $m^H(t) = (\sigma_Y(t) - \frac{\lambda_H a_H \kappa_H(t)\theta_H(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F},\, -\frac{\lambda_F a_F \kappa_F(t)\theta_F(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F},\, 0)^\top$.

We now turn to interpretation **D**. Our analysis until now, as well as our empirical tests, have not required a complete specification of information sets of the agents, all the necessary information has been captured by the Radon-Nikodym derivatives θ_H and θ_F. There derivatives may come from various economic settings; here, for brevity, we consider only the case of incomplete information where the agents observe all economic processes defined in Section 2, but do not have complete information about their dynamics. The description of the setting is intendedly dense, and we refer the reader to Basak (2000) for more detail. We consider two subcases: **D1** and **D2**. Under interpretation **D1**, the agents believe that there are only two independent innovations, w_i and w_i^*, $i \in \{H, F\}$. The innovation process w_i of each agent i is such that given his perceived mean growth of the Home output process, μ_i, the observed Home output process has the dynamics $dY(t) = \mu_i(t)Y(t)dt + \sigma_Y(t)Y(t)dw_i$. Similarly, the innovation w_i^* is such that given μ_i^* the Foreign output has dynamics $dY^*(t) = \mu_i^*(t)Y^*(t)dt + \sigma_Y(t)Y^*(t)dw_i^*$. The differences in opinion pertaining to the underlying innovation process are induced by differences in the agents' priors, which they may or may not update as new information arrives. The case of Bayesian updating is an example of a filtering problem, the details of which need not concern us here since the the optimization problem is well-known to be independent of the inferencing problem. The outcome of the inferencing problem is the country-specific drift processes μ_i and μ_i^*, which we treat as exogenous. Note that the agents differ in their assessment of the underlying innovations and the drift terms, but agree on the volatilities, which they may deduce from the quadratic variations of the observed processes. The sub-interpretation **D2** is similar to **D1**, except that now the agents believe that there are three independent innovations, w_i, w_i^* and w_i^θ, $i \in \{H, F\}$, that the parameters they estimate depend on. The dynamics of the Home and Foreign output processes are perceived to be as above. Additionally, the agents observe a public signal, s, perceived to be driven by the third innovation process, following $ds(t) = \mu_{si}(t)s(t)dt + \sigma_s(t)s(t)dw_i^\theta(t)$, $i \in \{H, F\}$, the drift component of which is not observable.

The innovation processes of agent i bear the following relationship to the "true" underlying Brownian motions: $dw_i(t) = dw(t) + \frac{\mu_Y(t)-\mu_i(t)}{\sigma_Y(t)}dt$, $dw_i^*(t) = dw^*(t) + \frac{\mu_Y^*(t)-\mu_i^*(t)}{\sigma_Y^*(t)}dt$ (interpretations **D1**-**D2**), and $dw_i^\theta(t) = dw^\theta(t) + \frac{\mu_{si}(t)-\mu_{si}(t)}{\sigma_s(t)}dt$ (interpretation **D2**). By Girsanov's theorem, the innovation process $\vec{w}_i \equiv (w_i, w_i^*, w_i^\theta)^\top$ (the last component is absent under interpretation **D1**) is a Brownian motion under the probability measure i, $i \in \{H, F\}$. We can then identify $\vec{\kappa}_H(t)$ in the representation of the Radon-Nikodym derivatives of H with respect to P, θ_H, to be $-(\frac{\mu_Y(t)-\mu_H(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t)-\mu_H^*(t)}{\sigma_Y^*(t)}, 0)^\top$, (interpretation **D1**), $-(\frac{\mu_Y(t)-\mu_H(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t)-\mu_H^*(t)}{\sigma_Y^*(t)}, \frac{\mu_s(t)-\mu_{sH}(t)}{\sigma_s(t)})^\top$, (interpretation **D2**). Analogously, $\vec{\kappa}_F = -(\frac{\mu_Y(t)-\mu_F(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t)-\mu_F^*(t)}{\sigma_Y^*(t)}, 0)^\top$ (interpretation **D1**), $\vec{\kappa}_F = -(\frac{\mu_Y(t)-\mu_F(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t)-\mu_F^*(t)}{\sigma_Y^*(t)}, \frac{\mu_s(t)-\mu_{sF}(t)}{\sigma_s(t)})^\top$ (interpretation **D2**). Hence, under the true measure P, $d\theta_i(t) = \vec{\kappa}_i(t)\theta_i(t)d\vec{w}(t)$, $i \in \{H, F\}$.

