

# The *Pre*-Producers

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## Abstract

While a start-up firm waits for its sales to materialize, it is a “pre-producer”. This waiting period represents a special kind of entry cost. This paper studies how such entry costs influence the several stages of an industry’s life cycle. [PRELIMINARY]

## 1 Introduction

Pre-production is a period during which a new firm has not yet generated any sales. Such firms are often found in young industries. The automobile industry, the pharmaceutical industry, and the numerous activities in which the recent wave of dot.com companies tried to engage in all contained many pre-producers. Many of these pre-producers folded without generating any sales. Carroll and Hannan (2000) report that at the tail end of the 19th century some 90% of all automobile pre-producers never sold a single car – the median pre-production time was less than a year while some pre-producers were at it for more than 10 years.

Pre-production is an investment in the form of a waiting time. There are several reasons why we expect such waiting times to arise in practice.

1. *Research and product-development lags.*—Among about one hundred semi-conductor firms, the average waiting time to first-product shipment of was 21 months with a standard deviation of 11 months (Schoonhoven *et al.* 1990 Table 1, reproduced in the Appendix). More generally, the lag in the average research project – including the research projects of established firms – tends to be about the same, somewhere between 1 to 2 years (Pakes and Schankerman 1984). Mansfield *et al.*(1981) report somewhat shorter delays for how long it takes firms to copy the inventions of others. In some industries the lag is much longer: The average time from a drug’s first worldwide patent application to its approval by the FDA is now about 13 years (Dranove and Meltzer 1994).

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2. *S-shaped sales-age relation.*—Even when sales do appear, they may initially amount to no more than a trickle. The more complex the activity, the more likely it is that its learning curve will be S-shaped, with low outputs early on followed by a sharp rise in productivity (Jovanovic and Nyarko 1995). In such cases the period proceeding the sudden rise in productivity will look like a pre-production episode. See Figure 10 in the Appendix.

In this paper I present some simple models of pre-production, and then analyze in detail one that relates closely to the model in Jovanovic (1982). It is an industry life-cycle model in which the entry cost is zero and in which the production cost consists entirely of a foregone-earnings cost. The model also resembles Loury (1979) and Lee and Wilde (1980) where inventions arrive at a random dates but where the first inventor gets the whole market. I add a continuum of players but remove the winner-take-all assumption by constraining each firm’s capacity, so that early producers do drive some but not all of the laggards out of the market, and the process takes time. The model also relates to Filson and Franco (2000) and Irigoyen (2003) where the pre-production period is a stint with an employer who pays less but who in return trains his workers.<sup>1</sup> Finally, other ways of endogenizing the pre-production period are studied in Jovanovic and Rousseau (2001) and Jovanovic (2003).

## 2 Three benchmark cases

Let us start with three easy cases: (i) no waiting time, (ii) equal waiting times, (iii) unequal but known waiting times.

### 2.1 No waiting time

Let us start with the case in which there is no pre-production period. Let the demand curve be  $p = D(q)$  where  $p$  is price and  $q$  is quantity. Suppose a firm can produce a unit of output with a unit of entrepreneurial time. If he chooses to go elsewhere, the entrepreneur can earn an income of  $w$  per unit of time. This foregone income is equivalent – here and in all other models I consider below – to a marginal cost of production. There is an infinite supply of such potential entrants.<sup>2</sup>

Suppose that the industry comes into being at date zero. The supply curve for this industry is infinitely elastic at  $w$ , and therefore the equilibrium price  $p^* = w$ , and the equilibrium output is  $q^*$ , at all dates. Figure 1 illustrates this. As long

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<sup>1</sup>Table 2 of Schoonhoven *et al* shows that the founder’s level of prior work experience significantly reduces the time it takes to generate sales.

<sup>2</sup>We should think of  $w$  not as the wage, but as the earnings foregone by those who wish to be “in the game.” Filson and Franco (2002) and Irigoyen (2003) study equilibrium models in which entrepreneurs learn some of their skills by working in firms that offer training and pay lower wages than do firms that do not offer such training.

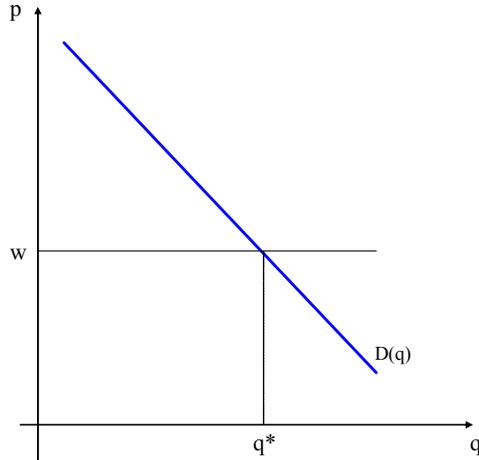


Figure 1: Equilibrium without pre-production

as neither  $w$  nor  $D(\cdot)$  change, the industry remains at rest, and has no interesting dynamics.

## 2.2 Equal waiting times, $z$

Suppose now that all firms must wait before they generate sales. The common waiting period is  $z$  and the rate of interest is  $r$ . Figure 2 describes the time-path of each firm's output. Assuming that time is continuous, the free-entry condition then says that

$$\frac{w}{r} = e^{-rz} \frac{p}{r},$$

so that the equilibrium price is now

$$p = e^{rz} w \tag{1}$$

Entrants require a mark-up over cost of  $(e^{rz} - 1)$ .

The introduction of a common waiting time does 3 things

1. It delays production by  $z$  periods,
2. It raises the equilibrium price from  $w$  to  $e^{rz}w$ , and
3. It produces no dynamics in  $p$  and  $q$ , and gives rise to no exits.

This model has implications that are almost identical to one in which the costs were direct, in terms of goods. I.e., if all firms had to pay a common dollar cost of entry  $\phi$ , where  $\phi = \frac{1}{r}(e^{rz} - 1)$ .

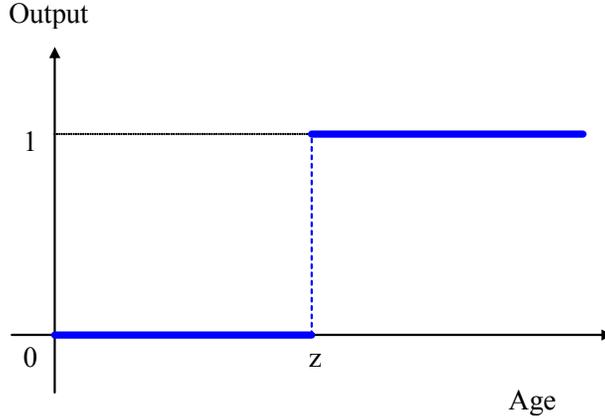


Figure 2: Output as a function of the firm's age

### 2.3 Unequal but known waiting times, with firms in fixed supply

There are good reasons to expect some prior differences in  $z$ 's. The background of the founder – the founder's prior work experience, e.g., and, especially, prior experience start-up companies – are significant predictors of performance.

In order to get a well-defined upward sloping supply curve of entrepreneurs, we must limit their number. Let the total mass of potential firms be  $n_0$ . We seek a marginal firm equilibrium where the best (i.e., lowest- $z$ ) firms draw rents on their superior quality. Let  $z_m$  be the highest- $z$  firm that enters. Figure 3 illustrates this. Then the time path of output is

$$q(t, z_m) = \min(n_0 F[t], n_0 F[z_m])$$

which means that industry price is

$$p(t, z_m) = D(q[t, z_m]).$$

The marginal entrant must be indifferent between entering and staying out:

$$\frac{w}{r} = e^{-rz_m} \frac{1}{r} D(n_0 F(z_m)), \quad (2)$$

so that

$$p = e^{rz_m} w. \quad (3)$$

EQUILIBRIUM is the pair of numbers  $(p, z_m)$  solving (2) and (3). Note

1. The similarity between (1) and (3). In (1) it is the common  $z$  that matters, whereas in (3) it is the marginal  $z$ ,

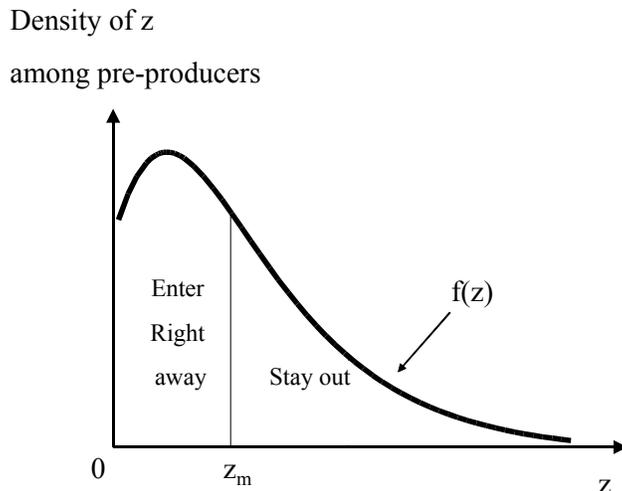


Figure 3: The marginal entrant equilibrium  $z_m$

2. No exit takes place, though industry output now grows smoothly. Once again we cannot explain the temporary presence of firms that have never sold and will never sell any output.
3. Pre-producers now experience obsolescence of their ideas even before they start to produce. I.e.,  $p$  declines as they wait to become producers.

The time path of prices is drawn in Figure 4. This model has implications that are almost identical to one in which firms face a firm-specific dollar cost of entry  $\phi(z) = \frac{1}{r}(e^{rz} - 1)$ . It makes more sense in mature industry in which the technological uncertainty has been resolved, but in which there still is variation in, e.g., the characteristics of the firm's founder.<sup>3</sup>

### 3 Heterogeneous and unknown waiting times

We now introduce differences among firms in the  $z$ 's. A potential entrant has a pre-production period of length  $z$  which, as before, tells us the age at which that entrant will start to be productive. The output of that firm is

$$y_a = \begin{cases} 0 & \text{if } a < z, \\ 1 & \text{if } a \geq z. \end{cases} \quad (4)$$

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<sup>3</sup>Table 3 of Schoonhoven et al (1990) shows that the founder's prior experience with other start-ups is especially significant in reducing  $z$ .

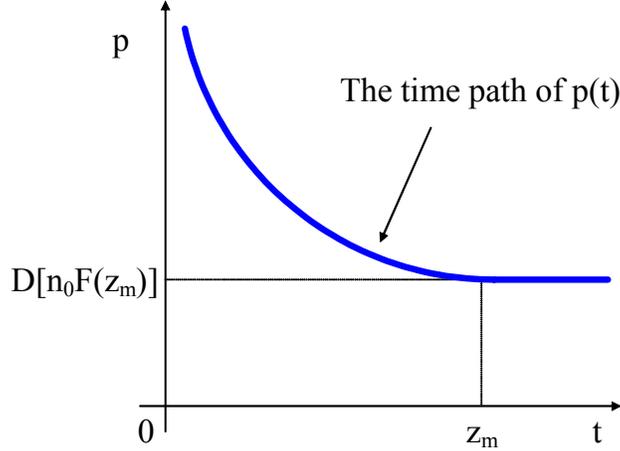


Figure 4: A plot of the function  $p(t)$  when firms are *ex-ante* different

Ex-ante firms are all the same, believing themselves to be a random draw from the distribution  $F(z)$ . The realized  $z$ 's differ, however, and they are uncorrelated.<sup>4</sup> There is no aggregate uncertainty.

The assumption that  $z$  is not known before entry and it may have considerable variance makes the most sense in a new industry in which many approaches look similar before the fact. Each  $z$  can be viewed as the waiting time resulting from the use of a slightly different technique or model design. In a new industry, many designs and techniques may look equally plausible *ex-ante*.<sup>5</sup> As the best technique(s) have been identified, they are the only ones that entrants would try and as a result the dispersion of  $F$  would be smaller, but it would not disappear entirely because  $z$  has been found to also depend on the characteristics of the firm founders, e.g., and these too vary considerably in any industry.

Until it has realized its  $z$ , the firm updates its future prospects using Bayes Rule.

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<sup>4</sup>This firm-specific parameter  $z$  reveals itself – in part gradually – to pre-producers in much the same way as the firm's efficiency revealed itself to firms in the model in Jovanovic (1982). Both models assume that the most efficient firms (in this case the lowest- $z$  firms) cannot serve the entire market

<sup>5</sup>More generally, let  $x$  be a technique, and  $G(z, x)$  the distribution of  $z$  resulting from that technique. Let  $H(\theta)$  be the distribution of techniques among the entrants. Then we may say that

$$F(z) = \int G(z, x) dH(x).$$

If  $x$  has a lot of dispersion, so then will  $z$ .

Define the hazard rate of  $F$  as

$$h(z) = \frac{f(z)}{1 - F(z)}.$$

There is no entry cost, and the only production cost is a foregone-earnings costs. Alternative earnings for every firm are  $w$  per unit of time. Any entrant can exit at any time. In the equilibria we shall look at here, re-entry will never be optimal because price will never increase. If, as will turn out, everyone were to enter at date 0 and if no one were to exit, the number of firms becoming productive by date  $t$  would be  $f(t)$  and the production hazard would be  $h(t)$ .