Applying Itô's lemma to (A.1) and using the dynamics of θ_H and θ_F, we obtain

$$\begin{aligned} d\xi(t) &= -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t) + \frac{\lambda_H a_H \vec{\kappa}_H(t)^\top\theta_H(t)d\vec{w}(t) + \lambda_F a_F \vec{\kappa}_F(t)^\top\theta_F(t)d\vec{w}(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}\xi(t) \\ &\quad - \frac{\lambda_H a_H(1, 0, 0)\vec{\kappa}_H(t)\theta_H(t)\sigma_Y(t) + \lambda_F a_F(1, 0, 0)\vec{\kappa}_F(t)\theta_F(t)\sigma_Y(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}\xi(t)dt\,. \end{aligned}$$

The last term is again due to the quadratic covariation of the process Y with θ_H and θ_F. Substituting the expressions for $\vec{\kappa}_H$ and $\vec{\kappa}_F$ from above, we arrive at the statement in the Proposition. The Home market price of risk is given by $m^H(t) = (\sigma_Y(t), 0, 0)^\top - \frac{\lambda_H a_H \vec{\kappa}_H(t)\theta_H(t) + \lambda_F a_F \vec{\kappa}_F(t)\theta_F(t)}{\lambda_H\theta_H(t)a_H + \lambda_F\theta_F(t)a_F}$.

The Foreign interest rate is determined through an analogous procedure, via an application of Itô's lemma to (A.2) and then an identification of r^F with the ensuing drift term. The diffusion term yields the Foreign market price of risk: $m^F(t) = (0, \sigma_Y^*(t), 0)^\top$ (interpretation **A**), $m^F(t) = (0, \sigma_Y^*(t), -\frac{\lambda_H(1-a_H)\kappa_H(t)\theta_H(t) + \lambda_F(1-a_F)\kappa_F\theta_F(t)}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)})^\top$ (interpretation **B**), $m^F(t) = (-\frac{\lambda_H(1-a_H)\kappa_H(t)\theta_H(t)}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}, \sigma_Y^*(t) - \frac{\lambda_F(1-a_F)\kappa_F(t)\theta_F(t)}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}, 0)^\top$ (interpretation **C**), $m^F(t) = (0, \sigma_Y^*(t), 0)^\top - \frac{\lambda_H(1-a_H)\vec{\kappa}_H(t)\theta_H(t) + \lambda_F(1-a_F)\vec{\kappa}_F(t)\theta_F(t)}{\lambda_H\theta_H(t)(1-a_H) + \lambda_F\theta_F(t)(1-a_F)}$ (interpretation **D**).

Finally, we obtain the uncovered interest rate parity relation through an application of Itô's lemma to both sides of a no-arbitrage restriction $\xi^*(t) = \xi(t)/q(t)$ and matching the corresponding dt terms. This procedure yields $-r^F(t) = -r^H(t) - \mu_q(t) + \sigma_q(t)^\top(m^H(t) + \sigma_q(t))$. *Q.E.D.*

Proof of Proposition 3. We first focus on the composition of x_i and the form of σ. We also report the form of the auxiliary matrix $\mathcal{I}$, whose role is to make the modify the dimension of the vectors m and $\vec{\kappa}_i$ to make the matrices involved in the multiplications conformable.

(i) Under interpretation **A**, one can only determine the investment in the composite (world) stock market, and not in individual stock markets. Position in the composite security is of form (21). We do not provide details for this interpretation, and refer the reader to Zapatero (1995) or Cass and Pavlova (2003).

(ii) Under interpretation **B**, four independent investment opportunities are required to dynamically complete financial markets. We take these to be represented by the Home stock S, the Foreign stock S^*, the Home bond B and the world bond B^W. Hence, x_i has three components: $x_i = (x_i^S, x_i^{S^*}, x_i^B)$. From the dynamics of S, S^* and B, (13)–(15), and the definitions of θ_H and θ_F we identify the matrix of the investment opportunity set:

$$\sigma(t) = \frac{1}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} \alpha q(t)\sigma_Y(t) & (1-\alpha)\sigma_Y^*(t) & 0 \\ \alpha q(t)\sigma_Y(t) & (1-\alpha)\sigma_Y^*(t) & 0 \\ (\alpha-1)\sigma_Y(t) & (1-\alpha)\sigma_Y^*(t) & 0 \end{pmatrix}$$

$$+ \frac{A(t)\theta(t)}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} 1-\alpha \\ -\alpha q(t) \\ 1-\alpha \end{pmatrix} \mathcal{I}\left(\vec{\kappa}_H(t) - \vec{\kappa}_F(t)\right)^\top, \qquad \text{(A.6)}$$

with $\mathcal{I}$ is simply a 3×3 identity matrix under this interpretation.