*Producers and pre-producers.*—Producers are all alike, as are the pre-producers in the ex-ante sense. If no producer exits,  $q$  denotes both output and the number of producers. Let  $n(t)$  denote the number of firms. Of these,  $q(t)$  are producers, and the rest are pre-producers. The instantaneous conditional probability that a pre-producer becomes a producer is just  $h(t)$ . Thus the differential equation for  $q$  is

$$\frac{dq}{dt} = (n[t] - q[t]) h(t) \quad \text{if } p > w \quad (5)$$

If  $n(t)$  decreases there is exit.

*Pre-producers' values.*—First define

$$v(z, t) = \int_z^\infty e^{-r(s-t)} p(s) ds \quad (6)$$

to be the date- $t$  present value of becoming productive and starting to sell output at date  $z$ . The critical function is the expected present value that a pre-producer gets for remaining in the industry at date  $t$ . First Call this  $V(t)$ . Then

$$V(t) \equiv \frac{1}{1 - F(t)} \int_t^\infty v(z, t) dF(z). \quad (7)$$

*The production hazard,  $h$ .*—The nature of equilibrium depends very much on the form of  $F$  or, rather, of  $h$ . There seems to be no particular reason why  $h$  should be increasing or decreasing. The notion that a pre-production period exists, that output is increasing with the waiting time, all that is already captured by (4). The distribution  $F$  is related only to the heterogeneity among the pre-producers. If  $F$  is bell-shaped or even uniform,  $h$  will initially be increasing. If the rate of increase is strong enough, it will induce pre-producers to hang around even though  $p(t)$  is declining. After a while, however, pre-producers will generally start to exit. An estimate of  $h$  for semi-conductor start-up firms is in Figure 8

### 3.1 Equilibrium

Equilibrium consists of two functions,  $p(t)$  and  $q(t)$ , describing the time path of the product price and the quantity supplied in the market. From these two functions, we can solve for all the other variables of interest.<sup>6</sup> We look for conditions under which we can find an equilibrium in which we also determine the total mass of entrants,  $n_0$ , and the date,  $\tau$  (possibly infinite) at which exit starts. The following definition places restrictions on the 4 unknowns:  $p(\cdot)$ ,  $q(\cdot)$ ,  $n_0$ , and  $\tau$ . Once these variables are determined, we shall be able to solve for  $n(t)$  separately. The conditions are

1. *Demand = Supply:*

$$p(t) = D(q[t]). \quad (8)$$

2. *Free entry.*—Since  $p(t)$  will be strictly decreasing, the free-entry condition holds with equality only at date 0 and it reads

$$V(0) = \frac{w}{r}, \quad (9)$$

3. *Free exit.*—For those values of  $t$  for which  $V(t) > \frac{w}{r}$  there is no exit. Exit at  $t$  requires that

$$V(t) = \frac{w}{r} \quad (10)$$

Exit begins at date  $\tau$  and ends at date  $T$ . The last firm to exit at  $T$  must be indifferent, and  $p(t)$  does not change after that. Evaluated at  $T$ , (6) reads

$$v(z, T) = \frac{1}{r} p(T) e^{-r(z-T)}$$

and so  $T$  satisfies

$$V(T) = \frac{\frac{1}{r} p(T)}{1 - F(T)} \int_T^\infty e^{-r(z-T)} dF(z) = \frac{w}{r} \quad (11)$$

4. For  $t \in [0, \tau]$  where there is no exit

$$q(t) = n_0 F(t) \quad (12)$$

These are 4 restrictions – (8), (9), (10), and (12) – in the 4 unknowns.

#### 3.1.1 Illustration of equilibrium when $h$ is bell-shaped

We will illustrate what happens when  $h(a)$  is bell-shaped – roughly as in Figure 8. A non-monotone  $h$  is the generalization of a time-to-build of a deterministic length, say  $T$  where the hazard would be zero everywhere except at  $a = T$ , where it would be infinite. Figure 5 shows a bell-shaped hazard in its third panel, with a peak occurring