(iii) Under interpretation **C**, there are only two independent sources of uncertainty, and hence three securities are sufficient to dynamically complete financial markets. We take them to be S, S^* and B^W. Hence, x_i has two components: $x_i = (x_i^S, x_i^{S^*})$. From (13)–(14) and the definitions of θ_H and θ_F we identify the matrix of the investment opportunity set:

$$\sigma(t) = \frac{1}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} \alpha q(t)\sigma_Y(t) & (1-\alpha)\sigma_Y^*(t) \\ \alpha q(t)\sigma_Y(t) & (1-\alpha)\sigma_Y^*(t) \end{pmatrix}$$

$$+ \frac{A(t)\theta(t)}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} 1-\alpha \\ -\alpha q(t) \end{pmatrix} \left[\mathcal{I}\left(\vec{\kappa}_H(t) - \vec{\kappa}_F(t)\right)\right]^\top, \text{ with } \mathcal{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \qquad \text{(A.7)}$$

(iv) To address interpretation **D**, we again consider subcases **D1** and **D2**. Under interpretation **D1**, three securities dynamically complete markets: for instance, S, S^* and B^W, and hence x_i has two components: $x_i = (x_i^S, x_i^{S^*})$. Under interpretation **D2**, there is an additional nonredundant security, B, and hence x_i has three components: $x_i = (x_i^S, x_i^{S^*}, x_i^B)$. The matrix σ is then obtained using (13)–(14) under interpretation **D1** to yield an expression in (A.7) and (13)–(15) under interpretation **D2** to yield an expression in (A.6). Of course, $\vec{\kappa}_H$ and $\vec{\kappa}_F$ used in the expressions for σ are interpretation-**D**-specific.

From now on we focus solely on interpretations **B**–**D**. Before we proceed to trading strategies, we derive the interest rate r on the world bond B^W, and the market price of risk m. Both are identified from the representation of the state price density $\hat{\xi}$, $d\hat{\xi}(t) = -r(t)\hat{\xi}(t)dt - m(t)\hat{\xi}(t)d\vec{w}(t)$. Using the definition of $\hat{\xi}$ and (A.1)–(A.2), we obtain

$$r(t) = \frac{\alpha\, r^H(t)q(t) + (1-\alpha)\, r^F}{\alpha\, q(t) + 1 - \alpha} \quad \text{and} \quad m(t) = \frac{\alpha\, m^H(t)\, q(t) + (1-\alpha)\, m^F(t)}{\alpha\, q(t) + 1 - \alpha}.$$

(i) Trading strategies. It can be verified that matrix σ is invertible under interpretations **B**-**D**. We can then appeal to the martingale representation methodology (Cox and Huang (1989), Karatzas, Lehoczky, and Shreve (1987)) to transform the dynamic optimization problem of Home (Foreign) country to a static problem of maximizing the objective in (4) (in (5)) subject to the static budget constraint $E[\int_0^T (\xi(t)C_i(t) + \xi^*(t)C_i^*(t))dt] = W_i(0)$, $i = H$ $(i = F)$. The first-order conditions for this optimization are

$$\frac{\theta_i(t)a_i}{C_i(t)} = \frac{1}{\lambda_i}\xi(t), \qquad \frac{\theta_i(t)(1-a_i)}{C_i^*(t)} = \frac{1}{\lambda_i}\xi^*(t), \qquad i \in \{H, F\}. \tag{A.8}$$

Note that the multipliers on the static countries' budget constraints are the reciprocals of the planner's weights λ_H and λ_F.

The optimal trading strategy of agent i is identified from the stochastic integral representation $M_i(t) = M_i(0) + \int_0^t \psi_i(s)^\top d\vec{w}(s)$ of the martingale $M_i(t) \equiv E_t\left[\int_0^T (\hat{\xi}(t)C_i(t) + \xi(t)C_i^*(t))dt\right]$ by modifying the standard argument to account for multiple goods (see, e.g., Karatzas and Shreve (1998), Theorem 7.3):

$$\sigma(t)^\top x_i(t) = \mathcal{I}\, m(t) + \mathcal{I}\, \frac{\psi_i(t)}{\hat{\xi}(t)W_i(t)}\,, \tag{A.9}$$

where $W_i(t)$ is time-t optimal wealth of agent i, $W_i(t) = \frac{1}{\hat{\xi}(t)}E_t[\int_t^T (\xi(s)C_i(s) + \xi^*(s)C_i^*(s))ds]$, and m, σ are interpretation-specific. In our model, $M_i(t) = E_t[\int_0^T (\xi(s)C_i(s) + \xi^*(s)C_i^*(s))ds] = E_t[\int_0^T (\lambda_i\theta_i(s)a_i + \lambda_i\theta_i(s)(1-a_i))ds] = \int_0^t \lambda_i\theta_i(s)ds + \lambda_i\theta_i(t)(T-t)$, and hence $dM_i(t) = \lambda_i\vec{\kappa}_i(t)\theta_i(t)(T-t)d\vec{w}(t)$. The diffusion coefficient of $M_i(t)$, $\vec{\kappa}_i(t)\theta_i(t)(T-t)$, is identified with $\psi_i(t)$. Note also that

$$\hat{\xi}(t)W_i(t) = E_t[\int_t^T (\xi(s)C_i(s) + \xi^*(s)C_i^*(s))ds] = \lambda_i\theta_i(t)(T-t)\,. \tag{A.10}$$

Plugging this together with $\psi_i(t)$ into (A.9) we arrive at the expression in Proposition 3.

(ii) Home bias. We now examine conditions under which consumers' portfolios exhibit a home bias. To do so, we need to sign the first and second components of the vector $(\sigma(t)^\top)^{-1}\mathcal{I}\,\vec{\kappa}_i(t)$, $i \in \{H,\, F\}$. Under interpretation **B**, these two components are equal to zero since $\vec{\kappa}_i(t) = (0,\, 0,\, \kappa_i)$, and hence fractions of wealth both Home and Foreign consumers invest in stocks of their respective countries is exactly as in the mean-variance portfolio. Under interpretation **C**, from (A.7),

$$\begin{aligned}(\sigma(t)^\top)^{-1} \quad &= \quad \frac{1}{det(\sigma(t))}\frac{1}{\alpha q(t) + 1 - \alpha} \\ &\times \begin{pmatrix} (1-\alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t) & -(1-\alpha)\sigma_Y^*(t) + A(t)\theta(t)(1-\alpha)\kappa_F(t) \\ -\alpha q(t)\sigma_Y(t) + A(t)\theta(t)\alpha q(t)\kappa_H(t) & \alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1-\alpha)\kappa_H(t) \end{pmatrix},\end{aligned}$$

where $det(\sigma(t))$ denotes the determinant of $\sigma(t)$. Note that

$$
\begin{aligned}
det(\sigma(t)) &= \big[\alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1-\alpha)\kappa_H(t)\big]\big[(1-\alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t)\big] \\
&\quad -\alpha q(t)\big[\sigma_Y(t) - A(t)\theta(t)\kappa_F(t)\big](1-\alpha)\big[\sigma_Y^*(t) - A(t)\theta(t)\kappa_H(t)\big] \\
&\Rightarrow \begin{cases} > 0 & \text{if} \quad \kappa_H(t) > 0,\, \kappa_F(t) > 0, \\ < 0 & \text{if} \quad \kappa_H(t) < 0,\, \kappa_F(t) < 0, \end{cases}
\end{aligned}
$$

It then follows that if $\kappa_H(t) > 0$, $\kappa_F(t) > 0$ (consumer confidence) then the first component of the vector $(\sigma(t)^\top)^{-1}\mathcal{I}\,\vec{\kappa}_H(t)$ and the second component of $(\sigma(t)^\top)^{-1}\mathcal{I}\,\vec{\kappa}_F(t)$ are unambiguously positive. On the other hand, if $\kappa_H(t) < 0$, $\kappa_F(t) < 0$ (catching up with the Joneses), the sign of the first component of the vector $(\sigma(t)^\top)^{-1}\mathcal{I}\,\vec{\kappa}_H(t)$ coincides with the sign of $\big[(1-\alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t)\big]$. Similarly, the sign of the second component of $(\sigma(t)^\top)^{-1}\mathcal{I}\,\vec{\kappa}_F(t)$ coincides with the sign of $\big[\alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1-\alpha)\kappa_H(t)\big]$. Finally, under interpretation **D**, the sign of either component of $(\sigma(t)^\top)^{-1}\mathcal{I}\,\vec{\kappa}_i(t)$ is ambiguous.