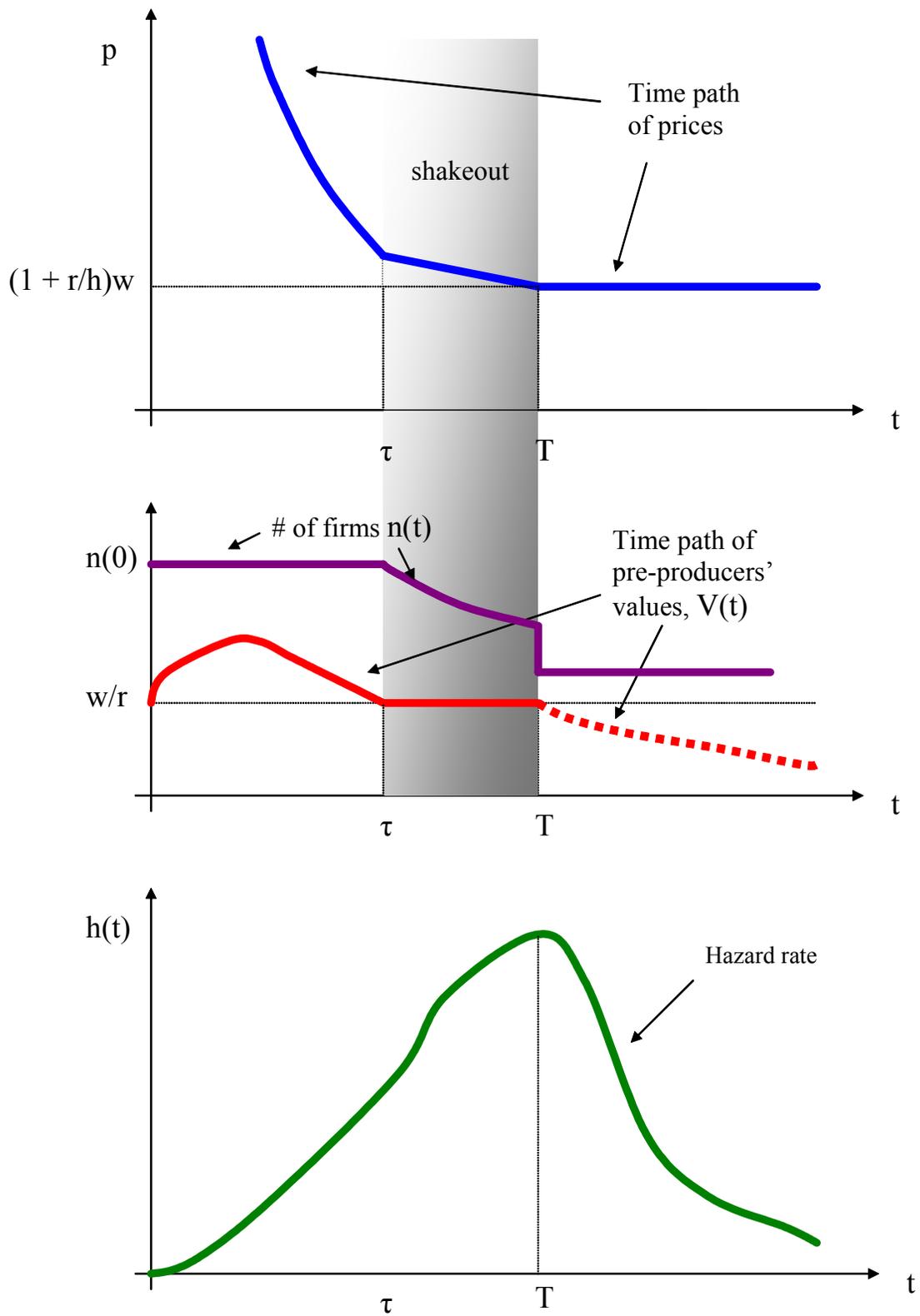


Figure 5: The nature of equilibrium when the hazard is bell shaped.

at date  $T$ . In the empirical counterpart depicted in Figure 8, the peak is at 30 months.

If demand is stable, no pre-producer can remain in the industry beyond point  $T$ . If some did stay,  $p$  would continue to fall, and so would  $h$ , thereby reducing  $V$  below  $w/r$ , as the dashed line of the middle panel shows. No firm wishes to keep trying if the chances of succeeding are getting slimmer at the same time that the rewards from success – i.e.,  $p_t$  – are declining. Therefore the model predicts that no pre-producers should remain after the hazard rate becomes permanently negatively sloped. Figure 8 gives some estimates of  $h$  which do indeed contain a negatively-sloped portion of  $h$ . The difficulty is that the constant-demand version of this model predicts that we should never be able to observe the negatively-sloped region. Presumably the data behind the hazard rate in Figure 8 were generated in a period when  $D(Q)$  was shifting to the right. Note the equilibrium has the following noteworthy features

1. Price declines monotonically from  $D(0)$  to  $D(n_0 F[\tau])$ . At this point the price path has a kink, becoming more shallow, as firms begin to exit
2. A fraction of at most  $1 - F(T)$  firms exits
3. If all the  $1 - F(T)$  firms do exit at  $T$ , the mean waiting time in pre-production (which can correspond, e.g., to the 11 month average in Schoonhoven *et al.* is

$$\int_0^T t dF(t) + T(1 - F[T]) \quad (13)$$

### 3.2 Formal arguments

We first establish a condition under which exit cannot start right away.

**Proposition 1** *Exit will not begin right away if*

$$w > \left( \frac{h(0)}{r + h(0)} \right) D(0) \quad (14)$$

**Proof.** Sufficient for this is that  $V'(0) > 0$ . The following result is therefore useful:

$$V'(0) \geq w - \frac{h(0)}{r} (D[0] - w) \quad (15)$$

Since  $v(z, t) = \int_z^\infty e^{-r(s-t)} p(s) ds$ ,

$$v_2(z, t) = rv(z, t).$$

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<sup>6</sup>Entry  $T$  periods before output can be sold occurs in Helpman and Trajtenberg (1998) when intermediate-goods producers become productive before the demand for their output materializes

From (7),

$$\begin{aligned} V'(t) &= h(t) [V(t) - v(t, t)] + \frac{1}{1 - F(t)} \int_t^\infty v_2(z, t) dF(z) \\ &= h(t) [V(t) - v(t, t)] + \frac{r}{1 - F(t)} \int_t^\infty v(z, t) dF(z) \end{aligned}$$

Then

$$\begin{aligned} V'(0) &= h(0) \left[ \frac{w}{r} - v(0, 0) \right] + r \int_0^\infty v(z, 0) dF(z) \\ &= w + h(0) \left[ \frac{w}{r} - v(0, 0) \right] \end{aligned}$$

Now  $v(0, 0) \leq \frac{D(0)}{r}$  and (15) follows. ■

An implication of (15) is that exit cannot commence right away if gestation lags are important or if, in other words, delays are inevitable and  $h(0)$  is either identically zero or quite small. In the extreme case, there is a time-to-build delay and  $h(a) = 0$  and no firm would ever have entered if it expected to exit before the time-to-build delay was over. Next we derive a condition that  $p(T)$ .

### 3.2.1 The region on which exit happens continuously

UNFINISHED AND INCOMPLETE. As we shall see in Section 4, the planner's problem is generally maximized by a piecewise continuous  $n(t)$ . There will in general be time intervals on which  $n$  will decline continuously. On such intervals we can solve for  $p$  explicitly.

We first solve for  $p(t)$ , then for  $q(t)$ , and then for the number of firms  $n(t)$  and for the number of exits,  $x(t)$ .

*Industry price.*—Using (10) and (6), we can solve for the entire time path of prices. The resulting time path is given in Proposition 1:

**Proposition 2** For  $t \in [\tau, T]$

$$p(t) = \left( 1 + \frac{1}{h(t)} \left[ r + \frac{h'(t)}{h(t)} \right] \right) w. \quad (16)$$

**Proof.** The proof consists of differentiating (10) twice with respect to  $t$ . Differentiating it once leads to

$$h(t) \left( \frac{1}{1 - F(t)} \int_t^\infty v(z, t) dF(z) - v(t, t) \right) + \frac{r}{1 - F(t)} \int_t^\infty v(z, t) dF(z) = 0. \quad (17)$$

Substituting into (17) from (10), we have

$$h(t) \left( \frac{w}{r} - v(t, t) \right) + r \left( \frac{w}{r} \right) = 0. \quad (18)$$

Rearranging (18), we have

$$v(t, t) = \left(1 + \frac{r}{h(t)}\right) \frac{w}{r}. \quad (19)$$

But (6) implies that  $v(t, t) = \int_t^\infty e^{-r(s-t)} p(s) ds$  which, together with (19) implies that

$$\int_t^\infty e^{-r(s-t)} p(s) ds = \left(1 + \frac{r}{h(t)}\right) \frac{w}{r}. \quad (20)$$

Differentiating (20), again w.r.t.  $t$ , yields

$$-p(t) + r \left(1 + \frac{r}{h(t)}\right) \frac{w}{r} = -\frac{h'(t)}{(h[t])^2} w,$$

which implies (16). ■

As long as  $h < \infty$ ,  $p(t)$  must exceed  $w$  so that the pre-producer is compensated for waiting around and foregoing the income  $w$  that it could earn elsewhere.<sup>7</sup> We now state the 3 restrictions that characterize  $\tau$  and  $T$

**Proposition 3**  $\tau$  satisfies

$$D(n_0 F[\tau]) = \left(1 + \frac{1}{h(\tau)} \left[r + \frac{h'(\tau)}{h(\tau)}\right]\right) w \quad (21)$$

**Proof.** Since  $F$  is continuous and since producers never exit,  $p(\tau)$  is continuous at  $\tau$ . The right limit is obtained by evaluating (16) at  $\tau$ , while the left limit is  $D(n_0 F[\tau])$ . The two must be equal, hence (21). ■

Note that (21) is a restriction on  $n_0$  and  $\tau$  jointly, and we need a further restriction to solve for these two unknowns. On the interval  $(\tau, T)$ ,  $p(t)$  is given by (16). At  $T$  all the remaining firms exit, and  $T$  satisfies

$$h'(T) = 0. \quad (22)$$

Moreover,  $n_0$  is characterized by the free-entry condition (9). Thus (9), (21) and (22) are 3 equations in the 3 unknowns  $n_0$ ,  $\tau$ , and  $T$ .

*Industry output and the number of firms.*—Industry output  $n_0 F(t)$  for  $t < \tau$ . Then it grows more slowly as firms start to exit for  $t$ , and  $p(t)$  is given by (16) and  $q(t) = D^{-1}(p[t])$ . Finally at  $T$  output stops growing altogether. Having gotten  $q(t)$ , we can then calculate  $n(t)$  as follows. For  $t < T$  For all  $t \in [0, T)$  we solve for  $n(t)$  from (5).

$$n(t) = q(t) + \frac{1}{h(t)} \frac{dq(t)}{dt} \quad (23)$$

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<sup>7</sup>If  $p(t)$  were not monotonically decreasing, one can imagine periods in which it could fall below  $w$  without inducing all preproducers to exit. That would be because the option of becoming a producer and later making an income more than  $w$  might induce them to remain in the game.

Since  $dq/dt$  is not defined at  $T$ , we solve for the mass of firms exiting at  $T$  as

$$\lim_{t \nearrow T} n(t) = D^{-1} \left( \left[ 1 + \frac{r}{h(T)} \right] w \right)$$

because by (16)  $p(T) = \left[ 1 + \frac{r}{h(T)} \right] w$ .

## 4 Planner's problem

There are no external effects here, and firms are negligible in size. The planner's solution should therefore coincide with equilibrium. The planner chooses the time-path of output and of the number of firms to maximize the discounted consumer's plus producer's surplus. I will restrict the planner to non-increasing sequences  $n_t$ , which means that all entry is at the outset. As we noted earlier, this makes sense if  $h$  is increasing. Moreover, since producers are homogeneous, it will never be optimal for the planner to withdraw any producers from the industry. Only pre-producers will be withdrawn. The problem is

$$\max_{(n_t, q_t)_{t=0}^{\infty}} \left\{ \int_0^{\infty} e^{-rt} \left( \int_0^{q_t} D(s) ds - wn(t) \right) dt \right\}$$

subject to

$$\frac{dq}{dt} = h(t) (n[t] - q[t]); \quad q(0) = 0$$

and to

$$\frac{dq}{dt} \geq 0 \quad \Leftrightarrow \quad q(t) \leq n(t).$$

The Hamiltonian in terms of a current-value multiplier  $\lambda(t)$  is

$$\mathbf{H}(n, q, \lambda, t) = e^{-rt} \left[ \int_0^{q_t} D(s) ds - wn(t) + \lambda(t) h(t) (n[t] - q[t]) \right]$$

Except for the infinite horizon, this problem fits the Maximum Principle formulation as described in, e.g., Kamien and Schwartz (2000 Section 14). The correspondence becomes exact when  $F$  has a bounded support, which is to say when  $F(t_{\max}) = 1$  for some finite  $t_{\max}$ . The saddle point conditions are  $\frac{\partial \mathbf{H}}{\partial n} \leq 0$  and  $\frac{\partial \mathbf{H}}{\partial q} = e^{-rt} (\lambda'[t] - r\lambda[t])$ . They imply

$$\lambda(t) h(t) \leq w \tag{24}$$

and, except at points of discontinuity of  $n(t)$ ,

$$D(q[t]) - \lambda(t) h(t) = r\lambda(t) - \lambda'(t) \tag{25}$$

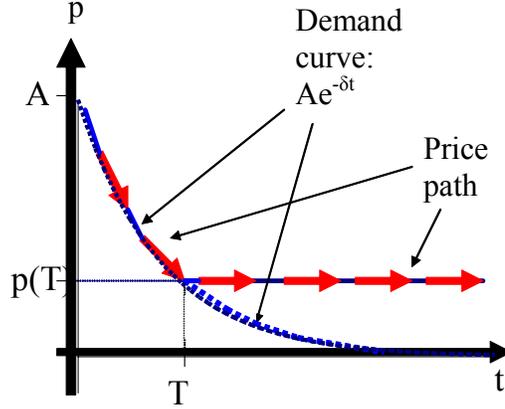


Figure 6: The price path in the uniform case

The solution for  $n(t)$  is piecewise continuous and may have jumps, as the uniform example below suggests. For those values of  $t$  for which (24) holds as an equality, we have

$$\lambda(t) = \frac{w}{h(t)} \quad \text{and} \quad \lambda'[t] = -\frac{w}{(h[t])^2} h'(t)$$

which together with (25) imply (16).

TO BE CONTINUED

#### 4.0.2 Example: $F(z) = z$ , and $D(q) = Ae^{-\delta q}$

With  $F(z) = z$  for  $z \in [0, 1]$ ,  $f(z) = 1$  and  $h(z) = \frac{1}{1-z}$ . This example does not involve a bell-shaped  $h(t)$  and the exit interval collapses to a singleton,  $T$ . That is, we seek an equilibrium in which all pre-producers will remain in the industry up to the time when a mass-exodus takes place at date  $\tau = T < 1$ . At that date, every remaining pre-producer exits and every producer stays in the industry permanently. Then (11) reads

$$\begin{aligned} V(T) &= \frac{w}{r} \\ &= \frac{\frac{1}{r} p(T)}{1-T} \int_T^1 e^{-r(z-T)} dz \\ &= \frac{p(T)}{r^2(1-T)} (1 - e^{-r(1-T)}) \end{aligned}$$