(iii) International CAPM. In an arbitrage-free market, the risk premium on stock j is related to the market price of risk in the following way (e.g., Karatzas and Shreve (1998), Theorem 4.2):

$$
\frac{E_t(dS^j(t)/dt)}{S^j(t)} - r(t) = \sum_{k=1}^{3} \sigma_k^j(t) m_k(t)\,,
$$

where the diffusion coefficients σ_1^j, σ_2^j and σ_3^j are the loadings of stock S^j on Brownian motions w, w^* and w^θ, respectively, and m_k are components of the market price of risk vector m. On the other hand,

$$
Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{d\hat{\xi}(t)}{\hat{\xi}(t)}\right) = \frac{Cov_t(dS^j(t),\, d\hat{\xi}(t)}{S^j(t)\hat{\xi}(t)} = -\sum_{k=1}^{3} \sigma_k^j(t) m_k(t) dt\,.
$$

Hence,

$$
\frac{E_t(dS^j(t))}{S^j(t)} - r(t)dt = -Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{d\hat{\xi}(t)}{\hat{\xi}(t)}\right) \tag{A.11}
$$

From (A.10), $W_i(t) = \lambda_i\theta_i(t)(T-t)/\hat{\xi}(t)$, $i \in \{H,\, F\}$. Hence, the aggregate wealth is

$$
W(t) = W_H(t) + W_F(t) = \frac{T-t}{\hat{\xi}(t)}\left[\lambda_H\theta_H(t) + \lambda_F\theta_F(t)\right]\,.
$$

Applying Itô's lemma and rearranging,

$$
\frac{d\hat{\xi}(t)}{\hat{\xi}(t)} = -\frac{dW(t)}{W(t)} + \frac{\lambda_H d\theta_H(t) + \lambda_F d\theta_F(t)}{\lambda_H\theta_H(t) + \lambda_F\theta_F(t)}\,. \tag{A.12}
$$

Then,

$$
\begin{aligned}
Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{d\hat{\xi}(t)}{\hat{\xi}(t)}\right) &= -Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{dW(t)}{W(t)}\right) + Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{\lambda_H d\theta_H(t)}{\lambda_H\theta_H(t) + \lambda_F\theta_F(t)}\right) \\
&\quad + Cov_t\left(\frac{dS^j(t)}{S^j(t)},\, \frac{\lambda_F d\theta_F(t)}{\lambda_H\theta_H(t) + \lambda_F(t)\theta_F}\right), \qquad S^j \in \{S,\, S^*\}\,.
\end{aligned}
$$

Combining this with (A.11), we obtain the required expression. *Q.E.D.*

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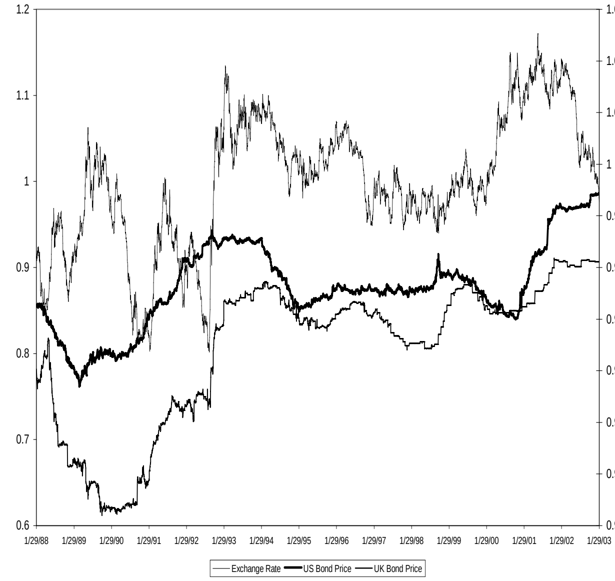
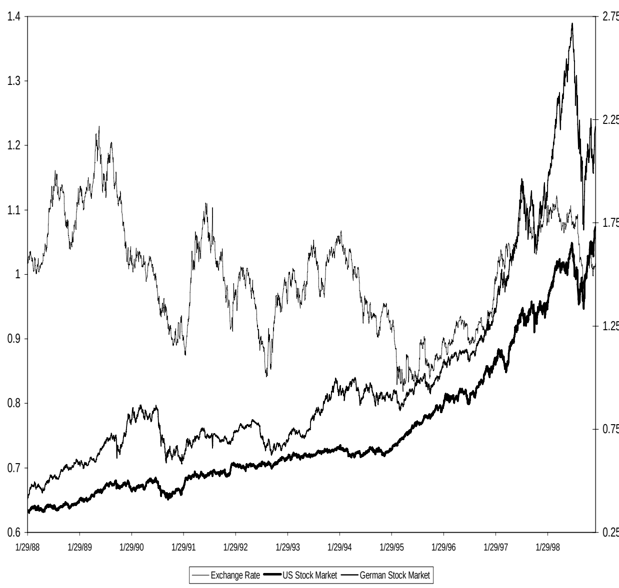
(a) The US and the UK stock market indexes and the dollar-pound exchange rate