Therefore

$$p(T) = \frac{r(1-T)}{1 - e^{-r(1-T)}} w$$

## Output and the # of Firms

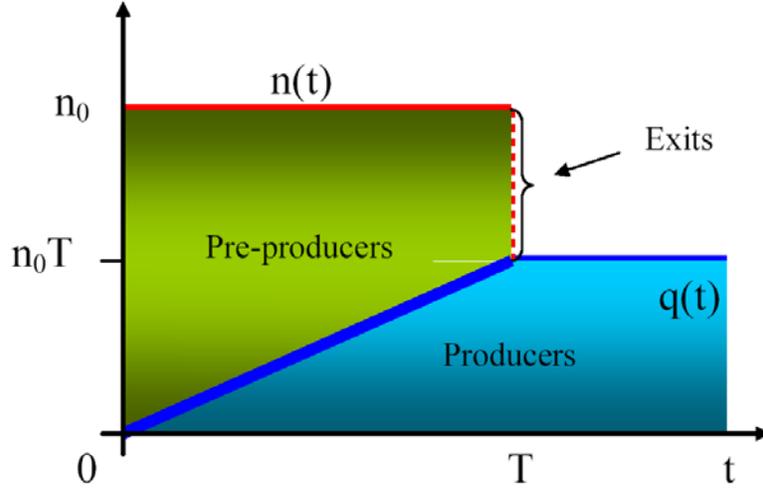


Figure 7: Evolution of  $n(t)$  and  $p(t)$  in the uniform case

The gross markup  $\frac{r(1-T)}{1-e^{-r(1-T)}} > 1$  and converges to unity as  $T \rightarrow 1$ . Industry output is

$$q(t) = n_0 \min(t, T)$$

and

$$p(t) = Ae^{-\delta n_0 \min(t, T)}.$$

The price path is portrayed in Figure 6. The time-path of the numbers of firms and of the numbers of producers and pre-producers is plotted in Figure 7. Therefore  $T$  solves

$$\frac{r(1-T)}{1-e^{-r(1-T)}}w = Ae^{-\delta n_0 T}. \quad (26)$$

To eliminate  $n_0$  we use the free-entry condition. Conditional on  $z$  and  $T$  the value of entry is

$$\varepsilon(z) = \begin{cases} A \int_z^T e^{-(r+\delta n_0)t} dt + e^{-rT} \frac{1}{r} p(T) & \text{if } z < T \\ e^{-rT} \frac{w}{r}. & \text{if } z \geq T \end{cases}$$

But  $z$  is not known at the start and free entry implies

$$\begin{aligned} \frac{w}{r} &= \int_0^1 \varepsilon(z) dz \\ &= e^{-rT} \left[ T \frac{p(T)}{r} + (1-T) \frac{w}{r} \right] + A \int_0^T \left[ \int_z^T e^{-(r+\delta n_0)t} dt \right] dz \end{aligned} \quad (27)$$

Now

$$\int_z^T e^{-(r+\delta n_0)t} dt = \frac{1}{r + \delta n_0} (e^{-(r+\delta n_0)z} - e^{-(r+\delta n_0)T})$$

so that

$$\begin{aligned} \int_0^T \left[ \int_z^T e^{-(r+\delta n_0)t} dt \right] dz &= \frac{1}{r + \delta n_0} \int_0^T (e^{-(r+\delta n_0)z} - e^{-(r+\delta n_0)T}) dz \\ &= - \left( \frac{1}{r + \delta n_0} \right) T e^{-(r+\delta n_0)T} + \left( \frac{1}{r + \delta n_0} \right)^2 (1 - e^{-(r+\delta n_0)T}) \end{aligned}$$

Substituting into (27) it reads

$$\frac{w}{r} = e^{-rT} \left[ T \frac{p(T)}{r} + (1 - T) \frac{w}{r} \right] + A \left( \frac{1}{r + \delta n_0} \right)^2 (1 - e^{-(r+\delta n_0)T}) - A \left( \frac{1}{r + \delta n_0} \right) T e^{-(r+\delta n_0)T}, \quad (28)$$

We now have two equations – (26) and (28) – for our two unknowns  $T$  and  $n_0$ . The solutions must satisfy  $T \leq 1$  and

$$V(t) > \frac{w}{r} \quad \text{for } t < T. \quad (29)$$

(iii) *Checking* (29).—Conditional on  $z \geq \tau$ , the value at date  $\tau < T$  of staying on as a pre-producer is

$$\varepsilon(z, \tau) = \begin{cases} A \int_z^T e^{-(r+\delta n_0)(t-\tau)} dt + e^{-r(T-\tau)} \frac{1}{r} p(T) & \text{if } z \in [\tau, T] \\ e^{-r(T-\tau)} \frac{w}{r} & \text{if } z \in (T, 1] \end{cases}$$

Therefore the unconditional value for a pre-producer at date  $\tau$  is

$$\begin{aligned} \frac{1}{1 - \tau} \int_{\tau}^1 \varepsilon(z, \tau) dz &= e^{-r(T-\tau)} \left[ \frac{(T - \tau) p(T)}{1 - \tau} \frac{1}{r} + \frac{(1 - T) w}{1 - \tau} \frac{1}{r} \right] + \frac{A}{1 - \tau} \int_{\tau}^T \left[ \int_z^T e^{-(r+\delta n_0)(t-\tau)} dt \right] dz \\ &\equiv V(\tau) \end{aligned}$$

Now

$$\begin{aligned} \int_{\tau}^T \left[ \int_z^T e^{-(r+\delta n_0)(t-\tau)} dt \right] dz &= \frac{1}{r + \delta n_0} \int_{\tau}^T (e^{-(r+\delta n_0)(z-\tau)} - e^{-(r+\delta n_0)(T-\tau)}) dz \\ &= - \left( \frac{1}{r + \delta n_0} \right) (T - \tau) e^{-(r+\delta n_0)(T-\tau)} + \left( \frac{1}{r + \delta n_0} \right)^2 (1 - e^{-(r+\delta n_0)(T-\tau)}) \end{aligned}$$

Therefore

$$\begin{aligned} V(\tau) &= e^{-r(T-\tau)} \left[ \frac{(T - \tau) p(T)}{1 - \tau} \frac{1}{r} + \frac{(1 - T) w}{1 - \tau} \frac{1}{r} \right] + \\ &\quad + \frac{A}{1 - \tau} \left[ - \left( \frac{1}{r + \delta n_0} \right) (T - \tau) e^{-(r+\delta n_0)(T-\tau)} + \left( \frac{1}{r + \delta n_0} \right)^2 (1 - e^{-(r+\delta n_0)(T-\tau)}) \right] \end{aligned}$$

*Comparative dynamics.*—We now vary  $A$  and study how that affects the shakeout. High  $A$ 's lead to more dramatic price declines and a larger fraction of firms exiting.  
TO BE CONTINUED

**Figure 3. Graph of hazard estimates for first product shipment (smoothed over 12-month intervals).**

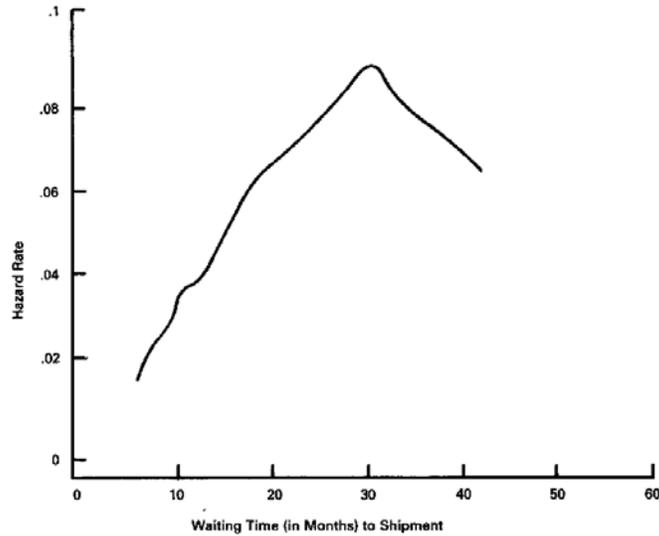


Figure 8: Schoonhoven et al’s Fig. 3 – an estimate of the production hazard  $h(t)$ .

## 5 The production hazard and the exit hazard

The theory of this paper takes  $F$  and its hazard  $h$  as given. But the definition of equilibrium is simpler if  $h$  is increasing because all the entry will take place in the very beginning. Thus the production hazard is one thing we want to measure. The second property of the model (and a consequence of having an initially low hazard  $h(0)$  that later rises sharply) is that no exit takes place for a while. Therefore we also want to measure the exit hazard. Finally, other measures may relate to the theory in this paper, namely, the period in a firm’s life before it generates profits and before it pays dividends.

*The production hazard  $h(a)$ .*—The evidence from the semi-conductor industry is that  $h$  does indeed rise. Figure (8) reproduces Schoonhoven *et al*’s Figure 3. This figure gives a non-parametric estimate of  $h(a)$  where  $a$  is the age of the firm since its founding, and it is measured in months. Note that at the mean of  $t = 21$  months, the hazard rate is still rising, and the peak, at  $t = 30$ , is almost one standard deviation to the right of the mean. Thus for the semi-conductor firms,  $h'(z) > 0$  is realistic. The model predicts that no pre-producer should have continued operations beyond the point  $t = 30$  where the hazard peaks. The Appendix reports the data in greater detail.

Some caveats about Figure 8. First, it is probable that pre-production begins even before the firm is founded – before starting his firm, the founder has probably been tinkering in his garage for a while. Second, established firms sometimes diversify into

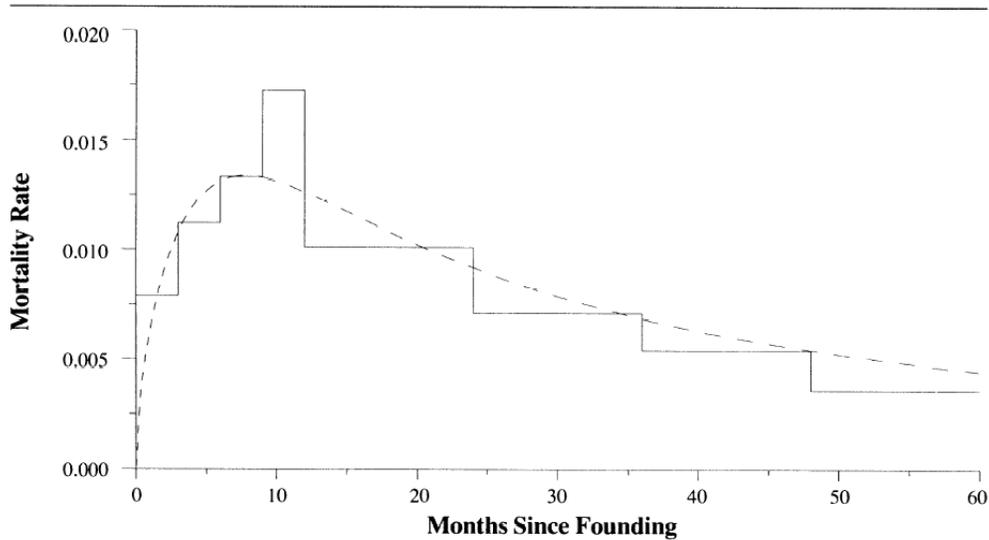


Figure 1. Mortality Rate for Newly Founded Business Organizations: Upper Bavaria, 1985 to 1990

Note: The step-function is the life-table rate estimator. The dashed curve is estimated using the proportional log-logistic rate model (without covariates). The number of cases is 1,794.

Figure 9: Bruderl *et al.*'s estimate of the exit hazard

new products, and then they are pre-producers only in part. Their activities are not covered in these data. The model applies to all activities that involve a sacrifice of earnings; Eisenhardt and Schoonhoven (1990) analyze the effects of various forms of the founder's prior experience on the performance of the firm.

*Exits.*—The three benchmark cases of section 2 have no exit. In the model of section 3, pre-producers exit. The exit hazard is either infinite at  $T$  or becomes positive only at date  $T$ . Bruderl *et al.* (1992) (present exit hazards for firms as a function of their ages since founding. We see that exits are common even in the first 4 months, and that they peak in months 9-12.

*Dividend payments.*—The preproduction period is certainly one during which a firm does not generate a cash flow and, if it is publicly owned, it will not pay any dividends. Pre-dividend periods are considerably longer than pre-production periods. Even if we measure age as of the time that a firm first lists, it takes firms a long time before they pay dividends. A year after listing, only 28% of firms, and even 10 years after listing, only 51% of firms do (Pastor and Veronesi, forthcoming, Table 1)..

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## 6 Appendix

A summary of Schoonhoven *et al*'s data on semi-conductor start-ups is in Table 1, taken from their paper. Time is measured in months.

Table 1

<b>Descriptive Statistics for Variables Used in Analysis of Waiting Time to First Product Shipment</b>					
<b>Variable</b>	<b>Mean</b>	<b>S.D.</b>	<b>Min.</b>	<b>Max.</b>	<b>N</b>
Waiting time to first shipment	21.07	11.19	1	53.0	98
Knowledge created*	4.27	3.29	0	10.0	97
Knowledge synthesized†	5.79	3.66	0	10.0	96
Minimum micron size	1.96	1.45	0.2	10.0	97
Mean years semiconductor industry experience	11.00	6.70	0	27.7	97
% Founders with start-up experience	0.38	0.39	0	1.0	97
% Founders with joint work experience	0.63	0.45	0	1.0	97
Log average monthly expenditures‡	-1.89	1.10	-5.0	1.1	85
% Venture capital ownership	21.03	27.46	0	89.0	96
% Outside investors on founding board	0.38	0.33	0	1.0	98
Number of competitors	23.77	16.66	0.0	61.0	95
Mimetic isomorphism	20.73	3.16	13.0	30.0	98
<b>Frequencies</b>			<b>No</b>	<b>Yes</b>	
Shipped first product			9	89	
Marketing & manufacturing positions in founding structure			82	15	
Founded in Silicon Valley region			29	69	
* Values: 0 = Low new knowledge creation; 5 = Moderate knowledge creation; 10 = Extensive knowledge creation.					
† Values: 0 = Low synthesis; 5 = Moderate synthesis; 10 = Extensive synthesis.					
‡ Measured as natural log of millions of dollars.					