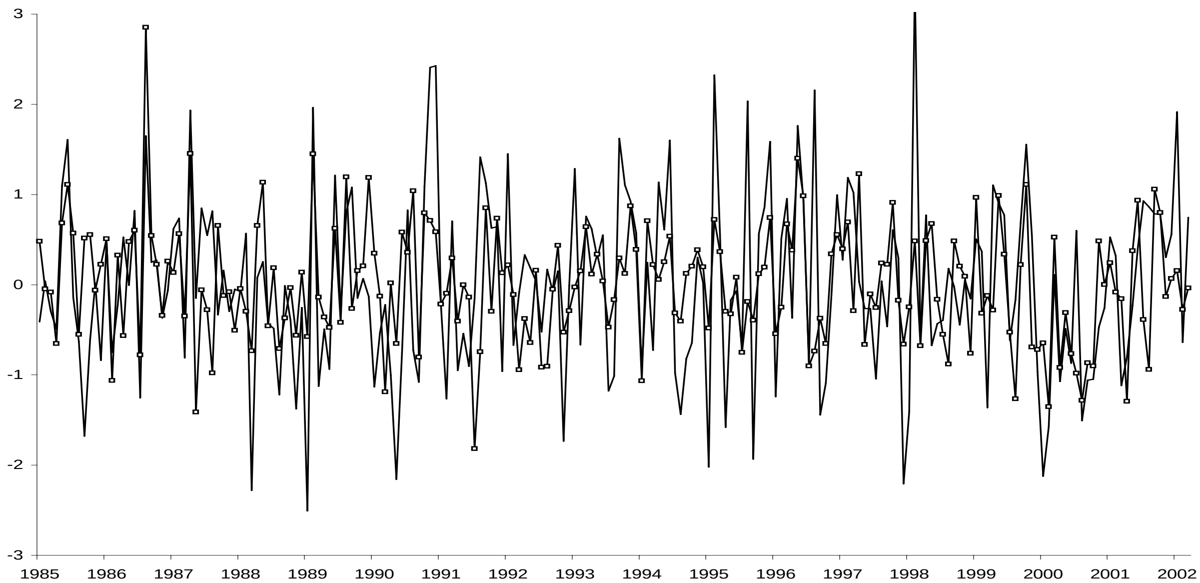
(b) The US and the UK three-month zero-coupon government bond prices and the dollar-pound exchange rate