Figure 10 is taken from Eisenhardt and Schoonhoven (1990) and it deals with the same sample that Schoonhoven *et al.* (1990) analyzed. The figure provides indirect evidence on S-shaped sales among the same sample of semi-conductor firms. Recall that the mean startup-time (i.e., the period during which the firm has no sales at all) is 21 months. By the 36'th month, many firms seem to have over \$20 million in sales and by year 5 since founding most well-managed firms do. This provides some support for the theoretical approximation of the sales-age relation portrayed in Figure 2.

## 7 Heterogeneous and unknown $z$ 's: Discrete time

We still maintain the convention that

$$y_a = \begin{cases} 0 & \text{if } a < z \\ 1 & \text{if } a \geq z. \end{cases}$$

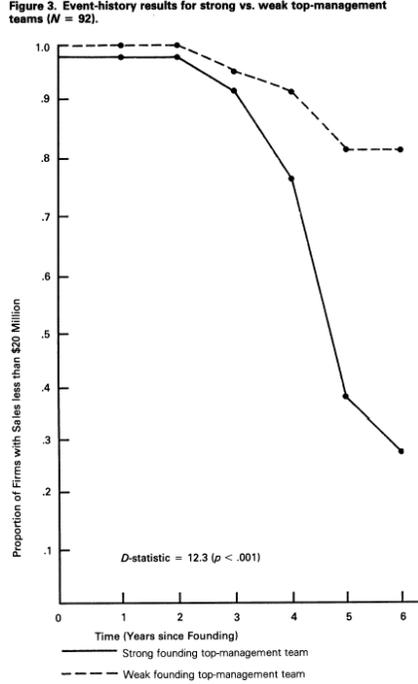


Figure 10: A reproduction of Figure 3 of Eisenhardt and Schoonhoven (1990).

Then  $z$  is the (integer) date at which production starts. The distribution  $F(z)$  is now over integers  $z = 0, 1, \dots$ , with probabilities  $f_z$  and hazard rate

$$h_z = \frac{f_z}{1 - F_{z-1}}.$$

This is the probability of coming up with something *today* given that nothing has happened yet. Let  $\beta = 1/(1 + r)$

### 7.0.3 Equilibrium

Equilibrium consists of two sequences,  $p_t$  and  $n_t$ . This is the price and the number of firms. If we have these two functions, we can get all the other variables of interest. We look for conditions under which we can find an equilibrium in which

1. Producers are better off remaining in the industry, i.e.,

$$p_t \geq w \tag{30}$$

for all  $t$ , and

2. Pre-producers are indifferent between staying in the industry and producing elsewhere:

$$\frac{w}{1-\beta} = \frac{1}{1-F_{t-1}} \sum_{z=t}^{\infty} v(z,t) f_z, \quad (31)$$

where

$$v(z,t) = \sum_{s=z}^{\infty} \beta^{s-t} p_s. \quad (32)$$

#### 7.0.4 Law of motion for $q_t$

Here we need a convention on timing. Exit,  $x_t$ , occurs right after production at date  $t$ , and before the existing pre-producers have discovered whether they will become productive in the following period.

Of the  $n_t$  firms in the industry,  $q(t)$  are producers, and  $n(t) - q(t)$  are pre-producers. The instantaneous conditional probability that a pre-producer becomes a producer is just  $h(t)$ . Then we have the differential equation.

$$q_{t+1} - q_t = (n_t - q_t - x_t) h_t, \quad (33)$$

from which we have

$$n_t = q_t + \frac{1}{h_t} (q_{t+1} - q_t). \quad (34)$$

Finally we need to solve for exits. Since price declines,  $n$  cannot increase. Exits,  $x$ , are

$$x_t = n_t - q_t - \frac{1}{h_t} (q_{t+1} - q_t). \quad (35)$$

### 7.1 Example 3: $f_z = \lambda(1-\lambda)^z$

This example is a limiting case. The hazard is

$$\frac{f_z}{\sum_{s=z}^{\infty} f_s} = \frac{\lambda(1-\lambda)^z}{\lambda \sum_{s=z}^{\infty} (1-\lambda)^s} = \frac{(1-\lambda)^z}{\frac{1}{\lambda}(1-\lambda)^z} = \lambda.$$

From (32), if price is constant at  $p$ ,

$$v(z,t) = \frac{p\beta^{z-t}}{1-\beta}.$$

Since  $1 - F_{t-1} = \lambda \sum_{s=t}^{\infty} (1-\lambda)^s = \frac{1}{\lambda}(1-\lambda)^t$ , (31) then reads

$$\frac{w}{1-\beta} = \frac{\lambda}{(1-\lambda)^t} \sum_{z=t}^{\infty} \frac{p\beta^{z-t}}{1-\beta} \lambda(1-\lambda)^z,$$

i.e.,

$$\begin{aligned} w &= \lambda^2 p \sum_{z=t}^{\infty} \beta^{z-t} (1-\lambda)^{z-t} \\ &= \frac{\lambda^2 p}{1-\beta(1-\lambda)} \end{aligned}$$

so that

$$p = \frac{1-\beta(1-\lambda)}{\lambda^2} w$$

Note that for  $\beta < 1$ ,

$$p > \frac{1}{\lambda} w$$

and that

$$\lim_{\beta \rightarrow 1} p = \frac{1}{\lambda} w$$

Initial mass of entrants,  $n_0$ , must be sufficient to guarantee that the first producers can supply the steady state quantity  $D^{-1}(p)$

$$n_0 = \frac{1}{\lambda} D^{-1} \left( \frac{1-\beta(1-\lambda)}{\lambda^2} w \right).$$

Thereafter, (34) tells us that since output is constant,

$$x_0 = n_0 - q_0 = (1-\lambda) n_0$$

everyone else exits, so that  $n_t = n_0$  for all  $t$ .

Everyone starts out as a pre-producer, but only a fraction  $\lambda$  manage to produce anything. Carroll and Hannan (2000) report that for the automobile industry only eleven percent of the pre-producers eventually sold any cars. In the present example, it implies we should set  $\lambda = 0.11$ .