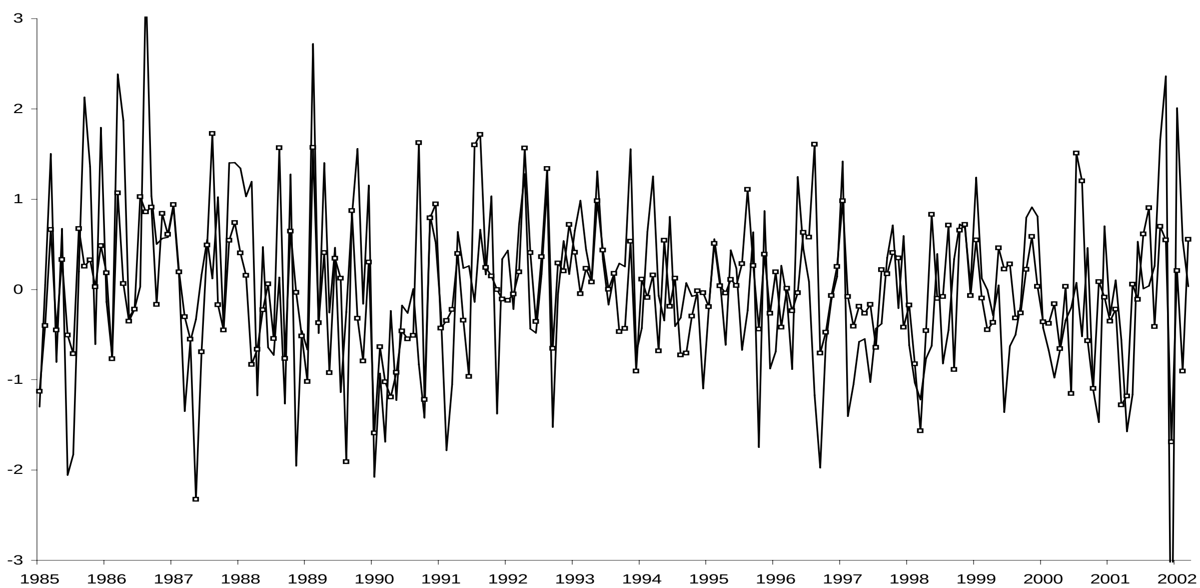
(c) The US and German stock market indexes and the dollar-mark exchange rate

(d) The US and German three-month zero-coupon government bond prices and the dollar-mark exchange rate

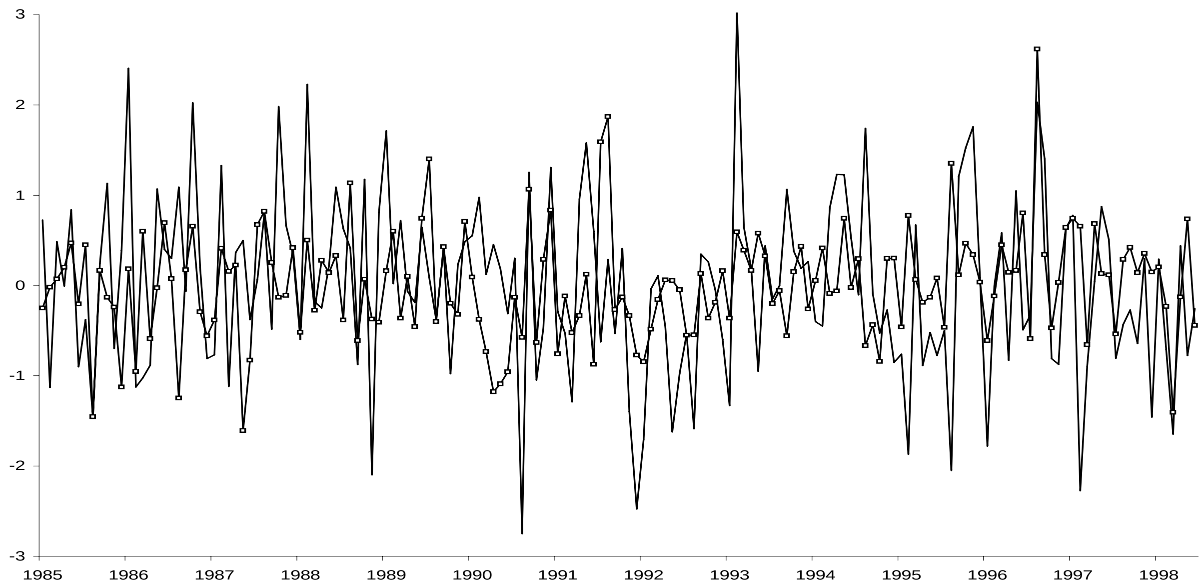
Figure 1: Asset prices and exchange rates. The exchange rate is measured in the left axis. In panels (a) and (c) the stock indexes are measured in the right vertical axis. In panels (b) and (d), the bond prices are measured in the left vertical axis. For demonstrative purposes, all prices are normalized so that the average exchange rate is equal to one.



(a) Estimated US output shocks vs. actual US output innovations



(b) Estimated UK output shocks vs. actual UK output innovations



(c) Estimated German output shocks vs. actual German output innovations

Figure 2: Latent factors corresponding to supply shocks vs. actual output innovations. In all panels, the latent factor is the schedule marked with squares and output innovations is the solid plot.